Chapter 4

The First Law of Thermodynamics

THE FIRST LAW OF THERMODYNAMICS

FOR A CONTROL MASS UNDERGOING A CYCLE FOR A CHANGE IN STATE OF A CONTROL MASS

INTERNAL ENERGY

A THERMODYNAMIC PROPERTY

ENTHALPY

THE THERMODYNAMIC PROPERTY

SPECIFIC HEATS

CONSTANT-VOLUME

CONSTANT-PRESSURE

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEAT OF IDEAL GASES

THE FIRST LAW AS A RATE EQUATION CONSERVATION OF MASS

Introduction to 1st Law of Thermodynamics

The first law of thermodynamics, also known as the conservation of energy principle. It states that "Energy can neither be created nor destroyed; it can only change its form."

Total energy of an isolated system in all its form remains constant.

The first law of thermodynamics cannot be proved mathematically but no process in nature is known to have violated the first law of thermodynamics.

It is the relation of energy balance and is applicable to any kind of system (open or closed) undergoing any kind of process.

First Law Applied to a Cyclic Process – Joule's Experiment

- Cyclic Process: "A process is cyclic if the initial and final states of the system executing the process are identical."
- A system represented by a state point 1 undergoes a process 1-a-2, and comes back to initial state following the path 2-b-1.
- All properties of the system are restored, when the initial state is reached.
- During the execution of these processes:

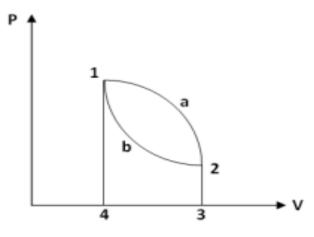


Fig. 2.1 Cyclic process

- Area 1-a-2-3-4-1 represents the work done by the system (W₁) during expansion process 1-a-2.
- Similarly area 2-3-4-1-b-2 gives work supplied to the system (W₂) during compression process 2-b-1.
- iii. Area 1-a-2-b-1 represents the net work $(W_1 W_2)$ delivered by the system.
- Since the system regains its initial state, there is no change in the energy stored by the system.
- For a cyclic process, the First Law of Thermodynamics can be stated as follows:
 "When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings."

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Mathematically,

$$\oint \delta Q = \oint \delta W - - - - - - (2.1)$$

Joule's Experiment

The first law can be illustrated by considering the following experiment (Fig. 2.2).

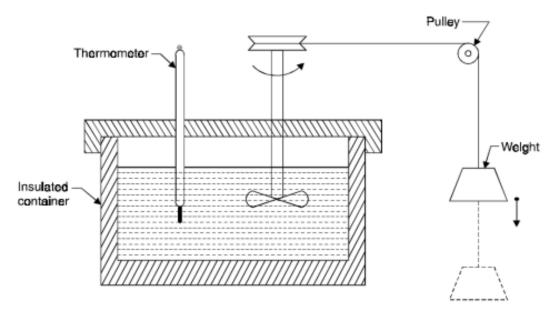


Fig. 2.2 Joule's paddle-wheel experiment

- A known mass of water is taken into a rigid and well insulated container provided with a paddle wheel.
- The insulation is provided to prevent any heat interaction with surroundings.
- The work input to the paddle wheel is measured by the fall of weight while the corresponding temperature rise of the liquid in the insulated container is measured by the thermometer.
- Joule conducted a number of experiments involving different types of work interactions and found that the work expended was proportional to increase in thermal energy, i.e.

$$Q \propto W$$

$$\therefore Q = \frac{W}{J}$$

$$\therefore W = JQ$$

 $Q \propto W$

Where.

J = Joule's equivalent or mechanical equivalent of heat
In SI system of units, both heat and work are measured in Joules.

THE FIRST LAW OF THERMODYNAMICS FOR A CONTROL MASS UNDERGOING A CYCLE

The First Law is often called the conservation of energy law.

The first law of thermodynamics states that
 during any cycle a system (control mass)
 undergoes, the cyclic integral of the heat is
 proportional to the cyclic integral of the work.

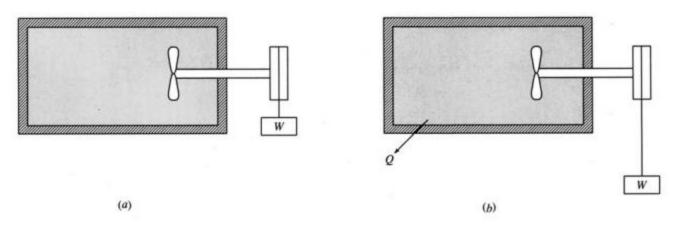


Fig. 4-1

The first law can be illustrated by considering the following experiment. Let a weight be attached to a pulley-paddle-wheel setup, such as that shown in Fig.4-1a. Let the weight fall a certain distance thereby doing work on the system, contained in the tank shown, equal to the weight multiplied by the distance dropped. The temperature of the system (the fluid in the tank) will immediately rise an amount ΔT . Now, the system is returned to its initial state (the completion of the cycle) by transferring heat to the surroundings, as implied by the Q in Fig. 4-1b. This reduces the temperature of the system to its initial temperature. The first law states that this heat transfer will be exactly equal to the work which was done by the falling weight.

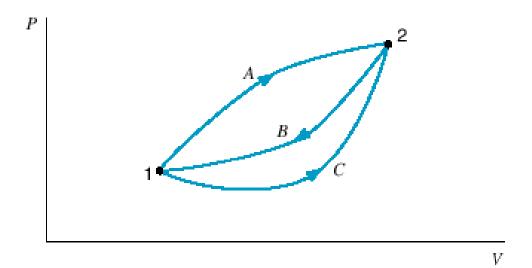
$$J \oint \delta Q = \oint \delta W \tag{5.1}$$

The symbol $\oint \delta Q$, which is called the cyclic integral of the heat transfer, represents the net heat transfer during the cycle, and $\oint \delta W$, the cyclic integral of the work, represents the net work during the cycle. Here, J is a proportionality factor that depends on the units used for work and heat.

$$\Sigma W = \Sigma Q$$

$$\oint \delta W = \oint \delta Q$$

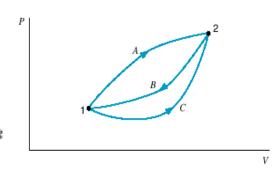
THE FIRST LAW OF THERMODYNAMICS FOR A CHANGE IN STATE OF A CONTROL MASS



$$\oint \delta Q = \oint \delta W$$

Considering the two separate processes, we have

$$\int_{1}^{2} \delta Q_{A} + \int_{2}^{1} \delta Q_{B} = \int_{1}^{2} \delta W_{A} + \int_{2}^{1} \delta W_{B}$$



