

Pattern recognition of fruit shape based on the concept of chaos and neural networks

T. Morimoto *, T. Takeuchi, H. Miyata, Y. Hashimoto

Department of Biomechanical Systems, Ehime University, Tarumi 3-5-7, Matsuyama 790-8566, Japan

Abstract

Shape evaluation of fruit is quite empirical and uncertain. In this study, a new technique is proposed to evaluate the fruit shape quantitatively using attractor, fractal dimension and neural networks. The shape of a fruit is usually described from its own profile information. A one-dimensional profile data consisting of radii between the centroid of the fruit and sampling points on the fruit profile was made to characterize the fruit shape. It includes six profiles of the fruit in different directions. The irregularities of one-dimensional profile data in several types of fruits were quantitatively measured by introducing the concepts of attractor and fractal dimension. A three-layer neural network was also used for identifying and tracing the one-dimensional profile data and then evaluating their irregularities. The relationships among identification errors, the shapes of attractors and fractal dimensions for several types of fruits were investigated. Significant correlations were observed in their relationships. The results showed that the uses of the attractor, the fractal dimension and the neural network allowed the complexity of the fruit shape to be quantitatively evaluated. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Quantitative evaluation for shape; Complex fruit shape; Attractor; Fractal dimension; Neural networks

1. Introduction

The shape of fruit is one of the most important factors for classifying and grading fruit. Until recently, various types of evaluation techniques for the fruit shape have been studied (Ding and Gunasekaran, 1994; Tao et al., 1995). However,

* Corresponding author. Tel.: +81-89-9469823; fax: +81-89-9469916.

E-mail address: morimoto@agr.ehime-u.ac.jp (T. Morimoto)

the potentials for quantitative evaluation of the fruit shape have not been fully exploited because the fruit shape is characterized by complexity and uncertainty.

In general, the shape of the fruit is basically determined by a human grader who makes an evaluation based on his own intuition and empirical knowledge. Hence, most of the evaluation results will become uncertain. Although neural networks have been widely used for evaluating the shape of fruits in recent years (Liao et al., 1993; Yang, 1993; Hatou et al., 1996; Nakano, 1997), their reference data (teaching signals) used for their training are also determined based on the intuition and empirical knowledge of human graders. From these findings, it seems that a more qualitative technique is necessary to evaluate the fruit shape precisely.

Here, the concept of chaos such as attractor and fractal dimension is introduced to quantitatively measure and evaluate the irregularity (or regularity) of the fruit shape. Chaos is a term used to describe randomness generated by simple rules (Farmer et al., 1983; Crutchfield et al., 1986; Hansell et al., 1997). Although chaos is irregular, it is deterministic. So, if the fruit shape can be judged to be chaos, its modeling and pattern recognition are possible with simple mathematical equations (for linear system) or neural networks (for nonlinear system). If it is judged to be random, in contrast, its modeling and pattern recognition theoretically become hard due to its randomness. It is known that fractal dimension is a measure for judging chaos, and it is a form index that is capable of characterizing the irregularity of the shape of an object (Orford and Whalley, 1983; Keller et al., 1987; Peitgen et al., 1992). From these findings, it seems to be very important to measure the irregularity of the fruit shape before performing the modeling and evaluation of the fruit. In recent years, computer-vision techniques based on fractal dimension have been developed for classifying different shapes of plants and fruits (Panigrahi et al., 1998; Valdez-Cepeda and Olivares-Sáenz, 1998).

It is known that neural networks are effective tools for identifying complex nonlinear systems (Isermann et al., 1997; Morimoto et al., 1997a,b). They perform arbitrary nonlinear mappings in patterns of information. In this study, as a new attempt, they are used for identifying the profiles of a fruit and then evaluating the fruit shape.

The aim of this study is to quantitatively evaluate the irregularity (or regularity) of the fruit shape using the concepts of attractor and fractal dimension and neural networks, aiming at more reliable and more sophisticated automated classification.

2. Methodology

2.1. Material

Tomato fruits (*Lycopersicon esculentum* Mill. cv. Momotaro) were used for experiment. Several shapes of tomatoes were used for their shape evaluation. Digital images of tomato in different directions were acquired with a CCD camera (Canon UCVIHi) and then only their profiles are extracted using an image-processing system. Each coordinate had a resolution of 320 (columns) \times 240 (row) with a maximum capacity of 256 levels (eight bits) of information.

