Uniqueness, answerhood, and pluralities of questions v2

Patrick D. Elliott
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1 Roadmap

- Uniqueness presuppositions of questions with singular *which*-phrases.
  - Plurality recap.
  - Dayal's (1996) analysis of the uniqueness presupposition and $\text{ANS}_{\text{Dayal}}$.
- Semantics of multiple questions.
  - Absence of uniqueness presupposition with multiple questions.
  - *Exhaustivity* and *pointwise uniqueness*.
  - Dayal's functional analysis.
  - The higher-order question analysis.
  - Pluralities of questions.

2 Uniqueness presupposition

Singular *which*-questions to carry a uniqueness presupposition.

(1) a. Which guest arrived?
b. Kramer arrived.
c. #Kramer and Jerry arrived.

(2) a. Which guests arrived?
b. Kramer arrived.
c. Kramer and Jerry arrived.

Simplex *wh*-phrases, on the other hand, do not.

(3) a. Who arrived?
b. Kramer arrived.
c. Kramer and Jerry arrived.

2.1 Plurality primer

I assume an ontology where singular NPs range over atomic individuals, and plural NPs range over sets of individuals (Schwarzschild 1996, Winter 2001, Champollion 2015, Vries 2015, etc.).[1]

(4) $[\![\text{guest}]\!]^w = \lambda x_e.\text{guest}_w(x) = \{\text{Kramer, Jerry, Elaine}\}$

The plural affix *-s* has the meaning in (5): it takes a predicate P (a set of individuals), and pluralizes it, returning $\mathbb{P}(P) - \{\emptyset\}$. N.b. that *-s* will turn out to have exactly the same denotation as the covert distributivity operator *Dist*.

(5) $[\![\text{-s}]\!] = [\![\text{Dist}]\!] = ...$

$... = \lambda P'_{et}.\lambda P_{et}.\exists x_e[P(x)] \wedge \forall x'[P(x) \rightarrow P'(x)]$

$... = \lambda P'_{et}.\lambda P_{et}.P \neq \emptyset \wedge P \subseteq P'$

[1] In the linguistics literature it's more common to treat pluralities as *i-sums* (going back to Link 1983), but the two approaches are completely isomorphic, and treating pluralities as *sets* will make it easier to bring out the parallels between pluralities and questions.

(6) $[\![\text{guests}]\!]^w = [\![\text{-s}]\!]([\![\text{guest}]\!]^w) = ...$

$... = \lambda X_{et}.\exists x_e[X(x)] \wedge \forall x'_e[X(x') \rightarrow \text{guest}_w(x')]$

$... = \lambda X_{et}.X \neq \emptyset \wedge X \subseteq \{x' \mid \text{guest}_w(x')\}$

$... = \{\{\text{Kramer}\}, \{\text{Jerry}\}, \{\text{Elaine}\},$
$\{\text{Kramer, Jerry}\}, \{\text{Kramer, Elaine}\}, \{\text{Jerry, Elaine}\}\}$
$\{\text{Kramer, Jerry, Elain}\}\}$

The plural definite article *the$_{pl}$* presupposes that there a maximal subset of the denotation of the plural NP, and picks it out. *Pluralities of individuals* are therefore of type $\langle e, t\rangle$.

(7) a. $Q_{et,t}$ is dom(the$_{\text{pl}}$) only if $\exists X_{et}[X \in Q \wedge \forall X'[X' \in Q \rightarrow X \subseteq X']]$

b. Whenever defined,
$\text{the}_{\text{pl}}(Q) = \iota X[X \in Q \wedge \forall X'[X' \in Q \rightarrow X \subseteq X']]$

(8) $[\![\text{the}_{\text{pl}} \text{ guests}]\!]^w = [\![\text{the}_{\text{pl}}]\!]([\![\text{guests}]\!]^w) = ...$

$... = \{\text{Kramer, Jerry, Elaine}\}$

I also assume that *conjoined proper names* denote plural individuals.

(9) $[\![\text{Kramer, Jerry and Elaine}]\!] = \{\text{Kramer, Jerry, Elaine}\}$

Distributive predicates are of type $\langle e, t\rangle$, whereas collective predicates are of type $\langle\langle e, t\rangle, t\rangle$.

(10) a. $[\![\text{smoke}]\!]^w = \lambda x_e.x \text{ smokes}_w$

b. $[\![\text{gather}]\!]^w = \lambda X_{et}.X \text{ gather}_w$

Singular DPs are of type e, and therefore can compose with distributive predicates, but not collective predicates. This is a good result.

(11) a. John smokes.

b. *John gathers.

Plural DPs are of type $\langle e, t\rangle$, and therefore can compose with collective predicates but not distributive predicates. This is a bad result, and we need to fix it.

(12) a. The boys smoke.

b. The boys gather.

A distributive operator Dist (which has the same denotation as the plural morpheme *-s*) is standardly used to shift distributive predicates to a higher type, allowing them to compose with a plural DP.

(13) $[\![\text{Dist}]\!] = [\![\text{-s}]\!] = ...$

$... = \lambda P'_{et}.\lambda P_{et}.\exists x_e[P(x)] \wedge \forall x'[P(x') \rightarrow P'(x')]$

$... = \lambda P'_{et}.\lambda P_{et}.P \neq \emptyset \wedge P \subseteq P'$

(14) $\text{Dist}([\![\text{smoke}]\!]^w) = ...$

$... = \lambda P_{et}.\exists x_e[\text{smoke}_w(x)] \wedge \forall x'[\text{smoke}_w(x') \rightarrow P(x')]$

$... = \lambda P_{et}.P \neq \emptyset \wedge P \subseteq S_w$

(15) $[\![\text{The guests smoke}]\!]^w = 1$ iff
$\{\text{Kramer, Jerry, Elaine}\} \subseteq \text{Dist}(S_w)$
...or equivalently...
$\exists x_e[\text{smoke}_w(x)] \wedge$
$\forall x'[\text{smoke}_w(x') \rightarrow x' \in \{\text{Kramer, Jerry, Elaine}\}]$

2.2 Dayal on the uniqueness presuppositions

Core idea: questions presuppose the existence of a maximally informative true answer. *Informativity* is understood in terms of entailment: P is more informative than Q iff $P \subset Q$.

