Disjunction, free choice, and the theory of scalar implicature

Innocent exclusion and beyond

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> Reading
>
> Please read the following and submit at least one question to me via email about the paper by **Wednesday 30 October** (next Friday's class is cancelled due to NELS):
>
> Danny Fox. 2018. Partition by exhaustification: comments on Dayal 1996. In Uli Sauerland & Stephanie Solt (eds.), *Proceedings of sinn und bedeutung 16* (ZASPiL 60), 403–434. Berlin: Leibniz-Centre General Linguistics

1 Introduction

- Last week, Roger gave an overview of the *grammatical theory of exhaustification*; its locus being the exhaustification operator exh, which we can think of as a silent, assertive counterpart of *only*.
- What is the semantic contribution of exh; is the classical semantics of *only* enough?
- Current formulations of exh in terms of *innocent exclusion* (Fox 2007) and more recently, *innocent inclusion* (Bar-Lev & Fox 2017), centre how to derive the following:
  - The implicatures of complex disjunctive sentences (see esp. Sauerland 2004).
  - Free choice inferences in the scope of existentials (Fox 2007).
- These phenomena also bear on the question of *how many alternatives to consider*, and ultimately reify a syntactic theory of alternativehood (Katzir 2008, Fox & Katzir 2011), according to which alternatives are at most as structurally complex as the prejacent.
- Fox (2007) argues that when faced with data from disjunction and free choice, we have two choices, we can either:
  - Complicate the overarching pragmatic principles according to which discourse participants operate (i.e., the Gricean Maxims).
  - Complicate the definition of exh, and maintain a *simple* theory of pragmatics.
- "we might think of exh as a syntactic device designed ('by a super-engineer') to facilitate communication in a pragmatic universe governed by [the maxim of quantity]. " (Fox 2007: p. 80)

2 Implicatures associated with disjunctive sentences

(1) Eleanor talked to Michael or Janet

a.	⇝ *Eleanor talked to Michael or Janet (or both)*	Basic inference
b.	⇝ *Eleanor didn't talk to Michael or Janet*	Scalar Implicature (SI)
c.	⇝ *The speaker doesn't know that Eleanor talked to Michael*	Ignorance Inferences
d.	⇝ *The speaker doesn't know that Eleanor talked to Janet*	

- (1a) just follows from the "literal" (logical) meaning of the sentence.
- (1c) and (1d) follow from general pragmatic principles (see below).

(2) **Maxim of Quantity**[1]
if S_1 and S_2 are both relevant to the topic of conversation, and S_1 is more informative than S_2, if the speaker believes that both are true, the speaker should utter S_1 rather than S_2.

Deriving the ignorance inferences via MQ:

- The speaker uttered $p \vee q$.
- Both p and q are *relevant*, and uttering either of p or q would have been more informative than uttering $p \vee q$.
- If we assume that the speaker is behaving in accordance with the MQ, then:
  - It can't be the case that the speaker believes that p is true, and;
  - It can't be the case that the speaker believes that q is true.

[1] Fox 2007: p. 73

Deriving the SI via exhaustification:

- In the literature on the *grammatical* theory of SIs, it is something of a mantra to claim that exh is the assertive counterpart of *only*. A basic meaning for *only* takes a set of alternatives Q, a prejacent p, and negates the truth-conditionally non-weaker alternatives to p.

(3) $\text{only}\ Q\ p \coloneqq \lambda w : p\ w\ .\ \forall q \in Q[p \not\subseteq q \rightarrow \neg\, q\ w]$

(4) Only Eleanor left.

(5) $\text{LF: only} \begin{Bmatrix} \text{Michael left} \\ \text{Janet left} \\ \text{Eleanor left} \end{Bmatrix} \text{(Eleanor left)}$

(6) Meaning: λw : Eleanor left in w . ¬ Michael left in w ∧ ¬ Janet left in w

- The exhaustivity operator does the same thing, except whereas with *only* the prejacent is *presupposed*, with exh the prejacent is asserted:

(7) $\text{exh}\ Q\ p \coloneqq \lambda w\ .\ \overbrace{p\ w}^{\text{basic inference}} \wedge \underbrace{\forall q \in Q[p \not\subseteq q) \rightarrow \neg\, q\ w]}_{\text{implicature}}$

- It's furthermore standard at this point to assume that exh, like *only*, is a focus sensitive operator.

(8) exh Eleanor left.

(9) $\text{LF: only} \begin{Bmatrix} \text{Michael left} \\ \text{Janet left} \\ \text{Eleanor left} \end{Bmatrix} \text{(Eleanor left)}$

(10) Meaning: $\lambda w\ .\ \overbrace{\text{Eleanor left in } w}^{\text{basic inference}} \wedge \underbrace{\neg\,\text{Michael left in } w \wedge \neg\,\text{Janet left in } w}_{\text{implicature}}$

- Let's now consider whether or not this is enough to derive the SI of our disjunctive sentence, repeated below:

(11) Eleanor talked to Michael or Janet
  a. p =Eleanor talked to Michael.
  b. q =Eleanor talked to Janet.

	p	q	$p \wedge q$	$p \vee q$	$p \veebar q$
w_1	1	1	1	1	0
w_2	1	0	0	1	1
w_3	0	1	0	1	1
w_4	0	0	0	0	0
	$\{w_1, w_2\}$	$\{w_1, w_3\}$	$\{w_1\}$	$\{w_1, w_2, w_3\}$	$\{w_2, w_3\}$

- Let's assume that the relevant set of possible worlds is $\{w_1, w_2, w_3, w_4\}$.

