Double Negation, Excluded Middle and Accessibility in Dynamic Semantics*

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Abstract

This paper addresses a recalcitrant problem for dynamic semantics: the inaccessibility of discourse referents from under double negation or from a negated left disjunct into a right disjunct. I propose that these discourse referents are made accessible by the discourse being interpreted in the context of designated formulae, which validate double negation elimination in a controlled way.

1 Puzzles of Accessibility

First-generation dynamic semantic theories were developed, in part, in order to capture the anaphoric dependencies in discourses like (1).

(1) John owns a car. It is parked in a weird place.

As Karttunen (1976, 366) put it:

> Let us say that the appearance of an indefinite noun phrase establishes a discourse referent just in case it justifies the occurrence of a corererential pronoun or a definite noun phrase later in the text.

So in (a natural interpretation of) (1), *a car* establishes a discourse referent, which is picked up by *it*. As Karttunen (1976) goes on to note, however, anaphoric dependencies created by indefinites don't always persist once introduced. To take just one example, negation closes off anaphoric dependencies. For example, (2) can't be interpreted with *it* dependent on *a car* if *a car* is in the scope of negation.

(2) John doesn't own a car. It is parked in a weird place.

Dynamic semantic theories generally have no problem accounting for these data. Concretely, let us look at the natural translations of (1) and (2) into dynamic predicate logic (DPL, Groenendijk and Stokhof (1991)), and their interpretations, in (3) and (4). DPL has the same syntax as classical predicate logic (PL) and its semantics is given in Figure 1.[1]

(3) $\exists x(Cx \wedge Ojx) \wedge Px$

$$[\![(3)]\!]^f_M = \{g \mid f[x]g \ \&\ g(x) \in \mathcal{I}(C) \ \&\ \langle \mathcal{I}(j), g(x)\rangle \in \mathcal{I}(O) \ \&\ g(x) \in \mathcal{I}(P)\}$$

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[1] This presentation is slightly idiosyncratic but is entirely equivalent to that in Groenendijk and Stokhof (1991).

$$\begin{aligned}
[\![Pt_1 \dots t_n]\!]^f_M &= \{g \mid f = g \;\&\; \langle [\![t_1]\!]^g_M, \dots, [\![t_n]\!]^g_M \rangle \in \mathcal{I}(P)\} \\
[\![t_1 = t_2]\!]^f_M &= \{g \mid f = g \;\&\; [\![t_1]\!]^g_M = [\![t_2]\!]^g_M\} \\
[\![\neg\phi]\!]^f_M &= \{g \mid f = g \;\&\; [\![\phi]\!]^g_M = \emptyset\} \\
[\![\phi \wedge \psi]\!]^f_M &= \Big\{h \mid \text{there's a } g : g \in [\![\phi]\!]^f_M \;\&\; h \in [\![\psi]\!]^g_M\Big\} \\
[\![\phi \vee \psi]\!]^f_M &= \{g \mid f = g \;\&\; [\![\phi]\!]^g_M \cup [\![\psi]\!]^g_M \neq \emptyset\} \\
[\![\phi \to \psi]\!]^f_M &= \Big\{g \mid f = g \;\&\; [\![\phi]\!]^g_M \subseteq \Big\{h \mid [\![\psi]\!]^h_M \neq \emptyset\Big\}\Big\} \\
[\![\exists x\phi]\!]^f_M &= \{h \mid \text{there's a } g : f[x]g \;\&\; h \in [\![\phi]\!]^g_M\} \\
[\![\forall x\phi]\!]^f_M &= \Big\{g \mid f = g \;\&\; \{h \mid g[x]h\} \subseteq \Big\{h \mid [\![\phi]\!]^h_M \neq \emptyset\Big\}\Big\}
\end{aligned}$$

Figure 1: Semantics of DPL

(4) $\neg\exists x(Cx \wedge Ojx) \wedge Px$

$$\Big[\!\Big[(4)\Big]\!\Big]^f_M = \{g \mid f = g \;\&\; \{h \mid g[x]h \;\&\; h(x) \in \mathcal{I}(C) \;\&\; \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\} = \emptyset \;\&\; g(x) \in \mathcal{I}(P)\}$$

