

# Disjunction and epistemic possibility

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## Contents

<b>1</b>	<b>TODO Say something more general about anaphora and simplification based inferences</b>	<b>1</b>
<b>2</b>	<b>Notices</b>	<b>1</b>
<b>3</b>	<b>First order EDS</b>	<b>2</b>
<b>4</b>	<b>Aspects of disjunction</b>	<b>3</b>
4.1	EDS disjunction . . . . .	3
4.2	Disjunction and triviality . . . . .	3
4.3	Internal staticity and Hurford's constraint . . . . .	6
<b>5</b>	<b>Modality and anaphora</b>	<b>8</b>
5.1	New data . . . . .	8
5.2	Epistemic modality in dynamic semantics . . . . .	9
5.2.1	Initial motivations . . . . .	9
<b>6</b>	<b>References</b>	<b>9</b>

**1 TODO Say something more general about anaphora and simplification based inferences**

**2 Notices**

Class next week is cancelled.

### 3 First order EDS

A concise presentation of EDS as a semantics for a first order calculus.

To appreciate the isomorphism between this presentation, and the previous presentation, consider that a trivalent relational semantics can be framed instead as a set of relations, each paired with one of three truth values.

- (1) Static semantics for atomic sentences:

$$[P(v_1, \dots, v_n)]^{w,g} = \begin{cases} \mathbf{defined} & g(v_1), \dots, g(v_n) \neq \#_e \\ \mathbf{true} & [P(v_1, \dots, v_n)]^{w,g} \text{ is } \mathbf{defined} \text{ and } \langle g(v_1), \dots, g(v_n) \rangle \in I_w(P) \end{cases}$$

- (2) Atomic sentences in EDS:

- a.  $\llbracket P(v_1, \dots, v_n) \rrbracket_+^w := \{ (g, h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h} \text{ is } \mathbf{true} \}$
- b.  $\llbracket P(v_1, \dots, v_n) \rrbracket_-^w := \{ (g, h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h} \text{ is } \mathbf{false} \}$
- c.  $\llbracket P(v_1, \dots, v_n) \rrbracket_u^w := \{ (g, h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h} \text{ is } \mathbf{undefined} \}$

- (3) Negative sentences:

- a.  $\llbracket \neg \phi \rrbracket_+^w := \llbracket \phi \rrbracket_-^w$
- b.  $\llbracket \neg \phi \rrbracket_-^w := \llbracket \phi \rrbracket_+^w$
- c.  $\llbracket \neg \phi \rrbracket_u^w := \llbracket \phi \rrbracket_u^w$

- (4) Conjunctive sentences:

- a.  $\llbracket \phi \wedge \psi \rrbracket_+^w := \llbracket \phi \rrbracket_+^w \circ \llbracket \psi \rrbracket_+^w$
- b.  $\llbracket \phi \wedge \psi \rrbracket_-^w := \llbracket \phi \rrbracket_-^w \circ \llbracket \psi \rrbracket_{+,-,u}^w \cup \llbracket \phi \rrbracket_{+,u}^w \circ \llbracket \psi \rrbracket_-^w$
- c.  $\llbracket \phi \wedge \psi \rrbracket_u^w := \llbracket \phi \rrbracket_+^w \circ \llbracket \psi \rrbracket_u^w \cup \llbracket \phi \rrbracket_u^w \circ \llbracket \psi \rrbracket_{+,u}^w$

- (5) Random assignment:

- a.  $\llbracket \varepsilon_v \rrbracket_+^w := \{ (g, h) \mid g[v]h \}$
- b.  $\llbracket \varepsilon_v \rrbracket_-^w := \emptyset$
- c.  $\llbracket \varepsilon_v \rrbracket_u^w := \emptyset$

- (6) Positive closure:

- a.  $\llbracket \dagger \phi \rrbracket_+^w := \llbracket \phi \rrbracket_+^w$

- b.  $\llbracket \dagger \phi \rrbracket_-^w := \{ (g, h) \mid g = h \wedge \llbracket \phi \rrbracket_+^w = \emptyset \wedge \llbracket \phi \rrbracket_-^w \neq \emptyset \}$
- c.  $\llbracket \dagger \phi \rrbracket_u^w := \llbracket \phi \rrbracket_u^w$

- (7) Existential quantification:  
 $\exists_v \phi := \varepsilon_v \wedge \phi$

## 4 Aspects of disjunction

### 4.1 EDS disjunction

- (8) Disjunctive sentences:
- a.  $\llbracket \phi \vee \psi \rrbracket_+^w := \llbracket \phi \rrbracket_+^w \circ \llbracket \psi \rrbracket_{+,-,u}^w \cup \llbracket \phi \rrbracket_{-,u}^w \circ \llbracket \psi \rrbracket_+^w$
  - b.  $\llbracket \phi \vee \psi \rrbracket_-^w := \llbracket \phi \rrbracket_-^w \circ \llbracket \psi \rrbracket_-^w$
  - c.  $\llbracket \phi \vee \psi \rrbracket_u^w := \llbracket \phi \rrbracket_-^w \circ \llbracket \psi \rrbracket_u^w \cup \llbracket \phi \rrbracket_u^w \circ \llbracket \psi \rrbracket_{-,u}^w$

### 4.2 Disjunction and triviality

Remember, we explained the apparent *external staticity* of disjunction (and its exceptions) by recourse to the independently-motivated requirement that each disjunct is a *real possibility*<sup>1</sup>.

For the following discussion, it will be important to make precise exactly what this means.

**Definition 4.1. Contingency requirement** (first attempt). A sentence of the form  $\phi \vee \psi$  is *felicitous* when asserted in a context  $c$  iff both  $c_w[\phi]$  and  $c_w[\psi]$  are non-empty proper subsets of  $c_w$ .<sup>2</sup>



A direct consequence: a disjunctive sentence is *infelicitous* in  $c$  if either disjunct is contextually trivial.

In a bivalent setting, the principle in (4.1) will have the desired effect.

- (9) Context: *Josie has just said “I have food poisoning”.*

<sup>1</sup>Plausibly, this can be made to follow from a global redundancy principle.

<sup>2</sup>We use  $c_w$  to indicate the worldly content of a file, and  $c_w[\phi]$  to indicate the worldly content of  $c[\phi]$ .

- a. ??Either Josie is sick or we'll eat out tonight.
- b. ??Either Josie is well or we'll stay home tonight.
- c. Either Josie is well by 7pm or we'll stay home tonight.

If we take presuppositions (anaphoric or otherwise) into account, we see that (4.1) is too strong. Consider our Stalnakerian bridge principle:

**Definition 4.2.** Update in EDS.

$$c[\phi] = \begin{cases} \bigcup_{(w,g) \in c} \{ (w, h) \mid (g, h) \in \llbracket \phi \rrbracket_+^w \} & \forall (w, g) \in c [\phi \text{ is } \mathbf{true} \text{ at } (w, g) \text{ or } \phi \text{ is } \mathbf{false} \text{ at } (w, g)] \\ \text{undefined} & \text{otherwise} \end{cases}$$

If either of the disjuncts is *presuppositional* (i.e., contains a pronoun/free variable), then the condition in (4.1) will only be met if the presupposition is satisfied throughout the context.<sup>3</sup>

This will place far-too-strong requirements on *bathroom disjunctions*, such as “Either there is no<sup>v</sup> bathroom, or it<sub>v</sub>’s upstairs” - the condition in (4.1) predicts that this should require *v* to be defined throughout the context.

The heart of the problem with the condition in (4.1) is that it ignores the possibility of dependencies between disjuncts.



The intuition behind the solution: we can isolate the parts of  $c[\phi \vee \psi]$  in which the disjunction is verified by the first disjunct, and the parts of  $c[\phi \vee \psi]$  in which the disjunction is verified by the second disjunct. Neither way of verifying the disjunction should be contextually trivial.<sup>4</sup>

In order to isolate the parts of the context corresponding to verifying each disjunct, we hold the positive extension of each disjunct constant.

- $c[\phi \vee \psi]_1 := \bigcup_{(w,g) \in c} \{ (w, h) \mid (g, h) \in \llbracket \phi \rrbracket_+^w \circ \llbracket \psi \rrbracket_{+,-,u}^w \}$
- $c[\phi \vee \psi]_2 := \bigcup_{(w,g) \in c} \{ (w, h) \mid (g, h) \in \llbracket \phi \rrbracket_{+,-,u}^w \circ \llbracket \psi \rrbracket_+^w \}$

Let's illustrate how this works:

- $w_{bu}$ : *b* is a bathroom upstairs.

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<sup>3</sup>Much the same point can be made for non-anaphoric presuppositions.

- $w_{bd}$ :  $b$  is a bathroom downstairs.
- $w_\emptyset$ : there are no bathrooms.

Initial file context  $c_\top$ :  $\{w_{bu}, w_{bd}, w_\emptyset\} \times g_\top$

- $c[\neg\exists_v[B(v)] \vee U(v)] = \{(w_{bu}, [v \rightarrow b]), (w_\emptyset, [])\}$
- $c[\neg\exists_v[B(v)] \vee U(v)]_1 = \{(w_\emptyset, [])\}$
- $c[\neg\exists_v[B(v)] \vee U(v)]_2 = \{(w_{bu}, [v \rightarrow b])\}$

**Definition 4.3. Contingency requirement** (second attempt). A sentence of the form  $\phi \vee \psi$  is *felicitous* when asserted in a context  $c$  iff neither way of verifying the disjunction would be trivial, i.e.:

- $c_w[\phi \vee \psi]_1$  is a non-empty proper subset of  $c$ .
- $c_w[\phi \vee \psi]_2$  is a non-empty proper subset of  $c$ .

Constraints that (4.3) places on the context in light of  $\neg\exists_v[B(v)] \vee U(v)$ .

- If the input context only contained worlds in which there are bathrooms,  $c_w[\phi \vee \psi]_1$  would be empty.
- If the input context only contained worlds in which there are bathrooms upstairs, then  $c_w[\phi \vee \psi]_2$  would not be a proper subset.
- Importantly, note that  $\{w_\emptyset, w_{bu}\}$  is compatible with the requirements in (4.3), but in such a context the disjunctive assertion would be informationally redundant.

This will give rise to the welcome prediction that “Either there is no bathroom or it’s upstairs” is felicitous  $c$ , just in case either:

- $c$  is compatible with (but doesn’t entail) there being no bathroom.
- $c$  is compatible with (but doesn’t entail) there being a bathroom upstairs.

For cases where there are no dependencies between disjuncts, (4.3) simply subsumes the initial formulation (hopefully).



How do we derive (4.3) from pragmatic reasoning about alternatives?

### 4.3 Internal staticity and Hurford's constraint

Anaphoric relations between disjuncts are disallowed (Groenendijk & Stokhof 1991), i.e., (10) presupposes that  $v$  is contextually defined.

(10) ??Either someone<sup>*v*</sup> is in the audience, or they<sub>*v*</sub>'re sitting down.

(11)  $\exists^v[A(v)] \vee S(v)$

In EDS, disjunction is internally dynamic, just like every other logical connective - how can we rule out (10)?

Let's compute its positive extension. First, some salient points:

- If the first disjunct is true, the second is guaranteed to be defined (i.e., have an empty  $u$ -extension, so we can ignore it).
- The first disjunct is *always* defined (we can ignore its  $u$ -extension).
- If the first disjunct is false, whether or not the second disjunct is true will depend on the input assignment.

$$(12) \quad \llbracket \exists^v[A(v)] \vee S(v) \rrbracket_+^w = \llbracket \exists_v[A(v)] \rrbracket_+^w \circ \llbracket S(v) \rrbracket_{+,-}^w \\ \cup \llbracket \exists_v[A(v)] \rrbracket_-^w \circ \llbracket S(v) \rrbracket_+^w$$

$$(13) \quad = \{ (g, h) \mid g[v]h \wedge g(v) \in I_w(A) \} \cup \{ (g, h) \mid g = h \wedge I_w(A) = \emptyset \wedge h(v) \in I_w(S) \}$$

So, "Either someone<sup>*v*</sup> is in the audience or they<sub>*v*</sub>'re sitting down", either introduces an audience member dref  $v$ , if there are audience members, or if there are no audience members asserts that  $v$  is sitting down.

Since we'll consider the conditions under which (11) is assertable, let's also compute its negative extension.

- If the first disjunct is false, whether or not the second disjunct is false will depend on the input assignment.

$$(14) \quad \llbracket \exists^v[A(v)] \vee S(v) \rrbracket_-^w = \llbracket \exists_v[A(v)] \rrbracket_-^w \circ \llbracket S(v) \rrbracket_-^w$$

$$(15) \quad = \{ (g, h) \mid g = h \wedge I_w(A) = \emptyset \wedge h(v) \notin I_w(S) \}$$

So, “Either someone<sup>*v*</sup> is in the audience or they’re sitting down” is *false* if there are no audience members and *v* isn’t sitting down.

Let’s consider what it would mean to assert (11) in a concrete context.

Recall our Stalnakerian bridge principle: What requirements does (11) place on the context? Every world assignment pair  $(w, g)$  should be s.t., either:

- *w* is an *audience-member world*.
- *w* is a non-audience member world, and  $g(v)$  is defined.

Let’s give a concrete example.

- $w_d$ : *a* is an audience member sitting down.
- $w_u$ : *a* is an audience member standing up.
- $w_{d'}$ : nobody is in the audience and *a* is sitting down.
- $w_{u'}$ : nobody is in the audience and *a* is standing up.

In a context  $\{w_d, w_u, w_{d'}, w_{u'}\} \times g_\top$ , (10) will be undefined due to the presupposition *if nobody is in the audience  $g(v)$  is defined*.

If we remove the worlds in which nobody is in the audience, then (10) is informationally redundant.

Let’s assume that we do someone manage to associate just the audienceless worlds with a discourse referent *a*.

- $c' := \{ (w_d, []), (w_u, []), (w_{d'}, [v \rightarrow a]), (w_{u'}, [v \rightarrow a]) \}$

Updating  $c'$  with (10) will give the following updated context. The only  $(w, g)$  which is eliminated is the one in which nobody is in the audience in and  $g(v)$  is standing up.

- $c'[\exists_v[A(v)] \vee S(v)] := \{ (w_d, [v \rightarrow a]), (w_u, [v \rightarrow a]), (w_{d'}, [v \rightarrow a]) \}$

Now, let’s consider the parts of the updated context that each disjunct is responsible for:

- $c'[\exists_v[A(v)] \vee S(v)]_1 := \{ (w_d, [v \rightarrow a]), (w_u, [v \rightarrow a]) \}$
- $c'[\exists_v[A(v)] \vee S(v)]_2 := \{ (w_d, [v \rightarrow a]), (w_{d'}, [v \rightarrow a]) \}$

Note that the resulting information states are overlapping, due to the existence of a world in which someone is in the audience sitting down  $w_d$  (i.e., the disjuncts are compatible).

(Singh 2008) argues that a disjunction is infelicitous if the disjuncts are contextually compatible.

(16) ??John is in Russia or Asia.

It follows that (10) can't convey that there is a person who is both in the audience and sitting down.

## 5 Modality and anaphora

### 5.1 New data



**A generalization that emerges from EDS:** an assertion of a sentence  $\phi$  containing an existential statement “a<sup>v</sup> linguist *P*-ed” introduces a dref  $v$  if the assertion is accepted and contextually entails the existence of a linguist that *P*-ed.

Interaction with modality draws this generalization into question.

Modal subordination and anaphora (Roberts 1989):

- (17) Maybe there's a<sup>v</sup> bathroom, and maybe it<sub>v</sub>'s upstairs.
- (18) Maybe there's a<sup>v</sup> bathroom. It<sub>v</sub> would be upstairs.
- (19) There might be a<sup>v</sup> bathroom, and it<sub>v</sub> might be upstairs.
- (20) There might be a<sup>v</sup> bathroom. It<sub>v</sub> would be upstairs.

Not possible with disjunction(?):

- (21) ??Maybe there's a<sup>v</sup> bathroom, or maybe it<sub>v</sub>'s upstairs.
- (22) ??There might be a<sup>v</sup> bathroom, or it<sub>v</sub> might be upstairs.

Surprisingly, conjunctive possibility statements can pattern with disjunction:

- (23) There might be no<sup>v</sup> bathroom, and it<sub>v</sub> might be upstairs.
- (24) Maybe there's no<sup>v</sup> bathroom, and maybe it<sub>v</sub>'s upstairs.
- (25) Either There's no<sup>v</sup> bathroom, or it<sub>v</sub>'s upstairs.

Note that this parallel is perhaps unsurprising, given modal theories of disjunction which validate  $\phi \vee \psi \vdash \Diamond\phi \wedge \Diamond\psi$  (Zimmermann 2000, Geurts 2005, Goldstein 2019).

Modal subordination with negation; data from (Hofmann 2019):

- (26) There is no<sup>v</sup> bathroom in this house. It<sub>v</sub> would be easier to find.



## 5.2 Epistemic modality in dynamic semantics

### 5.2.1 Initial motivations

The classical relational semantics for epistemic possibility (Kratzer 2012).

$$(27) \quad \llbracket \Diamond \phi \rrbracket^{w,f} := \mathbf{true} \text{ iff } \exists w' \in f(w) [\llbracket \phi \rrbracket^{w'} \text{ is } \mathbf{true}]$$

The parameter  $f$  is taken to be determined by the conversational context, and maps each world  $w$  to the set of worlds compatible with what is known in  $w$ .

Known by who? One salient possibility is the speaker.<sup>5</sup>

A fatal problem for this view ((Hawthorne 2004); data from (Rudin 2021)):

- (28)    a. Andrea: Paul might have been at the party last night.  
          b. Bertrand: You're wrong, he was in Barbados.

“The puzzle is that no way of pinning the relevant knowledge state down seems to be able to explain both why we are in a position to make the epistemic “might” claims we seem to be in a position to make, and also why it is often reasonable to disagree with “might” claims made by others” (Stalnaker 2014)

## 6 References

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<sup>5</sup>Yanovich calls this view “contextual solipsism” (Yanovich 2014).

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