An explanatory theory of presupposition projection

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1 Weak Kleene

Weak Kleene recipe

Where the classical semantics is silent, always return #.

One way of thinking of the third truth value, #, is as representing *undefinedness*.

This interpretation gives rise to a Weak Kleene logic.

Weak Kleene truth-tables

ϕ	$\neg \phi$
1	0
0	1
#	#

Figure 1: Negated formulas

$\phi \wedge \psi$

$\phi \backslash \psi$	1	0	#
1	1	0	#
0	0	0	#
#	#	#	#

Figure 2: Conjunctive formulas

$\phi \vee \psi$

$\phi \backslash \psi$	1	0	#
1	1	1	#
0	1	0	#
#	#	#	#

Figure 3: Disjunctive formulas

$\phi \rightarrow \psi$

$\phi \backslash \psi$	1	0	#
1	1	0	#
0	1	1	#
#	#	#	#

Figure 4: Conditional formulas

2 Strong Kleene (symmetric)

We can think of the third truth value, #, as representing *uncertainty whether 1 or 0*, which we can represent as the set $\{ 1, 0 \}$.

In order to explain the recipe, it will be helpful to think of our three truth-values as the following isomorphic three-membered set: $\{ \{ 1 \}, \{ 0 \}, \{ 1, 0 \} \}$, with $\{ 1 \}$ representing *definitely true*, $\{ 0 \}$ representing *definitiely false*, and $\{ 1, 0 \}$ representing *maybe true and maybe false*.

$$\{ \overbrace{\{ 1 \}}^{\text{true}}, \underbrace{\{ 0 \}}_{\text{false}}, \overbrace{\{ 1, 0 \}}^{\text{uncertain}} \}$$

The intuition behind our recipe will be as follows:

- Given a complex formula with an n-place truth-functional connective f, $\ulcorner f\ \phi_1 \ldots \phi_n \urcorner$.
- Assuming that I^{bi} gives the bivalent interpretation of f as a function, compute $\{ I^{bi}(f)\, t_1 \ldots t_n \mid t_1 \in [\![\phi_1]\!]^{tri}, \ldots, t_n \in [\![\phi_n]\!]^{tri} \}$.
- The result is the value of $[\![f\ \phi_1 \ldots \phi_n]\!]^{tri}$

2.1 Applying the Strong Kleene algorithm to conjunction

When the values of the arguments of the connective are $\{ 1 \}$ or $\{ 0 \}$, the algorithm will simply deliver the classical semantics. We can illustrate this with conjunction.

$$[\![p \wedge q]\!]^{tri} = \{ t \wedge u \mid t \in I(p) \wedge u \in I(q) \}$$

If $I(p)$ and $I(q)$ are singleton sets $\{ t \}$ and $\{ u \}$, this will obviously be equivalent to the classical semantics:

$$= \{ t \wedge u \}$$

What if one of $I(p)$ is $\{ 0, 1 \}$? The value of the conjunctive formula will differ depending on whether $I(q)$ is $\{ 1 \}$ or $\{ 0 \}$. Assuming that $I(q) = \{ 1 \}$:

(1) $I(p) = \{0,1\}, I(q) = 1$

a. $[\![p \wedge q]\!]^{tri} = \{ t \wedge u \mid t \in \{0,1\} \wedge u \in \{1\} \}$

b. $= \{ t \wedge 1 \mid t \in \{0,1\} \}$

c. $= \{0,1\}$

(2) $I(p) = \{0,1\}, I(q) = 0$

a. $[\![p \wedge q]\!]^{tri} = \{ t \wedge u \mid t \in \{0,1\} \wedge u \in \{0\} \}$

b. $= \{ t \wedge 0 \mid t \in \{0,1\} \}$

c. $= \{0\}$

Strong Kleene conjunction

- $\ulcorner \phi \wedge \psi \urcorner$ is *defined* if either (a) $[\![\phi]\!]$ is false, (b) $[\![\psi]\!]$ is false, or (c) both $[\![\phi]\!]$ and $[\![\phi]\!]$ are true.
- $\ulcorner \phi \wedge \psi \urcorner$ is *true* if both $[\![\phi]\!]$ and $[\![\psi]\!]$ are true.
- $\ulcorner \phi \wedge \psi \urcorner$ is *false* if either (a) $[\![\phi]\!]$ is false, or (b) $[\![\psi]\!]$ is false.

2.2 Disjunction in Strong Kleene semantics

Strong Kleene disjunction

- $\ulcorner \phi \vee \psi \urcorner$ is *defined* if either (a) $[\![\phi]\!]$ is true, (b) $[\![\psi]\!]$ is true, or (c) both $[\![\phi]\!]$ and $[\![\phi]\!]$ are false.
- $\ulcorner \phi \wedge \psi \urcorner$ is *false* if both $[\![\phi]\!]$ and $[\![\psi]\!]$ are false.
- $\ulcorner \phi \wedge \psi \urcorner$ is *true* if either (a) $[\![\phi]\!]$ is true, or (b) $[\![\psi]\!]$ is true.

2.3 Strong Kleene truth-tables

If we apply the strong Kleene algorithm to the classical connectives, substituting in $\{1, 0, \#\}$ for $\{\{1\}, \{0\}, \{1,0\}\}$, the result is the following truth-tables.

Strong Kleene truth-tables

The highlighted cells differ from weak Kleene.

