Possible witnesses

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1 Next week

- **Today:** more on modality and anaphora.
- **Next week:** no class (Patriot's day).
- **April 25:** introduction to dynamic plural logic (van den Berg 1996, Nouwen 2003).
- **May 2:** Filipe on postsuppositions.
- **May 9:** TBC.

2 Questions for today

How far can we get with the following generalization (which naturally emerges from EDS; see also (Mandelkern 2022)).

> **Witness generalization:** an assertion of a sentence ϕ containing an existential statement indexed v introduces a dref v if the assertion is accepted and contextually entails the existence of a witness to the existential statement.

Thanks to the interplay of positive/negative anaphoric information, and Strong Kleene semantics in EDS, introduction of a discourse referent tracks classical (contextual) entailment of a witness.

The existence of expressions which (arguably) explore contextual possibilities - *epistemic modals* - disrupts the witness generalization.

Some problematic data - most prominently, the *possible bathroom.*

(1) There might be a^v bathroom. It$_v$ might be upstairs.

The problem here is clear. (1) is a coherent discourse, whereas (2) is not.

Note that a subsequent pronoun is licensed (and therefore, a discourse referent introduced?) even though "There might be a^v bathroom" doesn't contextually entail a witness to the existential statement. After all, a claim about the *possibility* of there being a bathroom doesn't commit one to the actual existence of a bathroom.

Intriguingly, the possibility of a subsequent pronoun is *conditioned by the environment in which the pronoun occurs.*

(2) There might be a^v bathroom. ??It$_v$'s upstairs.

If the pronoun doesn't occur in a modalized environment, the discourse is incoherent - intuitively, this is because the familiarity presupposition of the pronoun isn't satisfied, since a witness isn't contextually entailed.

So, what's going on in (1)?

We'll explore the idea that modalized sentences can make variables *partially familiar* - something we've already seen when looking at the anaphoric potential of disjunctive sentences.

We'll formalize this using plain old Heimian file contexts, using an under-exploited aspect of their expressive power.

3 Veltman's test

The *locus classicus* for epistemic modality in dynamic semantics is Veltman's test semantics (Veltman 1996, Groenendijk, Stokhof & Veltman 1996).

Veltman's idea: a sentence $\Diamond\phi$ is an instruction to hypothetically update an information state c with ϕ, returning c unchanged if c can be consistently updated with ϕ, and the absurd state otherwise.

(3) c[it might be raining]

   a. Compute c[it's raining]; store the result as c'.
   b. Is c' are non-absurd information state? If so, return c.
   c. Otherwise, return c'.

3.1 Formalizing Veltman's *might*

An update semantics for a simple propositional fragment (Veltman 1996).

Definition 3.1. Test semantics for *might*.

$$c[\Diamond\phi] := \begin{cases} c & c[\phi] \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

In Veltman's update semantics, $\emptyset$ is the *absurd state*, i.e., the information state from which everything follows.

If we define update-semantic negation, we can treat *must* as the dual of *might*.

(4) $c[\neg\phi] := c - c[\phi]$

3.2 Epistemic *must* as the dual of *might*

Definition 3.2. Test semantics for *must*.

$$c[\Box\phi] := c[\neg\Diamond\neg\phi]$$

$$c[\Box\phi] := \begin{cases} c & c[\phi] = c \\ \emptyset & \text{otherwise} \end{cases}$$

In Veltman's terms, *must* ϕ is true in a context c if ϕ is *accepted* in c.

4 Modality and anaphora

Veltman's test semantics involves exploring properties of an information state; a modalized statement can't be judged true relative to an individual evaluation point.

In other words, incorporating epistemic modals into a simple update semantics renders the resulting system *non-distributive* (Rothschild & Yalcin 2016).

In order to keep the theory of anaphora lean and restrictive, EDS is *distributive* (but non-eliminative, just like DPL; (Groenendijk & Stokhof 1991)).

4.1 A terse presentation of EDS

(5) Atomic sentences:

a. $[\![P(v_1,\ldots,v_n)]\!]^w_+ := \{\,(g,h) \mid g = h \wedge |P(v_1,\ldots,v_n)|^{w,h} \text{ is } \mathbf{true}\,\}$

b. $[\![P(v_1,\ldots,v_n)]\!]^w_- := \{\,(g,h) \mid g = h \wedge |P(v_1,\ldots,v_n)|^{w,h} \text{ is } \mathbf{false}\,\}$

c. $[\![P(v_1,\ldots,v_n)]\!]^w_u := \{\,(g,h) \mid g = h \wedge |P(v_1,\ldots,v_n)|^{w,h} \text{ is } \mathbf{undefined}\,\}$

