Partition by Exhaustification

Question semantics and the logic of scalar strengthening

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> Squib deadline
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> For those of you registered for this class, the first squib (on presupposition) is due in today.

Homework

- Next week we'll be talking about plurality, and, specifically, *multiplicity inferences*.
- **Presupposition and implicature will collide**.
- You're required to read either of the following papers (free choice inference!). Send me at least one question via email by next Wednesday.
  - Benjamin Spector. 2007a. Aspects of the pragmatics of plural morphology: On higher-order implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*. London: Palgrave Macmillan UK
  - Uli Sauerland, Jan Anderssen & Kazuko Yatsushiro. 2005. The plural is semantically unmarked. In *Linguistic evidence empirical, theoretical and computational perspectives*. Berlin, Boston: De Gruyter
- I'll upload both papers to stellar later today.

1 Introduction to Partition semantics

- Groenendijk & Stokhof (1984) capture the difference between *indicatives* and *interrogatives* by treating the latter as partitions/propositional concepts.

(1) $[\![\text{it's raining}]\!] \coloneqq \{\, w \mid \text{raining}_w \,\}$

(2) $[\![\text{is it raining?}]\!] \coloneqq \lambda w\,.\,\{\, w' \mid \text{raining}_w = \text{raining}_{w'} \,\}$

- A partition divides up the logical space (i.e., the set of possible worlds) into mutually exclusive possibilities: "cells".
- Consider the following illustration:

	$w_{r,s}$	$w_{r,\neg s}$	$w_{\neg r,s}$	$w_{\neg r,\neg s}$
raining	1	1	0	0
sunny	1	0	1	0

- The meaning of the interrogative tells us, for any evaluation world, which cell in the partition it belongs to.

$$[\![\text{is it raining?}]\!] = \begin{bmatrix} w_{r,s} & \mapsto \{ w_{r,s}, w_{r,\neg s} \} \\ w_{r,\neg s} & \mapsto \{ w_{r,s}, w_{r,\neg s} \} \\ w_{\neg r,s} & \mapsto \{ w_{\neg r,s}, w_{\neg r,\neg s} \} \\ w_{\neg r,\neg s} & \mapsto \{ w_{\neg r,s}, w_{\neg r,\neg s} \} \end{bmatrix}$$

- The interrogative *is it sunny?* of course delivers a distinct partition:

$$[\![\text{is it raining?}]\!] = \begin{bmatrix} w_{r,s} & \mapsto \{ w_{r,s}, w_{\neg r,s} \} \\ w_{r,\neg s} & \mapsto \{ w_{r,\neg s}, w_{\neg r,\neg s} \} \\ w_{\neg r,s} & \mapsto \{ w_{r,s}, w_{\neg r,s} \} \\ w_{\neg r,\neg s} & \mapsto \{ w_{r,\neg s}, w_{\neg r,\neg s} \} \end{bmatrix}$$

- G&S's partition semantics extends straightforwardly to constituent questions:

(3) $[\![\text{who arrived}]\!] = \lambda w \,.\, \{ w' \mid \{ x \mid \mathsf{arrived}_w\ x \} = \{ x' \mid \mathsf{arrived}_{w'} \} \}$

- The G&S meaning for *who arrived* partitions the logical space according to the person(s) who arrived. Assume that the domain of individuals is Paul, Sophie.

$$\begin{aligned} &\{ w \mid \{ x \mid \mathsf{arrived}_w\ x \} = \{ \mathsf{Paul,Sophie} \} \} \\ &\{ w \mid \{ x \mid \mathsf{arrived}_w\ x \} = \{ \mathsf{Paul} \} \} \\ &\{ w \mid \{ x \mid \mathsf{arrived}_w\ x \} = \{ \mathsf{Sophie} \} \} \\ &\{ w \mid \{ x \mid \mathsf{arrived}_w\ x \} = \varnothing \} \end{aligned}$$

$$\begin{aligned} &\{ w \mid \text{Paul and Sophie arrived in } w \} \\ &\{ w \mid \text{only Paul arrived in } w \} \\ &\{ w \mid \text{only Sophie arrived in } w \} \\ &\{ w \mid \text{nobody arrived in } w \} \end{aligned}$$

1.1 Exhaustivity

- Two forms of exhaustivity (discussed by G&S):

(4) Dani knows who came.
  a. If x came, Dani believes that x came. *weak exhaustivity*
  b. If x came, Dani believes that x came; if x didn't come, then Dani believes that x didn't come. *strong exhaustivity*

- A partition semantics for questions directly captures the strongly exhaustive interpretation; the partition denoted by a question is a function from a world w to the *strongly exhaustive* answer to the question in w.

