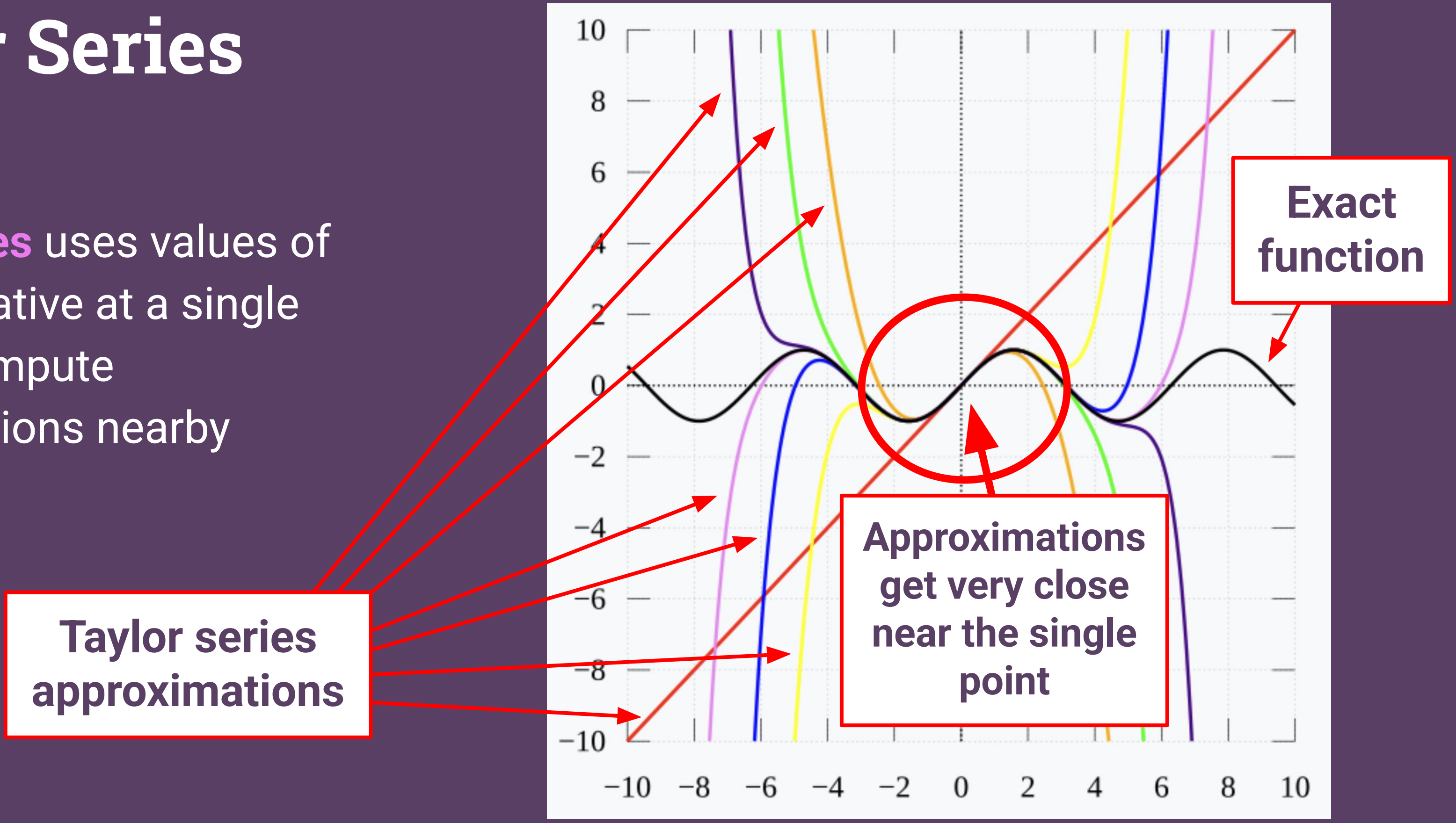
PH 366 Day 5:
Taylor Series of Electric Potential

22 Jan 2025

Taylor Series

Taylor series uses values of ***every*** derivative at a single point to compute approximations nearby



Derivatives of $f(z) = (1 + z)^p$

$$f(z) = (1+z)^p$$

$$f^{(n)}(z) = p(p-1)(p-2)\dots(p-n+1)(1+z)^{p-n}$$

```
def fn(z0, n, p=-1/2):
    product = 1
    for i in range(n):
        product *= (p - i)
    return product * (1 + z0) ** (p - n)
```

i = 0, then **i = 1, 2, 3, … , n-1**

product gets multiplied by **p,** then **(p-1)**, then **(p-2)**, … and finally **(p-n+1)**

Same as $\mathbf{f^{(n)}(z_0)}$

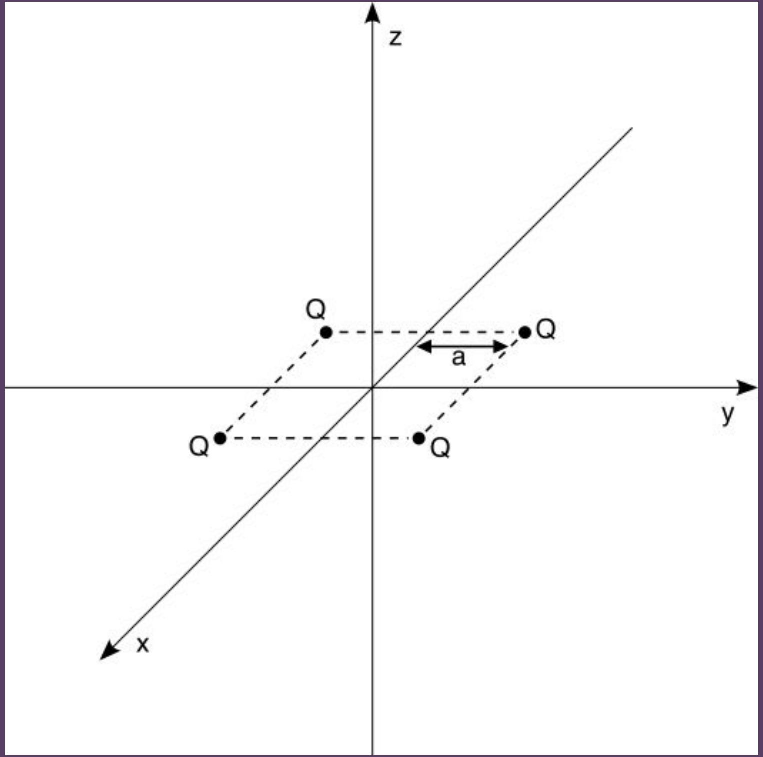
Setting up Taylor Series

If we have **z_0**, **$f^{(n)}(z_0)$**, and number of terms **N**, we can compute Taylor series:

$$f(z) = \sum_{n=0}^{n \leq N} \frac{f^{(\mathrm{n})}(z_0)}{n!}(z - z_0)^n$$

Electric Potential from Point Charges

$$V(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|}$$

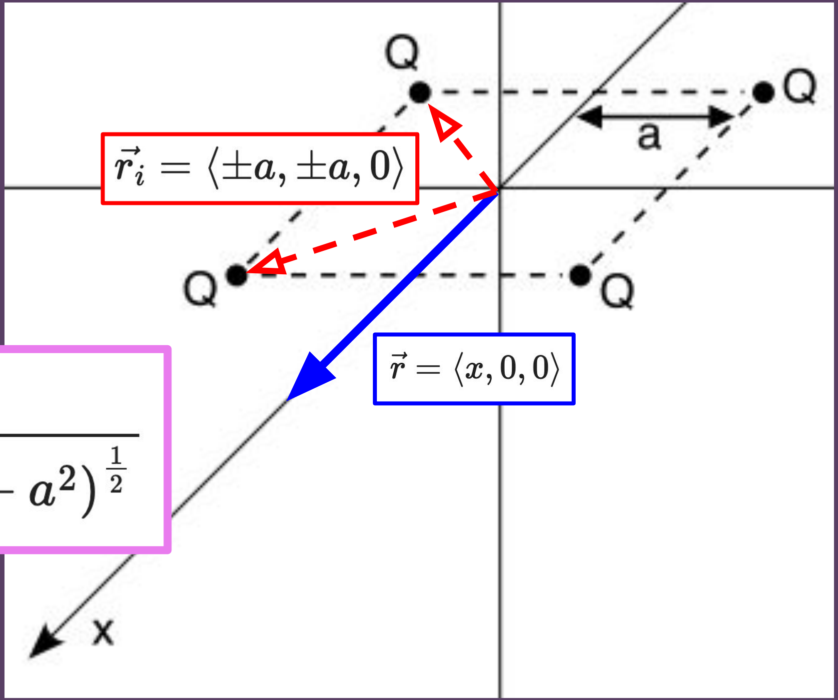


Electric Potential along x-axis

$$V(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|}$$

After some math...

$$V_x(x) = \frac{2kQ}{((x+a)^2 + a^2)^{\frac{1}{2}}} + \frac{2kQ}{((x-a)^2 + a^2)^{\frac{1}{2}}}$$



Substitutions to Use f(z)

$$V_x(x) = \frac{2kQ}{((x+a)^2+a^2)^{\frac{1}{2}}} + \frac{2kQ}{((x-a)^2+a^2)^{\frac{1}{2}}}$$

$$u := \left(\frac{x}{a}+1\right)^2, \ \ v := \left(\frac{x}{a}-1\right)^2$$

$$f(z) := (1+z)^p, \ \ p = -\frac{1}{2}$$

$$V_x(x) = \frac{2kQ}{a}(f(u)+f(v))$$

Taylor Series in Electric Potential

To set up Taylor series, we need:
u_0, v_0, $f^{(n)}(u_0)$, $f^{(n)}(v_0)$, and term limit
N

Approximating around $\mathbf{x_0 = 0}$ gives...

$u_0 = 1$, $v_0 = 1$

$f^{(n)}(u_0) = f^{(n)}(v_0) = f^{(n)}(1)$

(and we can choose value for **N**,
the higher the more precise)

$$V_x(x) = \frac{2kQ}{a}\left(f(u) + f(v)\right)$$

Replace **z** with **u**
and z_0 with u_0

Replace **z** with **v**
and z_0 with v_0

$$f(z) = \sum_{n=0}^{n \le N} \frac{f^{(\mathrm{n})}(z_0)}{n!}(z - z_0)^n$$

Coding Electric Potential from Taylor Series

```
set up constant parameters
set up x, u, v values

approximate f(u) and f(v) starting with the n=0 term

for n values from 1 through N:
    nth term of f(u) = fn(u0, n)/n! * (u - u0)^n
    nth term of f(v) = fn(v0, n)/n! * (v - v0)^n
    add each nth term to f(u) and f(v) approximations

Vx = 2kQ/a * (approx of f(u) + approx of f(v))
```