Negation and Disjunction in Discourse Representation Theory

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Abstract

Classical Discourse Representation Theory (DRT) predicts that an indefinite noun phrase cannot antecede an anaphoric element if the noun phrase is, but the anaphoric element is not, in the scope of a negation; the theory also predicts that no anaphoric links are possible between the two parts of a disjunction. However, it is well known that these predictions meet with counterexamples. In particular, anaphora is often possible if a double negation intervenes between antecedent and anaphoric element, and also if the antecedent not only occurs in the first part of a disjunction but also within the scope of a negation, while the anaphoric element is in the second part of the same disjunction. In this paper we argue that these recalcitrant phenomena are related and that a solution to the double negation problem will also provide us with a solution to the disjunction problem. We review the basic set-up of classical DRT and offer an extension (called 'Double Negation DRT') which validates the law of double negation. An adaptation of the standard DRT construction algorithm which transforms texts into Discourse Representation Structures is sketched and it is shown that the problems with negation and disjunction that led to the definition of our new version of DRT are properly dealt with.

I TWO PROBLEMS FOR DRT, AND A REDUCTION

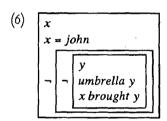
1.1 The double negation problem

In a now classic paper (Karttunen 1976) Karttunen noted that while a discourse referent cannot outlive a single negation or a single verb with an inherently negative implication (such as fail, neglect, or forget) it will not be blocked by a double negation. While in (1) the pronoun it cannot be interpreted as dependent on a question and in (2) the pronoun cannot depend on an answer, the definite in (3) may depend on the preceding indefinite and the it in (4) can be taken to refer to an umbrella. The anaphoric pronouns in (5) can likewise be interpreted as depending on the indefinite that precedes them, even though the latter is within the scope of two negations.¹

- (1) Bill didn't dare to ask a question. *The lecturer answered it.
- (2) John failed to find an answer. *It was wrong.
- (3) John didn't fail to find an answer. The answer was even right.

- (4) John didn't remember not to bring an umbrella, although we had no room for it.
- (5) It is not true that John didn't bring an umbrella. It was purple and it stood in the hallway.

Various authors² have pointed out that examples such as (3), (4), and (5) are a problem for dynamic theories of discourse such as Discourse Representation Theory (DRT, Kamp 1981; Kamp & Reyle 1993), File Change Semantics (FCS, Heim 1982, 1983), or Dynamic Predicate Logic (DPL, Groenendijk & Stokhof 1991). These theories correctly predict negation to be a plug with respect to anaphoric binding and thus fit the facts in (1) and (2),3 but they also incorrectly predict a double negation to be a double plug, not a plug unplugged as the facts in (3)-(5) would suggest. In DRT, for example, the discourse referent that is connected to an umbrella in the first sentence of (5) will land up in a Discourse Representation Structure (DRS) that is twice embedded to the main DRS and that will thus not be accessible for future anaphoric reference. An application of the DRT construction algorithm to the first sentence of (5) gives the DRS in (6) as an output, while it is the simpler DRS in (7) that would give the right predictions here. In the latter, but not in the former, the discourse referent y, which is connected to an umbrella, will be accessible from conditions in the main DRS.



(7)
$$\begin{array}{c} x & y \\ x = john \\ umbrella & y \\ x & brought & y \end{array}$$

Other formulations of the dynamic perspective are confronted with essentially the same difficulty. In DPL, the negation of a formula φ will act as a 'test', irrespective of the internal structure of φ , and so, since the first clause of (5) is of the form $\neg \varphi$, the anaphoric link between an umbrella and it is predicted to be impossible. In FCS we have that the first sentence in (5) does not succeed in extending the domain of the current file, while a new card for an umbrella is needed in order to establish the link between antecedent and anaphoric pronoun. In this paper we shall discuss the double negation problem (and the disjunction problem—see below) from a DRT perspective, but the reader will

have no difficulty in translating our proposed solutions to her favourite dynamic semantic framework.

While we think that Karttunen's data essentially show that double negations are holes for anaphoric linking and thus form a problem for standard dynamic accounts of anaphora, it should be noted that other data apparently point in an opposite direction. Consider (8a), for example, a text that is decidedly odd if the anaphoric pronoun is interpreted as depending on no guest, even though the latter occurs within the scope of a negation and no other relevant operators intervene between the would-be antecedent and its dependent element.

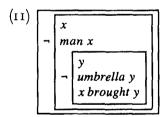
- (8a) It is not true that there is no guest at this wedding. ??He is standing right behind you.
- (8b) It is not true that there is no bride at this wedding. She is standing right behind you.

The oddity of (8a) should be contrasted, however, with the complete acceptability of (8b) and is due, we conjecture, to a *uniqueness* effect (cf. Evans 1977; Kadmon 1987). Given some highly unlikely context in which it is understood that at most one guest can be present at a wedding (8a) would be fine. We feel that it is precisely the unlikelihood of such contexts which explains the markedness of (8a). Note, however, that (8a) is still better than (9), its counterpart with one negation only.

(9) There is no guest at this wedding. *He is standing right behind you.