(12) Assume that the only alternative to ∨ is ∧:
  a. $\text{alt}_{p \vee q} = [\{w_1, w_2, w_3\} \mapsto \{\{w_1\}\}]$
  b. $\text{exh}\ (\text{alt}_{p \vee q})\ (p \vee q) = \{w_1, w_2, w_3\} \cap (W - \{w_1\})$
  $= \{w_1, w_2, w_3\} \cap \{w_2, w_3, w_4\}$
  $= \{w_2, w_3\} \equiv p \veebar q$

- According to the grammatical theory of alternatives (Katzir 2008, Fox & Katzir 2011), alternatives are all those constituents that are at most *as* syntactically complex as the prejacent.

- It follows that, for a disjunctive sentence [P or Q], we expect that both P and Q, the individual disjuncts, should count as alternatives. We call these alternatives the **Sauerland alternatives** (S-alternatives).

(13) Assume that ∨ also competes with the individual disjuncts – things go wrong.
  a. $\text{alt}_{p \vee q} = [\{w_1, w_2, w_3\} \mapsto \{\{w_1\}, \{w_1, w_2\}, \{w_1, w_3\}\}]$
  b. $\text{exh}\ (\text{alt}_{p \vee q})\ (p \vee q) = \{w_1, w_2, w_3\} \cap (W - \{w_1\}) \cap (W - \{w_1, w_2\}) \cap (W - \{w_1, w_3\})$
  $= \{w_1, w_2, w_3\} \cap \{w_2, w_3, w_4\} \cap \{w_3, w_4\} \cap \{w_2, w_4\}$
  $= \varnothing$

> The payoff
>
> Based on our current formulation of exh, in order to compute the SI for a disjunctive sentence, we must exclude the S-alternatives. This is at odds with the structural theory of alternatives.

- Note that, in order to compute the SI of a simple disjunctive sentence, we can solve this problem by fiat (i.e., by using a Horn scale), but this is *ad hoc*. We'll also see reason to believe, later on, that there are certain instances in which we *must* include the S-alternatives.

$$\mathsf{alt}_{Horn}\,(p \vee q) = \{\, p \wedge q \,\}$$

- This problem of which alternatives to exclude extends beyond the domain of disjunction.

(14) a. Who did John talk to?
Only Mary or SUE. $\rightsquigarrow$ *John talked to Mary or Sue, but not both, and nobody else*
b. Who did John talk to?
Only some GIRL. $\rightsquigarrow$ *John talked to exactly one girl*

- Starting with the problem posed by (15a), we can assume that the set of alternatives is provided by the question:

$$\mathsf{alt} = \{\, p \mid \exists x[p = \mathsf{j\ talked\ to}\ x] \,\} = \left\{\begin{array}{l}\mathsf{j\ talked\ to\ m}\\ \mathsf{j\ talked\ to\ s}\\ \dots\end{array}\right\}$$

- Our semantics for *only* delivers the wrong results, unless we exclude alternatives on an ad hoc basis. This is not a problem specific to exh then.

$$\mathsf{only}\ Q\ (\mathsf{j\ talked\ to\ m\ or\ s}) = \mathsf{j\ talked\ to\ m\ or\ s} \wedge \mathsf{j\ didn't\ talk\ to\ m} \wedge \mathsf{j\ didn't\ talk\ to\ s} = \varnothing$$

- A similar problem arises with (15b). Again, assuming the alternatives provided by the question are propositions of the form j talked to x, negating each of these alternatives contradicts the logical meaning of the existential answer.
- Note that in a Q-A scenario, the same inferences obtain in the absence of *only*:

(15) a. Who did John talk to?
Mary or SUE. $\rightsquigarrow$ *John talked to Mary or Sue, but not both, and nobody else*
b. Who did John talk to?
some GIRL. $\rightsquigarrow$ *John talked to exactly one girl*

2.1 Chierchia's puzzle

- We've seen reason to believe that, in order to compute the SI associated with a disjunctive sentence, the Sauerland alternatives must be excluded; all we want to exhaustify relative to is disjunctions Horn-mate *and*.
- Furthermore, The SI in (16a) can be derived via exh if we assume that *Eleanor did all of the homework*, is an alternative to *Eleanor did all of the homework*.

