Multiplicity

Plurality at the semantics-pragmatics interface

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1 The weak theory of plurality

- What does plural marking on nouns contribute to meaning? Naively: "[Plurality] just means 'more than one'" (Lasersohn 1995: ix)
- We'll refer to this view as the **strong theory of plurality**.
- The strong theory still has many champions; see e.g., Harbour (2014).[1]
- As has been noticed by numerous researchers (Sauerland, Anderssen & Yatsushiro 2005 cite Hoeksema 1983, van Eijck 1983, Schwarzchild 1996, and Beck & Sauerland 2000) this seems to get the facts right in simple cases, but faces problems in negative contexts.

(1) Some chairs are available.
$\rightsquigarrow$ there exist chairs X s.t. X are available and $\sharp X \geq 2$

(2) No chairs are available.
a. $\rightsquigarrow$ there are no chairs X s.t. X are available
b. $\not\rightsquigarrow$ there are no chairs X s.t. X are available and $\sharp X \geq 2$

- Taken at face value, the strong theory predicts that if *exactly one chair is available*, then (2) is true.
- The weak theory takes as its starting point sentences such as (2).
- In order to account for the truth-conditions of (2), it must be the case that plurality doesn't necessarily exclude the atoms.
- This leaves positive sentences as the residue. By far and away the most well-represented approach in the literature is to derive the *more than one* interpretation somehow via competition with the singular.

1.1 Implementing the weak theory

- Perhaps the most influential implementation of the weak theory is Sauerland 2003.
- Sauerland assumes that number features are of type $e \rightarrow e$.
- The feature PLUR is semantically vacuous (i.e., the identity function), whereas the feature SING is a *partial* identity function, that is defined iff its input has a cardinality of *one*.

(3) $[\![\text{PLUR}]\!] \coloneqq \text{id}$ $e \rightarrow e$

(4) $[\![\text{SING}]\!] \coloneqq \lambda x : \sharp x = 1 \,.\, x$ $e \rightarrow e$

- This gets the facts right for *no chairs are available*.
- But hang on a sec, doesn't that mean we lose an account of why, from an utterance of the following, we can infer that *more than one chair* is available?

(5) (Some) chairs are still available.

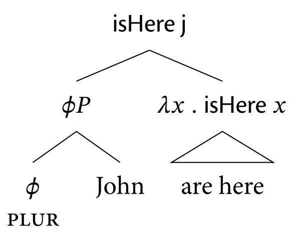
- Apparently, according to the weak theory, this should just mean that *one or more* chairs are available.
- Even worse, we predict the following to be fine.

(6) *John are here.

- The plural morphology on the predicate is licensed by the plural ϕP.
- Since plur is semantically vacuous – i.e., an *identity function* – compositionally nothing goes wrong.

[1] Harbour's work is a particularly interesting example (see, e.g., Harbour 2014). In order to account for generalisations concerning possible number systems cross-linguistically, it is crucial for Harbour that the plural is semantically strong.

(7)



1.2 *Maximize Presupposition* to the rescue

- Over the last couple of weeks, we've discussed mechanisms for strengthening the *at-issue* part of a sentence's meaning, through the lens of the exhaustivity operator exh.
- In the basic case, the logic was as follows: for an expression α, the strengthened meaning (the "implicature") can be derived by negating each alternative ψ to α, s.t., ψ is α-*excludable*.
- We defined our notion of *excludable* relative to logical relations between the at-issue meaning components of α and ψ.
- We began by considering a version of the exhaustivity operator according to which the *logically non-weaker* alternatives are excludable; in order to derive free choice inferences, we shifted to a more refined version of excludability – *innocent excludability*.
- We handily abstracted away from any cases involving sentences with *presuppositions* (the other half of this course).
- Relevant to this point, there are a number of expressions which seem to have equivalent at-issue meanings, but which differ in the logical strength of their presuppositional component:

weaker item	stronger item	differential presupposition
a	the	uniqueness
all/every	both	domain contains exactly two elements
believe	know	complement is true

- For such pairs, two kinds of interpretive effects have been observed.
  - The use of a presuppositionally weaker form in a context that does not satisfy the presupposition of its stronger complement tends to implicate that this presupposition is false. In the literature, these are commonly referred to as *anti-presuppositions*.[2]
- The following examples are taken from Marty (2017: p.,14):
- ⟨a, the⟩
- Anti-presupposition for (8a): ¬ [J has exactly one brother]

(8) John lives with a brother of his.
  a. Presupposition: ∅
  b. Assertion: J lives with a brother of his

(9) John lives with his brother.
  a. Presupposition: John has exactly one brother
  b. Assertion: J lives with a brother of his

- ⟨all, both⟩
- Anti presupposition for (10a): ¬ [J has exactly two children]

(10) John brought all his children.
  a. Presupposition: ∅
  b. Assertion: John brought all his children.

(11) John brought both his children.
  a. Presupposition: John has exactly two children.
  b. Assertion: John brought all of his children

- ⟨believe, know⟩
- Anti-presupposition for (12a): ¬[spkr is 6ft tall]

(12) John believes that I am six feet tall.
  a. Presupposition: ∅
  b. Assertion: J believes that spkr is 6ft tall.

(13) John knows that I am six feet tall.
  a. Presupposition: spkr is 6ft tall
  b. Assertion: J believes that spkr is 6ft tall

[2] A term coined by Kai von Fintel; introduced in Percus 2006.

- Note that these effects don't go through when it is clearly not the case that the speaker is opinionated:

(14) a. I have no idea whether John has only one or two brothers; however, I know for sure that he lives with a brother of his.
b. I don't know how many children John has, but he will bring them all.
c. I don't know how tall I am. John believes I am 6 feet tall.

- Second, use of the presuppositionally weaker item is perceived as "odd" when the presupposition of the stronger alternative is entailed by the common ground (Heim 1991).

(15) $C \subseteq$ there is exactly one sun ⟨a, the⟩
a. # A sun is shining.
b. The sun is shining.

(16) $C \subseteq$ John has exactly two parents ⟨all, both⟩
a. # John talked to all his parents.
b. John talked to both his parents.

(17) $C \subseteq$ 2+2 equals 4, and John just proved it ⟨believe, know⟩
a. # John believes that 2+2 equals 4.
b. John knows that 2+2 equals 4.

- Intuitively, this oddness obtains because the speaker does not take for granted the truth of the presuppositions of their (b) alternatives. Marty (2017) refers to these phenomena as *anti-presuppositional effects*.
- In order to capture these effects, Heim (1991) proposed an independent pragmatic principle, which has come to be known as *Maximize Presupposition!* (after Sauerland 2008).

> Maximize Presupposition (MP)
>
> ---
>
> If the following is true for any $\psi \in \mathsf{alt}\ \phi$, then the sentence ϕ cannot be felicitously uttered in context C:
> - ψ's presupposition *asymmetrically entails* ϕ's presupposition, and
> - ϕ and ψ are contextually equivalent.

- Informally, if there is an alternative ψ to a sentence ϕ that makes the same assertive contribution but has a stronger presupposition, then ψ should be preferred.
- If ψ is not used, then MP triggers the inference that ψ *could not have been used*.
- An example of MP-style reasoning:
  - Let ϕ and ϕ_π be two sentences with the same assertive content, which minimally differ in that ϕ_π carries an additional semantic presupposition π.
  - Suppose that a cooperative speaker chooses to utter ϕ instead of ϕ_π in a conversation.
  - Since the speaker did not presuppose π, MP predicts that the speaker shall not be in a position to presuppose π, e.g., because the speaker does not believe that π holds.
- This general reasoning process correctly accounts for *anti-presuppositional effects* discussed aboce.
- Furthermore, if one assumes that the speaker is opinionated about the truth of ϕ_π, this derives the anti-presuppositions initially discussed.

1.3 MP-effects as a reflex of grammatical exhaustification

- It has long been observed that anti-presuppositions are very similar to classical cases of scalar implicatures; the use of an *assertively* weaker sentence entails that the non-weaker alternatives are false.
- Furthermore, as observed by Magri (2009), the use of an assertively weaker sentence is perceived as odd in contexts where this sentence is contextually equivalent to an assertively stronger alternative.

(18) $C \subseteq$ the prof gave the same grade to every student ⟨some, all⟩
a. # The professor gave an A to some students.
b. The professor gave an A to all students.

(19) $C \subseteq$ John has three children of the same gender ⟨two, three⟩
a. # John has two sons.
b. John has three sons.

- Magri (2009) proposes that antipresuppositions are simply implicatures computed in the presuppositional dimension (he adopts a multi-dimensional approach to presupposition).

(20) $[\![[\mathsf{exh}_M\ \phi]]\!] \coloneqq \dfrac{\mathsf{exh}_P\ (\mathbb{P}\ \phi)}{\mathsf{exh}_A\ (\mathbb{A}\ \phi)}$

(21) **Strengthened presupposition**
a. $\mathsf{excl}_P\ \phi \coloneqq \{\psi \in \mathsf{alt}\ \phi \mid \mathbb{P}\ \phi \nrightarrow \mathbb{P}\ \psi\}$
b. $\mathsf{exh}_P\ \phi \coloneqq \mathbb{P}\ \phi \wedge \forall \psi \in \mathsf{excl}_P\ \phi[\neg\ \mathbb{P}\ \psi]$

(22) **Strengthened assertion**
a. $\mathsf{excl}_A\ \phi \coloneqq \{\psi \in \mathsf{alt}\ \phi \mid \mathbb{A}\ \phi \nrightarrow \mathbb{A}\ \psi\}$
b. $\mathsf{exh}_A\ \phi \coloneqq \mathbb{A}\ \phi \wedge \forall \psi \in \mathsf{excl}_P\ \phi[\neg\ \mathbb{A}\ \psi]$

- In order to capture both anti-presuppositional effects and oddness effects in the domain of implicature, Magri proposed two highly influential principles of implicature computation:
  - *Blindness*
  - *Mandatoriness*

Blindness

The notion of entailment relevant for the computation of excludable alternatives is *logical entailment* (rather than entailment relative to common knowledge).

Mandatoriness (Condition on Prunability)

Whenever an excludable alternative ψ to a sentence ϕ is contextually-equivalent to ϕ, the computation of the implicature associated with ψ is mandatory.

1.4 Back to the weak theory of plurality

- Sauerland (2003) claims that MP predicts the following contraint on the distribution of the singular vs. plural:

(23) Use the most specific agreement feature possible whose presupposition is satisfied.

- This captures the following contrast:

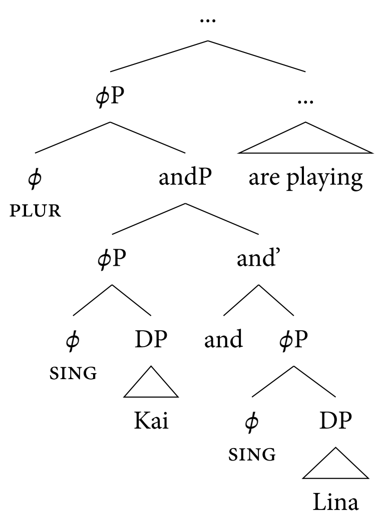
(24) a. *John are here.
b. John is here.

- We'll come back to how to derive the multiplicity inference for plural indefinites. In the meantime, both Sauerland (2003) and Sauerland, Anderssen & Yatsushiro (2005) present a raft of additional arguments for the weak theory of plurality, and **sauerland2003l**'s particular implementation.

1.4.1 Coordinations and Pronouns

- Sauerland (2003) argues that his proposal is independently necessary in order to account for environments in which number agreement is found.

(25) a. Kai and Lina are playing.
b. *Kai and Lina is playing.



- An alternative analysis would be to say that conjunction is inherently plural (Sauerland cites Vanek 1977), but notes that this proposal faces a problem with examples such as the following.

(26) a. Strawberries and cream is on the menu.
b. Beans and rice is a basic staple around here.

- Sauerland claims that singular agreement is possible just in case the denotation of the conjunction can be viewed as an atomic individual.
- Sauerland gives two more examples involving names:[3]

(27) a. Kai and Lina makes a good combination.
b. Tim and Sarah is a nice couple.