(6) Negative sentences:

a. $[\![\neg\phi]\!]^w_+ := [\![\phi]\!]^w_-$

b. $[\![\neg\phi]\!]^w_- := [\![\phi]\!]^w_+$

c. $[\![\neg\phi]\!]^w_u := [\![\phi]\!]^w_u$

(7) Conjunctive sentences:

a. $[\![\phi \wedge \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_+$

b. $$[\![\phi \wedge \psi]\!]^w_- := \begin{aligned}[t] & [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_{+,-,u} \\ & \cup [\![\phi]\!]^w_{+,u} \circ [\![\psi]\!]^w_- \end{aligned}$$

c. $$[\![\phi \wedge \psi]\!]^w_u := \begin{aligned}[t] & [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_u \\ & \cup [\![\phi]\!]^w_u \circ [\![\psi]\!]^w_{+,u} \end{aligned}$$

(8) Random assignment:

a. $[\![\varepsilon_v]\!]^w_+ := \{\,(g,h) \mid g[v]h\,\}$

b. $[\![\varepsilon_v]\!]^w_- := \emptyset$

c. $[\![\varepsilon_v]\!]^w_u := \emptyset$

(9) Positive closure:

a. $[\![\dagger\phi]\!]^w_+ := [\![\phi]\!]^w_+$

b. $[\![\dagger\phi]\!]^w_- := \{\,(g,h) \mid g = h \wedge |\phi|^{w,g} \text{ is } \mathbf{false}\,\}$

c. $[\![\dagger\phi]\!]^w_u := [\![\phi]\!]^w_u$

4.2 From EDS to EUS

Instead of interpreting sentences as *relations between assignments* (relative to an evaluation world), we'll interpret sentences as *updates on Heimian information states.*

We'll write updates using iconic infix notation $c[.]$; in order to carry over the key design features of EDS, we'll distinguish between the *positive*, *negative*, and *unknown* effects of sentences on information states, via $c[.]^{+,-,?}$ (EUS is a multivalent update semantics).

EDS can be straightforwardly lifted into a *multivalent update semantics* (EUS).

4.2.1 Atomic sentences in EUS

Atomic sentences update information states: $c[\phi]^+$ is the part of c at which ϕ is true, $c[\phi]^-$ is the part of c at which ϕ is false, and $c[\phi]^?$ is the part of c at which ϕ is unknown.

(10) $c[P(v_1 \dots v_n)]^+ := \{\, (w,g) \in c \mid |P(v_1,\dots,v_n)|^{w,g} \text{ is } \textbf{true} \,\}$

(11) $c[P(v_1 \dots v_n)]^- := \{\, (w,g) \in c \mid |P(v_1,\dots,v_n)|^{w,g} \text{ is } \textbf{false} \,\}$

(12) $c[P(v_1 \dots v_n)]^? := \{\, (w,g) \in c \mid |P(v_1,\dots,v_n)|^{w,g} \text{ is } \textbf{undefined} \,\}$

4.2.2 Bridge principle in EUS

Our bridge principle requires that the unknown part of the context is empty; $c_{\#}$ is the error state for context update.

(13) $c[\phi] := c[\phi]^+$ if $c[\phi]^? = \emptyset$ else $c_{\#}$

4.2.3 Negative sentences in EUS

Negation is defined exactly as in EDS.

(14) $c[\neg\phi]^+ := c[\phi]^-$

(15) $c[\neg\phi]^- := c[\phi]^+$

(16) $c[\neg\phi]^? := c[\phi]^?$

4.2.4 Random assignment in EUS

Random assignment is a tautology, which means that it is always true throughtout the information state.

(17) $c[\varepsilon_v]^+ := \{\, (w,h) \mid g[v]h \wedge (w,g) \in c \,\}$

(18) $c[\varepsilon_v]^- := \emptyset$

(19) $c[\varepsilon_v]^? := \emptyset$

4.2.5 Conjunction in EUS

Conjunction is defined just as in EDS, only instead of interpreting each cell in the Strong Kleene truth table as relational composition, we interpret each cell as a successive update.

(20) $c[\phi \wedge \psi]^+ := c[\phi]^+[\psi]^+$

(21) $c[\phi \wedge \psi]^- := c[\phi]^-[\psi]^{+,-,?} \cup c[\phi]^{+,?}[\psi]^-$

(22) $c[\phi \wedge \psi]^? := c[\phi]^?[\psi]^{+,?} \cup c[\phi]^+[\psi]^?$

4.2.6 Illustration: existential quantification

Let's see how this works briefly:

(23) $c[\varepsilon_v \wedge P(v)]^+$

(24) $c[\varepsilon_v]^+[P(v)]^+$

(25) $\{\, (w,h) \mid g[v]h \wedge (w,g) \in c \,\}\, [P(v)]^+$

(26) $\{\, (w,h) \mid g[v]h \wedge (w,g) \in c \wedge h_v \in I_w(P) \,\}$

Note that $c[\varepsilon_v \wedge P(v)]^?$ is clearly empty for any information state ?, since $c[\varepsilon_v]^?$ is empty for any information state, and $c[\varepsilon_v]^+[P(v)]^?$ is guaranteed to be empty, since v is defined throughout.

