Homework Signal 1

Week 1

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Collaborators. ChatGPT (for LaTeX styling and grammar checking)

1 Representing Signals

Problem 1. Sketch the following signals

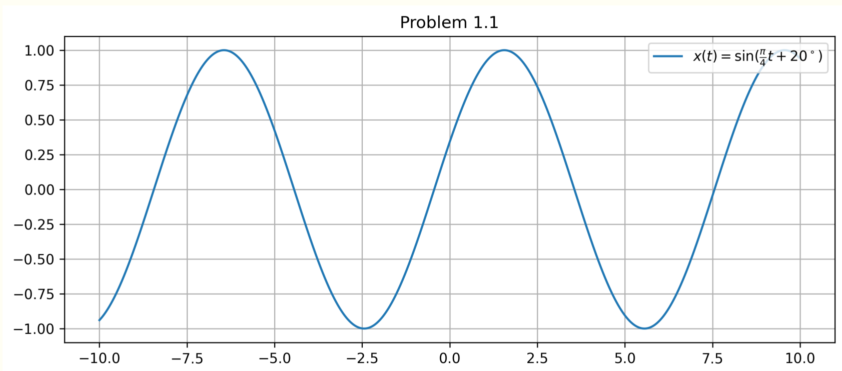
TO SUBMIT

1.1 $x(t) = \sin\left(\frac{\pi}{4}t + 20°\right)$

Solution. Using Python and Matplotlib to plot the signal $x(t) = \sin\left(\frac{\pi}{4}t + 20°\right)$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure()

t = np.arange(-10, 10, 0.01)
x = np.sin(np.pi/4 * t + np.pi/9)

plt.title("Problem 1.1")
plt.plot(t, x, label=r"$x(t) = \sin(\frac{\pi}{4}t + 20^\circ)$")
plt.legend(loc="upper right")
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:

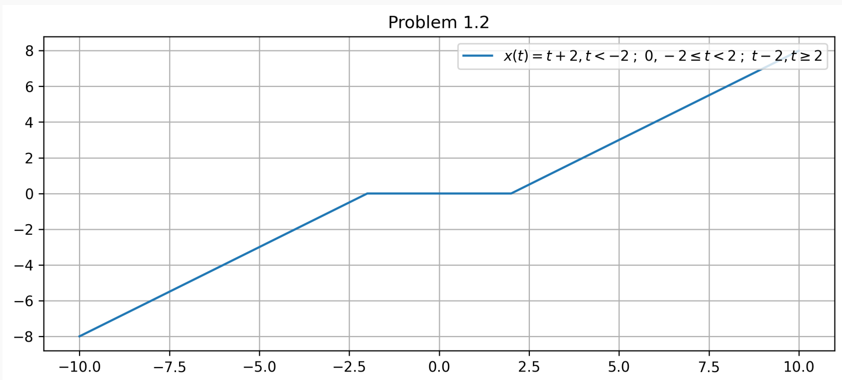


1.2 $x(t) = \begin{cases} t+2, & t \leq 2 \\ 0, & -2 \leq t \leq 2 \\ t-2, & t \geq 2 \end{cases}$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t)$:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)
x = np.piecewise(t, [t < -2, (t >= -2) & (t < 2), t >= 2], [lambda t: t + 2, 0, lambda t: t - 2])

plt.title("Problem 1.2")
plt.plot(t, x, label=r"$x(t) = t+2, t < -2 \;;\; 0, -2 \leq t < 2 \;;\; t-2, t \geq 2$")
plt.legend(loc="upper right")
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



