CROSSOVER AND THE DYNAMICS OF NEGATION[1]

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links

- Handout: `https://patrl.keybase.pub/handouts/nyu-handout.pdf`
- There's a manuscript covering most of the material in this handout, which you can find on lingbuzz: `https://ling.auf.net/lingbuzz/005311`
- Additional comments/questions are very much welcome. Contact me @ `pdell@mit.edu`

1 *Roadmap*

We'll be covering some extremely well-trodden ground, namely theories of anaphora to singular indefinites in the *dynamic interpretation* tradition (Heim 1982, Groenendijk & Stokhof 1991, Groenendijk, Stokhof & Veltman 1996, Beaver 2001, etc.)

This family of theories famously accounts for discourse anaphora, donkey anaphora, presupposition projection, epistemic modality and can be/has been extended to deal with a whole bunch of other phenomena.

We'll almost exclusively be focusing on Dynamic Semantics (DS) as a theory of *pronominal anaphora*, concentrating on discourse anaphora.

(1) A^1 philosopher arrived and she_1 immediately began to vape.

There are two main themes to this presentation.

- We'll discuss some well-known problems for DS, in the domain of *double negation*, and bathroom sentences.
- We'll also discuss a recent application of DS, to Weak Crossover (WCO) (i.e., Chierchia's 2020 NLS paper), and argue that it has some unresolved issues.

- As we'll see, the issues facing a dynamic account of WCO revolve around the same operators — negative operators — which have prove independently problematic for DS.

The problem of double negation is perhaps underdiscussed, but has been addressed in existing work. See especially Krahmer & Muskens (1995), Gotham (2019), Mandelkern (2020).

One of the main theoretical contributions here will be to provide a fresh take on this problem, extending Charlow's (2014, 2019) Dynamic Alternative Semantics (DAS). This will tap into a similar intuition to existing work, although much of the machinery we'll need will "come for free".

More speculatively, we'll use Chierchia's dynamic treatment of WCO as a jumping off point to refine DAS further.

Ultimately, we'll suggest that — contra received wisdom — negation doesn't wipe out all Discourse Referents (DRS), but only *indeterminate* DRS, such as those introduced by indefinites; *Determinate* DR, such as those introduced by definites, remain unscathed.

We'll get there by a rather circuitous route, winding through negation in orthodox dynamic frameworks, Chierchia's theory of crossover, and ending in an incremental refinement of DAS which addresses the problems we've seen.

2 *Dynamics and its discontents*

2.1 *Negation as a destructive operation*

Dynamic theories of anaphora (Heim 1982, Groenendijk & Stokhof 1991, etc.) treat negation is a *destructive operation*; DRS introduced in the scope of negation are obliterated. We'll see this in painstaking detail later, but the intuitive idea is clear.

This is motivated by data such as (2); anaphora *disambiguates* in favor of the wide scope reading of the indefinite.

(2) It's not true [that we invited a^x philosopher]. She_x 's waiting outside.
✗ $\neg > \exists$, ✓ $\exists > \neg$

The following examples are just plain unacceptable, since wide existential scope is independently ruled out.

(3) We invited no^x philosopher. *She$_x$'s waiting outside.[2]

(4) We didn't invite any^x philosopher. *she's waiting outside.[3]

A natural consequence of the destructive theory is that, once cast into the pit, DRS cannot be resurrected. This leads immediately to the first problem...

Problem 1: Double negation

It's long been known that the destructive theory makes bad predictions in the domain of *double negation* (see, e.g., Groenendijk & Stokhof 1991, Krahmer & Muskens 1995, van den Berg 1996, Gotham 2019 for discussion).[4]

(6) It's not true that we DIDN'T invite a philosopher. She's waiting outside!

(7) It's not true that we didn't invite ANY philosopher. She's waiting outside!

(8) We didn't invite NO philosopher. She's waiting outside!

In all of the above examples, it seems that a DR that has been "previously" wiped out by a negative operator can be resurrected be a higher negation operator.

In general, the kinds of logic we get from dynamic theory of meaning is *too non-classical*; there are certain classical equivalences, such as $\phi \equiv \neg\neg\phi$, which we'd like to maintain in a dynamic setting.

Problem 2: Bathroom sentences

In DS, it's typically assumed that the second disjunct is interpreted in the context of the *negation* of the first disjunct (Beaver 2001).

This is motivated by the Karttunen-Heim generalization concerning presupposition projection:

(9) Either Paul never vaped, or he stopped vaping.

Perhaps unsurprisingly, we find similar facts in the domain of anaphora.[5]

(10) Either they didn't invite any^x philosopher, or she$_x$'s waiting outside.

Note that if the second disjunct is interpreted in the context of the *negation* of the first disjunct, this problem boils down to the problem of double negation.

Now that we've familiarized ourselves with one of the fundamental problems associated with DS, involving negation, we'll shift gear and discuss Chierchia's (2020) dynamic treatment of WCO.

[2] This can be taken to illustrate the same point, if we assume that negative indefinites are decomposed into an existential in the scope of negation at Logical Form.

[3] Negative Polarity Items (NPIS) can be shown to independently license cross-sentential anaphora, as demonstrated by the following example.

(4) If [we invite any philosopher^x and she$_x$ ask a difficult question] we'll be held responsible.

[4] It's a little difficult to formulate naturalistic examples, and these seems to be at least in part because double negation is subject to rather poorly understood information-structural constraints. I won't address this today.

[5] These observations can be traced back to Barbara Partee's *bathroom* sentences.

3 *Chierchia's dynamic theory of* WCO

3.1 The problem of WCO

As famously observed by Postal (1971), bound variable interpretations are subject to apparently arbitrary structural restrictions:

(11) * Who^x does his$_x$ mother aggravate who?
LF: who (λx . x's mother aggravates x)

There's nothing deviant about the Logical Form (LF) in (11) — the following should be a totally felicitous question-answer pair:

(12) a. Who^x does his$_x$ mother aggravate?
b. #Tony Soprano^x's mother aggravates him$_x$.

Chomsky (1976) extended Postal's observations to constraints on bound variable interpretations with Quantificational Phrases (QPs).

(13) Everyone^x aggravates his$_x$ mother?
LF: everyone (λx . x aggravates x's mother)

(14) * his$_x$ mother aggravates everyone^x ?
LF: Everyone (λx . x's mother aggravates x)

Scope can feed binding

A tempting response: certain kinds of displacement/scope operations can't feed pronominal binding; concomitantly binding proceeds from thematic positions.

(15) a. ✓Who^x t_x aggravates his$_x$ mother?
b. ✗Who^x does his$_x$ mother aggravate t_x ?

As shown by many authors, such as Ruys (2000), this doesn't straightforwardly work — scope *can* feed binding.[6]

[6] In other words, a bound variable interpretation doesn't require surface c-command. See also Barker 2012 for extensive argumentation along these lines.

A paradigmatic case – *binding out of* DP:

(16) a. [Every boy^x 's mother] loves him$_x$.
b. [[Every boy^x 's mother]'s therapist] loves him$_x$.

(17) a. [Which boy^x 's mother] loves him$_x$?
b. [[Which boy^x 's mother]'s therapist] loves him$_x$?

See also – *inverse linking*.[7]

[7] Note that the bound variable interpretation is only available if the universal takes wide scope – this is compelling evidence that what is feeding binding here is really *scope*.

(18) [A friend of every capo^x] owes him$_x$ money. ✓ $\forall > \exists$; ✗ $\exists > \forall$

Another paradigmatic case – *binding into adjuncts* – a focus of Chierchia's paper.[8]

(20) Some cop interviewed every capo^x
[in the presence of his$_x$ lawyer].

$\exists > \forall; \forall > \exists$

(21) Which capo^x did some cop interview
[in the presence of his$_x$ lawyer]

A generalization that fits the facts as I've presented them: *scope can feed binding of a pronoun, just in case the base position of the scope-taker is to the* ***left*** *of the pronoun* (Shan & Barker 2006, Barker & Shan 2014).[9]

Why should it be that the interaction between displacement/scope and binding is subject to a leftness condition?

