The dynamic turn I: Update semantics

Patrick D. Elliott & Danny Fox

September 25, 2020

Homework

- If you haven't already, please submit your solutions to *last week's p-set*.
- You can find this week's p-set in the appendix of this handout. Please submit your work **before next week's class**. Unregistered students are also encouraged to attempt the p-set, especially since this will feed into our discussion next week.
- Please email this week's p-set directly to me @ `pdell@mit.edu`.

Readings

This will depend on how much progress we make today. I'll send out an announcement after class.

1 *Looking ahead*

The dynamic turn pt 1 propositional update semantics, and its applications to presupposition projection (Heim 1983), and epistemic modality (Veltman 1996, Groenendijk, Stokhof & Veltman 1996).

Explanatory approaches to projection George's (2007, 2008, 2014) *middle Kleene* — a more explanatory theory of presupposition projection and Fox (2013) on quantificational sentences.

Anaphora Anaphora in the dynamic tradition (Heim 1982, Groenendijk & Stokhof 1991, Dekker 1994). and explanatory theories of anaphora (Rothschild 2017, Elliott 2020a,b and Mandelkern 2020b,a) (n.b. very tentative!).

2 Recap: trivalent semantics and Stalnaker's bridge

The account of presupposition Danny has been outlining has the following basic ingredients.

- A "gappy" semantics for sentences; this gives rise to the notion of the *semantic presupposition* of a sentence.
- A notion of *assertion*, which tells us how to update a context c with the information conveyed by a sentence ϕ.
- The conditions under which update of a context c with ϕ is defined (*Stalnaker's bridge*).

2.1 Trivalence

Given a non-empty *set of possible worlds* W, a sentential meaning is a function $p : W \mapsto \{ 1, 0, \# \}$. Here's a simple example:

(1) $$[\![\text{Sarah's corgi is sleepy}]\!] = \begin{cases} 1 & \text{Sarah has a corgi \& Sarah's corgi is sleepy} \\ 0 & \text{Sarah has a corgi \& Sarah's corgi isn't sleepy} \\ \# & \text{otherwise} \end{cases}$$

In trivalent semantics, the *semantic presupposition of a sentence S* is the set of worlds w, such that $[\![S]\!]\ w$ is either true or false.[1]

Definition 2.1 (Semantic presupposition).

$$\phi^{\pi} \coloneqq \{ w \mid ([\![\phi]\!]\ w = 1) \vee ([\![\phi]\!]\ w = 0) \}$$

(2) $(\text{Sarah's corgi is sleepy})^{\pi} = \{ w \mid \text{Sarah has a corgi in } w \}$

2.2 Update and Stalnaker's bridge

Definition 2.2 (Stalnakerian update). The *update* induced by a a sentence ϕ on a context c is a partial function $c[\phi] : \mathscr{P}(W) \mapsto \mathscr{P}(W)$, defined as follows:[2]

$$c[\phi] \coloneqq \begin{cases} \{ w \mid w \in c \wedge [\![\phi]\!]\ w \} & c \subseteq \phi^{\pi} \\ \text{undefined} & \text{otherwise} \end{cases}$$

We say that the presuppositions of a sentence ϕ are *satisfied* with respect to a context c if $c[\phi]$ is defined.

[1] At the end of the previous handout, Danny alludes to the possibility of a principled trivalent semantics for generalized quantifiers and other expressions. We'll return to questions of projection in the sub-sentential domain once we move on to discuss explanatory approaches to presupposition projection in the following weeks.

Since we'll begin our discussion of dynamic semantics by introducing a semantics for a toy propositional fragment, questions of sub-sentential compositionality will be put to one side in this week's material.

[2] Note, this formulation of bridge is different to the one that Danny introduced — but equivalent. By defining the *semantic presupposition* of a sentence, we can describe Stalnaker's bridge as the requirement that the context c *entails* the semantic presupposition of ϕ. Since entailment in possible world semantics amounts to subsethood, this is equivalent to the requirement that ϕ is true or false (never #) at every world in c.

The advantage of this definition is that we can maintain Stalnaker's bridge irregardless of our assumptions concerning what kind of semantic objects *semantic presuppositions* and *contexts* are, just so long as we have a derivative notion of entailment.

Stalnakerian pragmatics allows us to define some potentially interesting notions, although their application will be limited to the level of the discourse. For example, *redundancy*.

Definition 2.3 (Redundancy)**.** A sentence ϕ is *redundant* with respect to a context c if (i) $c[\phi]$ is *defined* (i.e., $c \subseteq \phi^\pi$), and (ii) $c \subseteq \{\, w \mid [\![\phi]\!]\ w = 1 \,\}$.

I.e., ϕ is redundant in c if ϕ's presupposition is satisfied in c, and c entails the assertive content of ϕ.

(3) a. It's raining. (In fact) it's raining heavily.
  b. It's raining heavily. # (In fact) it's raining.

One thing that should already give us pause is that this account of *redundancy* doesn't straightforwardly generalize to conjunctive sentences (why?).

(4) a. It's raining and (in fact) it's raining heavily.
  b. # It's raining heavily and (in fact) it's raining.

2.3 Successive update

A (trivial?) observation: updating c with a sentence ϕ can make the presupposition of a sentence ψ redundant, thus ensuring that $c[\psi]$ is guaranteed to be defined.

