# Quantum Hardware
# - Part 1 -

2024/06/14

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# Outline

Part 1: Superconducting qubits
- Physics of the superconducting qubits
- Qubit control
- Quantum non-demolition measurement
- Two-qubit gate

Part 2: Device map and calibration data
- Hands-on: extracting the device information

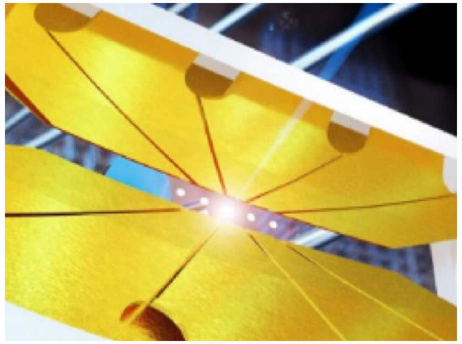
Part 3: Qubit scaling
- Modularity
- Microwave component development

Summary

# IBM Quantum System One

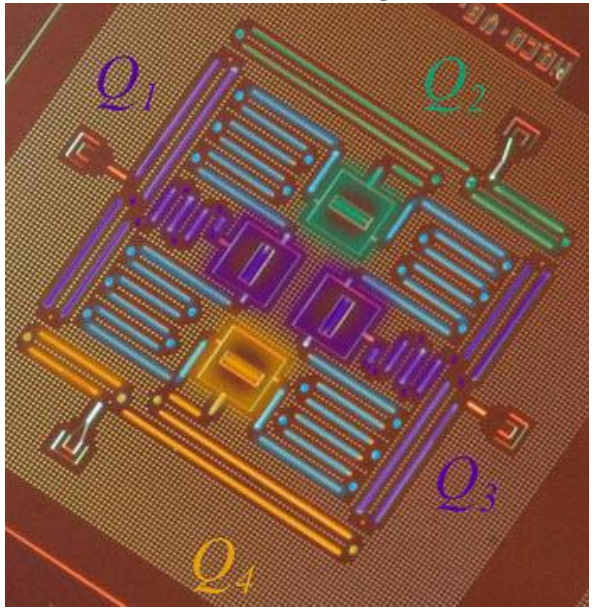
# Quantum computing technologies

## Ions



Credit: N. M. Linke et al., University of Maryland, 2017

## Superconducting circuits




Credit: A. D. Córcoles et al., IBM, 2015

## Neutral atoms

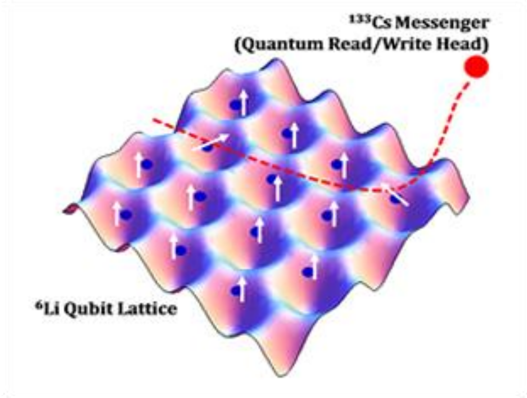



Image from Cheng Group, University of Chicago

## Photons

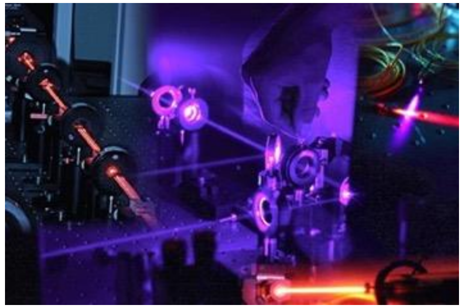


Image from the Centre for Quantum Computation & Communication Technology

## Solid-state defects (NV centers, phosphorous in Si)

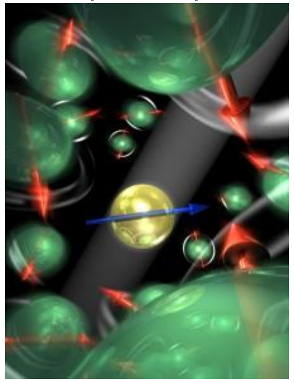
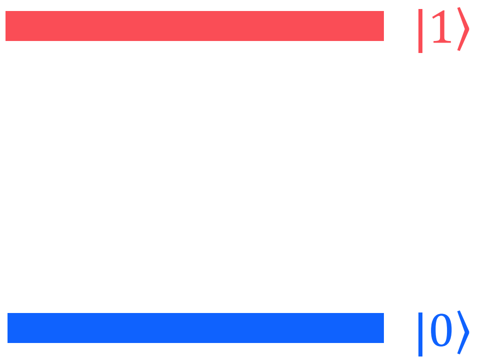