Dayal assumes that questions denote sets of possible answers.

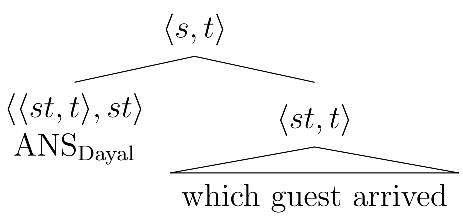
(16) $[\![\text{Which guest arrived}]\!]^w$
$= \{p_{st} \mid \exists x_e[\text{guest}_w(x) \wedge p = \lambda w'.x \text{ arrived}_{w'}]\}$

We can define an answerhood operator as follows:

(17) a. $Q_{st,t} \in \text{dom}(\text{ANS}_{\text{Dayal}_w})$ iff $\exists p_{st}[p \in Q \wedge p(w) \wedge \forall p'[(p' \in Q \wedge p'(w)) \rightarrow p \subseteq p']]$
b. Whenever defined,
$ANS_{\text{Dayal}_w}(Q) = \iota p[p \in Q \wedge \forall p'[(p' \in Q \wedge p'(w)) \rightarrow p \subseteq p']]$

N.b. that $\text{ANS}_{\text{Dayal}}$ is the counterpart of the_{pl} in the domain of propositions; both operators take a set of sets as their input, and return the unique maximal subset. The only difference is that $\text{ANS}_{\text{Dayal}}$ enforces the requirement that p is true with respect to the world of evaluation.[2]

We can assume that $\text{ANS}_{\text{Dayal}}$ is a covert operator in the LF of a question.



Now we have an answer for why singular *which*-questions carry a uniqueness presupposition – this comes from the combination of the semantics of the morphologically singular NP restrictor, and the presupposition that there is a unique maximally informative answer contributed by $\text{ANS}_{\text{Dayal}}$.

- Kramer arrived, Jerry arrived, and Elaine didn't arrive in w.

(18) $[\![\text{Which guest arrived}]\!]^w$
$$= \left\{ \begin{array}{l} \lambda w'.\ \text{Kramer arrived}_{w'} \\ \lambda w'.\ \text{Jerry arrived}_{w'} \\ \lambda w'.\ \text{Elaine arrived}_{w'} \end{array} \right\}$$

(19) $[\![\text{ANS}_{\text{Dayal}_w}]\!]^w([\![(18)]\!]) = \text{undefined}$

- Only Kramer arrived in w.

(20) $[\![\text{ANS}_{\text{Dayal}_w}]\!]^w([\![(18)]\!]) = \lambda w'.\text{Kramer arrived}_{w'}$

We correctly predict that questions with plural *which*-phrases don't carry a uniqueness presupposition:

- Kramer arrived, Jerry arrived, and Elaine didn't arrive in w.

(21) $[\![\text{Which guests arrived}]\!]^w$
$$= \left\{ \begin{array}{l} \lambda w'.\ \{\text{Kramer}\} \in \text{Dist}(\text{arrived}_{w'}) \\ \lambda w'.\ \{\text{Jerry}\} \in \text{Dist}(\text{arrived}_{w'}) \\ \lambda w'.\ \{\text{Elaine}\} \in \text{Dist}(\text{arrived}_{w'}) \\ \lambda w'.\ \{\text{Kramer, Jerry}\} \in \text{Dist}(\text{arrived}_{w'}) \\ \lambda w'.\ \{\text{Kramer, Elaine}\} \in \text{Dist}(\text{arrived}_{w'}) \\ \lambda w'.\ \{\text{Jerry, Elaine}\} \in \text{Dist}(\text{arrived}_{w'}) \\ \lambda w'.\ \{\text{Kramer, Jerry, Elaine}\} \in \text{Dist}(\text{arrived}_{w'}) \end{array} \right\}$$

(22) $\text{ANS}_{\text{Dayal}}([\![(21)]\!]) = \lambda w'.\{\text{Kramer, Jerry}\} \in \text{Dist}(\text{arrived}_{w'})$

[2] It's possible to define a single operator subsuming the definitions of the_{pl} and $\text{ANS}_{\text{Dayal}}$ in a natural way, by intensionalizing the_{pl}, but I refrain from doing this here.

Note furthermore that $\text{ANS}_{\text{Dayal}}$ also encodes an *existential* presupposition for *wh*-questions – this is because it is the parallel of the definite article, in the domain of propositions. Consider the following context:

No guests arrived in w.

(23) $[\![\text{Which guest arrived?}]\!]^w = \emptyset$
$\text{ANS}_{\text{Dayal}}(\emptyset)$ is undefined

As we discussed in a prrevious class, this seems like it could be a desirable result.

(24) a. John is wondering which guests arrived.
b. Hey, wait a minute! No guests arrived.

2.3 More applications of $\text{ANS}_{\text{Dayal}}$

$\text{ANS}_{\text{Dayal}}$ can also be used to account for maximality/minimality effects in *degree questions*.

