FUNCTIONAL READINGS II

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1 Recap: Engdahl's account of functional readings

THE PHENOMENA: questions with quantifiers allow complete answers specifying *functions* (Engdahl 1986); importantly, functional answers are irreducible to the Pair List (PL) reading (Groenendijk & Stokhof 1984).

(1) a. Which of his relatives does no Italian male love?
    b. His mother-in-law.
    c. #Giovanni, Maria; Paolo, Francesca.

ENGDAHL'S ANALYSIS: Engdahl derives functional readings on the basis of the following ingredients:

- A mechanism for shifting the restrictor of the *which* from a predicate of individuals to a predicate of skolem functions, $\mathbf{E}_n$.
- A polymorphic semantics for *which*.
- A "layered trace", consisting of a functional variable, and a covert pronoun, the latter of which may be semantically bound by an expression other than the *wh*-expression.

ROADMAP:

- Recap Engdahl's semantics for functional readings.
- A consideration of the answerhood conditions predicted by Engdahl's approach in more depth; this is important, since Heim's question denotations are roughly the same. In doing so, we'll address important questions raised by Jad and Adele.

1.1 Composing functional readings

A schematic LF:

(2) $\lambda p\ \boxed{\text{which } \mathbf{E}_1 \text{ relative of his}_1}\ \lambda f\ ?(p)$ no Italian male $\lambda x\ t_x$ love $t_f(x)$

Composing the restrictor:

$\mathbf{E}_n$ (def)

$$\mathbf{E} := \lambda R\,.\,\lambda f\,.\,\forall x[R(x)(f(x))]$$
$$: \langle\langle \mathtt{e}, \mathtt{et}\rangle, \langle\langle \mathtt{e}, \mathtt{e}\rangle, \mathtt{t}\rangle\rangle$$

(3) $[\mathbf{E}_1$ [relative of his$_1$]]

(4)

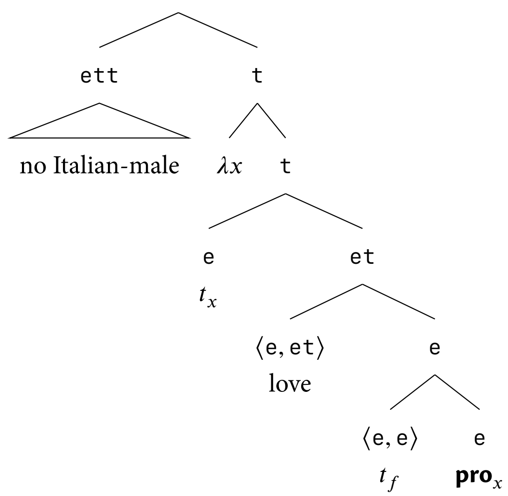
- $\langle \mathtt{ee}, \mathtt{t}\rangle$ — $\lambda f\,.\,\forall x[f(x)\ \textbf{relative-of}_{@}\ x]$
  - $\langle\langle \mathtt{e}, \mathtt{et}\rangle, \langle \mathtt{ee}, \mathtt{t}\rangle\rangle$ — $\mathbf{E}$
  - $\langle \mathtt{e}, \mathtt{et}\rangle$ — $\lambda y\,.\,\lambda x\,.\,x\ \textbf{relative-of}_{@}\ y$
    - λy
    - $\mathtt{et}$ — relative of his$_y$

Polymorphic *which*

$$[\![\text{which}]\!] := \lambda r\,.\,\lambda k\,.\,\exists x[r(x) \wedge k(x)]$$
$$: \langle \sigma\mathtt{t}, \sigma\mathtt{tt}\rangle$$

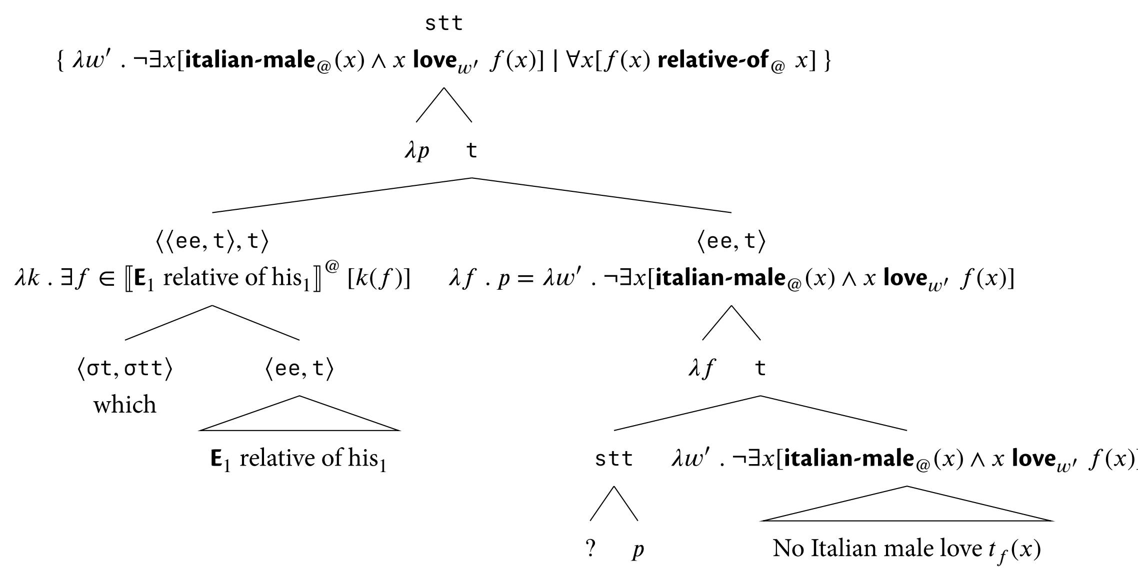
The *which*-phrase moves, leaving behind a functional trace — we insert a covert pronoun as the complement of the trace, semantically bound by *no Italian male*:

(5) $\lambda w' \, . \, \neg\exists x[\textbf{italian-male}_{@}(x) \wedge x \; \textbf{love}_{w'} \; f(x)]$



Now that we have the meaning of the TP, and the *wh* restrictor, we can compute the meaning of the question:

(6)



1.2 The pragmatics of functional answers

> Since Heim's theory will deliver more-or-less the same kind of question denotation as Engdahl's, it's worth stopping and considering whether or not the theory makes reasonable predictions when coupled with our theory of question pragmatics, before going any further.