Now consider another cycle in which the control mass changes from state 1 to state 2 by process C and returns to state 1 by process B, as before. For this cycle we can write

$$\int_{1}^{2} \delta Q_{C} + \int_{2}^{1} \delta Q_{B} = \int_{1}^{2} \delta W_{C} + \int_{2}^{1} \delta W_{B}$$

Subtracting the second of these equations from the first, we obtain

$$\int_{1}^{2} \delta Q_A - \int_{1}^{2} \delta Q_C = \int_{1}^{2} \delta W_A - \int_{1}^{2} \delta W_C$$

or, by rearranging,

$$\int_{1}^{2} (\delta Q - \delta W)_{A} = \int_{1}^{2} (\delta Q - \delta W)_{C}$$

$$(5.3)$$

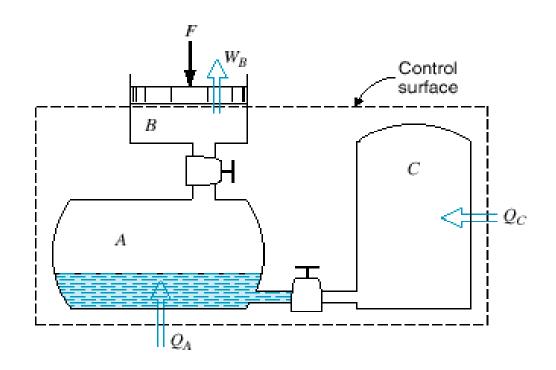
Since A and C represent arbitrary processes between states 1 and 2, the quantity $\delta Q - \delta W$ is the same for all processes between states 1 and 2. Therefore, $\delta Q - \delta W$ depends only on the initial and final states and not on the path followed between the two states. We conclude that this is a point function, and therefore it is the differential of a property of the mass. This property is the energy of the mass and is given the symbol E. Thus we can write

$$dE = \delta Q - \delta W$$

(5.4)

$$dE = \delta Q - \delta W$$

$$E_2 - E_1 = {}_{1}Q_2 - {}_{1}W_2 \tag{5.5}$$



$$\Delta$$
 Energy = $+$ in $-$ out

E = Internal energy + Kinetic energy + Potential energy

$$E = U + KE + PE$$

$$dE = dU + d(KE) + d(PE)$$

$$dE = dU + d(KE) + d(PE) = \delta Q - \delta W$$

$$\delta W = -F dx = -dKE$$

$$F = ma = m\frac{d\mathbf{V}}{dt} = m\frac{dx}{dt}\frac{d\mathbf{V}}{dx} = m\mathbf{V}\frac{d\mathbf{V}}{dx}$$

$$dKE = F dx = m\mathbf{V} d\mathbf{V}$$

$$\int_{KE=0}^{KE} dKE = \int_{\mathbf{V}=0}^{\mathbf{V}} m\mathbf{V} d\mathbf{V}$$
$$KE = \frac{1}{2}m\mathbf{V}^{2}$$

$$\delta W = -F dZ = -dPE$$
$$F = ma = mg$$

$$dPE = F dZ = mg dZ$$

$$\int_{PE_1}^{PE_2} d\, PE = m \int_{Z_1}^{Z_2} g \, dZ$$

$$PE_2 - PE_1 = mg(Z_2 - Z_1)$$

$$dE = dU + m\mathbf{V} d\mathbf{V} + mg dZ$$

$$E_2 - E_1 = U_2 - U_1 + \frac{m\mathbf{V}_2^2}{2} - \frac{m\mathbf{V}_1^2}{2} + mgZ_2 - mgZ_1$$

Similarly, substituting these expressions for kinetic and potential energy we have

$$dE = dU + \frac{d(m\mathbf{V}^2)}{2} + d(mgZ) = \delta Q - \delta W$$

Assuming g is a constant, in the integrated form of this equation,

$$U_2 - U_1 + \frac{m(\mathbf{V}_2^2 - \mathbf{V}_1^2)}{2} + mg(Z_2 - Z_1) = {}_1Q_2 - {}_1W_2$$

5. 5 ENTHALPY

Let us consider a control mass undergoing a quasiequilibrium constant-pressure process, as shown in Fig. 5.6.

$$_{1}Q_{2} = U_{2} - U_{1} + {_{1}W_{2}}$$

$$_1W_2 = \int_1^2 P \, dV$$

$$_{1}W_{2} = P \int_{1}^{2} dV = P(V_{2} - V_{1})$$

Since the pressure is constant,

$$_{1}Q_{2} = U_{2} - U_{1} + P_{2}V_{2} - P_{1}V_{1}$$

= $(U_{2} + P_{2}V_{2}) - (U_{1} + P_{1}V_{1})$

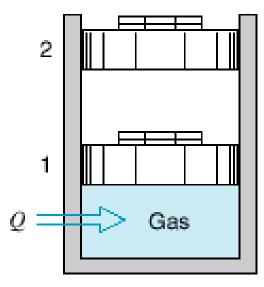


FIGURE 5.6 The constant-pressure quasi-equili brium process.

define a new extensive property, the enthalpy,

$$H \equiv U + PV$$

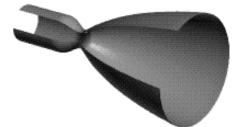
or, per unit mass, $h \equiv u + Pv$

$$h \equiv u + Pv$$

L specific enthalpy, h, and total enthalpy, H.]

(The significance and use of enthalpy is not restricted to the special process just described. Other cases in which this same combination of properties u + Pv appear will be developed later, notably in Chapter 6 in which we discuss control volume analyses.)

$$h = (1 - x)h_f + xh_g$$
$$h = h_f + xh_{fg}$$



E = Internal Energy

Q = Heat Transfer

T = Temperature

W = Work

p = Pressure

C_p = Heat Capacity

V = Volume

(constant pressure)

Define a new state variable that is a combination of other state variables.

Enthalpy =
$$H = E + pV$$

1st Law of Thermodynamics: $E_2 - E_1 = Q - W$

For a constant pressure process, the work is given: $W = p(V_2 - V_1)$

Substitute: $E_2 - E_1 = Q - p(V_2 - V_1)$

Re-group: $(E_2 + p V_2) - (E_1 + p V_1) = Q$

Heat Transfer at constant pressure: $Q = C_p (T_2 - T_1)$

Definition of Enthalpy: $(H_2 - H_1) = C_p (T_2 - T_1)$

Internal energy

- •Internal energy, kinetic and potential energies are extensive properties, because they depends on the mass of the system.
- •U designates the internal energy of a given mass of a substance, and u as the specific internal energy.
- •The values are given in relation to an arbitrarily assumed reference state, which, for water in the steam tables, \mathbf{u}_f is taken as zero for saturated liquid at the triple-point temperature, 0.01° C.

$$u = (1 - x) u_f + x u_g$$

$$u = u_f + x u_{fg}$$

5. 6 THE CONSTANT-VOLUME AND CONSTANT-PRESSURE SPECIFIC HEATS

$$\delta Q = dU + \delta W = dU + P dV$$

We find that this expression can be evaluated for two separate special cases.

