Negation and disjunction in dynamic semantics

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Introduction and recap

Logic of change: definitions

Sentences express *updates*, i.e., functions from possibilities (world-assignment pairs), to sets of assignments.

(1) **Random assignment:**
$[\![[x]]\!]^{w,g} = \{ h \mid h \text{ differs from } g \text{ at most at } x \}$ i.e., $\{ h \mid g[x]h \}$

(2) **Predication:**
$[\![[Sleep(x)]]\!]^{w,g} = \{ g \mid g(x) \in I(Sleep)(w) \}$

(3) **Negation:**
$[\![\neg\phi]\!]^{w,g} := \{ g \mid [\![\phi]\!]^{w,g} = \emptyset \}$

(4) **Dynamic conjunction:**
$[\![\phi\, ; \psi]\!]^{w,g} = \{ h \mid \exists i[i \in [\![\phi]\!]^{w,g} \wedge h \in [\![\psi]\!]^{w,i}] \}$

(5) Implication:

a. $\phi \rightarrow \psi := \neg(\phi\,;\neg\psi)$

b. $[\![\phi \rightarrow \psi]\!]^{w,g} = \{\, g \mid \neg\exists h[h \in [\![\phi]\!]^{w,g} \wedge [\![\psi]\!]^{w,h} = \emptyset] \,\}$

(6) Disjunction:

a. $\phi \vee \psi := \neg(\neg\phi\,;\neg\psi)$

b. $[\![\phi \vee \psi]\!]^{w,g} = \{\, g \mid [\![\phi]\!]^{w,g} \neq \emptyset \vee [\![\psi]\!]^{w,g} \neq \emptyset \,\}$

(7) Existential quantification:

a. $\exists x\phi := [x]\,;\phi$

(8) Universal quantification:

a. $\forall x\phi := [x] \rightarrow \phi$
(N.b., $\forall$ can equivalently be defined as the dual of $\exists$)

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- Negation is a test

The dynamics of negation

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- Negation is a test
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- Therefore, negation is externally static.

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- Therefore, disjunction is externally static.

Accessibility recap

	internally	externally	veridicality?
$\phi \wedge \psi$	dynamic	dynamic	veridical
$\neg\phi$	N/A	static	non-veridical
$\Diamond\phi$	N/A	static	non-veridical
$\phi \rightarrow \psi$	dynamic	static	non-veridical
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- Veridicality: A propositional operator is *veridical* iff it entails its prejacent.
- A generalization: Non-veridical operators are externally static

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- A generalization: Non-veridical operators are externally static
- Prediction: Anaphora to indefinites in the scope of non-veridical operators is not possible
- But there are many counterexamples in natural language...

Anaphora to antecedents in the scope of non-veridical operators

Some counterexamples to the generalization from classical dynamic semantics

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It's not the case that there isn't a bathroom in this house. It is upstairs.

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(20) Inter-speaker disagreement (Hofmann 2019)
A: *There isn't a bathroom in this house.*
B: *(What are you talking about?) It's upstairs.*

Double negation, discourse-inferences, and program disjunction: Contextual entailment

Double negation

(21) a. *It's not the case that there isn't [a bathroom]^x.*
b. *It$_x$ is upstairs.*

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The problem: Some of the inference rules of classical logic do not apply in dynamic semantics

Double negation elimination

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- In DS: $\neg\neg\phi \not\Leftrightarrow \phi$

No double negation elimination in DS

Existential statements in DS introduce Discourse Referents (DRS) via random assignment:

(24) There is [a bathroom]^x. $\rightsquigarrow$ $[x]\ ;\ [\mathit{Bathroom}(x)]$

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Let's see why in more detail:

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Negating *again* returns a test of whether the previous update (itself a test) has an empty output.

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The net result is that $\neg\neg\phi$ is always interpreted as a *test* of whether ϕ is true (i.e., has a non-empty output).

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Double negation in detail — no dynamic DNE

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The updates associated with (26) and (29) are different:

- (26) introduces a new variable *x* (pointing to a bathroom), but (29) only tests whether such an update is possible.

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Some takeaways

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Accessibility in DS is:

- Constrained by semantic representations; namely, the presence of negation.
- Not affected by truth-conditional inferences:
  The truth-conditional entailment of an existential statement does not enable anaphora
  (though maybe it should)

Double negation and anaphora

As we have seen, doubly-negated sentences *can* provide antecedents for anaphora:

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- Question: How could this problem be addressed?

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Consider the following possible entry for negation. Do you see any problems?

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Can you think of any alternative entries for negation that are possible to state in DPL?

(**??**) a. *There might be a bathroom in this house.*

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- Question: What is going on here?
- Another case, where a representational generalization does not work
- The truth-conditional inference of (a + b) allows for anaphora in (c)

Progam disjunction

Groenendijk & Stokhof (1991) noticed that natural language disjunction isn't always externally static. Here is their counterexample (slightly modified).

