# Negation and disjunction in dynamic semantics

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# Introduction and recap

# Logic of change: definitions

Sentences express *updates*, i.e., functions from possibilities (world-assignment pairs), to sets of assignments.

(1) **Random assignment:**
$[\![[x]]\!]^{w,g} = \{ h \mid h \text{ differs from } g \text{ at most at } x \}$ i.e., $\{ h \mid g[x]h \}$

(2) **Predication:**
$[\![[Sleep(x)]]\!]^{w,g} = \{ g \mid g(x) \in I(Sleep)(w) \}$

(3) **Negation:**
$[\![\neg\phi]\!]^{w,g} := \{ g \mid [\![\phi]\!]^{w,g} = \emptyset \}$

(4) **Dynamic conjunction:**
$[\![\phi \,;\, \psi]\!]^{w,g} = \{ h \mid \exists i[i \in [\![\phi]\!]^{w,g} \wedge h \in [\![\psi]\!]^{w,i}] \}$

## Logic of change: abbreviations

(5) Implication:

a. $\phi \rightarrow \psi := \neg(\phi\,;\neg\psi)$

b. $[\![\phi \rightarrow \psi]\!]^{w,g} = \{\, g \mid \neg\exists h[h \in [\![\phi]\!]^{w,g} \wedge [\![\psi]\!]^{w,h} = \emptyset] \,\}$

(6) Disjunction:

a. $\phi \vee \psi := \neg(\neg\phi\,;\neg\psi)$

b. $[\![\phi \vee \psi]\!]^{w,g} = \{\, g \mid [\![\phi]\!]^{w,g} \neq \emptyset \vee [\![\psi]\!]^{w,g} \neq \emptyset \,\}$

(7) Existential quantification:

a. $\exists x\phi := [x]\,;\phi$

(8) Universal quantification:

a. $\forall x\phi := [x] \rightarrow \phi$
(N.b., $\forall$ can equivalently be defined as the dual of $\exists$)

# The dynamics of negation

(9) *Pearl doesn't have a car.* $\rightsquigarrow$ $\neg([x];[Car(x)];[Have(p,x)])$

(10) $$[\![\neg([x];[Car(x)];[Have(p,x)])]\!]^{w,g} = \{\, g \mid \overbrace{[\![[x];[Car(x)];[Have(p,x)]]\!]^{w,g}}^{\text{hypothetical update}} = \emptyset \,\}$$
$$= \{\, g \mid \text{Pearl doesn't have a car in } w \,\}$$

- Negation is a test
- It specifies a hypothetical update, and then imposes the condition that this update has an empty output.
- Therefore, negation is externally static.

# The dynamics of epistemic possibility modals

(11) *Pearl might have a car.* $\rightsquigarrow$ $\Diamond([x]; [Car(x)]; [Have(p, x)])$

(12)

$$\llbracket \Diamond([x]; [Car(x)]; [Have(p, x)]) \rrbracket^{w,g} = \{\, g \mid \exists w' \in Dox_w, \underbrace{\llbracket [x]; [Car(x)]; [Have(p, x)] \rrbracket^{w',g}}_{\text{hypothetical update}} \neq \emptyset \,\}$$

$$= \{\, g \mid \text{Pearl might have a car in } w \,\}$$

- Epistemic possibility modals contribute a test
- Specify hypothetical update and impose the condition that update holds in some epistemically accessible world
- Therefore, epistemic possibility modals are externally static.

## The dynamics of disjunction

(13) *Either Pearl has a car, or Amethyst is happy* $\rightsquigarrow$
$([x];[Car(x)];[Have(p,x)]) \vee [Happy(a)]$

(14) $[\![([x];[Car(x)];[Have(p,x)]) \vee [Happy(a)]]\!]^{w,g}$

$$= \{\, g \mid \overbrace{[\![[x];[Car(x)];[Have(p,x)]]\!]^{w,g}}^{\text{hypothetical update 1}} \neq \emptyset \vee \overbrace{[\![Happy(a)]\!]^{w,g}}^{\text{hypothetical update 2}} \neq \emptyset \,\}$$

$$= \{\, g \mid \text{Pearl has a car in } w \text{ or Amethyst is happy in } w \,\}$$

- Disjunction is a test
- It specifies a hypothetical update for each disjunct, and imposes the condition that one or the other has a non-empty output.
- Therefore, disjunction is externally static.

