# Homework Optimize 2

Week 7

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## 1 Simplex Method & Two-Phase Method

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**TO SUBMIT**

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**Problem 4.** Formulate the following problem into LP and solve it. You can use any method you want.

Hamtaro likes to eat sunflower seed. He wants to eat it as much as possible. There are two types of sunflower seed: the regular one and the low-fat one. A gram of regular sunflower seed contains 0.5g of fat, 0.2g of protein, and 0.1g of fiber. A gram of low-fat sunflower seed contains 0.3g of fat, 0.3g of protein, and 0.15g of fiber.

Hamtaro should get at most 11 grams of fat and at most 8 grams of protein per day. To be healthy, he should get at least 3 grams of fiber per day. Find the maximum possible total amount of sunflower seed he can eat in a day.

**Solution.** Let $x_1$ be the grams of regular sunflower seed and $x_2$ be the grams of low-fat sunflower seed that Hamtaro eats in a day. The objective is to maximize the total amount of sunflower seed he can eat:

$$\max z = x_1 + x_2$$

The constraints based on fat, protein, and fiber intake are as follows:

$$
\begin{aligned}
\text{subject to} \quad 0.5x_1 + 0.3x_2 &\leq 11 \quad \text{(fat constraint)} \\
0.2x_1 + 0.3x_2 &\leq 8 \quad \text{(protein constraint)} \\
0.1x_1 + 0.15x_2 &\geq 3 \quad \text{(fiber constraint)} \\
x_1, x_2 &\geq 0 \quad \text{(non-negativity constraint)}
\end{aligned}
$$

Since, the third constraint is a 'greater than or equal to' type, we should use the two-phase method to solve this LP.

Re-write the standard form by introducing slack variables $s_1, s_2$, excess variable $e_1$, and artificial variable $a_1$:

$$
\begin{aligned}
\max \quad & z = x_1 + x_2 \\
\text{subject to} \quad & 0.5x_1 + 0.3x_2 + s_1 = 11 \\
& 0.2x_1 + 0.3x_2 + s_2 = 8 \\
& 0.1x_1 + 0.15x_2 - e_1 + a_1 = 3 \\
& x_1, x_2, s_1, s_2, e_1, a_1 \geq 0
\end{aligned}
$$

Using Two-Phase Method, we first solve Phase 1 by minimizing the sum of artificial variables:

$$\min w = a_1$$

Initial simplex tableau for Phase 1:

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | $a_1$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_1$ | 0.5 | 0.3 | 1 | 0 | 0 | 0 | 11 |
| $s_2$ | 0.2 | 0.3 | 0 | 1 | 0 | 0 | 8 |
| $a_1$ | 0.1 | 0.15 | 0 | 0 | −1 | 1 | 3 |
| $w$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

Becase we have to eliminate $a_1$ from the $w$-row, we perform the row operation:

$$R_w \leftarrow R_w - R_{a_1}$$

The updated tableau:

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | $a_1$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_1$ | 0.5 | 0.3 | 1 | 0 | 0 | 0 | 11 |
| $s_2$ | 0.2 | 0.3 | 0 | 1 | 0 | 0 | 8 |
| $a_1$ | 0.1 | 0.15 | 0 | 0 | $-1$ | 1 | 3 |
| $w$ | $-0.1$ | $-0.15$ | 0 | 0 | 1 | 0 | $-3$ |

Starting at basic feasible solution $(x_1, x_2, s_1, s_2, e_1, a_1) = (0, 0, 11, 8, 0, 3)$.

1. Entering variable: $x_2$ (most negative in $w$-row).

2. Leaving variable: $a_1$ (minimum ratio test: $3/0.15 = 20$).

3. Pivot on $(3, 2)$ to update tableau.

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | $a_1$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_1$ | 0.3 | 0 | 1 | 0 | 2 | $-2$ | 5 |
| $s_2$ | 0 | 0 | 0 | 1 | 2 | $-2$ | 2 |
| $a_1$ | $\frac{2}{3}$ | 1 | 0 | 0 | $-\frac{20}{3}$ | 1 | 20 |
| $w$ | 0 | 0 | 0 | 0 | 0 | $\frac{20}{3}$ | 0 |

Since there are no negative coefficients in the $w$-row and $w = 0$, we proceed to Phase 2 by removing the artificial variable $a_1$ and solving the original objective function.

Initial simplex tableau for Phase 2:

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | RHS |
|---|---|---|---|---|---|---|
| $s_1$ | 0.3 | 0 | 1 | 0 | 2 | 5 |
| $s_2$ | 0 | 0 | 0 | 1 | 2 | 2 |
| $x_2$ | $\frac{2}{3}$ | 1 | 0 | 0 | $-\frac{20}{3}$ | 20 |
| $z$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 |

Becase we have to adjust the $z$-row according to the current basis, we perform the row operation:

$$R_z \leftarrow R_z + R_{x_2}$$

The updated tableau:

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | RHS |
|---|---|---|---|---|---|---|
| $s_1$ | 0.3 | 0 | 1 | 0 | 2 | 5 |
| $s_2$ | 0 | 0 | 0 | 1 | 2 | 2 |
| $x_2$ | $\frac{2}{3}$ | 1 | 0 | 0 | $-\frac{20}{3}$ | 20 |
| $z$ | $-\frac{1}{3}$ | 0 | 0 | 0 | $-\frac{20}{3}$ | 20 |

Starting at basic feasible solution $(x_1, x_2, s_1, s_2, e_1) = (0, 20, 5, 2, 0)$.

1. Entering variable: $e_1$ (most negative in $z$-row).

2. Leaving variable: $s_2$ (minimum ratio test: $2/2 = 1$).

3. Pivot on $(2, 5)$ to update tableau.

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | RHS |
|---|---|---|---|---|---|---|
| $s_1$ | 0.3 | 0 | 1 | 0 | 0 | 3 |
| $s_2$ | 0 | 0 | 0 | $\frac{1}{2}$ | 1 | 1 |
| $x_2$ | $\frac{2}{3}$ | 1 | 0 | $\frac{10}{3}$ | 0 | $\frac{80}{3}$ |
| $z$ | $-\frac{1}{3}$ | 0 | 0 | $\frac{10}{3}$ | 0 | $\frac{80}{3}$ |

Then,

1. Entering variable: $x_1$ (most negative in $z$-row).

2. Leaving variable: $s_1$ (minimum ratio test: $3/0.3 = 10$).

3. Pivot on $(1,1)$ to update tableau.

| Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $e_1$ | RHS |
|---|---|---|---|---|---|---|
| $s_1$ | 1 | 0 | $\frac{10}{3}$ | 0 | 0 | 10 |
| $s_2$ | 0 | 0 | 0 | $\frac{1}{2}$ | 1 | 1 |
| $x_2$ | 0 | 1 | $-\frac{20}{9}$ | $\frac{10}{3}$ | 0 | 20 |
| $z$ | 0 | 0 | $\frac{10}{9}$ | $\frac{10}{3}$ | 0 | 30 |

Since there are no negative coefficients in the $z$-row, the optimal solution is reached:

$$\boxed{(x_1, x_2) = (10, 20) \text{ with } z = 30}.$$