Semantics in Generative Grammar

Notes from Class 1

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Contents

1 Reading for next week

- Chapters 1 and 2 of (Heim & Kratzer 1998), i.e., up to the end of **Executing the Fregean Program**.

2 Truth-conditions and compositionality

- Conjecture: to know the meaning of a sentence is to know its *truth-conditions* (Alfred Tarski).

(1) The sentence "snow is white" is true if and only if snow is white.

- More general schema (as long as both the object language and the meta-language are the same):

(2) The sentence "_____" is true if and only if _____

- Is this trivial/circular?
  - One way of understanding this conjecture: semantic competence involves "knowing" the associations between particular sentences and ways in which the world can be (as described by the meta-language).
  - Crucially, we know the truth-conditions of sentences we've never heard before.
  - Semantic competence can't amount to just learning associations of sentences-meanings, since the meanings of sentences relate to the meanings of their parts in systematic ways (Gottlob Frege).
- How are meanings put together? Frege's conjecture was that certain meanings are 'incomplete'; in his terms *unsaturated.*

  "Statements in general, just like equations or inequalities or expressions in Analysis, can be imagined to be split up into two parts; one complete in itself, and the other in need of supplementation, or "unsaturated." Thus, e.g., we split up the sentence "Caesar conquered Gaul" into " Caesar" and "conquered Gaul." The second part is "unsaturated" - it contains an empty place; only when this place is filled up with a proper name, or with an expression that replaces a proper name, does a complete sense appear. Here too I give the name "function" to what this "unsaturated" part stands for. In this case the argument is Caesar."
  – Frege

- Frege models the notion of an "unsaturated" meaning as a function looking for an argument.

3 Formal preliminaries

3.1 Sets

- A set A is a collection of objects called *members* or *elements* of the set; x is an element of A is written $x \in A$.
- Sets may have finitely, or infinitely many members.
- Sets are identified with their members; there is exactly one set with no members: the *empty set*, written $\emptyset$.

3.2 Set relations

- Two sets are equal $A = B$ if A and B have the same members.
  - The order in which members are written doesn't matter for set equality.
- A is a subset of B $A \subseteq B$, if $\forall x \in A$, $x \in B$.
- Which of the following statements (if any) are true?
  - For any set A, $\emptyset \in A$.
  - For any set A, $\emptyset \subseteq A$.
  - For any set A, $A \subseteq A$.
  - A is a *strict* subset of B, $A \subset B$, if $A \subseteq B$, and $A \neq B$.
  - A is a superset of B, $A \supseteq B$, if $B \subseteq A$.
    * Strict supersethood $A \sup B$ has the obvious definition (what is it?).

3.3 Set operations

- $x \in A \cap B$ (the intersection of A and B) iff $x \in A$ and $x \in B$.
- $x \in A \cup B$ (the union of A and B) iff $x \in A$ or $x \in B$.
- $x \in B - A$ (the complement of A in B) iff $x \in B$ and $x \notin A$.

3.4 Defining sets

- Sets can be defined extensionally by explicitly listing their members:

(3) $A := \{ a, b, c \}$

- Sets can be defined intensionally using abstraction notation:

(4) $A := \{ x : x \text{ is a cat} \}$

- $a \in \{ x \mid \phi \}$ iff $\phi[x \to a]$ is true.[1]

3.5 Questions

- If the x in $\{ x : x \text{ is a positive integer less than } 7 \}$ is a place-holder, why do we need it at all? Why don't we just write $\{ \_ : \_\text{is a positive integer less than } 7 \}$?
- Consider the following sets. Which one(s) corresponds to *the set of objects which don't like anything*, and which one corresponds to *the set of objects which nothing likes*?
  - $\{ x : \{ y : x \text{ likes } y \} = \emptyset \}$
  - $\{ x : \{ y : y \text{ likes } x \} = \emptyset \}$
  - $\{ y : \{ x : x \text{ likes y} \} = \emptyset \}$
- Why do we need the variable to the left of the colon? Why can't we just write $\{ x$ is a positive integer less than
- What does the following mean? $\{ \text{California} : \text{California is a western state} \}$
- What about the following? $\{ x : \text{California is a western state} \}$
- Evaluate whether the following is true, and show your reasoning: $29 \in \{ x : x \in \{ x : x \neq \emptyset \} \}$.