By inspecting the semantic clauses in Figure 1 we can see that the closing off of anaphoric dependencies is tied to negation. In DPL, the negation of any formula is a **test**: an identity relation on some set of assignments. Pretty much all dynamic semantic theories treat negation like this,[2] and hence they all have the property that $\neg\neg\phi$ is not generally equivalent to ϕ:

> [T]he law of double negation will not hold unconditionally. Consider a formula ϕ that is not a test. Negating ϕ results in the test $\neg\phi$, and a second negation, which gives $\neg\neg\phi$, does not reverse this effect [...] Hence, double negation is not in general eliminable. (Groenendijk and Stokhof, 1991, 62)

As has repeatedly been noted,[3] this failure of double negation elimination is problematic in that there are several examples where it seems that we would like a doubly-negated existential statement to behave more like its un-negated counterpart than these theories predict.

1.1 Double Negation

One class of examples concerns straightforward double negations, such as (5).

(5) It's not true that John doesn't own a car. It's (just) parked in a weird place.

The natural translation of (5) into DPL is given in (6). As the interpretation given shows, (6) does not captures the intended dependency, unlike (3).

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

$$\Big[\!\Big[(6)\Big]\!\Big]^f_M = \{g \mid f = g \;\&\; \{h \mid g[x]h \;\&\; h(x) \in \mathcal{I}(C) \;\&\; \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\} \neq \emptyset \;\&\; g(x) \in \mathcal{I}(P)\}$$

[2] An exception will be noted in Section 2.2.

[3] E.g. by Groenendijk and Stokhof (1990, 1991); Kamp and Reyle (1993); Krahmer and Muskens (1995).

1.2 Disjunction

Another class of examples concerns disjunctions like (7).

(7) Either John doesn't own a car, or it is parked in a weird place.

Given the most natural translation of (7) as shown in (8), the intended anaphoric dependency is not captured. Once again, other dynamic semantic theories are essentially the same at this point.

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

Note that in PL, (8) is equivalent to both (9) and (10).

(9) $\neg\exists x(Cx \wedge Ojx) \vee (\exists x(Cx \wedge Ojx) \wedge Px)$

(10) $\neg\exists x(Cx \wedge Ojx) \vee (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

Meanwhile, in DPL (8) is equivalent to (10) but not (9); and (9) *would* capture the intended dependency when interpreted in DPL. So, apparently, we again have a situation where the PL equivalence based on double negation would be desirable.

1.3 Uniqueness

However, there are examples that seem to show that we don't want ϕ to be *exactly* equivalent to $\neg\neg\phi$, such as (11)–(12).

(11) ??It's not true that John doesn't own a shirt. It's in the wardrobe.

(12) ??Either John doesn't own a shirt, or it's in the wardrobe.

Examples (11) and (12) sound strange in a way that their counterparts (5) and (7) respectively don't. The reason seems to be that these examples carry the implication[4] that, if John owns a car/shirt, then he owns exactly one. While that is a plausible (though possibly false) assumption in the case of cars, it is much less plausible in the case of shirts. No such implication is present in (1), or (13).

(13) John owns a shirt. It's in the wardrobe.

1.4 Plan

After very briefly reviewing the literature on this issue in Section 2, I will make a proposal for making discourse referents rendered inaccessible by (double) negation accessible again in Section 3. In Section 4 I will discuss how this proposal can be finessed in order to take account of the uniqueness effect. The proposal will be integrated into a compositional semantic system in Section 5. Section 6 concludes.

2 Previous Accounts

There have been a few attempts to address this issue in the literature, all of which approach it by doing something to the semantics of negation.

[4] I leave open the question of what exactly this 'implication' amounts to: whether entailment, presupposition, implicature, etc.

