

Continuation semantics and the split-scope signature

A note on Hirsch's (2017) argument that type-flexibility is insufficient

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Hirsch (2017) argues that *split scope* readings of conjunction can't be properly accounted for by positing a type-flexible meaning for *and*, and conversely that Conjunction Reduction (CR) *must* be available in order to account for the relevant data. In this note, we show that this argument doesn't go through, once we look beyond a movement-oriented theory of scope-taking – concretely, Barker's (2002) analysis of conjunction within *continuation semantics* (see also Barker & Shan 2014: chapter 7) *predicts* the split-scope signature, by virtue of independently motivated type-shifting operations.

1 Hirsch's argument

Hirsch's argument revolves around the sentence in (1). The key observation is that (1) has a reading which entails that there are two things that John refused to do: (i) *visit any city in Europe* and (ii) *visit any city in Asia*. Under this reading, *and* scopes above *refuse*, but the material in each conjunct must scope below *refuse*, since each conjunct contains an Negative Polarity Item (NPI) licensed by *refuse*.

- (1) John refused to visit [any city in Europe] and [any city in Asia]. (Hirsch 2017: p. 90)
 $\wedge > \text{refuse} > \text{any}$

Hirsch characterizes the “split-scope signature” as follows:¹

- (2) THE SPLIT-SCOPE SIGNATURE
And scopes above some operator, which the apparent Determiner Phrase (DP) conjuncts scope below. (Hirsch 2017: 90)

Hirsch’s account of the split-scope signature is to derive (1) from the Logical Form (LF) in (3), via deletion mechanisms. The details are unimportant for our purposes, but it should be clear enough how the LF in (3), if available, accounts for the split-scope signature:

¹As noted by Hirsch, Partee & Rooth (1983) already recognized the possibility split-scope readings of *disjunction*.

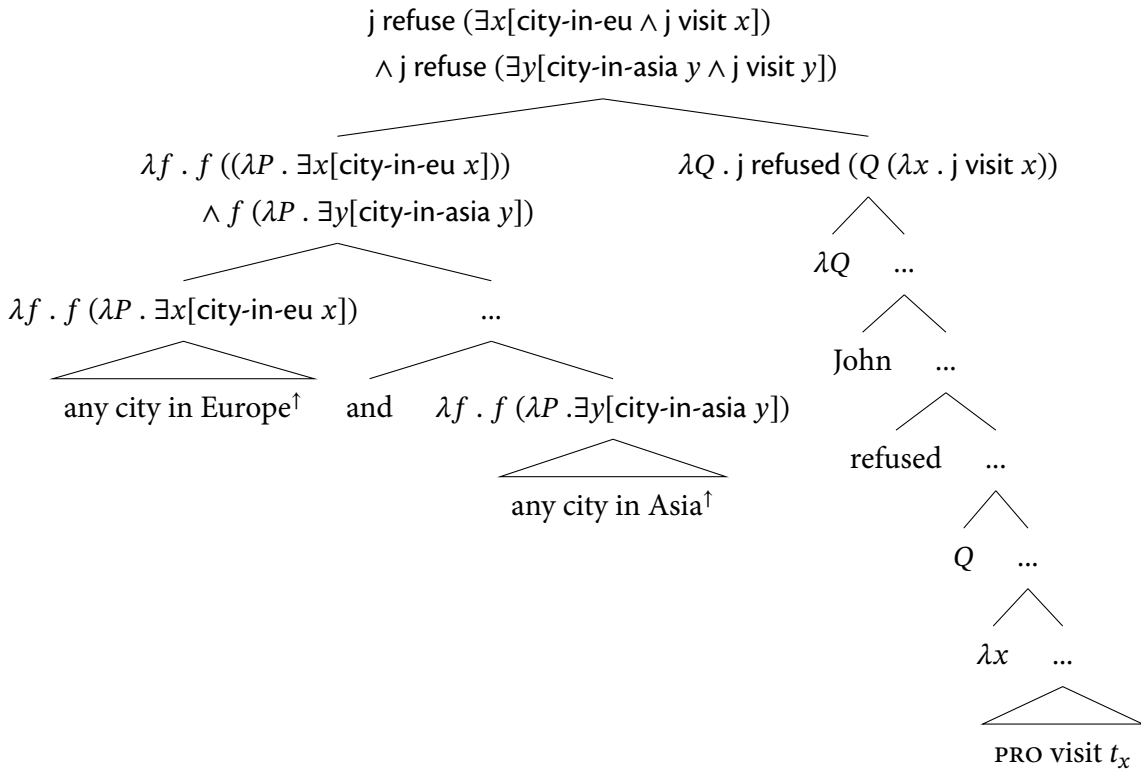
- (3) John λx [$\&P$ [$_{vP_1}$ x refused [any city in Europe λy PRO to visit y]
and [$_{vP_2}$ x refused [any city in Asia λz PRO to visit z]]]]

