Homework Signal 1

Week 1

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Collaborators. ChatGPT (for LaTeX styling and grammar checking)

1 Representing Signals

Problem 1. Sketch the following signals

TO SUBMIT

a) $x(t) = \sin \frac{\pi}{4} t + 20°$

Solution. Using Python and Matplotlib to plot the signal $x(t) = \sin \frac{\pi}{4} t + 20°$:

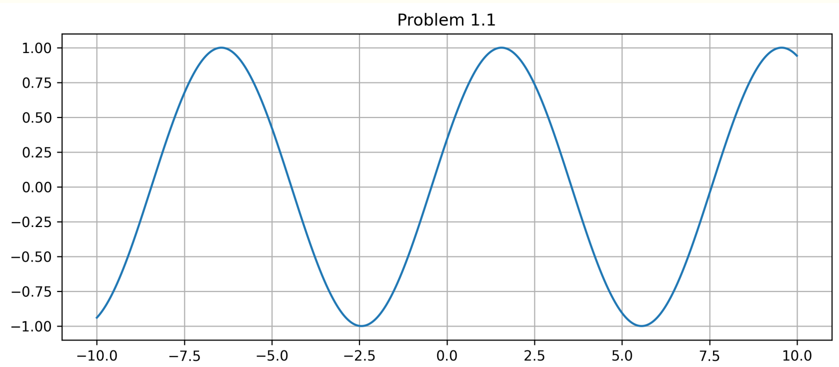
```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure()

t = np.arange(-10, 10, 0.01)
x = np.sin(np.pi/4 * t + np.pi/9)

plt.title("Problem 1.1")
plt.plot(t, x)
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



b) $x(t) = \begin{cases} t+2, & t \leq 2 \\ 0, & -2 \leq t \leq 2 \\ t-2, & t \geq 2 \end{cases}$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t)$:

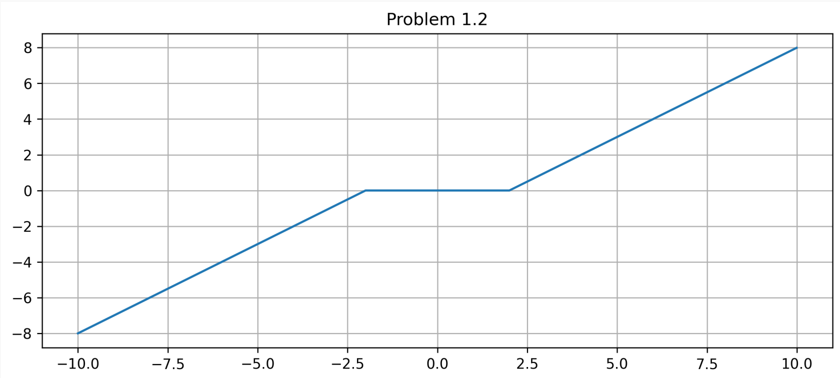
```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)
x = np.piecewise(t, [t < -2, (t >= -2) & (t < 2), t >= 2], [lambda t: t + 2, 0, lambda t: t - 2])

plt.title("Problem 1.2")
plt.plot(t, x)
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



