FROM DYNAMICS TO PSEUDO-DYNAMICS

KENY CHATAIN AND PATRICK D. ELLIOTT[1]

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[1] This presentation is based on Mandelkern 2020b,a, with the details recast in a way that is (hopefully) illuminating. We're grateful to Matt Mandelkern for sharing earlier versions of his work with us.

Download this handout:
https://patrl.keybase.pub/handouts/semtri-mandelkern.pdf

1 Background

1.1 Anaphors to indefinites

An indefinite can co-vary with a pronoun it does not out-scope:

(1) a. [A person came into the bar.] She ordered a mojito.
b. Harry owns some sheep and Bill vaccinates them.
c. Only one applicant [used a video resume] and it wasn't even posted on Youtube.

This is unexpected from the perspective of traditional FO or SO logic. The *donkey sentence* is the most clear-cut example of this peculiar property:

(2) Every farmer who owns a donkey is very fond of it.

(3) a. **DS:** reconceive meanings as context-change potentials ; allow lexical items to specify the DRs they introduce
A person$_{58}$ entered the bar.
$[13 \to \text{Kariman}] \to [13 \to \text{Kariman},\ 58 \to \text{Person X}]$
She ordered a mojito
b. **E-type** enrich the syntax of pronouns ; the pronoun elides some descriptive content.
⇝ *+ situation semantics upgrade (Elbourne 2005)*
A person entered the bar.
She~~<person who entered the bar>~~ ordered a mojito.

Criticism of E-type approaches:

- The bishop problem (a.k.a. the indiscernability of identicals)
- No official account of quantificational/modal subordination, cross-conjunction anaphors. [2]

[2] Rothschild & Mandelkern n.d. argue that attempts to bridge the empirical gap would remove E-type's main selling point, its classical meanings.

- Limited anaphoric possibilities (no anaphors to worlds, propositions, predicates)

1.2 Recent advances

Recent proposals (Rothschild 2017, Schlenker 2009) try to preserve the index-based flexibility of DS while restricting its expressive power and improving its empirical accuracy.

Empirical gaps

> **Rothschild/Schlenker's generalization**
>
> Anaphor to an indefinite is possible iff the local context entails that a witness for the indefinite exists.

Standard DS only validates the (⇒) part of RS's generalization:

(4) a. **Conjunction: ✓**
I have a brother$_i$. He$_i$ lives in Bordeaux.
LC(He lives in Bordeaux) ⊨ *I have a brother*

b. **Conditional: ✓**
If Patrick has a sister, she lives in Liverpool.
LC(he lives in Liverpool) ⊨ *Patrick has a sister*

c. **Disjunction: ✗**
Either it's false that Patrick has a sister or she lives in Liverpool.
LC(she lives in Liverpool) ⊨ *Patrick has a sister*

d. **Double Negation: ✗**
I didn't forget to send Nana a card. It just hasn't arrived yet.
LC(he lives in Liverpool) ⊨ *I sent Grandma a card* [3]

e. **Non-asserted antecedent + conditional: ✗**
Does Patrick has a sister? Idk but if the answer is yes, then she must be British.
LC(she be British) ⊨ *Patrick has a sister*

f. **Anti-presupposition?: ✗**
I thought Patrick didn't have a sister. So I was surprised when I met her.
LC(I met her) $\overset{?}{\vDash}$ *Patrick has a sister*

The Schlenker-Rothschild's generalization suggests a deep connection

[3] It's a bit tricky. *forget p* seems to mean *had to p not p*. But *had to p* can be locally accommodated as in:

(4) I didn't FORGET to buy bread. YOU were supposed to buy bread, not me.

between local contexts and anaphors. This is why many recent proposals focus on giving a unified account of presupposition projection and anaphor potential.

Expressive power

The main difficulty is providing an account which does not give too many degrees of freedom to lexical content (*contra* DS).

(6) **Unattested lexical items:**
  a. She entered and′ a person ordered a mojito. (reverse *and*)
  b. #Every′ farmer who owns a donkey gives it back rubs. (anaphorically inert *every*)
  c. #A′ woman came and she sat down (anaphorically inert indefinites[4])

There is a scale for expressiveness:

(7) The classicality scale
  a. **Purely classical:** context parameters are only modified by syncategorematic rules
  b. **Static:** context parameters can be modified by anything
  c. **Dynamic:** static + context parameters are outputted by constituents

Mandelkern's contribution: Against that background, we can isolate three innovations of Mandelkern's proposals

- Unification of local context with anaphors
- Validating RS's generalization and closing empirical gaps: double negation, disjunction, etc
- Static semantics

2 Proposal

The presentation here will be largely based on Mandelkern 2020b, with a couple of differences:

- Mandelkern provides a semantics for a first-order predicate calculus;[5]

[4] This is debated. Particularly tricky are the cases of Pseudo Noun Incorporation, which marginally introduce DRs. Some see this as evidence that anaphoric potential is lexically specified.

