# *The dynamic approach to anaphora*

*Patrick D. Elliott & Danny Fox*

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## 1 Background

*Dynamic semantics* was developed independently in the 1980s by Irene Heim (1982; *File Change Semantics*) and Hans Kamp (1981; *Discourse Representation Theory*).[1]

Today we'll be looking at an extremely influential offshoot of Heim's approach: Groenendijk & Stokhof's Dynamic Predicate Logic (DPL).

DPL is a nice theory to look at, since it idealizes away issues arising when giving a compositional semantics for *English*, and instead focuses on providing a dynamic interpretation for a well understood formal language — First Order Logic (FOL).

It also highlights many of the core properties of Heim's approach in an unusually clear and elegant fashion.[2]

> "The general starting point of the kind of semantics that DPL is an instance of, is that the meaning of a sentence does not lie in its truth conditions, but rather in the way it changes (the representation of) the information of the interpreter. The utterance of a sentence brings us from a certain state of information to another one. The meaning of a sentence lies in the way it brings about such a transition."
> (Groenendijk & Stokhof 1991: p. 43)

We've already seen a theory with character — namely, Veltman's (1996) propositional update semantics, which we used to model facts about presupposition projection.

Despite being arguably the simplest form of dynamic logic, we jumped the gun historically by starting with propositional update semantics — the original motivation for the dynamic approach to meaning is the logical relations that (singular) indefinites and pronouns enter into.

There are two main phenomena which suggest we need to re-think the logical relationship that indefinites and pronouns enter into: *discourse anaphora* and *donkey anaphora*.

[1] If you've read Yalcin (2013), recall that Heim's approach initiated the *dynamic interpretation* approach, and Kamp's approach initiated the *dynamic representation* approach.

[2] In the paper, Groenendijk & Stokhof justify our decision to ignore *dynamic representation* approaches in favour of dynamic interpretation; they prove an equivalence between DPL and Kamp's Discourse Representation Theory (DRT), which means that Kamp's representational language is strictly speaking unnecessary in order to account for the phenomena we're interested in.

### Discourse anaphora

Indefinites, unlike other quantificational expressions, can seemingly scope outside of their containing clause:

(1) A$^1$ philosopher attended the talk, and she$_1$ asked some difficult questions.

(2) a. *Every$^1$ philosopher attended the talk, and she$_1$ asked some difficult questions.
b. *No$^1$ philosopher attended the talk, and she$_1$ asked some difficult questions.

We might want to assign (1) the schematic Logical Form (LF) below:

(3) $\exists x[(\textsf{phil}\ x \land x\ \textsf{attended the talk}) \land x\ \textsf{asked some difficult questions}]$

This seems a bit odd, given that scope-taking is usually strictly clause-bound (May 1977), but maybe we have to bite the bullet, since indefinites are independently known to take exceptional scope:[3]

[3] Brasoveanu & Farkas 2011, Charlow 2014, a.o.

To illustrate, the following sentence has a reading that can be paraphrased with the given LF:

(4) If a relative of mine dies, i'll inherit a fortune.
$\exists x[\textsf{relative}\ x \land ((\textsf{dies}\ x) \to \textsf{i'll inherit a fortune})]$

We've deflated (1), but more problematically, the facts remain the same, even if the indefinite and pronoun are located in two different sentences.

(5) A$^1$ philosopher attended the talk. She$_1$ asked some difficult questions.

Perhaps we could posit that, implicitly, this discourse involves logical conjunction, but it's difficult to imagine how to analyze this case if we assume successive *assertion*.

Indefinites can even bind pronouns in multi-speaker discourses, where syntactic continuity is highly implausible:

(6) a. A: A$^1$ famous philosopher attended my talk.
b. B: Oh? Did she$_1$ ask any especially difficult questions.

Another fact which we'd like to capture: order seems to matter for successful anaphora:[4]

(8) a. A: She$_1$ attended my talk.
b. #B: Oh? Did a$^1$ famous philosopher ask any especially difficult questions?

[4] It's often suggested that ordering asymmetries in conjunctive sentences argue in favour of a dynamic approach:

(7) *She$_1$ attended my talk and a$^1$ famous philosopher asked some difficult questions.

This isn't a good argument — if binding can be fed by exceptional *quantificational* scope, then (7) is already ruled out as a violation of Weak Crossover (WCO) (Postal 1971; Charlow 2019 makes this same point).

One option, in order to capture discourse anaphora, could be to treat indefinites as *referring expressions*; we can see that this isn't going to work however, as soon as we embed an indefinite under some Downward Entailing (DE) operator.

Binding into its restrictor forces the indefinite to scope below negation,[5] in the following; this blocks discourse anaphora.

[5] Due to the *Binder Roof Constraint* (Brasoveanu & Farkas 2011).

(9) Nobody$^1$ bought $a_1$ picture of himself. $It_1$ was ugly.

We can account for this if we maintain our standard treatment of indefinites as existential quantifiers, and suppose that, minimally, discourse anaphora requires an existential entailment in the first sentence.

**Donkey anaphora**

As we saw, there is some wiggle room in accounting for discourse anaphora — the argument that we need to go beyond the classical was however most pressing in cases of cross-sentential anaphora, or anaphora across a multi-speaker discourse.

The argument that a classical semantics is insufficient is much more acute for *donkey anaphora*:

(10) If a$^1$ farmer owns a$^2$ donkey, $he_1$ feeds $it_2$ hay.

(11) Every farmer who owns a$^2$ donkey feeds $it_1$ hay.

How is anaphora licensed here? We might try the same trick we used last time, and posit exceptional quantificational scope of the indefinites; this results in the following LFS:

(12) a. $\exists x[\textsf{farmer} \wedge \exists y[\textsf{donkey}\ y \wedge (x\ \textsf{owns}\ y \rightarrow x\ \textsf{feeds-hay}\ y)]]$
b. $\exists y[\textsf{donkey} \wedge \forall x[(\textsf{farmer}\ x \wedge x\ \textsf{owns}\ y) \rightarrow (x\ \textsf{feeds-hay}\ y)]]$

This LFS certainly capture *one* reading of the previous sentences, but the most salient reading of (11) is that, every farmer $x$, is s.t., $x$ feeds every donkey that $x$ owns.

The LF in (12b) however predicts that (11) could be true if there's some donkey and a farmer that doesn't own it. This seems all wrong.

That the wide scope LF won't work becomes especially clear when we force the indefinite to take narrow scope:

(13) a. If any[1] of these books is censored, I won't buy $\text{it}_1$.
b. Everyone who bought any[1] of these books immediately returned $\text{it}_1$.

If we give the indefinite narrow scope however, we predict that binding shouldn't be successful; in each case the pronoun remains free:

(14) a. $(\exists x[\text{book } x \wedge \text{censored } x]) \rightarrow (\text{I won't buy } x)$
b. $\forall x[(\exists y[\text{book } y \wedge x \text{ bought } y]) \rightarrow x \text{ returned } y]$

Taken all together, discourse anaphora and donkey anaphora show that the logical relationship that holds between indefinites and pronouns isn't either of the following:

- Coreference.
- Quantifier and the variable it takes scope over.

It must, therefore be something else. Dynamic Semantics (DS) (and more narrowly, DPL) is the attempt to systematically generalize our semantic apparatus in order to account for these problematic cases.

In order to account for discourse anaphora, we'll also need to integrate this new approach to meaning into our pragmatic system — ultimately, we'll need a new notion of *information state*, which goes beyond a classical Stalnakerian setting (Stalnaker 1976).

Before we go into the technical details, we'll try to motivate the notion of *information* marshalled in a dynamic semantics for anaphora.

### *1.1 Referential information and subject matter*

**Worldly information and pronominal licensing**

The notion of *information state* we've been assuming — a set of possible worlds — is too limited in certain respects.

The sentences in (15a) and (15b) express the same information states.

(15) a. Andreea is married.
b. Andreea has a spouse.

But, we can detect a contrast in the following discourses; it seems, they're not intersubstitutable.

(16) a. Andreea is married. I saw ?them/Andreea's spouse yesterday.
b. Andreea has a husband. I saw them/Andreea's spouse yesterday.

One way of characterizing this contrast is to say that (16b) is "about" *Andreea's spouse* in a way that (16a) isn't.[6]

[6] In the literature on anaphora, the requirement that a pronoun have a nominal antecedent is often referred to as the *formal link condition*.

Heim (1982: p. 21) makes a similar point, using some (now, quite famous) examples attributed to Barbara Partee (p.c.):

(17) a. I dropped ten marbles and found all of them, except for one. It's probably under the sofa.
b. I dropped ten marbles and found only nine of them. ?It's probably under the sofa.

Heim (1982) is worth quoting directly on this point:

> "[…] we are compelled to conclude that the salience-shifting potential of an utterance is not predictable from its truth-conditions and the surrounding circumstances alone; it moreover depends on how the utterance is worded."

