

# Homework Signal 2

Week 2

6733172621 Patthadon Phengpinij

*Collaborators.* ChatGPT (for L<sup>A</sup>T<sub>E</sub>X styling and grammar checking)

## 1 Representing Signals

**Problem 1.** Evaluate the convolution of the following signals

a)  $\text{rect}\left(\frac{t-a}{a}\right) * \delta(t-b)$

**Solution.** From the sifting property of the delta function, we have:

$$f(t) * \delta(t-b) = f(t-b)$$

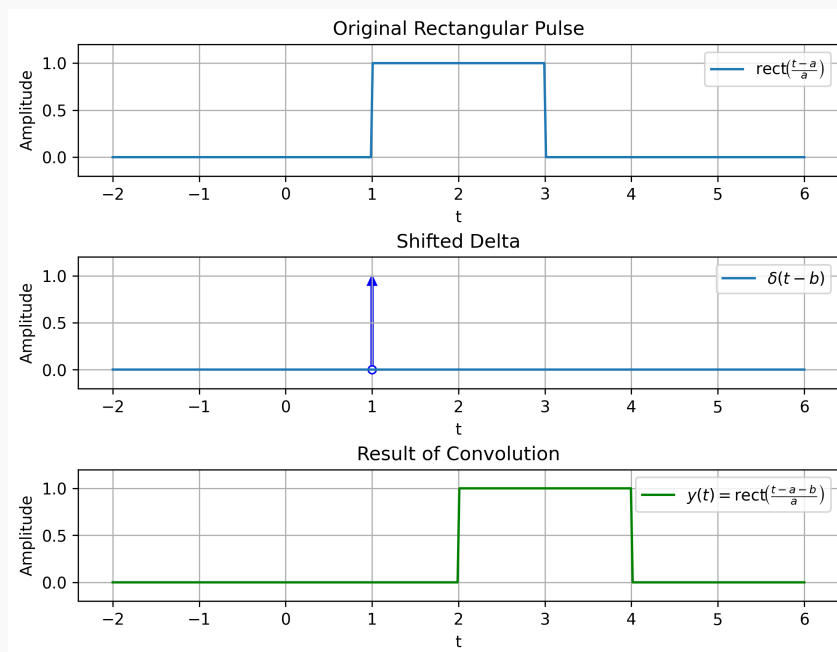
Applying this property to our problem, we get:

$$\text{rect}\left(\frac{t-a}{a}\right) * \delta(t-b) = \text{rect}\left(\frac{(t-b)-a}{a}\right) = \text{rect}\left(\frac{t-(a+b)}{a}\right)$$

Thus, the result of the convolution is:

$$\text{rect}\left(\frac{t-(a+b)}{a}\right)$$

Using Python to verify this result, we can implement the convolution and plot the results. The plot of the signal is shown below:



b)  $\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right)$

**Solution.** To evaluate the convolution of two rectangular functions, we start with the definition of the rectangular function:

$$\text{rect}\left(\frac{t}{a}\right) = \begin{cases} 1 & \text{if } |t| \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

The convolution of two functions  $f(t)$  and  $g(t)$  is defined as:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau$$

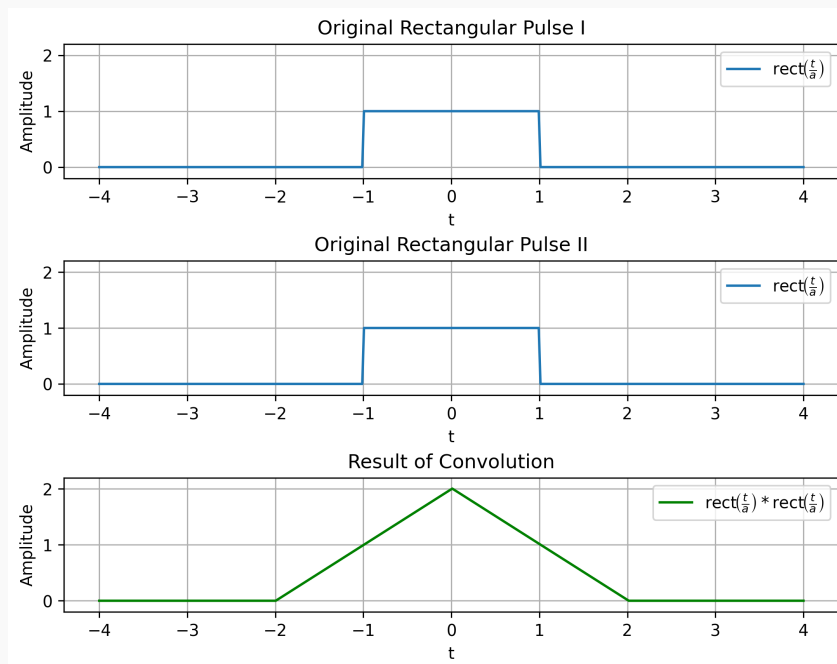
Applying this to our rectangular functions, we have:

$$\begin{aligned} \left(\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right)\right)(t) &= \int_{-\infty}^{\infty} \text{rect}\left(\frac{\tau}{a}\right) \text{rect}\left(\frac{t - \tau}{a}\right) d\tau \\ &= \int_{-\frac{a}{2}}^{\frac{a}{2}} \text{rect}\left(\frac{t - \tau}{a}\right) d\tau \\ &= \int_{\max(-\frac{a}{2}, t - \frac{a}{2})}^{\min(\frac{a}{2}, t + \frac{a}{2})} 1 d\tau \\ \left(\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right)\right)(t) &= \min\left(\frac{a}{2}, t + \frac{a}{2}\right) - \max\left(-\frac{a}{2}, t - \frac{a}{2}\right) \end{aligned}$$

Evaluating the limits, we find that the result is a triangular function:

$$\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right) = \begin{cases} 0 & |t| > a \\ t + a & -a \leq t < 0 \\ a - t & 0 \leq t \leq a \end{cases}$$

Using Python to verify this result, we can implement the convolution and plot the results. The plot of the signal is shown below:



c)  $t[u(t) - u(t-1)] * u(t)$

**Solution.** First, we define the functions involved in the convolution:

$$x(t) = t[u(t) - u(t-1)] = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

The convolution  $y(t) = x(t) * u(t)$  is given by:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau) d\tau$$

Evaluating the convolution integral, we find:

$$y(t) = \int_0^1 \tau \cdot u(t-\tau) d\tau$$

$$y(t) = \int_0^{\min(t,1)} \tau d\tau$$

Thus,

$$y(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \leq t < 1 \\ \frac{1}{2} & t \geq 1 \end{cases}$$

Using Python to verify this result, we can implement the convolution and plot the results. The plot of the signal is shown below:

