Plural Discourse Reference

Patrick D. Elliott

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1 PlDPL

1.1 Information states

A *state* is a set of partial assignments.

States assign variables to pluralities (i.e., sets of individuals) distributively:

(1) $G(x) = \{\, g(x) \mid g \in G \wedge g(x) \neq \star \,\}$

Definition 1.1. Assignment extension.

$$g \geq h \iff \forall x, h(x) \neq \star \rightarrow g(x) = h(x)$$

This induces an ordering on states by taking the cumulative generalization of assignment extension:

Definition 1.2. State extension.

$$G \geq H \iff \forall g \in G, \exists h \in H[g \geq h] \wedge \forall h \in H, \exists g \in G[g \geq h]$$

Some examples of states (H, G) s.t., $G \geq H$.

x
t
d
h

x	y
t	d
t	h
d	d
d	h
h	d
h	h

Relative to *matrices* G, H:

- $G(x)$ returns the set of all values in the column labelled x.
- $G \geq H$ holds iff for each row $g \in G$, there's a row $h \in H$, which is contained by g, and for each row $h \in H$, there's a row $g \in G$ which contains it.

As emphasized by van den Berg, it's often useful to think of *states* simply as assignments from variables to sets of individuals (for simple applications).

1.2 Atomic sentences

Predicates are interpreted collectively.

- $[P(x_1, \ldots, x_n)]^? = \{ (G, H) \mid G(x_1) = \emptyset \vee \ldots \vee G(x_n) = \emptyset \}$
- $[P(x_1, \ldots, x_n)]^+ = \{ (G, H) \mid G = H \wedge G(x_1), \ldots, G(x_n) \neq \emptyset \wedge (G(x_1), \ldots, G(x_n)) \in I(P) \}$
- $[P(x_1, \ldots, x_n)]^- = \{ (G, H) \mid G = H \wedge G(x_1), \ldots, G(x_n) \neq \emptyset \wedge (G(x_1), \ldots, G(x_n)) \notin I(P) \}$

The true output states are those for which $P(x_1, \ldots, x_n)$ is true in static plural logic.

Let's say that we're in a scenario where Tom loves Dick and Harry, and we interpret $L(x, y)$ at the following state G_1:

x
t

The sentence is undefined relative to this state since $G(y) = \emptyset$, but true relative to the following extended state G_2:

x	y
t	d
t	h

This is because $G(x) = \{ t \}$, and $G(y) = \{ d, h \}$ and $(\{ t \}, \{ d, h \}) \in I(L)$

1.3 Random assignment

We can recreate our notion of random assignment in a way parallel to our old logic:

- $[\varepsilon_x]^+ = \{ (G, H) \mid G(x) = \emptyset \wedge G[x]H \}$
- $[\varepsilon_x]^- = \emptyset$
- $[\varepsilon_x]^? = \{ (G, H) \mid G(x) \neq \emptyset \}$

We need to cash out exactly what $G[x]H$ means in a plural setting. The intuition is that H introduces plural values for x without creating dependencies by multiplying assignments in the input state. I.e.:

x	y
t	d
m	h

For each row in the input state we randomly assign values to z. Here's the result of doing random assignment relative to z with a domain $\{ t, d, h \}$.

x	y	z
t	d	t
m	h	t

x	y	z
t	d	d
m	h	d

x	y	z
t	d	h
m	h	h

x	y	z
t	d	t
t	d	d
m	h	t
m	h	d

x	y	z
t	d	t
t	d	h
m	h	t
m	h	h

x	y	z
t	d	d
t	d	h
m	h	d
m	h	h

x	y	z
t	d	t
t	d	d
t	d	h
m	h	t
m	h	d
m	h	h

1.4 Negation and closure

1.4.1 Negation

- $[\neg\phi]^+ = [\phi]^-$
- $[\neg\phi]^- = [\phi]^+$
- $[\neg\phi]^? = [\phi]^?$

1.4.2 Closure

- $[\dagger\phi]^+ := [\phi]^+$
- $[\dagger\phi]^- := \{\, (G,H) \mid G = H \wedge \neg\exists I[(G,I) \in [\phi]^+] \wedge \exists I[(G,I) \in [\phi]^-] \,\}$
- $[\dagger\phi]^? := [\phi]^?$

1.5 Dynamic generalized quantifiers

- $\phi_x = \{\, H(x) \mid (G,H) \in [\phi]^+ \,\}$
- $\langle\phi\rangle_x = \bigcup \{\, H(x) \mid (G,H) \in [\phi]^+ \,\} bigcup$

2 References