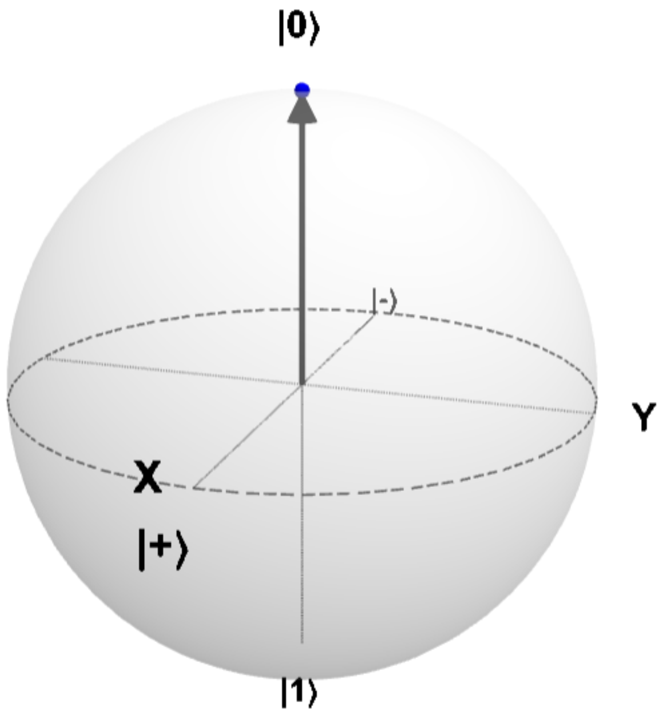
Image from Hanson Group, Delft

# Qubit: idea

Energy levels

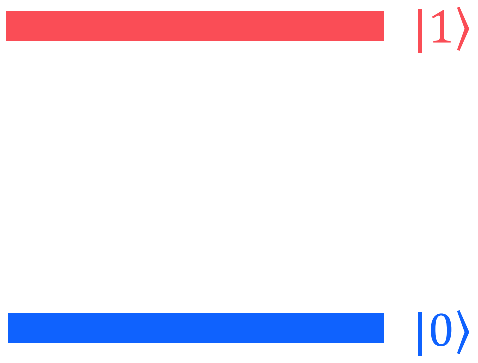



Bloch sphere

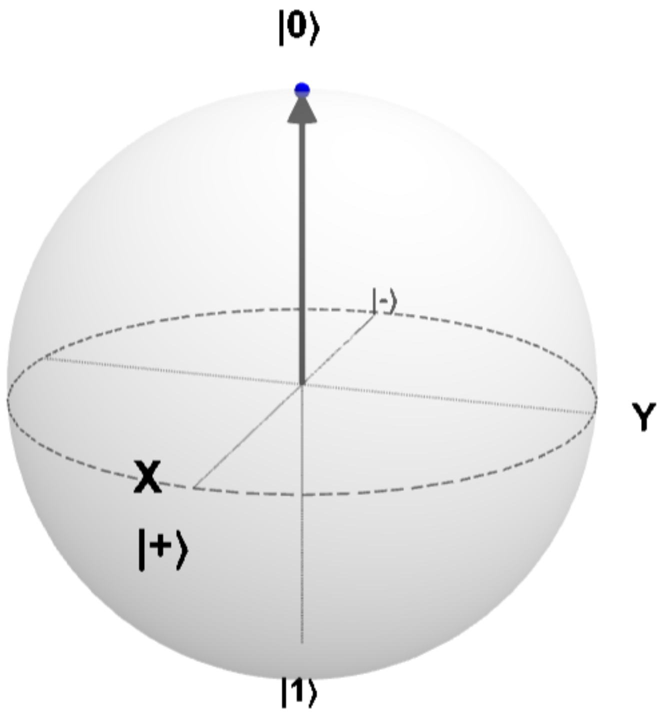


# Qubit: idea and reality

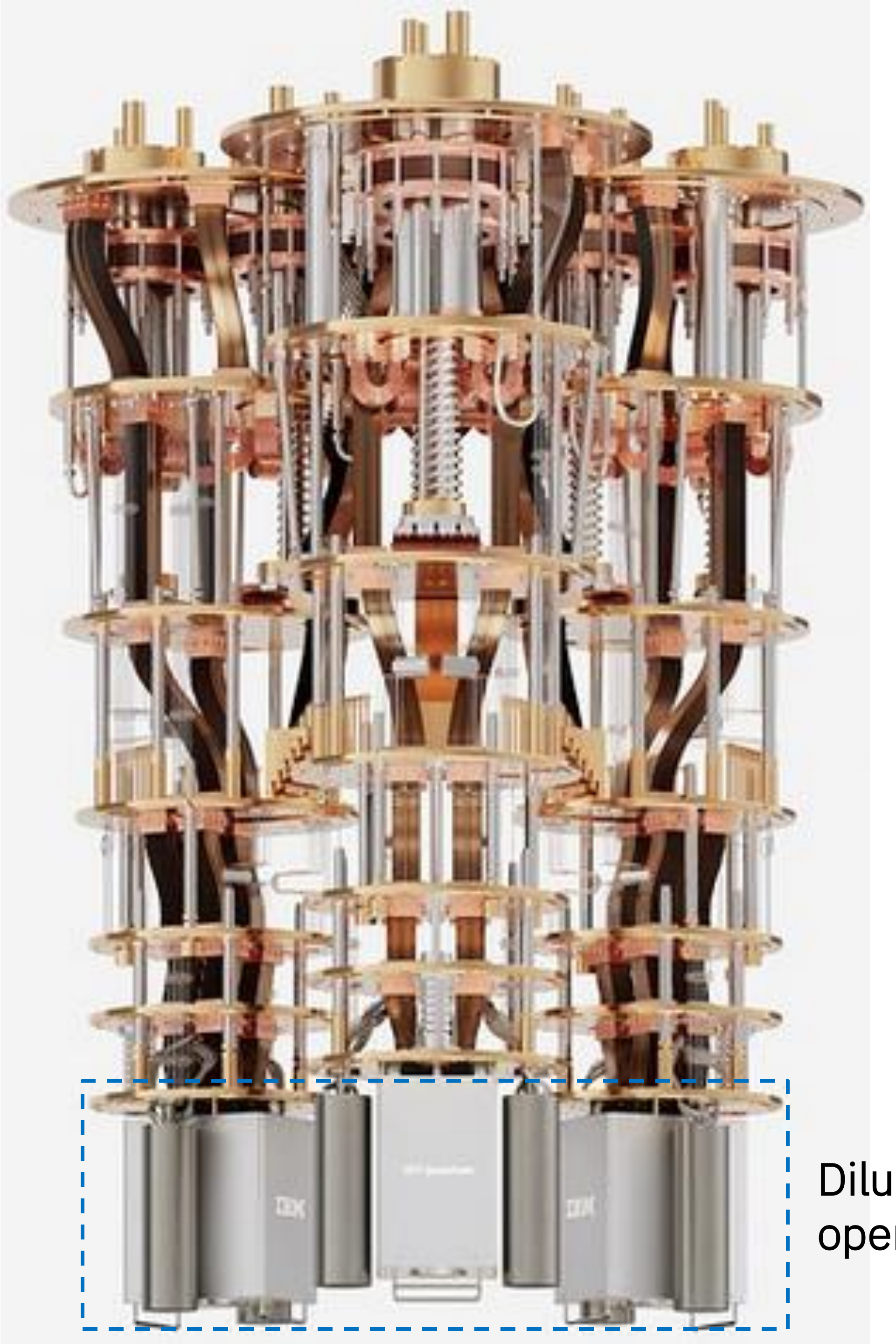
Energy levels




Bloch sphere

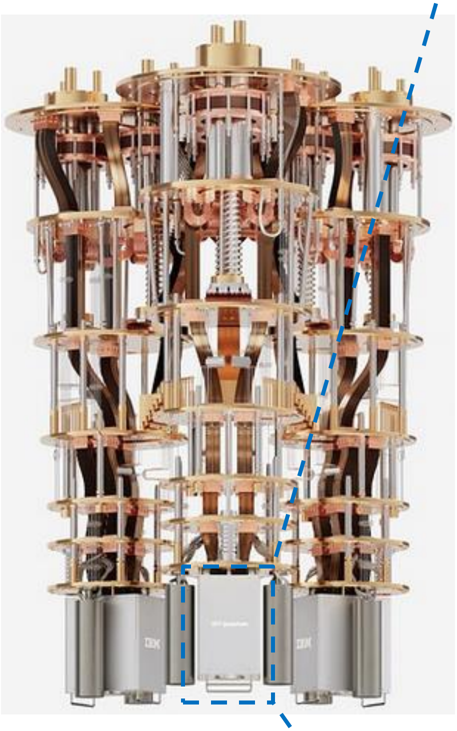


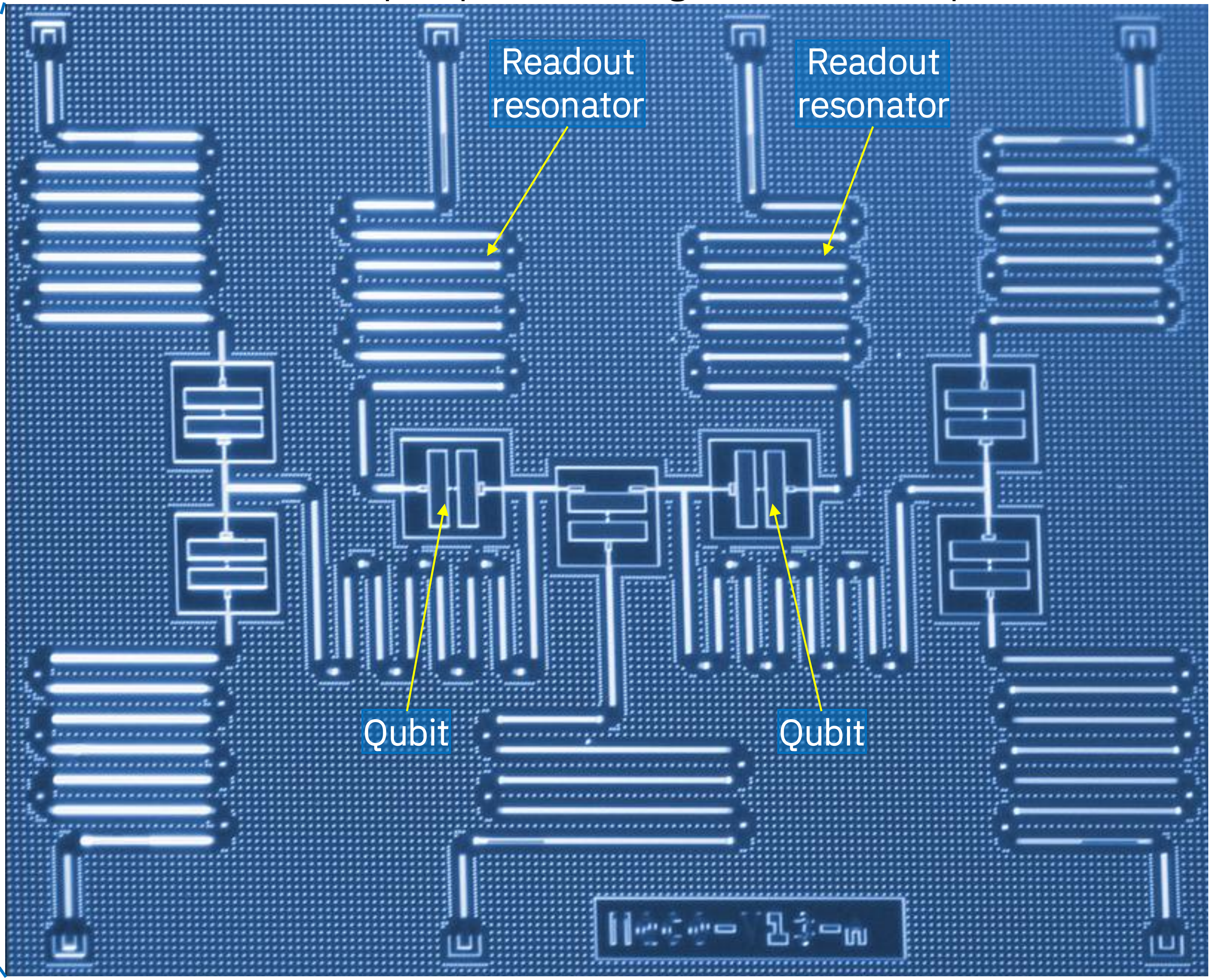

Real quantum computer

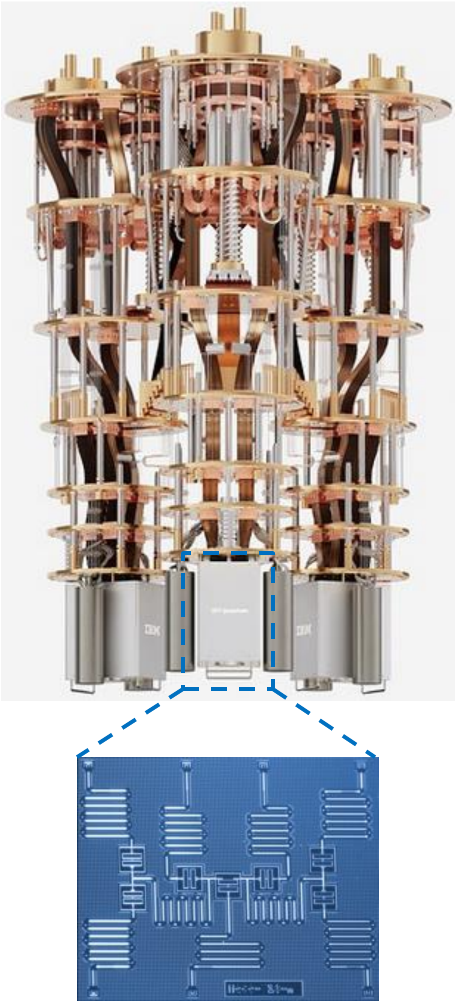
Dilution refrigerator
operated at 10 mK

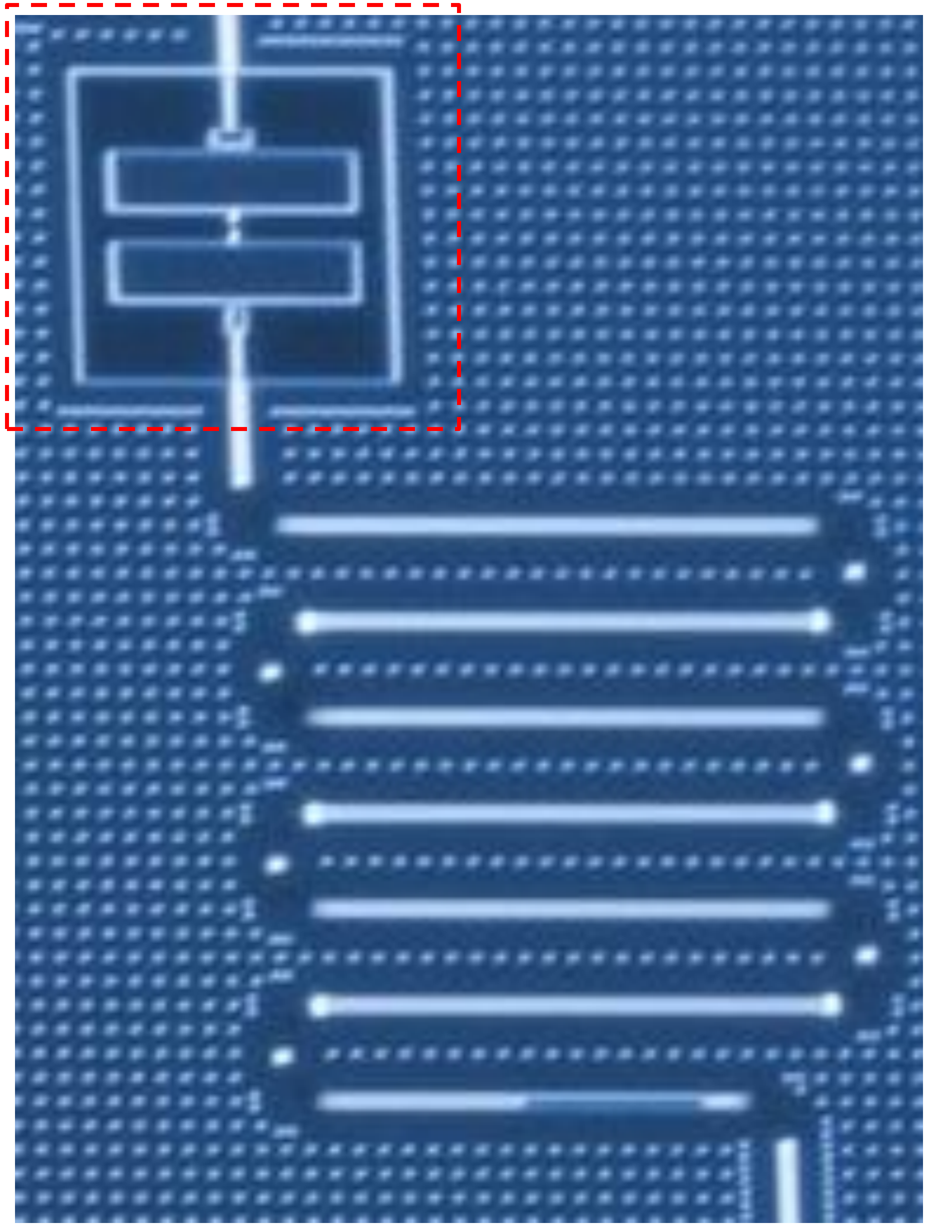
# Qubit chip: superconducting circuits on Si chip



