Decomposing context change

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Contents

1 Introduction

1.1 Next week

Topic: An alternative perspective on the logic of anaphora and information flow - *Externally Dynamic Dynamic Semantics.*

- **Obligatory reading**: My manuscript "Towards a principled logic of anaphora" (Elliott 2020).
- Optional readings:
  - Matt Mandelkern's extremely recent L&P paper on pseudo-dynamics (Mandelkern 2022).
  - Matthew Gotham's recent AC paper on negation and disjunction in DPL (Gotham 2019).
  - Daniel Rothschild's recent AC paper on trivalence and dynamic semantics (Rothschild 2017).

1.2 TODO Today's goals

2 Recap: DPL basics

In DPL, sentential meanings are functions from evaluation points (assignments) to sets of evaluation points.

This is use to capture two key features of information flow in dynamic semantics:

- Statefulness.
- Non-determinism.

Atomic formulas are *tests*.

Definition 2.1. DPL semantics for atomic formulas. For any model $M := (D, I)$

$$[\![t]\!]^g_M = \begin{cases} g_t & t \text{ is a variable} \\ I(t) & t \text{ is a constant} \end{cases}$$

$$[\![\mathrm{P}(\mathrm{t}_1,\ldots,\mathrm{t}_n)]\!]^g_M = \begin{cases} \{\, g \,\} & \langle [\![t_1]\!]^g_M\,, \ldots, [\![t_n]\!]^g_M \rangle \in I(P) \\ \emptyset & \text{otherwise} \end{cases}$$

Existential quantifiers induce *random assignment*:

Definition 2.2. DPL semantics for existential quantification.

$$[\![\exists x, \phi]\!]^g = \bigcup_{a \in D} \{\, g^{[x \to a]} \mid [\![\phi]\!]^{g^{[x \to a]}} \,\}$$

Definition 2.3. Truth in DPL. A formula ϕ is true with respect to an assignment g iff $[\![\phi]\!]^g \neq \emptyset$

Conjunction in DPL is *relational composition*; outputs of the first conjunct are fed pointwise into the second conjunct; the resulting outputs are gathered up.

Definition 2.4. Conjunctive formula in DPL.

$$[\![\phi \wedge \psi]\!]^g = \bigcup_{h \in [\![\phi]\!]^g} \{\, i \mid i \in [\![\psi]\!]^h \,\}$$

Fact 2.1. *Egli's theorem in DPL.*

$$\exists x, \phi \wedge \psi \iff \exists x[\phi \wedge \psi]$$

Negation in DPL is tests whether the embedded formula is classically false.

Definition 2.5. Negated formula in DPL.

$$[\![\neg\, \phi]\!]^g = \begin{cases} \{\, g \,\} & [\![\phi]\!]^g = \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

Double negation is therefore a test for classical truth:

Definition 2.6. Double negation in DPL.

$$[\![\neg\, \neg\, \phi]\!]^g = \begin{cases} \{\, g \,\} & [\![\phi]\!]^g \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

Corollary: double negation elimination isn't valid.

Material implication in DPL accounts for donkey anaphora. A different (but equivalent) presentation from last time's. Note that this relies on a classical equivalence.

Definition 2.7. Material implication in DPL.

$$\phi \rightarrow \psi \iff \neg\, (\phi \wedge \neg\, \psi)$$

Some facts can read off this definition:

- A implicational formula is a test (i.e., externally static; from the semantics of negation).
- The consequent is sensitive to anaphoric information introduced by the antecedent (from the semantics of conjunction).

Disjunction in DPL: again, unlike last time, we'll demonstrate this based on a classical equivalence:

Definition 2.8. Disjunction in DPL.

$$\phi \vee \psi \iff \neg\, (\neg\, \phi \wedge \neg\, \psi)$$

Some facts we can read off this definition:

- A disjunctive formula is a test (i.e., externally static; from the semantics of negation).
- Each disjunct is a test, so disjunctions are internally dynamic, despite being stated in terms of conjunction.

Finally, universal quantification in DPL:

Definition 2.9. Universal quantification in DPL.

$$\forall x, \phi \iff \neg\, \exists\, \neg\, \phi$$

3 Donkey anaphora in DPL

(1) If a farmer owns a donkey, he feeds it.

(2) $(\exists x[\exists y[\mathbf{farmer}(x) \wedge \mathbf{donkey}(y) \wedge \mathbf{owns}(x,y)]]) \rightarrow \mathbf{feeds}(x,y)$

First, let's rewrite the provided logical form using our definition for material implication.