1.4.2 Pronouns

- The second argument that Sauerland gives for his theory comes from pronouns.
- The structure Sauerland assumes for, e.g., (plural) *they* is as follows:

[3] Winter (2001) already discussed cases such as these in order to argue for a distinction between genuine pluralities and "impure atoms". This is consistent with Sauerland's proposal.

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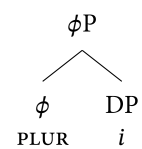

- According to Sauerland's approach, the exclusivity inference associated with *they* is derived via MP style reasoning, from competition with the singular pronouns.
- The prediction is, that is alternatives involving singular pronouns are independently unusable, then the exclusivity inference should no longer obtain.
- This accounts for uses of singular *they* when the speaker is ignorant of the gender of the referent (note that Sauerland does not consider the possibility of singular *they* used to refer to a non-binary individual – this will complicate the analysis).

(29) Some student left their umbrella.

- Singular forms can also be blocked by politeness considerations, in which case the plural form is used. This can be observed in, e.g., German.

(30) a. *Können Sie bitte etwas rücken!*
could they please a-little move
"Could you please move a little" (formal)
b. *Eure Majestät haben euren Silberlöffel geschluckt*
you majesty have your silver spoon
"Our majesty has swallowed our silver spoon"

1.4.3 Definites

- One unusual feature of Sauerland's analysis is that number features are only interpreted at the DP level – not on the noun itself.
- Sauerland explicitly argues for this on the basis of definites.

(31) The students wrote a paper.

- The **distributivity operator** $*$:

(32) $* P := \lambda X \,.\, \exists C[\text{cover } C\ X \wedge \forall x \in C[P\ x]]$

(33) cover $C\ X = 1$ iff $\bigoplus C = X$

- The distributivity operator can apply to nominal and verbal predicates equally.

(34) $[\![\text{student}]\!] := \{\,\text{Tom, Tina, Tanya}\,\}$

(35) $[\![* \text{ student}]\!] = \left\{ \begin{array}{c} \text{Tom, Tina, Tanya} \\ \text{Tom} \oplus \text{Tina, Tom} \oplus \text{Tanya, Tina} \oplus \text{Tanya} \\ \text{Tom} \oplus \text{Tina} \oplus \text{Tanya} \end{array} \right\}$

(36) $[\![\text{wrote a paper}]\!] = \left\{ \begin{array}{c} \text{Tom, Tanja,} \\ \text{Tom} \oplus \text{Tina} \end{array} \right\}$

(37) $[\![* \text{ wrote a paper}]\!] = \left\{ \begin{array}{c} \text{Tom, Tanya,} \\ \text{Tom} \oplus \text{Tina, Tom} \oplus \text{Tanya,} \\ \text{Tom} \oplus \text{Tina} \oplus \text{Tanya} \end{array} \right\}$

- According to the standard view (e.g., Bennett 1974, Link 1983, etc.) number on an NP is interpreted as $*$.
- According to such theories, what the definite article does is (a) presuppose that its complement has a unique maximal element X, and, if defined, returns X.
- The $*$-operator has no morphological reflex on the VP. The LF of *the students wrote the paper* is given below. It is predicted, on the standard theory, to be defined and true in a scenario where *Tom and Tanya wrote papers by themselves, and Tom and Tina collaborated on a paper.*

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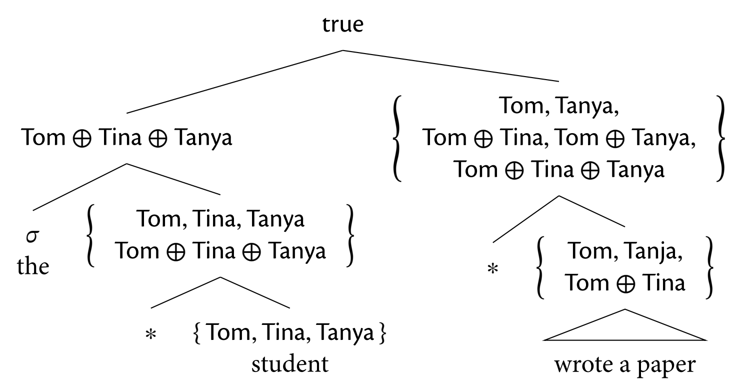