Another category of prima facie counterexamples to the double negation rule is formed by cases where the only plugs intervening between a possible antecedent and an anaphoric element are indeed two negations, but where the two still do not conspire to form an authentic double negation because they sandwich other material. We have in mind cases like (10), whose first sentence should be rendered as the DRS in (11).

(10) No man didn't bring an umbrella. *It was purple and it stood in the hallway.



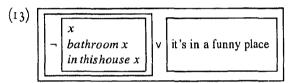
The truth conditions of (11) are exactly those of the predicate logical formula $\neg \exists x (man(x) \land \neg \exists y (umbrella(y) \land brought(x, y)))$ and (11) is as much a case of double negations as this predicate logical formula is. Since such apparent

counterexamples on closer examination thus turn out to be no counterexamples at all, it seems that we can take it to be a general rule that, as far as truth conditions and the possibility of anaphora are concerned, double negations in standard English behave as if no negation were present.

1.2 The disjunction problem

The double negation problem seems to be related to another problem that is also generally thought to be a hard nut for DRT and related theories. In (12)⁵ the pronoun *it* is naturally linked to *no bathroom*, while DRT and other dynamic theories predict no antecedent in one part of a disjunction to be accessible for a pronoun in the other part. If we apply the standard construction rules to this sentence we get the DRS which is given in (13), but in this DRS the pronoun *it* cannot be resolved as the referent x.

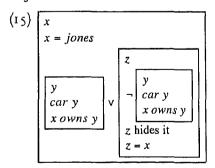
(12) Either there's no bathroom in this house, or it's in a funny place

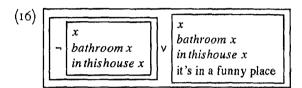


Kamp & Reyle (1993)⁶ remark that it is in fact the presence of a negative element in the first disjunct which seems to license the anaphora in (12),⁷ even though negations in themselves usually block the possibility of linking. If there is no such negative element, as in (14), coreference is impossible.

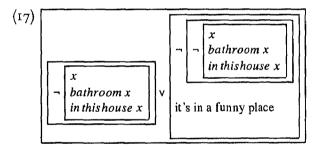
(14) ??Jones owns a car or he hides it

A second observation made by Kamp & Reyle is that sentences of the form A or B can in general be felicitously paraphrased as A or otherwise B and this leads to a proposal to let the DRT construction algorithm provide for the 'other case'. In (14) the 'other case' is the case where Jones does not own a car, and thus a revised form of the construction algorithm adds a condition to this effect to the second disjunct of the DRS for the sentence. The result is shown in (15).



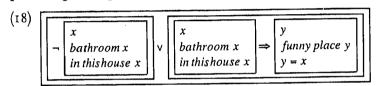


Kamp & Reyle's treatment of 'bathroom' sentences can perhaps be criticized for not being entirely precise, in the sense that their new construction rule does not seem to prescribe exactly what material is to be added to the second disjunct. Suppose that we take the rule to be that in constructing the DRS for a disjunction we should add the negation of the DRS for the first disjunct as a condition to the DRS for the second disjunct. Then the DRS associated with (14) would indeed be (15), but the DRS for (12) would be (17) instead of (16), i.e. we get a double negation where we want to negation at all.⁸

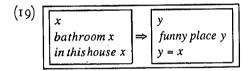


Note the structural similarity between the problem how to get from (17) to (16) and our previous problem how to obtain (7) from (6). In both cases we should like to be able to erase the double negation. An explicit rule to this effect would be very much ad hoc, however, and would be quite unlike all other DRT construction rules. It would have the useful property of being able to make certain referents accessible to certain pronouns (e.g. the referent x is accessible from it in (16) but not in (17)) but this very property would also make it theoretically suspicious for not being meaning preserving. If meanings determine context change potentials, as the dynamic perspective has it, then a rule to erase double negations that would change (6) into (7) (and (17) into (16)) cannot be meaning preserving since (6) gives a context which does not allow reference to y while (7) gives one which does.

There is another difficulty with Kamp & Reyle's proposed solution to the problem of 'bathroom' sentences: (16) simply does not have the truth conditions that (12) seems to have. Suppose there are in fact two toilets in the house, one of which is, and one of which is not in a strange place; then (12) is false according to our intuitions, but (16) is true since its second disjunct can be verified. We therefore turn to an earlier proposal from Roberts (1989), who renders (12) as (18). The idea here is that the material under the negation in the first disjunct is accommodated to provide an antecedent to the second disjunct. Since the first disjunct gives a negative answer to the question whether there is a toilet in the house, it is natural to interpret the second disjunct as pertaining to the possibility that there is one.

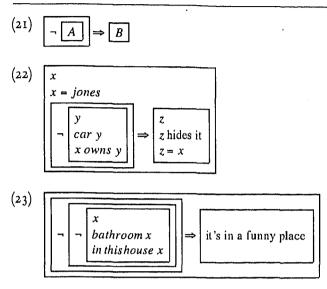


From a formal point of view it should be observed that, in a sense which will be made precise in the following section, (18) is equivalent to (19), the second disjunct of its only condition. And indeed, we feel that this is correct, since intuitively (12) is equivalent to (20).



