Partee conjunctions*

Projection and possibility

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Conjunctions of possibility statements sometimes pattern with disjunctive sentences with respect to presupposition projection and anaphoric accessibility — I dub such cases *Partee conjunctions.* Partee conjunctions pose a serious problem for any compositional account of anaphora/presupposition projection, given conventional assumptions about conjunction and possibility modals. I use this empirical puzzle to motivate a Bilateral Update Semantics (BUS), against the background of which I develop a new take on the dynamics of possibility statements. One of the core ideas is that *projection* in possibility statements is weaker than is often assumed — concretely, I assume that possibility modals are Strong Kleene existential quantifiers over information states. Furthermore, departing from existing integrations of modality and anaphora, I argue that possibility statements can potentially introduce discourse referents via a privileged tautology. The result is a system where possibility statements can introduce discourse referents that are merely *possible* rather than familiar in the Heimian sense.

1 Introduction

Barbara Partee famously observed that anaphora is possible in disjunctive sentences like (1) — henceforth: *Partee disjunctions.* Partee disjunctions are known to be problematic for first-generation dynamic theories of interpretation (Heim 1982, 1983a, Groenendijk & Stokhof 1991).

(1) Either there is no^x bathroom in this house, or it_x's in a funny place.

*I'm grateful to participants in my Spring 2022 MIT graduate seminar *Discourse referents inside and out*, where an earlier iteration of this material was first developed in detail. I'd also like to thank Amir Anvari, Enrico Flor, Martin Hackl, Filipe Hisao Kobayashi, Jacopo Romoli, and Yasu Sudo for their comments and feedback.
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Although one pay-off of the analysis developed later in this paper will be an account of Partee disjunctions, I'll primarily be concerned with a related phenomenon involving conjunctions of possibility statements, which I dub *Partee conjunctions*. Partee conjunctions are conjunctive possibility statements of the form $\Diamond\phi \wedge \Diamond\psi$. The key empirical observation is that Partee conjunctions can behave like disjunctions (2) with respect to anaphoric accessibility.[1]

(2) Maybe there's no^{x} bathroom in this house, and maybe it$_{x}$'s in a funny place.

An immediate problem for any account of (2) is that possibility statements typically can't be used to introduce familiar Discourse Referents (DRS) (in the sense of Heim 1982), as illustrated by the impossibility of anaphora in the discourse (3). This in itself is wholly unsurprising from the perspective of Heimian theories of discourse anaphora, which derive as a necessary condition for anaphora that a witness to the indefinite be (locally) contextually entailed. In (3), it seems that clear reason why anaphora isn't possible is that the first sentence in the discourse doesn't entail the existence of a bathroom.

(3) Maybe there's a^{x} bathroom in this house. #It$_{x}$'s in a funny place.

Looking at (2) in light of (3), it seems that what's crucial for anaphora is that the second conjunct is itself a possibility statement. This is a bit surprising in light of how dynamic theories of anaphora typically work — either the first conjunct introduces a familiar DR, or it doesn't.

In a sense, (2) is doubly disturbing, since the prejacent of the modal adverbial is a *negative* existential statement. There's independent reason to believe that negative statements don't introduce DRS, in order to account for data such as (31) (Groenendijk & Stokhof 1991).[2]

(4) There's no^{x} bathroom in this house. #It$_{x}$'s in a funny place.

It's important to mention at the outset that patterns of anaphoric accessibility exhibited in (1–2) closely parallel facts about presupposition projection, as illustrated by the minimal variants involving definite descriptions in (5–6). This is somewhat unsurprising given well known parallels between anaphoric accessibility and presupposition projection (van der Sandt 1992, Heim 1983b, Beaver 2001, Rothschild 2017). Ultimately, the analysis we develop for this data should be one in which the same mechanisms underlie both phenomena. A presuppositional Partee disjunction is provided in (5), and a presuppositional Partee conjunction in (6) — the phenomena are completely parallel.

(5) Either Brandon wasn't at dinner, or Lisa was there too.

[1] I haven't been able to find any discussion of exactly these kinds of cases in the existing literature.
[2] Sentences involving *double* negation make the generalization a bit more complicated than this — I'll discuss such cases later on.

(6) Maybe Brandon wasn't at dinner, and maybe Lisa was there too.

The presuppositional variant in (6) can be used to further explicate exactly why Partee conjunctions are so puzzling, given standard assumptions about presupposition projection. Firstly, it's typically assumed that possibility modals are *holes* (to use Karttunen's 1973 terminology). This is because in order to accept an assertion of a sentence such as (7), in which the prejacent of the possibility modal semantically presupposes that *someone stole the tarts*, we must typically accommodate that *someone stole the tarts*, unless already contextually entailed. The assumption here is that global accommodation is an accurate reflection of what is semantically presupposed.

(7) Maybe/It is possible that its the knave that stole the tarts.
$\Rightarrow$ *someone stole the tarts* (Beaver, Geurts & Denlinger 2021)

Secondly, it has been widely observed that the presupposition of a latter conjunct projects, unless it is contextually entailed by the initial conjunct (Karttunen 1976). If we put this together with what is assumed about projection out of the scope of possibility modals, it's quite obvious that we predict projection in cases such as (6) — if "maybe" is a hole, then the latter conjunct should presuppose that there is a bathroom, which is certainly *not* contextually entailed by the initial disjunct, a possibility statement.

Something that's notable about Partee conjunctions is that the modalized conjuncts are interpreted as *listing possibilities*. This parallels the role that disjunctive statements often play in discourse: an assertion of the sentence "Maybe there's no bathroom in this house, and maybe it's in a funny place" typically conveys that the speaker considers it possible that *there is no bathroom in this house*, and also considers it possible that *there is a bathroom in this house, and it's in a funny place*, and furthermore that the speaker isn't sure which is true. Strikingly, this completely parallels the inferences typically drawn from the disjunctive variant "Either there's no bathroom in this house, or it's in a funny place" — this modal/disjunction connection has been discussed at some length; see for example Zimmermann 2000, Geurts 2005, Simons 2005. I'll come back to this, but at this stage it suffices to note that it is likely not to be a coincidence that when conjunctions of possibility statements are interpreted as listing potentially mutually exclusive possibilities, anaphoric accessibility/presupposition projection patterns with disjunction.

At this stage, it's important to note that *listing possibilities* is not the only interpetation that a sentence of the form $\Diamond\phi \wedge \Diamond\psi$ can have. Consider the example in (8).

(8) Maybe there's a^x bathroom in this house, and maybe it$_x$'s in a funny place.

Intuitively, the second conjunct in (8) *elaborates* on the possibility introduced by the first conjunct. This phenomenon has been widely discussed in the literature under the rubric of *modal subordination* (Roberts 1989), and the availability of anaphora in (8) seems related to this reading. It is therefore not right to say that sentences of the form $\Diamond\phi \wedge \Diamond\psi$ *always* pattern with disjunction with respect to anaphoric accessibility/presupposition projection, but rather they pattern *variably* with disjunction/conjunction depending on how the sentence is interpreted; (8) patterns with conjunction in the sense that the second conjunct seems to pick up a DR introduced by the first conjunct. A desideratum of our analysis will be to account for this variable behaviour. I'll discuss the relationship with modal subordination at more length in the next section.

One of the main points of this paper will be to argue that, in order to understand the anaphoric potential of modalized sentences, it will be essential to recognize to notion of a *possible DR*, i.e., a DR which is only *partially* contextually familiar, in a way which I'll make precise by generalizing machinery developed in Heim 1982, 1983b,a. I'll also suggest that a mechanism of *modal subordination* (Roberts 1989) will not adequately account for Partee conjunctions, and that once we make the necessary adjustments to our theory, even examples such as (8) will be taken care of without modal subordination. In section 3, I develop a novel dynamic semantics which accounts for Partee conjunctions, as well as a raft of other cases assumed to be problematic for dynamic semantics. First however, I address the question of how exactly presuppositions project in modalized statements, as this will turn out to be crucial in accounting for Partee conjunctions.

2 Projection and possibility

As discussed in section 1, Partee conjunctions seem on the face of it problematic for a compositional account of presupposition projection, since it is standardly assumed that a sentence of the form $\Diamond\phi$ inherits the presuppositions of ϕ. This textbook view is not above suspicion however — it's generally known that sometimes we accommodate something stronger than what is semantically presupposed, and the evidence that has been marshalled for the projection properties of possibility statements comes from accommodation.

To illustrate, consider the pair of examples (9–10) (Fox 2013: p. 24). Upon accepting (9), we accommodate that *if John is a scuba diver, he has a wetsuit* — this weaker inference is exactly what (9) is predicted to presuppose by the standard theories of presupposition projection (e.g., Heim's 1983b *satisfaction theory*, the trivalent theories of Peters 1979, George 2008, 2014, etc.). Upon accepting (10) however we accommodate that *John has a car*, despite the fact that (10) presupposes (at least, according to standard theories) that *If John is a scuba diver, he has a car.* This is a typical example of the so-called *proviso problem* (Geurts 1996); one lesson to take from this is that what is accommodated is not always a reliable indicator of what is (semantically) presupposed.

(9) If John is a scuba diver, he will bring his wetsuit.

(10) If John is a scuba diver, he will bring his car.

Moreover, when (10) is placed in a discourse in which the weaker, conditional presupposition is contextually entailed, we observe satisfaction. This is illustrated by the discourse in (11).

(11) If John is a diver, he has a car, and if he is a *scuba* diver, he will bring his car.

What can be taken away from this discussion is that what is accommodated sometimes goes beyond what is semantically presupposed.[3] This raises the possibility of a reinterptretation of the evidence pertaining to projection in possibility statements. Let's entertain the possibility that $\Diamond\phi_\pi$ presupposes that $\Diamond\pi$[4] If this were the case, it would readily explain (12) (the presuppositional variant of (8)).