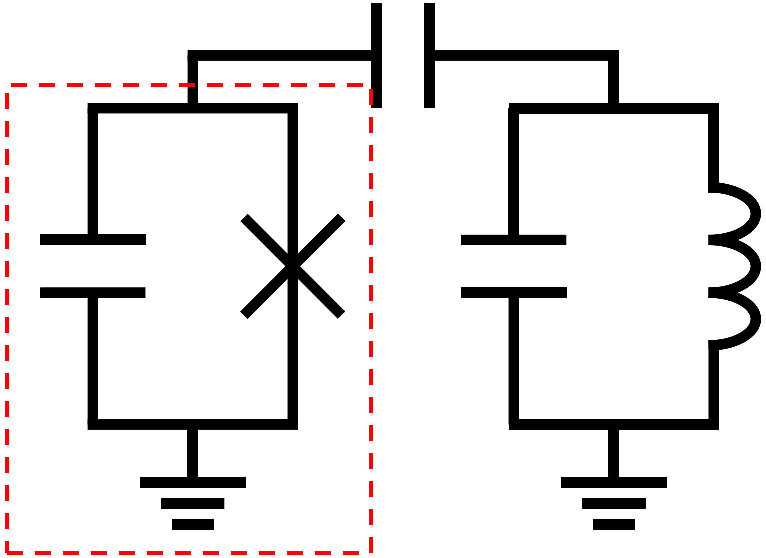


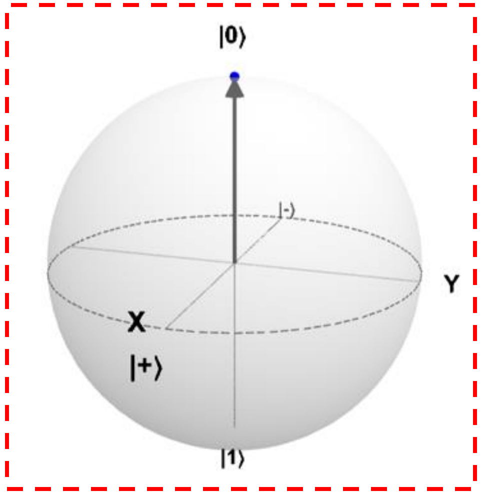
## Qubit and readout resonator





Equivalent circuits

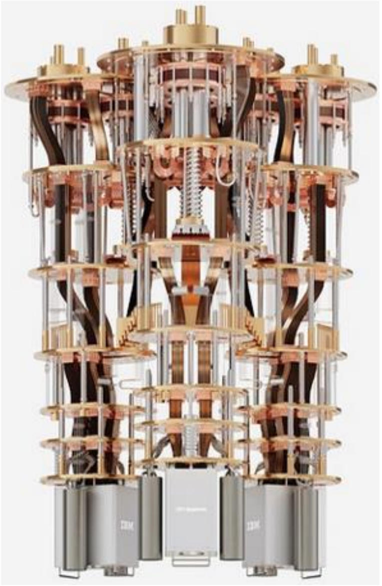


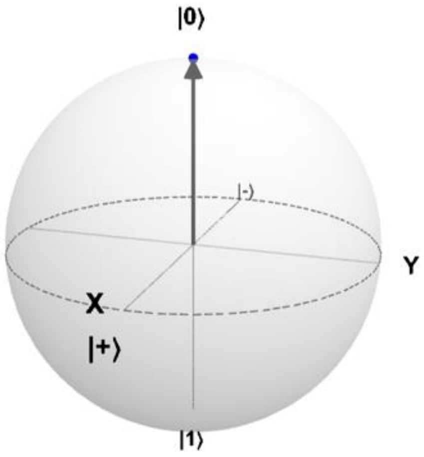


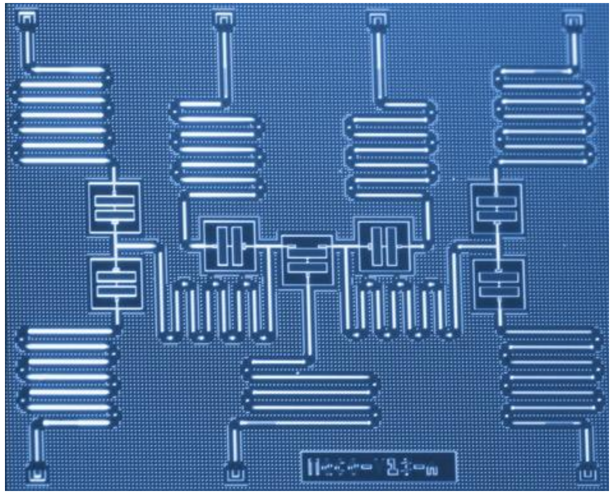
# Quantum computation flow

Control

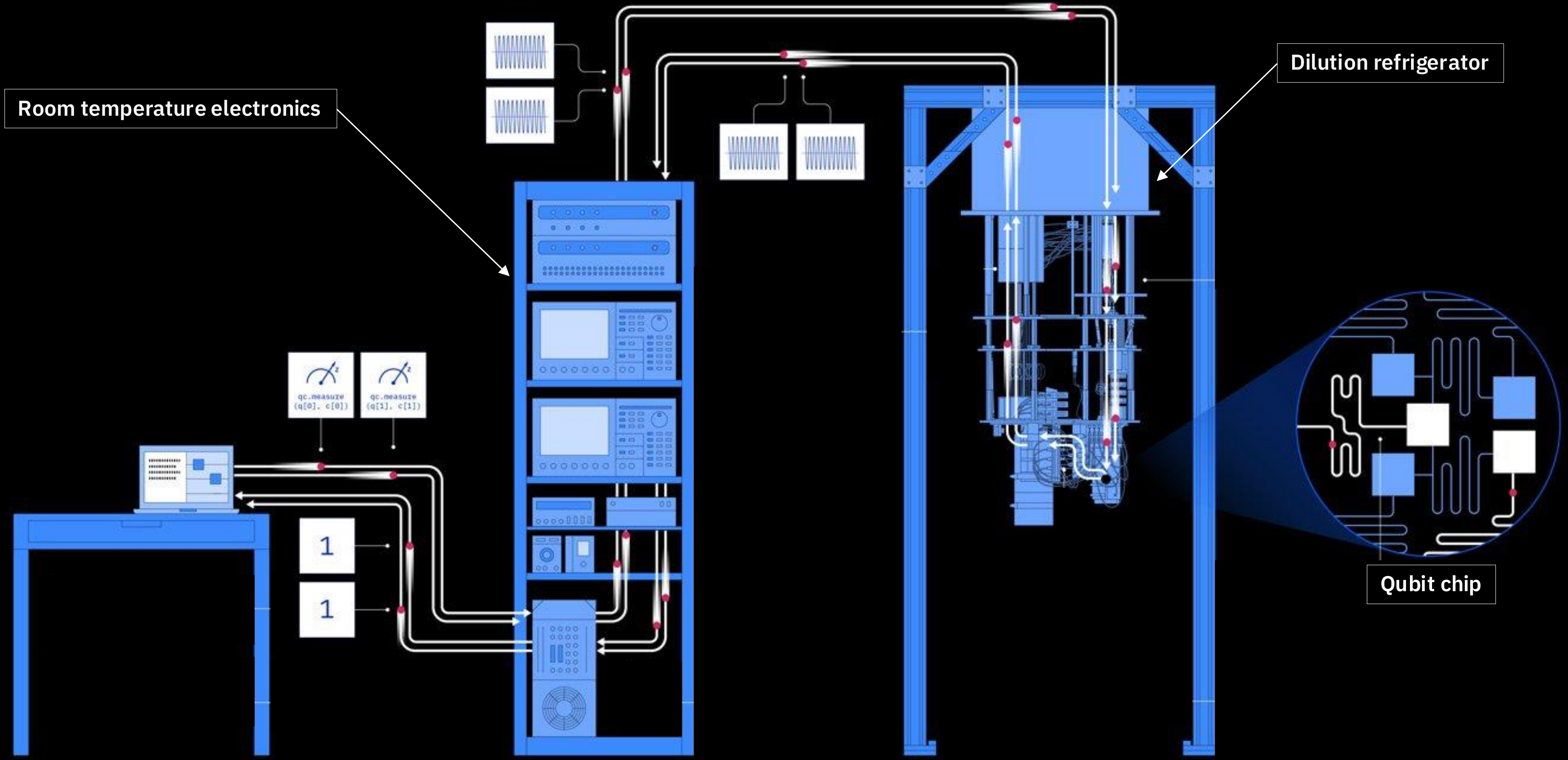
Readout

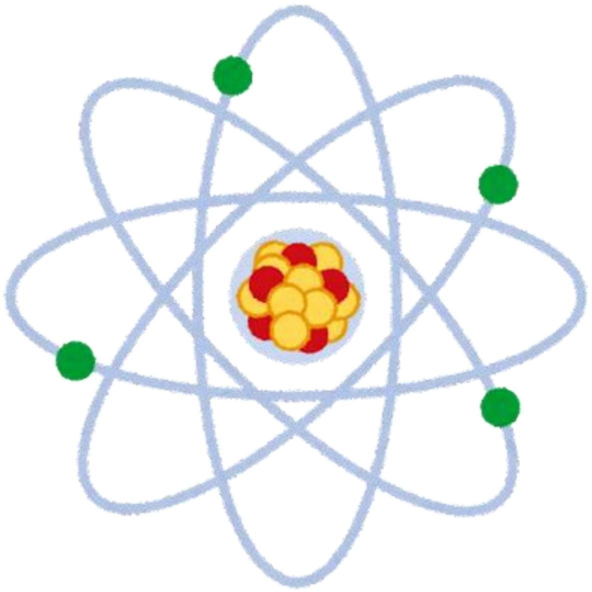


# Schematic of superconducting quantum computer



# How do real qubits behave: Natural atoms



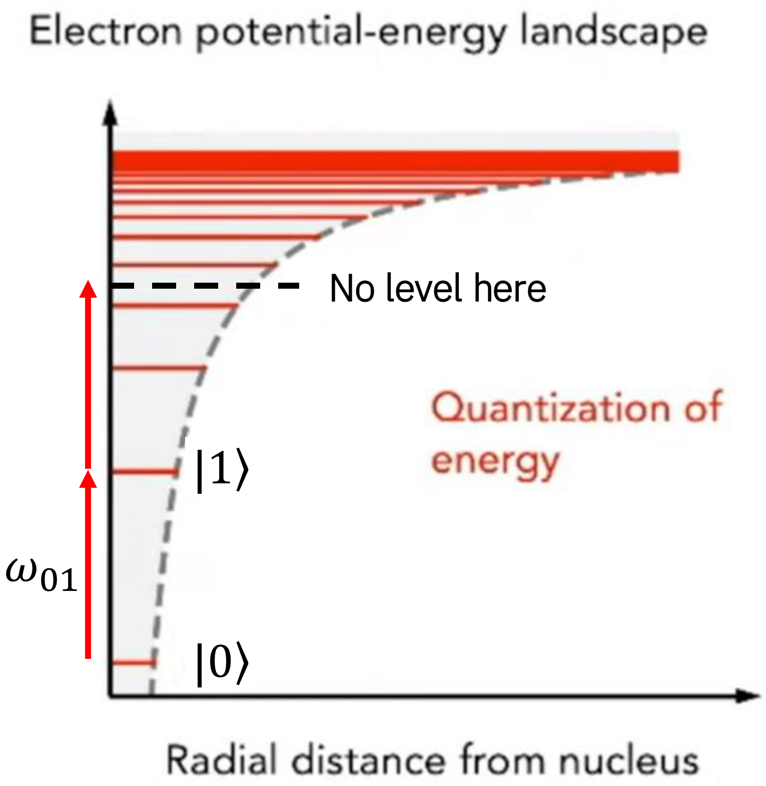
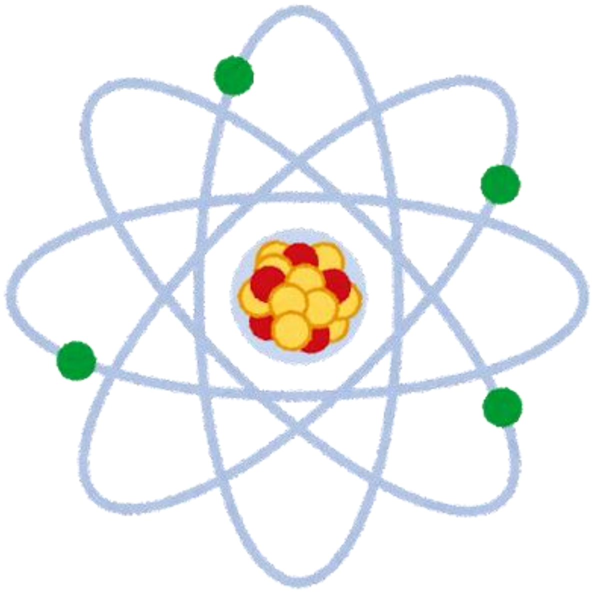