[5] This is a philosophy paper, lest we forget.

instead, we'll provide an semantics for a fragment of English, in the style of Montague 1970/Heim & Kratzer 1998.

- Mandelkern (2020b) uses a bidimensional theory of presupposition; we'll treat presuppositions as *definedness conditions*, as in Mandelkern (2020a).
- We'll assume that assignments can be genuinely *partial*, i.e., undefined for some n in the domain of indices.

2.1 The basics

In a classical setting, sentences are true relative to an evaluation point — formally, a world assignment pair.

(8) a. $[\![\text{Troy left}]\!]^{w,g} = \text{left}_w(\text{troy})$ $\quad\texttt{t}$

b. $[\![\text{Someone left}]\!]^{w,g} = \exists x[\text{left}_w(x)]$ $\quad\texttt{t}$

To set things up, consider what it means to assert a sentence ϕ, in a Stalnakerian setting.

We can think of a context c, following Stalnaker/Heim as consisting of a set of world-assignment pairs.[6]

Updating a context by asserting a sentence ϕ simply amounts to intersecting c with the with the points at which ϕ is true:[7]

(9) **Update (def.):** $c[\phi] := c \cap \{\, (w, g) \mid [\![\phi]\!]^{w,g} = 1 \,\}$

In a classical setting, it's easy to see that updating a context c with a sentence with an indefinite won't have any effect on the assignments in the context; it will simply wipe out worlds in which nobody left.

A classical semantics therefore fails to account for the fact that asserting a sentence with an indefinite introduces a Discourse Referent (DR).

At this point, we'd usually go ahead and shift to a *dynamic* semantics, where sentences directly denote actions on the context. Dynamic semantics unfortunately has some well known problems:

The bathroom problem

In pretty much every version of dynamic semantics, negation wipes out any

[6] Deparing from Mandelkern (2020b), we'll assume that assignments can potentially be partial.

[7] A classical Montagovian fragment together with a globally-defined update rule characterizes a *state system* (in the sense of Rothschild 2017), where we take the "context change potential" of a sentence ϕ to be $\lambda c \,.\, c[\phi]$. The state system is *van Bentham static*, since it is both distributive and eliminative. See Rothschild 2017 for discussion.

DRS introduced in its scope.

This is to count for the fact that indefinites are *inaccessible* under negation.

(10) a. #I didn't talk to anyone^1. They$_1$ looked angry.
b. #Nobody^1 walked in. They$_1$ sat down.

This makes the unfortunate prediction that double-negation elimination isn't valid. This seems wrong.

(11) a. It's not true that nobody^1 walked in. They$_1$ sat down right after.
b. It's wrong that I didn't talk to anyone^1. They$_1$'re over there.

A related problem is Partee's so-called bathroom sentence. First off, note that an indefinite in a first disjunct is inaccessible to the second.

(12) #Either there's a bathroom upstairs or it's downstairs.

Miraculously however, if an indefinite in the first disjunct is in the scope of negation, it is accessible.

(13) a. Either there isn't a^1 bathroom or it$_1$'s upstairs.
b. Either there is no^1 bathroom or it$_1$'s upstairs.

Naturally, we'd want to reduce this to the following:

(14) Either there isn't a bathroom or [there isn't not a^1 bathroom and] it$_1$'s upstairs.

This won't work in dynamic semantics however, due to the previous problem with double negation.

Mandelkern's strategy is different — he'll aim to maintain the classical picture, while accounting for DR-introducing potential via the logic of presupposition.

The resulting theory is dubbed *pseudo-dynamics*, and will account for the problems with double negation and disjunction.

2.2 Tweaking the semantics of indefinites

The witness presupposition

Mandelkern's core insight is that we can assign sentences with indefinites *witness presuppositions*, i.e.. disjunctive definedness conditions, which ensure that they only affect anaphoric potential if true.

(15) $$[\![\text{someone}^1 \text{ left}]\!]^{w,g} = \begin{cases} 1 & \mathsf{left}_w(g_1) \\ 0 & \neg\,(\exists x[\mathsf{left}_w(x)]) \\ \text{undefined} & \text{otherwise} \end{cases} \qquad \mathtt{t}$$

On Mandelkern's rendering, a sentence such as "someone^1 left" is *defined and true* if g_1 left in w, and *defined and false* if nobody left in w.

There are two equivalent ways of elucidating the presupposition:

- Either nobody left, or g_1 left.
- If anyone left, g_1 left.

The assertive contribution is just the classical semantics for the indefinite, i.e., *someone left*.

On the disjunctive rendering of the presupposition, note that the right disjunct entails the truth of the assertion, and the left disjunct entails the falsity of the assertion, hence the presentation in (15).

The effect of asserting a sentence with an indefinite

If we combine our global update rule for this semantics of the indefinite, it's easy to see that updating a context c with "someone^1 left" will knock out any assignments that are undefined for 1, and any world assignment pairs (w, g) if g_1 didn't leave in w.

