CHAPTER THREE
WORK AND HEAT

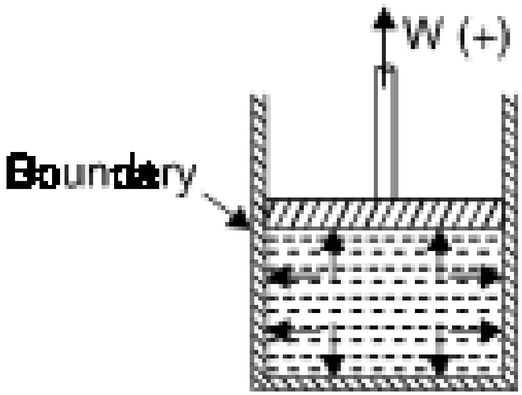
WORK

Work is said to be done when a *force moves through a distance*. If a part of the boundary of a system undergoes a displacement under the action of a pressure, the work done *W* is the product of the force (pressure × area), and the distance it moves in the direction of the force.

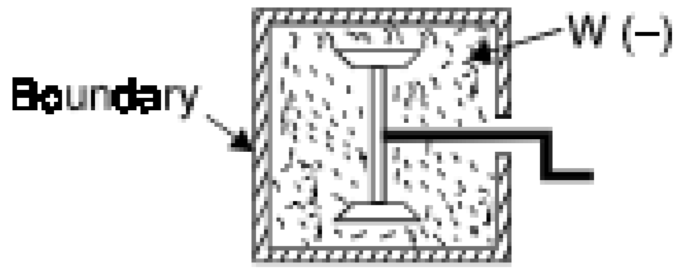
Fig. (*a*) illustrates this with the conventional piston and cylinder arrangement, the heavy line defining the   boundary of the system.

Fig. (*b*) illustrates another way in which work might be applied to a system.

A force is exerted by the paddle as it changes the momentum of the fluid, and since this force moves during rotation of the paddle work is done.



(a)



(b)

Work is a transient quantity which only appears at the boundary while a change of state is taking place within a system. Work is 'something' which appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.

Sign convention :

If the work is done *by* the system *on* the surroundings, *e.g.*, when a fluid expands pushing a piston outwards, the work is said to be *positive*.

Work output of the system = + W

If the work is done *on* the system *by* the surroundings, *e.g.*, when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be *negative*.

Work input to system $= -W$

HEAT

Heat (denoted by the symbol *Q*), may be, defined in an analogous way to work as follows : "*Heat is 'something' which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings*".

Heat, like work, is a transient quantity which only appears at the boundary while a change is taking place within the system.

It is apparent that neither δ*W* or δ*Q* are exact differentials and therefore any integration of the elemental quantities of work or heat which appear during a change from state 1 to state 2 must be written as

$$\int_1^2 \delta W = W_{1-2} \text{ or } {}_1W_2 \text{ (or } W\text{), and}$$

$$\int_1^2 \delta Q = Q_{1-2} \text{ or } {}_1Q_2 \text{ (or } Q\text{)}$$

Sign convention :

If the heat flows *into* a system *from* the surroundings, the quantity is said to be *positive* and, conversely, if heat flows *from* the system to the surroundings it is said to be *negative*.

In other words :

Heat received by the system = + Q
Heat rejected or given up by the system = – Q

Comparison of Work and Heat

Similarities :

(*i*) Both are *path functions and inexact differentials*.

(*ii*) Both are boundary phenomenon *i.e.,* both are recognized at the boundaries of the system as they cross them.

(*iii*) Both are associated with a process, not a state. Unlike properties, work or heat has no meaning at a state.

(*iv*) Systems possess energy, but not work or heat.

Dissimilarities :

(*i*) In heat transfer temperature difference is required.

(*ii*) In a stable system there cannot be work transfer, however, there is no restriction for the transfer of heat.

(*iii*) The sole effect external to the system could be reduced to rise of a weight but in the case of a heat transfer other effects are also observed.

QUASIEQUILIBRIUM WORK DUE TO A MOVING BOUNDARY

There are a number of work modes that occur in various engineering situations. These include the work needed to stretch a wire, to rotate a shaft, to move against friction, to cause a current to flow through a resistor, and to charge a capacitor. Many **of** these work modes are covered in other courses.

