From FCS to DPL (and back again)

Patrick D. Elliott

February 6, 2022

Contents

1 Next week

Topic: dynamic semantics as a theory of composition.

- **Obligatory reading**: Simon Charlow's recent paper "Static and dynamic exceptional scope" (Charlow 2020).

- **Optional readings**

    - Part I of Charlow's 2014 dissertation "On the semantics of exceptional scope" (Charlow 2014).

    - Groenendijk and Stokhof's 1990 paper "Dynamic Montague Grammar" (Groenendijk & Stokhof 1990).

2 FCS

2.1 Contexts, assignments, and variables

Definition 2.1. Contexts and assignments.

- A (file) context c is a set consisting of world-assignment pairs.

- An *assignment* is a total function from variables to individuals.

Definition 2.2. A variable x is *novel* with respect to a context c, iff:

$$\mathbf{nov}_c(x) := \forall g, g', w[g[x]g' \rightarrow ((w,g) \in c \iff (w,g'))]$$

Otherwise, x is *familiar* with respect to c.

N.b. *familiarity* is persistent, i.e., if $\mathbf{fam}_c(x)$, then for any $c' \subseteq c$, $\mathbf{fam}_x(c')$.

Core thesis of FCS: Both indefinite and definite DPs are interpeted as *variables*.

- Indefiniteness imposes a novelty check on the input context.

- Definiteness imposes a familiarity check on the input context.

(1) A^x woman walked in. $\Rightarrow \lambda c : \mathbf{nov}_c(x) . \{ (w,g) \in c \mid \mathbf{womanWhoWalkedIn}_w(g_x) \}$

(2) She$_x$ sat down. $\Rightarrow \lambda c : \mathbf{fam}_c(x) . \{ (w,g) \in c \mid \mathbf{satDown}_w(g_x) \}$

Definition 2.3. Heim's bridge principle. An assertion of sentence ϕ in a context c, if accepted, gives rise to an updated context $c[\phi]$, defined as follows:

$$c[\phi] := [\![\phi]\!] \, (c)$$

2.2 Example 1: discourse anaphora

A concrete example. Let's consider the flow of information in the following discourse:

(3) A^x woman walked in; she sat down.

- $w_\emptyset$: neither walked in nor sat down.

- w_a: a walked in and sat down; b did neither.

- w'_a: a walked and didn't sit; b didn't walk in but did sit down.

- w_b: b walked in and sat down; a did neither.

- w'_b: b walked in and sat down; a didn't walk in but did sit.

- $w_a b$: a and b both walked in and sat down.

Variables: x, y. The initial context is the Cartesian product of logical space and the set of assignments $g : \{ x, y \} \rightarrow \{ a, b \}$.

$$c_0 = \left\{ \begin{array}{cccc}
\left(w_\emptyset, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_\emptyset, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_\emptyset, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_\emptyset, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_a, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_a, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_a, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_a, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_{a'}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_{a'}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_{a'}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_{a'}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_b, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_b, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_b, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_b, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_{b'}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_{b'}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_{ab}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_{ab}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right)
\end{array} \right\}$$

Update with "A^x woman walked in".

- Novelty is trivially satisfied.

- We eliminate any (w, g), where g maps x to a woman who didn't walk in, in w.

$$c_1 = \left\{ \begin{array}{cc}
\left(w_a, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_a, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_{a'}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), & \left(w_{a'}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_b, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_b, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_{b'}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), & \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\[2ex]
\left(w_{ab}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), \left(w_{ab}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), & \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right)
\end{array} \right\}$$

Now let's update c_1 with "She$_x$ sat down".

- First, to check whether or not the update is defined, we check if x is familiar.

- All that's necessary to show that x is familiar is to find some world w which discriminates between assignments that differ only at x. There are plenty to choose from, but consider w_a, which is paired with $[x \to a, y \to a]$ but not $[x \to b, x \to a]$.

- Now we eliminate any $(w, g$, where g maps x to a woman who didn't sit down.

$$c_2 = \left\{ \begin{array}{c} \left(w_a, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), \left(w_a, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right) \\ \left(w_b, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), \left(w_b, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\ \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \\ \left(w_{ab}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}\right), \left(w_{ab}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}\right), \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}\right), \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix}\right) \end{array} \right\}$$

We're left just with the worlds in which someone walked in and sat down.

- In w_a, it's a who did so, so w_a is paired with assignments that map x to a.

- In w_b, it's b who did so, so w_b is paired with assignments that map x to b.

- In w_{ab}, it's both a and b who did so, so w_{ab} is paired with assignments that map a to b and assignments that map x to b.

