Disjunction, possibility, and hypothetical discourse referents

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Contents

1 Notices **2**

2 First order EDS **2**

3 Aspects of disjunction **3**
3.1 EDS disjunction 3
3.2 Disjunction, triviality, and ignorance 4
3.3 Simplification and anaphora 6
3.4 Internal staticity and Hurford's constraint 7

4 Modality and anaphora **9**
4.1 Some interactions between modals and anaphora 9
4.2 Epistemic modality in dynamic semantics 10
4.2.1 Initial motivations: disagreement 10
4.2.2 Sensitivity to local context 11
4.3 Test semantics 11
4.4 Hypothetical discourse referents 13

5 Integrating epistemic modality and EDS **14**
5.1 Lifting EDS into an update semantics 14
5.2 EDS as an update semantics 14
5.2.1 Atomic sentences 14
5.2.2 Negative sentences 14
5.2.3 Conjunctive sentences 15
5.2.4 Random assignment 15
5.2.5 Closure 15

5.2.6 Test semantics . . . 15
5.2.7 Pragmatics . . . 16
5.2.8 An alternative semantics for *might* . . . 16
5.2.9 Bathrooms and contextual entailment . . . 16
5.2.10 Conjunctive possibilities . . . 17

6 References **18**

1 Notices

Class next week is cancelled - I'll be giving a talk at the workshop *Dynamics in Logic and Language* (Tsinghua University) entitled "Disjunction in a predictive theory of anaphora".

I'll be talking about material related to the topic of this seminar, so please consider registering.

The registration URL is: `http://tsinghualogic.net/JRC/?page_id=4555`

On April 11, I plan to talk about either strong/weak readings of donkey sentences, or inquisitive dynamic semantics (let me know if you have a preference).

2 First order EDS

A concise presentation of EDS as a semantics for a first order calculus.

To appreciate the isomorphism between this presentation, and the previous presentation, consider that a trivalent relational semantics can be framed instead as a set of relations, each paired with one of three truth values.

(1) Static semantics for atomic sentences:

$$[P(v_1,\ldots,v_n)]^{w,g} = \begin{cases}\textbf{defined} & g(v_1),\ldots,g(v_n) \neq \#_e \\ \textbf{true} & [P(v_1,\ldots,v_n)]^{w,g} \text{ is } \textbf{defined} \text{ and } \langle g(v_1),\ldots,g(v_n)\rangle \in I_w(P)\end{cases}$$

(2) Atomic sentences in EDS:

a. $[\![P(v_1,\ldots,v_n)]\!]^w_+ := \{\, (g,h) \mid g = h \wedge |P(v_1,\ldots,v_n)|^{w,h} \text{ is } \textbf{true}\,\}$

b. $[\![P(v_1,\ldots,v_n)]\!]^w_- := \{\, (g,h) \mid g = h \wedge |P(v_1,\ldots,v_n)|^{w,h} \text{ is } \textbf{false}\,\}$

c. $[\![P(v_1,\ldots,v_n)]\!]^w_u := \{\, (g,h) \mid g = h \wedge |P(v_1,\ldots,v_n)|^{w,h} \text{ is } \textbf{undefined}\,\}$

(3) Negative sentences:

a. $[\![\neg\phi]\!]^w_+ := [\![\phi]\!]^w_-$

b. $[\![\neg\phi]\!]^w_- := [\![\phi]\!]^w_+$

c. $[\![\neg\phi]\!]^w_u := [\![\phi]\!]^w_u$

(4) Conjunctive sentences:

a. $[\![\phi \wedge \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_+$

b. $[\![\phi \wedge \psi]\!]^w_- := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_{+,-,u} \cup [\![\phi]\!]^w_{+,u} \circ [\![\psi]\!]^w_-$

c. $[\![\phi \wedge \psi]\!]^w_u := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_u \cup [\![\phi]\!]^w_u \circ [\![\psi]\!]^w_{+,u}$