(20) If there's a bathroom in this house it's in a funny place

How can we revise the DRT construction algorithm so that it gives (19) instead of (13) as an output for (12)? Here again we see that if we could but solve the double negation problem we would have a solution to the disjunction problem as well. For suppose that we would revise the construction algorithm so that at any time that a sentence disjunction A or B is encountered a condition of the form (21) (instead of the equivalent $A \vee B$) would be added to the current DRS, 12 then (22) would be the output for (14), but for (12) DRS (23) would be obtained. The first of these is indeed correct in the sense that the anaphoric link is predicted to be impossible, but in the second we have a double negation again where no negation is wanted. The problem how to get from (23) to (19) is formally similar to the problem how to get from (17) to (16) or indeed to the question how to get (7) from (6). 13 In this sense it can be said that the disjunction problem reduces to the double negation problem.



It thus seems that if we can revise the DRT language by adding a new negation which obeys the law of double negations (i.e. which allows for cancelling double negations) we may not only solve the problems that we have encountered with Karttunen's 'umbrella' sentences, but we may also be able to deal with 'bathroom' sentences. An attempt to carry out such a revision will be made in section 3 below, but first let us look into the syntax and semantics of the standard DRS language in some detail.

2 STANDARD DRT: THE FORMALITIES

The Double Negation DRT of the next section will be a generalization of standard DRT and for the sake of easy comparison we shall give concise versions of the most important DRT definitions in this one. In fact, we shall extend the standard syntax slightly and add a sequencing operator ';' which takes two DRSs and gives a complex DRS. 14 This addition, which seems natural in itself, in fact takes us already halfway from the standard set-up to the logic that is discussed in the next section, but nothing here hinges on the addition and a formalization of the core part of DRT can be obtained by simply omitting all reference to ';'.

One of the virtues of the DRS language is that DRSs are visually appealing. A disadvantage is that they take up a lot of space. This is especially annoying if one wants to talk about the DRS language and DRSs need to enter formal expressions in the metalanguage. For this reason we shall switch to a linear notation in this and in the next section, but in section 4, where we shall have occasion to discuss applications, we shall switch back to the easily readable DRS format again.

The basic ingredients of the DRS language are familiar from ordinary predicate logic; we have terms (constants or variables, the variables are also called discourse referents) and at least unary and binary predicate symbols. We use t to range over terms, P to range over unary predicate symbols, and R to range over binary ones. With the help of these ingredients we build up conditions (φ) and DRSs (K) by the following rules, which are presented in Backus Naur Form.

$$\varphi ::= Pt \mid t_1 R t_2 \mid t_1 = t_2 \mid \neg K \mid K_1 \lor K_2 \mid K_1 \Rightarrow K_2 K ::= [x_1 \dots x_n \mid \varphi_1, \dots, \varphi_m] \mid K_1 ; K_2$$

In the second clause it is to be understood that n or m may be equal to zero. The set of discourse referents $\{x_1, \ldots, x_n\}$ is called the *universe of* $[x_1, \ldots, x_n]$ $\varphi_1, \ldots, \varphi_m$ and the conditions $\varphi_1, \ldots, \varphi_m$ are the *conditions of* this DRS. As an example of a formula in this revised DRS language a linear alternative for (6) is given in (24) and an alternative for (19) is given in (25).

(24)
$$[x \mid x = john, \neg[|\neg[y| umbrella y, x brought y]]]$$

(25) $[|[x| bathroom x, in-this-house x] \Rightarrow [y| funny-place y, y - x]]$

Next, for each condition φ occurring in some DRS K' we are interested in the set $ACC(\varphi)$ of discourse referents that are accessible from φ (in K') and it will be expedient to define ACC(K) for each K that is a subDRS of K' as well. Our definition will be (intensionally) different from the standard one, but (extensionally) equivalent. Setting $ACC(K') = \emptyset$, we define ADR(K), the set of active discourse referents of any DRS K, by letting $ADR([x_1 \dots x_n | \varphi_1, \dots, \varphi_m]) = \{x_1, \dots, x_n\}$ and $ADR(K_1; K_2) = ADR(K_1) \cup ADR(K_2)$. The discourse referents accessible from any subDRS or condition in K' can be computed in a top-down way by the following rules. ¹⁵

(i) If ACC(¬K) = X then ACC(K) = X
 (ii) If ACC(K₁ ∨ K₂) = X then ACC(K₁) = X and ACC(K₂) = X
 (iii) If ACC(K₁ ⇒ K₂) = X then ACC(K₁) = X and ACC(K₂) = X ∪ ADR(K₁)
 (iv) If ACC([x₁ . . . xₙ | φ₁, . . ., φₙ]) = X then ACC(φᵢ) = X ∪ [x₁, . . ., xₙ]
 (1 ≤ i ≤ m)
 (v) If ACC(K₁; K₂) = X then ACC(K₁) = X and ACC(K₂) = X ∪ ADR(K₁)

In order to illustrate the procedure we compute the discourse referents accessible from y = x in (25): since $ACC((25)) = \emptyset$ by definition, we find with rule (iv) that the set of referents accessible from (25)'s only condition is \emptyset ; rule (iii) tells us that, since x is the only active discourse referent of this condition's antecedent, $ACC([y \mid funny-place y, y = x]) = \emptyset \cup \{x\} = \{x\}$ and a second application of rule (iv) shows that $ACC(y = x) = \{x, y\}$. If x occurs in some atomic condition (i.e. condition of the form Pt or t_1Rt_2 or $t_1 = t_2$) of K from

