

Homework 3

Week 4-5: Fisherface

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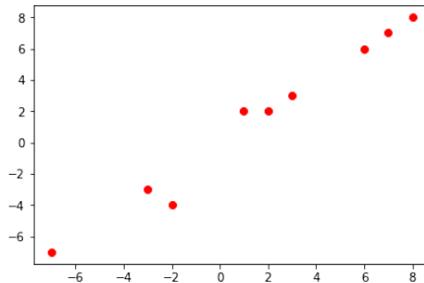
Collaborators. ChatGPT

1 Hello Soft Clustering (GMM)

Recall from HW1 we did K-means clustering. Fitting a GMM on a set of points can be considered as another method to do clustering but now with soft assignments.

Consider the same set of points we used in HW1

x	y
1	2
3	3
2	2
8	8
6	6
7	7
-3	-3
-2	-4
-7	-7



In class, we showed that we could fit a GMM on 1-dimensional data by using Expectation Maximization (EM). The algorithm for doing EM on N-dimensional GMM is very similar. The exact algorithm is as follows:

Initialization: Initialize the mixture weights, $\phi = \{m_j\}$, where j is the mixture number, means of each Gaussian, $\vec{\mu}_j$ (now a vector of N dimensions), and covariance matrices of each Gaussian, Σ_j .

Expectation: Find the soft assignments for each data point $w_{n,j}$ where n corresponds to the sample index.

$$w_{n,j} = \frac{p(x_n; \vec{\mu}_j, \Sigma_j) m_j}{\sum_j p(x_n; \vec{\mu}_j, \Sigma_j) m_j}$$

$w_{n,j}$ means the probability that data point n comes from Gaussian number j .

Maximization: Update the model parameters, ϕ , $\vec{\mu}$, Σ_j .

$$m_j = \frac{1}{N} \sum_n w_{n,j}$$

$$\vec{\mu}_j = \frac{\sum_n w_{n,j} \vec{x}_n}{\sum_n w_{n,j}}$$

$$\Sigma_j = \frac{\sum_n w_{n,j} (\vec{x}_n - \vec{\mu}_j)(\vec{x}_n - \vec{\mu}_j)^T}{\sum_n w_{n,j}}$$

The above equation is used for full covariance matrices. For our small toy example, we will use diagonal covariance matrices, which can be acquired by setting the off-diagonal values to zero. In other words, $\Sigma_{(i,j)} = 0$, for $i \neq j$.

T1. Here's problem statement for **T** problem.

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1 print("You can insert code like this")
```

You can insert sub-problems:

T1.1 Here's sub-problem 1.

T1.2 Here's sub-problem 2.

T1.3 Here's sub-problem 3.

T1.3.1 Here's sub-sub-problem 1.

T1.3.2 Here's sub-sub-problem 2.

Solution. The solution goes here.

OT1. Here's the problem statement for **OT** problem.

Solution. Here's the proof for the **OT** problem.

Proof. The proof inside the solution box goes here.