Note, c_2 is only true at worlds where the same woman both walked in and sat down.

Let's double check that y still isn't familiar in c_2 (remember, we only consider assignments which differ minimally wrt y).

2.3 Example 2: concurrent indefinites

Round two. Consider the following discourse:

(4)   A^x woman walked in. A^y woman sat down.

We already know what happens when we update the initial context with the first sentence, we get c_1. We know that y is *novel* in c_1, so let's update c_1 with "A^y woman sat down".

- We eliminate any (w, g), where g maps y to a woman who didn't sit down.

$$c_3 = \left\{ \begin{array}{c} \left(w_a, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix} \right) \\ \left(w_{a'}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix} \right) \\ \left(w_b, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix} \right) \\ \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix} \right), \left(w_{b'}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix} \right) \\ \left(w_{ab}, \begin{bmatrix} x \to a \\ y \to a \end{bmatrix} \right), \left(w_{ab}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix} \right), \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix} \right), \left(w_{ab}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix} \right) \end{array} \right\}$$

c_3 is true at worlds where the same individual both walked in and sat down (w_a, w_b, w'_b, w_{ab}), but also worlds in which distinct individuals walked in and sat down (w'_a).

FCS therefore doesn't derive *disjoint reference* out of the box, for concurrent, contra-indexed indefinites.

2.4 Informativity and implicature in FCS

What is the nature of the putative disjoint reference inference?

(5)   A woman walked in; a woman sat down. In fact, I later discovered that it was the same woman.

(6)   context: *the speaker is knows that that the same woman walked in and sat down, but isn't sure which woman walked in and sat down.*

(7)   ?? A^x woman walked in; A^y woman sat down.

(8) A^x woman walked in; she$_x$ sat down.

These facts suggest that disjoint reference should be derived as a *primary implicature* (in the sense of (Sauerland 2004)), i.e., the speaker is not certain that x and y are mapped to same individual.

In fact, this can be derived via a familiar Gricean reasoning process if "A^x woman walked in and the$_x$ woman sat down" is an alternative to "A^x woman walked in and a woman^y sat down", since the former is strictly more informative than the latter.

We can easily formalize *contextual informativity* in FCS as follows (definition from (Sudo 2022)):

Definition 2.4. Contextual informativity. A sentence ϕ is *contextually more informative* than a sentence ψ with respect to a context c iff:

$$\mathbf{W}(c[\phi]) \subset \mathbf{W}(c[\psi])$$

- Upon hearing "A^x woman walked; a^y woman sat down", the hearer notices an alternative "A^x woman walked in; the^y woman sat down", which would have been contextually more informative.

- The speaker didn't assert the alternative, so they must not be certain that it's true.

- A stronger, secondary implicature could be derived pragmatically (Sauerland 2004), or via an exhaustification operator geared to act on CCPs (Fox 2007).

Note that the judgement is that concurrent indefinites result in *oddness* in the context given in (8), which isn't necessarily expected on a pragmatic account.

This can arguably be understood as a special case of a phenomenon that applies to scalar implicatures more generally (Magri 2009).

2.5 Informational vs. dynamic redunancy

A contrast noted by Filipe last week:

(9)   A^x woman walked in; A^y woman sat down.

(10)   ??A^x woman walked in; A^y woman walked in.

Why does (9) sound OK (and give rise to a disjoint reference inference), whereas (10) sounds odd?

Conjecture: despite being *dynamically informative*, the second assertion in (10) is *informationally redundant*.

(11)   *Principle of non-redundancy:* an assertion is infelicitous if it is redundant.

We can define a number of different notions of redundancy within FCS. Here are two:

Definition 2.5. Informational redundancy. Assertion of ϕ with respect to a context c is *informationally redundant* iff:

$$\mathbf{W}(c[\phi]) = \mathbf{W}(c)$$

Definition 2.6. Dynamic redundancy. Assertion of ϕ with respect to a context c is *dynamically redundant* iff:

$$c[\phi] = c$$

Now, consider again our odd example, "A^x woman walked in. A^y woman walked in".

- The second assertion isn't *dynamically* redundant, since it cuts our world-assignment pairs where y gets mapped to someone who isn't a woman that walked in.

- It is, however, *informationally* redundant, since the informational contribution is the same as the first assertion.

- Note that the second assertion may not be informationally redundant if it were guaranteed that $x \neq y$ (e.g., via a disjointness inference).
  - The disjointness inference can't be derived in the same way as before, since the alternative "A^x woman walked in; the woman$_x$ walked in" also gives rise to a violation of the principle of redundancy.