(12) Maybe there's a bathroom in this house, and maybe the bathroom is in a funny place.

Putting aside modal subordination for the time being, this hypothesis is worth taking seriously, since treating possibility modals as presuppositional *filters* is more generally consistent with established generalizations concerning presupposition projection in complex sentences (Karttunen 1976). For example, in the conditional sentence (13) the presupposition of the consequent is filtered. This is consistent with the idea that the consequent presupposes that *it's possible that there is a bathroom*, which is contextually entailed by the conditional antecedent. In (14) the presupposition of the second disjunct is also filtered — here, the putative weak presupposition is contextually entailed by the negation of the first disjunct (assuming that *it's not certain there's no bathroom* is equivalent to *it's possible there's a bathroom*). This latter case is especially telling, since at least a naïve modal subordination approach would supply the information that *there is no bathroom* to the possibility modal.

(13) If it's possible there's a bathroom in this house,
then it's possible that the bathroom is in a funny place.

(14) Either it's certain there's no bathroom, or it's possible the bathroom is upstairs.

Singh (2008) argues for the same conclusion — weak projection with possibility modals — on the basis of parallel data. Singh: p. 75 also discusses (15) — a case in which only the putative weak presupposition is globally contextually entailed, and nothing stronger need be accommodated. This is especially compelling evidence that possibility modals are presupposition *filters*, since it's not possible to account for (15) via a mechanism of modal subordination.

(15) Context: *You see a man you don't know whistling at the bushes. You say to your friend:*
He might have lost his dog.

[3]See, e.g., Fox 2013, who suggests that the stronger inference should be derived from the weak presupposition via some pragmatic reasoning process (not spelled out). It's worth noting however that a prominent line of work argues that the semantics should generate both weak and strong presuppositions — see, e.g., Mandelkern 2016, Grove 2019a,b.

[4]In ϕ_π, π is understood as the semantic presupposition of ϕ.

The residue of this discussion is that if I'm right that possibility modals are presupposition filters, it is somewhat mysterious why an assertion of $\Diamond\phi_\pi$ leads us to accommodate π rather than $\Diamond\pi$, in a context where $\Diamond\pi$ isn't contextually entailed. In fact, this phenomenon should be treated as a special case of the proviso problem. I will leave it open exactly how to derive the stronger accommodated inference, but see especially Singh 2008: chapter 4–5 for discussion.

Before moving on to the analysis, in the following section I provide an independent argument that a mechanism of subordination account won't generalize to Partee conjunctions, due to independent constraints on complement anaphora.

2.1 Modal subordination and polarity switch

Modal subordination (Roberts 1989) describes configurations involving two (syntactically independent) modalized statements, where the prejacent of the latter modal operator is in some sense subordinate to the prejacent of the initial modal operator, as diagnosed by presupposition projection/anaphora — in (16) the antecedent of the pronoun *it* is provided by the prejacent of the first modal operator *might*.

(16) A wolf might come in. It would eat you first! (Roberts 2020: p. 1)

Moreover, von Fintel & Iatridou (2017) show that modal subordination allows for what they call "polarity switch", where the prejacent of a modal operator is subordinate to the *negation* of the prejacent of a previous modal operator. In (17), the prejacent of *will* is subordinate to the *negation* of the prejacent of the imperative operator, i.e., *you don't park there*. It's certainly tempting to appeal to the independent possibility of polarity switch, in order to subsume Partee conjunctions under the rubric of modal subordination as well.[5]

(17) Don't park there! You will be towed.
$\Rightarrow$ *If you park there you'll be towed*

One reason to be suspicious of this approach is that, as emphasized by Starr (2018), conjunction seems to be incompatible with polarity switch, and with complement anaphora more generally (Nouwen 2003). As initially observed by von Fintel & Iatridou, (18) can't mean the same thing as (17), but rather that in all worlds in which you *don't* park there, you'll be towed. (19) and (20) provide the relevant contrast for complement anaphora — in (19), *they* can pick up the congressmen who *don't* admire Kennedy, but this interpretation is no longer readily available with conjunction, as in (20).[6]

[5] I'm grateful to Sabine Iatridou (p.c.) for making this connection.
[6] To my knowledge, a satisfactory explanation as to why complement anaphora is limited in precisely this way hasn't been proferred. I leave this interesting question to future research.

(18) Don't park there and you'll be towed.
$\not\Rightarrow$ *If you park there you'll be towed*

(19) Few congressmen admire Kennedy. They think he's incompetent.
$\Rightarrow$ *the congressmen who don't admire Kennedy think he's incompetent*

(20) # Few congressmen admire Kennedy and they think he's incompetent.
$\not\Rightarrow$ *the congressmen who don't admire Kennedy think he's incompetent*

Partee conjunctions such as (14) may involve either discourse sequencing or sentential conjunction, and therefore display none of the hallmark brittleness of polarity switching modal subordination. Perhaps it's possible to resolve this complex paradigm and provide a theory of modal subordination which accounts for (13) and (14), but in the absence of such a theory I aim to develop a relatively simple dynamic account of epistemic modals which subsumes both cases.

Returning to the primary point of this section — weak projection from possibility statements — this will be an important component in our analysis of Partee conjunctions, and based on the motivations outlined in this section, I'll proceed to tailor the analysis of possibility modals in order to derive weak projection. Obviously, this will directly account for cases such as (12), but in order to account for Partee conjunctions in their full generality, we will need to further investigate the dynamics of possibility statements. In the following section, I begin to sketch a dynamic semantics for possibility modals which incorporates resources for dealing with anaphora.

3 Bilateral update semantics

One desideratum of the analysis developed here is that it should account for patterns of anaphoric accessibility in Partee conjunctions and beyond. Since Partee conjunctions arguably involve *discourse anaphora*, the semantics I'll develop here will be, out of necessity, dynamic. Concretely, I'll sketch an update semantics, according to which meanings manipulate information states (Heim 1982, 1983a, Veltman 1996, Groenendijk, Stokhof & Veltman 1996) but with a twist — following, e.g., Willer 2018 I'll argue that a *bilateral* update semantics is necessary, i.e., one in which we distinguish between the positive and negative updates associated with sentences.[7] I'll begin by outlining my background assumptions, taken from the existing dynamic literature.

3.1 Background

Following Heim (1982, 1983a), I model information states as files, i.e., sets of world-assignment pairs. I refer to each element in the file as a *possibility.* In order to straightforwardly model Heimian

[7] There is a small but significant existing dynamic literature exploiting the bilateral approach — see especially Krahmer & Muskens 1995, van den Berg 1996: chapter 2, and Elliott 2020.

familiarity for definites as a semantic presupposition, I adopt a partial presentation of Heim, with 'partial' assignments. Concretely, I make use of a privileged value $*$ (meant to be interpreted as the "unknown" individual), to emulate partiality. This means that when an assignment g maps a variable v to $*$, this is tantamount to g being 'undefined' for v. The initial assignment $g_\top$ will be frequently used to model a discourse in which no DRS have yet been introduced — this is simple the assignment that maps every variable to $*$.

Definition 3.1 (Assignments)**.** Given a stock of variables V, and a domain of individuals D, an assignment g is a total function from V to $D \cup \{ * \}$.

- If $g(v) \neq *$, we say that g is *defined* for v.
- The initial assignment $g_\top$ is not defined for any v.

Heimian states are rich informational objects, encoding as they do both worldly and anaphoric information. Given a Heimian state s, We can retrieve just the worldly information s_w (i.e., a classical information state) by taking the set of all the worlds which are part of some possibility in s, and likewise we can retrieve just the anaphoric information s_g by taking the set of all the assignments which are part of some possibility in s. The formal definitions are provided in definition 3.2, alongside another notion which will prove useful in the following discussion: an *initial state* is one which encodes no anaphoric information.

Definition 3.2 (Information states)**.** Given a set of individuals D, a stock of variables V, and the set of assignments $G : \{ g \mid g : V \to D \cup \{ * \} \}$, an *information state* $s \subseteq W \times G$.

- The *worldly information* of a state $s_w := \{ w \mid \exists g[(w, g) \in s] \}$.
- The *anaphoric information* of a state $s_g := \{ g \mid \exists w[(w, g) \in s] \}$.
- A state s is initial iff $s_g = \{ g_\top \}$.

3.2 Atomic sentences and negation

Here, I'll begin to develop BUS for a simple first order fragment. In classical update semantics, sentences express functions from states to states. The effect of applying the update conveyed by a sentence ϕ to a state s is often written as $s[\phi]$ — to be understood as: s updated with ϕ. Here, the bilateral twist is that sentences are associated with both a *positive* update (via $.[.]^+$) and a *negative* update (via $.[.]^-$). The initial intuition for an atomic sentence ϕ is that $s[\phi]^+$ returns all of the possibilities in s at which ϕ is defined and true, whereas $s[\phi]^-$ returns all of the possibilities in s at which ϕ is defined and false. I'll make the natural assumption (given partiality of assignments) that an atomic sentence $P(x_1, \ldots, x_n)$ is defined at a possibility (w, g) iff g is defined for $x_1, \ldots, x_n$, otherwise, truth/falsity of an atomic sentence relative to a possibility is classical.[8] This gives rise to a straightforward presentation of the BUS semantics for atomic sentences, outlined in definition 3.3.