Chierchia adopts an account couched in terms of a theory where anaphora proceeds from left-to-right, and potentially to non-c-commanding positions — Dynamic Semantics (DS).

The central move is to claim that the binder is, counterintuitively, always the *thematic* position of the QP, but, by dint of how anaphora is licensed in DS, the thematic position of the binder must be to the left of the pronoun.

In order to see how Chierchia's account works, it will be useful to start with a very brief technical excursus of DS.

3.2 Dynamic semantics: the basics

Propositions in DS map input assignments to sets of output assignments.

(22) Dynamic propositional type (def.)[10]
$\mathsf{D} \coloneqq \mathsf{g} \to \{\, \mathsf{g} \,\}$

Indefinites induce, in the dynamic parlance, *random assignment* — that is to say that the sentence takes an input assignment g and outputs a set of modified assignments. This is shown in (23).[11]

(23) Indefinites in DS
$[\![\text{A woman}^1 \text{ walked in}]\!] = \lambda g\,.\,\{\, g^{[1\to x]} \mid \textbf{woman}\ x \wedge \textbf{walked-in}\ x \,\}$ $\quad \mathsf{D}$

[8] A standard solution to this problem is to posit cascading/Larsonian shell structures for right-adjoined constituents, but this contradicts evidence for principle C effects.

(19) The lawyer interviewed him [in the presence of Tony's lawyer].

Similar paradoxes are documented extensively by Pesetsky (1995).

[9] Positing a linear component to the WCO generalization goes back to Chomsky's leftness condition.

[10] We write { a } for the type of a set of values of type a.

[11] Technically speaking, we'll assume that assignments are *partial* — that is to say, given a stock of variables $\mathbb{N}$, an assignment is a function whose domain is a subset of $\mathbb{N}$, and who codomain is the set of individuals. This means that a given assignment g may be *un*defined for a variable n.

We write $g^{[n\to x]}$ is the assignment that is just like g, other than mapping n to g — we assume that it is defined iff g_n is *undefined* (Heim's 1991 *novelty condition*).

Pronouns, on the other hand, induce a simple form of environment-sensitivity, as shown in (24).

(24) Pronouns in DS

$$[\![\text{She}_1 \text{ sat down}]\!] = \lambda g \,.\, \begin{cases} \{\, g \,\} & \textbf{sat-down}\ g_1 \\ \varnothing & \text{otherwise} \end{cases} \qquad \mathsf{D}$$

In order for pronouns to co-vary with indefinites in previous sentences, it is standard to define a dynamic sequencing operator (;), which threads dynamic propositions together by feeding the outputs of the first sentence into the second, pointwise, and gathering up the results (i.e., relation composition).

(25) Dynamic sequencing (def.)

$$p \mathbin{;} q \coloneqq \lambda g \,.\, \bigcup_{g' \in p\, g} \{\, g'' \mid g'' \in q\, g' \,\} \qquad ; : \mathsf{D} \to \mathsf{D} \to \mathsf{D}$$

It follows from the definition of dynamic sequencing that the input of the second sentence can be sensitive to the output of the first, but not vice versa.

In the dynamic parlance we say that, by dint of how dynamic sequencing is defined, in a sentence of the form $p \mathbin{;} q$, p is *accessible* to q, but not vice-versa.

3.3 *Chierchia's dynamic event semantics*

In order to get the notion of accessibility to do some work in the sentence-internal domain, Chierchia adopts a view of semantic composition that is fundamentally *conjunctive — neo-Davidsonian event semantics* (Castañeda 1967).

Some examples in a static setting:

(26) a. $[\![\text{it rained}]\!] = \exists e[\textbf{rain}\ e]$

b. $[\![\text{Shirley gave Jeff the book}]\!] = \exists e \begin{bmatrix} \textbf{agent}\ e = \textbf{shirley} \\ \wedge\ \textbf{theme}\ e = \textbf{the-book} \\ \wedge\ \textbf{goal}\ e = \textbf{jeff} \end{bmatrix}$

Following Chierchia, we can translate this basic setup into a dynamic setting by lifting event predicates into dynamic event predicates.

(27) Dynamically lifted verbal predicate

$$[\![\text{rain}]\!]^{\uparrow} \coloneqq\ = \lambda e g \,.\, \begin{cases} \{\, g \,\} & \textbf{rain}\ e \\ \varnothing & \text{otherwise} \end{cases} \qquad \mathsf{v} \to \mathsf{D}$$

In a static setting, thematic functions take individuals and return event predicates. These can be lifted into the current setting using an analogous recipe:

(28) Dynamically lifting a thematic function

$$\textbf{agent}^{\uparrow} := \; = \lambda g x e \,.\, \begin{cases} \{\, g \,\} & \textbf{agent}\ e = x \\ \varnothing & \text{otherwise} \end{cases} \qquad \mathsf{e} \to \mathsf{v} \to \mathsf{D}$$

We also need a composition rule for gluing together dynamic event predicates — *dynamic predicate modification*. This is simply an instantiation of dynamic sequencing, generalized in an analogous way to Partee & Rooth's (1983) generalized conjunction.

(29) Predicate modification in dynamic event semantics (def.)

$$f \,;\, g := \lambda e \,.\, f\ e \,;\, g\ e \qquad (\mathsf{v} \to \mathsf{D}) \to (\mathsf{v} \to \mathsf{D}) \to \mathsf{v} \to \mathsf{D}$$

Finally, we need a way of closing off the event variable, in order to get back a sentential meaning. In event semantics, this is typically done via an operation of existential closure. A dynamic version of this operation is defined below.

(30) Existential closure in dynamic event semantics

$$\varepsilon\ m := \lambda g \,.\, \bigcup_{e \in D_{\mathsf{v}}} m\ e\ g \qquad \varepsilon : (\mathsf{v} \to \mathsf{D}) \to \mathsf{D}$$

Indefinites, pronouns, and the Dynamic Predication Principle (DPP)

In order to illustrate the account of WCO it will only be necessary to discuss the semantics of indefinites and pronouns. Departing slightly from Chierchia here, are defined such that they may compose directly with thematic functions.

(31) $\textbf{pron}_n := \lambda r \,.\, \lambda e \,.\, \lambda g \,.\, r\ g_n\ e\ g \qquad (\mathsf{e} \to \mathsf{v} \to \mathsf{D}) \to \mathsf{v} \to \mathsf{D}$

Indefinites on the other hand denote functions from dynamic predicates to dynamic propositions. One important thing to note here is that they are *not* attributed any inherent anaphoric potential.

(32) $[\![\text{some boy}]\!] := \lambda k g \,.\, \bigcup_{\textbf{boy}\ x} k\ x\ g \qquad (\mathsf{e} \to \mathsf{D}) \to \mathsf{D}$

By dint of its type, it is clear that the indefinite is a scope-taker; they may not compose with thematic functions *in-situ*, but rather must be scoped out.

So, how are DRS introduced?

The Dynamic Predication Principle
DRS can only be introduced by predicates (Chierchia 2020: p. 32).

This is cashed out in the formal system via a superscript operator that applies to a thematic function, introducing a *determinate* DR relative to its individual argument.[12]

(33) DR introduction (def.)
$r^n := \lambda xeg \,.\, r\ x\ e\ g^{[n \rightarrow x]}$ $\qquad (\mathsf{e} \rightarrow \mathsf{v} \rightarrow \mathsf{D}) \rightarrow \mathsf{e} \rightarrow \mathsf{v} \rightarrow \mathsf{D}$

[12] Note that it's easy to define a version of the superscript operator that may apply to an individual or a quantifier, and therefore it's a crucial claim of the theory that such operators don't exist.

Some terminological clarification will be useful at this point:

- A dynamic proposition *introduces a DR n* if it is undefined for any input defined at n, and its outputs are defined at n.
- n is a *determinate* DR if every assignment in the output agrees on n (assuming they are defined at n).

  n is an *indeterminate* DR if the output assignments *dis*agree at n (again, assuming they are defined at n).