Conjecture: in natural language, dynamic informativity (e.g., introducing a discourse referent) isn't enough to obviate informational redundancy - the *principle of redundancy* should be understood relative to informational redundancy.

An aside: this finding addresses a potential issue with the Heimian notion of *novelty*, which predicts that tautological statements can't introduce discourse referents.

(12)   Something^x is self-identical; it$_x$ is John.

There are two possibilities for (12):

- Everything is self-identical, therefore "Something^x is self-identical" is informationally trivial and violates the principle of non-redundancy for any context.

- There is a way of understanding "Something^x is self-identical" which renders it informationally informative, in which case x becomes familiar and anaphora is possible.

Such sentences therefore don't seem to pose a serious threat to the Heimian notion of novelty/familiarity, contra (Heim 1982).

2.6 A loose end: definite descriptions

Recall that pronouns are variables with a *familiarity* presupposition.

We haven't said anything about definite descriptions, but FCS very naturally accounts for definite descriptions in the following way, e.g., for "the$_x$ woman sat down.":

$$(13) \quad \lambda c . \begin{cases} \{ \, (w,g) \in c \, . \, \mathbf{walkedIn}_w(g_x) \, \} & \forall (w,g) \in c[\mathbf{woman}_w(g_x)] \\ \text{undefined} & \text{else} \end{cases}$$

This accounts for the fact that definites behave like pronouns with an indefinite antecedent (we tailor our example here to show that uniqueness isn't required):

(14)   A^x woman walked in; The$_x$ woman sat down with another woman.

- Updating c with the first sentence eliminates world assignment pairs $(w,g$, where g_x isn't a woman who walked in in w.

- Updating with the second sentence is defined iff every world assignment pair $(w,g$ in the input is s.t. g_x is a woman.

- The requirement is satisfied, so now we eliminate (w,g)s where g_x didn't sit down with another woman in w.

    - Note that the presupposition of a definite description is a strictly stronger requirement than familiarity *per se.*

We can account for ϕ-features in a parallel fashion. A putative CCP for "She$_x$ sat down":

$$(15) \quad \lambda c . \begin{cases} \{ \, (w,g) \in c \mid \mathbf{satDown}_w(g_x) \, \} & \forall (w,g) \in c[\mathbf{prefersSheHer}_w(g_x)] \\ \text{undefined} & \text{else} \end{cases}$$

A problem with the approach to definite descriptions in FCS (the *familiarity theory* of definiteness) is that it fails to account for uniqueness inferences.

After all, there's a reason people believed that *uniqueness* is entailed by definite descriptions (the examples here are a variant of data discussed by (Mandelkern & Rothschild 2020)).

(16)   John went on a date; he liked the$_x$ woman, so he planned to see again.

(17)   John went on dates with a few different women; he liked the$_x$ woman, so he planned to see her again.

- In sum, FCS (and dynamic theories more generally) do excellently in accounting for the fact that definites don't require uniqueness if they have an indefinite antecedent.

- As it stands however, it is mysterious why definites require uniqueness when they don't have an antecedent. As emphasized by (Mandelkern & Rothschild 2020), this is a problem for theories of *accommodation* - in fact, the logic is very similar to the proviso problem.

2.7 Dynamic conjunction

Recall that *novelty-familiarity* is crucially defined relative to *files* rather than individual evaluation points.

There's no way to determine, just by looking at individual evaluation points, whether or not a variable is novel with respect to the file.

Since the novelty requirement is taken to be induced by a linguistic feature of particular sentences - (in)definiteness - there is apparently no escape from treating sentences as functions from files to files (i.e., CCPs).

We can define an operator to perform successive update. This turns out to just be function composition. We'll write it as ;, and it will play a prominent role in FCS.[1,2]

Definition 2.7. The dynamic sequencing operator ;.

$$p; q := \lambda F \, . \, q(p(F))$$

At this point, it will be useful to give a type for a CCP; let's use T. The dynamic sequencing operator is of type $T \to T \to T$.

The sequencing operator famously serves as the meaning of conjunction in FCS. Assuming a standard Kaynean syntax for conjunction:

(18)   and $\Rightarrow \lambda q \, . \, \lambda p \, . \, p; q$ $T \to T \to T$

A direct consequence of this definition is that successive update $c[\phi][\psi]$ is exactly equivalent to $c[\phi \text{ and } \psi]$.

Since dynamic sequencing is just function composition, it inherits the logical properties of function composition, e.g., it is associative. This means we'll often omit parentheses when sequencing.

---

[1] In more traditional mathematical notation, we'd write this as $\psi \circ \phi$. I prefer $\phi; \psi$, as it more directly reflects information flow in a dynamic setting.