To give a concrete example, where $\mathsf{dom} \coloneqq \{\ \mathsf{Xavier}, \mathsf{Yuna}, \mathsf{Zhaan}\ \}$, the stock of indices is $\{\ 1\ \}$, we can conceive of an initial context as follows:

- $W \coloneqq \{\ w_{xy}, w_x, w_y, w_\varnothing\ \}$ (subscripts indicate who left, exhaustively).
- $G \coloneqq \{\ g_\varnothing, [1 \to x], [1 \to y], [1 \to z]\ \}$ ($g_\varnothing$ is undefined for any index).

- $c = W \times G$

(16) $c[\text{someone}^1 \text{ left}] = c \cap \{ (w, g) \mid [\![\text{someone}^1 \text{ left}]\!]^{w,g} = 1 \}$
$= \{ (w_{xy}, [1 \to x]), (w_x, [1 \to x]), (w_{xy}, [1 \to y]), (w_y, [1 \to y]) \}$

One thing to note here is that, as Mandelkern acknowledges, what he describes as a "presupposition" isn't really what we would ordinarily describe as a presupposition, in a Stalnakerian setting.

Ordinarily, we think of presuppositions as *preconditions* on the context. This is formalized as Stalnaker's bridge:

(17) Stalnaker's bridge principle
Given ϕ_π a sentence that asserts ϕ and presupposes π,
$c[\phi_\pi]$ is defined iff π is true *throughout* c.

We have to abandon Stalnaker's bridge in order for Mandelkern's account to work, otherwise sentences with indefinites would frequently be undefined. Rather, we just toss out any assignments which are undefined relative to the index on the indefinite.

What we would think of as "ordinary" presuppositions are reintroduced by relativizing truth to a context parameter, as we'll see when we talk about definites.

2.3 *Tweaking the semantics of definites*

A more familiar presupposition

In Mandelkern's system, definites are genuinely presuppositional in the sentence of Heim-Stalnaker — they place *preconditions* on the context.

In order to capture this, we need to add a *context* parameter c to the interpretation function: $[\![.]\!]^{w,g,c}$.

Sentences which don't involve definites or other presuppositional expressions won't be sensitive to the context parameter, and its presence will essentially be vacuous.

The semantics of a sentence with a definite, however will impose a requirement that it be defined throughout the context.[8]

[8] We use $*$ here to range over possible semantic values.

(18) $$[\![\text{they}_1 \text{ sat down}]\!]^{w,g,c} = \begin{cases} 1 & \text{sat-down}_w(g_1) \wedge \forall(*, g) \in c[g_1 \text{ is defined}] \\ 0 & \text{sat-down}_w(g_1) \wedge \forall(*, g) \in c[g_1 \text{ is defined}] \\ \text{undefined} & \text{otherwise} \end{cases}$$

There's a sense in which Mandelkern's semantics for definites, which presumably carries over to presuppositional expressions more generally, *semanticizes* the bridge principle.

Revising update

We now need to redefine update in the obvious way — the context parameter of the sentence is identified with the context c which is being updated:

(19) Update (second attempt): $c[\phi] \coloneqq c \cap \{\, (w, g) \mid [\![\phi]\!]^{w,g,c} = 1 \,\}$

It's crucial here that we *don't* assume Stalnaker's bridge principle. If the sentence is undefined relative to $(w, g) \in c$, we simply toss them aside.

It's easy to see however that if we try to update a context c, which includes assignments that are undefined at 1, with the sentence "they$_1$ sat down", the result will be the empty set.

This is because $[\![\text{they}_1 \text{ sat down}]\!]$ undefined throughout c, since the conditions it places on c as a whole will never be satisfied.

We're now in a position to see how Mandelkern's system achieves the basic results of, e.g., Heim's dynamic semantics.

- Updating a context c with "someone^1 left", filters out any gs where g_1 is undefined.
- An update of c with "they$_1$ sat down" is only licit if g_1 is defined throughout c.

2.4 Compositionality and the logical connectives

Sub-sentential compositionality

We haven't said anything about how sentences with indefinites/definites come to mean what they mean. Here we'll sketch a simple Montagovian fragment with the desired properties.

Indefinites simply denote existential quantifiers with a disjunctive presupposition.[9]

(20) $[\![\text{someone}^1]\!]^{w,g,c} \coloneqq \lambda k \ : \neg\, (\exists x[k(x)]) \vee k(g_1)\ .\ \exists x[k(x)]$ $\qquad$ ett

[9] Since they denote scope-takers, it's easy to combine Mandelkern's semantics with your favourite theory of scope-taking, be it Quantifier Raising (QR), continuation semantics, or whatever.