(5) Sarah has a corgi. Sarah's corgi is sleepy.

Stalnakerian pragmatics directly captures this, since successive assertions, if accepted, induce successive update of the common ground.

We can write a successive update of c with ϕ followed by ϕ' as $c[\phi][\psi]$.

$$c[\phi][\psi] \coloneqq (c[\phi])[\psi]$$

Note that in the following example Stalnaker's bridge is trivially satisfied here, since $P \cap Q \subseteq Q$. More generally, if $Q' \subseteq Q$, then $P \cap Q' \subseteq Q$.

$$c[\text{Sarah has a corgi}] = \overbrace{\{\, w \mid w \in c \wedge \text{Sarah has a corgi in } w \,\}}^{c'}$$

$$(\text{Sarah's corgi is sleepy})^\pi = c'$$

$$c'[\text{Sarah's corgi is sleepy}] = \begin{cases} \{\, w \mid w \in c' \wedge \text{Sarah's corgi is sleepy in } w \,\} & c \cap c' \subseteq c' \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$c'[\text{Sarah's corgi is sleepy}] = \{\, w \mid w \in c' \wedge \text{Sarah's corgi is sleep in } w \,\}$$

Now, note that the conjunctive sentence has the same projection properties, and importantly exhibits *linear asymmetries*:

(6) a. Sarah has a corgi and her corgi is sleepy.
 b. # Sarah's corgi is sleepy and she has a corgi.

(7) a. Sarah has a corgi. Her corgi is sleepy.
 b. # Her corgi is sleepy. Sarah has a corgi.

In dynamic semantics, we'll take this intuition seriously — the *meaning* of $\phi \wedge \psi$ will be successive assertion of ϕ and ψ, in that order.

In our first propostitional dynamic fragment, we'll extend this notion to the other connectives.

First, a historical note...

3 *Dynamic semantics*

3.1 *Empirical motivations for dynamic semantics*

Historically, dynamic semantics — independently developed by Irene Heim (1982) and Hans Kamp (1981) — was motivated by *anaphora to singular indefinites*.

Briefly, pronouns can co-vary with indefinites in preceding sentences:

(8) a. A^1 man walked in. He_1 sat down.
 b. # He_1 walked in. A^1 man sat down.

More generally, pronouns can co-vary with indefinites, even when not in their scope (*donkey pronouns*).

(9) Everyone [who bought a^1 new puppy during the pandemic] treasured it_1.

Both discourse and donkey anaphora are sensitive to the form of preceding sentences; not just classical content (the famous *marble* example is due to Barbara Partee).

(10) a. #I've found nine out of my ten marbles. It_1's under the couch.
cf. One^1 out of my 10 marbles is lost. It_1's under the couch.

b. *Josie is married. He_1's annoying.
cf. Josie has a^1 husband. He_1's annoying.

We'll discuss the dynamic approach to anaphora invented by Heim (1982) and subsequently put on firm logical foundations by Groenendijk & Stokhof (1991) in a couple of weeks time. This will involve adopting a slightly richer notion of *information states* than sets of possible worlds.

Broadly, there are two approaches to dynamics, distinguished by Yalcin (2013) as follows:

Dynamic representation Sentences denote instructions for updating a particular kind of representation, which is subsequently interpreted compositionally (the DRT tradition; Kamp 1981).

Dynamic interpretation The compositional value of a sentence is an instruction for updating a body of information (the Heimian tradition; Heim 1982).

In this class, I'll take the latter view as a given. As shown by Groenendijk & Stokhof (1991), the Heimian approach is in fact equivalent to the DRT approach, and "cuts out the middle man", so to speak. It also makes the relationship to Stalnakerian pragmatics clearer.

Subsequently, the remit of dynamic semantics was expanded to encompass theories of an extremely broad range of phenomena, including amongst others:

- Presupposition projection (Heim 1983, Beaver 2001, a.o.).
- Epistemic modality (Veltman 1996, Groenendijk, Stokhof & Veltman 1996, a.o.).
- Intervention effects (Honcoop 1998, a.o.).
- Conditionals (Gillies 2004, a.o.).
- Generalized quantifiers and discourse plurals (van den Berg 1996, a.o.)
- Scalar implicature (Sudo 2019, a.o.).
- Weak crossover (Chierchia 2020, Elliott 2020a).

- etc.

We'll start with presupposition projection, and subsequently, epistemic modality, since these topics only require reference to the simplest version of dynamic semantics — update semantics on a simple propositional calculus — and the Stalnakerian notion of content that we're already familiar with.

3.2 Towards an update semantics

Successive assertion patterns with *conjunction* wrt presupposition projection (Danny's handout from last week; Karttunen's generalization).

A natural way of cashing this out: a conjunctive sentence, in some sense, *is* a successive assertion.

(11) Conjunctive sentences in update semantics (def.)
$c[\phi \text{ and } \psi] \coloneqq c[\phi][\psi]$

We'll formalize this idea and extend it to the other connectives here.

