Continuation semantics and the split-scope signature

A note on Hirsch's (2017) argument that type-flexibility is insufficient

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Hirsch (2017) argues that *split scope* readings of conjunction can't be properly accounted for by positing a type-flexible meaning for *and*, and conversely that Conjunction Reduction (CR) *must* be available in order to account for the relevant data. In this note, we show that this argument doesn't go through – concretely, Barker's analysis of conjunction within *continuation semantics* (see also Barker & Shan 2014: chapter 7) *predicts* the split-scope signature, by virtue of independently motivated type-shifting operations.

1 Hirsch's argument

Hirsch's argument revolves around the sentence in (1). The key observation is that (1) has a reading which entails that there are two things that John refused to do: (i) *visit any city in Europe* and (ii) *visit any city in Asia*. Under this reading, *and* scopes above *refuse*, but the material in each conjunct must scope below *refuse*, since each conjunct contains an Negative Polarity Item (NPI) licensed by *refuse*.

(1) The split-scope signature
John refused to visit any city in Europe and any city in Asia. (Hirsch 2017: p. 90)
$\wedge >$ refuse $>$ any

Hirsch characterizes the "split-scope signature" as follows:[1]

(2) *And* scopes above some operator, which the apparent Determiner Phrase (DP) conjuncts scope below.

Hirsch's account of the split-scope signature is to derive (1) from the Logical Form (LF) in (3), via deletion mechanisms. The details are unimportant for our purposes, but it should be clear enough how the LF in (3), if available, accounts for the split-scope signature:

[1] As noted by Hirsch, Partee & Rooth (1983) already recognized the possibility split-scope readings of *dis*junction.

(3) John λx $[_{\&P}$ $[_{vP_1}$ x refused [any city in Europe λy PRO to visit y]
and $[_{vP_2}$ x refused [any city in Asia λz PRO to visit z]]]]

Hirsch shows in detail that the following denotation for *and* (a special case of Partee & Rooth's 1983 recursive definition) can't capture this reading, on the assumption that (1) in fact involves Quantificational Phrase (QP) conjunction:

(4) $[\![\text{and}_Q]\!] := \lambda Q_1 Q_2 P \,.\, Q_1\, P \wedge Q_2\, P$ $\quad ((e \to t) \to t) \to ((e \to t) \to t) \to (e \to t) \to t$

2 Enter *continuations*

2.1 Background

Hirsch's argument takes the standard-bearer for a type-flexible theory of conjunction to be Partee & Rooth's recursive definition. Subsequently, however, a generalization of Partee & Rooth's account has been proposed by Barker (2002) and Barker & Shan (2014) within the framework of *continuation semantics*. Continuation semantics provides a perspective on scope-taking that generalizes Montague's account of quantification, taking inspiration from work in computer science on using delimited continuations to model control flow (see, e.g., Danvy & Filinski 1992, Wadler 1994). For our purposes, continuation semantics amounts to the following conjecture: composition of *scopal* meanings is accomplished *in-situ*, mediated by three type-flexible operations – *lift* (↑), *lower* (↓), and Continuized Function Application (CFA) (S).

The definitions of lift/lower are straightforward, so we provide those first. Lift, defined in (5a), is simply a type-flexible version of Partee's (1986) Montague Lift. Lower, defined in (5b) is simply an instruction to feed a function m the identity function over truth-values as its argument. In a continuized fragment, the role of lift is to allow non-scope-takers to compose with scope-takers, and the role of lower is to get an ordinary value back from a scopal value.

(5) a. Lift (def)
$a^{\uparrow} := \lambda k \,.\, k\, a$ $\quad \uparrow : a \to (a \to t) \to t$

b. Lower (def)
$m^{\downarrow} := m\, (\lambda x \,.\, x)$ $\quad \downarrow : ((t \to t) \to t) \to t$

The definition of continuized CFA is a little more complicated. Informally, (6) says that CFA takes a *scopal* function and a *scopal* argument (in either order), unwraps the contained values and performs Function Application (FA) on them, while sequencing the quantificational parts from left-to-right.