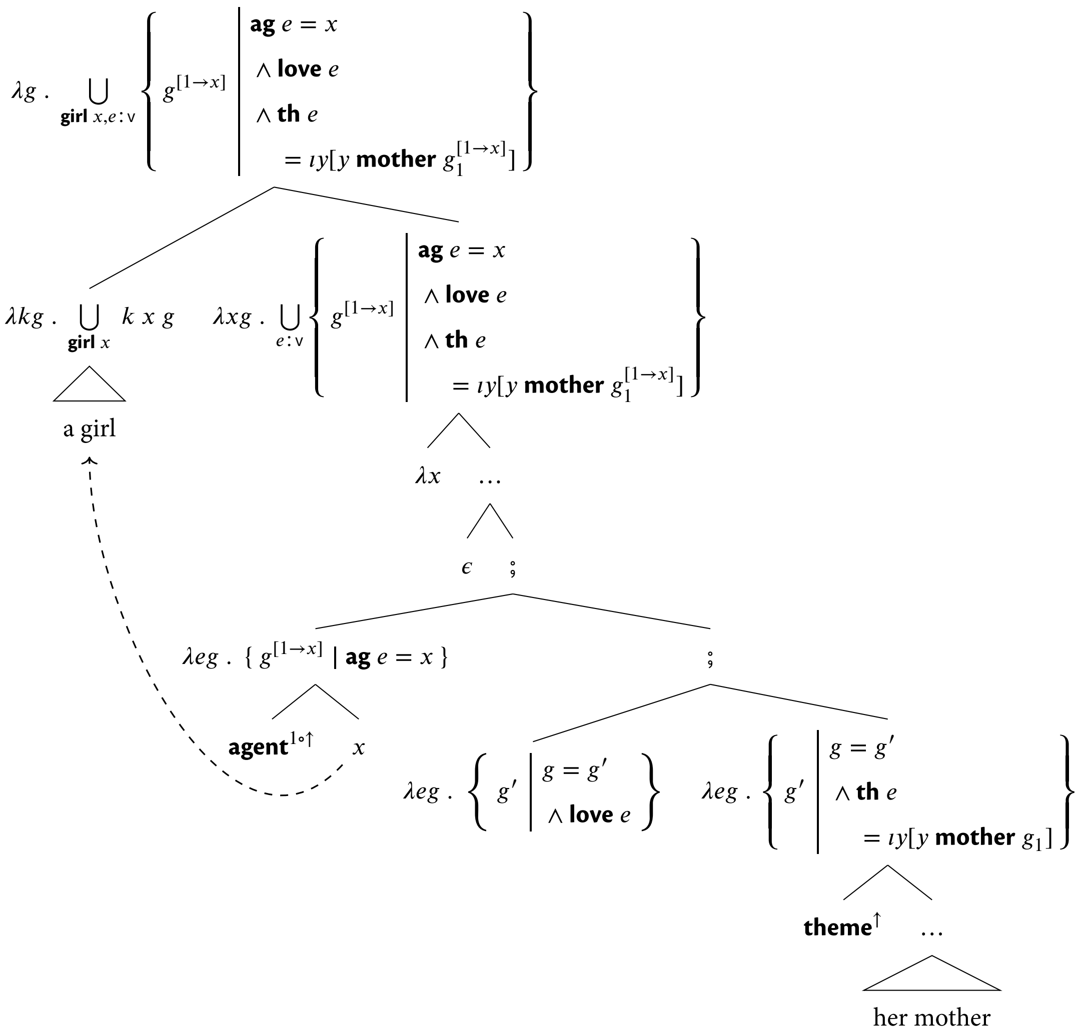
In classical DS, indefinites introduce indeterminate DRS by triggering random assignment. In Chierchia's system, x^n is guaranteed to introduce a determinate DR, since output assignments map n to the same individual: x.

Due to the DPP, only thematic positions may introduce DRS — scope never feeds DR-introduction. Anaphora therefore always respects the accessibility hierarchy initially established in the thematic core of the sentence.

Successful binding

(34) A[1] girl loves her_1 mother.

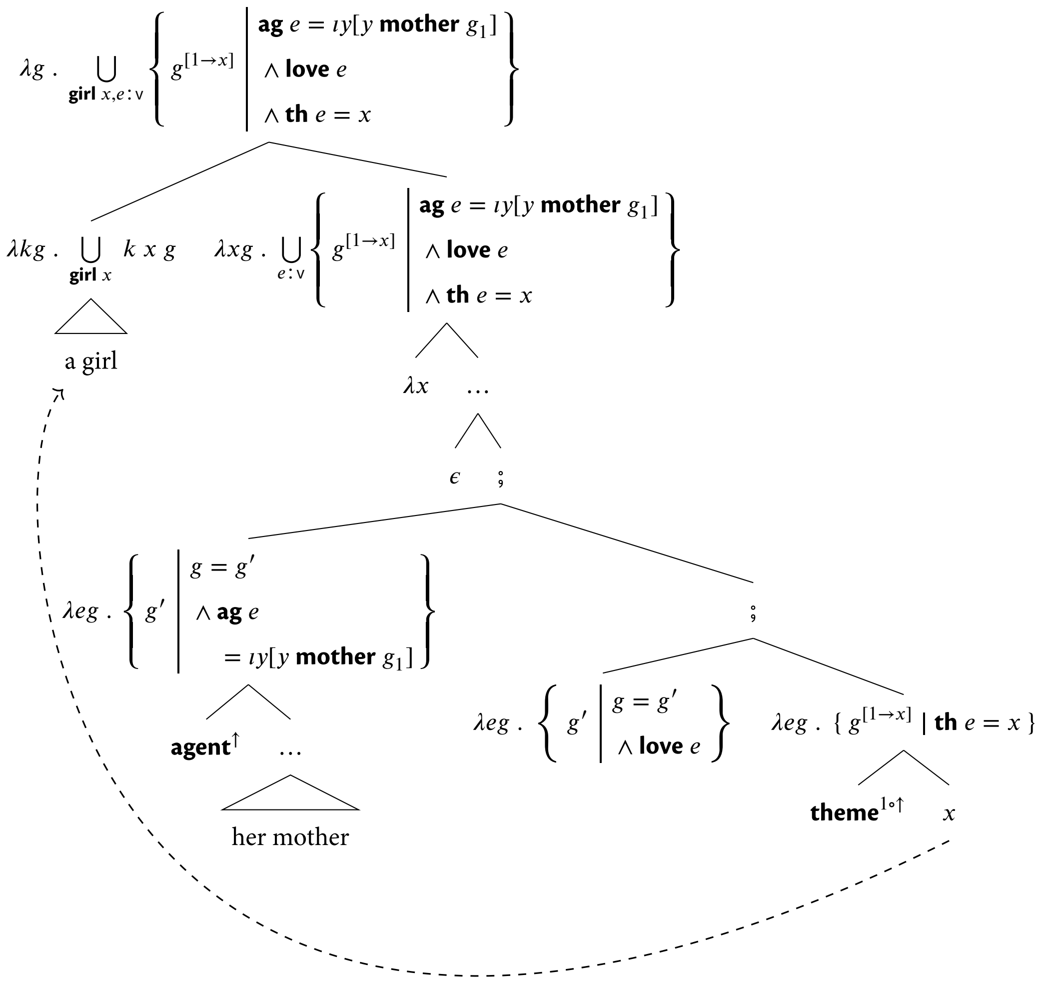
(35)



wco violation

(36) *Her$_1$ mother loves a^1 girl.

(37)



As discussed in detail in the paper, this account has the further advantage of straightforwardly accounting for binding into adjuncts, without recourse to shell-like descending structures.

(38) John [[loves every cat^1] [against it$_1$ s will]]. Chierchia 2020: 43

As discussed by Chierchia this follows straightforwardly from the system as laid out here, on the assumption that the adjunct is a right-adjoined dynamic event predicate.

3.4 *Negation strikes again: problems with Chierchia's account*

The problem for Chierchia's account is a simple one — as we've seen, certain operators, such as negation, render their arguments inaccessible for the purposes of anaphora.

