The dynamic turn I: Updates, satisfaction, and coherence

Patrick D. Elliott & Danny Fox

September 18, 2020

Homework

Since you have a presupposition p-set assigned this week, there'll be no p-set on dynamic semantics (yet). Expect a technical p-set next week! Your task for next Friday is straightforward:

- Finish reading Yalcin (2013), if you haven't already.

There are two supplementary readings: (i) Heim 1983, and (ii) Veltman 1996 (advanced). If you have any questions about the readings, I encourage you to use the slack, but you can also email (`pdell@mit.edu`) if you prefer.

The plan ahead

Our syllabus was rather sketchy; here i'll try to be a little more concrete concerning what I tentatively plan to cover in the coming weeks (as usual, this is subject to change):

- Today + next week: propositional update semantics, and its applications to presupposition projection (Heim 1983), and epistemic modality (Veltman 1996, Groenendijk, Stokhof & Veltman 1996).
- George's (2007, 2008, 2014) *middle Kleene* — a more explanatory theory of presupposition projection.
- Anaphora in the dynamic tradition (Heim 1982, Groenendijk & Stokhof 1991, Dekker 1994).
- Explanatory theories of anaphora (Rothschild 2017, Elliott 2020a,b and Mandelkern 2020b,a).

1 Recap: Stalnakerian pragmatics

The account of presupposition we've been developing has the following basic ingredients.

- A trivalent semantics for sentences; this gives rise to the notion of the *semantic presupposition* of a sentence.[1]
- A notion of *assertion*, which tells us how to update a context c with the information conveyed by a sentence ϕ.
- The conditions under which update of a context c with ϕ is defined (*Stalnaker's bridge*).

[1] There are various points at which Stalnaker sounds skeptical that we need a semantic notion of presupposition at all; we won't discuss this point today.

1.1 Trivalence

A sentential meaning is a function $p : W \mapsto \{ 1, 0, \# \}$. Here's a simple example:

(1) $$[\![\text{Sarah's corgi is sleepy}]\!] = \begin{cases} 1 & \text{Sarah has a corgi \& Sarah's corgi is sleepy} \\ 0 & \text{Sarah has a corgi \& Sarah's corgi isn't sleepy} \\ \# & \text{otherwise} \end{cases}$$

In trivalent semantics, the *semantic presupposition of a sentence S* is the set of worlds w, such that $[\![\text{S}]\!]\ w$ is either true or false.

Definition 1.1 (Semantic presupposition)**.**

$$\phi^{\pi} := \{ w \mid [\![\phi]\!]\ w = 1 \vee [\![\phi]\!]\ w = 0 \}$$

(2) $(\text{Sarah's corgi is sleepy})^{\pi} = \{ w \mid \text{Sarah has a corgi in } w \}$

1.2 Update and Stalnaker's bridge

The *update* induced by a a sentence ϕ is a partial function $c[\phi] : \mathscr{P}(W) \mapsto \mathscr{P}(W)$.

(3) Stalnakerian update (def.)
$$c[\phi] := \begin{cases} \{ w \mid w \in c \wedge [\![\phi]\!]\ w \} & c \subseteq \phi^{\pi} \\ \text{undefined} & \text{otherwise} \end{cases}$$

A (bivalent) proposition p is *redundant* wrt a context set c if $c \subseteq \{ w \mid p\ w \}$.

Stalnaker's bridge places a precondition on an update of c by ϕ — ϕ^{π} must be *redundant* wrt c.

1.3 Successive update

A (trivial?) observation: updating c with a sentence ϕ can make the presupposition of a sentence ψ redundant, thus ensuring that $c[\psi]$ is guaranteed to be defined.

(4) Sarah has a corgi. Sarah's corgi is sleepy.

Stalnakerian pragmatics directly captures this, since successive assertion gives rise to a successive update.

We can write a successive update of c with S followed by S' as $c[S][S']$.

