# An explanatory theory of presupposition projection

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#### 1 Weak Kleene

### Weak Kleene recipe

Where the classical semantics is silent, always return #.

One way of thinking of the third truth value, #, is as representing *undefined-ness*.

This interpretation gives rise to a Weak Kleene logic.

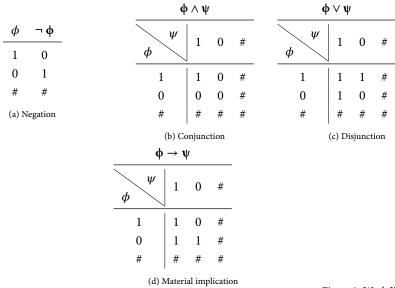


Figure 1: Weak Kleene/Bochvar truth-tables for the connectives

# 2 Strong Kleene (symmetric)

We can think of the third truth value, #, as representing *uncertainty whether 1* or 0, which we can represent as the set  $\{1,0\}$ .

In order to explain the recipe, it will be helpful to think of our three truth-values as the following isomorphic three-membered set:  $\{\{1\}, \{0\}, \{1,0\}\}$ ,

with  $\{1\}$  representing definitely true,  $\{0\}$  representing definitiely false, and  $\{1,0\}$  representing maybe true and maybe false.

$$\{\underbrace{1}_{\text{falso}}^{\text{true}}, \underbrace{0}_{\text{falso}}^{\text{uncertain}}, \underbrace{1}_{\text{falso}}^{\text{uncertain}}\}$$

The intuition behind our recipe will be as follows:

- Given a complex formula with an *n*-place truth-functional connective f,  $\lceil f \phi_1 ... \phi_n \rceil$ .
- Assuming that  $I^{bi}$  gives the bivalent interpretation of f as a function, compute  $\{I^{bi}(f) t_1 \dots t_n \mid t_1 \in \llbracket \phi_1 \rrbracket^{tri}, \dots, t_n \in \llbracket \phi_n \rrbracket^{tri} \}$ .
- The result is the value of  $[f \phi_1...\phi_n]^{tri}$

### 2.1 Applying the Strong Kleene algorithm to conjunction

When the values of the arguments of the connective are  $\{1\}$  or  $\{0\}$ , the algorithm will simply deliver the classical semantics. We can illustrate this with conjunction.

$$\llbracket p \wedge q \rrbracket^{tri} = \{ t \wedge u \mid t \in I(p) \wedge u \in I(q) \}$$

If I(p) and I(q) are singleton sets  $\{t\}$  and  $\{u\}$ , this will obviously be equivalent to the classical semantics:

$$= \{ t \wedge u \}$$

What if one of I(p) is  $\{0, 1\}$ ? The value of the conjunctive formula will differ depending on whether I(q) is  $\{1\}$  or  $\{0\}$ . Assuming that  $I(q) = \{1\}$ :

(1) 
$$I(p) = \{0, 1\}, I(q) = 1$$
  
a.  $[p \land q]^{tri} = \{t \land u \mid t \in \{0, 1\} \land u \in \{1\}\}\}$   
b.  $= \{t \land 1 \mid t \in \{0, 1\}\}$   
c.  $= \{0, 1\}$ 

(2) 
$$I(p) = \{0, 1\}, I(q) = 0$$
  
a.  $[p \land q]^{tri} = \{t \land u \mid t \in \{0, 1\} \land u \in \{0\}\}$   
b.  $= \{t \land 0 \mid t \in \{0, 1\}\}$   
c.  $= \{0\}$ 

#### Strong Kleene conjunction

- $\lceil \phi \land \psi \rceil$  is defined if either (a)  $\llbracket \phi \rrbracket$  is false, (b)  $\llbracket \psi \rrbracket$  is false, or (c) both  $\llbracket \phi \rrbracket$  and  $\llbracket \phi \rrbracket$  are true.
- $\lceil \phi \land \psi \rceil$  is *true* if both  $\llbracket \phi \rrbracket$  and  $\llbracket \psi \rrbracket$  are true.
- $\lceil \phi \land \psi \rceil$  is *false* if either (a)  $\llbracket \phi \rrbracket$  is false, or (b)  $\llbracket \psi \rrbracket$  is false.

#### 2.2 Disjunction in Strong Kleene semantics

#### Strong Kleene disjunction

- $\lceil \phi \lor \psi \rceil$  is defined if either (a)  $\llbracket \phi \rrbracket$  is true, (b)  $\llbracket \psi \rrbracket$  is true, or (c) both  $\llbracket \phi \rrbracket$  and  $\llbracket \phi \rrbracket$  are false.
- $\lceil \phi \land \psi \rceil$  is *false* if both  $\llbracket \phi \rrbracket$  and  $\llbracket \psi \rrbracket$  are false.
- $\lceil \phi \land \psi \rceil$  is *true* if either (a)  $\llbracket \phi \rrbracket$  is true, or (b)  $\llbracket \psi \rrbracket$  is true.

#### Strong Kleene truth-tables

If we apply the strong Kleene algorithm to the classical connectives, substituting in  $\{1,0,\#\}$  for  $\{\{1\},\{0\},\{1,0\}\}$ , the result is the following truthtables.

#### The role of linear order in presupposition projection

As discussed by Schlenker (2008), it's not clear that the projection generalization we've been assuming for disjunctive sentences is correct.

- a. Either this house has no bathroom, or the bathroom is upstairs.
  - b. Either the bathroom is upstairs, or (else) this house has no bathroom.

(Schlenker 2008: p. 185)

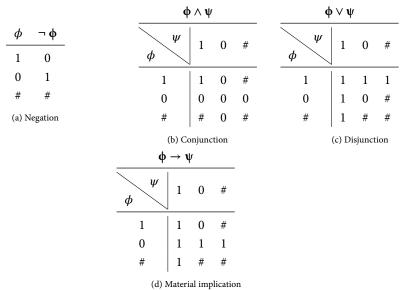


Figure 2: Strong Kleene truth-tables for the connectives

The entry we gave for disjunction in update semantics can capture the projection pattern illustrated by (3b), but not (??).

## 3 Middle Kleene/Peters (asymmetric)

Presupposition projection displays asymmetries based on *linear order*; something that strong Kleene fails to capture.

### References

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3).

# A Strong Kleene as an alternative semantics

Assume that t is the type of *bivalent* truth values.

(4) 
$$[stopped smoking]^w = \lambda x \cdot \begin{cases} \{1\} & smoked_w \ x \land \neg (smokes_w \ x) \\ \{0\} & smoked_w \ x \land smokes_w \ x \\ \{1,0\} & otherwise \end{cases}$$

(5) 
$$[and]^w := \lambda u \cdot \lambda t \cdot t \wedge u$$
  $t \to t \to t$ 

As in a standard alternative semantics, we just need two truth-values to massage composition.

(6) a. 
$$x^{\rho} := \{ x \}$$
  
b.  $m \otimes n := \{ x \land y \mid x \in m \land y \in n \}$   $\{ a \rightarrow b \} \rightarrow \{ a \} \rightarrow \{ b \}$ 

(7) Paul smoked and he stopped smoking.

 $\{t \land u \mid t \in [Paul \text{ smoked}]^{w \rho} \land u \in [Paul \text{ stopped smoking}]^{w}\}$ 