(??) a. *A professor or an assistant professor will attend the meeting of the university board.*

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- In DS, both disjuncts truth-conditionally entail an existential statement [x], but we do not get the anaphoric potential of an existential
- Once a test, always a test... Let's see why

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It's easy to understand why (**??**) is a counterexample — disjunction in DS is externally static.

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- Intuitively, *both* indefinites serve as antecedents to the anaphoric pronoun *she*.

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- Let's see how this works in more detail...

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- Discourse anaphora is licensed in the usual way.
- **Question:** does this account violate any constraints on semantic representations we've been assuming so far?

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Since $[\![\mathit{Cancelled}]\!]^{w,g} = \begin{cases} g & \text{there no meeting in } w \\ \emptyset & \text{otherwise} \end{cases}$

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This exposes a conceptual flaw with DS — both program and ordinary disjunction capture the truth-conditional contribution of disjunction, but the anaphoric information expressed is different.

Anaphoric information is not straightforwardly constrained by truth-conditional contribution — there isn't a 'recipe' to determine the anaphoric contribution of a given logical operator.

Towards a generalization

So what do these three cases have in common?

(29) a. *It's not the case that there isn't a bathroom.*
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And how are they different from cases with negation, modals, and disjunction that do not allow for anaphora?

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- On the left side, the existence of a witness to the existential *x* is contextually entailed
- On the right side, that is not the case
- This generalization will serve as motivation for the solution we propose tomorrow

Bathroom-sentences and modal subordination: *Local* contextual entailment

Bathroom disjunctions

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(43) Either there is no^{x} bathroom, or it$_{x}$'s upstairs.

(44) Either Steven didn't order a^{x} pizza, or it$_{x}$'s warming in the oven.

- We call such cases bathroom disjunctions (Roberts 1987, attributed to Barbara Partee)
- **Question:** Why do you think anaphora is possible here?

Bathroom disjunctions cont.

It's quite easy to see why DS doesn't capture bathroom disjunctions; DS disjunction simply tests whether either disjunct has a non-empty output at the input.

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(45) $$[\![\neg([x] \,;\, [Bathroom(x)]) \vee [Upstairs(x)]]\!]^{w,g}$$
$$= \{\, g \mid [\![\neg([x] \,;\, [Bathroom(x)])]\!]^{w,g} \neq \emptyset \vee [\![Upstairs(x)]\!]^{w,g} \neq \emptyset \,\}$$
$$= \{\, g \mid I(Bathroom)(w) \neq \emptyset \vee g(x) \in I(Upstairs)(w) \,\}$$

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(45) $$\begin{aligned}&[\![\neg([x]\,;[\mathit{Bathroom}(x)])\vee[\mathit{Upstairs}(x)]]\!]^{w,g}\\ &=\{\,g \mid [\![\neg([x]\,;[\mathit{Bathroom}(x)])]\!]^{w,g}\neq\emptyset\vee[\![\mathit{Upstairs}(x)]\!]^{w,g}\neq\emptyset\,\}\\ &=\{\,g \mid I(\mathit{Bathroom})(w)\neq\emptyset\vee g(x)\in I(\mathit{Upstairs})(w)\,\}\end{aligned}$$

Consequently, the interpretation of the variable in the second conjunct is dependent on the assignment of the context.

Classical equivalence 1: Interdefinability of disjunction and implication

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Intuitively, the two sentences *do* feel equivalent, but classical DS doesn't capture this.

- Here: $(\neg\phi \vee \psi) \not\Leftrightarrow (\phi \rightarrow \psi)$

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- Truth-conditional inferences cannot salvage this

Classical equivalences 2: Negating the first disjunct

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Under this approach: The negation of the first disjunct provides a *local context* of interpretation for the second disjunct

We would be taking advantage the classical equivalence in (48):

(48) $\phi \vee \psi \Leftrightarrow \phi \vee (\neg\phi \wedge \psi)$

Therefore:

(49) $\neg\phi \vee \psi \Leftrightarrow \phi \vee (\neg\neg\phi \wedge \psi)$

But again, this won't work, because once we negate the first disjunct, we have a test!

Bathroom disjunctions and double negation

- If we *were* to develop a system in which doubly-negated sentences are equivalent to their positive counterparts wrt to anaphoric information, either of this strategies could be viable.
- Accounting for bathroom disjunctions, then, is closely tied to the problematic status of negation in DS.

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So... Does this fall under our previous generalization: Does this entail the existence of a referent to the indefinite?

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- Putting the disjunction together with the conditional antecedent locally entails the truth of the disjunct that introduces a discourse referent.

