Anaphora: extensions and alternatives

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1 *Roadmap*

- We'll begin with a recap of where we're at with Dynamic Predicate Logic (DPL); this will be a good time for any remaining clarification questions. We'll also touch on some problems and possible extensions.

- Next, we'll discuss two big problems for first-generation dynamic approaches to anaphora (Heim 1982, Kamp 1981, Groenendijk & Stokhof 1991) — *double negation*, and *the proportion problem*.

- This will lead into a discussion of a recent alternative approach which aims to retain the good parts of Dynamic Semantics (DS), while being more explanatory and having superior empirical coverage — Mandelkern's (2020b, 2020a) *pseudo-dynamics*.[1]

- Next week we'll move away from anaphora, and DS; we'll be focusing on *implicature* for the last three classes of the semester.[2]

[1] I'll also touch on some recent, related work of mine Elliott 2020b,a, time permitting.

[2] If you haven't already, we'd encourage you to start thinking of possible squib topics.

2 *Recap and summary of DPL*

2.1 *Motivations*

Initial, empirical motivations: *discourse anaphora* and *donkey anaphora*:

Discourse anaphora

(1) A^1 philosopher attended the talk, and she$_1$ asked some difficult questions.

(2) a. A: A^1 famous philosopher attended my talk.
b. B: Oh? Did she$_1$ ask any especially difficult questions.

Donkey anaphora

(3) If a^1 farmer owns a^2 donkey, he$_1$ feeds it$_2$ hay.

(4) Every farmer who owns a^2 donkey feeds it$_1$ hay.

Conceptual motivation: a richer notion of context is necessary to track relative referential certainty.

The *formal link* condition indicates that we need a notion of *aboutness* above and beyond the classical notion of content.

(5) a. Andreea is married. I saw ?them/Andreea's spouse yesterday.
    b. Andreea has a husband. I saw them/Andreea's spouse yesterday.

(6) a. I dropped ten marbles and found all of them, except for one. It's probably under the sofa.
    b. I dropped ten marbles and found only nine of them. ?It's probably under the sofa. (Heim 1982: p. 21)

2.2 Technical summary

Syntax of DPL

Given the following:

- $\mathbb{V}$, a non-empty set of *variables*, $x_1, x_2, \ldots$.
- $\mathbb{C}$, a non-empty set of *individual constants*, $a, b, c, \ldots$.
- $\mathbb{P}_n$, a non-empty set of n-ary predicate symbols, $P, Q, \ldots$.
- $\mathbb{T}$, the set of terms: $\mathbb{V} \cup \mathbb{C}$.

A DPL language $\mathbb{L}$ is the smallest set where:

- If $P \in \mathbb{P}_n$, and $t_1, \ldots t_n \in \mathbb{T}$, then $P\ t_1 \ldots t_n \in \mathbb{L}$. — atomic sentences
- If $\phi \in \mathbb{L}$, then $\neg\, \phi \in \mathbb{L}$. — negated sentences
- If $\phi, \psi \in \mathbb{L}$, then $\phi \wedge \psi, \phi \vee \psi, \phi \rightarrow \psi \in \mathbb{L}$ — con/disjunctive & implicational sentences
- If $\phi \in L$, $x_n \in \mathbb{V}$, then $\exists x_n\ \phi, \forall x_n\ \phi \in \mathbb{L}$ — quantified sentences
- If $x_n \in \mathbb{V}$, then $\varepsilon x_n \in \mathbb{L}$ — random assignment

Semantics of terms

Constants:

$$[\![c]\!]^g \coloneqq I(c)$$

Variables:

$$[\![x_1]\!]^g \coloneqq \begin{cases} g_1 & g_1 \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Semantics of atomic sentences

$$[\![P\ t_1\ \ldots\ t_n]\!]^g \coloneqq \begin{cases} \{\, g\,\} & \langle [\![t_1]\!]^g, \ldots, [\![t_n]\!]^g \rangle \in I(P) \\ & \wedge\ [\![t_1]\!]^g, \ldots, [\![t_n]\!]^g \text{ are defined} \\ \varnothing & \langle [\![t_1]\!]^g, \ldots, [\![t_n]\!]^g \rangle \notin I(P) \\ & \wedge\ [\![t_1]\!]^g, \ldots, [\![t_n]\!]^g \text{ are defined} \\ \# & \text{otherwise} \end{cases}$$

Existentially quantified sentences

$$[\![\exists x_1\ \phi]\!]^g \coloneqq \bigcup_{x \in D} [\![\phi]\!]^{g^{[1 \to x]}}$$

Conjunctive sentences

$$[\![\phi \wedge \psi]\!]^g \coloneqq \bigcup_{g' \in [\![\phi]\!]^g} [\![\psi]\!]^{g'}$$

Random assignment

$$[\![\varepsilon x_n]\!]^g \coloneqq \{\ g^{[n \to x]} \mid x \in D\ \}$$

Negated sentences

$$[\![\neg\ \phi]\!]^g \coloneqq \begin{cases} \{\ g\ \} & [\![\phi]\!]^g = \varnothing \\ \varnothing & [\![\phi]\!]^g \neq \varnothing \\ \text{undefined} & \text{otherwise} \end{cases}$$

Disjunctive sentences

$$[\![\phi \vee \psi]\!]^g \coloneqq \begin{cases} \{\ g\ \} & [\![\phi]\!]^g \cup [\![\psi]\!]^g \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$$

Program disjunction

$$[\![\phi \veebar \psi]\!]^g \coloneqq [\![\phi]\!]^g \cup [\![\psi]\!]^g$$

Implicational sentences

$$[\![\phi \to \psi]\!]^g \coloneqq \begin{cases} \{\ g\ \} & \{\ g' \mid g' \in [\![\phi]\!]^g\ \} \\ & \subseteq \{\ g'' \mid \exists h [h \in [\![\psi]\!]^{g''}\ ]\ \} \\ \varnothing & \text{otherwise} \end{cases}$$

Universally quantified sentences

$$[\![\forall x_n\ \phi]\!]^g \coloneqq \begin{cases} \{\ g\ \} & \{\ g^{[n \mapsto x]} \mid x \in D\ \} \\ & \subseteq \{\ g' \mid \exists h [h \in [\![\phi]\!]^{g'}\ ]\ \} \\ \varnothing & \text{otherwise} \end{cases}$$

2.3 Some important results

True existential quantification can be dispensed with, and instead expressed in terms of random assignment and conjunction

Existential quantification and random assignment

$$\exists x_n\ \phi \Leftrightarrow \varepsilon x_n\ \wedge \phi$$

Egli's theorem and its corollary are validated — this is central to the account of discourse anaphora and donkey anaphora respectively:

Egli's theorem

$$\exists x_n\ \phi \wedge \psi \Leftrightarrow \exists x_n\ (\phi \wedge \psi)$$

Expressed in terms of random assignment, this is just associativity:

$$(\varepsilon x_n\ \wedge \phi) \wedge \psi \Leftrightarrow \varepsilon x_n\ \wedge (\phi \wedge \psi)$$

This means that...

(7) Someone[1] walked in and she$_1$ immediately sat down.
$\rightsquigarrow (\varepsilon x_1 \wedge\ W\ x_1) \wedge S\ x_1$

...can be rewritten as...

