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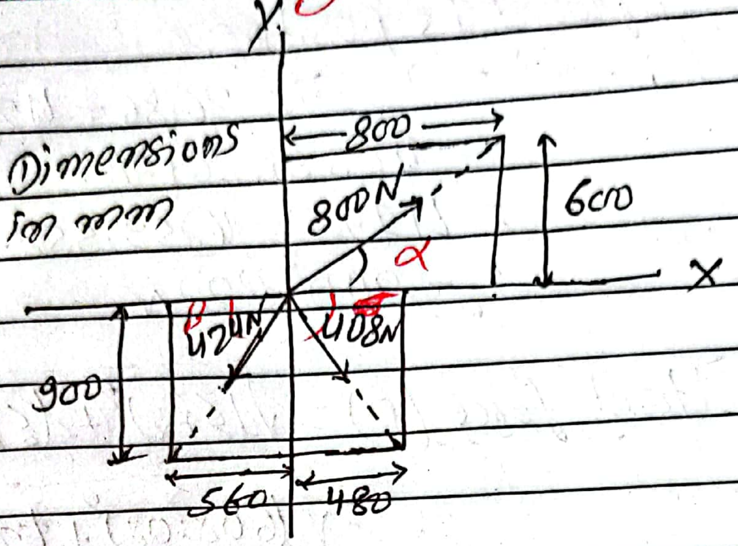
Q 2020 (1, a)

Determine the magnitude and direction of resultant of three forces acting at point 'O' as shown in figure.

Soln



Soln

Let α, β & σ are three angles ~~and~~ acting at a point 'O' and also F_1, F_2 & F_3 are the three forces.

$$\alpha = \tan^{-1}\left(\frac{0.6}{0.8}\right) = 36.86^\circ$$

$$\beta = \tan^{-1}\left(\frac{0.9}{0.56}\right) = 58.10^\circ$$

$$\sigma = \tan^{-1}\left(\frac{0.9}{0.48}\right) = 61.92^\circ$$

Now,

taking rightward is positive,

$$\Sigma F_x (\rightarrow +ve) = 800 \cos 36.86 - 424 \cos 58.10 + 408 \cos 61.92^\circ$$

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$= 640.808 - 224.04 + 192.04$

$= 608.08 N$

taking upward is positive,

$\Sigma F_y (\uparrow +ve) = 800 \sin 36.86 - 424 \sin 58.20 - 408 \sin 61.92$

$= 479.2 - 359.552 - 359.97$

$= -240.322 N$

$\therefore$ Resultant force $(F) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(608.08)^2 + (240.322)^2}$

$= 653.847 N$

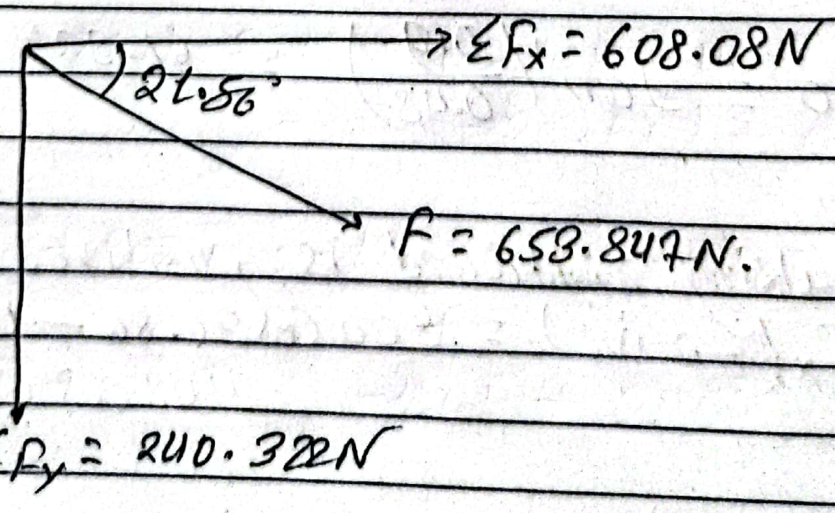
&

angle with horizontal,

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

$= \tan^{-1}\left(\frac{240.322}{608.08}\right)$

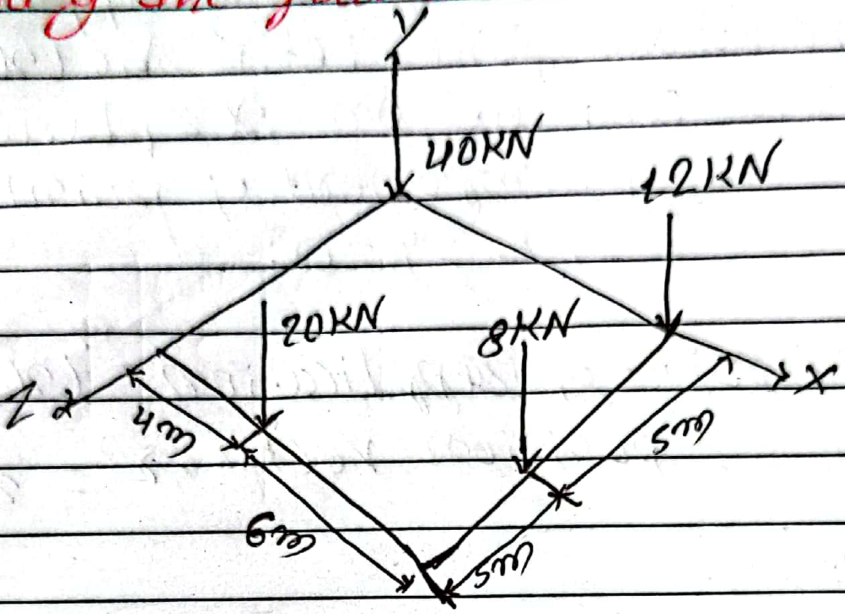
$= 21.56$



Q. 2020 (II. b)

A square foundation mat supports the four columns with the weight as shown. Determine the magnitude & point of application of the resultant of the four loads.

Sol^n:



Let $\vec{F_1} = -12\vec{j}$, $\vec{F_2} = -8\vec{j}$, $\vec{F_3} = -20\vec{j}$

& $\vec{F_4} = -40\vec{j}$

$F_1 = (10, 0, 0)$ ∴ $\vec{r_1} = 10\vec{i}$

$F_2 = (10, 0, 5)$ $\vec{r_2} = 10\vec{i} + 5\vec{k}$

$F_3 = (4, 0, 10)$ $\vec{r_3} = 4\vec{i} + 10\vec{k}$

$F_4 = (0, 0, 0)$ $\vec{r_4} = 0.$

Resultant of Column load, $\vec{R} = \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

$= -12\vec{j} - 8\vec{j} - 20\vec{j} - 40\vec{j}$

$= -80\vec{j}$

$\vec{i}\times\vec{j}=\vec{k}$ | $\vec{j}\times\vec{i}=-\vec{k}$
$\vec{j}\times\vec{k}=\vec{i}$ | $\vec{k}\times\vec{j}=-\vec{i}$
$\vec{k}\times\vec{i}=\vec{j}$ | $\vec{i}\times\vec{k}=-\vec{j}$

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Magnitude of resultant $|\vec{F}| = \sqrt{(-80)^2}$
$= 80\,kN$

Now, to find point of application of resultant force. Let it be (x, y, z). Since resultant lies on xz-plane,
$\therefore$ y component of point of application $= 0$
i.e. $y = 0$.

$\therefore$ point of application $= (x, 0, z)$
$\therefore$ position vector, $\vec{r} = x\vec{i} + z\vec{k}$

Now
Moment of resultant about O = sum of moment of individual forces about O
i.e.

$$\vec{r}\times\vec{F} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3} + \vec{r_4}\times\vec{F_4}$$

$$(x\vec{i}+z\vec{k})\times(-80\vec{j}) = 10\vec{i}\times(-12\vec{j}) + (10\vec{i}+5\vec{k})\times(-8\vec{j}) + (4\vec{i}+10\vec{k})\times(-20\vec{j}) + 0\times(-40\vec{j})$$

$$-80x\vec{k} - 80z(-\vec{i}) = -120\vec{k} - 80\vec{k} - 40(-\vec{i}) - 80\vec{k} - 200(-\vec{i}) + 0$$

or, $-80x\vec{k} + 80z\vec{i} = -120\vec{k} - 80\vec{k} + 40\vec{i} - 80\vec{k} + 200\vec{i}$

or, $-80x\vec{k} + 80z\vec{i} = -280\vec{k} + 240\vec{i}$

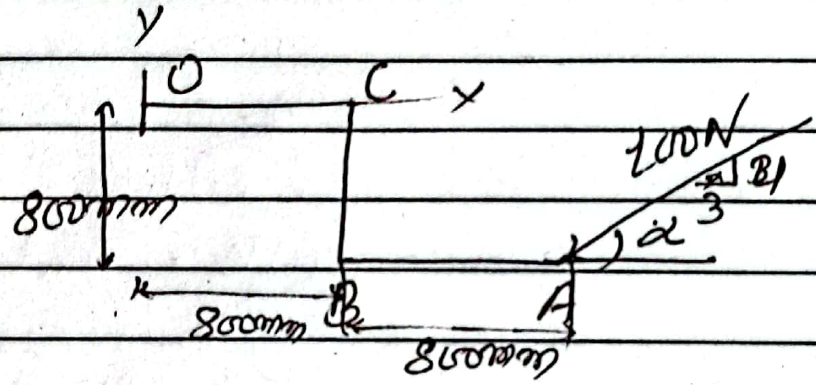
~~Comparing~~ Equating coeff. of $\vec{i}$ & $\vec{k}$,

$x = \frac{+280}{+80} = 3.5$ $\quad z = \frac{240}{80} = 3$

$\therefore x, y, z = (3.5, 0, 3)$ &

OR.

For the system of forces shown, determine its equivalent force moment system at the origin O.



Sol.

Let α be the angle,

$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

Equivalent force at the origin O = 100N

taking ACW is +ve,

Equivalent moment at origin O =

~~100sin~~ $F_x \times \left(\frac{1600}{1000}\right) - F_y\left(\frac{800}{1000}\right)$

$= 100\sin$

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Q. 2019 Fall (1.a)

What is applied mechanics? Write its importance and applications with real life examples.

=> Applied Mechanics is defined as that ~~Since~~ science which describes and predicts the conditions of rest or motion of bodies under the action of forces. It is divided into three parts: mechanics of rigid bodies, mechanics of deformable bodies, and mechanics of fluids. The mechanics of rigid bodies is subdivided into statics and dynamics, the former dealing with bodies at rest, the latter with bodies in motion.

Importance of Applied mechanics are formulating new ideas and theories, discovering and interpreting phenomena, and developing experimental and computational tools.

Application:-

2019 Fall (1.b)

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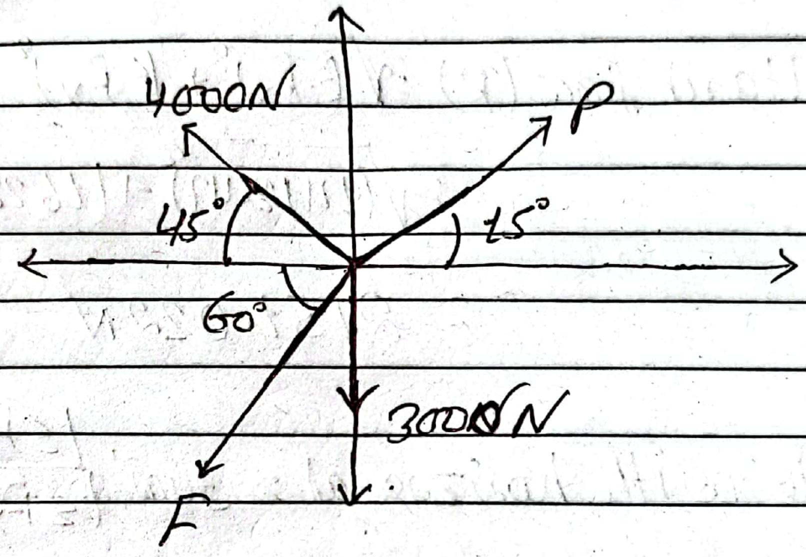
~~What is Applied Mechanics? write its importance and applications with real life examples.~~

Determine the magnitude and direction of resultant force in the following diagram. where $F = 2500N$ and $P = 2760N$

Soln



Given,

$F = 2500N$ & $P = 2760N$

Let $F_1 = 3000N$

$F_2 = 4000N$

& $\alpha = 15°, \beta = 45°, \sigma = 60°$ & $\theta = 90°$

taking Rightward is positive,

$\Sigma F_x (\rightarrow ve) = P\cos 15° - F_2 \cos 45° - F\cos 60 + F_1 \cos 90°$

$= 2760\cos 15 - 4000\cos 45° - 2500\cos 60 + 3000\cos 90°$

$= \cancel{2663.4 - 282.8 - 12}$

$= 2665.96 - 2828.43 - 1250 + 0$

$= -1412.47 N$

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taking upward is positive,

$$\Sigma F_y (\uparrow +ve) = 2760 \sin 15° + 4000 \sin 45° - 2500 \sin 60° - 3000 \sin 90°$$

$$= 714.34 + 2828.43 - 2165.06 - 3000$$

$$= -1622.29\,N$$

Resultant force (F) $= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

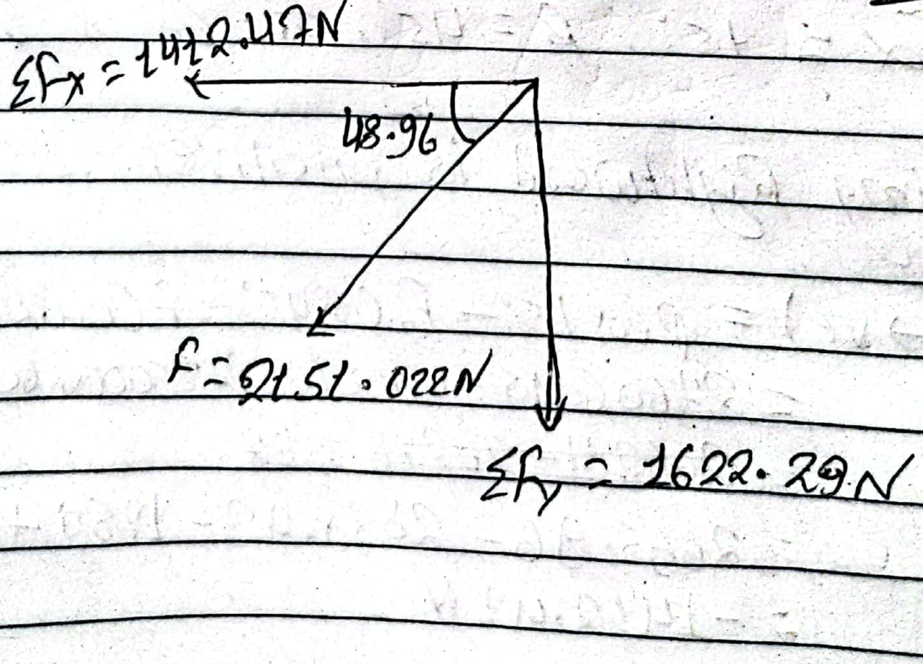
$$= \sqrt{(1412.47)^2 + (1622.29)^2}$$

$$= 2151.022\,N$$

Angle with horizontal $= \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

$$= \tan^{-1}\left(\frac{1622.29}{1412.47}\right)$$

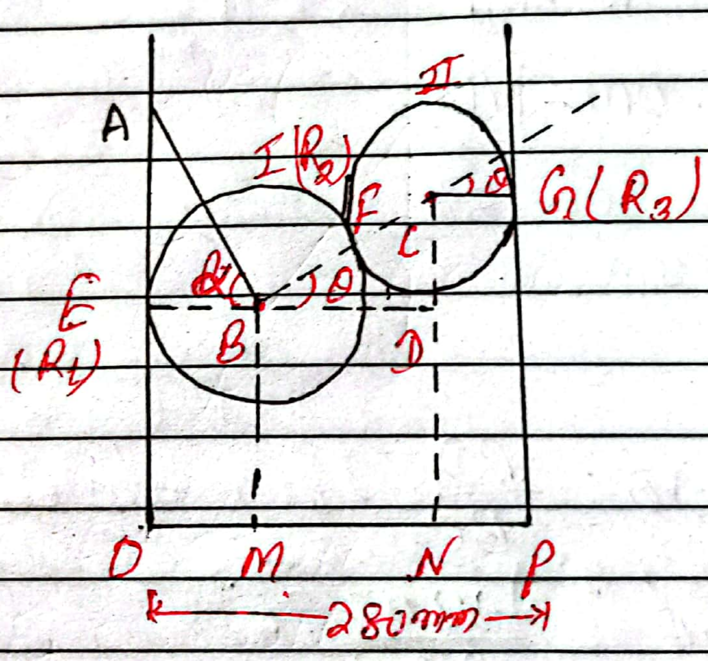
$$= 48.96°$$



8. 2019 Fall (2.a)

Two spheres I & II of radii 100 mm & 50 mm respectively are placed on a container as shown in the figure. Sphere I of mass 50kg is suspended with a string AB of length 200mm & ~~Sphere~~ sphere II of mass ~~25~~ 25kg is placed on top of it. Determine the tension in AB and the reaction from the container.

Soln



The three contact point E, F & G. Let R_1, R_2 and R_3 be the reaction developed at those contact point. Simple construction for the calculation of angle is shown in figure.

In ΔAEB

EB = radius of sphere I = 100cm

or, $\cos\alpha = \frac{EB}{AB}$

or, $\cos\alpha = \left(\frac{100}{200}\right)$ $[\because AB = 200cm]$

$\therefore \alpha = \cos^{-1}\left(\frac{100}{200}\right)$

$= 60°$

$BD = 280 - R_I - R_{II}$

$= 280 - OM - NP$

$= 280 - 100 - 50$

$= 180 cm$

<u>In ΔCBD</u>

$\cos\theta = \frac{BD}{BC}$

or, $\cos\theta = \frac{180}{R_I + R_{II}}$

or, $\theta = \cos^{-1}\left(\frac{180}{100+50}\right)$

$\therefore \theta = 29.92°$

FDB of Sphere II and I are shown below,

~~IIS~~

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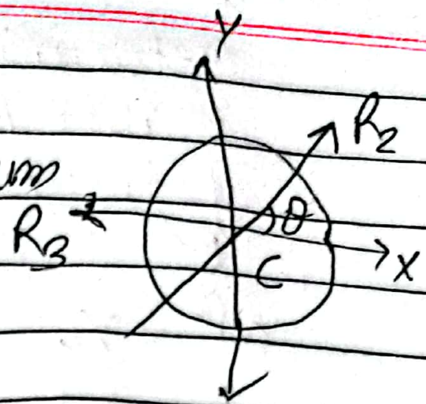
for

II weight $= 25 \times 9.81$

$= 245.25N$

Applying equation of equilibrium



$(+\uparrow)\ F_Y = 0$

or, $R_2 \sin\theta - 245.25 = 0$

or, $R_2 \sin 29.92' = 245.25$

or, $R_2 = \dfrac{245.25}{0.4988}$

or $R_2 = 491.68$

$(\rightarrow +ve) \Sigma F_x = 0$

$R_2 \cos\theta - R_3 = 0$

$R_3 = 491.68 \cos 29.92'$

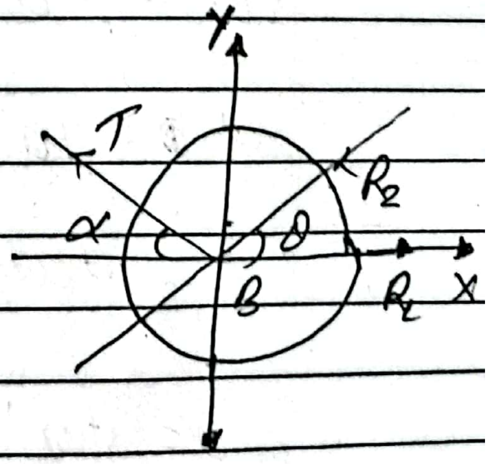
$= 426.15N$ &

for I

weight $= 50 \times 9.81$

$= 490N$

Let T be the tension on string AB



$\Sigma F_Y (\uparrow +ve) = 0$

$T\sin\alpha - R_2 \sin\theta - 490 = 0$

or, $T \sin 60 - 491.68 \sin 29.92' = 490$

or, $T = \dfrac{735.74}{\sin 60}$

$\therefore T = 849.56N$

&

$\Sigma F_x (\rightarrow +ve) = 0$

or $R_1 - T\cos\alpha - R_2 \cos\theta = 0$

or, $R_1 - 849.56 \cos 60° - 491.68 \cos 29.92 = 0$

or, $R_1 = 424.78 + 426.15$

$\therefore R_1 = 850.93N$

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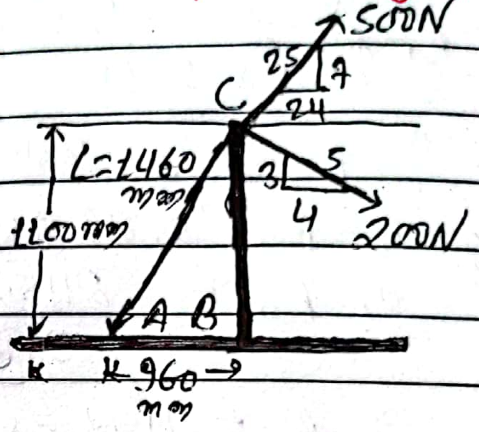
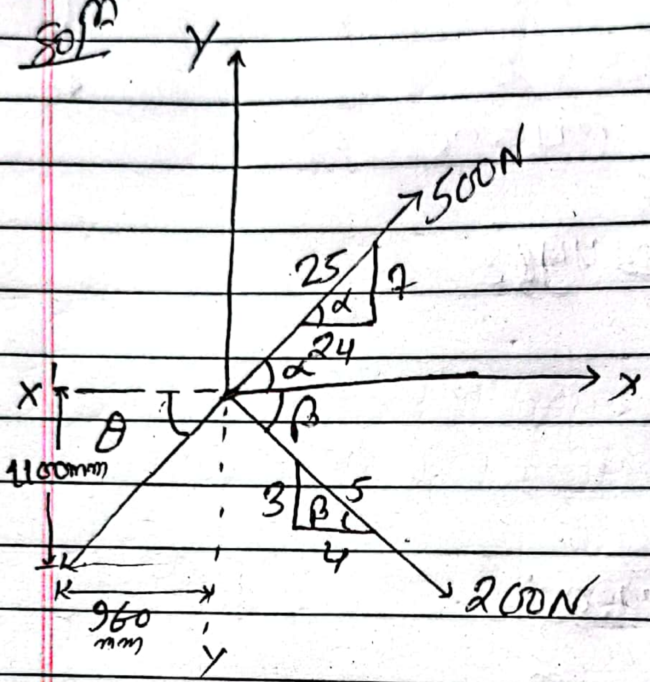
Q. 2019 spring (1.a)

Determine knowing that the tension in rope AC is 365N, determine the resultant of the three forces exerted at point C of post BC.



Sol^n



Let ~~αβ~~ θ, α & β are three angles,

$$\alpha = \tan^{-1}\left(\frac{7}{24}\right) = 16.26^\circ$$

$$\beta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\theta = \tan^{-1}\left(\frac{1100}{960}\right) = 48.89^\circ$$

taking ~~upward~~ rightward is positive,

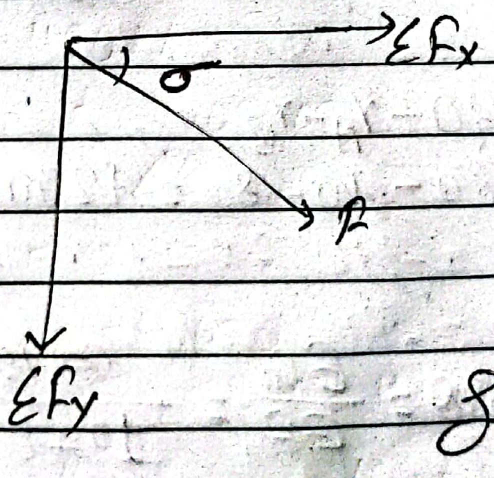
$$\Sigma F_x (\rightarrow +ve) = 500 \cos 16.26 - 365 \cos 48.89^\circ + 200 \cos 36.89$$
$$= 480 - 239.99 + 159.99$$
$$= 400 N$$

taking upward is positive,

$\Sigma F_y (\uparrow +ve) = 500 \sin 26.26 - 365 \sin 48.81 - 200 \sin 36.89$
$= 439.998 - 275.009 - 120.056$
$= -255.067 \, N$

Resultant force $(F) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$
$= \sqrt{(400)^2 + (255.067)^2}$
$= 474.404 \, N$

Angle with horizontal $(\sigma) = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$
$= \tan^{-1}\left(\frac{255.067}{400}\right)$
$= 32.582°$



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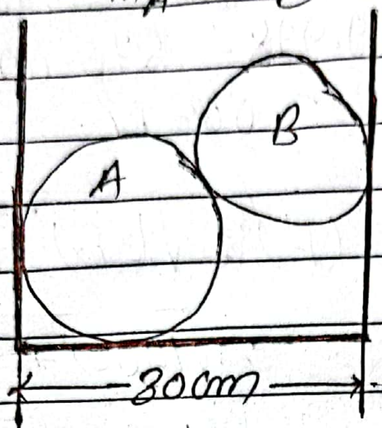
Q. 2019 Spring (1. b)

Determine the reactions at each contact point.

& $r_A = 10$ cm, $r_B = 8$ cm

$m_A = 10$ kg $m_B = 8$ kg.



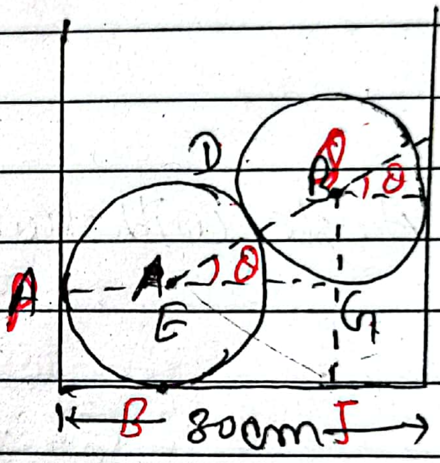
Soln

⇒ Weight of larger ball (W_A) =

$(W_A) = 10 \times 9.81 = 98.1$

weight of smaller ball

$(W_B) = 8 \times 9.81 = 78.48$



We have to determine the reactions developed from the contact points ~~A, B, C~~ P, Q, C & D.

$EG = QJ = 30 - r_A - r_B$

$= 30 - 10 - 8 = 12$ cm

In $\triangle EFG$

$\cos\theta = \frac{EG}{EF} = \frac{12}{10+8} = \frac{12}{18}$

$\therefore \theta = \cos^{-1}\left(\frac{12}{18}\right) = 48.18°$

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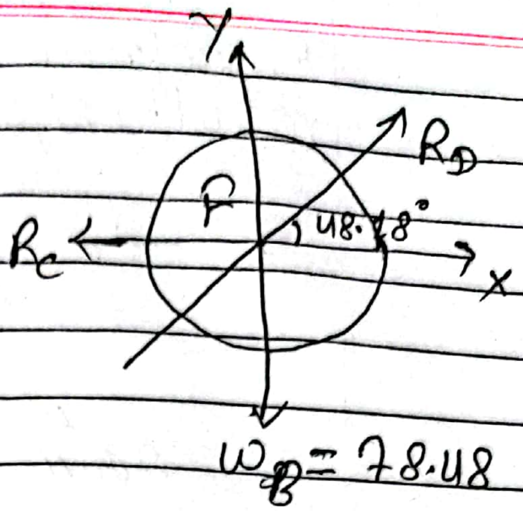
FBD of sphere B,



$\Sigma F_y (\uparrow +ve) = 0$

or, $R_D \sin 48.18 - 78.48 = 0$

or, $R_D = \frac{78.48}{\sin 48.18}$

$\therefore R_D = 105.307\ N$

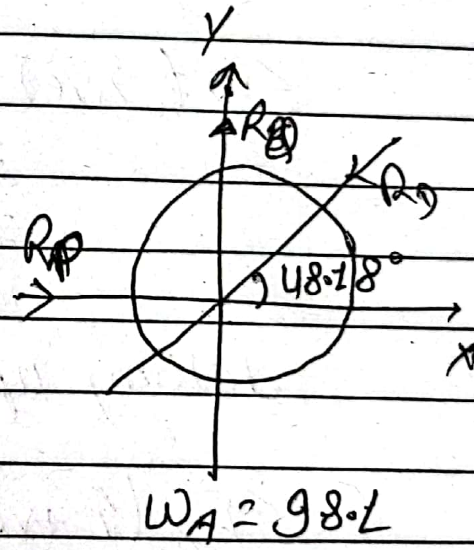
& $\Sigma F_x (\rightarrow +ve) = 0$

or, $R_D \cos 48.18 - R_C = 0$

or, $R_C = 105.307 \cdot \cos 48.18$

$\therefore R_C = 70.21 N$

FBD of sphere A



$\Sigma F_x (+\rightarrow ve) = 0$

or, $R_P - R_D \cos 48.18 = 0$

or, $R_P = 105.307 \cos 48.18$

$\therefore R_P = 70.21 N$

$\Sigma F_y (\uparrow +ve) = 0$

or, $R_Q - W_A - R_D \sin 48.18 = 0$

or, $R_Q - 98.1 - 105.307 \sin 48.18 = 0$

or, $R_Q = 98.1 + 78.47$

$\therefore R_Q = 176.57 N$

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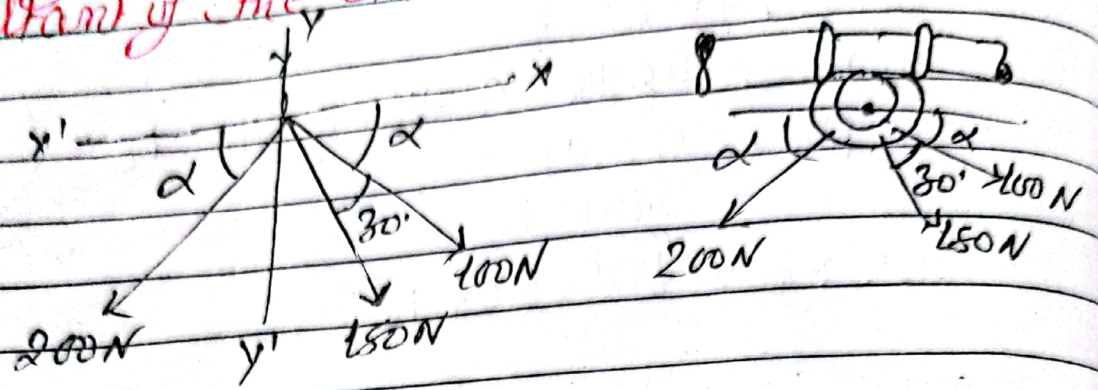
Q. 2018 Fall (1: a)

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.



Sol^n

Given, $\alpha = 35°$.

taking rightward is positive,

$$\Sigma F_x (\rightarrow +ve) = -200\cos 35° + 150\cos(80+35°) + 100\cos 35°$$
$$= -163.83 + 63.39 + 81.91$$
$$= -18.53 N$$

taking upward is positive,

$$\Sigma F_y (\uparrow +ve) = -200\sin 35° - 150\sin(80+35) - 100\sin 35°$$
$$= -114.72 - 135.95 - 57.36$$
$$= -308.03 N$$

Resultant force $(F) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(18.53)^2 + (308.03)^2}$$

$$= 308.58 N.$$

8.2018 fall (1.b)

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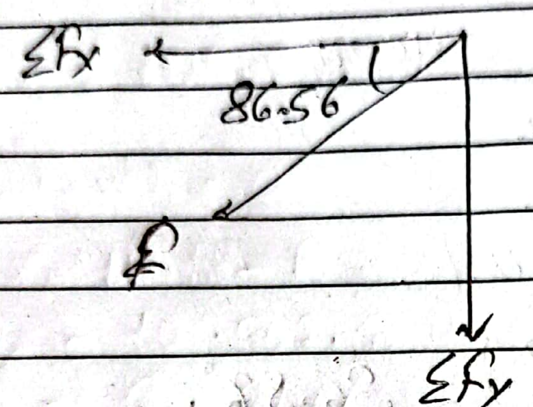
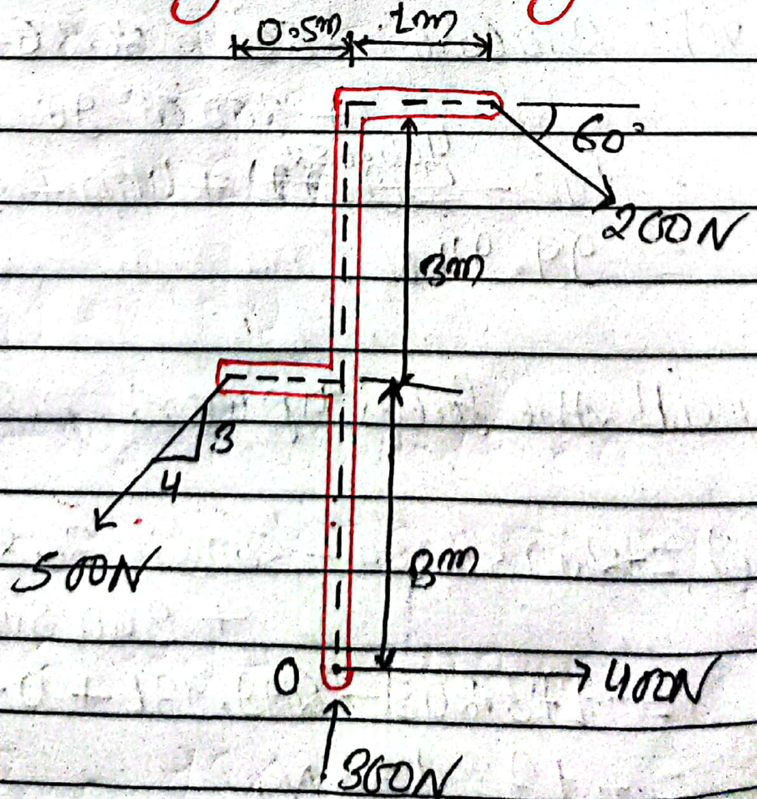
& angle with horizontal (θ) = $\tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

$= \tan^{-1}\left(\frac{308.03}{18.53}\right)$

$= 86.56°$



Q. Determine the resultant of the four forces acting on a body as shown.



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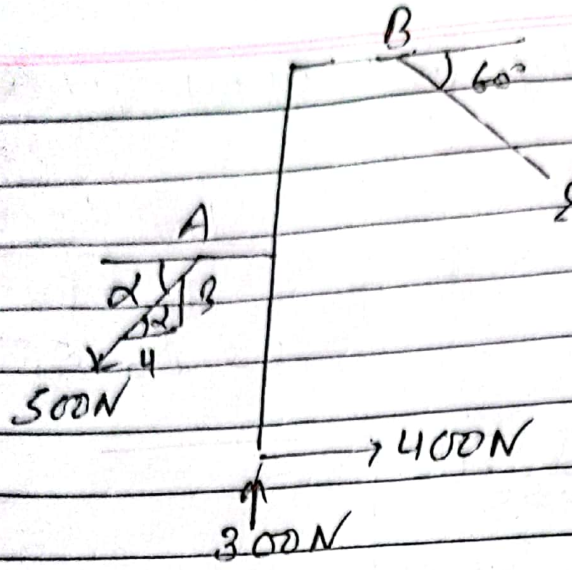
Soln



let α be the angle,

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.86°.$$

~~taking all force are upwar rightwar~~

Resolving all the forces rightward positive,

$$\Sigma F_x(\rightarrow +ve) = 200\cos 60° - 500\cos 36.86 + 400\cos 0 + 300\cos 90°$$

$$= 100 - \cancel{399.999}\ 400.058 + 400 + 0$$

$$= 99.94\,N$$

Resolving all the forces upward positive,

$$\Sigma F_y(\uparrow +ve) = -200\sin 60° - 500\sin 36.86 + 400\sin 0 + 300\sin 90°$$

$$= -173.205 - 299.931 + 0 + 300$$

$$= -173.136\,N.$$

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Resultant force $(F) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(99.94)^2 + (173.136)^2}$$

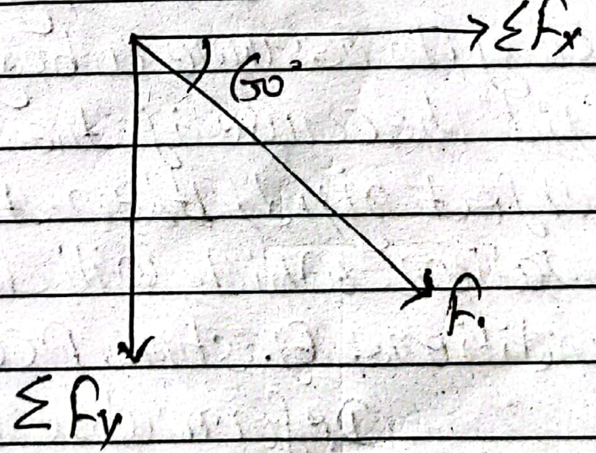
$$= 199.91\,N.$$

Angle with horizontal,

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{173.136}{99.94}\right)$$

$$= 60^\circ$$



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Q. 2018 Spring (1.a)

Discus about Concept of rigid, deformed and fluid bodies.

⇒ A rigid body is such type of body which does not deform even if very large force is applied on it. Most of the bodies considered in elementary mechanics are assumed to be rigid. Actual structures and machines, however, are never absolutely rigid and ~~deformations~~ deform under the loads to which they are subjected. But these deformations are usually small and do not appreciably affect the conditions of equilibrium or motion of the structure under consideration.

For the study of statics, it is necessary to assume a body as ~~per~~ "perfectly rigid" because if the body is not considered perfect rigid, ~~then~~ there will be some deformation which will lead to unstatic conditions. Eg. stone, RCC block.

Deformable body is such a body, which when on applying a force on it, there is appreciable change in the shape & size of it.

Q. 2018 Spring (1.b)

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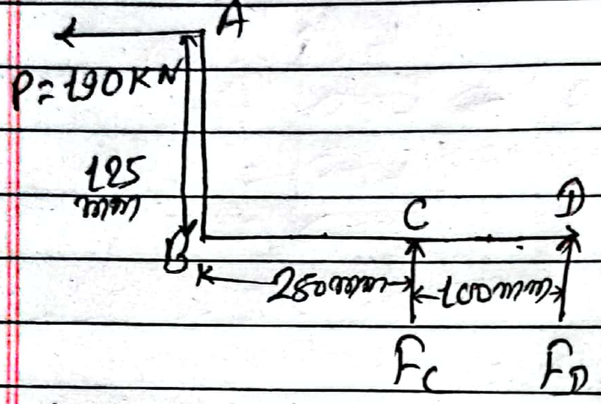
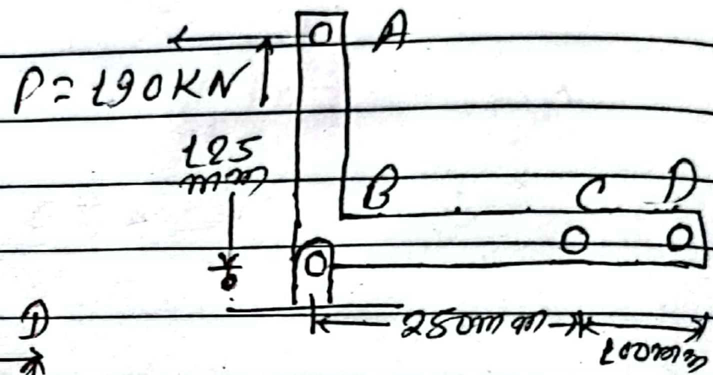
The 190 N horizontal force P acts on a bell crank as shown. Find the two equal vertical forces at C and D which are equivalent to the couple found at point B.

Soln





Let, F_C & F_D ~~are~~ be the equal vertical forces at C & D.

i.e. $\boxed{F_C = F_D = R}$

Couple at B = f.d. (force × distance)

or, $\cancel{(F_C \times 250) + (F_D \times} = 190 \times 125$

or, $F_C \times 250 + F_D \times (250 + 100) = 190 \times 125$

or, $250R + 350R = 23750$

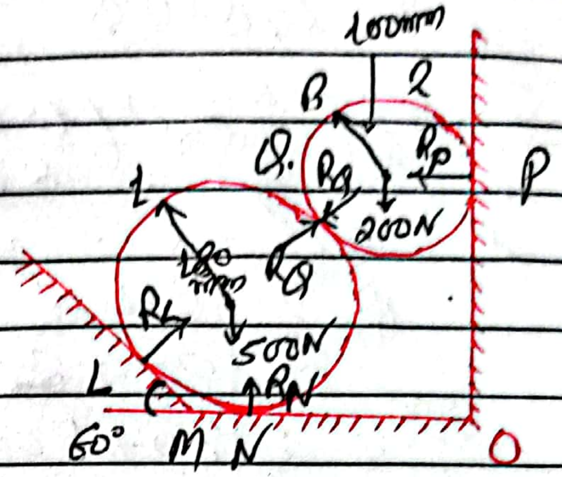
or, $600R = 23750$

$\therefore R = \frac{23750}{600}$

$= 39.583$ N $\cancel{8}$

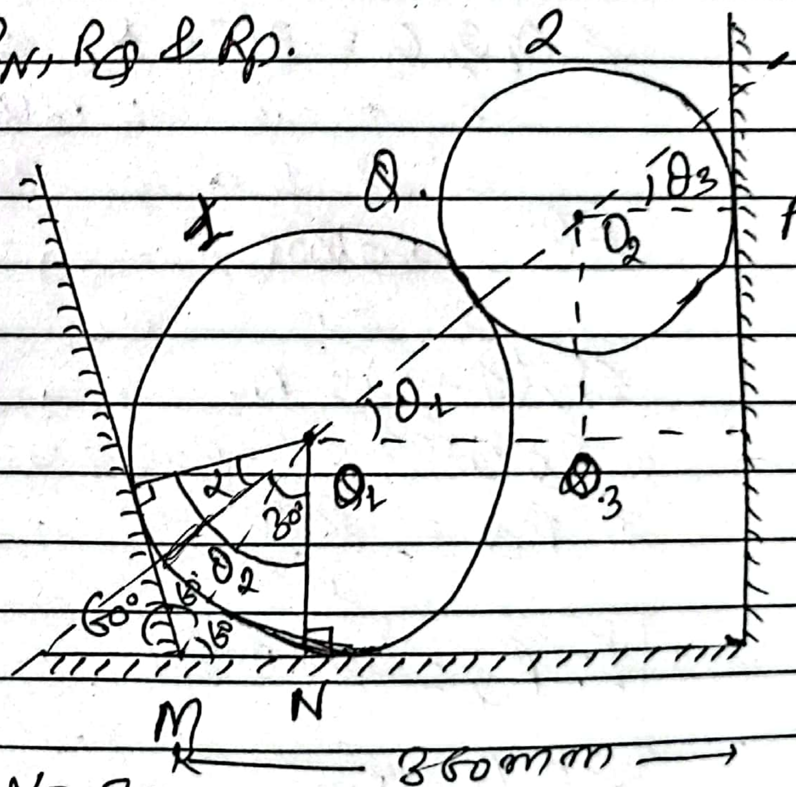
2018 Spring (2.a)

Determine the reaction on the sphere if the bottom channel is 360mm wide with one side vertical.



Soln

Here, we have to determine the reaction R_L, R_N, R_Q & R_P.



In ΔMO_1N, $\angle MO_1N = 30$.

$\tan 30° = \frac{MN}{O_1N}$

$MN = O_1N \tan 30° = 180 \times \tan 30° = 103.92\,mm$

$O_1O_3 = 360 - MN - O_2P = 360 - 103.92 - 100$

$O_1O_3 = 156.08\,mm$

In $\Delta O_1 O_2 O_3$

$$\cos\theta_1 = \frac{O_1O_3}{O_1Q + O_3Q} = \frac{156.08}{180+100} = \frac{156.08}{280}$$

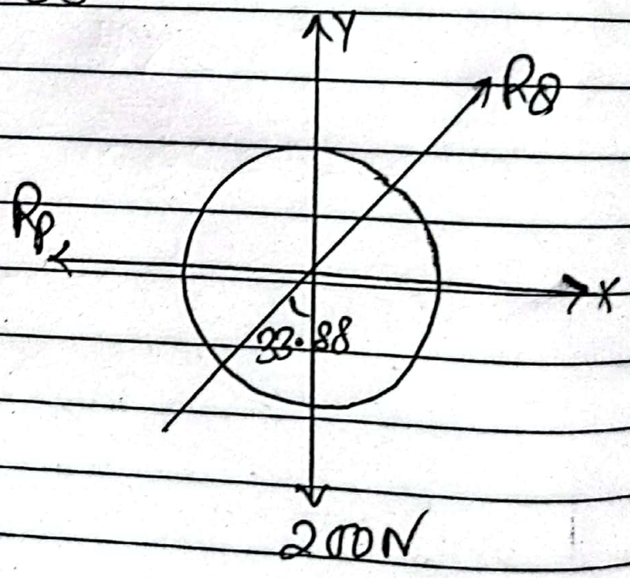
$$\therefore \theta_1 = \cos^{-1}\left(\frac{156.08}{280}\right) = 56.12^\circ$$

&

$$\theta_2 = 30 + \alpha$$
$$= 30 + \cancel{30}(90 - 60^\circ)$$
$$= 30 + 30$$
$$= 60^\circ$$

$$\angle O_1O_2O_3 = 90^\circ - \theta_1 = 90 - 56.12$$
$$= 33.88^\circ$$

FBD of sphere 2



$\Sigma F_y (\uparrow +ve) = 0$

or, $R_Q \cos 33.88 - 200 = 0$

or, $R_Q = \dfrac{200}{\cos 33.88}$

$\therefore R_Q = 240.9N$

$\Sigma F_x (\rightarrow +ve) = 0$

or $R_Q \cdot \sin 33.88 - R_p = 0$

or $R_p = 240.9 \sin 33.88$

$\therefore R_p = 434.23$ N.

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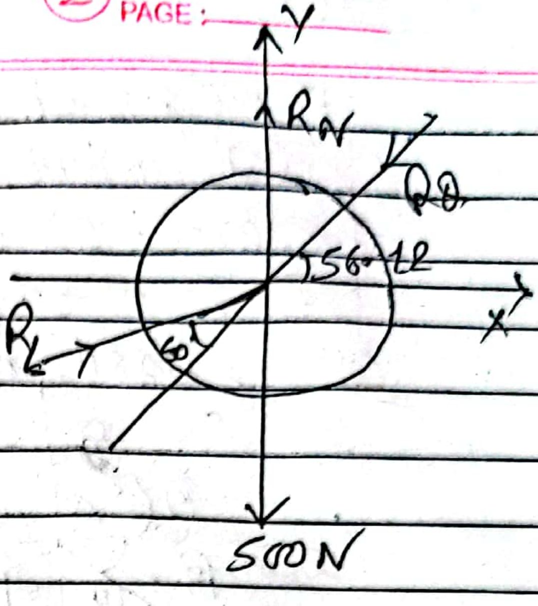
FBD of sphere 1



$\Sigma F_x (\rightarrow +ve) = 0$

$-R_Q \cos 56.12 + R_L \sin 60° = 0$

or, $R_L \sin 60° = R_Q \cos 56.12$

or, $R_L = \dfrac{240.9 \cos 56.12}{\sin 60}$

$\therefore R_L = 155.065N$

$\Sigma F_y (\uparrow +ve) = 0$

or, $-R_Q \sin 56.12 - 500 + R_N + R_L \cos 60° = 0$

or, $-240.9 \sin 56.12 - 500 + R_N + 155.065 \cos 60 = 0$

or, $-199.99 - 500 + R_N + 77.5325 = 0$

or, $R_N - 622.4575 = 0$

$\therefore R_N = 622.4575 N$

$\therefore$ $R_Q = 240.9N$

$R_P = 134.23N$

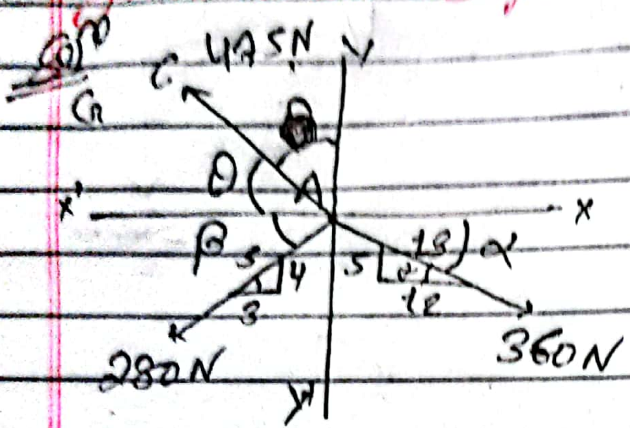
$R_L = 155.068N$

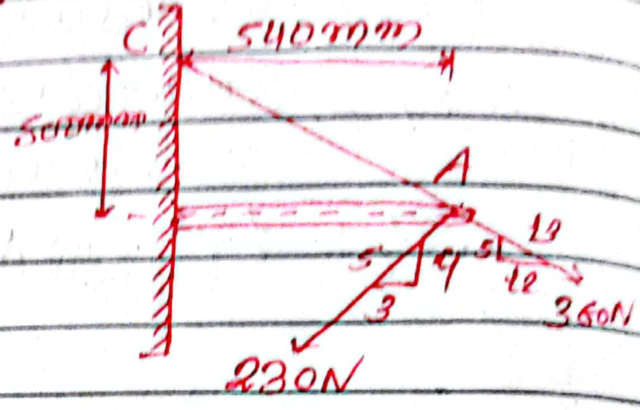
$R_N = 622.4575N$

Q. 2019 fall (1.a) 1

Knowing that tension in cable AC is 475N, determine the resultant of the three force exerted at point A of the beam AB. Figure given is on 2D plane.

Sol^n





Sol^n Given,

Tension in cable AC = 475N

Let θ, β & α be the three angles

$\theta = \tan^{-1}\left(\frac{500}{540}\right) = 42.797°$

$\beta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

$\alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$

Taking right ward is positive,

$\Sigma F_x (\rightarrow +ve) = -475 \sin 42.797 - 280 \cos 53.13 + 360 \cos 22.62$

$= -848.53 - 183.999 + 322.307$

$= -322.72 - 1838.00 + 332.307$

$=$

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Taking rightward is positive,

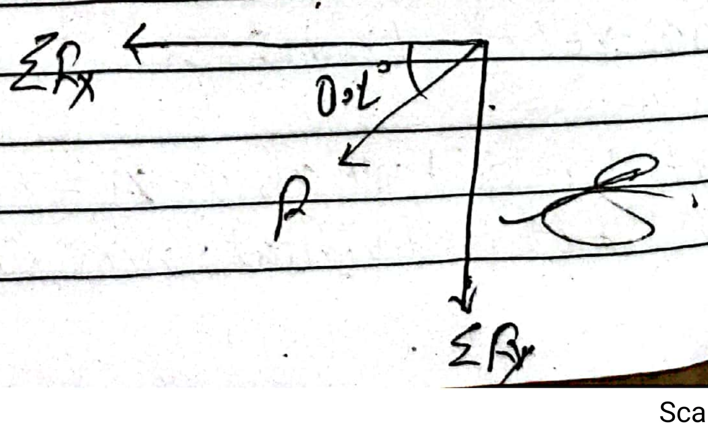
$\Sigma F_x (\rightarrow +ve) = 360 \cos 22.62 - 475 \cos 42.797$
$- 280 \cos 53.13°$
$= 332.0307 - 348.54 - 138.00$
$= -154.23$

Taking upward is positive,
$\Sigma F_y (\uparrow +ve) = -360 \sin 22.62 + 475 \sin 42.797$
$- 280 \cancel{\cos} \sin 53.13°$
$= -138.46 + 322.72 - 183.999$
$= -0.261 \, N$

Resultant force $(P) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$
$= \sqrt{(154.23)^2 + (0.261)^2}$
$= 154.23 \, N$

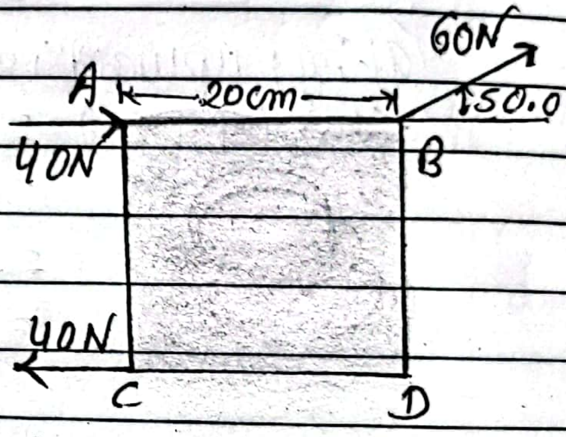
&

Direction of resultant (σ) with horizontal,
$= \tan^{-1}\left(\frac{\Sigma F_{\cancel{x}}}{\Sigma F_x}\right)$
$= \tan^{-1}\left(\frac{0.261}{154.23}\right)$
$= 0.1°$



Q. 2017 fall (1.b)

A force and a couple act (figure 1.b) on a square plate of side 20cm. Replace the given force and couple with a single force applied at a point located on line AB. Also determine the distance from A to the point of application of the force.



Sol^n

let
$\vec{F_1} = 60\vec{i}$

$F = (20, 0, 0)$
position vector,
$\vec{r} = 20\vec{i}$

Resultant of force $\vec{R} = \vec{F} = 60\vec{i}$

Magnitude of resultant $\vec{R} = |\vec{F}| = \sqrt{(60)^2}$
$= 60$ Ncm

Now to find point of application of resultant force, let it be (x, y, z). since resultant lies on x-plane.
$\therefore$ y & z component of point of application $= 0$
i.e. $y = 0$ & $z = 0$.
point of application $= (x, 0, 0)$
position vector $(\vec{r'}) = x\vec{i}$

Again, Moment of resultant about A = sum of moment of individual forces about A.

Q. 2017 Spring (1. a)

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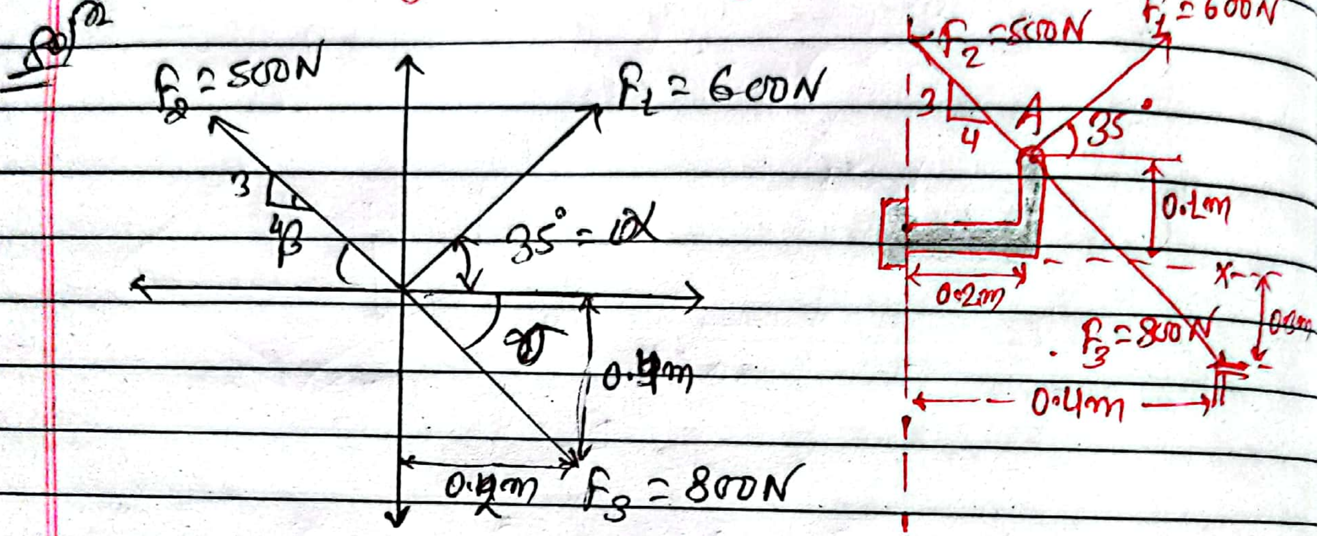
Three forces F_1, F_2 & F_3, all ~~one~~ of which act on point A of the bracket, are specified as shown in figure below. Determine the magnitude & direction of resultant of these forces.

Soln



Let α, β & σ are three angles,

$\alpha = 35°$

$\beta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

$\sigma = \tan^{-1}\left(\frac{0.4}{0.2}\right) = 63.43°$

Taking rightward is positive

$\Sigma F_x (\rightarrow +ve) = F_1 \cos\alpha - F_2 \cos\beta + F_3 \cos\sigma$

$= 600 \cos 35° - 500 \cos 36.87° + 800 \cos 63.43°$

$= 491.49 - 399.99 + 357.71$

$= 449.21 N$

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Taking upward is positive,

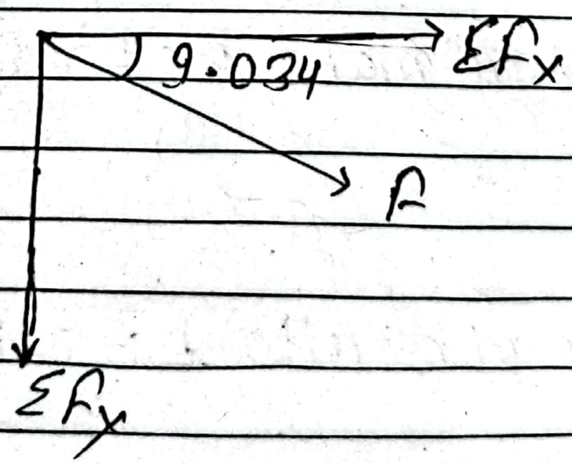
$$\Sigma F_y (\uparrow +ve) = F_1 \sin 35 + F_2 \sin \beta - F_3 \sin \sigma$$
$$= 600 \sin 35^\circ + 500 \sin 36.87 - 800 \sin 63.43$$
$$= 344.15 + 300 - 715.57$$
$$= -71.42 \, N$$

Resultant force $(F) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(449.22)^2 + (71.42)^2}$$
$$= 454.85 \, N$$

Direction of resultant force

$$(\theta) = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= \tan^{-1}\left(\frac{71.42}{449.22}\right)$$
$$= 9.034^\circ$$



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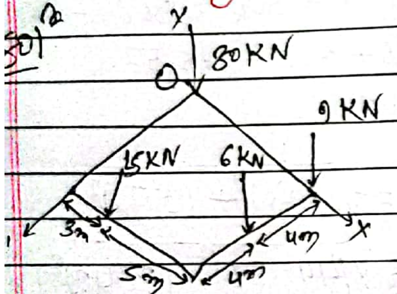
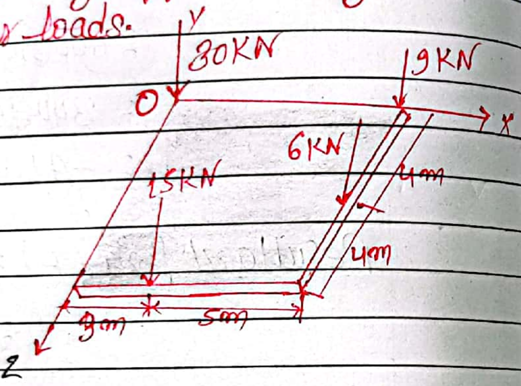
. 2017 Spring (1.b)

A Square foundation mat supports the four Columns with the weight as shown. Determine the magnitude & point of application of the resultant of the four loads.

Sol^n





let $\vec{F_1} = \cancel{8}-9\vec{j}$, $\vec{F_2} = -6\vec{j}$, $\vec{F_3} = -15\vec{j}$

& $\vec{F_4} = -80\vec{j}$

$F_1 = (8,0,0)$ | $\vec{r_1} = 8\vec{i}$
$F_2 = (8,0,4)$ | $\vec{r_2} = 8\vec{i} + 4\vec{k}$
$F_3 = (3,0,8)$ | $\vec{r_3} = 3\vec{i} + 8\vec{k}$
$F_4 = (0,0,0)$ | $\vec{r_4} = 0.$

Resultant of mat $= \vec{R} = \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$
$= -9\vec{j} - 6\vec{j} - 15\vec{j} - 80\vec{j}$
$= -60\vec{j}$

Magnitude of resultant $\vec{F} = |\vec{F}| = \sqrt{(60)^2}$
$= 60$ ~~N/m~~ KN.

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Now, to find point of application of resultant force, let it be (x,y,z). Since resultant lies on xz-plane,

$\therefore$ y component of point of application $= 0$
i.e. $y = 0$

point of application $= (x, 0, z)$
position vector $(\vec{r}) = x\vec{i} + z\vec{k}$

Again,

Moment of resultant about O = Sum of moment of individual forces about O.

i.e.
$$\vec{r}\times\vec{F} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3} + \vec{r_4}\times\vec{F_4}$$

$$(x\vec{i}+z\vec{k})\times(-60\vec{j}) = 8\vec{i}\times(-9\vec{j}) + (8\vec{i}+4\vec{k})\times(-6\vec{j}) + (3\vec{i}+8\vec{k})\times(-15\vec{j}) + 0\times(-80\vec{j})$$

or $-x60\vec{k} + 60z\vec{i} = -72\vec{k} - 48\vec{k} - 24(-\vec{i}) - 45\vec{k} - 120(-\vec{i}) + 0$

or, $-x60\vec{k} + 60z\vec{i} = -72\vec{k} - 48\vec{k} + 24\vec{i} - 45\vec{k} + 120\vec{i}$

or, $-x60\vec{k} + 60z\vec{i} = -165\vec{k} + 144\vec{i}$.

Equating coeff. of x, z,

$x = \frac{165}{60} = 2.75$ | $z = \frac{144}{60} = 2.4$

point of application $(x, y, z) = (2.75, 0, 2.4)$

Q. 2017 Spring (1.b) OR.

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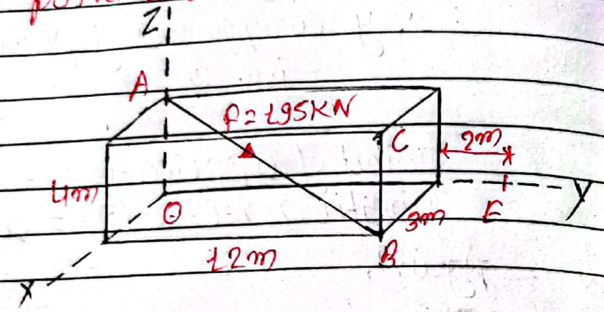
A force F acts at point A as shown in the figure. Determine the moment of the force F about point C and axis CE.

Soln



Given force (F) = 195KN

Moment of ~~resultant~~ the force F about point C,

$\vec{M_C} = \vec{r} \times \vec{F}$

$\therefore \vec{M_c} = \quad \times$

O (0, 0, 0)

A (0, 0, 4)

B (3, 12, 0)

C (3, 12, 4)

Now,

$\vec{R} = \vec{T_{AB}}$

$= \frac{\vec{AB}}{|\vec{AB}|} \times T_{AB}$ ——— (1)

$\vec{AB} = \vec{OB} - \vec{OA} = 3\vec{i} + 12\vec{j} - 4\vec{k}$.

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From eqn (1) becomes,

$\vec{R} = \frac{(3\vec{i} + 12\vec{j} - 4\vec{k})}{\sqrt{3^2 + 12^2 + (-4)^2}} \times 195$

$= (3\vec{i} + 12\vec{j} - 4\vec{k}) \times \frac{195}{13}$

$= (3\vec{i} + 12\vec{j} - 4\vec{k}) \times 15$

$= 45\vec{i} + 180\vec{j} - 60\vec{k}$

Again,

Moment of the force F about point C

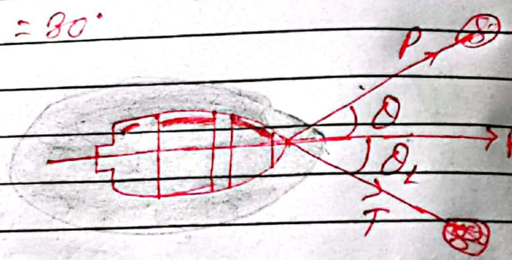
$= \vec{M_c} = \vec{r} \times \vec{F}$

$= \vec{r_{AC}} \times \vec{F}$

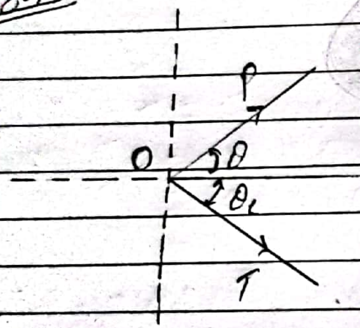
$\vec{r_{AC}} =$

Q.2016 Fall (1.a)

The boat is be pulled onto the shore using two ropes. If the resultant force is to be F_1 (= 80N), directed along the keel a-a as shown, determine the magnitudes of forces T & P acting in each rope and the angle θ of P so that the magnitude of P is a minimum. T acts at θ from the keel as shown. Take $\theta_1 = 30°$



Solⁿ



Given,

$\theta_1 = 30°$ / $\Sigma F_x = 80$ & $\Sigma F_y = 0$

Taking rightward is positive,

$\Sigma F_x (\rightarrow +ve) = P\cos\theta + T\cos\theta_1$

or, $80 = P\cos\theta + T\cos 30°$

or, $80 = P\cos\theta + T(0.866)$ — (i)

Taking upward is positive,

$\Sigma F_y (\uparrow +ve) = P\sin\theta - T\sin 30°$

$0 = P\sin\theta - T/2$

Now,

~~Resultant force (F_1) = $\sqrt{P\cos\theta +}$~~

or, $0 = 2P\sin\theta - T$

$T = 2P\sin\theta$ — (ii)

putting this value in eqⁿ (i), we get,

$80 = P\cos\theta + 2P\sin\theta.(0.866)$

$80 = P\cos\theta + 1.732P\sin\theta$

$80 = P(\cos\theta + 1.732\sin\theta)$

For magnitude of P to be minimum, then, $\cos\theta + 1.732\sin\theta$ must be maximum. So,

$\frac{d}{d\theta}(\cos\theta + 1.732\sin\theta) = 0$

or, $-\sin\theta + 1.732\cos\theta = 0$

or, $1.732\cos\theta = \sin\theta$

or, $1.732 = \frac{\sin\theta}{\cos\theta}$

or, $\tan\theta = 1.732$

or, $\theta = \tan^{-1}(1.732)$

∴ $\theta = 59.999°$

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or, $P(\cos 60 + 1.732 \sin 60) = 80$

or, $P(\cancel{\cos}\ 0.5 + 1.732 \times 0.866) = 80$

or, $P(0.5 + 1.4999) = 80$

or, $P = \frac{80}{1.99991}$

$\therefore P = 40\,N$

putting $P = 40\,N$ in eq^n (i), we get,

$T = 2 \times 40 \sin 60$

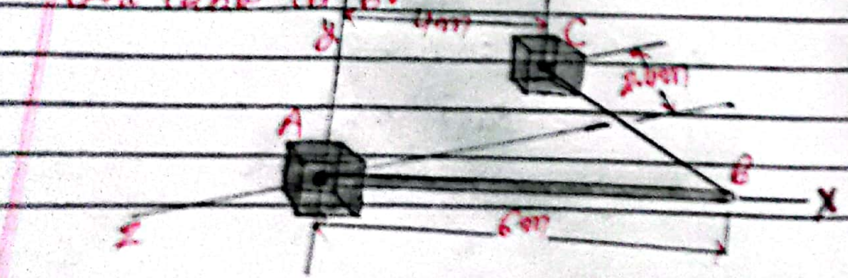
or, $T = 80 \times 0.866$

or, $T = 69.28\,N$

$\therefore P = 40\,N$

& $T = 69.28\,N$ Ans

Q. The 6-m beam AB has a fixed end A. A steel cable is stretched from the free end B of the beam to a point C located on the vertical wall. If the tension in the cable is 2.5 KN, determine the moment about A of the force exerted by the cable at B.



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Soln

Given,

Tension in the cable $T_{BC} = 2.5\,KN$
$= 2500\,N$

Magnitude of moment $(M_A) = ?$

Now,

Resultant $\vec{R} = \vec{T_{BC}}$

$= \frac{\vec{BC}}{|\vec{BC}|} \cdot T_{BC}$ — (1)

Coordinates are,

A(0, 0, 0)

B(6, ~~..~~, 0)

C(0, 2.4, -4)

O(0, 0, 0)

$\vec{OC} = 0\vec{i} + 2.4\vec{j} - 4\vec{k}$

$\vec{OB} = 6\vec{i}$

$\vec{BC} = \vec{OC} - \vec{OB} = -6\vec{i} + 2.4\vec{j} - 4\vec{k}$

From eq^n (1) becomes,

$$\vec{R} = \frac{-6\vec{i} + 2.4\vec{j} - 4\vec{k}}{\sqrt{(-6)^2 + (2.4)^2 + (-4)^2}} \times 2500$$

$$= (-6\vec{i} + 2.4\vec{j} - 4\vec{k}) \times \frac{2500}{7.6}$$

$$= (-6\vec{i} + 2.4\vec{j} - 4\vec{k}) \times 328.947$$

$$= -1973.68\vec{i} + 789.47\vec{j} - 1315.79\vec{k}$$

Moment of ~~Resultant~~ about A of the force exerted by the cable B. $= \vec{M_A} = \vec{r} \times \vec{F}$

$= \vec{AB} \times \vec{R}$

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$6\vec{i}$

~~$= (-6\vec{i} + 2.4\vec{j} - 11\vec{k}) \times (-1973.68\vec{i} + 789.47\vec{j} - 1315.79\vec{k})$~~

$= \vec{AB} \times \vec{R}$

~~$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 0 \\ -1973 & & \end{vmatrix}$~~

$= \vec{AB} \times \vec{R}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 0 \\ -1973.68 & 789.47 & -1315.79 \end{vmatrix}$$

$= 0\vec{i} + [6 \times (-1315.79) - 0]\vec{j} + (789.47 \times 6)\vec{k}$

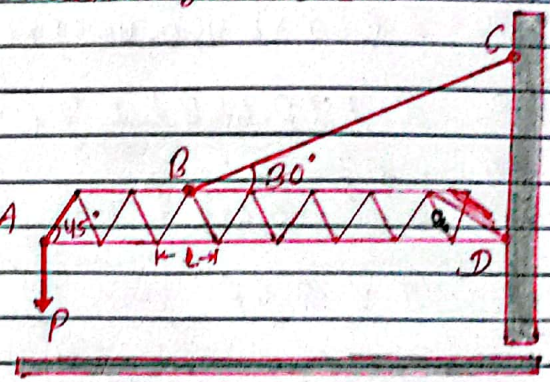
$= 7894.74\vec{j} + 4736.82\vec{k}$

Magnitude of moment $M_A = \sqrt{(7894.74)^2 + (4736.82)^2}$

$= 9206.76\ \text{Nm}$

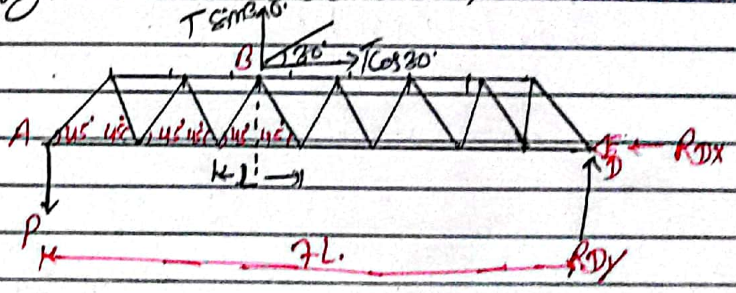
Q. 2016 Spring (1.a)

The truss structure shown carries a load 'p' and is supported by a cable, BC, and pinned at D to the wall. Determine the force in the cable BC and the reaction force at D.



Soln

The figure can be drawn as,



Let, tension on the cable BC be "T"

Height of triangle, $\tan 45° = \frac{h}{L/2}$

$1 = \frac{h}{L/2}$

$\therefore h = \frac{L}{2}$

Q. 2015 Fall (1.a)

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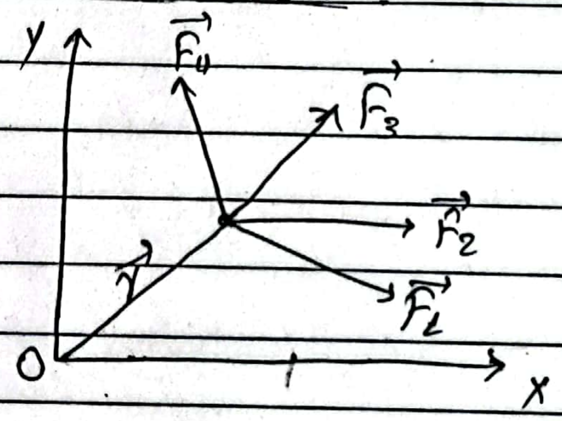
What is resolution of a force into force and a couple? Explain about Varignon's theorem with suitable example.

⇒ The ~~pross~~ process of transforming one force applied at one point, into a force and a couple at some other point is known as resolving a force into a force and a couple.

Varignon's Theorem :-



Moment of resultant of several concurrent forces about given point 'O' is equal to sum of moments of the various forces about same point 'O'.

$$\boxed{\vec{r} \times \vec{F} = \vec{r} * \vec{F_1} + \vec{r} \times \vec{F_2} + \vec{r} \times \vec{F_3} + \vec{r} \times \vec{F_4}}$$

In above figure, forces $\vec{F_1}, \vec{F_2}, \vec{F_3}$ & $\vec{F_4}$ are concurrent forces (i.e. they are meeting at a point A).

∴ Resultant of these forces,

$$\boxed{\vec{F} = \vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}}$$

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position vector of point of concurrency $= \vec{r}$

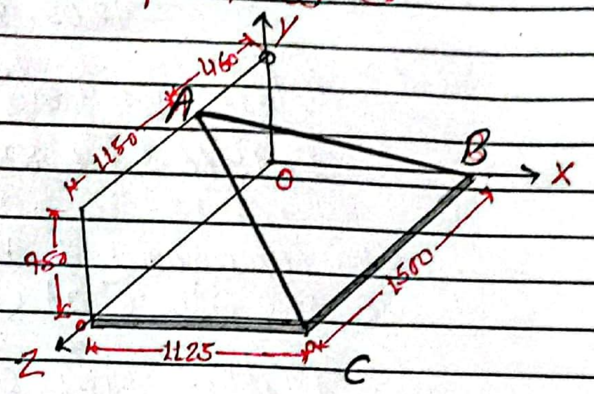
$\therefore$ According to Varignon's theorem,

$$\boxed{\vec{r}\times\vec{F} = \vec{r}\times\vec{F_1} + \vec{r}\times\vec{F_2} + \vec{r}\times\vec{F_3} + \vec{r}\times\vec{F_4}}$$

Q. 2015 Fall (1.b)

Knowing that the tension in the cable AC is 2000N, determine the component of the force exerted on the plane at C.

Soln



Given,

Tension in the cable $T_{AC} = 2000N$

Now,

$$\text{Resultant } \vec{R} = \frac{\vec{CA}}{|\vec{CA}|} \times T_{AC}$$

$$= \frac{\vec{CA}}{|\vec{CA}|} \times 2000 \quad \text{--- (1)}$$

Cordinates are,

O(0, 0, 0)

A(0, 780, 460)

B(1125, 0, 0)

C(1125, ~~1500~~ 0, 1500)

$\vec{OA} = 780\vec{j} + 460\vec{k}$

$\vec{OC} = 1125\vec{i} + 1500\vec{k}$

$$\vec{CA} = \vec{OA} - \vec{OC} = (780\vec{j} + 460\vec{k}) - (1125\vec{i} + 1500\vec{k})$$

$$= -1125\vec{i} + 780\vec{j} - 1040\vec{k}$$

$$|\vec{CA}| = \sqrt{(-1125)^2 + (780)^2 + (-1040)^2} = 1705.79 \text{ mm}$$

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From eqn ① becomes

$$\vec{R} = \frac{-1125\vec{i} + 750\vec{j} - 1040\vec{k}}{1705.79} \times 2000$$

$$= (-1125\vec{i} + 750\vec{j} - 1040\vec{k}) \times 1.172$$

$$= -1318.5\vec{i} + 881.25\vec{j} - 1218.88\vec{k}$$

∴ The component of the forces ~~are~~ exerted on the plane at C is

$(T_{CA})_x = -1318.5N$

$(T_{CA})_y = 881.25N$

$(T_{CA})_z = -1218.88N$ &

Q. 2015 Spring (1.a)

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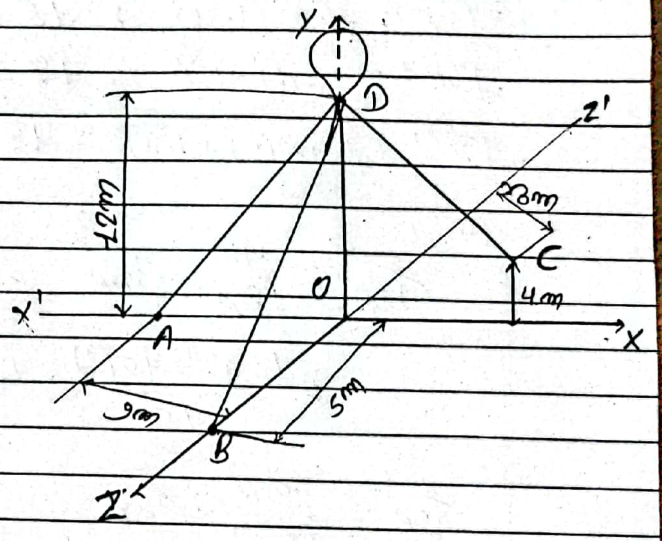
Three cables DA, DB, & DC are used to tie down a balloon at D as shown in figure 1C. Knowing that the balloon exerts 640N force at D, determine the tension in each cable.



Soln

The forces applied at D are $\vec{T}_{DA}$, $\vec{T}_{DB}$ & $\vec{T}_{DC}$ & F = 640N.

The coordinates of different points are,

A(-9, 0, 0)
B(0, 0, 5)
C(3, 0, -4)
D(0, 12, 0)

$\vec{DA} = -9\vec{i} - 12\vec{j} - 0\vec{k}$
$\vec{DB} = 0\vec{i} - 12\vec{j} + 5\vec{k}$
$\vec{DC} = 3\vec{i} - 12\vec{j} - 4\vec{k}$

$|\vec{DA}| = \sqrt{(-9)^2 + (-12)^2} = 15$
$|\vec{DB}| = \sqrt{(-12)^2 + (5)^2} = 13$
$|\vec{DC}| = \sqrt{(3)^2 + (-12)^2 + (-4)^2} = 13$

Now,

$$\vec{T_{DA}} = \frac{\vec{DA}}{|\vec{DA}|} \cdot T_{DA}$$
$$= \frac{(-9\vec{i} - 12\vec{j})}{15} \times T_{DA} = -0.6\vec{i}\,T_{DA} - 0.8\vec{j}\,T_{DA}$$

$$\vec{T_{DB}} = \frac{\vec{DB}}{|\vec{DB}|} \cdot T_{DB}$$
$$= \frac{(-12\vec{j} + 5\vec{k})}{13} \cdot T_{DB} = -0.92\vec{j}\,T_{DB} + 0.38\vec{k}\,T_{DB}$$

$$\vec{T_{DC}} = \frac{\vec{DC}}{|\vec{DC}|} \cdot T_{DC}$$
$$= \frac{(3\vec{i} - 12\vec{j} - 4\vec{k})}{13} \cdot T_{DB} =$$
$$= 0.23\vec{i}\,T_{DB} - 0.92\vec{j}\,T_{DB} - 0.33\vec{k}\,T_{DB}.$$

Applying equilibrium Condition.

$\Sigma\vec{F} = 0$