- The standard theory furthermore predicts that a singular noun can't be used in this scenario, since it has no unique maximum.
- Why can't *The students wrote a paper* be used in a scenario where exactly one student wrote a paper?
- Nothing goes wrong semantically. The standard theory must appeal to pragmatic factors similar to the ones that Sauerland appeals to – namely, don't use the plural if using the singular would result in an equivalent meaning.[4]
- Contrast now Sauerland's semantics for number, according to which $*$ can be applied to any predicate, and is *never* pronounced.
- Sauerland observes that, in order to account for the cases below, the standard theory must say that plural marking is *ambiguous* between $*$ and the cumulativity operator $**$ (Beck & Sauerland 2000).

(39) a. The daughters of the defense players/Bill and James...
b. The residents of these cities...
c. The winners of a gold medal at the 1992 and 1996 olympics...

(40) $**$ daughter $X\ Y$ iff there are both:

a. a cover C_X of X, s.t. $\forall x \in C_X \exists y \leq Y[\text{daughter } x\ y]$
b. a cover C_Y of Y, s.t. $\forall y \in C_Y \exists x \leq X[\text{daughter } x\ y]$

(41) a. daughter $= \{\,\langle \text{DB, Bill}\rangle, \langle \text{DJ, James}\rangle\,\}$
b. $**$ daughter $= \{\,\langle \text{DB, Bill}\rangle, \langle \text{DJ, James}\rangle, \langle \text{DB} \oplus \text{DJ, Bill} \oplus \text{James}\rangle\,\}$

[4] Unlike MP however, arguably this follows directly from existing Gricean principles, such as *brevity*.

- Singular nouns allow cumulative readings too:

(42) a. Every daughter of the defense players is watching the game.
b. Every winner of a gold medal at these events can be proud.
c. Every resident of these cities has a bicycle.

- Sauerland concludes that plural marking cannot be consistently interpreted as either $*$ or $**$; the standard theory does not have an account of why a cumulated noun in a definite description must be plural.
- However, according to Sauerland's theory, this straightforwardly follows – the number head above the DP must contain the [PLUR] feature, since [SING] is ruled out here.

(43) PLUR the ([$**$ daughter] of Bill and James)

1.5 Evidence for the weak theory

- The data in this section is from Sauerland, Anderssen & Yatsushiro (2005).

1.5.1 Mixed reference

(44) a. You are welcome to bring your children.
b. #You are welcome to bring your child.
c. You are welcome to bring your child or your children.

(45) a. Every boy should invite his sister to the party.
b. #Every boy should invite his sister to the party.

1.5.2 Indefinites in DE contexts

- Number marking in a DE context doesn't affect truth-conditions:

(46) a. Kai hasn't found any eggs.
b. Kai has found no eggs.

(47) a. Without artificial ingredients.
b. If John had eaten any apples from the basket, there would be at least one/#two less in the basket.

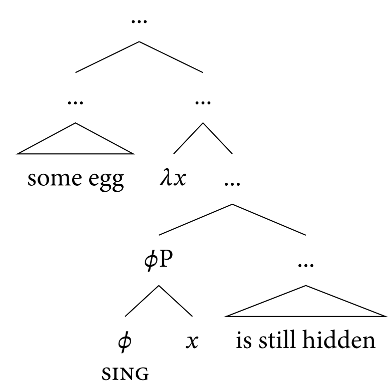
- In a UE context, number marking on an indefinite *does* affect truth-conditions.

(48) a. Some eggs are still hidden.
b. Some egg is still hidden.

(49) Kai couldn't find certain/some eggs.

- The analysis:

(50)



- The presupposition introduced by SING is *locally accommodated* in the nuclear scope of the GQ.
- What about the plural feature?

