Homework 3

Week 3

6733172621 Patthadon Phengpinij

Collaborators. ChatGPT (for LaTeX styling and grammar checking)

---

1 Week 3: Intro to Discrete Random Variable

Problem 1. A complex bet on a race has a payout value, X, determined by which tier of horse wins. The probability mass function (PMF) is given below:

Payout Value, k (\$)	0	10	50	100
pmf $P(X = k)$	0.65	?	0.10	0.05

a) What is the probability of winning a \$10 payout, $P(X = 10)$?
b) Calculate the cumulative distribution function value at 50, $F(50)$.

Solution. Because, the total PMF value for all possible outcomes is 1,

$$\sum_{\text{all}k} P(X = k) = 1$$

Thus, we can find the missing PMF value:

$$\begin{aligned} 1 &= P(X = 0) + P(X = 10) + P(X = 50) + P(X = 100) \\ 1 &= 0.65 + P(X = 10) + 0.10 + 0.05 \\ P(X = 10) &= 1 - 0.65 - 0.10 - 0.05 \\ P(X = 10) &= \boxed{0.20} \textbf{ a).} \end{aligned}$$

The cumulative distribution function (CDF) value at 50, $F(50)$, is the sum of the PMF values for all outcomes less than or equal to 50:

$$\begin{aligned} F(50) &= P(X = 0) + P(X = 10) + P(X = 50) \\ F(50) &= 0.65 + 0.20 + 0.10 \\ F(50) &= \boxed{0.95} \textbf{ b).} \end{aligned}$$

TO SUBMIT

Problem 5. A long-shot horse named "Hopeful" has only a 5% chance of winning any given race. Let X be the number of races he runs until he achieves his first win.

a) What is the probability that Hopeful wins on his 4th race?
b) What is the probability that his first win occurs after the 2nd race?

Solution. The probability of "Hopeful" winning the 4th race is the same with the probability of winning– $\boxed{0.05}$. **a).**

The probability that his first win occurs after the 2nd race is the same with the probability of losing the first 2 races,

$$P(\text{first win occurs after 2nd race}) = (0.95 \times 0.95) = \boxed{0.9025}\ \textbf{b).}$$

TO SUBMIT

Problem 7. A star jockey has a 30% chance of winning any given race. Let X be his number of wins in the 3 morning races, and Y be his number of wins in the 2 afternoon races. Assume his performance is independent between races. Let $Z = X + Y$ be his total wins for the day.

a) What is the distribution of Z and what are its parameters?
b) What is the probability that he wins exactly 2 races all day, $P(Z = 2)$?

Solution. a). The distribution of Z is a binomial distribution with parameters $n = 5$ (the total number of races) and $p = 0.3$ (the probability of winning any given race).
The probability mass function (PMF) of a binomial distribution is given by

$$P(Z = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Thus, the probability that he wins exactly 2 races all day is

$$P(Z = 2) = \binom{5}{2} (0.3)^2 (0.7)^3 = 10 \cdot 0.09 \cdot 0.343 = \boxed{0.3087}\ \textbf{b).}$$

TO SUBMIT

Problem 12. A trainer has two horses. Horse A is in a race where its probability of winning is 0.1. Horse B is in a separate race where its probability of winning is 0.2. Let Z be the total number of wins for the trainer. Find the probability mass function (PMF) for Z.

Solution. Let $X_A \sim \text{Bernoulli}(0.1)$ for Horse A, $X_B \sim \text{Bernoulli}(0.2)$ for Horse B.
Thus, $Z = X_A + X_B$ with possible values: $z \in \{0, 1, 2\}$.

- $P(Z = 0) = P(A \text{ loses})P(B \text{ loses}) = (1 - 0.1)(1 - 0.2) = 0.9 \cdot 0.8 = 0.72$
- $P(Z = 2) = P(A \text{ wins})P(B \text{ wins}) = 0.1 \cdot 0.2 = 0.02$
- $P(Z = 1) = 1 - P(Z = 0) - P(Z = 2) = 1 - 0.72 - 0.02 = 0.26$

So the PMF is:

$$\boxed{P(Z = z) = \begin{cases} 0.72, & z = 0 \\ 0.26, & z = 1 \\ 0.02, & z = 2 \\ 0, & \text{otherwise.} \end{cases}}$$