> "The slogan 'You know the meaning of a sentence if you know the conditions under which it is true', is replaced by this one: 'You know the meaning of a sentence if you know the change it brings about in the information state of anyone who accepts the news conveyed by it'." (Veltman 1996)

A (non-presuppositional) update semantics for a simple propositional language (after Veltman 1996 and Heim 1983).[3]

Definition 3.1 (Syntax of a simple propositional language)**.** A language $\mathscr{L}$ is the smallest set, where:[4]

- $\mathscr{A} \subseteq \mathscr{L}$, where $\mathscr{A}$ is a non-empty finite set of atomic formulas $p, q, \ldots$.
- if $\phi \in \mathscr{L}$, then $\neg\, \phi \in \mathscr{L}$.
- if $\phi, \psi \in \mathscr{L}$, then $(\phi \wedge \psi), (\phi \rightarrow \psi), (\phi \vee \psi) \in \mathscr{L}$.

Definition 3.2 (Model)**.** A model M is a pair $\langle W, I \rangle$ consisting of a non-empty set of possible worlds W, and an evaluation function $I : \mathscr{A} \mapsto \mathscr{P}(W)$ from atomic sentences of the language to subsets of W (i.e., bivalent propositions/information states).

Definition 3.3 (Information state)**.** An *information state*[5] (also called a *context*) is any subset of W_M. The set of possible information states is therefore $\mathscr{P}(W)$, where:

[3] Strangely, Veltman gives a symmetric semantics for conjunction his his original paper. We'll need the Heimian connectives to account for the data of interest to us here — this was later rectified in Groenendijk, Stokhof & Veltman (1996).

[4] This is just a concise statement of the syntax of propositional logic.

[5] Veltman (1996) gives an algebraic characterization of update systems which remains neutral regarding the ontology of information states themselves. Next time, we'll be consider an update system with a different notion of information state, but as we'll see, the algebraic properties of the update system will remain largely in place.

- $\varnothing$ is the *absurd information state*.
- W is the *ignorance state* (i.e., the space of logical possibilities).

N.b. the role of the *absurd information state* will be the same as in Stalnakerian pragmatics. It is an idealization representing the point at which a discourse crashes.

For example, a successful update of c by p, followed by an update of c with $\neg p$ will result in the absurd information state.

(12) It's raining. # It's not raining.

Naturally, discourse participants will ordinarily reject an assertion that would lead to the absurd information state.

Providing a *dynamic semantics* consists of recursively defining an *update function* $.[.] : \mathscr{L} \mapsto \mathscr{P}(W) \mapsto \mathscr{P}(W)$ which maps sentences of our language to functions from information states to information states.[6]

Think back to the slogan of dynamic semantics (taken from Veltman) — the meaning of a sentence *is* the effect it has on the context.

Definition 3.4 (Basic expressions)**.**

$$c[p] \coloneqq c \cap I(p)$$

Updating a context c with p involves subtracting worlds from c where p is false. Since, at this point we're dealing with a completely bivalent semantics, we don't need to say anything about bridge; every atomic sentence is assumed to be either true or false at every point.

Definition 3.5 (Negated formulas)**.**

$$c[\neg \phi] \coloneqq c - c[\phi]$$

To update a context c with a negated formula $\neg \phi$: (i) let $c' = c[\phi]$, (ii) do $c - c'$.

Definition 3.6 (Conjunctive formulas)**.**

$$c[\phi \wedge \psi] \coloneqq c[\phi][\psi]$$

To update a context c with a conjunctive formula $\phi \wedge \psi$: (i) let $c' = c[\phi]$, (ii) do $c'[\psi]$. This is identical to successive assertion.

Definition 3.7 (Disjunctive formulas)**.**

$$c[\phi \vee \psi] \coloneqq c[\phi] \cup c[\neg\psi][\psi]$$

To update a context c with a disjunctive formula $\phi \vee \psi$: (i) let $c' = c[\phi]$, let (ii) let $c'' = c[\neg \phi]$, (iii) do $c''[\psi]$ and union the result with c'.

[6] In fact, we only need to give a semantics for conjunctive and negated formulas. Disjunction and material implication can be defined in terms of conjunction and negation under classical equivalence:

- $\phi \vee \psi \coloneqq \neg(\neg \phi \wedge \neg \psi)$
- $\phi \rightarrow \psi \coloneqq \neg(\phi \wedge \neg \psi)$

See the first exercise in this week's problem set (the appendix) for more.

Definition 3.8 (Conditional formulas)**.**

$$c[\phi \to \psi] \coloneqq c - (c[\phi] - c[\phi][\psi])$$

To update a context c with a conditional formula $\phi \to \psi$: (i) let $c' = c[\phi]$, (ii) let $c'' = c[\phi][\psi]$, (iii) do $c - (c' - c'')$.

We can *staticize* Veltman's fragment by taking the *proposition expressed by* p to be $W[p]$, i.e., the logical space updated with p. $[\![p]\!] \coloneqq W[p]$.[7]

$$\begin{aligned}
[\![p]\!] &= I(p) \\
[\![\neg\,\phi]\!] &= W - [\![\phi]\!] \\
[\![\phi \wedge \psi]\!] &= [\![\phi]\!] \cap [\![\psi]\!] \\
[\![\phi \vee \psi]\!] &= [\![\phi]\!] \cup [\![\psi]\!]
\end{aligned}$$

As an exercise in this week's problem set, i've asked you informally prove the above equivalences.