If an indefinite takes scope *above* negation, however, anaphora is possible, even if its surface position is below negation.

This is illustrated in (39) — in fact, anaphora *disambiguates* in favor of the wide scope reading.

(39) It's not true that a man^1 walked in. He$_1$ sat down. $\exists > \neg$

This would seem to suggest that the indefinite may introduce a DR at its scope site, rather than at its thematic position, in conflict with the DPP.

There are two options here:

- The indefinite is permitted to introduce a DR at its scope site; we correctly account for the interaction with negation, but predict that such expressions should always obviate WCO.
- Alternatively, one could maintain the DPP, in which case the interaction with negation remains puzzling.

In order to understand exactly why this issue arises, we'll consider the status of negation in DS in somewhat more detail.

The standard move in DS is to adopt an *externally static* entry for negation — i.e., to define negation in such a way that it is guaranteed to return a test.

(40) Externally static negation (def.)

$$\textbf{not}\ m \coloneqq \lambda g\ .\ \begin{cases} \{\,g\,\} & m\,g = \varnothing \\ \varnothing & \text{otherwise} \end{cases} \qquad \mathsf{D} \to \mathsf{D}$$

If we consider the LF for a sentence involving an indefinite taking wide quantificational scope over negation, as in (41), we can see exactly how the problem will arise.

(41) Some man λx [not [x^1 walked in]]
$[\![\text{some man}]\!]\ (\lambda x g\ .\ \{\, g' \mid g' = g \wedge \{\, g^{[1 \to x]} \mid \textbf{walked-in}\ x \,\} = \varnothing \,\})$

The only remaining option is to assume that indefinites can introduce DRS at their scope sites.

Naturally, this gives rise to the prediction that, if an indefinite can dynamically outscope negation, it obviates WCO.

Dynamic scope and WCO obviation

This prediction has some initial plausibility since, as pointed out by Chierchia (2020: 49), specific indefinites, which generally take wide scope over negation, seem to ameliorate WCO, as illustrated by the examples below (judgments due to Chierchia 2020):

(42) His$_1$ father hates { a boy^1 I know | a friend^1 of mine | a certain boy^1 }

This response will however not work in the general case. Much like indefinites, *wh-expressions* are externally dynamic, as illustrated by the sentence in (43):

(43) I know [which boy^1 Mary invited to the party
and why he$_1$ was invited].

Furthermore, scoping a *wh*-expression over negation feeds anaphora, as illustrated by example (44).

(44) I know which boy^1 Mary didn't invite __ to the party,
and why he$_1$ was excluded.

For other scope takers too, just so long as the thematic position is in the scope of negation, any DRS introduced will be rendered inaccessible.[13]

[13] Thanks to Filipe Hisao Kobayashi (p.c.) for drawing my attention to examples such as this.

We can construct an example in which this predicts the impossibility of a universal binding into an adjunct, just so long as the adjunct takes scope above negation.

(45) John [decided [not to keep each of these puppies^1]
[after it$_1$ peed on the carpet]].

$\forall > \textbf{decide} > \neg$

(45) has a salient reading which can be paraphrased as follows: *Each puppy is such that, after it peed on the carpet, John decided not to keep it.*.

Surprisingly, Chierchia predicts that anaphora should be impossible here. This is because a DR is introduced in the position corresponding to the *theme* of *keep*, rather than at the scope site of the universal.

The solution, ultimately, will involve rethinking the dynamic plumbing on which Chierchia's account is built, specifically regarding the interactions between indefinites and negation.

We'll get part-way towards a solution by first tackling the general problem of double negation.

4 The analysis

4.1 Dynamic alternative semantics

As groundwork for the final proposal, I'll lay out a Dynamic Alternative Semantics (DAS) based directly on Charlow's (2014, 2019) work, drawing attention to its salient features.[14]

[14] Charlow demonstrates how to give the semantics outlined here a monadic grounding in terms of `State.Set`. This won't be relevant for our purposes, but it's worth noting that the monadic fragment accounts for exceptionally-scoping indefinites, and is thereby independently motivated.

Propositions in DAS

Given a sentential meaning, classical dynamic theories such as Groenendijk & Stokhof (1991) distinguish between assignments in/not in the output; in DAS, we distinguish between *true-tagged* assignments and *false-tagged* assignments, in addition to assignments simply absent from the output.

A proposition in DAS, after Charlow 2014:

(46) $[\![\text{some}^1 \text{ philosopher left}]\!] = \lambda g\ .\ \{\, (\textbf{left}\ x, g^{[1\to x]}) \mid \textbf{philosopher}\ x) \,\}$

Equivalently:

(47) $$= \lambda g\ .\ \{\, (\top, g^{[1\to x]}) \mid \textbf{philosopher}\ x \wedge \textbf{left}\ x \,\} \\ \cup \{\, (\bot, g^{[1\to x]}) \mid \textbf{philosopher}\ x \wedge \neg\,(\textbf{left}\ x) \,\}$$

This distinction is encoded in the *type* of a proposition in DAS, which we'll abbreviate as $\mathtt{T}$:[15]

[15] Here, g is the type of assignments, $\{\,.\,\}$ is the constructor for set types, and $*$ is the constructor for pair types, so $\mathtt{T}$ is the type of a function from assignments to sets of pairs of truth-values and assignments.

(48)

$\mathtt{D}$	$\coloneqq \mathtt{g} \to \{\,\mathtt{g}\,\}$	classical DS
$\mathtt{T}$	$\coloneqq \mathtt{g} \to \{\,\mathtt{t} * \mathtt{g}\,\}$	dynamic alternative semantics

Propositions in DAS encode the same information as standard dynamic propositions; in fact, we can define an operation $\mathscr{D}$ which shifts a DAS proposition back into its dynamic counterpart.

Definition 4.1 ($\mathscr{D}$ takes a DAS proposition and returns its standard dynamic counterpart.).

$$\begin{aligned}\mathscr{D}\ m &= \lambda g\ .\ \{\ g \mid (g, \top) \in (m\ g)\ \} \\ \mathscr{D} &: \mathsf{T} \to \mathsf{D}\end{aligned}$$

It's helpful to think of propositions in DAS as *partializing* dynamic semantics,[16] in the following sense: if we (equivalently) conceive of classical dynamic propositions as mapping pairs of assignments to either true or false, DAS propositions map pairs of assignments $\langle g, g' \rangle$ to:

- *true* if $(\top, g')$ is in the output,
- *false* $(\bot, g')$ is in the output,
- *undefined* if $(*, g')$ is absent from the output.[17]

[16] See van den Berg (1996: ch. 2) for relevant discussion.

[17] Here, and elsewhere I'll use $*$ as a "wildcard" ranging over possible semantic values.

An initial DAS fragment

Now that we have an understanding of the difference between DAS and more orthodox dynamic theories, we can lay out a semantics for a small fragment of English, including predicates, indefinites and pronouns, negation, and conjunction.

Predicates map individuals to dynamic propositions.

(49) $[\![\text{vaped}]\!] \coloneqq \lambda x g\ .\ \{\ (\textbf{vaped}\ x, g)\ \}$ $\qquad \mathsf{e} \to \mathsf{T}$

Indefinites compose with dynamic predicates, and induce an *indeterminate* output by via a set of alternatives.

(50) $[\![\text{some}^n\ \text{philosopher}]\!] \coloneqq \lambda k g\ .\ \bigcup_{x \in \textbf{philosopher}} k\ x\ g^{[n \to x]}$ $\qquad (\mathsf{e} \to \mathsf{T}) \to \mathsf{T}$

The result of composing an indefinite with a predicate is illustrated below:[18]

(51) a. $[\![\text{some}^n\ \text{philosopher vapes}]\!]$
b. $= [\![\text{some}^1\ \text{philosopher}]\!]\ \ \textbf{A}\ \ [\![\text{vapes}]\!]$
c. $= \lambda g\ .\ \{\ (\textbf{vapes}\ x, g^{[1 \to x]}) \mid \textbf{philosopher}\ x\ \}$ $\qquad \mathsf{T}$

[18] **A** indicates (overloaded) bi-directional function application.

