# *Disjunction*
## *in a predictive theory of anaphora*

Patrick D. Elliott https://patrickdelliott.com

April 4, 2022

MIT TLLM III

https://patrickdelliott.com/pdf/tllm-slides.pdf

# *Introduction*

**Big question:** what kind of dynamic logic underlies anaphoric dependencies in natural language?

**The dream:** Existential's are special (they introduce discourse referents; Karttunen 1976); otherwise all we need is a Strong Kleene semantics for the truth-functional operators, independently motivated by presupposition projection (Peters 1979, George 2007, 2008, 2014, Fox 2013).

# Roadmap ii

- An orthodox dynamic theory (DPL) — an elegant logic of anaphora, but the relationship to presupposition projection is obscure.
- A Strong Kleene dynamic logic: EDS.
    - The central idea: let's embed a Strong Kleene semantics in a dynamic setting by computing three DPL-style meanings (corresponding to *true*, *false*, and *unknown*) in tandem.
- Case study: disjunction, with a focus on Partee disjunctions, and the role of ignorance in constraining anaphora.

Sentences are relations between assignments; captures *statefulness* and *non-determinism* (Groenendijk & Stokhof 1991, Charlow 2020).

The standard bearer of DPL — existential quantification.

(1) $[\![\exists_v P(v)]\!]^w = \{\,(g, h) \mid g[v]h \wedge h_v \in I_w(P)\,\}$

Anaphoric information (encoded in assignments) is passed from left-to-right via relational composition ($\circ$) AKA dynamic conjunction.

A relational presentation of DPL.

(2) $[\![P(v_1, \dots, v_n)]\!]^w := \{ (g, h) \mid g = h \wedge h(v_1), \dots, h(v_n) \in I_w(P) \}$

(3) $[\![\varepsilon_v]\!] := \{ (g, h) \mid g[v]h \}$

(4) $[\![\phi \wedge \psi]\!]^w := [\![\phi]\!]^w \circ [\![\psi]\!]^w$

(5) $[\![\neg\phi]\!]^w := \{ (g, h) \mid g = h \wedge [\![\phi]\!]^w = \varnothing \}$

(6) $\exists_v \phi := \varepsilon_v \wedge \phi$

(7) $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi)$

(8) $\phi \rightarrow \psi := \neg(\phi \wedge \neg\psi)$

Truth is all or nothing in DPL — $\phi$ is true at $(w, g)$ iff $\{ h \mid (g, h) \in [\![\phi]\!]^w \} \neq \varnothing$.

# Some properties of DPL

Egli's theorem: $\exists_v \phi \wedge \psi \iff \exists_v(\phi \wedge \psi)$.

Egli's corollary: $\exists_v \phi \to \psi \iff \forall_v(\phi \to \psi)$

Double negation elimination isn't valid, since a negated sentence is *always* a test, and a positive sentence may introduce a dref.

Disjunction is a test (externally static), and the meaning of disjunction tests individual disjuncts (internally static). Motivation:

(9) # Either there's a$^v$ bathroom or it$_v$'s upstairs.

(10) # Either there's a$^v$ bathroom,
or we're in the wrong house; it$_v$'s upstairs.

# *Empirical challenges*

Partee disjunctions are famously problematic for orthodox theories like DPL (see, e.g., Krahmer & Muskens 1995, Gotham 2019)

(11) Either there is no$^{v}$ bathroom, or it$_{v}$'s upstairs.

Clear intuition that this relates to presupposition projection (Beaver 2001), but not clear how to cash this out, since disjunction is internally static (but see Rothschild 2017).

(12) Either Enrico never smoked, or he stopped smoking.

Desideratum: existential truth conditions (contra, e.g., Krahmer & Muskens 1995, Gotham 2019)

No uniqueness (Mandelkern & Rothschild 2020: p. 94):

(13) Either Sue didn't buy a$^{\upsilon}$ sage plant, or she bought eight others along with it$_{\upsilon}$.

