IN THE BAG

November 24, 1977

To the Problems Editor:

Enclosed is a solution to problem no. 58 . . . .

. . . We have taken the liberty of extending the problem to other counting bases in the hope that this would further stimulate some interest on the part of your readers. While our results resolve the problem as stated, the consideration of other counting bases seems to lead to some interesting questions; e.g., is there a simple formula for obtaining an Armstrong number of more than one digit when the counting base is even? (We have given such a result when the counting base is odd.) . . .

Sincerely,

James V. Peters

for Richard O'Sullivan
and James V. Peters,
St. Bonaventure University

58. (Gary Wernsing, Ithaca) An Armstrong number is an n-digit number equal to the sum of the nth powers of its digits. Examples: $371 = 3^3 + 7^3 + 1^3$ and $8208 = 8^4 + 2^4 + 0^4 + 8^4$. Prove that there are a finite number of Armstrong numbers.

An n-digit Armstrong number must be $\geq 10^{n-1}$ since its leading digit is non-zero. At the same time, the sum of the nth powers of the digits cannot exceed $n9^n$. It is simple to verify, using logarithms, that $10^{n-1} > n9^n$ for $n \geq 61$. This implies that no Armstrong number can contain 61 or more digits. Consequently, there are only finitely many such numbers.

More generally, if numbers are represented in terms of a counting base b, then $b^{n-1} > n(b-1)^n$ for n sufficiently large. For example, in binary ($b = 2$) no Armstrong number can contain $n = 3$ or more digits. This limits the search to 1, 10, 11. At the other extreme, as $b \to \infty$ so does n. The search for Armstrong numbers with respect to successively larger counting bases is, at least, non-vacuous. Indeed, it is not difficult to show that if b is odd, then aa is a two-digit Armstrong number when $a = (b+1)/2$. Moreover, aa cannot be an Armstrong number with respect to any base $b' > b$. To show this we simply note that

$$\left(\frac{b+1}{2}\right)b' + \frac{b+1}{2} = 2\left(\frac{b+1}{2}\right)^2$$

implies $b = b'$. Consequently there are an infinite number of *distinct* Armstrong numbers when one considers all counting bases simultaneously.

— *from bottom of page 3* —
Now you do!: Would you believe 50?

PRI
/
David R
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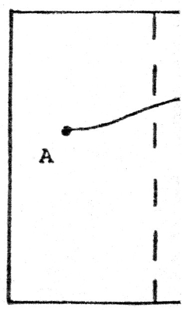
In an episode of the
had a chance to win a

Prize
Car
Stove
Bank

Molly, the contestant,
were all different. Her
time she selected a digit
in the position in whic
named all of the digits ir
Assuming Molly calls
of her winning each of t
bility that Molly will wi
upon the "Fundamental
as follows: Suppose task
A is completed, task B c
then be consecutively p
extended similarly to any
in the drawing below tl
point A to point B by go
path is depicted on the d



The following describe
together with the probabi

I. Molly needs only four

In this case, she must
number of ways of selecti
$10 \times 9 \times 8 \times 7 = 5040$
digits from the four av
$24/5040 = .004762$.