[2] (Chierchia 1995) goes as far as to suggest that a semantics is *dynamic* iff the meaning of conjunction is function composition. We'll see many reasons to reject this characterization in the following classes.

$$(\phi; \psi); \pi \iff \phi; (\psi; \pi)$$

Both of the following are equivalent:

(19) A^x man walked in and he$_x$ sat down. He$_x$ started fidgeting.

(20) A^x man walked in. He$_x$ sat down and he$_x$ started fidgeting.

Similarly, dynamic sequencing is not necessarily commutative (although some CCPs commute). This reflects the fact that some CCPs can set up the preconditions for other CCPs to apply.

(21) A^x man walked in and he$_x$ sat down.

(22) ?He$_x$ walked in and a^x man sat down.

FCS predicts the CCP of (22) to be undefined for *any* file context.

It's often assumed that FCS (and dynamic semantics more generally) explains why discourse anaphora displays a left-to-right bias, as illustrated by (22).

> It's important to note that all of the results stated so far must be assessed in light of a compositional proposal with sufficient resources for scope-taking. We'll come back to this in a couple of weeks time.

Before talking about subsentential compositionality (and donkey anaphora!) in FCS, we'll consider a partial variant of FCS.

3 TODO say something about false files (this relates to triviality)

4 Partial FCS

4.1 File contexts in partial FCS

Recall the apparent problem Heim observed with informationally trivial updates.

(23) Something^x is identical to itself. It$_x$ is John.

As we've discussed, this arguably isn't a problem for the Heimian notion of novelty, due to the independently motivated pragmatic principle against redundant assertions.

Nevertheless, Heim took this problem seriously. It is now common to adopt a variant of FCS which uses partial assignments, as suggested by Heim (Heim 1983). See, e.g., (Dekker 1996, Yalcin 2013, Rothschild & Mandelkern 2017, Mandelkern 2020, Heim 2017, Elliott 2020).[3, 4]

A partial assignment is a *partial* function from variables to individuals, i.e., a function whose domain is a subset of the stock of variables. This includes the empty/initial assignment $g_\top$, whose domain is the empty set.

In a partial setting, its natural to model the initial/ignorance file context in the following way:

$$c_\top = W \times \{\, g_\top \,\}$$

In partial FCS, familiarity is cashed out as a universal requirement on the domains of assignments in the file context.

4.2 Novelty and familiarity in partial FCS

Definition 4.1. Familiarity in partial FCS.

$$\mathbf{fam}_c(x) \iff \forall (*, g) \in c[x \in \mathbf{dom}(g)]$$

Novelty, just as before, is the other side of the same coin.

Definition 4.2. Novelty in partial FCS.

$$\mathbf{nov}_c(x) \iff \neg\, \mathbf{fam}_c(x)$$

---

[3] Another possible resolution, which to my knowledge hasn't been explored in any depth, is to adopt a non-Lewisian logical foundation in which certain tautologies can be distinguished, such as truthmaker/situation semantics (Fine 2017c,a,b).

[4] In her dissertation, Heim addresses this issue by giving files an additional domain parameter (Heim 1982).

4.3 (In)definiteness in partial FCS

Just as before, pronouns are interpeted as variables with a familiarity presupposition: the CCP for "she$_\text{x}$ sat down":

$$(24) \quad \lambda c \,.\, \begin{cases} \{\, (w, g) \in c \mid \mathbf{satDown}_w(g_x) \,\} & \mathbf{fam}_c(x) \\ \text{undefined} & \text{else} \end{cases}$$

Indefinites are a little different, since they must extend the domain of assignments in the input context. The CCP for "A$^\text{x}$ woman walked in":

$$(25) \quad \lambda c \,.\, \begin{cases} \{\, (w, g^{[x \to a]}) \mid (w, g) \in c \wedge \mathbf{womanWalkedIn}_w(a) \,\} & \mathbf{nov}_c(x) \\ \text{undefined} & \text{else} \end{cases}$$

(Note: this is awfully similar to what we'll end up with as the semantics of existential quantifiers in DPL, modulo the novelty requirement)

There's a way of mapping FCS file contexts to partial FCS file contexts:

Definition 4.3. Mapping from an FCS file context c to a partial FCS file context.

$$\{\, (w, g') \mid (w, g) \in c \wedge \forall v \in \mathbf{dom}(g)[(\mathbf{nov}_c(v) \to v \notin \mathbf{dom}(g')) \wedge (\mathbf{fam}_c(v) \to g_v = g'v)] \,\}$$

It can be shown that the CCPs provided for partial FCS correspond to mapping FCS contexts updated by our previous CCPS to partial FCS file contexts. I'll leave this as a homework exercise.