Definition 3.3 (Atomic sentences in BUS)**.**

$$s[P(x_1, \ldots, x_n)]^+ := \{\, (w, g) \in s \mid P(x_1, \ldots, x_n) \text{ is defined and true at } (w, g) \,\}$$
$$s[P(x_1, \ldots, x_n)]^- := \{\, (w, g) \in s \mid P(x_1, \ldots, x_n) \text{ is defined and false at } (w, g) \,\}$$

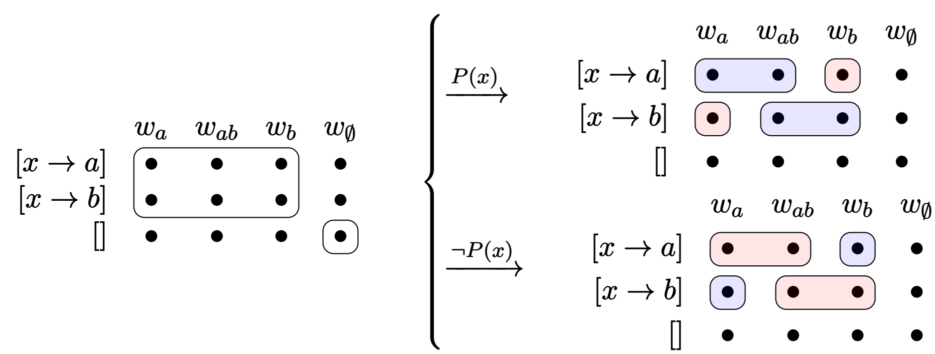
Adopting a bilateral semantics makes it almost trivial to state the meaning of negation as a flip-flop operator. Later, I'll show that the entry for negation in definition 3.4 pays dividends, since it makes Double Negation Elimination (DNE) valid.

Definition 3.4 (Negation in BUS)**.**

$$s[\neg\phi]^+ := s[\phi]^-$$
$$s[\neg\phi]^- := s[\phi]^+$$

With this much in hand, it's possible to visualize the dynamics of simple atomic sentences. I diagram information states as regions of possibility graphs, with worlds on the horizontal axis and assignments on the vertical axis (following Dotlail & Roelofsen 2019, 2021), as in fig. 1; the blue region indicates the output of the positive update, and the red region indicates the output of the negative update. As shown, negation simply flip-flops the two regions.

Figure 1: Dynamics of simple sentences.
Subscripts on worlds exhaustively indicate which individuals are P



[8] For the sake of exposition, I assume that atomic sentences consist only of predicate symbols and variables. It's straightforward to complicate the definitions given here to accommodate individual constants — I leave this as an exercise to the reader.

3.3 Partiality and assertion

The attentive reader will have noticed that in the input state in fig. 1, x is not *familiar* (in the Heimian sense) — that is to say that there are possibilities in the input state at which x is not defined. Heimian familiarity requires that x be defined at *every* possibility in the input state — see definition 3.5.

Definition 3.5 (Familiarity)**.** A variable v is *familiar* at information state s iff $\forall g \in s_g$, $g(v)$ is defined.

Nevertheless, the positive/negative updates associated with an open sentence $P(x)$ are defined. This is a departure from Heimian update semantics, in which the update conveyed by a sentence is practically identified with the effect of asserting a sentence. In Heimian update semantics, free variables are used to model anaphoric pronouns, and if familiarity isn't satisfied, the result is an error state. In order to mirror the results on Heimian update semantics in BUS, it's necessary to posit a *bridge principle*, which will turn out to be a straightforward and natural generalization of what von Fintel calls "Stalnaker's bridge". Concretely, in order for a sentence ϕ to be assertable relative to a body of information s, ϕ must be *contextually bivalent*. See definition 3.6; non-assertability is modeled by returning the *absurd state* $\emptyset$.

Definition 3.6 (Assertion in BUS (first attempt))**.** The effect of asserting a sentence ϕ relative to a body of information s, $s[\phi]$:

$$s[\phi] := s[\phi]^+ \text{ if } s = s[\phi]^+ \cup s[\phi]^- \text{ else } \emptyset$$

For the purposes of atomic sentences, the definition in definition 3.6 amounts to the requirement that $s[\phi]^+$ and $s[\phi]^-$ jointly partition s. It should be obvious that $s[P(x)]$ only results in a non-absurd state if x is familiar at s.

Once we consider existentials, which we turn to in the next section, it will turn out that the bridge principle in definition 3.6 is too stringent and will need to be redefined in terms of a natural relation between states — *subsistence* (Groenendijk, Stokhof & Veltman 1996) — which itself will end up being central to the semantics of possibility modals.

3.4 Existential statements

3.4.1 Introducing anaphoric information

Existential statements have a slightly more complicated definition than in some varieties of dynamic semantics, given the bilateral nature of BUS. The core intuition will be simply that an existential statement $\exists_x \phi$ introduces anaphoric information just in case a *witness* to the existential is entailed (see Elliott 2020, Mandelkern 2022 for versions of the same idea). In order to keep the semantics terse, I'll make use of an auxiliary notion of *random assignment* for DR introduction (Groenendijk & Stokhof 1991), which is defined here as an operation on information states. Random assignment ε_x is a privileged tautology which makes a variable x familiar by indeterministically extending every assignment in the input state.[9]

Definition 3.7 (Random assignment)**.**

$$s[\varepsilon_x] := \{\, (w,h) \mid g[x]h, (w,g) \in s \,\}$$

The positive update associated with an existential statement is given in (21). The positive update does what existential statements ordinary do in dynamic semantics, namely, it introduces anaphoric information.

(21) $s[\exists_x \phi]^+ := s[\varepsilon_x][\phi]^+$

3.4.2 Anaphoric information and subsistence

The negative update associated with an existential statement, on the other hand, is going to be defined in a comparatively unusual way, and will require a little setting up. The idea, informally, will be that a sentence such as "it's not the case that anyone is here" always just returns the points in s where nobody is here, without introducing the people who aren't here as DRS. Before I get to the definition, I'll need to talk about the notion of *subsistence* (Groenendijk, Stokhof & Veltman 1996), which will play a crucial role here and in many other parts of the analysis.

In order to define subsistence, I'll first define a notion of *state extension*, defined derivatively in terms of assignment extension. The base notion — assignment extension — captures the idea that an assignment g' *extends* an assignment g if it encodes more information about the values of variables. This is extended to possibilities by simply applying extension to the assignment part. Finally, extension is extended to states in the natural way — s' extends s if every possibility in s' is an extension of a possibility in s. The formalities are laid out in definition 3.8.

Definition 3.8 (Extension)**.** *Extension* ($\leq$) is overloaded as a relation between assignments, possibilities and states.

- assignment extension: $g \leq g'$ iff g' agrees with g at every value that g is defined for.
- possibility extension: $(w,g) \leq (w',g')$ iff $g \leq g'$ and $w' = w$.
- state extension: $s \leq s'$ iff every possibility in s' is an extension of some possibility in s.

[9] $g[x]h$ holds iff g and h differ only in the value they assign to x, and $h(x) \neq *$.

State extension partially orders states according to both worldly *and* anaphoric information — a state s' extends a state s if s' is at least as informative as s, in terms of both candidate worlds, and the values of variables. For example, considering the states in (22), $s_1 \leq s_2$, $s_1 \leq s_3$, and $s_2 \leq s_3$, but not vice versa.

(22) a. $s_1 := \{ (w_{ab}, [x \rightarrow a]), (w_a, [x \rightarrow a]) \}$
b. $s_2 := \{ (w_a, [x \rightarrow a]) \}$
c. $s_3 := \{ (w_a, [x \rightarrow a, y \rightarrow b]) \}$

Now that I've established extension for Heimian states, I can define subsistence. Subsistence is overloaded as both (a) a relation between possibilities and states, and (b) a relation between states, and I'll ultimately make crucial use of both. The first notion is simple: a possibility i subsists in a state s iff there is a possibility in s that extends i' — following Groenendijk, Stokhof & Veltman, I'll call such a possibility a *descendant* of i in s. A state s subsists in a state s' iff every possibility in s' is an extension of a possibility in s (i.e., s' extends s), and for every possibility in s subsists in s'. This is laid out explicitly in definition 3.9.

Definition 3.9 (Subsistence)**.** Subsistence is overloaded as both (i) a relation between possibilities and states, and (ii) a relation between states.

- $i \prec s$ iff there is a $i' \in s$ such that $i \leq i'$ (n.b. that i' is called a *descendant* of i in s).
- $s \prec s'$ iff $s \leq s'$, and every possibility $i \in s$ is such that there is a possibility $i \prec s'$.

The intuition behind state subsistence is that it provides a partial order on states based *strictly* on anaphoric informativity. Looking back at (22), s_1 doesn't subsist in s_2, since there is a possibility in s_1 that doesn't have a *descendant* (i.e., a possibility that extends it) in s_2. s_2 doesn't subsist in s_1 because extension is a pre-requisite for subsistence. s_2 however *does* subsist in s_3, since every possibility in s_2 has a descent in s_3, and s_3 extends s_2. Crucially, s_3 is *strictly anaphorically at least as informative as* s_2. Typically, if s subsists in s', s' is just like s except it encodes more information about the values of variables.

I'll now use the notion of possibility subsistence to state the negative update associated with an existential statement — the relevant clause is stated in (23). The intuition, to provide a simplified example, is that we first update the input state s with "someone$_x$ is here" giving rise to s', and we also update s with "someone$_x$ isn't here", giving rise to s''. We then just keep the possibilities in s that subsist in s'' but not s'. We should be left only with possibilities from s in which nobody is here.

(23) $s[\exists_x \phi]^- := \{ i \in s \mid i \not\prec s[\varepsilon_x][\phi]^+ \text{ and } i \prec s[\varepsilon_x][\phi]^- \}$

It will be helpful to see a concrete illustration, which I provide in the next section.