(6) Continuized Function Application (CFA) (def.)
$m \mathrm{S} n := \lambda k \,.\, m\, (\lambda x \,.\, n\, (\lambda y \,.\, k\, (x \mathbin{\mathrm{A}} y)))$
$\mathrm{S} : (((a \to b) \to t) \to t) \to ((a \to t) \to t) \to (b \to t) \to t$
$((a \to t) \to t) \to (((a \to b) \to t) \to t) \to (b \to t) \to t$

Show how this deals with quantifiers

2.2 Conjunction in continuation semantics

Within continuation semantics, conjunction receives an extremely natural definition, given in (**??**); *and* takes two scope-takers as m and n, and a *continuation argument* k, and feeds k into m and n. As shown in detail by Barker this subsumes all of the cases covered by Partee & Rooth's generalized conjunction.

(7) Conjunction in continuation semantics (def.)
$[\![\text{and}]\!] := \lambda mnk\,.\,n\,k \wedge m\,k$ $\quad\quad ((\mathsf{a} \to \mathsf{t}) \to \mathsf{t}) \to ((\mathsf{a} \to \mathsf{t}) \to \mathsf{t}) \to (\mathsf{a} \to \mathsf{t}) \to \mathsf{t}$

Note that, in the following, we'll abbreviate types of the form $(\mathsf{a} \to \mathsf{t}) \to \mathsf{t}$ as follows:

(8) $\mathsf{C}\,\mathsf{a} := (\mathsf{a} \to \mathsf{t}) \to \mathsf{t}$

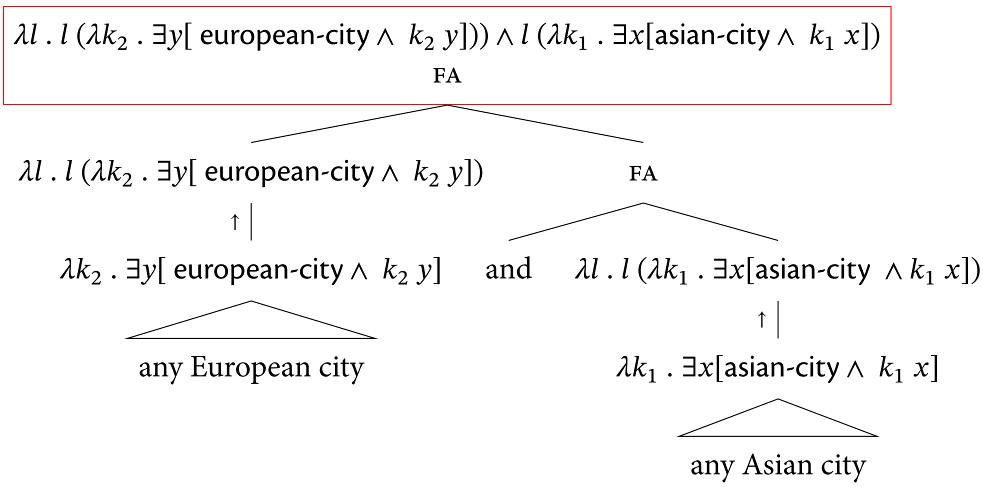
Give some examples

2.3 Accounting for the split-scope signature

Continuation semantics accounts for the split-scope signature straightforwardly, on the assumption that quantifiers can themselves be lifted (this is independently necessary to account for inverse scope readings). We'll make a number of simplifying assumptions here, but nothing in the analysis will crucially hinge upon them. For example, we'll treat NPI *any* simply as an existential quantifier, which is licensed as long as it takes scope within the argument of *refuse*.

The first step is to lift *any European city* and *any Asian city*, returning two higher-order scope-takers. Since *and* can conjoin anything scopal, it can conjoin the resulting meanings. The result is shown in the figure below:

Figure (1): Lift the NPIS and conjoin



The result of conjoining the lifted NPIS is a higher-order scope taker. In order to get this to compose with the rest of the embedded clause, we lift *visit* twice, and do higher-order CFA. Informally, the result is that the semantic contribution of *visit* is distributed between the two conjunctions. Next, we *internally lower* the result, in order to set the scope of the NPIS.

Figure (2): Compose the embedded clause and internally lower

$$\boxed{\lambda l \,.\, l\,(\exists y[\text{ european-city } \wedge \text{visit } y])) \wedge l\,(\exists x[\text{asian-city } \wedge \text{visit } x])}$$

...