4.4 Partial FCS is non-eliminative

One immediate consequence of this is that we lose an (appealing?) logical property of Heim's original FCS, which it shares with classical Stalnakerian pragmatics - *eliminativity* (Groenendijk & Stokhof 1991b).

Definition 4.4. ϕ is *eliminative* iff $c[\phi] \subseteq c, \forall c$

Sentenes with indefinites convey non-eliminative updates; in FCS proper all updates are eliminative.

This is important, since eliminativity plays an important role in characterizing exactly what it means for a semantics to be "dynamic", together with *distributivity*.

We'll talk more about this next week (Heim's FCS is eliminative but non-distributive; DPL is distributive but non-eliminative) (van Benthem 1989).

4.5 Presupposition in partial FCS

Something interesting we can note about this partial variant: the novelty/familiarity presupposition is cashed out as a universal requirement on every evaluation point in the domain.

It seems tempting to treat sentences as being simply partial functions from *evaluation points* to "files", allowing a Stalnakerian bridge principle to take care of the universal requirement on the file context. What we would end up with is a system where sentences denote something less expressive than a CCP.

Unfortunately this is incompatible with the semantics of logical vocabulary, and the account of donkey anaphora, which involve positing operators which operate on CCPs.

We'll resurrect this idea when we discuss Elliott's /externally-dynamic dynamic semantics in a couple of weeks time (Elliott 2020).

5 FCS as a theory of composition

We'll switch back to FCS proper now, although the partial variant will be mentioned from time to time.

5.1 The indefinite article

A remarkable property of FCS is that the indefinite article itself can be given a CCP (i.e., be treated as a sentence).

$$(26) \quad \mathrm{a}^x \Rightarrow \lambda c \, . \begin{cases} c & \mathbf{nov}_c(x) \\ \text{undefined} & \text{else} \end{cases} \qquad\qquad T$$

A natural consequence is a treatment of the restrictor and nuclear scope of a determiner as open formulas, just like sentences with pronouns.

$$(27) \quad t_x \text{ man} \Rightarrow \lambda c : \mathbf{fam}_c(x) \, . \, \{ \, (w, g) \in c \mid \mathbf{man}_w(g_x) \, \}$$

$$(28) \quad t_x \text{ sat down} \Rightarrow \lambda c : \mathbf{fam}_c(x) \, . \, \{ \, (w, g) \in c \mid \mathbf{satDown}_w(g_x) \, \}$$

By conjecture, dynamic sequencing is generally available as a composition principle. The logical form Heim assumed for a sentences with indefinites in FCS.

```
              T
              ;
          ╱       ╲
        T           T
        ;       $t_x$ walked in
      ╱   ╲
    T       T
   $a^x$  $t_x$ man
```
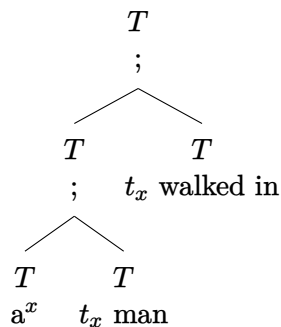
How to derive this syntactically? One possibility:

- A determiner indexed x moves to the edge of the DP, leaving behind a trace t_x.

- The DP inherits its index from the D head.

- The DP moves to the edge of the sentence, leaving behind a trace t_x.

A design feature of FCS is that sentences with indefinites are essentially equivalent to introducing a novel variable, and sequentially adding information to it.

(29) A^x man walked in.

(30)   There is someone^x. He$_x$ is a man. He$_x$ walked in.

This property is responsible for the validity of *Egli's theorem* in dynamic theories of meaning, including FCS. Egli's theorem states that sentences of the following kind are equivalent.

$$A^x \text{ man [walked in and sat down]} \iff A^x \text{ man walked in and he}_x \text{ sat down}$$

5.2 Compositionality

There are many reasons to be dissatisfied with FCS as a theory of subsentential compositionality.

Every natural language constituent is treated as either (i) a CCP, or (ii) an operator on CCPs. This is a somewhat inflexible mold.

- To what extent is it compatible with concurrent developments in formal semantics, in the Montagovian tradition?

- What do predicates mean in FCS?

- Does dynamic semantics really commit us to specific syntactic transformations? Why?

  We'll talk more about dynamic approaches to subsentential compositionality next week.

15

5.3 Donkey anaphora and universal quantification

5.3.1 Interpreting universal statements

(31) Every cat died.

In FCS, universal quantifiers are higher-order operators on CCPs, just like other logical vocabulary.