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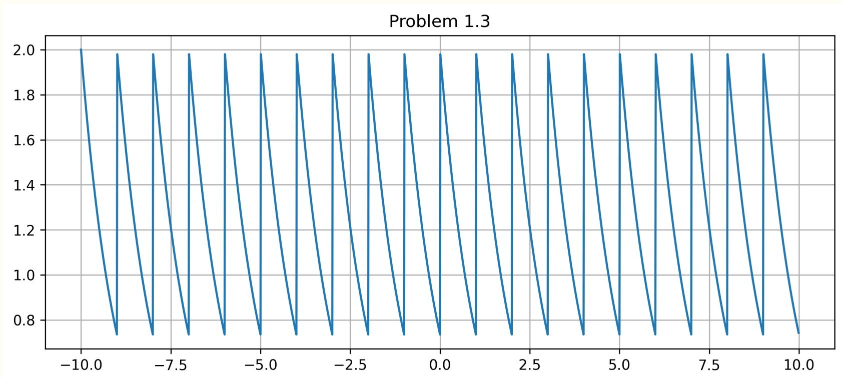
c) $x(t) = 2e^{-t}, 0 \leq t < 1$ and $x(t+1) = x(t), \forall t$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t) = 2e^{-t}, 0 \leq t < 1$ and $x(t+1) = x(t), \forall t$:

```python
import matplotlib.pyplot as plt
import numpy as np

def x3(t):
    if t >= 1:
        return x3(t - 1)
    if t < 0:
        return x3(t + 1)
    return 2 * (np.e ** (-t))

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)
x3_vectorize = np.vectorize(x3)
x = x3_vectorize(t)

plt.title("Problem 1.3")
plt.plot(t, x)
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:

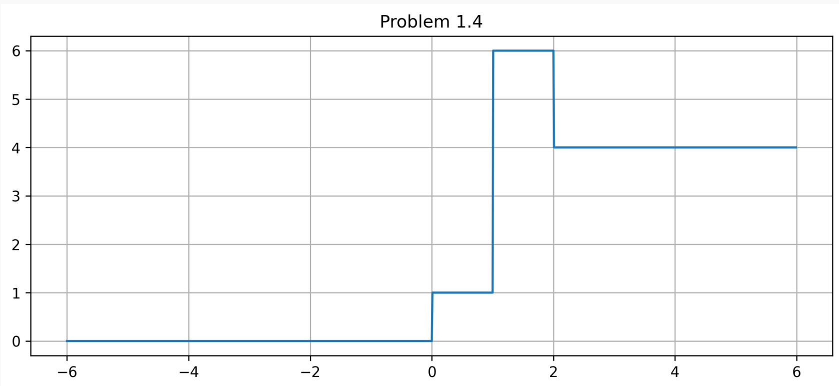


d) $x(t) = u(t) + 5u(t-1) + 2u(t-2)$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t) = u(t) + 5u(t-1) + 2u(t-2)$:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-6, 6, 0.01)

u1 = unit_signal_vectorize(t)
u2 = unit_signal_vectorize(t - 1)
u3 = unit_signal_vectorize(t - 2)

x = u1 + 5 * u2 - 2 * u3

plt.title("Problem 1.4")
plt.plot(t, x)
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



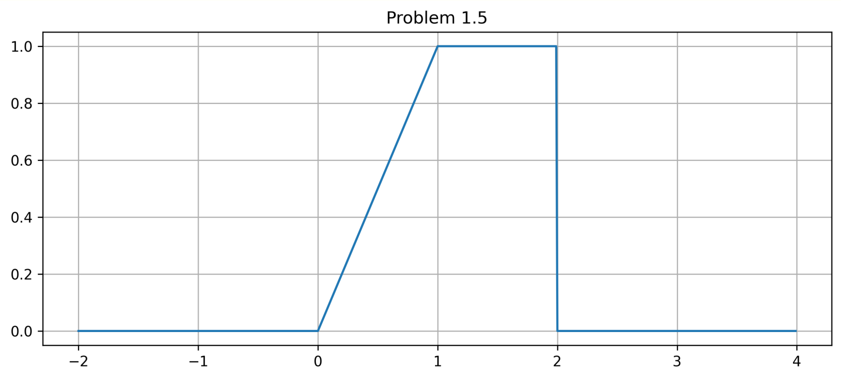
TO SUBMIT

e) $x(t) = r(t) - r(t-1) - u(t-2)$

Solution. Using Python and Matplotlib to plot the piecewise signal $x(t) = r(t) - r(t-1) - u(t-2)$:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

def ramp_signal(t):
    return t * unit_signal(t)

unit_signal_vectorize = np.vectorize(unit_signal)
ramp_signal_vectorize = np.vectorize(ramp_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-2, 4, 0.01)

r1 = ramp_signal_vectorize(t)
r2 = ramp_signal_vectorize(t - 1)
u1 = unit_signal_vectorize(t - 2)

x = r1 - r2 - u1

plt.title("Problem 1.5")
plt.plot(t, x)
plt.grid(True)
plt.show()
```