(8) $\rightsquigarrow \varepsilon x_1 \wedge\ (W\ x_1 \wedge S\ x_1)$

Egli's corollary

$$(\exists x_1\ \phi) \rightarrow \psi \Leftrightarrow \forall x_1\ (\phi \rightarrow \psi)$$

Expressed in terms of random assignment:

$$(\varepsilon x_1 \wedge \phi) \rightarrow \psi \Leftrightarrow \forall x_1 (\phi \rightarrow \psi)$$

This means that...

(9) If someone[1] walked in then she$_1$ immediately sat down.
$\rightsquigarrow (\varepsilon x_1 \wedge W\ x_1) \rightarrow S\ x_1$

...can be rewritten as...

(10) $\rightsquigarrow \forall x_1\ (W\ x_1 \rightarrow S\ x_1)$

Similar, the following sentence:

(11) Everyone[1] who owns a[2] donkey cares for it$_2$.
$\rightsquigarrow \forall x_1 ((\varepsilon x_2\ \wedge D\ x_2 \wedge O\ x_1\ x_2) \rightarrow C\ x_1\ x_2)$

...can rewritten as...

(12) $\rightsquigarrow \forall x_1 \forall x_2 ((D\ x_2 \wedge O\ x_1\ x_2) \rightarrow C\ x_1\ x_2)$

Accessibility

Due to the semantics of the operator, various *accessibility* results are derived.

Assuming that indefinites scope within their containing sentences, an indefinites in prior conjuncts are accessible to pronouns in subsequent conjuncts, but not vice versa — conjunction is both *externally and internally dynamic*:

(13) a. Someone^1 walked in and she$_1$ immediately sat down.
b. # She$_1$ immediately sat down, and someone^1 walked in.

Negation renders any indefinites within its scope inaccessible as antecedents to subsequent pronouns — negation is *externally static*:

(14) # It's not true that anyone^1 walked in. She$_1$ immediate sat down.

An indefinite in either disjunct is inaccessible as an antecedent to a pronoun in the other disjunct, and to pronouns in subsequent sentences — disjunction is both internally and externally static.[3]

[3] Stone disjunctions are an exception to this generalization.

(15) a. # Either a^1 philosopher attended this talk or she$_1$'s waiting outside.
b. # Either she$_1$'s waiting outside, or a^1 philosopher attended this talk.

(16) a. # Either a^1 philosopher attended this talk or nobody did.
She$_1$ enjoyed it.
b. # Either nobody attended this talk, or a^1 philosopher did.
She$_1$ enjoyed it.

Implicational sentences are *internally dynamic* (since they allow for donkey anaphora, asymmetrically), but *externally static*:

(17) a. If a^1 philosopher attended this talk, then she$_1$ enjoyed it.
b. # If she$_1$ enjoyed this talk, then a^1 philosopher will feel inspired.

(18) a. # If a^1 philosopher attended this talk, then it was a success,
but she$_1$ complained about it later.
b. # If this talk was a success, then it inspired a^1 philosopher,
but she$_1$ complained about it later.

Donkey cataphora?

One interesting nuance — Chierchia 1995: p. 192 notes that cataphora is surprisingly good in certain implicational sentences (see also Barker & Shan 2008 for similar remarks).

(19) ? If John overcooks it$_1$, then a^1 hamburger usually tastes bad.

Elliott & Sudo (2019) note that sentences like (19) become unacceptable in an episodic context, under the bound reading:

(20) #If John overcooked it$_1$, then a^1 hamburger tasted bad.

They took this to suggest that something special is responsible for cataphora in (19), such as reference to a kind.[4]

[4] This class of examples is however very poorly understood, and a topic ripe for future research.

See Elliott & Sudo (2019) for an argument that (something like) cataphoric dynamic binding is possible, but only with definite antecedents.

Quantificational subordination

Universally quantified sentences are also predicted to be externally static, although this is apparently not the case:

(21) Every woman bought a^1 book. Most of them read it$_1$ immediately.

But note that this is not generally possible:

(22) #Every woman bought a^1 book. Tom borrowed it$_1$ later.

In fact, (21) is part of a systematic class of exceptions. What seems crucial here is that the first sentence establishes a functional relationship between *women* and *book*, which is picked up (in some sense) in the second sentence.

This phenomena is known as *quantificational subordination*, and has often been taken to be one motivation a powerful extension of DPL — *plural dynamic semantics* see, e.g., van den Berg 1996, Brasoveanu 2007.[5]

[5] That said, there have been some attempts to capture quantificational subordination without significantly extending the expressive power of DPL. See, e.g., Gotham 2019b.

Modal subordination

Background: as we've seen in previous classes, modals are *holes* for presupposition projection:

(23) Paul might have stopped smoking. *presupposes: Paul used to smoke*

There are some surprising exceptions to this generalization, involving discourses with successive modals:

(24) Paul might have started smoking,
and he might have subsequently stopped. *presuppostionless*

Based on our naive projection generalizations, this is predicted to nevertheless presuppose that *Paul used to smoke*, since *Paul might have started smoking* doesn't entail the presuppostion.

In order to make sense of this, Roberts (1989) famously proposed that subsequent modals can be anaphoric on a tentative update associated with a previous modal:[6]

[6] Implementation details differ, but this is essentially the idea. See Kibble (1994) for a presentation which sticks fairly close to the spirit of DPL.

(25) Paul might^1 have started smoking,
and he might$_1$ have subsequently stopped.

Relevant to our purposes, modal subordination gives rise to a broad class of exceptions to the accessibility generalizations discussed above.

(26) If a^1 philosopher attended this talk, then it was a success.
She would have enjoyed it$_1$.

(27) Paul doesn't own a^1 truck, but it$_1$ would be a Ford Bronco.

It seems that other operators can license anaphoric modals, which in turn can exceptionally license anaphora — Geurts (2019) refers to this phenomenon as *piggyback anaphora*, and it's of course outside of the remit of the simple system we've developed here.

This is worth bearing in mind when assessing the adequacy of the accessibility generalizations.

3 *Beyond* DPL

3.1 Double negation and bathroom sentences

Double negation

In DS, negation is a *destructive* operation; it obliterates any Discourse Referents (DRS) in its scope since, the output state of the contained sentence is, essentially, closed.

This makes a pretty strong prediction; *double negation elimination* should *not* be valid, unlike in a classic setting.

We can illustrate this be giving a concrete example:

(28) It's not true that nobody left. $\neg\,(\neg\,\exists x_1\ L\ x_1)$

Let's compute the meaning of the sentence in DS:[7]

[7] As usual, we ignore undefinedness since there are no free variables.