# Accessibility recap

| | internally | externally | veridicality? |
|---|---|---|---|
| $\phi \wedge \psi$ | dynamic | dynamic | veridical |
| $\neg\phi$ | N/A | static | non-veridical |
| $\Diamond\phi$ | N/A | static | non-veridical |
| $\phi \rightarrow \psi$ | dynamic | static | non-veridical |
| $\phi \vee \psi$ | static | static | non-veridical |

- Veridicality: A propositional operator is *veridical* iff it entails its prejacent.
- A generalization: Non-veridical operators are externally static
- Prediction: Anaphora to indefinites in the scope of non-veridical operators is not possible
- But there are many counterexamples in natural language...

# Anaphora to antecedents in the scope of non-veridical operators

Some counterexamples to the generalization from classical dynamic semantics

(15) Double negation (Karttunen 1976, Krahmer & Muskens 1995)
*It's not the case that there isn't a bathroom in this house. It is upstairs.*

(16) Discourse-inferences
*There might be a bathroom in this house. In fact, I just remembered that's the case. It is upstairs.*

(17) Program disjunction (Groenendijk & Stokhof 1991)
*A professor or an assistant professor will attend the meeting of the university board.*
*She will report to the faculty.*

(18) 'Bathroom-disjunctions' (Evans 1977, Barbara Partee)
*Either there isn't a bathroom in this house, or it's upstairs.*

(19) Modal subordination (Roberts 1987)
*There might be a bathroom in this house. It would be upstairs.*

(20) Inter-speaker disagreement (Hofmann 2019)
A: *There isn't a bathroom in this house.*
B: *(What are you talking about?) It's upstairs.*

# Double negation, discourse-inferences, and program disjunction: Contextual entailment

# Double negation

(21) a. *It's not the case that there isn't [a bathroom]$^x$.* $\rightsquigarrow$ $\neg(\neg([x]; [\mathit{Bathroom}(x)]))$

b. *It$_x$ is upstairs.* $\rightsquigarrow$ $[\mathit{Upstairs}(x)]$

(22) a. *There is [a bathroom]$^x$* $\rightsquigarrow$ $[x]; [\mathit{Bathroom}(x)]$

b. *It$_x$ is upstairs.* $\rightsquigarrow$ $[\mathit{Upstairs}(x)]$

- Question: Why do you think anaphora is possible here?
- The (a) sentences make the anaphora in (b) possible.
- But only the translation in (19) predicts this.

**The problem:** Some of the inference rules of classical logic do not apply in dynamic semantics

# Double negation elimination

Classically, it's usually assumed that a doubly-negated sentence is equivalent to its positive counterpart.

- Classical double negation elimination: $\neg\neg\phi \Leftrightarrow \phi$

(23) a. It's raining.
b. It's not the case that it isn't raining.

Surpisingly, this equivalence breaks down in Dynamic Semantics (DS), as we'll show in detail, due to the dynamics of negation.

- In DS: $\neg\neg\phi \not\Leftrightarrow \phi$

# No double negation elimination in DS

Existential statements in DS introduce Discourse Referents (DRS) via random assignment:

(24) There is [a bathroom]$^x$. $\rightsquigarrow [x]\ ;\ [Bathroom(x)]$

- Recall, however, that negative statements in DS are *tests*; **once a test, always a test**.
- This means that a *doubly* negated statement is also a test. Ipso facto, doubly negated statements don't introduce DRS, and the classical equivalence doesn't go through.

(25) It's not the case that there is [a bathroom]$^x$. $\rightsquigarrow \neg(\neg([x]; [Bathroom(x)]))$

Let's see why in more detail:

# Double negation in detail

(26) $$[\![ [x] \,;\, [Bathroom(x)] ]\!]^{w,g} = \{\, h \mid g[x]h, h(x) \in I(Bathroom)(w) \,\}$$

Negating this once returns a test of whether the previous update has an empty output:

(27) $$\begin{aligned}[\![ \neg([x] \,;\, [Bathroom(x)] ]\!]^{w,g} &= \{\, g \mid \{\, h \mid g[x]h, h(x) \in I(Bathroom)(w) \,\} = \emptyset \,\} \\ &= \{\, g \mid I(Bathroom)(w) = \emptyset \,\}\end{aligned}$$

Negating *again* returns a test of whether the previous update (itself a test) has an empty output.