(25) *Context: the answerer knows that George read three books.*
a. How many books did George read?
b. #George read two books.
c. George read three books.
d. George read four books (*false*)

(26) *Context: the answerer knows that three eggs are sufficient to make an omelette*
a. How many eggs are sufficient to make an omelette?
b. Two eggs are sufficient to make an omelette. (*false*)
c. Three eggs are sufficient to make an omelette.
d. #Four eggs are sufficient to make an omelette.

(27) $[\![\text{How many books did George read?}]\!]^w$
$= \{p_{st} \mid \exists d[d \in \mathbb{N} \wedge$
$p = \lambda w'.\text{George read}_{w'}\ d\text{-many books}]$

$$= \left\{ \begin{array}{l} \dots \\ \lambda w'.\ \text{George read}_{w'}\ \text{one book } ① \\ \lambda w'.\ \text{George read}_{w'}\ \text{two books } ② \\ \lambda w'.\ \text{George read}_{w'}\ \text{three books } ③ \\ \lambda w'.\ \text{George read}_{w'}\ \text{four books } ④ \\ \dots \end{array} \right\}$$

④ ⊆ ③ ⊆ ② ⊆ ①, and ④ is false, therefore $\text{ANS}_{\text{Dayal}}$ is defined, and returns the proposition *that George read three books* as the maximally informative true answer.

(28) $[\![\text{How many eggs are sufficient to make an omelette?}]\!]^w$
$= \{p_{st} \mid \exists d[d \in \mathbb{N} \wedge$
$p = \lambda w'.d\text{-many eggs are sufficient to make an omelette}_{w'}]$

$$= \left\{ \begin{array}{l} \dots \\ \lambda w'.\ \text{one egg is sufficient...}_{w'}\ ① \\ \lambda w'.\ \text{two eggs are sufficient...}_{w'}\ ② \\ \lambda w'.\ \text{three eggs are sufficient...}_{w'}\ ③ \\ \lambda w'.\ \text{four eggs are sufficient...}_{w'}\ ④ \\ \dots \end{array} \right\}$$

This time the direction of entailment is reversed: ① ⊆ ② ⊆ ③ ⊆ ④. ① and ② are false, therefore $\text{ANS}_{\text{Dayal}}$ is defined, and it returns the proposition *that three eggs are sufficient to make an omelette* as the maximally informative true answer.

Furthermore, $\text{ANS}_{\text{Dayal}}$ derives the sensitivity of degree questions to negation, as a matter of the semantics of questions.

(29) #How many books didn't George read?

In an out-of-the-blue context, there is no upper bound to the number of books d, s.t. George *didn't* read d-many books, therefore $\text{ANS}_{\text{Dayal}}$ is not defined for (29), since for any given answer of the form *George didn't read d-many books*, there exists a more informative answer *George didn't read d′-many books*, s.t., $d' > d$.

If we explicitly supply an upper-bound, the question (as predicted) becomes better:

(30) Out of these ten books, how many didn't George read?

See Abrusán (2014) for an overview of these issues.

2.4 Uniqueness shift with multiple questions

Central problem with Multiple Questions (MQs): singular *which*-phrases no longer carry uniqueness presuppositions.

(31) a. Which woman is dating which man?
b. Elaine is dating Jerry.
c. Elaine is dating Jerry, and Susan is dating George.

First attempt at a denotation for a multiple question: set of possible answers. (This was generally the standard assumption in the literature until Dayal 1996.)

(32) $[\![\text{Which woman is dating which man?}]\!]^w$
$= \{p_{st} \,|\, p(w) \wedge \exists x, y[\text{woman}_w(x) \wedge \text{man}_w(y) \wedge p = \lambda w'.x \text{ is dating}_{w'}\ y]\}$

(33) $[\![\text{Which woman is dating which man?}]\!]^w$

$$= \left\{ \begin{array}{l} \lambda w'.\ \langle\text{Elaine, Jerry}\rangle \in \text{dating}_{w'} \\ \lambda w'.\ \langle\text{Susan, George}\rangle \in \text{dating}_{w'} \\ \lambda w'.\ \langle\text{Elaine, George}\rangle \in \text{dating}_{w'} \\ \lambda w'.\ \langle\text{Susan, Jerry}\rangle \in \text{dating}_{w'} \end{array} \right\}$$

Elaine is dating Jerry and Susan is dating George in w.

(34) $[\![\text{ANS}_{\text{Dayal}_w}]\!]^w([\![(33)]\!]) = \text{undefined}$

Applying Dayal's answerhood operator to the simple meaning in (33) gets **the wrong result**.

Two possible solutions:

(1) Re-define $\text{ANS}_{\text{Dayal}}$ to get the right result in this special case.

(2) Complicate the semantic value of the multiple question, s.t. $\text{ANS}_{\text{Dayal}}$ gets the right result.

Overwhelmingly, people pursue the second possibility. I don't know of any work pursuing the first, and it's even doubtful that one *could* re-define $\text{ANS}_{\text{Dayal}}$ to get the right result for MQs, while maintaining the explanation for the uniqueness presupposition.

3 Multiple questions

3.1 Exhaustivity and pointwise uniqueness

(35) **Exhaustivity**
"...a question with a wh in subject and a wh in object position presupposes that a list answer will exhaustively pair every member of the subject term, but not

necessarily every member of the object term."

(Dayal 1996, p. 105).

(36) a. Speaker A: We're organizing singles tennis games between men and women. There are three men interested in playing against women, namely Bill, Mike and John. But there are four women interested in playing against men, namely Mary, Sue, Jane and Sarah.
b. Speaker B: So, which man is playing against which woman?

(37) a. Speaker A: We're organizing singles tennis games between men and women. There are four men interested in playing against women, namely Harry, Bill, Mike and Jonn. But there are only three women interested in playing against men, namely Mary, Sue and Jane.
b. Speaker B: #So, which man is playing against which woman?