2.1 Separating Negation from Closure

Negation as defined in Figure 1 can be decomposed into three operators $\bullet$, $\sim$ and !, as defined in (14). ! closes off anaphoric dependencies, $\sim$ is (revised) negation and $\bullet$ checks that its operand is a test. N.B. $\simeq$ is DPL equivalence, i.e. $\phi \simeq \psi \Leftrightarrow$ for all M and f, $[\![\phi]\!]^f_M = [\![\psi]\!]^f_M$. An ancillary notion is *satisfaction equivalence*: $\phi \simeq_s \psi \Leftrightarrow$ for all M and f, $[\![\phi]\!]^f_M \neq \emptyset$ just in case $[\![\psi]\!]^f_M \neq \emptyset$ (Groenendijk and Stokhof, 1991, 56).

(14)
$$[\![\bullet\phi]\!]^f_M = \left\{ g \mid f = g \;\&\; g \in [\![\phi]\!]^f_M \right\}$$
$$[\![\sim\phi]\!]^f_M = \left\{ g \mid g \notin [\![\phi]\!]^f_M \right\}$$
$$[\![!\phi]\!]^f_M = \{ g \mid f = g \;\&\; [\![\phi]\!]^g_M \neq \emptyset \}$$

Facts 1 and 2 then follow.

Fact 1. $\sim\sim\phi \simeq \phi$

Fact 2. $\bullet\sim!\phi \simeq \neg\phi$

Given the equivalence noted in Fact 1, we could imagine that *not* as expressed in (5) is translated as $\sim$, but as expressed in (2) is translated as $\neg$ (or, equivalently, $\sim$ augmented with $\bullet$ and !). Groenendijk and Stokhof (1990) and Rothschild (2017) both make suggestions somewhat like this, and in systems that are sufficiently different from DPL for the decomposition to be achieved with two operators rather than three.

Nevertheless, such an approach immediately raises the question of why there is no reading of, say, (2) in which *not* is translated just with the revised negation. In the system of Groenendijk and Stokhof (1990) the result would be an interpretation equivalent to 'It's not true that John owns a car which is parked in a weird place', and in that of Rothschild (2017) the result would be an interpretation equivalent to 'there is something which is not a car owned by John, and which is parked in a weird place' (which is also what we'd get using just $\sim$ as defined above). Needless to say, neither of these is a possible interpretation of (2). Both Groenendijk and Stokhof (1990) and Rothschild (2017) make suggestions[5] about how to avoid such interpretations, but these are incomplete. Furthermore, both theories require additional assumptions to account for the disjunction cases outlined in Section 1.2—in neither theory does the decomposition of negation achieve this alone. Nor do they have anything to say about the uniqueness effect.

2.2 Bilateralism

The approach adopted by Krahmer and Muskens (1995), when adapted from Discourse Representation Theory (DRT, Kamp and Reyle (1993)) to DPL, is to give formulae both positive (verifying) and negative (falsifying) extensions, with negation amounting to reversal of these. Double negation elimination then follows immediately. Examples like (7) are taken care of by a tweak to the semantics of $\vee$ making $\phi \vee \psi$ equivalent to $\neg\phi \rightarrow \psi$. This makes (8) equivalent to (15) which, given that the system now has double negation elimination, is equivalent to (16).

(15) $\neg\neg\exists x(Cx \wedge Ojx) \rightarrow Px$

(16) $\exists x(Cx \wedge Ojx) \rightarrow Px$

[5] Rather more developed in the case of Groenendijk and Stokhof (1990) than Rothschild (2017).

Formula (16) is not equivalent to (9) (or (10)): interpreted in DPL, (9) means 'either John does not own a car, or he owns a car that is parked in a weird place', while (16) means 'every car John owns is parked in a weird place'. The difference is reminiscent of the difference between strong and weak readings of donkey sentences. Given the uniqueness effect it seems to be somewhat moot, however. Krahmer and Muskens (1995, 359) note the uniqueness effect but do not account for it. In any case, their semantics for $\vee$ could be tweaked in a different way to get the 'weak' reading instead of the 'strong' one they do get (and defend).