(29) $$[\![\neg\,(\neg\,\exists x_1\ L\ x_1)]\!]^g = \begin{cases}\{\,g\,\} & [\![\neg\,\exists x_1\ L\ x_1]\!]^g = \varnothing\\ \varnothing & \text{otherwise}\end{cases}$$

(30) $$= \begin{cases}\{\,g\,\} & [\![\exists x_1\ L\ x_1]\!]^g \neq \varnothing\\ \varnothing & \text{otherwise}\end{cases}$$

$$(31) \quad = \begin{cases} \{\, g \,\} & \{\, g^{[1 \to x]} \mid x \in I(L) \wedge x \in D \,\} \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$$

$$(32) \quad = \begin{cases} \{\, g \,\} & \exists x[x \in D \wedge x \in I(L)] \\ \varnothing & \text{otherwise} \end{cases}$$

If we just take the conditions under which the doubly negated sentence is *true*, then this is equivalent to the conditions under which its positive counterpart are true; namely, just so long as $I(L) \neq \varnothing$:

$$(33) \quad [\![\exists x_1 \; L \; x_1]\!]^g = \{\, g^{[1 \to x]} \mid x \in I(L) \wedge x \in D \,\}$$

However, if we compare (32) and (33), we can see that the output states are *not* the same; the doubly-negated sentence is a *test*, whereas its positive counterpart introduces x_1 as a DR.

It was already noted by Groenendijk & Stokhof that this is a problem, and indeed it seems to make bad predictions for anaphora.

(34) It's not true that NO[1] philosopher registered; she$_1$'s sitting at the back.

Anaphora from doubly-negated sentences seems to be subject to poorly understood constraints; Gotham (2019a) (see also Krahmer & Muskens 1995) claims that there is an associated uniqueness inference.[8]

[8] The following examples are based on Gotham 2019a.

(35) *Context: The speaker knows that John owns more than one shirt.*
  a. John owns a[1] shirt. It$_1$'s in the wardrobe.
  b. ?? It's not true that John DOESN'T own a[1] shirt; It$_1$'s in the wardrobe!

As I note in Elliott 2020b, anaphora from under double negation is compatible with a plural pronoun, just so long as it picks up a *maximal* DR; uniqueness is just a special case of maximality.

(36) John doesn't own no[1] shirt. They$_1$'re in the wardrobe.

The validity of Double Negation Elimination (DNE) with respect to anaphora might be taken to show that DPL strays too far from the classical; if Gotham is correct however, we might not want to reinstate $\neg(\neg\phi) \Leftrightarrow \phi$ wholesale.

We'll ultimately come back to this issue when we discuss Mandelkern's approach.

Bathroom sentences

There's a related problem with involving disjunctive sentence.

First, think back to the Heim/Karttunen projection generalization for disjunctive sentences.

(37) Either there is no bathroom, or the bathroom is upstairs.

(37) is presuppositionless, because the presupposition of the second disjunct (that *there is a bathroom*), is locally satisfied; in update semantics, a subsequent disjunct is interpreted relative to the *negation* of the initial disjunct.

We can make a completely parallel observation with anaphora.

(38) Either there is no^1 bathroom, or it$_1$'s upstairs.

The entry for disjunction we've given here, based on Groenendijk & Stokhof (1991), is both externally *and* internally dynamic, so it has no chance at all of accounting for the possibility of anaphora in (38).[9]

[9] Stone disjunction doesn't help either — despite being externally dynamic, it's internally static.

Our entry for disjunction predicts that the test imposed by (38) is passed if the union of output states of the first and second disjuncts is non-empty; the disjunctive sentence should therefore inherit the definedness conditions of *it's upstairs*, which contains a free variable.

An intuitive thought is that a subsequent disjunct is interpreted in the context of the *negation* of the first, just like in our update semantic entry for disjunction (Beaver 2001), so the problem of (38) is reduced to accounting for anaphora in the following:

(39) Either there is no^1 bathroom,
or (there isn't no^1 bathroom and) it$_1$'s upstairs.

This, naturally, reduces the problem of anaphora in bathroom sentences to the problem of DNE more generally.

Similarly, Gotham (2019a) claims that anaphora in bathroom sentences comes with an associated uniqueness inference.

(40) *Context: the speaker knows that, if John owns any shirts, he owns more than one.*
\# Either John has no^1 shirt, or it$_1$'s in the wardrobe.

This receives a natural explanation, if the account of anaphora in bathroom sentences is parasitic on the account of DNE.

The data is somewhat unclear however; Krahmer & Muskens (1995) develop an account of bathroom sentences that ascribes them universal truth-conditions, just like donkey sentences; for them, (38) is true just so long as there is no bathroom that *isn't* upstairs (there may be multiple bathrooms).

In essence, the idea is to reduce the bathroom sentence to the following paraphrase; due to Egli's corollary, the predicted truth conditions are universal.

(41) Either there is no^1 bathroom,
or (if there isn't no^1 bathroom, then) it$_1$'s upstairs.

There are a number of accounts of DNE and bathroom sentences in the literature which depart to a lesser or greater extent from DPL: see especially Krahmer & Muskens 1995, who develop a version of Discourse Representation Theory (DRT) which distinguishes between the positive and negative contribution of a sentence. We'll discuss a more recent proposal later on.[10]

[10] For anyone who's interested in this problem (I think it would make an *excellent* squib topic), there's an extremely useful discussion in van den Berg (1996: ch. 2).

3.2 Generalized quantifiers and the proportion problem

DPL with quantifiers

DPL is quite limited in its expressive power — we're not in a position to analyze the broader range of environments in which donkey anaphora is possible.

(42) Most [people who see a^1] [corgi pet it$_1$].

(43) Few [people who see a^1] [corgi pet it$_1$].

(44) Usually, a [person who sees a^1 corgi] [pets it$_1$].

Generalization: quantificational expressions are *internall dynamic*; anaphora is licensed from the restrictor to the scope (but not vice versa).

We'll try to naively extend DPL to generalized quantifiers; in so doing, we'll encounter an interesting (but solvable) problem.

In order to account for determiners, we first need to minimally extend the syntax of $\mathbb{L}$, in the following way:

- Let $\mathbb{Q}$ be a non-empty set of *determiners*.
- If $Q \in \mathbb{Q}$, $x_n \in V$, $\phi, \psi \in \mathbb{L}$, then $Q x_n\ \phi\ \psi \in \mathbb{L}$.

Note that we're treating determiners as (two-place) *sentential* operators, which come with a binding index.

Unselected semantics for quantified sentences

N.b. the semantics that we'll give for quantified sentences is called an *unselective* semantics, for reasons which will become clear.

Let's assume that the valuation function I maps determiners to conservative binary relations between sets of individuals (i.e., generalized quantifiers).

Remember our semantics for universal sentences, in terms of subsethood? This will help us give a general recipe for quantified statements:

Unselective semantics for quantified sentences

To compute the output of a quantified sentence $Q_n\ \phi\ \psi$, we must compute two sets: (i) the *restrictor set* is the outputs of the restrictor ϕ interpreted in the context of n-indexed random assignment. (ii) the *matrix set* is the set of inputs that make the matrix sentence ψ dynamically true. The quantified sentence $Q_n\ \phi\ \psi$ is a test that checks whether a set-theoretic relation delivered by the valuation function holds between these two sets.

$$\begin{aligned}&[\![Qx_n\ \phi\ \psi]\!]^g\\ &:= \begin{cases}\{\, g\,\} & [\![\varepsilon x_n \wedge \phi]\!]^g\ I(Q)\ \{\, g' \mid \exists h[h \in [\![\psi]\!]^{g'}\,]\,\}\\ \varnothing & \text{otherwise}\end{cases}\end{aligned}$$

We can check that this makes the right predictions for donkey anaphora in universal statements.