 Constant volume, for which the work term (P dV) is zero, so that the specific heat (at constant volume) is

$$C_v = \frac{1}{m} \left(\frac{\delta Q}{\delta T} \right)_v = \frac{1}{m} \left(\frac{\partial U}{\partial T} \right)_v = \left(\frac{\partial u}{\partial T} \right)_v$$
 (5.14)

2. Constant pressure, for which the work term can be integrated and the resulting PV terms at the initial and final states can be associated with the internal energy terms, as in Section 5.5, thereby leading to the conclusion that the heat transfer can be expressed in terms of the enthalpy change. The corresponding specific heat (at constant pressure) is

$$C_p = \frac{1}{m} \left(\frac{\delta Q}{\delta T} \right)_p = \frac{1}{m} \left(\frac{\partial H}{\partial T} \right)_p = \left(\frac{\partial h}{\partial T} \right)_p$$
 (5.15)

A very important relation between the constant-pressure and constant-volume specific heats of an ideal gas may be developed from the definition of enthalpy:

$$h = u + Pv = u + RT$$

Differentiating and substituting Eqs. 5.20 and 5.24, we have

$$dh = du + R dT$$

$$dh = du + R dT$$

$$C_{p0} dT = C_{v0} dT + R dT$$

Therefore,

$$C_{p0} - C_{v0} = R (5.27)$$

ENERGY BALANCE FOR CLOSED SYSTEMS

Net energy transfer by heat, work, and mass

$$\frac{\dot{E}_{\text{in}} - E_{\text{out}}}{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}} = \underbrace{\frac{\Delta E_{\text{system}}}{\text{Change in internal, kinetic, potential, etc., energies}}}_{\text{Change in internal, kinetic, potential, etc., energies}} \text{System undergoing any system undergoing any system undergoing any}} \\
\frac{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \text{balance in}}_{\text{Energy balance in}}$$

The total quantities are related to the quantities per unit the rate, form the rate, form quantities per unit the rate form (kJ) and $\Delta E = (dE/dt)\Delta t$ (kJ)

 $e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}}}$ (kJ/kg) Energy balance per unit mass basis $\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}}$ or $\delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}}$ Energy balance in $W_{\text{net,out}} = \dot{Q}_{\text{net,in}}$ or $\dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$ Energy differential balance for form

cycle

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}}$$
 or $Q - W = \Delta E$ $Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$ $W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$

Energy balance when sign convention is used: (i.e., heat

input and work output are positive; heat output and work

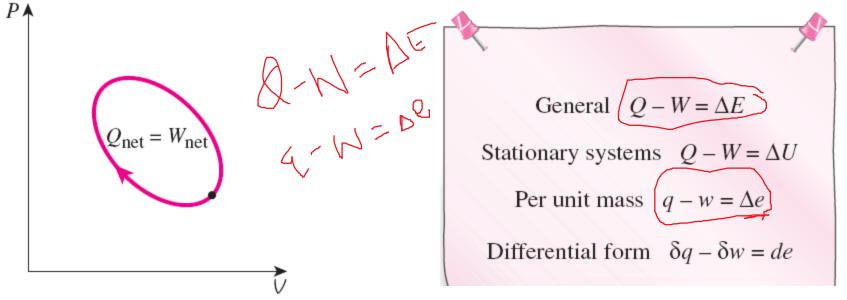


FIGURE 4-11

For a cycle $\Delta E = 0$, thus Q = W.

Various forms of the first-law relation for closed systems when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be

Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. *Q* is *to* the system and *W* is *from* the system.

$$E_{\text{in}} - E_{\text{out}} = \underbrace{\Delta E_{\text{system}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q - W = \Delta U + \Delta E + \Delta P E^{-0}$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process

$$W_{e, \text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e, \text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$

SPECIFIC HEATS

Specific heat at constant volume, c_{v} : The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p : The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

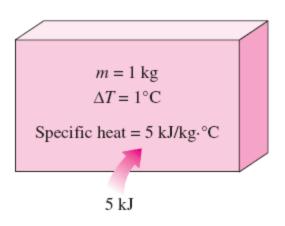


FIGURE 4-18

Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).

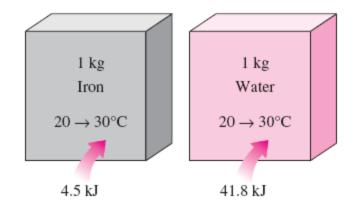
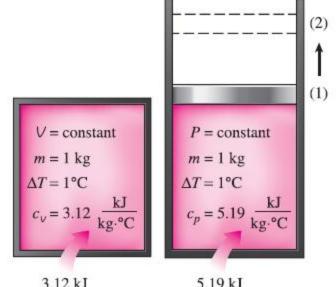


FIGURE 4-17

It takes different amounts of energy to raise the temperature of different substances by the same amount.



3.12 kJ

5.19 kJ

$$c_{V} = \left(\frac{\partial u}{\partial T}\right)_{V}$$
= the chage in internal energy with temperature at constant volume
$$c_{p} = \left(\frac{\partial h}{\partial T}\right)_{p}$$
= the chage in enthalpy with temperature at constant pressure

Formal definitions of c_v and c_p .

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

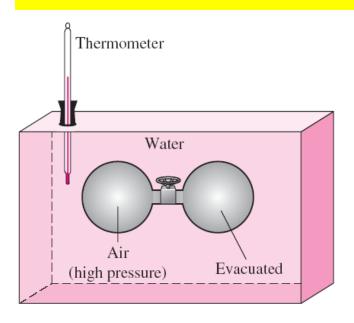


FIGURE 4-22

Schematic of the experimental apparatus used by Joule.

Joule showed using this experimenta

$$u = u(T)$$

$$h = h(T)$$

$$c_{v} = c_{v}(T)$$

$$c_{p} = c_{p}(T)$$

$$h = u + P \lor P \lor = RT$$

$$h = u + RT$$

$$u = u(T) \ h = h(T)$$

$$du = c_{\lor}(T) \ dT \ dh = c_{p}(T) \ dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) \ dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

For ideal gases, u, h, c_v , and c_p vary with

Internal energy and enthalpy change of an ideal gas³¹

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

Reversible Constant Volume (or Isochoric) Process (v = constant) :

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle-wheel work input. It will be assumed that 'constant volume' implies zero work unless stated otherwise.

Fig. 4.5 shows the system and states before and after the heat addition at constant volume.