(5) Dani knows who came.
  $\lambda w \,.\, \mathsf{d}\ \mathsf{know}_w\ \{ w' \mid \{ x \mid \mathsf{came}_w\ x \} = \{ x \mid \mathsf{came}_{w'}\ x \} \}$
  a. If only Yasu came in w, then:
     $[\![5]\!]\ w = \mathsf{d}\ \mathsf{know}_w\ \{ w' \mid \{ x \mid \mathsf{came}_{w'}\ x \} = \{ \mathsf{y} \} \}$
  b. If only Andy came in w, then:
     $[\![5]\!]\ w = \mathsf{d}\ \mathsf{know}_w\ \{ w' \mid \{ x \mid \mathsf{came}_{w'}\ x \} = \{ \mathsf{a} \} \}$
  c. If only Yasu and Andy came in w, then:
     $[\![5]\!]\ w = \mathsf{d}\ \mathsf{know}_w\ \{ w' \mid \{ x \mid \mathsf{came}_{w'}\ x \} = \{ \mathsf{y}, \mathsf{a}, \mathsf{y} \oplus \mathsf{a} \} \}$
  d. If nobody came in w, then:
     $[\![5]\!]\ w = \mathsf{d}\ \mathsf{know}_w\ \{ w' \mid \{ x \mid \mathsf{came}_{w'}\ x \} = \varnothing \}$

- A strongly exhaustive interpretation seems to be motivated, at least for the interpretation of *know*, as noted by G&S:

(6) a. Dani believes that Yasu and Andy are here.
 b. Only Andy is here.
 c. $\not\Rightarrow$ Dani knows who is here.

- As famously observed by Heim (1994), there are certain predicates for which this doesn't seem right.

(7) Dani is surprised by who came.
 a. ✓ If x came, Dani is surprised that x came.
 b. ??? If x came, Dani is surprised that x came; if x didn't come, then Dani is surprised that x didn't come.

(8) Context: *only Andy came, and Dani is surprised that Andy came. Yasu didn't come, and this doesn't surprise Dani one bit.*

 a. Dani is surprised by who came.
 b. # Dani isn't surprised by who came.

1.2 From Hamblin sets to partitions

1.2.1 Background on equivalence classes

- For any set, X, if we have some notion of equivalence (called an *equivalence relation* ($\sim$)) for the members $x \in X$, then we may divide up X into equivalence classes – subsets of X. Two elements x and x' belong to the same equivalence class only if they satisfy the equivalence relation.
- Let's say that we have a set of individuals, and our equivalence relation is *eye color*. In other words:

$$x \sim x' \text{ iff eye-color } x = \text{eye-color } x'$$

- For each individual y, their equivalence class is the following set:

$$\{ x \in X \mid x \sim y \}$$

- This maps, e.g., Britta to the set of blue-eyed individuals, if she has blue eyes, and Jeff to the set of brown-eyed individuals, if he has brown eyes.
- The set of equivalence classes is therefore the following set

$$\bigcup_{y \in X} \{ X' \mid X' = \{ x \in X \mid x \sim y \} \}$$

- It can be shown that the set of equivalence classes on X, relative to some equivalence relation, is always a *partition* of X. For example, *eye color* partitions D_e into cells where everyone has the same eye color.

1.2.2 Hamblin sets provide equivalence relations

- As pointed out by Fox, and many others, it's easy to retrieve a *partition of the logical space (i.e., the set of possible worlds W)* from a Hamblin set Q, by taking the Hamblin set to provide an equivalence relation on W.
- The idea, informally, is that two worlds w and w' are equivalent wrt a question Q, iff, each answer in Q is mapped to the same truth-value in both worlds.

(9) $w \sim w'$ iff $\forall p \in Q[p\ w = p\ w']$

- Once we have an equivalence relation, we can compute the set of equivalence classes of W. For each world in w, its equivalence class is the set of worlds which map each answer in Q to the same truth-value.
- Gathering together the resulting equivalence classes is guaranteed to partition the logical space (the set of equivalence classes is always a partition).

(10) $\mathsf{PART}_Q\ W \coloneqq \bigcup_{w' \in W} \{ w \in W \mid \forall p \in Q[p\ w = p\ w'] \}$

- Fox gives a simple example of how this works, when the Hamblin set simply contains two logically independent propositions:

$$Q = \{ p, q \}$$

- $\mathsf{PART}_Q\ W$ will map the logical space to the set of equivalence classes under the equivalence relation: $w \sim w'$ iff $p\ w = p\ w' \land q\ w = q\ w'$. This results in the following partition:

$$\{ p \land \neg q, q \land \neg p, p \land q, \neg p \land \neg q \}$$

- Let's see how this works in a slightly more involved case, by applying this reasoning to the Hamblin denotation of a constituent question:

$$[\![\text{who arrived}]\!] = \{ p \mid \exists x[p = \{ w \mid \mathsf{arrived}_w\ x \}] \}$$

$$= \left\{ \begin{array}{l} \{ w \mid \mathsf{arrived}_w\ \mathsf{p} \} \\ \{ w \mid \mathsf{arrived}_w\ \mathsf{s} \} \\ \{ w \mid \mathsf{arrived}_w\ \mathsf{p} \oplus \mathsf{s} \} \end{array} \right\}$$