3.4.3 Illustrating the dynamics of existential statements

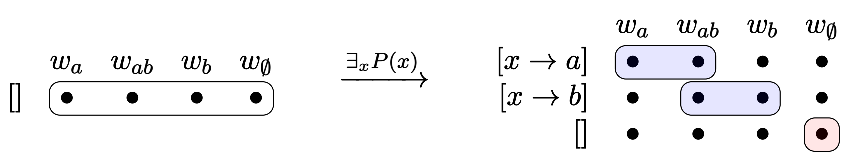
Definition 3.10 (Existential statements in BUS)**.**

$$s[\exists_x \phi]^+ := s[\varepsilon_x][\phi]^+$$
$$s[\exists_x \phi]^- := \{\, i \in s \mid i \not\prec s[\varepsilon_x][\phi]^+ \text{ and } i \prec s[\varepsilon_x][\phi]^- \,\}$$

A concrete illustration of the dynamics of existential statements is provided in fig. 2. I'll informally walk through exactly what is being shown. The information state to be updated is an *initial state* (which *we*l call s), in which it isn't known which individuals (of a, b) are P.

- $s[\exists_x Px]^+$ eliminates worlds in which no individuals are P, and furthermore adds the information that x is a P via random assignment.
- $s[\exists_x Px]^-$ picks out just those points in s which (a) don't subsist in the output of the positive update, and (b) subsist in $s[\exists_x \neg Px]^+$ (not shown here). These are exactly those points at which no individuals are P.

Figure 2: Dynamics of existential statements. Subscripts on worlds exhaustively indicate the individuals that are P.



I noted earlier that I'd eventually have to revise the simplified notion of assertion, stated in definition 3.1, and simple existential statements provide the immediate motivation for doing so. definition 3.1 requires that the positive/negative updates be a contextual partition — here, that is trivially not the case, since the output states contain possibilities that aren't in the input state, thanks to the fact that existential statements introduce anaphoric information in the positive update. Intuitively, assertion should demand that the positive/negative updates be a contextual partition *modulo additional anaphoric information*. In order to make this precise, we need to restate assertion using the notion of state subsistence instead of simple equality. See definition 3.11.

Definition 3.11 (Assertion in BUS (final))**.** The effect of asserting a sentence ϕ relative to a body of information s, $s[\phi]$:

$$s[\phi] := s[\phi]^+ \text{ if } s \prec s[\phi]^+ \cup s[\phi]^- \text{ else } \emptyset$$

In light of definition 3.11, note that the existential statement $\exists_x P(x)$ is assertable in an initial state — an initial state subsists in the union of the positive and negative updates induced by the existential statement. This is important — simple existential statement doesn't place any familiarity requirement on states. For simplicity, I won't encode Heimian novelty into our fragment, but this could be easily incorporated as a presuppositional requirement on existential statements.[10]

Although not the main focus of this paper, a couple of results pertaining mainly to negation are worth mentioning. Firstly, a negative existential statement won't introduce anaphoric information (Groenendijk & Stokhof 1991), due to the fact that the negative update of an existential statement only ever returns a subset of the possibilities in the input state. This captures the impossibility of anaphora in a discourse such as (24), assuming, as is standard, that each sentence is interpreted as a successive update.

(24) John doesn't own a^x car. #It$_x$'s parked outside.

Secondly, due to the flip-flop negation, DNE is obviously valid. An upshot of this is that BUS directly captures anaphora in discourses such as (25), unlike many other dynamic systems (but see Krahmer & Muskens 1995, Gotham 2019, Hofmann 2019 for various ways of setting up a dynamic system that validates DNE).

(25) John doesn't own NO^x car. It$_x$'s parked outside!

Much more needs to be said about the role of negation in dynamic semantics, but see Elliott 2020 for a more in-depth exploration of this topic, using a pared down semantics more akin to Groenendijk & Stokhof's (1991) Dynamic Predicate Logic (DPL).

3.5 Complex sentences and Partee disjunctions

The meanings of the logical connectives in BUS are related to the Strong Kleene logic of indeterminacy in a systematic way, following Elliott 2020. The relationship is easiest to illustrate by going through a concrete case — here, I'll concentrate on disjunction. In three-valued logic, the Strong Kleene semantics for disjunction imposes *weak* verification conditions, and *strong* falsification conditions. In other words, to verify a disjunctive sentence, it is sufficient to verify a single disjunct — the truth of the other disjunct may be unknown — however, to falsify a disjunctive sentence, both disjuncts must be falsified. This is schematized in fig. 3.

[10]See, e.g., van den Berg's 1996: p. 95 definition for *safe* random assignment.

$\phi \vee \psi$	ψ_+	ψ_-	$\psi_?$
ϕ_+	$+$	$+$	$+$
ϕ_-	$+$	$-$	?
$\phi_?$	$+$	?	?

Figure 3: Strong Kleene disjunction

In order to translate fig. 3 into BUS, we must first derivatively define $s[\phi]^?$ — the set of possibilities in s at which ϕ is undefined. This is stated in definition 3.12, using the notion of possibility subsistence. The set of possibilities in s at which ϕ is undefined are those that don't have a descendent in the positive or negative update of s by ϕ.[11]

Definition 3.12.

$$s[\phi]^? = \{\, i \in s \mid i \not\prec s[\phi]^+ \cup s[\phi]^- \,\}$$

Using this auxiliary notion, the truth schema in fig. 3 straightforwardly translates into the following schema for updates in BUS. The crucial bridging step is that each cell in fig. 3 is interpreted as an instruction to successively apply updates.

Definition 3.13 (Disjunction in BUS)**.**

$$\begin{aligned} s[\phi \vee \psi]^+ &:= s[\phi]^+[\psi]^{+,-,?} \cup s[\phi]^{-,?}[\psi]^+ \\ s[\phi \vee \psi]^- &:= s[\phi]^-[\psi]^- \end{aligned}$$

It will be easiest to see the workings of definition 3.13 by considering how it accounts for *Partee disjunctions*. Consider the simple example in (26), which I assume has the logical form in (27).

(26) Either there's no^x bathroom, or it$_x$'s in a funny place.

(27) $\neg\exists_x B(x) \vee F(x)$

Since $F(x)$ is an atomic sentence, $s[F(x)]^{+,-,?}$ partitions s. This means I can perform the following simplification when computing the the first part of positive update of the disjunctive sentence, i.e., verifying the disjunctive sentence by verifying the first disjunct:

(28) $s[\neg\exists_x B(x)]^+[F(v)]^{+,-,?} = S[\neg\exists_x B(x)]^+ = \{\, (w, g) \in s \mid I_w(B) = \emptyset \,\}$

[11] It's worth noting that the notion of assertion can be (equivalently) stated in a different way using the notion in definition 3.12.

(1) $s[\phi] := s[\phi]^+$ if $s[\phi]^? = \emptyset$ else $\emptyset$

Since the first disjunct is bivalent, the second part of the positive update is a simple successive update, i.e., verifying the disjunctive sentence by verifying the second disjunct:

(29) $s[\neg\exists_x B(x)]^-[F(v)]^+ = s[\exists_x B(x) \wedge F(x)]^+$

Now, the positive update of the disjunctive sentence is simply the union of (28) and (29):

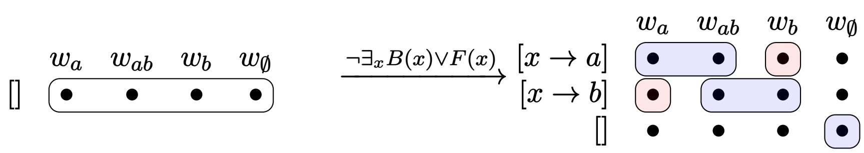
(30) $s[\neg\exists_x B(x) \vee F(x)]^+ = s[\neg\exists_x B(x)]^+ \cup s[\exists_x B(x) \wedge F(x)]^+$

The negative update is even easier to compute, since it is a simple successive update:

(31) $s[\neg\exists_x B(x) \vee F(x)]^- = s[\exists_x B(x)]^+[F(x)]^- = s[\exists_x B(x) \wedge \neg F(x)]^+$

Taking (30) and (31) together, it's easy to see that $\neg\exists_x B(x) \vee U(x)$ is bivalent, since for any information state s, the sentence partitions s into the parts which verify the disjunction, (possibilities where either no bathroom exists, or a bathroom upstairs exists), and the parts which falsify the disjunction (all other possibilities). The effect of updating an initial state with (27) is diagrammed in fig. 4.

Figure 4: Bathrooms exist in all worlds except null subscript; subscripts indicate which of the bathrooms is in a funny place.



There are some interesting things to note about the positive update shown above. Note that if the disjunctive sentence is asserted and accepted in the initial ignorance state indicated here, the resulting state is crucially *not* one in which x is familiar. This is because no anaphoric information is introduced alongside the *no bathrooms* world. This correctly predicts that anaphora should not be possible in (32), since there are some possibilities at which an open sentence such as $P(x)$ would be undefined. Intuitively, this is because at the point at which the second sentence is asserted, there is typically no contextual certainty that a bathroom exists.

(32) Either there is no^x bathroom, or it$_x$'s in a funny place. #It$_x$'s in the basement.

However, the positively updated state in fig. 4 does contain some anaphoric information — one might say that x is *partially familiar*, since there are some assignments in the updated state at which x is defined. Partial familiarity will be an important notion in accounting for Partee conjunctions. Importantly, this predicts that upon asserting a Partee disjunction like (26), if it's subsequently contextually entailed that a bathroom exists, anaphora should be possible. This is borne out — consider the discourse in (33), on the assumption that all renovated houses have bathrooms.[12]

(33) **A:** Either there's no^x bathroom, or it$_x$'s upstairs.
B: This house has been recently renovated, so you'll find it$_x$ on the right.