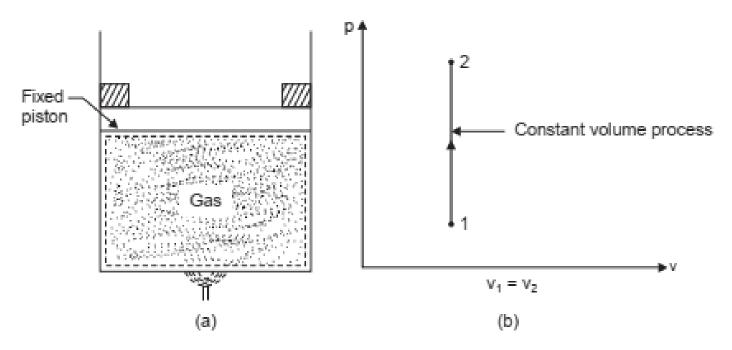


Fig. 4.5. Reversible constant volume process.

Considering mass of the working substance unity and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W \qquad ...(4.26)$$

The work done

$$W = \int_{1}^{2} p \, dv = 0 \text{ as } dv = 0.$$

$$Q = (u_2 - u_1) = c_v(T_2 - T_1) \qquad ...[4.27 (a)]$$

where $c_n = \text{Specific heat at constant volume}$.

For mass, m, of working substance

$$Q = U_2 - U_1 = mc_v(T_2 - T_1) \qquad ...[4.27 \ (b)]$$

[:: mu = U]

Reversible Constant Pressure (or Isobaric) Process (p = constant).

It can be seen from Fig. 4.5 (b) that when the boundary of the system is *inflexible* as in a constant volume process, then the pressure rises when heat is supplied. Hence for a constant pressure process, the boundary must move against an external resistance as heat is supplied; for instance a gas [Fig. 4.6 (a)] in a cylinder behind a piston can be made to undergo a constant pressure process. Since the piston is pushed through a certain distance by the force exerted by the gas, then the work is done by the gas on its surroundings.

Fig. 4.6 shows the system and states before and after the heat addition at constant pressure.

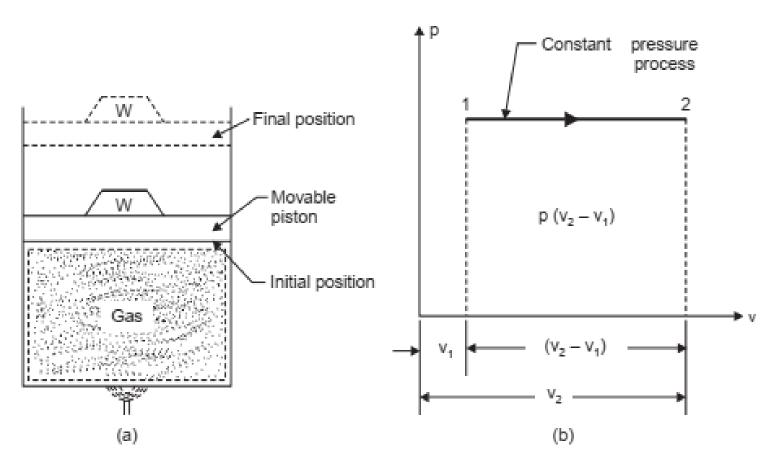


Fig. 4.6. Reversible constant pressure process.

Considering unit mass of working substance and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W$$
 The work done,
$$W = \int_1^2 p dv = p(v_2 - v_1)$$

$$\therefore \qquad Q = (u_2 - u_1) + p(v_2 - v_1) = u_2 - u_1 + pv_2 - pv_1$$

$$= (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1$$
 [: $h = u + pv$]
 $Q = h_2 - h_1 = c_p (T_2 - T_1)$...(4.28)

or

where h = Enthalpy (specific), and

 $c_{_{\scriptscriptstyle \mathcal{D}}}=$ Specific heat at constant pressure.

For mass, m, of working substance

$$Q = H_2 - H_1 = mc_p (T_2 - T_1)$$
 ...[4.28 (a)]
[:: $mh = H$]

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant):

A process at a constant temperature is called an isothermal process. When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall. In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the working substance continuously during the process.

Fig. 4.7 shows the system and states before and after the heat addition at constant temperature.

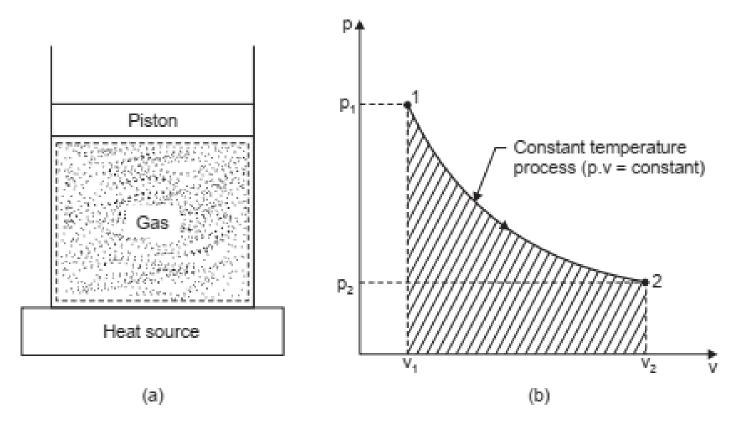


Fig. 4.7. Reversible isothermal process.

Considering unit mass of working substance and applying first law to the process

$$\begin{split} Q &= (u_2 - u_1) + W \\ &= c_v \left(T_2 - T_1 \right) + W \\ &= 0 + W \end{split} \qquad \begin{bmatrix} \because & T_2 = T_1 \end{bmatrix} \end{split}$$

The work done, $W = \int_{1}^{2} p dv$

In this case pv = constant or $p = \frac{C}{v}$ (where C = constant)

$$W = \int_{v_1}^{v_2} C \frac{dv}{v} = C[\log_e v]_{v_1}^{v_2} = C \log_e \frac{v_2}{v_1}$$

The constant C can either be written as p_1v_1 or as p_2v_2 , since

$$p_1v_1 = p_2v_2 = constant, C$$

i.e., $W = p_1v_1 \log_e \frac{v_2}{v_1}$ per unit mass of working substance

 $W = p_2 v_2 \log_e \frac{v_2}{v_1}$ per unit mass of working substance

$$Q = W = p_1 v_1 \log_e \frac{v_2}{v_1} \qquad ...(4.29)$$

For mass, m, of the working substance

or

$$Q = p_1 V_1 \log_e \frac{V_2}{V_1} \qquad ... [4.29 \ (a)]$$

or
$$Q = p_1 V_1 \log_e \frac{p_1}{p_2}$$
 $\left[\because \frac{V_2}{V_1} = \frac{p_1}{p_2} \right]$...[4.29 (b)]

4. Reversible Adiabatic Process (pv) = constant):

An adiabatic process is one in which no heat is transferred to or from the fluid during the process. Such a process can be reversible or irreversible. The reversible adiabatic non-flow process will be considered in this section.