Our definition of the semantics of the DRT language may at first blush seem different from the one given in Kamp (1981), or Kamp & Reyle (1993), although in fact (modulo our addition of the sequencing operator, which will have relational composition as its semantics)¹⁶ it will be equivalent. For reasons of conciseness and easy generalization we shall give a definition inspired by the one given in Groenendijk & Stokhof (1991).¹⁷ Let $M = \langle D, I \rangle$ be a first-order model with domain D and interpretation function I and let f, g, and h range over finite assignments, i.e. finite partial maps from the set of discourse referents into D. Define $||t||^f$ to be f(x) if t is the discourse referent x and $x \in \text{dom}(f)$, and define $||t||^f$ to be I(c) if t is the constant c. If f(x) is undefined $||x||^f$ will also remain undefined. Write $f(x_1, \ldots, x_n)g$ if $f \subseteq g$ and $\text{dom}(g) = \text{dom}(f) \cup \{x_1, \ldots, x_n\}$. We define the extension $||\varphi||$ of a condition φ to be a set of assignments and the extension ||K|| of a DRS K to be a binary relation between assignments by means of the following induction. ¹⁸

Definition (DRT semantics)

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 \begin{aligned} \|Pt\| & - \|f\| \|t\|^f \text{ is defined } \& \|t\|^f \in I(P)\} \\ & - \|f\| \|t\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \langle \|t_1\|^f, \|t_2\|^f \rangle \in I(R)\} \\ & \|t_1 - t_2\| & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ and } \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_1\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f\} \\ & - \|f\| \|t_2\|^f \text{ are defined } \& \|t_1\|^f - \|t_2\|^f + \|t_2\|^f +
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A proper DRS K is true iff the empty map \varnothing is an element of the domain of ||K||. The present definition gives us the possibility to define a natural notion of equivalence in meaning: two DRSs or conditions are called equivalent iff their extensions coincide. It is easy to see that $[|\neg K_1| \lor [|K_1 \Rightarrow K_2|]]$ is equivalent with $K_1 \Rightarrow K_2$, and hence that Roberts' (18), discussed in the previous section, is equivalent with the simpler (19). The reader may also note that $K \lor [x_1 \ldots x_n \mid \varphi_1, \ldots, \varphi_m]$ and $K \lor [x_1 \ldots x_n \mid \neg K, \varphi_1, \ldots, \varphi_m]$ are equivalent, provided that x_1, \ldots, x_n do not occur in K. This means that the revised construction rule which led to the construction of (17) in the previous section would give an output that is not semantically different from the output we get from the standard DRT construction rules (for example, (17) is in fact equivalent with (13)). Similarly, since $[|\neg K_1| \Rightarrow K_2]$ is equivalent with $K_1 \lor K_2$, adopting the rule that led to the construction of (22) and (23) would have no semantic effects either ((23) is also equivalent with (13)). There is a semantic difference between

(6) and (7) though, and since we want a version of DRT in which double negations can be cancelled we shall define a new negation in the next section.

3 DOUBLE NEGATION DRT

The basic problem with negation in standard DRT is that it is not a flip-flop operation like its cousin in ordinary logic. Even the very syntax of negation discourages flip-flop behaviour: if K is a DRS, ¬K is a condition and there is no comparable operator which takes us from conditions to DRSs again. In our variant of DRT—Double Negation DRT—we remedy this and let the negation ¬K of a DRS K itself be a DRS. This is our only addition and we have removed the original negation, so that the syntax of Double Negation DRT looks as follows.

$$\varphi ::= Pt \mid t_1 R t_2 \mid t_1 - t_2 \mid K_1 \lor K_2 \mid K_1 \Rightarrow K_2 K ::= [x_1 ... x_n \mid \varphi_1, ..., \varphi_m] \mid K_1; K_2 \mid \neg K$$

We interpret this language by borrowing a technique from partial logic. Conditions will as before have an extension which consists of a set of partial assignments, but with each DRS K two relations between assignments will be associated, its extension $||K||^+$ and its anti-extension $||K||^-$. In the definition below we give the semantics of Double Negation DRT. The idea is that all conditions, except those of the form $K_1 \vee K_2$, have a semantics that does not differ from the one given in the previous set-up and that the semantics of $K_1 \vee K_2$ is no different from that of $\sim K_1 \Rightarrow K_2$. The extension of a non-negated DRS K is as before, but its anti-extension is defined to be equal to the extension of $[|\neg K|]$ in the previous set-up. Negation is now indeed a flip-flop operator and switches between extensions and anti-extensions.

Definition (Double Negation DRT semantics)

$$||-K||^+$$
 = $||K||^-$ = $||K||^+$

As before, two conditions are said to be equivalent iff their extensions coincide. DRSs K_1 and K_2 are equivalent iff $||K_1||^+ = ||K_2||^+$ and $||K_1||^- = ||K_2||^-$. It is immediate that $\sim K$ is equivalent with K, whence the name 'Double Negation DRT'.