Unlike conjunction however, a quantificational determiner does not trigger simple successive updates, but does something more complex.

One way of cashing out the meaning of quantificational deteminers in FCS is to assume a logical form like the one given in (32), where the indefinite is silent.

(32) every $[\text{a}^x \text{ cat}]$ $[t_x \text{ died}]$

We'll need an auxiliary equivalence relation on assignments - we'll say that assignments are *file-equivalent* iff they agree on all familiar variables.[5, 6]

Definition 5.1. file-equivalence for assignments. Given a file context c, file-equivalence for two assignments g, g' is defined as follows.

$$g \sim_c g' \iff \forall v[\mathbf{fam}_c(v) \to (g_v = g'_v)]$$

We'll use this to ensure that the quantifier ranges over all possible values for **novel** variables.

Now, the meaning of *every* (definition taken from (Heim 2017)).

(33) $\mathbf{every}(p)(q) := \lambda c . \left\{ (w, g) \in c \; \middle| \; \begin{array}{l} \forall g'[g' \sim_c g \wedge (w, g') \in p(c) \\ \to \exists g''[g'' \sim_{p(c)} g' \wedge (w, g'') \in q(p(c))]] \end{array} \right\}$

What's happening here? Let's go through it step-by-step.

- Tentatively update c with with the restrictor; return $p(c)$.

- Tentatively update $p(c)$ with the nuclear scope; return $q(p(c))$.

- Now, for each $(w, g) \in c$, run the following test:

---

[5]The presentation here is based on discussion in (Heim 2017).
[6]Note that, in a partial setting, two assignments are file-equivalent simply if they are extensionally equivalent functions (since novel variables aren't in the domain).

- Take every c-equivalent g', such that (w, g') in $p(c)$.
  - See if you can find a $p(c)$-equivalent g'' in $p(q(c))$.
- Only keep those world assignment pairs $(w, g) \in F$ that pass the test.

5.3.2 A concrete example

In order to understand how this works, it's worth going through a concrete example.

(34)   every $[a^x$ cat$]$ $[t_x$ died$]$

Consider the following logical space W, with domain a, b:

- $w_\emptyset$: a, b are cats; neither died.

- w_a: a is a cat and b is a dog; both died.

- w_b: a, b are cats; only b died.

- w_{ab}: a, b are cats, both died.

Updating $c_\top$ with "every cat died" should eliminate world-assignment pairs involving $w_\emptyset, w_b$.

Given a stock of variables x, y, the initial state is given by taking the product of W and the set of possible assignments:

$$\left\{ \begin{bmatrix} x \to a \\ y \to a \end{bmatrix}, \begin{bmatrix} x \to a \\ y \to b \end{bmatrix}, \begin{bmatrix} x \to b \\ y \to a \end{bmatrix}, \begin{bmatrix} x \to b \\ y \to b \end{bmatrix} \right\}$$

(35) $c_\top$[every cat died]
$$= \left\{ (w, g) \in c_\top \;\middle|\; \begin{array}{l} \forall g'[g' \sim_{c_\top} g \wedge (w, g') \in p(c_\top) \\ \to \exists g''[g'' \sim_{p(c_\top)} g' \wedge (w, g'') \in q(p(c_\top))]] \end{array} \right\}$$

Every variable is novel in $c_\top$ so this can be simplified as follows:

(36) $c_\top$[every cat died]
$$= \left\{ (w, g) \in c_\top \;\middle|\; \begin{array}{l} \forall g'[(w, g') \in p(c_\top) \\ \to \exists g''[g'' \sim_{p(c_\top)} g' \wedge (w, g'') \in q(p(c_\top))]] \end{array} \right\}$$

$c_\top [\text{every cat died}]$

$$= \left\{ (w, g) \in c_\top \;\middle|\; \begin{array}{l} \forall g'[(w, g') \in c_\top \wedge \mathbf{cat}_w(g'_x) \\ \to \exists g''[g'' \sim_x g' \wedge (w, g'') \in c_\top \wedge \mathbf{catThatDied}_w(g''_x)]] \end{array} \right\}$$

What this says is that $(w, *)$ is retained in the output file context iff, every assignment g' paired with w that maps x to a cat is s.t. we can find an assignment g'' that maps x to the same value, and g'' maps x to a cat that died in w.

This will clearly succeed for $(w_a, *), (w_{ab}, *)$, and fail for $(w_\emptyset, *), (w_b, *)$

5.3.3 Extension to donkey anaphora

One of the original motivations for FCS was giving a semantic account of pronouns in sentences such as the following.