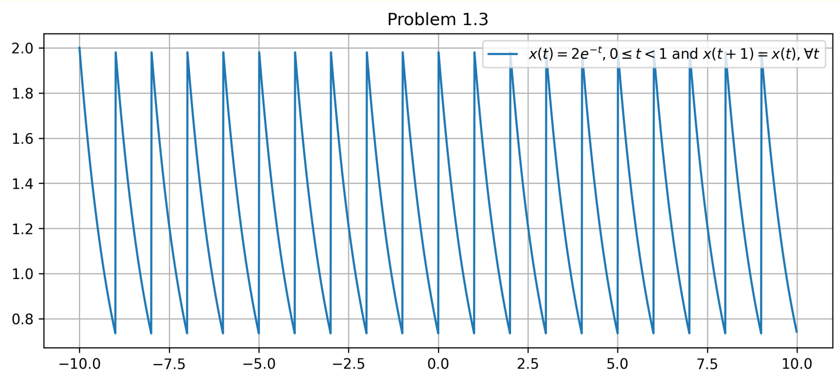
TO SUBMIT

1.3 $x(t) = 2e^{-t}, 0 \leq t < 1$ and $x(t+1) = x(t), \forall t$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t) = 2e^{-t}, 0 \leq t < 1$ and $x(t+1) = x(t), \forall t$:

```
import matplotlib.pyplot as plt
import numpy as np

def x3(t):
    if t >= 1:
        return x3(t - 1)
    if t < 0:
        return x3(t + 1)
    return 2 * (np.e ** (-t))

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)
x3_vectorize = np.vectorize(x3)
x = x3_vectorize(t)

plt.title("Problem 1.3")
plt.plot(t, x, label=r"$x(t) = 2e^{-t}, 0 \leq t < 1 \
    text{ and } x(t + 1) = x(t), \forall t$")
plt.legend(loc="upper right")
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:

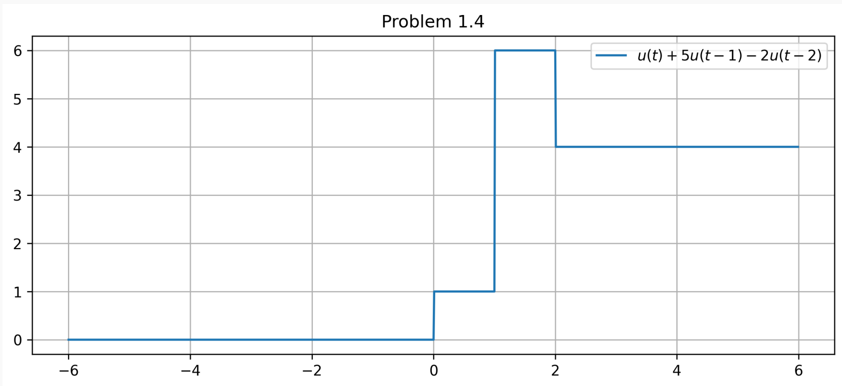


1.4 $x(t) = u(t) + 5u(t-1) + 2u(t-2)$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t) = u(t) + 5u(t-1) + 2u(t-2)$:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-6, 6, 0.01)

u1 = unit_signal_vectorize(t)
u2 = unit_signal_vectorize(t - 1)
u3 = unit_signal_vectorize(t - 2)

x = u1 + 5 * u2 - 2 * u3

plt.title("Problem 1.4")
plt.plot(t, x, label=r"$u(t) + 5u(t-1) - 2u(t-2)$")
plt.legend(loc="upper right")
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



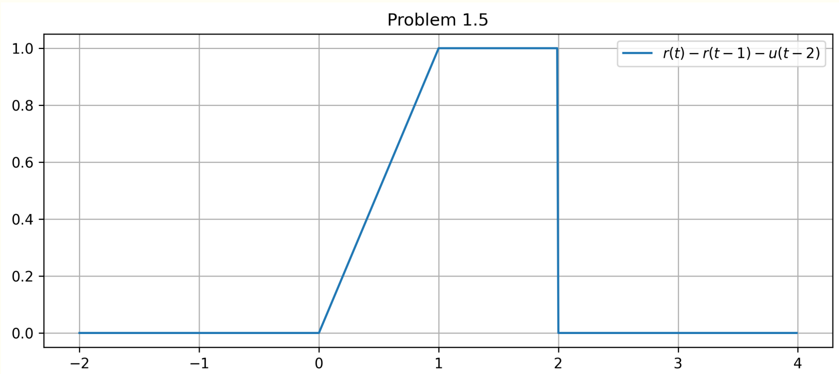
TO SUBMIT

1.5 $x(t) = r(t) - r(t-1) - u(t-2)$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t) = r(t) - r(t-1) - u(t-2)$:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

def ramp_signal(t):
    return t * unit_signal(t)

unit_signal_vectorize = np.vectorize(unit_signal)
ramp_signal_vectorize = np.vectorize(ramp_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-2, 4, 0.01)

r1 = ramp_signal_vectorize(t)
r2 = ramp_signal_vectorize(t - 1)
u1 = unit_signal_vectorize(t - 2)

x = r1 - r2 - u1

plt.title("Problem 1.5")
plt.plot(t, x, label=r"$r(t) - r(t-1) - u(t-2)$")
plt.legend(loc="upper right")
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



Problem 2. Determine whether each of following signals is periodic, and if so, find its period.

2.1 $x(t) = \sin\left(\frac{\pi}{3}t\right) + \cos\left(\frac{8\pi}{3}t\right)$

Solution. Consider each part of the signal separately:

$$\sin\left(\frac{\pi}{3}t\right) \text{ has a period of } T_1 = \frac{2\pi}{\frac{\pi}{3}} = 6$$

$$\cos\left(\frac{8\pi}{3}t\right) \text{ has a period of } T_2 = \frac{2\pi}{\frac{8\pi}{3}} = \frac{3}{4}$$

Considering the least common multiple of the two periods:

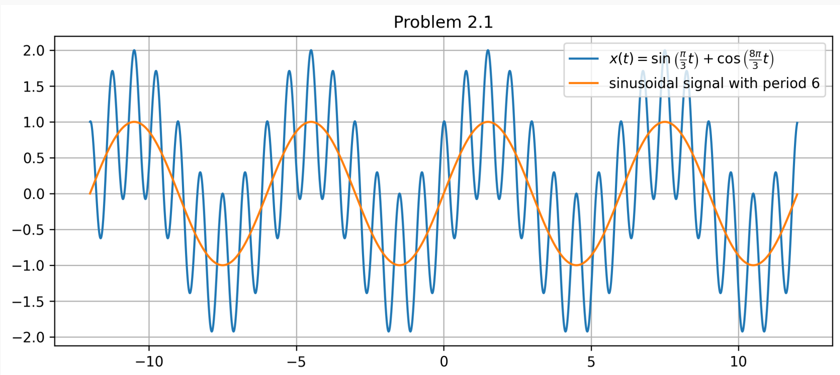
$$T = \text{lcm}(T_1, T_2) = \text{lcm}(6, \frac{3}{4}) = 6$$

Thus, the signal $x(t) = \sin\left(\frac{\pi}{3}t\right) + \cos\left(\frac{8\pi}{3}t\right)$ $\boxed{\textbf{is periodic}}$ with a period of $\boxed{T = 6}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-12, 12, 0.01)

x1 = np.sin(np.pi/3 * t)
x2 = np.cos(8*np.pi/3 * t)

x = np.sin(np.pi/3 * t)

plt.title("Problem 2.1")
plt.plot(t, x1 + x2, label=r"$x(t) = \sin\left(\frac{\pi}{3}t\right) + \cos\left(\frac{8\pi}{3}t\right)$")
plt.plot(t, x, label="sinusoidal signal with period 6")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



2.2 $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(j\frac{5\pi}{6}t\right)$

Solution. Consider each part of the signal separately:

$$\exp\left(j\frac{7\pi}{6}t\right) \text{ has a period of } T_1 = \frac{2\pi}{\frac{7\pi}{6}} = \frac{12}{7}$$

$$\exp\left(j\frac{5\pi}{6}t\right) \text{ has a period of } T_2 = \frac{2\pi}{\frac{5\pi}{6}} = \frac{12}{5}$$

Considering the least common multiple of the two periods:

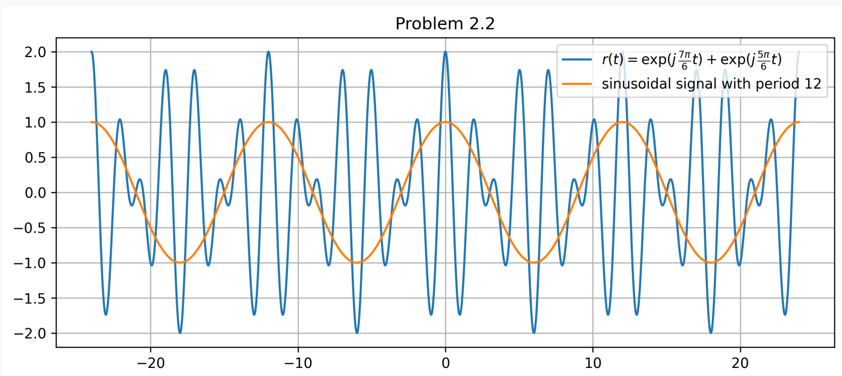
$$T = \operatorname{lcm}(T_1, T_2) = \operatorname{lcm}\left(\frac{12}{7}, \frac{12}{5}\right) = \frac{12}{1} = 12$$

Thus, the signal $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(j\frac{5\pi}{6}t\right)$ $\boxed{\textbf{is periodic}}$ with a period of $\boxed{T = 12}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-24, 24, 0.01)

x1 = np.exp(1j * 7*np.pi/6 * t)
x2 = np.exp(1j * 5*np.pi/6 * t)

x = np.exp(1j * np.pi/6 * t)

plt.title("Problem 2.2")
plt.plot(t, x1 + x2, label=r"$r(t) = \exp(j\,\frac{7\pi}{6}t) + \exp(j\,\frac{5\pi}{6}t)$")
plt.plot(t, x, label="sinusoidal signal with period 12")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



2.3 $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(\frac{5\pi}{6}t\right)$

Solution. Consider each part of the signal separately:

$$\exp\left(j\frac{7\pi}{6}t\right) \text{ has a period of } T_1 = \frac{2\pi}{\frac{7\pi}{6}} = \frac{12}{7}$$

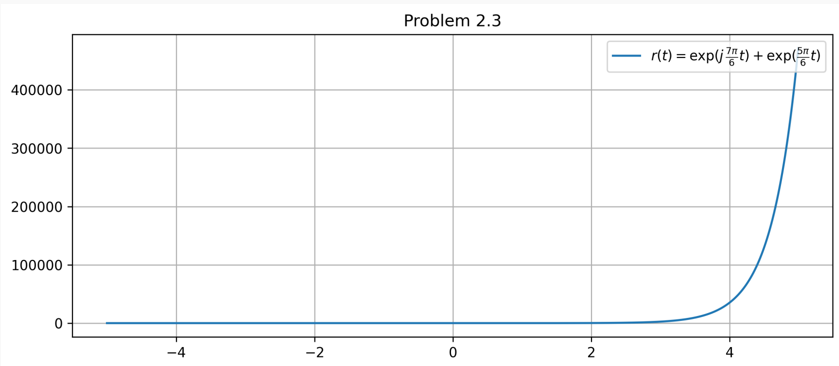
$$\exp\left(\frac{5\pi}{6}t\right) \text{ has no period since it is not a sinusoidal function. (non-periodic signal)}$$

Thus, the signal $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(\frac{5\pi}{6}t\right)$ $\boxed{\textbf{is non-periodic}}$ since one part of the signal is non-periodic.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-5, 5, 0.01)

x1 = np.exp(1j * 7*np.pi/6 * t)
x2 = np.exp(5*np.pi/6 * t)

plt.title("Problem 2.3")
