Intensional Dynamic Semantics

Anaphora to counterfactual content

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Recap and Introduction

Bilateral negation

Gives a dynamic semantics for negation as a flip/flop operator:

(1) **Negation in bilateral dynamic semantics**

a. $[\![\neg\phi]\!]^w_+ = \lambda g\,.\,[\![\phi]\!]^w_-\,(g)$

b. $[\![\neg\phi]\!]^w_- = \lambda g\,.\,[\![\phi]\!]^w_+\,(g)$

- Allows for dynamic double negation elimination
- Indefinites introduce a dref if the existence of a witness is entailed

Anaphora to antecedents in the scope of non-veridical operators

With bilateral dynamic semantics, are able to predict anaphora and truth conditions for:

(2) Double negation (Karttunen 1976, Krahmer & Muskens 1995)
It's not the case that there isn't a bathroom in this house. It is upstairs.

(3) Program disjunction (Groenendijk & Stokhof 1991)
A professor or an assistant professor will attend the meeting of the university board.
She will report to the faculty.

(4) 'Bathroom-disjunctions' (Evans 1977, Barbara Partee)
Either there isn't a bathroom in this house, or it's upstairs.

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(4) 'Bathroom-disjunctions' (Evans 1977, Barbara Partee)
Either there isn't a bathroom in this house, or it's upstairs.

But how about these:

(5) Modal subordination (Roberts 1987)
There might be a bathroom in this house. It would be upstairs.

(6) Discourse-inferences
There might be a bathroom in this house. In fact, I just remembered that's the case. It is upstairs.

(7) Inter-speaker disagreement (Hofmann 2019)
A: *There isn't a bathroom in this house.*
B: *(What are you talking about?) It's upstairs.*

Anaphora to hypothetical content

If bilateral semantics states truth-conditional conditions on dref introduction for a given utterance, we we run into a lookahead problem

(8) *Mary might have a car*

a. *It would be parked outside.*

b. *...so she may need a place to park it*

c. *...and Max claimed they saw it*

- But: Indefinites under disjunction (or modals) can introduce a dref to a subset of possibilities in the context
- i.e. they make it partially familiar

Towards modal subordination

(9) *Mary might own a car.* $\rightsquigarrow$ $might([x];[car(x)];[own(m,x)])$

(10) *It would be red.*

A model: In all worlds: a, b are cars. a is red, b is not red.

- w_{ab}: *Mary* owns a, b
- w_a: *Mary* owns a
- w_b: *Mary* owns b
- $w_\emptyset$: *Mary* does not own anything

$$(w_{ab}, [])$$

$$(w_a, [])$$

$$(w_b, [])$$

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- Assertion: Update under uncertainty

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$$\begin{tikzcd}
 & (w_{ab}, [x \mapsto a]) \\
(w_{ab}, []) \arrow[ur, dash] \arrow[dr, dash] & \\
 & (w_{ab}, [x \mapsto b]) \\
(w_a, []) \arrow[dr, dash] & \\
 & (w_a, [x \mapsto a]) \\
(w_b, []) \arrow[dr, dash] & \\
 & (w_b, [x \mapsto b]) \\
(w_\emptyset, []) \arrow[dr, dash] & \\
 & (w_\emptyset, [])
\end{tikzcd}$$

- Assertion: Update under uncertainty
- Weak epistemic modal similar to disjunction: Partial familiarity

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$$
\begin{array}{l}
(w_{ab},[]) \\
\\
(w_a,[]) \\
(w_b,[]) \\
(w_\emptyset,[])
\end{array}
\qquad
\begin{array}{l}
\left.\begin{array}{l}
(w_{ab},[x \mapsto a]) \\
(w_{ab},[x \mapsto b]) \\
(w_a,[x \mapsto a]) \\
(w_b,[x \mapsto b])
\end{array}\right\} \\
(w_\emptyset,[])
\end{array}
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- Assertion: Update under uncertainty
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- We may interpret the prejacent of *“would”* wrt the possibilities where it is defined