We can compute the negative update of the sentence in much the same fashion:

(27) $c[\varepsilon_v \wedge P(v)]^-$

(28) $c[\varepsilon_v]^+[P(v)]^-$

(29) $\{\, (w,h) \mid g[v]h \wedge (w,g) \in c \wedge h_v \notin I_w(P) \,\}$

4.2.7 Closure in EUS

One way of formulating positive closure in EUS is as follows:

(30) $c[\dagger\phi]^+ := c[\phi]^+$

(31) $c[\dagger\phi]^- := \{\, (w,g) \in c \mid (w,*) \notin c[\phi]^+ \wedge (w,*) \in c[\phi]^- \,\}$

(32) $c[\dagger\phi]^? := c[\phi]^?$

Applying closure to (29), we get:

(33) $c[\dagger(\varepsilon_v \wedge P(v))]^- = \{\, (w,g) \in c \mid I_w(P) = \emptyset \,\}$

5 Adding consistency tests to EUS

Adding consistency tests to EUS is not trivial. Let's start with a naive implementation of (the positive update) of Veltman's test:

(34) $c[\Diamond\phi]^+ = c$ if $c[\phi]^+ \neq \emptyset$ else $\emptyset$

This says that *might* ϕ is true at c if there's a non-empty part of c at which ϕ is true.

5.0.1 Epistemic contradictions

One of the calling cards of the dynamic treatment of modality is that it derives *epistemic contradictions* (Yalcin 2007).

(35) ??It's raining and it might not be raining.

We'll focus on the anaphoric case here (Groenendijk, Stokhof & Veltman 1996).

(36) ??Someone^v is hiding in the closet and they$_v$ might not be hiding in the closet.

(37) $\exists_v H(v) \wedge \Diamond \neg H(v)$

(38) ??Someone^v hiding in the closet might not be hiding in the closet.

(39) $\exists_v (H(v) \wedge \Diamond \neg H(v))$

(40) $c[\exists_v H(v)]^+ = \{\, (w,h) \mid (w,g) \in c \wedge g[v]h \wedge h_v \in I_w(H) \,\}$

(41) $c[\Diamond \neg H(v)]^+ = c$ if $c[H(v)]^- \neq \emptyset$ else $\emptyset$

(42) $c[\exists_v H(v)]^+[\Diamond \neg H(v)]^+ = c$ if $\{\, (w,h) \mid (w,g) \in c \wedge g[v]h \wedge h_v \in I_w(H) \,\}\, [H(v)]^- \neq \emptyset$ else $\emptyset$

(43) $c[\exists_v H(v)]^+[\Diamond \neg H(v)]^+ = \emptyset, \forall c$

5.0.2 Disjunctive epistemic contradictions

A virtue of EUS (over, e.g., (Groenendijk, Stokhof & Veltman 1996)) is that it also accounts for a disjunctive variant of epistemic contradictions.

(44) ??Either there's no^v bathroom upstairs, or it$_v$ might not be upstairs.
$\neg \exists_v U(v) \vee \Diamond \neg U(v)$

N.b. this doesn't seem like a Hurford's constraint violation.

As soon as we spell out the local context of the second disjunct (46) it's apparent that we have an epistemic contradiction:

(45) There's no bathroom upstairs.

(46) There's a bathroom upstairs and it's possible it's not upstairs.

Let's consider the EUS semantics for disjunction.

(47) $c[\phi \vee \psi]^+ := c[\phi]^+[\psi]^{+,-,?} \cup c[\phi]^{-,?}[\psi]^+$

(48) $c[\phi \vee \psi]^- := c[\phi]^-[\psi]^-$

(49) $c[\phi \vee \psi]^? := c[\phi]^?[\psi]^{-,?} \cup c[\phi]^-[\psi]^?$

Now consider the semantics of the disjuncts:

(50) $c[\neg \exists_v U(v)]^+ = \{ (w,g) \in c \mid I_w(U) = \emptyset \}$

(51) $c[\neg \exists_v U(v)]^- = \{ (w,h) \mid (w,g) \in c \wedge g[v]h \wedge h_v \in I_w(U) \}$

(52) $c[\Diamond \neg U(v)]^+ = c$ if $c[U(v)]^- \neq \emptyset$ else $\emptyset$

The problem here is that one way of verifying the disjunction: namely, if the first disjunct is false, and the second is true, turns out to be an epistemic contradiction and therefore trivial.

There's therefore only one way of verifying the disjunction - namely, if the first disjunct is true.

The disjunction as a whole ends up being contextually equivalent to the first disjunct "there is no bathroom", and therefore redundant.

6 Impossibility and necessity

So far, we've only established the positive extension of a modalized sentence:

(53) $c[\Diamond\phi]^+ = c$ if $c[\phi]^+ \neq \emptyset$ else $\emptyset$

What does it mean to assert "it's not possible that ϕ". Intuitively, this should be a test on c which checks whether there are any $(w,g) \in c$ that satisfy ϕ. If there aren't any, return c, else return $\emptyset$.

(54) Negative extension for modalized sentences (first attempt):
$c[\Diamond\phi]^- = c$ if $c[\phi]^+ = \emptyset$ else $\emptyset$

This won't be quite enough however, given the partiality inherent in EUS.