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- Double negation, discourse-inferences, and program disjunction: The existence of a referent is entailed in the context
- Bathroom-disjunction and modal subordination: The existence of a referent is entailed in the local context
- Upcoming: another case showing that this inference of local contextual entailment can be accommodated

Inter-speaker disagreement: Accommodation of contextual entailment

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- But truth-conditionally, B's utterance implies that there is a bathroom
- A possible result: The inference of contextual entailment of a witness can be accommodated

Accessibility and licensing — When pronouns are possible

Contextual entailment of a witness

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Accounts for the basic generalizations about veridical operators, and out counterexamples

Inference and form

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- The notion of accessibility in DS relies too much on linguistic form,
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(53) *David has a spouse. He's also invited.*

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DS posits a richer notion of information state which is sensitive to the presence/absence of an indefinite.

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- But we also need truth-conditional inferences to affect accessibility.
- Modal subordination shows that any externally static operator can provide antecedents under the right conditions.
- We may want to get rid of externally static operators altogether, and derive anaphoric accessibility from the truth-conditional properties of our sentential operators

Truth-conditions of DN and BR anaphora

Gotham 2019 on uniqueness

- Gotham (2019) pushes back against the conjecture that $\neg\neg\phi$ is truth-conditionally and anaphorically equivalent to ϕ, offering the following contrast (p. 144):

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- Gotham (2019) pushes back against the conjecture that $\neg\neg\phi$ is truth-conditionally and anaphorically equivalent to ϕ, offering the following contrast (p. 144):

(54) a. John owns a shirt. It's in the wardrobe.
 b. ??It's not true that John doesn't own a shirt. It's in his wardrobe.

- Gotham's empirical claim:
  - Let's assume that it's contextually entailed that *if someone owns shirts, they own more than one*.
  - (54a) is perfectly acceptable, since the existential statement is compatible with John owning multiple shirts.
  - (54b) is odd, because unlike its positive counterpart, the doubly-negated statement entails that *John owns exactly one shirt*.

A little historical background: prior to the development of DS, the main account of donkey and discourse anaphora was the E-type theory of Evans 1977, according to which pronouns are interpreted as *definite descriptions*.

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Sage plant sentences and the E-type theory

A little historical background: prior to the development of DS, the main account of donkey and discourse anaphora was the E-type theory of Evans 1977, according to which pronouns are interpreted as *definite descriptions*.

(55) John owns a shirt. [It=**the shirt**] is in the wardrobe.

One of the main arguments against Evans E-type theory was provided by Heim 1982; Definite descriptions semantically encode uniqueness, Heim's *sage plant* sentences involve donkey/discourse anaphora but are logically incompatible with uniqueness.

(56) a. Steven ordered a^x pizza, and he_x ordered eight others along with it.
b. If Steven orders a^x pizza, he orders eight others along with it_x.

Sage plant sentences cont.

Although not the focus of our course, it’s worth noting that the E-type account was subsequently improved to accommodate the absence of uniqueness by Heim 1990, but limited to donkey anaphora.

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Nevertheless, sage plant sentences are extremely useful for testing for (semantically-encoded) uniqueness inferences (Mandelkern & Rothschild 2020).

We can use sage plant sentences to show that doubly-negated sentences don’t semantically encode uniquess:

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Nevertheless, sage plant sentences are extremely useful for testing for (semantically-encoded) uniqueness inferences (Mandelkern & Rothschild 2020).

We can use sage plant sentences to show that doubly-negated sentences don't semantically encode uniquess:

(57) It's not true that Steven didn't order a^x pizza,
and furthermore he ordered eight others along with it$_x$.

Takeaway: uniqueness (or lack thereof)

Sage plant sentences suggest that doubly-negated existential statements have the same truth-conditions as their positive counterparts.

We don't however deny that there is a *reading* that entails (perhaps implicates?) uniqueness.

This is muddied by the fact that singular indefinites in positive contexts often implicate uniqueness (Spector 2007; cf. Gotham).

(58) Steven ordered a pizza. $\Rightarrow$ *Steven ordered exactly one pizza (as far as I know)*

We won't have anything concrete to say in this course about how to derive this (optional) inference, or whether doubly-negated sentences have more robust uniqueness inferences than their positive counterparts.

Truth-conditions of bathroom disjunctions

- We haven't addressed the question of what the actual *truth-conditions* of bathroom disjunctions
- An analysis should not just account for the possibility of anaphora, but also get the truth-conditions right.
- In the following two slides, we'll argue against two notable claims concerning the truth-conditions of bathroom disjunctions:
    - Uniqueness entailments (Gotham 2019).
    - Universal readings (Krahmer & Muskens 1995).

Bathroom disjunctions and uniqueness

- Similarly to double-negation, Gotham (2019) suggests that bathroom sentences are associated with uniqueness inferences, on the basis of the following contrast (Gotham's judgement).

(59) a. John owns a shirt and it's in his wardrobe.
b. ???Either John doesn't own a shirt, or it's in his wardrobe.