(38) **Pointwise uniqueness (functionhood):**
"We can say that an appropriate answer to a multiple wh question pairs each member of the subject term with a member of the object term. This pairing can be one-one or many-one, but crucially not one-many. The contrast that shows this is quite sharp and calls for an explanation. "

(Dayal 1996, p. 108)

(39) a. Which student read which book?
b. John read Moby Dick, and Bill read Moby Dick too.
John and Bill both read Moby Dick.
c. #John read Moby Dick and War and Peace.

Engdahl's example:

(40) a. Which table ordered which wine?
b. Table A ordered the Ridge Zinfandel, Table Bordered the Chardonay and Table C ordered the Rose and the Bordeaux.

"I argued that acceptable violations of bijectivity, such as [(40)], typically involve situations in which most of the pairings respect bijectivity and are therefore amenable to a pragamatic explanation. The questioner in [(40-a)], for example, probably expects each table to have ordered a single wine. Knowing that questions are usually exhaustive requests for information, a cooperative interlocuter may provide an answer which includes pairings which violate bijectivity, implicitly denying the questioner's presupposition."

(Dayal 1996, p. 108)

3.2 MQs as questions about functions

Dayal's (1996) idea is that the restrictor of the higher *wh*-phrase supplies the *domain* of the function, and the restrictor of the lower *wh*-phrase supplies the *range* of the function.

Here, f is a *skolem function* – a function from an individual to an individual. The first prominent work to use skolem functions to analyse *wh*-questions was Engdahl (1986), who used them to analyse so-called "functional readings".

(41) a. Which woman does every boy like?
b. His mother.

Engdahl argued that in questions such as (41), the *wh*-phrase can range over skolem functions from boys women:

(42) $[\![(41)]\!]^w = \{p_{st} \mid \exists f_{\langle e,e \rangle}[\text{Range}(f) = \text{woman}_w \wedge p = \lambda w'.\forall x[\text{boy}_{w'}(x) \rightarrow x \text{ likes}_{w'}\ f(x)]]\}$

(43) $[\![\text{Which woman is dating which man?}]\!]^w$
$= \{p_{st} \mid \exists f_{\langle e,e \rangle}[\text{Dom}(f) = \text{woman}_w \wedge \text{Range}(f) = \text{man}_w \wedge p = \bigcap\{p' \mid p'(w) \wedge \exists x[p' = \lambda w'.x \text{ is dating}_{w'}\ f(x)]\}]\}$

$$= \left\{ \begin{array}{l} \bigcap \left\{ \begin{array}{l} \lambda w'.\langle \text{Elaine}, f_1(\text{Elaine}) \rangle \in \text{dating}_{w'} \\ \lambda w'.\langle \text{Susan}, f_1(\text{Susan}) \rangle \in \text{dating}_{w'} \end{array} \right\} \\ \bigcap \left\{ \begin{array}{l} \lambda w'.\langle \text{Elaine}, f_2(\text{Elaine}) \rangle \in \text{dating}_{w'} \\ \lambda w'.\langle \text{Susan}, f_2(\text{Susan}) \rangle \in \text{dating}_{w'} \\ \ldots \end{array} \right\} \\ \ldots \end{array} \right\}$$

$$= \left\{ \begin{array}{l} (\lambda w'.\langle \text{Elaine}, \text{Jerry} \rangle \in \text{dating}_{w'} \wedge \\ \langle \text{Susan}, \text{George} \rangle \in \text{dating}_{w'}), \\ (\lambda w'.\langle \text{Elaine}, \text{Jerry} \rangle \in \text{dating}_{w'} \wedge \\ \langle \text{Susan}, \text{Jerry} \rangle \in \text{dating}_{w'}) \end{array} \right\}$$

- f_1(Elaine) = Jerry
- f_1(Susan) = George
- f_2(Elaine) = Jerry
- f_2(Susan) = Jerry
- ...

$\text{ANS}_{\text{Dayal}}$ applied to the set containing the propositions *that Elaine dates Jerry and Susan dates George* and *that Elaine dates Jerry and that Susan dates Jerry*, returns the unique, maximally informative proposition: *that Elaine dates Jerry and Susan dates George*. If *pointwise uniqueness* fails to hold, $\text{ANS}_{\text{Dayal}}$ is undefined. To see why, consider the following context:

Elaine is dating Jerry in w; Susan is dating Jerry AND George in w.

(44) $[\![\text{Which woman is dating which man?}]\!]^w$

$$= \left\{ \begin{array}{l} \bigcap \left\{ \begin{array}{l} \lambda w'.\langle \text{Elaine}, f_1(\text{Elaine}) \rangle \in \text{dating}_{w'} \\ \lambda w'.\langle \text{Susan}, f_1(\text{Susan}) \rangle \in \text{dating}_{w'} \end{array} \right\} \\ \bigcap \left\{ \begin{array}{l} \lambda w'.\langle \text{Elaine}, f_2(\text{Elaine}) \rangle \in \text{dating}_{w'} \\ \lambda w'.\langle \text{Susan}, f_2(\text{Susan}) \rangle \in \text{dating}_{w'} \end{array} \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} (\lambda w'.\langle \text{Elaine}, \text{Jerry} \rangle \in D_{w'} \wedge \\ \langle \text{Susan}, \text{George} \rangle \in \text{dating}_{w'}) \\ (\lambda w'.\langle \text{Elaine}, \text{Jerry} \rangle \in D_{w'} \wedge \\ \langle \text{Susan}, \text{Jerry} \rangle \in \text{dating}_{w'}) \end{array} \right\}$$

$\text{ANS}_{\text{Dayal}}$ applies to the set above is crucially undefined, since there is no *unique* maximally informative proposition in the set: both propositions are true, and neither is more informative than the other.