The plot of the signal is shown below:



Problem 2. Determine whether each of following signals is periodic, and if so, find its period.

a) $x(t) = \sin\left(\frac{\pi}{3}t\right) + \cos\left(\frac{8\pi}{3}t\right)$

Solution. Consider each part of the signal separately:

$$\sin\left(\frac{\pi}{3}t\right) \text{ has a period of } T_1 = \frac{2\pi}{\frac{\pi}{3}} = 6$$

$$\cos\left(\frac{8\pi}{3}t\right) \text{ has a period of } T_2 = \frac{2\pi}{\frac{8\pi}{3}} = \frac{3}{4}$$

Considering the least common multiple of the two periods:

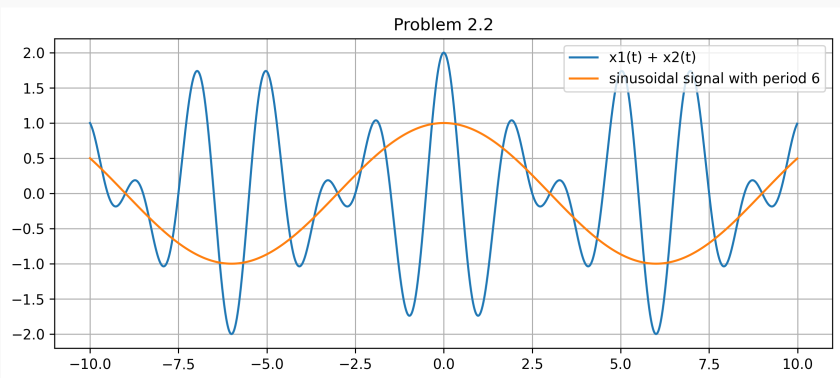
$$T = \text{lcm}(T_1, T_2) = \text{lcm}(6, \frac{3}{4}) = 6$$

Thus, the signal $x(t) = \sin\left(\frac{\pi}{3}t\right) + \cos\left(\frac{8\pi}{3}t\right)$ is periodic with a period of $T = 6$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)

x1 = np.sin(np.pi/3 * t)
x2 = np.cos(8*np.pi/3 * t)

x = np.sin(np.pi/3 * t)

plt.title("Problem 2.1")
plt.plot(t, x1 + x2, label="x1(t) + x2(t)")
plt.plot(t, x, label="sinusoidal signal with period 6")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



b) $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(j\frac{5\pi}{6}t\right)$

Solution. Consider each part of the signal separately:

$$\exp\left(j\frac{7\pi}{6}t\right) \text{ has a period of } T_1 = \frac{2\pi}{\frac{7\pi}{6}} = \frac{12}{7}$$

$$\exp\left(j\frac{5\pi}{6}t\right) \text{ has a period of } T_2 = \frac{2\pi}{\frac{5\pi}{6}} = \frac{12}{5}$$

Considering the least common multiple of the two periods:

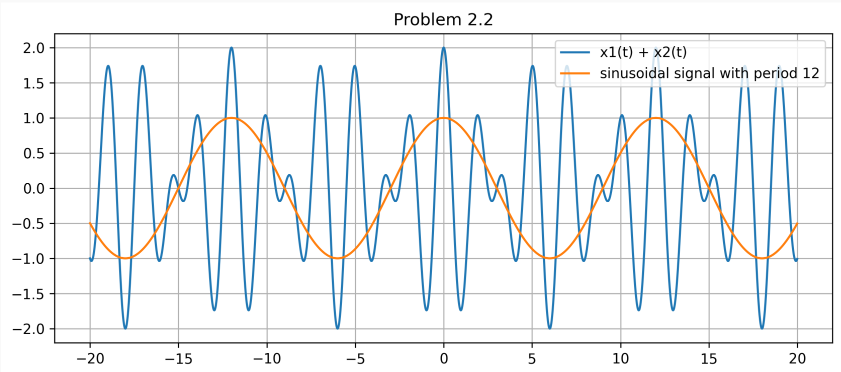
$$T = \operatorname{lcm}(T_1, T_2) = \operatorname{lcm}\left(\frac{12}{7}, \frac{12}{5}\right) = \frac{12}{1} = 12$$

Thus, the signal $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(j\frac{5\pi}{6}t\right)$ is periodic with a period of $T = 12$.