(45) Everyone who sees a^1 corgi pets it$_1$

We'll translate this into a quantified sentence as follows:[11]

(46) $\mathbf{every}_1\ (\varepsilon x_2 \wedge\ S\ x_1\ x_2)\ (P\ x_1\ x_2)$

(47) $$\begin{aligned}&[\![\mathbf{every}_1\ (\varepsilon x_2 \wedge\ S\ x_1\ x_2)\ (P\ x_1\ x_2)]\!]^g\\ &= \begin{cases}\{\, g\,\} & [\![\varepsilon x_1 \wedge \varepsilon x_2 \wedge S\ x_1\ x_2]\!]^g \subseteq \{\, g' \mid \exists h[h \in [\![P\ x_1\ x_2]\!]^{g'}\,]\,\}\\ \varnothing & \text{otherwise}\end{cases}\end{aligned}$$

In order to see if the test is passed, we first compute the restrictor set — this gives back the set of modified assignments $g^{[1\mapsto x, 2\mapsto y]}$, such that x saw y.

(48) $[\![\varepsilon x_1 \wedge \varepsilon x_2 \wedge S\ x_1 x_2]\!]^g = \{\, g^{[1\mapsto x, 2\mapsto y]} \mid \langle x, y\rangle \in I(S)\,\}$

Now we compute the matrix set — this gives back the set of assignments g' defined for $1, 2$, such that g'_1 petted g'_2.

[11] As discussed by Heim (1982), these logical forms can be constructed compositionally by scoping out the *determiner*.

(49) $\{\, g' \mid \exists h[h \in [\![P\ x_1\ x_2]\!]]\,\} = \{\, g' \mid g' \neq \#, \langle g'_1, g'_2 \rangle \in I(P)\,\}$

In order for the restrictor set to be a subset of the matrix set, it must be the case that each modified assignment $g^{[1\mapsto x, 1\mapsto y]}$ in the restrictor set is s.t. x petted y; if this does not hold for some assignment in the restrictor set, then the subsethood relation fails to hold.

(50) $\forall h \in \{\, g^{[1\mapsto x, 2\mapsto y]} \mid \langle x, y \rangle \in I(S)\,\} \rightarrow \langle h_1, h_2 \rangle \in I(P)$
$\rightsquigarrow \forall \langle x, y \rangle \in I(S)[\langle x, y \rangle \in I(P)]$

This elegant semantics for quantified sentences is essentially the semantics given for adverbs of quantification in Groenendijk & Stokhof 1991 and (implicitly) in Heim 1982, but it runs into two well-known problems: the *proportion problem* and the distinction between *weak and strong readings*.

The proportion problem

Now, let's consider what happens when we combine donkey anaphora with the determiner *most*:[12]

(51) Most people who see a corgi pet it.
$\rightsquigarrow \mathbf{most}_1\ (\epsilon x_2 \wedge C\ x_2 \wedge S\ x_1\ x_2)\ (P\ x_1\ x_2)$

[12] We assume here that *most* means *more than half*, although this is of course a simplification.

Let's compute the restrictor set relative to an input g, and the matrix set as usual:

(52) Restrictor set relative to g:
$\{\, g^{[1\mapsto x, 2\mapsto y]} \mid y \in I(C) \wedge \langle x, y \rangle \in I(S)\,\}$

(53) Matrix set:
$\{\, g' \mid g'_1, g'_2 \neq \# \wedge \langle g'_1, g'_2 \rangle \in I(P)\,\}$

For the test imposed by the quantified sentence to be successful, more than half $\langle x, y \rangle$ pairs, s.t. y is a corgi and x sees y, should be such that x pets y.

As many have remarked[13], it's easy to come up with scenarios to demonstrate that this gets the truth-conditions of the English sentence wrong.

[13] See, e.g., Partee 1984, Kadmon 1987, Rooth 1987, and Heim 1990.

Let's say that three people — Sarah, Josie, and Alex — saw corgis:

- Sarah went to a dog park, and saw 10 corgis ($c_1, \ldots, c_{10}$) — she petted all of them.
- Josie and Alex each saw one corgi (c_1 and c_2 respectively), but didn't pet them.

We can list all the $\langle x, y\rangle$ pairs such that y is a corgi, and x saw y. I've highlighted those pairs also in a petting relationship:

$$\left\{\begin{array}{c} \langle j, c_1\rangle, \langle a, c_2\rangle, \\ \underline{\langle s, c_1\rangle, \langle s, c_2\rangle, \langle s, c_3\rangle, \langle s, c_4\rangle, \langle s, c_5\rangle,} \\ \underline{\langle s, c_6\rangle, \langle s, c_7\rangle, \langle s, c_8\rangle, \langle s, c_9\rangle, \langle s, c_{10}\rangle,} \end{array}\right\}$$

It's pretty clear then, that our truth conditions predict that the sentence should be true, but it's intuitively false in this scenario.

(54) Most people who see a^1 corgi pet it$_1$.

The problem amounts to the fact that we end up quantifying over *person-corgi pairs*, rather than individuals. This is known as the *proportion problem*.

There's a (technical) fix to this, which relates to another problem with DPL:

Weak vs. strong donkeys

Consider the classic donkey sentence below — our entry for first-order universal quantification, and also *every* as a generalized quantifier predict it to have strong, universal truth conditions.[14]

(55) Every^1 farmer who owns a^2 donkey beats it$_2$.
$\rightsquigarrow$ $\textbf{every}_1\ (\epsilon x_2 \wedge D\ x_2\ \wedge F\ x_1 \wedge O\ x_1\ x_2)\ (B\ x_1\ x_2)$

Concretely, we predict this to be true iff each farmer is s.t. they beat each donkey that they own.

However, donkey sentences can receive a so-called "weak" reading too. Consider the following context from Chierchia 1995:

The farmers under discussion are all part of an anger management program, and they are encouraged by the psychotherapist involved to channel their aggressiveness towards their donkeys (should they own any) rather than towards each other. The farmers scrupulously follow the psychotherapist's advice.

(56) ...every farmer^{11} who owns a^2 donkey beats it$_2$.

In the context, this is true just so long as each farmer is s.t. they beat some donkey that they own.

Even more convincingly, there are donkey sentences for which the weak reading is the most salient:

[14] Apologies for the animal cruelty; I regrettably need to use this example to repeat Chierchia's reasoning.

(57) Every person who has a^{11} dime will put it$_1$ in the meter.

(58) Yesterday, every person who had a^1 credit card paid his bill with it$_1$.

The unselective analysis can't account for this reading.

The solution to both of these problems involves formulating a *selective* semantics for generalized quantifiers that relates sets of individuals rather than information states.

We won't go through how to do this in class, but see, e.g., Chierchia 1995 and Kanazawa 1994.[15]

[15] Probably the simplest way of doing this is by formulating an object language abstraction operator λ_n, and defining dynamic GQs as relating functions from individuals to dynamic propositions, e.g.

(59) Most farmers who own a^2 donkey beat it$_2$.
most $(\lambda_1\ (\epsilon x_2 \wedge D\ x_2 \wedge F\ x_1\ \wedge O\ x_1\ x_2))$
$(\lambda_3\ (B\ x_3\ x_2))$

4 *Pseudo-dynamics*

4.1 *The e-type approach*

Probably the most prominent alternative to DS is the *e-type*/description-theoretic approach to pronominal anaphora, according to which pronouns elide descripive content.[16]

[16] See Elbourne 2013 for the most up-to-date reference on this line of research. Uniqueness concerns are dealt with by relativizing uniqueness to a *situation*.

