Statistical Inference Project Assignment 1 - Question 1

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is \(1/\lambda \) and the standard deviation is also also \(1/\lambda \). According to the Central Limit Theorem, The mean or the sum of a random sample of a large enough size from an arbitary distribution have **approximately** normal distribution.

The sample mean \(\bar X \) is approximately normal \(N(\mu, \sigma^2/n) \)

We generated a thousand simulation of the averages of 40 exponentials where \(\lambda \) = **0.2** and the **mean** of the exponentials is equal to \(1/\lambda \) which is **5**.

```
set.seed(10)
lambda = 0.2
nosim <- 1000 #number of simulations
n <- 40 #number of exponential variables
mean_expo <- vector(mode="numeric", length=nosim)
sd_expo <- vector(mode = "numeric", length = nosim)

for(i in 1:nosim){
    sample <- rexp(n,lambda)
    mean_expo[i] <- mean(sample)
    sd_expo[i] <- sd(sample)
}
data <- data.frame(1:nosim, mean_expo, sd_expo)
```

1. We show where the distribution is centered at and compare it to the theoretical center of the distribution.

From the properties of the normal distribution which is a symmetrical distribution, it means that median is equal to mean.

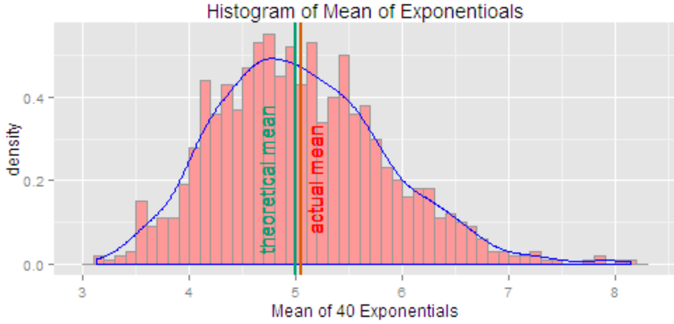
```
mean_data<- mean(data$mean_expo) #calculate mean of the samples
median_data <- median(data$mean_expo) #calculate median
print(paste("The mean of this sampling is", round(mean_data,3),",the median of the sampling is",
round(median_data,3), "and the mean of the exponentials is ", 1/lambda ,sep = " "))
```

```
## [1] "The mean of this sampling is 5.045 ,the median of the sampling is 4.987 and the mean of
the exponentials is  5"
```

This shows that the mean and median of the distribution of the sampling are almost equal and are close to the mean of the exponential distribution \((1/\lambda) \) which is 5. So we can say that this distribution is **centered**.

We also shows the distribution in the following graph:

```
library(ggplot2)
ggplot(data, aes(x=mean_expo))+
    geom_histogram(binwidth = .1,aes(y = ..density..),colour="#999999", fill="#FF9999")+
    geom_density(color = "blue") +xlab("Mean of 40 Exponentials") +
    ggtitle("Histogram of Mean of Exponentioals") +
    geom_vline(xintercept=mean_data,color="#D55E00", size = 1.0) +
    geom_text(aes(x=mean_data, label="\nactual mean", y=0.2), vjust = 0.5, colour="red",
angle=90)+
    geom_vline(xintercept = 1/lambda, color = "#009E73", size =1.0)+
    geom_text(aes(x=1/lambda, label="\ntheoretical mean", y=0.2), vjust = -0.5,colour="#009E73",
angle=90)
```



This graph shows that actual mean of the sampling data (red line) is close to the theoretical mean (green line).

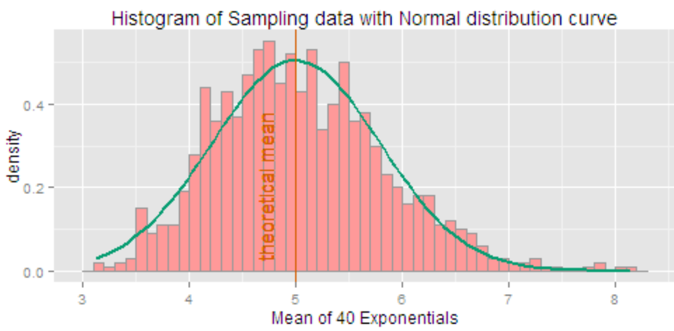
2. We show how variable it is and compare it to the theoretical variance \(1/(n\lambda^2) \) of the distribution.

```
Sample_SD <- mean(data$sd_expo) #get the sample standard deviation
variance_data <- (Sample_SD^2/n) #variance of the samples
theoretical_variance <- 1/(lambda^2*n) #theoretical variance
print(paste("Variance of the sampling is",round(variance_data,3), "and theoretical variance
is",theoretical_variance, ",which are really close.", sep = " "))
```

```
## [1] "Variance of the sampling is 0.61 and theoretical variance is 0.625 ,which are really
close."
```

3. We show that the distribution is approximately normal or not.

```
ggplot(data,aes(x = mean_expo)) +
    geom_histogram(binwidth = 0.1,aes(y = ..density..), fill = "#FF9999", colour ="#999999") +
    xlab("Mean of 40 Exponentials")+
    stat_function(fun=dnorm,colour = "#099E73",size = 1.0,arg = list(mean = 1/lambda,sd = 1/
(lambda*sqrt(n))))+
    geom_vline(xintercept = 1/lambda, color = "#D55E00", size =0.5)+
    geom_text(aes(x=1/lambda, label="\ntheoretical mean", y=0.2), vjust = -0.5,colour="#D55E00",
angle=90)+
    ggtitle("Histogram of Sampling data with Normal distribution curve")
```



From this graph, it shows the sampling data with the Normal distribution curve N(\(1/\lambda \), \(1/(n\lambda^2) \)) (green line) and we can say that the sample data is quite close to be approximated as normal distribution.

4. We evaluate the %95 coverage of the confidence interal \(\bar X \pm 1.96 S /\sqrt{n} \) where \(S \) is the sample standard deviation.

```
coverage95 <- round(mean_data +c(-1,1)*1.96 * (Sample_SD)*sqrt(1/n),4)
print(paste("The 95% coverage of the confidence interval are", coverage95[1], "and", coverage95
[2], sep = " "))
```

```
## [1] "The 95% coverage of the confidence interval are 3.514 and 6.5761"
```

We check if this values are true for this example by showing that 95% of the data are in between the covarage value.

```
norm_coverage <- subset(data, mean_expo > coverage95[1] & mean_expo < coverage95[2])
check_coverage <-nrow(norm_coverage)/nosim*100 #number should be around 0.95
print(paste("In this case, it actually covers ", check_coverage, "% of the data", sep = ""))
```

```
## [1] "In this case, it actually covers 95.3% of the data"
```