2.2. One-dimensional profile data

In order to identify the fruit shape by neural networks and introduce the concept of chaos and fractal dimension, several profiles of the fruit, which represents its own shape, are converted to a one-dimensional profile data like a time-series. Fig. 1 shows the procedure of obtaining the one-dimensional profile data. First, several (six) digital images of a fruit in different direction (j) are obtained by turning the fruit at the angle of 15° . Second, only the profile of each image is extracted. Third, the centroid of each image is calculated from the average values of x - and y -coordinates of all boundary points on the profile, and then radius, $x_j(i)$, between the boundary point i and the centroid of the fruit image is calculated along the profile, as shown in Fig. 1(a) ($i = 0, 1, \dots, N-1$, N ; data number, j ; direction, $j = 1, \dots, M$). Finally, the boundary radii in different directions are connected each other and expressed as a one-dimensional data like a time series, $X(k) = \{x_j(i)\} = \{x_1(0), \dots, x_1(N), x_2(0), \dots, x_2(N), \dots, x_M(0), \dots, x_M(N)\} = \{X(1), X(2), \dots, X(TN)\}$, as shown in Fig. 1 (b) (TN ; Total data number, $k = 1, \dots, TN$). This one-dimensional profile data are used for the shape evaluation and modelling.

2.3. Concept of chaos

Chaos is a term that means randomness generated by simple mathematical equations. It is noted that chaos is deterministic though it is irregular. If the one-dimensional data can be judged to be chaos, its modeling and qualitative

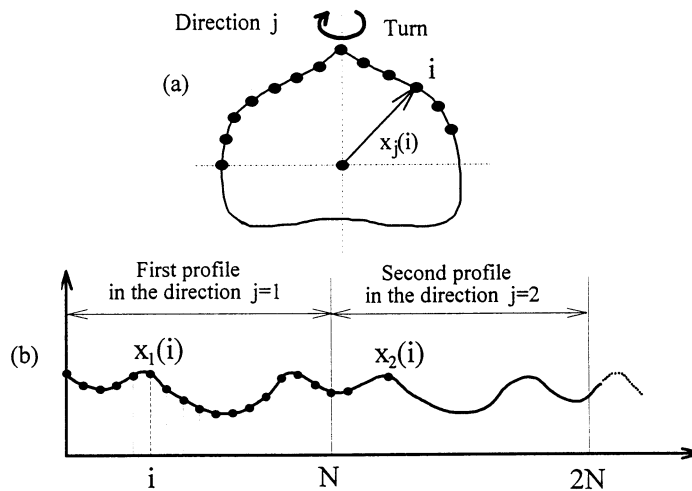


Fig. 1. Procedure of obtaining a one-dimensional profile data consisting of radii in different directions. (a) is fruit boundary tracing. The $x_j(i)$ is the radius between the boundary point i and the centroid of the fruit in a certain direction j . (b) is a one-dimensional profile data consisting of radii between the centroid and the boundary points of the fruit in several profiles in different directions ($j = 1, \dots, M$).

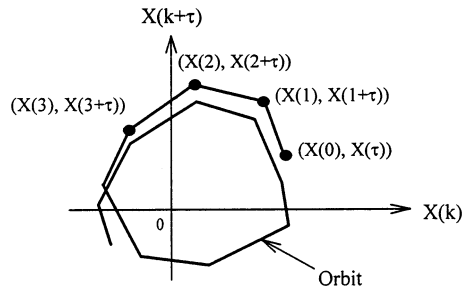


Fig. 2. An attractor in the case of two-dimensional phase plane of $X(k)$ and $X(k + \tau)$.

evaluation becomes possible using deterministic and comparatively simple rules. In this study, topological and geometrical ways, which are known as attractor and fractal dimension, are introduced to quantitatively judge the chaos nature and to measure the complexity and irregularity of the fruit shape (Farmer et al., 1983; Crutchfield et al., 1986). Quantitative features of the fruit shape are well captured by these tools (Orford and Whalley, 1983; Keller et al., 1987; Peitgen et al., 1992).

2.3.1. Attractor

Attractor is a geometric and topological form that characterizes the complexity of the morphological variation of an object (one-dimensional data) in the phase space (Crutchfield et al., 1986). The chaotic property (complexity) of the data can be visualized by drawing the attractor. The attractor is what the behavior of a system settles down to, or is attracted to. Many shapes of attractors can be described according to the irregularities of the shapes of objects.