(28) $$\begin{aligned}[\![ \neg\neg([x] \,;\, [Bathroom(x)]) ]\!]^{w,g} &= \{\, g \mid \{\, g \mid I(Bathroom)(w) = \emptyset \,\} = \emptyset \,\} \\ &= \{\, g \mid I(Bathroom)(w) \neq \emptyset \,\}\end{aligned}$$

The net result is that $\neg\neg\phi$ is always interpreted as a *test* of whether $\phi$ is true (i.e., has a non-empty output).

# Double negation in detail — no dynamic DNE

(26) $\llbracket [x]\,;[Bathroom(x)] \rrbracket^{w,g} = \{\, h \mid g[x]h, h(x) \in I(Bathroom)(w) \,\}$

(15) $\llbracket \neg\neg([x]\,;[Bathroom(x)]) \rrbracket^{w,g} = \{g \mid \{g \mid I(Bathroom)(w) = \emptyset\} = \emptyset\}$
$= \{g \mid I(Bathroom)(w) \neq \emptyset\}$

The updates associated with (26) and (15) are different:

- (26) introduces a new variable $x$ (pointing to a bathroom), but (15) only tests whether such an update is possible.
- They are truth-conditionally equivalent
- But their anaphoric potential is different

# Some takeaways

In DS:

- Existential statements have existential truth-conditions and introduce DRS.
- Doubly-negated existential statements have existential truth-conditons — like their positive counterparts — but *don't introduce* DRS.

Accessibility in DS is:

- Constrained by semantic representations; namely, the presence of negation.
- Not affected by truth-conditional inferences:
  The truth-conditional entailment of an existential statement does not enable anaphora
  (though maybe it should)

# Double negation and anaphora

As we have seen, doubly-negated sentences *can* provide antecedents for anaphora:

15 a. *It's not the case that there isn't [a bathroom]$^x$.*
b. *It$_x$ is upstairs.*

As already noted by Groenendijk & Stokhof (1991), this is problematic for accessibility in DS.

This is a more general pattern:

(29) Steven doesn't not own a$^x$ shirt — he's wearing it$_x$ right now!

(30) a. Q: Did Amethyst not buy any$^x$ pizza?
b. A: No (*she didn't not buy any pizza*), it$_x$'s heating up in the oven.

- **Question:** Can you think of more examples?
- **Question:** How could this problem be addressed?

# Externally dynamic negation?

In order for doubly-negated sentences to be externally dynamic in DS, negation itself would have to be externally dynamic.

Given the connection between non-veridicality and external staticity in DS, it's not really clear how an externally dynamic negation could be defined.

Consider the following possible entry for negation. Do you see any problems?

(31) $[\![\neg\phi]\!]^{w,g} = \{ h \mid h \notin [\![\phi]\!]^{w,g} \}$

Can you think of any alternative entries for negation that are possible to state in DPL?

(16) a. *There might be a bathroom in this house.*

b. *In fact, I just remembered that's the case.*

c. *#It is upstairs.*

d. *It is upstairs.*

Modals introduce an externally static operator into the representation, but a subsequent assertion to the truth of the modal prejacent allows for anaphora to indefinites in the modal prejacent

- Question: What is going on here?
- Another case, where a representational generalization does not work
- The truth-conditional inference of (a + b) allows for anaphora in (c)

# Progam disjunction

Groenendijk & Stokhof (1991) noticed that natural language disjunction isn't always externally static. Here is their counterexample (slightly modified).

(17) a. *A professor or an assistant professor will attend the meeting of the university board.*

b. *She will report to the faculty.*

- **Question:** What is going on here?
- **Question:** What is the antecedent of "*she*"?