Image: Z. Minev, IBM, 2022

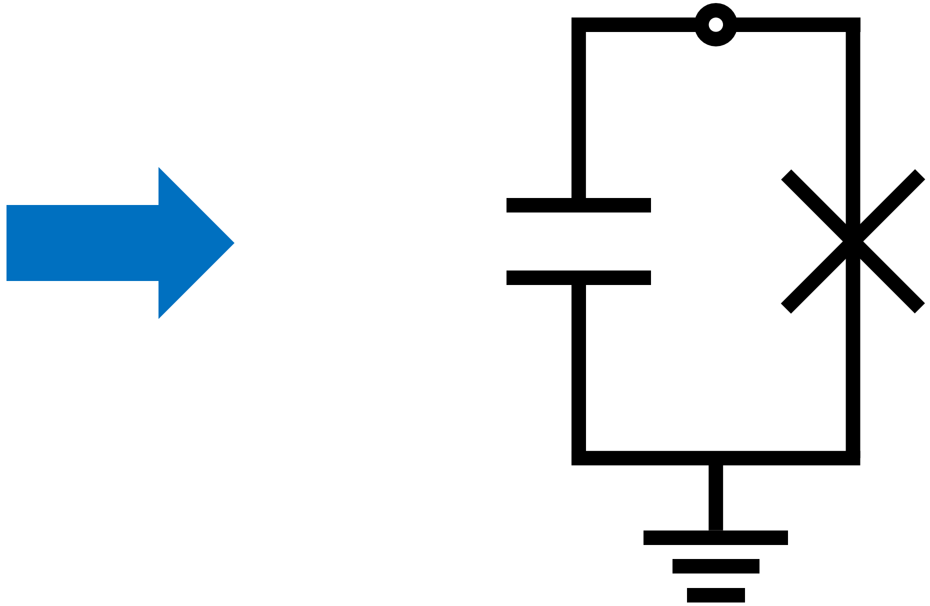
Qubit

- Multi energy levels
- Quantized (Discrete)
- Anharmonicity

# Artificial atoms = superconducting qubits

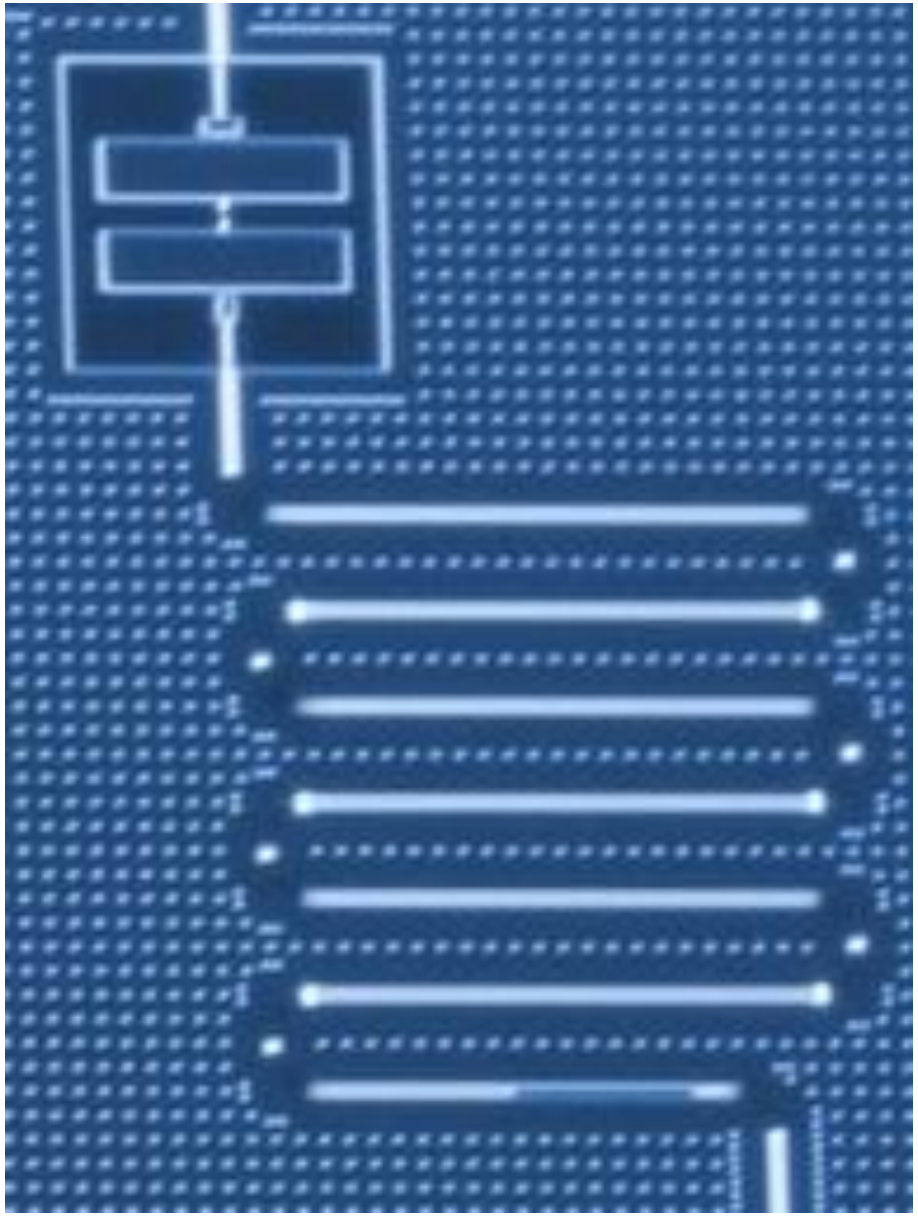


Natural atoms



Artificial atoms

# Superconducting qubit



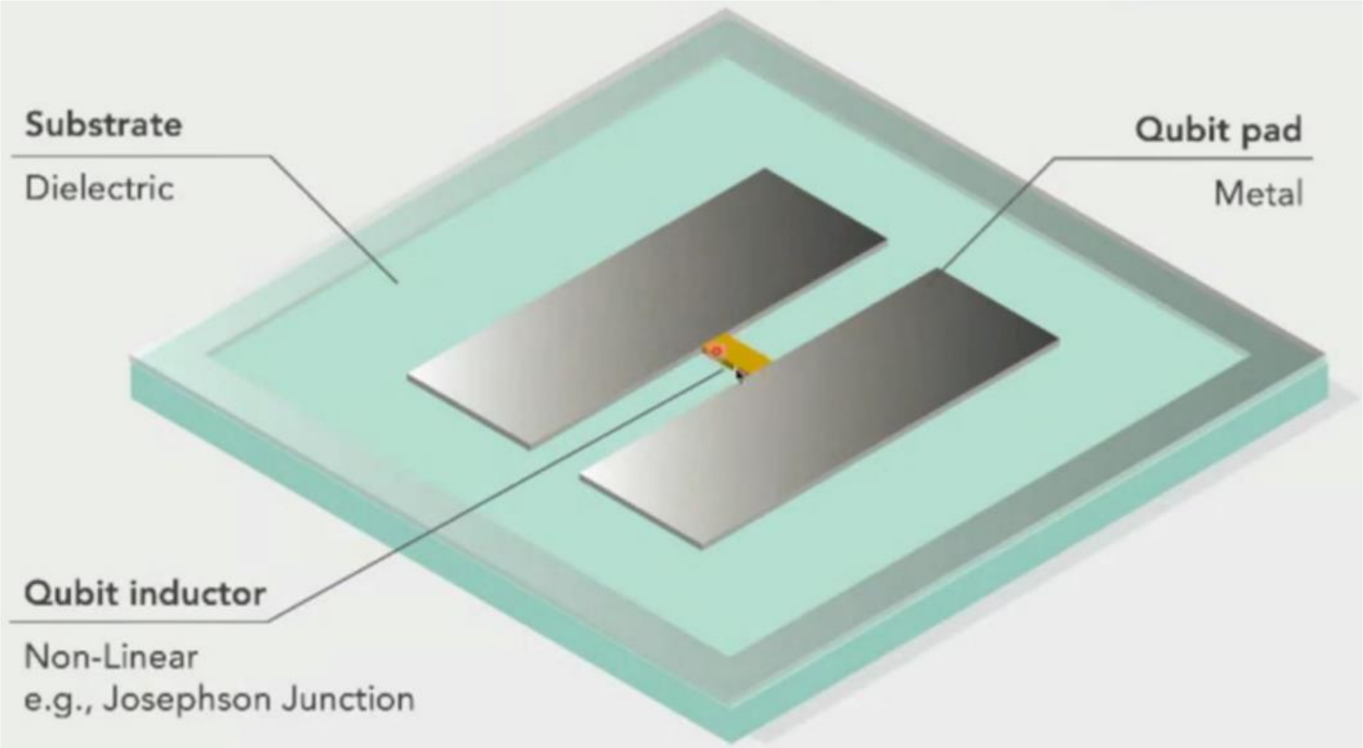
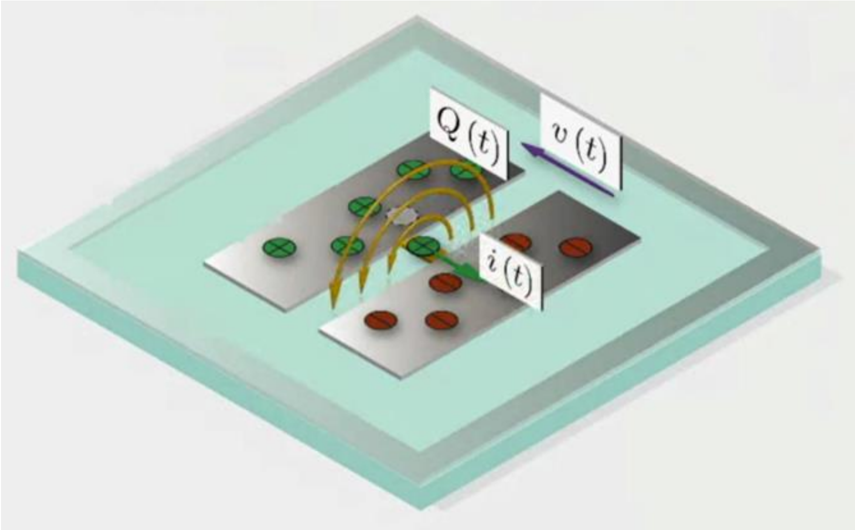
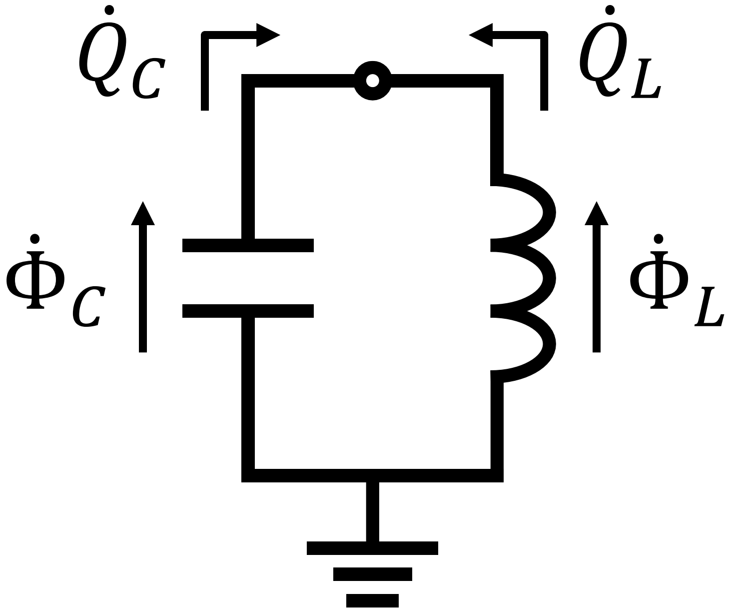



Image: Z. Minev
https://youtu.be/eZJjQGu85Ps?si=LrAlGV4CZOa-qHY3

# Electromagnetic oscillator




Universal relationships

$$\dot{Q} = I$$

$$\dot{\Phi} = V$$

Capacitance, Inductance relationships

$$Q = CV \left(= C\dot{\Phi}\right)$$

$$\Phi = LI \left(= L\dot{Q}\right)$$

Kirchhoff's voltage law

$$\dot{\Phi}_C = \dot{\Phi}_L$$

$$\Rightarrow \Phi_C = \Phi_L \equiv \Phi$$

Kirchhoff's current law

$$\dot{Q}_C + \dot{Q}_L = 0$$

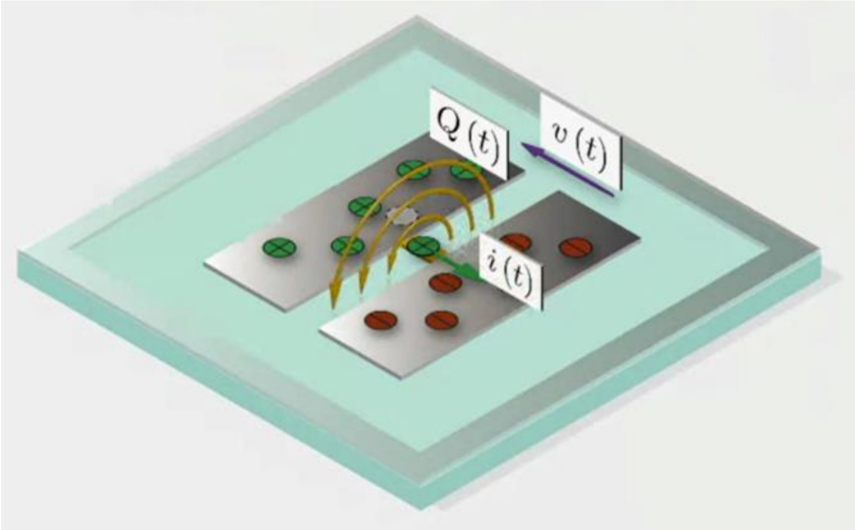
$$\Rightarrow C\ddot{\Phi}_C + \frac{\Phi_L}{L} = C\ddot{\Phi} + \frac{\Phi}{L} = 0$$

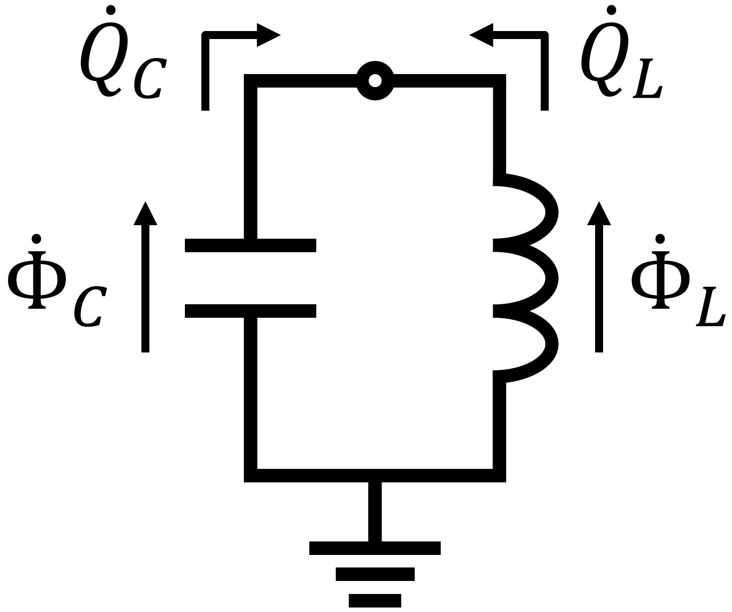
$$\Rightarrow \ddot{\Phi} = -\omega_0^2 \Phi, \omega_0 = \frac{1}{\sqrt{LC}}$$

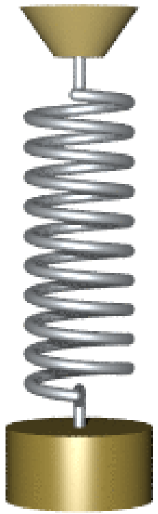
$$\Rightarrow \Phi = \Phi_0 \mathrm{e}^{-i\omega_0 t}$$

Harmonic oscillator of resonance frequency $\omega_0$

# Electromagnetic oscillator



Consider an LC circuit with a linear inductor $L$. The following formula is derived from Kirchhoff's law.

$$C\ddot{\Phi} + \frac{\Phi}{L} = 0$$

$$\Phi = \Phi_0 \mathrm{e}^{-i\omega_0 t}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

In the LC circuit, magnetic flux $\Phi$ oscillates as a harmonic oscillator of resonance frequency $\omega_0$