(60) A person entered the bar.
[She Δ] ordered a mojito. $\Delta =$ person who entered the bar

(61) Every farmer who owns a donkey
cares for [it Δ]. $\Delta =$ donkey that the farmer owns

There are some well-known problems for the e-type approach, e.g.:

Bishop sentences

Bishop sentences present a puzzle for the e-type approach involving the indiscernability of individuals:

(62) If a^1 bishop meets a^2 bishop, he^1 kisses him$_2$.

On the e-type approach, this should involve a failure of uniqueness, even when relativizing to situations:

(63) If a bishop meets a bishop,
[he Δ_1] kissed [him Δ_2] $\Delta_1 = \Delta_2 =$ the bishop that the bishop meets

There are various complications that one could entertain, but DS captures the data in (62) straightforwardly – it's just variable binding.

There's also no official theory of discourse anaphora in the e-type approach — when one tries to extend the e-type approach to discourse anaphora, one ends up with something suspiciously similar to DS.[17]

[17] See Rothschild & Mandelkern (2017) for discussion of this point.

I won't spend time on the e-type approach in this class, but rather, we'll move on to discuss a recent alternative approach which maintains many of the attractive features of DS.

4.2 *The witness generalization*

Mandelkern's approach tries to preserve the index-based flexibility of DS while restricting its expressive power and improving its empirical accuracy.[18]

[18] I'm grateful to both Matt Mandelkern and Keny Chatain for discussion of this material; this section is based partially on a presentation I gave with Keny earlier this year.

> **The witness generalization**
>
> Anaphora to an indefinite is possible iff the *local context* entails that a witness for the indefinite exists. (Rothschild & Mandelkern 2017)

Standard DS only validates the (⇒) part of the witness generalization:

(64) **Conjunction:** ✓DS
I have a^1 sister. She$_1$ lives in East Boldon.
LC(She$_1$ lives in East Boldon) ⊨ *I have a sister*

(65) **Conditional:** ✓DS
If Keny has a^1 brother, he$_1$ lives in Bordeaux.
LC(he$_1$ lives in Bordeaux) ⊨ *Keny has a*^{1} *brother*

(66) **Disjunction:** ✗DS
Either Keny doesn't have a^1 brother, or he$_1$ lives in Bordeaux. **LC**(he$_1$ lives in Bordeaux) ⊨ *Keny has a*^{1} *brother*

(67) **Double Negation:** ✗DS
I don't own no^1 shirt. It$_1$'s in the wardrobe.
LC(it$_1$'s in the wardrobe) ⊨ *I own a*^{1} *shirt*

(68) **Non-asserted antecedent + conditional:** ✗DS
Does Keny have a^1 brother? If so, then he$_1$ must be French.
LC(he$_1$ be French) ⊨ *Keny has a*^{1} *brother*

(Can you think of other cases?)

The witness generalization suggests a deep connection between anaphoric licensing and presupposition projection — assuming that a theory of local contexts is implicated in an account of presupposition projection.[19]

[19] Both trivalent and dynamic theories offer such a notion.

4.3 The explanatory challenge to DS

The main difficulty is providing an account which does not give too many degrees of freedom to lexical content (*contra* DS).

(69) **Unattested lexical items:**
  a. She entered and′ a person ordered a mojito. (reverse *and*)
  b. #Every′ farmer who owns a donkey gives it back rubs. (anaphorically inert *every*)
  c. #A′ woman came and she sat down (anaphorically inert indefinites[20])

DS is maybe[21] more expressive than needed for natural language

[20] This is debated. Particularly tricky are the cases of Pseudo Noun Incorporation, which marginally introduce DRs. Some see this as evidence that anaphoric potential is lexically specified.

[21] The usual disclaimers apply: communicational pressures shaping lexical inventories and lack of broad-scale cross-linguistic surveys.

Mandelkern's contribution: Against that background, we can isolate three innovations of Mandelkern's proposal:

1. A unification of conditions on presupposition satisfaction and anaphoric licensing..
2. Validating the witness generalization and (concomitantly) closing empirical gaps: double negation, disjunction, etc.
3. A static semantics (and therefore, more restrictive) semantics.

4.4 The proposal

The basics

In a classical setting, sentences are true relative to an evaluation point — formally, a world assignment pair.

(70) a. $[\![\text{Troy left}]\!]^{w,g} = \mathsf{left}_w(\mathsf{troy})$
  b. $[\![\text{Someone left}]\!]^{w,g} = \exists x[\mathsf{left}_w(x)]$

To set things up, consider what it means to assert a sentence ϕ, in a Stalnakerian setting.

We can think of a context c, following Stalnaker/Heim as consisting of a set of world-assignment pairs.[22]

[22] Deparing from Mandelkern (2020b), we'll assume that assignments can potentially be partial.

Ignoring presupposition, updating a context by asserting a sentence ϕ simply amounts to intersecting c with the with the points at which ϕ is true:

(71) **Update (def.):** $c[\phi] \coloneqq c \cap \{\, (w, g) \mid [\![\phi]\!]^{w,g} = 1 \,\}$

In a classical setting, it's easy to see that updating a context c with a sentence with an indefinite won't have any effect on the assignments in the context; it will simply wipe out worlds in which nobody left.

A classical semantics therefore fails to account for the fact that asserting a sentence with an indefinite introduces a DR.

At this point, we'd usually go ahead and shift to a *dynamic* semantics, where sentences directly denote actions on the context. Instead, we'll take a different tack.

The witness presupposition

Mandelkern's core insight is that we can assign sentences with indefinites *witness presuppositions*, i.e., disjunctive definedness conditions, which ensure that they only affect anaphoric potential if true. We do this using a trivalent semantics.

N.b. the witness presupposition will need to be stipulated, but as we'll see, not much else will have to be.

(72) $$[\![\text{someone}^1 \text{ left}]\!]^{w,g} = \begin{cases} 1 & \mathsf{left}_w(g_1) \\ 0 & \neg\,(\exists x[\mathsf{left}_w(x)]) \\ \text{undefined} & \text{otherwise} \end{cases}$$

On Mandelkern's rendering, a sentence such as "someone^1 left" is *defined and true* if g_1 left in w, and *defined and false* if nobody left in w.

There are two equivalent ways of elucidating the presupposition:

- Either nobody left, or g_1 left.
- If anyone left, g_1 left.

The assertive contribution is just the classical semantics for the indefinite, i.e., *someone left*.

On the disjunctive rendering of the presupposition, note that the right disjunct entails the truth of the assertion, and the left disjunct entails the falsity of the assertion, hence the presentation in (72).

The effect of asserting a sentence with an indefinite

In order to make sense of the effect of asserting a sentence with an indefinite, we're going to need to revise ourr notion of an information state — this will be an interesting departure from the version of DPL that I've presented.

Mandelkernian information states

Given a model, consisting of:

- A non-empty set of individuals D.
- A non-empty set of variables V
- A non-empty set of worlds W.