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 & (w_{ab}, [x \mapsto a]) \arrow[r, dash] & (w_{ab}, [x \mapsto a]) \\
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 & (w_{ab}, [x \mapsto b]) & \\
(w_a, []) \arrow[dr, dash] & & \\
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(w_b, []) \arrow[dr, dash] & & \\
 & (w_b, [x \mapsto b]) & \\
(w_\emptyset, []) \arrow[dr, dash] & & \\
 & (w_\emptyset, []) &
\end{tikzcd}
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- We may interpret the prejacent of *"would"* wrt the possibilities where it is defined

Anaphora to counterfactual content

But once we look at anaphora with counterfactual antecedents, the lookahead problem becomes more serious:

(11) *Mary doesn't have a car*

a. *It would be parked outside.*

b. *...so she doesn't need a place to park it.*

c. *...even though Max claimed they saw it.*

In bilateral dynamic semantics, negative updates do not make indefinites in their scope familiar (not even partially)

Counterfactual anaphora — a problem

(12) *Mary does not a car.*

(13) *It would be red.*

A model: In all worlds: a, b are cars. a is red, b is not red.

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Counterfactual anaphora — a problem

(12) *Mary does not a car.* $\rightsquigarrow$ $\neg([x];[car(x)];[own(m,x)])$

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- Interpreting the prejacent of *"would"* wrt the possibilities where it is defined does not work any more
- Question: What do we need to make this work?

Recipe for counterfactual anaphora (Hofmann 2022)

1. "Universal context:" Look at the full universe of possibilities U (all possible worlds)

$$(w_{ab}, [])$$

$$(w_a, [])$$

$$(w_b, [])$$

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Recipe for counterfactual anaphora (Hofmann 2022)

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To update with: *Mary doesn’t own a car.*

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2. Anaphoric potential and local context
   - For each world $w \in U$ s.t. Mary owns a car in w, we want the assignment that is paired with to be updated with x s.t. x points to a car that Mary owns in w
   - Interpret discourse variables relative to possible worlds (Stone 1999, Brasoveanu 2010b)

$$
\begin{array}{lcl}
 & & (w_{ab}, [x \mapsto a]) \\
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 & & (w_{ab}, [x \mapsto b]) \\
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   - Interpret discourse variables relative to possible worlds (Stone 1999, Brasoveanu 2010b)
3. Truth conditions and global context
   - Keep track of which options are taken to be possible candidates for the possible world
   - Speaker commitments

$$
\begin{array}{lll}
 & (w_{ab}, [x \mapsto a]) & \\
(w_{ab}, []) & & \\
 & (w_{ab}, [x \mapsto b]) & \\
(w_a, []) & & \\
 & (w_a, [x \mapsto a]) & \\
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 & (w_b, [x \mapsto b]) & \\
(w_\emptyset, []) & & \\
 & (w_\emptyset, []) &
\end{array}
$$

Intensional dynamic semantics

We state an intensional system, in which…

- Sentential operators introduce relations over sets of possible worlds
- Drefs / discourse variables are interpreted relative to sets of possible worlds
- We allow counterfactual possibilities that are distinct from the global context set

…based on e.g. Stone (1999), Stone & Hardt (1999), Brasoveanu (2010a,b), Hofmann (2019, 2022)

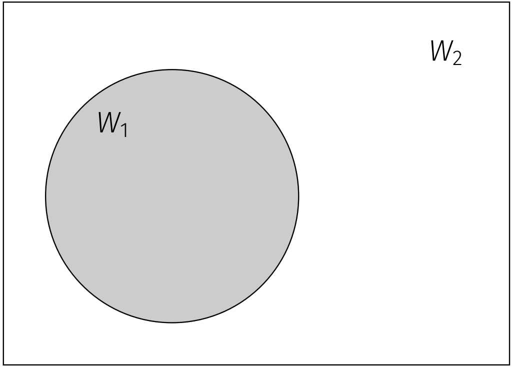
Intensional dynamic semantics

The basic idea pt. 1 — Propositional operators

We treat negation as an operator over propositions (sets of possible worlds)

(14) S: There isn't a bathroom

- Two propositions: W_1 and W_2
- W_1: Set of worlds where there isn't a bathroom
- W_2: Set of worlds where there is a bathroom
- Opposites of each other (Complement Relation)
- Speaker committed to W_1