- The Hamblin set Q provides an equivalence relation – $w \sim w'$ iff every proposition of the form x $\mathsf{arrived}$, maps w and w' to the same truth-value.

$$w \sim w' \text{ iff } \forall p \in [\![\text{who arrived}]\!]\ [p\ w = p\ w']$$

- This equivalence relation can be used to divide up the logical space W into equivalence classes/cells, i.e., just those worlds that "agree on" exactly who arrived.

$$\bigcup_{w' \in W} \{ p \mid p = \{ w \in W \mid \forall p \in [\![\text{who arrived}]\!]\ [p\ w = p\ w'] \} \}$$

- This gives us the following set:

$$\left\{ \begin{array}{ll} \left\{ w \left| \begin{array}{l} \mathsf{arrived}_w\ \mathsf{p} = 1 \\ \mathsf{arrived}_w\ \mathsf{s} = 0 \\ \mathsf{arrived}_w\ \mathsf{p} \oplus \mathsf{s} = 0 \end{array} \right. \right\}, & \left\{ w \left| \begin{array}{l} \mathsf{arrived}_w\ \mathsf{p} = 0 \\ \mathsf{arrived}_w\ \mathsf{s} = 1 \\ \mathsf{arrived}_w\ \mathsf{p} \oplus \mathsf{s} = 0 \end{array} \right. \right\} \\ \left\{ w \left| \begin{array}{l} \mathsf{arrived}_w\ \mathsf{p} = 1 \\ \mathsf{arrived}_w\ \mathsf{s} = 1 \\ \mathsf{arrived}_w\ \mathsf{p} \oplus \mathsf{s} = 1 \end{array} \right. \right\}, & \left\{ w \left| \begin{array}{l} \mathsf{arrived}_w\ \mathsf{p} = 0 \\ \mathsf{arrived}_w\ \mathsf{s} = 0 \\ \mathsf{arrived}_w\ \mathsf{p} \oplus \mathsf{s} = 0 \end{array} \right. \right\} \end{array} \right\}$$

$$= \left\{\begin{array}{l}\text{only Paul arrived}\\ \text{only Sophie arrived}\\ \text{Paul and Sophie arrived}\\ \text{Nobody arrived}\end{array}\right\}$$

2 Partition by exhaustification

2.1 An arithmetic problem arising from question-partition mismatch

- Fox's insight is that, questions are typically answered by uttering a proposition in the Hamblin set.
- Propositions in the Hamblin set are nevertheless too weak to identify a cell in the corresponding partition.

(11) a. Question: Who (of Yasu, Andy, Daniele) is here?
b. Answer: Andy is here.

- The proposition *Andy is here* is consistent with multiple cells in the corresponding logical partition: (i) *only Andy is here*, (ii) *only Andy and Yasu are here*, (iii) *only Andy and Daniele are here*, and (iv) *everyone is here*.
- However, if we *exhaustify* the proposition "Andy is here", relative to the alternatives provided by the Hamblin set, then the result is the (strengthened) proposition *only Andy is here*.
- Fox points out that this gives rise to what he calls an "arithmetic problem" – since the number of cells in the logical partition often outstrips the number of propositions in the answer set, then there will often be cells which simply *can't* be identified by exhaustifying a member of the Hamblin set.
- For example, in the context just given, there is no member in the Hamblin set, which, once strengthened, picks out the cell where *nobody is here*.

2.2 Dayal's presupposition

- In an extremely influential book, Dayal (1996) proposed that interrogatives carry a particular kind of presupposition – Fox demonstrates that, once this presupposition is adopted, the arithmetic problem no longer arises.
- contexts which satisfy Dayal's presupposition are exactly those where each member of the Hamblin set can pick out a cell of the (contextual) partition via exhaustification.
- Let's see how this works.

(12) **Dayal's presupposition**
a. $\mathsf{Ans}_D\ Q \coloneqq \lambda w : \exists p \in Q[p = \mathsf{MaxInf}\ Q\ w] \,.\, \mathsf{MaxInf}\ Q\ w$
b. $\mathsf{MaxInf}\ Q\ w \coloneqq \begin{cases} p & p\ w \wedge \forall q \in Q[q\ w \rightarrow p \subseteq q] \\ \sharp & \text{else} \end{cases}$

- If the presupposition of Ans_D is met, then, given a Stalnakerian context set C (representing the common ground), every world $w \in C$ will be such that $\mathsf{MaxInf}\ Q\ w = p$, for some $p \in Q$.
- The flip side of the presupposition being satisfied, will be that every cell in the partition of C will be identifiable via (the strengthened meaning of) each member of the Hamblin set.
- Imagine that we have a Hamblin set consisting of two logically independent propositions $Q = \{ p, q \}$
- And, let's say that C initially consists of the following worlds. For Dayal's presupposition to be satisfied in the context set, for every world, there must be a unique maximally informative element of Q.

