

Homework Signal 4

Week 4

6733172621 Patthadon Phengpinij

Collaborators. ChatGPT (for L^AT_EX styling and grammar checking)

1 Continuous-Time Fourier Transform (CTFT)

Problem 1. Find the Fourier transform of the following signals in terms of $X(j\omega)$, the Fourier transform of $x(t)$ ($\mathcal{F}\{x(t)\} = X(j\omega)$)

1.1 $x(-t)$

Solution. Using the Time-scaling property of the Fourier transform, we have:

$$\textbf{Time-scaling: } \mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

Substituting $a = -1$ into the time-scaling property, we get:

$$\mathcal{F}\{x(-t)\} = \frac{1}{|-1|} X\left(\frac{j\omega}{-1}\right)$$

Therefore, we can express the Fourier transform of $x(-t)$ as:

$$\boxed{\mathcal{F}\{x(-t)\} = X(-j\omega)}$$

1.2 $x_e(t) = \frac{x(t)+x(-t)}{2}$

Solution. Using the Time-scaling and Linearity properties of the Fourier transform, we have:

$$\textbf{Time-scaling: } \mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$\textbf{Linearity: } \mathcal{F}\{ax_1(t) + bx_2(t)\} = aX_1(j\omega) + bX_2(j\omega)$$

Considering the signal $x_e(t) = \frac{x(t)+x(-t)}{2}$, we can find its Fourier transform as follows:

$$\begin{aligned} \mathcal{F}\{x_e(t)\} &= \mathcal{F}\left\{\frac{x(t) + x(-t)}{2}\right\} \\ &= \frac{1}{2} (\mathcal{F}\{x(t)\} + \mathcal{F}\{x(-t)\}) \\ \mathcal{F}\{x_e(t)\} &= \frac{1}{2} \left(X(j\omega) + \frac{1}{|-1|} X\left(\frac{j\omega}{-1}\right) \right) \end{aligned}$$

Therefore, we can express the Fourier transform of $x_e(t)$ as:

$$\boxed{\mathcal{F}\{x_e(t)\} = \frac{X(j\omega) + X(-j\omega)}{2}}$$

$$1.3 \ x_o(t) = \frac{x(t) - x(-t)}{2}$$

Solution. Using the Time-scaling and Linearity properties of the Fourier transform, we have:

$$\textbf{Time-scaling: } \mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$\textbf{Linearity: } \mathcal{F}\{ax_1(t) + bx_2(t)\} = aX_1(j\omega) + bX_2(j\omega)$$

Considering the signal $x_o(t) = \frac{x(t) - x(-t)}{2}$, we can find its Fourier transform as follows:

$$\begin{aligned} \mathcal{F}\{x_o(t)\} &= \mathcal{F}\left\{\frac{x(t) - x(-t)}{2}\right\} \\ &= \frac{1}{2} (\mathcal{F}\{x(t)\} - \mathcal{F}\{x(-t)\}) \\ \mathcal{F}\{x_o(t)\} &= \frac{1}{2} \left(X(j\omega) - \frac{1}{|-1|} X\left(\frac{j\omega}{-1}\right) \right) \end{aligned}$$

Therefore, we can express the Fourier transform of $x_o(t)$ as:

$$\mathcal{F}\{x_o(t)\} = \frac{X(j\omega) - X(-j\omega)}{2}$$

Problem 2. Let $\mathcal{F}\{x(t)\} = X(j\omega) = \text{rect}((\omega - 1)/2)$. Find Fourier transform of

TO SUBMIT

$$2.1 \ x(-2t + 4)$$

Solution. From the Time-scaling and Time-shifting properties of the Fourier transform, we have:

$$\textbf{Time-scaling: } \mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$\textbf{Time-shifting: } \mathcal{F}\{x(t - t_0)\} = e^{-j\omega t_0} X(j\omega)$$

Combining these two properties, we can find the Fourier transform of $x(at - b)$.

$$\mathcal{F}\{x(at - b)\} = \mathcal{F}\left\{x\left(a\left(t - \frac{b}{a}\right)\right)\right\} = \frac{1}{|a|} e^{-j\omega \frac{b}{a}} X\left(\frac{j\omega}{a}\right)$$