This kind of nuanced prediction is beyond the remit of classical dynamic theories of anaphora, such as DPL, where disjunction blocks introduction of *any* anaphoric information by brute force. See Elliott 2020 for discussion. In the next section, I'll introduce the basic idea behind Veltman's (1996) *test semantics* for epistemic modals, before showing how to adapt Veltman's ideas into BUS.

3.6 Epistemic modals

Epistemic modals express possibilities relative to a body of information — in update semantics, this is handily provided by the input state. Veltman (1996) suggested that a possibility statement can be thought of as what he dubs a "test" — updating s with $\Diamond\phi$ amounts to testing whether or not s is consistent with ϕ; if it is, s is returned unchanged, otherwise, a failure state of some kind is returned. See, e.g., Groenendijk, Stokhof & Veltman 1996, Willer 2013, Yalcin 2007, Mandelkern 2019 for a variations on Veltman's idea.[13]

Important evidence for the idea that epistemic modals are interpreted relative to a compositionally-provided body of information comes from sentences such as (34) and (35).

(34) # It's not raining, but maybe it's raining.

(35) # Maybe it isn't raining, and it's raining.

The central intuition behind the BUS semantics for epistemic modals is that they are Strong Kleene existential quantifiers over the input state, in other words, they impose weak verification conditions and strong falsification conditions.[14] This can be straightforwardly stated as in definition 3.14.

[12] See Rothschild 2017, and Elliott 2020 for a generalization of this point to other kinds of complex sentences.

[13] This approach to epistemic modals may seem somewhat exotic to those more familiar with classical approaches to modal semantics based on accessibility relations (see, e.g., Kratzer 2012). For a reconciliation of the dynamic view and the classical view, see Mandelkern 2019.

[14] Strong Kleene existential quantification is a generalized form of Strong Kleene disjunction. The simplest way of understanding the schema is relative to set of truth values; verification is weak, and falsification is strong.

- $\exists\,\{\mathbf{t}, \mathbf{f}, ?\} = \mathbf{t}$

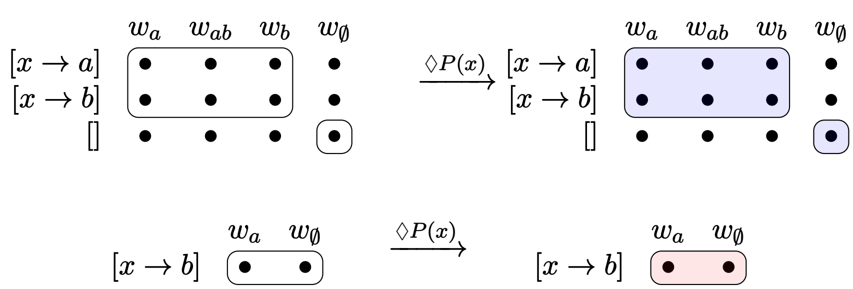
Starting with the positive update, epistemic modals in BUS have a test semantics — concretely, the positive update associated with a modalized sentence is a *weak consistency test* — $s[\Diamond\phi]^+$ tests s to see whether or not there are any possibilities in $s[\phi]^+$. N.b., that this test may be successful, even if $s[\phi]^?$ is non-empty (in this sense, the consistency test is weak). The negative update, on the other hand, imposes a more stringest test on s — the test is passed if there is no way of verifying ϕ at s), and ϕ is false throughout s.

Definition 3.14 (Possibility modals in BUS (first attempt))**.**

$$s[\Diamond\phi]^+ = s \text{ if } s[\phi]^+ \neq \emptyset \text{ else } \emptyset$$
$$s[\Diamond\phi]^- = s \text{ if } s \prec s[\phi]^- \text{ else } \emptyset$$

Note that possibility statements have a special status in update semantics — they are assessed relative to the *entire state* rather than individual possibilities in the state. In BUS, this means that $s[\Diamond\phi]^+$ and $s[\Diamond\phi]^-$ are mutually exclusive. An example of the dynamics of a possibility statement is illustrated in fig. 5. Here, an important prediction of BUS is exemplified — weak projection in possibility statements. Note that the test imposed by $.[\Diamond P(x)]^+$ is still passed, even though $P(x)$ is only defined at some possibilities in the input state.

Figure 5: Updating with possibility statements.



Strikingly, weak projection means that BUS can account for discourses such as (36) *without* a distinct mechanism of modal anaphora!

(36) a. Maybe there's a bathroom.
b. Maybe the bathroom in a funny place.

- $\exists\{\mathbf{f}, \mathbf{f}, ?\} = ?$
- $\exists\{\mathbf{f}, \mathbf{f}, \mathbf{f}\} = \mathbf{f}$

How does this work? At s, (36a) tests whether $s[\exists_x B(x)]^+$ is non-empty — this will be the case just in case there are at least some *bathroom* possibilities. I haven't stated a concrete treatment of definite descriptions, but assume that *the bathroom is in a funny place* is defined only at *bathroom* possibilities. Due to weak projection, $s[\Diamond$ *the bathroom is in a funny place*$]$ will just test whether there is at least one possibility in s at which the embedded sentence is defined and true. There is guaranteed to be at least one *bathroom* possibility so long as (36a) is accepted. Given the discussion of section 2, I'll take this to be a desirable outcome.

The semantics outlined here constitutes a naïve integration of Veltman's test semantics for epistemic modals into BUS, with the proviso that projection is weaker than is typically assumed. There is however a glaring issue with the semantics stated in definition 3.14, that will hinder an account of Partee conjunctions in their full generality — namely, the test semantics outlined here guarantees that possibility statements will be anaphorically inert, i.e., unable to introduce DRS. This is because a test semantics for modals is fundementally one according to which modalized statements are *uninformative* — modalized statements merely return the input state unchanged, if the test is passed. In the next section, I argue that there is independent evidence that modalized statements introduce anaphoric information. I'll use this to motivate a slightly modified version of the semantics in definition 3.14, which will ultimately underlie my account of Partee conjunctions.

4 Possible discourse referents

4.1 On the anaphoric potential of modalized statements

In this section, I'll argue that modalized statements can introduce anaphoric information, by analogy with the behavior we observed with disjunctive sentences in discourse. In section 3, I stated a semantics for disjunction that was tailored to allow disjunctive sentences to introduce anaphoric information — but, due to pragmatic constraints on asserting disjunctive sentences, they typically only make variables *partially* familiar. Crucial evidence for this came from the fact that, if a witness for the existential disjunct was subsequently contextually entailed, anaphora becomes possible (Rothschild 2017, Elliott 2020).

Much the same observation can be made for possibility statements. An assertion of a sentence of the form $\Diamond\phi$ is typically also leaves $\neg\phi$ open as a possibility. Nevertheless, over the course of a discourse, the truth of ϕ may be subsequently contextually entailed. If ϕ is an existential statement, then contextual entailment can feed anaphora. The most straightforward illustration of this is provided by the discourse in (37), on the assumption that recently renovated houses have bathrooms. In (37b), the antecedent of the conditional statement contextually entails the existence of a bathroom in this house — given the independently known fact that the consequent is interpreted relative to the antecedent, anaphora becomes possible, since the existence of a bathroom is (locally) contextually entailed. A disjunctive variation is provided in (38).

(37) a. Maybe there's a^x bathroom in this house.
b. If this house has recently been renovated, then it's upstairs.

(38) a. Maybe there's a^x bathroom in this house.
b. Either this house hasn't been renovated, or it$_x$'s upstairs.

The fact that modalized sentences can introduce DRS is even more readily apparent when we consider the behavior of necessity statements. Anaphora is readily possible in a discourse such as (39), where the prejacent of the necessity modal introduces a bathroom DR. In a sense, this is unsurprising, since it has often been argued that $\Box\phi$ entails ϕ.[15] Nevertheless, despite the fact that we haven't yet given a concrete proposal for the semantics of necessity modals, it's easy to see that a test semantics for *must* can't account for the data in (39) as genuine anaphora, since tests can't introduce anaphoric information.[16]

(39) a. There must be a^x bathroom in this house — we haven't seen it$_x$ because it$_x$'s in a funny place.

In the next section, I remedy the inability of modals to introduce anaphoric information in an intuitive way — the basic idea will be that $s[\Diamond\phi]^+$, rather than returning $\Diamond$ unchanged if the test passes, updates the input state with a privileged tautology — something akin to $\phi \vee \neg\phi$.

4.2 Refining possibility statements

According to the original definition in definition 3.14, if $s[\Diamond\phi]^+$ is non-empty, it returns s. In order to account for the anaphoric potential of modalized sentences, I'll propose that if $s[\Diamond\phi]^+$ is non-empty, it returns a state s', such that $s \prec s'$, i.e., a strictly anaphorically more informative state. I accomplish this by taking the *union* of the positive/negative/unknown updates associated with ϕ. The final definition for possibility modals is given in definition 4.1.[17] In fig. 6, I've provided a visualization of the difference between a standard test semantics for possibility statements and the refined semantics given in definition 4.1, which allows possibility statements to selectively introduce anaphoric informatiion.

Definition 4.1 (Possibility modals in BUS (final version)).

$$s[\Diamond\phi]^+ = s[\phi]^{+,-,?} \text{ if } s[\phi]^+ \neq \emptyset \text{ else } \emptyset$$
$$s[\Diamond\phi]^- = s[\phi]^{+,-,?} \text{ if } s \prec s[\phi]^- \text{ else } \emptyset$$

[15] See, e.g., von Fintel & Gillies 2010.
[16] As an exercise, the reader may consider the adapting the definition given in definition 3.14, by taking $\Box$ to be the dual of $\Diamond$.
[17] $s[\phi]^{+,-,?}$ is to be understood as $s[\phi]^+ \cup s[\phi]^- \cup s[\phi]^?$.