	$w_{p,q}$	$w_{p,\neg q}$	$w_{\neg p,q}$	$w_{\neg p,\neg q}$
p	1	1	0	0
q	1	0	1	0

  - ✗ In $w_{p,q}$, there is no such proposition; p and q are equally informative.
  - ✓ In $w_{p,\neg q}$, there is such a proposition: p.
  - ✓ In $w_{\neg p,q}$, there is such a proposition: q.
  - ✗ In $w_{\neg p,\neg q}$, there is no such proposition; both p and q are false.

- Since Dayal's presupposition isn't met in this context, we must *accommodate*, i.e., minimally shrink C such that Dayal's presupposition is satisfied.
- What is left is the revised context set $C' = \{ w_{p,\neg q}, w_{\neg p,q} \}$.
- Now, if we partition this revised context set relative to Q, the result is simply $\{\{ w_{p,\neg q} \}, \{ w_{\neg q,p} \}\}$. This corresponds exactly to what we get if we *exhaustify* each proposition in the Hamblin set.
- Fox suggests that this provides a more general solution to the arithmetic problem raised earlier.

(13) **Dayal's presupposition as an answer to the arithmetic challenge**
If A is a context set that satisfies the presupposition of $\mathsf{Ans}_D\ Q$, then every cell in $\mathsf{Part}_C\ Q\ A$ is identifiable by (the exhaustification of) a member of Q:
$\forall C \in \mathsf{Part}_C\ Q\ A[\exists p \in Q[[\mathsf{Exh}\ Q\ p = C]]]$

3 Evidence for Dayal's presupposition

3.1 Existence, uniqueness, and maximality

- Assumption: *wh*-expressions with singular restrictors range over atomic individuals *only*.
- Consequence: the Hamblin-set denoted by a singular *which*-question, such as "which linguist is here?" will be the set of propositions of the form *x is here*, where *x* is a linguist.
- The propositions in the resulting Hamblin set are logically independent:

$$[\![\text{which linguist is here?}]\!] = \left\{ \begin{array}{l} \text{Yasu is here} \\ \text{Dani is here} \\ \text{Andy is here} \end{array} \right\}$$

- For Dayal's presupposition to be true in a context set C, it must be the case that for every world $w \in C$, there is some member of the Hamblin set that is a unique, maximally informative true answer.
- This is equivalent to the requirement that every cell in the partition induced by the question by identifiable by an exhaustified member of the Hamblin set.
- The exhaustified answers – which correspond to the contextual partition, if Dayal's presupposition is met – are given below:

$$\left\{ \begin{array}{ll} \text{only Yasu is here} & (= \mathsf{Exh}\ Q\ (\text{Yasu is here})) \\ \text{only Dani is here} & (= \mathsf{Exh}\ Q\ (\text{Dani is here})) \\ \text{only Andy is here} & (= \mathsf{Exh}\ Q\ (\text{Andy is here})) \end{array} \right\}$$

- This means that the presupposition imposed by Ans_d will only be satisfied in a context where *there exists a unique linguist who is here*.

- Let's now move on to consider plural and simplex *wh*-expressions.
- Assumption: the domain of simplex and plural *wh*-expressions is *closed under sum formation*.
- Consequence: the Hamblin set denoted by a question such as "which linguists are here" no longer consists of a set of logically independent propositions.

$$[\![\text{which linguists are here}]\!] = \left\{ \begin{array}{c} \text{here y, here d, here a} \\ \text{here y} \oplus \text{a, here y} \oplus \text{d, here d} \oplus \text{a} \\ \text{here y} \oplus \text{d} \oplus \text{a} \end{array} \right\}$$

- Since *to be here* is a distributive predicate, the answer set is, in fact, closed under conjunction:

$$= \left\{ \begin{array}{c} \text{Yasu is here, Dani is here, Andy is here} \\ \text{Yasu and Dani are here, Yasu and Andy are here, Dani and Andy are here} \\ \text{everyone is here} \end{array} \right\}$$

- Applying $\lambda p \,.\, Exh\ Q\ p$ to each member of the set results in the following set. Note that exhaustification of the answer everyone is here is vacuous, since there are no excludable alternatives.

$$= \left\{ \begin{array}{c} \text{only Yasu is here, only Dani is here, only Andy is here} \\ \text{only Yasu and Dani are here,} \\ \text{only Yasu and Andy are here,} \\ \text{only Dani and Andy are here} \\ \text{everyone is here} \end{array} \right\}$$

- The prediction of Dayal's presupposition is that the presupposition of this question will only be satisfied in a context where either: (i) there is a unique linguist who is here, or (ii) there is a unique, maximal group of linguists who are here.
- The (accurate) prediction then, is that the following answer should convey a maximality inference:

(14) a. Question: Which linguists (out of Yasu, Dani, and Andy) are here?
b. Answer: Yasu and Andy. $\rightsquigarrow$ *Dani isn't here*

- Fox suggests that the same reasoning applies to simplex *wh*-questions – essentially, the suggestion is to treat *who* as meaning the same thing as *which people*.
- The prediction, then, is that Dayal's presupposition should "knock out" the cell in the logical partition where nobody is here, just as with the plural *which*-question.
- Elliott, Nicolae & Sauerland (2016) observe that this would (erroneously) predict that the question "who is here?" should be a presupposition failure in a context compatible with nobody being here.

(15) a. Question: Who is here?
b. Answer: Nobody.

- It's even easier to see this in an embedded context.

(16) For each day of the week, David knows who was here.

- If we assume universal projection from out of the scope of *each day of the week*, Fox (and Dayal) predict that this should presuppose that, on each day of the week, at least one person was here.
- Our judgement is that (16) is judged felicitous in a context where, on, e.g. Tuesday, nobody was here, and David knows that nobody was here on Tuesday.
- We'll come back to this in our discussion of higher-order readings.

3.2 Weak islands

- The argument from weak islands is based on the observation that, depending on the logical properties of the domain that the *wh*-expression ranges over, there are certain environments in which a question cannot have a maximally informative true answer. In such environments, the question is judged to be unacceptable.

(17) a. Tell me how fast you drove?
b. *Tell me how fast you didn't drive?

- Fox's discussion of this contrast is based on Fox & Hackl (2007).
- F&H assume that the domain of degrees is *densely ordered*; consequently degree questions denote infinite sets of propositions, densely ordered by entailment.

(18) a. $[\![\text{how fast did you drive?}]\!] = \{\, p \mid \exists d \in D[p = \{\, w \mid \mathsf{yourSpeed}_w \geq d \,\}] \,\}$
b. $[\![\text{how fast didn't you drive?}]\!] = \{\, p \mid \exists d \in D[p = \{\, w \mid \mathsf{yourSpeed}_w < d \,\}] \,\}$

- We won't go into the details here, but the observation is that (18a) *can* have a unique maximally informative true answer, namely the proposition: $\{\, w \mid \mathsf{yourSpeed}_w \geq d^* \,\}$, where $d*$ is the addressee's actual driving speed.
- (18b) can *never* have a maximally informative true answer, since, assuming that the domain of degrees is densely ordered, there is no smallest degreegreater than d^*.

3.3 Cell Identification

- Fox note that, if Dayal's presupposition is met, so is the constraint **Cell Identification (CI)**:

> **Cell Identification**
>
> A question Q and a context-set A meet CI if
> $\forall C \in \mathsf{Part}_C\ Q\ A[\exists p \in Q[[\mathsf{Exh}\ Q\ p]_A = C]]$

4 Challenges for Dayal's presupposition

4.1 *Mention some*

- Not every question presupposes the existence of a *unique* maximally-informative true answer.
- A notable exception to this is *mention some* (MS) questions.

(19) Mary knows where we can get gas in Cambridge.
a. Mary knows one location where we can get the gas. *mention some*
b. Mary knows all locations where we can get the gas. *mention all*

- Dayal's presupposition here seems to demand *too much* of the question denotation.

4.2 Higher order readings

- Assumption: *wh*-expressions range over the domain of individuals (perhaps closed under sum-formation).
- Now, consider the Hamblin set predicted for the following modalised question.

(20) What are we required to read for this class?

$$\left\{\begin{array}{l} \square \text{ we read War and Peace} \\ \square \text{ we read Brothers Karamazov} \\ \square \text{ we read War and Peace and Brothers Karamazov} \end{array}\right\}$$

- Dayal's presupposition predicts that, if there is nothing in particular that we are required to read, the question should be undefined.
- Spector (2007b) asks us to imagine a context such as the following:
- *There is no particular thing x s.t. we are required to read x, but there are still requirements pertaining to reading. For example, imagine that it would be sufficient to read all the French books or all the Russian books in order to satisfy the requirement, but it is up to us which ones we choose.*
- He observes that, in such a context, (20) is askable.
- Consider a modalised question under an embedding predicate in this context:

(21) Mary knows what we are required to read.

- Intuitively, this can be true if we are subject to the following reading requirement: *we must read all the French books, or all the Russian books*, and Mary knows this.
- Our standard assumptions – i.e., quantification over plural individuals, plus Dayal's presupposition – can't derive this.