Some important logical properties of update semantics:

Eliminativity For any sentence ϕ, $c[\phi] \subseteq c$.

Distributivity For any sentence ϕ, $c[\phi] = \bigcup_{w \in c} (\{\, w \,\}\,[\phi])$

- Informally, eliminativity says that updating an information state c with ϕ always results in either (a) a stronger information state, or (b) the absurd state. This ensures that updates can't *remove* information.
- Distributivity says that updating a context c with ϕ is equivalent to updating each world $w \in c$ with ϕ, and gathering up the results. This ensures that the result of an update is only ever sensitive to properties of individual points, rather than the context as a whole. This is an important constraint on update semantics.

We won't go into this in detail here, but van Benthem (1986) famously proves that any dynamic semantics that is *eliminative* and *distributive* admits of a static reformulation; in other words, for any sentence ϕ, we can model $c[\phi]$ as $c \cap [\![\phi]\!]$.[8]

As we modify the fragment to account for presupposition and epistemic modals, we'll see ways in which eliminativity and distributivity fail.[9]

[7] As Veltman (1996) remarks, since *updates* are functions from information states to information states, it would probably make more sense to write $[\phi](c)$ for the update of c by ϕ. We'll however follow much of the existing literature by sticking to the classical $c[\phi]$ notation, which has the advantage of making it easier to reason about successive updates ($[\psi]([\phi](c))$ vs. the more iconic $c[\phi][\psi]$).

[8] See Rothschild & Yalcin 2016, 2017 for detailed discussion of this point.

[9] Concretely, distributivity will fail once we discuss epistemic modals. Eliminativity will fail when we come round to discussing anaphora.

Before doing so, it's worth noting that even the simplest, bivalent formulation of propositional update semantics allows us to formulate a notion of *redundancy* which will do some empirical work.

Informally, a sentence ϕ is redundant in c if the result of computing some derivative update $c'[\psi]$ is redundant.

(13) a. It's raining and (in fact) it's raining heavily.
b. #It's raining heavily and (in fact) it's raining.
(14) a. If it's raining then it's raining heavily. #If it's raining heavily then it's raining.

We won't be discussing *redundancy* in depth in this class, but I'll finish this section by noting an interesting problem uncovered by Mayr & Romoli (2016):

(15) Either Paul isn't married or he is (married) and he lives in London.

As noted by Mayr & Romoli, the logical form of the sentence is $\ulcorner \neg \phi \vee (\phi \wedge \psi)$. If we apply the recipes for the logical connectives in dynamic semantics, we predict the following update:

$$c[\neg \phi \vee (\phi \wedge \psi)] = c[\neg \phi] \cup c[\neg \neg \phi][\phi][\psi]$$

Mayr & Romoli (2016) present a fairly involved solution, which we won't go into here. See also Sudo (under revision) for a solution which makes use of a stricter notion of redundancy, framed in terms of situation semantics.

3.3 *Update semantics and presupposition projection*

Heim (1983) was the first to demonstrate that update semantics can account for the Karttunen-Peters projection generalizations.

First we need to supplement our update semantics with presuppositions.

Definition 3.9 (Model with trivalence)**.** A model M is a pair $\langle W, I \rangle$ consisting of a non-empty set of possible worlds W, and a valuation function $I : \mathscr{A} \mapsto F$, where F is the set of total mappings $f : W \mapsto \{1, 0, \#\}$ from worlds to (trivalent) truth-values.

We only need to change the definition of our basic update operation to incorporate Stalnaker's bridge.

Definition 3.10 (Basic expressions (partial semantics))**.**

$$c[p] \coloneqq \begin{cases} c \cap \{\, w \mid I(p)(w) = 1 \,\} & \forall w' \in c[I(p)(w') = 1 \vee I(p)(w') = 0] \\ \text{undefined} & \text{otherwise} \end{cases}$$

Treating the update induced by an atomic sentence as a *partial* function imbues the entire logic with a notion of presupposition.

The difference between the approach here, and a classical trivalent semantics is that, due to the incremental nature of updates induced by complex sentences, the definedness conditions on update can be satisfied locally.

(16) a. $\overbrace{\text{Sarah stopped smoking}}^{p}$

b. $I(p) = \lambda w \,.\, \begin{cases} 1 & \text{Sarah smoked in } w \text{ and doesn't smoke in } w \\ 0 & \text{Sarah smoked in } w \text{ and still smokes in } w \\ \# & \text{otherwise} \end{cases}$

Equivalently:

(17) $I(p) = \lambda w \,.\, \begin{cases} \text{defined} & \text{Sarah smoked in } w \\ \text{true} & \text{Sarah doesn't smoke in } w \end{cases}$

As before, the semantic presupposition of a sentence p, p^{π} is the set of worlds in which $I(p)$ is defined.

We can use this abbreviation to simplify our update rule further:

Definition 3.11 (Basic expression (revised))**.**

$$c[p] \coloneqq \begin{cases} c \cap \{\, w \mid I(p)(w) = 1 \,\} & c \subseteq p^{\pi} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Together with our update rules, this accounts for the Karttunen-Peters projection generalizations.