(16) Eleanor did some of the homework.
a. $\rightsquigarrow$ *Eleanor didn't do all of the homework* SI

- What about the following (noticed by Chierchia 2004):

(17) Eleanor did the reading or some of the homework.
a. $\not\rightsquigarrow$ *Eleanor didn't do the reading or all of the homework*
b. $\rightsquigarrow$ *Eleanor didn't do all of the homework*

- If we exclude the S-alternatives, the set of alternatives we end up with is as follows.

$$\left\{\begin{array}{l}\mathsf{Eleanor\ did\ the\ reading\ or\ some\ of\ the\ homework}\\ \mathsf{Eleanor\ did\ the\ reading\ or\ all\ of\ the\ homework}\\ \mathsf{Eleanor\ did\ the\ reading\ and\ some\ of\ the\ homework}\\ \mathsf{Eleanor\ did\ the\ reading\ and\ all\ of\ the\ homework}\end{array}\right\}$$

- *Eleanor did the reading or all of the homework* is truth-conditionally non-weaker than *Eleanor did the reading or some of the homework.*
- If *Eleanor did the reading or all of the homework* is an alternative to *Eleanor did the reading or some of the homework*, we can derive the implicature in (17a) via exh, which together with the basic meaning, entails that Eleanor didn't do the reading, but did some but not all of the homework. This is clearly **too strong**.
- In the general case:

(18) Let U be an utterance of p *or* q where q has a stronger alternative q':
a. Problem 1: to avoid the implicature of $\neg p$.
b. Problem 2: to derive the implicature of $\neg q'$

In order to derive the SI for Chierchia's sentence, we'll see that ultimately we need to *include* the Sauerland alternatives, contradicting our earlier finding.

2.2 Fox's algorithm: *innocent exclusion*

Fox's insight

There is something in the meaning of *only* (and hence, in the meaning of exh) "designed" to avoid contradictions.

" If such a theory is correct, we might think of exh as a syntactic device designed ('by a super-engineer') to facilitate communication in a pragmatic universe governed by [the maxim of quantity]. " (Fox 2007: p. 80)

- We need an algorithm, that takes an alternative set Q, and a prejacent p, and figures out the maximal set of alternatives that are *excludable* (i.e., each of which can be negated) in such a way that doesn't lead to inconsistencies.
- We can start by taking the *subsets* of the alternatives Q that are p*–consistent*. These are the subsets of alternatives Q', the negations of which are consistent with p:

$$\mathsf{consistent}_p\ Q = \{\, Q' \subseteq Q \mid (\bigcap \{\, \neg q \mid q \in Q' \,\} \cup \{\, p \,\}) \neq \varnothing \,\}$$

- Next we exclude the p-consistent subsets that aren't *maximal*:

$$\mathsf{maxConsistent}_p Q = \{\, Q' \in \mathsf{consistent}_p\ Q \mid \neg\exists Q''[Q' \subset Q'' \wedge Q'' \wedge \in \mathsf{consistent}_p\ Q] \,\}$$

- Finally, the set of *innocently excludable* alternatives IE are just those alternatives that are in every maximal p-consistent subset of alternatives.

$$\mathsf{IE}_p Q = \{\, q \mid \forall Q' \in \mathsf{maxConsistent}_p\ Q[q \in Q'] \,\}$$

- The meaning of *only*/exh can now be stated simply in terms of IE. Rather than negating the *non-truth-conditionally weaker* alternatives, the exhaustification operator negates the *innocently excludable* alternatives.

$$\mathsf{exh}\ Q\ p = \lambda w\ .\ p\ w \wedge \forall q \in \mathsf{IE}_p\ Q[\neg\, q\ w]$$

Application to disjunction

- Let's apply this algorithm directly to our disjunctive sentence:

(19) Eleanor talked to Michael or Janet
  a. p =Eleanor talked to Michael.
  b. q =Eleanor talked to Janet.

- This time, we're not going to exclude the S-alternatives – we'll include everything, and then run Fox's algorithm.

	p	q	$p \wedge q$	$p \vee q$	$p \veebar q$
w_1	1	1	1	1	0
w_2	1	0	0	1	1
w_3	0	1	0	1	1
w_4	0	0	0	0	0
	$\{w_1, w_2\}$	$\{w_1, w_3\}$	$\{w_1\}$	$\{w_1, w_2, w_3\}$	$\{w_2, w_3\}$

- Assume that $p \vee q$ also competes with each of the disjuncts.

$$\mathsf{alt}_{p \vee q} = [\{\, w_1, w_2, w_3 \,\} \mapsto \{\{\, w_1 \,\}, \{\, w_1, w_2 \,\}, \{\, w_1, w_3 \,\}\}]$$

- Let's start by taking the subsets of $\mathsf{alt}_{p\vee q}$ that are p-consistent:

$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, p \wedge q \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1, w_2 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, p \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1, w_3 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, q \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1 \,\}, \{\, w_1, w_2 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, p, p \wedge q \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1 \,\}, \{\, w_1, w_3 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, q, p \wedge q \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1, w_2 \,\}, \{\, w_1, w_3 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} = \varnothing \qquad \{\, p, q \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1, w_2 \,\}, \{\, w_1, w_3 \,\}, \{\, w_1 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} = \varnothing \qquad \{\, p, q, p \wedge q \,\}$$

- As illustrated, any subset of alternatives that contains both p and q will fail to be p–consistent.
- Now let's just keep the *maximal* p-consistent sets:

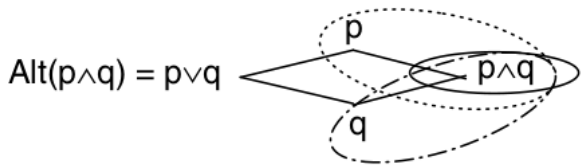
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1 \,\}, \{\, w_1, w_2 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, p, p \wedge q \,\}$$
$$(\bigcap \{\neg\, q \mid q \in \{\{\, w_1 \,\}, \{\, w_1, w_3 \,\}\}\} \cup \{\{\, w_1, w_2, w_3 \,\}\} \neq \varnothing \qquad \{\, q, p \wedge q \,\}$$

- Now we compute the set of *innocently excludable alternatives* by just keeping those alternatives that are in *every* maximal p-consistent set. Here, there is only one:

$$\mathsf{IE}_{p \vee q}\ \{\{\, w_1 \,\}, \{\, w_1, w_2 \,\}, \{\, w_1, w_3 \,\}\} = \{\{\, w_1 \,\}\} \equiv \{\, p \wedge q \,\}$$

- We've now solved the problem of simple disjunctive sentences without stipulating that the disjuncts aren't alternatives, since the only *innocently excludable* alternative is the conjunctive one.

- The maximally p−consistent subsets are circled with a dotted line, the one *innocently excludable* alternative is circled with a solid line.



Application to question-answer pairs

- As we saw earlier, we ran into the same problem of *which alternatives to exclude* in cases involving question-answer pairs.

(20) Who did Michael talk to?
  a. SOME woman. ⇝ *Michael talked to exactly one woman*

- We can assume that the syntax gives us the following representation for the answer, where the set of alternatives is provided by the question (I write, e.g., $\mathsf{j} \wedge \mathsf{t}$ for Michael talked to Janet and Tehani):

(21) exh_Q [Some woman ~~Michael talked to *t*~~]

$$Q = \{\, p \mid \exists X[p = \mathsf{m\ talked\ to}\ X] \,\}$$

- If the domain of quantification is $\{\mathsf{janet}, \mathsf{tehani}, \mathsf{eleanor}\}$, then the Hamblin set is as follows. Note that we're assuming that the alternatives are closed under disjunction – this can be accomplished by assuming that *who* ranges over atoms and pluralities; see Dayal 1996, Fox 2018. More on this next time.

$$[\![\text{who did Michael talk to?}]\!] = Q = \left\{ \begin{array}{c} \mathsf{j}, \mathsf{t}, \mathsf{e} \\ \mathsf{j} \wedge \mathsf{t}, \mathsf{j} \wedge \mathsf{e}, \mathsf{t} \wedge \mathsf{e} \\ \mathsf{j} \wedge \mathsf{t} \wedge\ \mathsf{e} \end{array} \right\}$$

- The maximal p-consistent subsets of alternatives are as follows, where p = $\mathsf{michael\ talked\ to\ some\ woman}$. Crucially, no maximally p-consistent subset contains all of the following alternatives: (a) *Michael talked to Janet*, (b) *Michael talked to Tehani*, (c) *Michael talked to Eleanor*.

$$\left\{ \begin{array}{c} \{\mathsf{j}, \mathsf{t}, \mathsf{j} \wedge \mathsf{t}, \mathsf{j} \wedge \mathsf{e}, \mathsf{t} \wedge \mathsf{e}, \mathsf{j} \wedge \mathsf{t} \wedge\ \mathsf{e}\} \\ \{\mathsf{j}, \mathsf{e}, \mathsf{j} \wedge \mathsf{t}, \mathsf{j} \wedge \mathsf{e}, \mathsf{t} \wedge \mathsf{e}, \mathsf{j} \wedge \mathsf{t} \wedge\ \mathsf{e}\} \\ \{\mathsf{t}, \mathsf{e}, \mathsf{j} \wedge \mathsf{t}, \mathsf{j} \wedge \mathsf{e}, \mathsf{t} \wedge \mathsf{e}, \mathsf{j} \wedge \mathsf{t} \wedge\ \mathsf{e}\} \end{array} \right\}$$

- In order to compute the innocently excludable alternatives, we take the intersection of the maximal excludable sets of alternatives. We end up with the set of alternatives of the form $\mathsf{m\ talked\ to}\ X$, where X is a plurality of women (i.e., not including atomic women!):

$$\mathsf{IE}\ Q = \{\, \mathsf{j} \wedge \mathsf{t}, \mathsf{j} \wedge \mathsf{e}, \mathsf{t} \wedge \mathsf{e}, \mathsf{j} \wedge \mathsf{t} \wedge\ \mathsf{e} \,\}$$

- The result of exhaustifying *Michael talked to some woman* relative to the set of innocently excludable alternatives, then, is the SI that *Michael didn't talk to a plurality of women*. When conjoined with the basic meaning, this entails that Michael talked to exactly one woman. This seems correct!

