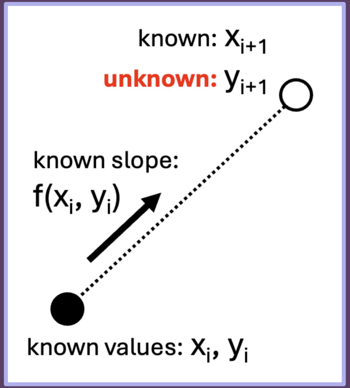
PH 366 Day 4:
Taylor Series

15 Jan 2025

Euler Method (Runge-Kutta) vs. Taylor Series

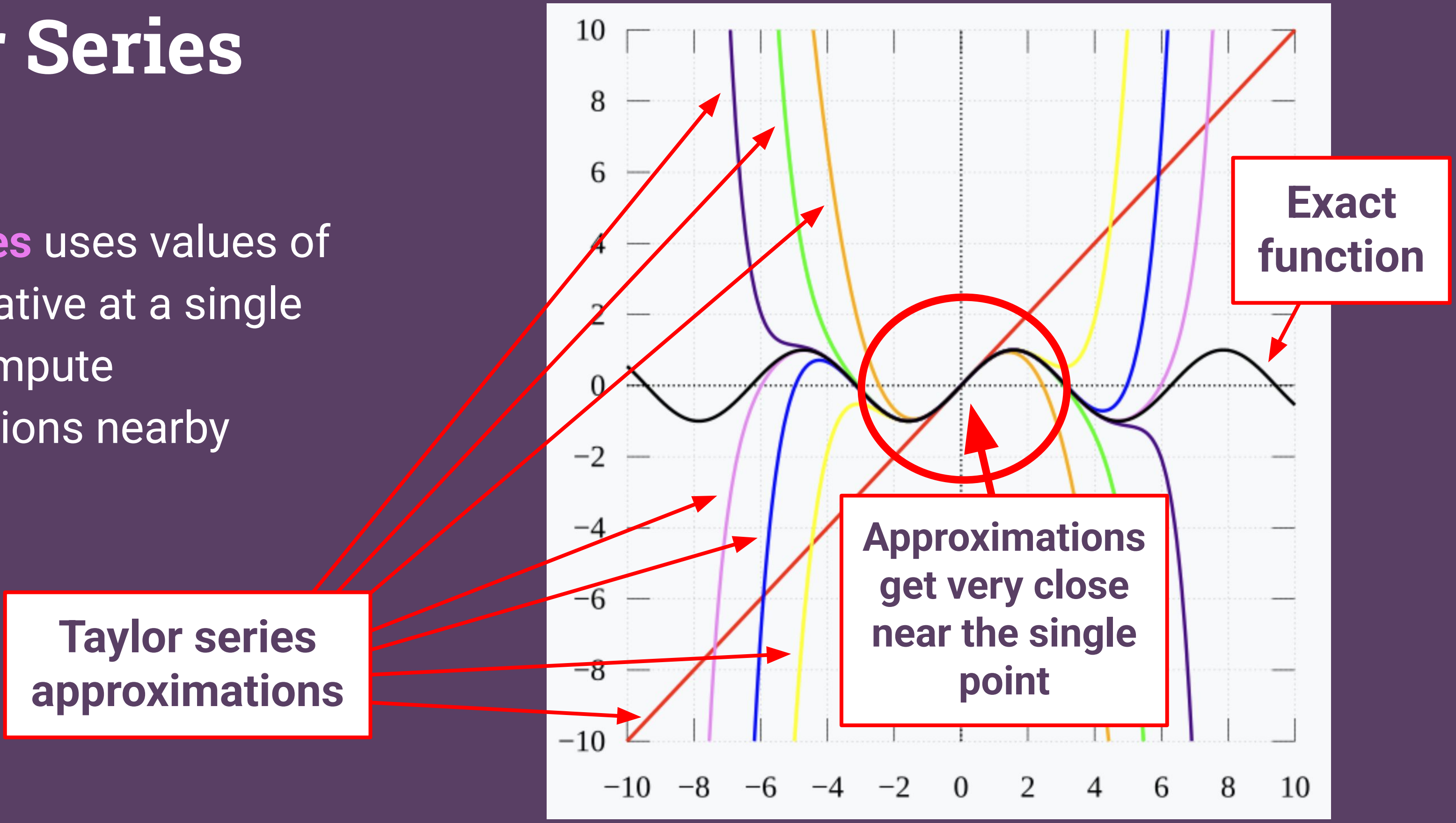
Euler and **Runge-Kutta** use values of first derivative **everywhere** to compute approximations far away



Taylor series uses values of **every** derivative at a single point to compute approximations nearby

Taylor Series

Taylor series uses values of **every** derivative at a single point to compute approximations nearby



Taylor Series Formula

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

Taylor Series Example

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

$f(z) = (1 + z)^p$

$$f^{(\mathrm{n})}(z) = p(p-1)(p-2)\ldots(p-n+1)(1+z)^{p-n}$$

Taylor series setup:

$p = -1$

$z_0 = 1$

term n=0 term n=1 term n=2 ...

$$f(z) = (1+z_0)^p + p(1+z_0)^{p-1}(z-z_0) + p(p-1)(1+z_0)^{p-2}(z-z_0)^2 + \ldots$$