plt.plot(t, x1 + x2, label=r"$r(t) = \exp(j\,\frac{7\pi}{6}t) + \exp(\frac{5\pi}{6}t)$")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



Problem 3. Determine whether the following signals are power or energy signals or neither. Justify your answers

3.1 $x(t) = A\sin(t), -\infty < t < \infty$

Solution. Consider the energy of the signal:

$$\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |A\sin(t)|^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} \sin^2(t) dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} \frac{1-\cos(2t)}{2} dt \\
&= \lim_{N\to\infty} \frac{A^2}{2} \left[t - \frac{\sin(2t)}{2} \right]_{-N}^{N} \\
&= \lim_{N\to\infty} \frac{A^2}{2} (N - (-N)) \\
&= \lim_{N\to\infty} A^2 N \\
E &= \infty
\end{aligned}$$

The integral diverges, so the energy is infinite.

Now, consider the power of the signal:

$$\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} A^2 N \\
P &= \frac{A^2}{2}
\end{aligned}$$

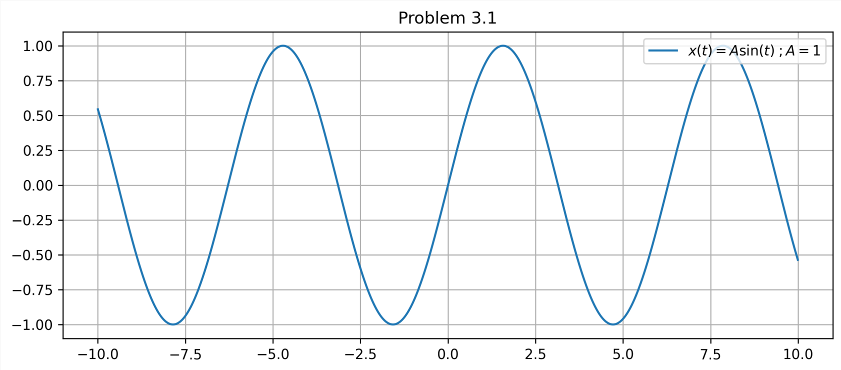
The integral converges to a finite value, so the power is finite.

Thus, the signal $x(t) = A\sin(t)$ is $\boxed{\textbf{a power signal}}$ with power $\boxed{P = \frac{A^2}{2}}$.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)
x = np.sin(t)

plt.title("Problem 3.1")
plt.plot(t, x, label=r"$x(t) = A\sin(t) \;; A = 1$")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



3.2 $x(t) = A(u(t-a) - u(t+a))$, $a > 0$

Solution. Consider the energy of the signal:

$$\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |A(u(t-a) - u(t+a))|^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} |u(t-a) - u(t+a)|^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} (u(t-a) - u(t+a))^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} (u(t+a) - u(t-a)) dt \\
&= \lim_{N\to\infty} A^2 \int_{-a}^{a} 1 dt \\
&= \lim_{N\to\infty} A^2 (a - (-a)) \\
E &= 2aA^2
\end{aligned}$$

The integral converges to a finite value, so the energy is finite.

Now, consider the power of the signal:

$$\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} 2aA^2 \\
P &= 0
\end{aligned}$$

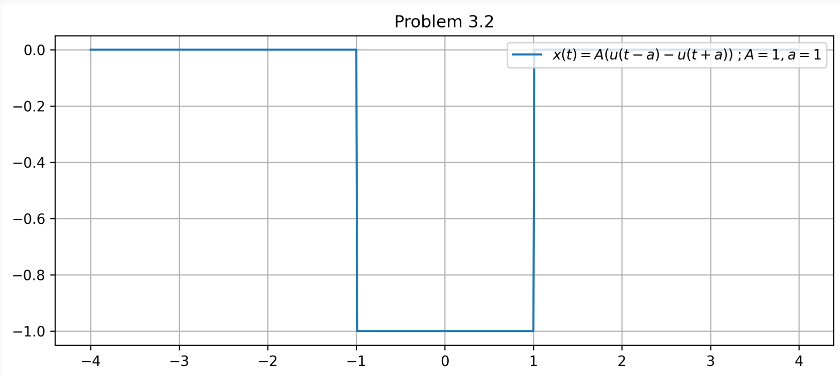
The integral converges to 0, so the power is 0.

Thus, the signal $x(t) = A(u(t-a) - u(t+a))$ is $\boxed{\textbf{a energy signal}}$ with energy $\boxed{E = 2aA^2}$.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-4, 4, 0.01)
x = unit_signal_vectorize(t - 1) - unit_signal_vectorize(t + 1)

plt.title("Problem 3.2")
plt.plot(t, x, label=r"$x(t) = A\left(u(t-a) - u(t+a)\right) \;; A = 1, a = 1$")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



3.3 $x(t) = \exp(-at)u(t)$, $a > 0$

Solution. Consider the energy of the signal:

$$\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |\exp(-at)u(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} \exp(-2at)\,(u(t))^2\,dt \\
&= \lim_{N\to\infty} \left[-\frac{1}{2a}\exp(-2at)\right]_0^N \\
E &= \lim_{N\to\infty} \left(-\frac{1}{2a}\exp(-2aN) + \frac{1}{2a}\right) = \frac{1}{2a}
\end{aligned}$$

The integral converges to a finite value, so the energy is finite.

Now, consider the power of the signal:

$$
\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} \frac{1}{2a} \\
P &= 0
\end{aligned}
$$

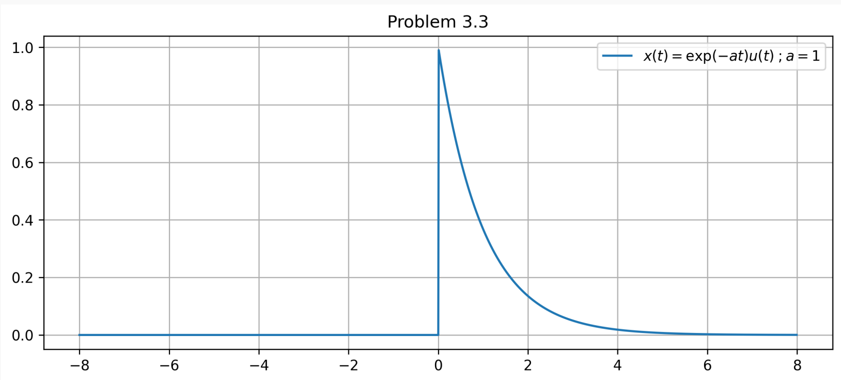
The integral converges to 0, so the power is 0.

Thus, the signal $x(t) = \exp(-at)u(t)$, $a > 0$ is $\boxed{\textbf{a energy signal}}$ with energy $\boxed{E = \frac{1}{2a}}$.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-8, 8, 0.01)
x = np.exp(-t) * unit_signal_vectorize(t)

plt.title("Problem 3.3")
plt.plot(t, x, label=r"$x(t) = \exp(-at)u(t) \;; a = 1$")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



3.4 $x(t) = A\exp(bt)u(t)$, $b > 0$

Solution. Consider the energy of the signal:

$$\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |A\exp(bt)u(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} |A\exp(bt)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} A^2\exp(2bt) dt \\
&= \lim_{N\to\infty} A^2 \left[\frac{1}{2b}\exp(2bt)\right]_0^N \\
&= \lim_{N\to\infty} A^2 \left(\frac{1}{2b}\exp(2bN) - \frac{1}{2b}\right) \\
E &= \infty
\end{aligned}$$

The integral diverges, so the energy is infinite.

Now, consider the power of the signal:

$$\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} A^2 \left(\frac{1}{2b}\exp(2bN) - \frac{1}{2b}\right) \\
P &= \infty
\end{aligned}$$

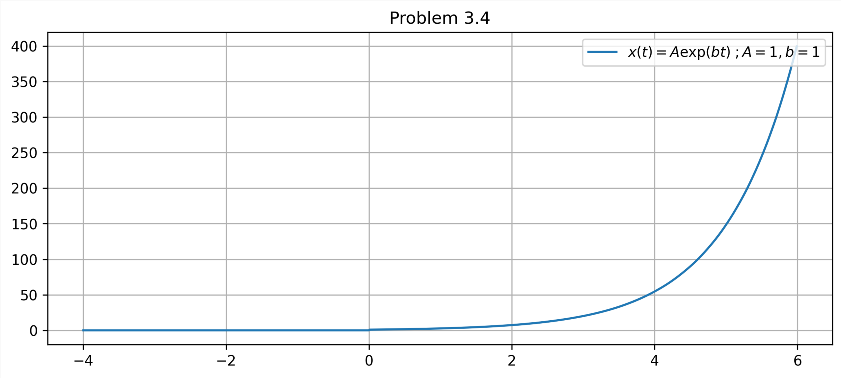
The integral diverges, so the power is infinite.

Thus, the signal $x(t) = A\exp(bt)u(t)$, $b > 0$ is $\boxed{\textbf{neither a energy nor a power signal}}$.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-4, 6, 0.01)
x = np.exp(t) * unit_signal_vectorize(t)

plt.title("Problem 3.4")
plt.plot(t, x, label=r"$x(t) = A\exp(bt) \;; A = 1, b = 1
    $")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

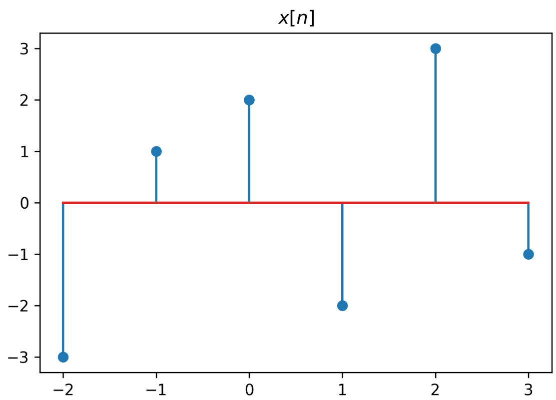
The plot of the signal is shown below:



Problem 4. For the discrete time signal $x[n]$ shown in Figure below, sketch each of the following

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

plt.stem(t, x_t)
plt.title(r"$x[n]$")
plt.show()
```

With the resulting plot shown below:



Solution. By using Python, we can create a function to transform the signal based on the given transformation function:

```python
def transform_signal(x, n, f):
    """Return x[f(n)] for any discrete-time signal x[n].
    """
    f_n = f(n)
    f_n_int = f_n[np.floor(f_n) == f_n]

    x_new = np.zeros(f_n_int.shape[0], dtype=float)

    idx = 0
    for i, val in enumerate(f_n):
        if int(val) == val:
            x_new[idx] = x[i]
            idx += 1

    return x_new, f_n_int
```

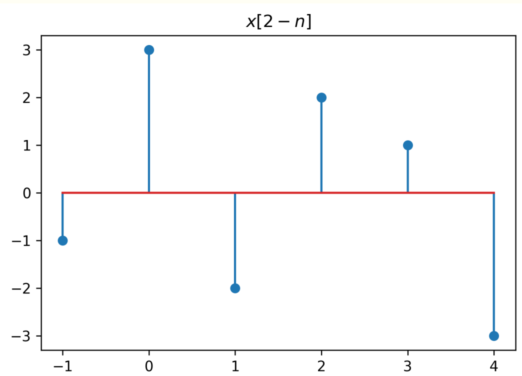
TO SUBMIT

4.1 $x[2-n]$

Solution. Using Python and Matplotlib to plot the signal $x[2-n]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: 2 - x)

plt.stem(t, x_t)
plt.title(r"$x[2 - n]$")
plt.show()
```

With the resulting plot shown below:

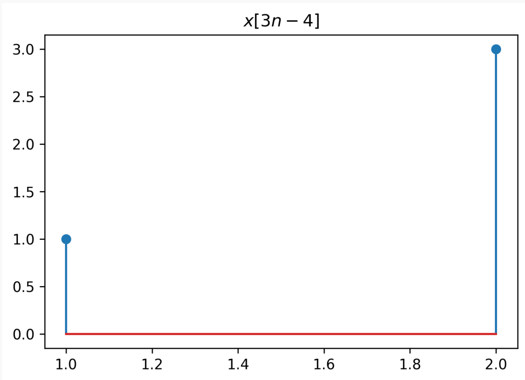


4.2 $x[3n-4]$

Solution. Using Python and Matplotlib to plot the signal $x[3n-4]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: (x + 4) / 3)

plt.stem(t, x_t)
plt.title(r"$x[3n - 4]$")
plt.show()
```

With the resulting plot shown below:



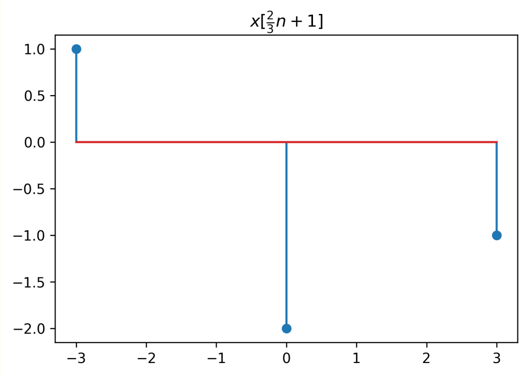
TO SUBMIT

4.3 $x\left[\frac{2}{3}n+1\right]$

Solution. Using Python and Matplotlib to plot the signal $x[\frac{2}{3}n+1]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: (x - 1) * 3 /
    2)

plt.stem(t, x_t)
plt.title(r"$x[\frac{2}{3}n + 1]$")
plt.show()
```

With the resulting plot shown below:

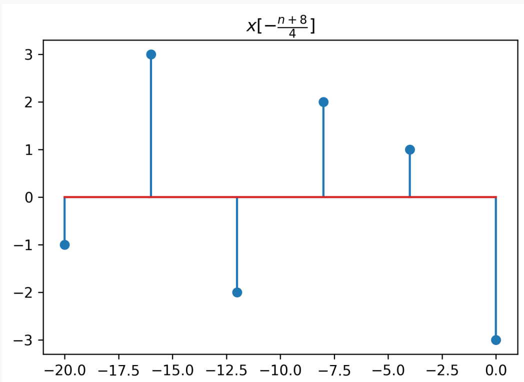


4.4 $x\left[-\frac{n+8}{4}\right]$

Solution. Using Python and Matplotlib to plot the signal $x\left[-\frac{n+8}{4}\right]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: (-4 * x) - 8)

plt.stem(t, x_t)
plt.title(r"$x[-\frac{n + 8}{4}]$")
plt.show()
```

With the resulting plot shown below:



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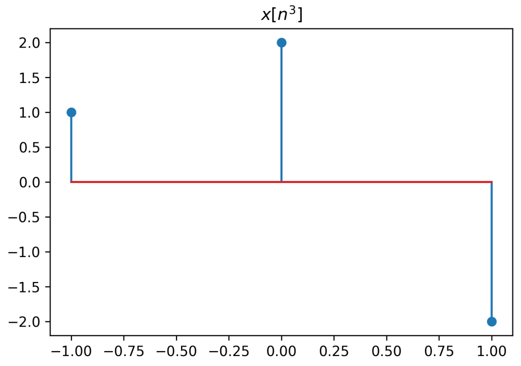
4.5 $x[n^3]$

Solution. Using Python and Matplotlib to plot the signal $x[n^3]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: np.cbrt(x))

plt.stem(t, x_t)
plt.title(r"$x[n^3]$")
plt.show()
```

With the resulting plot shown below:



4.6 $x[2 - n] + x[3n - 4]$

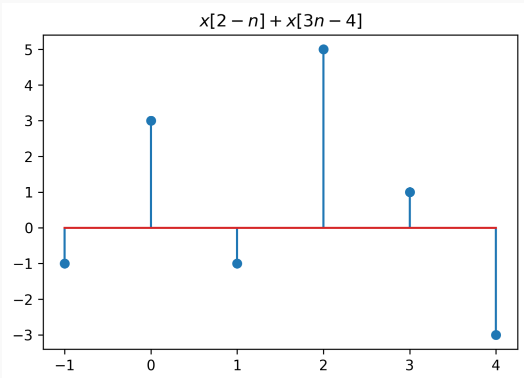
Solution. Introduce a helper function to add two signals:

```python
def add_discrete_signals(x1, t1, x2, t2):
    """Return x1[n] + x2[n] for any discrete-time signals
     x1[n] and x2[n]."""
    t = np.union1d(t1, t2)
    x = np.zeros(t.shape[0], dtype=float)

    for i, val in enumerate(t):
        if val in t1:
            x[i] += x1[np.where(t1 == val)[0][0]]
        if val in t2:
            x[i] += x2[np.where(t2 == val)[0][0]]

    return x, t
```

Using Python and Matplotlib to plot the signal $x[n^3]$:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t_1, t_1 = transform_signal(x_t, t, lambda x: 2 - x)
x_t_2, t_2 = transform_signal(x_t, t, lambda x: (x + 4) /
    3)

x_t, t = add_discrete_signals(x_t_1, t_1, x_t_2, t_2)

plt.stem(t, x_t)
plt.title(r"$x[2-n] + x[3n-4]$")
plt.show()
```

With the resulting plot shown below:



Problem 5. Determine whether each of following signals is periodic, and if so, find its period.