To find the Fourier transform of $x(-2t + 4)$, we have $a = -2$ and $b = -4$. Applying the combined properties and substituting $\mathcal{F}\{x(t)\}$, we get:

$$\begin{aligned} \mathcal{F}\{x(-2t + 4)\} &= \frac{1}{|a|} e^{-j\omega \frac{b}{a}} X\left(\frac{j\omega}{a}\right) \\ &= \frac{1}{|-2|} e^{-j\omega \frac{-4}{-2}} X\left(\frac{j\omega}{-2}\right) \\ &= \frac{1}{2} e^{-2j\omega} \text{rect}\left(\frac{\frac{\omega}{-2} - 1}{2}\right) \\ \mathcal{F}\{x(-2t + 4)\} &= \frac{1}{2} e^{-2j\omega} \text{rect}\left(\frac{-\omega - 2}{4}\right) \end{aligned}$$

Because, rect is an even function, we can express the Fourier transform of $x(-2t + 4)$ as:

$$\mathcal{F}\{x(-2t + 4)\} = \frac{1}{2} e^{-2j\omega} \text{rect}\left(\frac{\omega + 2}{4}\right)$$

TO SUBMIT

2.2 $(t-1)x(t-1)$

Solution. First, define a new signal $y(t) = tx(t)$. Then, we can express $(t-1)x(t-1)$ as:

$$(t-1)x(t-1) = y(t-1)$$

Now, using the Time-shifting property of the Fourier transform, we have:

$$\textbf{Time-shifting: } \mathcal{F}\{x(t-t_0)\} = e^{-j\omega t_0} X(j\omega)$$

Applying to $y(t-1)$, we get:

$$\begin{aligned} \mathcal{F}\{y(t-1)\} &= e^{-j\omega \cdot (1)} Y(j\omega) \\ &= e^{-j\omega} \mathcal{F}\{y(t)\} \\ \mathcal{F}\{y(t-1)\} &= e^{-j\omega} \mathcal{F}\{tx(t)\} \end{aligned}$$

Next, consider the differentiation of $\mathcal{F}\{x(t)\}$:

$$\begin{aligned} \frac{d}{d\omega} \mathcal{F}\{x(t)\} &= \frac{d}{d\omega} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} x(t) \frac{d}{d\omega} (e^{-j\omega t}) dt \\ &= \int_{-\infty}^{\infty} x(t) (-jte^{-j\omega t}) dt \\ &= -j \int_{-\infty}^{\infty} tx(t) e^{-j\omega t} dt \\ \frac{d}{d\omega} \mathcal{F}\{x(t)\} &= -j \mathcal{F}\{tx(t)\} \\ \mathcal{F}\{tx(t)\} &= j \frac{d}{d\omega} X(j\omega) \end{aligned}$$

Substituting $\mathcal{F}\{x(t)\}$ into the equation, we get:

$$\begin{aligned} \mathcal{F}\{tx(t)\} &= j \frac{d}{d\omega} X(j\omega) \\ &= j \frac{d}{d\omega} \left(\text{rect} \left(\frac{\omega-1}{2} \right) \right) \\ &= j \frac{d}{d\omega} (u(\omega) - u(\omega-2)) \\ \mathcal{F}\{tx(t)\} &= j (\delta(\omega) - \delta(\omega-2)) \end{aligned}$$

Therefore, we can express the Fourier transform of $(t-1)x(t-1)$ as:

$$\boxed{\mathcal{F}\{(t-1)x(t-1)\} = je^{-j\omega} (\delta(\omega) - \delta(\omega-2))}$$

TO SUBMIT

2.3 $t \frac{dx(t)}{dt}$

Solution. First, define a new signal $y(t) = \frac{dx(t)}{dt}$. Then, we can express $t \frac{dx(t)}{dt}$ as:

$$t \frac{dx(t)}{dt} = ty(t)$$

Now, using the Differentiation in frequency property (proved in the previous problem) of the Fourier transform, we have:

$$\textbf{Differentiation in frequency: } \mathcal{F}\{ty(t)\} = j \frac{d}{d\omega} X(j\omega)$$