(3) $\neg\, (\exists x[\exists y[\mathbf{farmer}(x) \wedge \mathbf{donkey}(y) \wedge \mathbf{owns}(x,y)]] \wedge \neg\, \mathbf{feeds}(x,y))$

Now, let's compute the semantic value of each conjunct with respect to an arbitrary assignment g.

(4) $[\![\exists x, y[\mathbf{farmer}(x) \wedge \mathbf{donkey}(y) \wedge \mathbf{owns}(x, y)]]\!]^g$
$= \{\, g^{[x \to f, y \to d]} \mid f \text{ is a farmer who owns donkey} d \,\}$

(5)
$$[\![\neg\ \mathbf{feeds}(x, y)]\!]^g = \begin{cases} \{\, g \,\} & g_x \text{ doesn't feed } g_y \\ \emptyset & \text{otherwise} \end{cases}$$

Now we can compute the semantic value of the entire conditional statement.

(6) $[\![.]\!]^g = \begin{cases} c & \{\, g^{[x \to f, y \to d]} \mid f \text{ is a farmer who owns donkey } d \text{ and } f \text{ didn't feed } d \,\} = \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

Note: the conditional statement is falsified as soon as we can find a farmer-donkey pair a, b such that a owns b and a didn't feed b.

Fact 3.1. *Egli's corollary in DPL.*

$$\exists x, \phi \to \psi \iff \forall x[\phi \to \psi]$$

Via Egli's corollary:

(7) $(\exists x[\exists y[\mathbf{farmer}(x) \wedge \mathbf{donkey}(y) \wedge \mathbf{owns}(x, y)]]) \to \mathbf{feeds}(x, y)$
$\iff \forall x, \forall y[(\mathbf{farmer}(x) \wedge \mathbf{donkey}(y) \wedge \mathbf{owns}(x, y)) \to \mathbf{feeds}(x, y)]$

In this way we derive universal readings for donkey sentences.

3.1 Challenges

The readings DPL predicts for donkey anaphora are often too strong (exactly the same problem arises in FCS) (Chierchia 1995, Kanazawa 1994, Champollion, Bumford & Henderson 2019).

Context: *we're discussing how dinner attendees paid for their meal.*

(8) If an^x attendee had a^y credit card, they$_x$ paid with it$_y$.

Truth conditions predicted by DPL, via Egli's corollary:

(9) $\forall x, \forall y[(\textbf{attendee}(x) \wedge \textbf{card}(y) \wedge \textbf{had}(x,y)) \to \textbf{paidWith}(x,y)]$

Intuitively, (8) can be true in the provided context, if, e.g., Alex has a chase sapphire and an apple card, and Justin just has an apple card - Alex paid with the sapphire, and Justin paid with apple card.

In other words, the sentence can be true, even if we can find an attendee-card pair, such that the attendee didn't pay with that particular card.

The truth-conditions in (9) however, require that every attendee paid with every credit card that they had with them!

In other words, then, sometimes we want to be able to generate *weak readings* for donkey anaphora, depending on the context; something along the lines of: every attendee a is such that either (i) a has a credit card c and paid with c, or (ii) a has no credit card.

Kanazawa discusses in detail how the availability of weak/strong readings isn't totally free, but seems to be affected by monotonicity properties.

(10) I doubt that [if anyone has a donkey, they (ever) feed it].

Here, the weak reading is much more plausible: *it's considered false that anyone with at least one donkey feeds any of their donkeys.* The strong reading would predict that: *it's considered false that anyone with at least one donkey feeds **all** of their donkeys.*

Next week, we'll develop an alternative logical perspective on anaphora which derives weak existential readings by default.

I'll tentatively suggest that we might want to try to derive strong readings via independently motivated pragmatic strengthening mechanisms (Bar-Lev 2018, Bar-Lev & Fox 2017).

Although we won't discuss the dynamic treatment of generalized quantifiers until after the break, we quickly run into some hairy issues when we stray beyond first-order quantification.

Consider the following example, adapted from Kanazawa:

(11) At least two farmers who own a donkey fed it.

We might imagine a semantics for (11) which assigns it the following truth-conditions: there are at least two pairs (x, y), s.t., x is a farmer who owns y and x fed y (in the spirit of FCS and DPL treatments of donkey anaphora).