In the following section I will present an account of pronoun accessibility in sentences like (5) and (7) in a unilateral semantics (in fact, without changing the semantics of DPL at all), which moreover accounts for the uniqueness effect.

3 Double Negation and Excluded Middle

The non-equivalence of ϕ and $\neg\neg\phi$ in DPL is reminiscent of the situation in intuitionistic logic (IL). Now, the parallel is by no means exact, since in IL this non-equivalence can be expressed as $\phi \dashv\!\!\not\vdash \neg\neg\phi$, whereas in DPL it can't really be brought out directly in terms of entailment or derivability. Nevertheless, it's worth looking at what one needs to add to IL in order to get the equivalence back.[6] Famously, adding any of (17)–(19) to IL gets you classical logic:

(17) $\neg\neg\phi \vdash \phi$ (double negation elimination)

(18) $\dfrac{\Gamma, \neg\phi \vdash \bot}{\Gamma \vdash \phi}$ (reductio ad absurdum)

(19) $\vdash \phi \vee \neg\phi$ (excluded middle)

This invites the following thought: could there be a way to achieve (something like) the double negation property for dynamic semantics by adding (something like) excluded middle? And could that help to resolve the issues we've identified with pronoun accessibility? The answer is *yes*, but it doesn't involve the standard DPL disjunction. Rather, it involves 'program disjunction' (Groenendijk and Stokhof, 1991, 88), defined in (20).

(20) Extend the language of DPL with the following clauses:

a. If ϕ and ψ are formulae, then $\phi \cup \psi$ is a formula.

b. $[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$

Like $\vee$, $\cup$ is internally static, but unlike $\vee$ it is externally dynamic. External dynamicity is crucial to the equivalence shown in Fact 3, which shows something of the extent to which (this form of) excluded middle gets us (something like) the double negation property in DPL.[7]

Fact 3. *If* $\phi \simeq \phi \wedge \phi$ *then* $(\phi \cup \neg\phi) \wedge \neg\neg\phi \simeq \phi$

In DPL $\phi \cup \neg\phi$ is a tautology, but there are many semantically distinct tautologies in DPL. Consequently, DPL does not have the property that ϕ is equivalent to $T \wedge \phi$ for any DPL tautology T and formula ϕ. So much can be seen from Fact 3. The relevance of this for us is that indefinites made inaccessible by double negation can be made accessible again on the assumption that the discourse is interpreted in the context of a specific tautology, namely, an appropriate instances of excluded middle with $\cup$.

[6] The connection is deeper than there is space to get into here. See (Ranta, 1994, 74–75) and Fernando (2001).

[7] Groenendijk and Stokhof (1991, 63–64) discuss the conditions under which $\wedge$ is idempotent. In this paper we're concerned about the case where $\phi := \exists x(Cx \wedge Ojx)$, and in that case $\phi \simeq \phi \wedge \phi$.

For example, if we assume that the discourse in (5) is interpreted in the context of $\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)$ and so augment (6) to (21), the interpretation we end up with is equivalent to (3), as shown below.

(21) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

$$\begin{aligned} [\![(21)]\!]^f_M &= \left\{ g \mid \begin{array}{l} g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \ \&\ [\![\exists x(Cx \wedge Ojx)]\!]^g_M \neq \emptyset \\ \&\ g(x) \in \mathcal{I}(P) \end{array} \right\} \\ &= \left\{ g \mid g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \ \&\ g(x) \in \mathcal{I}(P) \right\} \\ &= [\![(3)]\!]^f_M \end{aligned}$$

In the same way, if we augment (8) to (22), the interpretation we end up with is satisfaction-equivalent to (9), as shown below.

(22) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

$$\begin{aligned} [\![(22)]\!]^f_M &= \left\{ h \mid \begin{array}{l} \text{there's a } g : g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \\ \&\ h \in [\![\neg\exists x(Cx \wedge Ojx) \vee Px]\!]^g_M \end{array} \right\} \\ &= \left\{ g \mid (f = g \ \&\ [\![\exists x(Cx \wedge Ojx)]\!]^g_M = \emptyset) \text{ or } \left(g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \ \&\ g(x) \in \mathcal{I}(P) \right) \right\} \end{aligned}$$

Note that in simple positive cases like (3) and single-negation cases like (4), adding the instance of excluded middle does nothing bad, thanks to Facts 4 and 5.