$\text{PRO}^{\uparrow 2}$ $\quad \lambda l \,.\, l\,(\lambda k_2 \,.\, \exists y[\text{ european-city } \wedge k_2\,(\text{visit } y)])) \wedge l\,(\lambda k_1 \,.\, \exists x[\text{asian-city} \wedge\ k_1\,(\text{visit } x)])$

$\circledast$

$\text{visit}^{\uparrow 2}$ $\quad \lambda l \,.\, l\,(\lambda k_2 \,.\, \exists y[\text{ european-city } \wedge k_2\ y])) \wedge l\,(\lambda k_1 \,.\, \exists x[\text{asian-city} \wedge\ k_1\ x])$

Now we compose the result with refuse and the subject via Scopal Function Application (SFA), and lower the result.

Figure (3): Compose the matrix clause and lower

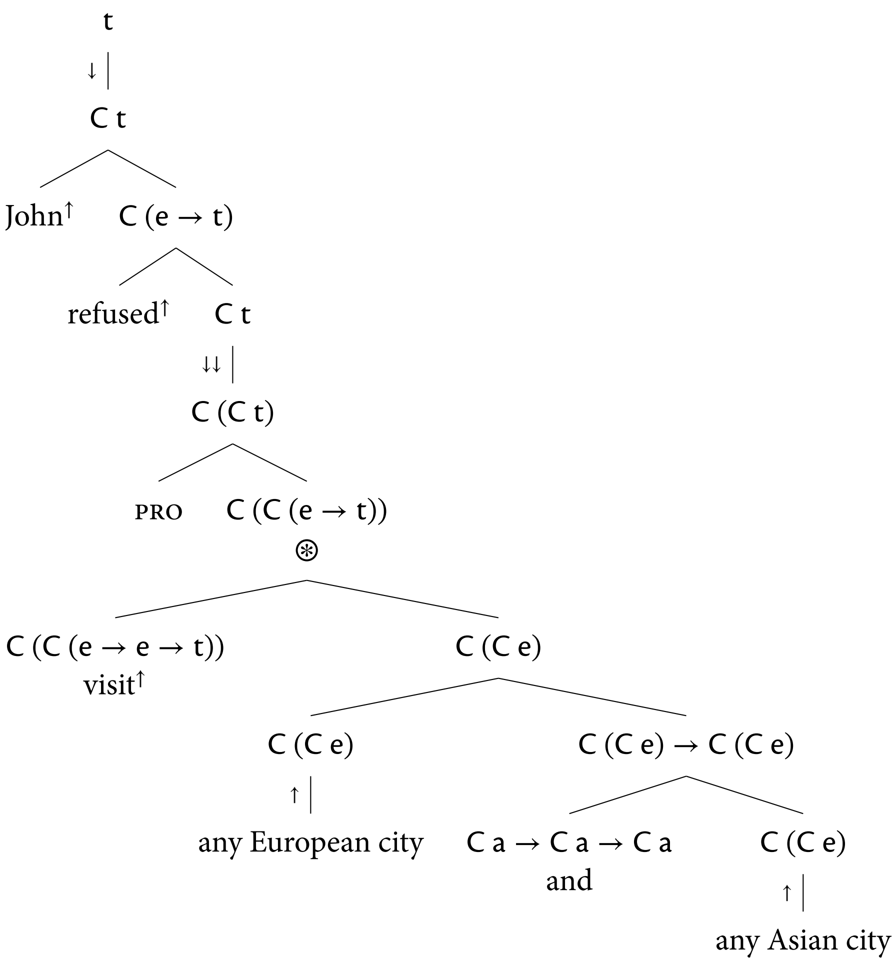
$$\boxed{\text{j refuse } \exists y[\text{ european-city } \wedge \text{j visit } y] \wedge \exists x[\text{asian-city } \wedge \text{visit } x]}$$

$\circledast$

$\text{John}^{\uparrow}$ $\quad \circledast$

$\text{refuse}^{\uparrow}$ $\quad \lambda l \,.\, l\,(\exists y[\text{ european-city } \wedge \text{visit } y])) \wedge l\,(\exists x[\text{asian-city } \wedge \text{visit } x])$

Zooming out, the graph of the semantic computation is given below:



It's worth noting that Hirsch (2017) does briefly consider and ultimately dismiss a similar solution, which involves type-lifting the NPIs, and allowing QR of the resulting complex quantifier formed by and_{Q} to leave behind a higher type (i.e., a quantificational trace), thus giving rise to semantic reconstruction. Hirsch ultimately rejects this solution, on the basis that such a derivation disentangles syntactic position and scope, which has been argued to be undesirable based on evidence that semantic reconstruction of a moved expression feeds condition C. This consideration is completely irrelevant here, since scope-taking does not logically entail *movement* in continuation semantics.