The basic idea pt. 2 — Drefs for individual concepts

Intuitively, we want the mapping from a variable to an object to differ by world. Based on Carnap (1947), Stone (1999), we use discourse referents for individual concepts, i.e. functions from worlds to individuals.

$$x \mapsto \begin{bmatrix} w_{ab} \mapsto & a \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \qquad x \mapsto \begin{bmatrix} w_{ab} \mapsto & b \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix}$$

The basic idea pt. 3 — Relative variable update

Individual drefs map to an individual for all worlds in which their referent exists, and to an indeterminate value $\#_e$ in all other worlds (building on Stone 1999, Brasoveanu 2006)

- Indefinites introduce drefs in relation to their local propositional context

(15) $g[W : x]h$ iff:

h is an update of g with x relative to W, iff

The basic idea pt. 3 — Relative variable update

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- g is an update of h that differs at most wrt the value assigned to x

The basic idea pt. 3 — Relative variable update

Individual drefs map to an individual for all worlds in which their referent exists, and to an indeterminate value $\#_e$ in all other worlds (building on Stone 1999, Brasoveanu 2006)

- Indefinites introduce drefs in relation to their local propositional context

(15) $g[W : x]h$ iff:
- $g[x]h$
- $\forall w \in W : h(x)(w) \neq \#$

h is an update of *g* with *x* relative to *W*, iff

- *g* is an update of *h* that differs at most wrt the value assigned to *x*
- for each world *w* in *W*, $h(x)(w)$ doesn't map to $\#$ (but an individual)

The basic idea pt. 3 — Relative variable update

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(15) $g[W : x]h$ iff:
- $g[x]h$
- $\forall w \in W : h(x)(w) \neq \#$
- $\forall w \notin W : h(x) = \#$

h is an update of g with x relative to W, iff

- g is an update of h that differs at most wrt the value assigned to x
- for each world w in W, $h(x)(w)$ doesn't map to $\#$ (but an individual)
- for each world w not in W, $h(x)(w)$ maps to $\#$

Atomic formulas — Relative variable update

Updates are binary relations between input assignment g and output h.

Relative variable update

$[\![[\dagger x]]\!]^W = \lambda g.\lambda h.g[W : x]h$

(Here, I use the † symbol to draw a parallel to Patrick’s system from yesterday.)

Asserting "There is a bathroom"

A discourse-context is a pair: $\langle W, g\rangle$, where $W \subseteq U$ is a context set, and g a partial variable assignment.

Asserting “There is a bathroom”

A discourse-context is a pair: $\langle W, g\rangle$, where $W \subseteq U$ is a context set, and g a partial variable assignment.

- Initial context w/o information: $W = U$, $g = []$

$$\left\langle \begin{array}{r} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}\,, \\ [] \end{array} \right\rangle$$

Asserting "There is a bathroom"

A discourse-context is a pair: $\langle W, g\rangle$, where $W \subseteq U$ is a context set, and g a partial variable assignment.

- Initial context w/o information: $W = U$, $g = []$
- $[\![[\dagger x]; [bathroom(x)]]\!]^{W'} = \lambda g.\lambda h.g[x]h \wedge \forall w' \in W' : h(x) \in I(bathroom)(w')$
- Assertion: $\langle W, g\rangle + [\dagger x]; [bathroom(x)] = \langle W \cap W', h\rangle$,
  where $[\![[\dagger x]; [bathroom(x)]]\!]^{W'}(g)(h) = 1$

$$
\begin{tikzcd}
 & \left\langle \{ w_{ab}, w_a, w_b \}, \left[x \mapsto \begin{bmatrix} w_{ab} \mapsto & a \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \right] \right\rangle \\
\left\langle \{ w_{ab}, w_a, w_b, w_\emptyset \}, [] \right\rangle \arrow[ur, no head] \arrow[dr, no head] & \\
 & \left\langle \{ w_{ab}, w_a, w_b \}, \left[x \mapsto \begin{bmatrix} w_{ab} \mapsto & b \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \right] \right\rangle
\end{tikzcd}
$$

Note that we actually need W' to be the largest set, s.t. $[\![[\dagger x]; [bathroom(x)]]\!]^{W'}(g)(h) = 1$