Pronouns also compose with dynamic predicates introducing input sensitivity; given the input g and an index n they feed in g_n and saturate the result with g.

(52) $[\![\text{she}_1]\!] \coloneqq \lambda k g \,.\, k\ g_1\ g$ $\quad (\mathsf{e} \to \mathsf{T}) \to \mathsf{T}$

(53) a. $[\![\text{she}_1 \text{ vapes}]\!]$
b. $= [\![\text{she}_1]\!]\ \mathbf{A}\ [\![\text{vapes}]\!]$
c. $= \lambda g \,.\, \{\, (\textbf{vapes}\ g_1, g) \,\}$ $\quad \mathsf{T}$

Conjunction involves feeding the outputs of the first conjunct pointwise into the second, and gathering up the results (i.e., relational composition). The contained truth-values are conjoined.

(54) $[\![\text{and}]\!] \coloneqq \lambda m \,.\, \lambda n \,.\, \lambda g \,.\, \{\, (t \wedge u, g'') \mid (u, g'') \in (n, g') \mid (t, g') \in m\ g \,\}$ $\quad \mathsf{T} \to \mathsf{T} \to \mathsf{T}$

This allows us to account for cross-sentential anaphora straightforwardly, as in orthodox dynamic semantics; indefinites induce a branching output, and conjunction passes each output into the input-sensitive second conjunct:

(55) a. $[\![\text{some}^1 \text{ philosopher walked in and she}_1 \text{ sat down}]\!]$
b. $= [\![\text{some}^1 \text{ philosopher walked in}]\!]\ \mathbf{A}\ ([\![\text{and}]\!]\ \mathbf{A}\ [\![\text{she}_1 \text{ sat down}]\!])$
c. $= \lambda g \,.\, \{\, (t \wedge u, g'') \mid (u, g'') \in [\![\text{she}_1 \text{ sat down}]\!]\ g' \mid (t, g') \in [\![\text{some}^1 \text{ philospher walked in}]\!]\ g \,\}$
d. $= \lambda g \,.\, \{\, (\textbf{vapes}\ x \wedge u, g'') \mid (u, g'') \in [\![\text{she}_1 \text{ sat down}]\!]\ g^{[1 \to x]} \mid \textbf{philosopher}\ x \,\}$
e. $= \lambda g \,.\, \{\, (\textbf{vapes}\ x \wedge \textbf{sat-down}\ x, g^{[1 \to x]}) \mid \textbf{philosopher}\ x \,\}$

Destructive negation in DAS

Finally, this brings us to destructive negation in DS; this is easy to define in a way parallel to negation in orthodox dynamic theories; the outputs of m are existentially closed, meaning that any anaphoric information introduced cannot be passed on further.

(56) $[\![\text{not}]\!]\ m \coloneqq \lambda g \,.\, \{\, \neg \exists g'[(\top, g') \in m\ g], g \,\}$ $\quad \mathsf{T} \to \mathsf{T}$

We can see this by composing negation with an existential statement:

(57) a. $[\![\text{it's not true that any philosopher}^1 \text{ vapes}]\!]$
b. $= [\![\text{not}]\!]\ \mathbf{A}\ [\![\text{any philosopher}^1 \text{ vapes}]\!]$
c. $= [\![\text{not}]\!]\ \mathbf{A}\ (\lambda g \,.\, \{\, (\textbf{vapes}\ x, g^{[1 \to x]}) \mid \textbf{philosopher}\ x \,\})$
d. $= \lambda g \,.\, \{\, (\neg\, \exists g'[(\top, g') \in \{\, (\textbf{vapes}\ x, g^{[1 \to x]}) \mid \textbf{philosopher}\ x \,\}], g) \,\}$
e. $= \lambda g \,.\, \{\, (\neg\, \exists x[\textbf{philosopher}\ x \wedge \textbf{vapes}\ x], g) \,\}$ $\quad \mathsf{T}$

Note that the result of composing an existential statement, or indeed any statement in negation, is a *test* (to use Groenendijk & Stokhof's terminology) – given an input g, the output is always $\{\, (t, g) \,\}$, where $t = \top$ if no philosopher vaped, and $t = \bot$ if some philosopher vaped.

It's already easy to see that *two* negations won't allow for subsequent anaphora

— in fact, the inner negations cancel out, and the result is a closure operator; (58) tags the input *true* if a philosopher vapes, and *false* otherwise.

(58) $[\![\text{it's not true that no philosopher vapes}]\!] = \lambda g\ .\ \{\, (\exists x[\textbf{philosopher}\ x \wedge \textbf{vapes}\ x], g) \,\}$ $\quad \mathsf{T}$

Why not externally dynamic negation?

One thing we might want to consider is just making negation externally dynamic. This is easy to define in DAS:

(59) $[\![\text{not}_{\text{dy}}]\!] \coloneqq \lambda m\ .\ \lambda g\ .\ \{\, (\neg\, t, g') \mid (t, g') \in m\ g \,\}$ $\quad \mathsf{T} \to \mathsf{T}$

There's an independent reason why having such an entry for negation might be conceptually desirable — it's a straightforward lifting of *classical* negation into a dynamic setting; each truth value t, such that $(t, *)$ is in the output of $m\ g$ is negated, and the results are gathered up.[19]

[19] Showing this is a little bit involved, but the basic observation is that T is simply the result of applying a general recipe for dynamic types to $\mathtt{t}$; let's call this *type constructor* S.

(60) $\mathsf{S}\ \mathsf{a} \coloneqq \mathsf{g} \to \{\, \mathsf{a} * \mathsf{g} \,\}$

S is a *functor*, which just means that we can lift functions of type $\mathsf{a} \to \mathsf{b}$ into functions of type $\mathsf{S}\ \mathsf{a} \to \mathsf{S}\ \mathsf{a}$, in a logically well-behaved way. The relevant lifting operation, **map**, is given below:

(61) $\textbf{map}\ f\ m \coloneqq \lambda g\ .\ \bigcup_{(x,g') \in m\ g} \{\, (f\ x, g') \,\}$ $\quad (\mathsf{a} \to \mathsf{b}) \to \mathsf{S}\ \mathsf{a} \to \mathsf{S}\ \mathsf{b}$

We can derive $[\![\text{not}_{\text{dy}}]\!]$ by applying **map** to classical negation.

Unfortunately, this will fail to fulfill one of our empirical desiderata — when an indefinite occurs in the scope of a negative operator, it is inaccessible for subsequent anaphora. With not_{dy}, indefinites always outscope negation.

(62c) introduces a philosopher who doesn't vape as a DR.

(62) a. $[\![\text{not}_{\text{dy}}]\!]\ \textbf{A}\ [\![\text{some}^1\ \text{philosopher vapes}]\!]$
b. $= [\![\text{not}_{\text{dy}}]\!]\ (\lambda g\ .\ \{\, (\textbf{vapes}\ x, g^{[1 \to x]}) \mid \textbf{philosopher}\ x \,\})$
c. $= \lambda g\ .\ \{\, (\neg\, (\textbf{vapes}\ x), g^{[1 \to x]}) \mid \textbf{philosopher}\ x \,\}$

Perhaps counter-intuitively, I'm going to argue that we can get away with retaining not_{dy} as an entry for natural language negation, but this will necessitate tweaking the semantics of indefinites we've been assuming.

4.2 Making DAS more classical

Let's think a little more about the properties of our system that give rise to the bad predictions with externally dynamic negation.

Intuitively, this is because the semantics for indefinites doesn't discriminate between true- and false-tagged assignments in the output. Assume that $\textbf{linguist} = \{\, \textbf{andy}, \textbf{dani}, \textbf{yasu} \,\}$, and only Dani vapes.

