

Plurality and collective predicate

Handout 2

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1 Reading

- Chapter 2 of (Winter 2001) (read this before next week's class).

2 The problem of collective predication

Generalized boolean conjunction accounts for the following kinds of equivalences - this is called *distributive predication*.

- (1) Mary and John slept.
 \iff Mary slept and John slept.
- (2) Most women or most men are tall.
 \iff Most women are tall or most men are tall.

- (3) Neither the milkman nor the postman arrived.
 \iff Neither did the milkman arrive, nor did the postman.

This is because generalized boolean conjunction, at its base, is defined in terms of boolean conjunction: **and** : $T \rightarrow T \rightarrow T$.

Remember, we invoked *Montague lift* to convert expressions such as **Mary** and **John** into expressions with a boolean type.

- (4) **Mary**[↑] \sqcap **John**[↑](**sleep**)
 (5) $((\lambda P . P(\mathbf{Mary})) \sqcap (\lambda P . P(\mathbf{John}))) (\mathbf{sleep})$
 (6) $(\lambda P . P(\mathbf{Mary}) \wedge P(\mathbf{John})) (\mathbf{sleep})$
 (7) **sleep**(**Mary**) \wedge **sleep**(**John**)

Generalized boolean conjunction predicts these kinds of equivalences for *all* NPs, and for *all* kinds of coordination.

Note that this general strategy doesn't just apply to binary logical operators, such as **and**, and **or**, but can be applied to unary logical operators, such as **not**.

Sometimes, we can analyze negation as a sentential operator of type $T \rightarrow T$:

- (8) It's not the case that John slept.
 (9) **not**(**sleep**(**John**)))

But what about cases like the following?

- (10) John doesn't sleep.
not_?(**sleep**)(**John**)
 (11) John isn't tall.
not_?(**tall**)(**John**)

There are various syntactic moves we could make to try to analyze all of the above cases in terms of sentential negation. But, what about the following kind of case

- (12) John and not Mary slept.

Exercise: give a recursive definition for *generalized boolean negation*. Use boolean negation $\text{not} : T \rightarrow T$ as the base of the recursion. Show how it accounts for 9, 10, 11. Finally, propose a Logical Form and analysis for 12. You'll need to use Montague lift!

There are however many simple and complex predicates that do not give rise to *distributive predication*.

- (13) Mary and John weigh exactly 140kg.
 \nRightarrow Mary weighs exactly 140 kg, and John weighs exactly 140kg.

Note that some theories assume that (13) is *ambiguous* between a “distributive reading”, on which distributive predication is valid, and a “collective reading”, on which it is not.

- (14) Mary and John met.
 \nRightarrow *Mary met and John met.

More complicated examples muddy the water, since certain distributive inferences are valid:

- (15) Mary and the postman or the milkman met.
 \iff Mary and the postman met, or Mary and the milkman met.

Other problematic examples:

- (16) An american and a Russian played a duo together.
 (17) Every American and every Russian spoke English to each other.
 (18) The Americans and the Russians fought each other.
 (19) Two Americans and three Russians made an excellent basketball team.

Note that here, the presence of *and* is crucial for obtaining collective effects.

- (20) The Americans or the Russians fought each other.
 \iff The Americans fought each other, or the Russians fought each other
 (21) Neither the Americans nor the Russians fought each other.
 \iff Neither did the Americans fight each other, nor did the Russians.
 (22) The Americans and the Russians fought each other.
 \nRightarrow The Americans fought each other, and the Russians fought each other.

Questions at this point:

1. What are the sources of the distributive/collective distinction?
2. Is the boolean assumption inadequate for conjunctive coordination?

3 A type-theoretic approach to plural individuals

3.1 Collective predicates

Verbs:

- (23) Lexically collective: *gather*, *disperse*, *meet* (when used intransitively), *outnumber* (both arguments).
- (24) Lexically “mixed” collective/distributive: the subject argument of *write*, *lift*, *eat*, and *carry*.
- (25) *together* modification: *sing together*, *live together*, *write NP together*.
- (26) *between them* modification: *grade over 200 papers between them*, *ate 15 pizzas between them*.
- (27) Reciprocal modification: *like each other*, *look at one another*.

Adjectives:

- (28) Lexically collective: *numerous*, *similar*, *alike*, *parallel*, *antagonistic*, *equivalent*, *neighboring*.
- (29) *together* modification: *happy together*, *irritating together*.
- (30) Reciprocal modification: *nice to each other*, *fond of one another*.

Predicative constructions with nominals:

- (31) Group denoting nominals: *seem a big group*, *be the organizing committee*, *be a nice couple*.
- (32) Relational nominals: *be brothers*, *sisters*, *friends*.
- (33) Nominals modified by collective adjectives: *be numerous people*, *similar students*, *parallel lines*.
- (34) Reciprocal possessives: *be teachers of each other*, *be admirers of one another*.

A broader notion of distributivity with plural NPs:

- (35) The old members of this institution retired last year.
- (36) Every old member of this institution retired last year.
- (37) The old members of this institution constitute the managing board.
- (38) Every old member of this institution constitutes the managing board.

3.2 Plural individuals

Fundamental hypothesis: a plural individual is a *set of individuals*.

$$(39) \quad \text{John, Mary and Sue} = \{ \mathbf{J}, \mathbf{M}, \mathbf{S} \}$$

$$(40) \quad \text{The boys} = \{ \mathbf{J}, \mathbf{B}, \mathbf{H} \}$$

How do we model sets of individuals in the Simply-Typed λ -calculus? Recall, the denotation of an expression of type $E \rightarrow T$ is a function $f : \mathbf{Dom}_E \mapsto \mathbf{Dom}_T$.