In the definition of accessibility a little care must be taken for the following reason. Clearly, in $[x|man\,x]$; $[y|umbrella\,y,xowns\,y]$ the first occurrence of x should be accessible to the condition x owns y. (Note that DRS is equivalent to $[x\,y|man\,x,umbrella\,y,xowns\,y]$.) But in $\sim [x|man\,x]$; $[y|umbrella\,y,xowns\,y]$ this should not be the case, while in $\sim \sim [x|man\,x]$; $[y|umbrella\,y,xowns\,y]$ the accessibility should be restored again. To get this right we not only define the set of active discourse referents of a given DRS this time, we also define its set of passive discourse referents. The following clauses do the job.

$$ADR([x_1 \dots x_n \mid \varphi_1, \dots, \varphi_m]) = \{x_1, \dots, x_n\}$$

$$PDR([x_1 \dots x_n \mid \varphi_1, \dots, \varphi_m]) = \emptyset$$

$$ADR(K_1; K_2) = ADR(K_1) \cup ADR(K_2)$$

$$PDR(K_1; K_2) = \emptyset$$

$$ADR(\sim K) = PDR(K)$$

$$PDR(\sim K) = ADR(K)$$

Accessibility in K can now be defined in a straightforward way by setting $ACC(K) = \emptyset$ and computing the accessible discourse referents of subDRSs and subconditions with the help of the following rules.

- (i) If $ACC(K_1 \vee K_2) = X$ then $ACC(K_1) = X$ and $ACC(K_2) = X \cup PDR(K_1)$
- (ii) If $ACC(K_1 \Rightarrow K_2) = X$ then $ACC(K_1) X$ and $ACC(K_2) X \cup ADR(K_1)$
- (iii) If $ACC([x_1 ... x_n \mid \varphi_1, ..., \varphi_m]) X$ then $ACC(\varphi_i) = X \cup \{x_1, ..., x_n\}$ $(1 \le i \le m)$
- (iv) If $ACC(K_1; K_2) X$ then $ACC(K_1) X$ and $ACC(K_2) X \cup ADR(K_1)$
- (v) If $ACC(\sim K) X$ then ACC(K) = X

Again, an occurrence of x in an atomic condition φ in K is said to be *free in K* iff $x \notin ACC(\varphi)$. An occurrence of x in a condition ψ is *free in \psi* iff it is free in $[| \psi]$. A DRS K is *proper* iff no occurrence of a discourse referent in K is free in K. A proper DRS is *true* if \emptyset is an element of the domain of its extension, *false* if \emptyset is an element of the domain of its anti-extension. The following lemma is of practical importance.

MERGING LEMMA

$$[x_1 \ldots x_n \mid \varphi_1, \ldots, \varphi_m]$$
; $[y_1 \ldots y_k \mid \psi_1, \ldots, \psi_j]$ is equivalent with

 $[x_1 \ldots x_n y_1 \ldots y_k \mid \varphi_1, \ldots, \varphi_m, \psi_1, \ldots, \psi_j],$ provided no referent in y_1, \ldots, y_k is free in any of $\varphi_1, \ldots, \varphi_m$.

4 APPLICATIONS

Since we want to show in this section how our new version of DRT deals with the kind of sentences that we have encountered in the first section, we must make clear how its construction algorithm works. Fortunately we can borrow many rules from the standard approach. The basic set-up is as follows (compare the following rule for the global structure of DRS construction with that of Kamp & Reyle 1993: 86).

CONSTRUCTION ALGORITHM	
Input:	a discourse $S_1,, S_n$ the empty DRS $K_0 = $
For $i = 1$ to n do:	
(i)	Let $K_i^* = K_{i-1}$; S_i . 20 Go to (ii).
(ii)	Keep on applying construction rules to each reducible condition of
	K_i^* until a DRS K_i is obtained that only contains irreducible
	conditions.

Applying one step of this algorithm to (5), reprinted as (26) below, gives (27) as an output.

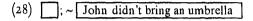
(26) It is not true that John didn't bring an umbrella. It was purple and it stood in the hallway.

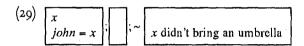
In (27) we encounter a negation and a proper name. For these we have construction rules that are slightly different from their standard variants. They are formulated as follows.

NEGATION RULE. Upon encountering any form of linguistic negation, prefix the DRS that the condition containing the negation belongs to with ~ and remove the linguistic negation.

PROPER NAME RULE. Upon encountering a proper name α , replace α with a new discourse referent x and prefix the entire DRS under construction with $[x \mid \alpha = x]$.

This exhausts our changes to the construction algorithm. An application of the negation rule to (27) gives (28) and a subsequent application of the proper name rule (29). In the latter we may (if we wish) merge $[x \mid john = x]$ and the empty DRS [] to $[x \mid john = x]$, according to the merging lemma of the previous section. This gives (30) and with a second application of the negation rule we obtain (31).