Non-universal readings:

(14) Either John has no$^{\upsilon}$ credit card, or he paid with it$_{\upsilon}$.
*True if John has a Chase card and a BOA card, and he paid with his Chase card.*

The availability of ∃-readings of Partee disjunctions mirrors their availability in donkey sentences (Chierchia 1995, Kanazawa 1994, Champollion, Bumford & Henderson 2019).

(15) If John has a$^{v}$ credit card, he paid with it$_{v}$.
*True if John has a Chase card and a BOA card, and he paid with his Chase card.*

N.b. DPL only derives ∀-readings, due to Egli's corrolary:

$$\exists_x \phi \to \psi \iff \forall_x (\phi \to \psi)$$

(16) No way is there NO$^{v}$ bathroom; it$_{x}$'s upstairs!

Insurmountable for DPL without invoking dref accommodation, since $\neg\neg\phi$ is a test, but would sacrifice an account of the formal link condition.

(17) a. Andreea has a$^{v}$ spouse. That's him$_{v}$ over there.

b. # Andreea is married. That's him$_{v}$ over there.

Clear contrast between these two kinds of cases.

# Program disjunction

(18) Either a$^{v}$ linguist was sitting in the front,
or a$^{v}$ philosopher was.
She$_{v}$ asked a very interesting question.

The relevant disjunction operator can of course be defined in DPL (see Groenendijk & Stokhof 1991), but the availability of this operator would overgenerate anaphoric possibilities elsewhere.

# Rothschild's observation

If the truth of the disjunct containing an indefinite is later (locally) contextually entailed, anaphora becomes possible (Rothschild 2017). Elliott (2020) shows that this generalizes to conditionals and other complex sentences.

(19) a. Either it's a weekday,
or $\text{a}^{v}$ critic is watching our play.

b. If it's Saturday today,
I want $\text{them}_{v}$ to give us a good review.

A hunch: complex sentences can give the *illusion* of external staticity, given the conversational backgrounds against which they can be felicitously uttered.

# *First-order EDS*

# EDS

EDS $:=$ *Externally-dynamic Dynamic Semantics/ Existential Dynamic Semantics* (developed in detail in Elliott 2020)

Signature logical properties:

- $\neg\neg\phi \iff \phi$
- Egli's equivalence only holds with respect to the *positive information* conveyed by a sentence.
- De Morgan's equivalences hold ($\neg\phi \vee \neg\psi \iff \neg(\phi \wedge \psi)$).
- Logical operators are an embedding of Strong Kleene semantics in a dynamic setting (predictiveness).

*Assignments* are functions from variables to $D \cup \#_e$, where $D$ is the domain of individuals, and $\#_e$ is a privileged value, corresponding to the 'unknown' individual.

Trivalent static semantics for atomic sentences:

$$[P(v_1, \dots, v_n)]^{w,g}$$
$$= \begin{cases} \textbf{defined} & g(v_1), \dots, g(v_n) \neq \#_e \\ \textbf{true} & [P(v_1, \dots, v_n)]^{w,g} \text{ is } \textbf{defined} \\ & \text{and } \langle g(v_1), \dots, g(v_n) \rangle \in I_w(P) \end{cases}$$

For recursively define $[\![.]\!]^w_+$, $[\![.]\!]^w_-$, $[\![.]\!]^w_?$ for each sentence of EDS; each is a DPL-style relational meaning.

For atomic sentences, we also guarantee that the ?-extension is a test (c.f. partial DPL; van den Berg 1996).

(20) $[\![P(v_1, \dots, v_n)]\!]^w_+ :=$
$\{ (g, h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h}$ is **true** $\}$

(21) $[\![P(v_1, \dots, v_n)]\!]^w_- :=$
$\{ (g, h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h}$ is **false** $\}$

(22) $[\![P(v_1, \dots, v_n)]\!]^w_? :=$
$\{ (g, h) \mid g = h \wedge |P(v_1, \dots, v_n)|^{w,h}$ is **undefined** $\}$

# Truth and falsity

- $|\phi|^{w,g}$ is **true** if $\{\, h \mid (g, h) \in [\![\phi]\!]^{w}_{+} \,\} \neq \varnothing$
- $|\phi|^{w,g}$ is **false** if $|\phi|^{w,g}$ is not **true**
  and $\{\, h \mid (g, h) \in [\![\phi]\!]^{w}_{-} \,\} \neq \varnothing$
- $|\phi|^{w,g}$ is **neither** otherwise

N.b. an open sentence $\phi$ can be **neither** at $g$ if $g$ is undefined for any free variable in $\phi$.

