Actual and hypothetical discourse referents

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1 Some data

A generalization that emerges from EDS: an assertion of a sentence ϕ containing an existential statement "a^v linguist P-ed" introduces a dref v if the assertion is accepted and contextually entails the existence of a linguist that P-ed.

Interaction with modality draws this generalization into question.

Modal subordination and anaphora (Roberts 1989):

(1) Maybe there's a^v bathroom, and maybe it$_v$'s upstairs.

(2) There might be a^v bathroom, and it$_v$ might be upstairs.

Modal subordination with negation; data from (Hofmann 2019):

(3) There is no^v bathroom in this house. It$_v$ would be easier to find.

Not possible with disjunction:

(4) ??Maybe there's a^v bathroom, or maybe it$_v$'s upstairs.

(5) ??There might be a^v bathroom, or it$_v$ might be upstairs.

Surprisingly, conjunctive possibility statements can pattern with disjunction:

(6) There might be no^v bathroom, and it$_v$ might be upstairs.

(7) Maybe there's no^v bathroom, and maybe it$_v$'s upstairs.

(8) Either There's no^v bathroom, or it$_v$'s upstairs.

Note that this parallel is perhaps unsurprising, given modal theories of disjunction which validate $\phi \vee \psi \vdash \Diamond\phi \wedge \Diamond\psi$ (Zimmermann 2000, Geurts 2005, Goldstein 2019).

2 Epistemic modality in dynamic semantics

2.1 Test semantics

The *locus classicus* is Veltman's test semantics (Veltman 1996).

Veltman's idea: a sentence $\Diamond\phi$ is an instruction to hypothetically update a context c with ϕ, returning c unchanged if c can be consistently updated with ϕ, and the absurd state otherwise.

(9) c[it might be raining]
  a. Compute c[it's raining]; store the result as c'.
  b. Is c' are non-absurd information state? If so, return c.
  c. Otherwise, return c'.

An update semantics for a simple propositional fragment (Veltman 1996).

Definition 2.1. Test semantics for *might*.

$$c[\Diamond\phi] := \begin{cases} c & c[\phi] \neq \emptyset \\ \phi & \text{otherwise} \end{cases}$$

In Veltman's update semantics, $\emptyset$ is the *absurd state*, i.e., the information state from which everything follows.

If we define update-semantic negation, we can treat *must* as the dual of *might*.

(10) $c[\neg\phi] := c - c[\phi]$

Definition 2.2. Test semantics for *must*.

$$c[\Box\phi] := c[\neg\Diamond\neg\phi]$$

$$c[\Box\phi] := \begin{cases} c & c[\phi] = c \\ \emptyset & \text{otherwise} \end{cases}$$

In Veltman's terms, *must* ϕ is true in a context c if ϕ is *accepted* in c.

Note that test semantics for *must* is **strong** (von Fintel & Gillies 2010, 2021).

Definition 2.3. Entailment in update semantics.

ϕ entails $\psi \iff \forall c[c[\phi] = c \rightarrow c[\psi] = c]$

If $c[\Box\phi] = c$ then $c[\phi] = c$, simply by the update rule for $\Box\phi$.

3 Modals introduce hypothetical discourse referents

Conjecture: asserting "possibly ϕ" is analogous to asserting "ϕ or not ϕ".

In a classical setting $\phi \vee \neg\phi$ is of course informationally trivial, but in a dynamic setting (specifically, in the context of EDS), $\phi \vee \neg\phi$, can introduce anaphoric information.

A consideration of the disjunctive case will help give a feel for the explanation.

(11) Either there's a^v bathroom, or there's no^v bathroom.

(12) $\exists_v[B(v)] \vee \neg\exists_v[B(v)]$

Predicted (positive) meaning in EDS is as follows. Note:

- Since the disjuncts are mutually exclusive, we can ignore the case of both disjuncts being true.
- Since neither disjunct contains a free variable, we can ignore the case of either disjunct being undefined (ignoring the novelty condition for simplicity).
- There are therefore two ways of dynamically verifying the disjunction to consider:
  - The first disjunct is true, and the second is false (a bathroom dref).[1]
  - The first disjunct is false and the second is true (no bathroom dref).