By using Python and Matplotlib, we can visualize the periodicity of the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-20, 20, 0.01)

x1 = np.exp(1j * 7*np.pi/6 * t)
x2 = np.exp(1j * 5*np.pi/6 * t)

x = np.exp(1j * np.pi/6 * t)

plt.title("Problem 2.2")
plt.plot(t, x1 + x2, label="x1(t) + x2(t)")
plt.plot(t, x, label="sinusoidal signal with period 12")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



c) $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(\frac{5\pi}{6}t\right)$

Solution. Consider each part of the signal separately:

$$\exp\left(j\frac{7\pi}{6}t\right) \text{ has a period of } T_1 = \frac{2\pi}{\frac{7\pi}{6}} = \frac{12}{7}$$

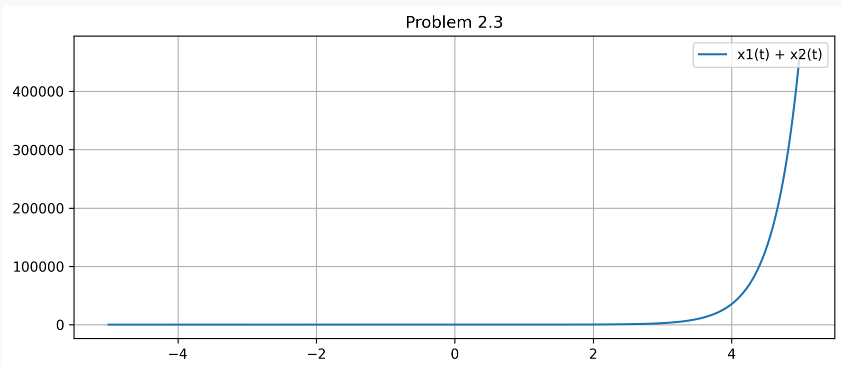
$$\exp\left(\frac{5\pi}{6}t\right) \text{ has no period since it is not a sinusoidal function. (non-periodic signal)}$$

Thus, the signal $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(\frac{5\pi}{6}t\right)$ is non-periodic since one part of the signal is non-periodic.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-5, 5, 0.01)

x1 = np.exp(1j * 7*np.pi/6 * t)
x2 = np.exp(5*np.pi/6 * t)

plt.title("Problem 2.3")
plt.plot(t, x1 + x2, label="x1(t) + x2(t)")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



Problem 3. Determine whether the following signals are power or energy signals or neither. Justify your answers

a) $x(t) = A\sin(t), -\infty < t < \infty$

Solution. Consider the energy of the signal:

$$\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |A\sin(t)|^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} \sin^2(t) dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} \frac{1-\cos(2t)}{2} dt \\
&= \lim_{N\to\infty} \frac{A^2}{2} \left[t - \frac{\sin(2t)}{2}\right]_{-N}^{N} \\
&= \lim_{N\to\infty} \frac{A^2}{2} (N - (-N)) \\
&= \lim_{N\to\infty} A^2 N \\
E &= \infty
\end{aligned}$$

The integral diverges, so the energy is infinite.