(37)   Every philosopher who had a^y question asked it$_y$.

The extension to donkey anaphora is straightforward - the schema for universal quantification ensures that the quantifier ranges over all possible values for all novel variables in the input context.

Logical form for (37):

- Every [a^x philosopher] [t_x asked it$_y$]

The FCS approach to donkey anaphora has some well known problems, which we'll return to in future classes, namely:

- Weak readings (Chierchia 1995, Kanazawa 1994).

- The proportion problem

(38)   Everyone who had a credit card used it.

(39)   Most men who own a donkey beat it.

Predicted reading: most man-donkey-he-owns pairs (m, d) are s.t. m beats d.

Falsifying scenario: *99 men own one donkey and don't beat it; one man owns 200 and beats them all.*

5.4 TODO negation and disjunction in FCS

5.5 Open questions

We've gone through the basics of FCS, arguably the first real dynamic semantic proposal.

Some questions to keep in mind:

- What makes FCS *dynamic*, as opposed to a more classic semantics that treats sentence meanings as functions from evaluation points to truth-values?

- Is the additional expressive power afforded by CCPs in the semantics warranted?

- Is *novelty* as a file-level notion unavoidable?

- How does logical vocabulary such as negation and disjunction interact with anaphora? Does FCS have a satisfying story (hint: it does not).

- Does dynamic semantics inescapably wed us to a non-standard view of subsentential composition?

6 DPL

6.1 Random assignment

Dynamic Predicate Logic (DPL) provides a dynamic interpretation schema for a simple first-order calculus.

(Groenendijk & Stokhof 1991a) give a relational presentation of DPL - formulas are interpreted relative to an evaluation point (a pair of assignments) and return a boolean value.

Here, I give an isomorphic presentation which emphasizes two key features of the DPL interpretation schema: (i) statefulness, and (ii) non-determinism.

Formulas are interpreted relative to an evaluation point (an assignment) and return a *set of evaluation points*.

Definition 6.1. DPL semantics for atomic formulas. For any model $M := (D, I)$

$$[\![t]\!]^g_M = \begin{cases} g_t & t \text{ is a variable} \\ I(t) & t \text{ is a constant} \end{cases}$$

$$[\![P(t_1,\ldots,t_n)]\!]^g_M = \{\, h \mid g = h \wedge \langle [\![t_1]\!]^g_M, \ldots, [\![t_n]\!]^g_M \rangle \in I(P) \,\}$$

An example:

(40) $\llbracket \mathtt{Swim}(x) \rrbracket^g = \begin{cases} \{\, g \,\} & g_x \text{ swims} \\ \emptyset & \text{otherwise} \end{cases}$

Atomic formulas in general are **tests**, i.e., identity relations on assignments.

Existential quantifiers induce what is known as **random assignment** in the dynamic literature.

Definition 6.2. DPL semantics for existential quantification.

$$\llbracket \exists x, \phi \rrbracket^g = \bigcup_{a \in D} \{\, g^{[x \to a]} \mid \llbracket \phi \rrbracket^{g^{[x \to a]}} \,\}$$

An example, where $D = \{\, a, b, c \,\}$:

(41) $\llbracket \exists x [\mathtt{Swim}(x)] \rrbracket^g = \{\, g^{[x \to a]} \mid a \text{ swims} \,\} \cup \{\, g^{[x \to b]} \mid b \text{ swims} \,\} \cup \{\, g^{[x \to c]} \mid c \text{ swims} \,\}$

Given a single evaluation point g, existential quantification gives rise to a *potentially non-deterministic output.*

Definition 6.3. Truth in DPL. A formula ϕ is *true* with respect to an assignment g iff $\llbracket \phi \rrbracket^g \neq \emptyset$

6.2 Complex formulas

Conjunction in DPL is intepreted as *relational composition.*

Definition 6.4. Conjunctive formula in DPL.

$$\llbracket \phi \wedge \psi \rrbracket^g = \bigcup_{h \in \llbracket \phi \rrbracket^g} \{\, i \mid i \in \llbracket \psi \rrbracket^h \,\}$$

This allows the indeterminate output induced by an existential quantifier to be picked up by a matching variable outside of its ordinary scope. Let's again assume that our domain consists of three individuals $\{\, a, b, c \,\}$.