Let $\{X(k)\} = \{x_j(i)\} = X(0), X(1), \dots, X(TN)$ be the one-dimensional profile data of the boundary radius consisting of several profiles of the fruit (TN: total data number). Attractor is a geometric form plotted and drawn in the k -dimensional phase space of $(X(k), X(k + \tau), X(k + 2\tau), \dots, X(k + [(N - k)/\tau]\tau))$ ($k = 0, 1, \dots, TN, \tau$; time lag). Fig. 2 shows an attractor in the case of two-dimensional phase space: $(X(k), X(k + \tau))$. An attractor is usually described by a combination of unstable periodic orbits. It can be seen that the shape of the attractor reflects the morphological feature of the object.

2.3.2. Fractal dimension

Mandelbrot (1977) introduced the concept of fractal to describe complicated shapes that don't have integer dimension but a fractal one. Fractal patterns are characterized by their self-similarities at all scales. They have partial correlation over long series. Here, fractal dimension D is used as a form index to understand the irregularity, complexity and periodicity of the jagged curve representing the shape of an object (Keller et al., 1987; Peitgen et al., 1992). How does the fractal dimension measure the irregularity of the data (fruit shape)?

Suppose that $L(\tau)$ is the total length of the curve of the one-dimensional profile data, and τ is a measured length of the curve. It is found that the value of $L(\tau)$,

which depends on τ , becomes large with increasing the irregularity of the curve, as shown in Fig. 3. When the curve has statistically self-similarity, the following relationship can be obtained.

$$L(\tau) \propto \tau^{-D} \quad (1)$$

From Higuchi's method (Higuchi, 1988), the fractal dimension, D , can be described as follows:

$$D = -\log L(\tau)/\log \tau \quad (2)$$

where

$$L(\tau) = 1/\tau \sum_{m=1}^k L_m(\tau) \quad (3)$$

$$L_m(\tau) = \sum_{i=1}^{Mn} \{|X(m+i\tau) - X(m+(i-1)\tau)| \cdot \text{Norm}/t\}$$

$$\text{Norm} = (TN - 1)/[(TN - m)/\tau]\tau$$

$$Mn = [(TN - m)/\tau]$$

$[]$: Gauss' notation

where Norm represents the normalization factor for the curve length.

The fractal dimension is given by a slope, $-D$, of the best fitting linear regression line plotted in the doubly logarithmic scales of $L(\tau)$ and τ . It is known that if the data has fractal geometry indicating self-similarity, then the relationship between $\log(L(\tau))$ and $\log(\tau)$ is expressed by a linear line in their logarithm space (Orford and Whalley, 1983; Panigrahi et al., 1998). A linear line can be obtained using the method of least-squares.

When the data is expressed in a two-dimensional plane, the value of fractal dimension should be at least 1.0 but must not exceed 2.0. The value of fractal dimension increases with the complexity of the data (Peitgen et al., 1992). When the D value becomes larger (near 2.0), the fruit shape is judged to be more complex and irregular. Thus, the fractal dimension is utilized as a means by which irregular morphological profiles can be quantified. In the paper, we evaluated the fruit shape in the manner that more irregular shaped fruits are worse ones.

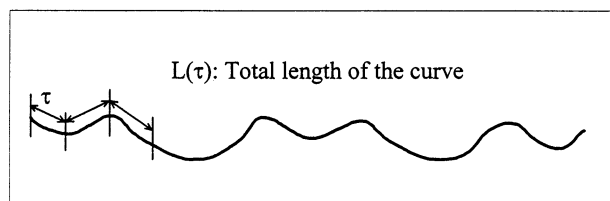


Fig. 3. Relationship between the basic length τ and the total length $L(\tau)$ of the curve.

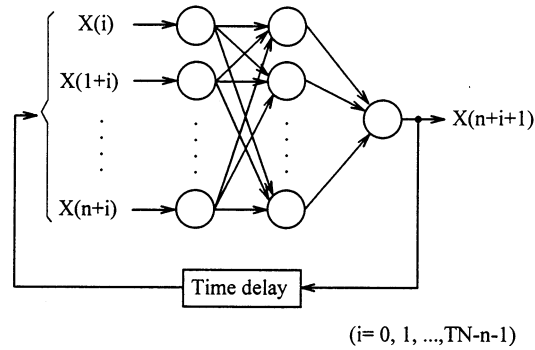


Fig. 4. A three-layer neural network used for identifying the profiles of the fruit (fruit shape).