The set of possible assignments G is the set of all mappings g, whose domain is V, and whose codomain is $D \cup \{ \# \}$, where # stands in for *undefined*.

An *information state* $c \subseteq W \times G$, a set of world-assignment pairs, where:

- The product of W and G is the *initial information state* (n.b., given a stock of variables, this will contain all possible assignments, both defined and undefined): $c_\top \coloneqq W \times G$
- $\varnothing$ is the *absurd information state*. $c_\bot \coloneqq \varnothing$

If we combine our global update rule for this semantics of the indefinite, it's easy to see that updating a context c with "someone^1 left" will knock out any assignments that are undefined for 1, and any world assignment pairs (w, g) if g_1 didn't leave in w.

To give a concrete example, where $\mathsf{dom} \coloneqq \{ \mathsf{Xavier, Yuna, Zhaan} \}$, the stock of indices is $\{ 1 \}$, we can conceive of an initial context as follows:

- $W \coloneqq \{ w_{xy}, w_x, w_y, w_\varnothing \}$ (subscripts indicate who left, exhaustively).
- $G \coloneqq \{ g_\varnothing, [1 \to x], [1 \to y], [1 \to z] \}$ ($g_\varnothing$ is undefined for any index).
- $c = W \times G$

(73) $$c[\text{someone}^1 \text{ left}] = c \cap \{ (w, g) \mid [\![\text{someone}^1 \text{ left}]\!]^{w,g} = 1 \}$$
$$= \{ (w_{xy}, [1 \to x]), (w_x, [1 \to x]), (w_{xy}, [1 \to y]), (w_y, [1 \to y]) \}$$

One thing to note here is that, as Mandelkern acknowledges, what he describes as a "presupposition" isn't really what we would ordinarily describe as a presupposition, in a Stalnakerian setting.

Ordinarily, we think of presuppositions as *preconditions* on the context. This is formalized as Stalnaker's bridge:

(74) Stalnaker's bridge principle
Given ϕ_π a sentence that asserts ϕ and presupposes π,
$c[\phi_\pi]$ is defined iff π is true *throughout* c.

We have to abandon Stalnaker's bridge in order for Mandelkern's account to work, otherwise sentences with indefinites would frequently be undefined. Rather, we just toss out any assignments which are undefined relative to the index on the indefinite.

What we would think of as "ordinary" presuppositions are reintroduced by relativizing truth to a context parameter, as we'll see when we talk about definites.

A more familiar presupposition

In Mandelkern's system, definites are genuinely presuppositional in the sentence of Heim-Stalnaker — they place *preconditions* on the context.

In order to capture this, we need to add a *context* parameter c to the interpretation function: $[\![.]\!]^{w,g,c}$.

Sentences which don't involve definites or other presuppositional expressions won't be sensitive to the context parameter, and its presence will essentially be vacuous.

The semantics of a sentence with a definite, however will impose a requirement that it be defined throughout the context.[23]

[23] We use $*$ here to range over possible semantic values.

(75) $$[\![\text{they}_1 \text{ sat down}]\!]^{w,g,c} = \begin{cases} 1 & \textsf{sat-down}_w(g_1) \wedge \forall(*, g) \in c[g_1 \text{ is defined}] \\ 0 & \textsf{sat-down}_w(g_1) \wedge \forall(*, g) \in c[g_1 \text{ is defined}] \\ \text{undefined} & \text{otherwise} \end{cases}$$

There's a sense in which Mandelkern's semantics for definites, which presumably carries over to presuppositional expressions more generally, *semanticizes* the bridge principle.

Revising update

We now need to redefine update in the obvious way — the context parameter of the sentence is identified with the context c which is being updated:

(76) Update (second attempt): $c[\phi] \coloneqq c \cap \{ (w, g) \mid [\![\phi]\!]^{w,g,c} = 1 \}$

It's crucial here that we *don't* assume Stalnaker's bridge principle. If the sentence is undefined relative to $(w, g) \in c$, we simply toss them aside.

It's easy to see however that if we try to update a context c, which includes assignments that are undefined at 1, with the sentence "they$_1$ sat down", the result will be the empty set.

This is because $[\![\text{they}_1 \text{ sat down}]\!]$ undefined throughout c, since the conditions it places on c as a whole will never be satisfied.

We're now in a position to see how Mandelkern's system achieves the basic results of, e.g., Heim's dynamic semantics.

- Updating a context c with "someone^1 left", filters out any gs where g_1 is undefined.
- An update of c with "they$_1$ sat down" is only licit if g_1 is defined throughout c.

Sub-sentential compositionality

We haven't said anything about how sentences with indefinites/definites come to mean what they mean. Here we'll sketch a simple Montagovian fragment with the desired properties.

Indefinites simply denote existential quantifiers with a disjunctive presupposition.

(77) $[\![\text{someone}^1]\!]^{w,g,c} \coloneqq \lambda k \;:\; \neg\,(\exists x[k(x)]) \vee k(g_1)\;.\;\exists x[k(x)]$

Predicates and proper names simply receive their ordinary denotations:

(78) a. $[\![\text{Xavier}]\!]^{w,g,c} = \textsf{xavier}$
b. $[\![\text{swims}]\!]^{w,g,c} = \lambda x\;.\;\textsf{swims}_w(x)$

Definites denote individuals with familiarity presuppositions:

(79) $$[\![\text{they}_1]\!]^{w,g,c} \coloneqq \begin{cases} g_1 & \forall g' \in c,\; g'_1 \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

We can supplement this with a rule of Heim & Kratzer's undefinedness-sensitive rule for function application, which essentially gives us a weak Kleene logic (i.e., undefinedness always projects):

(80) Function application:

$$\left[\!\!\left[\begin{array}{c} \cdots \\ \wedge \\ \alpha \quad \beta \end{array} \right]\!\!\right]^{w,g,c} := \begin{cases} [\![\alpha]\!]^{w,g,c}([\![\beta]\!]^{w,g,c}) & [\![\alpha]\!]^{w,g,c} : \langle\sigma,\tau\rangle, [\![\beta]\!]^{w,g,c} : \sigma, \\ & [\![\alpha]\!]^{w,g,c}, [\![\beta]\!]^{w,g,c} \text{ are defined} \\ [\![\beta]\!]^{w,g,c}([\![\alpha]\!]^{w,g,c}) & [\![\alpha]\!]^{w,g,c} : \sigma, [\![\beta]\!]^{w,g,c} : \langle\sigma,\tau\rangle, \\ & [\![\alpha]\!]^{w,g,c}, [\![\beta]\!]^{w,g,c} \text{ are defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

If we compose an indefinite with a predicate, we can see how we get the sentential meanings we've been assuming. Since undefinedness is guaranteed to project:

(81) $[\![\text{someone}^1 \text{ left}]\!]^{w,g,c}$
$= [\![\text{someone}^1]\!]^{w,g,c}([\![\text{left}]\!]^{w,g,c})$
$=$ if $\neg\exists x[\mathsf{left}_w(x)] \vee \mathsf{left}_w(g_1)$ then $\exists x[\mathsf{left}_w(x)]$ else undefined

The presupposition is that nobody left or g_1 left, and the assertion is that someone left. To reiterate, this gives rise to the following predictions:

defined and true if g_1 left or nobody left, and someone left.
(since only the first disjunct is compatible with, and in fact entails the assertion, this can be simplified to: $\mathbf{g_1}$ **left**)

defined and false if g_1 left or nobody left, and nobody left.
(since only the second disjunct is compatible with, and in fact equivalent to the assertion, this can be simplified to: **nobody left**)

undefined otherwise.