Just to illustrate, consider an initial context $c_\top := W \times \{ g_\top \}$.

- $c_\top[P(v)]^+ = \emptyset$
- $c_\top[P(v)]^- = \emptyset$
- $c_\top[P(v)]^? = c_T$

We want "it's not possible that ϕ" to ensure that (i) there are no $(w, g) \in c$ that satisfy ϕ, and (ii) some $(w, g) \in c$ satisfies $\neg\,\phi$.

(55) Negative extension for modalized sentences (second attempt):
$c[\Diamond\phi]^- = c$ if $c[\phi]^+ = \emptyset \wedge c[\phi]^- \neq \emptyset$ else $\emptyset$

What about presupposition projection? We'll come back to this later.

Now that we have the negative and positive extension of $\Diamond\phi$, we should be able to define $\Box$ as the dual of $\Diamond$.

(56) $\Box\phi := \neg\,\Diamond\,\neg\,\phi$

Let's figure out exactly what this predicts.

(57) $c[\Box\phi]^+ = c[\Diamond\,\neg\,\phi]^-$

(58) $= c$ if $c[\neg\,\phi]^+ = \emptyset \wedge c[\neg\phi]^- \neq \emptyset$ else $\emptyset$

(59) $= c$ if $c[\phi]^- = \emptyset \wedge c[\phi]^+ \neq \emptyset$ else $\emptyset$

I.e., "must ϕ" makes sure that c is inconsistent with $\neg\phi$ and consistent with ϕ.

It follows that (i) "must ϕ" dynamically entails "might ϕ", and ϕ.

7 The anaphoric potential of modalized sentences

Consider the following minimal pair, instantiating a modal variant of Rothschild's observation (i'm assuming that Andreea wearing a ring contextually entails that she has a husband).[1]

(60) a. Andreea might have a^v husband. If she's wearing a ring, I'll ask about him_v.
b. Andreea might be married. ??If she's wearing a ring, I'll ask about him_v.

What this seems to indicate is that, when uttered against c, $\Diamond\exists_v H(v)$ allows ϕ to introduce anaphoric information *only relative to the worlds in c at which there is an H*, but still retains worlds at which there is no H. More formally:

(61) $$c[\Diamond\exists_v H(v)]^+ = \begin{cases} \{\,(w,h) \mid (w,g) \in c \wedge g[v]h \wedge h_v \in I_w(H)\,\} \cup \{\,(w,g) \in c \mid I_w(H) = \emptyset\,\} & \exists w \in c_w[I_w(H) \neq \\ \emptyset & \text{otherwise} \end{cases}$$

In Heimian pragmatics, familiarity is typically *all or nothing* — a variable v is either familiar relative to a file context c, in which case v is defined at every $g \in c_a$, otherwise it isn't familiar, in which case typically it is /un/defined at every $g \in c_a$.

Modalized existential statements make variables *partially familiar.*

We can tweak our semantics for $\Diamond$ to predict this behaviour. The idea is as follows: when updating an information state c with $\Diamond\phi$, first:

- Check whether there is some part of c at which ϕ is true (consistency check).
- Take the union of $c[\phi]^+$, $c[\phi]^-$, and $c[\phi]^?$.

7.1 New entry for might

(62) $$c[\Diamond\phi]^+ = \begin{cases} c[\phi]^+ \cup c[\phi]^- \cup c[\phi]^? & c[\phi]^+ \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

[1] Thanks to Filipe for help with this data.

7.2 Illustration

To illustrate concretely how this works, consider the following file context:

- $c_1 := \{ w_a, w_b, w_\emptyset \} \times g_\top$

Updating c_1 with "Andreea might have a^v husband", first checks wheher the true update is non-empty.

Since the test is passed, we take the union of the true, false and unknown updates, resulting in the following updated file context:

- $c_2 := \{ (w_a, [v \to a]), (w_b, [v \to b]), (w_\emptyset, g_\top) \}$

Note that v *isn't familiar*, but it might become familiar if it becomes a contextual certainty that Andreea has a husband (i.e., if $w_\emptyset$ is eliminated).

Just in case "Andreea is wearing a ring" contextually entails "Andreea has a husband", asserting "Andreea is wearing a ring" relative to c_3 will result in an updated file context in which v is familiar.

Given (62) we make an interesting prediction. The following sentence should be able to make v partially familiar:

(63) Andreea might not have a^v husband.

(64) $$c[\Diamond \neg \exists_v H(v)]^+ = \begin{cases} \{ (w,h) \mid \exists (w,g) \in c[(w,h) \in \{ (w,g) \} [\neg \exists_v H(v)]^+ \vee (w,h) \in \{ (w,g) \} [\neg \exists_v H(v)]^-] \} \\ \emptyset \end{cases}$$

Note that this is equivalent to:

(65) $$c[\Diamond \neg \exists_v H(v)]^+ = \begin{cases} \{ (w,h) \mid \exists (w,g) \in c[(w,h) \in \{ (w,g) \} [\exists_v H(v)]^- \vee (w,h) \in \{ (w,g) \} [\exists_v H(v)]^+] \} & \exists(v \\ \emptyset & \text{oth} \end{cases}$$

In other words, (i) "Andreea might have a^v husband", and (ii) "Andreea might not have a^v husband" impose different consistency tests on c, but they introduce the same anaphoric information if the test is passed.