2.4. Neural networks

Neural networks are used for identifying and tracing the one-dimensional profile data. The fruit shape is also evaluated based on the identification error through estimation. Here, a one-dimensional profile data is defined as a time series and a dynamic behavior. Usually, neural networks perform arbitrary nonlinear mappings in patterns of information. For identification of a dynamic system, therefore, arbitrary feedback loops that produce time histories of the data are necessary in the network (Isermann et al., 1997). Fig. 4 shows a three-layer neural network used for identifying the one-dimensional profile data. Cybenko (1989) showed that a 3-layer neural network with one hidden layer allowed any continuous function to be successfully identified. From our experiments, the same results were also obtained.

Now, we have the one dimensional profile data, representing M sorts of profiles, $X(k) = \{X(0), X(1), \dots, X(n), X(n+1), \dots, X(TN)\}$ (n , system parameter number, $n < TN$). In the method, next points, $X(n+i+1)$, on the profile are in turn estimated from the data set of the previous points, $(X(i), \dots, X(n+i))$, as follows ($i = 0, 1, \dots, TN - n + 1$):

$$X(n+1) = f(X(0), X(1), \dots, X(n))$$

$$X(n+2) = f(X(1), X(2), \dots, X(n+1))$$

.....

$$X(TN) = f(X(TN - n - 1), X(TN - n), \dots, X(TN - 1))$$

where $f(\cdot)$ is a nonlinear function given by a three-layer neural network shown in Fig. 4. It is noted that the reason we use historical input and output data is to describe the dynamic characteristics of the system.

The most important task for determining the model structure is to choose the system parameter number. Here, the system parameter number, n , and the hidden neuron number of the neural network, were determined based on the Akaike's information criterion (AIC). AIC can be usually defined as follows (Akaike, 1974):

$$\text{AIC} = -2 \log (\text{maximum likelihood}) + 2n \quad (4)$$

Assuming that the prediction error is given by Gaussian noise, AIC can be expressed in the simple form (Nakamizo, 1992; Ljung and Glad, 1994):

$$\text{AIC} = \log [(1 + 2n/\text{TN})V] \quad (5)$$

$$V = \sum \varepsilon^2(k), \quad \varepsilon^2(k) = (X(k) - \hat{X}(k))^2$$

where $\hat{X}(k)$ is a prediction of $X(k)$, and TN is total data number.

The learning algorithm of the neural network is error backpropagation. It is to modify the values of the weights and the biases of the neural network with the generalized delta rule until the network output yields the desired output (Rumelhart et al., 1986; Vogl et al., 1988, etc.).

3. Preliminary experiments

3.1. Ideal elliptic bodies

Before applying our technique to real systems, preliminary experiments were carried out using ideal elliptic bodies. Fig. 5 shows three kinds of ideal elliptic bodies (A, B and C) which look like rugby balls and are characterized by different ratios of H/L (H : height, L : length). The value of H was kept constant. They have elliptic shapes only in their side views and one round shape in their front views. By turning the object one-fourth revolution (90°), therefore, we can obtain shapes from ellipse to round. The values of H/L were 0.856 for A, 0.695 for B and 0.629 for C, respectively. In this study, it was assumed that round fruits with smooth surface are better ones. That is, the closer the value of H/L gets to 1 (round), the better the shape of the fruit becomes. Among them, therefore, the body A is judged to be the best and the body C the worst.

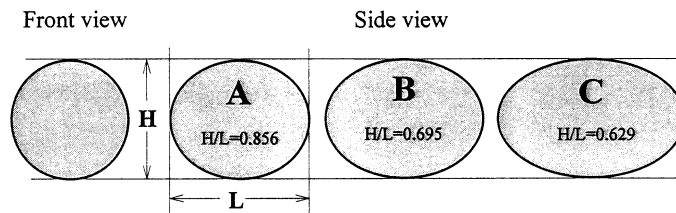


Fig. 5. Three kinds of ideal elliptic bodies, A, B, and C, which have different ratios of H/L only in their side views (H : height, L : length).

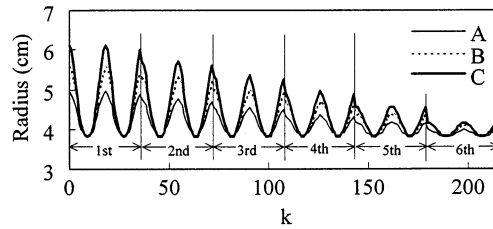


Fig. 6. One-dimensional profile data for three types of elliptic bodies (A, B, and C), which all consist of six profiles.