Now, consider the power of the signal:

$$\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} A^2 N \\
P &= \frac{A^2}{2}
\end{aligned}$$

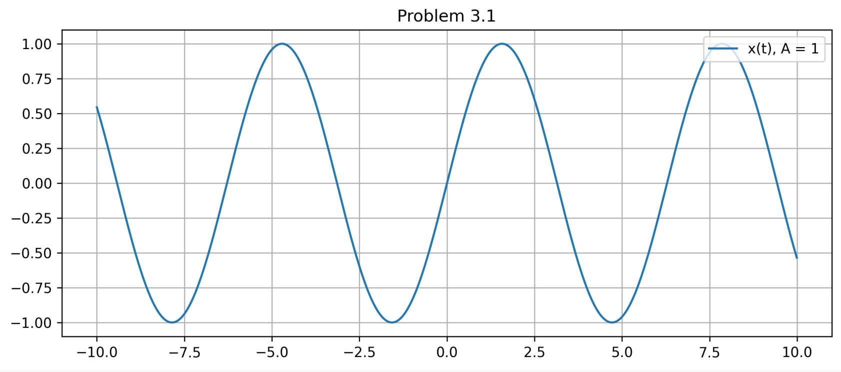
The integral converges to a finite value, so the power is finite.

Thus, the signal $x(t) = A\sin(t)$ is **a power signal** with power $P = \frac{A^2}{2}$.

By using Python and Matplotlib, we can visualize the signal:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(10, 4))

t = np.arange(-10, 10, 0.01)
x = np.sin(t)

plt.title("Problem 3.1")
plt.plot(t, x, label="x(t), A = 1")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



b) $x(t) = A(u(t-a) - u(t+a))$, $a > 0$

Solution. Consider the energy of the signal:

$$
\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |A(u(t-a) - u(t+a))|^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} |u(t-a) - u(t+a)|^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} (u(t-a) - u(t+a))^2 dt \\
&= \lim_{N\to\infty} A^2 \int_{-N}^{N} (u(t+a) - u(t-a)) dt \\
&= \lim_{N\to\infty} A^2 \int_{-a}^{a} 1 dt \\
&= \lim_{N\to\infty} A^2 (a - (-a)) \\
E &= 2aA^2
\end{aligned}
$$

The integral converges to a finite value, so the energy is finite.

Now, consider the power of the signal:

$$
\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} 2aA^2 \\
P &= 0
\end{aligned}
$$

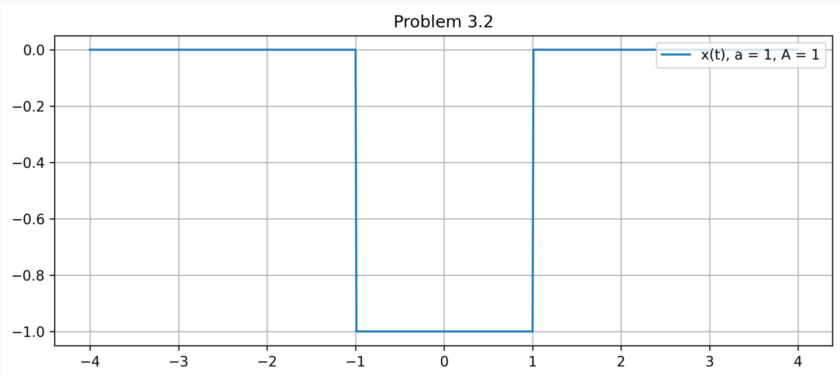
The integral converges to 0, so the power is 0.

Thus, the signal $x(t) = A(u(t-a) - u(t+a))$ is **a energy signal** with energy $E = 2aA^2$.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-4, 4, 0.01)
x = unit_signal_vectorize(t - 1) - unit_signal_vectorize(
    t + 1)

plt.title("Problem 3.2")
plt.plot(t, x, label="x(t), a = 1, A = 1")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



c) $x(t) = \exp(-at)u(t)$, $a > 0$

Solution. Consider the energy of the signal:

$$
\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |\exp(-at)u(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} |\exp(-at)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} \exp(-2at) dt \\
&= \lim_{N\to\infty} \left[-\frac{1}{2a}\exp(-2at)\right]_0^N \\
&= \lim_{N\to\infty} \left(-\frac{1}{2a}\exp(-2aN) + \frac{1}{2a}\right)
\end{aligned}
$$

$$E = \frac{1}{2a}$$

The integral converges to a finite value, so the energy is finite.

