Hello Haskell!

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Lambda calculus summary

- The lambda calculus is a *formal system* (i.e., a logic) for reasoning about *functions*.
- In the lambda calculus, computation is modelled as a form of simplification, using the following rules:
  - β-reduction. $(\lambda x.f(x))(y) \Rightarrow f(y)$
  - α-conversion. $\lambda x.x \Rightarrow \lambda y.y$
  - η-reduction. $\lambda x.f(x) \Rightarrow f$
- Haskell can be thought of as a kind of lambda calculi, where *running a program* amounts to reducing a complex expression until we reach normal form.
- Reduction doesn't always converge on a normal form; sometimes expressions diverge; this corresponds to *non-terminating computations* (imagine, for example, a program implementing a timer that runs indefinitely).

$$(\lambda xyz.xz(yz))(\lambda x.z)(\lambda x.a)$$

1. Curry arguments: $(\lambda x.\lambda y.\lambda z.xz(yz))(\lambda x.z)(\lambda x.a)$
2. α-conversion: $(\lambda x.\lambda y.\lambda z_1.xz_1(yz_1))(\lambda x.z)(\lambda x.a)$
3. β-reduce: $(\lambda y.\lambda z_1.(\lambda x.z)z_1(yz_1))(\lambda x.a)$
4. β-reduce: $\lambda z_1.(\lambda x.z)z_1((\lambda x.a)z_1)$
5. β-reduce: $\lambda z_1.(\lambda x.z)z_1 a$
6. β-reduce: $\lambda z_1.za$
7. Normal form!

Expressions and declarations

- Everything you write in Haskell is either an *expression* or a *declaration*.
  - Expressions can be values, functions, functions applied to values, etc.
  - Declarations are bindings that allow us to name complex expressions.

Here are some examples of expressions in Haskell:

```
1
1 + 1
"Icarus"
```

The REPL

- The GHCi REPL stands for the *Glasgow Haskell Compiler interactive Read-Eval-Print-Loop*.
- It allows us to evaluation Haskell expressions directly without the need to save the program in a source file.
- There are a few different ways to get a GHCi instance:
    - In the browser: `https://tryhaskell.org/`
    - By installing GHC and running `ghci` in the terminal.

REPL cont.

- When we type an expression into the REPL it automatically evaluates it for us.
- The following expressions are already in normal form, so they simply evaluate to themselves.

```
Prelude> 1
1
Prelude> "Icarus"
"Icarus"
```

Evaluating arithmetic expressions

- GHCi can be used as a basic calculator by inputting arithmetic expressions.
- Complex expressions are evaluated until we reach normal form:

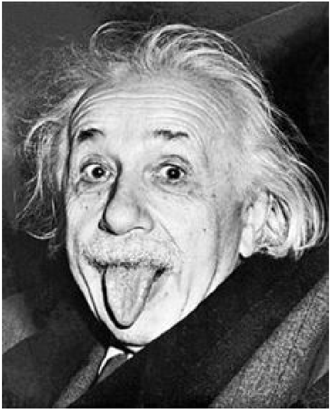
```
Prelude> ((1 + 2) * 3) + 100
109
```

- Note that GHCi doesn't show us any of the intermediate steps.
- N.b. expressions that can be reduced are called redexes (i.e., *reducible expressions*).

Functions

- *Functions* in haskell are particular kinds of expressions, which play a very important role.
- Just like mathematical functions, they map inputs to outputs, in a determinate fashion.
- A Haskell function always evaluates to the same result when given the same argument values.
    - This property is known as *referential transparency*, and makes Haskell programs extremely straightforward to reason about.
    - For those of you with some experience programming in an imperative language like C, this is quite a departure! In imperative languages, evaluating a line of code might affect the state in a way which changes subsequent evaluations.