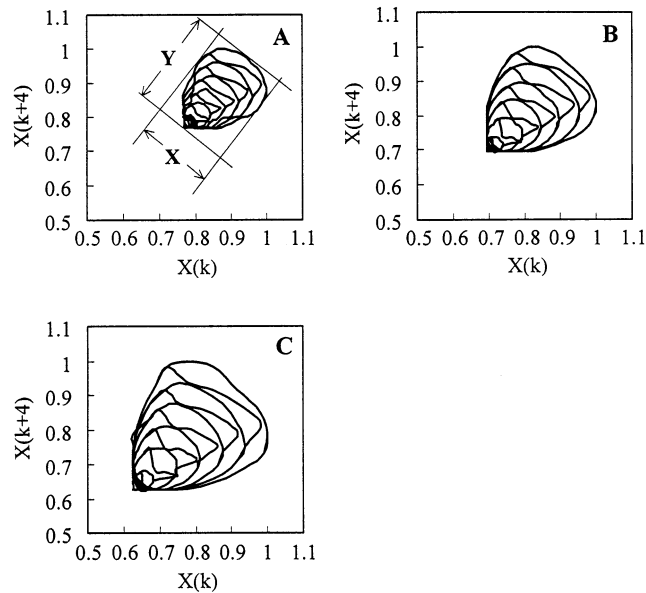


Fig. 7. Attractors of three types of one-dimensional profile data (A, B, and C) shown in Fig. 6.

3.2. One-dimensional profile data of the ideal elliptic bodies

Fig. 6 shows one-dimensional profile data obtained from the three elliptic bodies, A, B and C, shown in Fig. 5. Each data includes six profiles, which are obtained by turning the object at an angle of 15° . Since the data number of one profile is $N=36$, the total data number is $TN=216$. The value of the radius becomes gradually small as the k value increases, because the shape of the object gradually shifts from ellipse (side view) to round (front view) by turning it.

3.3. Attractor of the ideal elliptic bodies

Fig. 7 shows attractors of three types of one-dimensional profile data, A, B and C, shown in Fig. 6, which are all plotted in the two-dimensional phase plane of $X(k)$ and $X(k + 4)$. For comparison, all data were normalized. The time lag is $\tau = 4$. There are significant differences in their sizes (areas). In order to evaluate the shape of the attractor quantitatively, furthermore, the ratio, X/Y (X : width, Y : length) were introduced. It can be seen that the value of X/Y is smaller in the shape A than in the shape C. This is because the fluctuation of the data is smaller in the shape A than in the shape C.

Fig. 8 (a) and (b) show the relationships between H/L of the ellipse and the area of the attractor and between H/L of the ellipse and X/Y of the attractor. As mentioned above, it was assumed that the closer the value of H/L gets to 1 (round), the better the shape of the fruit becomes.

From the figures, it is found that both the area and the value of X/Y of the attractor markedly decrease as the value of H/L increases. Therefore, it can be seen that the ratio, X/Y , of the attractor is able to use as an evaluator for measuring the shape of the fruit.

3.4. Identification result

Next, three types of one-dimensional profile data were identified using the neural network. Fig. 9 shows the comparison between the estimated profile data and the real one in the ideal elliptic body A. The system parameter number, n , and the hidden neuron number were determined to be 20 and 20, respectively. It is found that the estimated data agree well with the real data. This means that a three-layer neural network is effective for tracing and modelling several profiles of the fruit.

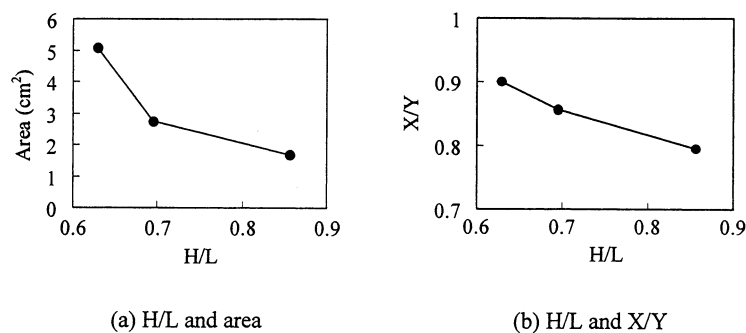


Fig. 8. Relationships between H/L for ellipses and the areas for attractors and between H/L for ellipses and X/Y for attractors.