Fact 4. *If* $\phi \simeq \phi \wedge \phi$ *then* $(\phi \cup \neg\phi) \wedge (\phi \wedge \psi) \simeq \phi \wedge \psi$

Fact 5. *If* $\phi \simeq \phi \wedge \phi$ *then* $(\phi \cup \neg\phi) \wedge (\neg\phi \wedge \psi) \simeq \neg\phi \wedge \psi$

Before we move on, I want to note two things about this treatment of disjunction. Firstly, binding is predicted to be symmetric, i.e. *either it's parked in a weird place, or John doesn't own a car* is predicted to be just as good as (7).

Secondly, in either case the interpretation amounts to 'either John doesn't own a car, or some car he owns is parked in a weird place'. As noted above, this take on the truth conditions of (7) is disputed by Krahmer and Muskens (1995).

Both of these bugs/features follow from the (independently-given) semantics of $\vee$ in DPL. Both would be changed on the assumption that *p or q* is translated into DPL not as $p \vee q$ but as $\neg p \rightarrow q$. Nevertheless, as noted above, the strong vs. weak issue at least is somewhat moot given the uniqueness effect, to be discussed next.

4 Uniqueness

So far, the uniqueness effect is not accounted for. Intuitively, what it seems that this effect amounts to is a restriction on anaphoric licensing: while a simple indefinite *a P* licenses subsequent pronouns, one under double negation only does so on the assumption that there is exactly one *P*. With that in mind, let us reflect on what program disjunction does in cases of an existential statement and its negation.

$$[\![\exists x Px \cup \neg\exists x Px]\!]^f_M = \begin{cases} \text{the set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P, \text{ if there are any} \\ \{f\} \ \text{ otherwise} \end{cases}$$

If we want the anaphoric dependency to be passed on only in the case of uniqueness, then the input context for our unaugmented formulae should look like this instead:

$$\begin{cases}\text{the (singleton) set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P\text{, if there's exactly one}\\ \{f\} \text{ otherwise}\end{cases}$$

That effect can be achieved by the introduction of an operator ℩, defined as follows.

(23) Extend the language of DPL with the following clauses:

a. If ϕ is a formula, then $\text{℩}\phi$ is a formula.

b. $\llbracket \text{℩}\phi \rrbracket^f_M = \begin{cases} \llbracket \phi \rrbracket^f_M & \text{if } \left|\llbracket \phi \rrbracket^f_M\right| = 1 \\ \{f\} & \text{otherwise} \end{cases}$

Or, equivalently,

$$\llbracket \text{℩}\phi \rrbracket^f_M = \left\{g \mid g \in \llbracket \phi \rrbracket^f_M \;\&\; \left|\llbracket \phi \rrbracket^f_M\right| = 1\right\} \cup \{g \mid f = g \;\&\; |\llbracket \phi \rrbracket^g_M| \neq 1\}$$

Note that $\text{℩}\phi$ is also a DPL tautology (for any ϕ). I will henceforth refer to formulae of the form $\text{℩}\phi$ as instances of 'unique excluded middle' (UEM).[8] If we now use UEM for our augmentations of (6) and (8) instead of excluded middle with $\cup$, we get the uniqueness effect, as seen below in (24) and (25) respectively.

