Introduction to dynamic semantics

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Introduction

Pronouns as variables

Bound pronouns: Pronouns can be bound by operators

(1) *Every*^{x} *linguist admires their*$_x$ *mentor.* $\rightsquigarrow$
$\forall x(linguist(x) \rightarrow \exists y(mentor(y, x) \wedge admire(x, y)))$
For each x who is a linguist, x admires x's mentor

- The denotation of *their*$_x$ co-varies with the value(s) x of the quantifier

Referential pronouns: They can also be *free*

(2) *She*$_x$ *is happy.* $\rightsquigarrow$ $happy(x)$
$g(x)$ is happy, where g is contextually salient

- The denotation of *She*$_x$ is given by a contexually salient variable assignment (Heim & Kratzer 1998)

Challenge I: donkey anaphora

Donkey sentences — a puzzle for static semantics (Evans 1977).

(3) *If a farmer owns a donkey, they feed it.*
For each farmer *x* who owns a donkey *y*, *x* feeds *y*

What is a descriptively adequate Logical Form?

1. $\exists x\,\overbrace{(\exists y(\mathit{farmer}(x) \wedge \mathit{donkey}(y) \wedge \mathit{own}(x,y)))}^{\text{scope of existential}} \rightarrow \mathit{feed}(x,y)$
   Problem: Pronoun in consequent is unbound
2. $\exists x\,\overbrace{(\exists y(\mathit{farmer}(x) \wedge \mathit{donkey}(y) \wedge \mathit{own}(x,y) \rightarrow \mathit{feed}(x,y)))}$
   Problem: Existentials scope over conditional — lose universal force of conditional

We want the existential quantifier to be interpreted in scope of conditional antecedent and bind pronouns in conditional consequent — impossible in classical static semantics.

Challenge II: discourse anaphora

(4) *A*^{x} *philosopher attended the class.*
She$_x$ *asked a difficult question.*
Some philosopher x is s.t., x attended the class and x asked a difficult question.

- Semantically, the pronoun in the second sentence seems to behave as a *bound variable*, bound by the indefinite in the first sentence.
- It's difficult to provide a paraphrase for the second, open sentence independently from the surrounding discourse; the pronoun doesn't have a determinate referent.

Dynamic semantics

Utterances of sentences update the information that is available in a discourse context

(5) a. *A*^{x} *philosopher attended the class.*
$g(x)$ points to some philosopher who attended the class
b. *She*$_x$ *asked a difficult question.*
$g(x)$ asked a difficult question

- Discourse referents: Indefinites introduce salient referents into the discourse which serve as antecedents for anaphoric pronouns (Karttunen 1976)
- Formally: Existential statements update variable assignments (Kamp 1981, Heim 1982, Groenendijk & Stokhof 1991)
- The meaning of an utterance is its Context-Change Potential, which combines truth-conditions and discourse effect (Heim 1982)

Historical background: (First-generation) dynamic semantics

Discourse anaphora and donkey sentences (and related observations) lead to the *dynamic turn* in formal semantics in the 80s — represented most prominently in the work of Irene Heim and Hans Kamp.

There are two main strands of work in dynamic approaches to anaphora (Chierchia 1995); in this class, we'll build on theories of dynamic *semantics* (Heim 1982: chapter 3; cf. Kamp 1981).

The core insights of Dynamic Semantics (DS) were crystalized in Groenendijk & Stokhof's (1991) Dynamic Predicate Logic (DPL).

Dynamic semantics

Updates

In truth-conditional semantics, sentences are interpreted as true/false relative to some parameters of the interpretation function (a world, and variable assignment, etc.).

$$[\![\phi]\!]^{w,g} = 1, \text{ iff } \ldots$$

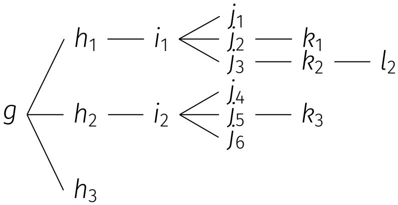
In DS, sentences are interpreted as non-deterministic updates of a discourse state.

$$[\![\phi]\!]^{w,g} = \{h \mid \ldots\}$$

Our discourse states are variable assignments

Update

$[\![\phi]\!]^{w,g} = \{h \mid \ldots\}$



Updates continuously update the information stored in our discourse state

- *Random assignment* non-deterministically adds new variable assignments
- *Tests* remove assignments by imposing conditions on their interpretation

The dynamic core: Random assignment

The operation *random assignment* is at the heart of a dynamic semantics for existential statements.