In a discourse, participants must keep track of more than just *what is true/false*, but additionally, *what has been mentioned*.

**Referential information growth**

In order to capture this idea, we'll need a new notion of *information*. It's easiest to illustrate how this works with a concrete example.

(18) *Context: we're playing a guessing game. You have to guess who I'm thinking about.*
a. I'm thinking of a$^1$ man.
b. He$_1$ was a mathematician.
c. He$_1$ made important contributions to computer science.
d. He$_1$ is British.
e. He$_1$ worked in Bletchley Park, and died in 1952.

As an idealization, we'll model an information state in which nothing has been said using the unique initial assignment $\{ g_\varnothing \}$.[7]

[7] Throughout, we'll assume that assignments are partial.

We can informally think of the pragmatic contribution of each of the assertions in (18) as inducing a shift in referential information, which we can represent as a *set of partial assignments*.

(19) $$\{\,[\,]\,\} \xrightarrow{\text{I'm thinking of a}^1\text{ man}} \left\{\begin{array}{l}[1 \to \textsf{einstein}]\\ [1 \to \textsf{feynman}]\\ [1 \to \textsf{gödel}]\\ [1 \to \textsf{church}]\\ [1 \to \textsf{hoare}]\\ [1 \to \textsf{türing}]\end{array}\right\}$$

(20) $$\left\{\begin{array}{l}[1 \to \textsf{einstein}]\\ [1 \to \textsf{feynman}]\\ [1 \to \textsf{gödel}]\\ [1 \to \textsf{church}]\\ [1 \to \textsf{hoare}]\\ [1 \to \textsf{türing}]\end{array}\right\} \xrightarrow{\text{He}_1\text{ was a mathematician}} \left\{\begin{array}{l}[1 \to \textsf{gödel}]\\ [1 \to \textsf{church}]\\ [1 \to \textsf{hoare}]\\ [1 \to \textsf{türing}]\end{array}\right\}$$

(21) $$\left\{\begin{array}{l}[1 \to \textsf{gödel}]\\ [1 \to \textsf{church}]\\ [1 \to \textsf{hoare}]\\ [1 \to \textsf{türing}]\end{array}\right\} \xrightarrow{\text{He}_1\text{ made contributions to comp sci}} \left\{\begin{array}{l}[1 \to \textsf{hoare}]\\ [1 \to \textsf{türing}]\end{array}\right\}$$

(22) $$\left\{\begin{array}{l}[1 \to \textsf{hoare}]\\ [1 \to \textsf{türing}]\end{array}\right\} \xrightarrow{\text{He}_1\text{ worked in Bletchley Park}} \left\{\,[1 \to \textsf{türing}]\,\right\}$$

In a classical Stalnakerian setting, the “points” that make up an information state are *possible worlds*, i.e. states of complete information.

Here, our points are *assignments*: they represent states of complete information regarding what has been mentioned, but not omniscience about everything that *will* ever be mentioned over the course of the discourse!

As such, the kind of information state modelled by a set of assignments doesn’t just *shrink*, but it also *expands* when an indefinite introduces a new variable (we can see this in the first step).[8]

[8] Technically, this will mean that the kind of conversation system characterized by DPL is *non-eliminative* unlike propositional update semantics (see Rothschild & Yalcin 2017).

## 2 *A dynamic semantics for a simple predicate calculus*

Note: the dynamic semantics I give here is a somewhat idiosyncratic presentation of Groenendijk & Stokhof’s (1991) DPL, with ingredients from van den Berg 1996: ch. 2.

It’s close enough that I’ll simply call it DPL, although one should bear in mind that there are some minor differences.[9]

[9] First and foremost, the modelling of familiarity via partial assignments.

### 2.1 Syntax

We can characterize the meanings of sentences of English by first giving a compositional translation procedure into a well-understood logical language $\mathbb{L}$, and then providing a semantics for $\mathbb{L}$.[10]

[10] This is known as *indirect* interpretation (Montague 1973), in contrast to *direct* interpretation theories (Heim & Kratzer 1998). This is only done here for convenience; $\mathbb{L}$ is dispensable.

Just as in our discussion of update semantics (Veltman 1996), we'll develop an indirect semantics to illustrate the basic tenets of dynamic semantics for anaphora.

Since we're interested in the semantic contribution of, e.g., definite and indefinite NPs, we need something more syntactically complex than propositional logic. Following, Groenendijk & Stokhof (1991), we'll use a simple first order predicate calculus.[11]

[11] This will allow us to nicely abstract away from the niceties of giving a compositional fragment of English; FOL is probably the syntactically simplest language we can get away with while still reasonably approximating natural language.

You should be familiar with translating English sentences into expressions of FOL from intro semantics.[12]

[12] If we're serious, it's of course desirable to make this translation procedure compositional, but this is largely mechanical.

(23) a. Some philosopher is here. $\rightsquigarrow \exists x[P\ x \wedge H\ x]$
b. Every linguist has met Chomsky. $\rightsquigarrow \forall x[L\ x \rightarrow M\ x\ c]$
c. She$_1$ is bored. $\rightsquigarrow B\ x_1$

Here's a terse specification of the syntax of FOL:[13]

[13] Intuitively, *variables* will stand as proxy for *traces* and *pronouns*; *individual constants* for names/definite descriptions and *predicate symbols* for verbs/adjectives etc.

**Definition 2.1** (Syntax of FOL)**.** Given the following:

- $\mathbb{V}$, a non-empty set of *variables*, $x_1, x_2, \ldots$.
- $\mathbb{C}$, a non-empty set of *individual constants*, $a, b, c, \ldots$.
- $\mathbb{P}_n$, a non-empty set of $n$-ary predicate symbols, $P, Q, \ldots$.
- $\mathbb{T}$, the set of terms: $\mathbb{V} \cup \mathbb{C}$.

A first-order language $\mathbb{L}$ is the smallest set where:

- If $P \in \mathbb{P}_n$, and $t_1, \ldots t_n \in \mathbb{T}$, then $P\ t_1 \ldots t_n \in \mathbb{L}$. atomic sentences
- If $\phi \in \mathbb{L}$, then $\neg\,\phi \in \mathbb{L}$. negated sentences
- If $\phi, \psi \in \mathbb{L}$, then $\phi \wedge \psi, \phi \vee \psi, \phi \rightarrow \psi \in \mathbb{L}$ con/disjunctive & implicational sentences
- If $\phi \in L$, $x_n \in \mathbb{V}$, then $\exists x_n\ \phi, \forall x_n\ \phi \in \mathbb{L}$ quantified sentences

This will give us everything we need to reason about the kinds of datapoints we introduced at the beginning of this handout.

### 2.2 Semantics I: From terms to atomic sentences

In order to give a dynamic semantics for FOL, we'll recursively define a (partial!) interpretation function relative to an assignment $[\![.]\!]^{g,M}$ (but we'll suppress the model parameter).

For now, a model is simply a tuple $\langle D, I \rangle$, where $D$ is a non-empty set of *individuals*, and $I$ is an valuation function, which:

- maps individual constants $c \in \mathbb{C}$ to individuals in $D$,
- and maps $n$-ary predicate symbols $P \in \mathbb{P}_n$ to $n$-tuples of individuals.

**Semantics of terms**

The interpretation of individual constants is given by the valuation function. For any individual constant $c \in \mathbb{C}$:

$$[\![c]\!]^g \coloneqq I(c)$$

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The interpretation of variables, on the other hand, is assignment dependent. For any variable $x_1 \in \mathbb{V}$:

$$[\![x_1]\!]^g \coloneqq \begin{cases} g_1 & g_1 \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

N.b. that this makes $[\![.]\!]^g$ a *partial* function.

We can now give a dynamic semantics for atomic sentences.

The idea here is that $[\![.]\!]^g$ is a function from a sentence of $\mathbb{L}$ to an *output state*: a set of assignment functions. Since we're not going to attempt to incorporate an account of presupposition,[14] we're assuming the principle of the excluded middle is valid.

[14] We'll perhaps talk about this next week.

**Semantics of atomic sentences**

$$[\![P\ t_1 \dots t_n]\!]^g \coloneqq \begin{cases} \{\ g\ \} & \langle [\![t_1]\!]^g, \dots, [\![t_n]\!]^g \rangle \in I(P) \\ & \wedge [\![t_1]\!]^g, \dots, [\![t_n]\!]^g \text{ are defined} \\ \varnothing & \langle [\![t_1]\!]^g, \dots, [\![t_n]\!]^g \rangle \notin I(P) \\ & \wedge [\![t_1]\!]^g, \dots, [\![t_n]\!]^g \text{ are defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

N.b. that if $[\![.]\!]^g$ is undefined for any term in an atomic sentence, undefinedness *projects*; the definedness conditions induced by variables permeates a weak Kleene logic.