This is then predicted to be true if there are two farmers, Brian and Sally - Brian has just one donkeys, but Sally has two - Sally beats both of her donkeys, and Brian doesn't beat his at all. But, the sentence is clearly false in the context.

4 Accessibility in DPL

DPL negation *closes off* the anaphoric information introduced by the formula it combines with (it is a test).

This makes a raft of predictions for how anaphora interacts with logical operators.

DPL negation is externally static.

(12) John doesn't own a^x shirt. ?It$_x$'s in his closet.

(13) John doesn't own a^x certain shirt. It$_x$'s (nevertheless) in his closet.

Recall, double negation is a test for classical truth, therefore double negation also closes off anaphoric information introduced by the embedded formula.

This doesn't seem quite right, as Groenendijk and Stokhof themselves observe (Groenendijk & Stokhof 1991).[1]

(14) Huh? Amy didn't bring **NO dish** to the pot luck; **it**'s right there on the table!

(15) John doesn't own **NO shirt**. **It**'s in his closet!

In other words, the *non-classicality* of DPL has an empirical reflex.[2]

Krahmer and Muskens suggest that although DPL makes bad predictions in this domain, the logic of anaphoric information nevertheless shouldn't validate double-negation elimination (Krahmer & Muskens 1995).

They suggest that examples such as (15) imply that John owns *exactly one* shirt, i.e., the uniqueness inference that dynamic accounts were exactly tailored to avoid.

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[1] Note that double-negation isn't so natural in English, except for perhaps when understood as an answer to an implicit question $?(\neg\phi)$.

[2] One of my primary goals next week will be to sketch an approach to the logic of anaphoric information that is more classical, thereby skirting these issues.

Simon Charlow (p.c.) points out that uniqueness doesn't seem like quite the right characterization of the putative inference associated with (15), but rather *maximality*.

(16) John doesn't own **NO shirt**. **They**'re in his closet!

However, I'm not sure there is a strong contrast with the positive counterpart, so this may be an independent factor:

(17) John does own **a shirt**; moreover, **they**'re in his closet.

We won't have the resources to provide a detailed analysis of such cases until we tackle plurality, in several weeks time.

Relatedly, recall that DPL disjunction is both externally static *and* internally static. In many cases, these predictions are good.

(18) ?Either a^x farmer left, or they$_x$ whistled.

(19) ?It's not the case [that any^x farmer left or that they$_x$ whistled].

(20) Either a^x philosopher is here, or the party is dull; ?They$_x$ are causing a scene.

There are challenges to both halves of this generalization. Let's start with the challenge to internal staticity, since it relates to the discussion of double negation.

(21) Either there isn't a^x bathroom, or it$_x$'s upstairs.

Intuitively, this relates to the interaction between disjunction and presupposition projection.

(22) Either Enrico never smoked, or he stopped smoking.

(23) Either Enrico doesn't smoke, or Amir smokes too.

On the dynamic approach to presupposition projection (satisfaction theory), we account for this by interpreting the second disjunct in the local context of the first.

Intuitively, we'd like to be able to give the same account to (21), but we can't due to the behavior of negation in DPL!

Krahmer and Muskens make similar claims regarding uniqueness inferences for (21), although Matt Mandelkern observed that this can't be generally true (cited p.c. in (Gotham 2019)):

(24) Either Sue didn't have a drink last night, or she had a second drink right after it.

Groenendijk and Stokhof themselves observe a counter-example to externally-staticity:

(25) Either a^x philosopher was in the audience, or a^x linguist was; (Either way) they$_x$ enjoyed it.

DPL at least has the resources to give an entry for G&S disjunctions, but it involves positing an ambiguity.

Definition 4.1. Program disjunction in DPL.

$$[\![\phi \cup \psi]\!]^g = [\![\phi]\!]^g \cup [\![\psi]\!]^g$$

Can we combine both phenomena?

(26) Either a philosopher was in a room with no bathroom, or a linguist was trying to find it; (Either way) they were deeply unhappy with the situation.

This provides a window into the explanatory challenge discussed in the next section.

4.1 The explanatory challenge

Arguably, the "core" of DPL (and file change semantics for that matter) is the semantics of existential quantification and conjunction.

It's true that we have to stipulate that information flows from the left conjunct to the right, but arguably this isn't so bad (and as we'll see later, we can rid ourselves of this *lexical* stipulation by building a left-to-right bias into the compositional procedure).