(5) Random assignment:

a. $[\![\varepsilon_v]\!]^w_+ := \{\, (g, h) \mid g[v]h \,\}$

b. $[\![\varepsilon_v]\!]^w_- := \emptyset$

c. $[\![\varepsilon_v]\!]^w_u := \emptyset$

(6) Positive closure:

a. $[\![\dagger\phi]\!]^w_+ := [\![\phi]\!]^w_+$

b. $[\![\dagger\phi]\!]^w_- := \{\, (g, h) \mid g = h \wedge [\![\phi]\!]^w_+ = \emptyset \wedge [\![\phi]\!]^w_- \neq \emptyset \,\}$

c. $[\![\dagger\phi]\!]^w_u := [\![\phi]\!]^w_u$

(7) Existential quantification:
$\exists_v \phi := \varepsilon_v \wedge \phi$

3 Aspects of disjunction

3.1 EDS disjunction

(8) Disjunctive sentences:

a. $[\![\phi \vee \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_{+,-,u} \cup [\![\phi]\!]^w_{-,u} \circ [\![\psi]\!]^w_+$

b. $[\![\phi \vee \psi]\!]^w_- := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_-$

c. $[\![\phi \vee \psi]\!]^w_u := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_u \cup [\![\phi]\!]^w_u \circ [\![\psi]\!]^w_{-,u}$

3.2 Disjunction, triviality, and ignorance

Remember, we explained the apparent *external staticity* of disjunction (and it exceptions) by recourse to the independently-motivated requirement that each disjunct is a *real possibility*[1].

For the following discussion, it will be important to make precise exactly what this means.

Definition 3.1. Contingency requirement (first attempt). A sentence of the form $\phi \vee \psi$ is *felicitous* when asserted in a context c iff both $c_w[\phi]$ and $c_w[\psi]$ are non-empty proper subsets of c_w.[2]

> A direct consequence: a disjunctive sentence is *infelicitous* in c if either disjunct is contextually trivial.

In a bivalent setting, the principle in (3.1) will have the desired effect.

(9) Context: *Josie has just said "I have food poisoning".*
  a. ??Either Josie is sick or we'll eat out tonight.
  b. ??Either Josie is well or we'll stay home tonight.
  c. Either Josie is well by 7pm or we'll stay home tonight.

If we take presuppositions (anaphoric or otherwise) into account, we see that (3.1) is too strong.

Consider our Stalnakerian bridge principle:.

Definition 3.2. Update in EDS.

$$c[\phi] = \begin{cases} \bigcup\limits_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_+ \,\} & \forall (w,g) \in c[\phi \text{ is } \textbf{true} \text{ at } (w,g) \text{ or } \phi \text{ is } \textbf{false} \text{ at } (w,g)] \\ \text{undefined} & \text{otherwise} \end{cases}$$

If either of the disjuncts is *presuppositional* (i.e., contains a pronoun/free variable), then the condition in (3.1) will only be met if the presupposition is satisfied throughout the context.[3]

[1] Plausibly, this can be made to follow from a global redundancy principle.
[2] We use c_w to indicate the worldly content of a file, and $c_w[\phi]$ to indicate the worldly content of $c[\phi]$.
[3] Much the same point can be made for non-anaphoric presuppositions.

This will place far-too-strong requirements on *bathroom disjunctions*, such as "Either there is no^v bathroom, or it$_v$'s upstairs" - the condition in (3.1) predicts that this should require v to be defined throughout the context.

The heart of the problem with the condition in (3.1) is that it ignores the possibility of dependencies between disjuncts.

> The intuition behind the solution: we can isolate the parts of $c[\phi \vee \psi]$ in which the disjunction is verified by the first disjunct, and the parts of $c[\phi \vee \psi]$ in which the disjunction is verified by the second disjunct. Neither way of verifying the disjunction should be contextually trivial.[4]

In order to isolate the parts of the context corresponding to verifying each disjunct, we hold the positive extension of each disjunct constant.

- $c[\phi \vee \psi]_1 := \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_{+,-,u} \,\}$
- $c[\phi \vee \psi]_2 := \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_{+,-,u} \circ [\![\psi]\!]^w_+ \,\}$

Let's illustrate how this works:

- w_{bu}: b is a bathroom upstairs.
- w_{bd}: b is a bathroom downstairs.
- $w_\emptyset$: there are no bathrooms.

Initial file context $c_\top$: $\{\, w_{bu}, w_{bd}, w_\emptyset \,\} \times g_\top$

- $c[\neg\exists_v[B(v)] \vee U(v)] = \{\, (w_{bu}, [v \to b]), (w_\emptyset, []) \,\}$
- $c[\neg\exists_v[B(v)] \vee U(v)]_1 = \{\, (w_\emptyset, []) \,\}$
- $c[\neg\exists_v[B(v)] \vee U(v)]_2 = \{\, (w_{bu}, [v \to b]) \,\}$

Definition 3.3. Contingency requirement (second attempt). A sentence of the form $\phi \vee \psi$ is *felicitous* when asserted in a context c iff neither way of verifying the disjunction would be trivial, i.e.:

- $c_w[\phi \vee \psi]_1$ is a non-empty proper subset of c.
- $c_w[\phi \vee \psi]_2$ is a non-empty proper subset of c.