A way of seeing this, is that our semantics for $\Diamond \phi$ essentially tests ϕ against c, and if the test passes asserts $\phi \vee \neg \phi$.

This seems to make the right predictions.

(66) Andreea might not have a^v husband,
but if she's wearing a ring, I'll ask about him$_v$.

As long as we define $\Box$ as the dual of $\Diamond$ this explanation should carry over to cases like the following:[2]

(67) I'm not certain that Andreea has a^v husband,
but if she's wearing a ring, I'll ask about him$_v$.

The explanation relies on the following fact (just in case $\Box\phi := \neg\Diamond\neg\phi$):

- $\neg\Box\exists_v H(v) \iff \Diamond\neg\exists_v H(v)$

7.3 Epistemic modals and projection

How do presuppositions project through epistemic modals?

The received wisdom is that epistemic modals are *holes* (in the sense of (Karttunen 1973)), on the basis of examples such as the following.

(68) Enrico might have stopped smoking. $\rightsquigarrow$ *Enrico smoked in the past*

(69) Perhaps the bathroom is upstairs. $\rightsquigarrow$ *There is a bathroom*

(70) Maybe Talin is at the party too. $\rightsquigarrow$ *someone else is at the party*

Typically, the evidence is based on *what we accommodate* on the basis of a modalized sentence containing a presupposition trigger.

But, we know that *what is accommodated* isn't always a reliable guide to what sentence semantically presupposes (Beaver & Zeevat 2007, von Fintel 2008, Geurts 1996, Fox 2013, Mandelkern 2016).

7.3.1 Filtration diagnostics

Filtration diagnostics indicate that the presuppositions project *existentially* in $\Diamond\phi$ - in other words, if ϕ presupposes π, then "possibly ϕ" presupposes "possibly π";[3] none of the examples in (71-73) inherit presuppositions from the consequent.

(71) If it's possible that Enrico was a smoker, it's possible that he has stopped smoking.

[2] Thanks to Filipe for bringing up this data.

[3] For reasons unknown to me, *it's possible that* is the only instantiation of $\Diamond$ that comfortably embeds in the antecedent of a conditional.

(72) If it's possible there's a bathroom, then it's possible the bathroom is upstairs.

(73) If it's possible that Geordie is at the party, then maybe Talin is at the party too.

One possible response is that in all such cases, the presupposition in the consequent is *locally accommodated* within the scope of the existential modal, but the following examples speak against local accommodation; the examples in (71-73) inherit their presupposition from the consequent.

(74) If it's possible that Enrico arrived early, it's possible that he stopped smoking.

(75) If it's possible that this house was renovated, then it's possible the bathroom is upstairs.

(76) If it's possible that the dresscode is casual, then maybe Talin is at the party too.

The same point can be made using disjunctions:

(77) Either it's impossible that Enrico wasn't a smoker, or it's possible that he stopped.

(78) Either it's impossible that there's a bathroom, or it's possible that the bathroom is upstairs.

(79) Either it's impossible that Geordie is at the party, or maybe Talin is at the party too.

With respect to what various dynamic proposals for epistemic modals predict - we haven't encoded non-anaphoric presuppositions explicitly into our grammar, but it's easy to see what the predictions would be were we to do so.

Veltman's test semantics perform a consistency test on the *entire information state*; this straightforwardly predicts that $c[\Diamond\phi]$ is only defined if $c[\phi]$ is defined (i.e., presuppositions project).

Our revised consistency test for *might* however checks consistency at single evaluation points, which predicts existential projection.

7.3.2 Existential projection and conjunctive possibility statements

Recall our puzzling sentence:

(80) Maybe there is no bathroom, and maybe it's upstairs.
$\Diamond \neg \exists_v B(v) \wedge \Diamond U(v)$

Now that we've established (a) the potential of modalized sentence to introduce (partially familiar) variables, (b) existential projection, we're in a position to explain (80).

Consider an information state c_1 consisting of the following worlds paired with the initial assignment $g_\top$:

- w_d: there's a bathroom b downstairs.
- w_u: there's a bathroom b upstairs.
- $w_\emptyset$: there's no bathroom.

First, let's figure out how to compute the conjunctive update:

(81) $c_1[\Diamond \neg \exists_v B(v) \wedge \Diamond U(v)]^+ = c_1[\Diamond \neg \exists_v B(v)]^+[\Diamond U(v)]^+$

Now, we'll update c_1 with the first conjunct.