Analogy with a mechanical oscillator

Magnetic flux → Position: $\Phi \mapsto x$

Inductance → Spring constant: $\frac{1}{L} \mapsto k$

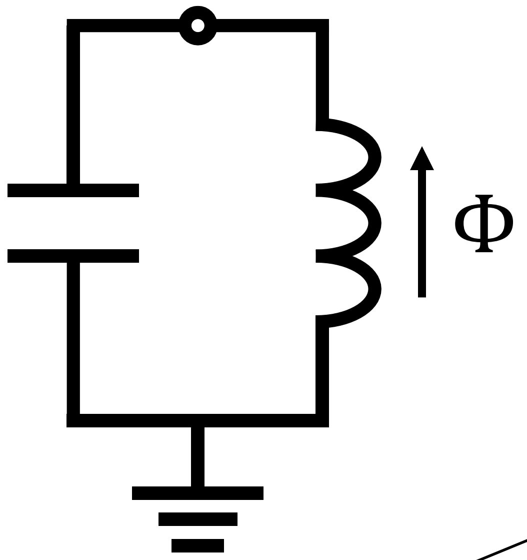
Capacitance → Mass: $C \mapsto m$

Voltage → Velocity: $\dot{\Phi}(= V) \mapsto v$

Charge → Momentum: $Q(= CV) \mapsto p(= mv)$

Equation of motion: $C\ddot{\Phi} + \frac{\Phi}{L} = 0 \mapsto F = ma$

# Lagrangian and Hamiltonian



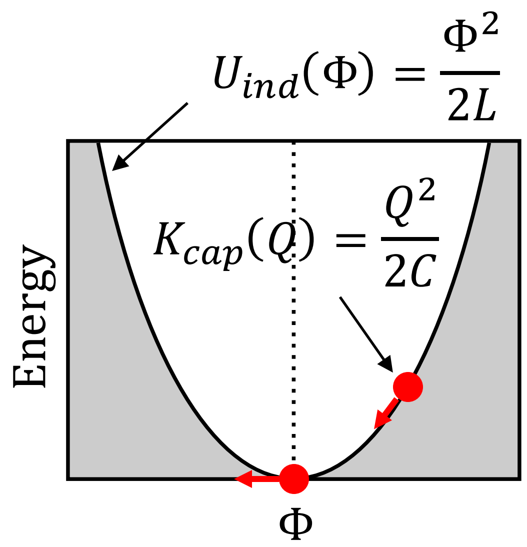

Lagrangian = Kinetic energy − Potential energy

$$\mathcal{L}(\Phi, \dot{\Phi}) = K_{cap}(\dot{\Phi}) - U_{ind}(\Phi)$$

$$= \frac{C\dot{\Phi}^2}{2} - \frac{\Phi^2}{2L} \qquad \mapsto \frac{1}{2}mv^2 - \frac{1}{2}kx^2$$

Euler-Lagrange equation (= equation of motion)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Phi}}\right) - \frac{\partial \mathcal{L}}{\partial \Phi} = 0 \qquad \mapsto F = ma$$




Canonically conjugate variable of $\Phi$

$$\frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi} = Q$$

$$C\ddot{\Phi} + \frac{\Phi}{L} = 0$$

Legendre transformation: Lagrangian → Hamiltonian

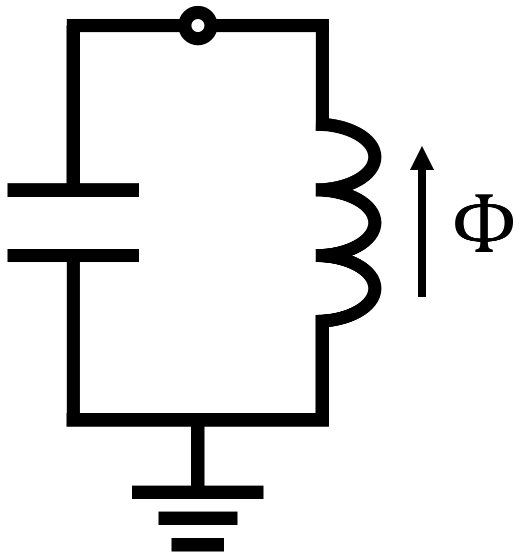
$$\mathcal{H}(\Phi, Q) = Q\dot{\Phi} - \mathcal{L}(\Phi, \dot{\Phi}) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Kinetic energy + Potential energy

Canonical equation (= equation of motion)

$$\dot{\Phi} = \frac{\partial \mathcal{H}}{\partial Q} = \frac{Q}{C}, \qquad \dot{Q} = -\frac{\partial \mathcal{H}}{\partial \Phi} = -\frac{\Phi}{L}$$

# Hamiltonian dynamics and phase space




Hamiltonian is denoted using $\alpha(t)$ as follows

$$\mathcal{H} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} = \frac{1}{2}\hbar\omega_0(\alpha^*\alpha + \alpha\alpha^*) = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

$$\alpha(t) = \sqrt{1/2\hbar Z}[\Phi(t) + iZQ(t)] = \alpha(0)e^{-i\omega_0 t}$$

$$Z = \sqrt{L/C}$$

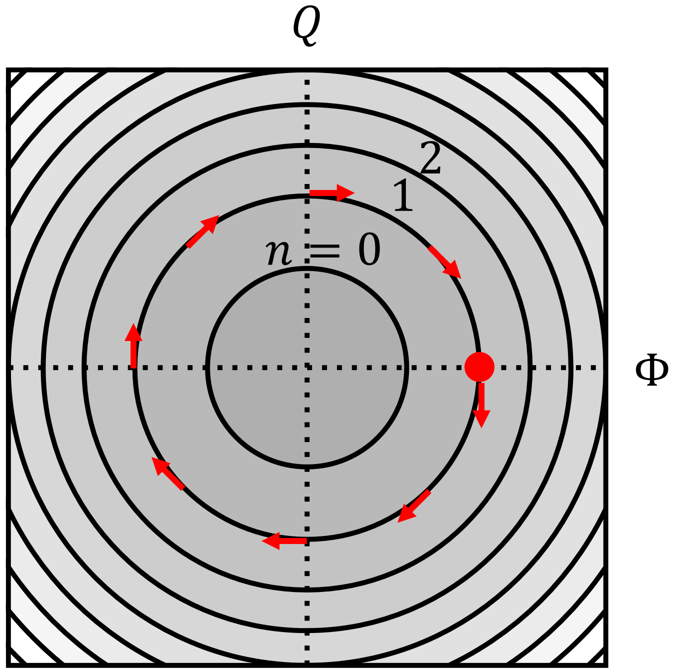
$\alpha(t)$ is a classical analog of bosonic ladder operator

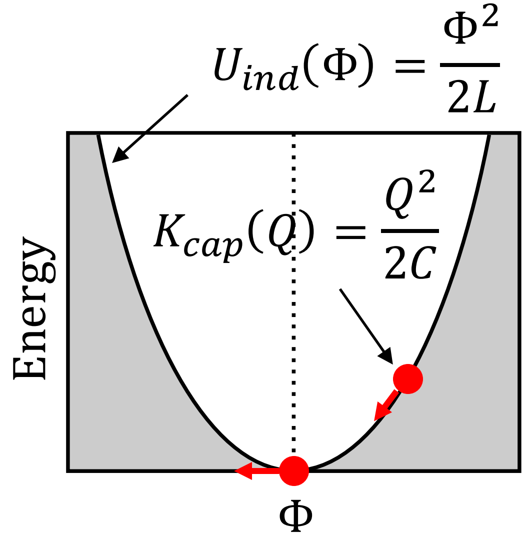
Hamiltonian → Total energy

$\alpha(t)$ : point in phase space

$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\begin{cases} \dot{\Phi} = \dfrac{\partial \mathcal{H}}{\partial Q} = \dfrac{Q}{C} \\ \dot{Q} = -\dfrac{\partial \mathcal{H}}{\partial \Phi} = -\dfrac{\Phi}{L} \end{cases}$$



# The classical and quantum oscillator

| | Classical | Quantum |
|---|---|---|
| Hamiltonian | $\Phi(t)$<br>$Q(t)$<br>$\mathcal{H} = \frac{\Phi^2}{2L} + \frac{Q^2}{2C} = \frac{1}{2}\hbar\omega_0(\alpha^*\alpha + \alpha\alpha^*)$ | $\hat{\Phi}$<br>$\hat{Q}$<br>$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \frac{1}{2}\hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right)$ |
| Phase space | $\alpha(t) = \sqrt{\frac{1}{2\hbar Z}}[\Phi(t) + iZQ(t)]$<br>$\alpha(t) = \alpha(0)e^{-i\omega_0 t}$<br>$\Phi(t) = \sqrt{\frac{\hbar Z}{2}}\left(\alpha^*(t) + \alpha(t)\right)$<br>$Q(t) = i\sqrt{\frac{\hbar}{2Z}}\left(\alpha^*(t) - \alpha(t)\right)$ | $\hat{a} = \sqrt{\frac{1}{2\hbar Z}}\left(\hat{\Phi} + iZ\hat{Q}\right)$<br>$\hat{a}(t) = \hat{a}(0)e^{-i\omega_0 t}$<br>$\hat{\Phi} = \Phi_{zpf}\left(\hat{a}^\dagger + \hat{a}\right)$<br>$\hat{Q} = iQ_{zpf}\left(\hat{a}^\dagger - \hat{a}\right)$ |
| Commutation | $\{\alpha, \alpha^*\} = 1/i\hbar$ | $\left[\hat{a}, \hat{a}^\dagger\right] = 1$ |

Zero-point fluctuation

$$\Phi_{zpf} = \sqrt{\frac{\hbar Z}{2}}$$

$$Q_{zpf} = \sqrt{\frac{\hbar}{2Z}}$$

# Energy levels of the quantum harmonic oscillator

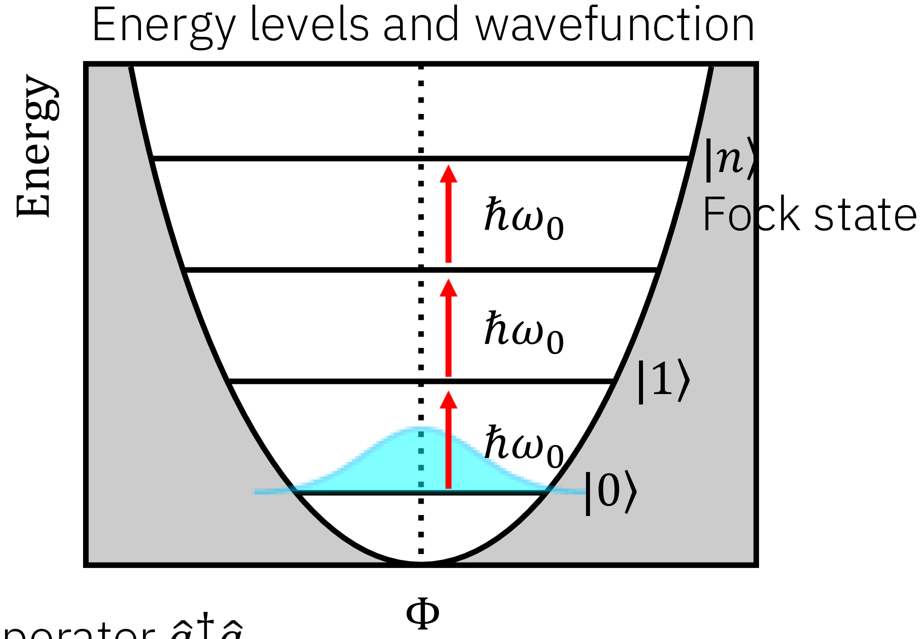
Hamiltonian (~Total energy) of the quantized LC circuit

$$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \frac{1}{2}\hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right) = \hbar\omega_0\left(\underline{\hat{a}^\dagger\hat{a}} + \frac{1}{2}\right)$$

$$\hat{a} = \sqrt{\frac{1}{2\hbar Z}}\left(\hat{\Phi} + iZ\hat{Q}\right),\ Z = \sqrt{L/C}$$

$\hat{N}$: photon number

Energy levels are equally spaced, so it cannot be used as a qubit!