Random assignment:

$[\![[x]]\!]^{w,g} = \{h \mid g[x]h\}$

Given an assignment g and a variable x, it randomly and *non-deterministically* assigns a value to $g(x)$.

- The set of assignments h that differ from g at most wrt x:

$$\{h \mid g[x]h\}$$

Illustration: Random assignment

- $[x]$

$$g := \begin{bmatrix} x \to \text{Garnet} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}$$

$$h_1 := \begin{bmatrix} x \to \text{Garnet} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}$$

$$h_2 := \begin{bmatrix} x \to \text{Amethyst} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}$$

$$h_3 := \begin{bmatrix} x \to \text{Pearl} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}$$

Pronouns and conditions

In DS, we maintain that pronouns are always interpreted as variables.

(6) *She$_x$ danced* $\rightsquigarrow$ $[Danced(x)]$

This update is an atomic formula, interpreted according to the definition for Predication

Predication:

$$[\![[Danced(x)]]\!]^{w,g} = \{\, g \mid g(x) \text{ danced in } w \,\}$$
$$\text{i.e. } \{\, g \mid g(x) \in I(Danced)(w) \,\}$$

Illustration: Test updates

Assume that only **Pearl** danced in w.

$$g := \begin{bmatrix} x \rightarrow \text{Garnet} \\ y \rightarrow \text{Amethyst} \\ z \rightarrow \text{Pearl} \\ \ldots \end{bmatrix}$$

Illustration: Test updates

Assume that only **Pearl** danced in w.

- $[x]$

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Assume that only **Pearl** danced in *w*.

- $[x]$
- $[Danced(x)]$

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Illustration: Test updates

Assume that only **Pearl** danced in w.

- $[x]$
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$$h_3 := \begin{bmatrix} x \rightarrow \text{Pearl} \\ y \rightarrow \text{Amethyst} \\ z \rightarrow \text{Pearl} \\ \dots \end{bmatrix} \qquad i_1 := \begin{bmatrix} x \rightarrow \text{Pearl} \\ y \rightarrow \text{Amethyst} \\ z \rightarrow \text{Pearl} \\ \dots \end{bmatrix}$$

An update with an atomic formula is a test:
Given an input assignment g, it always either returns $\{g\}$ (singleton set containing the input assignment) or $\emptyset$.

(7) a. $[\![[Danced(x)]]\!]^{w,[x \rightarrow \text{Pearl}]} = \{ [x \rightarrow \text{Pearl}] \}$
b. $[\![[Danced(x)]]\!]^{w,[x \rightarrow \text{Amethyst}]} = \emptyset$

Dynamic conjunction

Since sentences no longer evaluate to truth-values, in DS the idea that conjunction is boolean must be abandoned.

Instead, relative to an input assignment, sentences return a *set* of output assignments.

Dynamic conjunction builds on the intuition that subsequent utterances are interpreted in conjunction.

Pearl came in. She danced. Pearl came in and she danced.

Dynamic conjunction is interpreted as relational composition: We combine two relations ϕ, ψ to form a new relation $\phi; \psi$, by threading the output of ϕ as the input of ψ.

- $[x]$

$$h_1 := \begin{bmatrix} x \to \text{Garnet} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}$$

$$g := \begin{bmatrix} x \to \text{Garnet} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix} \qquad h_2 := \begin{bmatrix} x \to \text{Amethyst} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}$$

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Relational composition

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- $[x]$
- $[Danced(x)]$

$$g := \begin{bmatrix} x \rightarrow \text{Garnet} \\ y \rightarrow \text{Amethyst} \\ z \rightarrow \text{Pearl} \\ \ldots \end{bmatrix}$$

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Relational composition

Dynamic conjunction is interpreted as relational composition: We combine two relations ϕ, ψ to form a new relation $\phi; \psi$, by threading the output of ϕ as the input of ψ.

