

Semantics in Generative Grammar

Notes from Class 1

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1 Reading for next week

- Chapters 1 and 2 of (Heim & Kratzer 1998), i.e., up to the end of **Executing the Fregean Program**.

2 Truth-conditions and compositionality

- Conjecture: to know the meaning of a sentence is to know its *truth-conditions* (Alfred Tarski).

(1) The sentence “snow is white” is true if and only if snow is white.

- More general schema (as long as both the object language and the meta-language are the same):

(2) The sentence “_____” is true if and only if _____

- Is this trivial/circular?
 - One way of understanding this conjecture: semantic competence involves “knowing” the associations between particular sentences and ways in which the world can be (as described by the meta-language).
 - Crucially, we know the truth-conditions of sentences we’ve never heard before.
 - Semantic competence can’t amount to just learning associations of sentences-meanings, since the meanings of sentences relate to the meanings of their parts in systematic ways (Gottlob Frege).
- How are meanings put together? Frege’s conjecture was that certain meanings are ‘incomplete’; in his terms *unsaturated*.

“Statements in general, just like equations or inequalities or expressions in Analysis, can be imagined to be split up into two parts; one complete in itself, and the other in need of supplementation, or “unsaturated.” Thus, e.g., we split up the sentence “Caesar conquered Gaul” into “Caesar” and “conquered Gaul.” The second part is “unsaturated” - it contains an empty place; only when this place is filled up with a proper name, or with an expression that replaces a proper name, does a complete sense appear. Here too I give the name “function” to what this “unsaturated” part stands for. In this case the argument is Caesar.”

– Frege

- Frege models the notion of an “unsaturated” meaning as a function looking for an argument.

3 Formal preliminaries

3.1 Sets

- A set A is a collection of objects called *members* or *elements* of the set; x is an element of A is written $x \in A$.
- Sets may have finitely, or infinitely many members.
- Sets are identified with their members; there is exactly one set with no members: the *empty set*, written \emptyset .

3.2 Set relations

- Two sets are equal $A = B$ if A and B have the same members.
 - The order in which members are written doesn't matter for set equality.
- A is a subset of B $A \subseteq B$, if $\forall x \in A, x \in B$.
- Which of the following statements (if any) are true?
 - For any set A , $\emptyset \in A$.
 - For any set A , $\emptyset \subseteq A$.
 - For any set A , $A \subseteq A$.
 - A is a *strict* subset of B , $A \subset B$, if $A \subseteq B$, and $A \neq B$.
 - A is a superset of B , $A \supseteq B$, if $B \subseteq A$.
 - * Strict superset $A \supset B$ has the obvious definition (what is it?).

3.3 Set operations

- $x \in A \cap B$ (the intersection of A and B) iff $x \in A$ and $x \in B$.
- $x \in A \cup B$ (the union of A and B) iff $x \in A$ or $x \in B$.
- $x \in B - A$ (the complement of A in B) iff $x \in B$ and $x \notin A$.

3.4 Defining sets

- Sets can be defined extensionally by explicitly listing their members:

$$(3) \quad A := \{a, b, c\}$$

- Sets can be defined intensionally using abstraction notation:

$$(4) \quad A := \{x : x \text{ is a cat}\}$$

- $a \in \{x \mid \phi\}$ iff $\phi[x \rightarrow a]$ is true.¹

3.5 Questions

- If the x in $\{x : x \text{ is a positive integer less than } 7\}$ is a place-holder, why do we need it at all? Why don't we just write $\{_ : _ \text{ is a positive integer less than } 7\}$?
- Consider the following sets. Which one(s) corresponds to *the set of objects which don't like anything*, and which one corresponds to *the set of objects which nothing likes*?

$$- \{x : \{y : x \text{ likes } y\} = \emptyset\}$$

$$- \{x : \{y : y \text{ likes } x\} = \emptyset\}$$

$$- \{y : \{x : x \text{ likes } y\} = \emptyset\}$$

- Why do we need the variable to the left of the colon? Why can't we just write $\{x \text{ is a positive integer less than } 7\}$?
- What does the following mean? $\{\text{California} : \text{California is a western state}\}$
- What about the following? $\{x : \text{California is a western state}\}$
- Evaluate whether the following is true, and show your reasoning: $29 \in \{x : x \in \{x : x \neq \emptyset\}\}$.