(30)
$$\begin{bmatrix} x \\ john = x \end{bmatrix}$$
; ~ $\begin{bmatrix} x \text{ didn't bring an umbrella} \end{bmatrix}$

(31)
$$\begin{bmatrix} x \\ john = x \end{bmatrix}$$
, ~~ $\begin{bmatrix} x \\ x \text{ brought an umbrella} \end{bmatrix}$

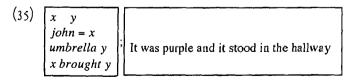
At this crucial point we may cancel the double negation, with (32) as a result, and an application of the standard rule for indefinites brings us to (33). Now the Merging Lemma can be applied, so that we get (34)

(32)
$$\begin{bmatrix} x \\ john = x \end{bmatrix}$$
; x brought an umbrella

(33)
$$\begin{bmatrix} x \\ john = x \end{bmatrix}$$
; umbrella y x brought y

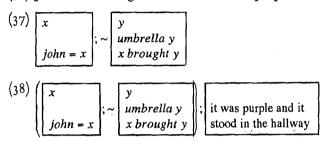
$$\begin{array}{c}
x \quad y \\
john = x \\
umbrella \ y \\
x \ brought \ y
\end{array}$$

Since there are no more reducible conditions now, the construction algorithm prescribes attaching a new DRS with the second sentence of our discourse as its only condition. The result is given in (35). Clearly, since y is accessible from this new condition, both occurrences of it can be resolved as y.



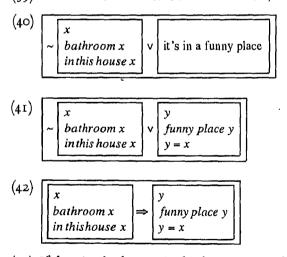
This shows that our version of DRT treats double negations as holes for anaphora.²¹ That it treats single negations as plugs can be illustrated from the treatment of (36).²² Since the only difference between the first sentence of (26) and that of (36) is that the latter lacks a negation, it is obvious that the construction algorithm outputs (37) instead of (33) for this sentence. This DRS can no further be reduced and if the second sentence of (36) is added, as in (38), we find that the two occurrences of it cannot be resolved as y since the latter referent is not accessible.

(36) John didn't bring an umbrella. *It was purple and it stood in the hallway.



This brings us to the treatment of 'bathroom' sentences. Supposing that the construction algorithm assigns (40) to (12) (here reprinted as (39)), we see that these sentences no longer form a problem. Since x is an active discourse referent of $[x \mid bathroom x$, in this house x], it is a passive discourse referent of its negation. This means that it will be accessible from the second disjunct, so that we can resolve it as x. The result is shown in (41). Note that this last DRS is equivalent to (19) (reprinted as (42)), so that (39) is predicted to be equivalent with (43).

(39) Either there's no bathroom in this house, or it's in a funny place



(43) If there's a bathroom in this house it's in a funny place

Our semantics for disjunction does not treat both disjuncts on a par. It predicts that anaphoric links with an antecedent in the second disjunct and a dependent element in the first disjunct are out, but that links with an antecedent in the first disjunct and an anaphoric pronoun in the second are acceptable in certain circumstances. In particular, sentences like (44) are predicted to be unacceptable if it is to be dependent on a bathroom only and not on previous or accommodated context.

(44) Either it's on the first floor or there is no bathroom in this house

This is correct in our opinion, as (44) is very strange if no previous mention of a bathroom has occurred. Note, however, that a symmetric treatment of both disjuncts can be obtained using the following alternative semantics for disjunction.²³

$$\begin{split} \|K_1 \vee K_2\| &= \{f \mid \forall g (\langle f, g \rangle \in \|K_1\|^- \to \exists h \langle g, h \rangle \in \|K_2\|^+) \ \& \\ \forall g (\langle f, g \rangle \in \|K_2\|^- \to \exists h \langle g, h \rangle \in \|K_1\|^+)\} \end{split}$$

A final word on representations. In this paper we have used a representation language that extends the familiar DRT language and for some discourses the DRS that we obtain after applying the construction algorithm will not be equivalent to a DRS of the old language. Thus while the DRS for the first sentence of (26) turned out to be part of the old language, the DRS in (37) could not be so reduced. Theoretically there is no problem here, but since discourses with an alternation of negated and non-negated sentences tend to get rather long DRSs and also for the sake of comparison with the standard DRT set-up, we may nevertheless want to use the old forms. To this end we may reintroduce the 'old' DRT negation into the new language, simply by defining $\neg K$ to be an abbreviation of $K \Rightarrow \sim [\ |\],^{24}$ and by noting that this leads to the following semantics.

$$\|\neg K\| = \{f \mid \neg \exists g \langle f, g \rangle \in \|K\|^+\}$$

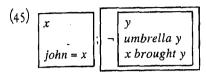
We now have the following useful lemma which has a simple proof.