4.2.1 The shift to higher-order quantification

- In order to reconcile this reading with Dayal's presupposition, Spector suggests that we allow *wh*-expressions to range over upward-entailing GQs.
- This would result in the following Hamblin-set:[1]

$$\left\{\begin{array}{ll} \square\,(\{\{w\},\{w,b\}\}\ (\lambda x\ .\ \text{we read } x)) & (= w^{\uparrow}) \\ \square\,(\{\{w\},\{w,b\}\}\ (\lambda x\ .\ \text{we read } x)) & (= b^{\uparrow}) \\ \square\,(\{\{w,b\}\}\ (\lambda x\ .\ \text{we read } x)) & (= w^{\uparrow} \wedge b^{\uparrow}) \\ \square\,(\{\{w\},\{b\},\{w,b\}\}\ (\lambda x\ .\ \text{we read } x)) & (= w^{\uparrow} \vee b^{\uparrow}) \\ \square\,(\{\varnothing,\{w\},\{b\},\{w,b\}\}\ (\lambda x\ .\ \text{we read } x)) & (= \top) \end{array}\right\}$$

- Why just the UE generalised quantifiers? If we dispense with this restriction, we'd expect that a modalised question should be felicitous in a context where: *we're not subject to any particular reading requirements, but we're forbidden from reading the German books*:

(22) a. Question: What are we required to read?
b. Answer: # We're required to read none of the German books.

- There is a DE quantifier which corresponds to a maximally informative true answer in this context, namely: *none of the German books* (extensionally, the set of sets that contain no German books).

[1] The set of all upward entailing quantifiers living on D_e

4.2.2 Sensitivity to negative islands

- In a positive context, modalised questions are ambiguous between an (ordinary) reading about individuals, and a higher-order reading.

(23) a. What are you required to read for this class?
b. War and Peace or Brothers Karamazov. $\Box > \vee; \vee > \Box$

- Assuming that *wh*-expressions have the capacity to range over GQs, it's surprising that, in a negative question, we don't find a corresponding ambiguity:

(24) a. What did you not read for this class?
b. War and Peace or Brothers Karamazov. $*\neg > \vee; \vee > \neg$

- Fox notes that, if the unattested reading were available, the answer in (24b) should convey no speaker ignorance about what was read. However, (24b) obligatorily conveys that the speaker doesn't know if they didn't read War and Peace, and doesn't know if they didn't read Brothers Karamazov.
- The answer therefore sounds somewhat odd, or biases an interpretation according to which the speaker has additional information, but doesn't want to share it.
- Much like with degree questions, the presence of a higher modal makes the higher order reading available again – this parallels the obviation effect we observe in the domain of degree questions.

(25) a. What are you not allowed to read for this class?
b. War and Peace or Brothers Karamazov. $\neg > \vee; \vee > \neg$

(26) How fast are we not allowed to drive?

- Dayal's presupposition however doesn't predict the *un*availability of a higher-order question in a negative context. To see why consider the predicted Hamblin set.

(27) What didn't you read?

$$= \left\{ \begin{array}{l} \neg \text{ you read w} \\ \neg \text{ you read b} \\ \neg \text{ you read w} \wedge \text{b} \\ \neg \text{ you read w} \vee \text{b} \end{array} \right\}$$

- In a context in which you read neither book, there is a unique, maximally-informative true answer to the question – namely *you read neither w nor b*. Dayal's presupposition is satisfied.
- If we want a uniform explanation for negative islands, we'd like to extend our explanation to these cases too. The current formulation of Dayal's presupposition doesn't do that.

Interim summary

- Mention some interpretations of questions show that there are cases in which Dayal's presupposition *demands too much*.
- The negative-island sensitivity of higher-order questions shows that there are cases in which Dayal's presupposition *demands too little*.

5 Question-Partition Matching

- Fox notes that we can rule out the unattested higher-order reading in a negative context by strengthening Dayal's presupposition. The new constraint (which subsumes Dayal's presupposition) is called **Question Partition Matching** (QPM).

> Question Partition Matching (QPM)
>
> A question Q and a context-set A meet QPM if they meet **Cell Identification** (CI) and **Non-Vacuity** (NV):
> – CI: $\forall C \in \mathsf{Part}_C\ Q\ A[\exists p \in Q[[\mathsf{Exh}\ Q\ p]_A = C]]$
> – NV: $\forall p \in Q[\exists C \in \mathsf{Part}_C\ Q\ A[[\mathsf{Exh}\ Q\ p]_A = C]]$
> Informally, this says that: (i) the exhaustification of each answer in the Hamblin set must correspond to a cell in the contextual partition, and furthermore (ii) each cell in the contextual partition must correspond to the exhaustification of an answer in the Hamblin set.