[1] $\phi[x \to a]$ is to be understood as the statement ϕ, where all occurrences of the variable x in ϕ have been replaced with a.

3.6 Exercise

In each case, say whether or not the equality holds:

1. $\{a\} = \{b\}$
2. $\{x : x = a\} = \{a\}$
3. $\{x : x \text{ is green}\} = \{y : y \text{ is green}\}$
4. $\{x : x \text{ likes } a\} = \{y : y \text{ likes } b\}$
5. $\{x : x \in A\} = A$
6. $\{x : x \in \{y : y \in B\}\} = B$
7. $\{x : \{y : y \text{ likes } x\} = \emptyset\} = \{x : \{x : x \text{ likes } x\} = \emptyset\}$

3.7 Functions

- An *ordered pair* of x, y, written $< x, y >$.
  - $< x, y > \neq < y, x >$ (if $x \neq y$).
- A *relation* is a set of ordered pairs.
- A *function* is a special kind of relation.

(5) **Functions (def.):** A relation f is a *function* iff it satisfies the following condition: for any x: if there are y, z s.t., $< x, y > \in f$, and $< x, z > \in f$, then $y = z$.

- The *domain* of a function f is $\{x : \text{there is a } y \text{ such that } < x, y > \in f\}$.
- The *range* of f is $\{x : \text{there is a } y \text{ such that } < y, x > \in f\}$.
- When A is the domain, and B the range of f, f is a function "from A onto B, written $f : A \rightarrow B$".

(6) **Mathematical notation for functions (def.):**
$f(x) :=$ the unique y s.t. $< x, y > \in f$

- $f(x)$ is pronounced "f applied to x", or "f of x". We'll usually write $f(x) = y$ instead of (equivalently) $< x, y > \in f$.
- Two different ways of writing functions extensionally:

$$f := \{ < a, b >, < c, b >, < d, e > \}$$

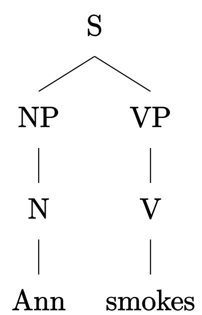
$$f := \begin{bmatrix} a \rightarrow b \\ c \rightarrow b \\ d \rightarrow e \end{bmatrix}$$

- Different ways of writing function *intensions*:

(7) Let f_{+1} be that function f s.t. $f : \mathbb{N} \rightarrow \mathbb{N}$, and for every $n \in \mathbb{N}$, $f(n) = n + 1$.

4 Executing the Fregean program

(8)



- Fregean denotation of a sentence: a truth value; a member of the set $\{0, 1\}$.
- Another word for a Fregean denotation: an *extension.*
- Extensions of proper names: individuals. The extension of "Ann" is **Ann**.

4.1 Components of a fragment

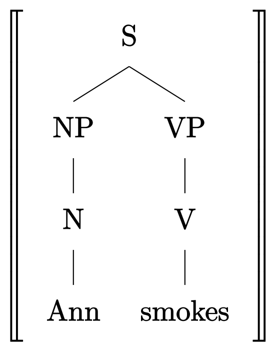
1. An inventory of denotations ("the model").
   - Things that linguistic expressions can get mapped to.
2. The lexicon.
   - Denotations for each expressions which may occupy a terminal node.
3. Semantic rules.

- Rules that allow us to compute the determine the denotation for each kind o non-terminal node.

4. The *interpretation function* $[\![.]\!]$ is a function from the set of linguistic expression onto the set of possible denotations.
   - By giving a total definition of the interpretation function for (i) lexical items, and (ii) non-terminal nodes, we provide the lexicon and the semantic rules respectively.

4.2 A concrete example

Exercise: prove the following claim:



= 1 iff Ann smokes

References

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar* (Blackwell Textbooks in Linguistics 13). Malden, MA: Blackwell. 324 pp.