(22) $[\![$ exh_Q [Some woman ~~Michael talked to *t*~~] $]\!]$

$$\begin{aligned} = \exists x[\mathsf{woman}\ x \wedge \mathsf{m\ talked\ to}\ x] &\wedge \neg (\mathsf{j} \wedge \mathsf{t}) \\ &\wedge \neg (\mathsf{j} \wedge \mathsf{e}) \\ &\wedge \neg (\mathsf{t} \wedge \mathsf{e}) \\ &\wedge \neg (\mathsf{j} \wedge \mathsf{t} \wedge\ \mathsf{e}) \end{aligned}$$

Application to Chierchia's puzzle

Consider again Chierchia's puzzling example:

(23) Eleanor did the reading or some of the homework.
  a. ⇝̸ *Eleanor didn't do the reading or all of the homework*
  b. ⇝ *Eleanor didn't do all of the homework*

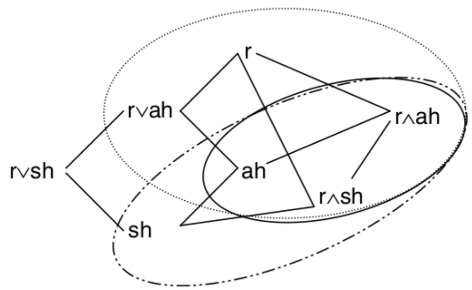
The puzzle here is how to derive the implicature in (23b) without also deriving the implicature in (23a), without making ad-hoc assumptions about alternatives.

- If we cash out alternativehood in terms of syntactic complexity, it follows that alternativehood should be *transitive*. In other words, if $q \in \mathsf{alt}\ p$ and $q' \in \mathsf{alt}\ q$, then it should follow that $q' \in \mathsf{alt}\ p$

- Under this assumption, we can readily enumerate the alternatives to *Eleanor did the reading or all of the homework* (which we'll abbreviate as $r \vee \exists h$).

$$\text{alt}\,(r \vee \exists h) = \left\{ \begin{array}{lr} r \wedge \exists h & \text{conjunctive alt} \\ r, \exists h & \text{S-alts} \\ r \vee \forall h & \text{universal alt} \\ \forall h & \text{S-alt of universal alt} \\ r \wedge \forall h & \text{conjunctive alt of S-alt} \end{array} \right\}$$

- We can order these alternatives relative to truth-conditional strength:



- As illustrated in the diagram, the maximal p-consistent subsets of alternatives are as follows,
  where p = Eleanor did the reading or some of the homework:

$$\left\{ \begin{array}{c} \{r \wedge \exists h, r, r \vee \forall h, \forall h, r \wedge \forall h\} \\ \{r \wedge \exists h, \exists h, \forall h, r \wedge \forall h\} \end{array} \right\}$$

- The intersection of the maximal p-consistent subsets gives us the set of innocently excludable alternatives, namely:

$$\text{IE} = \{r \wedge \exists h, \forall h, r \wedge \forall h\}$$

- When we exhaustify *Eleanor did the reading or some of the homework* relative to the innocently excludable alternatives, we derive the following inferences:
  - Eleanor did the reading or some (perhaps all) of the homework (or both). basic inference
  - Eleanor didn't do all of the homework. SI
  - Eleanor didn't do both the reading and some or all of the homework. SI
  - Strengthened meaning:
    Eleanor did either the reading, or some but not all of the homework, but not both, and she didn't do all of the homework.

- We've solved Chierchia's puzzle! For a disjunctive sentence of the form $p \vee q$, if q has a truth-conditionally stronger alternative q', then q' (unlike q) is an *innocently excludable alternative* and can be negated.

Embedding disjunction under universal quantifiers

- There are certain environments in which conjoining a disjunctive sentence with the negations of its disjuncts *doesn't* lead to a contradiction – specifically, embedding under an upward-monotone operator O:

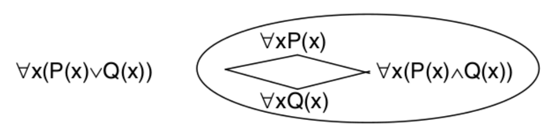
(24) $O\,(p \vee q) \wedge \neg\, O\, p \wedge \neg\, O\, q \wedge \neg\, O\,(p \wedge q)$

- I.e., if we take O to be a universal modal: p or q are enough to fulfil the requirements, but it's not required that p, and it's not required that q, and it's not required that $p \wedge q$.
- What are the facts?

(25) You're required to talk to Mary or Sue.
  a. ⇝ You're not required to talk to Mary.
  b. ⇝ You're not required to talk to Sue.
  c. ⇝ You're not required to talk to Mary and Sue.

(26) Every friend of mine has a dog or a cat.
  a. ⇝ Not every friend of mine has a dog.
  b. ⇝ Not every friend of mine has a cat.
  c. ⇝ Not every friend of mine has a cat and a dog.

- Since embedding under universals allows for consistent exclusion of all the individual disjunct alternatives, so our algorithm predicts the following set of innocently excludable alternatives. In fact there is only one maximal p-consistent subset of alternatives.



$$\text{IE} = \{\forall x[P\,x], \forall x[Q\,x], \forall x[P\,x \wedge Q\,x]\}$$

- Exhaustifying, e.g., *every friend of mine has a dog or a cat* relative to the innocently excludable alternatives results in the following strengthened meaning:

$$\forall x[P\,x \vee Q\,x] \wedge \neg\forall x[P\,x] \wedge \neg\forall x[Q\,x]$$

3 Recursive exhaustification and the problem of free choice

3.1 The problem of *free choice permission*

(27) You're allowed to eat the cake or the ice cream.
  a. *You're allowed to eat the cake* — Free Choice inferences
  b. *You're allowed to eat the ice cream*

The FC inferences don't logically follow from the most obvious LF we might assign to (27):

(28) allowed [[you eat the cake] or [you eat the ice cream]] $\Diamond (p \vee q)$

Assuming a possible world semantics for modality, this just means something like: *there is a world consistent with the rules in which $p \vee q$ is true*.