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$

 $O = (u_2 - u_1) + W$

or

 $W = (u_1 - u_2)$ for any adiabatic process

...(4.30)

Eqn. (4.30) is true for an adiabatic process whether the process is reversible or not. In an adiabatic expansion, the work done W by the fluid is at the expense of a reduction in the internal energy of the fluid. Similarly in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid.

For an adiabatic process to take place, perfect thermal insulation for the system must be available

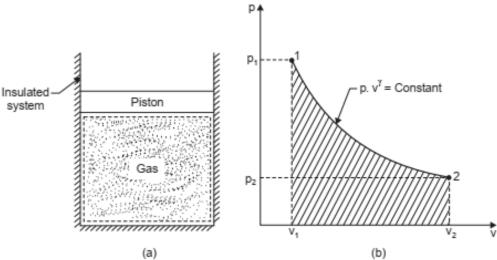


Fig. 4.8. Reversible adiabatic process.

To derive the law $pv^{\gamma} = constant$:

To obtain a law relating p and v for a reversible adiabatic process let us consider the nonflow energy equation in differential form,

$$dQ = du + dW$$

For a reversible process

$$dW = pdv$$

$$dQ = du + pdv = 0$$

(Since for an adiabatic process Q = 0)

Also for a perfect gas

$$pv = RT \text{ or } p = \frac{RT}{v}$$

Hence substituting,

$$du + \frac{RTdv}{v} = 0$$

Also

$$u = c_n T$$
 or $du = c_n dT$

$$c_v dT + \frac{RTdv}{v} = 0$$

The work done is given by the shaded area, and this area can be evaluated by integration.

$$i.e., W = \int_{v_1}^{v_2} p \, dv$$

or

Therefore, since $pv^{\gamma} = \text{constant}$, C, then

$$W = \int_{v_1}^{v_2} C \frac{dv}{v^{\gamma}} \qquad \left[\because p = \frac{C}{v^{\gamma}} \right]$$

$$\begin{split} W &= C \int_{v_1}^{v_2} \frac{dv}{v^\gamma} = C \left| \frac{v^{-\gamma+1}}{-\gamma+1} \right|_{v_1}^{v_2} \\ &= C \left(\frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{1-\gamma} \right) = C \left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma-1} \right) \end{split}$$

The constant in this equation can be written as $p_1v_1^{\gamma}$ or as $p_2v_2^{\gamma}$. Hence,

$$W = \frac{p_1 v_1^{\gamma} v_1^{-\gamma+1} - p_2 v_2^{\gamma} v_2^{-\gamma+1}}{\gamma - 1} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

i.e.,
$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$
...(4.32)

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$
 ...(4.33)

5. Polytropic Reversible Process ($pv^n = constant$):

It is found that many processes in practice approximate to a reversible law of form pv^n = constant, where n is a constant. Both vapours and perfect gases obey this type of law closely in many non-flow processes. Such processes are internally reversible.

We know that for any reversible process,

$$W = \int p \, dv$$

For a process in $pv^n = constant$, we have

$$p = \frac{C}{v^n}$$
 , where C is a constant

$$W = C \int_{v_1}^{v_2} \frac{dv}{v^n} = C \left| \frac{v^{-n+1}}{-n+1} \right| = C \left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1} \right)$$

$$i.e.\,, \qquad \qquad W = C\left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1}\right) = \frac{p_1v_1^{-n}v_1^{-n+1} - p_2v_2^{-n}v_2^{-n+1}}{n-1}$$

(since the constant C, can be written as $P_1v_1^n$ or as $p_3v_2^n$)

i.e., Work done,
$$W = \frac{p_1v_1 - p_2v_2}{n-1}$$
 ...(4.39)

or
$$W = \frac{R(T_1 - T_2)}{n - 1}$$
 ...(4.40)

Eqn. (4.39) is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n$$
 ...(4.41)

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \tag{4.42}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \dots (4.43)$$

Heat transfer during polytropic process (for perfect gas pv = RT) :

Using non-flow energy equation, the heat flow/transfer during the process can be found,

$$i.e.,$$
 $Q = (u_2 - u_1) + W$

$$= c_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n-1}$$

i.e.,
$$Q = \frac{R(T_1 - T_2)}{n - 1} - c_v (T_1 - T_2)$$

Also
$$c_v = \frac{R}{(\gamma - 1)}$$

On substituting,

1.6.

or

....

$$Q = \frac{R}{n-1}(T_1 - T_2) - \frac{R}{(\gamma - 1)}(T_1 - T_2)$$

$$\begin{split} Q &= R(T_1 - T_2) \left(\frac{1}{n-1} - \frac{1}{\gamma - 1} \right) \\ &= \frac{R \left(T_1 - T_2 \right) (\gamma - 1 - n + 1)}{(\gamma - 1)(n-1)} = \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma - 1)(n-1)} \end{split}$$

$$Q = \frac{(\gamma - n)}{(\gamma - 1)} \frac{R(T_1 - T_2)}{(n - 1)}$$

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W \qquad \left[\because W = \frac{R(T_1 - T_2)}{(n - 1)}\right] \qquad \dots (4.44)$$

Process	Index n	Heat added	$\int_{I}^{2} p dv$	p, v, T relations	Specific heat, c
Constant pressure	n = 0	$c_p(T_2-T_1)$	$p(v_2-v_1)$	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$	c _p
Constant volume	71: = ∞	$c_r(T_2-T_1)$	0	$\frac{T_1}{T_2} = \frac{p_1}{p_2}$	c _r
Constant temperature	n =1	$p_1v_1\log_e \frac{v_2}{v_1}$	$p_1v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\frac{p_1v_1 - p_2v_2}{\gamma - 1}$	$p_1v_1^{\gamma} = p_2v_2^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$ $= \left(\frac{p_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$	0
Polytropic	n = n	$c_n(T_2 - T_1)$ $= c_v \left(\frac{\gamma - n}{1 - n}\right)$ $\times (T_2 - T_1)$ $= \frac{\gamma - n}{\gamma - 1} \times \text{work}$ done (non-flow)	$\frac{p_1v_1 - p_2v_2}{n-1}$	$p_1v_1^n = p_2v_2^n$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$ $= \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	$c_n = c_U \left(\frac{\gamma - n}{1 - n} \right)$

Note. Equations must be used keeping dimensional consistence.



FIGURE 5-5

Conservation of mass principle for an ordinary bathtub.

Conservation of Mass Principle

The **conservation of mass principle** for a control volume can be expressed as: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt . That is,

$$\begin{pmatrix} \text{Total mass entering} \\ \text{the CV during } \Delta t \end{pmatrix} - \begin{pmatrix} \text{Total mass leaving} \\ \text{the CV during } \Delta t \end{pmatrix} = \begin{pmatrix} \text{Net change in mass} \\ \text{within the CV during } \Delta t \end{pmatrix}$$

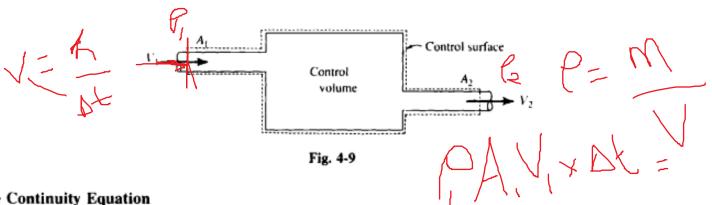
or

$$m_{\rm in} - m_{\rm out} = \Delta m_{\rm CV} \qquad (kg) \tag{5-8}$$

where $\Delta m_{\rm CV} = m_{\rm final} - m_{\rm initial}$ is the change in the mass of the control volume during the process (Fig. 5–5). It can also be expressed in *rate form* as

$$\dot{m}_{\rm in} - \dot{m}_{\rm out} = dm_{\rm CV}/dt$$
 (kg/s) (5-9)

where $\dot{m}_{\rm in}$ and $\dot{m}_{\rm out}$ are the total rates of mass flow into and out of the control volume, and $dm_{\rm CV}/dt$ is the time rate of change of mass within the control volume boundaries. Equations 5–8 and 5–9 are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.



The Continuity Equation

Consider a general control volume with an area A_1 where fluid enters and an area A_2 where fluid leaves, as shown in Fig. 4-9. It could have any shape and any number of entering and exiting areas, but we will derive the continuity equation using the geometry shown. Conservation of mass requires that

$$\begin{pmatrix} \text{Mass entering} \\ \text{control volume} \end{pmatrix} - \begin{pmatrix} \text{Mass leaving} \\ \text{control volume} \end{pmatrix} = \begin{pmatrix} \text{Change in mass} \\ \text{within control volume} \end{pmatrix} = \Delta m_{\text{c.v.}}$$

$$(4.54)$$

The mass that crosses an area A over a time increment Δt can be expressed as $\rho AV\Delta t$ where $V\Delta t$ is the distance the mass particles travel and $AV\Delta t$ is the volume swept out by the mass particles. Equation (4.54) can thus be put in the form

$$\frac{\rho_1 A_1 V_1 \Delta t - \rho_2 A_2 V_2 \Delta t}{\Delta t} = \Delta m_{c,v}.$$
 (4.55)

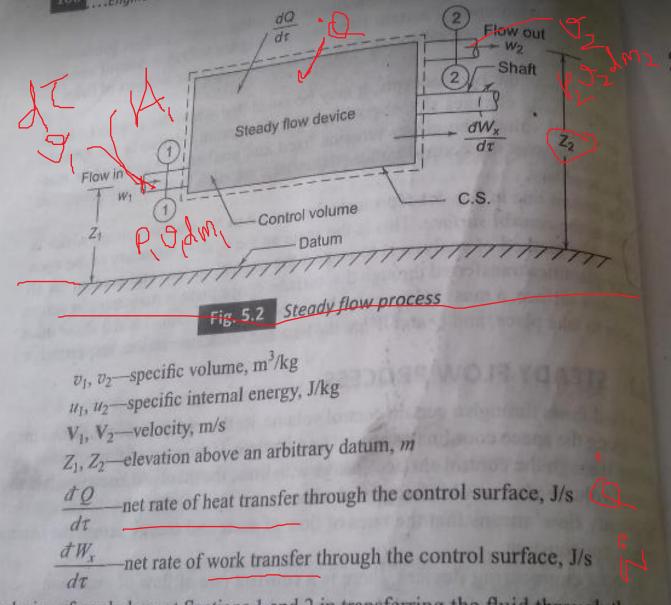
where the velocities V_1 and V_2 are perpendicular to the areas A_1 and A_2 , respectively. We have assumed the velocity and density to be uniform over the two areas, a good assumption for the turbulent flows most often encountered entering and leaving the devices of interest.

If we divide by Δt and let $\Delta t \rightarrow 0$, the derivative results and we have the *continuity equation*,

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 = \frac{dm_{ov}}{dt} \tag{4.56}$$

For the steady-flow situation, in which the mass in the control volume remains constant, the continuity equation reduces to

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{4.57}$$



exclusive of work done at Sections 1 and 2 in transferring the fluid through the control surface.

τ—time, s.

Subscripts 1 and 2 refer to the inlet and evit sections

By the conservation of mass, if there is no accumulation of mass within the control volume, the mass flow rate entering must equal the mass flow rate leaving, or $w_1 = w_2$ or $\frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2}$ This equation is known as the equation of continuity. (5.2)

5.3.2 Energy Balance

In a flow process, the work transfer may be of two types: the external work and the flow work. The external work refers to all the work transfer across the control surface other than that due to normal fluid forces. In engineering thermodynamics, the only kinds of external work of importance are shear (shaft or stirring) work and electri-

al work. In Fig. 5.2 the only external work occurs in the form of shaft work, W_{κ} flow work, as discussed in Sec. 3.4, is the displacement work done by the fluid of mass dm_1 at the inlet Section 1 and that of mass dm_2 at the exit Section 2, which $E = p_1 v_1 dm_1$) and $(+ p_2 v_2 dm_2)$ respectively. Therefore, the total work transfer is

$$W = W_x - p_1 v_1 dm_1 + p_2 v_2 dm_2$$
 (5.3)

n the rate form,

$$\frac{dW}{d\tau} = \frac{dW_x}{d\tau} - p_1 v_1 \left(\frac{dm_1}{d\tau}\right) + p_2 v_2 \frac{dm_2}{d\tau}$$

$$\frac{dW}{d\tau} = \frac{dW_x}{d\tau} - w_1 p_1 v_1 + w_2 p_2 v_2 \qquad (5.4)$$

ince there is no accumulation of energy, by the conservation of energy, the total rate f flow of all energy streams entering the control volume must equal the total rate of ow of all energy streams leaving the control volume. This may be expressed in the

where equations:
$$\frac{1}{\sqrt{C_1}} = \frac{dQ}{d\tau} = \frac{dW}{d\tau}$$
The equations:
$$\frac{dW}{d\tau} = \frac{dW}{d\tau} = \frac{dW}{d\tau}$$
The equations:
$$\frac{dW}{d\tau} = \frac{dW}{d\tau}$$
The equations:
$$\frac{dW}{d\tau}$$

$$w_1 e_1 + \frac{dQ}{d\tau} = w_2 e_2 + \frac{dW_x}{d\tau} - w_1 p_1 v_1 + w_2 p_2 v_2$$

$$w_1 e_1 + w_1 p_1 v_1 + \frac{d Q}{d \tau} = w_2 e_2 + w_2 p_2 v_2 + \frac{d W_x}{d \tau}$$
 (5.5)

where e_1 and e_2 refer to the energy carried into or out of the control volume with it mass of fluid.

it mass of fluid.