There are a number of different ways of understanding the explanatory challenge to dynamic semantics is considering other classical equivalences we might have used to define material implication.

For example, we might have easily tried to define a "funny" material implication in terms of disjunction, via the following classical equivalence:

(27) $\phi \rightsquigarrow \psi \iff \neg\, \phi \vee \psi$

Taking our existing definition for disjunction, this gives us the following definition:

(28) $\phi \rightsquigarrow \psi \iff \neg\,(\neg\,\neg\,\phi \wedge \neg\,\psi)$

Recall: in DPL, negation is a test for classical falsity, and double-negation is a test for classical truth.

Our semantics for "funny" material implication is testing whether the following is classically false: a test for whether ϕ is classically true conjoined with a test for whether ψ is classically false (in fact, all this definition does is doubly-negate the antecedent, relative to DPL material implication).

Let's consider what this predicts for our example (2).

(29) $[\![.]\!]^g = \begin{cases} c & \{\, g \mid \exists a, b[a \text{ is a farmer who owns a donkey } b] \wedge g_f \text{ doesn't feed } g_d \,\} = \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

This be false just in case there is a farmer who owns a donkey, and whatever value g furnishes for f doesn't feed the value furnished for d (and false otherwise).

5 The relationship between DPL and FCS

See especially (Dekker 1996) for a detailed comparison between DPL and FCS.

5.1 Background: Partial FCS

Recall, if we switch to a partial setting, we can check novelty/familiarity at individual evaluation points.

Definition 5.1. Novelty and familiarity with respect to an assignment g.

$$\mathbf{nov}_g(x) \iff x \notin \mathbf{dom}(g)$$

$$\mathbf{fam}_g(x) \iff x \in \mathbf{dom}(g)$$

We speculated that this might allow us to make FCS distributive, but we ran into a stumbling block when considering the meanings of logical operators.

Let's address this by showing how to incorporate the insights of DPL into FCS (we'll call the resulting system *pointwise FCS*).

As a necessary first step, we need to treat evaluation points as *world-assignment pairs*, rather than assignments, in order for DPL to realistically model natural language meanings.

(30) $$[\![\text{it's raining}]\!]^{w,g} = \begin{cases} \{\, g \,\} & \mathbf{satDown}_w(g_x) \\ \emptyset & \text{otherwise} \end{cases}$$

Information states in pointwise FCS:

Definition 5.2. Information states in pointwise FCS. An information state is a set of pairs (w, g), where w is a possible world, and g is a partial assignment. The initial information state is $W \times g_\emptyset$.

Definition 5.3. Bridge principle in pointwise FCS.

$$c[\phi] = \begin{cases} \bigcup\limits_{(w,g)\in c} \{\, (w, g') \mid g' \in [\![\phi]\!]^{w,g} \,\} & \forall (w,g) \in c[\phi \in \mathbf{dom}([\![.]\!]^{w,g})] \\ \text{undefined} & \text{otherwise} \end{cases}$$

We keep the DPL semantics for existentials (already implicit in our presentation of partial DPL), but incorporate a novelty check in order to avoid the downdate problem (see also (Dekker 1996)).

(31) $$[\![\text{someone}^x \text{ walked in}]\!]^{w,g} = \begin{cases} \bigcup\limits_{a\in D} \{\, g^{[x\to a]} \mid \mathbf{walkedIn}_w(a) \,\} & \mathbf{nov}_g(x) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Sentences with definites are tests:

(32) $$[\![\text{she}_x \text{ sat down}]\!]^{w,g} = \begin{cases} \{\, g \,\} & \mathbf{fam}_g(x) \wedge \mathbf{satDown}_w(g_x) \\ \emptyset & \mathbf{fam}_g(x) \wedge \neg\, \mathbf{satDown}_w(g_x) \\ \text{undefined} & \text{otherwise} \end{cases}$$

The semantics of logical operators can be taken wholesale from DPL, e.g.:

(33) $$[\![\phi \text{ and } \psi]\!]^{w,g} = \bigcup_{h\in[\![\phi]\!]^{w,g}} \{\, i \mid i \in [\![\psi]\!]^{w,h} \,\}$$

The contribution of *and* is classical with respect to worldly information.

Intuitively, switching to partial pointwise DPL takes a classical setting, and replaces falsity with an empty output set, and gives many different ways of being true.

Note that the way that novelty/familiarity interact with the logical operators looks suspiciously like presupposition projection. This is something the current system doesn't capture (see (Rothschild 2017), also next week).