In fact this is equivalent to the following, which is clearly weaker than the conjunction of the FC inferences of (27):

(29) You're allowed to eat the cake or you're allowed to eat the ice cream. $\Diamond p \vee \Diamond q$

The abstract form of the problem is as follows:

(30) a. $\Diamond (p \vee q)$
  b. $\Diamond p \vee \Diamond q$ — $a \equiv b$
  c. Free Choice: $\Diamond p \wedge \Diamond q$

Evidence that FC is a scalar implicature

Scalar implicatures disappear in certain downward entailing environments:

(31) Nobody talked to Michael or Janet.

(32) a. *Negation of SI: $\exists x[x$ talked-to-m $\wedge\ x$ talked-to-j$]$
  b. Negation of standard meaning: $\neg\exists x[x$ talked-to-m $\vee\ x$ talked-to-j$]$

Plausibly, this is because computing the SI in such an environment results in a logically *weaker* meaning: *Nobody talked to Michael and not Janet, and nobody talked to Janet and not Michael, perhaps someone talked to both Michael and Janet*.

The exhaustivity operator must be subject to an economy condition, such that it isn't inserted in an LF if it weakens the global meaning of the sentence. We won't cash out exactly how to state such a condition here, but see Fox & Spector (2018).

The key observation here is that, just like an SI, FC inferences disappear in certain DE environments:

(33) No one is allowed to eat the cake or the ice cream.

(34) a. *negation of FC: $\neg\exists x[\Diamond P\ x \wedge \Diamond Q\ x]$
  b. negation of standard meaning: $\neg\exists x[\Diamond P\ x \vee \Diamond Q\ x]$

If the FC was indeed negated, (33) should be *true* in a situation in which everyone is allowed to eat one of the two desserts, but nobody has free choice, i.e., in a situation where nobody is allowed to decide which dessert to eat. This seems far too weak.

If the FC inference is a SI, the logic is the same as in the basic case: the negation of the FC inferences is *weaker* than the negation of the basic meaning, and therefore an economy condition on the distribution of exh rules out its insertion in such a context.

3.2 Recursive exhaustification

- Recall that ignorance inferences associated with, e.g., disjunctive sentences are computed on the basis of the MQ.
- We've been assuming that exh can be freely prepended to any sentence. One factor which may bias towards inserting an exhaustivity operator is if it avoids otherwise undesirable ignorance inferences. Fox captures this idea with the following principle:

(35) *Recursive parsing strategy*
  If a sentence S has an undesirable ignorance inference, parse it as exh (Alt(S)) S.

- Consider the following:

(36) I ate the cake or the ice cream.

- If we decide not to insert exh, and therefore not to compute any SI, the ignorance inference generated on the basis of the MQ is that *speaker doesn't know what she ate*, only that it includes the cake, the ice cream, or both. Now, depending on the context, this may be an "undesirable" thing to infer (i.e., if it's common ground that the speaker knows what she ate).
- To avoid this, we parse the disjunctive sentence with exh:

(37) exh $C\ (c \vee i)$
  where $C =$ alt $(c \vee i)$

- As we've already determined, the parse in (38) gives rise to the exclusive inference as an SI.
- If we're in a context in which it's common ground that the speaker knows what she ate, this still leaves us with an undesirable ignorance inference: *the speaker ate the cake or the ice cream but not both, and she doesn't know whether she ate cake, and she doesn't know whether she ate ice cream*.
- We might try to avoid the undesirable ignorance inference by prepending exh again, in accordance with the recursive parsing strategy:

(38) $\mathsf{exh}\ C'\ (\mathsf{exh}\ C\ (c \vee i))$
where $C = \mathsf{alt}\ (\mathsf{exh}\ C\ (c \vee i))$

- It turns out that recursive exhaustification has *no effect*, given our algorithm for computing the strengthened meaning.
- To see this, let's first lay out the set of alternatives to the sentence: $\mathsf{exh}\ C\ (c \vee i)$

$$C' = \left\{ \begin{array}{lll} \mathsf{exh}\ C\ (c \vee i) & = (c \vee i) \wedge \neg (c \wedge i) & \\ & = (c \wedge \neg i) \vee (i \wedge \neg c) & \\ \mathsf{exh}\ C\ c & & = c \wedge \neg i \\ \mathsf{exh}\ C\ i & & = i \wedge \neg c \\ \mathsf{exh}\ C\ (c \wedge i) & & = p \wedge q \end{array} \right\}$$

- Two things to observe here:
  - The fourth alternative is already excluded by the prejacent, and hence can be ignored.
  - The first alternative is equivalent to the disjunction of the second and the third. In other words:

$$\mathsf{exh}\ C\ (c \vee i) \equiv (\mathsf{exh}\ C\ c) \vee (\mathsf{exh}\ C\ i)$$

- The relevant alternatives are therefore $c \wedge \neg i$ and $i \wedge \neg c$. If the first is excluded the second must be true, and vice versa. Therefore, neither is *innocently excludable*, and hence the meaning does not change with a second level of exhaustification.