Applying to $ty(t)$, we get:

$$\mathcal{F}\{ty(t)\} = j \frac{d}{d\omega} Y(j\omega) = j \frac{d}{d\omega} \mathcal{F}\{y(t)\} = j \frac{d}{d\omega} \mathcal{F}\left\{\frac{dx(t)}{dt}\right\}$$

Next, using the Differentiation in time property of the Fourier transform, we have:

$$\textbf{Differentiation in time: } \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega X(j\omega)$$

Substituting into the equation, we get:

$$\begin{aligned} \frac{d}{d\omega} \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} &= \frac{d}{d\omega} (j\omega X(j\omega)) \\ &= \frac{d}{d\omega} \left(j\omega \cdot \text{rect}\left(\frac{\omega-1}{2}\right) \right) \\ &= \frac{d}{d\omega} (j\omega (u(\omega) - u(\omega-2))) \\ &= j\omega \frac{d}{d\omega} (u(\omega) - u(\omega-2)) + (u(\omega) - u(\omega-2)) \frac{d}{d\omega} (j\omega) \\ &= j\omega (\delta(\omega) - \delta(\omega-2)) + \text{rect}\left(\frac{\omega-1}{2}\right) \cdot (j) \\ &= j\omega\delta(\omega) - j\omega\delta(\omega-2) + j\text{rect}\left(\frac{\omega-1}{2}\right) \\ &= 0 - j(2)\delta(\omega-2) + j\text{rect}\left(\frac{\omega-1}{2}\right) \\ \frac{d}{d\omega} \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} &= -j(2)\delta(\omega-2) + j\text{rect}\left(\frac{\omega-1}{2}\right) \end{aligned}$$

Thus, substituting back, we have:

$$\begin{aligned} \mathcal{F}\{ty(t)\} &= j \frac{d}{d\omega} \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} \\ &= j \cdot \left[-j(2)\delta(\omega-2) + j\text{rect}\left(\frac{\omega-1}{2}\right) \right] \\ \mathcal{F}\{ty(t)\} &= 2\delta(\omega-2) - \text{rect}\left(\frac{\omega-1}{2}\right) \end{aligned}$$

Therefore, we can express the Fourier transform of $t \frac{dx(t)}{dt}$ as:

$$\mathcal{F}\left\{t \frac{dx(t)}{dt}\right\} = 2\delta(\omega-2) - \text{rect}\left(\frac{\omega-1}{2}\right)$$

TO SUBMIT

2.4 $x(2t - 1)e^{-j2t}$

Solution. Using the Time-scaling, and Time-shifting properties of the Fourier transform, we have:

$$\text{Time-scaling} + \text{Time-shifting: } \mathcal{F}\{x(at - b)\} = \frac{1}{|a|} e^{-j\omega \frac{b}{a}} X\left(\frac{j\omega}{a}\right)$$

Define a new signal $y(t) = x(2t - 1)$. Then, we can express $x(2t - 1)e^{-j2t}$ as:

$$\begin{aligned} Y(j\omega) &= \mathcal{F}\{y(t)\} \\ &= \mathcal{F}\{x(2t - 1)\} \\ &= \frac{1}{|2|} e^{-j\omega \frac{1}{2}} X\left(\frac{j\omega}{2}\right) \\ Y(j\omega) &= \frac{1}{2} e^{-j\frac{\omega}{2}} X\left(\frac{j\omega}{2}\right) \end{aligned}$$

Now, using the Frequency-shifting property of the Fourier transform, we have:

$$\text{Frequency-shifting: } \mathcal{F}\{x(t)e^{j\omega_0 t}\} = X(j(\omega - \omega_0))$$

Applying to $y(t)e^{-j2t}$, we get:

$$\begin{aligned} \mathcal{F}\{y(t)e^{-j2t}\} &= \mathcal{F}\{y(t)e^{j(-2)t}\} \\ &= Y(j(\omega - (-2))) \\ &= Y(j(\omega + 2)) \\ \mathcal{F}\{y(t)e^{-j2t}\} &= \frac{1}{2} e^{-j\frac{\omega+2}{2}} X\left(\frac{j(\omega+2)}{2}\right) \end{aligned}$$