5.1 Addressing undergeneration

- To see how QPM rules out higher-order questions in a negative context, consider again the denotation of a negative higher-order question:

(28) What didn't you read?

$$\begin{Bmatrix} \neg \text{ you read w} \\ \neg \text{ you read b} \\ \neg \text{ you read w} \wedge \text{b} \\ \neg \text{ you read w} \vee \text{b} \end{Bmatrix}$$

- The result of exhaustification is as follows:

$$\begin{Bmatrix} \checkmark\ (\neg \text{ you read w}) \wedge \text{you read b} \\ \checkmark\ (\neg \text{ you read b}) \wedge \text{you read w} \\ \text{✗}\ (\neg \text{ you read w} \wedge \text{b}) \wedge (\text{you read w} \vee \text{b}) \\ \checkmark\ \neg\ (\text{you read w} \vee \text{b}) \end{Bmatrix}$$

- The exhaustification of the negative conjunctive answer gives back the proposition *that you read War and Peace or Brothers Karamazov, but not both*. This can never pick out a cell in a contextual partition – it overlaps with both the first and the second exhaustified answer.

5.2 Addressing overgeneration

- Recall that *mention some* interpretations suggested that the requirements imposed by Dayal's presupposition were *too strong*. Now, we've strengthened Dayal's presupposition to QPM, so the problem should be even worse.
- Fox suggests that we can address this worry if, rather, then formulating Exh in terms of *maximal informativity*, we use the *innocent inclusion* formulation of Exh due to Bar-Lev & Fox (2017).

5.2.1 Background on Innocent Inclusion

- *Innocent Inclusion* is basically a refinement of Fox's innocent exclusion algorithm (which we discussed last week). It has two primary virtues:

– it can derive free choice inferences without using recursive exhaustification; only one layer of Exh is required.

– it can derive universal free choice inferences, as in ().

(29) Every boy is allowed to eat ice cream or cake.
  a. $\rightsquigarrow$ every boy is allowed to eat ice cream.
  b. $\rightsquigarrow$ every boy is allowed to eat cake.

(30) No student is required to solve problem A and problem B.
  a. $\rightsquigarrow$ No student is required to solve problem A.
  b. $\rightsquigarrow$ No student is required to solve problem B:w

- Procedure for applying Exh^{II+IE} to a proposition p:
  - Negate the set of *innocently excludable* members of $\mathsf{Alt}(p)$.
    - To get the *innocently excludable* members of $\mathsf{Alt}(p)$, we gather the maximal members of $\mathscr{P}(\mathsf{Alt}(p))$ that can be negated consistently with p, and intersect them.
  - Assert the set of *innocently includable* members of $\mathsf{Alt}(p)$.
    - To get the *innocently includable* members of $\mathsf{Alt}(p)$, we gather the maximal members of $\mathscr{P}(\mathsf{Alt}(p))$ that can be asserted consistently with p and the negation of the innocently excludable alternatives, and intersect them.

(31) Every boy is allowed to eat ice cream or cake. $\forall x \Diamond (Px \vee Qx)$
  a. *Every boy is allowed to eat ice cream.* $\forall x \Diamond Px$
  b. *Every boy is allowed to eat cake.* $\forall x \Diamond Qx$

- First, let's compute the set of alternatives:

$$\begin{aligned}&\mathsf{Alt}(\forall x \Diamond (Px \vee Qx))\\ &= \left\{ \begin{array}{l} \overbrace{①\ \forall x \Diamond (Px \vee Qx)}^{\text{prejacent}}, \overbrace{②\ \forall x \Diamond Px, ③\ \forall x \Diamond Qx}^{\text{universal disjunctive alts}}, \overbrace{④\ \forall x \Diamond (Px \wedge Qx)}^{\text{universal conjunctive alt}} \\ \underbrace{⑤\ \exists x \Diamond (Px \vee Qx)}_{\text{existential alt}}, \underbrace{⑥\ \exists x \Diamond Px, ⑦\ \exists x \Diamond Qx}_{\text{existential disjunctive alts}}, \underbrace{⑧\ \exists x \Diamond (Px \wedge Qx)}_{\text{existential conjunctive alt}} \end{array} \right\}\end{aligned}$$

- Let's gather together the maximal subsets of $\mathsf{Alt}(\forall x \Diamond (Px \vee Qx))$ that can be negated consistently with $\forall x \Diamond (Px \vee Qx)$).

$$\{ ②\ \forall x \Diamond Px, ③\ \forall x \Diamond Qx, ④\ \forall x \Diamond (Px \wedge Qx), ⑧\ \exists x \Diamond (Px \wedge Qx) \}$$

$$\{ ②\ \forall x \Diamond Px, ⑥\ \exists x \Diamond Px, ④\ \forall x \Diamond (Px \wedge Qx), ⑧\ \exists x \Diamond (Px \wedge Qx) \}$$

$$\{ ③\ \forall x \Diamond Qx, ⑦\ \exists x \Diamond Qx, ④\ \forall x \Diamond (Px \wedge qx), ⑧\ \exists x \Diamond (px \wedge qx) \}$$

$$\mathrm{IE}(\forall x \Diamond (Px \vee Qx)) = \{ ④\ \forall x \Diamond (Px \wedge Qx), ⑧\ \exists x \Diamond (Px \wedge Qx) \}$$