The specific energy
$$e$$
 is given by
$$e = e_k + e_p + u = \frac{V_i^2}{2} + Zg + u$$
(5.6)

where
$$h = u + pv$$
.

And, since

Dividing Equation (5.7) by $\frac{dm}{d\tau}$
 $h_1 + \frac{V_1^2}{2} + Z_1g + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2} + Z_2g + \frac{dW_x}{dm}$

The steady flow energy equations (S.F.E.F.)

Equations (5.7) and (5.8) are known as steady flow energy equations (S.F.E.E.) Equations (5.7) and (5.8) are known as single stream of fluid leaving the come a single stream of fluid entering and a single stream of fluid entering and a single stream of fluid leaving the come a single stream of huid entering and some h volume. All the terms in Equation (5.7) represent energy flow per unit time (1.7), whereas the terms in Equation (5.7) represent energy flow per unit time (1.7). The basis of energy flow per unit mass is usually more convenient when only a single stream of fluid enters and leaves a control volume. When more than one fluid stre is involved the basis of energy flow per unit time is more convenient.

Equation (5.8) can be written in the following form:

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ($m_{\rm CV}$ = constant). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it. For a garden hose nozzle in steady operation, for example, the amount of water entering the nozzle per unit time is equal to the amount of water leaving it per unit time.

When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, the mass flow rate in. The conservation of mass principle for a general steady-flow system with multiple inlets and outlets can be expressed in rate form as (Fig. 5–7)

Steady flow:
$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad (kg/s)$$
 (5–18)

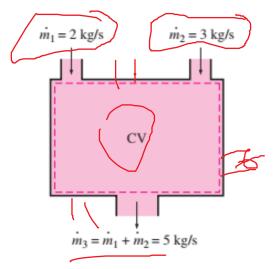


FIGURE 5-7

Conservation of mass principle for a two-inlet-one-outlet steady-flow system.

It states that the total rate of mass entering a control volume is equal to the total rate of mass leaving it.

Many engineering devices such as <u>nozzles</u>, <u>diffusers</u>, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet). For these cases, we denote the inlet state by the subscript <u>1</u> and the outlet state by the subscript <u>2</u>, and drop the summation signs. Then Eq. 5–18 reduces, for <u>single-stream steady-flow systems</u>, to

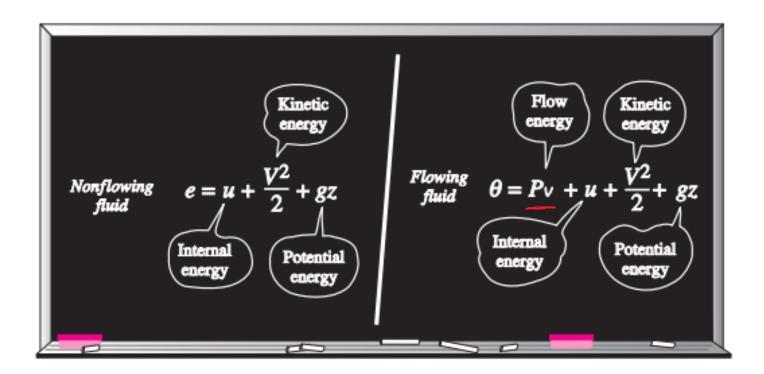
Steady flow (single stream):
$$\dot{m}_1 = \dot{m}_2 \rightarrow \left(\rho_1 V_1 A_1 = \rho_2 V_2 A_2\right)$$
 (5–19)

Total Energy of a Flowing Fluid

As we discussed in Chap. 2, the total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies (Fig. 5–14). On a unit-mass basis, it is expressed as

$$e = u + ke + pe = u + \frac{V^2}{2} + gz$$
 (kJ/kg) (5-25)

where V is the velocity and z is the elevation of the system relative to some external reference point.



Energy Transport by Mass

Noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass m is simply $m\theta$, provided that the properties of the mass m are uniform. Also, when a fluid stream with uniform properties is flowing at a mass flow rate of \dot{m} , the rate of energy flow with that stream is $\dot{m}\theta$ (Fig. 5–15). That is,

Amount of energy transport:
$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right)$$
 (kJ) (5–28)

Rate of energy transport:
$$\dot{E}_{\text{mass}} = \dot{m}\theta = h \left(h + \frac{V^2}{2} + gz\right)$$
 (kW) (5–29)

Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

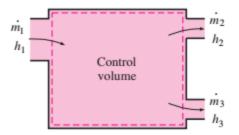


FIGURE 5-19

Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time). пош а зумет тетат сонмат читид а меацу-пом ргоссом.

The mass balance for a general steady-flow system was given in Sec. 5-1 as

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad \text{(kg/s)}$$

The mass balance for a single-stream (one-inlet and one-outlet) steady-flow system was given as

$$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$
 (5-32)

where the subscripts 1 and 2 denote the inlet and the exit states, respectively, ρ is density, V is the average flow velocity in the flow direction, and A is the cross-sectional area normal to flow direction.

During a steady-flow process, the total energy content of a control volume remains constant ($E_{\rm CV}$ = constant), and thus the change in the total energy of the control volume is zero ($\Delta E_{\rm CV}$ = 0). Therefore, the amount of energy entering a control volume in all forms (by heat, work, and mass) must be equal to the amount of energy leaving it. Then the rate form of the general energy balance reduces for a steady-flow process to

$$\underline{\dot{E}_{in} - \dot{E}_{out}} = \underbrace{dE_{system}/dt}_{0 \text{ (steady)}} = 0$$
Rate of net energy transfer by heat, work, and mass Rate of change in internal, kinetic, potential, etc., energies

Of

Energy balance:
$$\dot{E}_{in} = \dot{E}_{out}$$
 (kW) (5-34)

Rate of net energy transfer in by heat, work, and mass by heat, work, and mass

Noting that energy can be transferred by heat, work, and mass only, the energy balance in Eq. 5–34 for a general steady-flow system can also be written more explicitly as

$$\dot{Q}_{\rm in} + \dot{W}_{\rm in} + \sum_{\rm in} \dot{m}\underline{\theta} = \dot{Q}_{\rm out} + \dot{W}_{\rm out} + \sum_{\rm out} \dot{m}\underline{\theta}$$
 (5–35)

OF

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)$$
for each inlet
$$(5-36)$$