2.4 An unattested scope reading

(9) Some company hired a maid and a cook. ✓ $\exists > \wedge$; ✗ $\wedge > \exists$

3 *Only* and the split-scope signature

Hirsch (2017) makes a parallel argument for split-scope for pre-DP *only*, intended to rule out the possibility that *only* can compose directly with a DP. The examples Hirsch considers are those like the following:

(10) You're only required to read three books. $\text{only} > \Box > \exists$

In this section, I'll show that the analysis of *and* in continuation semantics (Barker 2002, Barker & Shan 2014: chapter 7), which involves re-executing a continuation, can be extended in a reasonably straightforward fashion to *only*, thus accounting for the cross-categorial nature of *only*, and *predicting* split-scope by virtue of the availability of *lift*.

3.1 Compositional focus semantics

Here, we make concrete our assumptions concerning the consequences of F-marking for the compositional semantics. We'll mostly follow Rooth (1992), but with a slightly different compositional regime, inspired by Charlow 2014: chapter 5. We'll assume that semantic values are *pairs*, consisting of the (a) the ordinary-semantic value, and (b) the focus-semantic value. This is captured by the focus type-constructor F, given in (12). Some example focus semantic values are given in (13).[2]

(12) $\mathsf{F}\,\mathsf{a} \coloneqq (\mathsf{a}, \{\,\mathsf{a}\,\})$

(13) a. $[\![\text{John}]\!] = (\mathsf{j}, \{\,j\,\})$ $\mathsf{F}\,\mathsf{e}$

b. $[\![\text{John}_\text{F}]\!] = (\mathsf{j}, \{\,x \mid x \in \mathsf{alt}\,\mathsf{j}\,\})$ $\mathsf{F}\,\mathsf{e}$

c. $[\![[_\text{VP}\ \text{hug John}_\text{F}\ ]]\!] = (\lambda y\,.\,y\ \mathsf{hug}\ \mathsf{j}, \{\,\lambda y\,.\,y\ \mathsf{hug}\ x \mid x \in \mathsf{alt}\,\mathsf{j}\,\})$ $\mathsf{F}\,(\mathsf{e} \to \mathsf{t})$

On this compositionalization, it's natural to treat *only* as seeking a *scope-taker* associated with some alternatives, and returning a plain scope-taker.[3] We give the denotation we assume for *only*

[2] We use the metalanguage function alt here as a "black box", intended to stand in for your favorite theory of alternatives.

On this way of compositionalizing Roothian focus semantics, it's even possible to assign the focus feature F a denotation directly:

(11) $[\![\text{F}]\!] \coloneqq \lambda x\,.\,(x, \{\,x' \mid x' \in \mathsf{alt}\,x\,\})$ $\mathsf{a} \to \mathsf{F}\,\mathsf{a}$

Although we abstract away from the details, composition in this framework can proceed by lifting non-F-marked things into pairs, where the focus-semantic value is just a singleton set, and composition proceeds via FA of the ordinary-semantic values, and Pointwise Function Application (PFA) of the focus-semantic values. See Charlow 2014: chapter 5

[3]

Something about how we abstract away here from the presupposition vs. assertion of only sentences in (14):

(14) only $(m, \mathbb{m}) := \lambda k \,.\, m\ k \wedge \forall m' \in \mathbb{m}[(m\ k \nrightarrow m'\ k) \rightarrow \neg\,(m'\ k)]$

$\mathsf{F}\,(\mathsf{C}\ \mathsf{a}) \rightarrow \mathsf{C}\ \mathsf{a}$

n order to feed *only* in the kind of meanings it's looking for, we'll need to lift *lift*, introduced in the previous section, into a function on pairs or O-semantic and O-semantic values.

(15) $(x, \mathbb{y})^{\uparrow_F} := (x^{\uparrow}, \{\, y^{\uparrow} \mid y \in \mathbb{y} \,\})$

Let's now see how this entry for *only* derives its flexibility:

(16) a. Mary hugged only JOHN.

b. Mary only hugged JOHN.