6 Arguments for CCPs

Hopefully we've managed to address a naive version of the idea that basic dynamic semantics commits us to wholesale CCPs.

Adopting the insights of DPL reduces the expressive power of the system (although not in a way that is completely satisfactory).

I only know of two arguments for CCPs in the literature.

On the worldly side, Veltman's "test" semantics for epistemic modals can only be stated in an update semantics where meanings are functions from contexts (qua sets of worlds) to contexts (Veltman 1996).[3]

Veltman's idea is that updating a context c with a modalized sentence *maybe* ϕ, is an instruction to tentatively compute $c[\phi]$ and test whether the resulting context is non-empty.

Obviously, this can't be cashed out by testing individual evaluation points; whether $\{ w \} [\phi]$ is empty or non-empty tells us nothing about whether $c[\phi]$ is empty or non-empty, assuming a non-trivial logical space.

Veltman's proposal has been *extremely* influential in the literature, and we'll consider it in a lot more detail when we come round to discussing modality and modal subordination.

On the anaphoric side, Charlow argues that cumulative readings of modified numerals can be derived straightforwardly in an update semantics, where *maximality* is computed relative to the entire input context, rather than individual evaluation points (Charlow 2016).

(34) Exactly three boys saw exactly five movies.

Charlow presents his update-theoretic analysis as an alternative to Brasoveanu's *post-suppositional* account (Brasoveanu 2013).

We'll discuss modified numerals, and related concepts (plurality!, quantification!) in several weeks time, as we introduce second-generation dynamic theories.

[3] Veltman's semantics is roughly as follows, where a CCP is understood to be a function from a set of worlds to a set of worlds.

(1) $c[\text{maybe } \phi] = \{ w \in c \mid c[\phi] \neq \emptyset \}$

The main empirical achievements of this semantics is an account of the oddness of so-called "epistemic contradictions" (when combined with naturally stateable notions of *consistency* and *coherence*).

(2) ?It's raining, but it might not be raining.

(3) ?It might not be raining, but it's raining.

7 Bonus round: definite descriptions, uniqueness, and sub-contexts

One potential reason to retain CCPs is that it might allow us to have a better account of definite descriptions.[4]

As we've discussed, definite descriptions act just like (restricted) variables when they have an indefinite antecedent, as the familiarity theory predicts. In the absence of an indefinite antecedent however, they give rise to a uniqueness inference. See (Mandelkern & Rothschild 2020) for a detailed assessment of the empirical landscape.

(35) A^x woman walked in; The$_x$ woman sat down with another woman.

(36) John went on a date; he liked the$_x$ woman, so he planned to see her$_x$ again.

(37) John went on dates with a few different women; ??he liked the$_x$ woman, so he planned to see her$_x$ again.

A tentative suggestion: in a setting with CCPs, we can minimally refactor the semantics of indefinites such that they create *sub-contexts*; definite descriptions require uniqueness, but when uniqueness is assessed relative to a sub-context, the requirement dissipates.

We'll ignore worldly information for convenience. In sub-contextual FCS, Sentential meanings are *functions from sets of assignments to sets of sets of assignments.*

Sentences with indefinites create singleton sub-contexts for different valuations of a given variable.

(38) $c[\text{Some}^v \text{ man walked in}] = \{\, \{\, g^{[v \to x]} \,\} \mid g \in c \land \mathbf{manWhoWalkedIn}(x) \,\}$

Since the output is a set of sets of assignments, we need a bridge principle.

Definition 7.1. Bridge principle in sub-contextual FCS. An assertion of ϕ in context c is interpreted as follows:

$$\bigcup c[\phi]$$

Definite descriptions have a complex meaning. They impose both *familiarity* and *uniqueness.*

- Familiarity: "The$_v$ man" requires that every assignment in the input context furnishes v with a *man* value.

[4] The logic here is very much inspired by (Charlow 2016).

- Uniqueness: "The$_v$ man" requires that the value of v is *determinate* in the input context.

(39) c[the$_v$ man sat down]

$$= \begin{cases} \{\, \{\, g \,\} \mid g \in c \wedge \mathbf{satDown}(g_v) \,\} & \forall g \in c[\mathbf{man}(g_v)] \wedge \neg \exists g, g'[g_v \neq g'_v] \\ \text{undefined} & \text{otherwise} \end{cases}$$

If the input context $c = \{\, [v \rightarrow m_1], [v \rightarrow m_2] \,\}$, where m_1, m_2 are men, update with a definite will be undefined, since the value of v is indeterminate.