5.1 $x[n] = \sin\left(\frac{\pi n}{4} + \frac{\pi}{8}\right)$

Solution. Consider the signal:

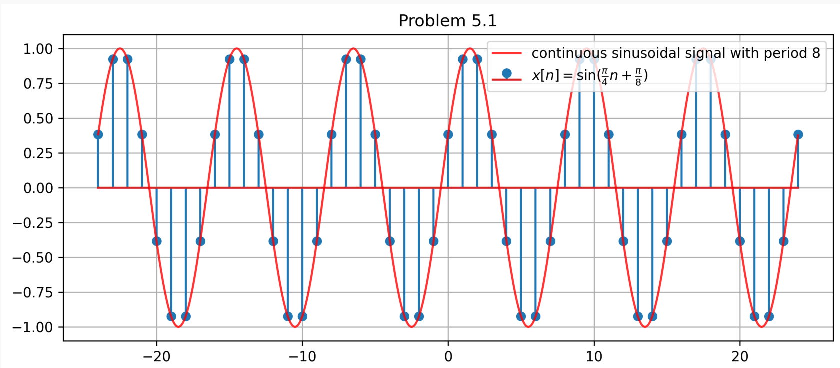
$$\sin\left(\frac{\pi n}{4} + \frac{\pi}{8}\right) \text{ has a period of } N = \frac{2\pi}{\frac{\pi}{4}} = 8$$

Thus, the signal $x[n] = \sin\left(\frac{\pi n}{4} + \frac{\pi}{8}\right)$ $\boxed{\textbf{is periodic}}$ with a period of $\boxed{N = 8}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-24, 25, 1)
x = np.sin(np.pi/4 * t + np.pi/8)

tc = np.arange(-24, 24, 0.01)
xc = np.sin(np.pi/4 * tc + np.pi/8)

plt.title("Problem 5.1")
plt.stem(t, x, label=r"$x[n] = \sin(\frac{\pi}{4} n + \frac{\pi}{8})$")
plt.plot(tc, xc, "r", alpha=0.8, label="continuous sinusoidal signal with period 8")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



5.2 $x[n] = \sin\left(\frac{3\pi n}{4}\right) + \sin\left(\frac{\pi}{3}n\right)$

Solution. Consider the signal:

$$\sin\left(\frac{3\pi n}{4}\right) \text{ has a period of } N_1 = \frac{2\pi}{\frac{3\pi}{4}} = \frac{8}{3}$$

$$\sin\left(\frac{\pi n}{3}\right) \text{ has a period of } N_2 = \frac{2\pi}{\frac{\pi}{3}} = 6$$

Considering the least common multiple of the two periods:

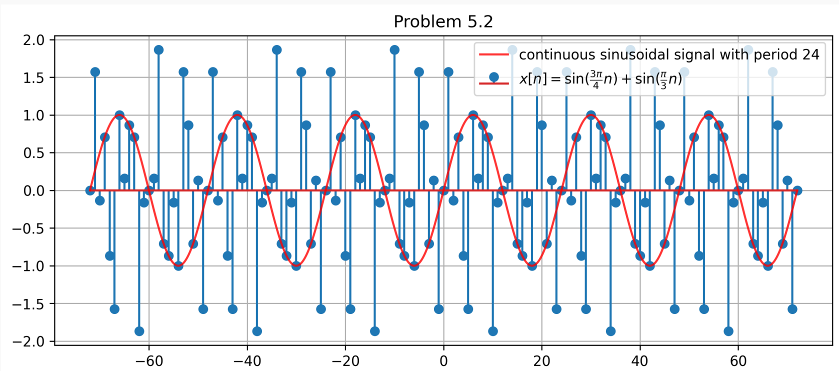
$$N = \text{lcm}(N_1, N_2) = \text{lcm}\left(\frac{8}{3}, 6\right) = \text{lcm}\left(\frac{8}{3}, \frac{18}{3}\right) = \frac{72}{3} = 24$$

Thus, the signal $x[n] = \sin\left(\frac{3\pi n}{4}\right) + \sin\left(\frac{\pi}{3}n\right)$ **is periodic** with a period of $\boxed{N = 24}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-72, 73, 1)
x = np.sin(3 * np.pi/4 * t) + np.sin(np.pi/3 * t)

tc = np.arange(-72, 72, 0.01)
xc = np.sin(np.pi/12 * tc)

plt.title("Problem 5.2")
plt.stem(t, x, label=r"$x[n] = \sin(\frac{3\pi}{4}n) + \
    sin(\frac{\pi}{3}n)$")
plt.plot(tc, xc, "r", alpha=0.8, label="continuous
    sinusoidal signal with period 24")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



5.3 $x[n] = \sin\left(\frac{3\pi n}{4}\right)\sin\left(\frac{\pi}{3}n\right)$

Solution. Using the product-to-sum identities, we can rewrite the signal as:

$$x[n] = \sin\left(\frac{3\pi n}{4}\right)\sin\left(\frac{\pi}{3}n\right) = \frac{1}{2}\left[\cos\left(\frac{3\pi n}{4} - \frac{\pi n}{3}\right) - \cos\left(\frac{3\pi n}{4} + \frac{\pi n}{3}\right)\right]$$

Consider the signal:

$$\cos\left(\frac{3\pi n}{4} - \frac{\pi n}{3}\right) = \cos\left(\frac{5\pi n}{12}\right) \text{ has a period of } N_1 = \frac{2\pi}{\frac{5\pi}{12}} = \frac{24}{5}$$

$$\cos\left(\frac{3\pi n}{4} + \frac{\pi n}{3}\right) = \cos\left(\frac{13\pi n}{12}\right) \text{ has a period of } N_2 = \frac{2\pi}{\frac{13\pi}{12}} = \frac{24}{13}$$

Considering the least common multiple of the two periods:

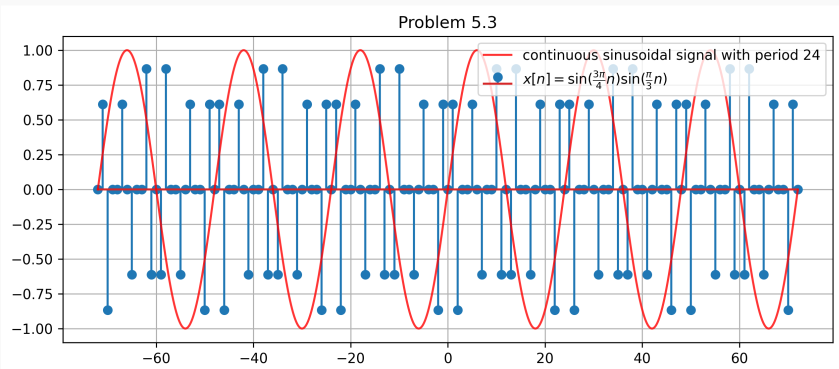
$$N = \operatorname{lcm}(N_1, N_2) = \operatorname{lcm}\left(\frac{24}{5}, \frac{24}{13}\right) = \frac{24}{\gcd(5, 13)} = 24$$

Thus, the signal $x[n] = \sin\left(\frac{3\pi n}{4}\right)\sin\left(\frac{\pi}{3}n\right)$ $\boxed{\textbf{is periodic}}$ with a period of $\boxed{N = 24}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-72, 73, 1)
x = np.sin(3 * np.pi/4 * t) * np.sin(np.pi/3 * t)

tc = np.arange(-72, 72, 0.01)
xc = np.sin(np.pi/12 * tc)

plt.title("Problem 5.3")
plt.stem(t, x, label=r"$x[n] = \sin(\frac{3\pi}{4}n) \sin
    (\frac{\pi}{3}n)$")
plt.plot(tc, xc, "r", alpha=0.8, label="continuous
    sinusoidal signal with period 24")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



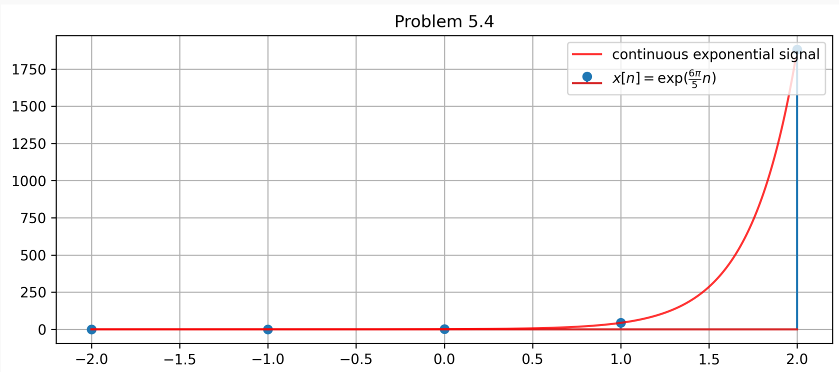
5.4 $x[n] = \exp\left(\frac{6\pi}{5}n\right)$