- This checks for consistency with there being no bathroom (satsified by $(w_\emptyset, g_\top)$)...
- ...and updates each $(w, g) \in c_1$ pointwise with the information that there is/isn't a bathroom, giving rise to an updated information state c_2

(82) $c_1[\Diamond \neg \exists_v B(v)]^+$

a. $= \{ (w_d, []) \} [\neg \exists_v B(v)]^{+,-,?}$
$\cup \{ (w_u, []) \} [\neg \exists_v B(v)]^{+,-,?}$
$\cup \{ (w_\emptyset, []) \} [\neg \exists_v B(v)]^{+,-,?}$

b. $= \emptyset \cup \{ (w_d), [v \to b] \} \cup \emptyset$
$\cup \emptyset \{ (w_u), [v \to b] \} \cup \emptyset$
$\cup \{ (w_\emptyset), [] \} \cup \emptyset \cup \emptyset$

c. $\{ (w_d, [v \to b]), (w_u, [v \to b]), (w_\emptyset, []) \} := c_2$

Now we can update c_2 with the second conjunct.

First, we perform the consistency test. This just requires that $U(v)$ is true at one of the evaluation points in c_2. The test succeeds, since:

(83) $\{ (w_u, [v \to b]) \} [U(v)]^+ = \{ (w_u, [v \to b]) \}$

Now, we compute the information introduced by the modalized second conjunct - since the consistency test is passed, the modalized second conjunct introduces no information:

(84) $c_2[\Diamond U(v)]^+$

a. $= \{ (w_d, [v \to b]) \} [U(v)]^{+,-,?}$
$\cup \{ (w_u, [v \to b]) \} [U(v)]^{+,-,?}$
$\cup \{ (w_\emptyset, []) \} [U(v)]^{+,-,?}$

b. $= \emptyset \cup \{ (w_d, [v \to b]) \} \cup \emptyset$
$\cup \{ (w_u, [v \to b]) \} \cup \emptyset \cup \emptyset$
$\cup \emptyset \cup \emptyset \cup \{ (w_\emptyset, []) \}$

c. $\{ (w_d, [v \to b]), (w_u, [v \to b]), (w_\emptyset, []) \} := c_3$

In this context, the following would be equivalent:

(85) There might be no^v bathroom, it$_v$ might be downstairs, and it$_v$ might be upstairs.

(86) There might be a^v bathroom, and it$_v$ might be upstairs.

Note that we predict *weak, existential truth conditions* for conjunctive possibility statements like this. This seems correct.

(87) Maybe Sarah didn't buy a^v drink, and maybe she bought another drink right after it$_v$.

(88) Mary Sarah bought a^v drink, and maybe she bought another drink right after it$_v$.

7.3.3 Impossible discourse referents

A loose end - saying what the negative extension of a modalized statement is, given (62). In a multivalent system, we have some freedom.

It looks like we probably want to maintain existential projection.

(89) There might be a^v bathroom, but it's impossible/there's no way that it$_v$'s upstairs.

(90) ??There might be a^v bathroom, but it$_v$'s not upstairs.

And we want to still allow modalized sentences to introduce anaphoric information under negation.

(91) It's not possible that there's no^v bathroom; it$_v$'s upstairs!
$\neg \Diamond \neg \exists_v B(v) \wedge U(v)$

(92) There must be a^v bathroom; I just saw it$_v$!
$\Box \exists_v B(v)$

Our semantics for $\neg \Diamond \phi$ - $\neg \Diamond$ imposes an two checks:

- No possibility in c is consistent with ϕ.
- Some possibility in c is consistent with $\neg \phi$.

(93) $c[\Diamond\phi]^- = \begin{cases} \bigcup\limits_{(w,g)\in c} \{\, (w,h) \mid (w,h) \in \{\, (w,g) \,\} [\phi]^{+,-,u} \,\} & c[\Diamond\phi]^+ = \emptyset \wedge \exists(w,g) \in c[\{\, (w,g) \,\} [\phi]^- \neq \emptyset] \\ \emptyset & \text{otherwise} \end{cases}$

This directly accounts for (92). First, let's decide how to compute the conjunctive update.

(94) $c[\neg \Diamond \exists_v B(v)]^+[U(v)]^+ \iff c[\Diamond\neg\exists_v B(v)]^-[U(v)]^+$

The update induced by the first conjunct is only non-empty if there are no non-bathroom worlds in c. If there are no non-bathroom words, then we update all the bathroom worlds with a bathroom discourse referent v. This makes v familiar, if we assume bivalence.

Defining the unknown extension of *might* is now straightforward.

(95) $c[\Diamond\phi]^? = \begin{cases} c & c[\Diamond\phi]^+ = \emptyset \wedge c[\Diamond\phi]^- = \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

We also straightforwardly define strong *must* as the dual of *might*.

8 Dividing up possiblities

We can define a special kind of update in terms of EUS that explores possibilities in a pointwise fashion (as in EDS); Instead of $c[.]$, we'll use $c\,\langle . \rangle$.