Now is a good time to talk about the *pragmatic* component of the theory — we'd like to understand how the kind of semantic object delivered by the theory, in tandem with an update rule, can capture the referential flow of information as discussed in the previous section.

So far, we've only talked about sentences which stand proxy for English sentences with pronouns and names, but this will still allow us to do some interesting work.

## *2.3 Pragmatics I: Update*

Does it even make sense to talk about pragmatics in the current setting, without introducing possible worlds?

**Yes.** — although, "pragmatics" is a bit of a misnomer. As a temporary idealization, we can assume that discourse participants are omniscient regarding (non-linguistic) worldly facts, we can still use the machinery we're developing to model *(un)certainty of reference*.[15]

[15] We're already idealizing away to a significant extent in standard Stalnakerian setting; it will be, in any case, easy to reintroduce possible worlds at a later point.

Let's take an *information state* to simply be a set of assignments:

**Information states**

An information state (also called a *context*) is a set of assignments, where:

- $\varnothing$ is the *absurd information state*.
- $\{\ g_\varnothing\ \}$ is the *ignorance state*.

N.b. that $g_\varnothing$ is the unique *initial assignment*, whose domain is $\varnothing$.

Just as in orthodox Stalnakerian pragmatics, we'll define an update operation, which will be a partial function from information states to information states, subject to the bridge principle.

**Update**

The *update* of induced by a sentence $\phi$ is a partial function from information states to information states. We write $c[\phi]$ for the update of $c$ by $\phi$.
Strikingly, this is almost *identical* to the update rule we used in a static setting, only instead of worlds, the relevant *points* are assignments.

$$c[\phi] := \begin{cases} \bigcup_{g \in c} [\![\phi]\!]^g & c \subseteq \phi^\pi \\ \text{undefined} & \text{otherwise} \end{cases}$$

Stalnakerian *update* will do immediate work in deriving Heim's (1991) notion of *familiarity* for pronouns.

First, we need to cash out the notion of semantic presupposition in the current setting.

**Definition 2.2** (Semantic presupposition)**.** The semantic presupposition of a sentence $\phi$ is written as $\phi^\pi$:

$$\phi^\pi := \{ g \mid [\![\phi]\!]^g \text{ is defined} \}$$

Now, let's consider the interpretation of a sentence with a free pronoun/variable.[16]

$$\text{She}_1 \text{ left} \rightsquigarrow [\![Lx_1]\!]^g := \begin{cases} \{ g \} & g_1 \in I(L) \land g_1 \text{ is defined} \\ \varnothing & g_1 \notin I(L) \land g_1 \text{ is defined} \\ \# & \text{otherwise} \end{cases}$$

We can now clearly see that the semantic presupposition of "she$_1$ left" is the set of assignments which have 1 in their domain.

$$(L\ x_1)^\pi = \{ g \mid g_1 \text{ is defined} \}$$

According to the bridge principle, then, $c[L\ x_1]$ is only defined if, for every $g \in c$, $g_1$ is defined; this is equivalent to Heim's *familiarity* condition on definites.

[16] Since we're giving an indirect semantics in which pronouns are translated as variables, we can use these terms interchangeably.

More generally, we can say that a variable $x_1$ is *familiar* in a context $c$, if every assignment in $c$ is defined at 1. A sentence with a free pronoun/variable $x_1$ therefore presupposes that $x_1$ is familiar.

A straightforward result is that $\{ g_\varnothing \} [L\ x_1]$ is *undefined*.

To see how a familiarity presupposition can be *satisfied*, next we'll give a semantics for existentially-quantified sentences, but first a brief remark on accommodation.

In Stalkarian pragmatics, we talked about a process that minimally changes the context set, s.t., the presuppositions of a sentence are satisfied: *accommodation*.[17]

[17] As we discussed, there are several ways of conceiving of accommodation, but this will do for our purposes.

Can accommodation sweep in and rescue a case where the familiarity presupposition isn't satisfied?

(24) *Context: nothing has been said:*
# She$_1$ left.

It seems not. The sentences discussed under the rubric of the *formal link condition* make the same point.

This is because the speaker has no grounds on which to grow the context in any particular way in order to satisfy the familiarity presupposition. There are, however, certain factors which may allow us to do so, such as deixis.

(25) *Context: pointing at Susan*
She$_1$'s leaving.

The fact that *the familiarity presupposition* can't (easily) be accommodated arguably makes anaphora a more straightforward object of study than presupposition.

Dynamic semantics bets on anaphora and presupposition projection as involving the same mechanisms, so this might make us optimistic that anaphora can independently motivated the kinds of processes we posited in order to account for presupposition projection.

## *2.4 Semantics II: Existentially quantified sentences*

The semantics of existentials, along with conjunction, is at the heart of what makes DPL a *dynamic* theory of meaning, so pay close attention.

**Existentially quantified sentences**

In order to compute the value of an existentially quantified sentence relative to $g$, we take each individual $x$, and compute the value of the contained sentence relative to the modified assignment $g^{[1 \to x]}$, then we gather up the results.

$$[\![\exists x_1\ \phi]\!]^g := \bigcup_{x \in D} [\![\phi]\!]^{g^{[1 \to x]}}$$

In essence, existentials do two things in DPL:

- The guarantee satisfaction of familiarity by introducing Discourse Referents (DRS).
- They induce uncertainty regarding the identity of a DR.

In order to illustrate, let's go through a concrete example.

(26) Someone[1] left. $\exists x_1\ L\ x_1$

We'll assume a model with the following properties:

- $D = \{\ \text{paul}, \text{sophie}, \text{yasu}\ \}$
- $I(L) = \{\ \text{paul}, \text{sophie}\ \}$.

(27) Someone[1] left $\rightsquigarrow [\![\exists x_1\ L\ x_1]\!]^g = \bigcup_{x \in D} [\![L\ x_1]\!]^{g^{[1 \to x]}}$

For each individual $x$, we compute $[\![L\ x_1]\!]^{g^{[1 \to x]}}$, and then gather up the results.

(28) a. $[\![L\ x_1]\!]^{g^{[1 \to \text{paul}]}} = \{\ g^{[1 \to \text{paul}]}\ \}$ $(\text{paul} \in I(L))$
b. $[\![L\ x_1]\!]^{g^{[1 \to \text{sophie}]}} = \{\ g^{[1 \to \text{sophie}]}\ \}$ $(\text{sophie} \in I(L))$
c. $[\![L\ x_1]\!]^{g^{[1 \to \text{yasu}]}} = \varnothing$ $(\text{yasu} \notin I(L))$

(29) $$\begin{aligned}[\![\exists x_1\ L\ x_1]\!]^g &= \{\ g^{[1 \to x]} \mid x \in I(L) \wedge x \in D\ \} \\ &= \{\ g^{[1 \to \text{paul}]}, g^{[1 \to \text{sophie}]}\ \}\end{aligned}$$

**Existential quantifiers scopally commute**

It's important to define existential quantification in the way that we do to ensure that existential quantifiers scopally commute.

$$\exists x_1\ (\exists x_2\ \phi) \Leftrightarrow \exists x_2\ (\exists x_1\ \phi)$$

We'll illustrate with a concrete example.

(30) Someone[1] criticized someone[2]. $\exists x_1\ (\exists x_2\ C\ x_1\ x_2)$

Assume a model with the following properties:

- $D = \{\ \text{paul}, \text{sophie}, \text{yasu}\ \}$
- $I(C) = \{\ \langle\text{yasu}, \text{paul}\rangle, \langle\text{sophie}, \text{paul}\rangle\ \}$

(31) Someone[1] critized someone[2] $\rightsquigarrow [\![\exists x_1\ (\exists x_2\ C\ x_1\ x_2)]\!]^g = \bigcup_{x \in D} [\![\exists x_2\ C\ x_1\ x_2]\!]^{g^{[1 \to x]}}$

For each individual $x$, we compute $[\![\exists x_2\ C\ x_1\ x_2]\!]^{g^{[1 \to x]}}$:

(32) a. $[\![\exists x_2\ C\ x_1\ x_2]\!]^{g^{[1\to\text{paul}]}} = \{\ g^{[1\to\text{paul},2\to y]} \mid C\ p\ y\ \} = \varnothing$
b. $[\![\exists x_2\ C\ x_1\ x_2]\!]^{g^{[1\to\text{sophie}]}} = \{\ g^{[1\to\text{sophie},2\to y]} \mid C\ p\ y\ \} = \{\ g^{[1\to\text{sophie},2\to\text{paul}]}\ \}$
c. $[\![\exists x_2\ C\ x_1\ x_2]\!]^{g^{[1\to\text{yasu}]}} = \{\ g^{[1\to\text{yasu},2\to y]} \mid C\ p\ y\ \} = \{\ g^{[1\to\text{yasu},2\to\text{paul}]}\ \}$

We can compute the output state of the sentence by gathering these up:

(33) $[\![\exists x_1\ \exists x_2\ C\ x_1\ x_2]\!]^g = \{\ g^{[1\to x,2\to y]} \mid C\ x\ y\ \} = \{\ g^{[1\to\text{sophie},2\to\text{paul}]}, g^{[1\to\text{yasu},2\to\text{paul}]}\ \}$

**Indeterminacy**

Notice how this captures *relative certainty of reference*.

(34) $\{\ g_\varnothing\ \}\,[\exists x_1\ \exists x_2\ C\ x_1\ x_2] = \left\{ \begin{bmatrix} 1 & \mapsto\text{sophie} \\ 2 & \mapsto\text{paul} \end{bmatrix}, \begin{bmatrix} 1 & \mapsto\text{yasu} \\ 2 & \mapsto\text{paul} \end{bmatrix} \right\}$

An update by "someone[1] critized someone[2]" results in an information state, where it's *certain* that $x_2$ gets mapped to *Paul* (since Paul is the only person who has been critized), whereas it's *uncertain* whether $x_1$ gets mapped to *Sophie* or *Yasu*, since both critized Paul.

A Discourse Referent (DR) is just another way to describe a familiar variable.

We can distinguish between two kinds of DR:

- A DR $x_n$ is *determinate* in a context $c$ if every assignment in $c$ maps $x_n$ to the same individual.

- A DR $x_n$ is *indeterminate* in a context $c$ otherwise.

As soon as a DR has been introduced/rendered familiar, we can think of the goal of a discourse as making the DR determinate.

### 2.5 Pragmatics II: Satisfying familiarity

The semantics we've given for existentially quantified sentences *guarantees* that the familiarity presupposition of a subsequent, co-indexed free pronoun will be satisfied.

This is because, a sentence with a free pronoun $x_n$ presupposes that $x_n$ is defined throughout $c$, and an existentially quantified sentence $\exists x_n\ \phi$ *guarantees* that $x_n$ is defined throughout $c$ (as long as there is at least one verifier).

To consider why, let's give an illustration. Assume a model with the following properties:

- $D = \{\ \text{paul}, \text{sophie}, \text{yasu}\ \}$
- $I(C) = \{\ \langle \text{sophie}, \text{paul} \rangle, \langle \text{yasu}, \text{paul} \rangle\ \}$
- $I(F) = \{\ \text{paul}, \text{sophie}\ \}$.

Consider the following discourse.

(35) Someone$^1$ critized Paul. They$_1$ are French.

Updating the initial context with the first sentence results in the following output state:

(36) $\{\ g_\varnothing\ \}\,[\exists x_1\ C\ x_1\ p] = \{\ [1 \mapsto \text{sophie}], [1 \mapsto \text{yasu}]\ \}$

Updating the resulting context with "they$_1$ are French" filters out those assignments which don't map $x_1$ to a French person; it is guaranteed to be defined, since an existential statement guarantees satisfaction of familiarity.

(37) $\{\ [1 \mapsto \text{sophie}], [1 \mapsto \text{yasu}]\ \}\,[F\ x_1] = \{\ [1 \mapsto \text{sophie}]\ \}$

Note, furthermore, that the existential sentence introduces an *indeterminate* DR; the subsequent sentence with a co-indexed pronoun makes the DR *determinate* by supplying further information.

This should give us a clue as to how to define conjunction, and indeed the other connectives.

## 2.6 *Semantics III: Conjunction and Egli's theorem*

**Conjunctive sentences**

To compute the output set of a conjunctive sentence, we feed the outputs of the first conjunct into the second pointwise, and gather up the results.

$$[\![\phi \wedge \psi]\!]^g := \bigcup_{g' \in [\![\phi]\!]^g} [\![\psi]\!]^{g'}$$

Note: since either conjunct can be undefined, we want to make sure that the grand union operator in the meta-language projects undefinedness.

Interestingly, unlike in propositional update semantics, in dynamic semantics we're clearly distinguishing between the semantic value of a sentence and the *update* it induces.

Update is a *pragmatic* notion in this setting, so we don't define conjunction in terms of successive update.

This actually makes the system more restrictive.[18] In order to see why, consider a different direction for a dynamic semantics: we could've decided to define an interpretation function *relative to an information state* $c$: $[\![.]\!]^c$.[19]

[18] Unlike Veltman's update semantics, DPL is *distributive*.

[19] This is more like what Heim (1982) does; in general, a semantics which treats meanings as transitions from information states to information states is called an *update semantics*.

We would give a semantics for a sentence with a free variable in such a framework as follows. Note that the universal requirement imposed by the bridge principle must be built directly into the semantics.

(38) $$[\![L\ x_1]\!]^c := \begin{cases} \{\, g \mid g \in c, g_1 \in I(L) \,\} & \forall g' \in c[g'_1 \text{ is defined}] \\ \text{undefined} & \text{otherwise} \end{cases}$$

In DPL, this is factored out into the update rule.

**Egli's theorem**

Now that we've defined existential quantification and conjunction, we're in a position to illustrate arguably the core property of dynamic semantics: *Egli's theorem*.

This basically says that an existential in an initial conjunct can take scope over subsequent conjuncts.

**Egli's theorem**

$$\exists x_n\ \phi \wedge \psi \Leftrightarrow \exists x_n\ (\phi \wedge \psi)$$

Let's go through a concrete case.

(39) Someone[1] walked in and she$_1$ sat down. $\exists x_1\ W\ x_1 \wedge S\ x_1$

We compute the meaning of the conjunctive sentence by feeding the outputs of the first conjunct into the second, pointwise, and gathering up the results:

(40) a. $[\![\exists x_1\ W\ x_1 \wedge S\ x_1]\!]^g = \bigcup_{g' \in [\![\exists x_1\ W\ x_1]\!]^g} [\![S\ x_1]\!]^{g'}$

b. $= \{\, g'' \mid g'' \in [\![S\ x_1]\!]^{g'} \mid g' \in [\![\exists x_1\ W\ x_1]\!]^g \,\}$

c. $= \{\, g'' \mid g'' \in [\![S\ x_1]\!]^{g'} \mid g' \in \{\, g^{[1 \mapsto x]} \mid x \in D \wedge x \in I(W) \,\} \,\}$

d. $= \{\, g^{[1 \mapsto x]} \mid x \in I(S) \wedge x \in I(W) \wedge x \in D \,\}$

Now let's check equivalence according to Egli's theorem:

(41) Someone[1] who walked in sat down. $\exists x_1\ (W\ x_1 \wedge S\ x_1)$

(42) a. $[\![\exists x_1\ (W\ x_1 \wedge S\ x_1)]\!]^g = \bigcup_{x \in D} [\![W\ x_1 \wedge S\ x_1]\!]^{g^{[1 \mapsto x]}}$

b. $\bigcup_{x \in D} \left( \bigcup_{g' \in \{\, g^{[1 \mapsto x]} \mid x \in I(W) \,\}} \{\, g' \mid g'_1 \in I(S) \,\} \right)$

c. $\{\, g^{[1 \mapsto x]} \mid x \in I(W) \wedge x \in I(S) \wedge x \in D \,\}$

In line with Egli's theorem, the result is the same.

**Random assignment**

Now that we've defined conjunction, we'll see it's possible to get at the heart of DPL's treatment of existential quantification.[20]

[20] This section is based on van den Berg 1996: chapter 2.

In order to do so, we'll add an additional clause to our specification of the syntax of DPL for the *random assignment operator*:

- If $x_n \in \mathbb{V}$, then $\varepsilon x_n \in \mathbb{L}$.

Sentences of the form $\ulcorner \varepsilon x_n \urcorner$ will be interpreted via *random assignment*:

**Random assignment**

Random assignment induced by $\epsilon x_n$ introduces a completely indeterminate DR $x_n$.

$$[\![\varepsilon x_n]\!]^g := \{\, g^{[n \to x]} \mid x \in D \,\}$$

N.b. random assignment is *never false*; it is essentially a distinguished tautology that introduces a DR.

Random assignment doesn't add to the expressive power of DPL, it's simply a different way of defining existential quantification:

(43) $\exists x_n\ \phi \Leftrightarrow \epsilon x_n \wedge \phi$

Let's give a concrete illustration:

(44) Someone[1] left. $\exists x_1\ L\ x_1$

(45) $[\![\exists x_1\ L\ x_1]\!]^g = \{\, g^{[1 \to x]} \mid x \in I(L) \wedge x \in D \,\}$

(46) Someone[1] left. $\varepsilon x_n \wedge L\ x_1$

(47) a. $[\![\varepsilon x_n \wedge L\ x_1]\!]^g = \bigcup_{g' \in [\![\varepsilon x_1]\!]^g} [\![L\ x_1]\!]^{g'}$

b. $= \bigcup_{g' \in \{\, g^{[1 \to x]} \mid x \in D \,\}} [\![L\ x_1]\!]^{g'}$

c. $= \{\, g^{[1 \to x]} \mid x \in I(L) \wedge x \in D \,\}$

Now that we've defined random assignment, we can see that Egli's theorem is just associativity of conjunction:

**Egli's theorem (alt.)**

$$(\varepsilon x_n \wedge \phi) \wedge \psi \Leftrightarrow \varepsilon x_n \wedge (\phi \wedge \psi)$$

One advantage of random assignment is that it allows us to systematically divorce DR introduction from indefinites.

Once we give a semantics for equality statments, we can allow names to introduce DRS:

**Equality statements**

Equality statements are intepreted as tests:

$$[\![t_1 = t_2]\!]^g := \begin{cases} \{\, g \,\} & [\![t_1]\!]^g = [\![t_2]\!]^g \wedge [\![t_1]\!]^g, [\![t_2]\!]^g \text{ are defined} \\ \varnothing & [\![t_1]\!]^g \neq [\![t_2]\!]^g \wedge [\![t_1]\!]^g, [\![t_2]\!]^g \text{ are defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

We can take advantage of this to make sense of the idea that definites can introduce DRS.

(48) Susan[1] left. $\quad \varepsilon x_1 \wedge x_1 = s \wedge L\, x_1$

(49) a. $[\![\varepsilon x_1 \wedge x_1 = s \wedge L\, x_1]\!]^g = \{\, g^{[1\to x]} \mid x = \text{susan} \wedge x \in I(L) \wedge x \in D \,\}$
b. $= \{\, g^{[1\to \text{susan}]} \mid \text{susan} \in I(L) \,\}$

Note that definites introduce fully *determinate* DRS.

(50) Context: *Susan left*:
$\{\, g_\varnothing \,\}$ [Susan[1] left] $= \{\, [1 \mapsto \text{susan}] \,\}$

Why might this be desirable?

**Novelty**

Another amendment we might like to make is an account of Heim's (1991) *novely condition*,

The novelty condition is often taken to ban index re-use.[21] It captures disjointness effects:

(51) *Someone[1] walked in and someone[1] sat down.

Our definition for existential quantification/random assignment is *destructive*; $g^{[1\to x]}$ is implicitly assumed to be defined even if $g_1$ is defined.

In order to capture novelty in a way that percolates through our system, we can make a very small amendment, and simply assume that $g^{[1\to x]}$ is defined iff $g_1$ is *undefined*.

To illustrate, $[1 \mapsto \text{paul}]^{[1\mapsto \text{sophie}]}$ is *undefined*, but $[2 \mapsto \text{paul}]^{[1\mapsto \text{sophie}]}$ is defined.

This means that, e.g., *random assignment* carries a presupposition.

[21] Although, as we'll see when we discuss Stone disjunctions, it is actually distinct from such a principle.

**Random assignment (revised)**

Random assignment induced by $\epsilon x_n$ introduces a completely indeterminate DR $x_n$.

$$[\![\varepsilon x_n]\!]^g := \begin{cases} \{\, g^{[n\to x]} \mid x \in D \,\} & g_n \text{ is undefined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Via the bridge principle, an indefinite indexed $n$ presupposes that $n$ is undefined throughout $c$.

We don't need to make any special assumptions about indices in the syntax; we can simply assume that articles and pronouns are freely assigned indices; possible indexings are constrained by the need to satisfy novelty and familiarity.

## 2.7 *Semantics IV: Negation and accessibility*

If an indefinite can antecede a pronoun, we say that the indefinite is *accessible* to the pronoun.

It can be observed that negation renders antecedents *inaccessible* to subsequent pronouns.

**Case 1:** a subsequent pronoun *disambiguates* the scope of an indefinite:

(52) It's not true that some$^1$ philosopher is in the audience.
She$_1$'s waiting outside. ✗ $\neg > \exists$, ✓$\exists > \neg$

**Case 2:** negative indefinites don't license (singular) pronominal anaphora.

(53) No philosopher is in the audience. She's waiting outside.

**Case 3:** Negative Polarity Items (NPIS) don't license prononimal anaphora.

(54) It's not true that any$^1$ philosopher is in the audience.
# She$_1$'s waiting outside.

- *Case 1* will follow straightforwardly from the semantics of negation we'll propose here.
- *Cases 2 & 3* will follow if (i) we assume that negative indefinites are decomposes into a negative and existential component, and (ii) NPIS are simply existentials in a DE environment.

**Negated sentences**

$$[\![\neg\, \phi]\!]^g := \begin{cases} \{\, g \,\} & [\![\phi]\!]^g = \varnothing \\ \varnothing & [\![\phi]\!]^g \neq \varnothing \\ \text{undefined} & \text{otherwise} \end{cases}$$

**The idea:** when a sentence $\phi$ is negated, we *test* whether the output of $[\![\phi]\!]^g$ is the absurd information state. If it is, the test is passed, and we return the input assignment $\{\, g \,\}$; if the output set is non-empty, the test fails, and we return the absurd information state; otherwise, the result is undefined (thus ensuring the familiarity presupposition projects through negation).
N.b. the similarity to Veltman's *test semantics* for epistemic modals.

Another way to see what negation is doing involves definiting dynamic *truth* and *falsity*.

**Definition 2.3** (Truth and falsity)**.** We'll write $|.|^g$ for the classical truth-value of a sentence, defined as follows:

$$|\phi|^g = \begin{cases} 1 & [\![\phi]\!]^g \neq \varnothing \\ 0 & [\![\phi]\!]^g = \varnothing \\ \# & \text{otherwise} \end{cases}$$

A negated sentence tests the classical-truth value of the contained sentence.

More technically, a *test* in DPL is a sentence that always outputs either the singleton set of the input assignment, or the absurd information state.

Similarly, in update semantics a test was a sentence that always outputs either the input state or the absurd information state.

A disanalogy is that, in update semantics, tests can be sensitive to properties of an *information state*; in DPL, tests can only be sensitive to individual points (i.e., assignments).

Interestingly, tests were only useful in update semantics for analyzing modalized sentences; in DPL, tests are completely prevalent; based on what we've seen so far, only existential statements are non-tests.

**An illustration**

To see how negation renders an indefinite *inaccessible*, it will pay to go through a concrete example. Let's imagine that $D = \{\, \mathsf{pat} \,\}$, and $I(P) = \{\, \mathsf{pat} \,\}$, $I(A) =$

$\varnothing$ and $I(W) = \{\, \text{pat} \,\}$

(55) It's not true that some[1] philosopher is in the audience.
$\neg\, (\exists x_1\ (P\ x_1 \wedge A\ x_1))$
# She$_1$'s waiting outside.
$W\ x_1$

Let's begin by updating the initial context with the first sentence:

(56) $\{\, g_{\varnothing} \,\}\, [\neg\, (\exists x_1\ (P\ x_1 \wedge A\ x_1))]$

Since the input assignment is a singleton set, we can simply interpret the result of feeding $g_{\varnothing}$ in as the input.

(57) $$[\![\neg\, (\exists x_1\ (P\ x_1 \wedge A\ x_1))]\!]^{g_{\varnothing}} = \begin{cases} \{\, g_{\varnothing} \,\} & [\![\exists x_1\ (P\ x_1\ \wedge A\ x_1)]\!]^{g_{\varnothing}} = \varnothing \\ \varnothing & [\![\exists x_1\ (P\ x_1\ \wedge A\ x_1)]\!]^{g_{\varnothing}} \neq \varnothing \\ \text{undefined} & \text{otherwise} \end{cases}$$

If we compute the output state of the contained sentence with respect to the initial assignment, we can see that the result is the absurd information state.

(58) $[\![\exists x_1\ (P\ x_1\ \wedge A\ x_1)]\!]^{g_{\varnothing}} = \{\, g_{\varnothing}^{[1 \mapsto x]} \mid x \in I(P) \wedge x \in I(A) \wedge x \in D \,\} = \varnothing$

This means that the test imposed by negation is passed, which means that we just get back the ignorance context.

(59) $\{\, g_{\varnothing} \,\}\, [\neg\, (\exists x_1\ (P\ x_1 \wedge A\ x_1))] = \{\, g_{\varnothing} \,\}$

We don't need to bother to compute the result of asserting "she$_1$'s waiting outside"; it's clear that the familiarity presupposition of the second sentence won't be satisfied.

Despite the fact that its motivation is on firm footing, there are some well known problems with negation in DS involving *double negation* and *disjunction*. We'll come back to this later today.

## *2.8 Semantics V: Disjunction*

**Disjunction**

**Disjunctive sentences**

Disjunctive sentences are tests: we take the union of the output states of both disjuncts — if the result is non-empty, the test is passed; if the result is empty the test is failed (and the result is undefined otherwise).

$$[\![\phi \vee \psi]\!]^g \coloneqq \begin{cases} \{\, g \,\} & [\![\phi]\!]^g \cup [\![\psi]\!]^g \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$$

N.b. this predicts that disjunction, unlike conjunction is *internally static* — this means that an indefinite in an initial disjunct isn't accessible as a pronoun in a subsequent disjunct.

(60) ?Either a$^1$ philosopher is outside or she$_1$ is in the audience.

There's an immediate problem with this semantics for disjunction, it fails to account for *Stone disjunctions*.

(61) Either a$^1$ philosopher is outside or a$^1$ linguist is; she$_1$ is smoking.

In order to account for such cases, Groenendijk & Stokhof (1991) posit another possible entry for natural language disjunction: *program disjunction*:

**Program disjunction**

Program disjunction simply gathers up the outputs of the two disjuncts relative to the input assignment.

$$[\![\phi \veebar \psi]\!]^g \coloneqq [\![\phi]\!]^g \cup [\![\psi]\!]^g$$

Let's see how program disjunction accounts for Stone disjunctions in a concrete case.

(62) Either a$^1$ philosopher is outside or a$^1$ linguist is.
$\rightsquigarrow \exists x_1[P\ x_1 \wedge O\ x_1] \veebar \exists x_1[L\ x_1 \wedge O\ x_1]$

(63) a. $[\![\exists x_1[P\ x_1 \wedge O\ x_1] \veebar \exists x_1[L\ x_1 \wedge O\ x_1]]\!]^g$
b. $= \{\, g^{[1 \mapsto x]} \mid x \in I(P) \wedge x \in I(O) \wedge x \in D \,\}$
$\cup \{\, g^{[1 \mapsto y]} \mid y \in I(L) \wedge x \in I(O) \wedge x \in D \,\}$

(64) Context: *Chomsky, Foucault, and Lacan are outside smoking*
a. $\{\, g_\varnothing \,\}\,[\exists x_1[P\ x_1 \wedge O\ x_1] \veebar \exists x_1[L\ x_1 \wedge O\ x_1]]$
b. $= \{\, [1 \mapsto \textsf{lacan}], [1 \mapsto \textsf{foucault}] \,\} \cup \{\, [1 \mapsto \textsf{chomsky}] \,\}$
c. $= \{\, [1 \mapsto \textsf{lacan}], [1 \mapsto \textsf{foucault}], [1 \mapsto \textsf{chomsky}] \,\}$

There are a couple of interesting things to note about Stone disjunctions:

- In order to account for Stone disjunctions and subsequent anaphora using program disjunction, its crucial that the two indefinites — *a linguist* and *a philosopher* — bear the same index. This is nevertheless compatible with novelty, since both indefinites are interpreted relative to the same input assignment.[22]
- If the indefinites were contra-indexed, the familiarity presupposition of a subsequent co-indexed pronoun would fail to be satisfied.

[22] This account of Stone disjunctions is however incompatible with an account of novelty that bans index re-use with indefinites, such as the account in Heim 1982.

Essentially, we end up treating Stone disjunctions as existential statements with a complex restrictor.

Note that this correctly predicts that negation renders the Stone disjunction inaccessible for a subsequent pronoun:

(65) *Neither a$^1$ philosopher nor a$^1$ linguist is outside. He$_1$'s inside smoking.

There are some interesting open questions regarding disjunction, e.g.:

- Under what conditions does natural language *or* express DPL disjunction vs. Stone disjunction?

As we'll see later today, there is still a problem: neither of these entries capture *Partee disjunctions*.

## 3 *Donkey sentences and Egli's corollary*

Remember, one of the central empirical motivations for dynamic semantics was an account of *donkey sentences*.

(66) If Sarah sees a$^1$ corgi, she pets it$_1$.

(67) Everyone who sees a$^1$ corgi pets it$_1$.

In order to get there, we'll first need to give a semantics for material implication, and universal quantification.

**Implication**

**Implicational sentences**

Implicational sentences are tests: we check whether each assignment in the output state of the antecedent makes the consequent (dynamically) true. If so, the test is passed; if there is however some assignment in the output state which makes the consequent (dynamically) false, the test is failed (and the result is undefined otherwise).

$$[\![\phi \rightarrow \psi]\!]^g := \begin{cases} \{\, g \,\} & \forall g'[g' \in [\![\phi]\!]^g \rightarrow ([\![\psi]\!]^{g'} \neq \varnothing)] \\ \varnothing & \text{otherwise} \end{cases}$$

This entry for implication is *externally static* (since it's a test), but *internally dynamic*, since assignments in the output state of the antecedent are passed in as the input of the consequent.

We can see that external staticity is desirable; anaphora is only possible from out of a conditional if the existential scopes out.

(68) If a[1] philosopher is in the audience, I won't talk.
She$_1$ asks annoying questions. ✓ ∃ > if, ✗ if > ∃

(69) If I talk for too long, a[1] philosopher will ask an annoying question.
She$_1$'s in the audience. ✓∃ > if, ✗ if > ∃

This is even easier to see via an NPI:

(70) If any[1] philosopher is in the audience, I won't talk.
\# She$_1$ asks annoying questions.

**Donkey sentences**

Let's illustrate how the semantics of implication, in tandem with existentials, accounts for donkey sentences.

(71) If Sarah sees a corgi, she pets it.
$(\exists x_1 C\ x_1) \rightarrow P\ x_1$[23]

(72) $[\![(\exists x_1\ C\ x_1) \rightarrow P\ x_1]\!]^g = \begin{cases} \{\, g \,\} & \forall g'[g' \in [\![\exists x_1\ C\ x_1]\!]^g \rightarrow ([\![P\ x_1]\!]^{g'} \neq \varnothing)] \\ \varnothing & \text{otherwise} \end{cases}$

Since we're assuming the excluded middle for predicates, and there are no free pronouns, the result will always be defined.

(73) $= \begin{cases} \{\, g \,\} & \forall g'[g' \in \{\, g^{[1 \rightarrow x]} \mid x \in I(C) \wedge x \in D \,\} \rightarrow (g'_1 \in I(P))] \\ \varnothing & \text{otherwise} \end{cases}$

[23] To simplify the computation, we'll treat *corgi seen by Sarah* ($C$), and *petted by Sarah* ($P$) as syntactically simplex predicates. This is a harmless idealization.

(74) $$= \begin{cases} \{ g \} & \forall x[x \in I(C) \to x \in I(P)] \\ \varnothing & \text{otherwise} \end{cases}$$

Note that our theory predicts *strong, universal* truth-conditions for donkey sentences; i.e., for (71) to be true, Sarah must pet every corgi that she sees.

This is most certainly a good prediction in this instance, although as we'll see later today, this semantics will lead to problems further down the line.

We can give an alternative rendering of the semantics for implicational sentences in terms of a subsethood relation between the output of the antecedent, and the assignments that make the consequent dynamically true:

**Implicational sentences (alt)**

$$[\![\phi \to \psi]\!]^g := \begin{cases} \{ g \} & \{ g' \mid g' \in [\![\phi]\!]^g \} \subseteq \{ g'' \mid \exists h[h \in [\![\psi]\!]^{g''} ] \} \\ \varnothing & \text{otherwise} \end{cases}$$

We'll informally demonstrate here how this alternative definition works.

(75) If Sarah sees a[1] corgi, she pets it$_1$.
$(\exists x_1\ C\ x_1) \to P\ x_1$

- To see whether the test is passed, we first compute the output set of the antecedent.
- $[\![\exists x_1\ C\ x_1]\!]^g = \{ g^{[1 \to x]} \mid x \in I(C) \wedge x \in D \}$
- If the input assignment is $g_\varnothing$, this might be, e.g., the set of assignments mapping 1 to corgis Sarah saw: $\{ [1 \to b], [1 \to c] \}$
- Next, we compute the set of assignments which make the consequent dynamically true:
- $\{ g'' \mid \exists h[h \in [\![P\ x_1]\!]^{g''} ] \} = \{ g'' \mid g''_1 \neq \# \wedge g''_1 \in I(P) \}$
- This is simply any assignment $g''$ defined at 1, s.t. $g''_1$ was petted by Sarah.
- For the former set to be a subset of the latter, each of the corgis Sarah saw must be petted by her.