Solution.
Consider the signal, this is a exponential signal, which have not imaginary part, thus it $\boxed{\textbf{is not periodic}}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-2, 3, 1)
x = np.exp(6 * np.pi/5 * t)

tc = np.arange(-2, 2, 0.01)
xc = np.exp(6 * np.pi/5 * tc)

plt.title("Problem 5.4")
plt.stem(t, x, label=r"$x[n] = \exp(\frac{6\pi}{5}n)$")
plt.plot(tc, xc, "r", alpha=0.8, label="continuous
    exponential signal")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



5.5 $x[n] = \exp\left(j\frac{5\pi}{6}n\right)$

Solution. Consider the signal:

$$\exp\left(j\frac{5\pi}{6}n\right) \text{ has a period of } N = \frac{2\pi}{\frac{5\pi}{6}} = \frac{12}{5}$$

We need to make the period an integer, thus we can multiply the period by 5:

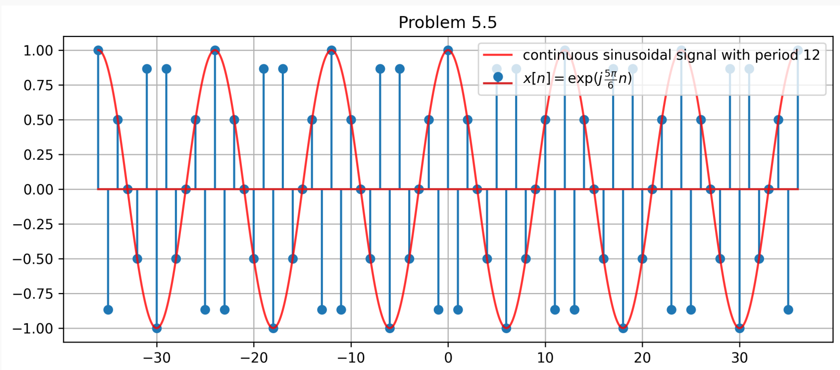
$$N = 5 \cdot \frac{12}{5} = 12$$

Thus, the signal $x[n] = \exp\left(j\frac{5\pi}{6}n\right)$ $\boxed{\textbf{is periodic}}$ with a period of $\boxed{N = 12}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-36, 37, 1)
x = np.exp(1j * 5 * np.pi/6 * t)

tc = np.arange(-36, 36, 0.01)
xc = np.exp(1j * np.pi/6 * tc)

plt.title("Problem 5.5")
plt.stem(t, x, label=r"$x[n] = \exp(j\,\frac{5\pi}{6}n)$")
plt.plot(tc, xc, "r", alpha=0.8, label="continuous sinusoidal signal with period 12")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



5.6 $x[n] = \sum_{m=-\infty}^{\infty} [\delta[n-2m] + 2\delta[n-3m]]$

Solution. Consider the signal, using the properties of the delta function:

1. The first term $\sum_{m=-\infty}^{\infty} \delta[n-2m]$ is a periodic signal with period of 2, since:

$$\sum_{m=-\infty}^{\infty} \delta[n-2m] = \sum_{m=-\infty}^{\infty} \delta[n+2-2m]$$

2. The second term $2\sum_{m=-\infty}^{\infty} \delta[n-3m]$ is a periodic signal with period of 3, since:

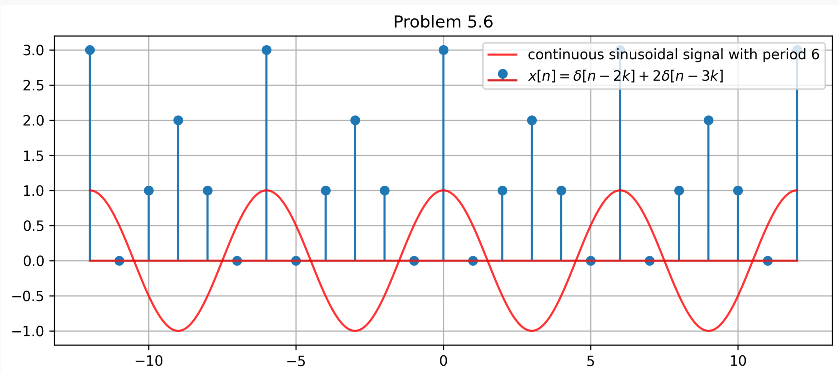
$$2\sum_{m=-\infty}^{\infty} \delta[n-3m] = 2\sum_{m=-\infty}^{\infty} \delta[n+3-3m]$$

Thus, the overall signal $x[n]$ $\boxed{\textbf{is periodic}}$ with a fundamental period of $\mathrm{lcm}(2,3) = \boxed{6}$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-12, 13, 1)
x1 = np.isin(np.mod(t, 2), [0]).astype(float) # delta[n-2k]
x2 = np.isin(np.mod(t, 3), [0]).astype(float) # delta[n-3k]

x = x1 + 2 * x2

tc = np.arange(-12, 12, 0.01)
xc = np.exp(1j * np.pi/3 * tc)

plt.title("Problem 5.6")
plt.stem(t, x, label=r"$x[n] = \delta[n-2k] + 2\delta[n-3k]$")
plt.plot(tc, xc, "r", alpha=0.8, label="continuous sinusoidal signal with period 6")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

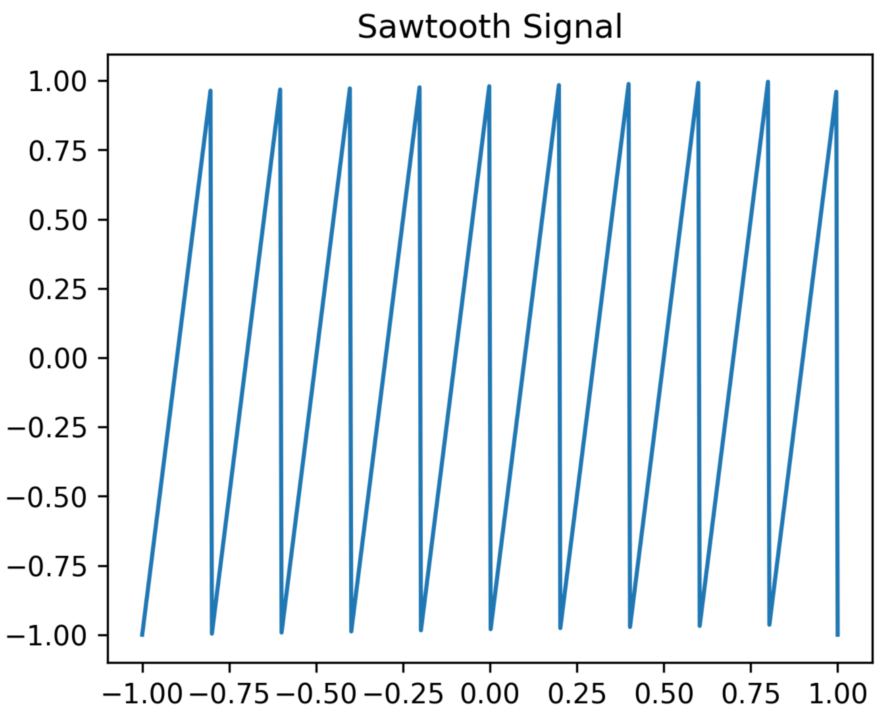
The plot of the signal is shown below:



Problem 6. Signal transformations: Study the sawtooth function in the figure below. Apply reflection, scaling, shifting operations to the signal and plot the transformed signals compared with the original sawtooth signal.

```
import numpy as np
from scipy import signal

fig = plt.figure(figsize=(5, 4))

t = np.linspace(-1, 1, 500)
sawtooth = signal.sawtooth(2 * np.pi * 5 * t)

plt.title("Sawtooth Signal")
plt.plot(t, sawtooth)
plt.show()
```

The plot of the signal is shown below:

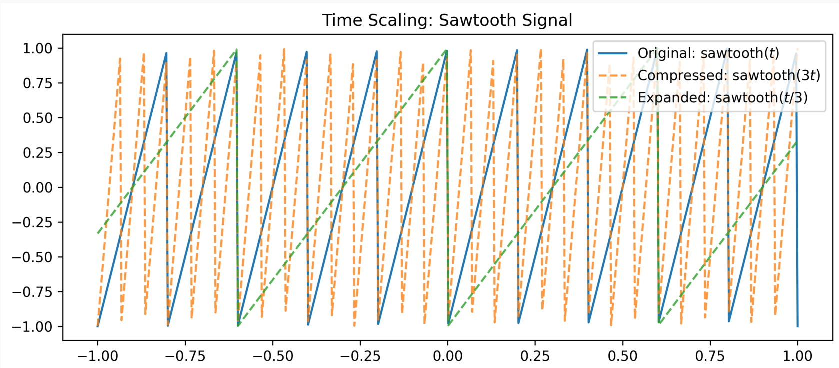


6.1 **time scaling**: scaling factor = 3 and 1/3

Solution. Using Python and Matplotlib to plot the time-scaled signals:

```python
import matplotlib.pyplot as plt
import numpy as np
from scipy import signal

fig = plt.figure(figsize=(10, 4))

t = np.linspace(-1, 1, 500)
sawtooth = signal.sawtooth(2 * np.pi * 5 * t)

# scaling factor = 3 and 1/3
## TODO : writing code for time scaling

t1 = t * 3
sawtooth1 = signal.sawtooth(2 * np.pi * 5 * t1)

t2 = t * (1/3)
sawtooth2 = signal.sawtooth(2 * np.pi * 5 * t2)

plt.title("Time Scaling: Sawtooth Signal")
plt.plot(t, sawtooth, label=r"Original: sawtooth$(t)$", alpha=1)
plt.plot(t, sawtooth1, label=r"Compressed: sawtooth$(3t)$", linestyle="--", alpha=0.8)
plt.plot(t, sawtooth2, label=r"Expanded: sawtooth$(t/3)$", linestyle="--", alpha=0.8)
plt.legend(loc="upper right")
plt.show()
```

With the resulting plot shown below:

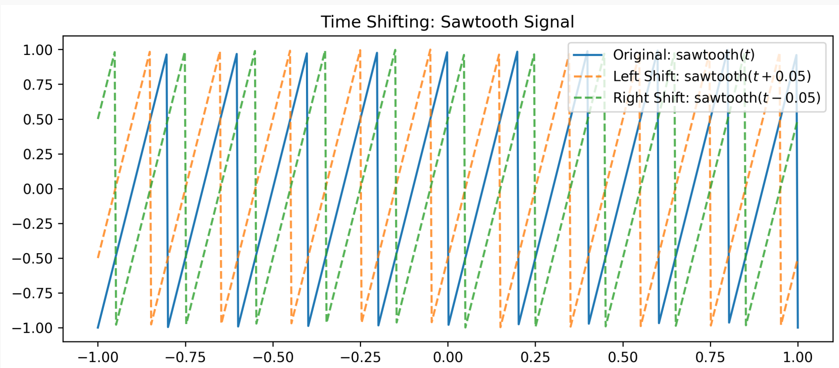


6.2 **time shifting**: shifting amount $= \pm 0.05$

Solution. Using Python and Matplotlib to plot the time-shifted signals:

```python
import matplotlib.pyplot as plt
import numpy as np
from scipy import signal

fig = plt.figure(figsize=(10, 4))

t = np.linspace(-1, 1, 500)
sawtooth = signal.sawtooth(2 * np.pi * 5 * t)

## TODO : writing code for time shifting
# shifting t to the left and right 0.05 units

t1 = t + 0.05
sawtooth1 = signal.sawtooth(2 * np.pi * 5 * t1)

t2 = t - 0.05
sawtooth2 = signal.sawtooth(2 * np.pi * 5 * t2)

plt.title("Time Shifting: Sawtooth Signal")
plt.plot(t, sawtooth, label=r"Original: sawtooth$(t)$", alpha=1)
plt.plot(t, sawtooth1, label=r"Left Shift: sawtooth$(t + 0.05)$", linestyle="--", alpha=0.8)
plt.plot(t, sawtooth2, label=r"Right Shift: sawtooth$(t - 0.05)$", linestyle="--", alpha=0.8)
plt.legend(loc="upper right")
plt.show()
```

With the resulting plot shown below:

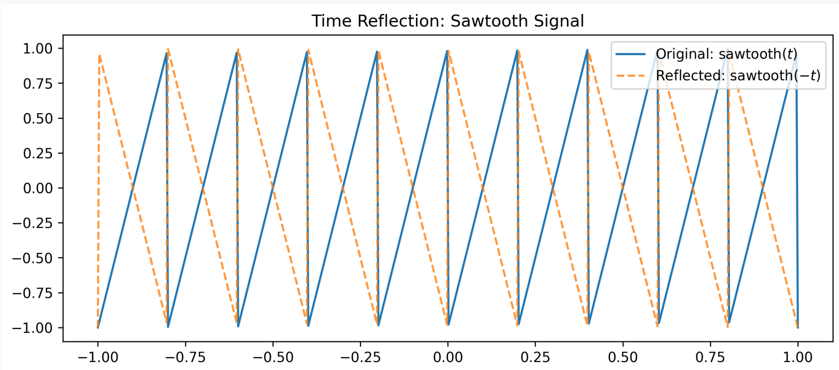


6.3 **time reflection**: reflecting over the y-axis

Solution. Using Python and Matplotlib to plot the time-reflected signals:

```python
import matplotlib.pyplot as plt
import numpy as np
from scipy import signal

fig = plt.figure(figsize=(10, 4))

t = np.linspace(-1, 1, 500)
sawtooth = signal.sawtooth(2 * np.pi * 5 * t)

## TODO : writing code for time Reflection

t1 = t
sawtooth1 = signal.sawtooth(2 * np.pi * 5 * (-1 * t))

plt.title("Time Reflection: Sawtooth Signal")
plt.plot(t, sawtooth, label=r"Original: sawtooth$(t)$", alpha=1)
plt.plot(t, sawtooth1, label=r"Reflected: sawtooth$(-t)$", linestyle="--", alpha=0.8)
plt.legend(loc="upper right")

plt.savefig("../images/problem_6_3.png", dpi=300, bbox_inches="tight")

plt.show()
```

With the resulting plot shown below:



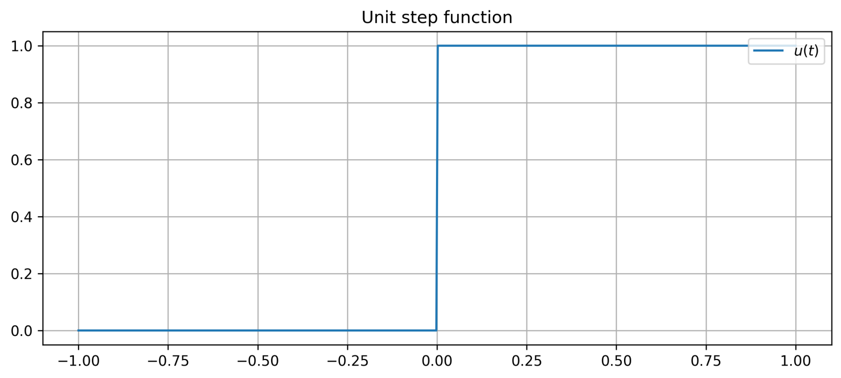
Problem 7. Elementary signals: study the ramp signal plotted in the example below. Plot these signals.

7.1 Writing code for plotting **Unit step function**

Solution. Using Python and Matplotlib to plot the unit step function:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.linspace(-1, 1, 500)

## TODO : writing code for plotting unit step function

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

x = unit_signal_vectorize(t)

plt.title("Unit step function")
plt.plot(t, x, label=r"$u(t)$", alpha=1)
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

With the resulting plot shown below:

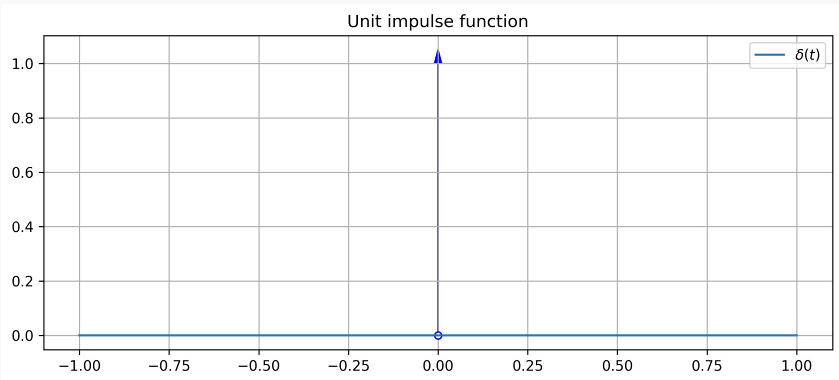


7.2 Writing code for plotting **Unit impulse function**

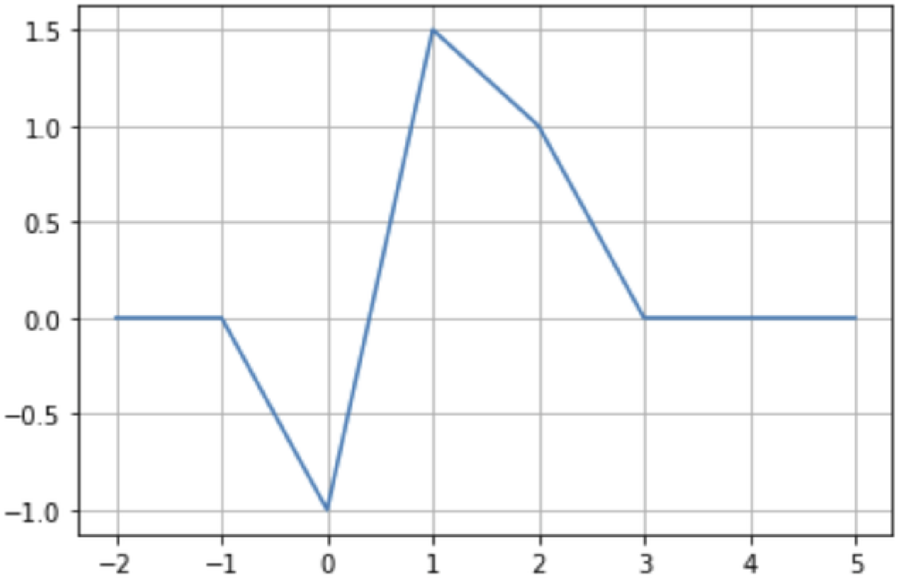
Solution. Using Python and Matplotlib to plot the unit impulse function:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.linspace(-1, 1, 500)

## TODO : writing code for plotting unit impulse function

x = np.zeros(t.shape[0])
x[t == 0] = 1

arrow = plt.arrow(0, 0, 0, 1, head_width=0.02,
    head_length=0.05, fc="blue", ec="blue")
origin = plt.plot(0, 0, color="blue", marker = "o",
    markerfacecolor="white", markersize=5)

plt.title("Unit impulse function")
plt.plot(t, x, label=r"$\delta(t)$", alpha=1)
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

With the resulting plot shown below:



Problem 8. Express the signal that shown in Figure below using Unit-ramp functions



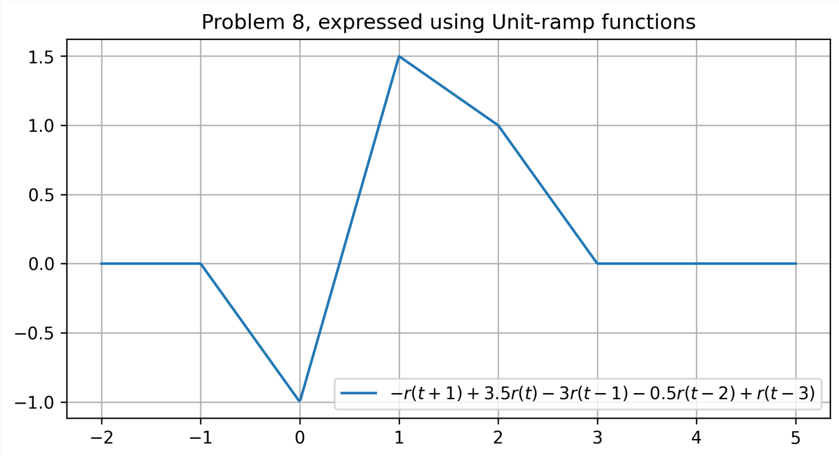
Solution. The signal can be expressed using unit-ramp functions as follows:

$$-r(t+1) + 3.5r(t) - 3r(t-1) - 0.5r(t-2) + r(t-3)$$

Using Python and Matplotlib to plot this unit-ramp function:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(5, 4))

t = np.linspace(-2, 5, 500)

x = np.zeros(t.shape[0])

x += ramp_signal_vectorize(t + 1) * (-1)            # -r(t+1)
x += ramp_signal_vectorize(t - 0) * (1 + 2.5)       # 3.5r(t)
x += ramp_signal_vectorize(t - 1) * (-2.5 + -0.5)   # -3r(t-1)
x += ramp_signal_vectorize(t - 2) * (0.5 + -1)      # -0.5r(t-2)
x += ramp_signal_vectorize(t - 3) * (1)             # r(t-3)

plt.title("Problem 8, expressed using Unit-ramp functions")
plt.plot(t, x, label=r"$-r(t+1)+3.5r(t)-3r(t-1)-0.5r(t-2)+r(t-3)$")
plt.legend(loc="lower right")
plt.grid(True)
plt.show()
```

With the resulting plot shown below:



Problem 9. Evaluate the following integrals

TO SUBMIT

9.1 $\int_{-\infty}^{\infty} \left(\frac{2}{3}t - \frac{3}{2}\right) \delta(t-1)\, dt$

Solution. Using the sifting property of the delta function, we have:

$$\int_{-\infty}^{\infty} \left(\frac{2}{3}t - \frac{3}{2}\right) \delta(t-1)\, dt = \left(\frac{2}{3}(1) - \frac{3}{2}\right)$$
$$= \frac{2}{3} - \frac{3}{2}$$
$$\int_{-\infty}^{\infty} \left(\frac{2}{3}t - \frac{3}{2}\right) \delta(t-1)\, dt = \boxed{-\frac{5}{6}}$$

9.2 $\int_{-\infty}^{\infty} (t-1)\delta\left(\frac{2}{3}t - \frac{3}{2}\right)\, dt$

Solution. Using the sifting property of the delta function, we have:

$$\int_{-\infty}^{\infty} (t-1)\delta\left(\frac{2}{3}t - \frac{3}{2}\right)\, dt = \int_{-\infty}^{\infty} (t-1)\delta\left(t - \frac{9}{4}\right) \cdot \left|\frac{d}{dt}\left(\frac{2}{3}t - \frac{3}{2}\right)\right|^{-1} dt$$
$$= \int_{-\infty}^{\infty} (t-1)\delta\left(t - \frac{9}{4}\right) \cdot \frac{3}{2} dt$$
$$= (\frac{9}{4} - 1) \cdot \frac{3}{2}$$
$$\int_{-\infty}^{\infty} (t-1)\delta\left(\frac{2}{3}t - \frac{3}{2}\right)\, dt = \frac{5}{4} \cdot \frac{3}{2} = \boxed{\frac{15}{8}}$$

TO SUBMIT

9.3 $\int_{-3}^{-2} \left[e^{(-t+1)} + \sin\left(\frac{2\pi t}{3}\right)\right] \delta\left(t - \frac{3}{2}\right)\, dt$

Solution. Because the argument of the delta function $t - \frac{3}{2}$ has its root at $t = \frac{3}{2}$, which is outside the integration limits of -3 to -2, the integral evaluates to zero:

$$\int_{-3}^{-2} \left[e^{(-t+1)} + \sin\left(\frac{2\pi t}{3}\right)\right] \delta\left(t - \frac{3}{2}\right)\, dt = \boxed{0}$$

9.4 $\int_{-3}^{2} \left[e^{(-t+1)} + \sin\left(\frac{2\pi t}{3}\right)\right] \delta\left(t - \frac{3}{2}\right)\, dt$

Solution. Using the sifting property of the delta function, we have:

$$\begin{aligned}
\int_{-3}^{2} \left[e^{(-t+1)} + \sin\left(\frac{2\pi t}{3}\right)\right] \delta\left(t - \frac{3}{2}\right)\, dt &= \left[e^{(-\frac{3}{2}+1)} + \sin\left(\frac{2\pi(\frac{3}{2})}{3}\right)\right] \\
&= e^{-\frac{1}{2}} + \sin(\pi) \\
&= e^{-\frac{1}{2}} + 0 \\
\int_{-3}^{2} \left[e^{(-t+1)} + \sin\left(\frac{2\pi t}{3}\right)\right] \delta\left(t - \frac{3}{2}\right)\, dt &= \boxed{e^{-\frac{1}{2}}}
\end{aligned}$$