(5) $c[S][S'] \coloneqq (c[S])[S']$

$$c[\text{Sarah has a corgi}] = \overbrace{\{\, w \mid w \in c \wedge \text{Sarah has a corgi in } w \,\}}^{c'}$$

$$c'[\text{Sarah's corgi is sleepy}] = \begin{cases} \{\, w \mid w \in c' \wedge \text{Sarah's corgi is sleep in } w \,\} & c \cap \{\, w \mid \text{Sarah has a corgi in } w \,\} \\ & \subseteq \{\, w \mid \text{Sarah has a corgi in } w \,\} \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$c'[\text{Sarah's corgi is sleepy}] = \{\, w \mid w \in c' \wedge \text{Sarah's corgi is sleep in } w \,\}$$

2 Dynamic semantics

2.1 Empirical motivations for dynamic semantics

Historically, dynamic semantics — independently developed by Irene Heim (1982) and Hans Kamp (1981) — was motivated by *anaphora to singular indefinites.*

Briefly, pronouns can co-vary with indefinites in preceding sentences:

(6) a. A^1 man walked in. He_1 sat down.
    b. #He_1 walked in. A^1 man sat down.

More generally, pronouns can co-vary with indefinites, even when not in their scope (*donkey pronouns*).

(7) Everyone [who bought a^1 new puppy during the pandemic] treasured it_1.

This phenomena is sensitive sensitive to the form of preceding sentences; not just classical content (the famous *marble* example is due to Barbara Partee).

(8) a. #I've found nine out of my ten marbles. It_1's under the couch.
cf. One out of my 10 marbles is lost. It_1's under the couch.
b. *Josie is married. He_1's annoying.
cf. Josie has a^1 husband. He_1's annoying.

We'll discuss the dynamic approach to anaphora in more depth in a couple of weeks time.

Subsequently, the remit of dynamic semantics was expanded to encompass theories of an extremely broad range of phenomena, including amongst others:

- Presupposition projection (Heim 1983, Beaver 2001, a.o.).
- Epistemic modality (Veltman 1996, Groenendijk, Stokhof & Veltman 1996, a.o.).
- Intervention effects (Honcoop 1998, a.o.).
- Conditionals (Gillies 2004, a.o.).
- Generalized quantifiers and discourse plurals (van den Berg 1996, a.o.)
- Scalar implicature (Sudo 2019, a.o.).
- Weak crossover (Chierchia 2020, Elliott 2020a).

We'll start with presupposition projection, and subsequently, epistemic modality, since these topics only require reference to the simplest version of dynamic semantics — update semantics on a simple propositional calculus — and the Stalnakerian notion of content that we're already familiar with.

2.2 Towards an update semantics

Successive assertion patterns with *conjunction* wrt presupposition projection (Danny's handout from last week; Karttunen's generalization).

A natural way of cashing this out: a conjunctive sentence induces successive update.

(9) Conjunctive sentences in update semantics (def.)
$c[S \text{ and } S'] \coloneqq c[S][S']$

> "The slogan 'You know the meaning of a sentence if you know the conditions under which it is true', is replaced by this one: 'You know the meaning of a sentence if you know the change it brings about in the information state of anyone who accepts the news conveyed by it'." (Veltman 1996)

A (non-presuppositional) update semantics for a simple propositional language (after Veltman 1996 and Heim 1983).[2]

Definition 2.1 (Syntax of a simple propositional language)**.** A language $\mathscr{L}$ is the smallest set, where:[3]

- $\mathscr{A} \subseteq \mathscr{L}$, where $\mathscr{A}$ is a non-empty finite set of atomic formulas $p, q, \ldots$.
- if $\phi \in \mathscr{L}$, then $\neg\, \phi \in \mathscr{L}$.
- if $\phi, \psi \in \mathscr{L}$, then $(\phi \wedge \psi), (\phi \rightarrow \psi), (\phi \vee \psi) \in \mathscr{L}$.

Definition 2.2 (Model)**.** A model M is a pair $\langle W, I \rangle$ consisting of a non-empty set of possible worlds W, and a valuation function $I : \mathscr{A} \mapsto \mathscr{P}(W)$ from atomic sentences of the language to subsets of W.