Interpreting variables in conditions

Here is our rule for predication:

Predication

$[\![[Sleep(x)]]\!]^W = \lambda g.\lambda h.g = h \wedge \forall w \in W : g(x)(w) \in I(Sleep)(w)$

The update with the atomic formula $[Sleep(x)]$ holds of g, h in a set of worlds W, iff

- $g = h$
- For each world w in W, $g(x)$ is a sleeper in w
- This is defined only if $g(x) \neq \#$ for each w

Asserting “It is upstairs.”

$$\left\langle \begin{array}{r} \{ w_{ab}, w_a, w_b, w_\emptyset \}, \\ [] \end{array} \right\rangle$$

Asserting "It is upstairs."

$[\dagger x]; [bathroom(x)]$

$$\left\langle \begin{matrix} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}, \\ \hfill [] \end{matrix} \right\rangle$$

$$\left\langle \{\, w_{ab}, w_a, w_b \,\}, \begin{bmatrix} x \mapsto \begin{bmatrix} w_{ab} \mapsto & a \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \end{bmatrix} \right\rangle$$

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Asserting “It is upstairs.”

$[\dagger x]; [\mathit{bathroom}(x)]\ ; [\mathit{upstairs}(x)]$

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Asserting "It is upstairs."

$[\dagger x]; [bathroom(x)]\ ; [upstairs(x)]$

- Assertion: $\langle W, g\rangle + [upstairs(x)] = \langle W \cap W', h\rangle$, where $[\![[upstairs(x)]]\!]^{W'}(g)(h) = 1$
  i.e. $g = h \wedge \forall w \in W : g(x)(w) \in I(upstairs)(w)$

$$\left\langle \{\, w_{ab}, w_a, w_b, w_\emptyset \,\},\ [] \right\rangle$$

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- Assuming that a is upstairs while b is not
- Assertion: Pick the largest set of worlds where update is true, and intersect
- Question: The local context of the antecedent was the proposition $\{\, w_{ab}, w_a, w_b \,\}$. In order for x to be defined in W', how do we choose W'?

Asserting “It is upstairs.”

$[\dagger x]; [\mathit{bathroom}(x)]\ ; [\mathit{upstairs}(x)]$

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Asserting “It is upstairs.”

$[\dagger x]; [\mathit{bathroom}(x)]\ ; [\mathit{upstairs}(x)]$

- Assertion: $\langle W, g\rangle + [\mathit{upstairs}(x)] = \langle W \cap W', h\rangle$, where $[\![[\mathit{upstairs}(x)]]\!]^{W'}(g)(h) = 1$
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- Assuming that a is upstairs while b is not
- Assertion: Pick the largest set of worlds where update is true, and intersect
- Question: The local context of the antecedent was the proposition $\{ w_{ab}, w_a, w_b \}$. In order for x to be defined in W', how do we choose W'?
  — $W' \subseteq \{ w_{ab}, w_a, w_b \}$

Anaphora to negated content

Propositional operators

Propositional operators state updates

Negation

$[\![[\neg\phi]]\!]^{W} = \lambda g.\lambda h.$

Disjunction

$[\![[\phi \vee \psi]]\!]^{W} = \lambda g.\lambda h.$

Conjunction

$[\![[\phi \wedge \psi]]\!]^{W} = \lambda g.\lambda h.$

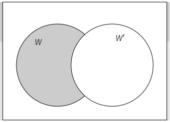
Propositional operators

Propositional operators state updates, by

- Specifying a relation over sets of worlds...