- If we take the prejacent together with the negation of the IE alternatives, we end up in a world where, either all boys are allowed to P and not Q, all boys are allowed to Q and not P, or some boys are allowed to P and not Q and some boys are allowed to Q and not P (no boys are allowed to P and Q), i.e.

$$\forall x \Diamond (Px \vee Qx) \wedge \neg \exists x \Diamond (Px \wedge Qx)$$

- Let's gather together the maximal subsets of $\mathsf{Alt}(\forall x \Diamond (Px \vee Qx))$ that can be asserted consistently with $\forall x \Diamond (Px \vee Qx)) \wedge \neg \exists x \Diamond (Px \wedge Qx)$.
- It turns out there is only one such set, consisting of all the non-IE alts:

$$\left\{ \begin{array}{l} ① \, \forall x \Diamond (Px \vee Qx), ② \, \forall x \Diamond Px, ③ \, \forall x \Diamond Qx, \\ ⑤ \, \exists x \Diamond (Px \vee Qx), ⑥ \, \exists x \Diamond Px, ⑦ \, \exists x \Diamond Qx \end{array} \right\}$$

- Asserting the II alts together with the prejacent and negation of the IE alts results in the following enriched meaning:

$$\forall x \Diamond (Px \vee Qx) \wedge \neg \exists x \Diamond (Px \wedge Qx) \wedge \forall x \Diamond Px \wedge \forall x \Diamond Qx$$

5.2.2 Accounting for *mention some*

- **Modification 1:** Answers in the Hamblin set are exhaustified via Exh^{IE+II}.
- **Modification 2:** The answerhood operator returns a set of propositions, rather than a unique proposition.
- Allowing for the possibility of the *wh*-expression ranging over higher order GQs, we get the following denotation for the mention some question below:

(32) Where can we buy gas?

(33) $= \{ p \mid \exists Q \in \mathsf{UGQ}\ L[p = \Diamond\ (Q\ (\lambda i\ .\ \text{we get gas at } i))] \}$

- Assume that there are three locations l_1, l_2, and l_3.
- The result is the following Hamblin set (Fox notes we must assume that the conjunctive alternatives are pruned).

$$= \left\{ \begin{array}{c} \Diamond\ l_1, \Diamond\ l_2, \Diamond\ l_3 \\ \Diamond\ (l_1 \vee l_2), \Diamond\ (l_1 \vee l_3), \Diamond\ (l_2 \vee l_3) \\ \Diamond\ (l_1 \vee l_2 \vee l_3) \end{array} \right\}$$

- Now we exhaustify each member of the set via Exh^{IE+II}

$$= \left\{ \begin{array}{ll} \Diamond\ l_1 \wedge \neg \Diamond\ l_2 \wedge \neg \Diamond\ l_3 & = (\mathsf{Exh}\ \Diamond\ Q\ l_1) \\ \Diamond\ l_2 \wedge \neg \Diamond\ l_1 \wedge \neg \Diamond\ l_3 & = (\mathsf{Exh}\ \Diamond\ Q\ l_2) \\ \Diamond\ l_3 \wedge \neg \Diamond\ l_1 \wedge \neg \Diamond\ l_2 & = (\mathsf{Exh}\ \Diamond\ Q\ l_3) \\ \Diamond l_1 \wedge \Diamond l_2 \wedge \neg \Diamond l_3 & = (\mathsf{Exh} \Diamond (l_1 \vee l_2)) \\ \Diamond l_1 \wedge \Diamond l_3 \wedge \neg \Diamond l_2 & = (\mathsf{Exh} \Diamond (l_1 \vee l_3)) \\ \Diamond l_2 \wedge \Diamond l_3 \wedge \neg \Diamond l_1 & = (\mathsf{Exh} \Diamond (l_2 \vee l_3)) \\ \Diamond l_2 \wedge \Diamond l_3 \wedge \Diamond l_1 & = (\mathsf{Exh} \Diamond (l_1 \vee l_2 \vee l_3)) \end{array} \right\}$$

- Note that the exhaustification of each proposition identifies a cell. If there is more than one location where one can get gas, the proposition that identifies the cell does not mention all the locations.
- The final step in accounting for mention-some is to allow the answerhood operator to return the set of propositions that entail the cell-identifier.

(34) $\mathsf{Ans}\ Q = \lambda w : \exists p \in Q[\mathsf{Exh}\ Q\ p\ w = 1]$
$\quad . \{ q \in Q \mid q\ w \wedge q \subseteq (\iota p \in Q[\mathsf{Exh}\ Q\ p\ w = 1]) \}$

- If there are two locations where we can gas, l_1 and l_2, the cell-identifier is $\Diamond\ (l_1 \vee l_2)$. Applying the reformulated answer-hood operator returns the set

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