Homework Signal 2

Week 2

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Collaborators. ChatGPT (for LaTeX styling and grammar checking)

1 Convolution

Problem 2. Determine the convolution $y(t) = h(t) * x(t)$ using Graphical Interpretation of the pairs of the signals shown

Solution. The convolution $y(t) = h(t) * x(t)$ can be determined graphically by following these steps:

1. Flip one of the signals, typically $h(t)$, to get $h(-\tau)$.
2. Shift the flipped signal by t to get $h(t - \tau)$.
3. For each value of t, calculate the area of overlap between $x(\tau)$ and $h(t - \tau)$.
4. The value of the convolution $y(t)$ at each t is the area of overlap calculated in the previous step.

Using Python to visualize and compute the convolution graphically, we can create an animation that demonstrates the convolution process step-by-step.

The resulting convolution $y(t)$ is shown in the gif files in my GitHub repository for this homework.

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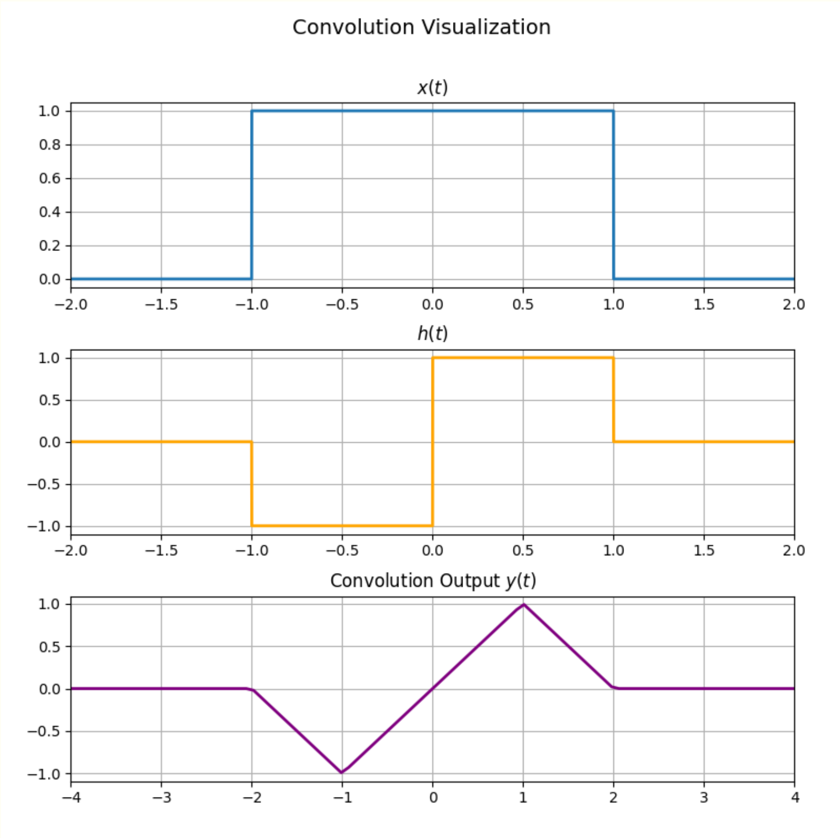
2.1 **Solution.**

Using Python to visualize and compute the convolution graphically, we can create an animation that demonstrates the convolution process step-by-step as shown in the gif files in Problem 2.1 Animation.

The plot of the signal is shown below:



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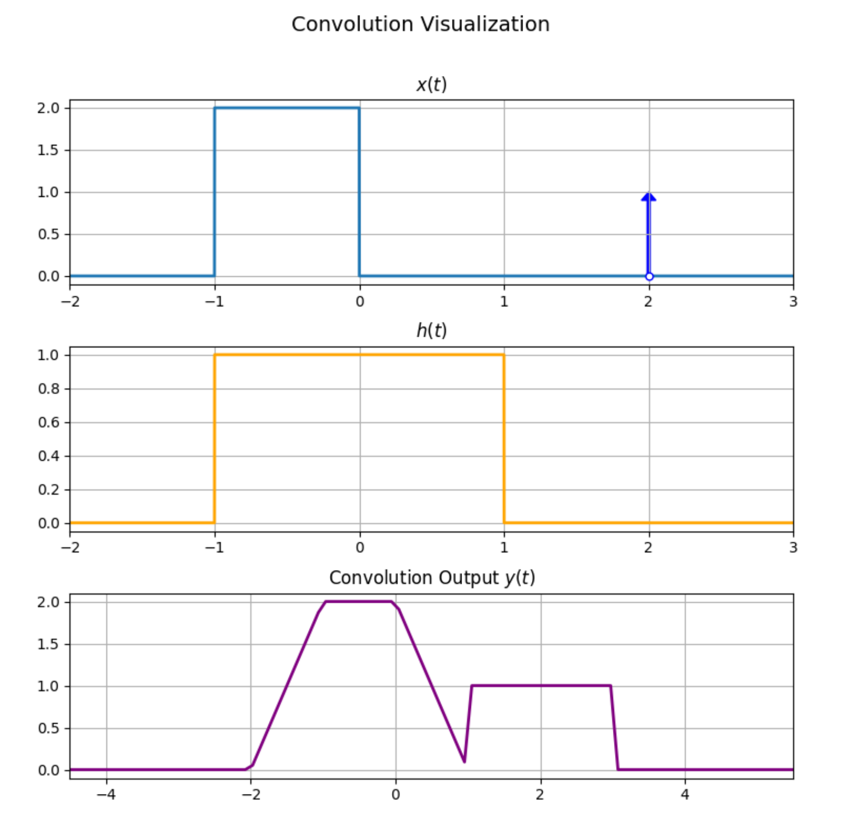
2.3 **Solution.**

