From types to typeclasses

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Next week we'll start implementing *linguistic* data structures using Haskell.

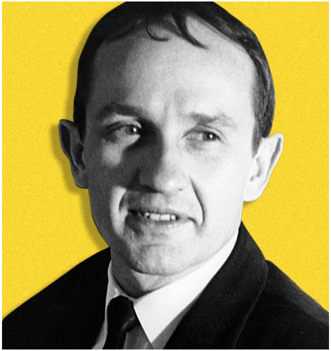
In preparation, you can read Chapter 4 of *Computational Semantics with Functional Programming* ”Formal syntax for fragments”.

Interlude: the indirect approach to semantics

In haskell, *sum types* can be used to model primitive types with fixed domains of entities:

```haskell
data E = John | Mary | Bill | Sue
```

In a sense, we're modelling a small fragment of English proper names "John", "Mary", "Bill", "Sue", interpreted as individual denoting constants.



”There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed I consider it possible to comprehend the syntax and semantics of both kinds of languages with a single natural and mathematically precise theory.” (Montague, Richard, 1970)

- Montague developed an influential technique for giving a denotational semantics for a *fragment* of a natural language, like English.
    - Montague's idea (inspired by the philosoher Gottlob Frege) was to first *translate* sentences of English into sentences of a logical language.
    - A denotational semantics could then be provided for the logical language quite straightforwardly.
    - This is called the *indirect approach* to compositional semantics.

Montague used *Intensional Logic* as the logical language, but more contemporary work in semantics typically uses the *Simply-Typed Lambda Calculus*, either implicitly or explicitly (Carpenter, Bob, 1998).

- every $\rightsquigarrow \lambda R.\lambda P.\mathbf{every}(R)(P) : (E \to T) \to T$
- and $\rightsquigarrow \lambda t.\lambda u.\mathbf{and}(t)(u) : T \to T \to T$
- Josie $\rightsquigarrow \mathbf{josie} : E$

The indirect approach cont.

$$\dfrac{\dfrac{\mathbf{every} : (E \to T) \to (E \to T) \to T \quad \mathbf{ling} : E \to T}{\mathbf{every(ling)} : (E \to T) \to T} \qquad \dfrac{\mathbf{love} :: E \to E \to T \quad \mathbf{haskell} :: E}{\mathbf{love(haskell)} : E \to T}}{\mathbf{every(ling)(love(haskell))} : T}$$

every · linguist · loves · Haskell

- The truth-conditions of the sentence are provided by the denotational semantics of the logical language.

$$[\![\mathbf{ling}]\!] = \{\, (x, \mathbf{True}) \mid x \text{ is a linguist} \,\} \cup \{\, (x, \mathbf{False}) \mid x \text{ isn't a linguist} \,\}$$

> *There is no important theoretical difference between natural languages and the artificial languages of programmers.*

- A modern (re-)rendering of Montague's conjecture:
  - English sentences can be systematically translated into *haskell expressions*.
  - Determining whether or not the sentence is true amounts to *normalization*.

```
and(even(4))(odd(2))
-- False
```

Typeclasses

Recall our basic type for individuals.

```
data E = John | Mary | Bill | Sue
```

We haven't given ghc any further information about this type, so there's not much we can do with it. See what happens if you evaluate the following:

```
John == John
```

What about the following:

```
True == True
```

The reason for the contrast here is that `Bool` by default is an instance of the type class `Eq`, which is the class of types that contain things that can be compared and determined to be equal in value.

```
type Bool :: *
data Bool = False | True
        -- Defined in ‘GHC.Types’
instance Eq Bool -- Defined in ‘GHC.Classes’
```

Since we didn’t explicitly say that `E` is an instance of `Eq`, ghc doesn’t assume that it is (you can try typing in `:i E` into ghci to verify this).

Likewise, try evaluating the following in ghci. What do you think is responsible for the different results?

```
ghci> John
```

```
ghci> True
```

Deriving typeclasses

We’ll learn later on how to declare our own typeclass instances, but in the meantime ghc has convenient mechanisms for automatically generating sensible typeclass instances for simple types.

```
data E = John | Mary | Bill | Sue deriving (Eq,Show)
```

- By default, an `Eq` instance for a sum type considers data constructors to be identical only to themselves.
- The default `Show` instance for a sum type simply converts the name of the data constructor into a printable string.

Constrained polymorphism

- Inspect the type of `id`.
- Now inspect the type of `(==)`, which is a function that tests for equality.
    - Polymorphism is used to constrain typeclasses.
    - The more typeclass constraints we add to a polymorphic type signature, the more assumptions the polymorphic function can make about its arguments.

- Typeclass constraints are applied to types using the `=>` syntax at the beginning of the type signature.
- Multiple typeclass constraints are separated by commas in parentheses.
- Typeclass constraints are interpreted *conjunctively*, e.g.,

```
f :: (Class1 a, Class2 a) => a -> a
```

This is interpeted as a *universal quantification* over types, where the *restriction* of the universal is provided by the typeclasses:

$$\forall t \in \mathbf{Typ}[(t \in \mathbf{Class1} \wedge t \in \mathbf{Class2}) \rightarrow f : (t \rightarrow t)]$$

- `=>` is a binding operator; it comes with a variable, and binds all matching variables in its scope.
- Typeclass constraints with different variables restrict different universal quantifiers.

```
f2 :: (Class1 a, Class2 b) => a -> b
```

$$\forall t, t' \in \mathbf{Type}[(t \in \mathbf{Class1} \land t' \in \mathbf{Class2}) \to f_2 : (t \to t')]$$

What do you think will happen if you declare the in a source file?

```
same :: Eq a => a -> b -> Bool
same a b = a == b
```

What about the following?

```
same2 :: (Eq a, Eq b) => a -> b -> Bool
same a b = a == b
```

Using typeclasses cont.

Remember that free type variables are *implicitly universally quantified*.

```
id :: a -> a
```

Informally, this means that the type of `id` is `a -> a`, for all `a` in the set of types.

Type class constraints restrict the universal quantification to just types which belong to particular classes:

```
(==) :: Eq => a -> a -> Bool
```

This means that the type of `(==)` is `a -> a -> Bool`, for all `a` that belong to the `Eq` class.

Combining typeclass restrictions

Typeclass restrictions can be combined. We've alluded to this before, but the typeclass `Show` is used to classify types whose inhabitants can be converted into strings (via the `show`) function.

What does the following function do?

```
func :: (Eq a, Show a) => a -> a -> String
func a b = if
  a == b
  then (show a) ++ " is equal to " ++ (show b)
  else "try again!"
```

Why do we need both typeclass constraints here?

Interlude: conditionals and tuples

Haskell has syntactic sugar for conditional statements like *if A then B*, which are conventionally written as follows:

```
if _condition then _expressionA else _expressionB
```

You can use conditionals anywhere where you could use `_expressionA` or `_expressionB` (the expressions must be of the same type).

What does the following function do?

```
toyFunc n = if even n then n + 1 else n - 1
```

Conditionals and syntactic sugar

It's important to remember that anything that isn't function-argument application in haskell is *syntactic sugar*.

As an exercise, implement conditional statements as a standard function:

```
cond :: Bool -> a -> a -> a
```

Test your answer by rewriting `toyFunc` using `cond`.

Conditionals and syntactic sugar: solution

```haskell
cond :: Bool -> a -> a -> a
cond True a _ = a
cond False _ b = b
```

We learned earlier about *lists* in haskell, of type `[a]`, for any type `a`.

```
myList1 :: [Int]
myList1 = [2,4,6,8]

myList2 :: [Char]
myList2 = "I'm a string"
```

The primary limitation of lists is that they can only contain *elements of the same type*.

A ubiquitous data structure in haskell used for elements of (potentially) distinct types is the *tuple*.

Tuples are a ubiquitous syntactic construct, defined in haskell as a special kind of type known as a *product type*.

Let's look at the data declaration for tuples:

```
(,) a b = (,) a b
```

- This is quite different from what we've seen so far.
  - The datatype declaration involves a function (called a *type constructor*) that takes two type arguments `a`, `b`.
  - Type constructors create types from types.
  - For example, `(,) Int String` is a distinct type from `(,) String Int`.
  - `(a,b)` is *syntactic sugar* for `(,) a b`.

Working with tuples

Consider some tuples:

```
("haskell", "rocks")
("haskell", 1)
```

We can write functions `fst` and `snd` using pattern matching to extract the elements of a tuple (these are provided already in the prelude).

```
fst :: (a,b) -> a
fst (a,b) = a
snd :: (a,b) -> b
snd (a,b) = b
```

N-tuples

Unlike lists, tuples have a *fixed number* of elements.

```
("Haskell", 1, "Rocks") :: (String, Int, String)
('a', 'b', "Hello", 73) :: (Char, Char, String, Int)
```

The `fst` and `snd` functions won't work for *n-tuples*, where $n > 2$; why not?

Tuples under the hood

Unlike lists, tuples in haskell aren't singly-linked. To see this, try evaluating the following:

```
ghci> (1,2,3) == ((1,2),3)
ghci> (1,2,3) == (1,(2,3))
```

In fact, a 2-tuple involves a distinct constructor to a 3-tuple.

```
ghci> (,,) 1 2 3
(1,2,3)
ghci> (,,,) 1 2 3 4
(1,2,3,4)
ghci> (,,) 1 2 3 4
-- type mismatch error
```

This explains why `fst` and `snd` don't work!

- Write a function `swap` that takes a tuple, and swaps the elements around.
- write a function `condTup` that takes a bool `t`, two tuples, `(a,b)`, `(c,d)`, and gives back a tuple of tuples `(a,c)` if `t` is true, and `(b,d)` otherwise (tip: think carefully about the type signature!).
- Write functions `fst5` and `snd5` that apply to 5-tuples. Is it possible to write an *unsafe* index function for tuples?

Solution

```haskell
swap :: (a,b) -> (b,a)
swap (a,b) = (b,a)

condTup :: Bool -> (a,a) -> (b,b) -> (a,b)
condTup True (a,b) (c,d) = (a,c)
condTup False (a,b) (c,d) = (b,d)

fst5 :: (a,b,c,d,e) -> a
fst5 (a,_,_,_,_) = a

snd5 :: (_,b,_,_,_) -> b
snd5 (_,b,_,_,_) = b
```

- Functions in Haskell strictly take one argument and return one result; sometimes that result is itself a function.
- When a function appears to take multiple arguments, in fact those arguments are *curried*, i.e., addition has the following type signature:

```
(+) :: Num a => a -> a -> a
```

Currying means that we can pass around the result of *partially applying* a function that takes multiple arguments.

```
ghci> myPartial = (+) 4
ghci> myPartial 6
10
```

Exercise: uncurrying

Exercise: write a function `myAddition` that takes a *tuple* as its sole argument.

Uncurrying: solution

```haskell
myAddition :: (Num a) => (a,a) -> a
myAddition (a,b) = a + b
```

Exercise: generalized (un)currying

This exercise is a bit harder:

- Part 1: write a function `myUncurry` of type `(a -> b -> c) -> (a,b) -> c`.
- Part 2: write a function `myCurry` of type `((a,b) -> c) -> a -> b -> c`.
- Part 3: now do the same thing, but for functions which take 3 arguments.
    - Is it possible to write a generalized function `myCurryN` that curries a function that takes `n` arguments?

Solution: generalized (un)currying

```
myUncurry f (a,b) = f a b
myCurry f a b = f (a,b)
myUncurry3 f (a,b,c) = f a b c
myUncurry3 f a b c = f (a,b,c)
```

Polymorphism

Type signatures in haskell can be (parametrically) polymorphic. Recall that typeclasses constrain what we can do with arguments to a polymorphic function.

- Try to write a function of type `a -> a` that does soemthing other than return the input value.
- There are two possible implementations of the function with type signature `a -> a -> a`. Write them both.
- How many implementations can can `a -> b -> b` have?

Combinators (remember those from the lambda calculus?) in haskell are polymorphic functions.

Function composition is an infix operator `f . g`.

Here's one way of writing its definition:

```
f . g = \x -> f $ g x
```

This will be useful in the following exercises.

- In all of the following cases, the goal is to make the program pass the type checker by modifying the `???` declaration, and it alone.

Type-kwon-do 1

```haskell
f :: Int -> String
f = undefined

g :: String -> Char
g = undefined

h :: Int -> Char
h = ???
```

Solution

```
h = g . f
```

Type-kwon-do 2

```haskell
data A
data B
data C

q :: A -> B
q = undefined

w :: B -> C
w = undefined

e :: A -> C
e = ???
```

Solution

```
e = w . q
```

Type-kwon-do 3

```haskell
data X
data Y
data Z

xz :: X -> Z
xz = undefined

yz :: Y -> Z
yz = undefined

xform :: (X, Y) -> (Z, Z)
xform = ???
```

Solution

```
xform (x,y) = (xz x, yz y)
```

```haskell
munge :: (x -> y)
  -> (y -> (w, z))
  -> x
  -> w
munge = ???
```

Solution

```
munge f g = fst . g . f
```

$\mathcal{F}in$

References

Carpenter, Bob (1998). *Type-Logical Semantics*, MIT Press.

Montague, Richard (1970). *Universal Grammar*.