Predicates and proper names simply receive their ordinary denotations:

(21) a. $[\![\text{Xavier}]\!]^{w,g,c} = \mathsf{xavier}$ $\qquad$ e

b. $[\![\text{swims}]\!]^{w,g,c} = \lambda x\ .\ \mathsf{swims}_w(x)$ $\qquad$ $\langle \mathtt{e}, \mathtt{t} \rangle$

Definites denote individuals with familiarity presuppositions:

(22)
$$[\![\text{they}_1]\!]^{w,g,c} \coloneqq \begin{cases} g_1 & \forall g' \in c,\ g'_1 \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

We can supplement this with a rule of Heim & Kratzer's undefinedness-sensitive rule for function application, which essentially gives us a weak Kleene logic (i.e., undefinedness always projects):

(23) Function application:

$$\left[\!\!\left[\begin{array}{c} \dots \\ \diagup\ \diagdown \\ \alpha \quad \beta \end{array} \right]\!\!\right]^{w,g,c} \coloneqq \begin{cases} [\![\alpha]\!]^{w,g,c}\,([\![\beta]\!]^{w,g,c}) & [\![\alpha]\!]^{w,g,c} : \langle \sigma, \tau \rangle, [\![\beta]\!]^{w,g,c} : \sigma, \\ & [\![\alpha]\!]^{w,g,c}, [\![\beta]\!]^{w,g,c} \text{ are defined} \\ [\![\beta]\!]^{w,g,c}\,([\![\alpha]\!]^{w,g,c}) & [\![\alpha]\!]^{w,g,c} : \sigma, [\![\beta]\!]^{w,g,c} : \langle \sigma, \tau \rangle, \\ & [\![\alpha]\!]^{w,g,c}, [\![\beta]\!]^{w,g,c} \text{ are defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

If we compose an indefinite with a predicate, we can see how we get the sentential meanings we've been assuming. Since undefinedness is guaranteed to project:

(24)
$$\begin{aligned} & [\![\text{someone}^1 \text{ left}]\!]^{w,g,c} \\ &= [\![\text{someone}^1]\!]^{w,g,c}\,([\![\text{left}]\!]^{w,g,c}) \\ &= \text{if } \neg\exists x[\mathsf{left}_w(x)] \vee \mathsf{left}_w(g_1) \text{ then } \exists x[\mathsf{left}_w(x)] \text{ else undefined} \end{aligned}$$

The presupposition is that nobody left or g_1 left, and the assertion is that someone left. To reiterate, this gives rise to the following predictions:

defined and true if g_1 left or nobody left, and someone left.
(since only the first disjunct is compatible with, and in fact entails the assertion, this can be simplified to: **$\mathbf{g_1}$ left**)

defined and false if g_1 left or nobody left, and nobody left.
(since only the second disjunct is compatible with, and in fact equivalent to the assertion, this can be simplified to: **nobody left**)

undefined otherwise.

Sentential compositionality and local contexts

One of the virtues of Mandelkern's approach is that it allows us to maintain a classical semantics for the logical connectives:

(25) a. $[\![\text{not}]\!]^{w,g,c} \coloneqq \lambda t \,.\, \neg\, t$ $\qquad \langle \mathtt{t}, \mathtt{t} \rangle$
b. $[\![\text{and}]\!]^{w,g,c} \coloneqq \lambda u \,.\, \lambda t \,.\, t \wedge u$ $\qquad \langle \mathtt{t}, \langle \mathtt{tt} \rangle \rangle$
c. $[\![\text{or}]\!]^{w,g,c} \coloneqq \lambda u \,.\, \lambda t \,.\, t \vee u$ $\qquad \langle \mathtt{t}, \langle \mathtt{tt} \rangle \rangle$

If we couple this with our rule of function application, this simply gives rise to a weak Kleene logic, which of course won't have the desired results.

In order to account for presupposition projection, this system can be supplemented with an independently motivated algorithm for determining local contexts (e.g., Schlenker 2009, 2010).[10]

[10] As we've seen, Mandelkern's fragment, together with a globally defined *update* rule gives rise to a state system, in the sense of Rothschild & Yalcin 2017, we could alternatively treat sentences as denoting their corresponding update, and define the logical connectives as in update semantics (1982, 1996), where conjunction is interpreted as successive update, etc. Mandelkern's semantics therefore can in principle be embedded within a dynamic semantics, and is neutral wrt how the projection behavior of connectives is to be derived.

In order to avoid introducing the details of Schlenker's theory, we'll simply define syncategorematic rules for determining local contexts in complex sentences.

Our rule for conjunction ensures that the second conjunct's context parameter is the context of utterance c updated with the first conjunct.

(26) Conjunction

$$\left[\!\!\left[\ {}_{\ldots\ldots}[\ \phi\ \ {}_{\ldots}[\ \text{and}\ \ \psi\ ]\ ]\ \right]\!\!\right]^{w,g,c} \coloneqq [\![\text{and}]\!]^{w,g,c}\,([\![\psi]\!]^{w,g,c[\phi]})([\![\phi]\!]^{w,g,c})$$

Our rule for disjunction ensures that the second disjunct's context parameter is the context of utterance c updated with the *negation* of the first disjunct.