Conjunction however threads the singleton sub-contexts in as the input to the sentence with the definite (note that this is essentially just the DPL semantics for conjunction, typed differently).

Definition 7.2. Conjunction in sub-contextual FCS.

$$c[p \text{ and } q] = \bigcup_{c' \in c[p]} c'[q]$$

This is highly speculative, and something we may revisit in a future class.

Still to-do: Something needs to be said about accommodation. We need an injunction against accommodating an arbitrary discourse referent.

8 Charlow's monadic grammar

The goal of Charlow's paper is to account for two properties of indefinites in a unified fashion:

- Unified binding scope.
- Unified quantificational scope.

We've talked a lot about to former, and in fact this is the bread and butter of first-generation dynamic theories.

The latter phenomenon we haven't discussed explicitly.

(40) If a rich relative of mine dies, I'll inherit a house. $\exists > \textbf{if}$

(41) If every rich relative of mine dies, I'll inherit a house. $*\forall > \textbf{if}$

Importantly, exceptional quantificational scope feeds exceptional binding scope (a desideratum of Charlow's account).

(42) If a^{x} rich relative of mine dies, I'll inherit a house. They^{x}'re terminally ill. $\exists > \textbf{if}; *\textbf{if} > \exists$

Orthodox dynamic theories based on DPL, of course capture this correlation, since material implication is externally static - they don't however explain *why* the indefinite can scope out of the conditional antecedent, whereas other quantifiers can't.

> "[...] the enrichment is grafted onto a base grammar, such that we are not required to rewrite our entire lexicon and rewire all our compositional operations." (Charlow 2020)

8.1 `State.Set`

Definition 8.1. The `State.Set` monad consists of *type constructor* D, together with functions $\eta : a \to D\ a$ (*unit*), and $\star : D\ a \to (a \to D\ b) \to D\ b$ (*bind*), defined as follows:

$$D\ a := g \to (a \times g)$$

$$\begin{aligned}
\eta &:= \lambda x \,.\, \lambda g \,.\, (x, g) & \qquad \eta : a \to D\ a \\
\star &:= \lambda m \,.\, \lambda k \,.\, \lambda g \,.\, \bigcup_{(x,h) \in m(g)} k(x)(h) & \qquad \star : D\ a \to (a \to D\ b) \to D\ b
\end{aligned}$$

Definition 8.2. Application in `State.Set`.

$$X * F := \lambda g \,.\, \bigcup_{(f,h) \in F(g)} \{\, (f(x), i) \mid (x, i) \in X(h) \,\}$$

Fact 8.1.

$$*(F)(X) \iff \star(F)(\lambda f \,.\, \star(X)(\lambda x \,.\, \eta(f(x))))$$

Definition 8.3. The monad laws.

1. **Left identity**: $\star(\eta(x))(f) \iff f(x)$.
2. **Right identity**: $\star(m(\lambda x \,.\, \eta(x))) \iff m$
3. **Associativity**: $\star(\star(m)(\lambda x \,.\, f(x)))(e) \iff \star(m)(\lambda x \,.\, \star(f(x))(e))$

8.2 TODO check laws (especially assoc)

8.3 The lexicon

(43) $\mathbf{a.ling}^v := \lambda g \,.\, \{\, (x, g^{[v \to x]}) \mid \mathbf{ling}(x) \,\}$ $\qquad$ $\mathbf{a.ling}^v : D\ e$

(44) $\mathbf{she}_v := \lambda g \,.\, \{\, (g_v, g) \,\}$ $\qquad$ $\mathbf{she}_v : D\ e$

Predicates have their ordinary (Montagovian) lexical entries:

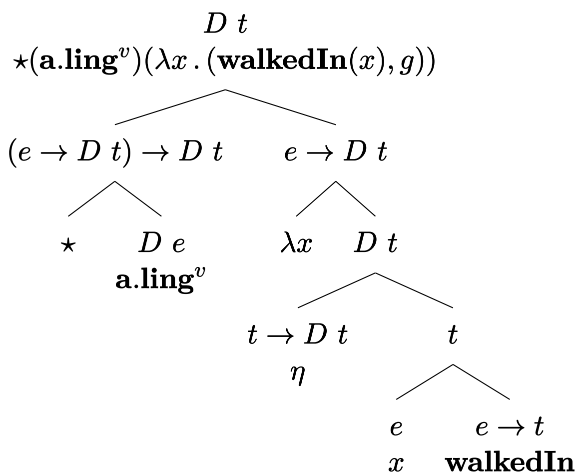
(45) $\mathbf{walkedIn} := \lambda x \,.\, \mathbf{walkedIn}(x)$

(46) $\mathbf{hugs} := \lambda x \,.\, \lambda y \,.\, \mathbf{hugs}(x)(y)$

There are two ways of composing meanings in Charlow's monadic grammar.

The monadic interface involves bind-shifting the indefinite, and scoping it over monadic unit.

(47)



Alternatively, we can adopt an in-situ interface via $*$:

(48)

- $D\ t$ — $\mathbf{a.ling}^v * \lambda g \,.\, \{\, (\lambda x \,.\, \mathbf{walkedIn}(x), g) \,\}$
  - $D\ e$ — $\mathbf{a.ling}^v$
  - $D\ e \to D\ t$
    - $D\ et \to D\ e \to D\ t$ — $*$
    - $D\ et$
      - $et \to D\ et$ — η
      - $e \to t$ — $\mathbf{walkedIn}$

The resulting sentential meanings pair classical truth-values with output assignments:

(49) $\star(\mathbf{a.ling}^v)(\lambda x \,.\, (\mathbf{walkedIn}(x), g))$

(50) $\implies \lambda g \,.\, \{\, (\mathbf{walkedIn}(x), g^{[v \to x]}) \mid \mathbf{ling}(x) \,\}$

(51) $\implies \lambda g \,.\, \{\, (\top, g^{[v \to x]}) \mid \mathbf{ling}(x) \wedge \mathbf{walkedIn}(x) \,\} \cup \{\, (\bot, g^{[v \to x]}) \mid \mathbf{ling}(x) \wedge \neg\, \mathbf{walkedIn}(x) \,\}$

As such, Charlow's monadic grammar is more expressive than DPL. It keeps track of both ways of being true, and ways of being false.

As such, we need a different entry for negation, in order to achieve the same results as DPL.

We can define an operator $.^{+}$ to get back a DPL meaning from a Charlowian meaning, by forgetting this extra structure.

Definition 8.4. Positive output operator in monadic dynamic semantics.

$$m^{+} := \lambda g \,.\, \{\, h \mid (h, \top) \in m(g) \,\}$$

Using this, we can define DPL-style negation (i.e., a test for classical falsity):

Definition 8.5. Negation in monadic dynamic semantics.

$$\mathbf{not} := \lambda m \,.\, \lambda g \,.\, \{\, (m^{+}(g) = \emptyset, g) \,\}$$

Note that the extra information afforded in Charlow's monadic grammar makes available an intriguing second possibility:

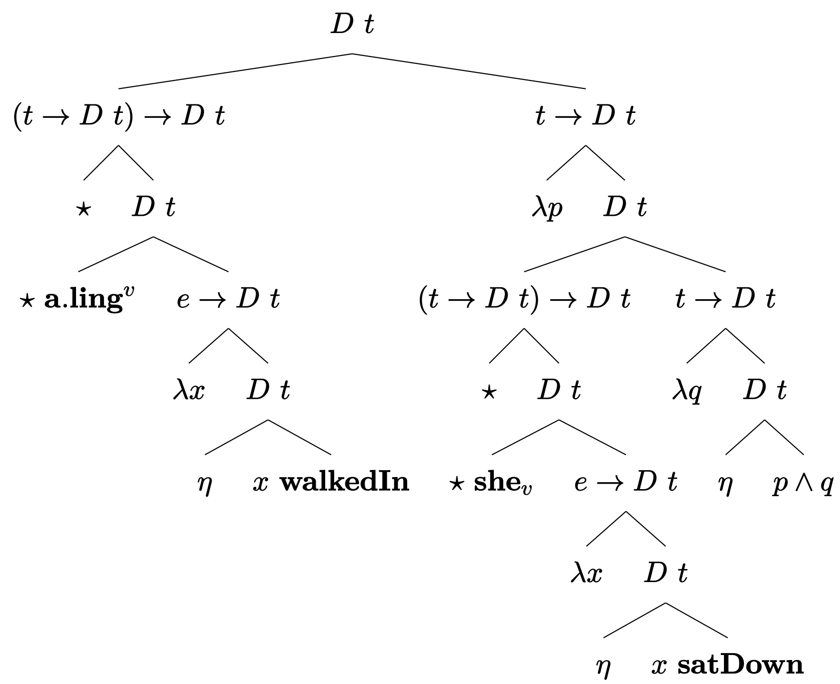
Definition 8.6. Externally dynamic negation in monadic dynamic semantics.

$$\mathbf{not}^e := \lambda m \,.\, \lambda g \,.\, \{\, (\neg\, t, h) \mid (t, h) \in m(g) \,\}$$

What predictions does this make for the interaction of anaphora and negation? We'll discuss this possibility more next week.