(63) a. $[\![\text{some}^1\ \text{linguist vapes}]\!] = \lambda g\ .\ \{\, (\textbf{vapes}\ x, g^{[1 \to x]}) \mid \textbf{linguist}\ x \,\}$
b. $= \{\, (\bot, g^{[1 \to a]}), (\top, g^{[1 \to d]}), (\bot, g^{[1 \to y]}) \,\}$ $\quad \mathsf{T}$

In a positive context, the presence of the false-tagged assignments doesn't matter. To be more concrete, this is because, if we conceive of a context c as a set of assignments, the update rule in DAS is as follows:

Definition 4.2 (Update in DAS)**.** Given a set of assignments c, and a sentence ϕ, the update of c by ϕ, written $c[\phi]$ is defined as follows:

$$c[\phi] \coloneqq \bigcup_{g \in c} \{ \, g' \mid (\top, g') \in [\![\phi]\!] \;\; g \, \}$$

Externally dynamic negation simply flips the polarity of the output assignments, resulting in a linguist DR who doesn't vape:

(64) a. $[\![\text{not}_{\text{dy}}]\!]$ **A** $[\![\text{a}^1 \text{ linguist vapes}]\!] = \lambda g \; . \; \{ \, (\neg \, (\textbf{vapes} \; x), g^{[1 \to x]}) \mid \textbf{linguist} \; x \, \}$
b. $= \{ \, (\top, g^{[1 \to a]}), (\bot, g^{[1 \to d]}), (\top, g^{[1 \to y]}) \, \}$

We can address this problem by refining the semantics of indefinites, such that they only introduce modified *true*-tagged assignments.

In order to do this, we'll introduce a couple of derivative notions.

Positive and negative extension

Positive and negative extension operators simply filter out the false- and true-tagged assignments from an output set, respectively.[20]

(65) Positive and negative extension (def)
a. $p^{+} \coloneqq \{ \, (\top, g') \mid (\top, g') \; \in p \, \}$ $\qquad + : \{ \, \mathtt{g}, \mathtt{t} \, \} \to \{ \, \mathtt{g}, \mathtt{t} \, \}$
b. $p^{-} \coloneqq \{ \, (\top, g') \mid (\top, g') \; \in p \, \}$ $\qquad - : \{ \, \mathtt{g}, \mathtt{t} \, \} \to \{ \, \mathtt{g}, \mathtt{t} \, \}$

[20] We'll only be making use of the positive extension for now, but the negative extension will come in handy for giving a concise semantics for disjunction.

Positive collapse

The positive collapse operations takes a DAS proposition m, and gives back a new proposition which just returns that true-tagged assignments in $m \; g$ if there are any, and $(\bot, g)$.

(66) Positive collapse (first attempt)

$$m^{\dagger} \coloneqq \lambda g \; . \; \begin{cases} (m \; g)^{+} & (m \; g)^{+} \neq \varnothing \\ \{ \, (\bot, g) \, \} & \text{otherwise} \end{cases} \qquad \mathtt{T} \to \mathtt{T}$$

We'll later tweak the definition of † in order to address the problems we observed with the dynamic theory of crossover, but we'll come back to this later.

Indefinites redefined

We can now redefine indefinites in terms of our original semantics + positive collapse.

Indefinites in DAS — second attempt

$$[\![\text{some}^1 \text{ linguist}]\!] = \lambda k \,.\, \left(\lambda g \,.\, \bigcup_{x \in \mathbf{ling}} k\ x\ g^{[1 \to x]}\right)^{\dagger}$$

The revised entry in action

Let's first check that the revised entry accounts for the same data as destructive negation was designed to capture — namely, the fact that negation renders indefinites inaccessible for subsequent anaphora.

Let's say that $\mathbf{linguists} = \{\, \mathbf{andy}, \mathbf{dani}, \mathbf{paul} \,\}$, and only Andy and Dani vape. The revised semantics for indefinites makes no difference in a positive context, since false-tagged assignments have no effect on update anyway.

(67) a. $[\![\text{some}^1 \text{ linguist vapes}]\!] = (\lambda g \,.\, \{\, (\top, g^{[1 \to a]}), (\top, g^{[1 \to d]}), (\bot, g^{[1 \to y]}) \,\})^{\dagger}$
b. $= \{\, (\top, g^{[1 \to a]}), (\top, g^{[1 \to d]}) \,\}$

Now, if we apply *externally dynamic negation*, this time we fail to erroneously introduce a DR, since there are no false-tagged assignments to be flipped; in fact, the sentence is predicted to be false in the given context.[21]

(68) $[\![\text{not}_{\text{dy}}]\!]\ \mathbf{A}\ [\![\text{some}^1 \text{ linguist vapes}]\!] = \{\, (\bot, g^{[1 \to a]}), (\bot, g^{[1 \to d]}) \,\}$

If the set of linguists is the same, and nobody vapes, we predict the existential statement to be false, and the negated statement to be true, without introducing any DRS. So far so good!

(69) *Context: nobody vapes*
a. $[\![\text{some}^1 \text{ linguist vapes}]\!] = \lambda g \,.\, \{\, (\bot, g) \,\}$
b. $[\![\text{not}_{\text{dy}}]\!]\ \mathbf{A}\ [\![\text{some}^1 \text{ linguist vapes}]\!] = \lambda g \,.\, \{\, (\top, g) \,\}$

Now the moment of truth; if only Andy and Dani vape, a second negation flips the polarity of the false-tagged assignments, thus *re*-introducing the DR.

(70) *Only Andy and Dani vape*
a. $[\![\text{some}^1 \text{ linguist vapes}]\!] = \lambda g \,.\, \{\, (\top, g^{[1 \to a]}), (\top, g^{[1 \to d]}) \,\}$
b. $[\![\text{not}_{\text{dy}}]\!]\ \mathbf{A}\ [\![\text{some}^1 \text{ linguist vapes}]\!] = \lambda g \,.\, \{\, (\bot, g^{[1 \to a]}), (\bot, g^{[1 \to d]}) \,\}$
c. $[\![\text{not}_{\text{dy}}]\!]\ \mathbf{A}\ ([\![\text{not}_{\text{dy}}]\!]\ \mathbf{A}\ [\![\text{some}^1 \text{ linguist vapes}]\!]) = \lambda g \,.\, \{\, (\top, g^{[1 \to a]}), (\top, g^{[1 \to d]}) \,\}$

[21] A sentence ϕ is g*−true* in DAS iff $\exists g' [(\top, g') \in [\![\phi]\!]\ g]$

We've a dynamic theory in which (i) negation roofs the dynamic scope of indefinites, and (ii) double-negation elimination is validated![22]

Fixing conjunction

The careful reader will have noticed that we need to tweak the entry for conjunction in DAS in order to avoid some bad results.

As it stands, we predict that anaphora should succeed in sentences like (71).[23]

(71) # It's false [that there's no[1] bathroom, and it$_1$'s upstairs].

This is because our existing entry for conjunction feeds each assignment that the first conjunct outputs, be it true-tagged, or false-tagged into the second conjunct.

It follows that a false-tagged assignment outputted by the first conjunct can license anaphora in the second, and the result of the entire conjunction can be flipped by negation, resulting in a true sentence.

The solution we adopt here is straightforward: conjunction only feeds outputs in the *positive collapse* of the first conjunct into the second.

(72) Revised dynamic conjunction (def.)

$$[\![\text{and}]\!] := \lambda n \,.\, \lambda m \,.\, \lambda g \,.\, \bigcup_{(t,g')\in m^{\dagger}\, g} \{\, (t \wedge u, g'') \mid (u, g'') \in n\, g' \,\} \qquad \mathsf{T} \to \mathsf{T} \to \mathsf{T}$$

This ensures that, in the problematic example (71), DR 1 is eliminated by the positive collapse of the first conjunct, and anaphora will fail.

Disjunction and bathroom sentences

In order to give a concise entry for disjunction which accounts for bathroom sentences, as below, we'll define the negative counterpart of positive collapse — *negative collapse*.

(73) Either there's no[1] bathroom or it$_1$'s upstairs.

Negative collapse retains false-tagged outputs, if there are any, and if not returns the input assignment paired with *true*.