Referential transparency



"Insanity is doing the same thing over and over and expecting different results." (Albert Einstein)

More on functions

There are a number of different ways of declaring functions in haskell. Here is the simplest way:

```
Prelude> triple x = x * 3
Prelude> triple 4
12
```

Function names always start with lower case letters in haskell. It's good practice to use descriptive function names, which conventionally use camel case, e.g.:

```
Prelude> multiplyByThree x = x * 3
Prelude> multiplyByThree 4
12
```

Abstractions

- Note that the equals sign = indicates that this is a *declaration* rather than an expression.
- Note that declarations are much like abstractions, in the sense that the variable(s) to the left of the = bind the corresponding variable(s) to the right.
- In fact it's also possible to define functions directly as abstractions, using the following syntax:

```
Prelude> triple = \x -> x * 3
Prelude> triple 4
12
Prelude> (\x -> x * 3) 4
12
```

How would we declare a function that has one parameter and words for al the following expressions?

```
pi * (5 * 5)
pi * (10 * 10)
pi * (2 * 2)
pi * (4 * 4)
```

Note that `pi` is an expression that is given by the Haskell `Prelude`. The prelude is a module (i.e., a set of declarations) that is implicitly imported by default.

Solution

```
Prelude> circleArea radius = pi * (radius * radius)
Prelude> circleArea 5
78.53981633974483
```

Note that as well descriptive function names, we can also use descriptive *variable* names; there's no reason (aside from brevity) that we have to use single letters as variable names.

Prefix vs. infix

As you've probably gathered, the syntax for *function application* in Haskell just involves whitespace, i.e.., `f x` means $f(x)$.

The arithmetic operators like + are *infix operators*; they can be used as ordinary functions by enclosing them in paretheses:

```
Prelude> 200 + 300
500
Prelude> (+) 200 300
500
Prelude> ((+) 200) 300
500
```

Declarations in the REPL

We can define functions and later use them with a single REPL session; the REPL has a limited form of state.

```
Prelude> y = 10
Prelude> x = 10 * 5 + y
Prelude> myResult = x * 5
Prelude> myResult
300
```

You can quit the REPL by typing `:q`; declarations won't persist between REPL sessions, so typing `myResult` in a new session will give you the following error:

```
Prelude> myResult
error: Variable not in scope: myResult
```

Declarations in source files

In order to get your declarations to persist, you need to write them into source files (called *modules*). Try saving the following as `learn.hs`.

```
module Learn where


y = 10
x = 10 * 5 + y
myResult = x * 5
```

You can now *load* the module in GHCi.

```
Prelude> :l learn.hs
Ok, one module loaded.
Prelude> myResult
300
```

Tips for writing source files

A module must always start with a module declaration `module MyModule where`; the module name should always start with a capital letter, unlike a function declaration.

White space and line-breaks are *significant*; the following won't compile; the second line should be indented:

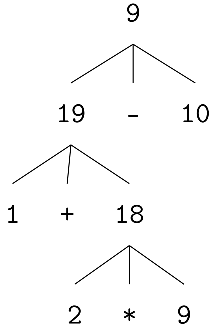
```
x = 10 *
5 + y
```

Comments are lines starting with a double dash.

```
-- a random declaration serving no apparent purpose:
x = 10 * 5 + y
```

Basic arithmetic

Basic arithmetic can help us get a feel for how haskell expressions are evaluated, e.g., `1 + 2 * 9 - 10`.



Associativity and precedence

Arithmetic infix operators in haskell:

- + : addition
- `-`: subtraction
- `*`: multiplication
- `/`: fractional division

You can get information about operator *associativity* and *precedence* using the `:info` command in GHCi.

```
Prelude> :i (+)
infixl 6 +
```

N.b. this will also give you information about the *type* of the expression. This won't be relevant yet, but will be important soon.

The `$` operator

This `$` is an important infix operator that is often used to write terse haskell code without parentheses. Here is its definition:

```
f $ a = f a
```

This is an `infixr` operator with the lowest possible precedence:

```
(2^) $ 2 + 2
(2^) (2 + 2)
```

`let` and `where`

- `let` is used to introduce an expression.
- `where` is a *declaration* that is bound in its containing syntactic construct.

```
printInc n = print plusTwo
  where plusTwo = n + 2

printInc n = let plusTwo = n + 2
               in print plusTwo
```

References