(27) Disjunction

$$\left\llbracket \begin{array}{c} \ldots \\ \diagup\ \diagdown \\ \phi \quad\ \ldots \\ \qquad \diagup\ \diagdown \\ \qquad \text{or} \quad \psi \end{array} \right\rrbracket^{w,g,c} \coloneqq \llbracket \text{or} \rrbracket^{w,g,c} (\llbracket \psi \rrbracket^{w,g,c[\text{not } \phi]})(\llbracket \phi \rrbracket^{w,g,c})$$

This brings us round to a discussion of negation:

2.5 *Keeping negation classical*

Matt's fragment allows us to maintain a classical treatment of negation, while maintaining the accessibility results of classical dynamic semantics *and* validating double negation.

Negation roofs the introduction of a discourse referent

Recall that a sentence "someone^1 left" presupposes that (a) either nobody left or g_1 left, and if defined, asserts (2) someone left. It follows that "someone^1 left" is defined and false simply if nobody left.

Since negation is classical "not ϕ" is true iff ϕ is defined and false.[11]

[11] We assume that negative indefinites involve negation scoping over an indefinite at LF.

(28) $$\llbracket \text{nobody}^1 \text{ left} \rrbracket^{w,g,c} = \begin{cases} 1 & \neg\, (\exists x[\mathsf{left}_w\ x]) \\ 0 & \mathsf{left}_w(g_1) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Updating a context with "nobody^1 left" therefore fails to filter out assignments at which 1 is undefined, and we correctly predict that an indefinite under the scope of negation is *inaccessible*.

Double negation elimination is valid

Since negation is classical, and presuppositions project through negation, "It's not the case that nobody^1 left" has the same definedness conditions as "Someone^1 left":

(29) $$[\![\text{it's not true that nobody}^1 \text{ left}]\!]^{w,g,c} = \begin{cases} 1 & \mathsf{left}_w(g_1) \\ 0 & \neg\,(\exists x[\mathsf{left}_w\ x]) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Given the previous rule for interpreting disjunctive sentences, an account of bathroom sentences naturally follows (modulo the concerns of Krahmer & Muskens 1995).

2.6 *Donkey sentences, weak/strong*

So far, the system has been *purely classical* (in the sense of the introduction). However, donkey sentences require a step outside the classical. To see this, let's look at current prediction with a standard denotation for *every*.

(30) a. Every farmer [who owns a donkey_{75}] [cherishes it_{75}].
b. $[\![\text{who owns a donkey}_{75}]\!]^{w,g,c} = \lambda x : \big(\exists y, \mathsf{donkey}(y) \wedge x \text{ owns } y\big) \rightarrow \mathsf{donkey}(g(75)) \wedge x \text{ owns } g(75).\ \exists y, \mathsf{donkey}(y) \wedge x \text{ owns } y$

It is plausible that projection out of restrictors is universal (every farmer must satisfy the presupposition). Predicted referential constraint:

(31) $\forall x \in \text{farmer}, \big(\exists y, \mathsf{donkey}(y) \wedge x \text{ owns } y\big) \rightarrow \mathsf{donkey}(g(75)) \wedge x \text{ owns } g(75)$
$\rightsquigarrow$ *there is a communal donkey*

The cause of the problem is that the assignment function does not vary with the value of x (i.e. the predicate's variable). Mandelkern proposed to have *every* introduce the needed assignment variation[12]

(32) $$[\![\text{every}]\!]^{w,g,c}\,(\text{restr})\,\big(\text{scope}\big)$$
$$= \big\{x \mid \exists g',\ g \prec g' \wedge \text{restr}(g')(c)(x) = 1\big\}$$
$$\subset \left\{ x \;\middle|\; \exists g',\ g \prec g' \quad \begin{array}{l} \wedge\, \text{restr}(g')(c)(x) = 1 \\ \wedge\, \text{scope}(g')(c[\text{restr}(\cdot)(c)(x)])(x) = 1 \end{array} \right\}$$

[12] The extension in assignment functions used in the denotation of *every* will look familiar from situation E-type theories. It makes an appearance in many non-classical static theories of anaphora (Onea 2013, Elbourne 2005)

To make this work, we need, first, a syncategorematic rule to abstract over assignment function and contexts.