Annihilation operator $\hat{a}$

$\hat{a}|0\rangle = 0$

$\hat{a}|1\rangle = |0\rangle$

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

Creation operator $\hat{a}^\dagger$

$\hat{a}^\dagger|0\rangle = |1\rangle$

$\hat{a}^\dagger|1\rangle = \sqrt{2}|2\rangle$

$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

$$\hat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & \ddots \end{pmatrix}$$

Photon number operator $\hat{a}^\dagger\hat{a}$

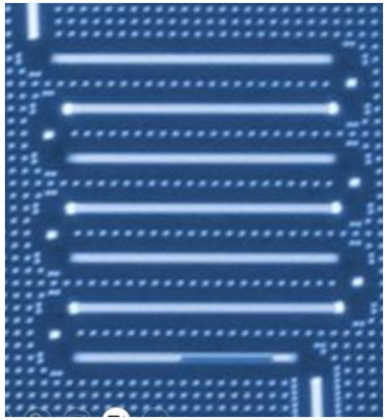
$\hat{a}^\dagger\hat{a}|0\rangle = 0$

$\hat{a}^\dagger\hat{a}|1\rangle = |1\rangle$

$\hat{a}^\dagger\hat{a}|n\rangle = n|n\rangle$

$$\hat{a}^\dagger\hat{a} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

Readout resonator is the linear LC



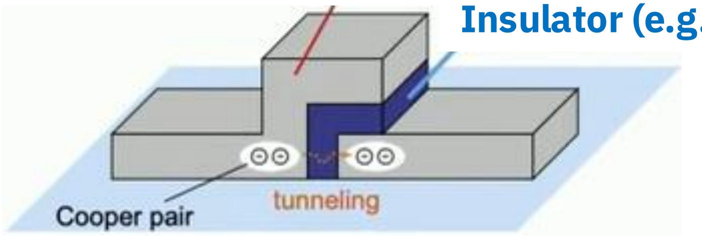
# The transmon qubit as a non-linear oscillator

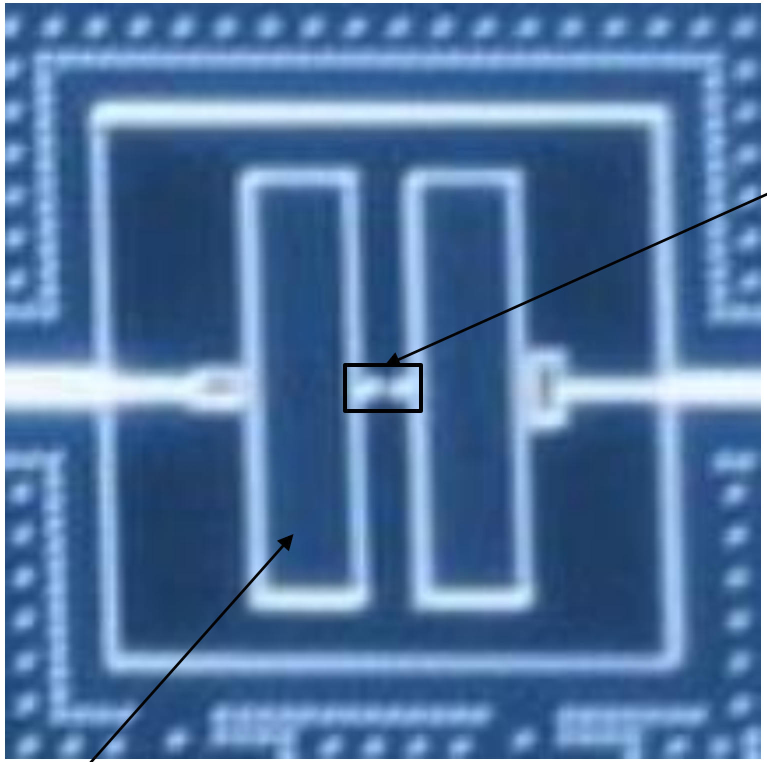
*Transmon: transmission-line shunted plasma oscillation qubit*
[https://arxiv.org/pdf/cond-mat/0703002](https://arxiv.org/pdf/cond-mat/0703002)

Image: S. Tamate

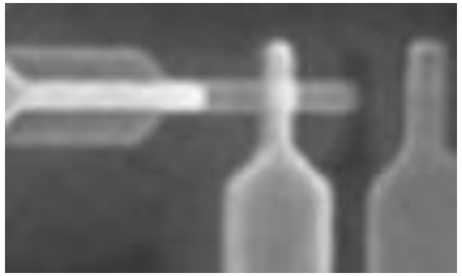
**Superconductor (e.g., Al)**

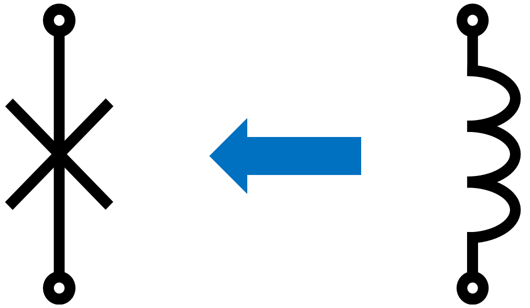
**Insulator (e.g., AlOx)**



**Nonlinear qubit inductor**
e.g., Josephson junction





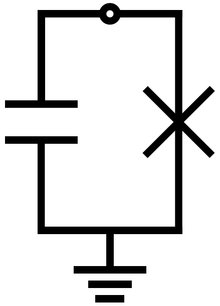
Josephson junction

Linear $L$

$\Phi = LI$

$U_{lin} = \frac{\Phi^2}{2L}$

Qubit



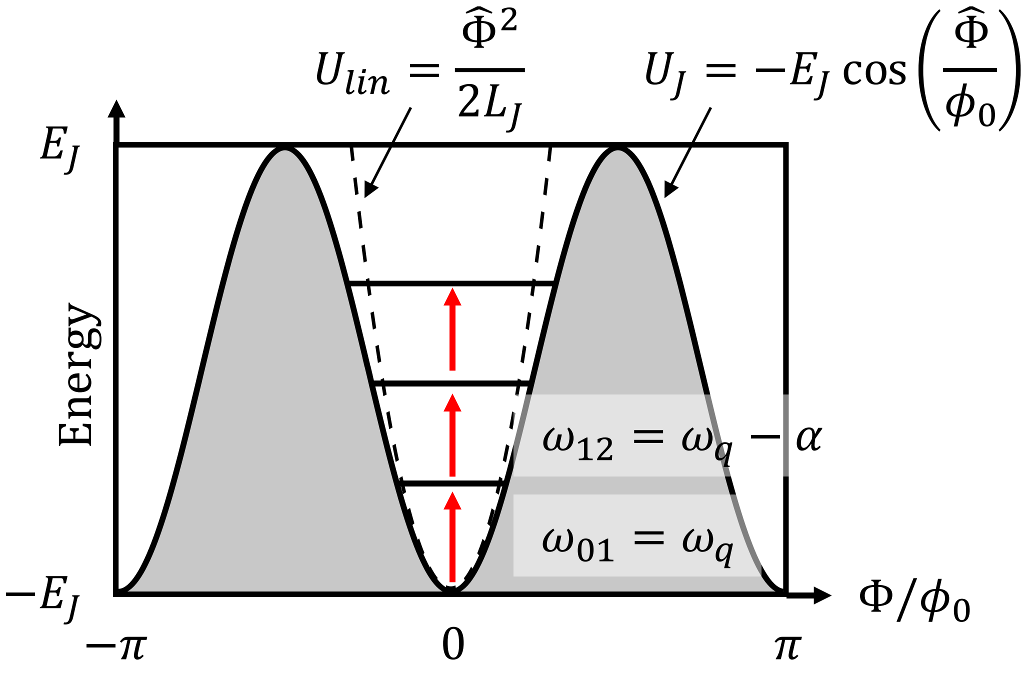
**Qubit Hamiltonian**

$$\hat{H} = \frac{\hat{Q}^2}{2C} - \underline{E_J \cos\left(\frac{\hat{\Phi}}{\phi_0}\right)}$$

Nonlinear potential

**Capacitor**
Superconducting material (Nb, TiN, Ta, etc.)

# Hamiltonian of the transmon qubit




Hamiltonian of the transmon qubit

$$\hat{H}^{RWA} = \hbar\omega_q \hat{a}^\dagger \hat{a} - \frac{\hbar\alpha}{2}\hat{a}^{\dagger 2}\hat{a}^2 = \underbrace{\hbar\omega_q \hat{N}}_{\text{Linear}} - \underbrace{\frac{\hbar\alpha}{2}\hat{N}(\hat{N}-1)}_{\text{Nonlinear}}$$

where $\alpha$ is anharmonicity, so the energy levels are anharmonic.

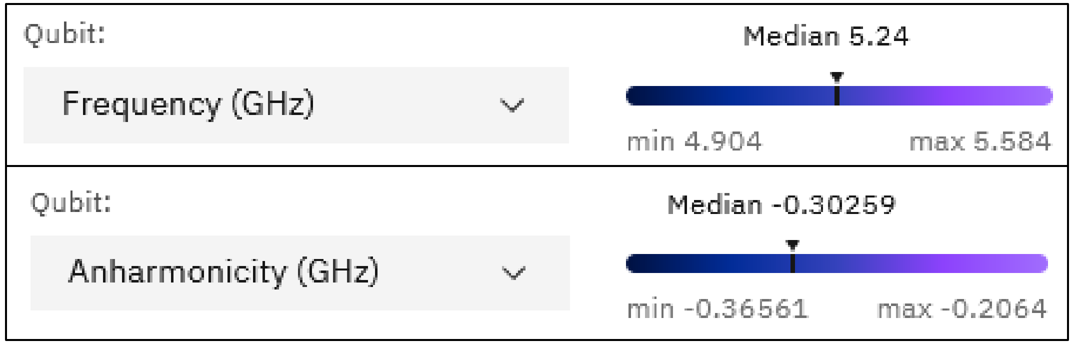
**The qubit acts as a two-level system due to the anharmonicity!**

Q. What is the frequency and the anharmonicity of the qubit of ibm_kawasaki?

Check out on the IBM quantum platform

https://quantum.ibm.com/

A. Qubit: Microwave frequency range




The qubit frequency corresponds to the following temperature

$$\omega_q \sim 5 \text{ GHz} \Leftrightarrow T_q \sim 0.25 \text{ K} \left(\hbar\omega_q = k_B T_q\right)$$

$\hbar\omega_q \gg k_B T$ must be satisfied for qubits not to be affected by thermal noise

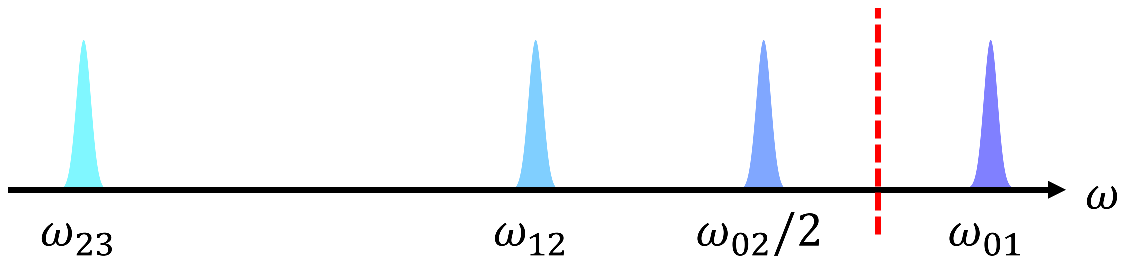
⇒ Necessity of 10mK order refrigerator

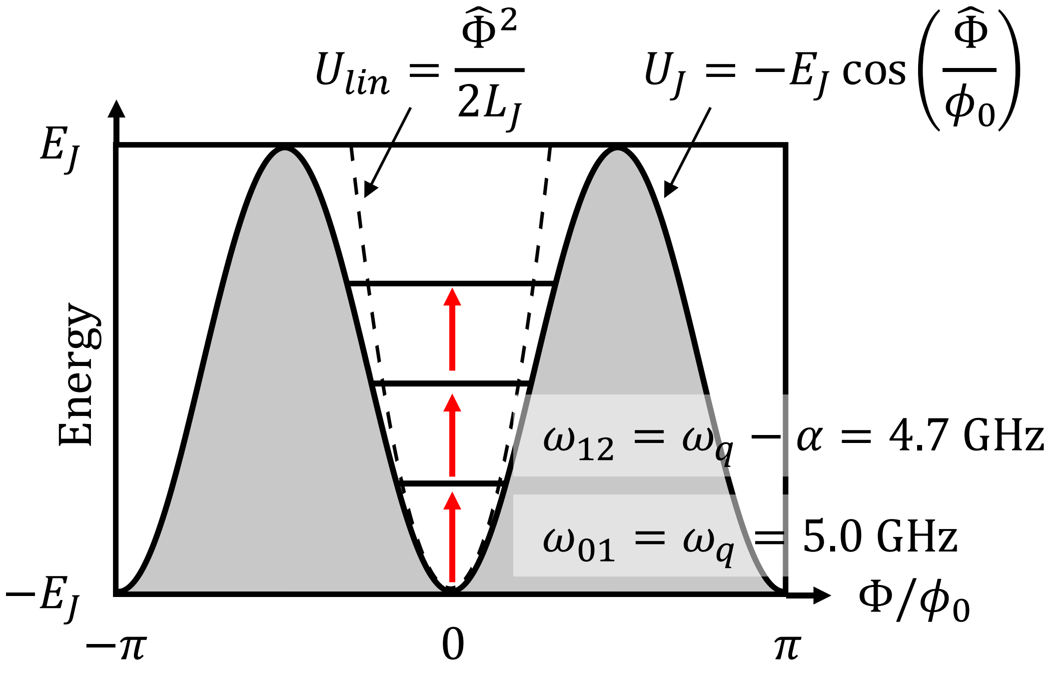
# Qubit transition spectrum

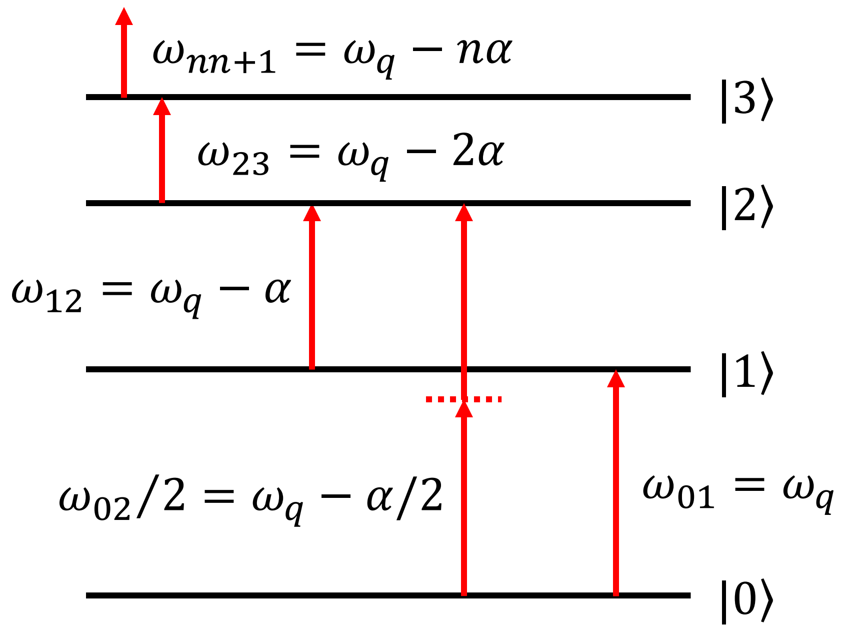
$$\hat{H}^{RWA} = \hbar\omega_q \hat{a}^\dagger \hat{a} - \frac{\hbar\alpha}{2}\hat{a}^{\dagger 2}\hat{a}^2 = \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}(\hat{N}-1)$$