Then, substituting $\mathcal{F}\{x(t)\}$ into the equation, we get:

$$\begin{aligned} \mathcal{F}\{y(t)e^{-j2t}\} &= \frac{1}{2} e^{-j\frac{\omega+2}{2}} X\left(\frac{j(\omega+2)}{2}\right) \\ &= \frac{1}{2} e^{-j\frac{\omega+2}{2}} \text{rect}\left(\frac{\frac{\omega+2}{2} - 1}{2}\right) \\ &= \frac{1}{2} e^{-j\frac{\omega+2}{2}} \text{rect}\left(\frac{\omega+2-2}{4}\right) \\ \mathcal{F}\{y(t)e^{-j2t}\} &= \frac{1}{2} e^{-j\frac{\omega+2}{2}} \text{rect}\left(\frac{\omega}{4}\right) \end{aligned}$$

Therefore, we can express the Fourier transform of $x(2t - 1)e^{-j2t}$ as:

$$\mathcal{F}\{x(2t - 1)e^{-j2t}\} = \frac{1}{2} e^{-j\frac{\omega+2}{2}} \text{rect}\left(\frac{\omega}{4}\right)$$

TO SUBMIT

2.5 $x(t) * x(t - 1)$

Solution. First, define a new signal $y(t) = x(t - 1)$. Then, we can express $x(t) * x(t - 1)$ as:

$$x(t) * x(t - 1) = x(t) * y(t)$$

Next, using the Time-shifting property of the Fourier transform, we have:

$$\textbf{Time-shifting: } \mathcal{F}\{x(t - t_0)\} = e^{-j\omega t_0} X(j\omega)$$

Substituting into the equation, we get:

$$\begin{aligned} Y(j\omega) &= \mathcal{F}\{y(t)\} \\ &= \mathcal{F}\{x(t - 1)\} \\ &= e^{-j\omega(1)} X(j\omega) \\ Y(j\omega) &= e^{-j\omega} X(j\omega) \end{aligned}$$

Now, using the Convolution property of the Fourier transform, we have:

$$\textbf{Convolution: } \mathcal{F}\{x_1(t) * x_2(t)\} = X_1(j\omega) \cdot X_2(j\omega)$$

Applying to $x(t) * y(t)$, we get:

$$\begin{aligned} \mathcal{F}\{x(t) * y(t)\} &= \mathcal{F}\{x(t)\} \cdot \mathcal{F}\{y(t)\} \\ &= X(j\omega) \cdot \mathcal{F}\{x(t - 1)\} \\ &= X(j\omega) \cdot e^{-j\omega(1)} X(j\omega) \\ \mathcal{F}\{x(t) * y(t)\} &= (X(j\omega))^2 \cdot e^{-j\omega} \end{aligned}$$

Lastly, substituting $\mathcal{F}\{x(t)\} = \text{rect}((\omega - 1)/2)$ back into the equation, we get:

$$\mathcal{F}\{x(t) * y(t)\} = \text{rect}^2\left(\frac{\omega - 1}{2}\right) \cdot e^{-j\omega}$$

But, since $\text{rect}(\cdot)$ is equal to either 0 or 1, we have:

$$\text{rect}^2\left(\frac{\omega - 1}{2}\right) = \text{rect}\left(\frac{\omega - 1}{2}\right)$$

Therefore, we can express the Fourier transform of $x(t) * x(t - 1)$ as:

$$\mathcal{F}\{x(t) * x(t - 1)\} = e^{-j\omega} \text{rect}\left(\frac{\omega - 1}{2}\right)$$

Problem 3.