Let's go through a concrete concrete case, in w_{cy} Sarah has a corgi and it's cute, in w_{cn} Sarah has a corgi and it's not cute, and in $w_{\varnothing}$ Sarah has no corgi. Recall also the rule for conditional statements.

$$c[\phi \rightarrow \psi] \coloneqq c - (c[\phi] - c[\phi][\psi])$$

(18) If Sarah has a corgi, then Sarah's corgi is cute.
$p \rightarrow q$, where $I(p) = q^{\pi}$

- $\{ w_{cy}, w_{cn}, w_{\varnothing} \} [p \rightarrow q] = \{ w_{cy}, w_{cn}, w_{\varnothing} \} - (\{ w_{cy}, w_{cn} \} - \{ w_{cy}, w_{cn}, w_{\varnothing} \} [p][q])$
- $= \{ w_{cy}, w_{cn}, w_{\varnothing} \} - (\{ w_{cy}, w_{cn} \} - \{ w_{cy}, w_{cn} \} [q])$
- $\{ w_{cy}, w_{cn} \} [q]$ is defined, since $\{ w_{cy}, w_{cn} \} = q^{\pi}$
- $= \{ w_{cy}, w_{cn}, w_{\varnothing} \} - (\{ w_{cy}, w_{cn} \} - \{ w_{cy} \})$
- $= \{ w_{cy}, w_{\varnothing} \}$

Update semantics gives rise to the following generalization for presupposition:

Definition 3.12 (Presupposition satisfaction)**.** The presupposition of ϕ is satisfied in c, if the following hold:

- $c \subseteq p^{\pi}$, if $\phi = p$
- The presupposition of ψ is satisfied in c,
  if $\phi = (\neg \psi)$.
- The presupposition of ψ is satisfied in c
  and the presupposition of χ is satisfied in $c[\psi]$,
  if $\phi = (\psi \wedge \chi)$.
- The presupposition of ψ is satisfied in c,
  and the presupposition of χ is satisfied in $c[\neg \psi]$
  if $\phi = (\psi \vee \chi)$.
- The presupposition of ψ is satisfied in c,
  and the presupposition of χ is satisfied in $c[\psi]$
  if $\phi = (\psi \rightarrow \chi)$.

Note that the proviso problem still lurks in the background here (Geurts 1996). Consider the following worlds: w_{hc} Sarah is here and has a cute corgi, w_c Sarah isn't here but has a cute corgi, w_h Sarah is here but has no corgi, $w_{\varnothing}$ Sarah isn't here and doesn't have a cute corgi.

(19) If Sarah is here, then Sarah's corgi is cute.
$p \rightarrow q$

- $\{ w_{hc}, w_c, w_h, w_{\varnothing} \} [p \rightarrow q]$
  $= \{ w_{hc}, w_c, w_h, w_{\varnothing} \} - (\{ w_{hc}, w_h \} - \{ w_{hc}, w_c, w_h, w_{\varnothing} \} [p][q])$
- $\{ w_{hc}, w_c, w_h, w_{\varnothing} \} - (\{ w_{hc}, w_h \} - \{ w_{hc}, w_h \} [q])$

- $\{ w_{hc}, w_h \} [q]$ is undefined since $\{ w_{hc}, w_h \} \nsubseteq q^\pi$
- How could we minimally modify the context such that the presupposition is satisfied? We can simply remove the world in which Sarah is here, and doesn't have a corgi.
- $\{ w_{hc}, w_c, w_\varnothing \} [p \to q] = \{ w_{hc}, w_c, w_\varnothing \} - (\{ w_{hc} \} - \{ w_{hc}, w_c, w_\varnothing \} [p][q])$
- $= \{ w_{hc}, w_c, w_\varnothing \} - (\{ w_{hc} \} - \{ w_{hc} \} [q])$
- $\{ w_{hc} \} [q]$ is defined, since $\{ w_{wc} \} \subseteq q^\pi$
- $= \{ w_{hc}, w_c, w_\varnothing \}$

There are various ways of tweaking the theory to account for proviso cases, requiring a more or less radical departure from standard update semantics: see e.g., Mandelkern 2016 and Grove 2019a,b for two recent approaches.

It's worth emphasizing that this is a problem specifically for the theory of *accommodation* — since we're just interested in presupposition projection here, we'll put this problem to one side.

3.4 The Rooth-Soames objection

Heim (1983) assumed that the classical semantics for the logical connectives *fully determined* the formulation of the update rules.

This was subsequently noted to be incorrect by Mats Rooth, p.c. to Irene Heim in 1987, as well as Soames 1989; a result that has motivated much recent work on presupposition projection.[10]

[10] This relates to Danny's objection to using higher-order functions to capture presupposition projection.

The most straightforward way of demonstrating this by defining an update rule for *backwards conjunction* ($\vartriangle$).

Definition 3.13 (Update rule for backwards conjunction)**.**

$$c[\phi \vartriangle \psi] \coloneqq c[\psi][\phi]$$

Backwards conjunction presumably isn't plausibly lexicalized in natural language, and it certainly doesn't characterize the meaning of *and*.

Otherwise, we'd predict the following sentence to be presuppositionless, contrary to fact:

(20) Josie's sister met her in London, and Josie has a sister.