$L_J = 14\ \text{nH}$ $\quad E_J = \frac{\phi_0^2}{L_J} = 12\ \text{GHz}$ $\quad \omega_0 = \sqrt{\frac{1}{LC}} = 2\pi \times 5.3\ \text{GHz}$

$C_J = 65\ \text{fF}$ $\quad E_C = \frac{e^2}{2C} = 0.3\ \text{GHz}\ (= \Delta_q = \alpha)$ $\quad \frac{Q_{zpf}}{2e} \sim 1$



# Restrict to qubit subspace

Fock number operator

$$\hat{a}^\dagger \hat{a} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

$$\hat{N} - \frac{1}{2}\hat{I} \mapsto -\frac{1}{2}\hat{Z}$$

Qubit Pauli Z operator

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

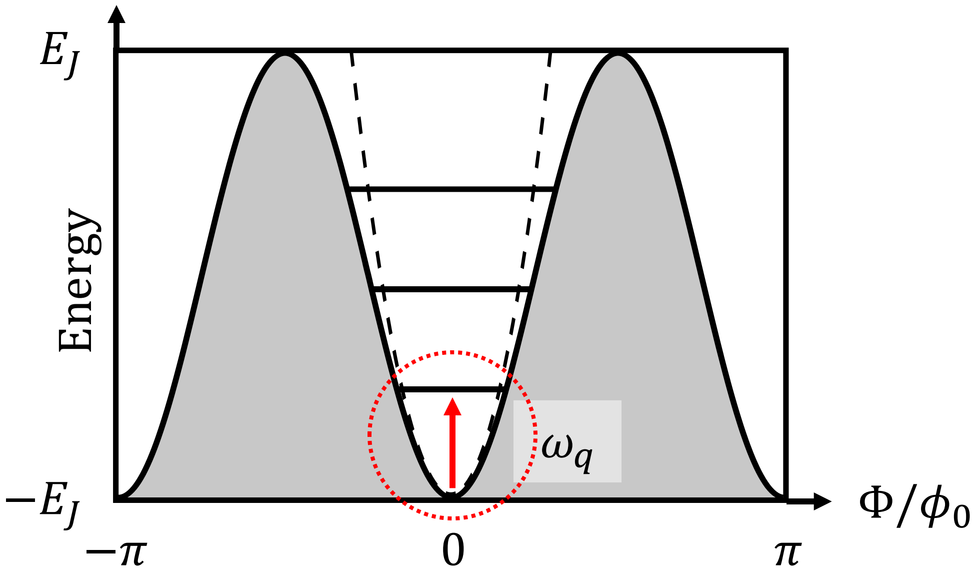
Annihilation operator

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

$$\hat{a} \mapsto \hat{\sigma} = \frac{1}{2}\left(\hat{X} + i\hat{Y}\right)$$

Qubit Pauli X and Y operators

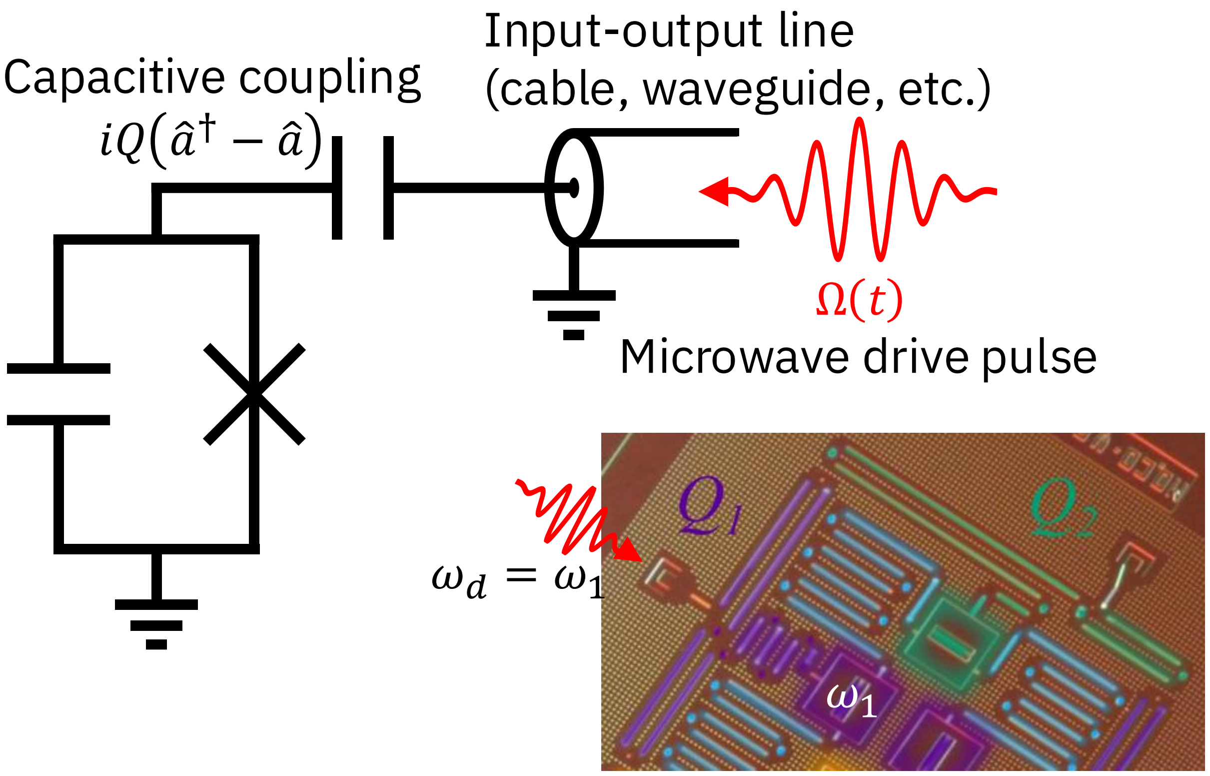
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$




Qubit Hamiltonian

$$\hat{H}_{\text{qubit}} = -\frac{1}{2}\hbar\omega_q \hat{Z}$$

# Qubit control




Qubit is controlled by microwave drive pulse

$$\hat{H}_{\text{drive}} = i\frac{\hbar}{2}\Omega(t)\left(\hat{\sigma}^\dagger - \hat{\sigma}\right) = \frac{\hbar}{2}\Omega(t)\hat{Y}$$

Drive pulse oscillates with $\omega_d$

$$\Omega(t) = \Omega_0 \sin(\omega_d t + \theta)$$

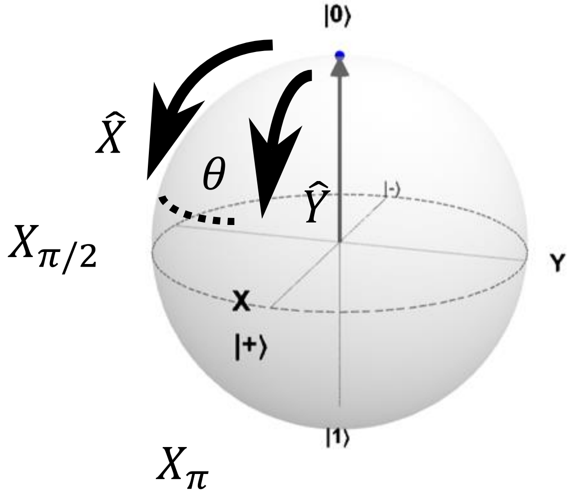
$$\hat{\sigma}(t) = \hat{\sigma}e^{-i\omega_d t}$$

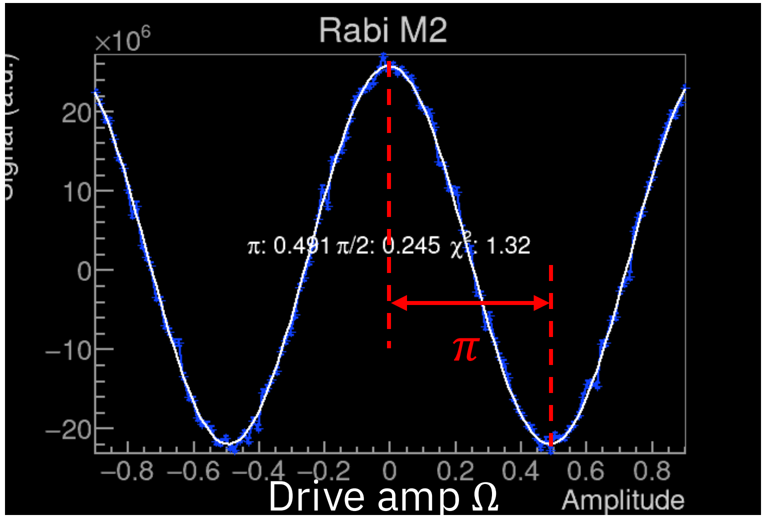
$$\hat{H}_{\text{drive}}^{RWA} = -\frac{\hbar}{4}\Omega_0\left(\hat{\sigma}^\dagger e^{-i\theta} + \hat{\sigma}e^{i\theta}\right)$$

Rabi rate $\Omega(t)$ and phase $\theta$ tune rotation speed and rotation axis, respectively

$$\hat{X} = \hat{\sigma}^\dagger + \hat{\sigma}$$

$$\hat{Y} = i\left(\hat{\sigma}^\dagger - \hat{\sigma}\right)$$

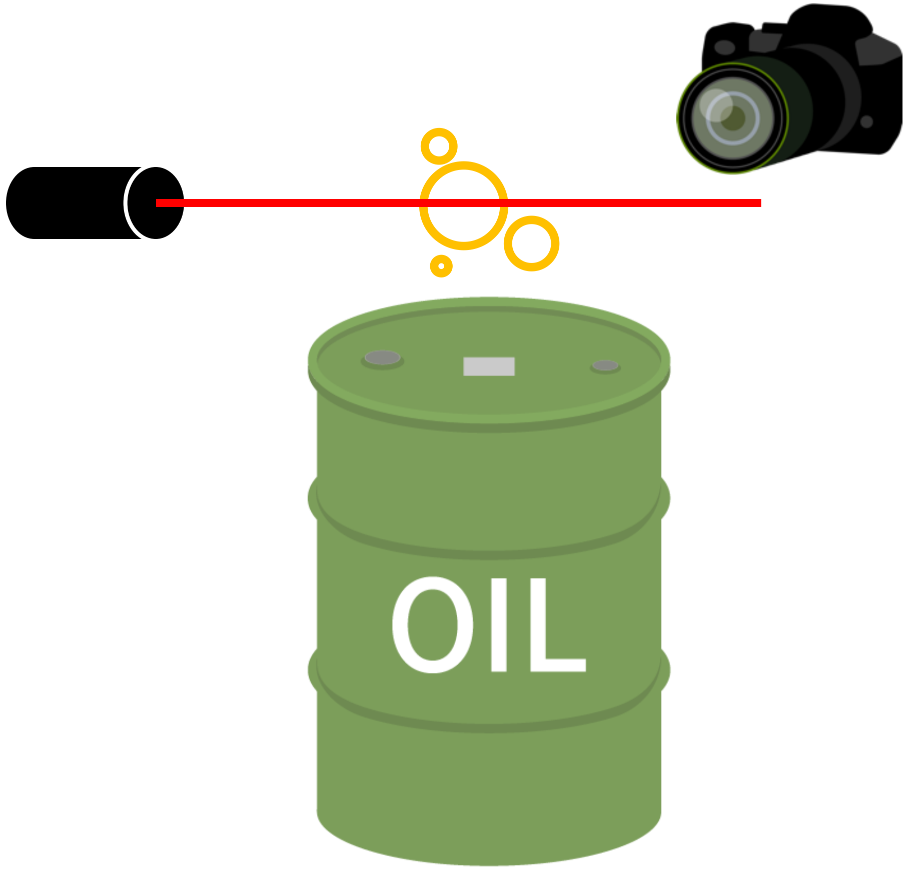


# Qubit measurement – two classes of measurements

Demolition
e.g., photon absorption


Non-demolition
e.g., dispersive measurement (QND)