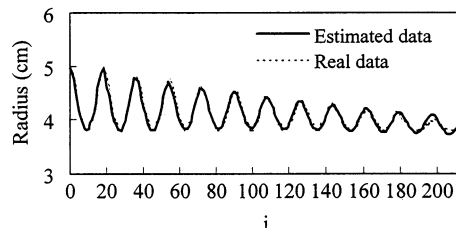


Fig. 9. Comparison between the estimated and real profile data in the case of the ideal elliptic body.

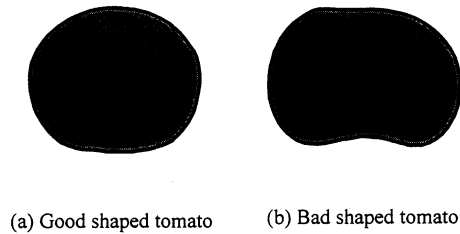


Fig. 10. Digital images of a good-shaped tomato and a bad-shaped tomato, obtained using a CCD camera. Here a good shape means higher regularity and symmetry in the shape.

4. Application

4.1. Digital images of tomatoes

Next, various shapes of tomatoes were used for shape evaluation. Fig. 10 shows examples of the digital images for a good shaped tomato and a bad shaped tomato measured by an image processing system and a CCD camera. The judgment of a good shape or a bad shape was performed from a viewpoint of the regularity of the fruit shape. In this study, it is assumed that fruits with higher regularity and symmetry in their shapes are better. Six images ($M = 6$) were obtained in each fruit by turning the fruit at an angle of 15° .

4.2. One-dimensional profile data of tomatoes

The profile of each fruit is extracted using an image processing system and then the coordinates of all points on the profile are obtained at the angle of 10° , as shown in Fig. 1. A centroid of the fruit image is calculated based on their coordinates. Then, the radii between the centroid and the profile points were obtained. Fig. 11 (a) and (b) indicate two types of one-dimensional profile data obtained from the good shaped and bad shaped fruits, respectively. Since they consist of six profiles, the total data number is $TN = 216$. Seeing the data of the good shaped fruit, similar partial changes are observed at several parts. It is also found that the amplitude of the fluctuation is much smaller in the good shaped fruit than in the bad shaped fruit.

4.3. Attractors of the tomato shapes

Fig. 12 (a) and (b) show attractors of the good shaped and bad shaped fruits, plotted in the two-dimensional phase plane of $X(k)$ and $X(k+4)$. The attractor of the good shaped fruit is longer and narrower than that of the bad shaped fruit. This characteristic is the same as that of the preliminary experiment.

In the good shaped fruit, most of orbits have a tendency to concentrate on a certain boundary and form a clear ellipse. It is also found that the empty area (EA) is

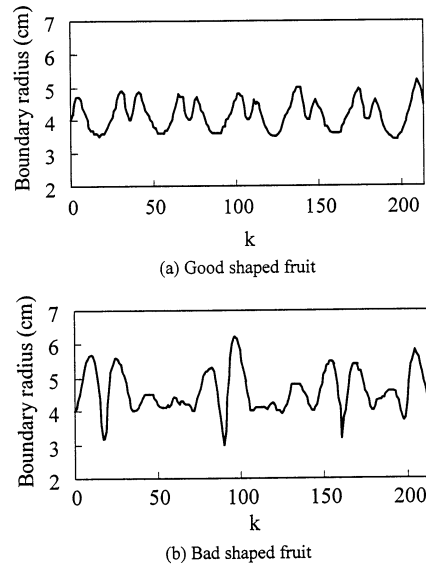


Fig. 11. One-dimensional profile data comprising boundary radii of 6 profiles of the fruit.

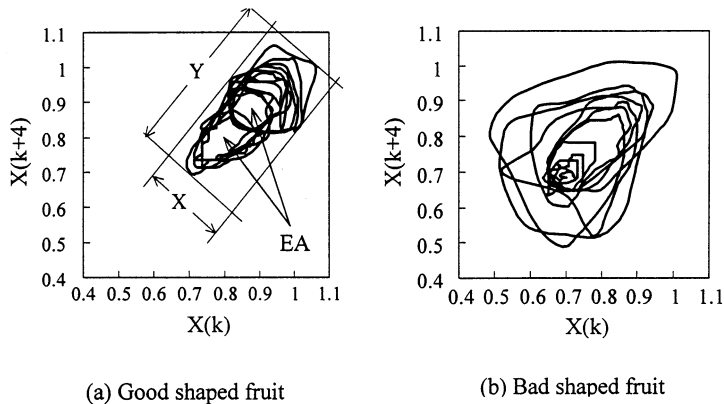


Fig. 12. Attractors of the good-shaped and the bad-shaped fruits, plotted in the two-dimensional phase plane: $X(k)$ and $X(k+4)$. EA represents the empty area.