**Universal quantification**

**Universally quantified sentences**

A universally quantified sentence is a *test* (in fact, it can be defined as the dual of the existential quantifier). It checks, for each individual $x$, if the contained sentence interpreted relative to $g^{[n \rightarrow x]}$ returns a non-empty set. If so, the test is passed.

$$[\![\forall x_n\ \phi]\!]^g := \begin{cases} \{\, g \,\} & \forall x[[\![\phi]\!]^{g^{[n \rightarrow x]}} \neq \varnothing] \\ \varnothing & \text{otherwise} \end{cases}$$

Like we did with implicational sentences, we can give an alternative formulation of the semantics of universal sentences in terms of subsethood:

**Universally quantified sentences (alt.)**

$$[\![\forall x_n\ \phi]\!]^g := \begin{cases} \{\, g \,\} & \{\, g^{[n \mapsto x]} \mid x \in D \,\} \subseteq \{\, g' \mid \exists h[h \in [\![\phi]\!]^{g'}\,] \,\} \\ \varnothing & \text{otherwise} \end{cases}$$

We can simplify this even further, using the definition of random assignment:

$$[\![\forall x_n\ \phi]\!]^g := \begin{cases} \{\, g \,\} & [\![\varepsilon x_n]\!]^g \subseteq \{\, g' \mid \exists h[h \in [\![\phi]\!]^{g'}\,] \,\} \\ \varnothing & \text{otherwise} \end{cases}$$

Let's see informally how this works:

(76) Everyone[1] who sees a corgi pets it$_1$.
$\forall x_1[(\varepsilon x_2 \wedge C\ x_2 \wedge S\ x_1\ x_2) \rightarrow P\ x_1 x_2]$

- To see whether the test is passed, we first compute the result of doing random assignment:
- $[\![\varepsilon_1]\!]^g = \{\, g^{[1 \mapsto x]} \mid x \in D \,\}$
- We now compute the set of assignments which make the contained sentence dynamically true:
- $\{\, g'' \mid \exists h[h \in [\![(\varepsilon x_2 \wedge C\ x_2 \wedge S\ x_1\ x_2) \rightarrow P\ x_1 x_2]\!]^{g''}\,] \,\}$
- This is all those assignments $g''$ that are defined at 1, such that $g''_1$ petted every corgi that $g''_1$ saw (from the semantics of implicational sentences).
- For the former set to be a subset of the latter, everyone in the domain must be s.t. they petted every corgi that they saw.

**Egli's corollary**

$$(\exists x_1\ \phi) \to \psi \Leftrightarrow \forall x_1\ (\phi \to \psi)$$

In terms of random assignment:

$$(\varepsilon x_1 \wedge \phi) \to \psi \Leftrightarrow \forall x_1(\phi \to \psi)$$

From Egli's corrolary our Corgi sentence is equivalent to universally quantifying over people and corgis:

(77) $\forall x_1[(\varepsilon x_2 \wedge C\ x_2 \wedge S\ x_1\ x_2) \to P\ x_1 x_2]$
$\Leftrightarrow \forall x_1 \forall x_2[(C\ x_2 \wedge S\ x_1\ x_2) \to P\ x_1\ x_2]$

## 4 *Problems, prospects, and extensions*

### 4.1 *Double negation and bathroom sentences*

**Double negation**

In DS, negation is a *destructive* operation; it obliterates any DRS in its scope since, the output state of the contained sentence is, essentially, closed.

This makes a pretty strong prediction; *double negation elimination* should *not* be valid, unlike in a classic setting.

We can illustrate this be giving a concrete example:

(78) It's not true that nobody left. $\neg\,(\neg\,\exists x_1\ L\ x_1)$

Let's compute the meaning of the sentence in DS:[24]

[24] As usual, we ignore undefinedness since there are no free variables.

(79) $[\![\neg\,(\neg\,\exists x_1\ L\ x_1)]\!]^g = \begin{cases} \{\,g\,\} & [\![\neg\,\exists x_1\ L\ x_1]\!]^g = \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

(80) $= \begin{cases} \{\,g\,\} & [\![\exists x_1\ L\ x_1]\!]^g \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

(81) $= \begin{cases} \{\,g\,\} & \{\,g^{[1\to x]} \mid x \in I(L) \wedge x \in D\,\} \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

$$(82) \quad = \begin{cases} \{ g \} & \exists x[x \in D \wedge x \in I(L)] \\ \varnothing & \text{otherwise} \end{cases}$$

If we just take the conditions under which the doubly negated sentence is *true*, then this is equivalent to the conditions under which its positive counterpart are true; namely, just so long as $I(L) \neq \varnothing$:

(83) $[\![\exists x_1 \; L \; x_1]\!]^g = \{ g^{[1 \to x]} \mid x \in I(L) \wedge x \in D \}$

However, if we compare (82) and (83), we can see that the output states are *not* the same; the doubly-negated sentence is a *test*, whereas its positive counterpart introduces $x_1$ as a DR.

It was already noted by Groenendijk & Stokhof that this is a problem, and indeed it seems to make bad predictions for anaphora.

(84) It's not true that NO philosopher registered; she$_1$'s sitting at the back.

Anaphora from doubly-negated sentences seems to be subject to poorly understood constraints; Gotham (2019) (see also Krahmer & Muskens 1995) claims that there is an associated uniqueness inference.[25]

[25] The following examples are based on Gotham 2019.

(85) *Context: The speaker knows that John owns more than one shirt.*
a. John owns a$^1$ shirt. It$_1$'s in the wardrobe.
b. ?? It's not true that John DOESN'T own a$^1$ shirt; It$_1$'s in the wardrobe!

As I note in Elliott 2020, anaphora from under double negation is compatible with a plural pronoun, just so long as it picks up a *maximal* DR. This is a puzzle for uniqueness.

(86) John doesn't own no$^1$ shirt. They$_1$'re in the wardrobe.

The validity of Double Negation Elimination (DNE) with respect to anaphora might be taken to show that DPL strays too far from the classical; if Gotham is correct however, we might not want to reinstate $\neg(\neg \phi) \Leftrightarrow \phi$ wholesale.

**Bathroom sentences**

There's a related problem with involving disjunctive sentence.

First, think back to the Heim/Karttunen projection generalization for disjunctive sentences.

(87) Either there is no bathroom, or the bathroom is upstairs.

(87) is presuppositionless, because the presupposition of the second disjunct (that *there is a bathroom*), is locally satisfied; in update semantics, a subsequent disjunct is interpreted relative to the *negation* of the initial disjunct.

We can make a completely parallel observation with anaphora.

(88) Either there is no$^1$ bathroom, or it$_1$'s upstairs.

The entry for disjunction we've given here, based on Groenendijk & Stokhof (1991), is both externally *and* internally dynamic, so it has no chance at all of accounting for the possibility of anaphora in (88).

Our entry for disjunction predicts that the test imposed by (88) is passed if the union of output states of the first and second disjuncts is non-empty; the disjunctive sentence should therefore inherit the definedness conditions of *it's upstairs*, which contains a free variable.

An intuitive thought is that a subsequent disjunct is interpreted in the context of the *negation* of the first, just like in our update semantic entry for disjunction (Beaver 2001), so the problem of (88) is reduced to accounting for anaphora in the following:

(89) Either there is no$^1$ bathroom,
or (there isn't no$^1$ bathroom and) it$_1$'s upstairs.

This, naturally, reduces the problem of anaphora in bathroom sentences to the problem of DNE more generally.

Similarly, Gotham (2019) claims that anaphora in bathroom sentences comes with an associated uniqueness inference.

(90) *Context: the speaker knows that, if John owns any shirts, he owns more than one.*
\# Either John has no$^1$ shirt, or it$_1$'s in the wardrobe.

This receives a natural explanation, if the account of anaphora in bathroom sentences is parasitic on the account of DNE.

The data is somewhat unclear however; Krahmer & Muskens (1995) develop an account of bathroom sentences that ascribes them universal truth-conditions, just like donkey sentences; for them, (88) is true just so long as there is no bathroom that *isn't* upstairs (there may be multiple bathrooms).

In essence, the idea is to reduce the bathroom sentence to the following paraphrase:

(91) Either there is no$^1$ bathroom, or (if there isn't no$^1$ bathroom, then) it$_1$'s upstairs.

There are a number of accounts of DNE and bathroom sentences in the literature which depart to a lesser or greater extent from DPL: see Rothschild 2017 and Mandelkern 2020b,a for a significant departure, and Krahmer & Muskens 1995, Gotham 2019, Elliott 2020 for accounts which more closely toe the line.[26]

[26] For anyone who's interested in this problem (I think it would make an *excellent* squib topic), there's an extremely useful discussion in van den Berg (1996: ch. 2).

## *4.2 Generalized quantifiers and the proportion problem*

### DPL with quantifiers

DPL is quite limited in its expressive power — we're not in a position to analyze the broader range of environments in which donkey anaphora is possible.

(92) Most people who see a$^1$ corgi pet it$_1$.

(93) Few people who see a$^1$ corgi pet it$_1$.

(94) Usually, a person who sees a$^1$ corgi pets it$_1$.