1.3 Answerhood conditions

Consider the final meaning of the question — as an idealization Heim (2012) assumes that NPs are interpreted *de re*:[1]

(7) $\{ \lambda w' . \neg\exists x[\textbf{italian-male}_{@}(x) \wedge x\ \textbf{love}_{w'}\ f(x)] \mid \forall x[f(x)\ \textbf{parent-of}_{@}\ x] \}$

Let's consider whether this returns sensible results, in light of some independently-motivated assumptions regarding question pragmatics.[2]

Consider the following scenario:

- $\textbf{italian-male}_{@} = \{$ **giovanni**, **paolo** $\}$
- $\textbf{parent-of}_{@} = \{ \langle$**maria**, **giovanni**$\rangle, \langle$**italo**, **giovanni**$\rangle, \langle$**francesca**, **paolo**$\rangle, \langle$**mateo**, **paolo**$\rangle, \ldots \}$
- $\textbf{mother-of}_{@} = \{ \langle$**maria**, **giovanni**$\rangle, \langle$**francesca**, **paolo**$\rangle \}$
- $\textbf{father-of}_{@} = \{ \langle$**italo**, **giovanni**$\rangle, \langle$**mateo**, **paolo**$\rangle \}$

To avoid questions of partiality, let's hold the domain of f constant as the Italian males in @. There are four possible functions from Italian males to their relatives.[3]

$$f_{\text{mother}} : \begin{bmatrix} \textbf{giovanni} \rightarrow \textbf{maria} \\ \textbf{paolo} \rightarrow \textbf{francesca} \end{bmatrix} \quad f' : \begin{bmatrix} \textbf{giovanni} \rightarrow \textbf{maria} \\ \textbf{paolo} \rightarrow \textbf{mateo} \end{bmatrix}$$

$$f'' : \begin{bmatrix} \textbf{giovanni} \rightarrow \textbf{italo} \\ \textbf{paolo} \rightarrow \textbf{francesca} \end{bmatrix} \quad f_{\text{father}} : \begin{bmatrix} \textbf{giovanni} \rightarrow \textbf{italo} \\ \textbf{paolo} \rightarrow \textbf{mateo} \end{bmatrix}$$

The resulting answer set in @:

$$= \begin{Bmatrix} \textbf{that no Italian male}_{@}\ x\ \textbf{loves}\ f_{\textbf{mother}}(x) \\ \textbf{that no Italian male}_{@}\ x\ \textbf{loves}\ f'(x) \\ \textbf{that no Italian male}_{@}\ x\ \textbf{loves}\ f''(x) \\ \textbf{that no Italian male}_{@}\ x\ \textbf{loves}\ f_{\textbf{father}}(x) \end{Bmatrix}$$

Extensionally:

[1] @ is used to refer to the utterance evaluation world.

[2] I'm grateful to Jad for pressing me to clarify this.

[3] Because the *which*-phrase is singular, the functions that the *which*-phrase quantifies over map Italian males to atomic individuals.

$$= \left\{\begin{array}{l} \text{① } \textbf{that giovanni doesn't love maria and paolo doesn't love francesca} \\ \text{② } \textbf{that giovanni doesn't love maria and paolo doesn't love mateo} \\ \text{③ } \textbf{that giovanni doesn't love italo and paolo doesn't love francesca} \\ \text{④ } \textbf{that giovanni doesn't love italo and paolo doesn't love mateo} \end{array}\right\}$$

A bit more abstractly:

$$= \left\{ \boxed{\neg (p \vee q)}, \boxed{\neg (p \vee q')}, \boxed{\neg (p' \vee q)}, \boxed{\neg (p' \vee q')} \right\}$$

All of the answers are logically independent, which means that, applying Dayal's presupposition[4], the question should presuppose that exactly one of these answers is true — i.e., each Italian male doesn't love exactly one his relatives.

[4] See Danny's handout from last time.

Jad noticed that this is actually at odds with one of Groenendijk & Stokhof's central observations.

Context: *Giovanni and Paolo are both on poor terms with their fathers, but Giovanni loves his mother, while Paolo is on poor terms with both of his parents. (both* $\neg(p' \vee q')$ *and* $\neg(p' \vee q)$ *hold)*

(8) a. Which of his parents does no Italian male love?
b. His father ($\neg(p' \vee q')$).

Groenendijk & Stokhof deem this to be a complete answer in the context. This is even easier to see in an embedded context:

(9) Jean knows which of his parents no Italian male loves.

As far as I can see, the question is however predicted to be deviant in this context, since Dayal's presupposition isn't satisfied. This is because both ④ and ③ are true.

As Jad pointed out to me, the problematic members of the answer set arise because we allow for "mixed" functions, mapping one Italian to his mother, and another to his father. If we can somehow ensure that these answers don't end up in the Hamblin set, we'll be OK.

One possible move, suggested by Groenendijk & Stokhof (1984), Chierchia (1992) is to contextually restrict functions quantified over by *which* to just the "natural" functions, e.g., *mother-of*, *father-of*, etc.

I don't think this is quite the whole story however, based on cases like the following:

Context: *Giovanni's mother criticized him, so he's on poor terms with her but not his father; Paolo's father criticized him, so he's on poor terms with him but not his mother*

(10) a. Which of his parents does no Italian male love?
b. The one who criticized him.

> In the following, we'll assume that this problem is solvable, and Engdahl's denotation is reasonable, but this issue deserves further investigation.

> The fact that functions can be characterized extensionally begs the question of *why* exactly we can't answer (10a) by saying "Giovanni doesn't love Maria, and Paolo doesn't love Francesca", which conveys exactly the same information as *his mother* on the *de re* construal. It's difficult to see how to block this if completely *de re* readings can be generated.

1.4 Covert pronouns

Recall that one of the crucial ingredients in Engdahl's account is the construction of a *layered trace* via insertion of a covert pronoun.

Adele pointed out last time that there's nothing in the system that forces the pronoun to be bound by a quantificational subject. We can generate "functional" readings with free covert pronouns. Heim (2012: p. 10) acknowledges this.

(11) Which book did John read?

Possible LF:

(12) λp [which $\mathbf{E}_1$ book] λf ?(p) John read $f(\mathbf{pro}_2)$

Heim asks us to imagine a context in which Mary is sufficiently salient, that a free pronoun might pick her out.

The we predict the following answer set:

(13) $\{\ \lambda w'\ .\ \textbf{john read}_{w'}\ f(\textbf{mary}) \mid \forall x[\textbf{book}_{@}(f(x))]\ \}$

For example, answers might be of the form *Mary's favourite book*, or *Mary's least favourite book*, but not *War & Peace*, unless there is some known connection between Mary and *War & Peace*.