For negative sentences, verification and falsification are flipped, presuppositions project.

(23) a. $[\![\neg\phi]\!]^w_+ := [\![\phi]\!]^w_-$

b. $[\![\neg\phi]\!]^w_- := [\![\phi]\!]^w_+$

c. $[\![\neg\phi]\!]^w_? := [\![\phi]\!]^w_?$

Immediate consequence: $\phi \iff \neg\neg\phi$

# Embedding Strong Kleene

Each cell in the Strong Kleene truth-table is interpreted as a *relational composition* (DPL conjunction) rather than a truth value.

| $\phi \wedge \psi$ | $[\![\psi]\!]^w_+$ | $[\![\psi]\!]^w_-$ | $[\![\psi]\!]^w_?$ |
|---|---|---|---|
| $[\![\phi]\!]^w_+$ | $\circ, +$ | $\circ, -$ | $\circ, ?$ |
| $[\![\psi]\!]^w_-$ | $\circ, -$ | $\circ, -$ | $\circ, -$ |
| $[\![\psi]\!]^w_?$ | $\circ, ?$ | $\circ, -$ | $\circ, ?$ |

**Figure 1:** Strong Kleene conjunction in EDS

$[\![\phi \wedge \psi]\!]^w_+$ is just the union of the true cells, $[\![\phi \wedge \psi]\!]^w_-$ the union of the false cells, and $[\![\phi \wedge \psi]\!]^w_?$ the union of the neither cells.

(24)
a. $[\![\phi \wedge \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_+$

b. $[\![\phi \wedge \psi]\!]^w_- := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_{+,-,?} \cup [\![\phi]\!]^w_{+,?} \circ [\![\psi]\!]^w_-$

c. $[\![\phi \wedge \psi]\!]^w_? := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_? \cup [\![\phi]\!]^w_? \circ [\![\psi]\!]^w_{+,?}$

We'll see how extending this strategy to disjunction helps in a moment, but first...

# Ingredients of existential quantification

Random assignment is a tautology that introduces a dref:

(25) a. $[\![\varepsilon_v]\!]^w_+ := \{ (g, h) \mid g[v]h \}$

b. $[\![\varepsilon_v]\!]^w_- := \varnothing$

c. $[\![\varepsilon_v]\!]^w_? := \varnothing$

Positive closure guarantees that if $\phi$ is false, no drefs are introduced (we'll use this to guarantee that $\exists$ doesn't commute with negation).

(26) a. $[\![\dagger\phi]\!]^w_+ := [\![\phi]\!]^w_+$

b. $[\![\dagger\phi]\!]^w_- := \{ (g, h) \mid g = h \wedge |\phi|^{w,g}$ is **false** $\}$

c. $[\![\dagger\phi]\!]^w_? := [\![\phi]\!]^w_?$

(27) $\exists_v\phi := \dagger(\varepsilon_v \wedge \phi)$

(28) a.
$$\begin{aligned}
[\![\varepsilon_v \wedge P(v)]\!]^w_+ &= [\![\varepsilon_v]\!]^w_+ \circ [\![P(v)]\!]^w_+ \\
&= \{ (g,h) \mid g[v]h \wedge h_v \in I_w(P) \} \\
[\![\varepsilon_v \wedge P(v)]\!]^w_- &= [\![\varepsilon_v]\!]^w_+ \circ [\![P(v)]\!]^w_- \\
&= \{ (g,h) \mid g[v]h \wedge h_v \notin I_w(P) \}
\end{aligned}$$

Thanks to positive closure, existential statements are negative tests.

(29) $[\![\dagger(\varepsilon_v \wedge P(v))]\!]^w_- = \{ (g,h) \mid g = h \wedge I_w(P) = \varnothing \}$

This guarantees that, e.g., $\neg\exists_v P(v)$ is a test, whereas $\neg\neg\exists_v P(v)$ isn't.

# Warmup: discourse anaphora

(30) $$[\![\exists_v P(v) \wedge Q(v)]\!]^w_+ = [\![\varepsilon_v \wedge P(v)]\!]^w_+ \circ [\![Q(v)]\!]^w_+$$

(31) $$\begin{aligned}[\![\exists_v P(v) \wedge Q(v)]\!]^w_- &= [\![\exists_v P(v)]\!]^w_+ \circ [\![Q(v)]\!]^w_- \\ &\quad \cup [\![\exists_v P(v)]\!]^w_- \circ [\![Q(v)]\!]^w_{+,-,?} \\ &= \{ (g, h) \mid g[v]h \wedge h_v \in I_w(P), \notin I_w(Q) \} \\ &\quad \cup \{ (g, h) \mid g = h \wedge I_w(P) = \varnothing \}\end{aligned}$$

Already possible to see: $\neg(\exists_v P(v) \wedge Q(v))$ is going to end up equivalent to $\neg \exists_v P(v) \vee \neg Q(v)$.

Negating discourse anaphora can introduce a dref — ignorance will be responsible for blocking subsequent anaphora, just as with disjunction (in contrast to negating a simple existential statement).

## Strong Kleene disjunction

We could define disjunction in EDS as $\neg(\neg\phi \wedge \neg\psi)$, like in DPL, but we don't need to (predictiveness)! SK already gives us the recipe for the semantics of disjunction.

| $\phi \vee \psi$ | $[\![\psi]\!]^w_+$ | $[\![\psi]\!]^w_-$ | $[\![\psi]\!]^w_?$ |
|---|---|---|---|
| $[\![\phi]\!]^w_+$ | $\circ, +$ | $\circ, +$ | $\circ, +$ |
| $[\![\psi]\!]^w_-$ | $\circ, +$ | $\circ, -$ | $\circ, ?$ |
| $[\![\psi]\!]^w_?$ | $\circ, +$ | $\circ, ?$ | $\circ, ?$ |

**Figure 2:** Strong Kleene disjunction in EDS

# Disjunction

(32) a. $$[\![\phi \vee \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_{+,-,?} \cup [\![\phi]\!]^w_{-,?} \circ [\![\psi]\!]^w_+$$

b. $$[\![\phi \vee \psi]\!]^w_- := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_-$$

c. $$[\![\phi \vee \psi]\!]^w_? := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_? \cup [\![\phi]\!]^w_? \circ [\![\psi]\!]^w_{-,?}$$

**Insight for Partee disjunction:** two ways of dynamically verifying a disjunction: (i) composing the negative extension of the first disjunct with the positive extension of the second; (ii) composing the positive extension of the first with the positive/negative/unknown extension of the second.

Positive extension:

(33) $[\![\neg\exists_v B(v) \vee U(v)]\!]^w_+$

a. $[\![\neg\exists_v B(v)]\!]^w_- \circ [\![U(v)]\!]^w_+ =$
$\{ (g, h) \mid g[v]h \wedge h_v \in I_w(B) \wedge h_v \in I_w(U) \}$

b. $[\![\neg\exists_v B(v)]\!]^w_+ \circ [\![U(v)]\!]^w_{+,-,?} =$
$\{ (g, h) \mid g = h \wedge I_w(B) = \varnothing \}$

c. $= \{ (g, h) \mid g[v]h \wedge h_v \in I_w(B) \wedge h_v \in I_w(U) \}$
$\cup \{ (g, h) \mid g = h \wedge I_w(B) = \varnothing \}$

If classically true, then if there is a bathroom, introduce a bathroom dref $v$, otherwise do nothing.

# Update in EDS

In order to understand Rothschild discourses, we need to define what it means to *update* an information state in the context of EDS.

In fact, we can adopt a very simple generalization of Stalnaker's bridge principle (von Fintel 2008) to file contexts (i.e. sets of world-assignment pairs; Heim 1982).