In order to account for determiners, we need to go beyond DPL. We can minimally extend the syntax of $\mathbb{L}$, in the following way:

- Let $\mathbb{Q}$ be a non-empty set of *determiners*.
- If $Q \in \mathbb{Q}$, $x_n \in V$, $\phi, \psi \in \mathbb{L}$, then $Qx_n\ \phi\ \psi \in \mathbb{L}$.

Note that we're treating determiners as (two-place) *sentential* operators, which come with a binding index.

### Semantics for quantified sentences

N.b. the semantics that we'll give for quantified sentences is called an *unselective* semantics, for reasons which will become clear.

Let's assume that the valuation function $I$ maps determiners to conservative binary relations between sets of individuals (i.e., generalized quantifiers).

Remember our semantics for universal sentences, in terms of subsethood? This will help us give a general recipe for quantified statements:

**Unselective semantics for quantified sentences**

To compute the output of a quantified sentence $Q_n\ \phi\ \psi$, we must compute two sets: (i) the *restrictor set* is the outputs of the restrictor $\phi$ interpreted in the context of $n$-indexed random assignment. (ii) the *matrix set* is the set of inputs that make the matrix sentence $\psi$ dynamically true. The quantified sentence $Q_n\ \phi\ \psi$ is a test that checks whether a set-theoretic relation delivered by the valuation function holds between these two sets.

$$[\![Qx_n\ \phi\ \psi]\!]^g$$
$$:= \begin{cases} \{\, g \,\} & [\![\varepsilon x_n \wedge \phi]\!]^g\ I(Q)\ \{\, g' \mid \exists h[h \in [\![\psi]\!]^{g'}\,]\,\} \\ \varnothing & \text{otherwise} \end{cases}$$

We can check that this makes the right predictions for donkey anaphora in universal statements.

(95) Everyone who sees a[1] corgi pets it$_1$

We'll translate this into a quantified sentence as follows:[27]

(96) $\mathbf{every}_1\ (\varepsilon x_2 \wedge\ S\ x_1\ x_2)\ (P\ x_1\ x_2)$

(97) $[\![\mathbf{every}_1\ (\varepsilon x_2 \wedge\ S\ x_1\ x_2)\ (P\ x_1\ x_2)]\!]^g$
$$= \begin{cases} \{\, g \,\} & [\![\varepsilon x_1 \wedge \varepsilon x_2 \wedge S\ x_1\ x_2]\!]^g \subseteq \{\, g' \mid \exists h[h \in [\![P\ x_1\ x_2]\!]^{g'}\,]\,\} \\ \varnothing & \text{otherwise} \end{cases}$$

In order to see if the test is passed, we first compute the restrictor set — this gives back the set of modified assignments $g^{[1\mapsto x, 2\mapsto y]}$, such that $x$ saw $y$.

(98) $[\![\varepsilon x_1 \wedge \varepsilon x_2 \wedge S\ x_1 x_2]\!]^g = \{\, g^{[1\mapsto x, 2\mapsto y]} \mid \langle x, y\rangle \in I(S)\,\}$

Now we compute the matrix set — this gives back the set of assignments $g'$ defined for $1, 2$, such that $g'_1$ petted $g'_2$.

(99) $\{\, g' \mid \exists h[h \in [\![P\ x_1\ x_2]\!]]\,\} = \{\, g' \mid g' \neq \#, \langle g'_1, g'_2\rangle \in I(P)\,\}$

In order for the restrictor set to be a subset of the matrix set, it must be the case that each modified assignment $g^{[1\mapsto x, 1\mapsto y]}$ in the restrictor set is s.t. $x$ petted $y$; if this does not hold for some assignment in the restrictor set, then the subsethood relation fails to hold.

This elegant semantics for quantified sentences is essentially the semantics given for adverbs of quantification in Groenendijk & Stokhof 1991 and (implic-

[27] As discussed by Heim (1982), these logical forms can be constructed compositionally by scoping out the *determiner*.

itly) in Heim 1982, but it runs into two well-known problems: the *proportion problem* and the distinction between *weak and strong readings*.

**The proportion problem**

Now, let's consider what happens when we combine donkey anaphora with the determiner *most*:[28]

[28] We assume here that *most* means *more than half*, although this is of course a simplification.

(100) Most people who see a corgi pet it. $\rightsquigarrow$
$\mathbf{most}_1\ (\varepsilon x_2 \wedge C\ x_2 \wedge S\ x_1\ x_2)\ (P\ x_1\ x_2)$

Let's compute the restrictor set relative to an input $g$, and the matrix set as usual:

(101) Restrictor set relative to $g$:
$\{\ g^{[1 \mapsto x, 2 \mapsto y]} \mid y \in I(C) \wedge \langle x, y \rangle \in I(S)\ \}$

(102) Matrix set:
$\{\ g' \mid g'_1, g'_2 \neq \# \wedge \langle g'_1, g'_2 \rangle \in I(P)\ \}$

For the test imposed by the quantified sentence to be successful, more than half $\langle x, y \rangle$ pairs, s.t. $y$ is a corgi and $x$ sees $y$, should be such that $x$ pets $y$.

As many have remarked[29], it's easy to come up with scenarios to demonstrate that this gets the truth-conditions of the English sentence wrong.

[29] See, e.g., Partee 1984, Kadmon 1987, Rooth 1987, and Heim 1990.

Let's say that three people — Sarah, Josie, and Alex — saw corgis:

- Sarah went to a dog park, and saw 10 corgis ($c_1, \ldots, c_{10}$) — she petted all of them.
- Josie and Alex each saw one corgi ($c_1$ and $c_2$ respectively), but didn't pet them.

We can list all the $\langle x, y \rangle$ pairs such that $y$ is a corgi, and $x$ saw $y$. I've highlighted those pairs also in a petting relationship:

$$\left\{ \begin{array}{c} \langle j, c_1 \rangle, \langle a, c_2 \rangle, \\ \underline{\langle s, c_1 \rangle, \langle s, c_2 \rangle, \langle s, c_3 \rangle, \langle s, c_4 \rangle, \langle s, c_5 \rangle,} \\ \underline{\langle s, c_6 \rangle, \langle s, c_7 \rangle, \langle s, c_8 \rangle, \langle s, c_9 \rangle, \langle s, c_{10} \rangle,} \end{array} \right\}$$

It's pretty clear then, that our truth conditions predict that the sentence should be true, but it's intuitively false in this scenario.

The problem amounts to the fact that we end up quantifying over *person-corgi pairs*, rather than individuals.

**Weak vs. strong donkeys**

Consider the classic donkey sentence below — our entry for first-order universal quantification, and also *every* as a generalized quantifier predict it to have strong, universal truth conditions.[30]

[30] Apologies for the animal cruelty; I regrettably need to use this example to repeat Chierchia's reasoning.

(103) Every[1] farmer who owns a[2] donkey beats it$_2$.
$\rightsquigarrow$ $\textbf{every}_1\ (\varepsilon x_2 \wedge D\ x_2\ \wedge F\ x_1 \wedge O\ x_1\ x_2)\ (B\ x_1\ x_2)$

Concretely, we predict this to be true iff each farmer is s.t. they beat each donkey that they own.

However, donkey sentences can receive a so-called "weak" reading too. Consider the following context from Chierchia 1995:

*The farmers under discussion are all part of an anger management program, and they are encouraged by the psychotherapist involved to channel their aggressiveness towards their donkeys (should they own any) rather than towards each other. The farmers scrupulously follow the psychotherapist's advice.*

(104) ...every farmer who owns a donkey beats it.

In the context, this is true just so long as each farmer is s.t. they beat some donkey that they own.

Even more convincingly, there are donkey sentences for which the weak reading is the most salient:

(105) Every person who has a[11] dime will put it$_1$ in the meter.

(106) Yesterday, every person who had a[1] credit card paid his bill with it$_1$.

The unselective analysis can't account for this reading.

The solution to both of these problems involves formulating a *selective* semantics for generalized quantifiers that relates sets of individuals rather than information states.

We won't go through how to do this in class (although if there's a demand for it, I can discuss how to solve this problem next week), but see, e.g., Chierchia 1995 and Kanazawa 1994.[31]

[31] Probably the simplest way of doing this is by formulating an object language abstraction operator $\lambda_n$, and defining dynamic GQs as relating functions from individuals to dynamic propositions, e.g.

(107) Most farmers who own a[2] donkey beat it$_2$.
$\textbf{most}\ (\lambda_1\ (\varepsilon x_2 \wedge D\ x_2 \wedge F\ x_1\ \wedge O\ x_1\ x_2))$
$(\lambda_3\ (B\ x_3\ x_2))$

**Next time**

Next week I'll discuss dynamic treatments of anaphora and *explanatory adequacy* — reading is Mandelkern 2020b.

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