- **Question:** What is the output of $[x]; [Danced(x)]$?

$$
\begin{tikzcd}[arrows=-]
 & h_1 := \begin{bmatrix} x \to \text{Garnet} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix} & \\
g := \begin{bmatrix} x \to \text{Garnet} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix} \arrow[ur] \arrow[r] \arrow[dr] & h_2 := \begin{bmatrix} x \to \text{Amethyst} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix} & \\
 & h_3 := \begin{bmatrix} x \to \text{Pearl} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix} \arrow[r] & i_1 := \begin{bmatrix} x \to \text{Pearl} \\ y \to \text{Amethyst} \\ z \to \text{Pearl} \\ \ldots \end{bmatrix}
\end{tikzcd}
$$

Relational composition

Dynamic conjunction is interpreted as relational composition: We combine two relations ϕ, ψ to form a new relation $\phi; \psi$, by threading the output of ϕ as the input of ψ.

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$$g := \begin{bmatrix} x \rightarrow \text{Garnet} \\ y \rightarrow \text{Amethyst} \\ z \rightarrow \text{Pearl} \\ \ldots \end{bmatrix}$$

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Conjunction as relational composition

(Dynamic) conjunction:

$[\![\phi\,;\psi]\!]^{w,g} = \{h \mid \text{there exists some } k\text{, s.t. } k \in [\![\phi]\!]^{w,g} \text{ and } h \in [\![\psi]\!]^{w,k}\}$

Dynamic semantics for existential statements

Indefinites in DS come with a variable x — an existential statement does random assignment of x to individuals that satisfy the scope in the world of evaluation.

Existential quantification (Abbreviation):

$\exists x(\phi) := [x]; \phi$

(8) a. *Someone*^{x} *walked in.* $\rightsquigarrow$

b. $\exists x(\mathit{Walked\text{-}In}(x)) =$

c. $[x]; [\mathit{Walked\text{-}In}(x)]$

d. $[\![[x]; [\mathit{Walked\text{-}In}(x)]]\!]^{w,g} := \{h \mid g[x]h \wedge h(x) \in I(\mathit{Walked\text{-}In})(w)\}$

Illustration: existential statements

- w_{ab}: a, b walked in.
- w_a: only a walked in.
- w_b: only b walked in.
- $w_\emptyset$: nobody walked in

(9) $$[\![[x]; [\mathit{Walked\text{-}In}(x)]]\!]^{w_{ab},g} = \{ g[x \to a], g[x \to b] \}$$

(10) $$[\![[x]; [\mathit{Walked\text{-}In}(x)]]\!]^{w_a,g} = \{ g[x \to a] \}$$

(11) $$[\![[x]; [\mathit{Walked\text{-}In}(x)]]\!]^{w_b,g} = \{ g[x \to b] \}$$

(12) $$[\![[x]; [\mathit{Walked\text{-}In}(x)]]\!]^{w_\emptyset,g} = \emptyset$$

Note that indefinites overwrite any existing information associated with x, therefore it's typically assumed that indefinites come with a variable that hasn't already been used (Heim's *novelty condition*).

Egli's theorem

Some immediate results of treating conjunction as relational composition:

- Conjunction is *asymmetric* but nevertheless associative ($(\phi \wedge \psi) \wedge \gamma \equiv \phi \wedge (\psi \wedge \gamma)$).
- Conjunctive discourse anaphora is equivalent to conjunction in the scope of an indefinite (Groenendijk & Stokhof call this property *Egli's theorem*).