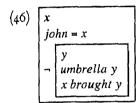
SINGLE NEGATION LEMMA

$$\begin{array}{ll} K\Rightarrow \sim [x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m] & \text{is equivalent with } K\Rightarrow [\,\,|\,\neg[x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m]] \\ \sim [x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m] \Rightarrow K & \text{is equivalent with } [\,\,|\,\neg[x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m]] \Rightarrow K \\ K;\sim [x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m] & \text{is equivalent with } K;[\,\,|\,\neg[x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m]] \Rightarrow K \\ \sim [x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m];K & \text{is equivalent with } [\,\,|\,\neg[x_1\ldots x_n\,|\,\varphi_1,\ldots,\varphi_m]];K \\ \end{array}$$

Since we can cancel double negations, and since we can trade disjunctions for implications via the equivalence between $K_1 \vee K_2$ and $\sim K_1 \Rightarrow K_2$, and in virtue of the properties of the construction algorithm, we can now reduce our new DRSs to the old ones. The procedure is illustrated for (37) below. To this DRS

the Single Negation Lemma applies, and we get (45). A last application of the Merging Lemma results in (46), the form that we are used to associating with the first sentence of (36).





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NOTES

- 1 Examples (1)-(4) are taken from Karttunen's original paper (Karttunen 1976).
- 2 E.g. Chierchia (1992), Groenendijk & Stokhof (1991), Kamp & Reyle (1993).
- 3 We assume that negative verbs such as fail and forget are analysed with the help of negations (see e.g. Karttunen & Peters 1979 for such an analysis).
- 4 Double negations in standard English are one of our two main concerns in this paper. Negative vrbs allow for easy
- construction of natural examples of double negations, but some also introduce problems that are orthogonal to our present interests (forget and deny, for example, are verbs of propositional attitude as well, of course). Therefore, in the rest of this paper we shall stick to straightforward examples of double negations, as the one in (5). See also note 21.
- 5 Roberts (1989) attributes this sentence to

- 6 For a discussion of the issue of accessibility in disjunctions see section 2.3.1 (pp. 185-190) of Kamp & Reyle (1993).
- 7 Roberts (1989) gives the following example (attributed to Berman), however:

Either there's a bathroom on the first floor, or it's on the second floor

We believe that in this example the indefinite noun phrase a bathroom gets a wide scope specific reading. Intuitively the speaker is committed to the existence of a bathroom. Note that the indefinite allows for subsequent anaphoric reference: we can continue with I keep forgetting exactly where it is, but it's easy to find, another sign that the indefinite has wide scope here.

- 8 In fact, in a sense that will be made precise in section 2, (17) and (13) are equivalent in meaning and the new rule that in constructing the DRS for a disjunction we should add the negation of the DRS for the first disjunct as a condition to the DRS for the second does not give us any output that is semantically different from the output of the original construction rule.
- 9 Loss of explanatory power would also be a consequence of a rule to this effect. Although the entailment John didn't fail to find an answer/John found an answer would come out valid under such a rule, this would only be a consequence of what we feel would be sleight of hand: the representation of the premise would be turned into the representation of the conclusion by mere stipulation.
- 10 An anonymous referee informs us that he would call (12) neither true nor false in these circumstances. This means that although his intuitions do not square with Kamp & Reyle's solution of rendering (12) as (16), they neither square with Roberts' (1989) proposal (discussed below) to treat (12) as (18), or with our

- rendering of (12) as the equivalent (19). Perhaps a uniqueness presupposition is involved here.
- 11 Roberts uses a modal box instead of an implication, to be quite precise, but this is immaterial for our present purposes.
- 12 In section 4 below we shall give a slightly different analysis of disjunctions. We shall not change the DRS construction rule for disjunctions, but the semantics for the symbol V will be altered in such a way that A or B will be semantically equivalent to if not A then B. In an earlier version of this paper our analysis of 'bathroom' sentences was based on Kamp & Reyle's analysis plus our solution to the double negation problem. We wish to thank Paul Dekker for insisting that the equivalence between A or B and if not A then B should be retained.
- 13 Various people, including Werner Saurer and one anonymous referee, have suggested that the relation between (23) and (19), (17) and (16), and (6) and (7) should be one of inference. In their proposals the DRT construction algorithm is enriched with an inferencing mechanism, so that drawing conclusions is an admissible processing rule. (7), for example, may be constructed from (6) in such approaches, since it will follow from (6) in some suitable DRS inferencing system (the one in Saurer 1993, for example). The principle challenge for theories along these lines, a challenge also noted by Saurer, is that of restricting overgeneration. While it is not difficult to let (19) follow from (23) and (7) from (6), it is difficult to do so and not have many unwanted inferences in the bargain. Consider Partee's 'marble' examples:
 - (a) I dropped ten marbles and found only nine of them. *It's probably under the sofa
 - (b) I dropped ten marbles and found all of them except one. It's probably under the sofa.

On a natural account of inference the first sentence of (b) will follow from the first sentence of (a). So a theory which allows inference as an acceptable processing rule will not be able to explain the difference in acceptability between (a) and (b).