Some problems with Dayal's proposal:

(3) Compositionality.

(4) Reconstruction.

(5) *De dicto* readings.

3.3 MQs as pluralities of questions

An alternative solution is to posit *higher-order question meaning* for multiple questions (Fox 2012, Kotek 2014) .

(45) $[\![\text{Which woman is dating which man}]\!]^w$
$= \{Q_{st,t} \mid \exists x[\text{woman}_w(x) \wedge$
$Q = \{p_{st} \mid p(w) \wedge \exists y[\text{man}_w(y) \wedge$
$p = \lambda w'.x \text{ is dating}_{w'}\ y]\}]\}$

$$= \left\{ \begin{array}{l} [\![\text{Which man is Elaine dating?}]\!]^w \\ [\![\text{Which man is Susan dating?}]\!]^w \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left\{ \begin{array}{l} \lambda w'.\ \text{Elaine is dating}_{w'}\ \text{Jerry} \\ \lambda w'.\text{Elaine is dating}_{w'}\ \text{George} \end{array} \right\} \\ \left\{ \begin{array}{l} \lambda w'.\ \text{Susan is dating}_{w'}\ \text{George} \\ \lambda w'.\ \text{Susan is dating}_{w'}\ \text{Jerry} \end{array} \right\} \end{array} \right\}$$

On the face of it, this doesn't seem to help, since $\text{ANS}_{\text{Dayal}}$ will still be undefined for the set in (45) (in a world w where Elaine is dating Jerry and Susan is dating George). This is simply because $\text{ANS}_{\text{Dayal}}$ is defined for arguments of type $\langle st, t\rangle$, whereas (45) is of a higher type: $\langle\langle st, t\rangle, t\rangle$.

We could simply define a new answerhood operator ANS_{MQ}.

Intuitively, ANS_{MQ} should give back the *intersection* of the result of $\text{ANS}_{\text{Dayal}}$ applied to each question belonging to the set denoted by the multiple question. Something like (46):

(46) $$\text{ANS}_{\text{MQ}}(Q) = \bigcap_{q \in Q} \text{ANS}_{\text{Dayal}}(q)$$

If we go down this route, then the *plurality of questions* theory faces a similar objection as Dayal's theory. What we are suggesting simply amounts to a construction specific rule.

3.3.1 Pluralizing $\text{ANS}_{\text{Dayal}}$

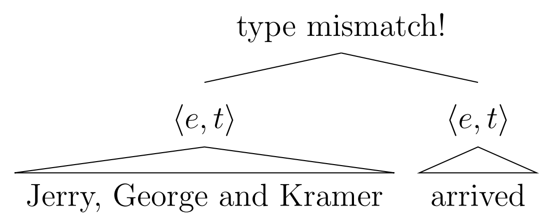
$\text{ANS}_{\text{Dayal}}$ is a (partial) function of type $\langle\langle\langle s,t\rangle, t\rangle, \langle s,t\rangle\rangle$: it takes a question (i.e. a set of propositions) as its input, returning a proposition (a member of that set).

Recall that if we adopt a *family of questions* analysis of multiple questions, then they denote the characteristic function of a set of questions, of type $\langle\langle\langle s,t\rangle, t\rangle, t\rangle$. Attempting to compose $\text{ANS}_{\text{Dayal}}$ with a multiple question will result in a type-mismatch, since $\text{ANS}_{\text{Dayal}}$ wants an argument of type $\langle\langle s,t\rangle, t\rangle$, whereas MQs denote sets of such elements.

This is the same problem as how to compose a plural DP with a distributive predicate. Recall:[3]

(47) $[\![\text{Jerry, George and Kramer}]\!]^w = \{\text{Jerry, George, Kramer}\}$

(48) $[\![\text{arrived}]\!]^w = \lambda x.x \in A_w$



The solution was to type-lift the predicate, such that it accepts arguments of a higher type, via the distributive operator D.

(49) $[\![\text{Dist}]\!]^w = \lambda P_{et}.\lambda Q_{et}.Q \neq \emptyset \wedge Q \subseteq P$

(50) $[\![\text{Dist(arrived)}]\!]^w = \lambda Q_{et}.Q \neq \emptyset \wedge Q \subseteq \{x \mid x \in A_w\}$

We can give a more general (intensionalized) definition of the distributivity operator for any potentially complex type $\sigma\tau$.

(51) $$[\![\text{Dist}]\!]^w = \lambda P_{\langle\sigma,\tau\rangle}.\lambda Q_{\langle\sigma,t\rangle}.\lambda w_\tau.\ \bigwedge_{q \in Q} P(q)(w)$$

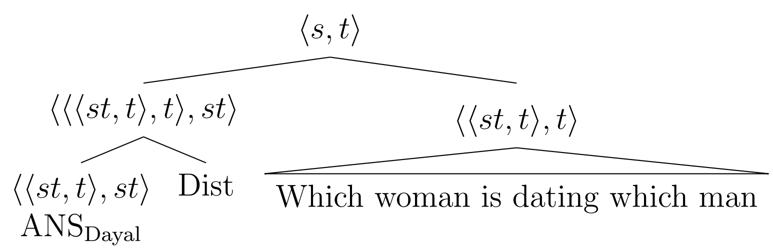
[3]Strictly speaking, a plural DP and a verb could compose via the rule of *predicate modification*, since both expressions denote sets of individuals, but this would fail to result in a sentence meaning of type t.

We can now apply our new, generalized distributivity operator to $\text{ANS}_{\text{Dayal}}$.