Nevertheless, staticizing backwards conjunction gives us...*logical conjunction*:[11]

[11] Because set intersection is a symmetric operation.

$$[\![\phi \vartriangle \psi]\!] \coloneqq W[\psi][\phi] = [\![\phi]\!] \cap [\![\psi]\!] \eqqcolon [\![\phi \wedge \psi]\!]$$

What this shows is that, even though there's a unique mapping from an update semantics to the corresponding classical semantics, there are many possible mappings from classical semantics to the corresponding update semantics.

> Optional exercise
>
> ---
>
> Give a "deviant" update semantics for the following connectives. Demonstrate in each case that your entry (i) is truth-conditionally equivalent to the classical counterpart, but (ii) fails to capture the Karttunen-Peters projection generalizations.
>
> - Disjunction.
> - Material implication.

What does this mean for update semantics as an *explanation* for presupposition projection?

I think it's wrong to conclude that this makes the account *completely* stipulative/non-explanatory in character.[12]

[12] Danny may disagree here!

- On the basis of a relatively small set of stipulations, update semantics delivers predictions for an infinite number of sentences.
- We can't really escape from stipulating the classical semantics that update semantics extends.

The objection is narrower, but simultaneously perhaps more interesting.

We have the hunch that it's no accident that the dynamic semantics of natural language conjunction is forwards conjunction $\wedge$ and not backwards conjunction $\vartriangle$. In theory there *could* be a principled story for why we converge upon this update rule rather than some other concievable update rule that is truth-conditionally adequate.

In other words, if we have a predictive algorithm for update rules, or something similar, perhaps we can get away with generating the same set of predictions as Heim's update semantics with fewer stipulations.

A big question in the current literature on presupposition projection is how exactly to accomplish this. A non-exhaustive list of references includes George 2007, 2008, 2014, Schlenker 2008, 2009, 2010, and Fox 2013.

In future classes, we plan to discuss George's *middle Kleene* algorithm in this light.[13]

[13] If we have time, i'll also discuss my own work on developing a predictive theory of anaphora using similar technical machinery (Elliott 2020a), as well as Mandelkern's recent (2020b, 2020a) work on this topic.

For the moment, we'll put this explanatory challenge to one side, and discuss an *independent* motivation for the expressive power afforded by update semantics — epistemic modals and epistemic contradictions.[14]

[14] In Veltman 1996, update semantics is also applied to cases of so-called "default reasoning", i.e.:

(21) a. P's are normally R.
b. x is P.
c. Presumably, x is R.

We won't cover this topic in this class, but I encourage those of you interested in reading further to do so.

3.5 *Veltman's test semantics and epistemic contradictions*

> "I will borrow from Veltman's work to show how the context sensitivity of [epistemic modal] words like 'might' and 'must' motivates a dynamic semantics. None of the alternative CCPs for connectives that have been suggested by Rooth and Soames would be compatible with this semantics, and it is hard to imagine how a relevantly different dynamic semantics could still get the facts right about the meanings of the epistemic modalities." (Beaver 2001)

An initial motivation for a dynamic treatment of epistemic modality: epistemic contradictions and order sensitivity.[15]

[15] Discussion of such *epistemic contradictions* has a long history in the philosophy of language literature, going back to Moore 1942.

(22) a. ? It might be raining, but it's not raining.
b. # It's not raining, but it might be raining.

Do we need to go beyond a classical semantics for modals to explain the oddness of (22)? Let's tentatively assume that an assertion of "it might be raining" is true if *it's raining* is compatible with the speaker's knowledge.

An argument against a pragmatic story: Yalcin's (2007) embedded cases:

(23) a. ? Suppose that [it might be raining but it's not raining].
b. ? Suppose that [it's not raining but it might be raining].

Difficult to see how a pragmatic explanation might extend to such cases.

As originally demonstrated by Veltman (1996), it's possible to state an elegant semantics for epistemic modality in update semantics that captures the oddness of epistemic contradictions.

We'll extend our simple propositional language with an additional unary operator, standing in for *might*: $\Diamond$.

Definition 3.14 (Test semantics for epistemic possibility)**.**

$$c[\Diamond\, p] \coloneqq \begin{cases} c & c[p] \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$$

The intuition

An assertion of "it might be raining" is a prompt to *tentatively* update the context set c with the information *it's raining*. If the update is successful (i.e., if it doesn't result in the absurd information state), simply return c unchanged.

Exercise

What happens if we *staticize* $\Diamond\, p$? What does this tell us about Veltman's proposal?

Discussion point

According to Veltman's theory, when we assert a modalized sentence one of two things can happen: (i) if the test is successful, the context is unchanged. (ii) if the test is unsuccessful, we find ourselves in the absurd state.
From a pragmatic point of view, what is the utility of epistemic modals then? Ideally, we need a pragmatic theory to go alongside Veltman's semantics.

Optional exercise

Can we state the meaning of epistemic *must* as the dual of $\Diamond$?

- If so, demonstrate that this delivers intuitively correct results.
- If not, show why not.

To show how this captures asymmetries in epistemic contradictions, first we will need some derivative notions.

Definition 3.15 (Consistency)**.** A sentence ϕ is consistent with respect to c, if

$c[\phi] \neq \varnothing$; a sentence ϕ is *consistent* simpliciter, if there is some information state c', s.t., $c'[\phi]$ is consistent.