(74) Negative collapse (first attempt)

$$m^{\ddagger} := \begin{cases} (m\, g)^{-} & (m\, g)^{-} := \varnothing \\ \{\, (\top, g) \,\} & \text{otherwise} \end{cases} \qquad \mathsf{T} \to \mathsf{T}$$

[22] The solution to the double negation problem is highly reminiscent of Krahmer & Muskens's analysis; I won't attempt a detailed comparison here, but what seems to be important is that the (independently motivated) mechanisms of alternative semantics introduce a kind of bilateralism.

[23] Mayr's (2020) recent SuB presentation claims that similar sentences are in fact acceptable. Since I haven't been able to replicate these claims, I'll assume that (71) should indeed be ruled out.

We can now give a concise entry for disjunction which accounts for bathroom sentences:

(75) Disjunction in DAS

$$[\![\text{or}]\!] := \lambda n\,.\,\lambda m\,.\,\lambda g\,.\,m\,g \cup \bigcup_{g' \in m^{\ddagger}\,g} n\,g' \qquad \mathsf{T} \to \mathsf{T} \to \mathsf{T}$$

The entry above feeds the output of the *negative collapse* of the first disjunct into the second disjunct pointwise, and gathers up the results.

Note that this (accurately) predicts that disjunction is internally static[24], since only false-tagged assignments are fed into the second disjunct.

[24] See Groenendijk & Stokhof (1991) for discussion.

(76) #Either there's a^1 bathroom or it_1's upstairs.

We also predict that (accurately) that (73) is *true* if there's no bathroom. To show this, we'll assume that the sentence is interpreted relative to the initial context $\{\, g_{\varnothing} \,\}$, where $g_{\varnothing}$ is undefined for every index $n \in \mathbb{N}$.[25]

[25] Note that the fact that the entry for disjunction is stated in terms of set union induces a kind of strong Kleene logic — the disjunction is true just in case one of the disjuncts is true.

(77) *Context: there is no bathroom*

a. $[\![\text{there is no}^1 \text{ bathroom}]\!]\ g_{\varnothing} = \{\, (\top, g_{\varnothing}) \,\}$

b. $[\![\text{there is no}^1 \text{ bathroom}]\!]^{\ddagger}\ g_{\varnothing} = \{\, (\top, g_{\varnothing}) \,\}$

c. $[\![\text{it}_1\text{'s upstairs}]\!]\ g_{\varnothing} = \#$

d. $([\![\text{there is no}^1 \text{ bathroom}]\!]\ \mathbf{A}\ ([\![\text{or}]\!]\ \mathbf{A}\ [\![\text{it}_1\text{'s upstairs}]\!]))\ g_{\varnothing}$
$= \{\, (\top, g_{\varnothing}) \,\}$

Now the main scenario of interest: there is a bathroom, and it's upstairs.

(78) Context: *There is exactly one bathroom (**b**) and it's upstairs.*

a. $[\![\text{there isn't a bathroom}^1]\!]\ g_{\varnothing} = \{\, (\bot, [1 \to \mathbf{b}]) \,\}$

b. $[\![\text{there isn't a bathroom}^1]\!]^{\ddagger}\ g_{\varnothing} = \{\, (\bot, [1 \to \mathbf{b}]) \,\}$

c. $[\![\text{it}_1\text{'s upstairs}]\!]\ [1 \to \mathbf{b}] = \{\, (\top, [1 \to \mathbf{b}]) \,\}$

d. $([\![\text{there isn't a bathroom}^1]\!]\ \mathbf{A}\ ([\![\text{or}]\!]\ \mathbf{A}\ [\![\text{it}_1\text{'s upstairs}]\!]))\ g_{\varnothing}$
$= \{\, (g, t) \mid (g, t) \in (\{\, (\bot, [1 \to \mathbf{b}]) \,\} \cup \{\, (\top, [1 \to \mathbf{b}]) \,\}) \,\}$
$= \{\, (\bot, [1 \to \mathbf{b}]), (\top, [1 \to \mathbf{b}]) \,\}$

Note that we predict that *negating* the disjunctive sentence in this context should reintroduce a bathroom DR. This seems correct:

(79) It's false that there's neither a^1 bathroom, nor is it_1 upstairs.
It_1's (in fact) in the basement.

Uniqueness/maximality

One potential worry for the analysis comes from uniqueness effects: as discussed by Gotham (2019), anaphora from under double negation seems to give rise to a uniqueness inference.

(80) a. John owns a shirt^1. It$_1$'s hanging up. The rest are in the closet.
b. John doesn't own no shirt^1. It$_1$'s hanging up. ??The rest are in the closet.

I don't have much to add here, other than to note that there seem to be exceptions to uniqueness, involving maximal reference, as illustrated in (81).[26]

[26] To my knowledge, this was first observed by Simon Charlow in unpublished work on negation.

(81) John doesn't own no shirt^1. They're in the closet.

Indeed, Gotham does not have a principled explanation for the uniqueness effect, and it seems reasonable to conclude that this phenomenon is still poorly understood.

See also Krahmer & Muskens (1995), who argue that bathroom sentences have *universal* readings. I don't take a stance on this issue here.

4.3 Back to crossover

We still haven't fixed our theory in a way that addresses the problem with Chierchia's account of WCO.

Currently, we don't distinguish between determinate and indeterminate DRS.

We'll address this problem by minimally tweaking just one of our helper functions — positive collapse.

Redefining positive collapse

The intuition behind revised the positive collapse operation is as follows:

- It takes a dynamic proposition, which could, in principle, output a heterogeneous set of true- and false-tagged assignments.
- If there are any verifiers, it simply filters out the falsifiers.
- If there *aren't* any verifiers it collapses the modified assignments associated with the falsifiers into a single output assignment, thereby eliminating any indeterminacy from the output.

(82) Positive collapse operation (revised def.)

$$m^{\dagger} \coloneqq \lambda g \,.\, \begin{cases} (m\ g)^{+} & (m\ g)^{+} \neq \varnothing \\ \{\, (\bot, \bigcap \{\, g \mid (g, \bot) \in (m\ g) \,\}) \,\} & \text{otherwise} \end{cases} \qquad \mathsf{D} \to \mathsf{D}$$

What does it mean to take the *intersection* of a set of assignments? We can take assignment functions to be equivalent to their graphs, i.e., sets of variable-value pairs. This is illustrated below for an assignment mapping the variables $\{ 1, 2, 3 \}$ to boys.

$$\begin{bmatrix} 1 \to \textbf{jeff} \\ 2 \to \textbf{troy} \\ 3 \to \textbf{abed} \end{bmatrix} =: \begin{Bmatrix} (1, \textbf{jeff}), \\ (2, \textbf{troy}), \\ (3, \textbf{abed}) \end{Bmatrix}$$

Intersecting two assignments therefore amounts to set intersection of their graphs.[27]

$$\bigcap \left\{ \begin{Bmatrix} (1, \textbf{troy}) \\ (2, \textbf{abed}) \\ (3, \textbf{shirley}) \end{Bmatrix}, \begin{Bmatrix} (1, \textbf{troy}) \\ (2, \textbf{abed}) \\ (3, \textbf{britta}) \end{Bmatrix}, \begin{Bmatrix} (1, \textbf{troy}) \\ (2, \textbf{abed}) \\ (3, \textbf{annie}) \end{Bmatrix} \right\} = \begin{Bmatrix} (1, \textbf{troy}) \\ (2, \textbf{abed}) \end{Bmatrix}$$

[27] I'm grateful to Yasu Sudo (p.c.) for suggesting this simple formulation in terms of graph intersection.

Just as in the previous section, it is straightforward to define a negative collapse operation $\ddagger$ — the negative counterpart of positive collapse. This simply returns the false-tagged assignments, if there are any; if not, it collapses the true-tagged assignments via intersection, thereby eliminating any indeterminacy.

(83) Negative collapse (def.)

$$m^{\ddagger} := \begin{cases} (m\ g)^{-} & (m\ g)^{-} \coloneqq \varnothing \\ \{ (\top, \bigcap \{ g \mid (g, \top) \in (m\ g) \}) \} & \text{otherwise} \end{cases} \qquad \mathsf{D} \to \mathsf{D}$$

With our revised collapse operators, everything else can remain as before.