(52) $[\![\text{Dist}]\!]^w = \lambda P_{\langle\langle st,t\rangle,st\rangle}.\lambda Q_{\langle\langle st,t\rangle,t\rangle}.\lambda w. \bigwedge_{q\in Q} P(y)(w)$

(53) $[\![\text{Dist}(\text{ANS}_{\text{Dayal}})]\!]^w$
$= \lambda Q_{\langle\langle st,t\rangle,t\rangle}.\lambda w. \bigwedge_{q\in Q} \text{ANS}_{\text{Dayal}}(q)(w)$

Our question LF for a multiple question now looks like this:



Applying $\text{Dist}(\text{ANS}_{\text{Dayal}})$ to the multiple question denotation achieves the desired result (I assume that Elaine is dating Jerry and Susan is dating George in the world of evaluation).

(54) $= \lambda w. \bigwedge_{q\in[\![\text{which woman dating which man}]\!]} \text{ANS}_{\text{Dayal}}(\text{q})(\text{w})$

(55) $= \lambda w.\text{ANS}_{\text{Dayal}}[\![\text{which man is Elaine dating}]\!](w) \bigwedge$
$\text{ANS}_{\text{Dayal}}[\![\text{which man is Susan dating}]\!](w)$

(56) $= \lambda w.$ Elaine is dating_w Jerry $\wedge$ Susan is dating_w George

3.3.2 Pluralities of questions, compositionally

In recent work, I give a compositional framework for deriving standard Hamblin/Karttunen question denotations and higher-order question meanings via two type-neutral operators: Q and C_{Q}.

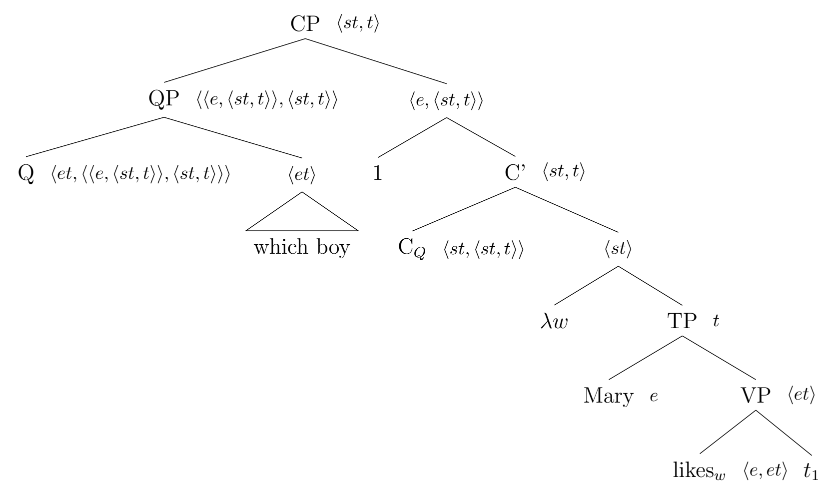
For any types τ and ρ:

(57) $[\![\text{Q}]\!] = \underbrace{\lambda X_{\tau t}.}_{\text{alt. gen.}} \underbrace{\lambda Y_{\langle\tau,\rho t\rangle}.}_{\text{scope}} \lambda Z_\rho.\exists x_\tau[X(x) = 1 \wedge Y(x)(Z) = 1]$

(58) $[\![\text{C}_Q]\!] = \lambda a_\rho.\lambda b_\rho.b = a$

I furthermore assume (following Charlow 2015) that *wh*-phrases denote *sets* (contra the standard treatment, as existential quantifiers).

(59) $[\![\text{which boy}]\!]^w = \{x_e \,|\, x \text{ is a boy}_w\}$



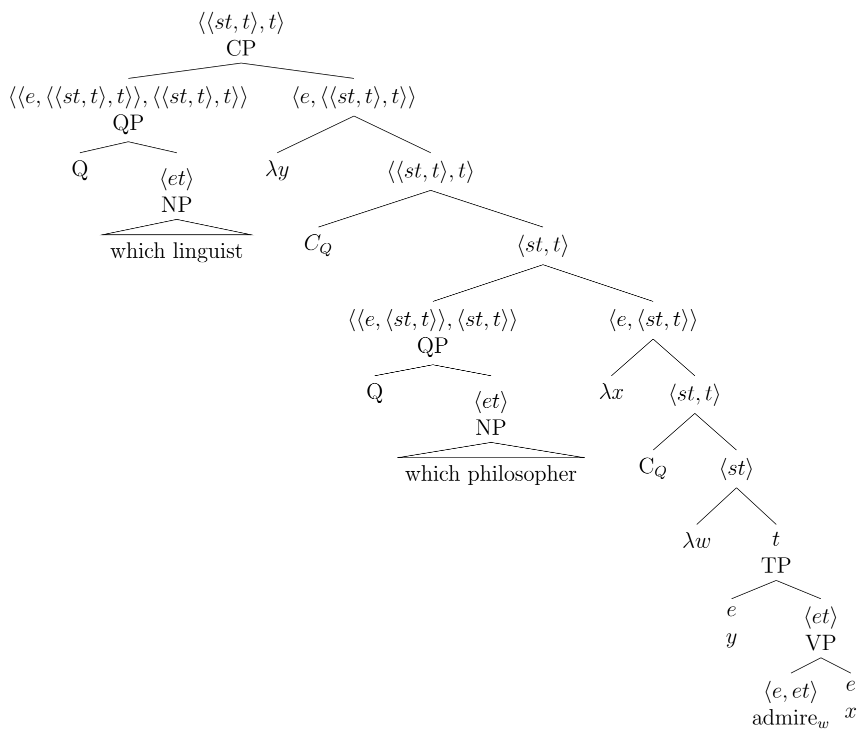
(60) $[\![\text{QP}]\!]^g = \lambda K_{\langle e,\langle st,t\rangle\rangle}.\lambda p_{st}.\exists x[\text{boy}(x) = 1 \wedge K(x)(p) = 1]$