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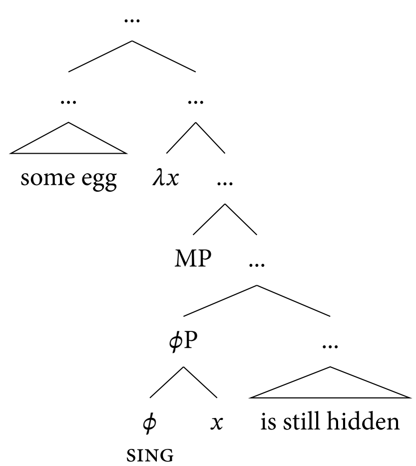

> Local *Maximize Presupposition*
>
> MP applies to the scope of an existential if this strengthens the entire utterance.

- Evidence for local maximise presupposition (based on Percus 2006):

(52) a. Every professor with exactly two students told **both** of his students to quit.
b. # Every professor with exactly two students told **all** of his students to quit.

- We see further parallels with scalar implicature:

(53) a. After every one of the professor's students had arrived, the professor asked all of them to leave again.
b. After every one of the professor's students had arrived, the professor asked some of them to leave again.
(# if the professor asked all the students to leave)

2 Spector's puzzle

- Spector is largely concerned with accounting for inferences associated with plural indefinites.
- We won't focus on all of the details of Spector's theory, but its flaws will be instructive.
- Like previous authors, Spector observes that the exclusivity inference obtains in a UE environment, and disappears in a DE environment.
- But further to this, Spector (2007) observes that when a plural indefinite appears in the scope of some *non-monotonic operator* such as *exactly one of my students*, we find something unexpected.

(54) a. Exactly one of my students has solved difficult problems.
b. Exactly one of my students has solved some difficult problems.

- (54a) is equivalent to none of the following sentences:

(55) a. Exactly one of my students has solved at least one difficult problem.
b. Exactly one of my students has solved two, or more than two, difficult problems.

- (55a) is true (taking the scope of *at least one* to be fixed below *exactly one*) in a situation where one student, say Jack, has solved exactly one difficult problem, and no other student solved any problem.
- (54a) however, suggests that the only student who has solved a problem has solved more than one problems.
- (55b) on the other hand, is true where one student, say Jack, has solved exactly two problems, and all other students have solved exactly one problem; but intuitively, (54a) entails that no student, apart from the unique one who solved problems, solved any problem.
- On the face of it then, (54a) is a probelm for both strong AND weak theories of plurality.
- The attested interpretation is as follows:

(56) One of my students has solved several difficult problems, and all other students have solved no difficult problems at all.

- Spector's account: recursive exhaustification.
- Spector assumes a so-called "minimal worlds" formulation of the exhaustivity operator, defined below:

(57) $\mathsf{exh}_{mw}\ \phi \coloneqq \{ w \mid \phi\ w \wedge \neg\exists w'[\phi\ w' \wedge w' <_S w] \}$

(58) $w' <_S w$ iff the members of $\mathsf{alt}\ \phi$ true in w' are a proper subset of the members of $\mathsf{alt}\ \phi$ true in w.

- Let's apply this exhaustivity operator to a concrete case: the exclusivity inference associated with disjunction.

(59) $\phi = p \vee q$

(60) $\mathsf{alt}\ \phi = \{ p, q, p \wedge q, p \vee q \}$

- Let's gather up the members of $\mathsf{alt}\ \phi$ true in each world:
  - $w_\varnothing = \varnothing$
  - $w_{p\neg q} = \{ p, p \vee q \}$
  - $w_{\neg pq} = \{ q, p \vee q \}$
  - $w_{pq} = \{ p, q, p \vee q, p \wedge q \}$
- This gives us the following ordering on worlds.

$$w_\varnothing <_S w_{p\neg q}; w_{\neg pq} <_S w_{p\wedge q}$$

- $\mathsf{exh}_{mw}\ \phi$ gathers up the set of ϕ-worlds which make as few of the alternatives true as possible without contradicting the prejacent. In this scenario this delivers the following set:

$$\{ w_{p\neg q}, w_{\neg pq} \} = p \vee q \wedge \neg\,(p \wedge q)$$

- Note that the minimal worlds formulation of the exhaustivity operator, like exh_{ie} is *contradiction free*, and consistent with considering the individual disjuncts as alternatives.
- In fact, Spector (2016) demonstrates that whenever the set of alternatives is semantically closed under conjunction, innocent exclusion and the minimal worlds formulation are *equivalent*.