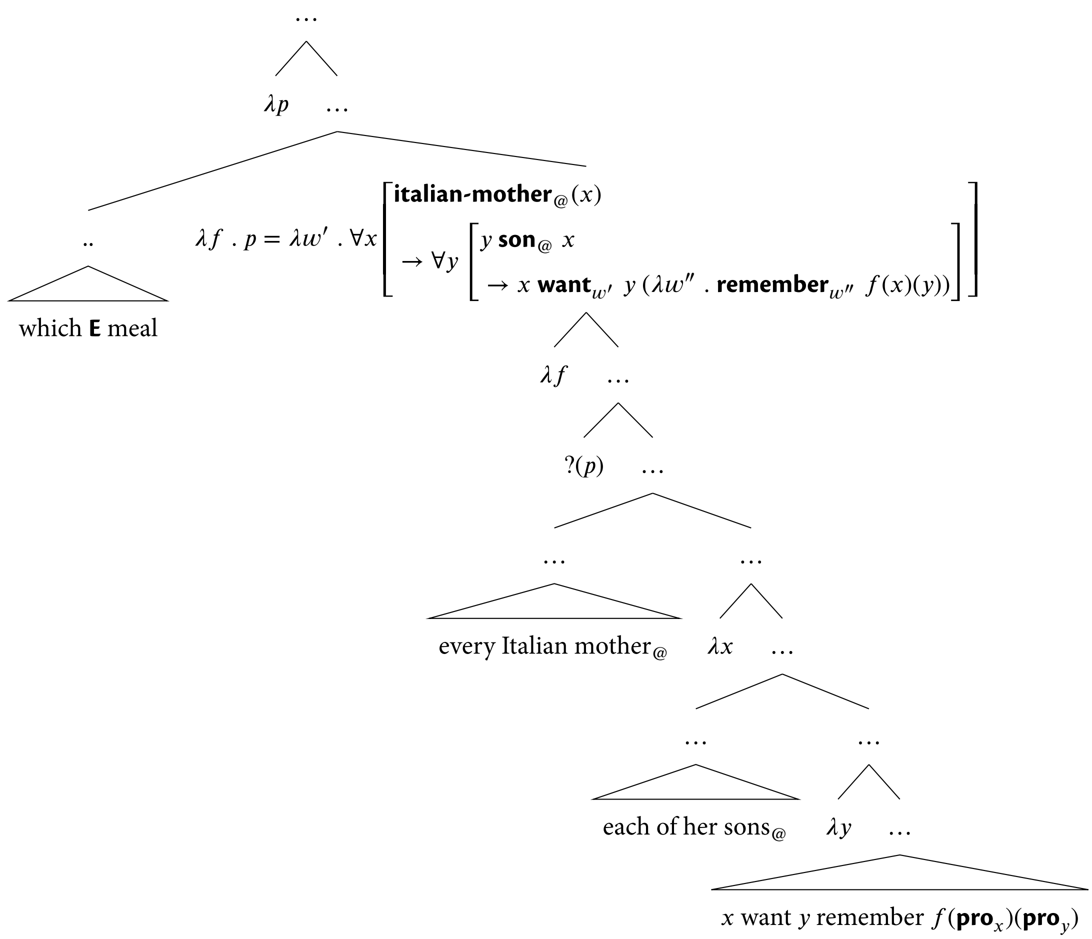
Heim suggests that there is nothing to worry about, since we already have independent reasons to believe that quantifier domains can be contextually restricted, and this reading is indistinguishable from contextual domain restriction.

Relatedly, it's worth mentioning that we want our theory to be able to account for functional readings involving multiple binders, as discussed by Engdahl.

(14) Which meal does every Italian mother
want each of her sons to remember fondly?
   a. His favourite dish by her.

In order to account for this reading, we must posit a syntactic structure involving a layered trace with *two* covert pronouns.

(15)



> **Homework exercise**
>
> As you'll have notice, the functional trace in (15) must be of type $\langle e, \langle e, e \rangle\rangle$, which means that the meaning of E must be adjusted accordingly.
>
> - Write the entry for **E** we need to account for the question in (15).
> - **Advanced:** It's possible to add potential binders indefinitely, which means that we need to an algorithm for defining new **E** operators. Write a recursive definition for **E** which accommodates a potentially infinite number of argument slots.

1.5 De dicto readings

Recall Kai's askability condition on questions: roughly, for a question ϕ to be askable relative to a context c, $[\![\phi]\!]^{w}$ should be identical across worlds in c.

For simple *which*-questions, this straightforwardly predicts that the restrictor set should be common ground, i.e., the following is assertable just in case it's common ground what the movies are (subject to contextual domain restriction):

(16) Which movie did you watch?

Let's consider whether this makes reasonable predictions for Engdahl's answer set. Consider again:

(17) a. Which of his parents does no Italian male love? *de re*
  b. $\{\ \lambda w'\ .\ \neg\exists x[\textbf{italian-male}_{@}(x) \wedge x\ \textbf{love}_{w'}\ f(x)] \mid \forall x[f(x)\ \textbf{parent-of}_{@}\ x]\ \}$

For the question to be askable, it should at least be the case that it's common ground who the Italian males are, and who their parents are.

Intuitively, however, the question is askable even if the asker has no idea who the Italian males are, or who their parents are, and a functional answer such as "his father", doesn't necessarily add any of this information to the common ground.

This is even easier to see in an embedded context:

(18) Gigi knows which of his parents no Italian male loves.

One way around this problem is to redefine **E** such that *which*-phrases quantify over functions from individuals to *individual concepts*. Here's a way of doing so:[5]

[5] The revised entry for **E** is looking for an *intensional* relation. I assume it composes with its argument via intensional function application.

(19) $\textbf{R} \coloneqq \lambda w\ .\ \lambda y\ .\ \lambda x\ .\ x\ \textbf{parent-of}_{w}\ y$ $\langle \mathtt{s}, \langle \mathtt{e}, \mathtt{et}\rangle\rangle$

(20) $[\![\textbf{E}]\!]^{w} \coloneqq \lambda R\ .\ \{\ f \mid \forall x \in D, w \in W[R(w)(x)(f(x)(w))]\ \}$
$\langle\langle \mathtt{s}, \langle \mathtt{e}, \mathtt{et}\rangle\rangle, \langle\langle \mathtt{e}, \mathtt{se}\rangle, \mathtt{t}\rangle\rangle$

(21) $[\![\textbf{E}\ 1\ \text{parent of his}_{1}]\!]^{w} = \{\ f \mid \forall x \in D, w \in D[f(x)(w)\ \textbf{parent-of}_{w}\ x]\ \}$
$\langle\langle \mathtt{e}, \mathtt{se}\rangle, \mathtt{t}\rangle$

What we get back is a set of functions that map individuals x, to functions that map worlds w, to relatives of x in w (i.e., *parent* individual concepts).