"It's raining and it's not raining" is *inconsistent*, since there is no information state c, such that updating c with this sentence will result in a non-absurd information state. This holds for all classical contradictions.

Concretely, if $[\![\phi]\!] = \varnothing$, then ϕ is inconsistent.

Let's return to one of the examples that motivated a dynamic semantics for epistemic modality.

(24) It's not raining outside, but it might be raining outside.
$\neg\, p \wedge \Diamond\, p$

A good result: (24) is *inconsistent*.

Before giving an informal proof, the intuition is as follows: updating an information state with the information that it's not raining is guaranteed to make a tentative update of "it's raining" fail.

- $c[\neg\, p \wedge \Diamond\, p] = c[\neg\, p][\Diamond\, p]$
- $= (c\ - I(p))[\Diamond\, p]$
- $= \begin{cases} c - I(p) & (c - I(p)[p] \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$
- $= \begin{cases} c - I(p) & ((c - I(p) \cap I(p)) \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$
- $(c - s) \cap s = \varnothing, \forall s$, hence $(\neg\, p \wedge \Diamond\, p)$ is *inconsistent*.

What about the other ordering, repeated in (25)? Although we didn't assign this a # diacritic, arguably there is something pragmatically marked about this sentence. There's an intuition that the speaker has changed their mind after uttering the first conjunct.

(25) ? It might be raining and it's not raining.
$\Diamond\, p \wedge \neg\, p$

In fact, we can construct variations of (25) which sound more natural:

(26) A: It might be raining.
B: It's not raining!

(27) It might be raining [...] it's not raining.

We can make sense of this in update semantics by using the notion of *coherence*.

Definition 3.16 (Support). An information state c *supports* a sentence ϕ iff:

$$c[\phi] = c$$

Other terms which are often used to mean the same thing: c *accepts* ϕ, c *incorporates* ϕ.

Definition 3.17 (Coherence). ϕ is coherent iff there is some non-absurd information state c, s.t., c *supports* ϕ.

Note that *coherence* implies *consistency*: if a non-absurd c supports ϕ, then $c[\phi]$ is consistent, and hence ϕ is consistent simpliciter.

Now we can ask ourselves, is (25) consistent/coherent?

- $c[\Diamond p \wedge \neg p] = c[\Diamond p][\neg p]$
- $= \begin{cases} c[\neg p] & c[p] \neq \varnothing \\ \varnothing[\neg p] & \text{otherwise} \end{cases}$
- $= \begin{cases} c - c[p] & c[p] \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

(25) is *consistent*, since as long as both p and $\neg p$ are c-consistent, then updating c with (25) will result in a non-absurd information state — namely, one that supports $\neg p$.

Now can can ask, is (25) *coherent*? The answer is no. For the test imposed by $\Diamond p$ to be successful in c, c cannot support $\neg p$, and for c to support $p \wedge q$, $c[p]$ must support q.[16]

[16] A sketch of a proof by contradiction:

- If $\Diamond p \wedge \neg p$ is coherent; there exists a c, s.t., $c[\Diamond p \wedge \neg p] = c$.
- If $c[\Diamond p \wedge \neg p] = c$,
  then $(c[\Diamond p])[\neg p] = c$,
  so by eliminativity $c[\Diamond p] = c[\neg p] = c$
- if $c[\Diamond p] = c$, then $c[p] \neq \varnothing$
- if $c[p] \neq \varnothing$, then $c - p \neq c$

Optional exericse

Recall that, due to presupposition projection facts, the update rule for disjunctive sentences is as follows:

$$c[\phi \vee \psi] \coloneqq c[\phi] \cup \; c[\neg\, \phi][\psi]$$

What does the theory predict for a sentence such as "either it's raining, or it might be raining"?

$$p \vee \Diamond\, p$$

What about the reverse order, "it might be raining, or it's raining"?

$$p \vee \Diamond\, p$$

Try to connect the results to your intuitions about what these sentences mean.

References

Beaver, David I. 2001. *Presupposition and Assertion in Dynamic Semantics*. 250 pp.

van Benthem, Johan. 1986. *Essays in Logical Semantics* (Studies in Linguistics and Philosophy). Springer Netherlands.

Chierchia, Gennaro. 2020. Origins of weak crossover - when dynamic semantics meets event semantics. *Natural Language Semantics* (28). 23–76.

Dekker, Paul. 1994. Predicate logic with anaphora. *Semantics and Linguistic Theory* 4. 79–95.

Elliott, Patrick D. 2020a. Crossover and accessibility in dynamic semantics. Massachusetts Institute of Technology.

Elliott, Patrick D. 2020b. Tetradynamics: an explanatory theory of anaphora. Unpublished manuscript. Massachusetts Institute of Technology.

Fox, Danny. 2013. Presupposition projection from quantificational sentences - Trivalence, local accommodation, and presupposition strengthening. In Ivano Caponigro & Carlo Cecchetto (eds.), *From grammar to meaning*, 201–232.

George, B. R. 2007. Predicting presupposition projection - Some alternatives in the strong Kleene tradition. unpublished manuscript. UCLA.

George, B. R. 2008. A new predictive theory of presupposition projection. In *Proceedings of SALT 18*, 358–375. Ithaca, NY: Cornell University.