- **Assumptions regarding plurality**: Spector assumes that singular NPs denote sets of atoms, and plural marking is interpreted as Landman's $*$-operator.

(61) $*\,P \coloneqq \lambda X\,.\,\forall x[(\mathsf{atom}\ x \wedge x \leq X) \rightarrow P\ x]$

- The singular, on the other hand, is assumed to be semantically vacuous.
- This directly accounts for the interpretation of indefinite plurals in DE contexts, but doesn't account for the exclusivity inference in UE contexts, or Spector's central puzzle.
- This proposal renders singular and plural indefinites *equivalent* (at least for distributive predictates).

(62) Jack saw a horse.
$\exists x \in \mathsf{horse}[\mathsf{j\ saw}\ x]$

(63) Jack saw horses.
$\exists X[\forall x[(\mathsf{atom}\ x \wedge x \leq X) \rightarrow x \in [\![\text{horse}]\!]] \wedge \mathsf{j\ saw}\ X]$

- Since $\lambda X\,.\,\mathsf{j\ saw}\ X$ is distributive, the following equivalence holds for any X:

$$\mathsf{j\ saw}\ X \Leftrightarrow \forall x[(\mathsf{atom}\ x \wedge x \leq X) \rightarrow \mathsf{j\ saw}\ x]$$

- The puzzle then, is how to derive a multiplicitly inference for the plural via competition with the singular. Since the two are equivalent, this shouldn't be possible.
- Spector suggests that the plural doesn't compete with the literal meaning of the singular alternative, but rather the pragmatically strengthened meaning. The pragmatically strengthened meaning of the singular alternative is taken to convey an *exactly one* inference.

2.1 Spector's assumptions regarding alternatives

- Spector makes the following assumptions regarding alternatives:

(64) a. Jack saw horses.
b. Jack saw a horse.
c. Jack saw several horses.

(65) a. alt (64a) = { Jack saw horses, Jack saw a horse }
b. alt (64b) = { Jack saw a horse, Jack saw horses, Jack saw several horses }
c. alt (64c) = { Jack saw several horses }

> Question
>
> What assumptions is Spector making regarding the relation of *alternativehood*? Can you spot anything problematic.

- We begin by computing the strengthened meaning of each sentence:
- The only alternative to the bare plural is the (equivalent) singular, which is of course not excludable, so exhaustification is vacuous.

(66) exh_{mw} (Jack saw horses) = Jack saw horses

- The singular indefinite can be exhaustified relative to *Jack saw several horses* – the minimal set of worlds consistent with the prejacent that make as many of the alternatives false as possible, are the ones in which Jack saw exactly one horse.

(67) exh_{mw} (Jack saw a horse) = Jack saw exactly one horse

- Exhaustification of (64c) is vacuous, since there are no non-equivalent alternatives.
- Now let's consider the alternatives to the exhaustification of the bare plural.

(68) exh_{mw} Jack saw horses

(69) $$\mathsf{alt}\,(68) = \left\{\begin{array}{r} \mathsf{exh}_{mw}\text{ (Jack saw horses)} \\ \mathsf{exh}_{mw}\text{ (Jack saw a horse)} \\ \mathsf{exh}_{mw}\text{ (Jack saw several horse)} \end{array}\right\} = \left\{\begin{array}{r} \text{Jack saw horses} \\ \text{Jack saw exactly one horse} \\ \text{Jack saw several horse} \end{array}\right\}$$

- What is set of worlds which makes as many of the exhaustified alternatives false, without contradicting the prejacent? The worlds in which Jack saw more than one horse, of course.
- Spector thereby derives the exclusivity inference.

2.2 Back to Spector's puzzle

- Recall:

(70) Exactly one of my students has solved difficult problems.

- The only alternative to (70) involves a singular indefinite:

(71) alt (70) = { Exactly one of my students solved a difficult problem }

- First, let's compute the strengthened meaning of the alternative. It competes with the following, which is logically non-weaker:

(72) Exactly one of my students has solved several difficult problems.

(73) exh_{mw} Exactly one of my students has solved a difficult problem
= exactly one of my students has solved a difficult problem
$\wedge\neg$ (exactly one of my students has solved several difficult problems)

- This is equivalent to:

(74) Exactly one of my students has solved exactly one difficult problem, and no other student has solved any problem at all.