We furthermore assume that variables over functions from individuals to individual concepts can be interpreted relative to a local intensional operator. The resulting *de dicto* question denotations are as follows:

(22) a. Which of his parents does no Italian male love? *de dicto*
  b. $\{\ \lambda w'\ .\ \neg\exists x[\textbf{italian-male}_{w'}(x) \wedge x\ \textbf{love}_{w'}\ f(w')(x)] \mid \forall x, w[f(x)(w)\ \textbf{parent-of}_{w}\ x]\ \}$

The resulting question denotation is simply as follows:

$$= \left\{ \begin{array}{r} \textbf{that no Italian male loves his mother} \\ \textbf{that no Italian males loves his father} \\ \dots \end{array} \right\}$$

This is a more satisfactory account of the functional reading under attitude verbs, and the askability conditions on questions with functional readings.

> At this point, we've hopefully convinced ourselves that Engdahl's account of functional answers, in terms of *wh*-quantifiers ranging over skolem functions, is a reasonable one. In the latter part of this class, we'll turn to Heim's reconstruction of Engdahl — the fundamental idea behind the question denotations won't change, but the role of partiality will be clarified, and we'll ground Engdahl's semantics in an independently motivated syntactic component.

2 *Prelude to Heim 2012: Projecting Partiality*

In the following we're going to be dealing with (potentially) partial functions, so let's be precise about how our semantic composition principles deal with potential partiality:

Function application (Heim & Kratzer 1998, von Fintel & Heim 2021):

$$\left[\!\!\left[\begin{array}{c} \dots \\ \wedge \\ \alpha \quad \beta \end{array} \right]\!\!\right]^{w,g} = \begin{cases} [\![\alpha]\!]^{w,g}([\![\beta]\!]^{w,g}) & [\![\beta]\!]^{w,g} \in \textbf{dom}([\![\alpha]\!]^{w,g}) \\ \text{undefined} & \text{else} \end{cases}$$

Predicate Abstraction:

$$\left[\!\!\left[\begin{array}{c} \dots \\ \wedge \\ n \quad \gamma \end{array} \right]\!\!\right]^{w,g} = \lambda x\ .\ \begin{cases} [\![\gamma]\!]^{w,g[x/n]} & \gamma \in \textbf{dom}([\![.]\!]^{w,g[x/n]}) \\ \text{undefined} & \text{else} \end{cases}$$

2.1 *Partiality in functional readings*

Consider again the meaning that Engdahl would ascribe to the question to capture the functional answer:

(23) $\{\, \lambda w' \,.\, \neg\exists x[\textbf{girl}_{@}(x) \wedge x\ \textbf{submit}_{w'}\ f(x)] \mid \forall x[f(x)\ \textbf{picture-of}_{@}\ x] \,\}$

Heim points out that elements of the answer set vary according to functions f, which, for *every* element x in the domain, map x to a picture of x.

As long as there is at least one individual in the domain which hasn't had its picture taken, no such functions will exist, so the denotation in (23) can't be quite right — in fact, the functions in question intuitively only need to be defined for *girls*.

Intuitively, the answers in the question denotation should vary according to *partial* functions. Let's change the definition of **E** so that it can handle partial functions:[6]

(24) $\textbf{E} \coloneqq \lambda R \,.\, \lambda f \,.\, \forall x[x \in \textbf{dom}(f) \rightarrow R(x)(f(x))]$ $\langle\langle \texttt{e}, \texttt{et}\rangle, \langle \texttt{ee}, \texttt{t}\rangle\rangle$

[6] The presupposition of f is accommodated within the restrictor of the universal.

As we've mentioned, the functions that the answers in the question denotation range over should (at least) be defined for all girls.

(25) Which picture of herself did no girl^{x} $\boxed{t_x \text{ submit } f(x)}$

Let's compute the meaning of TP (the scope site of the negative indefinite), taking into account the possibility that f is a *partial* function.

(26)

VP
$\lambda x : x \in \textbf{dom}(f) \,.\, x\ \textbf{submit}_{w'}\ f(x)$

- λx
- $\begin{cases} g_x\ \textbf{submit}_{w'}\ g_f(g_x) & g_x \in \textbf{dom}(g_f) \\ \text{undefined} & \text{else} \end{cases}$
  - x
  - ...
    - submit
    - $\begin{cases} g_f(g_x) & g_x \in \textbf{dom}(g_f) \\ \text{undefined} & \text{else} \end{cases}$
      - f
      - x

After abstraction triggered by movement of *no girl*, we have a partial function that is only defined for individuals already in the domain of f.

Now to compose *no girl* with its sister, a partial function.

(27) Which picture of herself did $\boxed{\text{no girl}^x\ t_x \text{ submit } f(x)}$

Assumption: presuppositions project *universally* from under negative indefinites.

We can independently motivate this assumption by looking at the behaviour of presuppositions under the scope of negative indefinites.

(28) No girl $\boxed{\text{parked her bicycle}}$. presupposes: *every girl has a bicycle*

$$(29)\quad \begin{cases} \textbf{no girl parked her bicycle} & \textbf{every girl has a bicycle} \\ \text{undefined} & \text{else} \end{cases}$$

Now we can compose the partial function denoted by the scope site with *no girl* — the resulting presupposition is that *every girl is in the domain of f*.

$$(30)\quad \begin{cases} \neg\exists x[\mathbf{girl}_@(x) \wedge x\ \mathbf{submit}_{w'}\ f(x)] & \forall x[\mathbf{girl}_@(x) \rightarrow x \in \mathbf{dom}(f)] \\ \text{undefined} & \text{otherwise} \end{cases}$$

$\lambda k\,.\,\neg\exists x[\mathbf{girl}_@(x) \wedge k(x)]$ $\qquad$ $\lambda x : x \in \mathbf{dom}\ x\,.\,x\ \mathbf{submit}_{w'}\ f(x)$

no girl $\qquad$ $\lambda x\ x$ submit $f(x)$

Now we have a question nucleus which denotes a partial proposition. If we compose the rest of the *wh*-question using Engdahl's machinery, the result is a set of "partial" propositions, which vary across potentially partial functions f mapping individuals to pictures of themselves, and are defined iff every actual girl is in the domain of f.