(13) Someone^x walked in and she$_x$ sat down.
$\exists x([\mathit{Walked\text{-}In}(x)])\ ;\ [\mathit{Sat\text{-}Down}(x)]$
$[x];\ [\mathit{Walked\text{-}In}(x)]\ ;\ [\mathit{Sat\text{-}Down}(x)]$

(14) Someone^x [t_x walked in and t_x sat down].
$\exists x([\mathit{Walked\text{-}In}(x)]\ ;\ [\mathit{Sat\text{-}Down}(x)])$
$[x];\ [\mathit{Walked\text{-}In}(x)]\ ;\ [\mathit{Sat\text{-}Down}(x)]$

Dynamic semantics affords a *sentential* semantics for indefinites themselves as simply conveying random assignment (Heim 1982, van den Berg 1996):

(15) *someone*^{x} $\rightsquigarrow$ $[x]$

The semantics for existential statements can be derived by conjoining the indefinite (via relational composition) with its scope:

(16) *Someone*^{x} t_x *left* $\rightsquigarrow$ $[x]\ ;\ [\mathit{Left}(x)]$

N.b., with this decomposition, Egli's theorem simply boils down to the associativity of relational composition!

Accessibility in conjunctions

Terminology: an indefinite is accessible to a pronoun if the indefinite can anaphorically bind the pronoun.

Core result of dynamic semantics: an indefinite in an initial conjunct may be accessible to a pronoun in a subsequent conjunct (but not vice versa); conjunction is internally dynamic (Groenendijk & Stokhof 1991).

(17) A woman^x walked in and she$_x$ sat down.
cf. #She$_x$ sat down and a woman^x walked in.

An indefinite in a conjunction may be accessible; conjunction is externally dynamic.

(18) Harry stood up and a woman^x sat down
...and then she$_x$ smiled.

Novelty

Based on the dynamic semantics outlined here, there is nothing preventing an existential statement from overwriting previously introduced referential information.

The following sentence is equivalent to asserting "a man^x sat down".

(19) A man^x walked in and a man^x sat down.

One possibility (entertained by Heim 1982) is to exploit a syntactic condition on indexation (what could it be?).

Indefinites must be paired with variables which haven't previously occurred.

Familiarity

Relatedly, nothing intrinsically prevents a pronoun from being used in a discourse in the absence of an indefinite antecedent.

Heim entertains that familiarity is a syntactic condition on variable names — pronouns must use variables that have already been introduced.

Negation in dynamic semantics

Negation

Negation in DS *tests* whether the prejacent has an empty output.

Negation:

$[\![\neg\phi]\!]^{w,g} = \{g \mid [\![\phi]\!]^{w,g} = \emptyset\}$

An immediate consequence is that negative statements never introduce anaphoric information in DS. Let's see why in more detail.

Illustration negation

If the embedded existential statement is classically true, then the output set is $\emptyset$, if it's classically false then the output set is just the singleton set of the input assignment.

- *Someone^x walked in:*
  $[\![[x]; [\mathit{Walked\text{-}In}(x)]]\!]^{w_{ab},[x\rightarrow \mathsf{Ruby}]} = \{\, [x \rightarrow \mathsf{Amethyst}], [x \rightarrow \mathsf{Beryl}] \,\}$
- *Nobody^x walked in:*
  $[\![\neg([x]; [\mathit{Walked\text{-}In}(x)])]\!]^{w_{ab},[x\rightarrow \mathsf{Ruby}]} = \emptyset$
- *Nobody^x walked in:*
  $[\![\neg([x]; [\mathit{Walked\text{-}In}(x)])]\!]^{w_{\emptyset},[x\rightarrow \mathsf{Ruby}]} = \{\, [x \rightarrow \mathsf{Ruby}] \,\}$

N.b. We'll typically assume that *nobody* is decomposed into an indefinite and sentential negation.

Accessibility and negation

An indefinite in the scope of negation is never accessible to a subsequent pronoun; negation is externally static.

This is because negative statements are semantically guaranteed to be *tests*.

(20) #Ruby didn't bake a^x cake and it$_x$ was delicious.

Implication, disjunction, and donkey anaphora

Implication and disjunction

By taking the semantics of indefinites, conjunction and negation as primitives, we can define the semantics of implication and disjunction via some routine equivalences.

Implication:

$\phi \to \psi := \neg(\phi; \neg\psi)$

Disjunction:

$(\phi \vee \psi) := \neg(\neg\phi; \neg\psi)$

N.b. not just any classical equivalence will do; it matters for anaphora which equivalences we use. In particular, since implications and disjunctions are negative statements, they will ultimately be tests.

Implications

Cashing out the equivalence, we end up with the following semantics for implication in DS:

(21) $[\![\phi \rightarrow \psi]\!]^{w,g} = \{g \mid$ for every k: if $k \in [\![\phi]\!]^{w,g}$, there exists some j, s.t. $j \in [\![\psi]\!]^{w,k}\}$

An implication tests that there is no assignment outputted by the antecendent that renders the output of the consequent empty.

This derives universal truth-conditions for sentences with donkey anaphora, as shown on the next slide.

Accessibility in implications

An indefinite in a conditional antecedent may be accessible to a pronoun in a subsequent consequent (but not vice versa); implications are internally dynamic.

(22) If a philosopher^{x} asked a question, then they$_{x}$ stood up.
cf. #If they$_{x}$ stood up, then a philosopher^{x} asked a question.

An indefinite in a conditional, on the other hand, isn't accessible to a pronoun outside of the conditions; implications are externally static (semantically, this means that they're always *tests*).

(23) If a woman^{x} smiles, then Harry is happy. #She$_{x}$ sat down.

(N.b., we have to be careful to control for the scope of the indefinite)

Donkey anaphora: illustration

- w_{ab}: a, b walked in, both sat down.
- w_{a}: a, b walked in, only a sat down.
- $w_{\emptyset}$: nobody walked in or sat down.

(24) *If someone^x walked in, they$_x$ sat down.*

$[\![([x]\,;[\mathit{WalkedIn}(x)]) \rightarrow [\mathit{SatDown}(x)]]\!]^{w_{ab},g}$

a. $= \left\{ g \;\middle|\; \neg\exists h \begin{bmatrix} g[x]h \wedge h(x) \text{ walked in}_w \\ \wedge\, h(x) \text{ didn't sit down}_w \end{bmatrix} \right\}$

b. $= g$

(25) $[\![([x]\,;[\mathit{WalkedIn}(x)]) \rightarrow [\mathit{SatDown}(x)]]\!]^{w_{a},g}$

a. $= \emptyset$ (assignment of x to **Beryl** falsifies the conditional)

(26) $[\![([x]\,;[\mathit{WalkedIn}(x)]) \rightarrow [\mathit{SatDown}(x)]]\!]^{w_{\emptyset},g}$

a. $= g$ (requirement trivially satisfied, since output of antecedent is $\emptyset$)

Egli's corrolary

- The property of universal readings for donkey anaphora is known as *Egli's corrolary* DPL.

(27) If someone^x walked in, she sat$_x$ down.

(28) Everyone who walked in sat down.

- If *everyone* is the dual of *someone*, then this follows as a theorem from the semantics of implication.
  - We don't show this in detail, but see if you can convince yourself that this holds.
- As a preview, this is ultimately a property that we'll lose when we refine DS for independent reasons.

Disjunction

Finally, the semantics we arrive at for disjunction based on the stated equivalence is the following:

(29) $[\![\phi \vee \psi]\!]^{w,g} = \{g \mid \text{there exists some } k\text{, s.t. } k \in [\![\phi]\!]^{w,g}\text{, or } k \in [\![\psi]\!]^{w,g}\}$

In plain English, disjunction is a test of whether each disjunct is classically true.

Note that no anaphoric information passes between the disjuncts, since the anaphoric contribution of each is essentially ‘closed off’ by negation.

Accessibility in disjunctions

Since disjunction is merely a test of the classical truth of each of the disjunctions, dynamic semantics predicts that disjunction is both internally static and externally static.

(30) #Either a woman^x walked in or she$_x$ sat down.

(31) Either Harry sat down or a woman^x walked in. #She$_x$ smiled.

(As usual, we need to be careful to control the scope of the indefinite.)

Bonus: modal operators

In a similar way to disjunction, modal statements can be treated as *tests*.