# Disjunction elimination

(17) a. *A professor or an assistant professor will attend the meeting of the university board.*

b. *She will report to the faculty.*

(32) a. (17*a*) $\rightsquigarrow$ ([$x$] ; [*ProfAttend*($x$)]) $\vee$ ([$x$] ; [*AsstAttend*($x$)])

b. (17*b*) $\rightsquigarrow$ [*ReportToFaculty*($x$)]

Another inference rule of classical logic:

- Disjunction elimination: If $\phi \rightarrow \chi$, and $\psi \rightarrow \chi$, then $(\phi \vee \psi) \rightarrow \chi$
- In DS, both disjuncts truth-conditionally entail an existential statement [$x$], but we do not get the anaphoric potential of an existential
- Once a test, always a test... Let's see why

# No dynamic disjunction elimination

It's easy to understand why (17) is a counterexample — disjunction in DS is externally static.

(17) a. A professor or an assistant professor will attend the meeting of the university board.
b. She will report to the faculty.

(33) $$[\![([x]\ ;\ [\mathit{ProfAttend}(x)]) \vee ([x]\ ;\ [\mathit{AsstAttend}(x)])]\!]^{w,g} = \{\, g \mid I(\mathit{ProfAttend})(w) \neq \emptyset \vee I(\mathit{AsstAttend})(w) \neq \emptyset \,\}$$

- Does not predict anaphora
- Intuitively, *both* indefinites serve as antecedents to the anaphoric pronoun *she*.

# Program disjunction

There is a way to account for program disjunctions within the constraints of classical DS, but it requires positing a new logical connective — program disjunction.

(34) $[\![\phi \cup \psi]\!]^{w,g} = [\![\phi]\!]^{w,g} \cup [\![\psi]\!]^{w,g}$

- Program disjunction simply involves computing an output set that is the *union* of the output sets of each disjunct.
- Note especially that program disjunction isn't (guaranteed to be) a test, which means that program disjunctions can pass on anaphoric information.
- Let's see how this works in more detail...

# Program disjunction in detail

(35) $[\![([x]\ ;\ [ProfAttend(x)]) \cup ([x]\ ;\ [AsstAttend(x)])]\!]^{w,g} =$
$\{\, h \mid g[x]h, h(x) \in I(ProfAttend)(w)\,\} \cup \{\, h \mid g[x]h, h(x) \in I(AsstAttend)(w)\,\}$
$\{\, h \mid g[x]h, h(x) \in I(ProfAttend)(w) \vee I(AsstAttend)(w)\,\}$

(36) $[\![(([x]\ ;\ [ProfAttend(x)]) \cup ([x]\ ;\ [AsstAttend(x)]))\ ;\ [Report(x)]]\!]^{w,g} =$
$\{\, h \mid g[x]h, h(x) \in I(ProfAttend)(w) \cup I(AsstAttend)(w), h(x) \in I(Report)(w)\,\}$

- So long as both indefinites carry the same variable index (in this case $x$), the output of the disjunction is guaranteed to be one in which $x$ is introduced as a discourse referent.
- Discourse anaphora is licensed in the usual way.
- **Question:** does this account violate any constraints on semantic representations we've been assuming so far?

# Program disjunction and external staticity

So, why not just say that natural language disjunction is *always* translated as program disjunction?

- Remember our motivation of external staticity:

(37) Either a professor is attending the meeting, or it was cancelled.
???She will report it to the faculty.

- Program disjunction predicts that anaphora is possible, at a world $w_a$ where the meeting took place and $a$, a professor, attended, because program disjunction amounts to an existential statement.

(38) $[\![([x]\,;\,\mathit{ProfAttend}(x)) \cup \mathit{Cancelled}]\!]^{w_a,g} = \{\, h \mid g[x]h, h(x) \in I(\mathit{ProfAttend})(w) \,\}$

Since $[\![\mathit{Cancelled}]\!]^{w,g} = \begin{cases} g & \text{there no meeting in } w \\ \emptyset & \text{otherwise} \end{cases}$

# Summing up program disjunction

We’re left in a rather uncomfortable position:

- When an “or” sentence involves two ‘parallel’ existential statements, it is translated as program disjunction.
- Otherwise, an “or” sentence is translated as externally static DS disjunction.

This exposes a conceptual flaw with DS — both program and ordinary disjunction capture the truth-conditional contribution of disjunction, but the anaphoric information expressed is different.

Anaphoric information is not straightforwardly constrained by truth-conditional contribution — there isn’t a ‘recipe’ to determine the anaphoric contribution of a given logical operator.

# Towards a generalization

So what do these three cases have in common?