(96) $c\,\langle\phi\rangle^+ := \bigcup\limits_{(w,g)\in c} \{\, (w,g) \,\} [\phi]^+$

(97) $c\,\langle\phi\rangle^- := \bigcup\limits_{(w,g)\in c} \{\, (w,g) \,\} [\phi]^-$

(98) $c\,\langle\phi\rangle^? := \bigcup\limits_{(w,g)\in c} \{\, (w,g) \,\} [\phi]^?$

To illustrate, consider the following information state c_1 (a, b are upstairs in w_u; b is downstairs in w_d):

(99) $\{\, (w_u, [v \to a]), (w_u, [v \to b]), (w_d, [v \to b]), (w_\emptyset, []) \,\} := c_1$

$U(v)$ is of course not assertable at c_1, since v isn't familiar at c_1 (thanks to $w_\emptyset$).

We can however explore each of the possibilities individually using $c\,\langle . \rangle^{+,-,?}$.

(100) $c\langle U(v)\rangle^{+} \iff \bigcup_{(w,g)\in c_1} \{ (w,g) \} [U(v)]^{+} = \{ (w_u, [v \to a]), (w_u, [v \to b]) \}$

(101) $c\langle U(v)\rangle^{-} \iff \bigcup_{(w,g)\in c_1} \{ (w,g) \} [U(v)]^{-} = \{ (w_d, [v \to b]) \}$

(102) $c\langle U(v)\rangle^{?} \iff \bigcup_{(w,g)\in c_1} \{ (w,g) \} [U(v)]^{?} = \{ (w_\emptyset, []) \}$

$c\langle U(v)\rangle^{+,-,?}$ induces a tripartition of c_1, since $U(v)$ is a test:

1. The maximal part of c_1 which $U(v)$ is defined and true.
2. the maximal part of c_1 at which $U(v)$ is defined and false.
3. The maximal part of c_1 at which $U(v)$ is undefined.

It's important to note that $c\langle\phi\rangle^{+,-,?}$ doesn't always partition c, since it can multiply possibilities (e.g., if ϕ is an existential statement). Consider the following information state:

(103) $\{ (w_u, []), (w_d, []), (w_\emptyset, []) \} := c_2$

Exploring each of the possibilities individually via $c\langle.\rangle^{+,-,?}$ introduces anaphoric information.

The result is still a tripartition of a new information state c_3.

(104) $c_2\langle\exists_v B(v)\rangle^{+} = \{ (w_u, [v \to a]), (w_u, [v \to b]), (w_d, [v \to b]) \}$
(105) $c_2\langle\exists_v B(v)\rangle^{-} = \{ (w_\emptyset, []) \}$
(106) $c_2\langle\exists_v B(v)\rangle^{?} = \emptyset$

The possiblity of introducing anaphoric information in a pointwise fashion is what underlies our semantics for *might.*

We can give a terse semantics of *might* in terms of $c\langle.\rangle$.

(107) $c[\Diamond\phi]^{+} = c\langle\phi\rangle^{+,-,?}$ if $c\langle\phi\rangle^{+} \neq \emptyset$ else $\emptyset$
(108) $c[\Diamond\phi]^{-} = c\langle\phi\rangle^{+,-,?}$ if $c\langle\phi\rangle^{+} = \emptyset \land c\langle\phi\rangle^{-} \neq \emptyset$ else $\emptyset$
(109) $c[\Diamond\phi]^{?} = c\langle\phi\rangle^{+,-,?}$ if $c\langle\phi\rangle^{+} = \emptyset \land c\langle\phi\rangle^{-} = \emptyset \land c\langle\phi\rangle^{?} \neq \emptyset$ else $\emptyset$

It's now easy to see that our semantics for *might* is in fact *presuppositional*; $\Diamond\phi$ presupposes at c that ϕ can be verified at some point in c, or ϕ can be falisifed at some point in c.

This explains why, despite it's fairly weak requirements on the context, $\Diamond U(v)$ is still infelicitious when there is no antecedent.

(110) ??If Andreea is wearing a ring, I might confront him.

9 Free choice

9.1 Free choice with anaphora

As we discussed last week, no theories of free choice can capture *free choice with anaphora.*

(111) It's possible that either there's no bathroom, or it's upstairs.
t's possible that there's no bathroom. t's possible that there's a bathroom upstairs.

Here, we'll show that by extending Goldstein's dynamic account (Goldstein 2019), we can capture free choice with anaphora within the current setting.

The idea will be that we can distinguish formally between *ways of verifying* a disjunctive sentence, tracking the truth of the first and second disjuncts respectively.

(112) $c[\phi \vee \psi]^1 = c[\phi]^+[\psi]^{+,-,?}$

(113) $c[\phi \vee \psi]^2 = c[\phi]^{+,-,?}[\psi]^+$

We'll enrich our semantics for disjunction by adding the requirement that *both verification strategies for disjunction are contextually viable*:

(114) $c[\phi \vee \psi]^+ = c[\phi \vee \psi]^1 \cup c[\phi \vee \psi]^2$ if $c[\phi \vee \psi]^1, c[\phi \vee \psi]^2 \neq \emptyset$ else $\emptyset$

Let's see how this combines with our entry for *might* to derive free choice with anaphora.