**Update** in EDS:

$$c[\phi] = \begin{cases} \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^w_+ \,\} & \forall (w,g) \in c \begin{bmatrix} |\phi|^{w,g} \text{ is } \textbf{true} \\ \text{or } |\phi|^{w,g} \text{ is } \textbf{false} \end{bmatrix} \\ \text{undefined} & \text{otherwise} \end{cases}$$

This derives Heimian familiarity for free variables, as a universal requirement on $c$.

An assertion of $\phi \vee \psi$ in $c$ is only felicitous if $\phi$ are $\psi$ are *real possibilities* in $c$.

**Contingency requirement:** A sentence of the form $c[\phi \vee \psi]$ is felicitous iff both $c_w[\phi]$ and $c_w[\psi]$ are non-empty proper subsets of $c_w$.

(34) $$[\![\exists_v P(v) \vee Q(a)]\!]^w_+ = \\ \{\,(g,h) \mid g[v]h \wedge h_v \in I_w(P)\,\} \\ \cup \{\,(g,h) \mid g = h \wedge I_w(P) = \varnothing \wedge a \in I_w(Q)\,\}$$

$P$ worlds are associated with a discourse referent $v$, and non-$P$ $Q$-worlds are not. non-$P$ non-$Q$ worlds are eliminated.

As long as $\exists_v P(v)$ doesn't contextually entail $Q(a)$ (Hurford 1974), the updated context must contain some non-$P$ worlds if contingency is satisfied.

Those non-$P$ worlds will not be associated with anaphoric information, and therefore a subsequent pronoun won't be licensed (by failure of Heimian familiarity).

However, if the the non-$P$ worlds are eliminated subsequently, then familiarity will be contextually satisfied for the variable $v$, since all $P$ worlds are associated with a dref $v$.

Rothschild discourses are thereby accounted for.

EDS disjunction is already a kind of program disjunction — if both disjuncts introduce a dref $v$, then $v$ will be defined throughout the updated context even if contingency is satisfied. A subsequent pronoun is predicted to be possible.

(35) $$\begin{aligned}&[\![\exists_v P(v) \vee \exists_v Q(v)]\!]^w_+ \\ &= \{\,(g,h) \mid g[v]h \wedge h_v \in I_w(P)\,\} \\ &\quad \cup \{\,(g,h) \mid g[v]h \wedge h_v \in I_w(Q)\,\}\end{aligned}$$

This explanation is compatible with encoding novelty via guarded random assignment (van den Berg 1996) thanks to Strong Kleene disjunction (details suppressed).

(36) $\exists^{v}[B(v)] \vee U(v)$

- If the first disjunct is true, the second is guaranteed to be defined (i.e., have an empty ?-extension, so we can ignore it).
- The first disjunct is always defined (we can ignore its ?-extension).
- If the first disjunct is false, whether or not the second disjunct is true will depend on the input assignment.

# The residue: apparent internal staticity ii

(37) $$[\![\exists^v[B(v)] \vee U(v)]\!]^w_+ = [\![\exists_v[B(v)]]\!]^w_+ \circ [\![U(v)]\!]^w_{+,-} \cup [\![\exists_v[B(v)]]\!]^w_- \circ [\![U(v)]\!]^w_+$$

(38) $$= \{ (g,h) \mid g[v]h \wedge g(v) \in I_w(B) \} \cup \{ (g,h) \mid g = h \wedge I_w(B) = \varnothing \wedge h(v) \in I_w(U) \}$$

(39) $$[\![\exists^v[B(v)] \vee U(v)]\!]^w_- = [\![\exists_v[B(v)]]\!]^w_- \circ [\![U(v)]\!]^w_-$$

(40) $$= \{ (g,h) \mid g = h \wedge I_w(B) = \varnothing \wedge h(v) \notin I_w(U) \}$$

To be assertable in $c$, every world assignment pair $(w, g)$ should be s.t., either $w$ is a $B$-world, or $w$ is a non-$B$ world, and $g(v)$ is defined. Given the contingency requirement $v$ must be defined in the non-$B$ parts of $c$. The sentence will be infelicitous in an *out-of-the-blue* context.