- 14 This is the same as the conjunction in Groenendijk & Stokhof (1991).
- 15 The following analogy seems relevant. Karttunen (1974) gives a set of rules which allow us to compute when a context (set of sentences) C_0 satisfies the presuppositions of a sentence S_0 . The idea is to associate a local context C with each of the subclauses of S_0 . Local contexts can be computed in a top-down way by setting $LC(S_0) C_0$ and proceeding to compute the local contexts of proper subclauses by means of the following rules.
 - (i) If LC(not S) C then LC(S) C
 - (ii) If LC(S or S') = C then LC(S) = C and LC(S') = $C \cup \{\text{not } S\}$
 - (iii) If LC(if S then S') C then LC(S) C and LC(S') $C \cup \{S\}$
 - C and LC(S') = $C \cup \{S\}$ (iv) If LC(S and S') = C then LC(S) = Cand LC(S') = $C \cup \{S\}$

 C_0 now satisfies the presuppositions of S_0 just in case the local context of each subclause of S_0 entails all presuppositions that are triggered at the level of that subclause. This results in Karttunen's well-known theory of presupposition projection, for which the reader may consult Karttunen (1974), Karttunen & Peters (1979), and the vast subsequent literature. The point that is relevant here is that the accessibility calculus for DRT and the calculus for presupposition projection in Karttunen's theory share important formal characteristics. Also compare the accessibility calculus for 'Double Negation DRT' which will be given in section 3 below. At several points in the paper we have used the terms plug and hole from Karttunen (1973) as an informal way to refer to operators which do and operators which do not block the possibility of anaphoric reference. The present observation suggests that our plugs for accessibility and Karttunen's plugs for presupposition projection may actually have more in common than just their names. Analogies between anaphora and presuppositions have been noted by various authors (see especially van der Sandt 1992).

- 16 This is the standard semantics of sequencing—see Pratt (1976).
- 17 See definition 26 in Groenendijk & Stokhof's (1991).
- 18 The definition that is given here is very close to Groenendijk & Stokhof's (1991) definition 26, but we follow the standard DRT set-up by
 - (i) using *finite* (and hence partial) assignments instead of total assignments,
 - (ii) disallowing what are called reassignments.

The use of finite assignments has been argued for extensively by Fernando (1992). As for (ii), note that our definition of $f[\vec{x}]g$ has the consequence that for example in $[x \mid donkey \mid x] \Rightarrow [x \mid grey \mid x]$ the occurrence of x in the universe of the second box has no effect at all and that the condition is equivalent to $[x \mid donkey \mid x] \Rightarrow [y \mid grey \mid x]$. In the Groenendijk & Stokhof set-up it will be equivalent to $[x \mid donkey \mid x] \Rightarrow [y \mid grey \mid y]$. Of course, conditions like $[x \mid donkey \mid x] \Rightarrow [x \mid grey \mid x]$ will not be generated by the standard DRT construction algorithm.

19 Note that the following alternative definition for the value of a disjunction would embody a version of the Kamp & Reyle theory of 'bathroom' sentences.

$$||K_1 \lor K_2|| = \{f \mid \exists g \langle f, g \rangle \in ||K_1||^+ \\ \lor \exists g h (\langle f, g \rangle \in ||K_1||^- \& \langle g, h \rangle \\ \in ||K_2||^+)\}$$

Note also that we are not committed to an asymmetric treatment of disjunction. See the discussion at the end of this paper.

- 20 A more precise account would have the syntactic analysis of S_i as the contents of the new box. Compare Kamp & Reyle (1993).
- 21 This means that, given our acceptance of

the Karttunen & Peters' analysis of forget as not remember, we have reduced an explanation of the acceptability of (a) to an explanation of the acceptability of (b).

- (a) John didn't forget to bring an umbrella. It's in the hallway.
- (b) John remembered to bring an umbrella. It's in the hallway.

As one anonymous referee points out, there is still a problem with (b), as it needs to be explained how a discourse entity that is introduced within the complement of an attitude verb can become accessible for subsequent anaphoric reference. However, this is a general problem for DRT, for standard DRT as well as for our version. While we do see ways for dealing with the question, we feel that the matter falls outside the scope of this paper. See also note 4.

- 22 In this respect our negation is different from the dynamic negations considered in Groenendijk & Stokhof (1990), van den Berg (1993), and Dekker (1993). While these negations correctly predict that a double negation does not block anaphora, they also wrongly predict that a single negation does not.
- 23 Note the analogy between the present discussion and the old discussion whether disjunction should be symmetric with

- respect to presupposition projection (see e.g. Karttunen 1973).
- 24 The negated box ~[[] denotes the empty relation and may thus stand proxy for the falsum \perp in our theory. $\neg K$ is thus defined as $K \Rightarrow \bot$, as it is in some versions of the standard set-up of DRT. Note that in the set-up of section 2 an implication $K_1 \Rightarrow K_2$ may also be defined as $\neg (K_1; [] \neg$ K2]) (see Groenendijk & Stokhof 1991), so that negation is not only definable from implication (and falsum), but implication is also definable from negation (and sequencing). This interdefinability is lost in our set-up, as $K_1 \Rightarrow K_2$ is not equivalent to $\sim (K_1; \sim K_2)$. For good reasons, as discourse referents that were created in the antecedent of an implication will not become accessible if that implication is negated, while discourse referents created under a negation will become accessible if delivered by a second negation.
 - (a) It is not true that John is happy if he owns a donkey. *It is grey.
 - (b) It is not true that John doesn't own a donkey. It is grey.

If the implication in the first sentence of (a) were of the form $\sim (K_1; \sim K_2)$, however, the first sentence itself would be equivalent to K_1 ; $\sim K_2$ and the discourse referent for a donkey would be accessible.

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