Figure 6: Standard dynamic theory of epistemic modals.

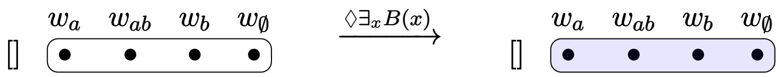
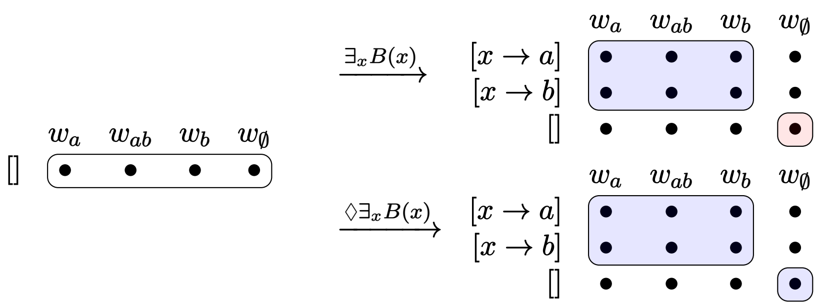


Figure 7: New dynamic theory of epistemic modals.



I've also modified the negative update associated with $\Diamond\phi$ in a similar fashion — as is standard, I'll assume that $\Box$ is the dual of $\Diamond$, so this will be essential in order to allow necessity statements to introduce anaphoric information. Concretely, the semantics for $\Box$ is given below (assuming that $\Box\phi \iff \neg\Diamond\neg\phi$). In order for $s[\Box\phi]^+$ to be non-empty, s must subsist in $s[\phi]^+$, which guarantees that $\Box\phi$ entails ϕ. In order for $s[\Box\phi]^-$ to be non-empty, $s[\phi]^-$ must be non-empty.

Definition 4.2 (Necessity modals in BUS)**.**

$$s[\Box\phi]^+ = s[\phi]^{+,-,?} \text{ if } s \prec s[\phi]^+ \text{ else } \emptyset$$
$$s[\Box\phi]^- = s[\phi]^{+,-,?} \text{ if } s[\phi]^- \neq \emptyset \text{ else } \emptyset$$

The semantics sketched here immediately accounts for the anaphoric potential of modalized sentences, discussed in the previous section. I'll illustrate with possibility modals — concretely, I'll show how to formally model the dialogue below:

(40) a. There might be a^x bathroom in this house. $\Diamond\exists_x B(x)$
b. This house has been renovated.
c. It$_x$'s upstairs! $U(x)$

Let's entertain an initial state s_1, where subscripts exhaustively indicate *bathrooms.*

(41) $s_1 := \{ w_{ab}, w_a, w_b, w_\emptyset \} \times \{ [] \}$

Consider an assertion of the sentence $\Diamond\exists_x B(x)$, against s_1. $\Diamond\exists_x B(x)$ is clearly contextually bivalent, so the bridge principle is satisfied, therefore in order to compose $s_1[\Diamond\exists_x B(x)]^+$, check whether $s_1[\exists_x B(x)]^+$ is non-empty. It is, therefore return $s_1[\exists_x B(x)]^{+,-}$, resulting in the following updated state s_2:[18]

(42) $s_2 := \overbrace{\{ (w_{ab}, [x \to a]), (w_{ab}, [x \to b]), (w_a, [x \to a]), (w_b, [x \to b]) \}}^{s_1[\exists_x B(x)]^+} \cup \overbrace{\{ (w_\emptyset, []) \}}^{s_1[\exists_x B(x)]^-}$

It should be clear that an assertion of $U(x)$ will not be assertable at s_2, since $s_2[U(x)]^? = \{ (w_\emptyset, []) \}$. I'll assume however that *this house has been renovated* is contextually equivalent to *there's a bathroom in this house*, in which case asserting *this house has been renovated* will eliminate non-bathroom possibilities, resulting in an updated state s_3. x is clearly familiar at s_3

(43) $s_3 := \{ (w_{ab}, [x \to a]), (w_{ab}, [x \to b]), (w_a, [x \to a]), (w_b, [x \to b]) \}$

In the next section, I'll show how the semantics outlined here accounts for Partee conjunctions.

4.3 Accounting for Partee conjunctions

4.3.1 Conjunction in BUS

Given the refined semantics for possibility modals given in definition 4.1, almost all of the necessary ingredients are in place to account for Partee conjunctions. One crucial component which has however not yet been mentioned is the semantics of conjunction. The strategy I adopt for conjunction will be exactly the same as the one that I used to motivate the BUS semantics for disjunction — namely, a dynamic generalization of the Strong Kleene logic of indeterminacy. The Strong Kleene schema for conjunction imposes *strong* verification conditions and *weak* falsification conditions — that is to say, to verify a conjunctive statement, both conjuncts must be verified, but in order to falsify a conjunctive statement, it is sufficient to falsify a single conjunct, even if the status of the other conjunct is unknown. The schema is illustrated in fig. 8, and the generalization to BUS is given in definition 4.3

$\phi \wedge \psi$	ψ_+	ψ_-	$\psi_?$
ϕ_+	$+$	$-$	$?$
ϕ_-	$-$	$-$	$-$
$\phi_?$	$?$	$-$	$?$

Figure 8: Strong Kleene conjunction

[18] $s_1[\exists_x B(x)]^?$ is just empty, since $\exists_x B(x)$ is bivalent.

Definition 4.3 (Conjunction in BUS).

$$s[\phi \wedge \psi]^{+} := s[\phi]^{+}[\psi]^{+}$$
$$s[\phi \wedge \psi]^{-} := s[\phi]^{-}[\psi]^{+,-,?} \cup s[\phi]^{+,?}[\psi]^{-}$$

The positive update associated with $\phi \wedge \psi$ preserves the idea, central to update semantics, that asserting a conjunctive sentence is tantamount to successive assertion of each conjunct, only in this instance we take the successive *positive* updates induced by each conjunct; discourse anaphora is captured in the usual way.

The negative update associated with $\phi \vee \psi$ will be less familiar, but the intuition is that a negative conunctive statement like $\neg(\phi \wedge \psi)$ is treated parallel to a disjunction of negative statements like $\neg\phi \vee \neg\psi$ — in fact, the semantics outlined here validates de Morgan's equivalences — right down to anaphora — as in (44). See Elliott 2020 for discussion.

(44) a. There's no way that there's a^{x} bathroom and it$_{x}$'s in the usual place.
b. Either there's no^{x} bathroom, or it$_{x}$ isn't in the usual place.

This has as a somewhat peculiar prediction that a negative conjunctive statement should, in principle, be capable of making a DR partially familiar, just like the corresponding disjunctive sentence. Tentatively, this seems to be nevertheless a welcome prediction, as diagnosed by contextual entailment:

(45) There's no way that there's a^{x} bathroom and it$_{x}$'s in the usual place.
If this house has been recently renovated, it$_{x}$'s in the basement.

4.3.2 Elaboration without modal anaphora

Recall from our discussion of Partee conjunctions, that there are two interpertations that conjunctions of possibility statements can have. Partee conjunctions *list possibilities*, much like disjunctions, as illustrated by (46), but the second modalized conjunct can also be interpreted as *elaborating* on the possibility made salient by the first conjunct.

(46) Maybe there's no^{x} bathroom in this house, and maybe it$_{x}$'s in a funny place.
$$\Diamond\neg\exists_{x}B(x) \wedge \Diamond F(x)$$

(47) Maybe there's a^{x} bathroom in this house, and maybe it$_{x}$'s in a funny place.
$$\Diamond\exists_{x}B(x) \wedge \Diamond F(x)$$

It will be easier to illustrate the account of Partee disjunctions by first showing how BUS accounts for the elaboration reading as a matter of course. Recall that the idea is that each possibility statement is simply a Strong Kleene existential quantifier over the input state, which furthermore can introduce anaphoric information via a privileged tautology. $\Diamond\exists_x B(x)$ tests whether there are at least some *bathroom* possibilities, and furthermore introduces *bathroom* DRS just at those *bathroom* possibilities. $\Diamond F(x)$ tests whether there are at least some $F(x)$ possibilities — since $\Diamond$ is stated in terms of Strong Kleene existential quantification, it doesn't care if $F(x)$ is undefined at some possibilities in the input state. In other words, $\Diamond F(x)$ only requires that x be *partially* familiar, for the purposes of verifying the modal statement. The positive update is stated formally (48). (47) is non-empty at s just in case there is some *bathroom in a funny place* possibility in s, and makes a bathroom DR partially familiar.

(48) $$s[\Diamond\exists_x B(x)]^+[\Diamond F(x)]^+ = s[\exists_x B(x)]^+ \cup s[\exists_x B(x)]^- \\ \text{if } s[\exists_x B(x) \wedge F(x)]^+ \neq \emptyset \text{ else } \emptyset$$

4.3.3 Extension to Partee conjunctions

The key to Partee conjunctions will be the equivalence in (49): essentially, taking the union of $s[\phi]^{+,-,?}$ is equivalent to taking the union of $s[\neg\phi]^{+,-,?}$. Based on the definition of negation, it should be obvious why this holds, namely: $s[\phi]^+ = s[\neg\phi]^-$, $s[\phi]^- = s[\neg\phi]^+$, and $s[\phi]^? = s[\neg\phi]^?$.