(24) $\text{℩}\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

$$\begin{aligned}\llbracket(24)\rrbracket^f_M &= \left\{ g \;\middle|\; \begin{array}{l} g \in \left\{h \mid h \in \llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M \;\&\; \left|\llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M\right| = 1\right\} \\ \quad \cup \left\{h \mid f = h \;\&\; \left|\llbracket \exists x(Cx \wedge Ojx) \rrbracket^h_M\right| \neq 1\right\} \\ \& \; \llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M \neq \emptyset \;\&\; g(x) \in \mathcal{I}(P) \end{array} \right\} \\ &= \left\{ g \;\middle|\; \begin{array}{l} \left(\llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M = \{g\} \;\&\; g(x) \in \mathcal{I}(P)\right) \\ \text{or } \left(f = g \;\&\; \left|\llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M\right| > 1 \;\&\; g(x) \in \mathcal{I}(P)\right) \end{array} \right\}\end{aligned}$$

'Either John owns exactly one car, which is parked in a weird place, or John owns more than one car and x is parked in a weird place' (with x free).

(25) $\text{℩}\exists x(Cx \wedge Ojx) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

$$\begin{aligned}\llbracket(25)\rrbracket^f_M &= \left\{ g \;\middle|\; \begin{array}{l} g \in \left\{h \mid h \in \llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M \;\&\; \left|\llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M\right| = 1\right\} \\ \quad \cup \left\{h \mid f = h \;\&\; \left|\llbracket \exists x(Cx \wedge Ojx) \rrbracket^h_M\right| \neq 1\right\} \\ \& \; \llbracket \neg\exists x(Cx \wedge Ojx) \rrbracket^f_M \cup \llbracket Px \rrbracket^f_M \neq \emptyset \end{array} \right\} \\ &= \left\{ g \;\middle|\; \begin{array}{l} \left(\llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M = \{g\} \;\&\; g(x) \in \mathcal{I}(P)\right) \\ \text{or } (f = g \;\&\; \llbracket \exists x(Cx \wedge Ojx) \rrbracket^g_M = \emptyset) \\ \text{or } \left(f = g \;\&\; \left|\llbracket \exists x(Cx \wedge Ojx) \rrbracket^f_M\right| > 1 \;\&\; g(x) \in \mathcal{I}(P)\right) \end{array} \right\}\end{aligned}$$

'Either John doesn't own a car, or he owns exactly one car, which is parked in a weird place, or he owns more than one car and x is parked in a weird place' (with x free). As noted above,

[8] In a previous version of this material, UEM was defined as a formula of the form $\text{℩}(\phi \cup \neg\phi)$. That was before I realised that this is equivalent to $\text{℩}\phi$.

this interpretation abolishes the distinction between weak and strong readings by making *a car* inaccessible as an antecedent to *it* if John owns more than one car.

As before, adding UEM to a simple positive formula like (3) doesn't change anything (Fact 6). Adding UEM to a single-negated sentence like (4) doesn't change anything either (Fact 7).

Fact 6. *If* $\phi \simeq \phi \wedge \phi$ *then* $\upharpoonleft\phi \wedge (\phi \wedge \psi) \simeq \phi \wedge \psi$

Fact 7. *If* $\phi \simeq \phi \wedge \phi$ *then* $\upharpoonleft\phi \wedge (\neg\phi \wedge \psi) \simeq \neg\phi \wedge \psi$

4.1 Uniqueness?

Matt Mandelkern (p.c.) has argued against building uniqueness into interpretations in this way, on the basis of examples like (26).

(26) Either Sue didn't have a drink last night, or she had a second drink right after it.

Clearly, (26) makes no sense if *it* is interpreted to mean the unique drink that Sue had last night, which is what the UEM-based analysis implies. On the other hand, if we stick with the analysis based on excluded middle with $\cup$, then (26) can mean 'either Sue didn't have a drink last night, or she did have a drink last night and had a second drink right after it', which is coherent.[9] The question remains, though, whether or not (26) can mean this.

Personally, I find (26) strange precisely because (I think) of the uniqueness implication, and so in what follows I will continue to use UEM. However, if this turns out not to be sustainable then we can switch to excluded middle with $\cup$ and look for another way to treat the uniqueness effect in examples like (11)–(12).

5 Composition

The obvious question that this treatment of double negation etc. raises is where these instances of UEM might come from. Answering that question requires moving from DPL to a dynamic semantic system that permits compositionality below the level of the clause. To that end I will move to compositional discourse representation theory (CDRT, Muskens (1996)). Here, the boxes of DRT are taken to be abbreviations of type-logical expressions of type $s \to s \to t$ (abbreviated T), where s is the type of states and discourse referents u_n are of type $s \to e$ (abbreviated E). The abbreviations are summarized in Figure 2.[10]

I will treat the instances of UEM as being introduced lexically by negation as a kind of projective content. That is to say, in addition to introducing $[\,|\,\neg D]$ in the standard dimension of meaning, instances of negation introduce $\upharpoonleft D$ in another dimension of meaning. The simplest way to do this is to make the result type of lexical entries not T but $T \times T$, with (some of) these instances of UEM in the first dimension and other content in the second.[11] In contrast to other kinds of projective content, however, I don't assume that these instances of UEM project very far, as can be seen from the lexical entries given to (*and*) and *or* in the lexicon shown in (27).[12]

[9] As Matt also pointed out, on the assumptions made by Krahmer and Muskens (1995) it has to mean that Sue had infinitely many drinks.

[10] Axioms ensure that discourse referents and states really do behave like variables and variable assignments.

[11] This can be done in a more principled way with a writer monad (Giorgolo and Asudeh, 2012). Thanks to Simon Charlow for pointing this out.

[12] (*and*) is a silent conjunction assumed to hold sentences together at the text level. Different projective behaviour might be required of overt *and*.

$$
\begin{array}{lcl}
\text{Conditions:} & & \\
R(\delta_1, \ldots, \delta_n) & \Longrightarrow & \lambda i.R(\delta_1(i), \ldots, \delta_n(i)) \\
\delta_m = \delta_n & \Longrightarrow & \lambda i.\delta_m(i) = \delta_n(i) \\
\neg D & \Longrightarrow & \lambda i.\neg\exists j.D(i)(j) \\
D \vee E & \Longrightarrow & \lambda i.\exists j.D(i)(j) \vee E(i)(j) \\
\text{DRSs:} & & \\
[u_1 \ldots u_n \mid C_1, \ldots, C_n] & \Longrightarrow & \lambda i.\lambda j.(\forall u.(u \neq u_1 \wedge \ldots \wedge u \neq u_n) \to u(i) = u(j)) \\
 & & \qquad \wedge\, C_1(j) \wedge \ldots \wedge C_n(j) \\
D \,;\, E & \Longrightarrow & \lambda i.\lambda k.\exists j.D(i)(j) \wedge E(j)(k) \\
D \cup E & \Longrightarrow & \lambda i.\lambda j.D(i)(j) \vee E(i)(j) \\
\upharpoonleft D & \Longrightarrow & \lambda i.\lambda j.(D(i)(j) \wedge \exists! k.D(i)(k)) \vee (i = j \wedge \neg\exists! k.D(j)(k))
\end{array}
$$

Figure 2: Abbreviations for CDRT (Muskens, 1996, 157) augmented with $\cup$ and $\upharpoonleft$

(27) $\textit{John} \rightsquigarrow \mathsf{j} : E \qquad \ldots\textit{not}\ldots, \textit{doesn't} \rightsquigarrow \lambda p^{T\times T}.\,\langle \pi_1(p) \,;\, \upharpoonleft(\pi_2(p)), [\,|\,\neg(\pi_2(p))]\rangle$

$\textit{own} \rightsquigarrow \lambda y^E.\lambda x^E.\,\langle [\,|\,], [\,|\,\mathsf{own}(x,y)]\rangle \qquad \textit{is parked}\ldots \rightsquigarrow \lambda x^E.\,\langle [\,|\,], [\,|\,\mathsf{parked}(x)]\rangle$

$\textit{a}^1 \rightsquigarrow \lambda P^{E\to T\times T}.\lambda Q^{E\to T\times T}.\,\langle [\,|\,], [u_1\,|\,] \,;\, \pi_1(P(u_1)) \,;\, \pi_2(P(u_1)) \,;\, \pi_1(Q(u_1)) \,;\, \pi_2(Q(u_1))\rangle$

$\textit{car} \rightsquigarrow \lambda x^E.\,\langle [\,|\,], [\,|\,\mathsf{car}(x)]\rangle \qquad (\textit{and}) \rightsquigarrow \lambda q^{T\times T}.\lambda p^{T\times T}.\,\langle [\,|\,], \pi_1(p) \,;\, \pi_2(p) \,;\, \pi_1(q) \,;\, \pi_2(q)\rangle$

$\textit{or} \rightsquigarrow \lambda q^{T\times T}.\lambda p^{T\times T}.\,\langle [\,|\,], \pi_1(p) \,;\, \pi_1(q) \,;\, [\,|\,\pi_2(p) \vee \pi_2(q)]\rangle \qquad \textit{it}_1 \rightsquigarrow u_1$

Assuming the structures for (5) and (7) shown in (28) and (29) respectively, their interpretations are shown below.

(28) $[[\ldots\text{not}\ldots[\,\text{John}\,[\,i\,[\,\text{doesn't}\,[[\,\text{a}^1\ \text{car}\,]\,[\,j\,[\,\text{t}_i\,[\,\text{own}\ \text{t}_j\,]]]]]]]]\,[\,(\text{and})\,[\,\text{it}_1\ \text{is parked}\ldots\,]]]$

$$
\begin{aligned}
&\rightsquigarrow \langle [\,|\,], \upharpoonleft[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)] \,;\, \upharpoonleft[\,|\,\neg[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)]]; \\
&\qquad [\,|\,\neg[\,|\,\neg[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)]], \mathsf{parked}(u_1)]\rangle \\
&\equiv \langle [\,|\,], \upharpoonleft[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)] \,;\, [\,|\,\neg[\,|\,\neg[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)]], \mathsf{parked}(u_1)]\rangle
\end{aligned}
$$

(29) $[[\,\text{John}\,[\,i\,[\,\text{doesn't}\,[[\,\text{a}^1\ \text{car}\,]\,[\,j\,[\,\text{t}_i\,[\,\text{own}\ \text{t}_j\,]]]]]]]\,[\,\text{or}\,[\,\text{it}_1\ \text{is parked}\ldots\,]]]$

$$
\rightsquigarrow \langle [\,|\,], \upharpoonleft[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)] \,;\, [\,|\,[\,|\,\neg[u_1 \mid \mathsf{car}(u_1), \mathsf{own}(\mathsf{j}, u_1)]] \vee [\,|\,\mathsf{parked}(u_1)]]\rangle
$$

Expanding the second projections of (28) and (29) according to the key in Figure 2 shows these interpretations to be equivalent to those given above for (24) and (25) respectively.

One thing to note about the lexicon in (27) is that the lexical entry for *or* predicts symmetric binding, as discussed at the end of Section 3. An alternative lexical entry predicting left-to-right binding only would be $\lambda q^{T\times T}.\lambda p^{T\times T}.\,\langle [\,|\,], \pi_1(p) \,;\, [\,|\,\pi_2(p) \vee (\pi_1(q) \,;\, \pi_2(q))]\rangle$.

6 Conclusion

In this paper I have suggested a way for discourse referents that are problematically inaccessible in standard dynamic semantics to be rendered accessible again, without needing to change the semantic clauses for the existing connectives of any dynamic semantic theory at all. I proposed that these discourse referents are made accessible by the discourse being interpreted in the context of designated formulae, either excluded middle with $\cup$ or UEM, which are tautologies in the sense of being always true according to the designated truth definition. The fact that they can nevertheless be leveraged to help with this (in)accessibility issue is simply another example of meaning in dynamic semantics going beyond truth conditions.

I also proposed a compositional system for the introduction of these formulae, treating them as a kind of projective content introduced by the lexical semantics of negative operators. This part of the story is necessarily (even) less secure than the general idea. A reviewer notes that this seems to predict that other languages could behave differently to English in terms of pronoun accessibility from under double negation or from a negated left disjunct into a right disjunct. It certainly would be welcome if UEM could be introduced more systematically, but at the moment I don't see a way to do this.

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