> Conclusion
>
> In a simple disjunctive sentence, recursive application of exh doesn't change the meaning.

- There is no way to avoid the undesirable ignorance inference. In fact "I ate cake or ice cream" is indeed an odd thing for the speaker to utter in a context where it's common ground that she knows what she ate.

3.3 Deriving free choice via recursive exhaustification

- Consider again the following sentence:

(39) You may eat the cake or the ice cream.

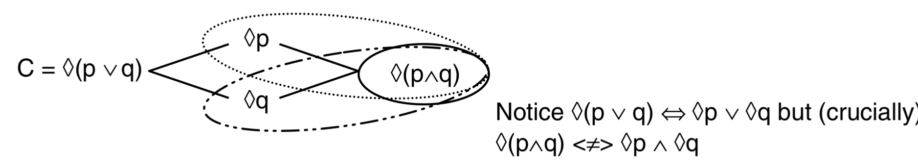
- According to the MQ, the following ignorance inferences are derived:
  - The speaker doesn't know that you may eat cake.
  - The speaker doesn't know that you may eat ice cream.
  - $\rightsquigarrow$ the speaker doesn't know what you may eat.
- Assuming we're in a context where the speaker can be assumed to be informed about what we may eat, the hearer will opt for a parse with exh:

(40) $\mathsf{exh}\ C$ you may eat the cake or the ice cream.

- i.e.:

$$\mathsf{exh}\ C\ (\Diamond\ (c \vee i))$$

- Assuming that the individual disjuncts count as alternatives, the set of alternatives to $\Diamond\ (c \vee i)$ is the following (where $p \vee q$ is subbed in for $c \vee i$):



- $\Diamond\ (c \wedge i)$ is the only proposition that may be innocently excluded, given our algorithm.
- The strengthened meaning is therefore:

$$\Diamond(c \vee i) \wedge \neg \Diamond\ (c \wedge i)$$

- This is consistent with free choice possibility (i.e., $\Diamond c \wedge \Diamond i$), but of course doesn't assert free choice.
- This strengthened meaning will give rise the the following ignorance inferences via MQ: *the speaker doesn't know what one is allowed to eat, only that it includes cake and ice cream, but not both.*
- The resulting ignorance inference might be implausible, in which case we can prepend exh again:

(41) $\mathsf{exh}\ C'\ (\mathsf{exh}\ C\ (\Diamond\ (c \vee i)))$

- *This* time, the second exhaustification has consequences. To see this, let's first compute the meanings of the various alternatives:

$$C' = \left\{ \begin{array}{ll} \text{exh } C\ (\Diamond(c \vee i)) & = \Diamond(c \vee i) \wedge \neg\Diamond\ (c \wedge i) \\ \text{exh } C\ (\Diamond\ c) & = \Diamond\ c \wedge \neg\ \Diamond i \\ \text{exh } C\ (\Diamond\ i) & = \Diamond i \wedge \neg\ \Diamond\ c \\ \text{exh } C\ (\Diamond\ (c \wedge i)) & = \Diamond\ (c \wedge i) \end{array} \right\}$$

- There are now two propositions in C' that can be innocently excluded, since negating $\Diamond\ c \wedge \neg\Diamond\ i$ does *not* necessarily include $\Diamond\ i \wedge \neg\ \Diamond\ c$, and vice versa.
- Hence:

$$\begin{aligned} \text{exh } C'\ (\text{exh } C\ (\Diamond\ (c\ \vee i))) = {} & \Diamond\ (c \vee i) \wedge \neg\ \Diamond\ (c \wedge i) \\ & \wedge \neg\ (\Diamond\ c \wedge \neg\Diamond\ i) \\ & \wedge \neg\ (\Diamond\ i \wedge \neg\Diamond\ c) \\ & \equiv \Diamond\ c \wedge \Diamond\ i \\ & \wedge \neg\ \Diamond\ (c \wedge i) \end{aligned}$$

- We've derived the free choice inference for disjunction embedded under an existential modal!

Next time

- A refinement of Fox's algorithm based on *universal free choice*:

(42) Every boy is allowed to eat ice cream or cake.
   a. $\rightsquigarrow$ every boy is allowed to eat ice cream.
   b. $\rightsquigarrow$ every boy is allowed to eat cake.

(43) No student is required to solve problem A and problem B.
   a. $\rightsquigarrow$ No student is required to solve problem A.
   b. $\rightsquigarrow$ No student is required to solve problem B:w

- Procedure for applying $\text{EXH}^{\text{IE+II}}$ to a proposition p:
  - Negate the set of *innocently excludable* members of Alt(p).
    - To get the *innocently excludable* members of Alt(p), we gather the maximal members of $\mathcal{P}$(Alt(p)) that can be negated consistently with p, and intersect them.
  - Assert the set of *innocently includable* members of Alt(p).
    - To get the *innocently includable* members of Alt(p), we gather the maximal members of $\mathcal{P}$(Alt(p)) that can be asserted consistently with p and the negation of the innocently excludable alternatives, and intersect them.