(15) a. *It's not the case that there isn't a bathroom.*
b. *It is upstairs.*

(16) a. *There might be a bathroom in this house.*
b. *In fact, I just remembered that's the case.*
c. *It is upstairs.*

(17) a. *A professor or an assistant professor will attend the meeting.*
b. *She will report to the faculty.*

And how are they different from cases with negation, modals, and disjunction that do not allow for anaphora?

(39) a. *There isn't a bathroom in this house.*
b. *#It is upstairs.*

(40) a. *There might be a bathroom in this house.*
b. *#It is upstairs.*

(41) a. *A professor attended the meeting or the meeting was cancelled.*
b. *#She will report to the faculty.*

- We need truth-conditional inferences to influence anaphoric accessibility
- On the left side, the existence of a witness to the existential $x$ is contextually entailed
- On the right side, that is not the case
- This generalization will serve as motivation for the solution we propose tomorrow

# Bathroom-sentences and modal subordination: *Local* contextual entailment

# Bathroom disjunctions

Recall that disjunction in DS is internally static — anaphoric dependencies between disjuncts are (predicted to be) impossible.

- Surprisingly, a pronoun in the second disjunct may be anaphoric to a negated existential in the first disjunct

(42) Either there is no$^x$ bathroom, or it$_x$'s upstairs.

(43) Either Steven didn't order a$^x$ pizza, or it$_x$'s warming in the oven.

- We call such cases bathroom disjunctions (Roberts 1987, attributed to Barbara Partee)
- **Question:** Why do you think anaphora is possible here?

# Bathroom disjunctions cont.

It's quite easy to see why DS doesn't capture bathroom disjunctions; DS disjunction simply tests whether either disjunct has a non-empty output at the input.

(44) $$[\![\neg([x]\,;[\mathit{Bathroom}(x)])\vee[\mathit{Upstairs}(x)]]\!]^{w,g}$$
$$=\{\,g\mid[\![\neg([x]\,;[\mathit{Bathroom}(x)])]\!]^{w,g}\neq\emptyset\vee[\![\mathit{Upstairs}(x)]\!]^{w,g}\neq\emptyset\,\}$$
$$=\{\,g\mid I(\mathit{Bathroom})(w)\neq\emptyset\vee g(x)\in I(\mathit{Upstairs})(w)\,\}$$

Consequently, the interpretation of the variable in the second conjunct is dependent on the assignment of the context.

# Classical equivalence 1: Interdefinability of disjunction and implication

There's a sense in which it isn't surprising that anaphoric dependencies are possible in bathroom disjunctions.

- Interdefinability of disjunction and implication in classical logic: $(\neg\phi \vee \psi) \Leftrightarrow (\phi \rightarrow \psi)$

So, we might have expected bathroom disjunctions to be equivalent to corresponding donkey sentences:

(45) a. *Either there's no bathroom or it's upstairs.*
b. *If there's a bathroom, then it's upstairs.*

Intuitively, the two sentences *do* feel equivalent, but classical DS doesn't capture this.

- Here: $(\neg\phi \vee \psi) \not\Leftrightarrow (\phi \rightarrow \psi)$

# No interdefinability of disjunction and implication

**Disjunction:** $(\phi \vee \psi) := \neg(\neg\phi; \neg\psi)$

i.e. $[\![\phi \vee \psi]\!]^{w,g} = \{g \mid$ there exists some $k$, s.t. $k \in [\![\phi]\!]^{w,g}$, or $k \in [\![\psi]\!]^{w,g}\}$

**Implication:** $(\phi \rightarrow \psi) := \neg(\phi; \neg\psi)$

i.e. $[\![\phi \rightarrow \psi]\!]^{w,g} = \{g \mid$ for every $k$: if $k \in [\![\phi]\!]^{w,g}$, there exists some $j$, s.t. $j \in [\![\psi]\!]^{w,k}\}$

**Disjunction with negated first disjunct:** $(\neg\phi \vee \psi) \Leftrightarrow \neg(\neg\neg\phi; \neg\psi)$

- Again, this boils down to the absence of double negation elimination
- Semantic representation (/logical form) prevents internal anaphora
- Truth-conditional inferences cannot salvage this

# Classical equivalences 2: Negating the first disjunct

It seems that anaphora is possible in bathroom disjunctions, because (in some sense), the second disjunct is interpreted in light of the negation of the first.