The resulting question denotation:

$$(31)\quad \left\{ \lambda w' \begin{array}{l} : \forall x[\mathbf{girl}_@(x) \rightarrow x \in \mathbf{dom}(f)] \\ .\ \neg\exists x[\mathbf{girl}_@(x) \wedge x\ \mathbf{submit}_{w'}\ f(x)] \end{array} \middle| \ \forall x[x \in \mathbf{dom}(f) \rightarrow f(x)\ \mathbf{picture\text{-}of}_@\ x] \right\}$$

As Heim (2012) observes, once we compute the question denotation, answers will never be partial propositions in a meaningful way.

Let's assume that the domain of $f_{\text{self-portrait}}$ is $\{\, x \mid \textbf{girl}_{@}(x) \,\}$, and furthermore that f maps each girl to her unique *self-portrait*.

The resulting proposition in the answer set will be total, since the presupposition is satisfied:

(32) $\lambda w' : \forall x[\textbf{girl}_{@}(x) \rightarrow x \in \textbf{dom}(f_{\text{self-portrait}})]$
$. \neg\exists x[\textbf{girl}_{@}(x) \wedge x \ \textbf{submit}_{w'} \ f_{\textbf{self-portrait}}(x)]$

Now let's imagine a different function f_{selfie}, and the domain of f_{selfie} is $\{\, x \mid \textbf{has-cellphone}_{@}(x) \,\}$; a set which only partially overlaps with the set of girls. f_{selfie} maps each individual in the domain to their unique selfie.

(33) $\lambda w' : \forall x[\textbf{girl}_{@}(x) \rightarrow x \in \textbf{dom}(f_{\text{selfie}})] \, . \ldots$

Since the presupposition is false, the resulting proposition will be undefined for every world in the domain.

Since every proposition is the resulting answer set is either total, or undefined for every world, we can rewrite the resulting question denotation as follows:

(34)
$$\left\{ \lambda w' \, . \neg\exists x[\textbf{girl}_{@}(x) \wedge x \ \textbf{submit}_{w'} \ f(x)] \;\middle|\; \begin{array}{l} \forall x[\textbf{girl}_{@}(x) \rightarrow x \in \textbf{dom}(f)] \\ \wedge \forall x[x \in \textbf{dom}(x) \rightarrow f(x) \ \textbf{picture-of}_{@} \ x] \end{array} \right\}$$
$$\cup \{ \, \lambda w' \, . \, \# \, \}$$

As Heim (2012) points out, the presence of the pathological element makes no difference for how the resulting Hamblin set partitions worlds in the context set.[7]

[7] Once we tweak our algorithm for partitioning based on a Hamblin set, in light of the possibility of partial propositions, we can demand that two worlds w and w' are cell-mates iff they are defined at and return the same truth value for every proposition in the Hamblin set.

> Disregarding the pathological element, the desired result is achieved — every answer in the question denotation involves a function whose domain includes all the girls in the actual world.

$$= \left\{ \begin{array}{c} \lambda w' \, . \neg\exists[\textbf{girl}_{@}(x) \wedge x \ \textbf{submit}_{w'} \ f_{\text{self-portrait}}(x)] \\[1em] \lambda w' \, . \neg\exists[\textbf{girl}_{@}(x) \wedge x \ \textbf{submit}_{w'} \ f_{\text{caricature}}(x)] \\[1em] \ldots \end{array} \right\}$$

$f_{\text{self-portrait}}$: a partial function whose domain is the actual girls, and which maps them all to their actual self-portraits; $f_{\text{caricature}}$: a partial function whose domain is the actual girls, and which maps them all to their actual caricatures...

> Having clarified the role of partiality in Engdahl's analysis, the goal now will be to refine and reconstruct the analysis in terms of independently motivated mechanisms for interpreting copies, thereby eliminating **E**.

3 *Heim's refinement*

3.1 *Background: the copy theory of movement*

> CRUCIAL ASSUMPTION: the restrictor of the *wh*-expression may be interpreted *in-situ*.

There are two ways of cashing out this conjecture; Heim adopts the second:

- At Logical Form, *which*-phrases are interpreted *in-situ* as definite descriptions (Rullmann & Beck 1998).
- Movement leaves behind a *copy*, which is converted into a *bound definite description* at LF (Fox 1999).

Independent motivation for the Rullmann & Beck 1998 conjecture: *which*-phrases sub-extracted from intensional contexts can be interpreted *de dicto*.

(35) John believes that there is unicorn.
Which unicorn$_i$ does John think that Mary tried to catch the$_i$ unicorn?

Cf. projection behaviour of definite descriptions under attitude verbs (Heim 1992) (modulo proviso inferences):

(36) John believes that there is a unicorn and
John thinks that Mary tried to catch the unicorn.

In order to interpret lower copies, we need two type-shifters: Partee's (1986) THE and IDENT.

IDENT is essentially a concretely-typed variant of ?:

(37) $\textbf{IDENT} \coloneqq \lambda x \,.\, \lambda y \,.\, y = x$ $\langle \texttt{e}, \texttt{et} \rangle$

THE is a covert definite determiner:

(38) $\textbf{THE} \coloneqq \lambda k \ :\ \exists! x[k(x)] \,.\, \iota x[k(x)]$ $\langle \texttt{et}, \texttt{e} \rangle$

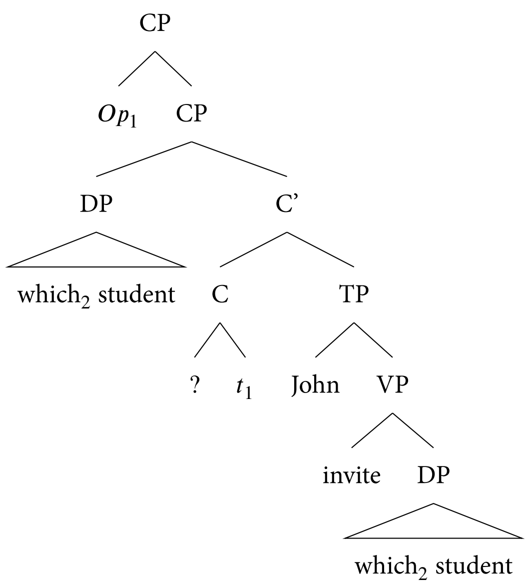
We'll also need to assume that *which* is interpreted as an *unrestricted* existential quantifier:

(39) $[\![\text{which}]\!] = \lambda k \,.\, \exists x[k(x)]$ $\texttt{ett}$

The structure delivered by the narrow syntax for a simple question:

(40) Which student did John invite?