# *Conclusion*

Just as it generates weak readings for Partee disjunctions, EDS generates weak readings for donkey sentences; Strong readings must be derived via an independent strengthening mechanism.

This is easy to see, since we predict the following equivalence, via Strong Kleene material implication:

(41) a. Either there is no$^{v}$ bathroom, or it$_{v}$'s upstairs.

b. If there is a$^{v}$ bathroom, it$_{v}$'s upstairs.

See Elliott 2020 for details.

EDS is intended to be a bona fide *evolution* of the logical tools developed by Groenendijk & Stokhof (1991).

We've drawn a tight connection between the Strong Kleene logic of presupposition projection and anaphora, solving several empirical problems along the way.

One key lesson is that we should pay attention to the role of independent pragmatic factors in constraining anaphoric possibilities.

**Thank you!**

- Beaver, David I. 2001. *Presupposition and Assertion in Dynamic Semantics*. CSLI Publications. 250 pp.
- Champollion, Lucas, Dylan Bumford & Robert Henderson. 2019. Donkeys under discussion. *Semantics and Pragmatics* 12(0). 1.
- Charlow, Simon. 2020. Static and dynamic exceptional scope. Unpublished manuscript. Accepted at journal of semantics.
- Chierchia, Gennaro. 1995. *Dynamics of meaning - anaphora, presupposition, and the theory of grammar*. Chicago: University of Chicago Press. 270 pp.

Elliott, Patrick D. 2020. Towards a principled logic of anaphora. lingbuzz/005562. MIT. Submitted to Semantics & Pragmatics.

Fox, Danny. 2013. Presupposition projection from quantificational sentences - Trivalence, local accommodation, and presupposition strengthening. In Ivano Caponigro & Carlo Cecchetto (eds.), *From grammar to meaning*, 201–232.

George, B. R. 2007. Predicting presupposition projection - Some alternatives in the strong Kleene tradition. unpublished manuscript. UCLA.

George, B. R. 2008. A new predictive theory of presupposition projection. In *Proceedings of SALT 18*, 358–375. Ithaca, NY: Cornell University.

George, B. R. 2014. Some remarks on certain trivalent accounts of presupposition projection. *Journal of Applied Non-Classical Logics* 24(1-2). 86–117.

Gotham, Matthew. 2019. Double negation, excluded middle and accessibility in dynamic semanttics. In Julian J. Schlöder, Dean McHugh & Floris Roelofsen (eds.), *Proceedings of the 22nd Amsterdam Colloquium*, 142–151.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

- Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases.* University of Massachusetts - Amherst dissertation.
- Hurford, James R. 1974. Exclusive or Inclusive Disjunction. *Foundations of Language* 11(3). 409–411.
- Kanazawa, Makoto. 1994. Weak vs. Strong Readings of Donkey Sentences and Monotonicity Inference in a Dynamic Setting. *Linguistics and Philosophy* 17(2). 109–158.
- Karttunen, Lauri. 1976. Discourse referents. In J. D. McCawley (ed.), *Syntax and semantics vol. 7*, 363–386. Academic Press.

# References v


- Krahmer, Emiel & Reinhard Muskens. 1995. Negation and Disjunction in Discourse Representation Theory. *Journal of Semantics* 12(4). 357–376.
- Mandelkern, Matthew & Daniel Rothschild. 2020. Definiteness projection. *Natural Language Semantics* 28(2). 77–109.
- Peters, Stanley. 1979. A truth-conditional formulation of Karttunen's account of presupposition. *Synthese* 40(2). 301–316.

- Rothschild, Daniel. 2017. A trivalent approach to anaphora and presupposition. In Alexandre Cremers, Thom van Gessel & Floris Roelofsen (eds.), *Proceedings of the 21st Amsterdam Colloquium*, 1–13.
- van den Berg, M. H. 1996. Some aspects of the internal structure of discourse. The dynamics of nominal anaphora.
- von Fintel, Kai. 2008. What Is Presupposition Accommodation, Again?*. *Philosophical Perspectives* 22(1). 137–170.