(61) $[\![\text{C'}]\!]^g = \lambda p_{st}.p = \lambda w.\text{Mary likes}_w\ g(1)$

(62) $[\![1\ \text{C'}]\!]^g = \lambda x_e.\lambda p_{st}.p = \lambda w.\text{Mary likes}_w\ x$

(63) $[\![\text{QP}]\!]([\![1\ \text{C'}]\!]^g) = \lambda p_{st}.\exists x[\text{boy}(x) = 1 \wedge p = \lambda w.\text{Mary likes}_w\ x]$
$= \{\text{that Mary likes John, that Mary likes Paul, etc.}\}$

Additional C_Q operators introduce an additional layer of nesting. This allows us to derive a *plurality of questions* meaning compositionally without defining any new operators.



(64) $[\![\text{which philosopher}_2\ t_1\ \text{admires}\ t_2]\!] = ...$
$... = \lambda p_{st}.\exists x[\text{philosopher}(x) = 1 \wedge p = \lambda w.y\ \text{admires}_w\ x]$

(65) $[\![\text{C}_Q]\!]([\![(64)]\!]) = \lambda Q_{\langle st,t\rangle}.Q = \lambda p_{st}.\exists x[\text{philosopher}(x) = 1 \wedge p = \lambda w.y\ \text{admires}_w\ x]$

(66) $[\ \lambda y\ [\![(65)]\!]] = ...$
$... = \lambda y.\lambda Q_{\langle st,t\rangle}.Q = \lambda p_{st}.\exists x[\text{philosopher}(x) = 1 \wedge p = \lambda w.y\ \text{admires}_w\ x]$

(67) $[\![\text{Q which linguist}]\!] = ...$
$... = \lambda K \in D_{\langle e,\langle\langle st,t\rangle,t\rangle\rangle}.\lambda Q_{\langle\langle st,t\rangle,t\rangle}.\exists y[\text{linguist}(y) = 1 \wedge K(y)(Q) = 1]$

Potential problem: island sensitivity.

Solution: large-scale pied-piping at LF.

3.3.3 Prediction: cumulative readings

(68) George and Jerry ordered the *Calzone*, the *Margharita*, and the *Hawaiian*.

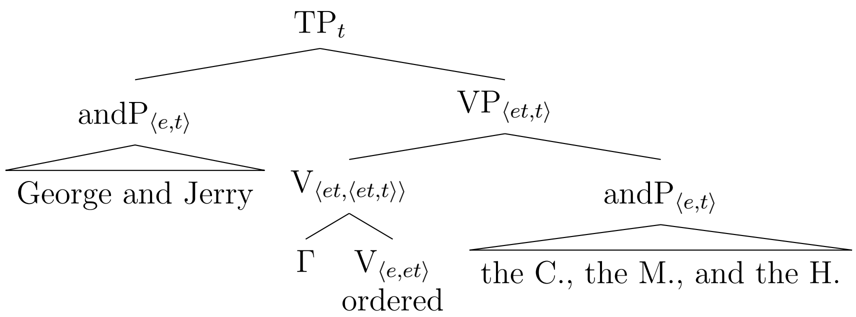
(73) is true in a scenario where George just ordered the *Calzone*, but Jerry was extra hungry, and ordered the *Margharita* and the *Hawaiian*.

To capture the *cumulative* reading, we need to posit another operator Γ in addition to Dist, which applies to relations. Γ is defined for any types σ, τ:

(69) $[\![\Gamma]\!] = ...$

$... = \lambda R_{\sigma,\tau t}.\lambda X_{\tau t}.\lambda Y_{\sigma t}.\forall x[(X(x) \rightarrow \exists x'[Y(x') \wedge R(x',x)]) \wedge (Y(x) \rightarrow \exists x''[X(x'') \wedge R(x,x'')])]$

$... = \lambda R_{\sigma,\tau t}.\lambda X_{\tau t}.\lambda Y_{\sigma t}.\forall x[(x \in X \rightarrow \exists x'[x' \in Y \wedge \langle x',x\rangle \in R]) \wedge (x \in Y \rightarrow \exists x''[x'' \in X \wedge \langle x,x''\rangle \in R])]$



(70) a. $[\![\text{G. and J.}]\!]^w = \{\text{G., J.}\}$
b. $[\![\text{C., M., and H.}]\!]^w = \{\text{C., M., H.}\}$
c. $[\![\text{ordered}]\!]^w = \lambda x_e.\lambda y_e.y \text{ ordered}_w\ x$

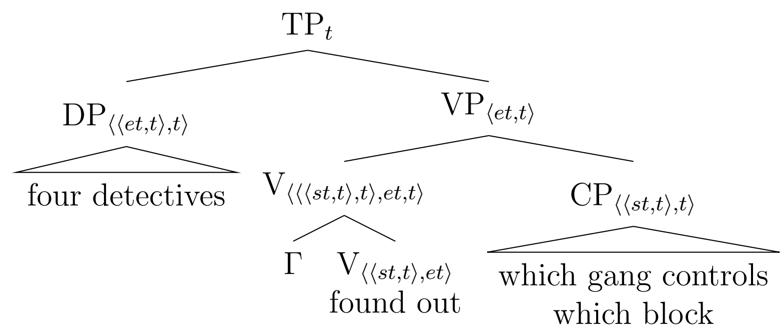
(71) $[\![\Gamma]\!]([\![\text{ordered}]\!]) = ...$
$... = \lambda X_{et}.\lambda Y_{et}.\forall x[(X(x) \to \exists x'[Y(x') \wedge$
$\text{ordered}_w(x', x)])\wedge$
$(Y(x) \to \exists x''[X(x'') \wedge$
$\text{ordered}_w(x, x'')])]$