(41)



Schematic algorithm for trace converstion at LF:

(42) a. λp [which$_2$ student] ?(p) John invite [which$_2$ student]

b. λp [which student] λ_2 ?(p) John invite [which 2 student]

$\Rightarrow$ *insert binder and variable*

c. λp [which ~~student~~] λ_2 ?(p) John invite [~~which~~ 2 student]

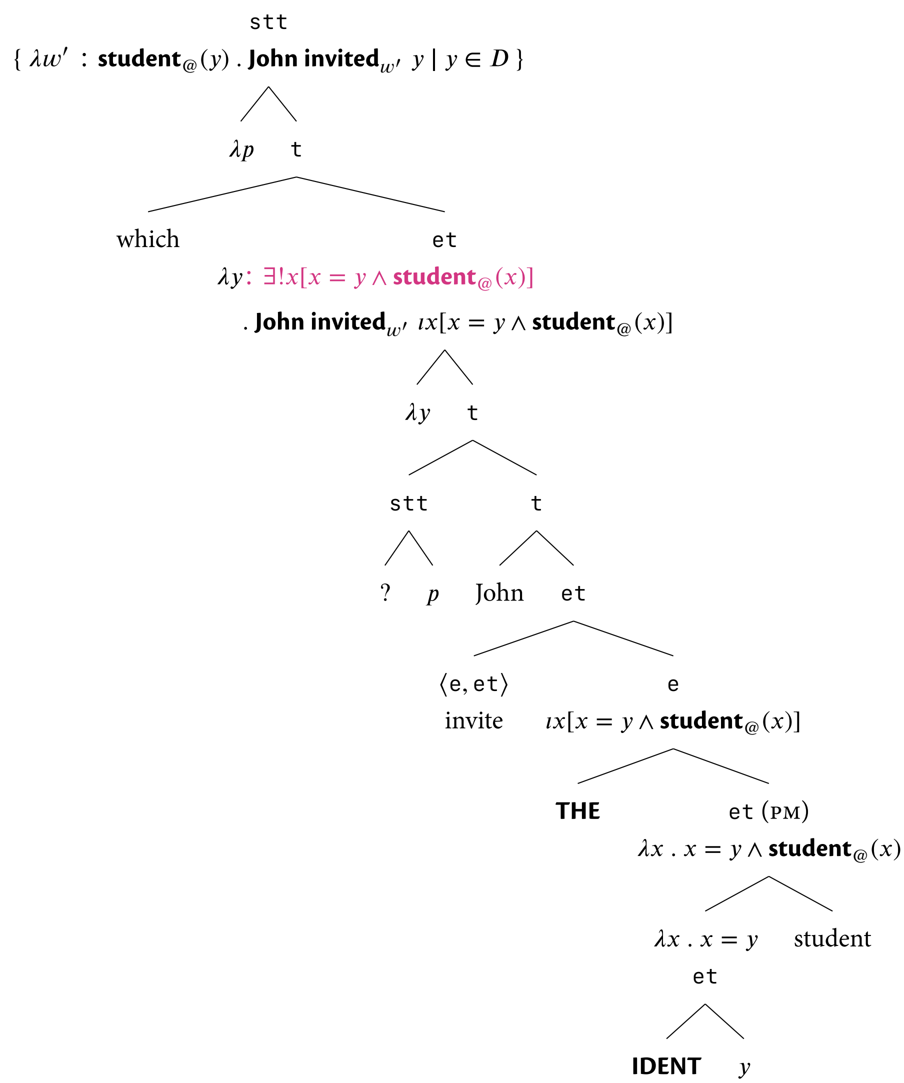
$\Rightarrow$ *delete higher restrictor and lower determiner*

d. λp which λ_2 ?(p) John invite [**THE** [**IDENT** 2] student]

$\Rightarrow$ *Rescue lower copy using type-shifters*

The resulting LF can now be interpreted; LF of the *which*-question post trace conversion:

(43)



> Since the restrictor of the *which*-phrase is interpreted *de re*, the resulting propositions in the question denotation are not really partial; rather, they are either *total* propositions, if y is a student in @, or the unique proposition undefined for any world.

(44) $\{ \lambda w' : \textbf{student}_{@}(y) . \textbf{John invited}_{w'}\ y \mid y \in D \}$

This is equivalent to:

(45) $\{\ \lambda w'\ .\ \textbf{John invited}_{w'}\ y \mid \textbf{student}_{@}(y)\ \} \cup \{\ \lambda w\ .\ \#\ \}$

As acknowledged by Heim, the proposal here is not obviously compatible with the *scope theory of intensionality*; the restrictor in the lower copy is interpreted *de re*, despite occurring within the scope of ?.

3.2 Functional readings via complex copies

(46) Which picture of herself did no girl submit?

The plan: generalize the basic theory to functional readings. We'll need to adopt polymorphic entries for *which*, and the type-shifters responsible for interpreting lower copies, as well as mechanisms for constructing something analogous to layered traces.

Which is a polymorphic existential quantifier, which will allow *which* to quantify over skolem functions.

(47) $[\![\text{which}]\!] := \lambda k\ .\ \exists x[k(x)]$ $\quad \sigma tt$

IDENT takes any value, and returns the (characteristic function of) the singleton set containing that value.

(48) $\textbf{IDENT} := \lambda x\ .\ \lambda y\ .\ y = x$ $\quad \langle \sigma, \sigma t \rangle$

THE is a polymorphic definite determiner.

(49) $\textbf{THE} := \lambda k\ :\ \exists! x[k(x)]\ .\ \iota x[k(x)]$ $\quad \langle \sigma t, \sigma \rangle$

We'll also need to allow for insertion of covert pronouns, in order to derive something corresponding to a *layered trace*.

The structure of the question (under the functional reading) delivered by the narrow syntax:

(50) λp [which picture of herself$_y$]$_2$
?(p) no girl λy y submit [which$_2$ picture of herself$_y$]

Post TC:

(51) λp which λf
?(p) no girl λy y submit [**THE** [**IDENT** f] picture of herself$_y$]

Rescue via insertion of covert pronoun:

(52) λp which λf
?(p) no girl λy y submit [**THE** [**IDENT** f(**pro**$_y$)] picture of herself$_y$]

Note immediately that the reflexive is *semantically bound* by *no girl*; the reflexive in the higher copy is simply deleted, along with the rest of the restrictor.

