An explanatory theory of presupposition projection

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1 Weak Kleene

Weak Kleene recipe

Where the classical semantics is silent, always return #.

One way of thinking of the third truth value, #, is as representing *undefinedness*.

This interpretation gives rise to a Weak Kleene logic.

ϕ	$\neg\phi$
1	0
0	1
#	#

(a) Negation

$\phi \wedge \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	0	0	#
#	#	#	#

(b) Conjunction

$\phi \vee \psi$

ϕ \ ψ	1	0	#
1	1	1	#
0	1	0	#
#	#	#	#

(c) Disjunction

$\phi \rightarrow \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	1	1	#
#	#	#	#

(d) Material implication

Figure 1: Weak Kleene/Bochvar truth-tables for the connectives

2 Strong Kleene (symmetric)

We can think of the third truth value, #, as representing *uncertainty whether 1 or 0*, which we can represent as the set $\{1, 0\}$.

In order to explain the recipe, it will be helpful to think of our three truth-values as the following isomorphic three-membered set: $\{\{1\}, \{0\}, \{1, 0\}\}$, with $\{ 1 \}$ representing *definitely true*, $\{ 0 \}$ representing *definitiely false*, and $\{ 1, 0 \}$ representing *maybe true and maybe false*.

$$\{ \overbrace{\{ 1 \}}^{\text{true}}, \underbrace{\{ 0 \}}_{\text{false}}, \overbrace{\{ 1, 0 \}}^{\text{uncertain}} \}$$

The intuition behind our recipe will be as follows:

- Given a complex formula with an n-place truth-functional connective f, $\ulcorner f\ \phi_1 \ldots \phi_n \urcorner$.
- Assuming that I^{bi} gives the bivalent interpretation of f as a function, compute $\{ I^{bi}(f)\, t_1 \ldots t_n \mid t_1 \in [\![\phi_1]\!]^{tri}, \ldots, t_n \in [\![\phi_n]\!]^{tri} \}$.
- The result is the value of $[\![f\ \phi_1 \ldots \phi_n]\!]^{tri}$

2.1 *Applying the Strong Kleene algorithm to conjunction*

When the values of the arguments of the connective are $\{ 1 \}$ or $\{ 0 \}$, the algorithm will simply deliver the classical semantics. We can illustrate this with conjunction.

$$[\![p \wedge q]\!]^{tri} = \{ t \wedge u \mid t \in I(p) \wedge u \in I(q) \}$$

If $I(p)$ and $I(q)$ are singleton sets $\{ t \}$ and $\{ u \}$, this will obviously be equivalent to the classical semantics:

$$= \{ t \wedge u \}$$

What if one of $I(p)$ is $\{ 0, 1 \}$? The value of the conjunctive formula will differ depending on whether $I(q)$ is $\{ 1 \}$ or $\{ 0 \}$. Assuming that $I(q) = \{ 1 \}$:

(1) $I(p) = \{ 0, 1 \}, I(q) = 1$
  a. $[\![p \wedge q]\!]^{tri} = \{ t \wedge u \mid t \in \{ 0, 1 \} \wedge u \in \{ 1 \} \}$
  b. $= \{ t \wedge 1 \mid t \in \{ 0, 1 \} \}$
  c. $= \{ 0, 1 \}$

(2) $I(p) = \{ 0, 1 \}, I(q) = 0$

a. $[\![p \wedge q]\!]^{tri} = \{ t \wedge u \mid t \in \{ 0, 1 \} \wedge u \in \{ 0 \} \}$

b. $= \{ t \wedge 0 \mid t \in \{ 0, 1 \} \}$

c. $= \{ 0 \}$

Strong Kleene conjunction

- $\ulcorner \phi \wedge \psi \urcorner$ is *defined* if either (a) $[\![\phi]\!]$ is false, (b) $[\![\psi]\!]$ is false, or (c) both $[\![\phi]\!]$ and $[\![\phi]\!]$ are true.
- $\ulcorner \phi \wedge \psi \urcorner$ is *true* if both $[\![\phi]\!]$ and $[\![\psi]\!]$ are true.
- $\ulcorner \phi \wedge \psi \urcorner$ is *false* if either (a) $[\![\phi]\!]$ is false, or (b) $[\![\psi]\!]$ is false.

2.2 Disjunction in Strong Kleene semantics

Strong Kleene disjunction

- $\ulcorner \phi \vee \psi \urcorner$ is *defined* if either (a) $[\![\phi]\!]$ is true, (b) $[\![\psi]\!]$ is true, or (c) both $[\![\phi]\!]$ and $[\![\phi]\!]$ are false.
- $\ulcorner \phi \wedge \psi \urcorner$ is *false* if both $[\![\phi]\!]$ and $[\![\psi]\!]$ are false.
- $\ulcorner \phi \wedge \psi \urcorner$ is *true* if either (a) $[\![\phi]\!]$ is true, or (b) $[\![\psi]\!]$ is true.

2.3 Strong Kleene truth-tables

If we apply the strong Kleene algorithm to the classical connectives, substituting in $\{ 1, 0, \# \}$ for $\{ \{ 1 \}, \{ 0 \}, \{ 1, 0 \} \}$, the result is the following truth-tables.

2.4 The role of linear order in presupposition projection

As discussed by Schlenker (2008), it's not clear that the projection generalization we've been assuming for disjunctive sentences is correct.

(3) a. Either this house has no bathroom, or the bathroom is upstairs.

b. Either the bathroom is upstairs, or (else) this house has no bathroom.

(Schlenker 2008: p. 185)

ϕ	$\neg\phi$
1	0
0	1
#	#

(a) Negation

$\phi \wedge \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

(b) Conjunction

$\phi \vee \psi$

ϕ \ ψ	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

(c) Disjunction

$\phi \rightarrow \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

(d) Material implication

Figure 2: Strong Kleene truth-tables for the connectives

The entry we gave for disjunction in update semantics can capture the projection pattern illustrated by (3b), but not (**??**).

3 *Middle Kleene/Peters (asymmetric)*

Presupposition projection displays asymmetries based on *linear order*; something that strong Kleene fails to capture.

References

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3).

A *Strong Kleene as an alternative semantics*

Assume that `t` is the type of *bivalent* truth values.

(4) $$[\![\text{stopped smoking}]\!]^w = \lambda x\,.\begin{cases}\{\,1\,\} & \mathsf{smoked}_w\ x \wedge \neg\,(\mathsf{smokes}_w\ x)\\ \{\,0\,\} & \mathsf{smoked}_w\ x \wedge \mathsf{smokes}_w\ x\\ \{\,1,0\,\} & \text{otherwise}\end{cases}$$

$\mathtt{e} \rightarrow \{\,\mathtt{t}\,\}$

(5) $[\![\text{and}]\!]^{w} \coloneqq \lambda u \,.\, \lambda t \,.\, t \wedge u$ $\quad\quad$ $\mathtt{t} \to \mathtt{t} \to \mathtt{t}$

As in a standard alternative semantics, we just need two truth-values to massage composition.

(6) a. $x^{\rho} \coloneqq \{\, x \,\}$ $\quad\quad$ $\mathtt{a} \to \{\, \mathtt{a} \,\}$

b. $m \circledast n \coloneqq \{\, x \mathbin{\text{A}} y \mid x \in m \wedge y \in n \,\}$ $\quad\quad$ $\{\, \mathtt{a} \to \mathtt{b} \,\} \to \{\, \mathtt{a} \,\} \to \{\, \mathtt{b} \,\}$

(7) Paul smoked and he stopped smoking.