(72) $[\![\text{TP}]\!]^w = 1$ iff...
$...\forall x[(x \in \{\text{C., M., H.}\} \to \exists x'[x' \in \{\text{G., J.}\} \wedge$
$\text{ordered}_w(x', x)])\wedge$
$(x \in \{\text{C., M., H.}\} \to \exists x''[x'' \in \{\text{G., J.}\} \wedge$
$\text{ordered}_w(x'', x)])]$

Responsive verbs can express relations between *individuals* and *questions*. If we analyze multiple questions as *pluralities of questions*, we predict that a sentence with a plural subject and a multiple question object should receive cumulative readings.

(73) *Context:*
There are four crack detectives in the city, investigating organized crime. The detectives are mavericks, preferring to work strictly solo. Between them, they manage to cover the entire city.

(74) Four detectives found out which gang controls which block.



(75) $[\![\text{four detectives}]\!] = \lambda P_{et,t}.\exists X_{et}[|X| = 4 \wedge \text{detectives}(X) \wedge P(X)]$

(76) $[\![\text{find out}]\!] = \lambda q_{st,t}.\lambda x_e.x \text{ found out}_w\ Q$

(77) $[\![\Gamma]\!]([\![\text{find out}]\!]) = ...$
$... = \lambda Q_{\langle st,t\rangle,t}.\lambda X_{et}.\forall q_{st,t}[Q(q) \to \exists x[X(x) \wedge x\text{found out } q]] \wedge$
$\forall x_e[X(x) \to \exists q_{st,t}[Q(q)\ \wedge\ x \text{ found out } q]]$

(78) $[\![\text{TP}]\!] = 1$ iff
$\exists X_{et}[|X| = 4\ \wedge\ \text{detectives}(X) \wedge$
$\forall q[q \in [\![\text{which gang controls which block}]\!]$
$\to \exists x[X(x)\ \wedge\ x \text{ found out } q]] \wedge$
$\forall x[x \in X \to \exists q[q \in [\![\text{which gang controls which block}]\!] \wedge$
$x \text{ found out } q]]]$

Note that cumulative readings seem to be available for other kinds of embedded questions too:

(79) *Context: same as (73)*
Four detectives found out [which block each gang controls].

(79) is a *wh*-question with universal quantifier. These also license PL readings, so perhaps we should analyze these as pluralities of questions too.

(80) a. Which block does each gang control?
b. The Brockley Boys control this block, and the South Man Syndicate control that one.

Surprisingly, *wh*-questions with a plural restrictor also license cumulative readings with *find out*:

(81) Four detectives found out [which gangs are at large].

(81) can be true in a situation where, e.g., detective A found out that gang A is at large, detective B that gang B is at large etc.

One way to account for this would be to model the meaning of a plural *wh*-question as a plurality of polar questions, although it's not clear how to derive this meaning compositionally.

(82) $[\![\text{which gangs are at large?}]\!] = ...$

$$... = \left\{ \begin{array}{l} [\![\text{is gang A at large?}]\!]^w \\ [\![\text{is gang B at large?}]\!]^w \\ [\![\text{is gang C at large?}]\!]^w \\ \text{...etc.} \end{array} \right\}$$

An additional open question is why only certain predicates such as *find out* license cumulative readings.

(83) *Distributive reading only with "know".*
a. Four detectives know which gang controls which block.
b. Four detectives know which block each gang controls.
c. Four detectives know which gangs are at large.

More places to look for evidence (see Fox 2012)

- Quantificational variability.
- Plural agreement.
- ???

References

Abrusán, Márta. 2014. *Weak island semantics*. First edition. Oxford studies in semantics and pragmatics 3. Oxford: Oxford University Press.

Champollion, Lucas. 2015. "Ten Men and Women Got Married Today: Noun Coordination and the Intersective Theory of Conjunction". *Journal of Semantics* (): ffv008. Visited on 02/11/2016.

Charlow, Simon. "On the semantics of exceptional scope". PhD thesis, New York University.

Dayal, Veneeta. 1996. *Locality in WH Quantification*. Red. by Gennaro Chierchia, Pauline Jacobson, and Francis J. Pelletier. Vol. 62. Studies in Linguistics and Philosophy. Dordrecht: Springer Netherlands. Visited on 08/31/2015.

Engdahl, Elisabet. 1986. "Constituent questions: The syntax and semantics of questions with special reference to Swedish. D". *D. Reidel, Dordrecht*.

Fox, Danny. 2012. *The semantics of questions*. Class notes, MIT seminar.

Kotek, Hadas. 2014. "Composing questions". PhD thesis, Massachusetts Institute of Technology.

Link, Godehard. 1983. "The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretic Approach". In *Formal Semantics - the Essential Readings*, ed. by P. Portner and B. H. Partee, 127–147. Blackwell.

Schwarzschild, Roger. 1996. *Pluralities*. Red. by Gennaro Chierchia, Pauline Jacobson, and Francis J. Pelletier. Vol. 61. Studies in Linguistics and Philosophy. Dordrecht: Springer Netherlands. Visited on 02/11/2016.

Vries, H. de. 2015. "Shifting sets, hidden atoms : the semantics of distributivity, plurality and animacy" (): 1–179. Visited on 02/11/2016.

Winter, Yoad. 2001. *Flexibility principles in boolean semantics: the interpretation of coordination, plurality, and scope in natural language*. Current studies in linguistics 37. Cambridge, Mass: MIT Press.