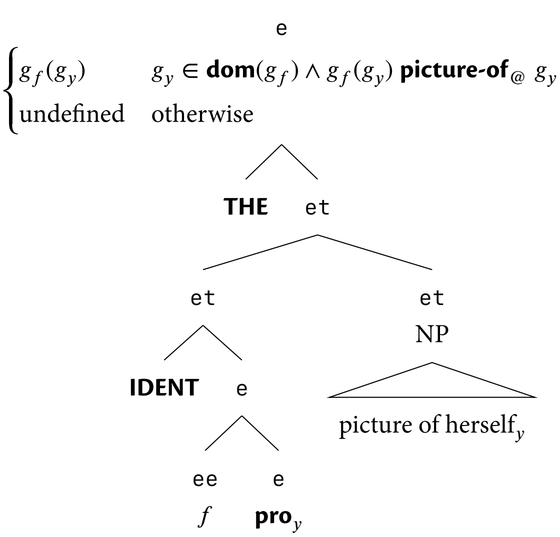
A PREDICTION(?): functional readings of questions should *always* feed condition C violations.

(53) Which picture of John did he show no girl?
a. ?The one she wanted to see the most.

Since, post TC:

(54) λp which λf
?(p) no girl λy he show [**THE** [**IDENT** f(**pro**$_y$)] picture of John]

The structure of the lower copy, post trace conversion + insertion of covert pronouns; **pro**$_y$ will eventually be semantically bound by the quantificational subject.

(55)


As composition proceeds, abstraction over y yields a partial function:

(56) $\lambda y : y \in \textbf{dom}(f) \land f(y)\ \textbf{picture-of}_{@}\ y\ .\ y\ \textbf{submitted}_{w'}\ f(y)$

VP

λy y submitted [**THE IDENT** $f(y)$ picture of y]

The presupposition projects universally through *no girl*:

(57) TP

$$\begin{cases} \neg\exists y[\textbf{girl}_{@}(y) \land y\ \textbf{submitted}_{w'}\ f(y)] & \forall y[\textbf{girl}_{@}(y) \to y \in \textbf{dom}(f) \land f(y)\ \textbf{picture-of}_{@}\ y] \\ \text{undefined} & \text{else} \end{cases}$$

No girl submitted [**THE IDENT** $f(y)$ picture of y]

Again, because the restrictor is interpreted *de re*, the propositions in the answer set are never partially defined.

(58) $\{\ \lambda w'\ .\ \neg\exists y[\textbf{girl}_{@}(y) \land y\ \textbf{submitted}_{w'}\ f(y)] \mid \forall y[\textbf{girl}_{@}(y) \to y \in \textbf{dom}(f) \land f(y)\ \textbf{picture-of}_{@}\ y]\ \} \cup$
$\{\ \lambda w'\ .\ \#\ \}$

3.3 Comparison with Engdahl

> One of the main differences between Heim 2012 and Engdahl 1986 is that, on Heim's approach, the reflexive in the restrictor really is (semantically) bound by its antecedent; on Engdahl's approach, the reflexive is *indirectly* bound by **E**.

Evidence for *direct* binding: ϕ-feature transmission (examples from Heim 2012: p. 12):

(59) Which picture of himself/*herself did no boy submit.
(60) Which relative of theirs did most people complain about?

(61) Which mistake that we have made will none of us ever forgive ourselves?

N.b., as Heim acknowledges, the force of this argument depends on the assumption that ϕ-features on bound pronouns/reflexives are determined configurationally (*feature transmission*; Kratzer 2009).

4 *Addendum: Functional readings without covert pronouns*

There's a line of work in Variable Free Semantics (VFS) generalizing a mechanism independently necessary to account for *paycheck pronouns* to functional readings of questions. See, especially Polly Jacobson's work (Jacobson 1999, 2000, 2014).

As shown by Charlow (2019a,b), Jacobson's innovations aren't proprietary to VFS. In the following, I'll attempt to reconstruct Jacobson's analysis of functional readings in a more standard, variable-full setting, based on techniques developed in Charlow 2019a.[8]

[8] This section benefited from discussion with Filipe Hisao-Kobayashi, who independently worked out something similar.

4.1 Paycheck pronouns

(62) Every philosopher spent his paycheck. Every linguist saved it.

Here, the pronoun it denotes a function that maps individuals to their paychecks.

> How do we account for this compositionally, are pronouns ambiguous? It turns out that we can capture paycheck pronouns by generalizing standard machinery for interpreting variables.

Instead of relativizing the interpretation function to an assignment parameter, we can equivalently enrich our denotations with outer assignment-arguments:

Old system (Heim & Kratzer 1998):

(63) $[\![\text{he}_1]\!]^g = g_1$ e

New system:

(64) $[\![\text{he}_1]\!] = \lambda g \,.\, g_1$ ge

We can define some compositional glue for threading assignments through composition.

Assignment sensitive Function Application (FA) replicates Heim & Kratzer's FA — it performs FA while keeping track of assignment-sensitivity.

(65) Assignment-sensitive FA:
$m \circledast n \coloneqq \lambda g \,.\, m(g)(n(g))$

Pure lifts any value into a trivially assignment sensitive value; this is a correlate of the fact that our old interpretation function was relativized to an assignment parameter, even in the absence of assignment-sensitivity.

(66) Pure:
$\rho \coloneqq \lambda x \,.\, \lambda g \,.\, x$ $\langle \sigma, g\sigma \rangle$

We use assignment-sensitive FA and *pure* to compose pronouns with non-assignment-sensitive expressions.

(67) John likes him_1

$\lambda g \,.\, \textbf{john likes}\ g_1$
$\circledast$
- $\lambda g \,.\, \textbf{john}$
  ρ John
- $\lambda g \,.\, \lambda y \,.\, y\ \textbf{likes}\ g_1$
  $\circledast$
  - $\lambda g \,.\, \textbf{likes}$
    ρ likes
  - $\lambda g \,.\, g_1$

We no longer need a syncategorematic rule of **pa!** (**PA!**) — an abstraction operator can be defined categorematically:

(68) Abstraction operator:
$\Lambda_n \coloneqq \lambda k \,.\, \lambda g \,.\, \lambda x \,.\, k(g^{[n \rightarrow x]})$ $\langle g\sigma, \langle g, e\sigma \rangle \rangle$

Binding involves insertion of an abstraction operator (Büring 2005):

(69) John Λ_1 spent his_1 paycheck.