(42) $\llbracket \exists x [\mathtt{WalkedIn}(x)] \wedge \mathtt{satDown}(x) \rrbracket^g$

(43) $= \bigcup_{h \in \llbracket \exists x [\mathtt{walkedIn}(x)] \rrbracket^g} \{\, i \mid i \in \llbracket \mathtt{satDown}(x) \rrbracket^h \,\}$

$$(44) \quad = \bigcup_{h \in \{\, g^{[x \to a]} \mid a \text{ walked in} \,\}} \{\, i \mid i = h \land h_x \text{ sat down} \,\}$$

$$(45) \quad = \{\, g^{[x \to a]} \mid a \text{ walked in and } a \text{ sat down} \,\}$$

7 References

References

van Benthem, Johan. 1989. Semantic Parallels in Natural Language and Computation. In H. -D. Ebbinghaus et al. (eds.), *Studies in Logic and the Foundations of Mathematics*, vol. 129 (Logic Colloquium'87), 331–375. Elsevier. `https://www.sciencedirect.com/science/article/pii/S0049237X08701332` (31 January, 2022).

Charlow, Simon. 2014. *On the semantics of exceptional scope.* New Brunswick: Rutgers University dissertation.

Charlow, Simon. 2020. Static and dynamic exceptional scope. Unpublished manuscript. Accepted at journal of semantics.

Chierchia, Gennaro. 1995. *Dynamics of meaning - anaphora, presupposition, and the theory of grammar.* Chicago: University of Chicago Press. 270 pp.

Dekker, Paul. 1996. The Values of Variables in Dynamic Semantics. *Linguistics and Philosophy* 19(3). 211–257.

Elliott, Patrick D. 2020. Towards a principled logic of anaphora. lingbuzz/005562. MIT. `https://ling.auf.net/lingbuzz/005562`. Submitted to Semantics & Pragmatics.

Fine, Kit. 2017a. A Theory of Truthmaker Content I: Conjunction, Disjunction and Negation. *Journal of Philosophical Logic* 46(6). 625–674.

Fine, Kit. 2017b. A Theory of Truthmaker Content II: Subject-matter, Common Content, Remainder and Ground. *Journal of Philosophical Logic* 46(6). 675–702.

Fine, Kit. 2017c. Truthmaker Semantics. In *A Companion to the Philosophy of Language*, 556–577. John Wiley & Sons, Ltd.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 71–120. London: Palgrave Macmillan UK.

Groenendijk, Jeroen & Martin Stokhof. 1990. *Dynamic Montague Grammar.* Report. `https://eprints.illc.uva.nl/id/eprint/1148/` (2 July, 2021).

Groenendijk, Jeroen & Martin Stokhof. 1991a. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

Groenendijk, Jeroen & Martin Stokhof. 1991b. Two theories of dynamic semantics. In *Proceedings of the European workshop on Logics in AI* (JELIA '90), 55–64. Berlin, Heidelberg: Springer-Verlag.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases.* University of Massachusetts - Amherst dissertation.

Heim, Irene. 1983. File Change Semantics and the Familiarity Theory of Definiteness. In *File Change Semantics and the Familiarity Theory of Definiteness*, 164–189. De Gruyter. `https://www.degruyter.com/document/doi/10.1515/9783110852820.164/html` (27 January, 2022).

Heim, Irene. 2017. Basics and beyond. Unpublished handout from Topics in Semantics. Massachusetts Institute of Technology.

Kanazawa, Makoto. 1994. Weak vs. Strong Readings of Donkey Sentences and Monotonicity Inference in a Dynamic Setting. *Linguistics and Philosophy* 17(2). 109–158.

Magri, Giorgio. 2009. *A theory of individual-level predicates based on blind mandatory implicatures. Constraint promotion for optimality theory.* Massachussetts Institute of Technology dissertation.

Mandelkern, Matthew. 2020. Witnesses. Unpublished manuscript. Oxford.

Mandelkern, Matthew & Daniel Rothschild. 2020. Definiteness projection. *Natural Language Semantics* 28(2). 77–109. `https://doi.org/10.1007/s11050-019-09159-2` (5 February, 2022).

Rothschild, Daniel & Matthew Mandelkern. 2017. Dynamic semantics and pragmatic alternatives. Lecture notes from a course taught at ESSLLI 2017. `http://danielrothschild.com/esslli2017/`.

Sauerland, Uli. 2004. Scalar implicatures in complex sentences. *Linguistics and Philosophy* 27(3). 367–391. `https://doi.org/10.1023/B:LING.0000023378.71748.db`.

Sudo, Yasu. 2022. Scalar implicatures with discourse referents. Unpublished manuscript. UCL. `https://ling.auf.net/lingbuzz/005602`.

Yalcin, Seth. 2013. Introductory Notes on Dynamic Semantics. Berkeley. `https://www.dropbox.com/s/4id64r39kj4c0q2/Yalcin%202013%20dynamic%20notes.pdf`.