The dynamic translucency of negation

The interesting part comes when alongside an indefinite, we also have some determinate DR introduced.

Consider the sentence "A boy^1 likes Annie^2" in a context where no boy in fact likes Annie. Let's start out with the meaning predicted by vanilla DAS semantics, and just take the false-tagged assignments:

(84) $\lambda g\ .\ \{ (\bot, g^{[1 \to \textbf{jeff}, 2 \to \textbf{annie}]}), (\bot, g^{[1 \to \textbf{troy}, 2 \to \textbf{annie}]}), (\bot, g^{[1 \to \textbf{abed}, 2 \to \textbf{annie}]}) \}$

To compute the meaning of "A boy^1 likes Annie^2" in this context, recall that we first check whether the positive extension is empty. Since it is, we take the grand intersection of the false-tagged assignments.

(85) Context: *No boys like Annie*
$[\![\text{A boy}^1 \text{ likes Annie}^2]\!] = \lambda g \,.\, \{\, (\bot, g^{[2\to\mathbf{a}]}) \,\}$

Negating the resulting meaning simply flips the resulting meaning, and the result is no longer a test! We correctly predict that determinate DRS can survive negation.

We can think of negation as being a dynamically *translucent* operator — it allows only determinate DRS to survive — namely, only those that all output assignments agree upon.

(86) Context: *No boys like Annie*
$[\![\text{not}_{\text{dy}}]\!] \ \mathbf{A}\ [\![\text{A boy}^1 \text{ likes Annie}^2]\!] = \lambda g \,.\, \{\, (\top, g^{[2\to\mathbf{a}]}) \,\}$

Applying the results to crossover

We can now solve the problem for Chierchia's account of crossover in terms of DAS.

Recall, to maintain Chierchia's account, we want to remove from indefinites the inherent ability to introduce a DR:

(87) Indefinites in DAS (final ver.)

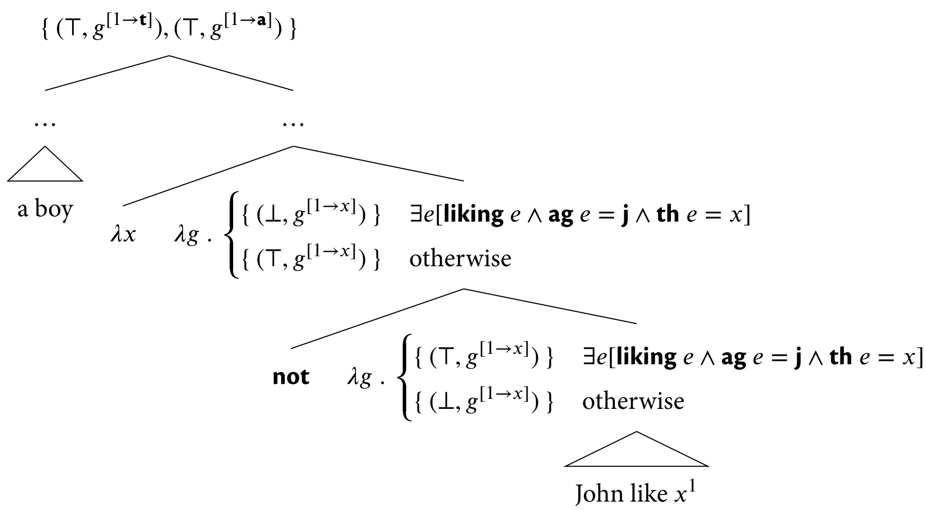
$$[\![\text{a boy}]\!] \coloneqq \lambda k \,.\, \left(\lambda g \,.\, \bigcup_{\mathbf{boy}\ x} k\ x\ g\right)^{\dagger}$$

Recall that examples where indefinites take scope over negation were problematic for Chierchia's account.

The insight here is that, if the indefinite takes wide scope, at the point in the derivation at which negation composes, the DR introduced by the thematic position is *determinate*, and therefore survives.

(88) *Context: John likes Troy and Abed but not Jeff*
A boy [John doesn't like $\_\_^1$]

(89)



Note that, since disjunction is defined in terms of negative collapse, there is a prediction that antecedents which introduce *determinate* DRS in the first disjunct will still be accessible from the second.

(90) Either John^1 is downstairs, or he$_1$'s upstairs.

It's not immediately clear what is responsible for the "bound" reading in (90) however; since the antecedent is a definite, this could merely be accidental coreference.

We can attempt to diagnose the possibility of binding from the first disjunct into the second by constructing an elliptical sentence with a sloppy reading, as in (91).

(91) Mary^1 thinks that either [her$_1$ brother]^2 is upstairs, or he$_2$'s outside, and Sally does Δ too.

5 *Extensions and related work*

Negation in a non-distributive setting

The fragment outlined in the latter part of this paper is *distributive* and *non-eliminative*, just like, e.g., Groenendijk & Stokhof's (1991) Dynamic Predicate Logic.

One may wonder whether either of these logical properties are crucial for validating double-negation elimination within a genuinely dynamic setting.

In related work, Elliott (2020) develops a dynamic semantics with non-distributive updates, which uses similar techniques to the fragment outlined here.

Briefly, sentential updates are analyzed as functions from an input state — a *proposition-assignment* pair — to a set of truth-value, world, assignment triples; the output of an update thereby distinguish between validating world-assignment pairs, and falsifying world-assignment pairs.

This allows for a semantics of indefinites which mirrors the strategy adopted here, as illustrated in (94) — indefinites only introduce indeterminate DRS in worlds in which there is a verifier.

(92) Update type (def.)
$\mathtt{u} \coloneqq (\{\ \mathtt{s}\ \}, \mathtt{g}) \to \{\ (\mathtt{t}, \mathtt{s}, \mathtt{g})\ \}$

(93) $[\![\text{Jeff left}]\!] = \lambda(c, g)\ .\ \{\ (\textbf{left}_w\ j, w, g) \mid w \in c\ \}$

(94) $$[\![\text{someone}^1\ \text{left}]\!] = \lambda(c, g)\ .\ \{\ (\top, w, g^{[1\to x]}) \mid \textbf{left}_w\ x, x \in \textbf{dom}, w \in c\ \} \cup \{\ (\bot, w, g) \mid \neg\exists x[\textbf{left}_w\ x], w \in c\ \}$$

Updates induced by sentences with epistemic modals end up being non-distributive, due to the adopting of the test semantics based on Veltman 1996 and Groenendijk, Stokhof & Veltman 1996. This is illustrated below.

(95) $$[\![\text{someone}^1\ \text{might have left}]\!] \coloneqq \lambda(c, g)\ .\ \begin{cases} \{\ (\top, w, g) \mid w \in c\ \} & \exists(\top, *, *) \in (94)\ (c, g) \\ \{\ (\bot, w, g) \mid w \in c\ \} & \text{otherwise} \end{cases}$$

Elliott also develops an anaphoric account of modal subordination, showing how an update semantics which validates double negation elimination addresses some pitfalls of existing dynamic frameworks which incorporate both anaphora and modality.

It seems, therefore that a dynamic semantics can be either distributive or non-distributive, and validate double negation elimination. What seems to be *crucial* is a distinction between, informally, *true* vs. *false* information in the output.

Mandelkern's pseudo-dynamics

In recent work, Mandelkern (2020) is also concerned with developing a fragment which achieves the basic results of DS while validating double negation elimination.

In contrast to the strategy pursued here, Mandelkern's semantics is static, and gives rise to eliminative updates.

Mandelkern's core insight is that a sentence with an indefinite such as "someone[1] left" comes with a disjunctive "witness presupposition", which is satisfied if (a) g_1 left, or (b) nobody left.

This is somewhat reminiscent to the semantics for indefinites proposed in the current setting, although one problematic aspect of Mandelkern's proposal is that he must stipulate that the presupposition of an indefinite is always automatically accommodated.

I leave a detailed comparison between these approaches to future work.

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