Table 1

Evaluations of attractors by the ratio (X/Y) of the attractor (X : width, Y : length)

Fruit	Shape	X/Y	Fruit	Shape	X/Y
1	Good	0.500	5	Bad	0.766
2	Good	0.524	6	Bad	0.712
3	Good	0.513	7	Bad	0.722
4	Good	0.568	8	Bad	0.753

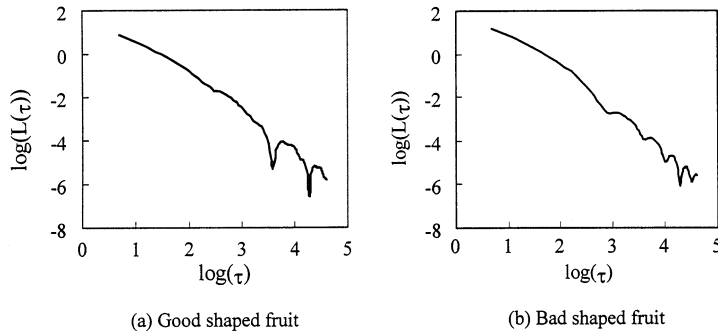


Fig. 13. Fractal dimensions in the cases of the good-shaped and the bad-shaped fruits.

can be observed at the inside of the ellipses. This feature is often observed in the chaotic data. On the other hand, orbits in the bad shaped fruit irregularly distribute on the whole plane.

Next, the ratio, X/Y , was calculated for easier evaluation of the attractor. Variables, X and Y , are the width and length of the attractor, respectively. The result is shown in Table 1. It is found that values of X/Y are smaller in the good shaped fruits than in the bad shaped fruits.

4.4. Fractal dimensions of tomato shapes

Next, fractal dimensions in all the one-dimensional data were calculated to quantitatively evaluate their irregularity. Fig. 13 (a) and (b) show changes in $\log(L(\tau))$, as a function of $\log(\tau)$, for the good shaped and bad shaped fruits, calculated from Eqs. 2 and 3. The fractal dimension is given by a slope, $-D$, of the best fitting linear regression line plotted in the doubly logarithmic scales of $L(\tau)$ and τ . The value of fractal dimension increases with the complexity of the data. In both cases, there is good linearity in their regression curves. In the good shaped fruit, however, significant drops of the curve are observed at several points. These drops are probably caused by the periodicity of the original data. It is confirmed that the one-dimensional profile data of the good shaped fruit in Fig. 11(a) is rather periodic than irregular.

Table 2 shows values of the fractal dimension in the good shaped and bad shaped fruits. The values of fractal dimension in the cases of the good shaped and bad shaped fruits were 1.414 ± 0.068 and 1.663 ± 0.106 , respectively. In general, the value of fractal dimension increases with the irregularity of the data. Especially, a value between 1.5 and 2.0 is a sign that the curve is very irregular (Peitgen et al., 1992). Therefore, it seems that these values reflect the irregularity of the fruit shape.

4.5. Identification results of fruit shape

Next, the system parameter number and the hidden neuron number were determined based on AIC. Fig. 14 shows the relationship between the system parameter number n and AIC. The value of AIC decreased with increasing system parameter number and the minimum value was reached at around the 20th. Over the 20th, however, it had a tendency to increase. This might be caused by a system property. So, the 20th system parameter number was adopted as the optimal value of the system parameter number.

Fig. 15 (a) and (b) show comparisons of the estimated one-dimensional profile data, calculated by the neural network, and the observed one for the good and bad shaped fruits. In both cases, the data used for identifications were four profiles, and three profiles of the fruit were estimated. The total data number is $TN = 36 \times 4 = 144$. As mentioned above, the system parameter number and the hidden neuron number were $n = 20$ and $n_h = 20$, respectively. From the figures, it is found that the

Table 2
Fractal dimensions (FD) in the good shaped and bad shaped fruits

Fruit	Shape	FD	Fruit	Shape	FD
1	Good	1.464	5	Bad	1.755
2	Good	1.378	6	Bad	1.667
3	Good	1.407	7	Bad	1.716
4	Good	1.425	8	Bad	1.514

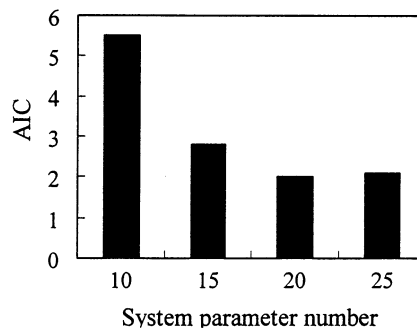


Fig. 14. Relationship between the system parameter number and AIC.