Using Python to visualize and compute the convolution graphically, we can create an animation that demonstrates the convolution process step-by-step as shown in the gif files in Problem 2.3 Animation.

The plot of the signal is shown below:



Problem 4. Find the convolution $y[n] = h[n] * x[n]$ of the following signals:

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4.1 $x[n] = \begin{cases} -1, -5 \leq n \leq -1 \\ 1, 0 \leq n \leq 4 \end{cases}$, $h[n] = 2u[n]$

Solution. Using the property of convolution with unit step function (Running sum):

$$y[n] = x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

Consider the value of $y[n]$:

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= x[n] * 2u[n] \\ y[n] &= 2 \sum_{k=-\infty}^{n} x[k] \end{aligned}$$

Calculating the convolution for different ranges of n:

- For $-5 \leq n < 0$:

$$\begin{aligned} y[n] &= 2 \sum_{k=-\infty}^{n} x[k] \\ &= 2 \sum_{k=-5}^{n} (-1) \\ &= 2 \cdot (-1)(n - (-5) + 1) \\ &= 2(-n - 6) \\ y[n] &= -2n - 12 \end{aligned}$$

- For $0 \leq n < 5$:

$$\begin{aligned} y[n] &= 2 \sum_{k=-\infty}^{n} x[k] \\ &= 2 \left[\sum_{k=-5}^{-1} x[k] + \sum_{k=0}^{n} x[k] \right] \\ &= 2 \left[\sum_{k=-5}^{-1} (-1) + \sum_{k=0}^{n} (1) \right] \\ &= 2 \left(-5 + (n + 1) \right) \\ &= 2 \left(n - 4 \right) \\ y[n] &= 2n - 8 \end{aligned}$$

Thus, the final result of the convolution is:

$$\boxed{y[n] = \begin{cases} -2n - 12 & -5 \leq n < 0 \\ 2n - 8 & 0 \leq n < 5 \\ 0 & \text{otherwise} \end{cases}}$$

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4.3 $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $h[n] = \delta[n] + \delta[n-1] + \left(\frac{1}{3}\right)^n u[n]$

Solution. Using the property of convolution with unit step function (Running sum):

$$y[n] = x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

and the shifting property of convolution:

$$x[n] * \delta[n - n_0] = x[n - n_0]$$

Consider the value of $y[n]$:

$$\begin{aligned}
y[n] &= x[n] * h[n] \\
&= x[n] * \left[\delta[n] + \delta[n-1] + \left(\frac{1}{3}\right)^n u[n]\right] \\
&= (x[n] * \delta[n]) + (x[n] * \delta[n-1]) + \left(x[n] * \left(\frac{1}{3}\right)^n u[n]\right) \\
y[n] &= \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1] + \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^k u[k] \left(\frac{1}{3}\right)^{n-k} u[n-k]
\end{aligned}$$

Calculating the convolution for different ranges of n:

- For $n = 0$:

$$\begin{aligned}
y[n] &= \left(\frac{1}{2}\right)^0 u[0] + \left(\frac{1}{2}\right)^{0-1} u[0-1] + \sum_{k=-\infty}^{0} \left(\frac{1}{2}\right)^k u[k] \left(\frac{1}{3}\right)^{0-k} u[0-k] \\
&= 1 + 0 + \left(\frac{1}{2}\right)^0 u[0] \left(\frac{1}{3}\right)^0 u[0] \\
&= 1 + 0 + 1 \\
y[n] &= 2
\end{aligned}$$

- For $n \geq 1$:

$$\begin{aligned}
y[n] &= \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1] + \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^k u[k] \left(\frac{1}{3}\right)^{n-k} u[n-k] \\
&= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} + \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{3}\right)^{n-k} \\
&= 3\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n \left[\sum_{k=0}^{n} \left(\frac{3}{2}\right)^k\right] \\
&= 3\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n \cdot (-2)\left(1 - \left(\frac{3}{2}\right)^{n+1}\right) \\
y[n] &= 6\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n
\end{aligned}$$

Thus, the final result of the convolution is:

$$\boxed{y[n] = \begin{cases} 2 & n = 0 \\ 6\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n & n \geq 1 \\ 0 & \text{otherwise} \end{cases}}$$