- **Exercise:** formulate a sage plant sentence to test whether or not uniqueness is semantically entailed in bathroom disjunctions.

Bathroom disjunctions and uniqueness cont.

Here's one we made earlier:

(60) Either Steven didn't order a pizza, or he ordered 8 others along with it.

This sentence is logically incompatible with the putative (conditional) uniqueness entailment *if Steven ordered a pizza then he ordered exactly one* (see also Mandelkern & Rothschild 2020 on definite descriptions).

Universal readings?

The truth-conditions of bathroom disjunctions have also been discussed by Krahmer & Muskens (1995), who suggest that bathroom disjunctions have a *universal* reading.

(61) Either there's no bathroom, or it's upstairs.
$\Rightarrow$ *every bathroom (if any) is upstairs*

(It's interesting to note that this is what we expect given the conditional equivalence, due to Egli's corrolary, i.e., $\phi \vee (\neg\phi \rightarrow \psi)$).

Krahmer & Muskens report that (61) is *false* as soon as there's a bathroom that isn't upstairs.

Exercise: come up with a concrete example paired with a context designed to test this claim.

Existential readings?

Elliott (2023) argues that bathroom disjunctions *can* in fact have existential readings, meaning that universal truth-conditions are too strong.

(62) *Context: we're wondering how Gabe paid for dinner.*
Either Gabe doesn't have a credit card, or he paid with it.

The intuition here is that the sentence is *true* just so long as Gabe paid with a credit card of his, even if he has other credit cards which he didn't pay with.

(N.b., existential readings are a natural expectation if we opt for the conjunctive equivalence, i.e., $\phi \vee (\neg\phi \wedge \psi)$).

Exercise: can you come up with a context in which the sentence involving bathrooms receives an existential reading?

Recommended reading

- Gotham. “Double Negation, Excluded Middle and Accessibility in Dynamic Semantics”. 2019.
- Krahmer & Muskens. “Negation and Disjunction in Discourse Representation Theory”. 1995

Questions?

References i

Elliott, Patrick D. 2023. **Disjunction in a Predictive Theory of Anaphora.** In Dun Deng et al. (eds.), *Dynamics in Logic and Language*, vol. 13524, 76–98. Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-25894-7_4. https://link.springer.com/10.1007/978-3-031-25894-7_4 (30 April, 2023).

Evans, Gareth. 1977. **Pronouns, Quantifiers, and Relative Clauses (I).** *Canadian Journal of Philosophy* 7(3). Publisher: [Taylor & Francis, Ltd., Canadian Journal of Philosophy] JSTOR: 40230703, 467–536.

Gotham, Matthew. 2019. **Double Negation, Excluded Middle and Accessibility in Dynamic Semantics.** In *Proceedings of the 22nd Amsterdam Colloquium*. Amsterdam.

Groenendijk, Jeroen & Martin Stokhof. 1991. **Dynamic Predicate Logic.** *Linguistics and Philosophy* 14(1). JSTOR: 25001418, 39–100.

Heim, Irene. 1982. ***The Semantics of Definite and Indefinite Noun Phrases.*** University of Massachusetts Amherst Doctoral dissertation.

- Heim, Irene. 1990. **E-Type pronouns and donkey anaphora.** *Linguistics and Philosophy* 13(2). 137–177. `https://doi.org/10.1007/BF00630732`. `https://doi.org/10.1007/BF00630732` (23 November, 2020).
- Hofmann, Lisa. 2019. **The Anaphoric Potential of Indefinites under Negation and Disjunction.** In *Proceedings of the 22nd Amsterdam Colloquium*, 181–190.
- Karttunen, Lauri. 1976. **Discourse Referents.** In *Notes from the linguistic underground*, 363–385. Brill. `https://doi.org/10.1163/9789004368859\_021`.
- Krahmer, Emiel & Reinhard Muskens. 1995. **Negation and Disjunction in Discourse Representation Theory.** en. *Journal of Semantics* 12(4). 357–376. (6 March, 2019).
- Mandelkern, Matthew & Daniel Rothschild. 2020. **Definiteness Projection.** *Natural Language Semantics* 28(2). 77–109. `https://doi.org/10.1007/s11050-019-09159-2`. `https://doi.org/10.1007/s11050-019-09159-2` (5 February, 2022).

Roberts, Craige. 1987. *Modal Subordination, Anaphora and Distributivity.* University of Massachusetts, Amherst Doctoral dissertation.

Rothschild, Daniel. 2017. A Trivalent Account of Anaphora and Presupposition. In *Proceedings of the 21st amsterdam colloquium*, vol. 21, 1–13. 21st Amsterdam Colloquium.

Spector, Benjamin. 2007. Aspects of the pragmatics of plural morphology - On higher-order implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*. London: Palgrave Macmillan UK.