Hirsch shows in detail that the following denotation for *and* (a special case of Partee & Rooth's 1983 recursive definition) can't capture this reading, on the assumption that (1) in fact involves Quantificational Phrase (QP) conjunction:

- (4) $\llbracket \text{and}_Q \rrbracket := \lambda Q_1 Q_2 P . Q_1 P \wedge Q_2 P$ $((e \rightarrow t) \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t) \rightarrow (e \rightarrow t) \rightarrow t$

Of particular interest to us, is that Hirsch considers (and ultimately dismisses), a derivation of the split scope reading involving *semantic reconstruction*. The LF is sketched in figure 1. Achieving split-scope via semantic reconstruction relies on some key assumptions: (i) the QPs can be type-lifted (via Montague LIFT) prior to conjunction, (ii) Quantifier Raising (QR) can leave behind a higher-type $((e \rightarrow t) \rightarrow t)$ intermediate trace.

Figure (1): Split-scope via semantic reconstruction



Since Hirsch is working in a framework in which scope-taking entails *movement*, Hirsch argues that this mechanism should be independently ruled out, since semantic reconstruction doesn't seem to be available in the absence of syntactic reconstruction. Evidence for this comes from the fact that, when an A-moved constituent reconstructs for scope, it necessarily also syntactically reconstructs, as diagnosed by condition C effects (see Romero 1998).

Add the other references here.

- (5) a. [A new theory by him₁] seems to Quine₁ [___ to be needed].
 b. * [A new theory by Quine₁] seems to him₁ [___ to be needed].

(Hirsch 2017: p. 95)

The argument against this derivation only goes through if scope-taking necessarily involves syntactic movement. In the next section, we'll overview a prominent theory of scope taking – *continuation semantics* – where this scope-taking doesn't necessitate movement.

2 Enter *continuations*

2.1 Background

Hirsch's argument takes the standard-bearer for a type-flexible theory of conjunction to be Partee & Rooth's recursive definition. Subsequently, however, a generalization of Partee & Rooth's account has been proposed by Barker (2002) and Barker & Shan (2014) within the framework of *continuation semantics*. Continuation semantics provides a perspective on scope-taking that generalizes Montague's account of quantification, taking inspiration from work in computer science on using delimited continuations to model control flow (see, e.g., Danvy & Filinski 1992, Wadler 1994). For our purposes, continuation semantics amounts to the following conjecture: composition of *scopal* meanings is accomplished *in-situ*, mediated by three type-flexible operations – *lift* (\uparrow), *lower* (\downarrow), and *Scopal Function Application* (SFA) (S).

The definitions of lift/lower are straightforward, so we provide those first. Lift, defined in (6a), is simply a type-flexible version of Partee's (1986) Montague LIFT. Lower, defined in (6b) is simply an instruction to feed a function m the identity function over truth-values as its argument. In a continuized fragment, the role of lift is to allow non-scope-takers to compose with scope-takers, and the role of lower is to get an ordinary value back from a scopal value.

- (6) a. Lift (def)
 $a^\uparrow := \lambda k . k \ a$ $\uparrow : a \rightarrow (a \rightarrow t) \rightarrow t$
 b. Lower (def)
 $m^\downarrow := m \text{ id}$ $\downarrow : ((t \rightarrow t) \rightarrow t) \rightarrow t$