ϕ	$\neg\phi$
1	0
0	1
#	#

Figure 5: Negated formulas

$\phi \wedge \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

Figure 6: Conjunctive formulas

$\phi \vee \psi$

ϕ \ ψ	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

Figure 7: Disjunctive formulas

$\phi \rightarrow \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

Figure 8: Conditional formulas

2.4 *The role of linear order in presupposition projection*

As discussed by Schlenker (2008), it's not clear that the projection generalization we've been assuming for disjunctive sentences is correct.

(3) a. Either this house has no bathroom, or the bathroom is upstairs.
   b. Either the bathroom is upstairs, or (else) this house has no bathroom.
(Schlenker 2008: p. 185)

The entry we gave for disjunction in propositional update semantics (after Beaver 2001) can capture the projection pattern illustrated by (3b), but not (**??**).

This is because, in update semantics, when we compute $c[\phi \vee \psi]$, the first disjunct ϕ updates the global input context c, but the second disjunct updates a modified context $c[\neg\,\phi]$.[1]

Schlenker suggests that there is a similar problem involving conditional sentences.

(4) a. If this house has a bathroom, then the bathroom is well hidden.
   b. If the bathroom is well hidden, then this house has a bathroom.

[1] As a reminder, here's the semantics for disjunctive formulas in propositional update semantics:

- $c[\phi \vee \psi] = c[\phi] \cup c[\neg\,\phi][\psi]$

(Schlenker 2008: p. 186)

He furthermore suggests that post-posing the antecedent makes no difference to the judgements.

(5) a. The bathroom is well hidden, if this house has a bathroom.
 b. Mary's doctor knows she is expecting a child, if she is pregnant.
(Schlenker 2008: p. 186)

3 *Middle Kleene/Peters (asymmetric)*

As we've seen, presupposition projection displays asymmetries based on *linear order*; something that strong Kleene fails to capture.

We need to adjust the strong Kleene algorithm to account for ordering asymmetries.

George's intuition: currying, motivated by considerations of compositionality, imposes an asymmetry between arguments based on evaluation order. Strong Kleene can be modified to be sensitive to evaluation order.

Middle Kleene truth-tables

N.b. the highlighted cells diverge from strong Kleene.

ϕ	$\neg\phi$
1	0
0	1
#	#

Figure 9: Negated formulas

$\phi \wedge \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	0	0	0
#	#	#	#

Figure 10: Conjunctive formulas

$\phi \vee \psi$

ϕ \ ψ	1	0	#
1	1	1	1
0	1	0	#
#	#	#	#

Figure 11: Disjunctive formulas

$\phi \rightarrow \psi$

ϕ \ ψ	1	0	#
1	1	0	#
0	1	1	1
#	#	#	#

Figure 12: Conditional formulas

References

Beaver, David I. 2001. *Presupposition and Assertion in Dynamic Semantics*. 250 pp.

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3).

A Strong Kleene as an alternative semantics

Assume that `t` is the type of *bivalent* truth values.

(6) $[\![\text{stopped smoking}]\!]^{w} = \lambda x \,.\, \begin{cases} \{\,1\,\} & \mathsf{smoked}_{w}\ x \wedge \neg\,(\mathsf{smokes}_{w}\ x) \\ \{\,0\,\} & \mathsf{smoked}_{w}\ x \wedge \mathsf{smokes}_{w}\ x \\ \{\,1,0\,\} & \text{otherwise} \end{cases}$

$\mathtt{e} \rightarrow \{\,\mathtt{t}\,\}$

(7) $[\![\text{and}]\!]^{w} \coloneqq \lambda u \,.\, \lambda t \,.\, t \wedge u$ $\qquad \mathtt{t} \rightarrow \mathtt{t} \rightarrow \mathtt{t}$

As in a standard alternative semantics, we just need two truth-values to massage composition.

(8) a. $x^{\rho} \coloneqq \{\,x\,\}$ $\qquad \mathtt{a} \rightarrow \{\,\mathtt{a}\,\}$

b. $m \circledast n \coloneqq \{\,x \mathbin{\mathsf{A}} y \mid x \in m \wedge y \in n\,\}$ $\qquad \{\,\mathtt{a} \rightarrow \mathtt{b}\,\} \rightarrow \{\,\mathtt{a}\,\} \rightarrow \{\,\mathtt{b}\,\}$

(9) Paul smoked and he stopped smoking.

$\{\,t \wedge u \mid t \in [\![\text{Paul smoked}]\!]^{w,\rho} \wedge u \in [\![\text{Paul stopped smoking}]\!]^{w}\,\}$

$\circledast$

$\begin{cases} \{\,1\,\} & \mathsf{smoked}_{w}\ \mathsf{p} \\ \{\,0\,\} & \text{otherwise} \end{cases}$

1 iff Paul smoked in w

Paul smoked

$\circledast$

$\{\,\lambda ut \,.\, t \wedge u\,\}$

and$^{\rho}$

$\begin{cases} \{\,1\,\} & \mathsf{smoked}_{w}\ p \wedge \neg\,(\mathsf{smokes}_{w}\ p) \\ \{\,0\,\} & \mathsf{smoked}_{w}\ p \wedge \mathsf{smokes}_{w}\ p \\ \{\,1,0\,\} & \text{otherwise} \end{cases}$

Paul stopped smoking

A.1 Strong Kleene for quantifiers

(10) $\{ (a \to b) \to c \} \to (a \to \{ b \}) \to \{ c \}$

A.2 Middle Kleene as an alternative semantics

Fill this is.