The energy balance relation just given is intuitive in nature and is easy to use when the magnitudes and directions of heat and work transfers are known. When performing a general analytical study or solving a problem that involves an unknown heat or work interaction, however, we need to assume a direction for the heat or work interactions. In such cases, it is common practice to assume heat to be transferred *into the system* (heat input) at a rate of \dot{Q} , and work produced by the system (work output) at a rate of \dot{W} , and then solve the problem. The first-law or energy balance relation in that case for a general steady-flow system becomes

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)$$
for each exit

for each inlet

(5–37)

Obtaining a negative quantity for \dot{Q} or \dot{W} simply means that the assumed direction is wrong and should be reversed. For single-stream devices, the steady-flow energy balance equation becomes

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$
 (5-38)

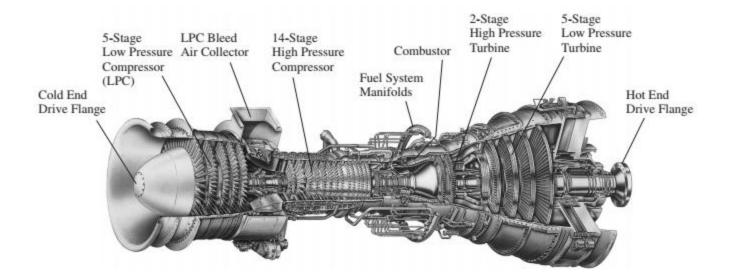
5-4 • SOME STEADY-FLOW ENGINEERING DEVICES

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance (Fig. 5–24). Therefore, these devices can be conveniently analyzed as steady-flow devices.

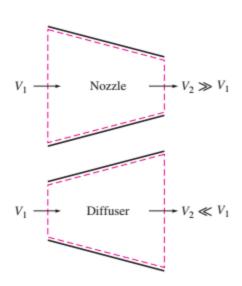
In this section, some common steady-flow devices are described, and the thermodynamic aspects of the flow through them are analyzed. The conservation of mass and the conservation of energy principles for these devices are illustrated with examples.

1 Nozzles and Diffusers

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses. A **nozzle** is a device that *increases the velocity* of a fluid at the expense of pressure. A **diffuser** is a device that *increases* the pressure of a fluid by slowing it down. That is, nozzles and diffusers perform opposite tasks. The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.



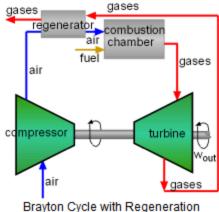
The rate of heat transfer between the fluid flowing through a nozzle or a diffuser and the surroundings is usually very small ($\dot{Q}\approx 0$) since the fluid has high velocities, and thus it does not spend enough time in the device for any significant heat transfer to take place. Nozzles and diffusers typically involve no work ($\dot{W}=0$) and any change in potential energy is negligible ($\Delta pe\cong 0$). But nozzles and diffusers usually involve very high velocities, and as a fluid passes through a nozzle or diffuser, it experiences large changes in its velocity (Fig. 5–25). Therefore, the kinetic energy changes must be accounted for in analyzing the flow through these devices ($\Delta ke\neq 0$).



2 Turbines and Compressors

In steam, gas, or hydroelectric power plants, the device that drives the electric generator is the turbine. As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as pumps and fans, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft. Therefore, compressors involve work inputs. Even though these three devices function similarly, they do differ in the tasks they perform. A fan increases the pressure of a gas slightly and is mainly used to mobilize a gas. A *compressor* is capable of compressing the gas to very high pressures. Pumps work very much like compressors except that they handle liquids instead of gases.



(a) An adjustable valve



(c) A capillary tube

FIGURE 5-29

Throttling valves are devices that cause large pressure drops in the fluid.

3 Throttling Valves

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid. Some familiar examples are ordinary adjustable valves, capillary tubes, and porous plugs (Fig. 5–29). Unlike turbines, they produce a pressure drop without involving any work. The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications. The magnitude of the temperature drop (or, sometimes, the temperature rise) during a throttling process is governed by a property called the *Joule-Thomson coefficient*, discussed in Chap. 12.

Throttling valves are usually small devices, and the flow through them may be assumed to be adiabatic ($q \cong 0$) since there is neither sufficient time nor large enough area for any effective heat transfer to take place. Also, there is no work done (w = 0), and the change in potential energy, if any, is very small ($\Delta pe \cong 0$). Even though the exit velocity is often considerably higher than the inlet velocity, in many cases, the increase in kinetic energy is insignificant ($\Delta ke \cong 0$). Then the conservation of energy equation for this single-stream steady-flow device reduces to

$$h_2 \cong h_1$$
 (kJ/kg) (5–41)

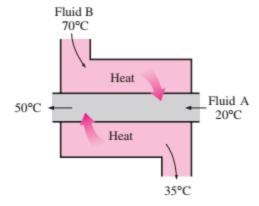


FIGURE 5-35

A heat exchanger can be as simple as two concentric pipes.

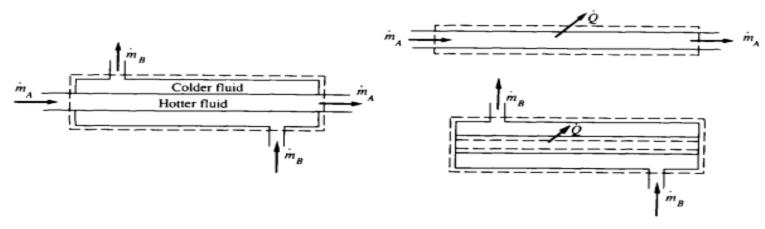
4b Heat Exchangers

As the name implies, **heat exchangers** are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.

The simplest form of a heat exchanger is a *double-tube* (also called *tube-and-shell*) *heat exchanger*, shown in Fig. 5–35. It is composed of two concentric pipes of different diameters. One fluid flows in the inner pipe, and the other in the annular space between the two pipes. Heat is transferred from the hot fluid to the cold one through the wall separating them. Sometimes the inner tube makes a couple of turns inside the shell to increase the heat transfer area, and thus the rate of heat transfer. The mixing chambers discussed earlier are sometimes classified as *direct-contact* heat exchangers.

The conservation of mass principle for a heat exchanger in steady operation requires that the sum of the inbound mass flow rates equal the sum of the outbound mass flow rates. This principle can also be expressed as follows: Under steady operation, the mass flow rate of each fluid stream flowing through a heat exchanger remains constant.

Heat exchangers typically involve no work interactions (w = 0) and negligible kinetic and potential energy changes ($\Delta ke \approx 0$, $\Delta pe \approx 0$) for each fluid stream. The heat transfer rate associated with heat exchangers depends on how the control volume is selected. Heat exchangers are intended for heat transfer between two fluids within the device, and the outer shell is usually well insulated to prevent any heat loss to the surrounding medium.



(b) Separated control volumes