Concentrating on (16a), the derivation proceeds quite straightforwardly – first, the F-marked DP *John* is lifted into a scope-taker.

Figure (4): hello

...

only $\left(\dfrac{[]}{\mathsf{j}}, \left\{ \dfrac{[]}{\mathsf{x}} \;\middle|\; x \in \mathsf{alt\ j} \right\}\right)$

$(\mathsf{j}, \{\, x \mid x \in \mathsf{alt\ j} \,\})$

John_{F}

3.2 Split scope

Let's assume that *three*$_{\text{F}}$ *papers* denotes a pair, consisting of its ordinary semantic value, and its focus-semantic value.

(17) $[\![\text{three}_{\text{F}}\ \text{papers}]\!] = \begin{pmatrix} \lambda k \,.\, \exists X[|X| = 3 \wedge \mathsf{papers}\ X], \\ \{\, \lambda k \,.\, \exists X[\,|X| = n \wedge \mathsf{papers}\ X] \mid n \in \mathbb{N} \,\} \end{pmatrix}$ $\mathsf{F}\,(\mathsf{C}\ \mathsf{a})$

We can analyze *only* as looking for a scope-taker associated with some (scopal) alternatives, and returning a scope-taker.

(18) only $(m, \mathbb{m}) := \lambda k \,.\, m\ k \wedge \forall m' \in \mathbb{m}[(m\ k \nrightarrow m'\ k) \rightarrow \neg\,(m'\ k)]$

$\mathsf{F}\,(\mathsf{C}\ \mathsf{a}) \rightarrow \mathsf{C}\ \mathsf{a}$

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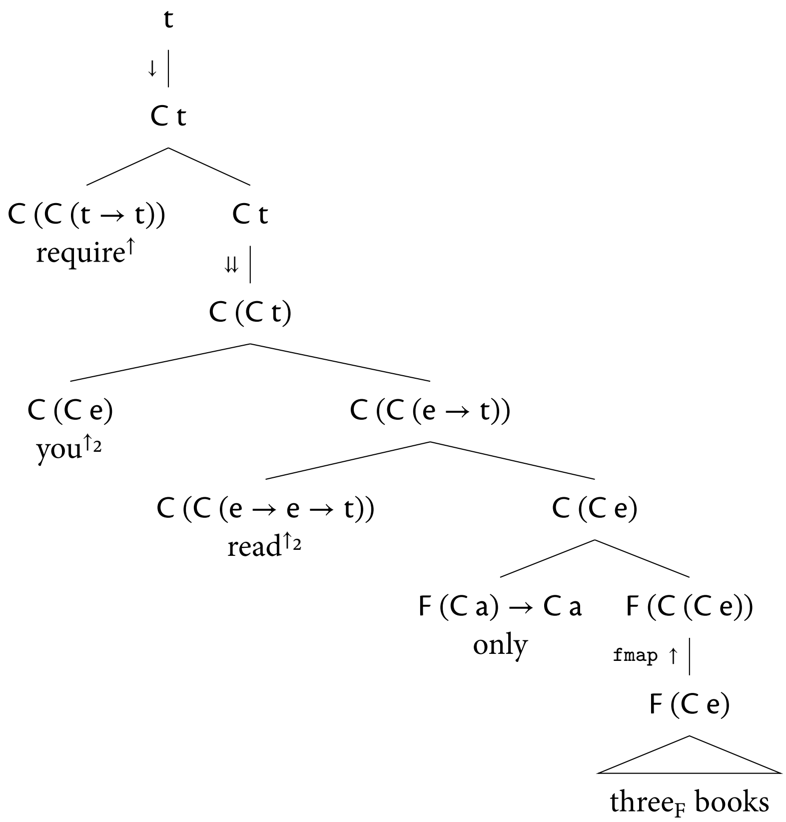
3.3 Deriving split scope

Figure (5): Graph of the derivation



(19) $\square\ \exists X[|X| = 3 \wedge \text{books } X \wedge \text{you read } X]$
$\wedge\ \forall p \in \text{alts}[((\square\ \exists X[|X| = 3 \wedge \text{books } X \wedge \text{you read } X]) \nrightarrow p) \rightarrow \neg\ p]$

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