(13) $$\lambda(w,g)\,.\,\{\, g^{[v\rightarrow x]} \mid \mathbf{bathroom}_w(x) \,\} \\ \cup \{\, g \mid \mathbf{bathroom}_w = \emptyset \,\}$$

An assertion of (13) relative to a context c will introduce a bathroom discourse referent at worlds $\in c$ where a bathroom exists, but otherwise leave the context unchanged, e.g.:

(14) $$\{\, (w_\emptyset, g), (w_1, g), (w_2, g), (w_{12}, g) \,\} \\ \Rightarrow \{\, (w_\emptyset, g), (w_{b_1}, g^{[v\rightarrow b_1]}), (w_{b_2}, g^{[v\rightarrow b_2]}), (w_{b_1,b_2}, g^{[v\rightarrow b_1]}), (w_{b_1,b_2}, g^{[v\rightarrow b_2]}) \,\}$$

[1]There's a subtlety here involving the novelty condition and downdate that we're glossing over here. Can you spot it?

N.b. we already account for the impossibility of anaphora in the following discourse, due to the universal presupposition introduced by the pronoun.[2]

(15) Either there's a_v bathroom, or there isn't a bathroom.
??It_v's upstairs.

4 Integrating epistemic modality and EDS

EDS can be framed as an update semantics quite easily. Let's start with first-order EDS.

4.1 First order EDS

A concise presentation of EDS as a semantics for a first order calculus.

(16) Static semantics for atomic sentences:

$$[P(v_1, \dots, v_n)]^{w,g} = \begin{cases} \textbf{defined} & g(v_1), \dots, g(v_n) \neq \#_e \\ \textbf{true} & [P(v_1, \dots, v_n)]^{w,g} \text{ is } \textbf{defined} \text{ and } \langle g(v_1), \dots, g(v_n) \rangle \in I_w(P) \end{cases}$$

(17) Atomic sentences in EDS:

$$[\![P(v_1, \dots, v_n)]\!]^w_+ := \{\, (g,h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h} \text{ is } \textbf{true} \,\}$$
$$[\![P(v_1, \dots, v_n)]\!]^w_- := \{\, (g,h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h} \text{ is } \textbf{false} \,\}$$
$$[\![P(v_1, \dots, v_n)]\!]^w_u := \{\, (g,h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,g} \text{ is } \textbf{undefined} \,\}$$

(18) Negative sentences:

$$[\![\neg\phi]\!]^w_+ := [\![\phi]\!]^-$$
$$[\![\neg\phi]\!]^w_- := [\![\phi]\!]^+$$
$$[\![\neg\phi]\!]^w_u := [\![\phi]\!]^u$$

(19) Conjunctive sentences:

$$[\![\phi \wedge \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_+$$

$$\begin{aligned} [\![\phi \wedge \psi]\!]^w_- := {} & [\![\phi]\!]^w_- \circ [[\![\psi]\!]^w_{+,-,u} \\ & \cup [\![\phi]\!]^w_{+,u} \circ [\![\psi]\!]^w_- \end{aligned} \qquad \begin{aligned} [\![\phi \wedge \psi]\!]^w_u := {} & [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_u \\ & \cup [\![\phi]\!]^w_u \circ [\![\psi]\!]^w_{+,u} \end{aligned}$$

[2]Given partial assignments, a standard bridge principle predicts that v should be defined at *every* assignment in the file context.

(20) Random assignment:
$[\![\varepsilon_v]\!]^w_+ := \{\, (g,h) \mid g[v]h \,\}$
$[\![\varepsilon_v]\!]^w_- := \emptyset$
$[\![\varepsilon_v]\!]^w_u := \emptyset$

(21) Positive closure:
$[\![\dagger\phi]\!]^w_+ := [\![\phi]\!]^w_+$
$[\![\dagger\phi]\!]^w_- := \{\, (g,h) \mid g = h \wedge [\![\phi]\!]^w_+ = \emptyset \wedge [\![\phi]\!]^w_- \neq \emptyset \,\}$
$[\![\dagger\phi]\!]^w_u := [\![\dagger]\!]^w_u$

To appreciate the isomorphism between this presentation, and the previous presentation, consider that a trivalent relational semantics can be framed instead as a set of relations, each paired with one of three truth values.

4.2 Lifting EDS into an update semantics

As usual, we'll model information states using Heimian files, supplemented with a failure state $\#_c$.

EDS can be lifted into a multivalent update semantics, where we define $c[.]_+$, $c[.]_-$, and $c[.]_u$.

(22) a. $c[\phi]_+ := \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_+ \,\}$
b. $c[\phi]_- := \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_- \,\}$
c. $c[\phi]_u := \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_u \,\}$

4.3 EDS as an update semantics

4.3.1 Atomic sentences

Atomic sentences in EDS update semantics induce a tripartition of the input file, since no anaphoric information can be introduced.