(33) $$[\![\text{X}]\!]^{w,g_0,c_0} \rightsquigarrow \lambda g.\lambda c.\,[\![\text{X}]\!]^{w,g,c}$$

The donkey sentence is then derived as such:

(34) a. **Restrictor:**

$$\{x \mid \exists g',\ g \prec g' \wedge [\![\lambda\, \text{g. farmer who owns a donkey}]\!]^{w,g,c}(g')(x)\}$$
$$= \{x \mid \exists g',\ g \prec g' \wedge \mathsf{farmer}(x) \wedge x \text{ owns } g'(75) \wedge \mathsf{donkey}(g'(75))\}$$
$$= \{x \mid \exists y, \mathsf{farmer}(x) \wedge x \text{ owns } y \wedge \mathsf{donkey}(y)\}$$

b. **Scope:**

$$\left\{ x \;\middle|\; \exists g',\ g \prec g' \quad \begin{array}{l} \wedge\, \mathsf{farmer}(x) \wedge x \text{ owns } g'(75) \wedge \mathsf{donkey}(g'(75)) \\ \wedge\, \mathsf{scope}(g')(c[x \text{ owns a donkey}])(x) = 1 \end{array} \right\}$$
$$= \{x \mid \exists g',\ g \prec g' \wedge \mathsf{farmer}(x) \wedge x \text{ owns } g'(75) \wedge \mathsf{donkey}(g'(75)) \wedge x \text{ cherishes } g(75)\}$$
$$= \{x \mid \exists y, \mathsf{farmer}(x) \wedge x \text{ owns } y \wedge \mathsf{donkey}(y) \wedge x \text{ cherishes } y\}$$

This derives the so-called ∃ reading of donkey anaphors

(35) (34a) ⊂ (34b)
iff
every farmer who owns a donkey owns a donkey that he cherishes

However, the more readily available reading is the ∀ reading, paraphrased as "every farmer who owns a donkey cherishes every donkey he owns". Matt Mandelkern points out that his system is sufficiently expressive to form the have the universal reading as well:

(36)
$$[\![\text{every}]\!]^{w,g,c}(\text{restr})(\text{scope})$$
$$= \{x \mid \forall g',\ g \prec g' \wedge \text{restr}(g')(c)(x) \neq 0\}$$
$$\subset \left\{ x \;\middle|\; \forall g',\ g \prec g' \quad \begin{array}{l} \wedge\, \text{restr}(g')(c)(x) \neq 0 \\ \wedge\, \text{scope}(g')(c[\text{restr}(\cdot)(c)(x)])(x) \neq 0 \end{array} \right\}$$

However, note that his denotation for the universal reading impose its own rules of projection: # values are explicitly manipulated. So it is unclear whether this part of Mandelkern's system can be derived from a non-stipulative system of presupposition projection (Schlenker 2009, Fox 2013)

Addendum Mandelkern does not take a stance on whether ∃ or ∀ readings is a matter of ambiguity or underdetermination. However, he makes the following remark. It is often said that *No* never gives rise to ∀ reading:

(37) No father who has a teenage son lends him the car on the weekends.
a. ≈ *no father who has teenage son lends* ***any*** *of his sons the car on the weekends*
b. ≠ *no father who has teenage son lends* ***all*** *of his sons the car on the weekends*

This suggests that the ∃ reading is basic and the ∀ reading is the result of strengthening in UE environments. However, Mandelkern notes that *no* behaves anaphorically like a negated indefinite:

(38) Either there is no bathroom or it is on the second floor $(\underbrace{\neg\exists}_{no} \vee \text{pro})$

If that is so, it is not, properly speaking, a GQ (insofar as indefinites aren't GQ in his theory). Mandelkern concludes that it should not be used as evidence for a putative "default" reading of GQ.

Definedness conditions. Since *every* abstracts over contexts, it is also responsible for defining projection behaviour. Looking back at the definition, we left no room for undefinedness. The following sentences are predicted to be false:

(39) a. Every donkey farmer loves it.
b. Every farmer loves the donkey he owns.

So we need to add some definedness conditions for *every*. Paraphrasing a little bit, these truth-conditions yield the correct presupposition projection behaviour:

(40) $[\![\text{every}_{13}]\!]^{w,g,c}\,(\text{restr})\,\big(\text{scope}\big) \neq \#$
iff
1) $\text{restr}(g)(c)(g(13)) \neq \#$
2) if $\text{restr}(g)(c)(g(13)) = 1$, then $\text{scope}(g)(c[\text{restr}(\cdot)(c)(g(13))])(g(13)) = 1$

There is some disconnect between these definedness conditions and the denotation of *every* and its actual denotation. This makes it dubious how well we can connect the proposal to predictive theory of presupposition projection. But note the following projection pattern:

(41) a. Every_{13} farmer owns a donkey_{75}.
b. 1) for all x, $\text{farmer}(g(13)) \neq \# \rightsquigarrow$ *trivial*
c. 2) for all x, if $\text{farmer}(g(13)) = 1$, then if x owns a donkey, $\text{donkey}(g(75)) = 1$

The second presupposition is very interesting, besause it gives a simple account of quantificational subordination:

(42) Every$_{13}$ farmer owns a donkey$_{75}$. Every$_{13}$ farmer cherishes it$_{75}$.