George, B. R. 2014. Some remarks on certain trivalent accounts of presupposition projection. *Journal of Applied Non-Classical Logics* 24(1-2). 86–117.

Geurts, Bart. 1996. Local satisfaction guaranteed: A presupposition theory and its problems. *Linguistics and Philosophy* 19(3). 259–294.

Gillies, Anthony S. 2004. Epistemic Conditionals and Conditional Epistemics. *Noûs* 38(4). 585–616.

Groenendijk, J. a. G., M. J. B. Stokhof & F. J. M. M. Veltman. 1996. *Coreference and modality*. OxfordBlackwell.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

Grove, Julian. 2019a. Satisfaction without provisos. lingbuzz/004914.

Grove, Julian. 2019b. *Scope-taking and presupposition satisfaction*. University of Chicago dissertation.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases*. University of Massachusetts - Amherst dissertation.

Heim, Irene. 1983. On the projection problem for presuppositions. In *Proceedings of WCCFL 2*, 114–125. Stanford University.

Honcoop, Martin. 1998. *Dynamic excursions on weak islands*. University of Leiden dissertation.

Kamp, Hans. 1981. A theory of truth and semantic representation. In Paul Portner & Barbara H. Partee (eds.), *Formal semantics: The essential readings*, 189–222. Blackwell.

Mandelkern, Matthew. 2016. Dissatisfaction Theory. *Semantics and Linguistic Theory* 26. 391–416.

Mandelkern, Matthew. 2020a. Pseudo-dynamics. Unpublished manuscript. Oxford.

Mandelkern, Matthew. 2020b. Witnesses. Unpublished manuscript. Oxford.

Mayr, Clemens & Jacopo Romoli. 2016. A puzzle for theories of redundancy - Exhaustification, incrementality, and the notion of local context. *Semantics and Pragmatics*.

Moore, George Edward. 1942. A Reply to My Critics. In Paul Arthur Schilpp (ed.), *The Philosophy of G. E. Moore*. Open Court.

Rothschild, Daniel. 2017. A trivalent approach to anaphora and presupposition. unpublished manuscript.

Rothschild, Daniel & Seth Yalcin. 2016. Three notions of dynamicness in language. *Linguistics and Philosophy* 39(4). 333–355.

Rothschild, Daniel & Seth Yalcin. 2017. On the Dynamics of Conversation. *Noûs* 51(1). 24–48.

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3).

Schlenker, Philippe. 2009. Local contexts. *Semantics and Pragmatics* 2.

Schlenker, Philippe. 2010. Local contexts and local meanings. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 151(1). 115–142.

Soames, Scott. 1989. Presupposition. In D. Gabbay & F. Guenthner (eds.), *Handbook of Philosophical Logic: Volume IV: Topics in the Philosophy of Language* (Synthese Library), 553–616. Dordrecht: Springer Netherlands.
Sudo, Yasutada. 2019. Implicatures with discourse referents.
van den Berg, M. H. 1996. Some aspects of the internal structure of discourse. The dynamics of nominal anaphora.
Veltman, Frank. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic* 25(3). 221–261.
Yalcin, Seth. 2007. Epistemic Modals. *Mind* 116(464). 983–1026.
Yalcin, Seth. 2013. Introductory Notes on Dynamic Semantics. Berkeley.

A Problem set (due before Friday 2 October)

A.1 Dynamic semantics and classical equivalence

In today's handout, we stated a dynamic semantics for negated sentences, conjunctive and disjunctive sentences, as well as material implications.

In fact, the only primitives we need are a dynamic semantics for negated and conjunctive sentences. We can define disjunction and material implication via classical equivalence, but not just any classical equivalences will do.

Exercise

Part 1: Informally prove the following equivalences:

- $c[\phi \vee \psi] \equiv c[\neg\,(\neg\,\phi \wedge \neg\,\psi)]$
- $c[\phi \rightarrow \psi] \equiv c[\neg\,(\phi \wedge \neg\,\psi)]$

Part 2: Provide formulas using only conjunction and negation that are classically equivalent to $\ulcorner \phi \vee \psi \urcorner$, $\ulcorner \phi \rightarrow \psi \urcorner$, which nevertheless aren't equivalent in propositional dynamic semantics. Demonstrate where the equivalence breaks down.
Part 3: Comment briefly on what this tells us about the explanatory potential of propositional dynamic semantics.

A.2 Staticization

As noted in today's handout, we can *staticize* a (bivalent) propositional update semantics by taking the *proposition expressed by p* to be $W[p]$, i.e., the logical space updated with p. $[\![p]\!] := W[p]$.

Exercise

Part 1: Prove the following equivalences (an informal demonstration is fine).

$$
\begin{aligned}
[\![p]\!] &= I(p) \\
[\![\neg \phi]\!] &= W - [\![\phi]\!] \\
[\![\phi \wedge \psi]\!] &= [\![\phi]\!] \cap [\![\psi]\!] \\
[\![\phi \vee \psi]\!] &= [\![\phi]\!] \cup [\![\psi]\!]
\end{aligned}
$$

Part 2: What is the staticization of a modalized sentence $\ulcorner \Diamond \phi \urcorner$? Comment on the significance of the result.