Homework Signal 2

Week 2

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Collaborators. ChatGPT (for LaTeX styling and grammar checking)

1 Convolution

Problem 1. Evaluate the convolution of the following signals

a) $\operatorname{rect}\left(\frac{t-a}{a}\right) * \delta(t-b)$

Solution. From the sifting property of the delta function, we have:

$$f(t) * \delta(t-b) = f(t-b)$$

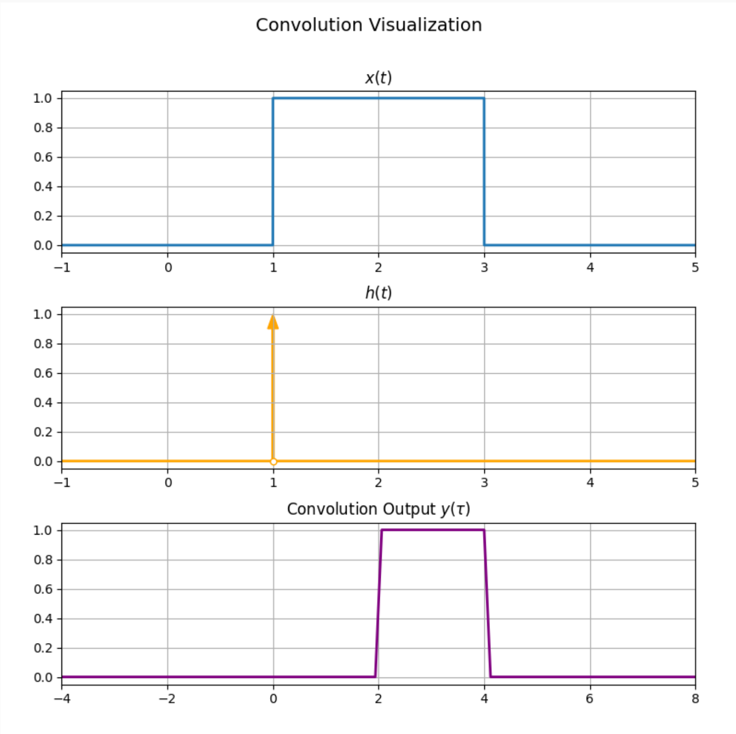
Applying this property to our problem, we get:

$$\operatorname{rect}\left(\frac{t-a}{a}\right) * \delta(t-b) = \operatorname{rect}\left(\frac{(t-b)-a}{a}\right) = \operatorname{rect}\left(\frac{t-(a+b)}{a}\right)$$

Thus, the result of the convolution is:

$$\boxed{\operatorname{rect}\left(\frac{t-(a+b)}{a}\right)}$$

Using Python to verify this result, we can implement the convolution and plot the results. The plot of the signal is shown below:



b) $\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right)$

Solution. To evaluate the convolution of two rectangular functions, we start with the definition of the rectangular function:

$$\text{rect}\left(\frac{t}{a}\right) = \begin{cases} 1 & \text{if } |t| \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

The convolution of two functions $f(t)$ and $g(t)$ is defined as:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\, d\tau$$

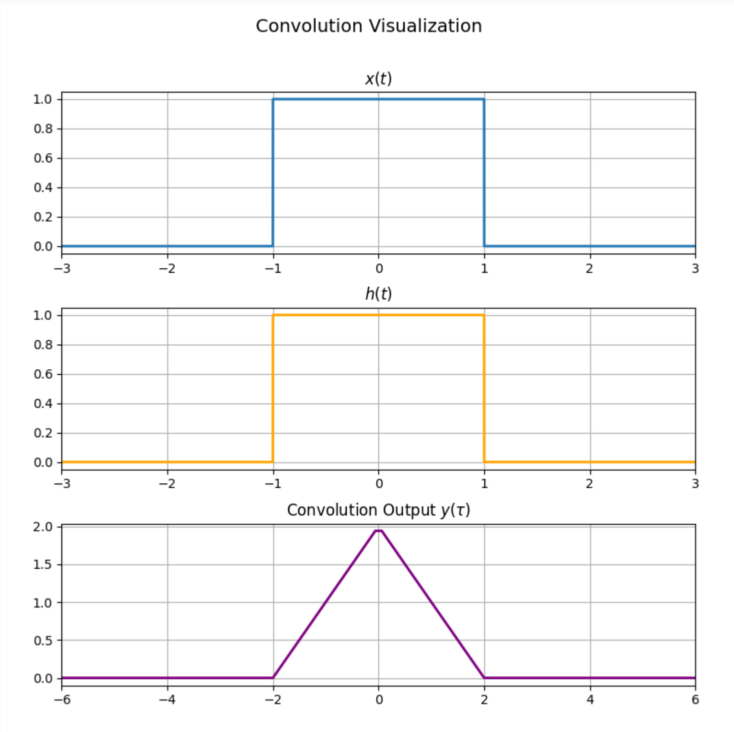
Applying this to our rectangular functions, we have:

$$\begin{aligned} (\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right))(t) &= \int_{-\infty}^{\infty} \text{rect}\left(\frac{\tau}{a}\right) \text{rect}\left(\frac{t-\tau}{a}\right) d\tau \\ &= \int_{-\frac{a}{2}}^{\frac{a}{2}} \text{rect}\left(\frac{t-\tau}{a}\right) d\tau \\ &= \int_{\max(-\frac{a}{2}, t-\frac{a}{2})}^{\min(\frac{a}{2}, t+\frac{a}{2})} 1\, d\tau \\ (\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right))(t) &= \min\left(\frac{a}{2}, t+\frac{a}{2}\right) - \max\left(-\frac{a}{2}, t-\frac{a}{2}\right) \end{aligned}$$

Evaluating the limits, we find that the result is a triangular function:

$$\boxed{\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right) = \begin{cases} 0 & |t| > a \\ t+a & -a \leq t < 0 \\ a-t & 0 \leq t \leq a \end{cases}}$$

Using Python to verify this result, we can implement the convolution and plot the results. The plot of the signal is shown below:



c) $t[u(t) - u(t-1)] * u(t)$

Solution. First, we define the functions involved in the convolution:

$$x(t) = t[u(t) - u(t-1)] = \begin{cases} 0 & t < 0 \\ t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \ge 0 \end{cases}$$

The convolution $y(t) = x(t) * u(t)$ is given by:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) u(t - \tau)\, d\tau$$

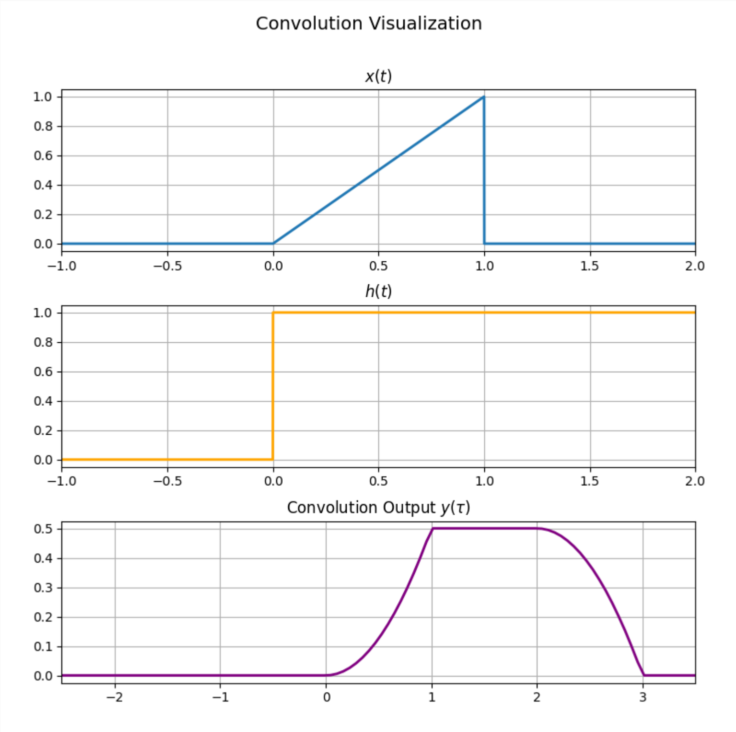
Evaluating the convolution integral, we find:

$$y(t) = \int_{0}^{1} \tau \cdot u(t - \tau)\, d\tau$$
$$y(t) = \int_{0}^{\min(t,1)} \tau\, d\tau$$

Thus,

$$\boxed{y(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \le t < 1 \\ \frac{1}{2} & t \ge 1 \end{cases}}$$

Using Python to verify this result, we can implement the convolution and plot the results. The plot of the signal is shown below:



Problem 2. Determine the convolution $y(t) = h(t) * x(t)$ using Graphical Interpretation of the pairs of the signals shown