3 Comparisons

Mandelkern's fragment has an extremely nice feature: it achieves the basic results of dynamic semantics while maintaining the validity of double negation elimination.

One might wonder what aspects of Mandelkern's system are *essential* for achieving this result? For example, how important is it that Mandelkern's system is *eliminative*?[13]

[13] A fragment is *eliminative* iff, for all sentences ϕ, $c[\phi] \subseteq c$. It's easy to see that this holds.

Elliott 2020 develops a system with roughly the same properties as Mandelkern's, for independent reasons.[14] Namely, it's a theory in which (a) existential quantifiers introduce discourse referents, (b) negation is classical; double-negation elimination is valid, and (b) negation renders indefinites inaccessible.

[14] As many of you know, I've been interested in addressing some of the problems with Chierchia's (2020) theory of crossover. You can find the manuscript here `https://ling.auf.net/lingbuzz/005311`. Comments welcome!

Elliott's account is sufficiently distinct that it's perhaps interesting to compare the two. As we'll see:

- Elliott's fragment is *non-eliminative*, but distributive, like, e.g., Groenendijk & Stokhof's (1991) Dynamic Predicate Logic (DPL).
- It doesn't rest on the logic of presupposition, but rather on a distinction between verifiers and falsifiers in the output.

I'll present a slightly different version of Elliott (2020) here for easy of comparison.

Sentences are interpreted relative to an *evaluation point* (a world assignment pair), and output a set of assignment-truth value pairs.

(43) $[\![\text{Xavier left}]\!]^{w,g} = \{\, (\text{left}_w(\text{xavier}), g) \,\}$ $\{\, \texttt{t} \cdot \texttt{g} \,\}$

We can think of the assignments in the output as having a "polarity" – in fact, we'll refer to outputted assignments as either being *truth-tagged* or *false-tagged*.

The key innovation here is the following semantics for indefinites:[15]

[15] The semantics suggested in Elliott 2020 is a little more sophisticated this, and actually returns the grand intersection of all modified assignments in the false case. This won't be important for our purposes.

(44) $$[\![\text{someone}^1 \text{ left}]\!]^{w,g} := \begin{cases} \{\, (1, g^{[1\to x]}) \mid \mathsf{left}_w(x), x \in \mathsf{dom} \,\} & \exists x[\mathsf{left}_w(x)] \\ \{\, (0, g) \,\} & \text{otherwise} \end{cases}$$

Elliott assumes that assignments are partial, and furthermore that we can think of the *initial state* as the product of the set of worlds in the common ground with the initial assignment $g_\varnothing$ (i.e., the unique assignment with an empty domain).

Update in Elliott's system is defined as follows: for each assignment g in c, we keep (i) all the worlds w in c which, when fed into the sentence with g give back a true tagged assignment, and (ii) pair w with that assignment.

(45) Update (def.): $$c[\phi] := \bigcup_{(w,g)\in c} \{\, (w, g') \mid (1, g') \in [\![\phi]\!]^{w,g} \,\}$$

To give a concrete example, assume:

- $\mathsf{dom} := \{\, Xavier, Yuna, Zhaan \,\}$
- $W := \{\, w_{xy}, w_x, w_y, w_\varnothing \,\}$, where subscripts indicate who left, understood exhaustively.
- $c = W \times g_\varnothing \equiv \{\, (w_{xy}, g_\varnothing), (w_x, g_\varnothing), (w_y, g_\varnothing), (w_\varnothing, g_\varnothing) \,\}$

(46) c[someone[1] left]
$$= \bigcup_{(w,g)\in c} \{\, (w, g') \mid (1, g') \in [\![\text{someone}^1 \text{ left}]\!]^{w,g} \,\}$$
$$= \left\{\, (w_{xy}, [1 \to x]), (w_x, [1 \to x]), (w_{xy}, [1 \to y]), (w_y, [1 \to y]) \,\right\}$$

Observe that, only worlds in which there is a verifier are retained, and in each world in which there is a verifier, a discourse referent is introduced. This is basically equivalent to DPL.

It's already easy to see that Elliott's system isn't eliminative, since eliminativity doesn't hold for an update with an indefinite (just like DPL).

Sentences with definites have a totally orthodox interpretation:

(47) $$[\![\text{they}_1 \text{ are outside}]\!]^{w,g} = \{\, (\mathsf{outside}_w(g_1), g) \,\}$$

Since assignments are partial, the sentence with the indefinite will be undefined if the evaluation assignment g is undefined for 1.

We can make Stalnaker's bridge explicit in our update rule — updating a context c with ϕ is only defined if ϕ is defined throughout c.

(48) Update (revised): $$c[\phi] \coloneqq \begin{cases} \bigcup_{(w,g)\in c} \{\, (w, g') \mid (1, g') \in [\![\phi]\!]^{w,g} \,\} & \forall (w,g) \in c, [\![\phi]\!]^{w,g} \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

It follows that an update with an indefinite will ensure that a successive update with a definite is defined.

