Quantification

Handout 2

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Contents

1 Reading

- Chapter 6 of *Invitation to Formal Semantics*, especially the sections on quantifiers.

2 Quantifiers: the basics

We'd translate the following sentence into predicate logic as $\forall x, \mathbf{smiled}(x)$

(1) Everybody smiled.

What, exactly, is the contribution of *everybody*? Informally, it looks like some kind of template:

(2) $\forall x \,.\, \_(x)$

The predicate **smiled** fills in the slot.

We'll implement this idea in the lambda calculus by using a functional abstraction:

(3) **everybody** $:= \lambda P \,.\, \forall x \,.\, P(x)$

That means that the type of *everybody* is $(E \to T) \to T)$, i.e., a function from predicates to truth-values.

This is the type of a **quantifier**; as we'll see later on, this can also be the type of NPs more generally!

We can define the other quantifiers from predicate logic in a similar way:

(4) **something** $:= \lambda P \,.\, \exists x \,.\, P(x)$

(5) **nothing** $:= \lambda P \,.\, \neg\exists x \,.\, P(x)$

3 The type of quanificational NPs

Quantificational NPs in general seem to pattern with type E expressions, in terms of their distribution.

(6) A singer loves Frida.

(7) Frida loves a singer.

See also: *someone*, *everybody*, *nobody*, *some lingust*, *at least one linguist*, *at most one linguist*, *no linguist*, *few linguists*, etc.

Can quantificational NPs be of type E? In fact, we can in fact show that they can't be.

3.1 Subset-to-superset inferences

(8) Susan came yesterday morning.
$\Rightarrow$ Susan came yesterday.

> Why does this hold?

Is the inference valid in the following case?

(9) At most one letter came yesterday morning. $\Rightarrow_?$ At most one letter came yesterday.

> **Exercise:** figure out which (if any) quantificational NPs validate subset to superset inferences.

3.2 Law of non-contradiction

(10) Mont Blanc is higher than 4000m, and Mont Blanc is not higher than 4000m.

(11) More than two mountains are higher than 4000m, and more than two mountains are not higher than 4000m.

> **Exercise:** again, figure out which (if any) quantificational NPs validate the law of non-contradiction.

4 Predicates of predicates

The solution, as we've alluded to, is to treat quantifiers as expressions of type $(E \to T) \to T$.

What does this mean for their meaning?

A helpful way to think about what quantifiers do exploits a correspondence between functions $f : \mathbf{Dom}_\sigma \mapsto \{\,\mathbf{true}, \mathbf{false}\,\}$, and *sets* of things in $\mathbf{Dom}_\sigma$.

We can define the following translation; $Set(f)$ is called the *set characterized by* f, and we can freely switch between sets and functions without losing information:

$$Set(f) = \{\, x \mid f(x) = \mathbf{true} \,\}$$

This allows us to think of predicates, for example, as denoting *sets of individuals.*

Quantifiers, on the other hand, denote *sets of predicates.*

As an example, consider *everything* and *nothing*:

(12) $[\![\textbf{everything}]\!] = \{\, f \mid \forall x \in \mathbf{Dom}_E, f(x) = \mathbf{true} \,\}$

Alternatively, if we think of the functions in the denotation of /everything as themselves sets:

(13) $[\![\textbf{everything}]\!] = \{\, X \mid \mathbf{Dom}_E \subseteq X \,\}$

> **Exercise:** what about *something* and *nothing*?

5 Determiners

Based on what we know about the type of NPs, and the type of quantifiers, we can conclude what the type of a determiner such as *some*, *no*, and *every* should be:

$$(E \to T) \to (E \to T) \to T$$

Determiners denote functions from predicates to quantifiers.

In quasi-predicate-logic notation, we can write the following:

(14) $\textbf{every} := \lambda R \,.\, \lambda P \,.\, \forall x[R(x) \to P(x)]$

6 Generalized quantifiers

What is the meaning of *every cat*?

In predicate logic, it would be something like the following:

(15) $\lambda P\,.\,\forall x[\mathbf{cat}(x) \rightarrow P(x)]$

What set of sets does this quantifier characterize?

(16) $\{\, P \subseteq \mathbf{Dom}_E \mid \{\, x \mid x \text{ is a cat} \,\} \subseteq P \,\}$

some dog:

(17) $\lambda P\,.\,\exists_x[\mathbf{dog}(x) \wedge P(x)]$

(18) $\{\, P \subseteq \mathbf{Dom}_E \mid \{\, x \mid x \text{ is a dog} \cap P \neq \emptyset \,\} \,\}$

We can derive *everything* and *something*, by replacing *cat* and *dog* with $\mathbf{Dom}_E$.

The strategy of characterizing quantifiers in terms of sets of sets generalizes to any quantificational expression.

(19) $[\![\mathbf{nothing}]\!] = \{\, P \subseteq \mathbf{Dom}_E \mid P = \emptyset \,\}$

(20) $[\![\mathbf{exactlyTwoThings}]\!] = \{\, P \subseteq \mathbf{Dom}_E \mid \mathbf{Card}(P) = 2 \,\}$

(21) $[\![\mathbf{atleastTwoThings}]\!] = \{\, P \subseteq \mathbf{Dom}_E \mid \mathbf{Card}(P) \geq 2 \,\}$

Sets of sets of entities are called **generalized quantifiers**.

Determiners characterize curried *relations* between sets, we call such relations between sets **determiner relations**.

(22) $[\![\mathbf{every}]\!] = \{\, (R, P) \mid R \subseteq P \,\}$

(23) $[\![\mathbf{some}]\!] = \{\, (R, P) \mid R \cap P \neq \emptyset \,\}$