3.1 Proof that $\mathcal{F}\{e^{-|t|}\} = \mathcal{F}\{\exp(-|t|)\} = \frac{2}{\omega^2 + 1}$

Solution. Using the definition of the Continuous-Time Fourier Transform (CTFT), we have:

$$\mathcal{F}\{x(t)\} = X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$

Substituting $x(t) = e^{-|t|}$ into the CTFT definition, we get:

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t} dt \\ &= \int_{-\infty}^0 e^t e^{-j\omega t} dt + \int_0^{\infty} e^{-t} e^{-j\omega t} dt \\ &= \int_{-\infty}^0 e^{(1-j\omega)t} dt + \int_0^{\infty} e^{-(1+j\omega)t} dt \\ &= \left[\frac{e^{(1-j\omega)t}}{1-j\omega} \right]_{-\infty}^0 + \left[\frac{-e^{-(1+j\omega)t}}{1+j\omega} \right]_0^{\infty} \\ &= \frac{1}{1-j\omega} + \frac{1}{1+j\omega} \\ &= \frac{(1+j\omega) + (1-j\omega)}{(1-j\omega)(1+j\omega)} \\ X(j\omega) &= \frac{2}{1+\omega^2} \end{aligned}$$

Therefore, we have proven that:

$$\boxed{\mathcal{F}\{e^{-|t|}\} = \frac{2}{\omega^2 + 1}} \quad \square.$$

3.2 Using the outcome obtained in Problem 3.1, Find the Fourier Transform of the given equation.

3.2.1 $\frac{d}{dt}(e^{-|t|})$

Solution. Using the Differentiation in time property of the Fourier transform, we have:

$$\text{Differentiation in time: } \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega X(j\omega)$$

Define a new signal $y(t) = e^{-|t|}$, applying the Differentiation in time property to $\frac{dy(t)}{dt}$, we get:

$$\begin{aligned} \mathcal{F}\left\{\frac{dy(t)}{dt}\right\} &= j\omega Y(j\omega) \\ &= j\omega \mathcal{F}\{y(t)\} \\ \mathcal{F}\left\{\frac{dy(t)}{dt}\right\} &= j\omega \mathcal{F}\{e^{-|t|}\} \end{aligned}$$

Substituting the result from Problem 3.1 into the equation, we get:

$$\boxed{\mathcal{F}\left\{\frac{d}{dt}(e^{-|t|})\right\} = \frac{2j\omega}{\omega^2 + 1}}$$

3.2.2 $\exp(3jt - |2t + 2|)$

Solution. First, define a new signal $x(t) = e^{-|t|}$ and $y(t) = x(2t + 2) = e^{-|2t+2|}$. Then, we can express $\exp(3jt - |2t + 2|)$ as:

$$\exp(3jt - |2t + 2|) = y(t)e^{j3t}$$

Using the Time-scaling and Time-shifting property, we have:

$$\textbf{Time-scaling + Time-shifting: } \mathcal{F}\{x(at - b)\} = \frac{1}{|a|} e^{-j\omega \frac{b}{a}} X\left(\frac{j\omega}{a}\right)$$

Substituting this property into the equation, we get:

$$\begin{aligned} Y(j\omega) &= \mathcal{F}\{y(t)\} \\ &= \mathcal{F}\{x(2t + 2)\} \\ &= \frac{1}{|2|} e^{-j\omega \frac{2}{2}} X\left(\frac{j\omega}{2}\right) \\ Y(j\omega) &= \frac{1}{2} e^{-j\omega} X\left(\frac{j\omega}{2}\right) \end{aligned}$$

Now, applying the Frequency-shifting property

$$\textbf{Frequency-shifting: } \mathcal{F}\{x(t)e^{j\omega_0 t}\} = X(j(\omega - \omega_0))$$

to $y(t)e^{j3t}$, we get:

$$\mathcal{F}\{y(t)e^{j3t}\} = Y(j(\omega - 3))$$

Then, substituting back and use the result from Problem 3.1, we have:

$$\begin{aligned} \mathcal{F}\{y(t)e^{j3t}\} &= Y(j(\omega - 3)) \\ &= \frac{1}{2} e^{-j(\omega-3)} X\left(\frac{j(\omega-3)}{2}\right) \\ &= \frac{1}{2} e^{-j(\omega-3)} \cdot \frac{2}{\left(\frac{\omega-3}{2}\right)^2 + 1} \\ &= \frac{e^{-j(\omega-3)}}{\frac{(\omega-3)^2}{4} + 1} \\ \mathcal{F}\{y(t)e^{j3t}\} &= \frac{4e^{-j(\omega-3)}}{(\omega-3)^2 + 4} \end{aligned}$$