Sentential compositionality and local contexts

One of the virtues of Mandelkern's approach is that it allows us to maintain a classical semantics for the logical connectives:

(82) a. $[\![\text{not}]\!]^{w,g,c} := \lambda t\,.\,\neg\, t$ $\langle \mathtt{t},\mathtt{t}\rangle$
b. $[\![\text{and}]\!]^{w,g,c} := \lambda u\,.\,\lambda t\,.\,t \wedge u$ $\langle \mathtt{t},\langle \mathtt{tt}\rangle\rangle$
c. $[\![\text{or}]\!]^{w,g,c} := \lambda u\,.\,\lambda t\,.\,t \vee u$ $\langle \mathtt{t},\langle \mathtt{tt}\rangle\rangle$

If we couple this with our rule of function application, this simply gives rise to a weak Kleene logic, which of course won't have the desired results.

In order to account for presupposition projection, this system can be supplemented with an independently motivated algorithm for determining local contexts (e.g., Schlenker 2009, 2010).[24]

[24] As we've seen, Mandelkern's fragment, together with a globally defined *update* rule gives rise to a state system, in the sense of Rothschild & Yalcin 2017, we could alternatively treat sentences as denoting their corresponding update, and define the logical connectives as in update semantics (1982, 1996), where conjunction is interpreted as successive update, etc. Mandelkern's semantics therefore can in principle be embedded within a dynamic semantics, and is neutral wrt how the projection behavior of connectives is to be derived.

In order to avoid introducing the details of Schlenker's theory, we'll simply define syncategorematic rules for determining local contexts in complex sentences.

Our rule for conjunction ensures that the second conjunct's context parameter is the context of utterance c updated with the first conjunct.

(83) Conjunction

$$\left[\!\left[\left[_{\ldots} \ \phi \ \left[_{\ldots} \ \text{and} \ \psi \right] \right] \right]\!\right]^{w,g,c} := [\![\text{and}]\!]^{w,g,c} \left([\![\psi]\!]^{w,g,c[\phi]}\right)\left([\![\phi]\!]^{w,g,c}\right)$$

Our rule for disjunction ensures that the second disjunct's context parameter is the context of utterance c updated with the *negation* of the first disjunct.

(84) Disjunction

$$\left[\!\left[\left[_{\ldots} \ \phi \ \left[_{\ldots} \ \text{or} \ \psi \right] \right] \right]\!\right]^{w,g,c} := [\![\text{or}]\!]^{w,g,c} \left([\![\psi]\!]^{w,g,c[\text{not } \phi]}\right)\left([\![\phi]\!]^{w,g,c}\right)$$

This brings us round to a discussion of negation:

4.5 Keeping negation classical

Matt's fragment allows us to maintain a classical treatment of negation, while maintaining the accessibility results of classical dynamic semantics *and* validating double negation.

Negation roofs the introduction of a discourse referent

Recall that a sentence "someone^1 left" presupposes that (a) either nobody left or g_1 left, and if defined, asserts (2) someone left. It follows that "someone^1 left" is defined and false simply if nobody left.

Since negation is classical "not ϕ" is true iff ϕ is defined and false.

(85) $$[\![\text{nobody}^1 \text{ left}]\!]^{w,g,c} = \begin{cases} 1 & \neg\,(\exists x[\mathsf{left}_w\ x]) \\ 0 & \mathsf{left}_w(g_1) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Updating a context with "nobody^1 left" therefore fails to filter out assignments at which 1 is undefined, and we correctly predict that an indefinite under the scope of negation is *inaccessible*.

Double negation elimination is valid

Since negation is classical, and presuppositions project through negation, "It's not the case that nobody^1 left" has the same definedness conditions as "Someone^1 left":

(86) $$[\![\text{it's not true that nobody}^1 \text{ left}]\!]^{w,g,c} = \begin{cases} 1 & \mathsf{left}_w(g_1) \\ 0 & \neg\,(\exists x[\mathsf{left}_w\ x]) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Given the previous rule for interpreting disjunctive sentences, an account of bathroom sentences naturally follows

There are number of complications regarding an extension of Mandelkern's theory to donkey sentences, but they won't concern us here. See the paper for details.

There are however two troubling features of Mandelkern's theory which may motivate us to consider alternatives:

- A "weird" (i.e., effortlessly accommodated) presupposition.
- A semanticized bridge principle.

Elliott 2020b,a independently developed a system with roughly the same properties as Mandelkern's, for independent reasons: *dynamic alternative semantics*.

4.6 *Comparison with Elliott 2020b,a*

Mandelkern's fragment has an extremely nice feature: it achieves the basic results of dynamic semantics while maintaining the validity of double negation elimination.

One might wonder what aspects of Mandelkern's system are *essential* for achieving this result? For example, how important is it that Mandelkern's system is *eliminative*?[25]

[25] A fragment is *eliminative* iff, for all sentences ϕ, $c[\phi] \subseteq c$. It's easy to see that this holds.

In Elliott's theory (a) existential quantifiers introduce discourse referents, (b) negation is classical; double-negation elimination is valid, and (b) negation renders indefinites inaccessible.

Elliott's *dynamic alternative semantics* is sufficiently distinct that it's perhaps interesting to compare the two. As we'll see:

- Elliott's fragment is *non-eliminative*, but distributive, like, e.g., Groenendijk & Stokhof's (1991) DPL.
- It doesn't rest on the logic of presupposition, but rather on a distinction between verifiers and falsifiers in the output.

Sentences are interpreted relative to an *evaluation point* (a world assignment pair), and output a set of assignment-truth value pairs.

(87) $[\![\text{Xavier left}]\!]^{w,g} = \{\, (\mathsf{left}_w(\mathsf{xavier}), g) \,\}$ $\qquad \{\, \mathtt{t} \cdot \mathtt{g} \,\}$

We can think of the assignments in the output as having a "polarity" – in fact, we'll refer to outputted assignments as either being *truth-tagged* or *false-tagged*.

The key innovation here is the following semantics for indefinites:[26]

[26] The semantics suggested in Elliott 2020b is a little more sophisticated this, and actually returns the grand intersection of all modified assignments in the false case. This won't be important for our purposes.

(88) $$[\![\text{someone}^1 \text{ left}]\!]^{w,g} := \begin{cases} \{\, (1, g^{[1\to x]}) \mid \mathsf{left}_w(x), x \in \mathsf{dom} \,\} & \exists x[\mathsf{left}_w(x)] \\ \{\, (0, g) \,\} & \text{otherwise} \end{cases}$$

Elliott assumes that assignments are partial, and furthermore that we can think of the *initial state* as the product of the set of worlds in the common ground with the initial assignment $g_\varnothing$ (i.e., the unique assignment with an empty domain).

Update in Elliott's system is defined as follows: for each assignment g in c, we keep (i) all the worlds w in c which, when fed into the sentence with g give back a true tagged assignment, and (ii) pair w with that assignment.