(49) $$s[\phi]^{+,-,?} = s[\neg\phi]^{+,-,?}$$

What this means is that the *anaphoric* contribution of $\Diamond\neg\phi$ is predicted to be identical to that of $\Diamond\phi$, even though the two sentences obviously impose different tests. In othe words, the positive update induced by $\Diamond\neg\exists_x B(x)$ will make x partially familiar just in case $\neg\exists_x B(x)$ isn't contextually entailed. Independently, it seems to be infelicitous to assert a sentence of the form $\Diamond\phi$ if ϕ is contextually entailed, as illustrated by the infelicity of the follow-up in (50). I'll assume that the principle in (51) holds, which should ideally be explained in terms of pragmatic principles.[19]

(50) Josie is wearing a wedding ring. # she might be married.

[19] For my purposes, there is no need to commit to exactly *why* $\Diamond\phi$ is ruled out when ϕ is contextually entailed, but it is nevertheless an interesting question, since a test semantics for epistemic modals renders them uninformative. It's tempting to invoke competition between the possibility and necessity modals, since a necessity modal is possible in the same context.

(1) Josie is wearing a wedding ring. She must be married.

It seems that we need a pragmatic principle like the following — given two contextually-equivalent sentences ϕ and ψ, ϕ is ruled out if ψ imposes a more stringent test on the input state. This is suspiciously reminiscent of Heim's (1991) principle *Maximize Presupposition!*.

(51) **Condition on assertion of** $\Diamond\phi$: ϕ is assertable at s iff $\not\models_s \phi$.

A perhaps surprising consequence of (51) is that $\Diamond\phi$ and $\Diamond\neg\phi$ become assertable relative to exactly the same set of information states — namely, those where both ϕ and $\neg\phi$ are open possibilities. And furthermore contribute exactly the same anaphoric information via a privileged tautology. Explaining exactly how $\Diamond\phi$ and $\Diamond\neg\phi$ nevertheless make intuitively distinct semantic contributions will be beyond the remit of this paper, but see especially Ciardelli, Groenendijk & Roelofsen 2014 on the idea that $\Diamond\phi$ makes salient or "highlights" ϕ.

With (51) in place, it's easy to see that in any state s at which $\Diamond\neg\exists_x B(x)$ is assertable, it will make x partially familiar, which is subsequently enough to satisfy the anaphoric presupposition of $\Diamond F(x)$. The predicted positive update of a Partee conjunction is given in (52): $\Diamond\neg\exists_x B(x) \wedge \Diamond F(x)$ performs two tests on the input state s — it ensures that there are some *no bathroom* possibilities, and also that there are some *bathroom in funny place* possibilities, as well as introducing a partially familiar bathroom DR.

(52) $$s[\Diamond\neg\exists_x B(x)]^+[\Diamond F(x)]^+ = s[\exists_x B(x)]^+ \cup s[\exists_x B(x)]^- \quad \text{if } s[\exists_x B(x)]^- \neq \emptyset \text{ and } s[\exists_x B(x) \wedge F(x)]^+ \neq \emptyset \text{ else } \emptyset$$

At this point, it's time to step back and consider the account of Partee conjunctions from a broader perspective. Arguably the most striking fact about Partee conjunctions is the parallelism with presupposition projection/anaphora in disjunctive sentences — the relevant examples are repeated below:

(53) Either there's no bathroom in this house, or it's in a funny place

(54) Maybe there's no bathroom in this house, and maybe it's in a funny place.

The parallelism is intuitively unsurprising, since in discourse, disjunctive sentences $\phi \vee \psi$, and conjunctions of possibility statements $\Diamond\phi \wedge \Diamond\psi$ play a similar role — they *list possibilities*, ϕ, ψ. What this suggests is that the projection behaviour we observe in complex sentences is indeliby linked to their semanto-pragmatic contribution. When we look at the positive updates induced by Partee disjunctions and Partee conjunctions, the connection becomes clear. Consider once more the positive update associated with a disjunctive sentence, (55):

(55) $$s[\phi \vee \psi]^+ := s[\phi]^+[\psi]^{+,-,?} \cup s[\phi]^{-,?}[\psi]^+$$

The update schema in (55) encodes how to *dynamically verify* a disjunctive sentence, in-line with the Strong Kleene strategy. We can distinguish the updated information state into the part associated with verification of the first disjunct $s[\phi \vee \psi]^+_\phi$, and the part associated with verification of the second disjunct, $s[\phi \vee \psi]^+_\psi$ — note that the parts may overlap, e.g., if there are worlds at which both disjuncts are true:

(56) $s[\phi \vee \psi]^{+}_{\phi} := s[\phi]^{+}[\psi]^{+,-,?}$

(57) $s[\phi \vee \psi]^{+}_{\psi} := s[\phi]^{+,-,?}[\psi]^{+}$

One of the ways of verifying the second disjunct — highlighted in Partee disjunctions, which express mutually exclusive possibilities — is by dynamically falsifying the first disjunct, and verifying the second disjunct in light of this. Pragmatic constraints on disjunctive sentences demand that the contribution of both disjuncts not be contextually trivial[20] — a consequence of this is that both the sets in (56) and (57) should be non-empty. In other words, an utterance of the form "Either there's no bathroom, or it's in a funny place" is infelicitous unless it's considered an open possibility (i.e., not contextually trivial) that *there is no bathroom*, and an open possibility that *there is a bathroom in a funny place*.

Possibility modals are different kinds of beasts — a sentence of the form $\Diamond\phi$, at s, directly tests whether $s[\phi]^{+}$ is non-empty, by dint of its semantic contribution. Furthermore, due to pragmatic factors, an utterance of the form $\Diamond\phi$ is infelicitous if it's a contextual certainty that ϕ. In tandem, the semantics and pragmatics ensures that an assertion of the $\Diamond\phi$ at s requires that ϕ is an open possibility at s. This means that a conjunction of possibility statements $\Diamond\phi \wedge \Diamond\psi$, much like a disjunctive sentence, ends up listing open possibilities.

5 Comparison with existing work

One of the most well known existing attempts to combine the dynamic approach to anaphoric information with epistemic modality is the fragment presented in Groenendijk, Stokhof & Veltman (GSV) 1996 (henceforth, I'll refer to the semantics outlined therein as GSV). In several respects, BUS builds upon the insights of GSV, making use of key notions such as *subsistence*. Unlike BUS, GSV is a more conventional update system, in which sentences express mappings from information states to information states. In definition 5.1, I present a simplified version of GSV for the same first-order fragment assumed in this paper, in which information states are Heimian; assignments are assumed to be total.[21]

Definition 5.1 (Semantics of GSV)**.** The semantics of GSV constitutes a recursive definition of $s[.]$.

- $s[P(x_1, \ldots, x_n)] := \{\, i \in s \mid P(x_1, \ldots, x_n) \text{ is true at } i \,\}$

[20] I'll remain neutral with regards to how to derive this, but it can plausibly be thought of as a *Manner* implicature (see Meyer 2016 for discussion).

[21] One of the key differences between the simplified system outlined here and that of Groenendijk, Stokhof & Veltman, is that I do not make use of the latter's distinction between assignments and *pegs*. I also opt for a simpler semantics for the existential quantifier based on random assignment and successive update. Both cases relate to subtleties that go beyond the scope of this short section.

The discussion here is deeply indebted to Chris Barker's 2021 NYU seminar "Dynamic semantics: from content to uptake", and especially his extremely lucid presentation of GSV: `https://github.com/cb125/Dynamics`.

- $s[\neg\phi] := \{\, i \mid i \not\prec s[\phi] \,\}$
- $s[\phi \wedge \psi] := s[\phi][\psi]$
- $s[\phi \vee \psi] := \{\, i \mid i \prec s[\phi] \text{ or } i \prec s[\neg\phi][\psi] \,\}$
- $s[\exists_x \phi] := \{\, (w, h) \mid (w, g) \in s, g[x]h \,\}\, [\phi]$
- $s[\Diamond\phi] := s \text{ if } s[\phi] \neq \emptyset \text{ else } \emptyset$

In this section, I'll comment on some of the most salient differences between BUS and GSV, as regards both their conceptual underpinnings and empirical predictions.

5.1 Disjunction and negation in GSV

In definition 5.1, I've stated a semantics for disjunction which accounts for the attested presupposition projection patterns in disjunctive sentences such as "There is no bathroom, or the bathroom is in a funny place" (once a satisfaction-based theory of presupposition projection is appropriately integrated). The latter disjunct is evaluated relative to the input state first updated with the *negation* of the initial disjunct (Beaver 2001), thus satisfying the existential presupposition of the latter disjunct. This semantics is in fact exactly what we get by defining disjunction in terms of GSV negation and conjunction as in (58), as alluded to by Groenendijk, Stokhof & Veltman.[22] It's important that GSV-conjunction is taken as a primitive; were conjunction defined in terms of GSV-disjunction (e.g., via $\phi \wedge \psi := \neg(\neg\phi \vee \neg\psi)$), GSV wouldn't capture even elementary facts about anaphora or projection in conjunctive sentences.

(58) $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi)$

In BUS, on the other hand, the question of whether conjunction or disjunction is primitive is in a sense irrelevant, since the core semantic contribution of the connectives is provided by the Strong Kleene interpretation schema; the Strong Kleene connectives are systematically lifted into BUS. BUS therefore arguably has a conceptual advantage over more orthodox dynamic systems such as GSV — there is but a single locus of stipulation concerning the dynamics of logical connectives.

Conceptual distinctions aside, an important fact about GSV is that it *doesn't* validate double-negation elimination. This is for the simple reason that the rule given for negative sentences in definition 5.1, stated in terms of subsistence, can only ever eliminate possibilities from the input state — the definition guarantees that negative sentences never introduce anaphoric information. Consequently the schema for disjunction, although intuitively correct for presupposition projection, won't help in accounting for the possibility of anaphora in Partee disjunctions. This is because pronouns don't merely carry an existential presupposition, but presuppose the presence of a familiar discourse referent.