- We now compute the meaning of (70) relative to its strengthened alternative, just like last time. The result is:

(75) Exactly one of my students solved at least one difficult problem, and its not the case that exactly one of my students has solved exactly one difficult problem and no other student has solved any difficult problem at all.

- This is equivalent to:

(76) Exactly one of my students has solved at least two difficult problems, and all other students have solved no difficult problem at all.

- Informal proof:
  - Suppose (75) is true: then there is a student, say Jack, who solved a difficult problem, and is such that all other students have solved no difficult problem.

- Suppose Jack solved exactly one difficult problem.
- Since the following is false – *exactly one student has solved exactly one difficult problem, and no other students has solved any difficult problem* – it follows that a student distinct from Jack has solved exactly one difficult problem, or Jack is the only one who has solved exactly one difficult problem but there are other students who have solved difficult problems.
- In both cases, there must be students different from Jack who have solved a difficult problem. This is a contradiction.
- Therefore, Jack has solved several difficult problems; no other student solved any difficult problem – it follows that a student distinct from Jack has solved exactly one difficult problem, or Jack is the only one who has solved exactly one difficult problem but there are other students who have solved difficult problems.
- In both cases, there must be students different from Jack who have solved a difficult problem. This is a contradiction.
- Therefore, Jack has solved several difficult problem; no other student solved any difficult problem.

2.3 Towards an account of Spector's puzzle

- Note that the results reported here are the result of ongoing joint work with Paul Marty.

- Recall:

(77) Exactly one of my students has solved difficult problems.
  a. one of my students has solved *more than one* difficult problem
  b. none of my other students have solved *one or more* difficult problems

- **Ingredient 1:** following Sauerland (2013), we treat *exactly* as a focus-sensitive operator.
- Exactly, much like, e.g., *only*, takes a proposition p that contains a focused element (such as a numeral), and returns (i) that p is true, and (ii) for every $q \in \mathsf{alt}\ p$ that is not entailed by p, $\neg\, q$ is true.

(78) Exactly/only [ONE$_F$ students came to the meetings].
  a. One student came to the meeting.
  b. $\neg$ [n students came to the meeting], for any numeral $n > 1$

- **Ingredient 2:** a number of researchers have noted that, in the scope of *only*:
  - implicatures *are* generated in the upward-entailing (UE) component (i.e., the prejacent).
  - implicatures *disappear* in the downward-entailing (DE) component (i.e., the negated alternatives).
- See, e.g., Gajewski & Sharvit (2012), Alxatib (2014), Bar-Lev (2018). Bar-Lev describes this as a "two-sided inference".
- We illustrate this phenomena here with a scalar expression *some* in the scope of *exactly/only*:

(79) Exactly/Only [ONE$_F$ student ate some of the cookies]
  a. <u>UE component: implicature</u>
  one student ate some *but not all* of the cookies
  b. <u>DE component: no implicature</u>
  $\neg$[n students ate some of the cookies], for any numeral $n >$ *one*

- We propose that the case in (77) is another instance of the above phenomenon.
- **Ingredient 3:** following Mayr (2015), we assume *predicate-level exhaustification*.

(80) $\mathsf{exh}\ f \coloneqq \lambda X\ .\ f\ X \wedge \forall g \in \mathsf{alt}\ f[f \nsubseteq g \rightarrow \neg\, g\ X]$

- a multiplicity implicature is generated in the UE-prejacent of *exactly*, delivering (77a), but not in its DE-alternatives, hence (77b).
- The intuition here is that exh can be rendered vacuous in these DE-alternatives as its working would otherwise weaken their meaning, (81).
- This should ultimately follow from the Economy condition constraining the distribution of exh (see Fox & Spector 2018 and others).

(81) Exactly [ONE_F student solved exh [difficult problems]]

a. one student solved exh [difficult problems]
⇒ one student solved *more than one* difficult problems

b. ¬[*n* student solved ~~exh~~ [difficult problems]], for any numeral $n >$ *one*
⇒ none of the other students have solved *one or more* difficult problems

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