Now, consider the power of the signal:

$$\begin{aligned} P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\ &= \lim_{N\to\infty} \frac{1}{2N} \frac{1}{2a} \\ P &= 0 \end{aligned}$$

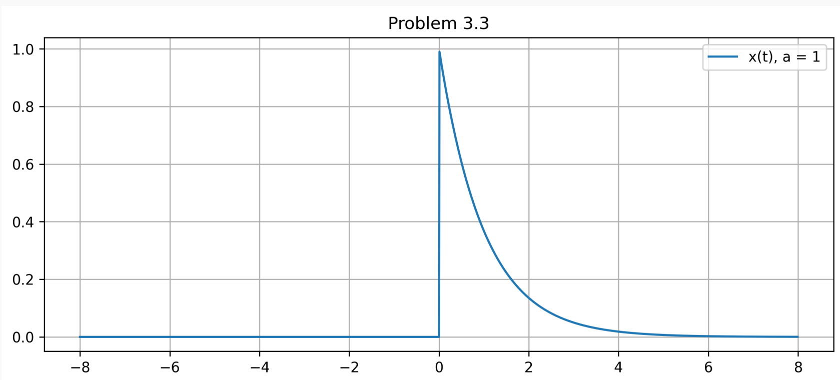
The integral converges to 0, so the power is 0.

Thus, the signal $x(t) = \exp(-at)u(t)$, $a > 0$ is **a energy signal** with energy $E = \frac{1}{2a}$.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-8, 8, 0.01)
x = np.exp(-t) * unit_signal_vectorize(t)

plt.title("Problem 3.3")
plt.plot(t, x, label="x(t), a = 1")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

The plot of the signal is shown below:



d) $x(t) = A\exp(bt)u(t)$, $b > 0$

Solution. Consider the energy of the signal:

$$
\begin{aligned}
E &= \lim_{N\to\infty} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{-N}^{N} |A\exp(bt)u(t)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} |A\exp(bt)|^2 dt \\
&= \lim_{N\to\infty} \int_{0}^{N} A^2\exp(2bt) dt \\
&= \lim_{N\to\infty} A^2 \left[\frac{1}{2b}\exp(2bt)\right]_0^N \\
&= \lim_{N\to\infty} A^2 \left(\frac{1}{2b}\exp(2bN) - \frac{1}{2b}\right) \\
E &= \infty
\end{aligned}
$$

The integral diverges, so the energy is infinite.

Now, consider the power of the signal:

$$
\begin{aligned}
P &= \lim_{N\to\infty} \frac{1}{2N} \int_{-N}^{N} |x(t)|^2 dt \\
&= \lim_{N\to\infty} \frac{1}{2N} A^2 \left(\frac{1}{2b}\exp(2bN) - \frac{1}{2b}\right) \\
P &= \infty
\end{aligned}
$$

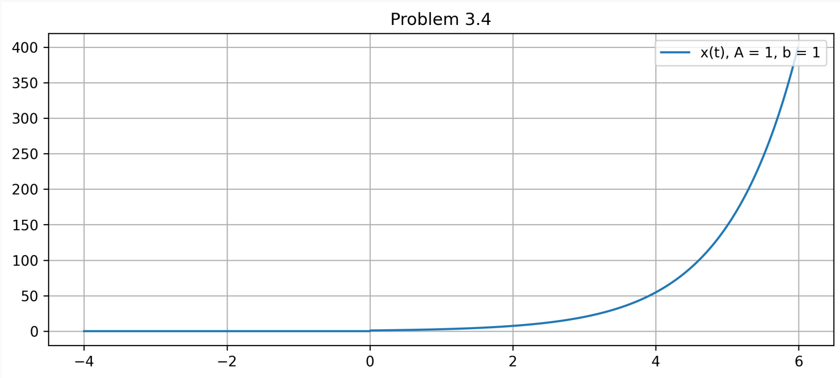
The integral diverges, so the power is infinite.

Thus, the signal $x(t) = A\exp(bt)u(t)$, $b > 0$ is **neither a energy nor a power signal**.