(46) *Either there's no bathroom, or it's not true that there's no bathroom and it's upstairs.*

**Under this approach:** The negation of the first disjunct provides a *local context* of interpretation for the second disjunct

We would be taking advantage the classical equivalence in (47):

(47) $\phi \vee \psi \Leftrightarrow \phi \vee (\neg\phi \wedge \psi)$

Therefore:

(48) $\neg\phi \vee \psi \Leftrightarrow \phi \vee (\neg\neg\phi \wedge \psi)$

But again, this won't work, because once we negate the first disjunct, we have a test!

- If we *were* to develop a system in which doubly-negated sentences are equivalent to their positive counterparts wrt to anaphoric information, either of this strategies could be viable.
- Accounting for bathroom disjunctions, then, is closely tied to the problematic status of negation in DS.

# A generalization

(18) *Either there isn't a bathroom in this house, or it's upstairs.*

So... Does this fall under our previous generalization: Does this entail the existence of a referent to the indefinite?

...What then?

- Bathroom-anaphora are possible, *if the negation of the first disjunct entails a witness to the existential* (Evans 1977, Barbara Partee).
- Here, the negation of "there is no bathroom" entails the existence of a bathroom, so anaphora is possible.
- Local context: Remember the idea that the negation of the first disjunct is assumed to provide a context of interpretation for the second disjunct
- A generalization: The existential is entailed in the local context of the second disjunct

# Disjunctive syllogism

As it happens, we can even formulate cases where a pronoun can be anaphoric to an indefinite inside of an (ordinarily externally static) disjunction.

(49) Either it's a holiday or a customer$^x$ will come in. And if it's not a holiday, they$_x$ will want to be served. (Rothschild 2017)

Classical inference rule:

- Disjunctive syllogism: $((\phi \vee \psi) \wedge \neg\phi) \Leftrightarrow \psi$
- In DS: $((\phi \vee \psi); \neg\phi) \not\Leftrightarrow \psi$
- We do get: $((\phi \vee \psi); \neg\phi) \Leftrightarrow \neg\neg\psi$

- It's often assumed that the consequent of a conditional, is interpreted in light of the antecedent.
- Putting the disjunction together with the conditional antecedent locally entails the truth of the disjunct that introduces a discourse referent.

# Modal subordination

# Modal subordination

If an anaphor is itself embedded under a modal, it can have an antecedent in an externally static context:

(19) *There might be a bathroom in this house. It would be upstairs.*

- This is also not predicted in our logic of change
- **Question:** Why is anaphora possible here?

There is an intuition that the modal statement is interpreted like an implication:

(50) *There might be a bathroom in this house. If there is a bathroom, then it would be upstairs.*

- We can adress this in light of out generalization of local contextual entailment of a referent

# Interim summary

Our generalizations so far:

- Double negation, discourse-inferences, and program disjunction: The existence of a referent is entailed in the context
- Bathroom-disjunction and modal subordination: The existence of a referent is entailed in the local context
- Upcoming: another case showing that this inference of local contextual entailment can be accommodated

# Inter-speaker disagreement: Accommodation of contextual entailment

# Inter-speaker disagreement

(20) A: *There isn't a bathroom in this house.*
B: *(What are you talking about?) It's upstairs.*

Here, we have an inference on the level of multi-speaker discourse: *B* disagrees with *A*.

- The semantic representation of externally static negation rules this out
- Interpreting *B*'s utterance as contradicting *A* cannot solve this, as long as we define accessibility on the level of semantic representations
- But truth-conditionally, B's utterance implies that there is a bathroom
- A possible result: The inference of contextual entailment of a witness can be accommodated

# Accessibility and licensing — When pronouns are possible

# Contextual entailment of a witness

- Double negation, discourse-inferences, and program disjunction
- Bathroom-disjunction and modal subordination
- Inter-speaker disagreement

The existence of a referent is entailed in the local context (this inference can be contributed by the utterance containing the anaphor)

| | internally | externally | veridicality? |
|---|---|---|---|
| $\phi \wedge \psi$ | dynamic | dynamic | veridical |
| $\neg\phi$ | N/A | static | non-veridical |
| $\Diamond\phi$ | N/A | static | non-veridical |
| $\phi \rightarrow \psi$ | dynamic | static | non-veridical |
| $\phi \vee \psi$ | static | static | non-veridical |