(70) $\lambda g \,.\, \textbf{John spent } (\textbf{paycheck-of John})$

$$\circledast$$

- $\lambda g \,.\, \textbf{John}$
  - ρ John
- $\lambda g \,.\, \lambda x \,.\, x \textbf{ spent } (\textbf{paycheck-of } x)$
  - Λ_1
  - $\lambda g \,.\, g_1 \textbf{ spent } (\textbf{paycheck-of } g_1)$
    - t_1 spent his$_1$ paycheck

So far, we've done nothing except for reconstruct the standard treatment of variables in model-theoretic terms.

> In order to account for paycheck pronouns, we give pronouns a *recursive* type-signature

$$\langle \mathsf{g}_1, \langle \ldots \langle \mathsf{g}_n, \mathsf{e} \rangle \rangle \rangle$$

The meaning of a pronoun doesn't change — it's something that returns a value based on an outer-layer of assignment sensitivity, it's just that sometimes the return value is itself assignment-sensitive.

(71) $[\![\text{pro}_n]\!] \coloneqq \lambda g \,.\, g_n$ $\langle \mathsf{g}_1, \langle \ldots \langle \mathsf{g}_n, \mathsf{e} \rangle \rangle \rangle$

A simple paycheck pronoun is a rigidly typed instantiation of this meaning; given an outer layer of assignment-sensitivity, it returns an assignment-sensitive value:

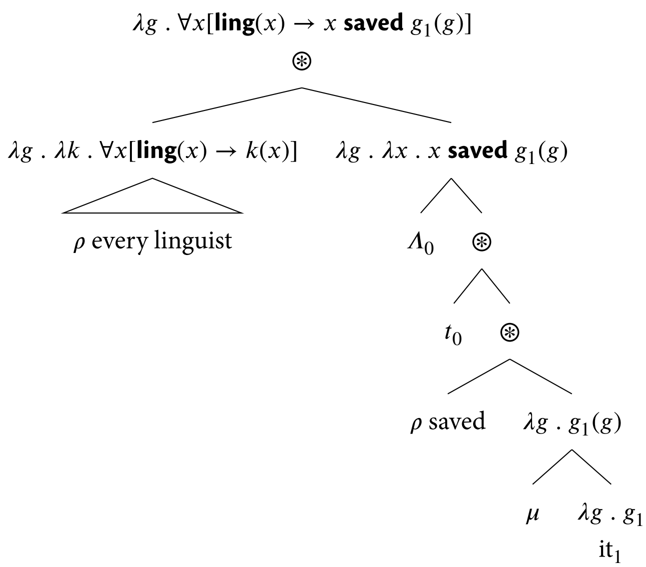
(72) Paycheck pronoun: $[\![\text{it}_1]\!] \coloneqq \lambda g \,.\, g_1$ $\langle \mathsf{g}, \mathsf{ge} \rangle$

In order to incorporate paycheck pronouns into semantic composition, we need one more type-shifter: a flattener (called *join*):

(73) $\mu \coloneqq \lambda i \,.\, \lambda g \,.\, i(g)(g)$ $\langle \langle \mathsf{g}, \mathsf{g}\sigma \rangle, \mathsf{g}\sigma \rangle$

Paycheck pronoun derivation:

(74)



If $g_1 = \lambda g\ .$**paycheck-of** g_0 (the value of "his$_0$ paycheck" on this theory), we'll get the paycheck reading.

Let's assume that traces, like pronouns, denote variables, and like pronouns, have a recursive type-signature.

This *predicts* the possibility of a functional reading, just in case the trace of the moved *wh*-expression has a paycheck denotation.

(75) λp Which picture of herself$_0$ Λ_1 ?(p) did no girl Λ_0 submit t_1?

Question nucleus:

(76) $\lambda g\ .\ \lambda i\ .\ p = \lambda w\ .\ \neg\exists x[\textbf{girl}_w(x) \wedge x\ \textbf{submit}_w\ i(g^{[0\to x]})]$

- Λ_1
- $\lambda g\ .\ p = \lambda w\ .\ \neg\exists x[\textbf{girl}_w(x) \wedge x\ \textbf{submit}_w\ g_1(g^{[0\to x]})]$
  - ρ ?(p)
  - $\lambda g\ .\ \neg\exists x[\textbf{girl}_w(x) \wedge x\ \textbf{submit}_w\ g_1(g^{[0\to x]})]$
    - ...
      - ρ no girl
    - $\lambda g\ .\ \lambda x\ .\ x\ \textbf{submit}_w\ g_1(g^{[0\to x]})$
      - Λ_0
      - $\lambda g\ .\ g_0\ \textbf{submit}_w\ g_1(g)$
        - t_0
        - $\lambda g\ .\ \lambda x\ .\ x\ \textbf{submit}_w\ g_1(g)$
          - ρ submitted
          - $\lambda g\ .\ g_1(g)$
            - μ
            - $\lambda g\ .\ g_1$
              - t_1

Now, we want the *wh*-expression to existentially quantify over assignment-sensitive description, i.e., $\lambda g\ .\ g_0$**'s selfie**, $\lambda g\ .\ g_0$**'s self-portrait**, etc.

If we compose the meaning ordinarily, the *wh*-restrictor will denote an assignment-sensitive predicate:

(77) $[\![\text{picture of herself}_0]\!]^w := \lambda g\ .\ \{\ x \mid\ x\ \textbf{picture-of}\ g_0\ \}$ $\langle \texttt{g}, \texttt{et} \rangle$

In order to get the functional reading, we need an operation that will push the assignment-sensitivity inwards. We may as well call it **E**:

(78) $\textbf{E}(P) := \{\ i \mid \forall g[i(g) \in P(g)]\ \}$

(79) $[\![\textbf{E}\ \text{picture of herself}_0]\!]^w\ :=\ \{\ i \mid\ \forall g[i(g) \in \{\ x \mid x\textbf{picture-of}_w\ g_0\ \}]\ \}$
$\langle \texttt{g}, \texttt{et} \rangle$

As usual, we can assume that *which* is a polymorphic existential determiner.

The resulting question denotation:

(80) $\{\ \lambda g w'\ .\ w'\ .\ \neg\exists x[\textbf{girl}_{w'}(x) \wedge x\ \textbf{submit}_{w'}\ i(g^{[0\to x]})] \mid \forall g[i(g) \in \{\ x \mid x\ \textbf{picture-of}_{w}\ g_0\ \}]\ \}$

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