[22] I leave this up to the reader to verify.

5.2 Epistemic contradictions

One of the main virtues of GSV is that it accounts for a variety of deviant sentences and discourses in terms of notions of *consistency* and *coherence*. Consider for example the discourses in (59), and (60).

(59) It isn't raining outside. # It might be raining outside. $\neg R \wedge \Diamond R$

(60) It might be raining outside. ? It's not raining outside. $\Diamond R \wedge \neg R$

I'll briefly survery the GSV account of the above cases, before showing that BUS maintains the essence of the GSV account, while generalizing to more complex cases involving disjunction. The key notions that GSV exploits are *consistency*, and *coherence*. Starting with consistency, the definition is given below in definition 5.2.

Definition 5.2 (Consistency in GSV)**.** A sentence ϕ is *consistent* iff there is some s, s.t., $s[\phi] \neq \emptyset$.

The intuition behind consistency is that a sentence must be in principle sincerely assertable; a sentence is *inconsistent* if it can never be sincerely asserted (i.e., there is no state that it can update resulting in a non-absurd state). (59) is clearly inconsistent, if we understand the discourse conjunctively. Recall that in GSV conjunction is a successive update; given a starting state s, $s' := s[\neg R]$ eliminates *raining outside* possibilities. The test imposed by $\Diamond R$ is therefore bound to fail for s', thereby always returning the absurd state $\emptyset$. Indeed, the intuition behind the oddness of (59) is that the hypothetical speaker has, in some sense, said something contradictory.

Note that, perhaps surprisingly, (60) *is* consistent, since we can find a state against which it can be sincerely asserted. Consider e.g., a diverse state s, where *raining outside* is possible but not certain. The test imposed by $\Diamond R$ will pass at s, and subsequently $\neg R$ will eliminate raining outside possibilities from s. This relates to an important distinction between the qualitative intuitions behind the unacceptability of (59) and (60). Imagine the following scenario: an ignorant speaker asserts "It might be raining outside"; they walk over to the window, look outside, and subsequently assert (thanks to their new information) "It's not raining outside". The discourse in (60) may be amnestied just in case the speaker acquires new information before asserting the second sentence. In order to capture the oddness of the discourse of (60) asserted relative to a single information state, GSV introduce a second notion of *coherence*, given below in definition 5.3.

Definition 5.3 (Coherence in GSV)**.** A sentence ϕ is *coherent* iff there is some non-empty state s, s.t., $s \prec s[\phi]$.

Even thought each sentence making up the discourse in (60) is individually coherent, the discourse understood as a conjunctive sentence is incoherent. This is because, for any s to subsist in $s' :=$

$s[\Diamond R]$, R must be a possibility in s, but then $\neg R$ will eliminate all R-possibilities from s'. It follows that no s can subsist in $s[\Diamond R][\neg R]$.

GSV is expressive enough to account for variations of (59) and (60) involving discourse-anaphoric dependencies.

(61) # Someone^x isn't hiding in the closet and they$_x$ might be hiding in the closet.
$$\exists_x \neg H(x) \wedge \Diamond H(x)$$

(62) ? Someone^x might be hiding in the closet and they$_x$ aren't hiding in the closet.
$$\exists_x \Diamond H(x) \wedge \neg H(x)$$

Consider the dynamics of (61). Given an information state s, the first conjunct eliminates possibilities where everyone is hiding the closet, and introduces the anaphoric information that x is *not hiding in the closet.*

(63) $s' := \{ (w, h) \mid g[x]h, h(x)$ isn't hiding in the closet in $w, (w, g) \in s \}$

The second conjunct takes the state s', and runs a test — does $s'[H(x)] \neq \emptyset$? This test will always fail, since every possibility in s' is guaranteed to map x to a non-closet-hider. The sentence is therefore inconsistent.

Similarly, consider the dynamics of (62). Given an information state s, the first conjunct introduces a DR x, and tests the resulting state, ensuring that there are some possibilities in which x is hiding in the closet. The second conjunct subsequently *eliminates* possibilities in which x is hiding in the closet, ensuring that the discourse in (62) is incoherent.

A virtue of BUS is that it extends the GSV account to encompass deviant discourses involving discourse anaphoric dependencies across disjunctions like (64).

(64) # Either there isn't anyone^x hiding, or they$_x$ might not be hiding. $\neg \exists_x H(x) \vee \Diamond \neg H(x)$

(65) # Either nobody is possibly hiding, or they're not hiding. $\neg \exists_x \Diamond H(x) \vee \neg H(x)$

I'll begin with a more detailed consideration of the status of (64). We'll begin by computing the negative update, which should be equivalent to the positive update associated with $\exists_x H(x) \wedge \Box H(x)$, by DNE and duals. Here, the necessity statement is vacuous, and so the negation of the disjunctive statement is equivalent to the negation of the first disjunct.

(66)
$$\begin{aligned} s[\neg \exists_x H(x) \vee \Diamond \neg H(x)]^- &= s[\exists_x H(x)]^+[\Box H(x)]^+ \\ &= s[\neg \exists_x H(x)]^- \end{aligned}$$

Now, I'll consider its positive update. Since the second disjunct doesn't introduce any anaphoric information, one way of dynamically verifying the disjunction is just by taking the positive update of the first disjunct. Alternatively, we can take dynamically verify the disjunction via $s[\exists_x H(x)]^+[\Diamond\neg H(x)]^+$ (i.e., the first disjunct is false, and the second is true). Note that this is always empty, since $\exists_x H(x) \wedge \Diamond\neg H(x)$ is inconsistent in GSV's sense. Therefore, the positive update ends up being equivalent to the positive update associated with the first disjunct. This is a clear violation of the pragmatic constraints on disjunctive assertions, plausibly related to the maxim of Manner (see the discussion in section 4.3.3).

(67) $s[\neg\exists_x H(x) \vee \Diamond\neg H(x)]^+ = s[\neg\exists_x H(x)]^+$

Now, turning to (65), I'll start with the associated negative update. This is easy to compute, since by DNE, it simply involves composing the positive update *someone might be hiding* with the positive update *they're hiding* — the result is equivalent to just *someone is hiding.*

(68) $s[\neg\exists_x \Diamond H(x) \vee \neg H(x)] = s[\exists_x \Diamond H(x)]^+[H(x)]^+ \quad = s[\exists_x H(x)]^+$

The positive update, as before, is more complicated. We begin by considering the possibility that the first disjunct is true — since the second disjunct introduces no anaphoric information, we can simplify to just the positive update associated with the first disjunct — namely, *nobody is possibly hiding.* The second possibility is that the first disjunct is false — in which case it introduces a discourse referent x and tests whether it's possible that x is hiding — and subsequently updates with the information that x isn't hiding. Note that the second verification strategy corresponds to an incoherent sentence in the sense of GSV, namely $\exists_x \Diamond H(x) \wedge \neg H(x)$. It's natural to speculate that this is what is responsible for the deviance of (65). In fact, we can state a special version of *coherence* for disjunctive sentences as follows:

Definition 5.4 (Coherence of disjunctive sentences)**.** A sentence $\phi \vee \psi$ is *coherent* iff there is some non-empty state s, s.t., $s \prec s[\phi]^+[\psi]^{+,-,?}$ and $s \prec s[\phi]^{+,-,?}[\psi]^+$.

The definition in definition 5.4 seems however rather ad-hoc, and it is unclear how to derive it from more general principles. I leave a more thorough investigation into coherence and disjunctive sentences to another occasion.

6 Conclusion

In this paper, I began with a novel puzzle for theories of anaphora/presupposition projection — modal conjunctions of the form $\Diamond\phi \wedge \Diamond\psi$ pattern *variably* with disjunction/conjunction depending on whether the sentence is interpreted as listing possibilies, or elaborating on a single possibility.

This puzzle, together with more familiar puzzles involving disjunction and double-negation, motivated a novel *bilateral* take on Heimian update semantics BUS. The general lesson is that we need a more nuanced understanding of the dynamics of discourse anaphora — it's generally assumed that pronouns place a requirement on the context that a discourse referent be *familiar*. Here, familiarity is derived by treating pronouns as presuppositional in the ordinary sense, together with a universal bridge principle. A natural consequence is that anaphoric presuppositions can be filtered just like ordinary presuppositions, leading to cases where pronouns only give rise to a requirement that a discourse referent be *partially* familiar. Partee conjunctions therefore constitute a case where we must take seriously the parallels between ordinary presuppositions and anaphoric presuppositions, and BUS is a theory where anaphora and presupposition are tightly connected — anaphoric accessibility conditions arise by dint of how presuppositions project according to the Strong Kleene logic of indeterminacy.

In classical dynamic semantics, logical operators can artifically 'gate' discourse referents — this feature has been widely exploited to account for accessibility generalizations. An important general conclusion of this paper is that such an approach misses a clear generalization — the possibility of discourse anaphora tracks contextual entailment of a witness to the indefinite much more closely than classical theories lead us to expect. Modalized statements consistute a clear instance of this: "Maybe there's a bathroom" doesn't typically license discourse anaphora, but, if the presupposition of a subsequent pronoun is sufficiently weakened *or* if the existence of a witness is subsequently contextually entailed, discourse anaphora becomes possible. This more general conclusion converges with the findings of some recent approaches to discourse anaphora — see, e.g., Mandelkern 2022, Hofmann 2022. The empirical scope of BUS remains somewhat limited, and an exciting prospect of future work is an extension of the ideas in BUS to a logic with the resources for describing, e.g., quantification and plurality. Assuming a tight connection between presupposition projection and anaphoric accessibility makes clear predictions, the vast majority of which have yet to be explored in detail.

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