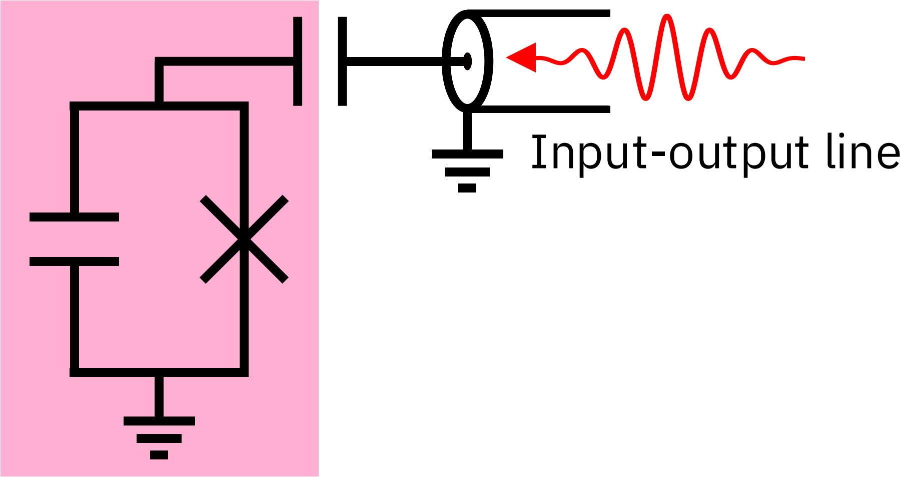



Ref. Zlatko Minev, Qiskit global summer school in 2020

# Circuit Quantum Electrodynamics dispersive measurement

## Direct measurement

- Demolition
- Qubit energy leak out
- Noise come in

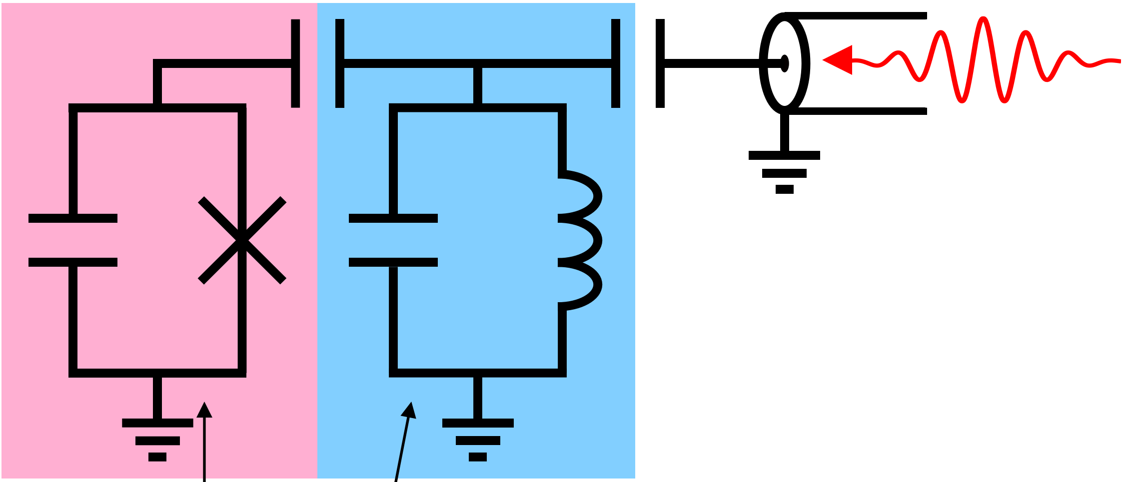



Qubit

## cQED dispersive measurement

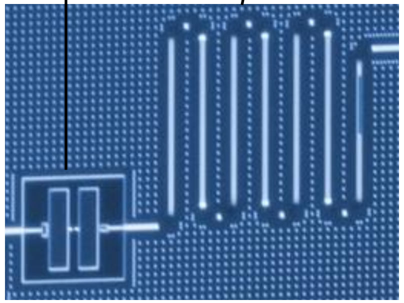
Isolate qubit through resonator

- Non-demolition: extracted from resonator
- Not leak out qubit energy
- Isolate noise

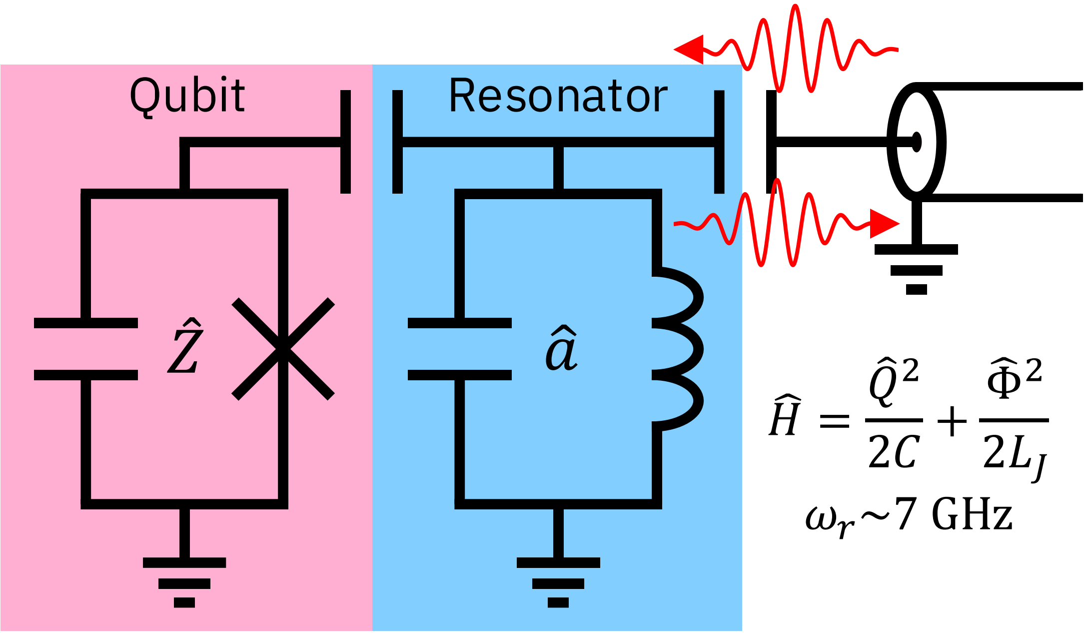


Qubit

Readout resonator: Linear LC circuit
(e.g., coplanar waveguide)



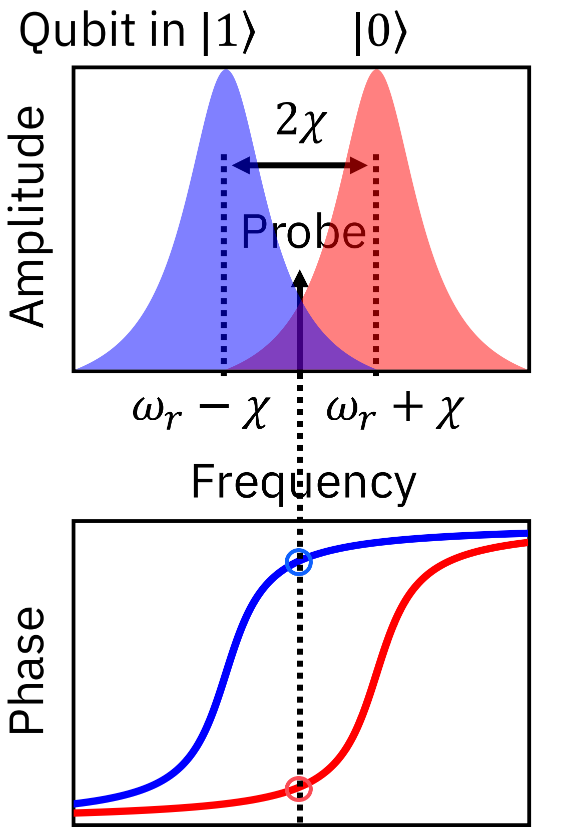
# Dispersive measurement




$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\left(\frac{\hat{\Phi}}{\phi_0}\right)$$
$\omega_q \sim 5\ \mathrm{GHz}$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L_J}$$
$\omega_r \sim 7\ \mathrm{GHz}$

Resonator response
on microwave irradiation

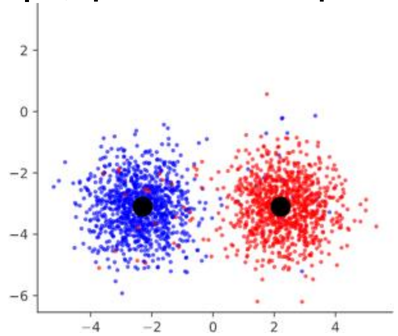



Hamiltonian including qubit – resonator interaction

$$\hat{H}_{\mathrm{eff}} = -\frac{1}{2}\hbar(\omega_q - \chi)\hat{Z} + \hbar\underline{(\omega_r + \chi\hat{Z})}\hat{a}^\dagger\hat{a}$$

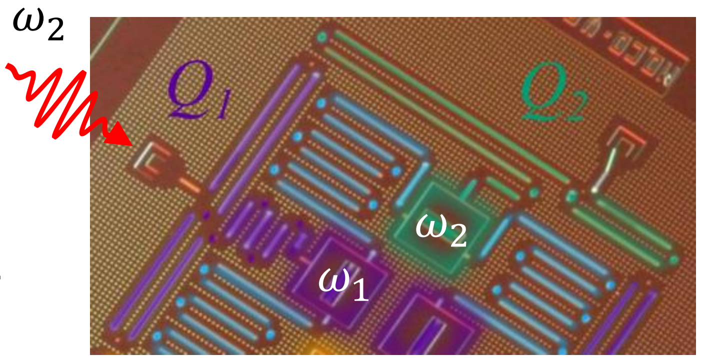
Resonator frequency is dispersive shifted depending on qubit state

$|0\rangle, |1\rangle$ plot on IQ plane




Ref. Qiskit Textbook 6.4 Circuit Quantum Electrodynamics

# Two-qubit gate – Cross resonance gate




- Cross resonance gate: basis of CNOT

Driving control qubit 1 with microwave at resonance frequency $\omega_2$ of target qubit 2

$$\tilde{H}_{\rm eff}^{\rm CR} = -\frac{\tilde{\omega}_1 - \tilde{\omega}_2}{2}\hat{Z}_1 + \frac{\Omega(t)}{2}\left(\hat{X}_2 - \frac{J}{2\Delta_{12}}\hat{Z}_1\hat{X}_2\right)$$

The entanglement is generated by the ZX interaction.

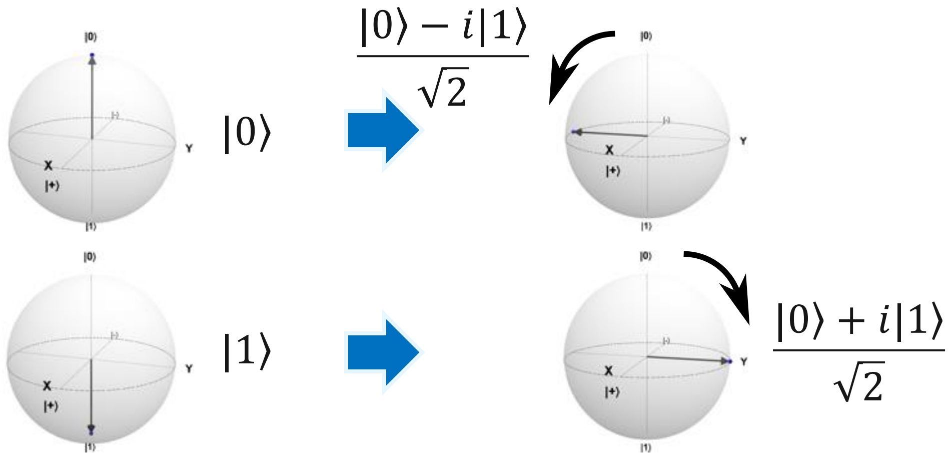
The max entanglement is reached, when qubit 1 is a 0/1 superposition, and CR gate of $\pi/2$ rotation is applied.

$$ZX_{\pi/2} = \exp\{-i(\pi/4)ZX\} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & i & 1 \end{pmatrix}$$

The direction of x-rotation is opposite depending on the state of control qubit

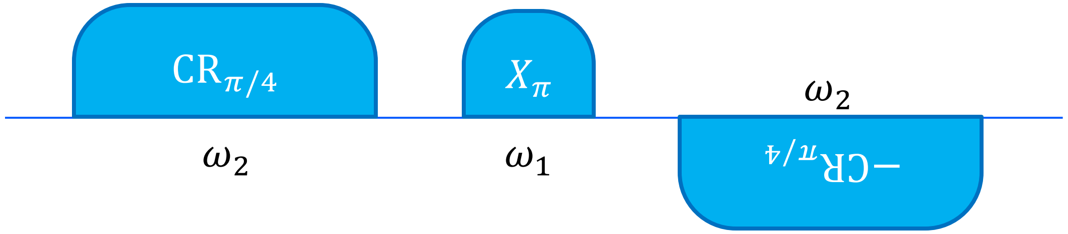
Control qubit 1

Target qubit 2




https://javafxpert.github.io/grok-bloch/

Echoed Cross Resonance (ECR) gate for removing dephasing



# Continued in Part 2