Definition 2.3 (Information state)**.** An *information state*[4] (also called a *context*) is any subset of W_M. The set of possible information states is therefore $\mathscr{P}(W)$, where:

- $\varnothing$ is the *absurd information state*.
- W is the *ignorance state* (i.e., the space of logical possibilities).

Providing a dynamic semantics consists of recursively defining an *update function* $.[.] : \mathscr{L} \mapsto \mathscr{P}(W) \mapsto \mathscr{P}(W)$[5]

Definition 2.4 (Basic expressions)**.**

$$c[p] \coloneqq c \cap I(p)$$

Updating a context c with p involves subtracting worlds from c where p is false.

Definition 2.5 (Negated formulas)**.**

$$c[\neg\, \phi] \coloneqq c - c[\phi]$$

To update a context c with a negated formula $\neg\, \phi$: (i) let $c' = c[\phi]$, (ii) do $c - c'$.

Definition 2.6 (Conjunctive formulas)**.**

$$c[\phi \wedge \psi] \coloneqq c[\phi][\psi]$$

To update a context c with a conjunctive formula $\phi \wedge \psi$: (i) let $c' = c[\phi]$, (ii) do $c'[\psi]$. This is identical to successive assertion.

[2] Strangely, Veltman gives a symmetric semantics for conjunction his his original paper. We'll need the Heimian connectives to account for the data of interest to us here — this was later rectified in Groenendijk, Stokhof & Veltman (1996).

[3] This is just a concise statement of the syntax of propositional logic.

[4] Veltman (1996) gives an algebraic characterization of update systems which remains neutral regarding the ontology of information states themselves. Next time, we'll be consider an update system with a different notion of information state, but as we'll see, the algebraic properties of the update system will remain largely in place.

[5] In fact, we only need to give a semantics for conjunctive and negated formulas. Disjunction and material implication can be defined in terms of conjunction and negation under classical equivalence:

- $\phi \vee \psi \coloneqq \neg\, (\neg\, \phi \wedge \neg\, \psi)$
- $\phi \rightarrow \psi \coloneqq \neg\, (\phi \wedge \neg\, \psi)$

Not all classical equivalences will work however — this is a corollary of the Rooth/-Soames objection to dynamic semantics, which we'll discuss in a later section. As an optional exercise, convince yourself that the definitions above are equivalent to the ones stated in the body of the text, and furthermore, that the following deviant definitions aren't:

- $\phi \vee \psi \coloneqq \neg\, (\neg\, \psi \wedge \neg\, \phi)$
- $\phi \rightarrow \psi \coloneqq \neg\, (\neg\, \psi \wedge \phi)$

Definition 2.7 (Disjunctive formulas)**.**

$$c[\phi \vee \phi] := c[\phi] \cup c[\neg\phi][\phi]$$

To update a context c with a disjunctive formula $\phi \vee \psi$: (i) let $c' = c[\phi]$, let (ii) let $c'' = c[\neg\, \phi]$, (iii) do $c''[\psi]$ and union the result with c'.

Definition 2.8 (Conditional formulas)**.**

$$c[\phi \rightarrow \psi] := c - (c[\phi] - c[\phi][\psi])$$

To update a context c with a conditional formula $\phi \rightarrow \psi$: (i) let $c' = c[\phi]$, (ii) let $c'' = c[\phi][\psi]$, (iii) do $c - (c' - c'')$.

We can *staticize* Veltman's fragment by taking the *proposition expressed by* p to be $W[p]$, i.e., the logical space updated with p. $[\![p]\!] := W[p]$.[6]

[6] As Veltman (1996) remarks, since *updates* are functions from information states to information states, it would probably make more sense to write $[\phi](c)$ for the update of c by ϕ. We'll however follow much of the existing literature by sticking to the classical $c[\phi]$ notation, which has the advantage of making it easier to reason about successive updates ($[\psi]([\phi](c))$ vs. the more iconic $c[\phi][\psi]$).

$$\begin{array}{lll}
[\![p]\!] & = \{ w \mid w \in W \wedge w \in I(p) \} & = I(p) \\
[\![\neg\, \phi]\!] & & = W - [\![\phi]\!] \\
[\![\phi \wedge \psi]\!] & = W[\phi][\psi] & = [\![\phi]\!] \cap [\![\psi]\!] \\
[\![\phi \vee \psi]\!] & & = [\![\phi]\!] \cup [\![\psi]\!] \\
[\![\phi \rightarrow \psi]\!] & & = (W - [\![\phi]\!]) \cup [\![\psi]\!]
\end{array}$$

Some important logical properties of update semantics:

Eliminativity For any sentence ϕ, $c[\phi] \subseteq c$.

