Homework Signal 3

Week 3

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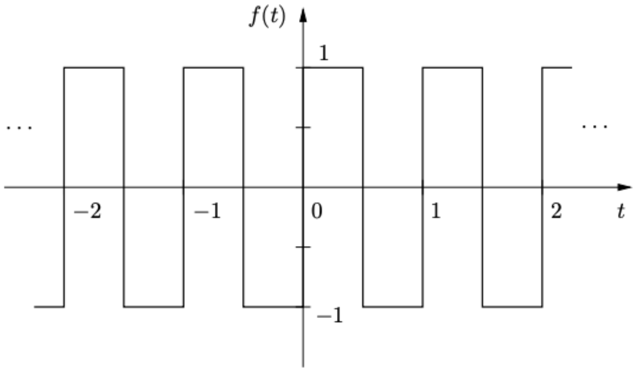
Collaborators. ChatGPT (for LaTeX styling and grammar checking)

1 Fourier Series

Problem 1. Find the Fourier series of the following periodic function:



Solution. The signal in the image is a periodic signum function with period $T = 1$ and amplitude $A = 1$. The function can be defined as:

$$x(t) = \begin{cases} 1, & 0 \leq t < 0.5 \\ -1, & 0.5 \leq t < 1 \end{cases}$$

To find the Fourier series coefficients, we use the formulas for a_0, a_n, and b_n:

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos\left(\frac{2\pi nt}{T}\right) dt$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin\left(\frac{2\pi nt}{T}\right) dt$$

Calculating a_0:

$$\begin{aligned} a_0 &= \frac{1}{1} \int_0^1 x(t)\, dt \\ &= \frac{1}{1} \left(\int_0^{0.5} 1\, dt + \int_{0.5}^1 -1\, dt \right) \\ &= (0.5 - 0.5) \\ a_0 &= 0 \end{aligned}$$

Calculating a_n:

$$\begin{aligned}
a_n &= \frac{2}{1}\int_0^1 x(t)\cos(2\pi nt)\,dt \\
&= 2\left(\int_0^{0.5}(1)\cdot\cos(2\pi nt)\,dt + \int_{0.5}^{1}(-1)\cdot\cos(2\pi nt)\,dt\right) \\
&= 2\left(\left[\frac{\sin(2\pi nt)}{2\pi n}\right]_0^{0.5} - \left[\frac{\sin(2\pi nt)}{2\pi n}\right]_{0.5}^{1}\right) \\
&= \frac{2}{2\pi n}\left[(\sin(\pi n)-\sin(0)) - (\sin(2\pi n)-\sin(\pi n))\right] \\
&= \frac{1}{\pi n}\left[0\right] \\
a_n &= 0
\end{aligned}$$

Calculating b_n:

$$\begin{aligned}
b_n &= \frac{2}{1}\int_0^1 x(t)\sin(2\pi nt)\,dt \\
&= 2\left(\int_0^{0.5}(1)\cdot\sin(2\pi nt)\,dt + \int_{0.5}^{1}(-1)\cdot\sin(2\pi nt)\,dt\right) \\
&= 2\left(\left[-\frac{\cos(2\pi nt)}{2\pi n}\right]_0^{0.5} - \left[-\frac{\cos(2\pi nt)}{2\pi n}\right]_{0.5}^{1}\right) \\
&= \frac{2}{2\pi n}\left[(-\cos(\pi n)+\cos(0)) - (-\cos(2\pi n)+\cos(\pi n))\right] \\
&= \frac{1}{\pi n}\left[2\left(1-\cos(\pi n)\right)\right] \\
&= \frac{2}{\pi n}\left(1-(-1)^n\right) \\
b_n &= \begin{cases} \frac{4}{\pi n}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}
\end{aligned}$$

Because, in term of a_0, a_n, b_n, we have:

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(2\pi nt) + b_n\sin(2\pi nt)\right)$$

and we found that $a_0 = 0$, $a_n = 0$, and $b_n = \frac{4}{\pi n}$ for odd n and 0 for even n. Therefore, the Fourier series representation of the signal is:

$$x(t) = \sum_{n=1, n \text{ odd}}^{\infty} \frac{4}{\pi n}\sin(2\pi nt)$$

Problem 2. Find the Fourier Series (FS) of the periodic function $x(t)$ which are provided as follows.

2.1 $x(t) = \frac{\pi t^3}{2};\ -1 < t < 1$

Solution.

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2.2 $x(t) = \pi - t;\ -\pi \leq t \leq \pi$

Solution.

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2.3 $x(t) = t^2 + \sin^3(\pi t);\ -1 \leq t \leq 1$

Solution.