Constraints that (3.3) places on the context in light of $\neg\exists_v[B(v)] \vee U(v)$.

- If the input context only contained worlds in which there are bathrooms, $c_w[\phi \vee \psi]_1$ would be empty.
- If the input context only contained worlds in which there are bathrooms upstairs, then $c_w[\phi \vee \psi]_2$ would not be a proper subset.
- Importantly, note that $\{ w_\emptyset, w_{bu} \}$ is compatible with the requirements in (3.3), but in such a context the disjunctive assertion would be informationally redundant.

This will give rise to the welcome prediction that "Either there is no bathroom or it's upstairs" is felicitous c, just in case either:

- c is compatible with (but doesn't entail) there being no bathroom.
- c is compatible with (but doesn't entail) there being a bathroom upstairs.

For cases where there are no dependencies between disjuncts, (3.3) simply subsumes the initial formulation (hopefully).

3.3 Simplification and anaphora

How do we derive (3.3) from pragmatic reasoning about alternatives?

Other cases where accounts based on formal alternatives break down.[5]

Free choice:

(10) You may write no^v term paper, or submit it$_v$ by midnight.

Note that a naïve free choice inference doesn't go through, since the pronoun becomes unbound.

(11) a. $\Longrightarrow$ *you may write no term paper*
b. $\not\Longleftarrow$ *you may submit it by midnight*
c. $\Longrightarrow$ *you may write a^v term paper and submit it$_v$ by midnight*

I don't know of any account of free choice which can accommodate this; especially hard for implicature accounts which rely on formal alternatives (Fox 2007, Bar-Lev & Fox 2017).

Other inferences which are accounted for in terms of simplification face similar problems (Crnič, Chemla & Fox 2015).

[5] Thanks to Yasu Sudo for discussion of this point.

(12) Everyone either didn't write a term paper or submitted it by midnight.

a. $\Longrightarrow$ *Some people didn't write a term paper.*

b. $\not\Longrightarrow$ *Some people submitted it by midnight.*

c. $\Longrightarrow$ *Some people wrote a term paper and submitted it by midnight.*

Related issues:

- Ignorance inferences associated with the antecedent and consequent of a conditional, in the presence of donkey anaphora.
- Ignorance inferences associated with a negated conjunctive sentence, in the presence of discourse anaphora.

Notice there's a related problem with propositional anaphora to a disjunct:

(13) Either there's no bathroom, or it's upstairs. If **the latter**, then we'll see it soon.

3.4 Internal staticity and Hurford's constraint

Anaphoric relations between disjuncts are disallowed (Groenendijk & Stokhof 1991), i.e., (14) presupposes that v is contextually defined.

(14) ??Either someone^v is in the audience, or they$_v$'re sitting down.

(15) $\exists^v[A(v)] \vee S(v)$

In EDS, disjunction is internally dynamic, just like every other logical connective - how can we rule out (14)?

Let's compute its positive extension. First, some salient points:

- If the first disjunct is true, the second is guaranteed to be defined (i.e., have an empty u-extension, so we can ignore it).
- The first disjunct is *always* defined (we can ignore its u-extension).
- If the first disjunct is false, whether or not the second disjunct is true will depend on the input assignment.

(16) $$\begin{aligned}[\![\exists^v[A(v)] \vee S(v)]\!]^w_+ = \; & [\![\exists_v[A(v)]]\!]^w_+ \circ [\![S(v)]\!]^w_{+,-} \\ & \cup [\![\exists_v[A(v)]]\!]^w_- \circ [\![S(v)]\!]^w_+\end{aligned}$$

(17) $= \{ (g,h) \mid g[v]h \wedge g(v) \in I_w(A) \} \cup \{ (g,h) \mid g = h \wedge I_w(A) = \emptyset \wedge h(v) \in I_w(S) \}$

So, "Either someone^v is in the audience or they$_v$'re sitting down", either introduces an audience member dref v, if there are audience members, or if there are no audience members asserts that v is sitting down.

Since we'll consider the conditions under which (15) is assertable, let's also compute its negative extension.

- If the first disjunct is false, whether or not the second disjunct is false will depend on the input assignment.

(18) $[\![\exists^v[A(v)] \vee S(v)]\!]^w_- = [\![\exists_v[A(v)]]\!]^w_- \circ [\![S(v)]\!]^w_-$

(19) $= \{ (g,h) \mid g = h \wedge I_w(A) = \emptyset \wedge h(v) \notin I_w(S) \}$

So, "Either someone^v is in the audience or they're sitting down" is *false* if there are no audience members and v isn't sitting down.

Let's consider what it would mean to assert (15) in a concrete context.

Recall our Stalnakerian bridge principle: What requirements does (15) place on the context? Every world assignment pair $(w, g$ should be s.t., either:

- w is an *audience-member world.*
- w is a non-audience member world, and $g(v)$ is defined.

Let's give a concrete example.

- w_d: a is an audience member sitting down.
- w_u: a is an audience member standing up.
- $w_{d'}$: nobody is in the audience and a is sitting down.
- $w_{u'}$: nobody is in the audience and a is standing up.

In a context $\{ w_d, w_u, w_{d'}, w_{u'} \} \times g_\top$, (14) will be undefined due to the presupposition *if nobody is in the audience $g(v)$ is defined.*

If we remove the worlds in which nobody is in the audience, then (14) is informationally redundant.

Let's assume that we do someone manage to associate just the audienceless worlds with a discourse referent a.

- $c' := \{ (w_d, []), (w_u, []), (w_{d'}, [v \rightarrow a]), (w_{u'}, [v \rightarrow a]) \}$

Updating c' with (14) will give the following updated context. The only (w, g) which is eliminated is the one in which nobody is in the audience in and $g(v)$ is standing up.

- $c'[\exists_v[A(v)] \vee S(v)] := \{ (w_d, [v \to a]), (w_u, [v \to a]), (w_{d'}, [v \to a]) \}$

Now, let's consider the parts of the updated context that each disjunct is responsible for:

- $c'[\exists_v[A(v)] \vee S(v)]_1 := \{ (w_d, [v \to a]), (w_u, [v \to a]) \}$
- $c'[\exists_v[A(v)] \vee S(v)]_2 := \{ (w_d, [v \to a]), (w_{d'}, [v \to a]) \}$

Note that the resulting information states are overlapping, due to the existence of a world in which someone is in the audience sitting down w_d (i.e., the disjuncts are compatible).

(Singh 2008) argues that a disjunction is infelicitous if the disjuncts are contextually compatible.

(20) ??John is in Russia or Asia.

It follows that (14) can't convey that there is a person who is both in the audience and sitting down.

4 Modality and anaphora

4.1 Some interactions between modals and anaphora

> **A generalization that emerges from EDS:** an assertion of a sentence ϕ containing an existential statement "a^v linguist P-ed" introduces a dref v if the assertion is accepted and contextually entails the existence of a linguist that P-ed.

Interaction with modality draws this generalization into question.

Modal subordination and anaphora (Roberts 1989):

(21) Maybe there's a^v bathroom, and maybe it$_v$'s upstairs.

(22) Maybe there's a^v bathroom. It$_v$ would be upstairs.

(23) There might be a^v bathroom, and it$_v$ might be upstairs.

(24) There might be a^v bathroom. It$_v$ would be upstairs.

Not possible with disjunction(?):

(25) ??Maybe there's a^v bathroom, or maybe it$_v$'s upstairs.

(26) ??There might be a^v bathroom, or it$_v$ might be upstairs.

Surprisingly, conjunctive possibility statements can pattern with disjunction:

(27) There might be no^v bathroom, and it$_v$ might be upstairs.

(28) Maybe there's no^v bathroom, and maybe it$_v$'s upstairs.

(29) Either There's no^v bathroom, or it$_v$'s upstairs.

Note that this parallel is perhaps unsurprising, given modal theories of disjunction which validate $\phi \vee \psi \vdash \Diamond\phi \wedge \Diamond\psi$ (Zimmermann 2000, Geurts 2005, Goldstein 2019).

Modal subordination with negation; data from (Hofmann 2019):

(30) There is no^v bathroom in this house. It$_v$ would be easier to find.

4.2 Epistemic modality in dynamic semantics

4.2.1 Initial motivations: disagreement

The classical relational semantics for epistemic possibility (Kratzer 2012).

(31) $[\![\Diamond\phi]\!]^{w,f} :=$ **true** iff $\exists w' \in f(w)[[\![\phi]\!]^{w'}$ is **true**$]$

The parameter f is taken to be determined by the conversational context, and maps each world w to the set of worlds compatible with what is known in w.

Known by who? One salient possibility is the speaker.[6]

A fatal problem for this view ((Hawthorne 2004); data from (Rudin 2021)):

(32) a. Andrea: Paul might have been at the party last night.
b. Bertrand: You're wrong, he was in Barbados.

> "The puzzle is that no way of pinning the relevant knowledge state down seems to be able to explain both why we are in a position to make the epistemic "might" claims we seem to be in a position to make, and also why it is often reasonable to disagree with "might" claims made by others" (Stalnaker 2014)

N.b. epistemic *must* is classically treated as the dual of *might*.

[6]Yanovich calls this view "contextual sollipsism" (Yanovich 2014).

4.2.2 Sensitivity to local context

Following data from (Rothschild 2021). Disjunction:

(33) Either John is here or he must be in China.
  a. Either John is here, or else it must be that he's not here and is in China.
  b. Either John is here, or he's in China.
  c. Either John is here, or some contextual body of information entails that he is in China.

Conditionals:

(34) If John isn't here, he must be in China.
  a. It must be that if John isn't here, he's in China.
  b. If John isn't here, he's in China.
  c. If John isn't here, then some contextual body of information entails that he is in China.

Conjunction:

(35) Either John is in the US, or else John is in France and Chloe must be with him.
  a. Either John is in the US, or else it must be that John is in France and Chloe is with him.
  b. Either John is in the US, or John is in France and Chloe is in France with him.
  c. Either John is in France and a contextual bodyof information entails that Chloe is with him, or else they're both in the US.

> The patters above aren't decisive evidence against the classical view, as Rothschild emphasizes. It's easy to set the context in a way that makes sense of these intuitions, in a classical setting. However, the regularity suggests that the classical account is missing a generalization.

4.3 Test semantics

The *locus classicus* is Veltman's test semantics (Veltman 1996).

Veltman's idea: a sentence $\Diamond\phi$ is an instruction to hypothetically update a context c with ϕ, returning c unchanged if c can be consistently updated with ϕ, and the absurd state otherwise.

(36) c[it might be raining]

a. Compute c[it's raining]; store the result as c'.

b. Is c' are non-absurd information state? If so, return c.

c. Otherwise, return c'.

An update semantics for a simple propositional fragment (Veltman 1996).

Definition 4.1. Test semantics for *might*.

$$c[\Diamond\phi] := \begin{cases} c & c[\phi] \neq \emptyset \\ \phi & \text{otherwise} \end{cases}$$

In Veltman's update semantics, $\emptyset$ is the *absurd state*, i.e., the information state from which everything follows.

If we define update-semantic negation, we can treat *must* as the dual of *might*.

(37) $c[\neg\phi] := c - c[\phi]$

Definition 4.2. Test semantics for *must*.

$$c[\Box\phi] := c[\neg\Diamond\neg\phi]$$

$$c[\Box\phi] := \begin{cases} c & c[\phi] = c \\ \emptyset & \text{otherwise} \end{cases}$$

In Veltman's terms, *must* ϕ is true in a context c if ϕ is *accepted* in c.

Note that test semantics for *must* is **strong** (von Fintel & Gillies 2010, 2021).

Definition 4.3. Entailment in update semantics.

ϕ entails $\psi \iff \forall c[c[\phi] = c \rightarrow c[\psi] = c]$

If $c[\Box\phi] = c$ then $c[\phi] = c$, simply by the update rule for $\Box\phi$.

4.4 Hypothetical discourse referents

Conjecture: asserting "possibly ϕ" is analogous to asserting "ϕ or not ϕ".

In a classical setting $\phi \vee \neg\phi$ is of course informationally trivial, but in a dynamic setting (specifically, in the context of EDS), $\phi \vee \neg\phi$, can introduce anaphoric information.

A consideration of the disjunctive case will help give a feel for the explanation.

(38) Either there's a^v bathroom, or there's no^v bathroom.

(39) $\exists_v[B(v)] \vee \neg\exists_v[B(v)]$

Predicted (positive) meaning in EDS is as follows. Note:

- Since the disjuncts are mutually exclusive, we can ignore the case of both disjuncts being true.
- Since neither disjunct contains a free variable, we can ignore the case of either disjunct being undefined (ignoring the novelty condition for simplicity).
- There are therefore two ways of dynamically verifying the disjunction to consider:
  - The first disjunct is true, and the second is false (a bathroom dref).
  - The first disjunct is false and the second is true (no bathroom dref).

(40) $$\lambda(w,g)\,.\,\{\, g^{[v\to x]} \mid \mathbf{bathroom}_w(x) \,\} \\ \cup \{\, g \mid \mathbf{bathroom}_w = \emptyset \,\}$$

An assertion of (40) relative to a context c will introduce a bathroom discourse referent at worlds $\in c$ where a bathroom exists, but otherwise leave the context unchanged, e.g.:

(41) $$\{\, (w_\emptyset, g), (w_1, g), (w_2, g), (w_{12}, g) \,\} \\ \Rightarrow \{\, (w_\emptyset, g), (w_{b_1}, g^{[v\to b_1]}), (w_{b_2}, g^{[v\to b_2]}), (w_{b_1,b_2}, g^{[v\to b_1]}), (w_{b_1,b_2}, g^{[v\to b_2]}) \,\}$$

N.b. we already account for the impossibility of anaphora in the following discourse, due to the universal presupposition introduced by the pronoun.

(42) Either there's a$_v$ bathroom, or there isn't a bathroom.
??It$_v$'s upstairs.

5 Integrating epistemic modality and EDS

EDS can be framed as an update semantics quite easily. Let's start with first-order EDS.

5.1 Lifting EDS into an update semantics

As usual, we'll model information states using Heimian files, supplemented with a failure state $\#_c$.

EDS can be lifted into a multivalent update semantics, where we define $c[.]_+$, $c[.]_-$, and $c[.]_u$.

(43) a. $c[\phi]_+ := \bigcup_{(w,g)\in c} \{ (w,h) \mid (g,h) \in [\![\phi]\!]^w_+ \}$

b. $c[\phi]_- := \bigcup_{(w,g)\in c} \{ (w,h) \mid (g,h) \in [\![\phi]\!]^w_- \}$

c. $c[\phi]_u := \bigcup_{(w,g)\in c} \{ (w,h) \mid (g,h) \in [\![\phi]\!]^w_u \}$

5.2 EDS as an update semantics

5.2.1 Atomic sentences

Atomic sentences in EDS update semantics induce a tripartition of the input file, since no anaphoric information can be introduced.

(44) $c[P(v_1,\ldots,v_n)]_+ := \{ (w,g) \in c \mid |P(v_1,\ldots,v_n)|^{w,g} \text{is } \textbf{true} \}$

(45) $c[P(v_1,\ldots,v_n)]_- := \{ (w,g) \in c \mid |P(v_1,\ldots,v_n)|^{w,g} \text{is } \textbf{false} \}$

(46) $c[P(v_1,\ldots,v_n)]_u := \{ (w,g) \in c \mid |P(v_1,\ldots,v_n)|^{w,g} \text{is } \textbf{undefined} \}$

5.2.2 Negative sentences

(47) $c[\neg\phi]_+ := c[\phi]_-$

(48) $c[\neg\phi]_- := c[\phi]_+$

(49) $c[\neg\phi]_u := c[\phi]_u$

5.2.3 Conjunctive sentences

(50) $c[\phi \wedge \psi]_+ := c[\phi]_+[\psi]_+$

(51) $c[\phi \wedge \psi]_- := c[\phi]_-[\psi]_+ \cup c[\phi]_-[\psi]_- \cup c[\phi]_-[\psi]_u$
$c[\phi]_+[\psi]_- \cup c[\phi]_u[\psi]_-$

(52) $c[\phi \wedge \psi]_u := c[\phi]_+[\psi]_u$
$c[\phi]_u[\psi]_+ \cup c[\phi]_u[\psi]_u$

5.2.4 Random assignment

(53) $c[\varepsilon_v]_+ := \{ (w,h) \mid \exists g[(w,g) \in c \wedge g[v]h] \}$

(54) $c[\varepsilon_v]_- := \emptyset$

(55) $c[\varepsilon_v]_- := \emptyset$

5.2.5 Closure

(56) $c[\dagger\phi]_+ := c[\phi]_+$

(57) $c[\dagger\phi]_- := \{ (w,g) \in c \mid c[\phi]_+ = \emptyset \wedge c[\phi]_- \neq \emptyset \}$

(58) $c[\dagger\phi]_u := c[\phi]_u$

5.2.6 Test semantics

Let's stick to Veltman's idea that a modalized statement, if true, adds no information to the CG.

(59) *Might* (first attempt):

a. $c[\Diamond\phi]_+ := \{ (w,g) \in c \mid c[\phi]_+ \neq \emptyset \}$

b. $c[\Diamond\phi]_- := \{ (w,g) \in c \mid c[\phi]_+ = \emptyset \wedge c[\phi]_- \neq \emptyset \}$

c. $c[\diamond\phi]_u := c[\phi]_u$

By definition, modalized sentences are *tests* on information states (they can't introduce any anaphoric information).

5.2.7 Pragmatics

What kind of bridge principle do we want for a trivalent update semantics?

Definition 5.1. Assertion. An assertion of ϕ in c, $c[\phi]$, is defined as follows:

$$c[\phi] := \begin{cases} c[\phi]_+ & c[\phi]_u = \emptyset \\ & \#_c \\ \text{otherwise} & \end{cases}$$

5.2.8 An alternative semantics for *might*

Let's start by specifying the positive contribution of *might*.

(60) *Might* (first attempt):

a. $c[\Diamond\phi]_+ := \begin{cases} c[\phi]_+ \cup c[\phi]_- & c[\phi]_+ \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

We'll come back to the negative update associated with $\diamond\phi$.

5.2.9 Bathrooms and contextual entailment

This has the virtue of accounting for a variant of Rothshchild discourses involving epistemic modals.

(61) Context: *It's common ground that a restaurant critic will be here on Monday, but it's not common ground what day it is.*

a. A: It's possible that a restaurant critic is here.

b. B: It's Monday, so they$_v$'re here right now.

(62) a. It's possible that a restaurant critic is here.

b. $\Diamond(\exists_v[C(v)])$

(63) $c[\Diamond(\exists_v[C(v)])]_+ = \begin{cases} c[\neg\exists_v[C(v)]]_+ \cup c[\exists_v[C(v)]]_+ & \exists w \in c[I_w(C) \neq \emptyset] \\ \emptyset & \text{otherwise} \end{cases}$

In plain English, "it's possible that a^v critic is here" is an instruction to take a file c, and:

- Check that there is at least one world where a critic is here (Veltman's consistency test).
- Update c with the information that there is a critic v, giving back c'.
- Update c with the information that there is no critic, giving back c''.
- Return $c \cup c'$.

If a subsequent update eliminates all non-critic worlds in c', then anaphora may subsequently be licensed (since familiarity will be satisfied).

5.2.10 Conjunctive possibilities

The following is an acceptable sentence.

(64) There might be no^v bathroom and it$_v$ might be upstairs.

Let's compute the positive contribution of the first sentence.

(65) $$c[\Diamond(\neg\exists_v[B(v)])]_+ = \begin{cases} c[\neg\exists_v[B(v)]]_+ \cup c[\exists_v[B(v)]]_+ & \exists w \in c[I_w(B) = \emptyset] \\ \emptyset & \text{otherwise} \end{cases}$$

This update ensures that c is consistent with their being no bathroom, and pairs bathroom worlds with bathroom drefs, and leaves non-bathroom worlds unchanged.

Let's move on to the second sentence.

(66) It$_v$ might be upstairs.

(67) $\Diamond(U(v))$

This update ensures that c is consistent with v being upstairs, and simply returns the union of the $v-$upstairs and v-not-upstairs worlds.

(68) $$c[\Diamond(U(v))]_+ = \begin{cases} c[U(v)]_+ \cup c[U(v)]_- & c[U(v)]_+ \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

We can now think through, what happens when we chain these updates together. That's pretty easy for conjunction.

(69) $$c[\Diamond(\neg\exists_v[B(v)]) \wedge \Diamond(U(v))]_+ = c[\Diamond(\neg\exists_v[B(v)])]_+[\Diamond(U(v))]_+$$

Oh no! something has gone wrong here. Concretely, the update expressed by the second sentence requires that the familiarity presupposition is met *throughout* the input context. If the test imposed by the first epistemic modal is successful, there are guaranteed to be worlds in which there are no bathrooms, and the second update will fail.

6 References

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