Accounts for the basic generalizations about veridical operators, and out counterexamples

# Inference and form

Our takeaways:

- The notion of accessibility in DS relies too much on linguistic form,
- It needs to be possible to have truth-conditional inferences affect accessibility

But: entailment of a referent alone is not enough

(51) *David is married. #He's also invited.*

(52) *David has a spouse. He's also invited.*

We also need an overt antecedent. (This is Heim's Formal Link Condition)

DS posits a richer notion of information state which is sensitive to the presence/absence of an indefinite.

# Conclusions

- We need discourse referents to be introduced explicitly by some (indefinite) DP.
- But we also need truth-conditional inferences to affect accessibility.
- Modal subordination shows that any externally static operator can provide antecedents under the right conditions.
- We may want to get rid of externally static operators altogether, and derive anaphoric accessibility from the truth-conditional properties of our sentential operators

# Truth-conditions of DN and BR anaphora

# Gotham 2019 on uniqueness

- Gotham (2019) pushes back against the conjecture that $\neg\neg\phi$ is truth-conditionally and anaphorically equivalent to $\phi$, offering the following contrast (p. 144):

(53) a. John owns a shirt. It's in the wardrobe.
  b. ??It's not true that John doesn't own a shirt. It's in his wardrobe.

- Gotham's empirical claim:
  - Let's assume that it's contextually entailed that *if someone owns shirts, they own more than one*.
  - (53a) is perfectly acceptable, since the existential statement is compatible with John owning multiple shirts.
  - (53b) is odd, because unlike its positive counterpart, the doubly-negated statement entails that *John owns exactly one shirt*.

A little historical background: prior to the development of DS, the main account of donkey and discourse anaphora was the E-type theory of Evans 1977, according to which pronouns are interpreted as *definite descriptions*.

(54) John owns a shirt. [It=**the shirt**] is in the wardrobe.

One of the main arguments against Evans E-type theory was provided by Heim 1982; Definite descriptions semantically encode uniqueness, Heim's *sage plant* sentences involve donkey/discourse anaphora but are logically incompatible with uniqueness.

(55) a. Steven ordered a$^x$ pizza, and he$_x$ ordered eight others along with it.
b. If Steven orders a$^x$ pizza, he orders eight others along with it$_x$.

# Sage plant sentences cont.

Although not the focus of our course, it's worth noting that the E-type account was subsequently improved to accommodate the absence of uniqueness by Heim1990, but limited to donkey anaphora.

Nevertheless, sage plant sentences are extremely useful for testing for (semantically-encoded) uniqueness inferences (Mandelkern & Rothschild 2020).

We can use sage plant sentences to show that doubly-negated sentences don't semantically encode uniquess:

(56) It's not true that Steven didn't order $a^x$ pizza,
and furthermore he ordered eight others along with $it_x$.

# Takeaway: uniqueness (or lack thereof)

Sage plant sentences suggest that doubly-negated existential statements have the same truth-conditions as their positive counterparts.

We don't however deny that there is a *reading* that entails (perhaps implicates?) uniqueness.

This is muddied by the fact that singular indefinites in positive contexts often implicate uniqueness (Spector2007; cf. Gotham).

(57) Steven ordered a pizza. $\Rightarrow$ *Steven ordered exactly one pizza (as far as I know)*

We won't have anything concrete to say in this course about how to derive this (optional) inference, or whether doubly-negated sentences have more robust uniqueness inferences than their positive counterparts.

# Truth-conditions of bathroom disjunctions

- We haven't addressed the question of what the actual *truth-conditions* of bathroom disjunctions
- An analysis should not just account for the possibility of anaphora, but also get the truth-conditions right.
- In the following two slides, we'll argue against two notable claims concerning the truth-conditions of bathroom disjunctions:
  - Uniqueness entailments (Gotham 2019).
  - Universal readings (Krahmer & Muskens 1995).

# Bathroom disjunctions and uniqueness

- Similarly to double-negation, Gotham (2019) suggests that bathroom sentences are associated with uniqueness inferences, on the basis of the following contrast (Gotham's judgement).