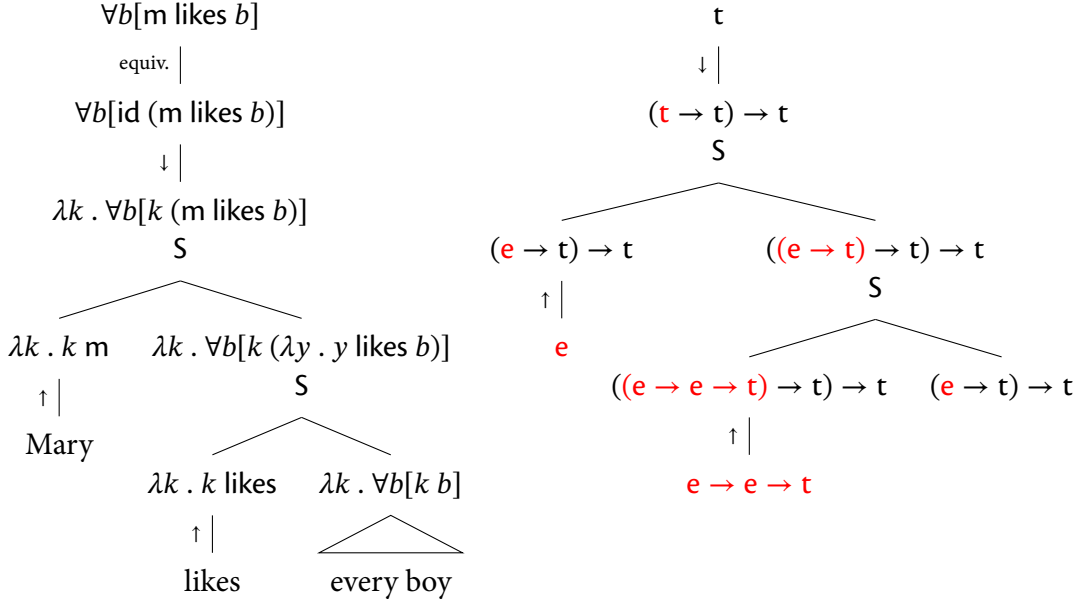
The definition of SFA is a little more complicated. Informally, (7) says that SFA takes a *scopal* function and a *scopal* argument (in either order), unwraps the contained values and performs Function Application (FA) on them, while sequencing the quantificational parts from left-to-right.²

- (7) Scopal Function Application (SFA) (def.)
 $m \ S \ n := \lambda k . m \ (\lambda x . n \ (\lambda y . k \ (x \ A \ y)))$
 $S : S : (((a \rightarrow b) \rightarrow t) \rightarrow t) \rightarrow ((a \rightarrow t) \rightarrow t) \rightarrow (b \rightarrow t) \rightarrow t$
 $S : ((a \rightarrow t) \rightarrow t) \rightarrow (((a \rightarrow b) \rightarrow t) \rightarrow t) \rightarrow (b \rightarrow t) \rightarrow t$

²The careful reader will notice that S in (7) has *two* type-signatures. This is because the S is defined in terms of vanilla Function Application (FA) – A. As in Heim & Kratzer 1998, A is overloaded in order to account for the possibility that the function appear to the left of its argument or vice versa.

A QP such as *every boy* in object position may compose *in-situ* via SFA; non-scope-takers must be systematically lifted. The scope of the QP is evaluated by saturating the continuation argument with the identity function (lower). This is illustrated for the simple sentence “Mary likes every boy” in figure 2. The type of the “contained” value is highlighted – on an intuitive level, continuation-semantic composition involves surrounding meanings with additional scopal scaffolding, in order to allow them to play nicely with scope-takers.

Figure (2): In-situ composition in continuation semantics: “Mary likes every boy”



2.2 Conjunction in continuation semantics

Within continuation semantics, conjunction receives an extremely natural definition, given in (8); *and* takes two scope-takers as m and n , and a *continuation argument* k , and feeds k into m and n . As shown in detail by Barker this subsumes all of the cases covered by Partee & Rooth’s generalized conjunction.

(8) Conjunction in continuation semantics (def.)

$$[\text{and}] := \lambda m n k . n k \wedge m k \quad ((a \rightarrow t) \rightarrow t) \rightarrow ((a \rightarrow t) \rightarrow t) \rightarrow (a \rightarrow t) \rightarrow t$$

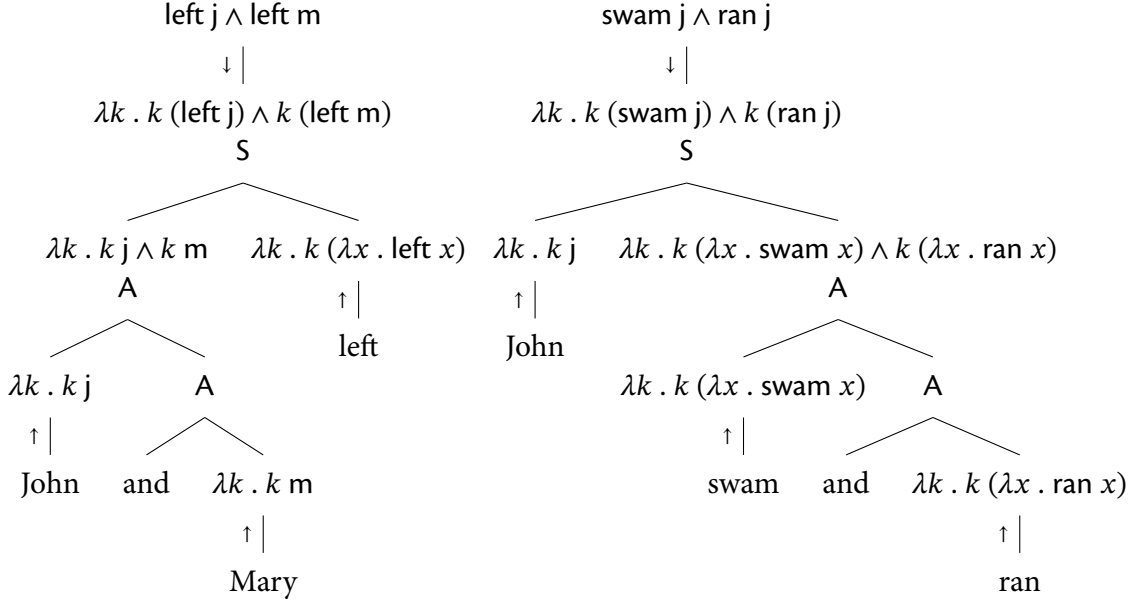
In continuation semantics, the types we’re dealing with will very quickly become rather unwieldy. In order to address this purely notational issue, we’ll abbreviate scopal types as in (9). This means that we can alternatively write the type signature for conjunction as $C \ a \rightarrow C \ a \rightarrow C \ a$.

(9) $C \ a := (a \rightarrow t) \rightarrow t$

It’s obvious the entry for *and* in (8) will be able to account for QP conjunction. We briefly show how it accounts for, e.g., proper name and predicate conjunction in figure 3. In general, unless

the coordinands are scope-takers, they must first be lifted before composing with *and* via vanilla FA.

Figure (3): Proper name conjunction (“John and Mary left”) and predicate conjunction (“John swam and ran”).

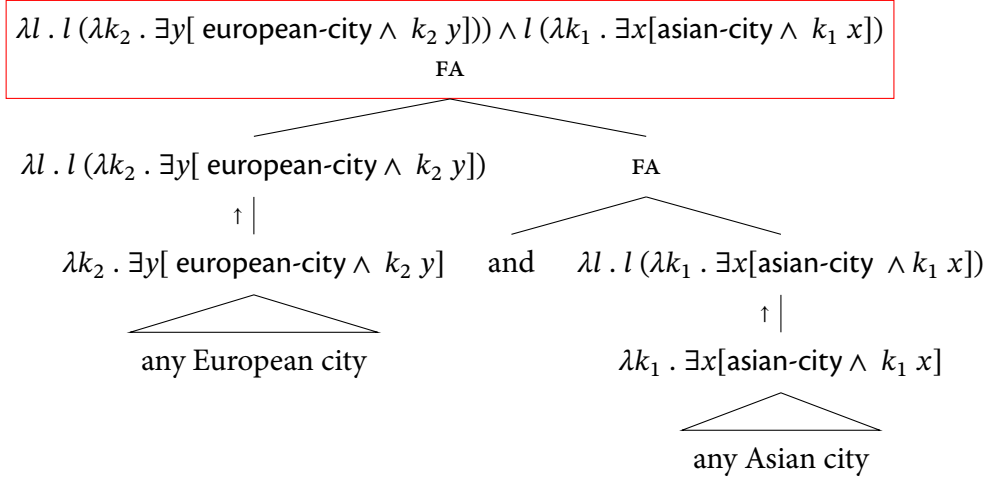


2.3 Accounting for the split-scope signature

Now that we understand how continuation semantics handles coordination, we’re in a position to see how it accounts for the split-scope signature straightforwardly, without positing movement. As with the putative LF involving semantic reconstruction, the crucial step will be to allow QPs to be lifted (this is independently necessary to account for inverse scope readings). We’ll make a number of simplifying assumptions here, but nothing in the analysis will crucially hinge upon them. For example, we’ll treat NPI *any* simply as an existential quantifier, which is licensed as long as it takes scope within the argument of *refuse*.

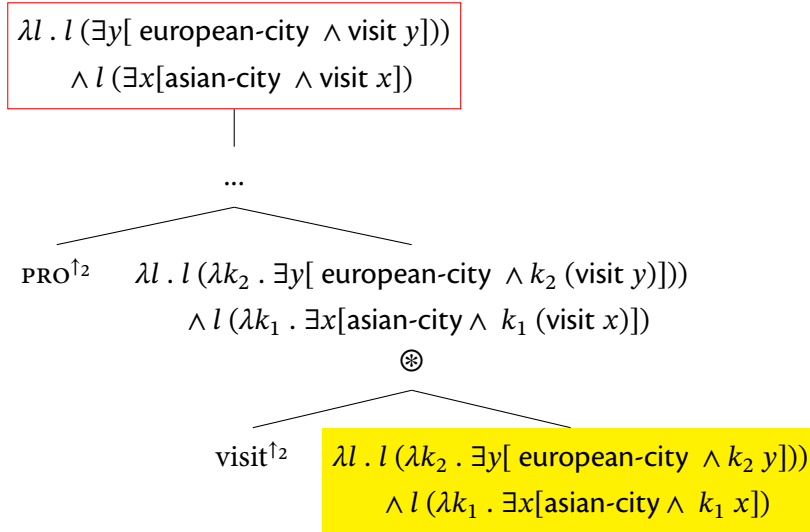
The first step is to lift *any European city* and *any Asian city*, returning two higher-order scope-takers. Since *and* can conjoin anything scopal, it can conjoin the resulting meanings. The result is shown in the figure below:

Figure (4): Lift the NPIS and conjoin



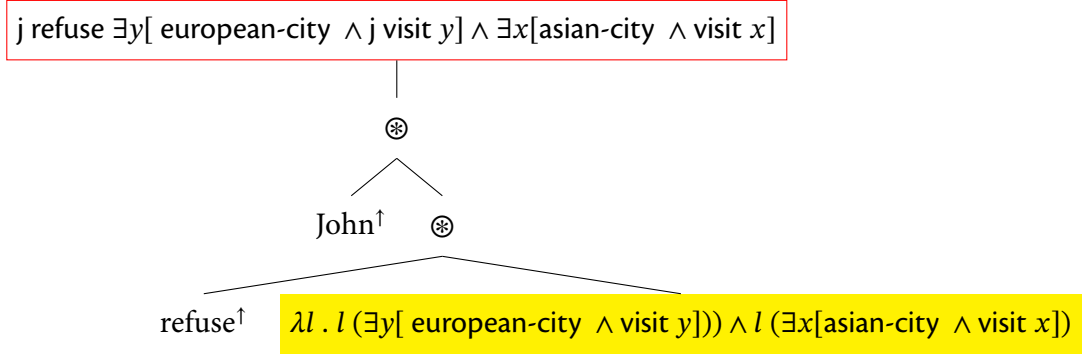
The result of conjoining the lifted NPIS is a higher-order scope taker. In order to get this to compose with the rest of the embedded clause, we lift *visit* twice, and do higher-order SFA. Informally, the result is that the semantic contribution of *visit* is distributed between the two conjunctions. Next, we *internally lower* the result, in order to set the scope of the NPIS.

Figure (5): Compose the embedded clause and internally lower

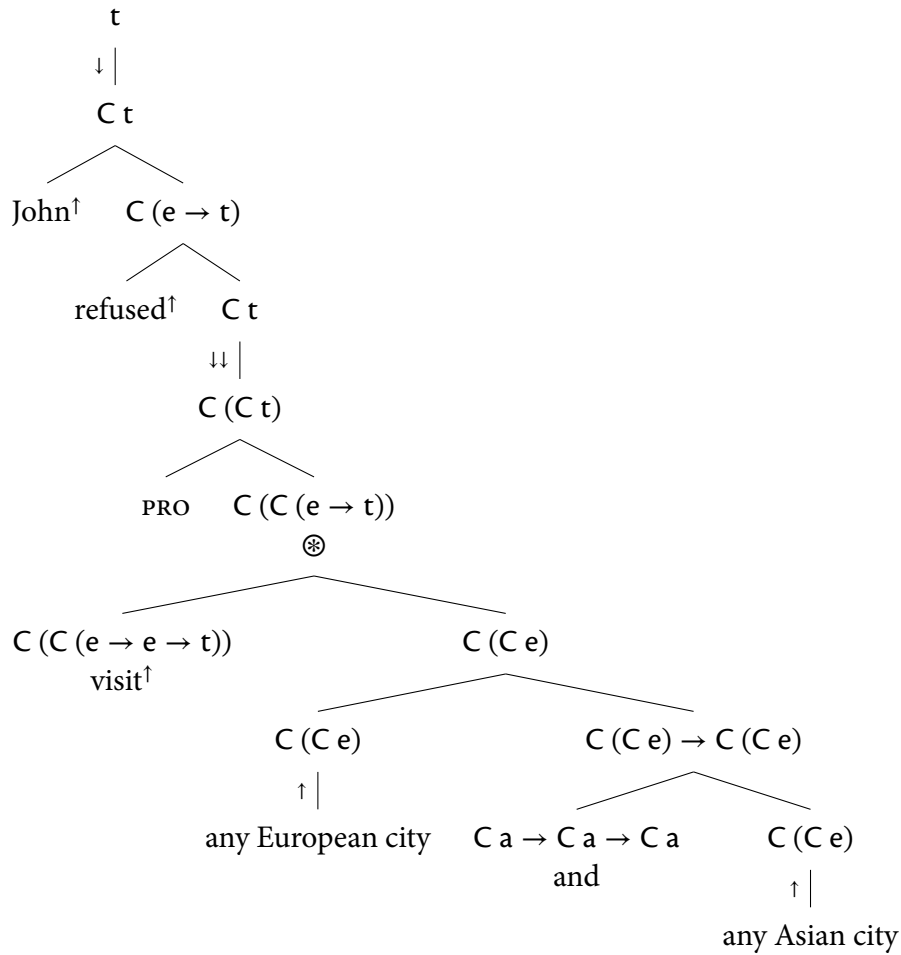


Now we compose the result with *refuse* and the subject via SFA, and lower the result.

Figure (6): Compose the matrix clause and lower



Zooming out, the graph of the semantic computation is given below:



It's worth noting that [Hirsch \(2017\)](#) does briefly consider and ultimately dismiss a similar solution, which involves type-lifting the NPIS, and allowing QR of the resulting complex quantifier formed by *and*_Q to leave behind a higher type (i.e., a quantificational trace), thus giving rise to

semantic reconstruction. [Hirsch](#) ultimately rejects this solution, on the basis that such a derivation disentangles syntactic position and scope, which has been argued to be undesirable based on evidence that semantic reconstruction of a moved expression feeds condition C. This consideration is completely irrelevant here, since scope-taking does not logically entail *movement* in continuation semantics.

2.4 An unattested scope reading

- (10) Some company hired a maid and a cook. $\checkmark \exists > \wedge; \times \wedge > \exists$

3 *Only* and the split-scope signature

[Hirsch \(2017\)](#) makes a parallel argument for split-scope for pre-DP *only*, intended to rule out the possibility that *only* can compose directly with a DP. The examples [Hirsch](#) considers are those like the following:

- (11) You're only required to read three books. only > \square > \exists

In this section, I'll show that the analysis of *and* in continuation semantics ([Barker 2002](#), [Barker & Shan 2014](#): chapter 7), which involves re-executing a continuation, can be extended in a reasonably straightforward fashion to *only*, thus accounting for the cross-categorical nature of *only*, and *predicting* split-scope by virtue of the availability of *lift*.

3.1 Compositional focus semantics

Here, we make concrete our assumptions concerning the consequences of F-marking for the compositional semantics. We'll mostly follow [Rooth \(1992\)](#), but with a slightly different compositional regime, inspired by [Charlow 2014](#): chapter 5. We'll assume that semantic values are *pairs*, consisting of the (a) the ordinary-semantic value, and (b) the focus-semantic value. This is captured by the focus type-constructor F, given in (13). Some example focus semantic values are given in (14).³

³We use the metalanguage function *alt* here as a “black box”, intended to stand in for your favorite theory of alternatives.

On this way of compositionalizing Roothian focus semantics, it's even possible to assign the focus feature F a denotation directly:

- (12) $\llbracket F \rrbracket := \lambda x . (x, \{ x' \mid x' \in \text{alt } x \})$ $a \rightarrow F a$

Although we abstract away from the details, composition in this framework can proceed by lifting non-F-marked things into pairs, where the focus-semantic value is just a singleton set, and composition proceeds via FA of the ordinary-semantic values, and Pointwise Function Application (PFA) of the focus-semantic values. See [Charlow 2014](#): chapter 5

$$(13) \quad F a := (a, \{a\})$$

$$(14) \quad \begin{array}{ll} \text{a. } \llbracket \text{John} \rrbracket = (j, \{j\}) & F e \\ \text{b. } \llbracket \text{John}_F \rrbracket = (j, \{x \mid x \in \text{alt } j\}) & F e \\ \text{c. } \llbracket [\text{VP hug John}_F] \rrbracket = (\lambda y . y \text{ hug } j, \{\lambda y . y \text{ hug } x \mid x \in \text{alt } j\}) & F (e \rightarrow t) \end{array}$$

On this compositionalization, it's natural to treat *only* as seeking a *scope-taker* associated with some alternatives, and returning a plain scope-taker.⁴ We give the denotation we assume for *only* in (15):

$$(15) \quad \text{only } (m, \mathbb{m}) := \lambda k . m \ k \wedge \forall m' \in \mathbb{m} [(m \ k \nrightarrow m' \ k) \rightarrow \neg (m' \ k)] \quad F (C a) \rightarrow C a$$

In order to feed *only* in the kind of meanings it's looking for, we'll need to lift *lift*, introduced in the previous section, into a function on pairs of o-semantic and o-semantic values.

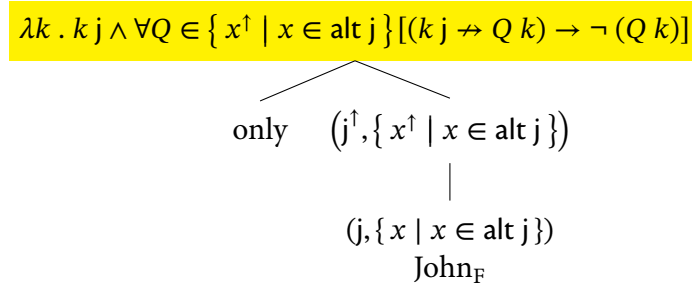
$$(16) \quad (x, y)^{\uparrow F} := (x^{\uparrow}, \{y^{\uparrow} \mid y \in y\})$$

Let's now see how this entry for *only* derives its flexibility:

- (17) a. Mary hugged only JOHN.
b. Mary only hugged JOHN.

Concentrating on (17a), the derivation proceeds quite straightforwardly – first, the F-marked DP *John* is lifted into pair consisting of lifted *John*, and lifted alternatives to *John*.

Figure (7): Composition of “only JOHN”

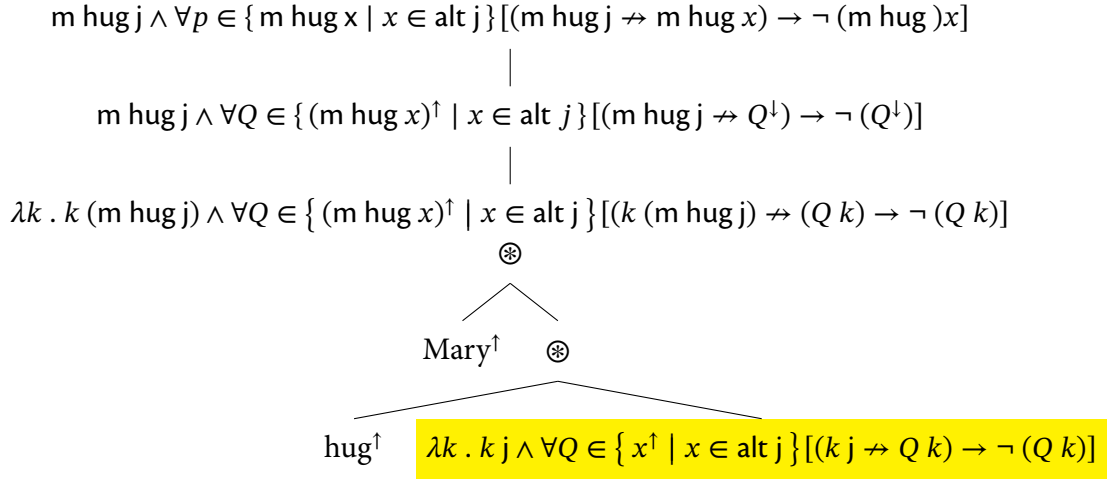


Now that we have a scope-taker of type $C e$, composition of the rest of the sentence proceeds straightforwardly – *lower* evaluates the scope of *only* at the sentential level.