(89) Update (def.): $$c[\phi] := \bigcup_{(w,g)\in c} \{\, (w, g') \mid (1, g') \in [\![\phi]\!]^{w,g} \,\}$$

To give a concrete example, assume:

- $\mathsf{dom} \coloneqq \{ \mathit{Xavier}, \mathit{Yuna}, \mathit{Zhaan} \}$
- $W \coloneqq \{ w_{xy}, w_x, w_y, w_\varnothing \}$, where subscripts indicate who left, understood exhaustively.
- $c = W \times g_\varnothing \equiv \{ (w_{xy}, g_\varnothing), (w_x, g_\varnothing), (w_y, g_\varnothing), (w_\varnothing, g_\varnothing) \}$

(90) $c[\text{someone}^1 \text{ left}]$
$$= \bigcup_{(w,g)\in c} \{ (w, g') \mid (1, g') \in [\![\text{someone}^1 \text{ left}]\!]^{w,g} \}$$
$$= \left\{ (w_{xy}, [1 \to x]), (w_x, [1 \to x]), (w_{xy}, [1 \to y]), (w_y, [1 \to y]) \right\}$$

Observe that, only worlds in which there is a verifier are retained, and in each world in which there is a verifier, a discourse referent is introduced. This is basically equivalent to DPL.

It's already easy to see that Elliott's system isn't eliminative, since eliminativity doesn't hold for an update with an indefinite (just like DPL).

Sentences with definites have a totally orthodox interpretation:

(91) $[\![\text{they}_1 \text{ are outside}]\!]^{w,g} = \{ (\mathsf{outside}_w(g_1), g) \}$

Since assignments are partial, the sentence with the indefinite will be undefined if the evaluation assignment g is undefined for 1.

We can make Stalnaker's bridge explicit in our update rule — updating a context c with ϕ is only defined if ϕ is defined throughout c.

(92) Update (revised): $c[\phi] \coloneqq \begin{cases} \bigcup_{(w,g)\in c} \{ (w, g') \mid (1, g') \in [\![\phi]\!]^{w,g} \} & \forall (w,g) \in c, [\![\phi]\!]^{w,g} \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$

It follows that an update with an indefinite will ensure that a successive update with a definite is defined.

Negation

As in Mandelkern's fragment, negation is totally classical; in essence, all it does is flip the polarity of the outputted assignments.[27]

(93) $[\![\text{not}]\!]^{w,g} \coloneqq \lambda m \,.\, \bigcup_{(t,g)\in m} \{ (\neg\, t, g) \}$

If, in our previous context, we instead update with the negative sentence, no discourse referent is introduced; the worlds in which discourse referents are introduced are all false-tagged, since the polarities of the outputs got flipped:

[27] This may not look totally classical, so you'll have to trust me when I say that it is. Essentially, all we're doing here is mapping classical negation through the functorial structure of a `State.Set` monad (i.e., applying it pointwise).

(94) $c[\text{nobody}^1 \text{ left}] = \{\, (w_\varnothing, g_\varnothing) \,\}$

It follows that double negation elimination will be classical. Flipping the polarities of the outputted assignments twice will cancel out.

The connectives

We give syncategorematic rules for the connectives below; note that the second conjunct is interpreted relative to the true-tagged output of the first conjunct; the second disjunct is interpreted relative to the false-tagged outputs of the first disjunct.

(95) $[\![\phi \text{ and } \psi]\!]^{w,g} := \bigcup_{(1,g')\in[\![\phi]\!]^{w,g}} [\![\psi]\!]^{w,g'}$

(96) $[\![\phi \text{ or } \psi]\!]^{w,g} := [\![\phi]\!]^{w,g} \cup \bigcup_{(0,g')\in[\![\phi]\!]^{w,g}} [\![\psi]\!]^{w,g'}$

Illustration: bathroom sentences

Let's see how this solves the bathroom sentence problem:

- $\mathsf{dom} := \{\, b \,\}$
- $W = \{\, w_b, w_\varnothing \,\}$
- $c = W \times g_\varnothing$

(97) $$\left[\!\!\left[\text{there's no}^1 \text{ bathroom or it}_1\text{'s upstairs}\right]\!\!\right]^{w,g}$$
$$= \left[\!\!\left[\text{there's no}^1 \text{ bathroom}\right]\!\!\right]^{w,g} \cup \bigcup_{(0,g')\in[\![\text{there's no bathroom}]\!]^{w,g}} [\![\text{it}_1\text{'s upstairs}]\!]^{w,g'}$$

We're computing the result of doing the following:

(98) $c[\text{there's no}^1 \text{ bathroom or it}_1\text{'s upstairs}]$

Step 1: interpret the sentence relative to $(w_b, g_\varnothing)$

If we feed this into the first disjunct, we get a false-tagged output. This will be ignored in the result of the update.

(99) $\left[\!\!\left[\text{there's no}^1 \text{ bathroom}\right]\!\!\right]^{w_b,g_\varnothing} = \{\, (0, [1 \rightarrow b]) \,\}$

Now if we feed the false-tagged output into the second disjunct, we get a true-tagged output, since anaphora succeeded. This means that this (w, g) will be retained by the update.

(100) $[\![\text{it}_1\text{'s upstairs}]\!]^{w_b,[1\to b]} = \{\, (1,[1 \to b]) \,\}$

(101) $\{\, (w_b, g_\varnothing) \,\}$ [there is no[1] bathroom or it_1's upstairs] $= \{\, (w_b, [1 \to b]) \,\}$

Step 2: interpret the sentence relative to $(w_\varnothing, g_\varnothing)$

If we feed this into the first disjunct we get a true-tagged output. This means that this (w, g) will be retained by the update.

(102) $[\![\text{there's no}^1\text{ bathroom}]\!]^{w_\varnothing,g_\varnothing} = \{\, (1, g_\varnothing) \,\}$

There are no false-tagged outputs, so the second disjunct is irrelevant here.

(103) $\{\, (w_\varnothing, g_\varnothing) \,\}$ [there is no[1] bathroom or it_1's upstairs] $= \{\, (w_\varnothing, g_\varnothing) \,\}$

Step 3: take the union

We predict that anaphora will be successful, and furthermore disjunction is (correctly) predicted to be externally static.

(104) c[there's no[1] bathroom or it_1's upstairs] $= \{\, (w_b, [1 \to b]), (w_\varnothing, g_\varnothing) \,\}$

4.7 Outlook

There's a clear intuition that Mandelkern and Elliott's approaches share some important features:

- On both theories, indefinites only introduce discourse referents if there is a verifier, and not otherwise.
- On Mandelkern's theory, the double-life of indefinites is handled via the Strawson logic; on Elliott's theory, what's crucial is distinguishing between true and false information in the output.[28]

Open questions: how do we characterize exactly what these approaches have in common? There is clearly a shared insight here.

As it stands, one could argue that Mandelkerns account is more explanatory than Elliott's as its grounded in an independently motivated theory of local contexts. Currently, i'm working on recasting the central ideas of dynamic alternative semantics using the middle Kleene semantics for connectives.

[28] In that sense, Elliott's approach is more closely related to previous approaches to double negation in dynamic semantics, such as Krahmer & Muskens 1995. Although elided here, this informational richness directly follows from the kind of monadic dynamics suggested by Charlow (2019).

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