A *set* of individuals can be modelled as a function from individuals to truth-values, where members of the set are mapped to **true**, and non-members are mapped to **false**. This is called the **characteristic function** of the set.

Here's the characteristic function of the set of *John*, *Mary*, and *Sue*:

$$\begin{bmatrix} \mathbf{J} & \mapsto \mathbf{true} \\ \mathbf{M} & \mapsto \mathbf{true} \\ \mathbf{S} & \mapsto \mathbf{true} \\ \mathbf{B} & \mapsto \mathbf{false} \\ \mathbf{H} & \mapsto \mathbf{false} \\ \dots & \end{bmatrix}$$

Here's the characteristic function of the set of *John*, *Bill*, and *Harry* (i.e., *the boys*):

$$\begin{bmatrix} \mathbf{J} & \mapsto \mathbf{true} \\ \mathbf{M} & \mapsto \mathbf{false} \\ \mathbf{S} & \mapsto \mathbf{false} \\ \mathbf{B} & \mapsto \mathbf{true} \\ \mathbf{H} & \mapsto \mathbf{true} \\ \dots & \end{bmatrix}$$

Our hypothesis will be that a *group-denoting NP* is translated to an *expression of type* $E \rightarrow T$

Since *characteristic functions* are ways of encoding sets, we can also translate familiar set-theoretic notions of *intersection* and *union* into operations on functions. Look familiar?

$$(41) \quad P_{ET} \sqcap Q_{ET} = \lambda x . P(x) \wedge Q(x)$$

$$(42) \quad P_{ET} \sqcup Q_{ET} = \lambda x . P(x) \vee Q(x)$$

Exercise: Give an informal proof that \sqcap and \sqcup deliver the same results as intersection and union.

If group-denoting NPs are of type $E \rightarrow T$, there's an obvious way to encode the selectional requirements of strictly collective predicates - we can translate them as expressions of type $(E \rightarrow T) \rightarrow T$.

| | Singular/Distributive | Plural/Collective |
|------------|-----------------------------------|---|
| Individual | E | $E \rightarrow T$ |
| Predicate | $E \rightarrow T$ | $(E \rightarrow T) \rightarrow T$ |
| Quantifier | $(E \rightarrow T) \rightarrow T$ | $((E \rightarrow T) \rightarrow T) \rightarrow T$ |

3.3 Collective predication in practice

Collective predicates denote *higher-order functions* from functions $f : \mathbf{Dom}_E \rightarrow \mathbf{Dom}_T$ to \mathbf{Dom}_T .

What does this mean? *met* takes a function P_{ET} , and maps it to true, just in case the individuals that P maps to true met each other.

Imagine that the set of individuals is $\{a, b, c\}$, and the only meetings that happened were between a, b and a, c . The denotation of $\mathbf{met}_{ET \rightarrow T}$ would be as follows:

$$\left[\begin{array}{l} [a \rightarrow \mathbf{t}] \\ [b \rightarrow \mathbf{t}] \\ [c \rightarrow \mathbf{t}] \end{array} \rightarrow \mathbf{f} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{t}] \\ [b \rightarrow \mathbf{t}] \\ [c \rightarrow \mathbf{f}] \end{array} \rightarrow \mathbf{t} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{t}] \\ [b \rightarrow \mathbf{f}] \\ [c \rightarrow \mathbf{t}] \end{array} \rightarrow \mathbf{t} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{f}] \\ [b \rightarrow \mathbf{t}] \\ [c \rightarrow \mathbf{t}] \end{array} \rightarrow \mathbf{f} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{t}] \\ [b \rightarrow \mathbf{f}] \\ [c \rightarrow \mathbf{f}] \end{array} \rightarrow \mathbf{f} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{f}] \\ [b \rightarrow \mathbf{t}] \\ [c \rightarrow \mathbf{f}] \end{array} \rightarrow \mathbf{f} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{f}] \\ [b \rightarrow \mathbf{f}] \\ [c \rightarrow \mathbf{t}] \end{array} \rightarrow \mathbf{f} \right. \\ \left[\begin{array}{l} [a \rightarrow \mathbf{f}] \\ [b \rightarrow \mathbf{f}] \\ [c \rightarrow \mathbf{f}] \end{array} \rightarrow \mathbf{f} \right]$$

Due to the equivalence between characteristic functions and sets, we can also write the denotation of a collective predicate as a *set of sets*. This will often be much more convenient. I.e., the following set of sets encodes the same information.

$$\{ \{a, b\}, \{a, c\} \}$$

As a starting point, we'll assume that plural definites such as “the boys”, and conjunctions of singular definites such as “John, Mary, and Sue”, are translated as expressions of type $E \rightarrow T$, and therefore denote sets of individuals.

(43) **theBoys**_{ET}

(44) **JohnMaryAndSue**_{ET}

The question (reflected in the title of this seminar), is how to derive this result compositionally.

If singular definites are of type E , then this explains the incompatibility of collective predicates with singular definites - the result will never be well-typed.

We'll continue to assume that predicates which give rise to *distributive inferences* are of type $E \rightarrow T$.

A question which we won't yet be able to answer is how distributive predicates compose with *group-denoting NPs*, giving rise to distributive inferences.

References

Winter, Yoad. 2001. *Flexibility principles in boolean semantics - the interpretation of coordination, plurality, and scope in natural language* (Current Studies in Linguistics 37). Cambridge Massachusetts: The MIT Press. 297 pp.