⁴

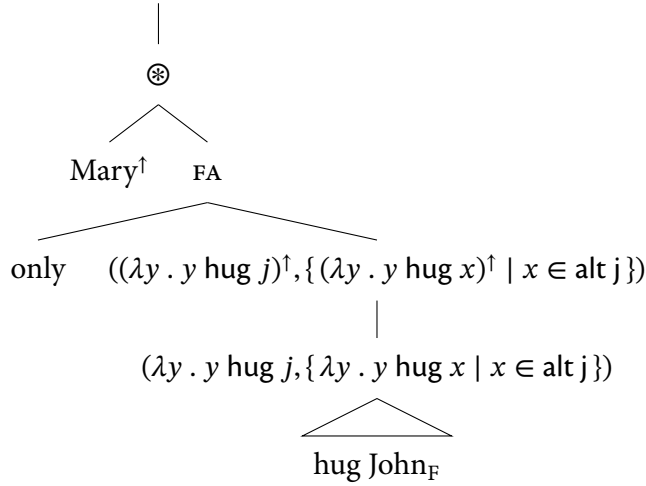
Something about how we abstract away here from the presupposition vs. assertion of only sentences

Figure (8): Scoping out “only JOHN”



The derivation of (17b) proceeds in a completely parallel fashion – JOHN introduces focus alternatives; the Verb Phrase (vp) is lifted before it composes with *only*.

Figure (9): Composition of vp *only*



3.2 Split scope

Let’s assume that *three_F papers* denotes a pair, consisting of its ordinary semantic value, and its focus-semantic value.

$$(18) \quad \llbracket \text{three}_F \text{ papers} \rrbracket = \left(\lambda k . \exists X [|X| = 3 \wedge \text{papers } X], \right. \\
 \left. \{ \lambda k . \exists X [|X| = n \wedge \text{papers } X] \mid n \in \mathbb{N} \} \right) \quad \text{F (C a)}$$

We can analyze *only* as looking for a scope-taker associated with some (scopal) alternatives, and returning a scope-taker.

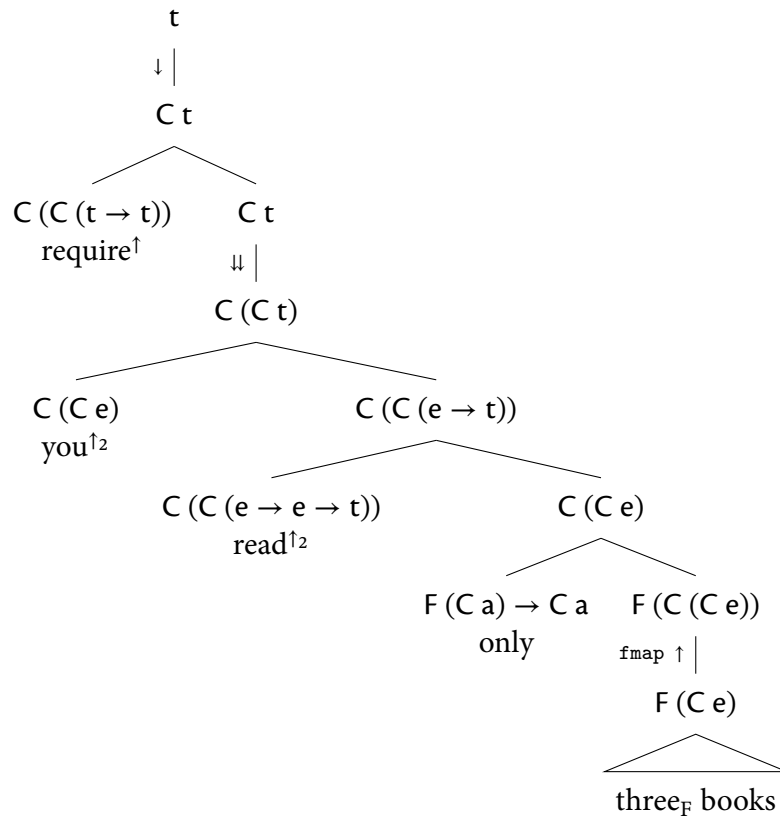
$$(19) \quad \text{only} (m, \mathbb{m}) := \lambda k . m \, k \wedge \forall m' \in \mathbb{m} [(m \, k \not\rightarrow m' \, k) \rightarrow \neg (m' \, k)]$$

$$F (C \, a) \rightarrow C \, a$$

I

3.3 Deriving split scope

Figure (10): Graph of the derivation



$$(20) \quad \Box \exists X [|X| = 3 \wedge \text{books } X \wedge \text{you read } X] \\ \wedge \forall p \in \text{alts} [(\Box \exists X [|X| = 3 \wedge \text{books } X \wedge \text{you read } X]) \not\rightarrow p] \rightarrow \neg p]$$

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