(44) Every boy is allowed to eat ice cream or cake. $\forall x \Diamond(Px \vee Qx)$
   a. *Every boy is allowed to eat ice cream.* $\forall x \Diamond Px$
   b. *Every boy is allowed to eat cake.* $\forall x \Diamond Qx$

- First, let's compute the set of alternatives:

$$\begin{aligned} & \text{Alt}(\forall x \Diamond(Px \vee Qx)) \\ & = \left\{ \begin{array}{l} \overbrace{①\ \forall x \Diamond(Px \vee Qx)}^{\text{prejacent}}, \overbrace{②\ \forall x \Diamond Px, ③\ \forall x \Diamond Qx}^{\text{universal disjunctive alts}}, \overbrace{④\ \forall x \Diamond(Px \wedge Qx)}^{\text{universal conjunctive alt}} \\ \underbrace{⑤\ \exists x \Diamond(Px \vee Qx)}_{\text{existential alt}}, \underbrace{⑥\ \exists x \Diamond Px, ⑦\ \exists x \Diamond Qx}_{\text{existential disjunctive alts}}, \underbrace{⑧\ \exists x \Diamond(Px \wedge Qx)}_{\text{existential conjunctive alt}} \end{array} \right\} \end{aligned}$$

- Let's gather together the maximal subsets of Alt($\forall x \Diamond(Px \vee Qx)$) that can be negated consistently with $\forall x \Diamond(Px \vee Qx)$).

$$\{\ ②\ \forall x \Diamond Px, ③\ \forall x \Diamond Qx, ④\ \forall x \Diamond(Px \wedge Qx), ⑧\ \exists x \Diamond(Px \wedge Qx)\ \}$$

$$\{\ ②\ \forall x \Diamond Px, ⑥\ \exists x \Diamond Px, ④\ \forall x \Diamond(Px \wedge Qx), ⑧\ \exists x \Diamond(Px \wedge Qx)\ \}$$

$$\{\ ③\ \forall x \Diamond Qx, ⑦\ \exists x \Diamond Qx, ④\ \forall x \Diamond(Px \wedge qx), ⑧\ \exists x \Diamond(px \wedge qx)\ \}$$

$$\text{IE}(\forall x \Diamond(Px \vee Qx)) = \{\ ④\ \forall x \Diamond(Px \wedge Qx), ⑧\ \exists x \Diamond(Px \wedge Qx)\ \}$$

- If we take the prejacent together with the negation of the IE alternatives, we end up in a world where, either all boys are allowed to P and not Q, all boys are allowed to Q and not P, or some boys are allowed to P and not Q and some boys are allowed to Q and not P (no boys are allowed to P and Q), i.e.

$$\forall x \Diamond(Px \vee Qx) \wedge \neg\exists x \Diamond(Px \wedge Qx)$$

- Let's gather together the maximal subsets of Alt($\forall x \Diamond(Px \vee Qx)$) that can be asserted consistently with $\forall x \Diamond(Px \vee Qx)) \wedge \neg\exists x \Diamond(Px \wedge Qx)$.
- It turns out there is only one such set, consisting of all the non-IE alts:

$$\left\{ \begin{array}{l} ①\ \forall x \Diamond(Px \vee Qx), ②\ \forall x \Diamond Px, ③\ \forall x \Diamond Qx, \\ ⑤\ \exists x \Diamond(Px \vee Qx), ⑥\ \exists x \Diamond Px, ⑦\ \exists x \Diamond Qx \end{array} \right\}$$

- Asserting the II alts together with the prejacent and negation of the IE alts results in the following enriched meaning:

$$\forall x \Diamond(Px \vee Qx) \wedge \neg\exists x \Diamond(Px \wedge Qx) \wedge \forall x \Diamond Px \wedge \forall x \Diamond Qx$$

3.4 The *only*-exh correspondence

(45) a. EXH^{IE} asserts that its prejacent is true and that all IE alternatives are false.
b. *only* presupposes that its prejacent is true and that all IE alternatives are false.

Revised version:

(46) a. EXH^{IE+II} asserts that all II alternatives are true and asserts that all IE alternatives are false.
b. *only* presupposes that all II alternatives are true and that all IE alternatives are false.

(47) We are only allowed to eat [ice cream or cake]$_F$
a. $\rightsquigarrow$ we are allowed to eat ice cream
b. $\rightsquigarrow$ we are allowed to eat cake

(48) Are we only allowed to eat ice cream or cake?
a. $\rightsquigarrow$ We are allowed to eat ice cream.
b. $\rightsquigarrow$ We are allowed to eat cake.

(49) Are we allowed to eat ice cream or cake?
a. $\neg \rightsquigarrow$ We are allowed to eat ice cream.
b. $\neg \rightsquigarrow$ We are allowed to eat cake.

4 Innocent Conclusion

- The relation between exhaustification and Groenendijk & Stokhof's (1984) partition semantics.

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