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Chapter 4 write up

This write up will cover the 3 sections that we didn't not cover as a class. Normal beta, and gamma distribution and all different, but challenging in their own way. Prior to this chapter I was not the best at integrals, but after completing this chapter I am starting to feel more confident and comfortable with my integration work.

The first section I am going to cover is the normal distribution, this distribution is the most widely used probability distribution. According to the textbook a random variable is said to have a normal probability distribution if $\sigma > 0$ and $-\infty < \mu < \infty$. This means that our mean must be a finite number that is within our bounds. This also means that our standard deviation must be greater than zero, or in other words non-negative. If our mean is in our bounds and our standard deviation is positive, then we can use our formula for normal distribution which is $f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y-\mu}{2\sigma^2}}$. This formula looks complex, and it is, but once we get our variance and expected then it's a simple plug and chug. In the textbook we are given that our expected (E(y)) is equal to our mean (μ) and our variance V(Y) is equal to our (μ^2) . On page 179 of the textbook, we are given a proof of this formula, but the important thing that can be derived from this long proof is that σ implies the spread of a graph and μ locates the center of a graph. Although I'm not going to break down the proof it is important to understand what our mean and standard deviation represents.

On page 180 we are given an example of normal distribution, this problem first starts off with defining our mean and standard deviation to be 0 and 1 respectively. It then wants us to find three things P (z > 2), P$(-2 \le Z \le 2)$, and P $(0 \le Z \le 1.73)$. We first start with P(z>2), since our mean is finite, and our standard deviation is above 0 we can just plug in and solve $f(2) = \frac{1}{1\sqrt{2\pi}} e^{-\frac{2-0}{2(1)^2}}$, which gives us 0.0228. The next part we must solve for is part b which is P$(-2 \le Z \le 2)$. Since our mean is 0 and mean is symmetrical, we can assume that we can solve for p(z) by doing 1 – 2(0.0228). This is because our probability total is always going to be 1 and we already solved for p (2). Once we solve it, we get .9544. The finale part wants us to find the probability for P $(0 \le Z \le 1.73)$. first, we can solve for A (0), which gives us .5, we then can solve for p (1.73) using our formula, which will then give us 0.0418. Next, we can just subtract .5 from 0.0418 and get our total probability which is .4582

The next topic that I am going to cover is the gamma distribution. This distribution mainly covers random variables that are always non-negative and distributions that are skewed to the right. According to the textbook, a random variable has a gamma distribution if α>0 and β>0. This means that our function has an original value of a but an altered value which is b. If these parameters are true, we can then use our Gamma distribution formula which is $f(y) = \int \frac{y^{\alpha-1} e^{\frac{-y}{\beta}}}{\beta^a \Gamma(a)}$ were $\int_0^\infty y^{a-1}\, e^{-y}$dy. This formula is very complex and is a steep climb up from normal distribution. I struggled very much with this chapter but tried my hardest to try and understand the proofs presented in this section. The function Γ(a) is know as the gamma function, a is generally known as the shape parameter, while β is known as the scale parameter. There are several proofs in this section that try and help break down the concept of the gamma distribution. The first proof of this section states if Y has a gamma distribution with parameters a and b then μ=E(y)=ab and σ^2=V(Y)=$a\beta^2$. This is proven in the textbook but essentially our expect is just aβ and our variance is just aβ^2.

An example of the gamma distribution is given on page 189 this example wants us to show that if a >0 and b> 0 what is our P (Y > a + b|Y > a) = P (Y > b). To start this problem, we first must apply condonation probability which gives us $\frac{P(Y>a+b)}{P(Y>a)}$. We can then apply our integral which gives us $\int_{a+b}^{\infty} \frac{1}{b} e^{-\frac{y}{b}}$ which gives us dy=$-e^{\left(-\frac{y}{\mathrm{b}}\right)}$. We can then plug in our limits and solve our integral, so $e^{-\frac{(a+b)}{\beta}}$. Next, we need to solve for the other side of our probability statement P(Y>a). This would look like this $\int_{a}^{\infty} \frac{1}{\beta} e^{-\frac{y}{\beta}} dy$ and once we take the integral it looks like this $e^{-\frac{a}{\beta}}$. Now we refer to our above formula and plug in our new answers, so $\frac{e^{-\frac{(a+b)}{\beta}}}{e^{-\frac{a}{\beta}}}$, which equals $e^{-\frac{b}{\beta}}$ this means that our probability of y is greater than our probability of b, so P(Y>b).

The last section that I am going to cover is the beta probability distribution. This distribution is defined over the closed interval $0 \leq y \leq 1$, and is often used as a model for proportions. In the book, it says that a random variable Y is said to have a beta probability distribution if $\alpha > 0 \ and \ \beta > 0$. It also must have a density function that is f(y)=$\frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}$, where P$(\alpha, \beta\ )$ = $\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\, dy$. This is another complex distribution, but this formula can be put simply as the distribution you use when you have lots of complex graphs that all have different shapes. The beta random variable is commonly called the incomplete beta function; this function is shown in the textbook as $F(y) = \int_0^y \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)} dt = 1_y(\alpha, \beta)$. This textbook shows several proofs for this distribution, but the main thing that it proves is that if $\alpha > 0 \ and \ \beta > 0$, then the expected is $\frac{\alpha}{\alpha+\beta}$ and the variance is $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

An example of this distribution is shown on page 196, this question gives us our $\alpha = 4$ and or $\beta = 2$ as well as our probability is 90% or .9. Although the above formulas were complex and difficult this problem is actually fairly easy. We first can plug in our values into the starting formula giving us $\frac{\Gamma(4+2)}{\Gamma(4)\Gamma(2)} y^3(1-y)$. Once we solve this, we can then plug it into our integral and solve this would look like this P(Y>.9) =$\int_{.9}^{\infty} f(y)dy = \int_{.9}^{1} 20(y^3 - y^4)dy$ = 20(.004) =0.08. This gives us the likely hood that 90% of livestock is sold, which according to the beta distribution is not very likely