Negation

As in Mandelkern's fragment, negation is totally classical; in essence, all it does it flip the polarity of the outputted assignments.[16]

[16] This may not look totally classical, so you'll have to trust me when I say that it is. Essentially, all we're doing here is mapping classical negation through the functorial structure of a `State.Set` monad (i.e., applying it pointwise).

(49) $$[\![\text{not}]\!]^{w,g} \coloneqq \lambda m \,.\, \bigcup_{(t,g)\in m} \{\, (\neg\, t, g) \,\}$$

If, in our previous context, we instead update with the negative sentence, no discourse referent is introduced; the worlds in which discourse referents are introduced are all false-tagged, since the polarities of the outputs got flipped:

(50) $$c[\text{nobody}^1 \text{ left}] = \{\, (w_\varnothing, g_\varnothing) \,\}$$

It follows that double negation elimination will be classical. Flipping the polarities of the outputted assignments twice will cancel out.

The connectives

We give syncategorematic rules for the connectives below; note that the second conjunct is interpreted relative to the true-tagged output of the first conjunct; the second disjunct is interpreted relative to the false-tagged outputs of the first disjunct.

(51) $$[\![\phi \text{ and } \psi]\!]^{w,g} \coloneqq \bigcup_{(1,g')\in[\![\phi]\!]^{w,g}} [\![\psi]\!]^{w,g'}$$

(52) $[\![\phi \text{ or } \psi]\!]^{w,g} \coloneqq [\![\phi]\!]^{w,g} \cup \bigcup_{(0,g') \in [\![\phi]\!]^{w,g}} [\![\psi]\!]^{w,g'}$

Illustration: bathroom sentences

Let's see how this solves the bathroom sentence problem:

- dom $\coloneqq \{ b \}$
- $W = \{ w_b, w_\varnothing \}$
- $c = W \times g_\varnothing$

(53) $\left[\!\left[\text{there's no}^1 \text{ bathroom or it}_1\text{'s upstairs}\right]\!\right]^{w,g}$
$= \left[\!\left[\text{there's no}^1 \text{ bathroom}\right]\!\right]^{w,g} \cup \bigcup_{(0,g') \in [\![\text{there's no bathroom}]\!]^{w,g}} [\![\text{it}_1\text{'s upstairs}]\!]^{w,g'}$

We're computing the result of doing the following:

(54) c[there's no[1] bathroom or it_1's upstairs]

Step 1: interpret the sentence relative to $(w_b, g_\varnothing)$

If we feed this into the first disjunct, we get a false-tagged output. This will be ignored in the result of the update.

(55) $\left[\!\left[\text{there's no}^1 \text{ bathroom}\right]\!\right]^{w_b,g_\varnothing} = \{ (0, [1 \rightarrow b]) \}$

Now if we feed the false-tagged output into the second disjunct, we get a true-tagged output, since anaphora succeeded. This means that this (w, g) will be retained by the update.

(56) $[\![\text{it}_1\text{'s upstairs}]\!]^{w_b,[1 \rightarrow b]} = \{ (1, [1 \rightarrow b]) \}$

(57) $\{ (w_b, g_\varnothing) \}$ [there is no[1] bathroom or it_1's upstairs] $= \{ (w_b, [1 \rightarrow b]) \}$

Step 2: interpret the sentence relative to $(w_\varnothing, g_\varnothing)$

If we feed this into the first disjunct we get a true-tagged output. This means that this (w, g) will be retained by the update.

(58) $[\![\text{there's no}^1 \text{ bathroom}]\!]^{w_\varnothing, g_\varnothing} = \{ (1, g_\varnothing) \}$

There are no false-tagged outputs, so the second disjunct is irrelevant here.

(59) $\{ (w_\varnothing, g_\varnothing) \}$ [there is no^1 bathroom or it$_1$'s upstairs] $= \{ (w_\varnothing, g_\varnothing) \}$

Step 3: take the union

We predict that anaphora will be successful, and furthermore disjunction is (correctly) predicted to be externally static.

(60) c[there's no^1 bathroom or it$_1$'s upstairs] $= \{ (w_b, [1 \to b]), (w_\varnothing, g_\varnothing) \}$

3.1 Outlook

There's a clear intuition that Mandelkern and Elliott's approaches share some important features:

- On both theories, indefinites only introduce discourse referents if there is a verifier, and not otherwise.
- On Mandelkern's theory, the double-life of indefinites is handled via the Strawson logic; on Elliott's theory, what's crucial is distinguishing between true and false information in the output.[17]

[17] In that sense, Elliott's approach is more closely related to previous approaches to double negation in dynamic semantics, such as Krahmer & Muskens 1995. Although elided here, this informational richness directly follows from the kind of monadic dynamics suggested by Charlow (2019).

Open question: how do we characterize exactly what these approaches have in common? There is clearly a shared insight here.

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