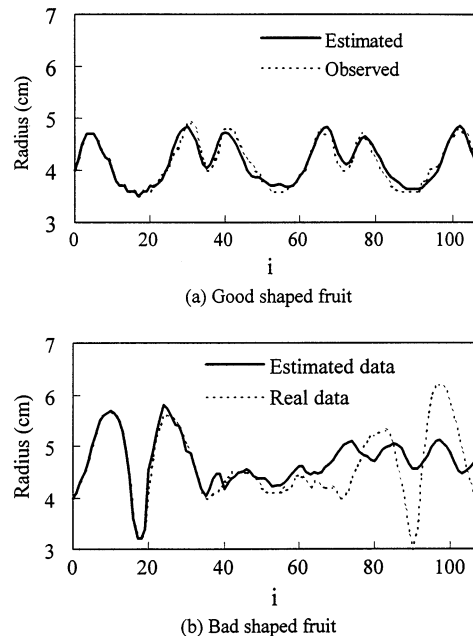


Fig. 15. Comparisons of the observed and estimated one-dimensional profile data calculated by the neural network for the good- and bad-shaped fruits.

estimated accuracy of the one-dimensional profile data is superior in the good shaped fruit to in the bad shaped fruit. This is probably due to the lower irregularity of the good shaped fruit, comparing with the bad shaped fruit. Through these identification procedures, their identification errors were obtained. The identification error became large with the irregularity of the data. This means that the identification error is also able to utilize as an evaluator for the irregularity of the data.

Modelling of the fruit shape is an important task for creating a database on fruit shapes. From the experiment, it was found that the neural network is able to identify and model the fruit shape, especially a comparatively good shaped fruit with higher regularity. In this case, values of the attractor and fractal dimension seem to be useful to judge the difficulty in identifying the complex shape of the fruit. For example, if the fruit shape is judged to be regular (or chaos) from the value of the fractal dimension, its modelling becomes easy with high reliability. If it is completely random, in contrast, its modelling theoretically becomes hard due to its randomness.

4.6. Relationships among attractor, fractal dimension and identification error

Finally, relationships among the shapes of attractors, fractal dimensions and identification errors were investigated. The ratio, X/Y , of the attractor increased

with the irregularity of the fruit. Identification error increased with the irregularity of the fruit shape. The fractal dimension also increased with the irregularity of the fruit shape. Fig. 16 shows the relationship between the identification error and the fractal dimension. It can be seen that the identification error increases with the fractal dimension. This result suggests that identification error can be also used for measuring the irregularity of the fruit shape. Therefore, it is found that the combinatorial uses of these three tools are important for more reliable and precise grading of the fruit shape.

5. Conclusions

In this study, the concepts of attractor and fractal dimension and neural networks were applied to the quantitative evaluation of fruit shape, aiming at a more precise grading of the fruit shape. A one-dimensional profile data consisting of six profiles in each fruit was used for evaluation. From the fundamental analysis of the attractor, there existed a close correlation between the ratio (X/Y) of the attractor (X ; width, Y ; length) and the irregularity of the one-dimensional profile data. The value of X/Y increased with the irregularity of the data. The fractal dimension also increased with the irregularity of the data (fruit shape). On the other hand, a three-layer neural network was effective to identify and trace the one-dimensional profile data. The identification error became large with the irregularity of the data. So, it seems that identification error is also able to utilize as one of the indicators for evaluating the fruit shape. It is also found that a three-layer neural network offers the possibility to model the fruit shape. This feature is useful in creating models of the fruit in a database system on fruit shapes. Thus, it seems that the combinatorial uses of attractor, fractal dimension and neural networks offer more reliable and more sophisticated classification.

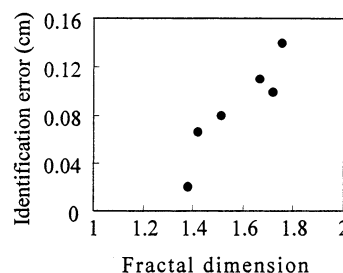


Fig. 16. Relationship between the fractal dimension and the identification error of the one-dimensional profile data.

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