(115) $\Diamond(\neg \exists_v B(v) \vee U(v))$

Recall: *might* imposes a requirement on c: there should be at least one possibility in c which is consistent with $\neg\exists_v B(v) \vee \psi$.

(116) $c[\Diamond(\neg \exists_v B(v) \vee U(v))]^+ \neq \emptyset$ if $c\,\langle\neg\exists_v B(v) \vee U(v)\rangle^+ \neq \emptyset$

We're now going to run into a problem $c\,\langle\neg\exists_v B(v) \vee U(v)\rangle^+ \neq \emptyset$ just in case we can find some possibility (w, g), s.t., $\{\,(w, g)\,\}\,[\neg \exists_v B(v) \vee U(v)]^+ \neq \emptyset$.

This will be impossible to meet, since for $\{\,(w, g)\,\}\,[\neg \exists_v B(v) \vee U(v)]^+ \neq \emptyset$, it must be true that both ways of verifying the disjunction are contextually viable at $\{\,(w, g)\,\}$, and this can never be the case.

We need to modify our entry for *might* yet again, but in a relatively harmless way. Instead of considering individual possibilities, we'll consider (sub-)information states.

In fact, I think we can get away with just modifying $c\,\langle.\rangle^{+,-,?}$.

(117) $c\langle\phi\rangle^{+} := \bigcup_{c' \in P(c) - \emptyset} c'[\phi]^{+}$

(118) $c\langle\phi\rangle^{-} := \bigcup_{c' \in P(c) - \emptyset} c'[\phi]^{-}$

(119) $c\langle\phi\rangle^{?} := \bigcup_{c' \in P(c) - \emptyset} c'[\phi]^{?}$

Now we can restate the requirement imposes by *might* on c - there should be at least one non-empty subset of c which is consistent with $\neg\exists_v B(v) \vee U(v)$.

This will hold just in case there is some part of c which contextually entails there's no bathroom, and some part of c which contextually entails that there's a bathroom upstairs.

9.2 Negative free choice with anaphora

(120) I'm not certain that John both bought a^v book and read it_v.
   a. I'm not certain that John bought a^v book.
   b. If John bought a book, I'm not certain that he read it_v.

We can require that both ways of falsifing a conjunction are possible.

10 TODO Going inquisitive

11 References

References

Beaver, David & Henk Zeevat. 2007. Accommodation. *The Oxford Handbook of Linguistic Interfaces.* `https://www.oxfordhandbooks.com/view/10.1093/oxfordhb/9780199247455.001.0001/oxfordhb-9780199247455-e-17` (5 September, 2019).

Fox, Danny. 2013. Presupposition projection from quantificational sentences - Trivalence, local accommodation, and presupposition strengthening. In Ivano Caponigro & Carlo Cecchetto (eds.), *From grammar to meaning*, 201–232.

Geurts, Bart. 1996. Local satisfaction guaranteed: A presupposition theory and its problems. *Linguistics and Philosophy* 19(3). 259–294. `https://doi.org/10.1007/BF00628201` (20 September, 2019).

Goldstein, Simon. 2019. Free choice and homogeneity. *Semantics and Pragmatics* 12(0). 23. `https://semprag.org/index.php/sp/article/view/sp.12.23` (4 March, 2022).

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

Groenendijk, Jeroen a. G., Martin J. B. Stokhof & Frank J. M. M. Veltman. 1996. Coreference and modality. In *The handbook of contemporary semantic theory* (Blackwell Handbooks in Linguistics), 176–216. Oxford: Blackwell. `https://dare.uva.nl/search?identifier=c655089e-9fae-4842-90da-30e46f37825b` (22 July, 2020).

Karttunen, Lauri. 1973. Presuppositions of compound sentences. *Linguistic Inquiry* 4(2). 169–193.

Mandelkern, Matthew. 2016. Dissatisfaction Theory. *Semantics and Linguistic Theory* 26. 391–416. `https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/26.391` (20 September, 2019).

Mandelkern, Matthew. 2022. Witnesses. *Linguistics and Philosophy.*

Nouwen, R. W. F. 2003. *Plural Pronominal Anaphora in Context : Dynamic Aspects of Quantification.* `http://localhost/handle/1874/630` (20 November, 2020).

Rothschild, Daniel & Seth Yalcin. 2016. Three notions of dynamicness in language. *Linguistics and Philosophy* 39(4). 333–355. `https://doi.org/10.1007/s10988-016-9188-1` (2 August, 2020).

van den Berg, M. H. 1996. Some aspects of the internal structure of discourse. The dynamics of nominal anaphora. `https://dare.uva.nl/search?arno.record.id=7073` (31 August, 2020).

Veltman, Frank. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic* 25(3). 221–261.

von Fintel, Kai. 2008. What Is Presupposition Accommodation, Again?*. *Philosophical Perspectives* 22(1). 137–170. `https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1520-8583.2008.00144.x` (18 November, 2020).

Yalcin, Seth. 2007. Epistemic Modals. *Mind* 116(464). 983–1026. `https://academic.oup.com/mind/article/116/464/983/951766` (18 September, 2020).