For example, *might* ϕ tests whether ϕ is classically true at some doxastically accessible world.

(32) $$[\![\Diamond\phi]\!]^{w,g} = \{g \mid \exists w' \in Dox_w, [\![\phi]\!]^{w',g} \neq \emptyset\}$$

Question: what predictions does this make for anaphora?

Conclusion and outlook

Accessibility summary

	internally	externally	veridicality?
$\phi \wedge \psi$	dynamic	dynamic	veridical
$\neg\phi$	N/A	static	non-veridical
$\Diamond\phi$	N/A	static	non-veridical
$\phi \rightarrow \psi$	dynamic	static	non-veridical
$\phi \vee \psi$	static	static	non-veridical

- **Question:** What do the externally static operators have in common?
- Veridicality: A propositional operator is *veridical* iff it entails its prejacent.
- A generalization: Non-veridical operators are externally static
- Prediction: Anaphora to indefinites in the scope of non-veridical operators is not possible
- The truth-functional contribution is baked into the context-change potential
- But there are many counterexamples in natural language...

Main takeaways

A dynamic logic for analyzing utterances of natural language sentences as updates.

- Random assignment [*x*]
- Test updates impose conditions, without introducing anaphoric information.
- Dynamic conjunction $\phi\,;\psi$ threads anaphoric information from ϕ to ψ.
- Negation as test, and derived notions (disjunction, implication)

Sentential operators that do not entail their operands are externally static

- As seen above, this captures the fact that indefinites in their scope (often) do not provide antecedents for subsequent anaphora
- A strong prediction is that this is never possible
- Tomorrow, we will consider some counterexamples to this generalization

The fragment so far

Sentences express *updates*, i.e., functions from possibilities (world-assignment pairs), to sets of assignments.

(33) **Random assignment:**
$[\![[x]]\!]^{w,g} = \{\, h \mid h$ differs from g at most at $x \,\}$ i.e., $\{\, h \mid g[x]h \,\}$

(34) **Predication:**
$[\![[\mathbf{Sleep}(x)]]\!]^{w,g} = \{\, g \mid g(x) \in I(Sleep)(w) \,\}$

(35) **Negation:**
$[\![\neg\phi]\!]^{w,g} := \{\, g \mid [\![\phi]\!]^{w,g} = \emptyset \,\}$

(36) **Dynamic conjunction:**
$[\![\phi \,;\, \psi]\!]^{w,g} = \{\, h \mid \exists i[i \in [\![\phi]\!]^{w,g} \wedge h \in [\![\psi]\!]^{w,i}] \,\}$

Logic of change: abbreviations

(37) Implication:

a. $\phi \rightarrow \psi := \neg(\phi \,;\, \neg\psi)$
b. $[\![\phi \rightarrow \psi]\!]^{w,g} = \{\, g \mid \neg\exists h[h \in [\![\phi]\!]^{w,g} \wedge [\![\psi]\!]^{w,h} = \emptyset] \,\}$

(38) Disjunction:

a. $\phi \vee \psi := \neg(\neg\phi \,;\, \neg\psi)$
b. $[\![\phi \vee \psi]\!]^{w,g} = \{\, g \mid [\![\phi]\!]^{w,g} \neq \emptyset \vee [\![\psi]\!]^{w,g} \neq \emptyset \,\}$

(39) Existential quantification:

a. $\exists x\phi := [x] \,;\, \phi$

(40) Universal quantification:

a. $\forall x\phi := [x] \rightarrow \phi$
(N.b., $\forall$ can equivalently be defined as the dual of $\exists$)

Recommended reading

- Gillies. “On Groenendijk and Stokhof’s “Dynamic Predicate Logic””. 2022.
  (a contemporary overview and critique of Groenendijk & Stokhof 1991)

Questions?

van den Berg, M. H. 1996. Some Aspects of the Internal Structure of Discourse. The Dynamics of Nominal Anaphora. Publisher: AmsterdamILLC. https://dare.uva.nl/search?arno.record.id=7073 (31 August, 2020).

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