By using Python and Matplotlib, we can visualize the signal:

```python
import matplotlib.pyplot as plt
import numpy as np

def unit_signal(t):
    return 1.0 if t >= 0 else 0.0

unit_signal_vectorize = np.vectorize(unit_signal)

fig = plt.figure(figsize=(10, 4))

t = np.arange(-4, 6, 0.01)
x = np.exp(t) * unit_signal_vectorize(t)

plt.title("Problem 3.4")
plt.plot(t, x, label="x(t), A = 1, b = 1")
plt.grid(True)
plt.legend(loc="upper right")
plt.show()
```

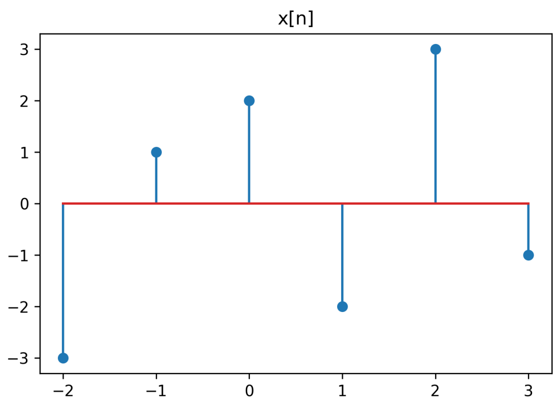
The plot of the signal is shown below:



Problem 4. For the discrete time signal $x[n]$ shown in Figure below, sketch each of the following

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

plt.stem(t, x_t)
plt.title('x[n]')
plt.show()
```

With the resulting plot shown below:



Solution. By using Python, we can create a function to transform the signal based on the given transformation function:

```python
def transform_signal(x, n, f):
    """Return x[f(n)] for any discrete-time signal x[n].
    """
    f_n = f(n)
    f_n_int = f_n[np.floor(f_n) == f_n]

    x_new = np.zeros(f_n_int.shape[0], dtype=float)

    idx = 0
    for i, val in enumerate(f_n):
        if int(val) == val:
            x_new[idx] = x[i]
            idx += 1

    return x_new, f_n_int
```

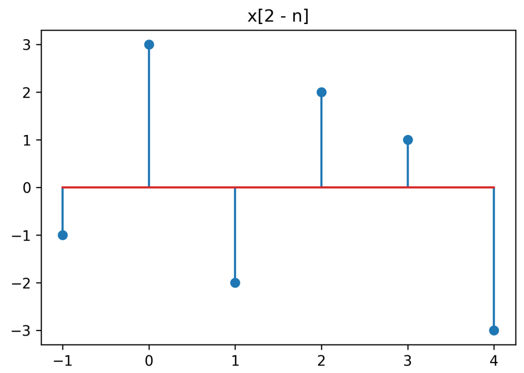
TO SUBMIT

a) $x[2-n]$

Solution. Using Python and Matplotlib to plot the signal $x[2-n]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: 2 - x)

plt.stem(t, x_t)
plt.title("x[2 - n]")
plt.show()
```

With the resulting plot shown below:

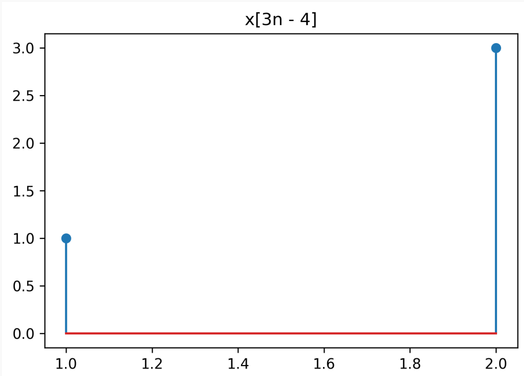


b) $x[3n-4]$

Solution. Using Python and Matplotlib to plot the signal $x[3n-4]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: (x + 4) / 3)