Negation

$[\![\neg\phi]\!]^W = \lambda g.\lambda h.\ \exists W' : W = \overline{W'}$



Disjunction

$[\![\phi \vee \psi]\!]^W = \lambda g.\lambda h.$

Conjunction

$[\![\phi \wedge \psi]\!]^W = \lambda g.\lambda h.$

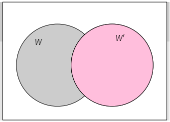
Propositional operators

Propositional operators state updates, by

- Specifying a relation over sets of worlds...
- and providing a local set of worlds for interpretation of their prejacent

Negation

$$[\![[\neg \phi]]\!]^{W} = \lambda g.\lambda h.\ \exists W' : W = \overline{W'}$$
$$\wedge\ h \in [\![\phi]\!]^{W'}(g)$$



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Conjunction

$$[\![[\phi \wedge \psi]]\!]^{W} = \lambda g.\lambda h.$$

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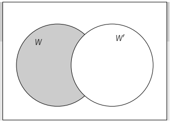
- Specifying a relation over sets of worlds...
- and providing a local set of worlds for interpretation of their prejacent

Negation

$$[\![[\neg\phi]]\!]^{W} = \lambda g.\lambda h.\ \exists W' : W = \overline{W'} \\ \wedge\ h \in [\![\phi]\!]^{W'}(g)$$



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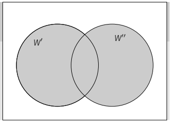

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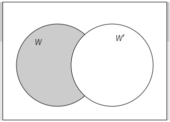

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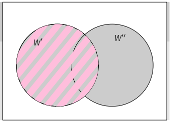

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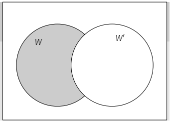

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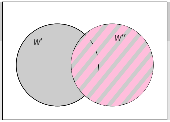

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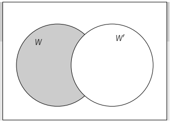

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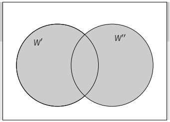

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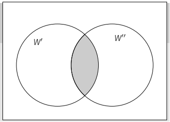

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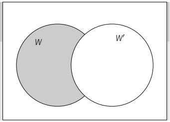

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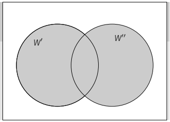

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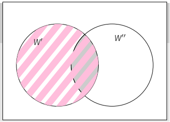

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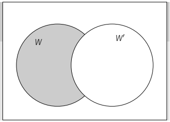

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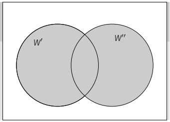

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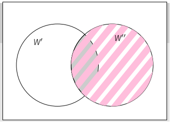

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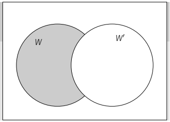

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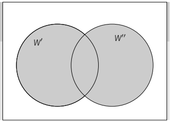

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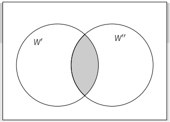

The counterfactual bathroom

- *There is no bathroom* $\rightsquigarrow \neg([\dagger x]; [bathroom(x)])$
- $[\![\neg([\dagger x]; [bathroom(x)])]\!]^W = \lambda g.\lambda h.\exists W' : W = \overline{W'} \wedge h \in [\![[\dagger x]; [bathroom(x)]]\!]^{W'}(g)$

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- $$\begin{aligned} [\![\neg([\dagger x]; [bathroom(x)])]\!]^{W} &= \lambda g.\lambda h.\exists W' : W = \overline{W'} \wedge h \in [\![[\dagger x]; [bathroom(x)]]\!]^{W'}(g) \\ &= \lambda g.\lambda h.\exists W' : W = \overline{W'} \wedge g[W' : x]h \wedge \forall w \in W' : h(x)(w) \in I(bathroom)(w) \end{aligned}$$

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- In other words: We are looking for a (maximal) set of worlds W, s.t. for all $w \in W'$, we can introduce an x that is a bathroom

$$\left\langle \begin{array}{r} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}\,, \\ [] \end{array} \right\rangle$$

$$\left\langle \quad , \left[x \mapsto \begin{bmatrix} w_{ab} \mapsto & a \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \right] \right\rangle$$

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- In other words: We are looking for a (maximal) set of worlds W, s.t. for all $w \in W'$, we can introduce an x that is a bathroom
- So that we can assert the opposite

$$\left\langle \begin{array}{r} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}, \\ [] \end{array} \right\rangle$$

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Veridical anaphora

$$\left\langle \begin{matrix} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}\,, \\ \hfill [] \end{matrix} \right\rangle$$

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- There is no bathroom. $\rightsquigarrow$ $\neg([\dagger x];[\mathit{bathroom}(x)])$

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Veridical anaphora

- There is no bathroom. $\rightsquigarrow$ $\neg([\dagger x];[bathroom(x)])$
- # It's upstairs $\rightsquigarrow$ $[upstairs(x)]$
- $[\![[upstairs(x)]]\!]^W = \lambda g.\lambda h.\forall w \in W : g = h \wedge h(x)(w) \in I(upstairs)(w)$

$$\left\langle \begin{array}{r} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}\,, \\ [] \end{array} \right\rangle$$

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- We want to choose $W = \{ w_{ab}, w_a \}$, bc we have an assignment that maps x to the upstairs bathroom a in all of these worlds
- But this cannot be asserted, because our context set of worlds is already $\{ w_\emptyset \}$, where there is no bathroom
- Contradiction!!

Counterfactual anaphora

$$\left\langle \begin{array}{r} \{\, w_{ab}, w_a, w_b, w_\emptyset \,\}\,, \\ [] \end{array} \right\rangle$$

Counterfactual anaphora

- There is no bathroom. $\rightsquigarrow$ $\neg([\dagger x];[\mathit{bathroom}(x)])$

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Counterfactual anaphora

- There is no bathroom. $\rightsquigarrow$ $\neg([\dagger x];[\mathit{bathroom}(x)])$
- It would be upstairs $\rightsquigarrow$ $\mathit{would}([\mathit{upstairs}(x)])$

would

$[\![\phi]\!]^{W} = \lambda g.\lambda h.\exists W' \wedge [\![\phi]\!]^{W'}$

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- Non-veridical embedding (ignore other aspects of semantics for *“would”* here)

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- It would be upstairs $\rightsquigarrow$ $\mathit{would}([\mathit{upstairs}(x)])$
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  $\lambda g.\lambda h.\exists W' \wedge g = h \wedge \forall w' \in W' : h(x)(w') \in I(\mathit{upstairs})(w')$

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- Non-veridical embedding (ignore other aspects of semantics for *“would”* here)

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- There is no bathroom. $\rightsquigarrow$ $\neg([\dagger x]; [bathroom(x)])$
- It would be upstairs $\rightsquigarrow$ $would([upstairs(x)])$
- $[\![would([upstairs(x)])]\!]^{W} = \lambda g.\lambda h.\exists W' \wedge [\![[upstairs(x)]]\!]^{W'}$
  $\lambda g.\lambda h.\exists W' \wedge g = h \wedge \forall w' \in W' : h(x)(w') \in I(upstairs)(w')$

$$\begin{tikzcd}
& \left\langle \{ w_\emptyset \}, \left[x \mapsto \begin{bmatrix} w_{ab} \mapsto & a \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \right] \right\rangle \arrow[r, dash] & \left\langle \{ w_\emptyset \}, \left[x \mapsto \begin{bmatrix} w_{ab} \mapsto & a \\ w_a \mapsto & a \\ w_b \mapsto & b \\ w_\emptyset \mapsto & \# \end{bmatrix} \right] \right\rangle \\
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\end{tikzcd}$$

- Non-veridical embedding (ignore other aspects of semantics for *"would"* here)
- How to choose W'?
  1. Content: $[\![would([upstairs(x)])]\!]^{W'}$
  2. Accessibility: $W' \subseteq \{ w_{ab}, w_a, w_b \}$
  3. Veridicality: No relation between W, W' specified
- As long as there is a non-contradictory interpretation or the discourse, we are good!

Double negation

It's not the case that there is no bathroom $\rightsquigarrow \neg\neg([\dagger x];[bathroom(x)])$

- $[\![\neg\neg([\dagger x];[bathroom(x)])]\!]^{W} = \lambda g.\lambda h.\exists W' : W = \overline{W'} \wedge h \in [\![\neg([\dagger x];[bathroom(x)]]\!]^{W'}(g)$
  $= \lambda g.\lambda h.\exists W' : W = \overline{W'} \wedge \exists W'' : W' = \overline{W'} \wedge h \in [\![[\dagger x];[bathroom(x)]]\!]^{W''}(g)$

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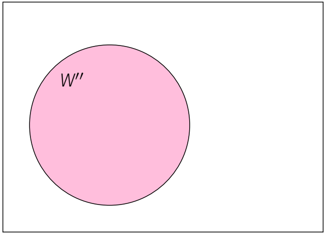

- $[\dagger x];[bathroom(x)]$ is interpreted in relation to W''

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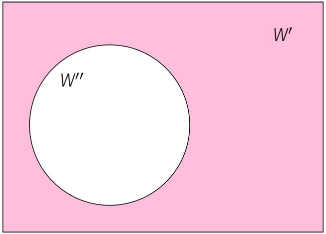

- $[\dagger x]; [bathroom(x)]$ is interpreted in relation to W''
- W'' is the complement of W'

Double negation

It's not the case that there is no bathroom $\rightsquigarrow \neg\neg([\dagger x];[\mathit{bathroom}(x)])$

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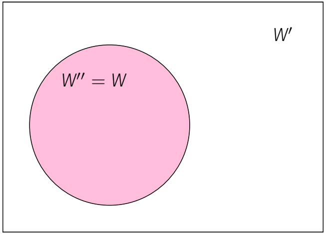

- $[\dagger x];[\mathit{bathroom}(x)]$ is interpreted in relation to W''
- W'' is the complement of W'
- W' is the complement of W, therefore $W'' = W$
- Boolean complementation allows for DNE

The optional bathroom

Either there is no bathroom, or it's upstairs $\rightsquigarrow (\neg([\dagger x];[bathroom(x)]) \vee [upstairs(x)])$

- $[\![(\neg([\dagger x];[bathroom(x)]) \vee [upstairs(x)])]\!]^{W}$
  $= \lambda g.\lambda h.\exists W', W'' : W = W' \cup W'' \wedge \ \exists i \ \in [\![\neg([\dagger x];[bathroom(x)])]\!]^{W'}(g) : h \ \in [\![[upstairs(x)]]\!]^{W''}(i)$
  $= \lambda g.\lambda h.\exists W', W'' : W = W' \cup W'' \ \wedge \exists i :$
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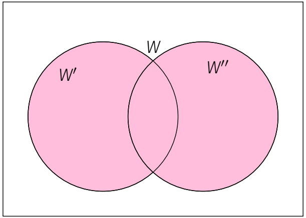

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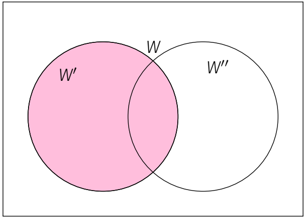

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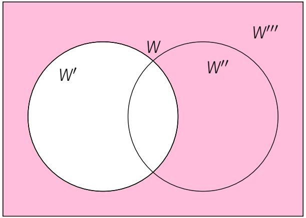

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- $[\dagger x]; [bathroom(x)]$ is interpreted in relation to W''': x is defined in all and only the W'''-worlds

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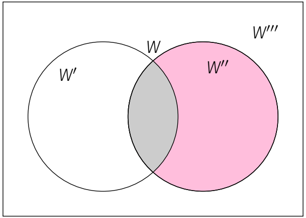

- W is the union of W', W''
- The first disjunct is interpreted in relation to W'
- Negation: W''' is the complement of W'
- $[\dagger x];[bathroom(x)]$ is interpreted in relation to W''': x is defined in all and only the W'''-worlds
- $[upstairs(x)]$ is interpreted in relation to W'': x is defined here only if $W'' \subseteq W'''$
- This is possible if the intersection of the disjunction is empty

Accessibility

Accessibility and veridicality

As soon as we choose an appropriate intensional representation of discourse variable mappings, anaphoric accessibility falls out from discourse consistency (the requirement that a discourse contain only non-contradictory information).

Now that our discourse stores the information about ...

- ...which worlds out drefs exist in (and do not)
- ...which worlds are considered to be possible candidates for the actual world, and which ones are part of hypothetical / counterfactual

...we can keep track of epistemic information about drefs:

- Actual drefs: speaker committed to existence
- Counterfactual drefs: speaker committed to non-existence
- Hypothetical drefs: speaker not committed either way

Explaining accessiblity, pt. I

- Drefs introduced under negation are *counterfactual*:
  Speaker committed to the non-existence of referent

(cf. hypothetical drefs in Stone 1999, Stone & Hardt 1999)

- Pronoun use is *factive*:
  Presupposes existence of referent
- Use of a pronoun is inconsistent with a counterfactual dref antecedent
- Speaker would be asserting non-existence and then presupposing existence

(16) *Mary doesn't have* [*a car*]^x.
It$_x$ *is red.*

Explaining accessiblity, pt. II

Content under negation can provide antecedents for anaphora, in case this doesn't create an inconsistent discourse:

- Antecedent is in a veridical context (double negation)

(17) *It's not true that Mary doesn't have* [*a car*]^x. *It*$_x$ *would be red.* → Factive anaphoric requirement satisfied globally

- Anaphor is also in a non-veridical context (modal subordination, bathroom disjunctions)

(18) *Mary doesn't have* [*a car*]^x. *It*$_x$ *would be red.* → Anaphoric requirement accommodated locally: True in worlds where local /embedded proposition is true (Heim 1992)

- Interpretation of the discourse segments containing the antecedent and anaphor does not require them to be consistent with each other (inter-speaker disagreement)

(19) A: *Mary doesn't have* [*a car*]^x. B: *Actually, it*$_x$*'s just parked in the back.* → Discourse segments inconsistent with each other, but that's okay

Summary and Conclusion

Conclusions

Counterfactual anaphora:

- As soon as we look at anaphora to counterfactual content, we need to consider refererence in possible worlds that are not considered as candidates for the actual world any more.

Intensional drefs:

- Variables in discourse are interpreted intensionally. For every discourse referent, the discourse state stores the information about which worlds the denoted objects exist in, and in which worlds they do not.

Sentential operators:

- The truth-functional meaning of sentential operators is static, dynamic updates with embedded content are interpreted relative to static sets of worlds.

Anaphoric accessibility:

- With these assumptions, we do not need separate constraints on accesibility, or bake this into the semantics of sentential operators. It can be explained in terms of a condition that a variable is defined in the set of worlds where it is interpreted, and discourse consistency.

Intensional logic of change

(20) **Atomic formulas:**

a. Relative variable update: $[\![[\dagger x]]\!]^{W} = \lambda g.\lambda h.g[W : x]h$

b. Predication: $[\![[Sleep(x)]]\!]^{W} = \lambda g.\lambda h.g = h \wedge \forall w \in W : g(x)(w) \in I(Sleep)(w)\}$

(21) Unary operators:

a. **Negation:** $[\![[\neg\phi]]\!]^{W} = \lambda g.\lambda h.\exists W' : W = \overline{W'} \wedge \ h \in [\![\phi]\!]^{W'}(g)$

b. *might:* $[\![[might\ \phi]]\!]^{W} = \lambda g.\lambda h.\exists W' : W \cap W' \neq \emptyset \wedge \ h \in [\![\phi]\!]^{W'}(g)$

c. *would:* $[\![[would\ \phi]]\!]^{W} = \lambda g.\lambda h.\exists W' : h \in [\![\phi]\!]^{W'}(g)$

(22) Binary operators

a. **Disjunction:** $[\![[\phi \vee \psi]]\!]^{W} = \lambda g.\lambda h.\exists W', W'' : W = W' \cup W'' \wedge \ \exists i \ \in [\![\phi]\!]^{W'}(g) : h \ \in [\![\psi]\!]^{W''}(i)$

b. **Conjunction:** $[\![[\phi \wedge \psi]]\!]^{W} = \lambda g.\lambda h.\exists W', W'' : W = W' \cap W'' \wedge \ \exists i \ \in [\![\phi]\!]^{W'}(g) : h \ \in [\![\psi]\!]^{W''}(i)$

c. Implication: $[\![[\phi \rightarrow \psi]]\!]^{W} =$
$\lambda g.\lambda h.\exists W', W'' : W = W' \cup W'' \wedge W'' \subseteq W' \wedge \ \exists i \ \in [\![\phi]\!]^{W'}(g) : h \ \in [\![\psi]\!]^{W''}(i)$