Therefore, we can express the Fourier transform of $\exp(3jt - |2t + 2|)$ as:

$$\boxed{\mathcal{F}\{\exp(3jt - |2t + 2|)\} = \frac{4e^{-j(\omega-3)}}{(\omega-3)^2 + 4}}$$

3.2.3 $\frac{1}{2\pi t^2 + 1}$

Solution. Consider the CFTF of $e^{-|t|}$ obtained in Problem 3.1:

$$\mathcal{F}\{e^{-|t|}\} = \frac{2}{\omega^2 + 1}$$

Using the Duality property of the Fourier transform, we have:

$$\textbf{Duality: } \mathcal{F}\{X(t)\} = 2\pi x(-\omega)$$

Define a new signal $y(t) = \frac{2}{t^2 + 1}$. Then, applying the Duality property to $y(t)$, we get:

$$\begin{aligned} Y(j\omega) &= \mathcal{F}\{y(t)\} \\ &= 2\pi x(-\omega) \\ &= 2\pi \mathcal{F}^{-1}\{X(t)\} \Big|_{t=-\omega} \\ &= 2\pi e^{-|-\omega|} \\ Y(j\omega) &= 2\pi e^{-|\omega|} \end{aligned}$$

Using the Time-scaling property of the Fourier transform, we have:

$$\textbf{Time-scaling: } \mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

We can rewrite the given signal as:

$$\frac{1}{2\pi t^2 + 1} = \frac{1}{2} \frac{2}{(\sqrt{2\pi}t)^2 + 1} = \frac{1}{2} y(\sqrt{2\pi}t)$$

Find $y(\sqrt{2\pi}t)$ by substituting $a = \sqrt{2\pi}$ into the Time-scaling property, we get:

$$\begin{aligned} \mathcal{F}\{y(\sqrt{2\pi}t)\} &= \frac{1}{|\sqrt{2\pi}|} Y\left(\frac{j\omega}{\sqrt{2\pi}}\right) \\ &= \frac{1}{\sqrt{2\pi}} \cdot 2\pi e^{-\left|\frac{\omega}{\sqrt{2\pi}}\right|} \\ \mathcal{F}\{y(\sqrt{2\pi}t)\} &= \sqrt{2\pi} e^{-\frac{|\omega|}{\sqrt{2\pi}}} \end{aligned}$$

Therefore, we can express the Fourier transform of $\frac{1}{2\pi t^2 + 1}$ as:

$$\boxed{\mathcal{F}\left\{\frac{1}{2\pi t^2 + 1}\right\} = \frac{\sqrt{2\pi}}{2} e^{-\frac{|\omega|}{\sqrt{2\pi}}}}$$

Problem 4. For each of the following Fourier transform ($X(j\omega)$), use Fourier transform properties to determine the corresponding time-domain signal ($x(t)$).

4.1 $X(j\omega) = u(\omega) - u(\omega - 2)$

Solution. Using the Duality property of the Fourier transform, we have:

$$\textbf{Duality: } \mathcal{F}\{X(jt)\} = 2\pi x(-\omega)$$

Thus, we can express the inverse Fourier transform of $X(\omega)$ as:

$$\begin{aligned} x(t) &= \mathcal{F}^{-1}\{X(j\omega)\} \\ &= \frac{1}{2\pi} \mathcal{F}\{X(t)\} \Big|_{t=-\omega} \\ &= \frac{1}{2\pi} \mathcal{F}\{u(t) - u(t-2)\} \Big|_{t=-\omega} \\ &= \frac{1}{2\pi} \mathcal{F}\left\{\text{rect}\left(\frac{t-1}{2}\right)\right\} \Big|_{t=-\omega} \\ x(t) &= \frac{1}{2\pi} \cdot 2e^{-j\omega} \cdot \frac{\sin(\omega)}{\omega} \\ x(t) &= \frac{1}{\pi} e^{-j\omega} \cdot \frac{\sin(\omega)}{\omega} \end{aligned}$$

4.2 $X(j\omega) = \cos(2\omega) \sin\left(\frac{\omega}{2}\right)$

Solution. Cooming Soon...