plt.stem(t, x_t)
plt.title("x[3n - 4]")
plt.show()
```

With the resulting plot shown below:



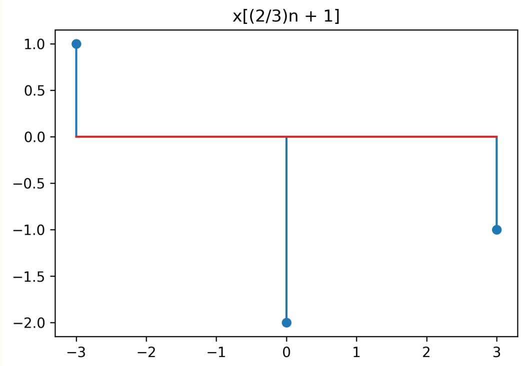
TO SUBMIT

c) $x\left[\frac{2}{3}n+1\right]$

Solution. Using Python and Matplotlib to plot the signal $x[\frac{2}{3}n+1]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: (x - 1) * 3 /
    2)

plt.stem(t, x_t)
plt.title("x[(2/3)n + 1]")
plt.show()
```

With the resulting plot shown below:

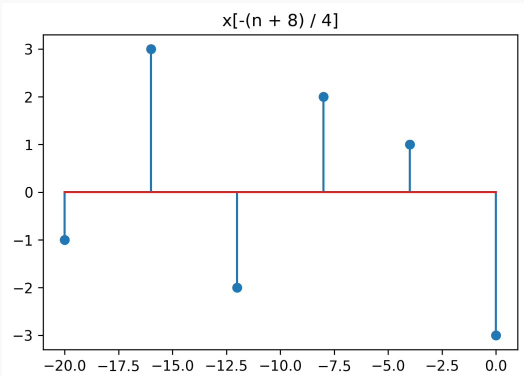


d) $x\left[-\frac{n+8}{4}\right]$

Solution. Using Python and Matplotlib to plot the signal $x\left[-\frac{n+8}{4}\right]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: (-4 * x) - 8)

plt.stem(t, x_t)
plt.title("x[-(n + 8) / 4]")
plt.show()
```

With the resulting plot shown below:



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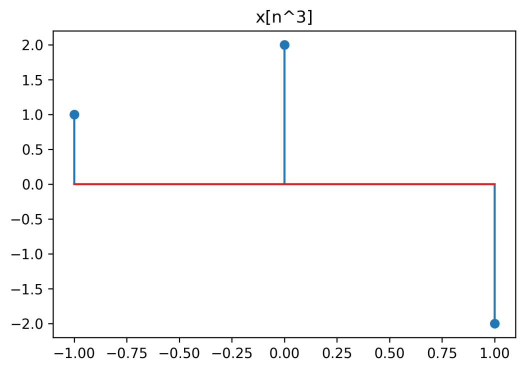
e) $x[n^3]$

Solution. Using Python and Matplotlib to plot the signal $x[n^3]$:

```python
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t, t = transform_signal(x_t, t, lambda x: np.cbrt(x))

plt.stem(t, x_t)
plt.title("x[n^3]")
plt.show()
```

With the resulting plot shown below:



f) $x[2-n] + x[3n-4]$

Solution. Introduce a helper function to add two signals:

```python
def add_discrete_signals(x1, t1, x2, t2):
    """Return x1[n] + x2[n] for any discrete-time signals
     x1[n] and x2[n]."""
    t = np.union1d(t1, t2)
    x = np.zeros(t.shape[0], dtype=float)

    for i, val in enumerate(t):
        if val in t1:
            x[i] += x1[np.where(t1 == val)[0][0]]
        if val in t2:
            x[i] += x2[np.where(t2 == val)[0][0]]

    return x, t
```

Using Python and Matplotlib to plot the signal $x[n^3]$:

```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure(figsize=(6, 4))

t = np.arange(-2, 4)
x_t = np.array([-3, 1, 2, -2, 3, -1])

x_t_1, t_1 = transform_signal(x_t, t, lambda x: 2 - x)
x_t_2, t_2 = transform_signal(x_t, t, lambda x: (x + 4) /
    3)

x_t, t = add_discrete_signals(x_t_1, t_1, x_t_2, t_2)

plt.stem(t, x_t)
plt.title("x[2-n] + x[3n-4]")
plt.show()
```

With the resulting plot shown below: