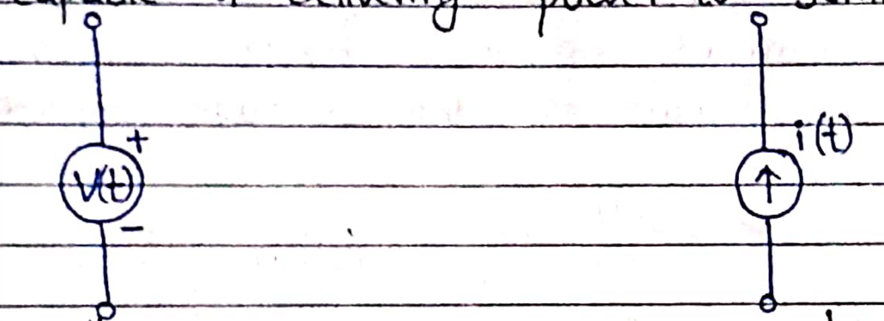
DC Circuit Analysis

N/w elements may be classified into four groups:
1. Active & passive elements
2. Unilateral & bilateral elements
3. Linear & Non Linear elements
4. Lumped & distributed elements

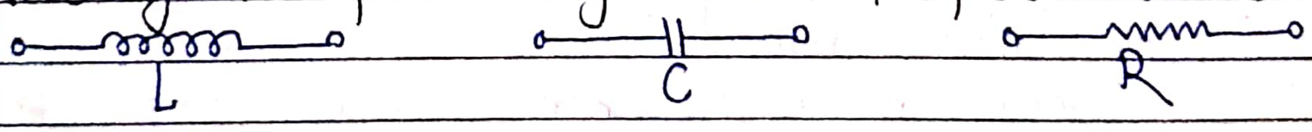
1.# Active & Passive Elements :-

Active Elements
Energy source (voltage & current source) are active element capable of delivering power to some external device.



voltage source current source

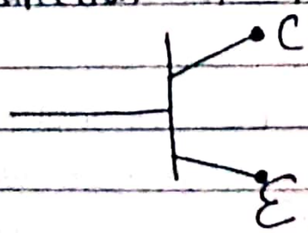
Passive Elements
Passive elements are those which are capable of only receiving the power. E.g. inductor, capacitor or resistor.



2.# Unilateral & bilateral elements :-

Unilateral elements are those whose properties change with the direction of its operation. E.g. diode,

transistor



Bilateral elements are those which properties are same in either direction. Example : transmission line.

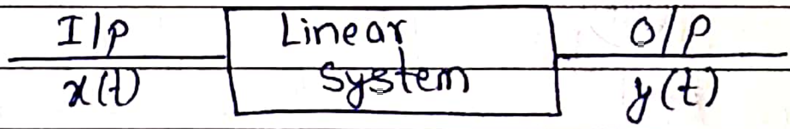
Linear & Non-Linear elements

An element is said to be linear, if it satisfy the linear voltage - current relationship. That means, the current through the element is multiplied by some constant 'a' & they results in multiplication of voltage across the element by the same constant.

Example: The current passing through the resistors is directly proportional to the voltage applied through it.

$V \propto I$

$V = IR$



In other words, a system is linear if the superposition theorem holds good. For input & output relation

$x_1 \rightarrow y_1$ & $x_2 \rightarrow y_2$

$ax_1 + bx_2 \rightarrow ay_1 + by_2$ ← Superposition theorem

Non - linear elements:

→ They are those which does not satisfy this relation.

Lumped & distributed elements:

The elements which are separated physically are known as lumped elements. E.g. three elements R, L & C are lumped.

Distributed el. are those which are not seperatable for analytical purpose. E.g. a transmission line has distributed resistor, inductor & capacitor along its length

Circuit elements [RLC] & their characters:
(ckt)

① Resistance parameter
② Inductance parameter
③ Capacitance parameter

1) Resistance Parameter :-

When the current flowing through the ~~circuit~~ material, the free electron moves through the material & collide with the atoms. This collision causes $e^{\ominus}$ lose some energy. This loss of energy per unit charge is the drop in potential ~~causes~~ across the material. The amount of energy lost by the $e^{\ominus}$ is related to the physical properties of material. There collision ~~is~~ restrict the moment of $e^{\ominus}$. This property of material to restrict the flow of $e^{\ominus}$ is called resistance denoted by 'R' & unit is ohm (Ω).

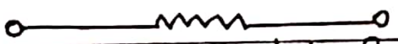


fig ⓐ symbol for resistance

According to ohm's law, the current is directly prop. to the voltage & inversely prop. to the total resistance of the circuit so

$$I = \frac{V}{R} \quad ; \quad i = \frac{v}{R}$$

we can write the above eq^n in terms of charge as under.

$$V = R\frac{dq}{dt}$$

$$i = \frac{v}{R} = G \cdot v \quad \text{when, } \frac{1}{R} = G$$

conductance of the conductor

** The unit of the conductance is mho ($\mho$) or siemen (S) or ohm^{-1}.

$P = VI$; $P = IR \cdot I = I^2 R$

$$P = I^2 R$$

Energy lost in time 't' is given by

$$W = \int_0^t P \cdot dt$$

So, $W = Pt = I^2 Rt$

$$\Rightarrow \boxed{W = I^2 Rt} \rightarrow \text{Joules}$$

2) Inductance Parameter:

By defination, the inductance is the 1 H when current through the coil change at the rate of 1A per sec induced 1 Volt across the coil.

fig. (b) symbol of inductor

* Current - Voltage relation:

$$v = L \frac{di}{dt}$$

where v = voltage through the inductor in volts

i = current through the inductor in ampere.

Re-writing above eq^n :

$$di = \frac{1}{L} v \, dt$$

Integrating both sides,

$$\int_{i(0)}^{i(t)} di = \frac{1}{L} \int_0^t v\, dt$$

$$i(t) - i(0) = 1/L \int_0^t v\, dt$$

$$i(t) = \frac{1}{L} \int_0^t v\, dt + i(0)$$

↳ initial current passing through inductor on coil

The power absorbed by inductor is

$$P = vi$$

$$= Li \frac{di}{dt} \text{ watt} \qquad \left[v = L\frac{di}{dt} \right]$$

The energy accepted by inductor is

$$\omega = \int_0^t P\, dt = \int_0^t P\, dt = \int_0^t Li \cdot \frac{di}{\cancel{dt}} \cdot \cancel{dt}$$

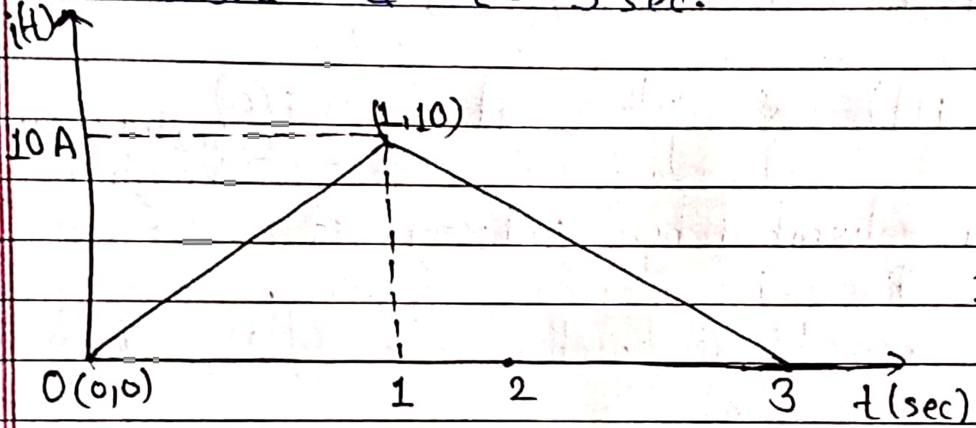
Note: $t = 0$, $i = 0$

$$= \int_0^t Li \cdot di = L \int_0^t i\, dt$$

$$\therefore W = \frac{1}{2} Li^2$$

Problems

Q. A current having a variation as shown in fig (a) below & applied to pure inductor having the value of 2H, calculate the voltage across the inductor at a time $t = 1$ sec & $t = 3$ sec.



$0 \le t \le 1 \rightarrow 1^{st}$ boundary condn
$1 \le t \le 3 \rightarrow 2^{nd}$ boundary cond

For 1st boundary condn:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{10 - 0}{1 - 0}(x - 0)$$

$$y = 10x$$

$$i(t) = 10t$$

$$i = 10t$$

$$\frac{di}{dt} = 10$$

$$v = L \cdot \frac{di}{dt}$$

$$= 2 \times 10$$

$\therefore$ $v = 20$ Volts

For 2nd boundary condn:

$$1 \le t \le 3$$

$x_1 = 1, y_1 = 10, x_2 = 3, y_2 = 0$

$$y - 10 = \frac{0 - 10}{3 - 1}(x - 1)$$

$$y - 10 = -5x + 5$$

$$y = -5x + 15$$

$$i(t) = -5t + 15$$

$$\frac{di}{dt} = -5$$

$$v = L \cdot \frac{di}{dt} = 2 \times (-5)$$

$$= -10V$$

Imp short note

Colour Coding

Carbon composition resister are very small in size. It is ~~different~~ difficult to print value in resister. That cannot read print value of human naked eyes. Therefore, colour bands are printed on one end of resister casting. This system of representing the resister value is called colour coding.

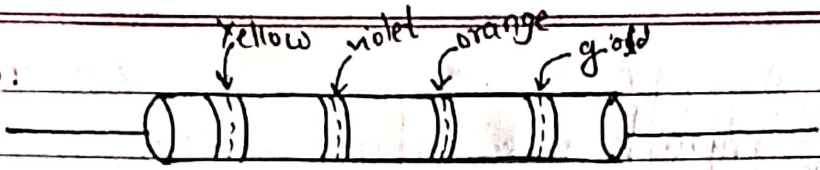
The first and the second band represent the first & second significant digit of resistance value respectively. The third band is multiplier. Sometimes the third band is golden or silver color. Then this colour represent the multiplying factor of 0.1 and 0.01 resp. The fourth band represent the resister tolerance.

Formula

$\boxed{\text{BBROY of Great Britian has a very Good Wife}}$

Colour	Significant digit	Multiplier	Tolerance
Black	0	$10^0 = 1$	—
Brown	1	$10^1 = 10$	—
Red	2	$10^2 = 100$	—
Orange	3	10^3	—
Yellow	4	10^4	—
Green	5	10^5	—
Blue	6	10^6	—
Violet	7	10^7	—
Gray	8	10^8	—
White	9	10^9	—
Golden	—	0.1	±5%
Silver	—	0.01	±10%
No colour	—	—	±20%

E.g:



$47 \times 10^3 \pm 5\%$ (2.3)

$= 47 + 2.3 = 49.3 \, k\Omega$

$= 47 - 2.3 = 44.7 \, k\Omega$

Q:1) A resistor has colour band sequence green, blue, orange and gold. Find the range in which its value must lie depending upon the manufacture tolerance to suit a circuit.

G B O g

$= 4.5$

solution:

1st significant	2nd significant	Multiplier	Tolerence
Green	Blue	Orange	gold
5	6	10^3	$= 5\%$

$5 \quad 6 \times 10^3 \pm 5\%$

$= 56k \pm 2.8$

$= 56 + 2.8$

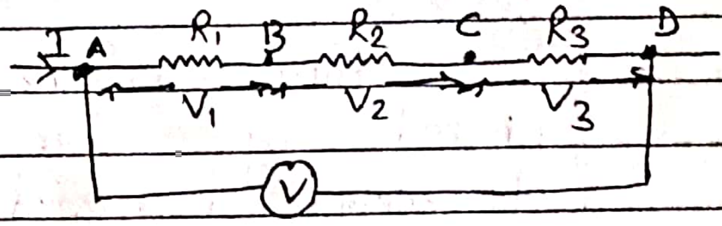
$= 58.8 \, k\Omega$

Series and Parallel Compilation of Resistance

1) Conductor in Series
2) Conductor in Parallel

1. Conductor in Series

$V = IR$ —— (i)



$V =$ sum of $(V_1+V_2+V_3)$
Pd across R_1, R_2 & R_3

$V = V_1 + V_2 + V_3$

$V = IR_1 + IR_2 + IR_3$ —— (ii)

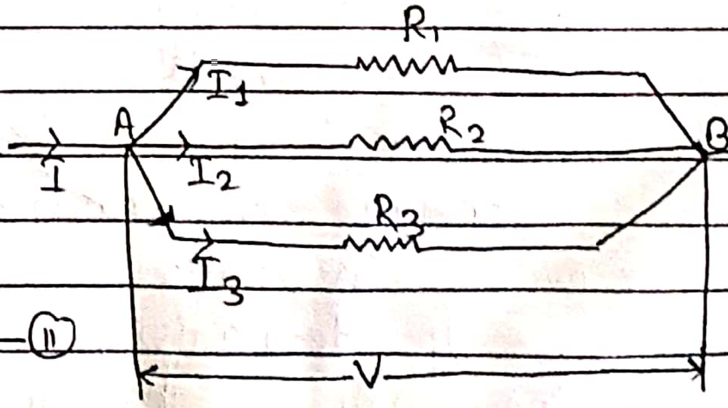
From eq^n (i) & (ii),

$IR = IR_1 + IR_2 + IR_3$

$\cancel{I}R = \cancel{I}(R_1 + R_2 + R_3)$

$[R = R_1 + R_2 + R_3]$

2. Conductor in Parallel



$I = \frac{V}{R}$ —— (1)

$I = I_1 + I_2 + I_3$

$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$ —— (11)

From eq^n (1) & (11),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{\cancel{V}}{R} = \cancel{V}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Concept of Power, Energy & its Calculations:

Energy is the capacity for doing work. Energy is nothing but store work. Energy may exist in many form as mechanical, chemical, electrical & so on.

Power is the rate of change of energy. It is denoted by either P or P. If the certain amount of energy is used over a certain unit of time then,

$$P = \frac{Energy}{time} = \frac{\omega}{t}$$

$$P = \frac{dw}{dt}$$

where dw = change in energy
dt = change in time

$$P = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt}$$

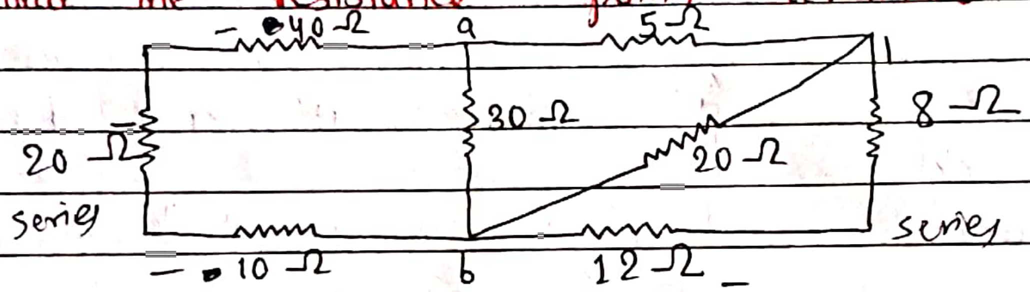
$$\therefore P = v \times i = watt$$

Energy is measured in joule, time in sec & power in watt.

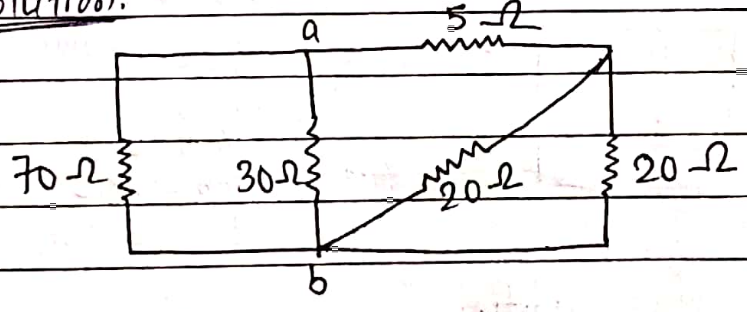
By defination,
One watt is the amount of power generated when one joule of energy is consumed in 1 sec. Thus, the number of joule consume in 1 sec is always equal to the number of watts. Amount of power less than 1 watt are usually express in fraction of watt. In the field of electronics via milliwatt (mw) & microwatt (μw). In electrical phased, kilowatt & megawatt (mw) are

common units. Radio & tv station also use large amount of power to transmit the signal.

Q. 2) Find the resistance point terminals a-b.



Solution:



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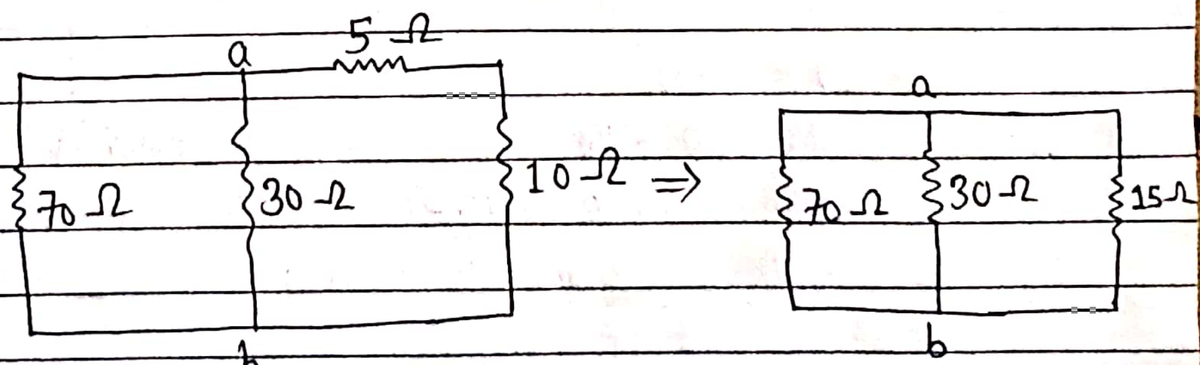


$$\frac{1}{R} = \frac{1}{70} + \frac{1}{30} + \frac{1}{10} = \frac{31}{210}$$

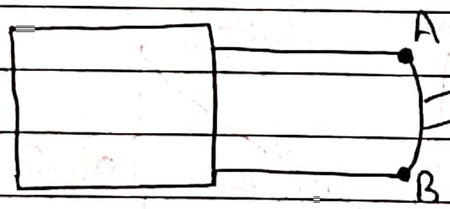
$$\therefore R = 6.77\ \Omega$$

Short and open ckt :

1) Short ckt
- a shorts, in a series circuit
- b shorts, in a parallel ckt

2) Open ckt
- (a) open in series ckt
- open in parallel ckt

1) Short ckt



Thick metallic wire

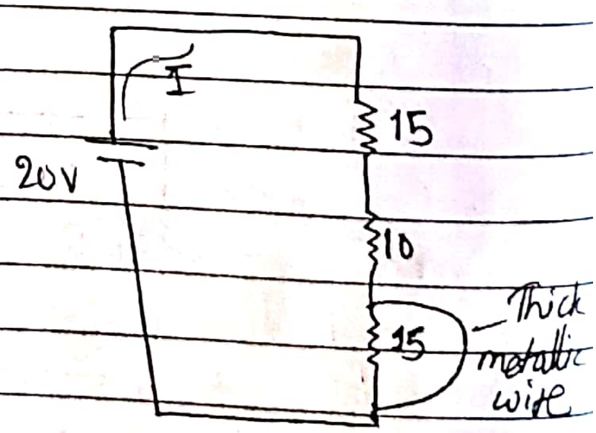
$R = 0$

$\cancel{V_{AB} = 0}$

$I_{ab} = I_{sc}$

$I_{sc} = \infty$

a) ~~for short ckt condition.~~



$V = 20V$

$R_1 = R_1 + R_2 + R_3$

$= 5 + 10 = 15\Omega$

$I = \frac{V}{R} = \frac{20}{15} = 1.33$ amp

$P = I^2 R = (1.33)^2 \times 15$

$= 26.66$ watt

Normal condition,

$V = 20V$

$R_T = R_1 + R_2 + R_3$

$= 5 + 10 + 15$

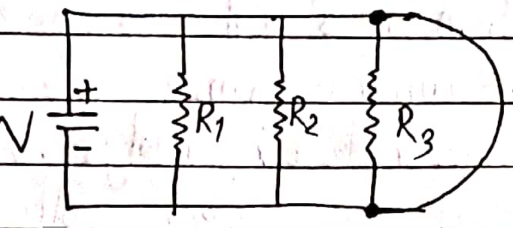
$= 30\Omega$

$I = \frac{V}{R_T} = \frac{20}{30} = 0.67$ amp

$P = I^2 R = (0.67)^2 \times 30$

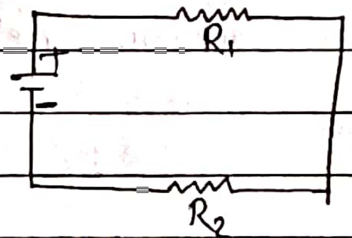
$= 13.26$ watt

b) Short in parallel circuit:

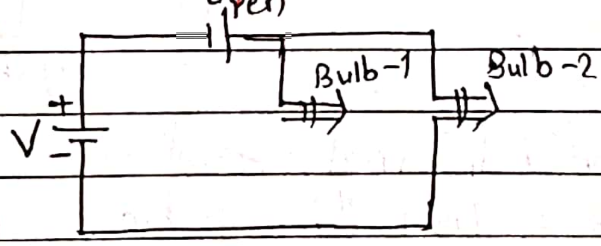


2) Open ckt

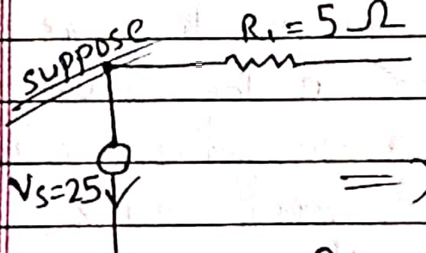
a) Opens, in a series circuit



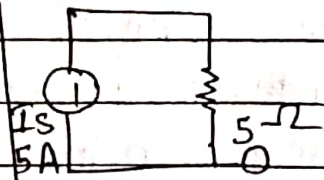
b) Opens, in a parallel ckt



III) Source Conversion [Source Transformation]



$\Rightarrow$



$I = 5A$

$R = 5\,\Omega$

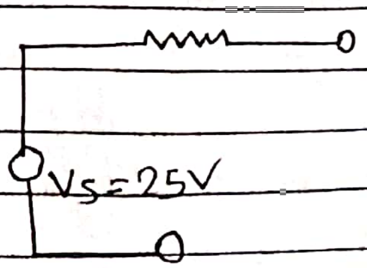
$V = I \cdot R$

$= 5 \times 5 = 25V$

$V = 25$

$R_1 = 5\,\Omega$

$I = \frac{V}{R} = \frac{25}{5} = 5A$



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→ Basically, energy Source are either voltage source or current source. Sometimes it is necessary to convert voltage source to current source & current source to voltage source in network analysis. Any practical voltage source consist of an ideal voltage source in a series with an internal resistance (impedence). Similarly, a practical current source consist of ideal current source in parallel with internal resistance (impedence) as shown below in fig a & b.

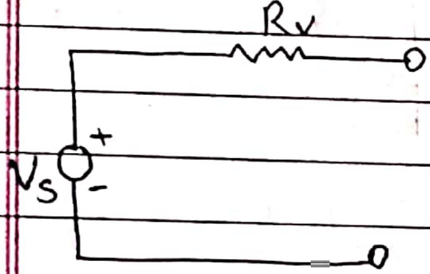

fig (a)

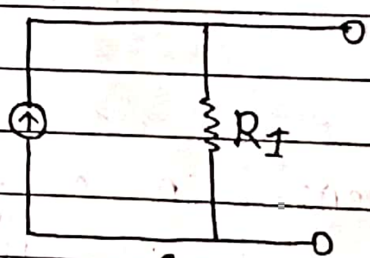

fig (b)

where, R_V = Internal resistance of voltage source (V_s)
R_I = Internal resistance of current source (I_s)

Any Source, whether it be a current source or voltage source derives current through its load resistance & the magnitude of current depend on the value of load resistance. In fig. c & d represents the practical voltage source & a practical current source connected to the same load resistance (R_L)

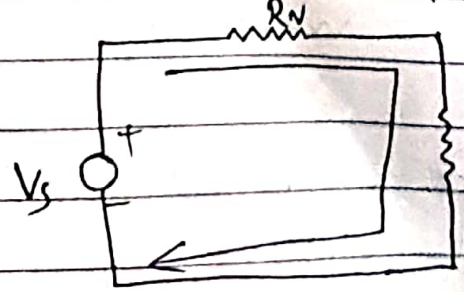

fig (c)

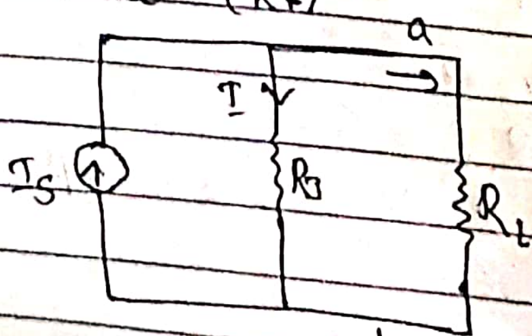

fig (d)

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The load current can be calculated by using Kirchoff's voltage law.

From fig (c),

$$I_V = \frac{V_S}{R_S + R_L} \quad \text{——— ①}$$

From fig (d), the load current can be calculated by using Kirchoff's current law as

$$I_L = I_S = \frac{R_I}{R_I + R_L} \quad \text{——— ②}$$

The above two source are said to be equal when they produce equal amount of current & voltage to the equal load resistance.

$$I_V = I_L$$

$$\frac{V_S}{R_V + R_L} = \frac{I_S \cdot R_I}{R_I + R_L} \quad \text{——— ③}$$

The current source, the voltage across the terminal a.b,

$$V_S = I_S \cdot R_I$$

Substituting V_S value in eqn ③,

$$\frac{I_S \cdot R_I}{R_V + R_L} = \frac{I_S \cdot R_I}{R_I + R_L}$$

$$R_V + R_L = R_I + R_L$$

$$\therefore \boxed{R_V = R_I}$$

Therefore, any practical voltage source having an ideal voltage (V_S) & internal series resistance (R_V) can be replaced by a current source $I = V_S / R_V$ in parallel with an internal resistance $R_I = R_V$.

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Energy Sources

Voltage & current sources :-

a) Ideal voltage & ideal current source
b) Independent or dependent voltage & current source.

a) Ideal voltage & ideal current source :

* Ideal voltage source :-

An ideal voltage source is one that maintain a constant terminal voltage no matter how much current is drawn from it.

E.g.

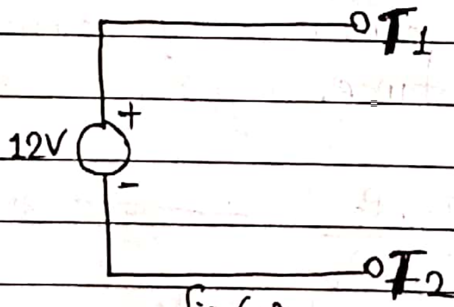


fig (a)

Terminal T_1 & T_2 whether we connect 1 MΩ or 1 kΩ resistor across ll terminals. In practice, it is difficult to construct an ideal voltage source. It is fact that all the real voltage source have some internal resistance that causes terminal voltage to be decreased if the current drawn large.

* Ideal current source :-

Ideal current source is the one that supplies the same current to any resistance connected across ~~the~~ its terminals. However, it is difficult to construct such current source because it has same internal resistance that increases the current. If is the resistance connected across the terminals is made too large.

b) Independent voltage & current source:
whose voltage and current doesn't depends on other ckt variable.

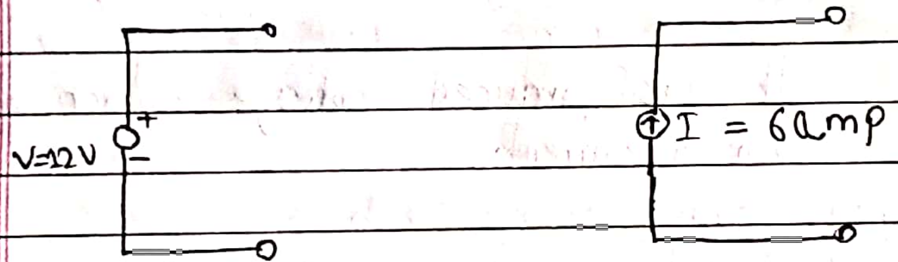
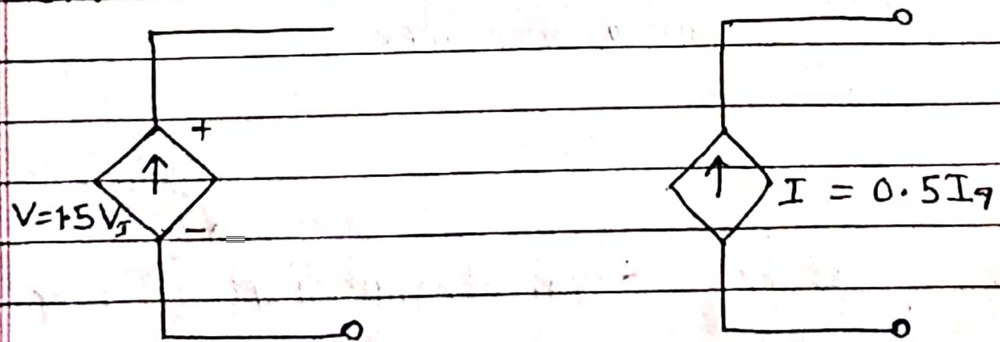


fig (a)

A dependent voltage and current source is that whose voltage & current depends on the other ckt variable.



Here are four types of dependent Sources:
① Voltage - dependent voltage source (VDVs)

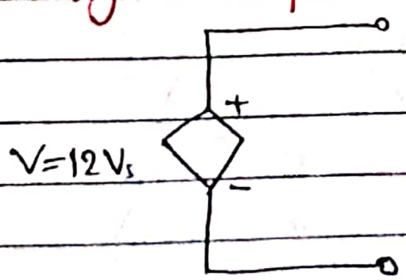
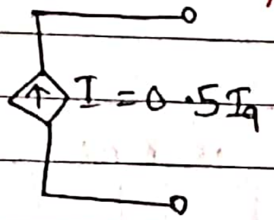


fig @

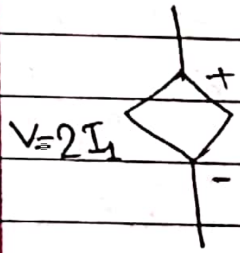
It produce voltage which depends on the voltage.

② Current-dependent current source (CDCS):



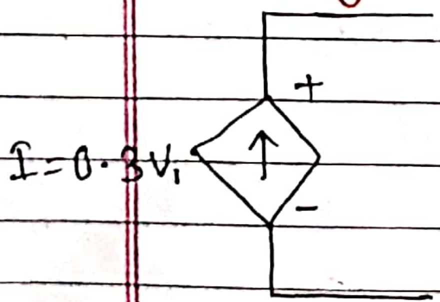
It produced current that depends on the current source.

③ Currents-dependent voltage source (CDVS):



It is produced voltage which depends on a current.

④ Voltage-dependent Current Source (VDCS):



It produce a currents which depends on a variable.

short note – 5 marks
numerical – 7 marks

V.V.imp

Star-Delta Transformation [Y] or [Δ]:

Y → wye Δ → mesh

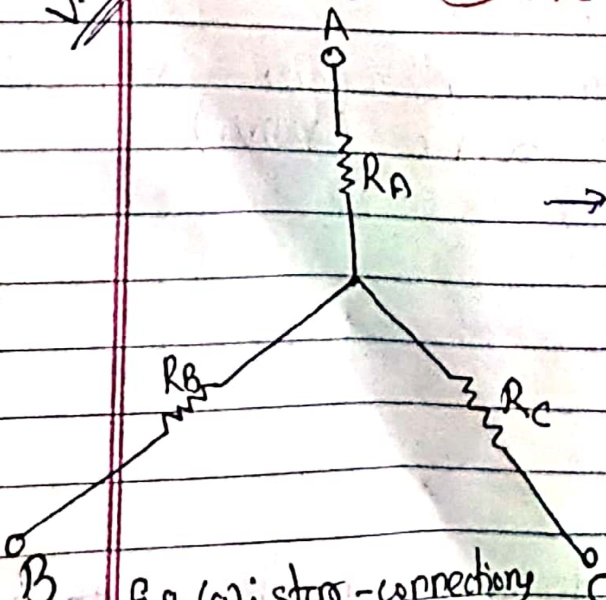


fig (a): star-connections

→

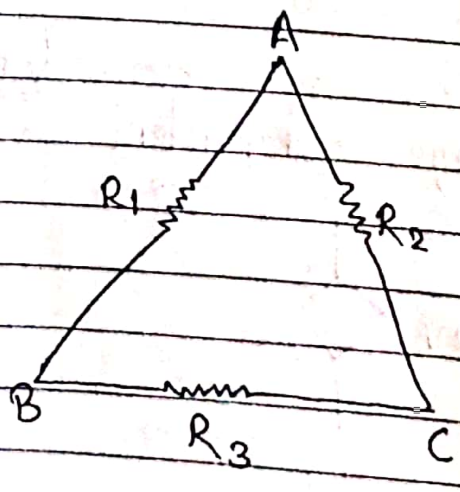


fig (b): Delta connections

These two arrangements will be electrically same if the resistance as measured betn any pair of terminals is the same on both the arrangements.

Star Connections :-

Now, consider the star-connections ckt, the register from the terminal AB, BC and CA.

$$R_{AB}(Y) = R_A + R_B$$

$$R_{BC}(Y) = R_B + R_C$$

$$\& \quad R_{CA}(Y) = R_C + R_A$$

Delta Connections :-

Resistance seen from the terminal AB, BC & CA (R_1 is parallel with series of R_2 & R_3).

$$R_{AB}(\Delta) = R_1 \parallel (R_2 + R_3)$$

$$= \frac{R_1 \times (R_2 + R_3)}{R_1 + R_2 + R_3}$$

$$R_{BC}(\Delta) = R_3 \parallel (R_1 + R_2)$$

$$= \frac{R_3 \times (R_1 + R_2)}{R_1 + R_2 + R_3}$$

$$R_{CA}(\Delta) = R_2 \parallel (R_1 + R_3)$$

$$= \frac{R_2 \times (R_1 + R_3)}{R_1 + R_2 + R_3}$$

Now,

if we equate the resistance of star(Y) to delta (Δ) ckt:

$$R_A + R_B = \frac{R_1 \times (R_2 + R_3)}{R_1 + R_2 + R_3} \quad \text{—— (i)}$$

$$R_B + R_C = \frac{R_3 \times (R_1 + R_2)}{R_1 + R_2 + R_3} \quad \text{—— (ii)}$$

$$R_C + R_A = \frac{R_2 \times (R_1+R_3)}{R_1+R_2+R_3} \quad \text{—— (III)}$$

Subtracting eqⁿ (II) from (I) & adding eqⁿ (III),

~~RA + RB − RB − RC + RC + RA =~~

$$R_A + \cancel{R_B} - \cancel{R_B} - \cancel{R_C} + \cancel{R_C} + R_A = \frac{R_1 \times (R_2+R_3)}{R_1+R_2+R_3} - \left[\frac{R_3 \times (R_1+R_2)}{R_1+R_2+R_3}\right] + \left[\frac{R_2 (R_1+R_3)}{R_1+R_2+R_3}\right]$$

$$\text{or } 2R_A = \frac{R_1R_2 + R_1R_3}{R_1+R_2+R_3} - \frac{R_1R_3 + R_2R_3}{R_1+R_2+R_3} + \frac{R_1R_2 + R_2R_3}{R_1+R_2+R_3}$$

$$\text{or } 2R_A = \frac{R_1R_2 + \cancel{R_1R_3} - \cancel{R_1R_3} - \cancel{R_2R_3} + R_1R_2 + \cancel{R_2R_3}}{R_1+R_2+R_3}$$

$$\text{or } \cancel{2}R_A = \frac{\cancel{2}R_1R_2}{R_1+R_2+R_3}$$

$$\boxed{\therefore R_A = \frac{R_1R_2}{R_1+R_2+R_3}} \quad \text{—— (IV)}$$

Delta to star transformation

Similarly,

$$R_B = \frac{R_1R_3}{R_1+R_2+R_3} \quad \text{—— (V)}$$

$$R_C = \frac{R_2R_3}{R_1+R_2+R_3} \quad \text{—— (VI)}$$

Delta to star transformation

Multiplying eq^n (IV) & (V), we get

$$R_A \times R_B = \frac{R_1 R_2}{R_1+R_2+R_3} \times \frac{R_1 R_3}{R_1+R_2+R_3}$$

$$= \frac{R_1^2 R_2 R_3}{(R_1+R_2+R_3)^2} \quad \text{--- (VII)}$$

Similarly,

$$R_B \times R_C = \frac{R_1 R_3}{(R_1+R_2+R_3)} \times \frac{R_2 R_3}{R_1+R_2+R_3}$$

$$= \frac{R_1 R_2 R_3^2}{(R_1+R_2+R_3)^2} \quad \text{--- (VIII)}$$

$$R_C \times R_A = \frac{R_2 R_3}{R_1+R_2+R_3} \times \frac{R_1 R_2}{R_1+R_2+R_3}$$

$$= \frac{R_1 R_2^2 R_3}{(R_1+R_2+R_3)^2} \quad \text{--- (IX)}$$

Now, adding eq^n (VII), (VIII) & (IX),

$$R_A \times R_B + R_B \times R_C + R_C \times R_A = \frac{R_1^2 R_2 R_3}{(R_1+R_2+R_3)^2} + \frac{R_1 R_2 R_3^2}{(R_1+R_2+R_3)^2} + \frac{R_1 R_2^2 R_3}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1 R_2 R_3 \cancel{(R_1+R_2+R_3)}}{(R_1+R_2+R_3)^{\cancel{2}}}$$

$$= \frac{R_1 \cdot R_2 \cdot R_3}{R_1+R_2+R_3} \quad \text{--- (X)}$$

$$R_A \cdot R_B + R_B \cdot R_C + R_C \cdot R_A = R_A \cdot R_3$$

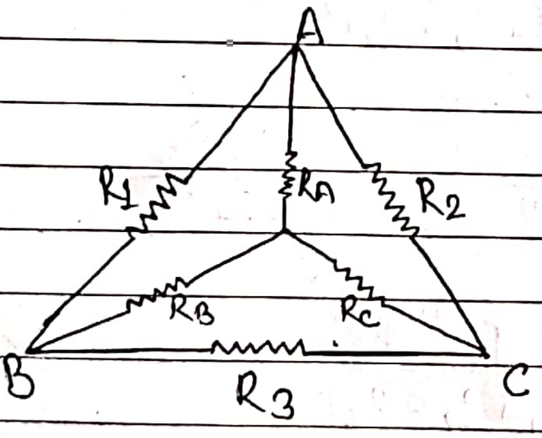
$$R_3 = \frac{R_A \cdot R_B + R_B \cdot R_C + R_C \cdot R_A}{R_A} \quad \text{—— (xi)}$$

$$R_2 = \frac{R_A \cdot R_B + R_B R_C + R_C \cdot R_A}{R_B} \quad \text{—— (xii)}$$

$$R_1 = \frac{R_A \cdot R_B + R_B \cdot R_C + R_C \cdot R_A}{R_C} \quad \text{—— (xiii)}$$

Star to delta transformation

* Star to delta transformation:



Delta to star:

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Star to delta,

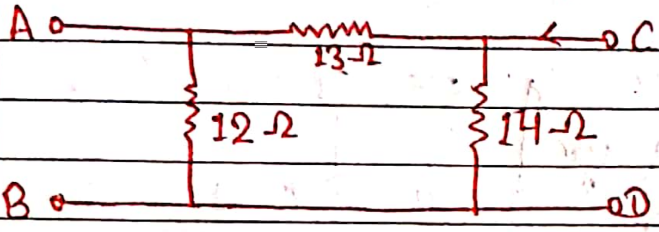
$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

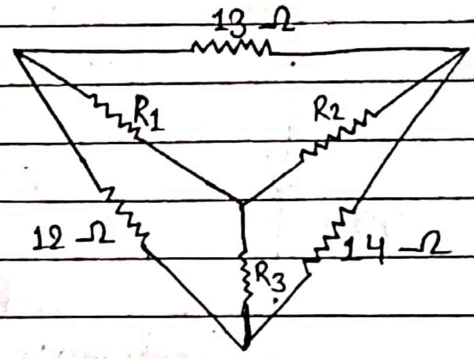
$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

Q. 1) Obtain the star connected equivalent for delta connected ckt:

PU



⟹

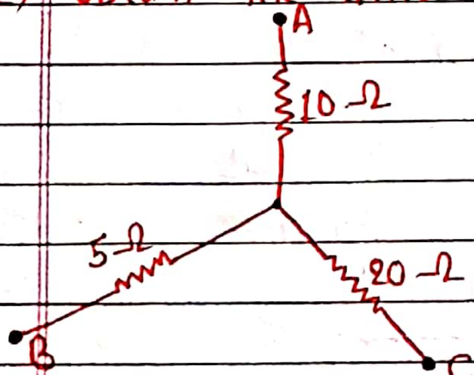


$$R_1 = \frac{12 \times 13}{12+13+14} = 4\,\Omega$$

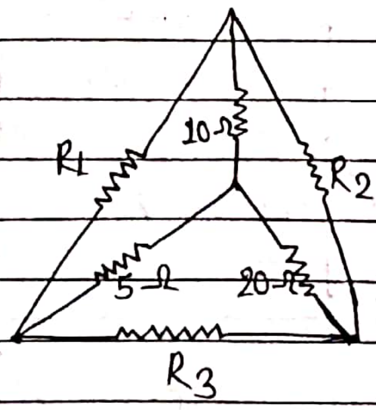
$$R_2 = \frac{13 \times 14}{12+13+14} = 4.67\,\Omega$$

$$R_3 = \frac{12 \times 14}{12+13+14} = 4.31\,\Omega$$

Q. 2) Obtain the delta connected equivalent for star connected ckt:



⟹

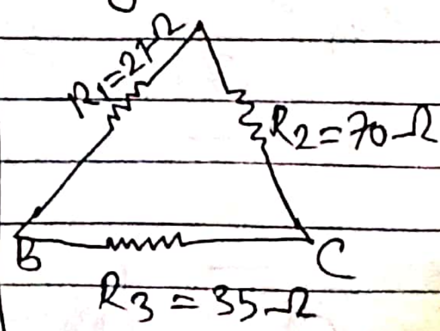


$$R_1 = \frac{10 \times 5 + 5 \times 20 + 20 \times 10}{20} = 21\,\Omega$$

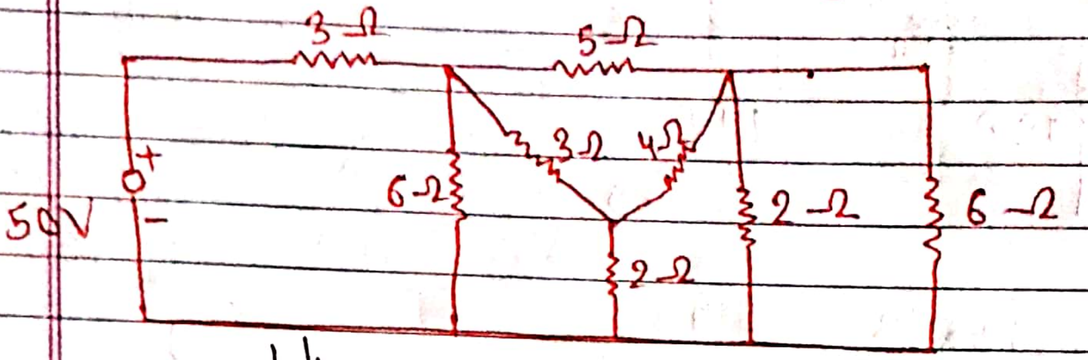
$$R_2 = \frac{350}{5} = 70\,\Omega$$

$$R_3 = \frac{350}{10} = 35\,\Omega.$$

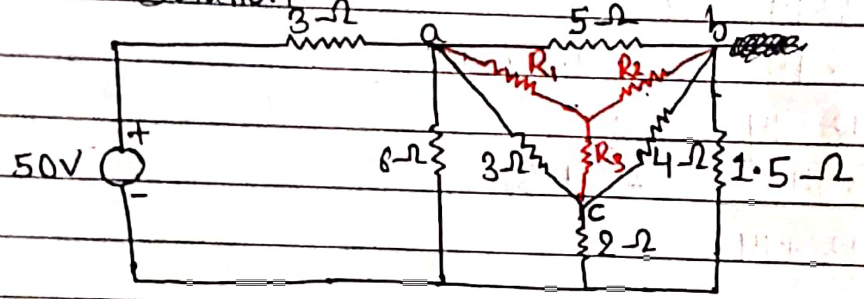
Finally,



Q. 3) Determine the current drawn by the ckt shown in fig.



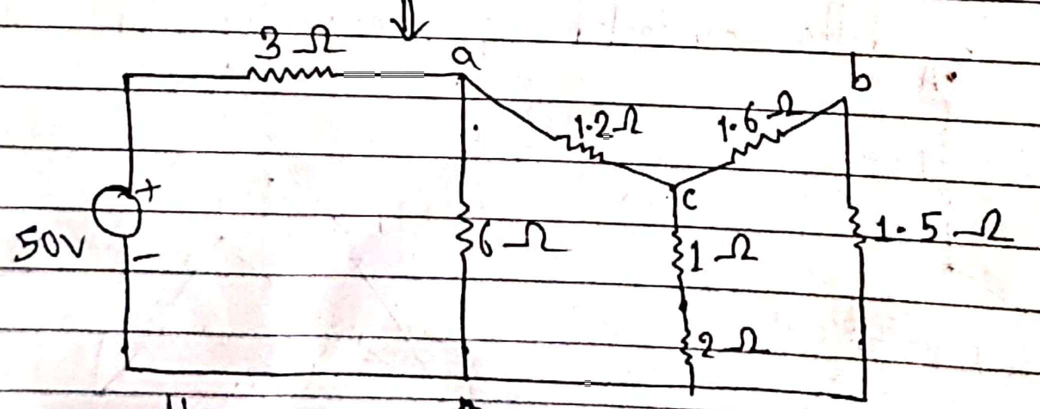
solution:-



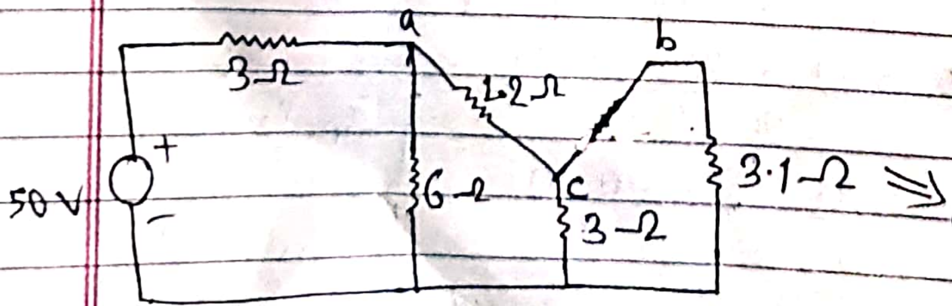
$$R_1 = \frac{5 \times 3}{3+4+5} = 1.2\ \Omega$$

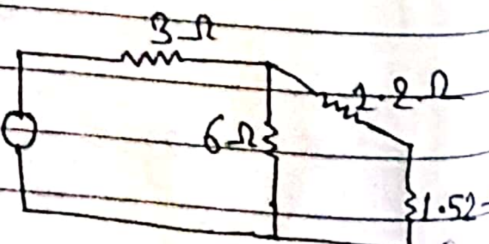
$$R_2 = \frac{4 \times 5}{12} = 1.6\ \Omega$$

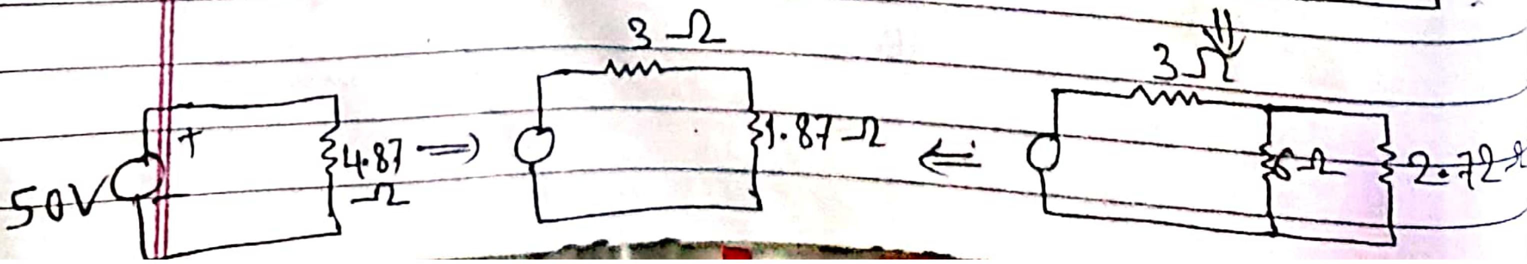
$$R_3 = \frac{3 \times 4}{12} = 1\ \Omega$$



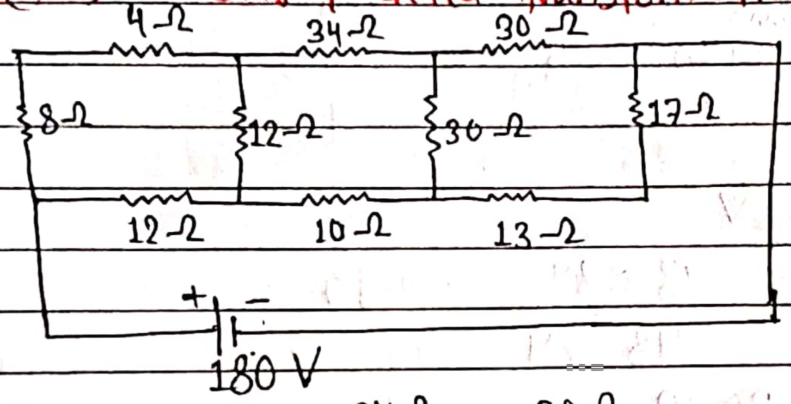
$$I = \frac{V}{R_T} = \frac{50}{4.87} = 10.27A$$



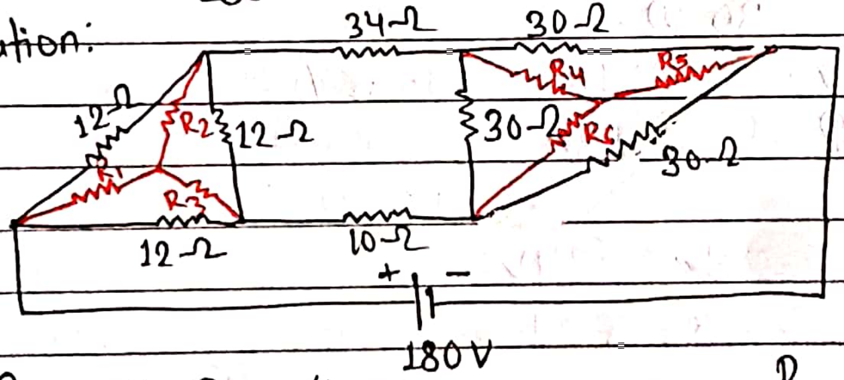




Q. 4) Determine the value of current in 10 Ω resistance in network (N/w). star / delta transformation ckt is given below:



solution:



$$R_1 = \frac{12 \times 12}{12+12+12} = 4\ \Omega$$

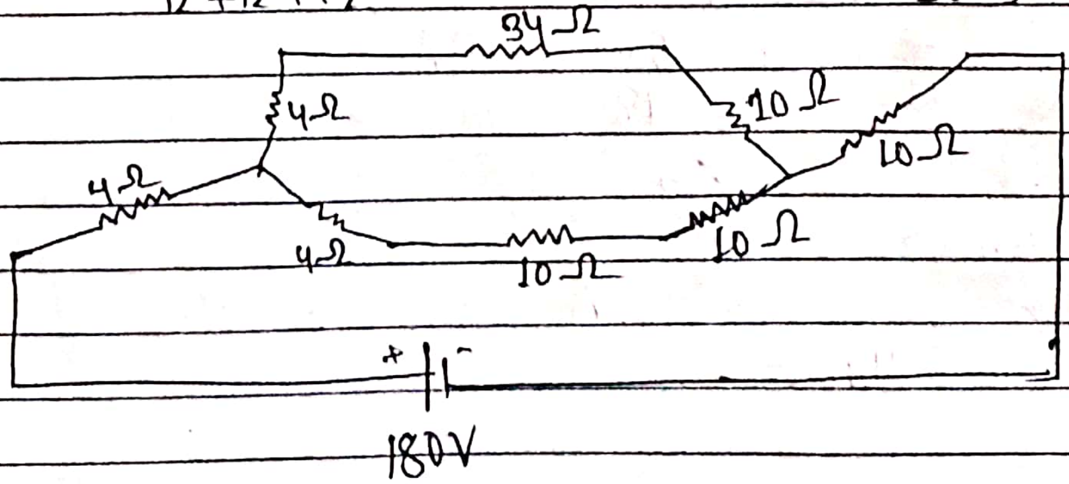
$$R_4 = \frac{30 \times 30}{30+30+30} = 10\ \Omega$$

$$R_2 = \frac{12 \times 12}{12+12+12} = 4\ \Omega$$

$$R_5 = \frac{30 \times 30}{30+30+30} = 10\ \Omega$$

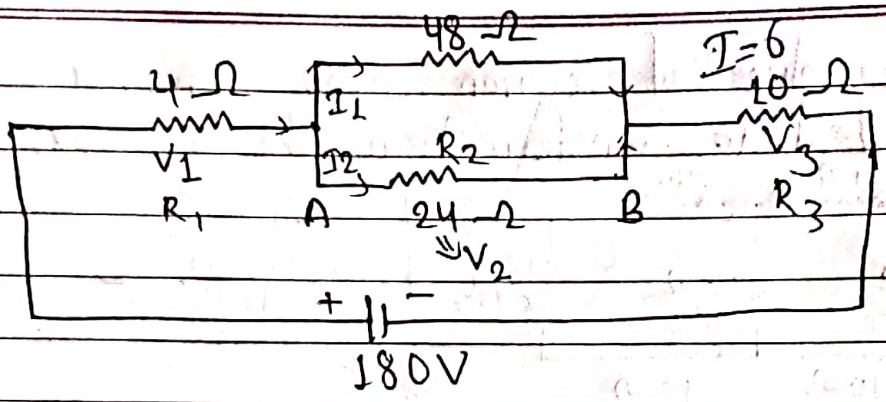
$$R_3 = \frac{12 \times 12}{12+12+12} = 4\ \Omega$$

$$R_6 = \frac{30 \times 30}{30+30+30} = 10\ \Omega$$



$V = IR$

$I = \frac{V}{R}$



$R_T = 4 + \frac{48 \times 24}{48 + 24} + 10$

$\therefore R_T = 30\ \Omega$

$\therefore I = \frac{V}{R} = \frac{18\cancel{0}}{3\cancel{0}} = 6\ A$

So,

$V_1 = IR_1 = 4 \times 6 = 24V$

$\cancel{V_2 = IR_2 = 6 \times 24 =}$

$V_3 = IR_3 = 6 \times 10 = 60\ V$

$V_2 = 180 - 60 - 24 = 96\ V$

4

- For 48 Ω resistance,

$V = I_1 R$ [V is same in ||]

$\Rightarrow 96 = I_1 R$

$\Rightarrow I_1 = \frac{96}{48} = 2A$

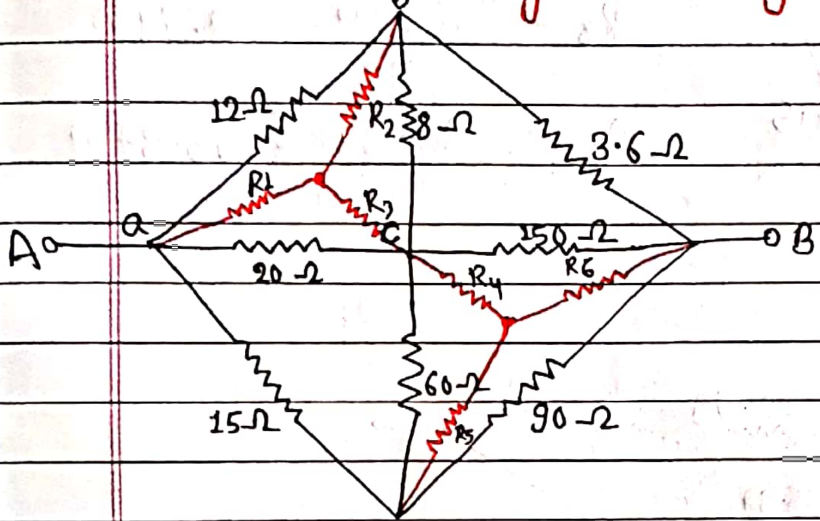
$\therefore I = I_1 + I_2$

or $6 = 2 + I_2$

$\Rightarrow I_2 = 4A$

$I_{10} = 4A$

Q.5) Resistance measured between terminals A and B of the ckt shown in fig below by star delta transformation:



$$R_1 = \frac{12 \times 20}{12+20+8} = 6\,\Omega$$

$$R_2 = \frac{12 \times 8}{12+20+8} = 2.4\,\Omega$$

$$R_3 = \frac{20 \times 8}{12+20+8} = 4\,\Omega$$

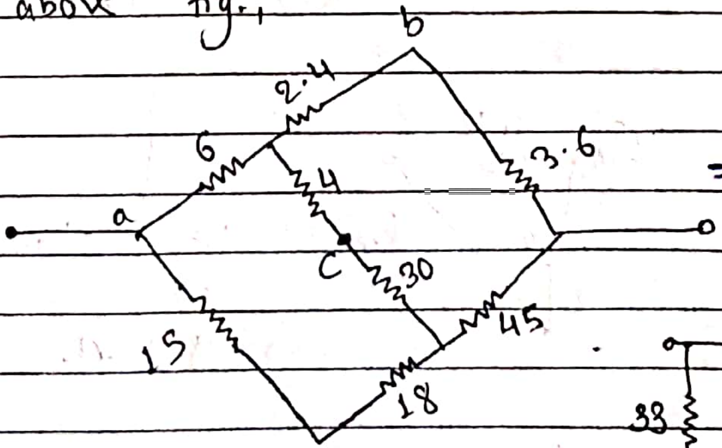
Similarly,

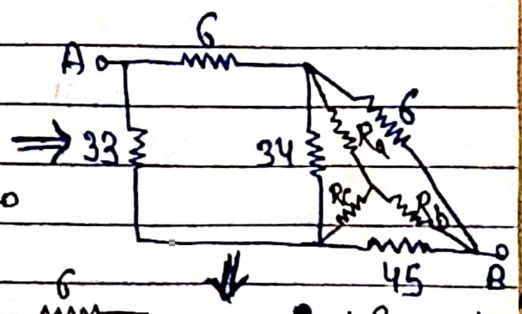
$$R_4 = \frac{150 \times 60}{60+90+150} = 30\,\Omega$$

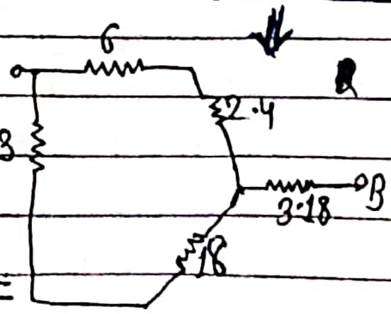
$$R_5 = \frac{60 \times 90}{60+90+150} = 18\,\Omega$$

$$R_6 = \frac{150 \times 90}{60+90+150} = 45\,\Omega$$

From above fig.,



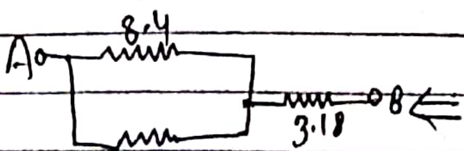




$R_a = 2.4\,\Omega$

$R_b = 3.18\,\Omega$

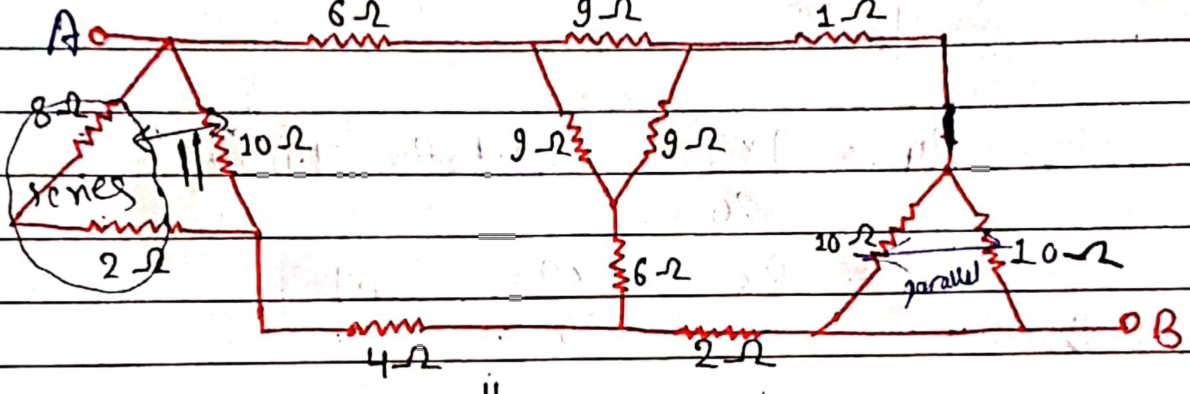
$R_c = 18\,\Omega$



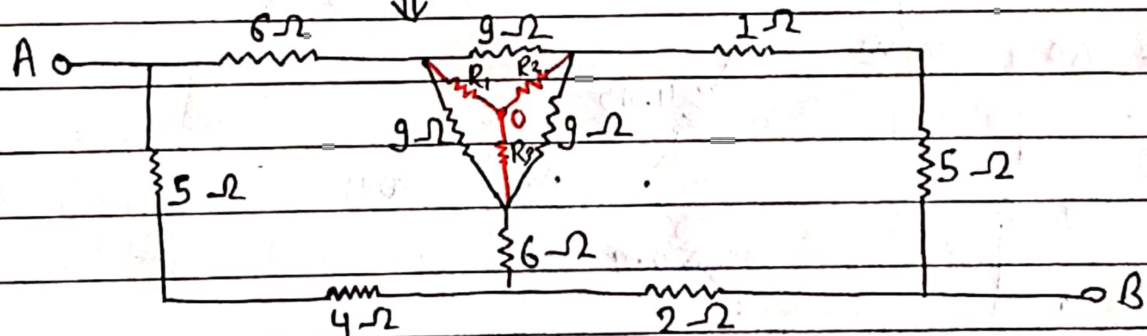
$$R_T = 3.18 + \frac{8.4 \times 51}{8.4+51}$$

$= 10.38\,\Omega$

Q.6) Calculate the equivalent resistance betn point A & B (by star-delta transformation):



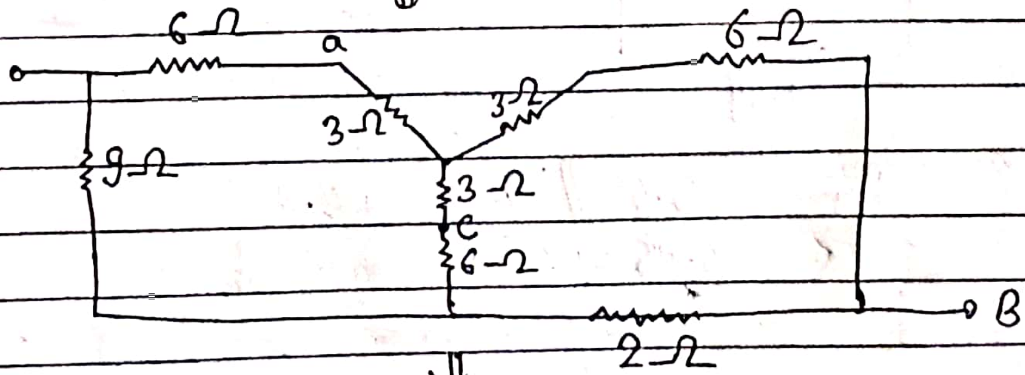
$\Downarrow$



$\Downarrow$

$R_1 = \dfrac{9 \times 9}{9+9+9} = 3\,\Omega$, $R_2 = \dfrac{9 \times 9}{9+9+9} = 3\,\Omega$, $R_3 = \dfrac{9 \times 9}{9+9+9} = 3\,\Omega$

$\Downarrow$



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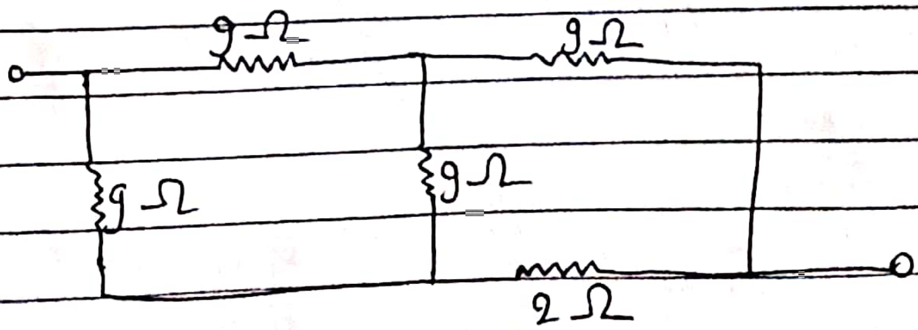
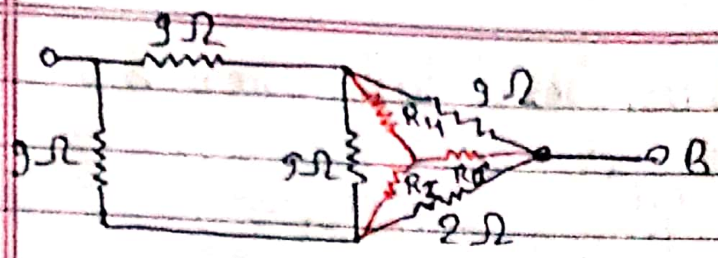


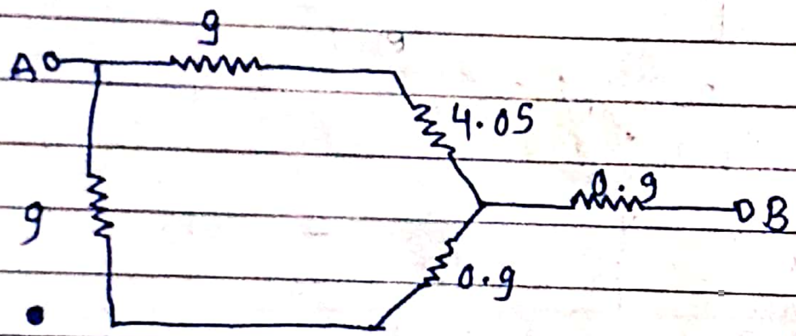


$$R_4 = \frac{9 \times 9}{20} = \cancel{0.9\,\Omega}\ 4.05\,\Omega$$

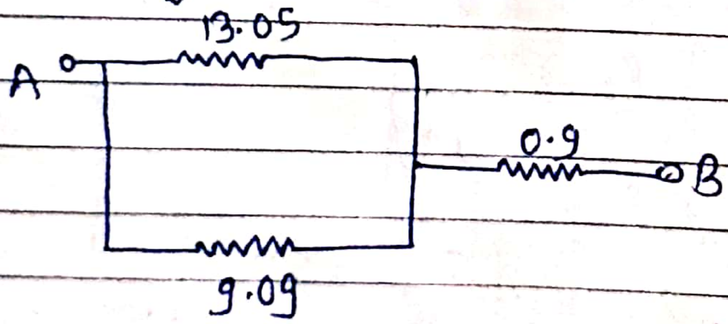
$$R_5 = R_6 = \frac{9 \times 2}{20} = 0.9$$

$R_T = 3.3$

⇓

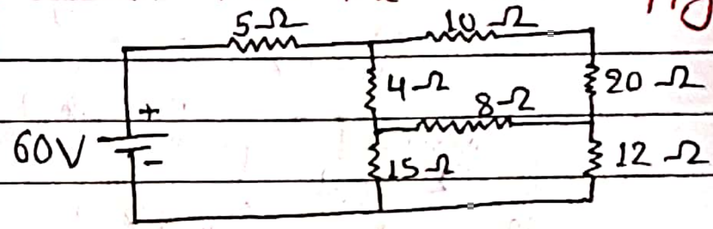


⇓



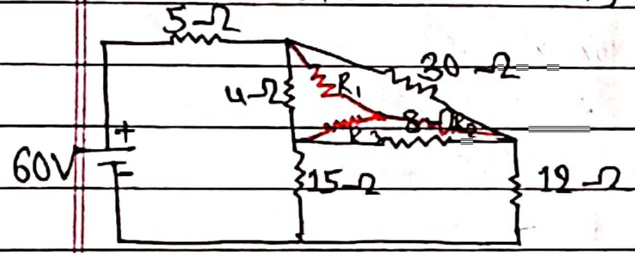
$$R_T = 13.05 \parallel 9.09 + 0.9$$
$$= 6.26\,\Omega$$

Q.7) Use the star-delta conversion to find the resistance as seen from the battery terminal & hence find the current supply by the battery.



Sol^n :-

Star / delta eq. ckt is

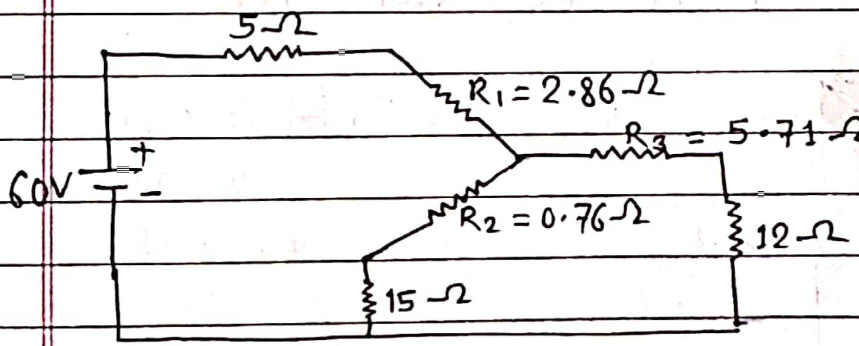


Considering star eq. to delta ckt,

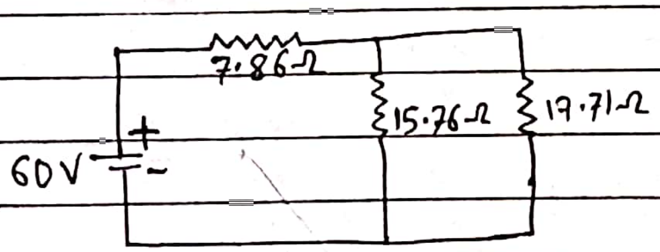
$$R_1 = \frac{30 \times 4}{30+4+8}\ \Omega = 2.86\ \Omega$$

$$R_2 = \frac{4 \times 8}{42} = 0.76\ \Omega,\quad R_3 = \frac{30 \times 8}{42} = 5.71\ \Omega$$

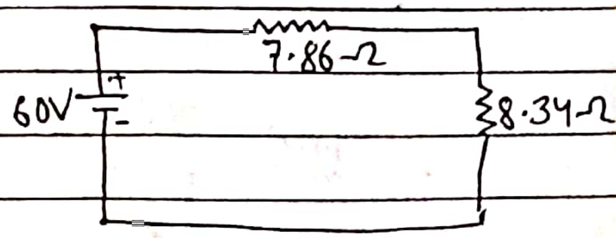
⇓



⇓



⇒

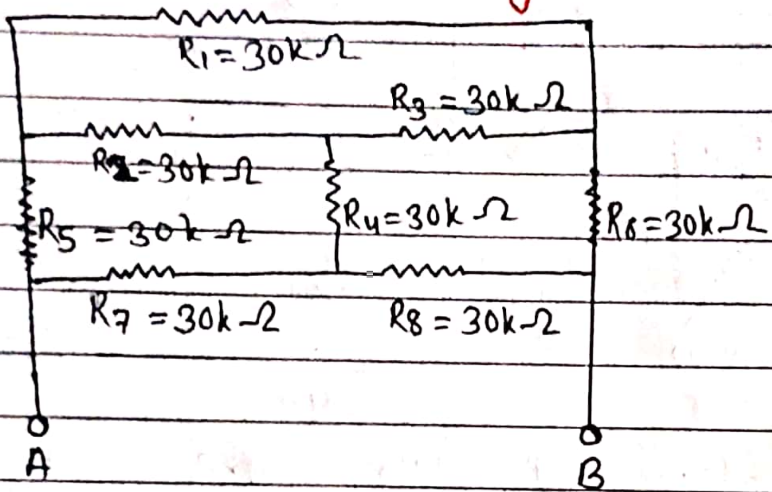


$$\therefore R_{eq} = (7.86 + 8.34)\ \Omega$$
$$= 16.2\ \Omega$$

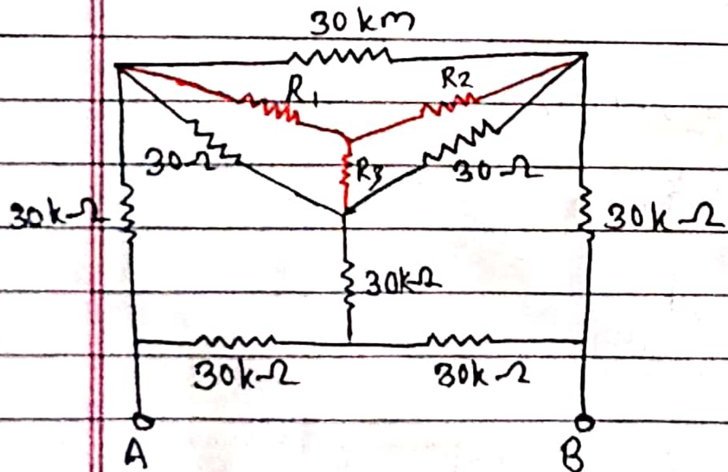
Now,

$$\text{Current} = \frac{V}{R_{eq}} = \frac{60\ V}{16.2\ \Omega} = 3.7 \text{ ampere}$$

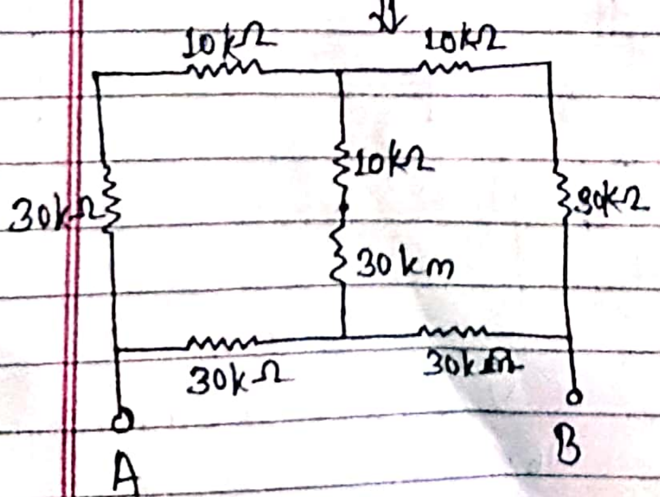
Q. 18) Find the equivalent resistance across the terminal AB for ckt shown in fig ⓐ below:

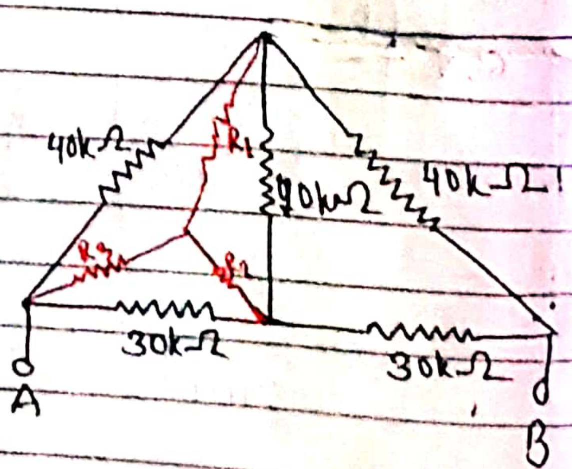


Soln :- Considering the eq. star ckt,

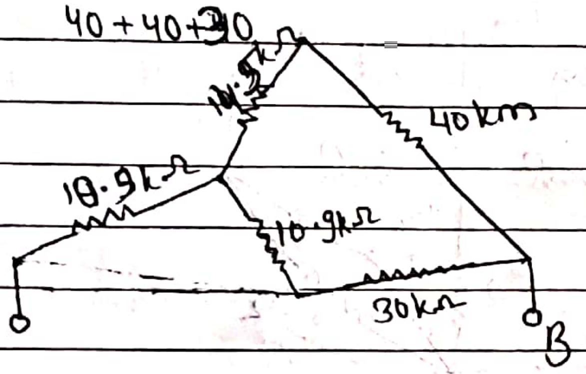


$$R_1 = R_2 = R_3 = \frac{30 \times 30}{30+30+30}$$
$$= 10\,k\Omega$$





$$R_1 = \frac{40 \times 40}{40+40+30}\ k\Omega = 14.5\,k\Omega \ ; \ R_2 = R_3 = \frac{30 \times 40}{40+40+30} = 10.9\,k\Omega$$



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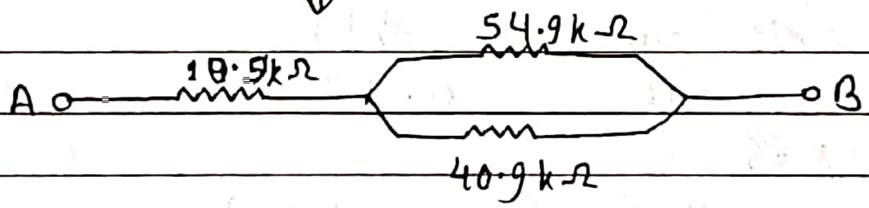

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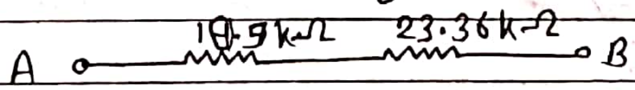
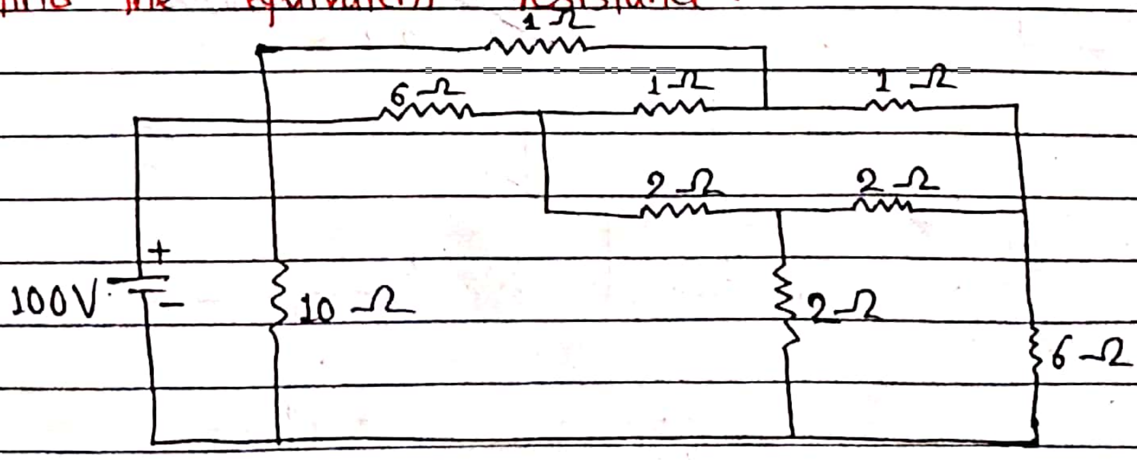
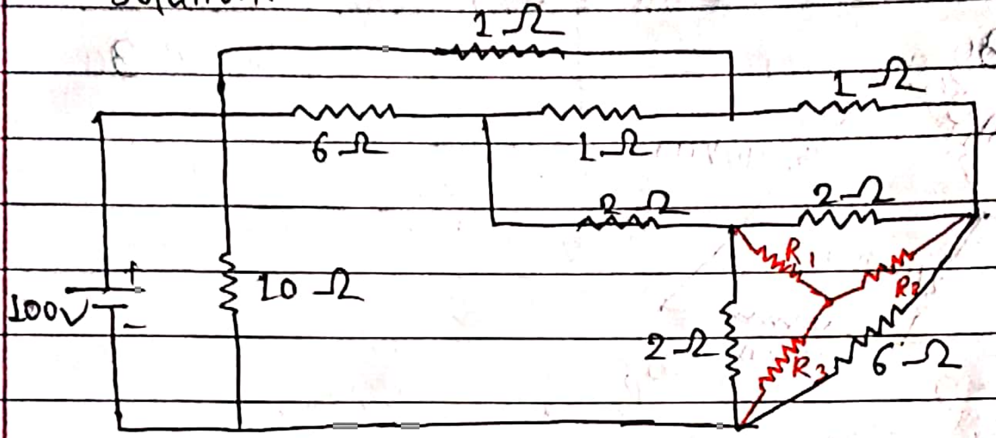


$$\therefore R_{eq} = (10.9 + 23.36)\ k\Omega$$
$$= 34.26\ k\Omega$$

Q.12 Find the equivalent resistance :



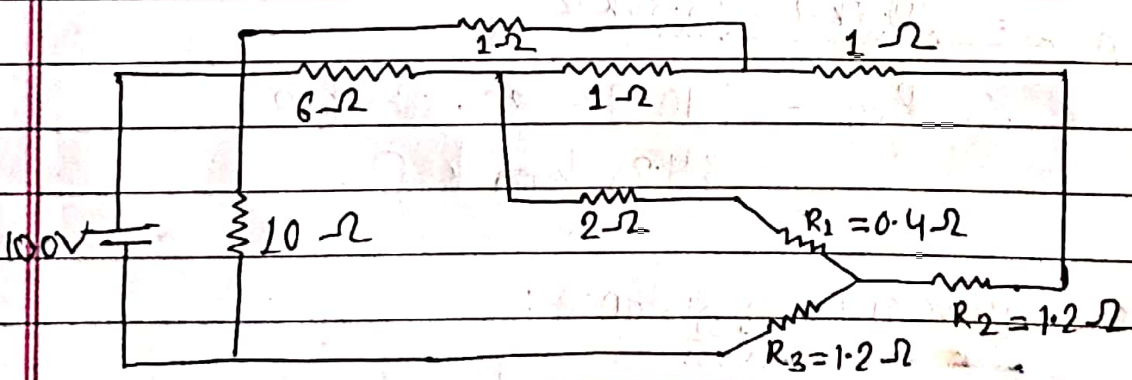
solution:



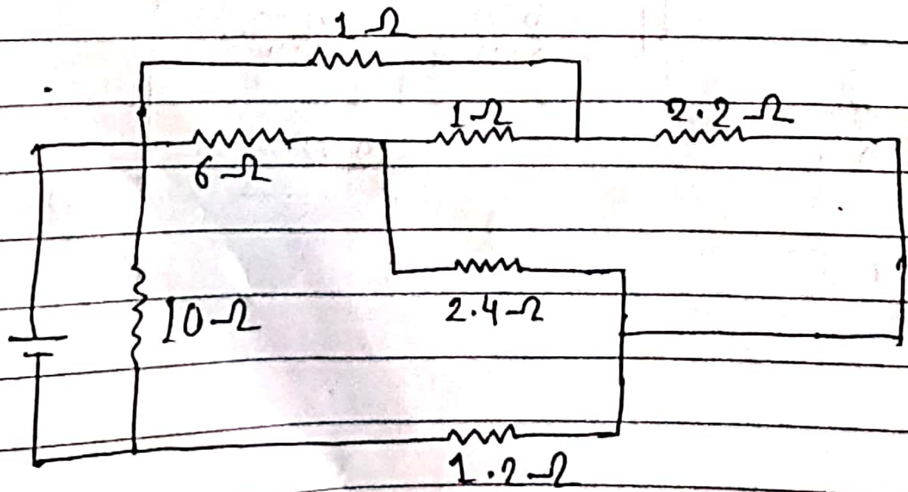
Considering the eq. star ckt,

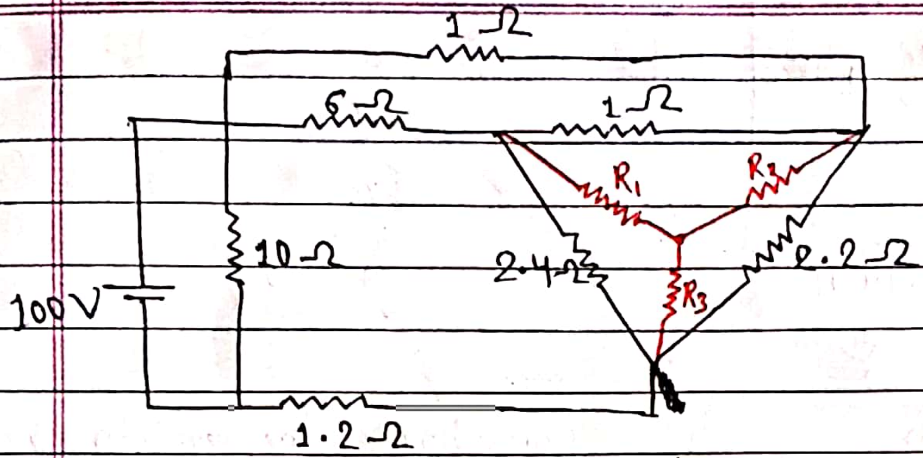
$$R_1 = \frac{2\times 2}{2+2+6}\,\Omega = 0.4\,\Omega$$

$$R_2 = R_3 = \frac{2\times 6}{2+2+6}\,\Omega = 1.2\,\Omega$$



$\Downarrow$



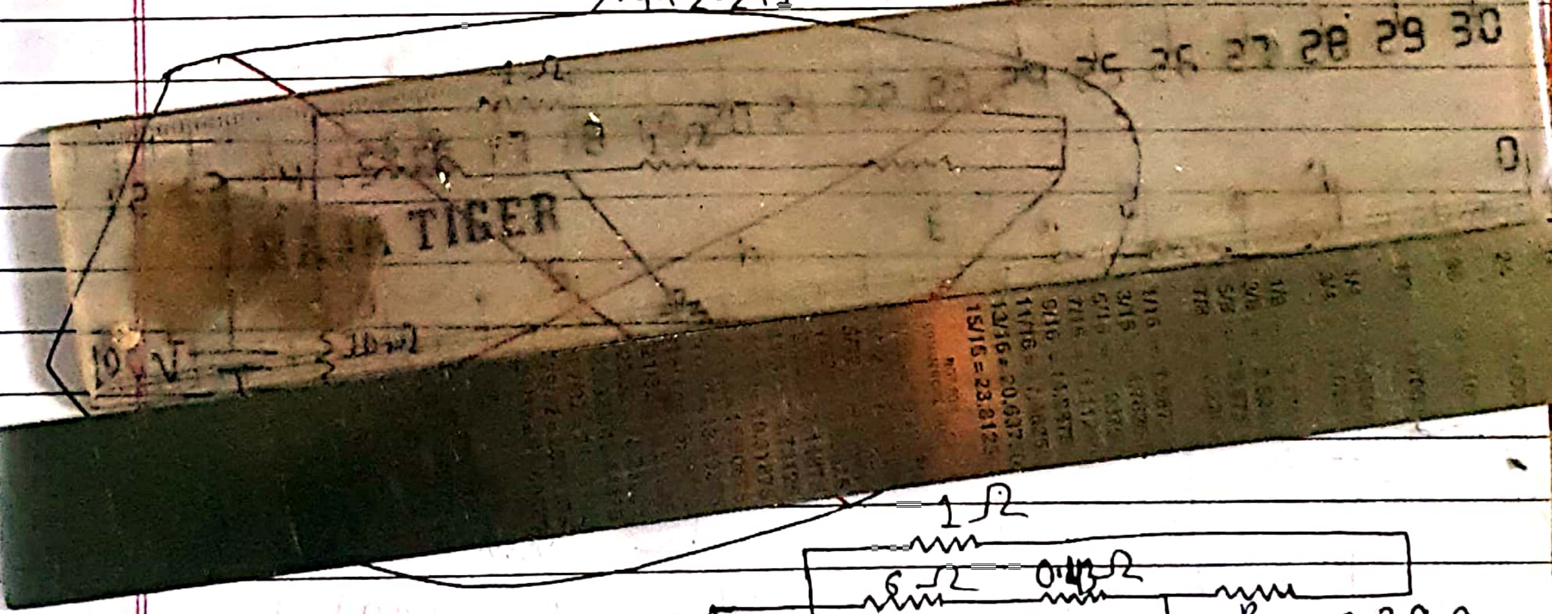


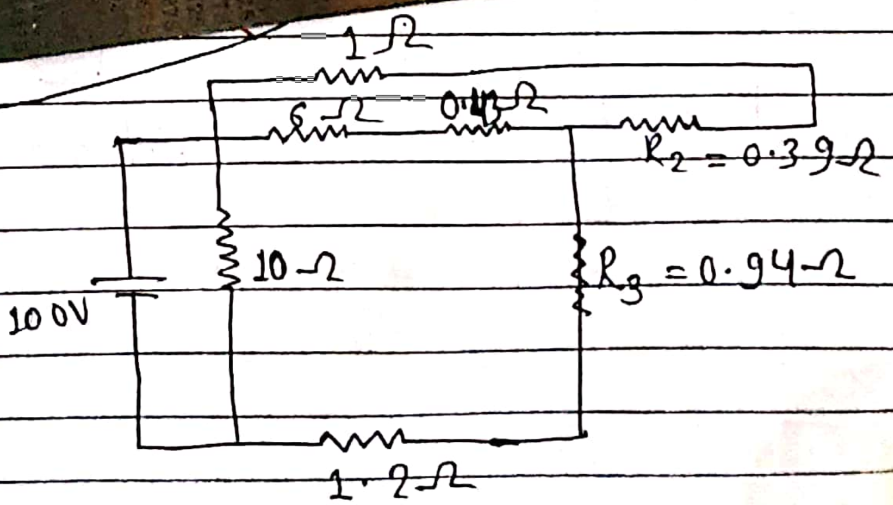
Considering the equivalent star ckt,

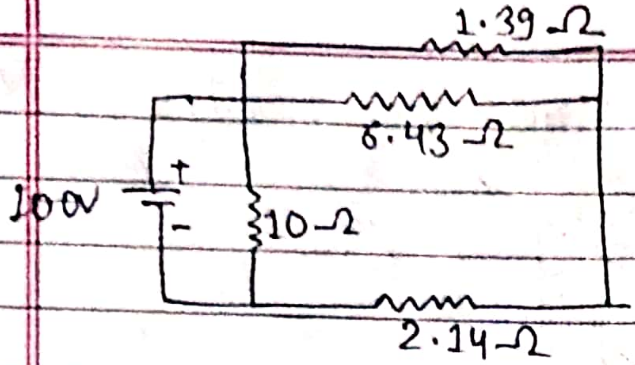
$$R_1 = \frac{1 \times 2.4}{2.4+2.2+1} \Omega = 0.43\Omega$$

$$R_2 = \frac{1 \times 2.2}{2.4+2.2+1} \Omega = 0.39\Omega$$

$$R_3 = \frac{2.2 \times 2.4}{2.4+2.2+1} \Omega = 0.94\Omega$$







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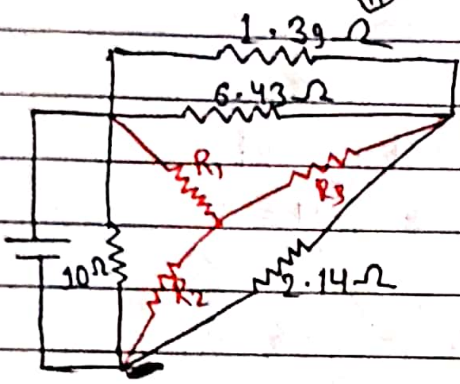
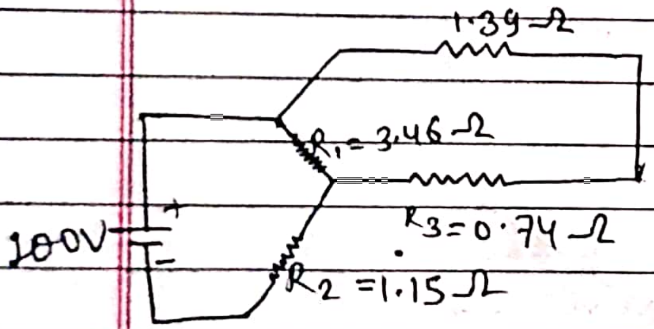


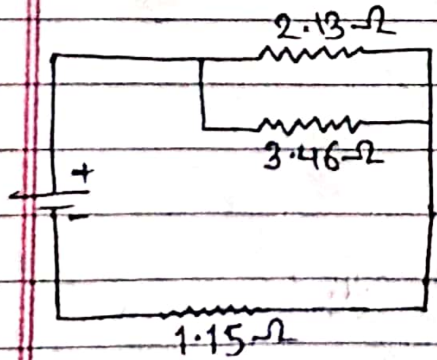
Considering the eq. star ckt,

$$R_1 = \frac{10 \times 6.43}{10 + 6.43 + 2.14}\ \Omega = 3.46\ \Omega$$

$$R_2 = \frac{10 \times 2.14}{10 + 6.43 + 2.14}\ \Omega = 1.15\ \Omega$$

$$R_3 = \frac{6.43 \times 2.14}{10 + 6.43 + 2.14}\ \Omega = 0.74\ \Omega$$





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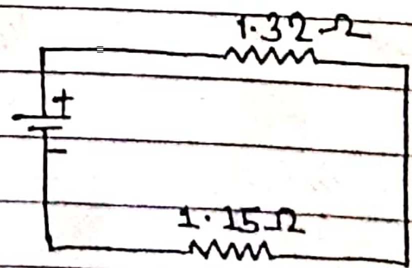
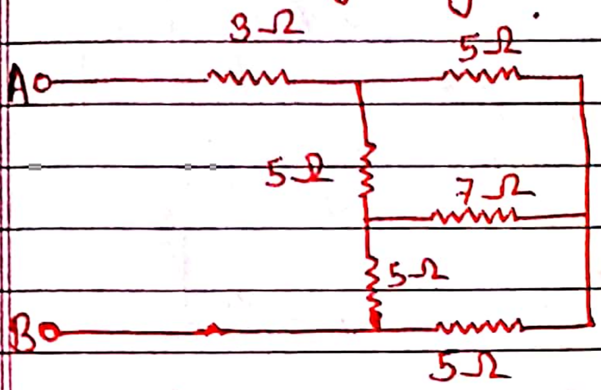


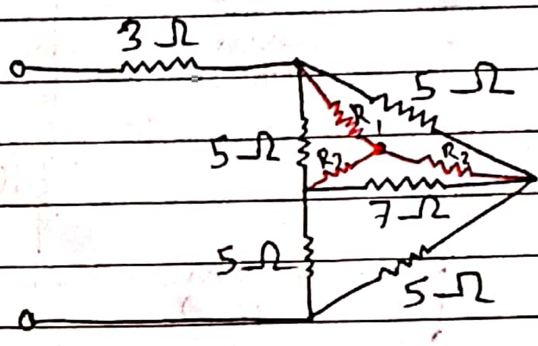
$$\therefore R_{eq} = (1.15 + 1.32)\ \Omega$$
$$= 2.47\ \Omega$$

Q.8 Find R_{AB} by using star to delta conversion techniques:

PU



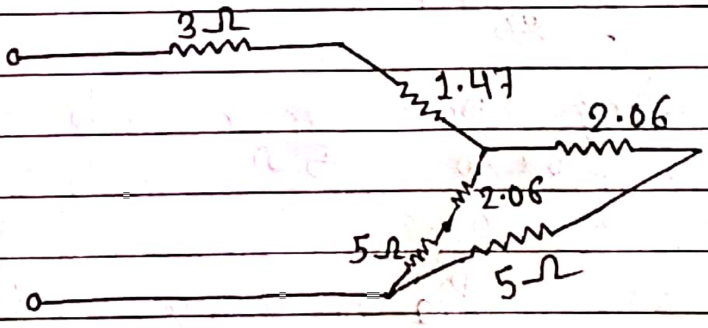
$\Rightarrow$



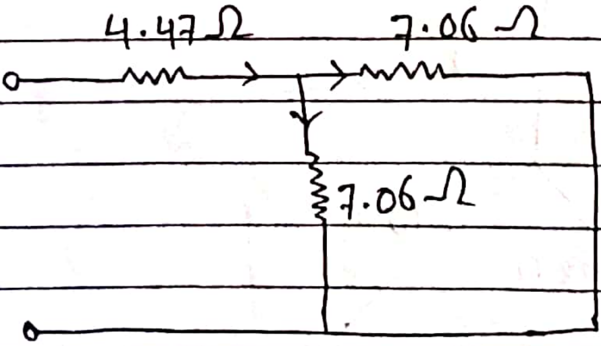
$$R_1 = \frac{5 \times 5}{5+5+7}\,\Omega = \frac{25}{17}\,\Omega = 1.47\,\Omega$$

$$R_2 = R_3 = \frac{5 \times 7}{5+5+7}\,\Omega = \frac{35}{17}\,\Omega = 2.06\,\Omega$$

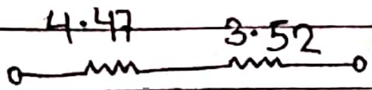
$\Downarrow$



$\Downarrow$

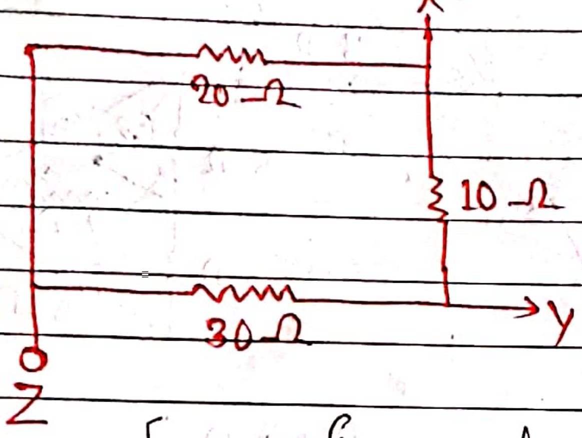


$\Rightarrow$

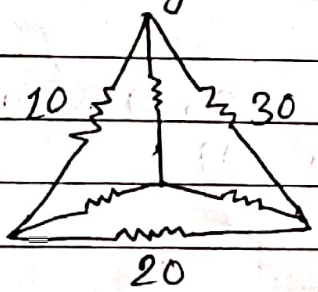


$\therefore R_T = 4.47 + 3.52$
$= 7.99\,\Omega$

Q. 9) PU Convert the following delta (Δ) N/w to star (Y) N/w.



from figure above,

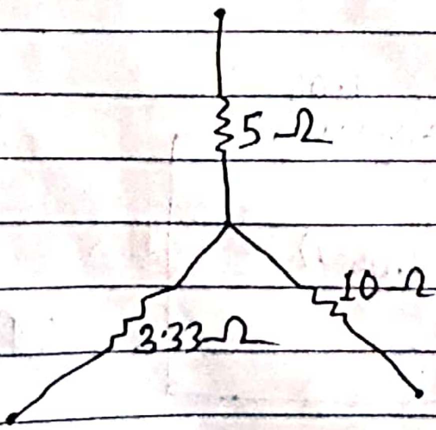


~~Consid~~

Converting above mesh into star network,

$R_1 = \frac{10 \times 30}{60} = 5\,\Omega$, $R_3 = \frac{20 \times 30}{60} = 10\,\Omega$

$R_2 = \frac{10 \times 20}{60} = 3.33\,\Omega$



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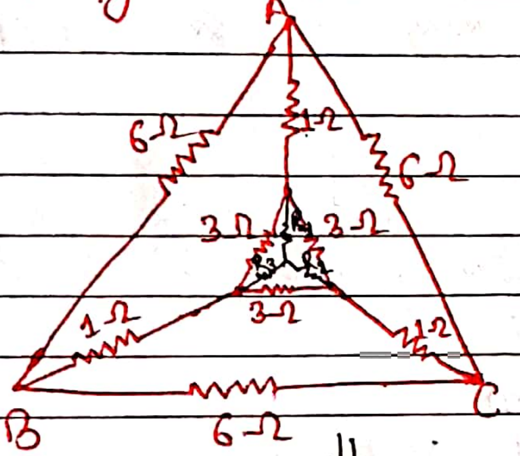
Q. 10) Determine the resistance betⁿ point A & B N/w shown in fig. below:

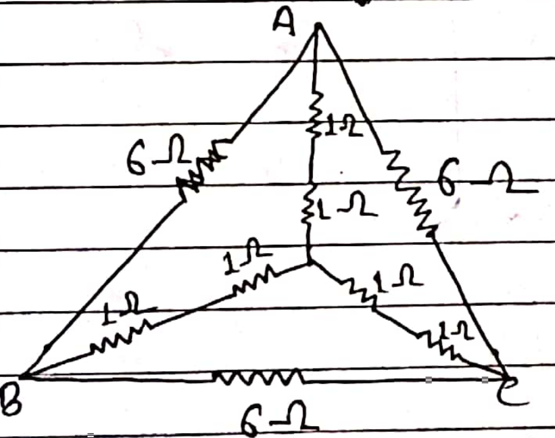


Considering the eq. star ckt;

$R_1 = \frac{3 \times 3}{3+3+3} = 1\Omega$

$R_1 = R_2 = R_3 = 1\Omega$

⇓



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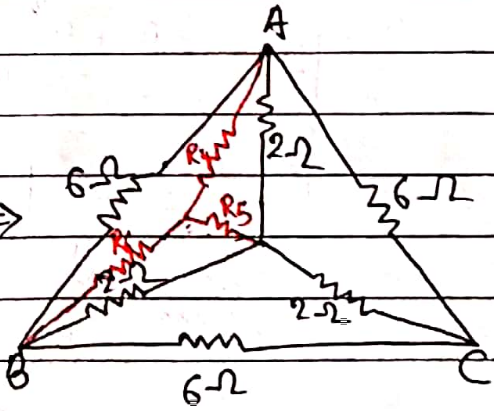
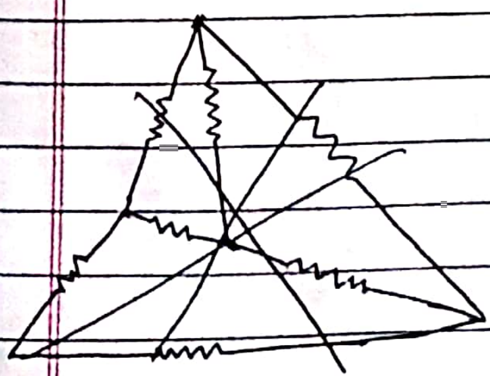


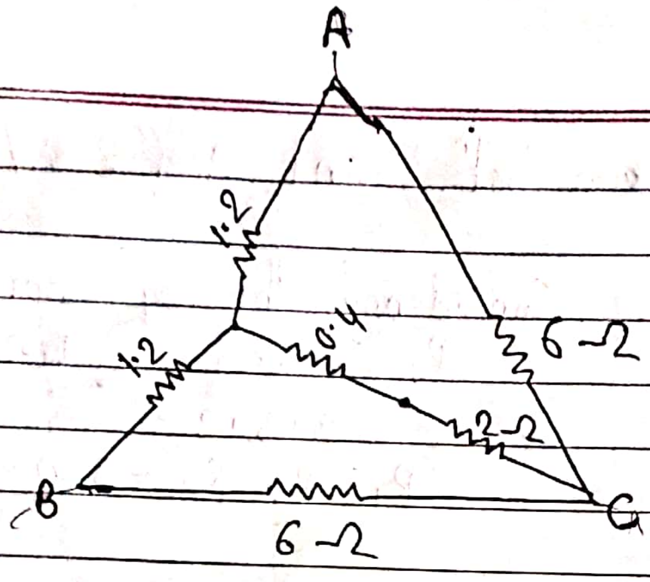
Considering the eq. star ckt;

$R_4 = \frac{6 \times 2}{6+2+2} = \frac{12}{10} = 1.2\Omega$

$R_5 = \frac{2 \times 2}{10} = 0.4\Omega$

$R_6 = \frac{6 \times 2}{10} = 1.2\Omega$





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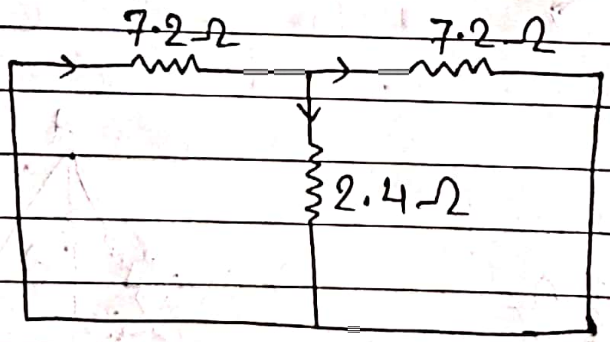


⇓

$$R_T = 7.2 + \frac{7.2 \times 2.4}{7.2 + 2.4}$$

$$= 9\,\Omega$$

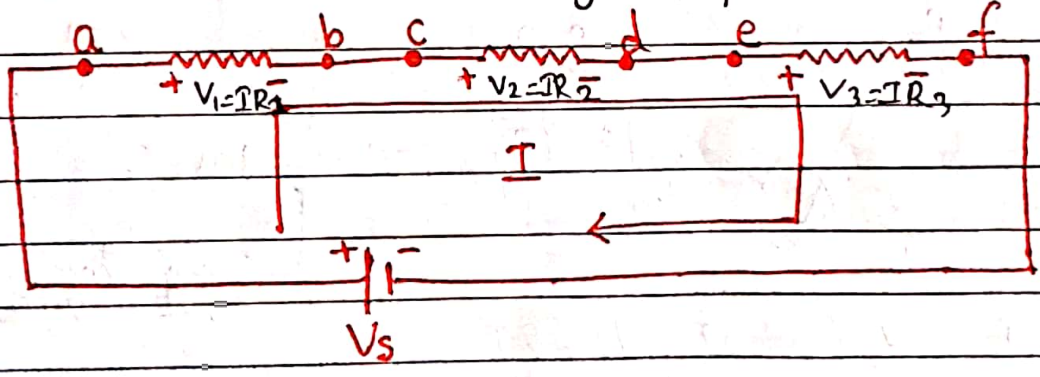
loop → closed path

Date
Page

Kirchhoff's Current & Voltage law

1) Kirchhoff's Voltage Law:-

It states that, algebric sum of the voltage around any closed path in a circuit is always zero (0). In any circuit, the voltage drop across the registers always have polarity opposite to the source polarity. When the current is passing through the resistor, there is loss of energy & therefore voltage drops. In any element, the current flow from higher potential to lower potential.

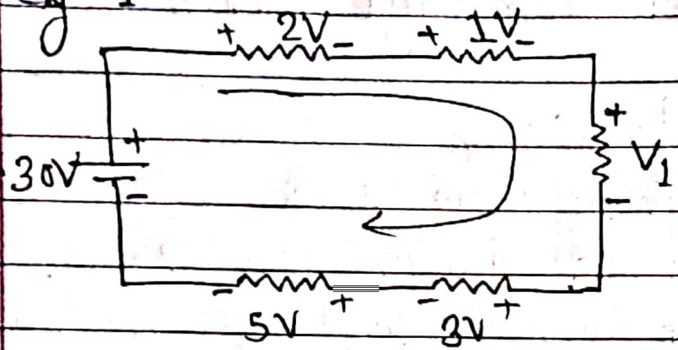


It is customary to take the direction of current I indicate in fig. above. It leaves the +ve terminal of the voltage source & enter into −ve terminal as the current passes in the circuit, ~~same~~ sum of the voltage drops around the loop is equal to the total voltage in that loop. The voltage at a point a, c & e are more than the voltage at points b, d & f.

So,

$$V_s = V_1 + V_2 + V_3$$

Eg. 1:

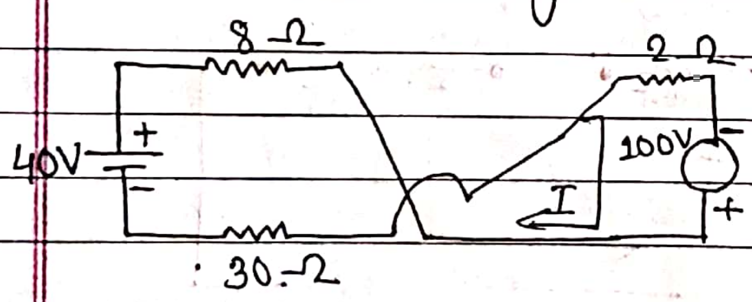


$V_s = V_1 + V_2 + V_3 + V_4 + V_5$

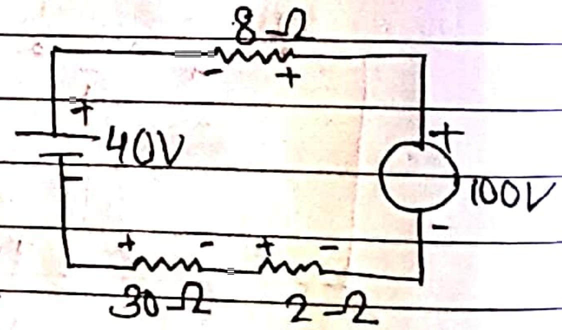
or $30 = 2 + 1 + V_1 + 3 + 5$

$\therefore V_1 = 19V$

Eg. 2) The given ckt in fig (a) below find
(a) The current I
(b) The voltage across $30\,\Omega$



$\Rightarrow$



$V_8 = 8I,\ V_{30} = 30I,\ V_2 = 2I$

Kirchoff's voltage law,

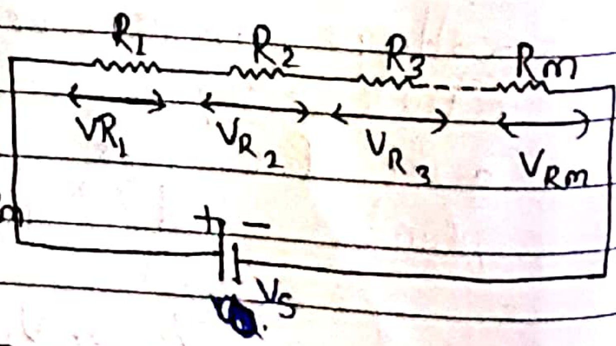
$100 = 8I + 30I + 2I + 40$

$\Rightarrow I = 1.5A$

$V_{30} = 30 \cdot I = 30 \times 1.5 = 45V$

Voltage division :-

$I = \dfrac{V_S}{R_1 + R_2 + R_3 + \cdots + R_m}$



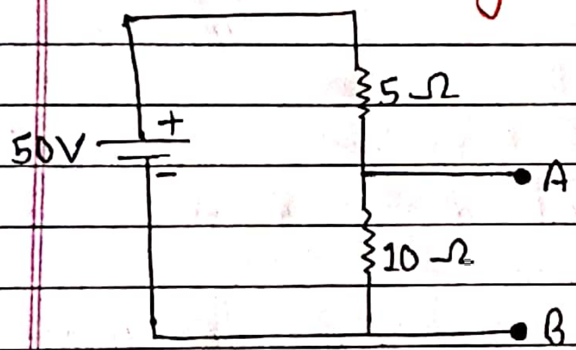
$V_{R_1} = IR_1,\ V_{R_2} = IR_2,\ V_{R_3} = IR_3, \ldots, V_{Rm} = IR_m$

$\therefore V_{Rm} = \dfrac{V_S \cdot R_m}{R_1 + R_2 + R_3 + \cdots + R_m}$

$$\boxed{V_m = \frac{R_m}{R_T} \cdot V_S}$$

where, V_m = voltage across m^{th} resistor
R_m = resistance across the voltage is to be determined
R_T = total series resistance

E.g. 1) What is the voltage across the 10 Ω resistor in fig. below

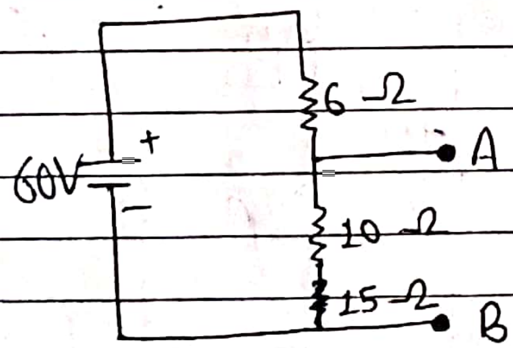


$V_{10} = ?$

$$V_m = \frac{R_m}{R_T} \cdot V_s$$

$$V_{10} = \frac{10}{15} \times 50$$

$$= 33.33 \text{ V}$$

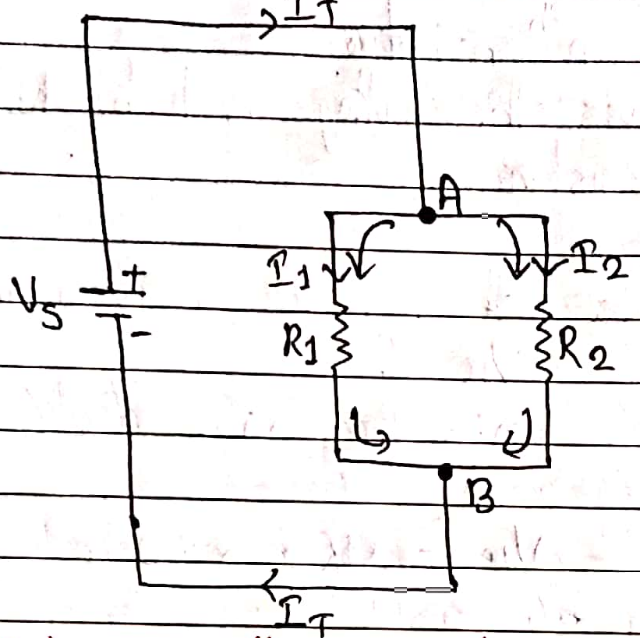


$$V_{AB} = \frac{(10+5)}{6+10+15} \times 60$$

$$= 48.39 \text{ V}$$

2) Kirchoff's Current Law :-

It states that, the sum of currents entering into any point is equal to the sum of the currents leaving that points. The point may be interconnection of two or more branches. In parallel circuit, node consists of two or more branches. Therefore the total current entering into the node is equal to the current leaving that node.



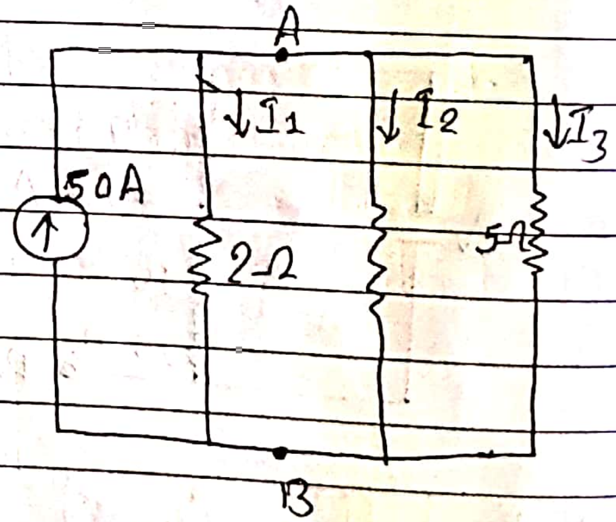
Consider the node A

$I_T = I_1 + I_2$

Consider the node B

$I_1 + I_2 = I_T$

Q. Determine the current in all registers in the circuit shown in fig. below:



$I_T = I_1 + I_2 + I_3$

or, $50 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

or $50 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

or $50 = V\left(\frac{1}{2} + \frac{1}{1} + \frac{1}{5}\right)$

$\Rightarrow V = 29.4V$

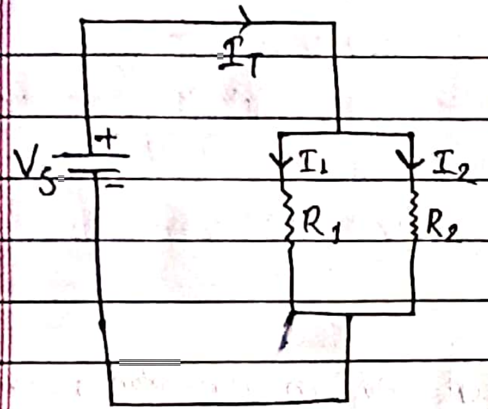
So,

$I_1 = \frac{V}{R_1} = \frac{29.41}{2} = 14.7A$

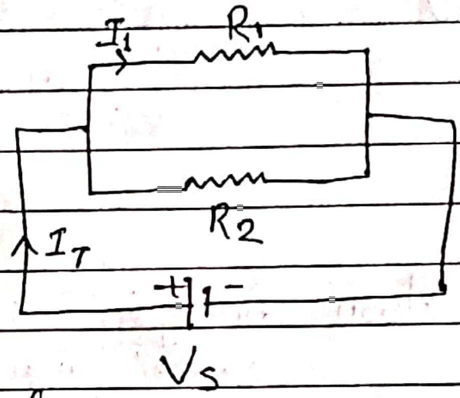
$I_2 = \frac{V}{R_2} = \frac{29.41}{1} = 29.41A$

$I_3 = \frac{V}{R_3} = \frac{29.41}{5} = 5.88A$

Current divisions



or



The voltage applied to each of resister is $= V_s$

The current passing through each of resister is given by

$$I_1 = \frac{V_s}{R_1} \quad \text{———} (\text{I}) \qquad \therefore V_s = I_1 \cdot R_1$$

$$I_2 = \frac{V_s}{R_2} \quad \text{———} (\text{II}) \qquad \therefore V_s = I_2 \cdot R_2$$

The total resistance of the given ckt :

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \quad \text{———} (\text{III})$$

Total current

$$I_T = \frac{V_s}{R_T} \quad \text{———} (\text{IV})$$

$$I_T = \frac{V_s}{\frac{R_1 R_2}{R_1 + R_2}}$$

$$I_T = V_s \cdot \frac{(R_1 + R_2)}{R_1 R_2}$$

$$I_T = \frac{I_1 \cdot \cancel{R_1} (R_1 + R_2)}{\cancel{R_1} R_2}$$

$$I_T = I_1 \frac{(R_1 + R_2)}{R_2}$$

$$\therefore \boxed{I_1 = I_T \cdot \frac{R_2}{R_1 + R_2}}$$

$$I_T = \frac{V_s \cdot (R_1 + R_2)}{R_1 R_2}$$

$$= \frac{I_2 \cdot \cancel{R_2} (R_1 + R_2)}{R_1 \cancel{R_2}}$$

$$I_T = \frac{I_2 (R_1 + R_2)}{R_1}$$

$$\boxed{\therefore I_2 = I_T \cdot \frac{R_1}{R_1 + R_2}}$$

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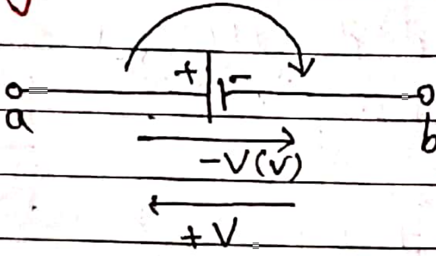
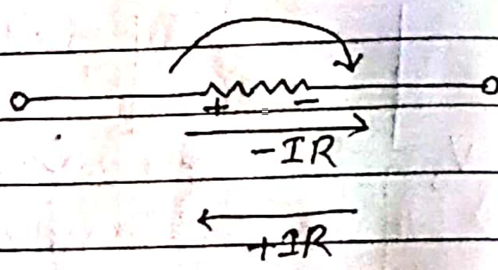
Sign Conversion:-

① 

② 

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Mesh Analysis

↳ used only in planar circuit; not in non-planar ckt.

Mesh is defined as a loop which does not contain any other loops within its.

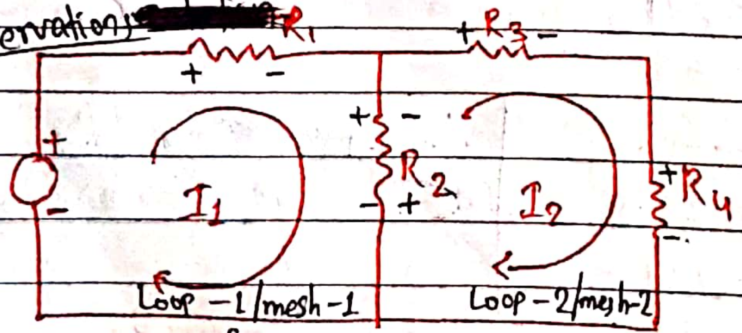
1) Mesh Analysis

Step 1: Check whether the ckt is planar or not.

Step 2: Select the mesh current (loop current).

Step 3: Finally, write the Kirchhoff's voltage law eqⁿ in terms of unknown & solving leads to final

observation:



fig(a)

solution:

Consider the loop-1

$$+V_s - I_1R_1 - R_2[I_1 - I_2] = 0$$

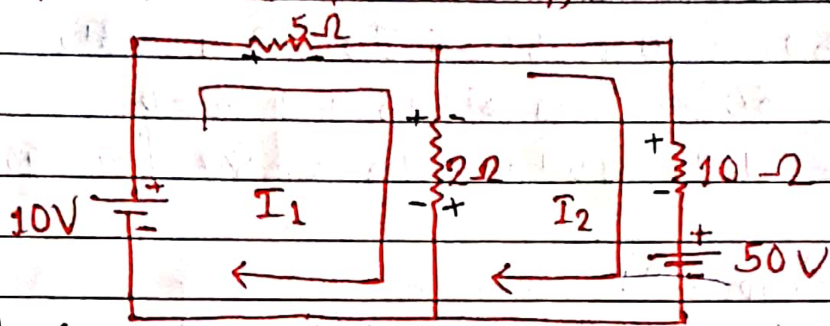
$$\Rightarrow I_1R_1 + R_2[I_1 - I_2] = V_s \quad \text{—— (I)}$$

Consider the loop-2

$$-R_3I_2 - R_4I_2 - R_2(I_2 - I_1) = 0$$

$$\Rightarrow R_3I_2 + R_4I_2 + R_2(I_2 - I_1) = 0 \quad \text{—— (II)}$$

Eg. 1) Write the mesh current eqⁿ in the ckt shown in fig below & determine the currents.



solution:

Consider loop-1:

$10V - 5I_1 - 2(I_1 - I_2) = 0$

$\Rightarrow -5I_1 - 2I_1 + 2I_2 = -10V$

$\Rightarrow 7I_1 - 2I_2 = 10$ —— (i)

Consider the loop-2:

$-10I_2 - 50V - 2[I_2 - I_1] = 0$

$\Rightarrow 10I_2 + 2I_2 - 2I_1 = -50$

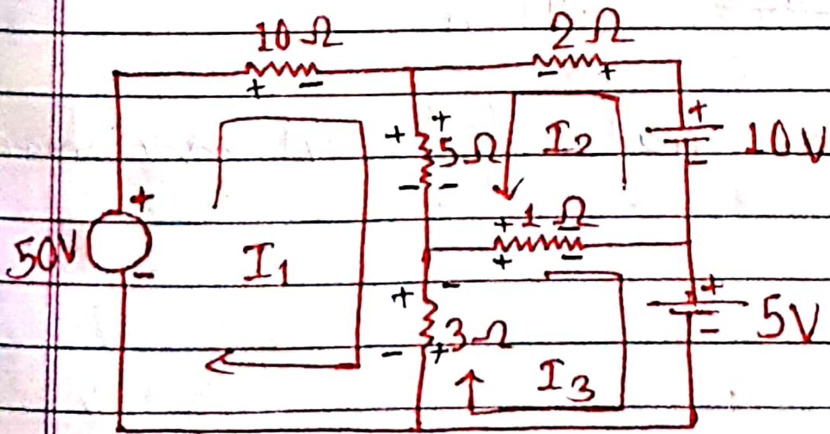
$\Rightarrow -2I_1 + 12I_2 = -50$ —— (ii)

Solving (i) & (ii),

$I_1 = 0.25\ A$

$I_2 = -4.125\ A$ (The current will flow in opposite direction of what we have assumed.)

Q.2) Determine the mesh current I_1, I_2 & I_3 in the ckt shown in fig below:



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Consider loop -1:

$50V - 10I_1 - 5(I_1+I_2) - 3(I_1-I_3) = 0$

$\Rightarrow 50 - 10I_1 - 5I_1 - 5I_2 - 3I_1 + 3I_3 = 0$

$\Rightarrow 50 - 18I_1 - 5I_2 + 3I_3 = 0$

$\Rightarrow 18I_1 + 5I_2 - 3I_3 = 50$ —— (I)

Consider loop - 2:

$10V - 2I_2 - 5(I_1+I_2) - 1(I_2+I_3) = 0$

or $10 - 2I_2 - 5I_1 - 5I_2 - I_2 - I_3 = 0$

or $10 - 8I_2 - 5I_1 - I_3 = 0$

$\Rightarrow 5I_1 + 8I_2 + I_3 = 10$ —— (II)

Consider loop - 3:

$-5V - 3(I_3-I_1) - 1(I_2+I_3) = 0$

or $-5 - 3I_3 + 3I_1 - I_2 - I_3 = 0$

or $-5 - 4I_3 + 3I_1 - I_2 = 0$

$\Rightarrow -3I_1 + I_2 + 4I_3 = -5$ —— (III)

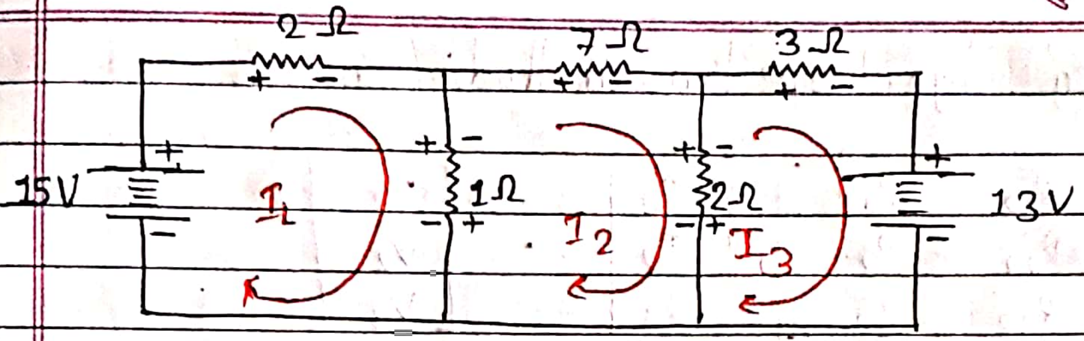
Solving (I), (II) & (III),

$I_1 = \cancel{4.12}\ 3.30\ A$

$I_2 = \cancel{1.92}\ -0.997\ A$

$I_3 = \cancel{4.82}\ 1.47\ A$

2015 PU Q.3) Calculate the mesh current of the ckt ~~given~~ shown in fig. below:



Consider loop-1:

$$15 - 2I_1 - 1(I_1 - I_2) = 0$$

or $$15 - 2I_1 - I_1 + I_2 = 0$$

$$\Rightarrow \quad 3I_3 - I_2 = 15 \quad \text{—— (i)}$$

Consider loop-2:

$$-1(I_2 - I_1) - 7I_2 - 2(I_2 - I_3) = 0$$

or $$-I_2 + I_1 - 7I_2 - 2I_2 + 2I_3 = 0$$

$$\Rightarrow \quad -I_1 + 10I_2 - 2I_3 = 0 \quad \text{—— (ii)}$$

Consider loop-3:

$$-13 - 2(I_3 - I_2) - 3I_3 = 0$$

or $$-13 - 2I_3 + 2I_2 - 3I_3 = 0$$

$$\Rightarrow \quad 2I_2 - 5I_3 = 13 \quad \text{—— (iii)}$$

Solving;

$$I_1 = 4.99 \text{ A}$$
$$I_2 = -0.022 \text{ A}$$
$$I_3 = -2.609 \text{ A}$$

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Mesh eq^n by Inspection method:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

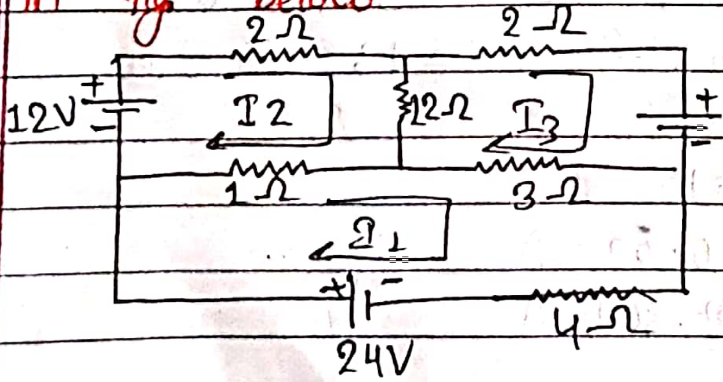
R_{11} = self resistance of mesh-1 [sum of resistance in mesh-1]
R_{22} = " " " mesh-2 [" " " " mesh-2]
R_{33} = " " " mesh-3 [" " " " mesh-3]

$R_{12} = R_{21} = -$ [sum of all resistance common to mesh 1 & 2]

$R_{13} = R_{31} = -$ [sum of all resistance common to mesh 1 & 3]

$R_{23} = R_{32} = -$ [sum of all resistance common to mesh 2 & 3]

Q. Determine the ~~current~~ voltage drop in 4Ω branch in the ckt shown in fig below.



$R_{11} = 1+3+4 = 8\,\Omega$
$R_{22} = 2+12+1 = 15\,\Omega$
$R_{33} = 2+3+12 = 17\,\Omega$

$R_{12} = R_{21} = -1\,\Omega$
$R_{13} = R_{31} = -3\,\Omega$
$R_{23} = R_{32} = -12\,\Omega$

Substituting Above values in matrix form:

$$\begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \\ -10 \end{bmatrix}$$

$V_1 = 24V$
$V_2 = 12V$
$V_3 = -10V$

$I_1 = 4.11\,A$, $I_2 = 2.71\,A$, $I_3 = 2.05\,A$

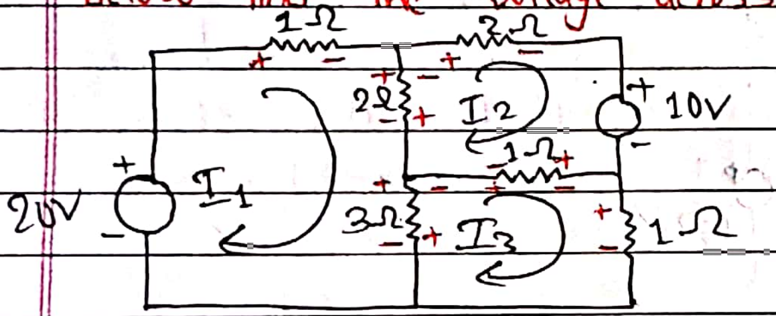
~~5.22, 1.044, 3.28~~

$V = I_1 R$

$= 4.11 \times 4$

$= 16.44$

Q. Determine the mesh currents in the ckt shown in fig (a) below. Find the voltage across the 3-Ω resistor.



Consider loop-1:

$20 - I_1 - 2(I_1 - I_2) - 3(I_1 - I_3) = 0$

or $20 - I_1 - 2I_1 + 2I_2 - 3I_1 + 3I_3 = 0$

or $20 - 6I_1 + 2I_2 + 3I_3 = 0$

$\Rightarrow 6I_1 - 2I_2 - 3I_3 = 20$ —— (i)

Consider loop-2:

$-10 - 1(I_2 - I_3) - 2(I_2 - I_1) - 2I_2 = 0$

or $-I_2 + I_3 - 2I_2 + 2I_1 - 2I_2 = 10$

$\Rightarrow 2I_1 - 5I_2 + I_3 = 10$ —— (ii)

Consider loop-3:

$-I_3 - (I_3 - I_2) - 3(I_3 - I_1) = 0$

or $I_2 - 2I_3 - 3I_3 + 3I_1 = 0$

$\Rightarrow 3I_1 + I_2 - 5I_3 = 0$ —— (iii)

Solving:

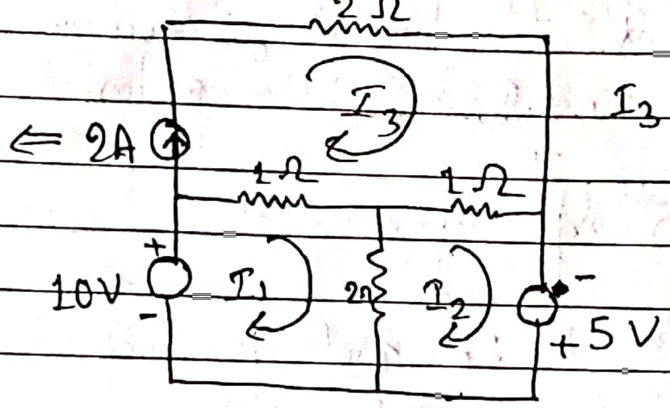
$I_1 = 5.22$ A

$I_2 = 0.746$ A

$I_3 = 3.28$ A

2) Mesh Analysis with current sources:

PU 2012 Q. Find the mesh current in the given fig below:



① $I_3 = -2A$

$I_3 = 2$ Amp

$\therefore \quad I_3 = 2\ A$

Consider loop - 1:

$10V - 1(I_1 - I_3) - 2(I_1 - I_2) = 0$

or $\quad 3I_1 - 2I_2 = 10 + 2 = 12$

$\therefore \quad 3I_1 - 2I_2 = 12 \quad \text{—— (I)}$

Consider loop -2:

$5V - 2(I_2 - I_1) - 1(I_2 - I_3) = 0$

or $\quad 5 - 2I_2 + 2I_1 - I_2 + 2 = 0$

$\Rightarrow \quad 2I_1 - 3I_2 + 7 = 0 \quad \text{—— (II)}$

Solving:

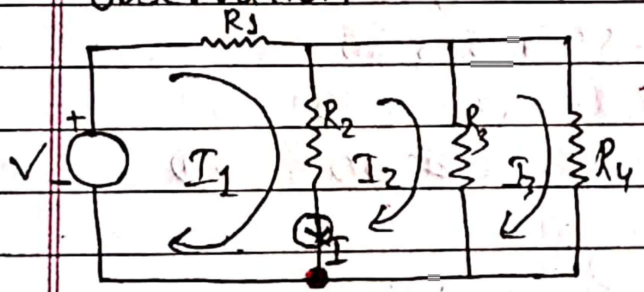
$I_1 = 10A \quad ; \quad I_2 = 9A$

3) Super mesh Analysis:

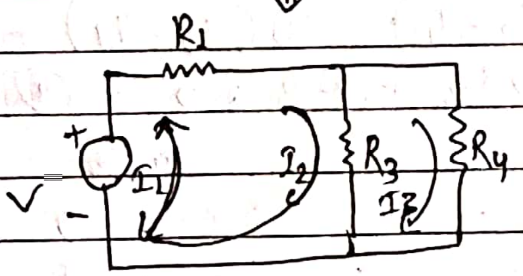
Suppose any branch in network has a current source than it is slightly difficult to apply Mesh analysis. One way to overcome this difficultly is by applying the super mesh technique. Here we have choose a kind of super mesh.

"A super mesh is constituded by two adjacent loops that have common current source."

Observation



Solution:- Write the combine eq^n of loop-1 & loop-2.



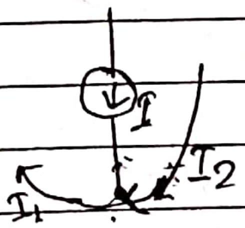
$$\Rightarrow V - I_1 R_1 - R_3(I_2 - I_3) = 0 \quad \text{—(I)}$$

Consider loop -3 :

$$-R_4 I_3 - R_3(I_3 - I_2) = 0 \quad \text{—(II)}$$

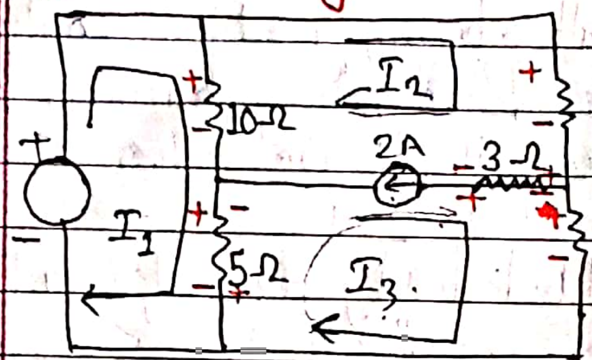
Other eq^n: $I + I_2 = I_1$

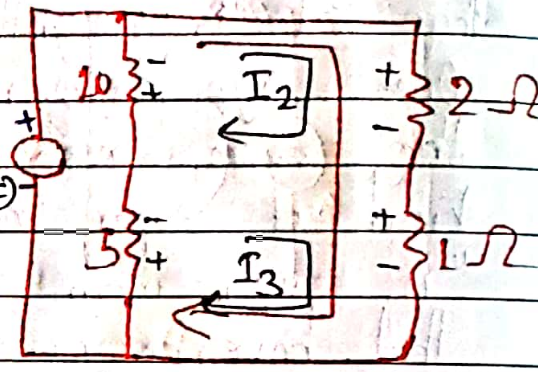
$$\Rightarrow I_1 - I_2 = I \quad \text{—(III)}$$



$\Rightarrow I + I_2 = I_1$

1) Q. Determine the current in a 5-Ω resistor in the network given in fig. below.





Consider loop -1:

$$50V - 10(I_1 - I_2) - 5(I_1 - I_3) = 0$$

or $\quad 50 - 10I_1 + 10I_2 - 5I_1 + 5I_3 = 0$

$\Rightarrow \quad 15I_1 - 10I_2 - 5I_3 = 50$

$\Rightarrow \quad 3I_1 - 2I_2 - I_3 = 10 \quad$ — (I)

Combine eq^n of loop -2 & loop -3:

$$-2I_2 - 1I_3 - 5(I_3 - I_1) - 10(I_2 - I_1) = 0$$

$\Rightarrow \quad -15I_1 + 12I_2 + 6I_3 = 0 \quad$ — (II)

Other eq^n:

$2 + I_3 = I_2 \Rightarrow I_2 - I_3 = 2 \quad$ — (III)

$I_1 = 19.99$
$I_2 = 17.33$
$I_3 = 15.33$

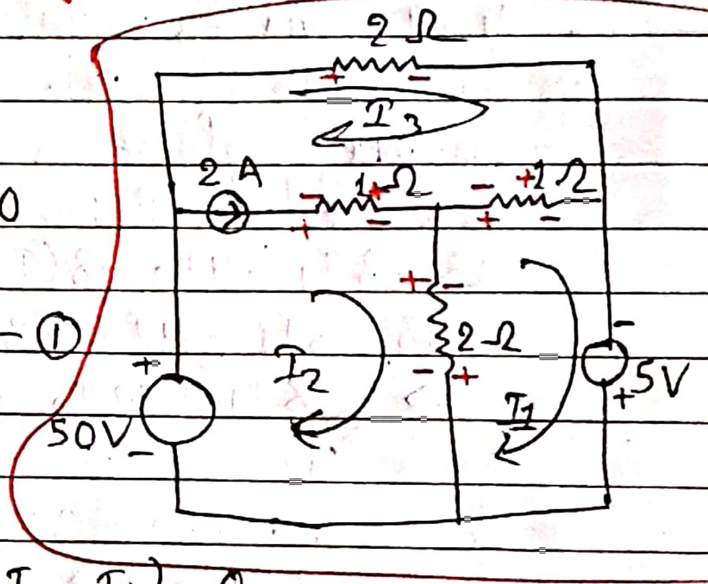
Current in 5Ω resistor

$= I_1 - I_3$

$= 19.99 - 15.33$

$= 4.66\ \Omega$

2) Q. Use the concept of mesh analysis to find the mesh current in a given circuit.



Consider loop -1:

$$5 - 2(I_1 - I_2) - 1(I_1 - I_2) = 0$$

or $5 - 2I_1 + 2I_2 - I_1 + I_3 = 0$

$$\Rightarrow \quad 3I_1 - 2I_2 - I_3 = 5 \quad \text{—} \ (I)$$

Combine eq^n of loop-2 & loop-3 ;

$$50 - 2(I_2 - I_1) - 2I_3 - 1(I_3 - I_1) = 0$$

or $50 - 2I_2 + 2I_1 - 2I_3 - I_3 + I_1 = 0$

or $3I_1 - 2I_2 - 3I_3 = -50$ — (II)

Other eq^n :

$\cancel{2 + I_2 = I_3}$ $\quad I_2 = 2 + I_3$

$\cancel{\Rightarrow I_2 - I_3 = -2}$ — (III)

Solving:

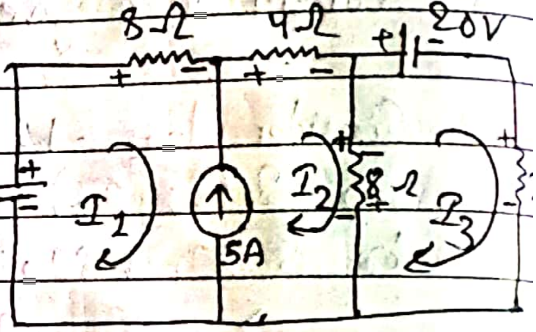
$I_1 = 27.83$ A

$I_2 = 25.5$ A

$I_3 = 27.5$ A

PU 2005, 2008, 2012 F

3) Q. Use concept of super mesh analysis, find the mesh current I_1, I_2 and I_3.



Combine eq^n of loop 1 & loop-2;

$$18 - 8I_1 - 4I_2 - 8(I_2 - I_3) = 0$$

or $18 - 8I_1 - 4I_2 - 8I_2 + 8I_3 = 0$

$$\Rightarrow \quad 8I_1 + 12I_2 - 8I_3 = 18 \quad \text{—— (I)}$$

Consider loop-3:

$$-8(I_3 - I_2) - 20 - 18I_3 = 0$$

or $-8I_3 + 8I_2 - 20 - 18I_3 = 0$

$$\Rightarrow \quad 8I_2 - 26I_3 = 20 \quad \text{—— (II)}$$

Other eq^n:

$$I + I_2 = I_1$$

$$\Rightarrow \quad 5 + I_2 = I_1$$

$$\Rightarrow \quad I_1 - I_2 = 5 \quad \text{—— (III)}$$

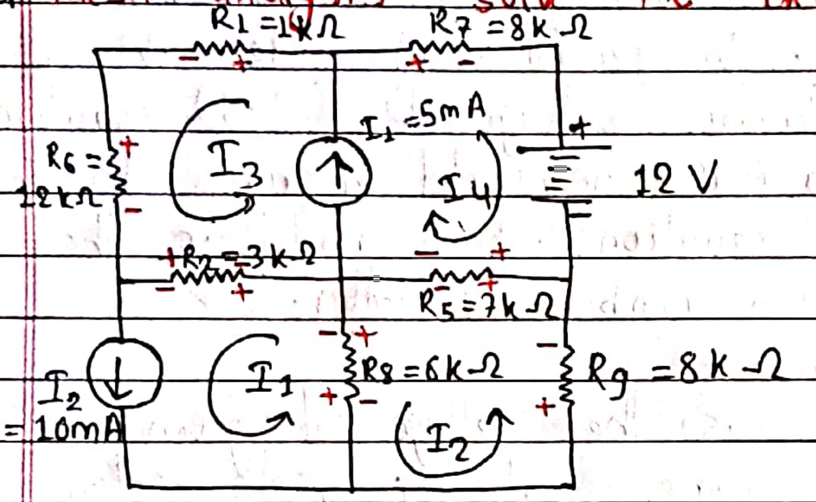
Solving:

$I_1 = 3.39$ A

$I_2 = -1.61$ A

$I_3 = -1.26$ A

4) fall 2014 Q. Determine the voltage across 3 k-Ω resistor. Using the super mesh analysis solve the ckt shown below:



Solving:
$I_1 = 10\ mA$
$I_2 = -5.34$
$I_3 = -19.6$
$I_4 = 24.6$

Solution:

$I_1 = 10\ mA$ — (I)

Consider loop-2:

$-8I_2 - 7(I_2 + I_4) - 6(I_2 - I_1) = 0$

or $-8I_2 - 7I_2 - 7I_4 - 6(I_2 - 10) = 0$

or $-8I_2 - 7I_2 - 7I_4 - 6I_2 + 60 = 0$

$\Rightarrow -21I_2 - 7I_4 = -60$

$\Rightarrow 21I_2 + 7I_4 = 60$ — (II)

Combine eq^n of loop-3 & Loop-4;

$-12 - 7(I_4 + I_2) - 3(I_3 - I_1) - I_3 - 12I_3 - 8I_4 = 0$

or $-12 - 7I_4 - 7I_2 - 3I_3 + 30 - I_3 - 12I_3 - 8I_4 = 0$

or $-7I_2 - 16I_3 - 15I_4 + 18 = 0$

$\Rightarrow 7I_2 + 16I_3 + 15I_4 = 18$ — (III)

~~Cons~~ Other eq^n:

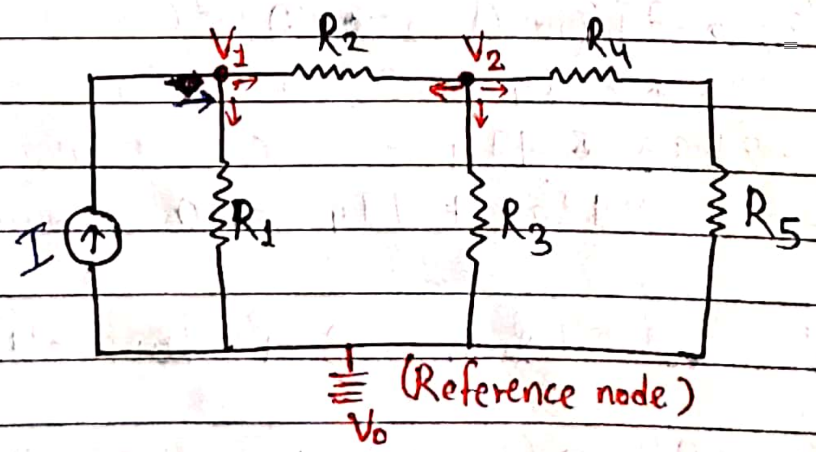
$I_3 + I_4 = 5$ — (IV)

Nodal Analysis

In simple circuit contaning any two nodes including the reference node. In general, N node ckt will require (N−1) unknown voltage & (N−1) equation. for example; a 10 nodes ckt require 9 unknown voltage and 9 equations. Each node in the ckt can be assigned a number or letter.

The node voltage is the voltage of given node with respect to one particular node called the reference node or Datum node or '0' zero potential node which can be assumed to be zero potential that's why it is also called zero potential node.

Observation:



Consider Node−1:

$$I = \frac{V_1 - V_0}{R_1} + \frac{V_1 - V_2}{R_2} \quad \text{——— (1)}$$

or

$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \quad \text{——— (1)} \qquad \boxed{V_0 = 0}$$

V_0 is assumed '0' always

or $\frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} = I$

$$\Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = I \quad \text{--- (I)}$$

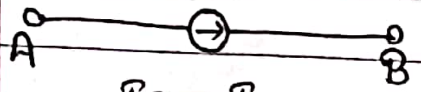
Consider Node - 2:

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - V_0}{R_4 + R_5} + \frac{V_2 - V_0}{R_3} = 0$$

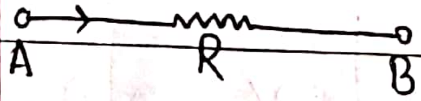
or $\frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_4+R_5} + \frac{V_2}{R_3} = 0$

or $-V_1\left(\frac{1}{R_2}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4+R_5}\right) = 0$ --- (II)

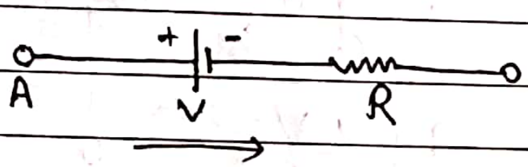
Simplification :

(i) 

$I_{AB} = I$

(ii) 

$$I_{AB} = \frac{V_A - V_B}{R}$$

(iii) 

$$I_{AB} = \frac{V_A - V_B - V}{R}$$

$$I_{AB} = \frac{V_B - V_A + V}{R}$$

Nodal Analysis Procedure:

1) Convert all the voltage source to current source & redraw the ckt diagram.

2) Identify all the nodes & choose a reference node.

3) Write the eqn for current following into & out of each node, with exceptions of reference node.

4) Solve the eqn to determine the node voltage & required the branch currents.

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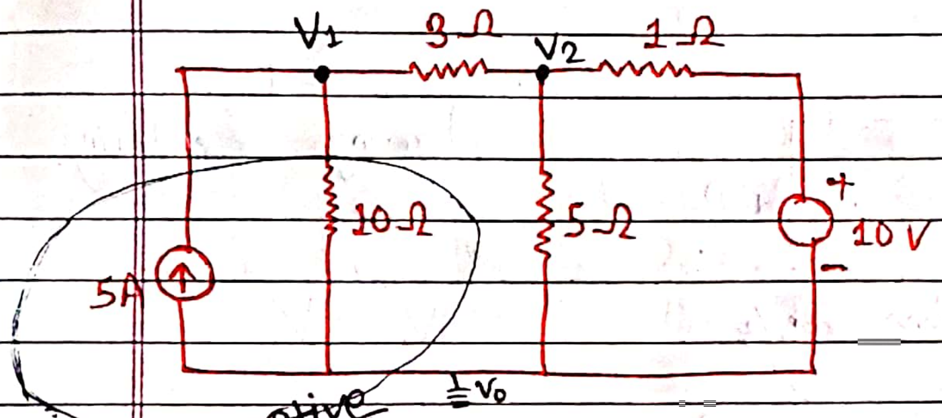
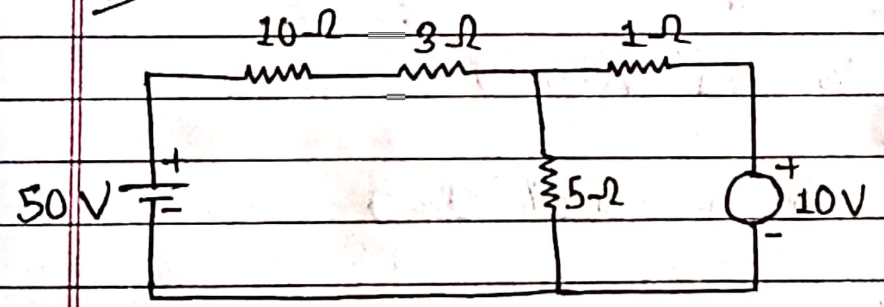
Q: Write the node voltage eq^n and determine the current in each branch for network in fig. below:

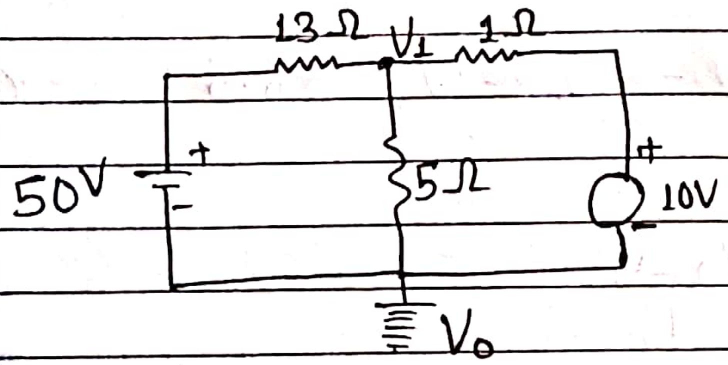


Alternative

$V = IR = 5 \times 10 = 50V$

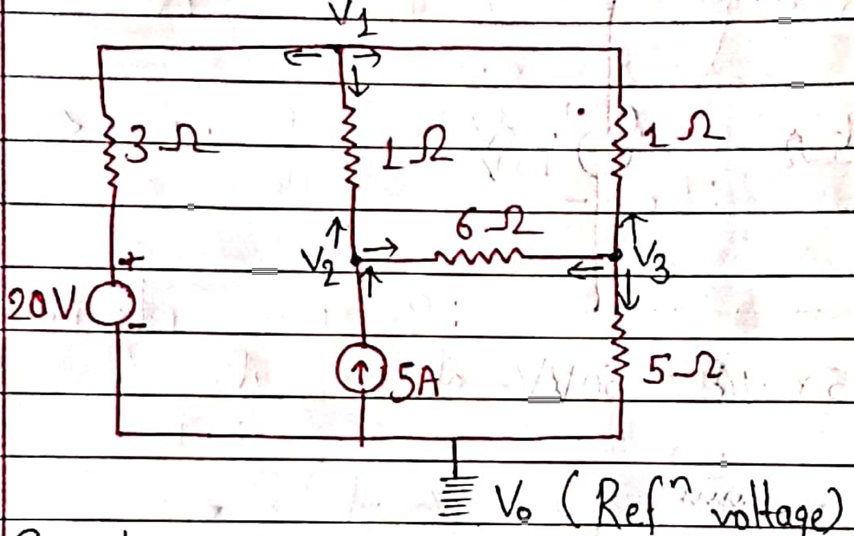


⇓



Q. Use the nodal analysis to find the power deissipt in $6\,\Omega$ resistor of the ckt shown in fig. below:

leaving = entering



Consider Node -1:

$$\frac{V_1-V_2}{1}+\frac{V_1-V_2}{1}+\frac{V_1-20-V_0}{3}=0$$

or $V_1-V_2+V_1-V_3+\frac{V_1}{3}-\frac{20}{3}=0$

or $V_1(2+0.33)-V_2-V_3=6.67$ —— (I)

Consider Node -2:

$$\frac{V_2-V_3}{6}+\frac{V_2-V_1}{1}=5 \quad \text{—— (II)}$$ [leaving = entering]

$\Rightarrow V_2-V_3+6V_2-6V_1=30$

$\Rightarrow -6V_1+7V_2-V_3=30$ —— (II)

Consider Node - 3:

$$\frac{V_3-V_1}{1}+\frac{V_3-V_0}{5}+\frac{V_3-V_2}{6}=0$$

or $V_3-V_1+\frac{V_3}{5}-\frac{V_0}{5}+\frac{V_3}{6}-\frac{V_2}{6}=0$

$$\text{or}\quad -V_1 - \frac{V_2}{6} + V_3\left(\frac{1}{1}+\frac{1}{5}+\frac{1}{6}\right) = 0$$

$$\text{or}\quad -V_1 - \frac{V_2}{6} + \frac{41}{30}V_3 = 0 \quad \text{—— (iii)}$$

Solving,

$V_1 = 23.12$ V

$V_2 = 26.98$ V

$V_3 = 20.21$ V

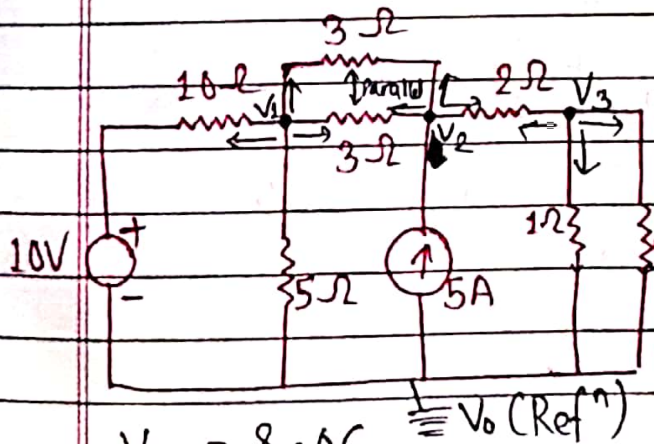
~~V = IR~~

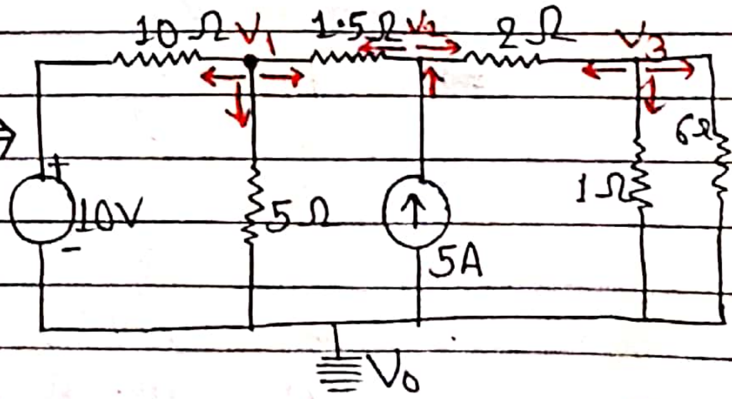
(highest voltage − lowest voltage)

$$P = I^2R \Rightarrow \frac{(V_2 - V_3)^2}{\cancel{R^2}} \times \cancel{R} = \frac{(26.98 - 20.21)^2}{6}$$

$\therefore P = 7.64$ watt

Q. Determine the voltage for each node for the ckt below:





$V_1 = 8.06$

$V_2 = 10.2$

$V_3 = 3.07$

Consider node-1:

$$\frac{V_1-10}{10}+\frac{V_1-V_2}{1.5}+\frac{V_1-V_0}{5}=0$$

or $$V_1\left(\frac{1}{10}+\frac{1}{1.5}+\frac{1}{5}\right)-V_2\cdot\frac{1}{1.5}=1$$

$$\Rightarrow V_1\cdot\frac{29}{30}-V_2\cdot\frac{2}{3}=1 \quad\text{—— (I)}$$

Consider node-2:

$$\frac{V_2-V_1}{1.5}+\frac{V_2-V_3}{2}=5$$

or, $$\frac{2}{3}(V_2-V_1)+\frac{1}{2}(V_2-V_3)=5$$

$$\Rightarrow -\frac{2}{3}V_1+1.16\times V_2-\frac{1}{2}V_3=5 \quad\text{—— (II)}$$

Consider node-3:

$$\frac{V_3-V_2}{2}+\frac{V_3-V_0}{6}+\frac{V_3-V_0}{1}=0$$

$$\Rightarrow V_2\left(-\frac{1}{2}\right)+V_3\left(\frac{1}{2}+\frac{1}{6}+1\right)=0$$

$$\Rightarrow V_2\cdot\frac{-1}{2}+V_3\cdot\frac{5}{3}=0 \quad\text{—— (III)}$$

Solving (I), (II) & (III);

$V_1 = 8.27\,V$

$V_2 = 10.42\,V$

$V_3 = 3.13\,V$

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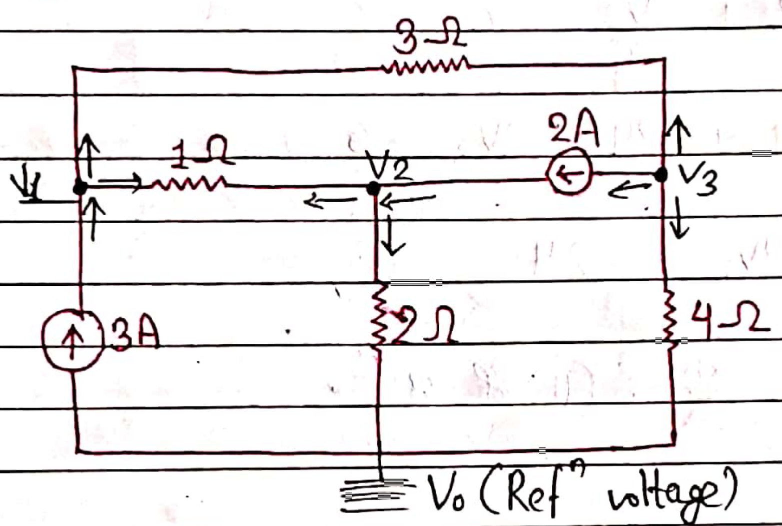
Page

Q. Use the nodal analysis to determine all the node voltage and all the element current for the network.



$V_1 = 6.7\ V$
$V_2 = 5.8\ V$
$V_3 = 0.4V$

Consider node -1:

$$\frac{V_1 - V_3}{3} + \frac{V_1 - V_2}{1} = 3$$

or $V_1\left(\frac{1}{3} + 1\right) - V_2 - V_3 \cdot \frac{1}{3} = 3$

or $V_1 \cdot \frac{4}{3} - V_2 \cdot 1 - V_3 \cdot \frac{1}{3} = 3$

$\Rightarrow 4V_1 - 3V_2 - V_3 = 9$ —— (I)

Consider node - 2:

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_0}{2} = 2$$

$\Rightarrow 2V_2 - 2V_1 + V_2 = 4$

$\Rightarrow 3V_2 - 2V_1 = 4$ — (II)

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In node - III,

$$\frac{V_3 - V_1}{3} + 2 + \frac{V_3}{4} = 0$$

$$\Rightarrow 4V_3 - 4V_1 + 24 + 3V_3 = 0$$

$$\Rightarrow 7V_3 - 4V_1 = -24 \quad \text{—— (III)}$$

Solving eq^n (I), (II) & (III),

$V_1 = 6.7V$
$V_2 = 5.8V$
$V_3 = 0.4V$

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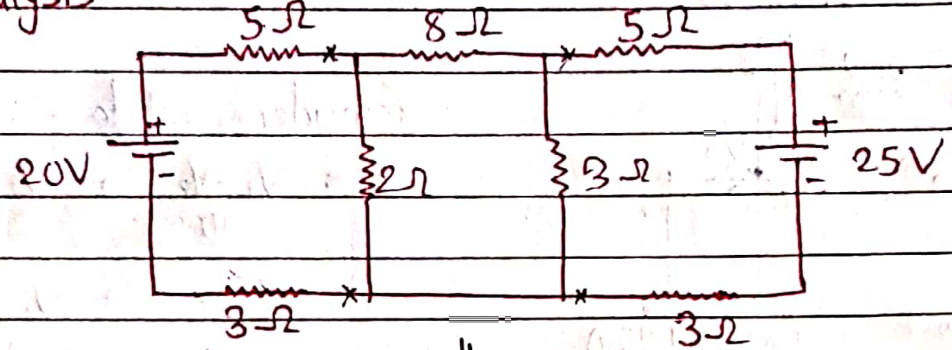
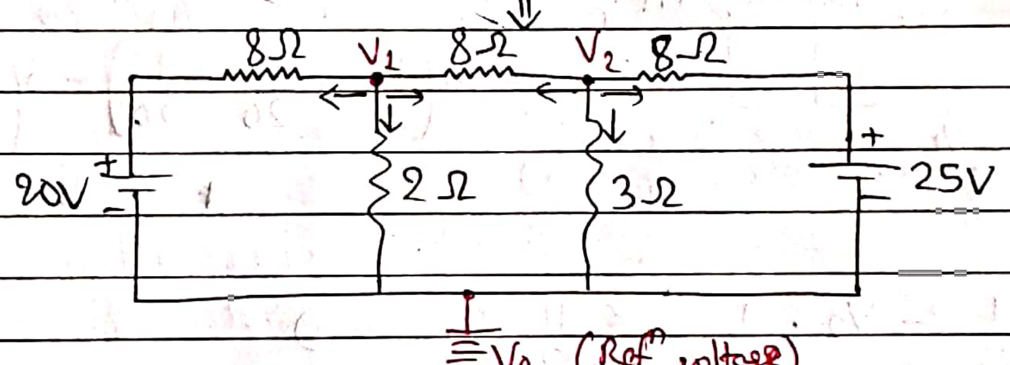
Q. Find the current across the 8Ω resistor using the nodal analysis.



$V_1 = 4.38V$
$V_2 = 6.29V$



$$I_8 = \frac{V_2 - V_1}{8} = \frac{6.29 - 4.38}{8}$$
$$= 0.24 \text{ amp}$$

Consider node-1:

$$\frac{V_1 - V_2}{8} + \frac{V_1 - V_0}{2} + \frac{V_1 - 20}{8} = 0$$

~~or~~ $\cancel{\frac{V_1}{8} - \frac{V_2}{8} + \frac{V_1}{2}}$

or $V_1 - 20 + 4V_1 + V_1 - V_2 = 0$

$\Rightarrow 6V_1 - V_2 = 20$ — ①

$I =$

In node II,

$$\frac{V_2 - V_1}{8} + \frac{V_2}{3} + \frac{V_2 - 25}{8} = 0$$

$\Rightarrow 3V_2 - 3V_1 + 8V_2 + 3V_2 - 75 = 0$

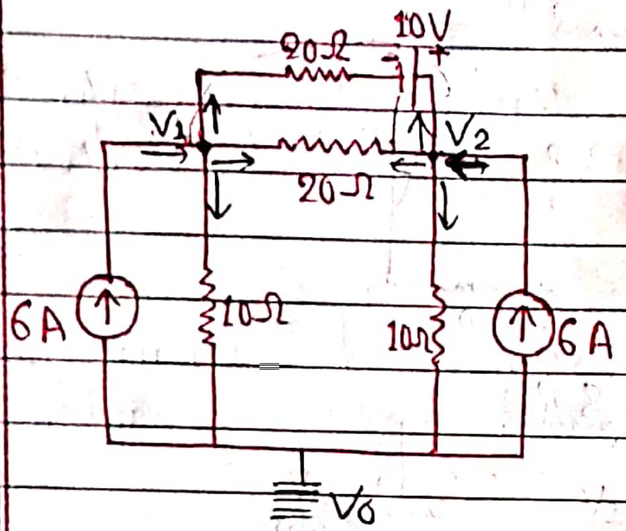
$\Rightarrow 14V_2 - 3V_1 - 75 = 0$ — (ii)

Solving,

$V_1 = 4.38V$

$V_2 = 6.29V$

2019 Q. Using nodal analysis method, find the current through each 10 Ω resistor.



Consider Node -1:

$$6 = \frac{V_1 - V_0}{10} + \frac{V_1 - V_2}{20} + \frac{V_1 - V_2 + 10}{20}$$

or $V_1\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{20}\right) - V_2\left(\frac{1}{20} + \frac{1}{20}\right) = 6 - \frac{1}{2}$

$$\Rightarrow V_1 \cdot \frac{1}{5} - V_2 \cdot \frac{1}{10} = \frac{11}{2} \quad \text{— (i)}$$

Consider Node - 2:

$$6 = \frac{V_2 - V_0}{10} + \frac{V_2 - V_1}{20} + \frac{V_2 - V_1 - 10}{20} \quad \text{— (ii)}$$

$$\cancel{\text{or } 6 = V_1\left(-\frac{1}{10} - \frac{1}{20}\right) + V_2\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right) - \frac{1}{2}}$$

$$\cancel{\Rightarrow V_1\left(-\frac{3}{20}\right) + V_2\left(\frac{1}{4}\right) = \frac{13}{2} \quad \text{— (i)}}$$

Solving (i) & (ii),

$V_1 = 58.33V$

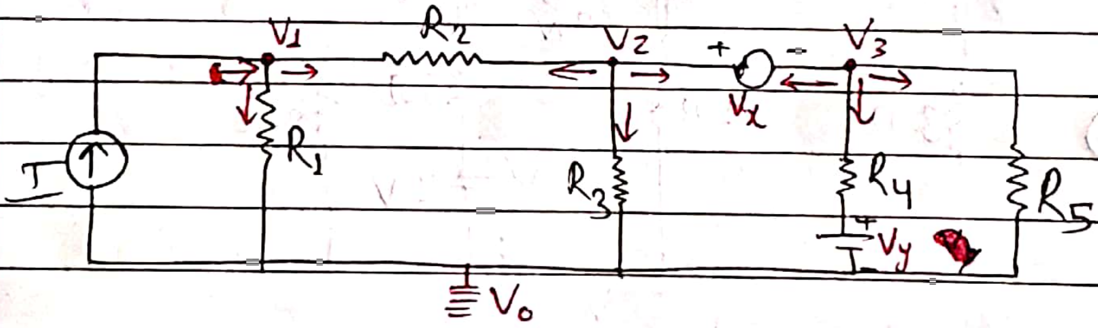
$V_2 = 61.67V$

Now,

$(I_{10})' = \frac{58.33}{10} = 5.83\ A$ | $(I_{10})'' = \frac{61.67}{10} = 6.16\ A$

Super Node Analysis

Suppose a network as a single voltage source in any branch, then it is difficult to apply the nodal analysis, one way to overcome this difficulty is to apply supernode techniques. In this method, the two adjuscent node that are connected by a voltage source are reduced to a single node & then the eqⁿ's are formed by applying the Kirchoff's current law as usual. This is explained with the help of fig. below:



Applying KCL at node -1:

$$I = \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_0}{R_1} \quad \text{—— (I)}$$

Due to the presence of voltage source V_x in beth node -2 & node -3, it is slightly difficult to find out the current. The super node techniques can convienently applied in this case. Accordingly, we can write the combined eqⁿ for node - 2 & node - 3 as under:

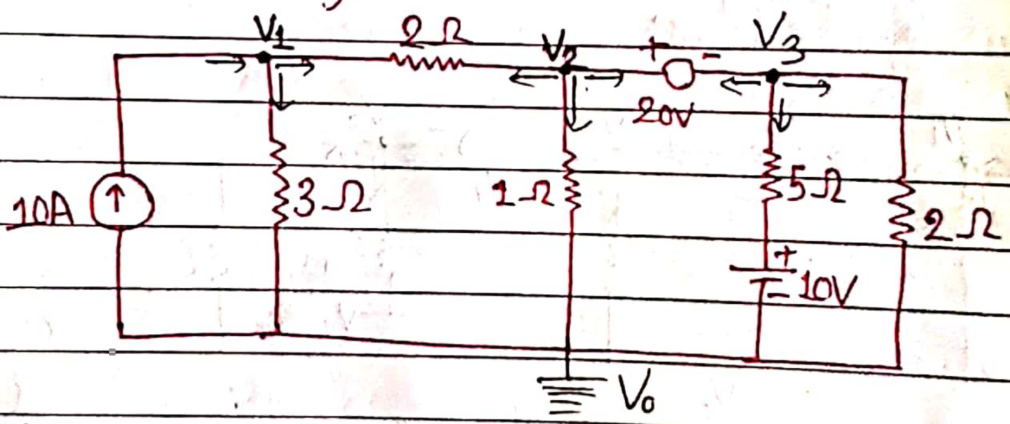
$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - V_0}{R_3} + \frac{V_3 - V_0}{R_5} + \frac{V_3 - V_Y}{R_4} = 0 \quad \text{—— (II)}$$

Other eq^n:

$$V_2 - V_3 = V_x \quad \text{—— (iii)}$$

from above theree eq^{n}s, we find the three unkonown voltage.

Q. Determine the current in 5Ω resistor for the ckt shown in fig. below:



Consider node -1:

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_0}{3} = 10 \quad \text{—— (1)}$$

or $V_1\left(\frac{1}{2}+\frac{1}{3}\right) - \frac{V_2}{2} = 10$

or $\frac{5}{6} V_1 - \frac{V_2}{2} = 10$ —— (1)

Combine eq^n of node -2 & node -3:

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_0}{1} + \frac{V_3 - V_0}{2} + \frac{V_3 - V_0 - 10}{5} = 0$$

or $V_1\left(-\frac{1}{2}\right) + V_2\left(\frac{1}{2}+1\right) + V_3\left(\frac{1}{2}+\frac{1}{5}\right) = 2$

or $-\frac{1}{2}V_1 + \frac{3}{2} \cdot V_2 + \frac{7}{10} \cdot V_3 = 2$ — (II)

Other eq^n:

$$V_2 - V_3 = 20 \quad \text{— (III)}$$

Solving,

$V_1 = 18 \cdot 95 \text{ V}$

$V_2 = 11 \cdot 58 \text{V}$

$V_3 = -8 \cdot 42 \text{ V}$