  (58) a. John owns a shirt and it's in his wardrobe.
  b. ???Either John doesn't own a shirt, or it's in his wardrobe.

- **Exercise:** formulate a sage plant sentence to test whether or not uniqueness is semantically entailed in bathroom disjunctions.

# Bathroom disjunctions and uniqueness cont.

Here's one we made earlier:

(59) Either Steven didn't order a pizza, or he ordered 8 others along with it.

This sentence is logically incompatible with the putative (conditional) uniqueness entailment *if Steven ordered a pizza then he ordered exactly one* (see also Mandelkern & Rothschild 2020 on definite descriptions).

The truth-conditions of bathroom disjunctions have also been discussed by Krahmer & Muskens (1995), who suggest that bathroom disjunctions have a *universal* reading.

(60) Either there's no bathroom, or it's upstairs.
$\Rightarrow$ *every bathroom (if any) is upstairs*

(It's interesting to note that this is what we expect given the conditional equivalence, due to Egli's corrolary, i.e., $\phi \vee (\neg\phi \rightarrow \psi)$).

Krahmer & Muskens report that (60) is *false* as soon as there's a bathroom that isn't upstairs.

**Exercise:** come up with a concrete example paired with a context designed to test this claim.

# Existential readings?

Elliott2023a argues that bathroom disjunctions *can* in fact have existential readings, meaning that universal truth-conditions are too strong.

(61) *Context: we're wondering how Gabe paid for dinner.*
Either Gabe doesn't have a credit card, or he paid with it.

The intuition here is that the sentence is *true* just so long as Gabe paid with a credit card of his, even if he has other credit cards which he didn't pay with.

(N.b., existential readings are a natural expectation if we opt for the conjunctive equivalence, i.e., $\phi \vee (\neg\phi \wedge \psi)$).

Exercise: can you come up with a context in which the sentence involving bathrooms receives an existential reading?

# Recommended reading

- Gotham. “Double Negation, Excluded Middle and Accessibility in Dynamic Semantics”. 2019.
- Krahmer & Muskens. “Negation and Disjunction in Discourse Representation Theory”. 1995

Questions?

Evans, Gareth. 1977. **Pronouns, Quantifiers, and Relative Clauses (I).** *Canadian Journal of Philosophy* 7(3). Publisher: [Taylor & Francis, Ltd., Canadian Journal of Philosophy] JSTOR: 40230703, 467–536.

Gotham, Matthew. 2019. **Double Negation, Excluded Middle and Accessibility in Dynamic Semantics.** In *Proceedings of the 22nd Amsterdam Colloquium*. Amsterdam.

Groenendijk, Jeroen & Martin Stokhof. 1991. **Dynamic Predicate Logic.** *Linguistics and Philosophy* 14(1). JSTOR: 25001418, 39–100.

Heim, Irene. 1982. ***The Semantics of Definite and Indefinite Noun Phrases.*** University of Massachusetts Amherst Doctoral dissertation.

Hofmann, Lisa. 2019. **The Anaphoric Potential of Indefinites under Negation and Disjunction.** In *Proceedings of the 22nd Amsterdam Colloquium*, 181–190.

Karttunen, Lauri. 1976. **Discourse Referents.** In *Notes from the linguistic underground*, 363–385. Brill. `https://doi.org/10.1163/9789004368859\_021`.

Krahmer, Emiel & Reinhard Muskens. 1995. Negation and Disjunction in Discourse Representation Theory. en. *Journal of Semantics* 12(4). 357–376. (6 March, 2019).

Mandelkern, Matthew & Daniel Rothschild. 2020. Definiteness Projection. *Natural Language Semantics* 28(2). 77–109. `https://doi.org/10.1007/s11050-019-09159-2`. `https://doi.org/10.1007/s11050-019-09159-2` (5 February, 2022).

Roberts, Craige. 1987. *Modal Subordination, Anaphora and Distributivity.* University of Massachusetts, Amherst Doctoral dissertation.

Rothschild, Daniel. 2017. A Trivalent Account of Anaphora and Presupposition. In *Proceedings of the 21st amsterdam colloquium*, vol. 21, 1–13. 21st Amsterdam Colloquium.