(23) $c[P(v_1,\ldots,v_n)]_+ := \{\, (w,g) \in c \mid |P(v_1,\ldots,v_n)|^{w,g}\text{is } \mathbf{true} \,\}$
(24) $c[P(v_1,\ldots,v_n)]_- := \{\, (w,g) \in c \mid |P(v_1,\ldots,v_n)|^{w,g}\text{is } \mathbf{false} \,\}$
(25) $c[P(v_1,\ldots,v_n)]_u := \{\, (w,g) \in c \mid |P(v_1,\ldots,v_n)|^{w,g}\text{is } \mathbf{undefined} \,\}$

4.3.2 Negative sentences

(26) $c[\neg\phi]_+ := c[\phi]_-$

(27) $c[\neg\phi]_- := c[\phi]_+$

(28) $c[\neg\phi]_u := c[\phi]_u$

4.3.3 Conjunctive sentences

(29) $c[\phi \wedge \psi]_+ := c[\phi]_+[\psi]_+$

(30) $c[\phi \wedge \psi]_- := c[\phi]_-[\psi]_+ \cup c[\phi]_-[\psi]_- \cup c[\phi]_-[\psi]_u$
$c[\phi]_+[\psi]_- \cup c[\phi]_u[\psi]_-$

(31) $c[\phi \wedge \psi]_u := c[\phi]_+[\psi]_u$
$c[\phi]_u[\psi]_+ \cup c[\phi]_u[\psi]_u$

4.3.4 Random assignment

(32) $c[\varepsilon_v]_+ := \{\, (w,h) \mid \exists g[(w,g) \in c \wedge g[v]h] \,\}$

(33) $c[\varepsilon_v]_- := \emptyset$

(34) $c[\varepsilon_v]_- := \emptyset$

4.3.5 Closure

(35) $c[\dagger\phi]_+ := c[\phi]_+$

(36) $c[\dagger\phi]_- := \{\, (w,g) \in c \mid c[\phi]_+ = \emptyset \wedge c[\phi]_- \neq \emptyset \,\}$

(37) $c[\dagger\phi]_u := c[\phi]_u$

4.3.6 Test semantics

Let's stick to Veltman's idea that a modalized statement, if true, adds no information to the CG.

(38) *Might* (first attempt):

a. $c[\Diamond\phi]_+ := \{\, (w,g) \in c \mid c[\phi]_+ \neq \emptyset \,\}$

b. $c[\Diamond\phi]_- := \{\, (w,g) \in c \mid c[\phi]_+ = \emptyset \wedge c[\phi]_- \neq \emptyset \,\}$

c. $c[\diamond\phi]_u := c[\phi]_u$

By definition, modalized sentences are *tests* on information states (they can't introduce any anaphoric information).

4.3.7 Pragmatics

What kind of bridge principle do we want for a trivalent update semantics?

Definition 4.1. Assertion. An assertion of ϕ in c, $c[\phi]$, is defined as follows:

$$c[\phi] := \begin{cases} c[\phi]_+ & c[\phi]_u = \emptyset \\ & \#_c \\ \text{otherwise} & \end{cases}$$

4.3.8 An alternative semantics for *might*

(39) *Might* (first attempt):

a. $c[\Diamond\phi]_+ := \begin{cases} c[\phi]_+ \cup c[\phi]_- & c[\phi]_+ \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

b. $c[\Diamond\phi]_- := \begin{cases} c[\phi]_+ \cup c[\phi]_- & c[\phi]_+ = \emptyset \wedge c[\phi]_- \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

c. $c[\Diamond\phi]_u := c[\phi]_u$

5 Advantages

5.1 Possible bathrooms

The following is a felicitous discourse.

(40) a. There might be no^v bathroom.
b. It$_v$ might be upstairs.

Let's compute the positive contribution of the first sentence.

(41) There might be no bathroom.

(42) $\Diamond(\neg\exists_v[B(v)])$

(43) $$c[\Diamond(\neg\exists_v[B(v)])]_+ = \begin{cases} c[\neg\exists_v[B(v)]]_+ \cup c[\neg\exists_v[B(v)]]_- & c[\phi]_+ \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

(44) $$c[\Diamond(\neg\exists_v[B(v)])]_+ = \begin{cases} c[\neg\exists_v[B(v)]]_+ \cup c[\exists_v[B(v)]]_+ & \exists w \in c[I_w(B) = \emptyset] \\ \emptyset & \text{otherwise} \end{cases}$$

In plain English, "there might be no^v bathroom" is an instruction to take a file c, and:

- Check that there is at least one world where no bathroom exists (Veltman's consistency test).
- Update c with the information that there is no bathroom, giving back c'.
- Update c with the information that there is a bathroom v, giving back c''.
- Return $c \cup c'$.

6 References

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