¹ $\phi[x \rightarrow a]$ is to be understood as the statement ϕ , where all occurrences of the variable x in ϕ have been replaced with a .

3.6 Exercise

In each case, say whether or not the equality holds:

1. $\{a\} = \{b\}$
2. $\{x : x = a\} = \{a\}$
3. $\{x : x \text{ is green}\} = \{y : y \text{ is green}\}$
4. $\{x : x \text{ likes } a\} = \{y : y \text{ likes } b\}$
5. $\{x : x \in A\} = A$
6. $\{x : x \in \{y : y \in B\}\} = B$
7. $\{x : \{y : y \text{ likes } x\} = \emptyset\} = \{x : \{x : x \text{ likes } x\} = \emptyset\}$

3.7 Functions

- An *ordered pair* of x, y , written $\langle x, y \rangle$.
 - $\langle x, y \rangle \neq \langle y, x \rangle$ (if $x \neq y$).
 - A *relation* is a set of ordered pairs.
 - A *function* is a special kind of relation.
- (5) **Functions (def.):** A relation f is a *function* iff it satisfies the following condition: for any x : if there are y, z s.t., $\langle x, y \rangle \in f$, and $\langle x, z \rangle \in f$, then $y = z$.
- The *domain* of a function f is $\{x : \text{there is a } y \text{ such that } \langle x, y \rangle \in f\}$.
 - The *range* of f is $\{y : \text{there is a } x \text{ such that } \langle x, y \rangle \in f\}$.
 - When A is the domain, and B the range of f , f is a function “from A onto B , written $f : A \rightarrow B$ ”.
- (6) **Mathematical notation for functions (def.):**
 $f(x) := \text{the unique } y \text{ s.t. } \langle x, y \rangle \in f$
- $f(x)$ is pronounced “ f applied to x ”, or “ f of x ”. We’ll usually write $f(x) = y$ instead of (equivalently) $\langle x, y \rangle \in f$.
 - Two different ways of writing functions extensionally:

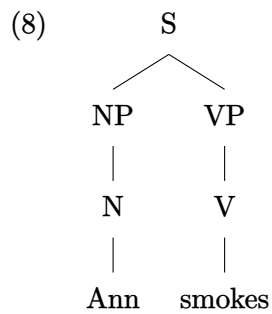
$$f := \{ \langle a, b \rangle, \langle c, b \rangle, \langle d, e \rangle \}$$

$$f := \begin{bmatrix} a \rightarrow b \\ c \rightarrow b \\ d \rightarrow e \end{bmatrix}$$

- Different ways of writing function *intensions*:

(7) Let f_{+1} be that function f s.t. $f : \mathbb{N} \rightarrow \mathbb{N}$, and for every $n \in \mathbb{N}$, $f(n) = n + 1$.

4 Executing the Fregean program



- Fregean denotation of a sentence: a truth value; a member of the set $\{0, 1\}$.
- Another word for a Fregean denotation: an *extension*.
- Extensions of proper names: individuals. The extension of “Ann” is **Ann**.

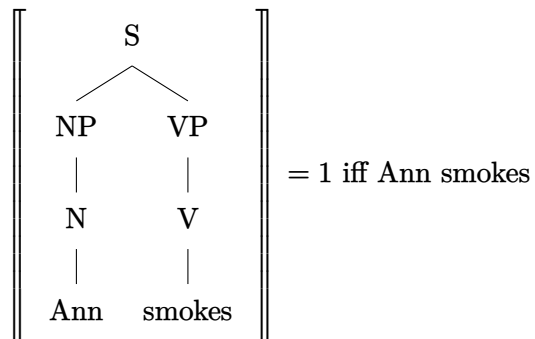
4.1 Components of a fragment

1. An inventory of denotations (“the model”).
 - Things that linguistic expressions can get mapped to.
2. The lexicon.
 - Denotations for each expressions which may occupy a terminal node.
3. Semantic rules.

- Rules that allow us to compute the determine the denotation for each kind o non-terminal node.
4. The *interpretation function* $\llbracket \cdot \rrbracket$ is a function from the set of linguistic expression onto the set of possible denotations.
- By giving a total definition of the interpretation function for (i) lexical items, and (ii) non-terminal nodes, we provide the lexicon and the semantic rules respectively.

4.2 A concrete example

Exercise: prove the following claim:



References

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar* (Blackwell Textbooks in Linguistics 13). Malden, MA: Blackwell. 324 pp.