Distributivity For any sentence ϕ, $c[\phi] = \bigcup_{w \in c} (\{ w \} [\phi])$

- Eliminativity says that updating a context c with any sentence ϕ results in an updated context c', s.t. $c' \subseteq c$; updates always shrink the context.
- Distributivity says that updating a context c with ϕ is equivalent to updating each world $w \in c$ with ϕ, and gathering up the results.

In fact, van Benthem (1986) proves that any dynamic semantics that is *eliminative* and *distributive* admits of a static reformulation; in other words, for any sentence ϕ, we can model $c[\phi]$ as $c \cap [\![\phi]\!]$.[7]

[7] See Rothschild & Yalcin 2016, 2017 for detailed discussion of this point.

As we modify the fragment to account for presupposition and epistemic modals, we'll see ways in which eliminativity and distributivity fail.

2.3 Update semantics and presupposition projection

Heim (1983) was the first to demonstrate that update semantics can account for the Karttunen-Peters projection generalizations.

First we need to supplement our update semantics with presuppositions.

A model M is a pair $\langle W, I\rangle$ consisting of a non-empty set of possible worlds W, and a valuation function $I : \mathscr{A} \mapsto F$, where F is the set of total functions $f : W \mapsto \{ 1, 0, \# \}$ from worlds to (trivalent) truth-values.

We only need to change the definition of our basic update operation to incorporate Stalnaker's bridge.

Definition 2.9 (Basic expression (revised))**.**

$$c[p] \coloneqq \begin{cases} c \cap \{ w \mid I(p)(w) = 1 \} & \forall w' \in c[I(p)(w') = 1 \vee I(p)(w') = 0] \\ \text{undefined} & \text{otherwise} \end{cases}$$

(10) a. $\overbrace{\text{Sarah stopped smoking}}^{p}$

b. $I(p) = \lambda w\ .\begin{cases} 1 & \text{Sarah smoked in } w \text{ and doesn't smoke in } w \\ 0 & \text{Sarah smoked in } w \text{ and still smokes in } w \\ \# & \text{otherwise} \end{cases}$

Equivalently:

(11) $I(p) = \lambda w\ .\begin{cases} \text{defined} & \text{Sarah smoked in } w \\ \text{true} & \text{Sarah doesn't smow in } w \end{cases}$

As before, the semantic presupposition of a sentence p, p^{π} is the set of worlds in which $I(p)$ is defined.

We can use this abbreviation to simplify our update rule further:

Definition 2.10 (Basic expression (revised))**.**

$$c[p] \coloneqq \begin{cases} c \cap \{ w \mid I(p)(w) = 1 \} & c \subseteq p^{\pi} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Together with our update rules, this accounts for the Karttunen-Peters projection generalizations.

Let's go through a concrete concrete case, in w_{cy} Sarah has a corgi and it's cute, in w_{cn} Sarah has a corgi and it's not cute, and in $w_{\varnothing}$ Sarah has no corgi.

(12) If Sarah has a corgi, then Sarah's corgi is cute.
$p \rightarrow q$, where $I(p) = q^{\pi}$

- $\{ w_{cy}, w_{cn}, w_{\varnothing} \} [p \rightarrow q] = \{ w_{cy}, w_{cn}, w_{\varnothing} \} - (\{ w_{cy}, w_{cn} \} - \{ w_{cy}, w_{cn}, w_{\varnothing} \} [p][q])$
- $= \{ w_{cy}, w_{cn}, w_{\varnothing} \} - (\{ w_{cy}, w_{cn} \} - \{ w_{cy}, w_{cn} \} [q])$
- $\{ w_{cy}, w_{cn} \} [q]$ is defined, since $\{ w_{cy}, w_{cn} \} = q^{\pi}$
- $= \{ w_{cy}, w_{cn}, w_{\varnothing} \} - (\{ w_{cy}, w_{cn} \} - \{ w_{cy} \})$
- $= \{ w_{cy}, w_{\varnothing} \}$

Update semantics gives rise to the following generalization for presupposition:

Definition 2.11 (Presupposition satisfaction)**.** The presupposition of ϕ is satisfied in c, if the following hold:

- $c \subseteq p^{\pi}$, if $\phi = p$
- The presupposition of ψ is satisfied in c,
  if $\phi = (\neg \psi)$.
- The presupposition of ψ is satisfied in c
  and the presupposition of χ is satisfied in $c[\psi]$,
  if $\phi = (\psi \wedge \chi)$.
- The presupposition of ψ is satisfied in c,
  and the presupposition of χ is satisfied in $c[\neg \psi]$
  if $\phi = (\psi \vee \chi)$.
- The presupposition of ψ is satisfied in c,
  and the presupposition of χ is satisfied in $c[\psi]$
  if $\phi = (\psi \rightarrow \chi)$.

Note that the proviso problem still lurks in the background here (Geurts 1996). Consider the following worlds: w_{hc} Sarah is here and has a cute corgi, w_c Sarah isn't here but has a cute corgi, w_h Sarah is here but has no corgi, $w_{\varnothing}$ Sarah isn't here and doesn't have a cute corgi.

(13) If Sarah is here, then Sarah's corgi is cute.
$p \rightarrow q$

- $\{ w_{hc}, w_c, w_h, w_{\varnothing} \} [p \rightarrow q]$
  $= \{ w_{hc}, w_c, w_h, w_{\varnothing} \} - (\{ w_{hc}, w_h \} - \{ w_{hc}, w_c, w_h, w_{\varnothing} \} [p][q])$

- $\{ w_{hc}, w_c, w_h, w_\varnothing \} - (\{ w_{hc}, w_h \} - \{ w_{hc}, w_h \} [q])$
- $\{ w_{hc}, w_h \} [q]$ is undefined since $\{ w_{hc}, w_h \} \nsubseteq q^\pi$
- How could we minimally modify the context such that the presupposition is satisfied? We can simply remove the world in which Sarah is here, and doesn't have a corgi.
- $\{ w_{hc}, w_c, w_\varnothing \} [p \rightarrow q] = \{ w_{hc}, w_c, w_\varnothing \} - (\{ w_{hc} \} - \{ w_{hc}, w_c, w_\varnothing \} [p][q])$
- $= \{ w_{hc}, w_c, w_\varnothing \} - (\{ w_{hc} \} - \{ w_{hc} \} [q])$
- $\{ w_{hc} \} [q]$ is defined, since $\{ w_{wc} \} \subseteq q^\pi$
- $= \{ w_{hc}, w_c, w_\varnothing \}$

Recall, this is a problem specifically for the theory of *accommodation* — since we're just interested in presupposition projection here, we'll put this problem to one side.

2.4 *The Rooth-Soames objection*

Heim (1983) assumed that the classical semantics for the logical connectives *fully determined* the formulation of the update rules.

This was subsequently noted to be incorrect by Mats Rooth, p.c. to Irene Heim in 1987, as well as Soames 1989; a result that has motivated much recent work on presupposition projection.

The most straightforward way of demonstrating this by defining an update rule for *backwards conjunction* ($\triangle$).

Definition 2.12 (Update rule for backwards conjunction)**.**

$$c[\phi \triangle \psi] \coloneqq c[\psi][\phi]$$

Backwards conjunction presumably isn't plausibly lexicalized in natural language, and it certainly doesn't characterize the meaning of *and*.

Otherwise, we'd predict the following sentence to be presuppositionless, contrary to fact:

(14) Josie's sister met her in London, and Josie has a sister.

Nevertheless, staticizing backwards conjunction gives us...*logical conjunction*:[8]

[8] Because set intersection is a symmetric operation.

$$[\![\phi \vartriangle \psi]\!] \coloneqq W[\psi][\phi] = [\![\phi]\!] \cap [\![\psi]\!] \eqqcolon [\![\phi \wedge \psi]\!]$$

What this shows is that, even though there's a unique mapping from an update semantics to the corresponding classical semantics, there are many possible mappings from classical semantics to the corresponding update semantics.

Optional exercise

Give a "deviant" update semantics for the following connectives. Demonstrate in each case that your entry (i) is truth-conditionally equivalent to the classical counterpart, but (ii) fails to capture the Karttunen-Peters projection generalizations.

- Disjunction.
- Material implication.

What does this mean for update semantics as an *explanation* for presupposition projection?

I think it's wrong to conclude that this makes the account *completely* stipulative/non-explanatory in character.[9]

[9] Danny may disagree here!

- On the basis of a relatively small set of stipulations, update semantics delivers predictions for an infinite number of sentences.
- We can't really escape from stipulating the classical semantics that update semantics extends.

The objection is narrower, but simultaneously perhaps more interesting.

We have the hunch that it's no accident that the dynamic semantics of natural language conjunction is forwards conjunction $\wedge$ and not backwards conjunction $\vartriangle$. In theory there *could* be a principled story for why we converge upon this update rule rather than some other concievable update rule that is truth-conditionally adequate.

In other words, if we have a predictive algorithm for update rules, or something similar, perhaps we can get away with generating the same set of predictions as Heim's update semantics with fewer stipulations.

A big question in the current literature on presupposition projection is how exactly to accomplish this. A non-exhaustive list of references includes George 2007, 2008, 2014, Schlenker 2008, 2009, 2010, and Fox 2013.

In future classes, we plan to discuss George's *middle Kleene* algorithm in this light.[10]

[10] If we have time, i'll also discuss my own work on developing a predictive theory of anaphora using similar technical machinery (Elliott 2020a), as well as Mandelkern's recent (2020b, 2020a) work on this topic.

For the moment, we'll put this explanatory challenge to one side, and discuss an *independent* motivation for the expressive power afforded by update semantics — epistemic modals and epistemic contradictions.

2.5 Evidence for updates in the semantics

> "I will borrow from Veltman's work to show how the context sensitivity of [epistemic modal] words like 'might' and 'must' motivates a dynamic semantics. None of the alternative CCPs for connectives that have been suggested by Rooth and Soames would be compatible with this semantics, and it is hard to imagine how a relevantly different dynamic semantics could still get the facts right about the meanings of the epistemic modalities." (Beaver 2001)

An initial motivation for a dynamic treatment of epistemic modality: epistemic contradictions and order sensitivity.[11]

[11] Discussion of such *epistemic contradictions* has a long history in the philosophy of language literature, going back to Moore 1942.

(15) a. ? It might be raining, but it's not raining.
b. # It's not raining, but it might be raining.

As originally demonstrated by Veltman (1996), it's possible to state an elegant semantics for epistemic modality in update semantics that captures this contrast.

We'll extend our simple propositional language with an additional unary operator, standing in for *might*: $\Diamond$.

Definition 2.13 (Test semantics for epistemic possibility)**.**

$$c[\Diamond\, p] \coloneqq \begin{cases} c & c[p] \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$$

The intuition

An assertion of "it might be raining" is a prompt to *tentatively* update the context set c with the information *it's raining*. If the update is successful (i.e., if it doesn't result in the absurd information state), simply return c unchanged.

Exercise

What happens if we *staticize* $\Diamond\, p$? What does this tell us about Veltman's proposal?

Optional exercise

Can we state the meaning of epistemic *must* as the dual of $\Diamond$?

- If so, demonstrate that this delivers intuitively correct results.
- If not, show why not.

To show how this captures asymmetries in epistemic contradictions, first we will need some derivative notions.

Definition 2.14 (Consistency)**.** A sentence ϕ is consistent with respect to c, if $c[\phi] \neq \varnothing$; a sentence ϕ is *consistent* simpliciter, if there is some information state c', s.t., $c'[\phi]$ is consistent.

"It's raining and it's not raining" is *inconsistent*, since there is no information state c, such that updating c with this sentence will result in a non-absurd information state. This holds for all classical contradictions.

Concretely, if $[\![\phi]\!] = \varnothing$, then ϕ is inconsistent.

Let's return to one of the examples that motivated a dynamic semantics for epistemic modality.

(16) It's not raining outside, but it might be raining outside.
$\neg\, p \wedge \Diamond\, p$

A good result: (16) is *inconsistent*.

Before giving an informal proof, the intuition is as follows: updating an information state with the information that it's not raining is guaranteed to make a tentative update of "it's raining" fail.

- $c[\neg\, p \wedge \Diamond\, p] = c[\neg\, p][\Diamond\, p]$
- $= (c\ - I(p))[\Diamond\, p]$
- $= \begin{cases} c - I(p) & (c - I(p)[p] \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

- $= \begin{cases} c - I(p) & ((c - I(p) \cap I(p)) \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$
- $(c - s) \cap s = \varnothing, \forall s$, hence $(\neg\, p \wedge \Diamond\, p)$ is *inconsistent*.

What about the other ordering, repeated in (17)? Although we didn't assign this a # diacritic, arguably there is something pragmatically marked about this sentence. There's an intuition that the speaker has changed their mind after uttering the first conjunct.

(17) ? It might be raining and it's not raining.
$\Diamond\, p \wedge \neg\, p$

In fact, we can construct variations of (17) which sound more natural:

(18) A: It might be raining.
B: It's not raining!

(19) It might be raining [...] it's not raining.

We can make sense of this in update semantics by using the notion of *coherence*.

Definition 2.15 (Support)**.** An information state c *supports* a sentence ϕ iff:

$$c[\phi] = c$$

Other terms which are often used to mean the same thing: *c accepts ϕ*, *c incorporates ϕ*.

Definition 2.16 (Coherence)**.** ϕ is coherent iff there is some non-absurd information state c, s.t., c *supports* ϕ.

Note that *coherence* implies *consistency*: if a non-absurd c supports ϕ, then $c[\phi]$ is consistent, and hence ϕ is consistent simpliciter.

Now we can ask ourselves, is (17) consistent/coherent?

- $c[\Diamond\, p \wedge \neg\, p] = c[\Diamond\, p][\neg\, p]$
- $= \begin{cases} c[\neg\, p] & c[p] \neq \varnothing \\ \varnothing[\neg\, p] & \text{otherwise} \end{cases}$

- $= \begin{cases} c - c[p] & c[p] \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

(17) is *consistent*, since as long as both p and $\neg\, p$ are c-consistent, then updating c with (17) will result in a non-absurd information state — namely, one that supports $\neg\, p$.

Now can can ask, is (17) *coherent*? The answer is no. For the test imposed by $\Diamond\, p$ to be successful in c, c cannot support $\neg\, p$, and for c to support $p \wedge q$, $c[p]$ must support q.[12]

[12] A sketch of a proof by contradiction:

- If $\Diamond\, p \wedge \neg\, p$ is coherent; there exists a c, s.t., $c[\Diamond\, p \wedge \neg\, p] = c$.
- If $c[\Diamond\, p \wedge \neg\, p] = c$,
  then $(c[\Diamond\, p])[\neg\, p] = c$,
  so by eliminativity $c[\Diamond p] = c[\neg\, p] = c$
- if $c[\Diamond p] = c$, then $c[p] \neq \varnothing$
- if $c[p] \neq \varnothing$, then $c - p \neq c$
- Therefore $c[\neg\, p] \neq c$

Optional exericse

Recall that, due to presupposition projection facts, the update rule for disjunctive sentences is as follows:

$$c[\phi \vee \psi] \coloneqq c[\phi] \cup\ c[\neg\, \phi][\psi]$$

What does the theory predict for a sentence such as "either it's raining, or it might be raining"?

$$p \vee \Diamond\, p$$

What about the reverse order, "it might be raining, or it's raining"?

$$p \vee \Diamond\, p$$

Try to connect the results to your intuitions about what these sentences mean.

References

Beaver, David I. 2001. *Presupposition and Assertion in Dynamic Semantics*. 250 pp.

van Benthem, Johan. 1986. *Essays in Logical Semantics* (Studies in Linguistics and Philosophy). Springer Netherlands.

Chierchia, Gennaro. 2020. Origins of weak crossover - when dynamic semantics meets event semantics. *Natural Language Semantics* (28). 23–76.

Dekker, Paul. 1994. Predicate logic with anaphora. *Semantics and Linguistic Theory* 4. 79–95.

Elliott, Patrick D. 2020a. Crossover and accessibility in dynamic semantics. Massachusetts Institute of Technology.

Elliott, Patrick D. 2020b. Tetradynamics: an explanatory theory of anaphora. Unpublished manuscript. Massachusetts Institute of Technology.

Fox, Danny. 2013. Presupposition projection from quantificational sentences - Trivalence, local accommodation, and presupposition strengthening. In Ivano Caponigro & Carlo Cecchetto (eds.), *From grammar to meaning*, 201–232.

George, B. R. 2007. Predicting presupposition projection - Some alternatives in the strong Kleene tradition. unpublished manuscript. UCLA.

George, B. R. 2008. A new predictive theory of presupposition projection. In *Proceedings of SALT 18*, 358–375. Ithaca, NY: Cornell University.

George, B. R. 2014. Some remarks on certain trivalent accounts of presupposition projection. *Journal of Applied Non-Classical Logics* 24(1-2). 86–117.

Geurts, Bart. 1996. Local satisfaction guaranteed: A presupposition theory and its problems. *Linguistics and Philosophy* 19(3). 259–294.

Gillies, Anthony S. 2004. Epistemic Conditionals and Conditional Epistemics. *Noûs* 38(4). 585–616.

Groenendijk, J. a. G., M. J. B. Stokhof & F. J. M. M. Veltman. 1996. *Coreference and modality*. OxfordBlackwell.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases*. University of Massachusetts - Amherst dissertation.

Heim, Irene. 1983. On the projection problem for presuppositions. In *Proceedings of WCCFL 2*, 114–125. Stanford University.

Honcoop, Martin. 1998. *Dynamic excursions on weak islands*. University of Leiden dissertation.

Kamp, Hans. 1981. A theory of truth and semantic representation. In Paul Portner & Barbara H. Partee (eds.), *Formal semantics: The essential readings*, 189–222. Blackwell.

Mandelkern, Matthew. 2020a. Pseudo-dynamics. Unpublished manuscript. Oxford.

Mandelkern, Matthew. 2020b. Witnesses. Unpublished manuscript. Oxford.

Moore, George Edward. 1942. A Reply to My Critics. In Paul Arthur Schilpp (ed.), *The Philosophy of G. E. Moore*. Open Court.

Rothschild, Daniel. 2017. A trivalent approach to anaphora and presupposition. unpublished manuscript.

Rothschild, Daniel & Seth Yalcin. 2016. Three notions of dynamicness in language. *Linguistics and Philosophy* 39(4). 333–355.

Rothschild, Daniel & Seth Yalcin. 2017. On the Dynamics of Conversation. *Noûs* 51(1). 24–48.

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3).

Schlenker, Philippe. 2009. Local contexts. *Semantics and Pragmatics* 2.

Schlenker, Philippe. 2010. Local contexts and local meanings. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 151(1). 115–142.

Soames, Scott. 1989. Presupposition. In D. Gabbay & F. Guenthner (eds.), *Handbook of Philosophical Logic: Volume IV: Topics in the Philosophy of Language* (Synthese Library), 553–616. Dordrecht: Springer Netherlands.

Sudo, Yasutada. 2019. Implicatures with discourse referents.

van den Berg, M. H. 1996. Some aspects of the internal structure of discourse. The dynamics of nominal anaphora.

Veltman, Frank. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic* 25(3). 221–261.

Yalcin, Seth. 2013. Introductory Notes on Dynamic Semantics. Berkeley.