8.4 Dynamic binding with static conjunction

(52)



Alternatively, this information can be pre-compiled into *and.* In fact, there's a standard way of lifting binary connectives.

Definition 8.7. Lift^2.

$$m\ R^{\uparrow}\ n := \lambda g \,.\, \bigcup_{(x,h)\in m} \{\, (x\ R\ y, i) \mid (y, i) \in n(h) \,\}$$

$$m\ R^{\uparrow}\ n \iff \star(m)(\lambda x \,.\, \star\,(n)(\lambda y \,.\, \eta(x\ R\ y)))$$

8.5 Other operators

Let's try lifting classical disjunction with lift^2:

(53) $m \vee^{\uparrow} n := \lambda g \,.\, \bigcup_{(t,h) \in m\ g} \{\, (t \vee u, h) \mid (u,i) \in n(h) \,\}$

What predictions does this make?

Just like in DPL, disjunction has to be defined via a DPL-style classical equivalence:

(54) $m \textbf{ or } n \iff \textbf{not}(\star(\textbf{not } m)(\lambda t \,.\, \star(\textbf{not } n)(\lambda u \,.\, \eta(t \wedge u))))$

(55) $= \lambda g \,.\, \{\, (\exists h[h \in m^+(g) \vee h \in n^+(g)], g) \,\}$

8.6 Exceptional scope

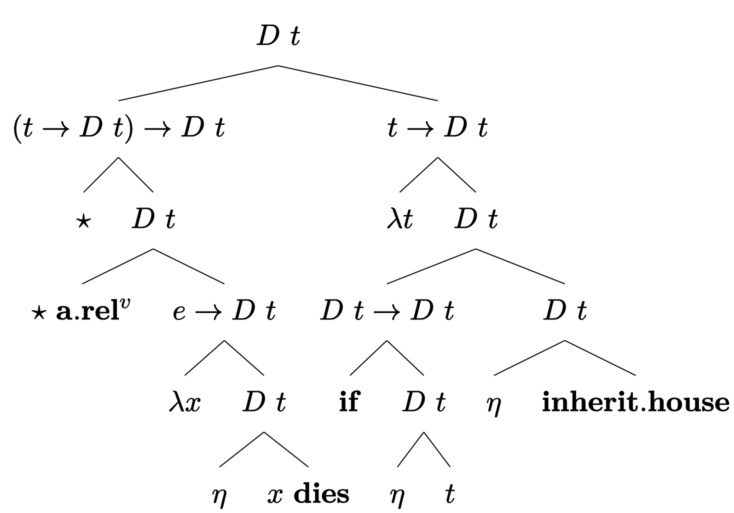
Material implication must also be defined via a DPL-style classical equivalence:

(56) $\textbf{if } m\ n \iff \textbf{not}(\star(m)(\lambda t \,.\, (\textbf{not } n)(\lambda u \,.\, \eta(t \wedge u))))$

(57) $= \lambda g \,.\, \{\, (\forall h[h \in m^+(g) \rightarrow \exists i[i \in n^+(h)]], g) \,\}$

Charlow's system accounts for exceptional quantificational/binding scope via island-pied-piping.

(58)



(59) $\star(\star(\textbf{a.rel}^v)(\lambda x \,.\, \eta(\textbf{dies}(x))))(\lambda t \,.\, \textbf{if}(\eta(t))(\eta(\textbf{inherit.house})))$

(60) $\Longrightarrow \star(\textbf{a.rel}^v)(\lambda x \,.\, \star(\eta(\textbf{dies}(x)))(\lambda t \,.\, \textbf{if}(\eta(t))(\eta(\textbf{inherit.house}))))$ assoc.

(61) $\Longrightarrow \star(\textbf{a.rel}^v)(\lambda x \,.\, \textbf{if}(\eta(\textbf{dies}(x)))(\eta(\textbf{inherit.house})))$

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