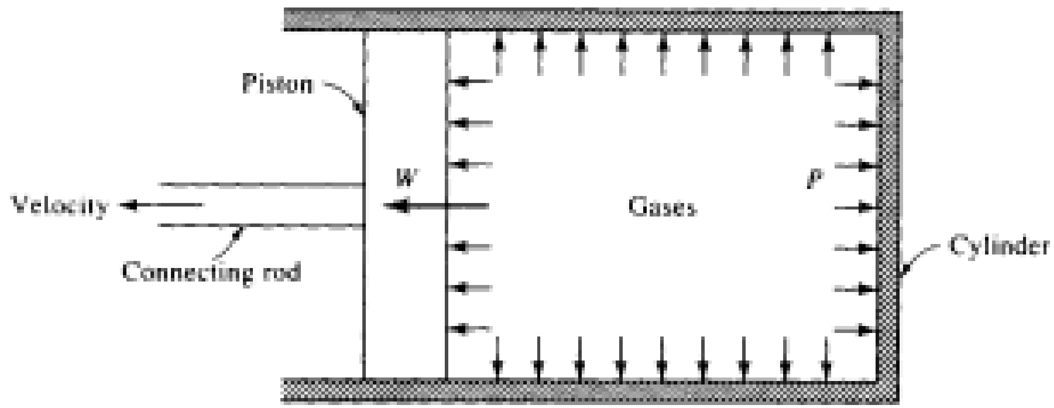
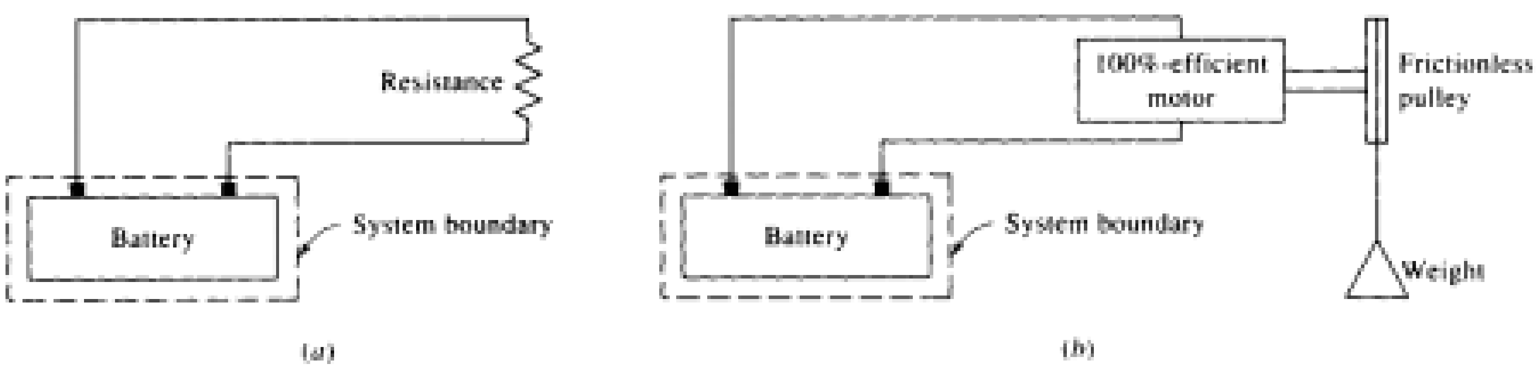


Fig. 3-1



Consider the piston-cylinder arrangement shown in Fig. 3-3. There is a seal to contain the gas in the cylinder, the pressure is uniform throughout the cylinder, and there are no gravity, magnetic, or electric effects. This assures us of a quasiequilibrium process, one in which the gas is assumed to pass through a series of equilibrium states. Now, allow an expansion of the gas to occur by moving the piston upward a small distance dl. The total force acting on the piston is the pressure times the area of the piston. This pressure is expressed as *absolute* pressure since pressure is a result of molecular activity; any molecular activity will yield a pressure which will result in work being done when the boundary moves. The infinitesimal work which the system (the gas) does on the surroundings (the piston) is then the force multiplied by the distance:

$$\delta W = PA\,dl \tag{3.2}$$

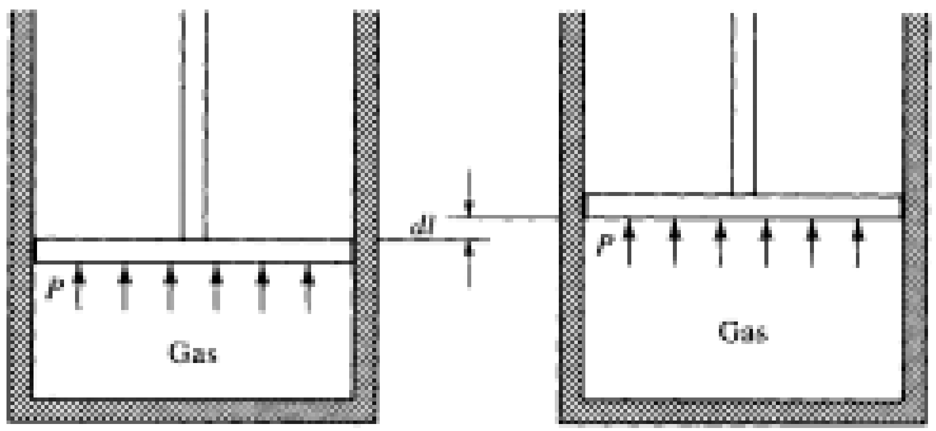


Fig. 3-3

The symbol δW will be discussed shortly. The quantity $A\,dl$ is simply dV, the differential volume, allowing (3.2) to be written in the form

$$\delta W = P\,dV \tag{3.3}$$

As the piston moves from some position l_1 to another position l_2, the above expression can be integrated to give

$$W_{1-2} = \int_{V_1}^{V_2} P\,dV \tag{3.4}$$

where we assume the pressure is known for each position as the piston moves from volume V_1 to volume V_2. Typical pressure-volume diagrams are shown in Fig. 3-4. The work W_{1-2} is the area under the P-V curve.

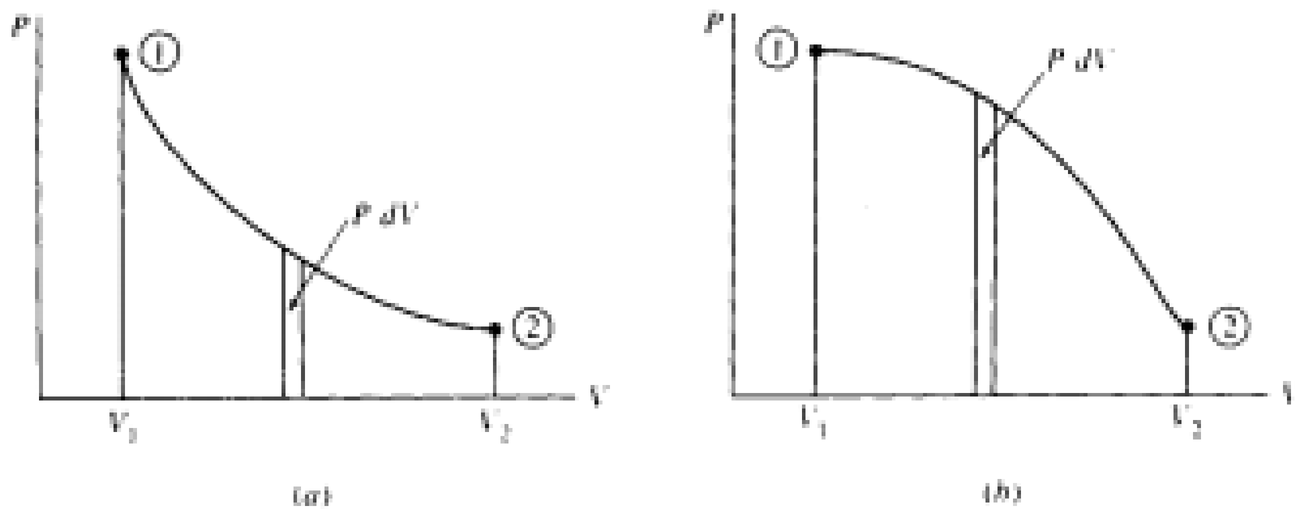


Fig. 3-4

2.20. REVERSIBLE WORK

Let us consider an ideal frictionless fluid contained in a cylinder above a piston as shown in Fig. 2.32. *Assume that the pressure and temperature of the fluid are uniform and that there is no friction between the piston and the cylinder walls.*

Let A = Cross-sectional area of the piston,

p = Pressure of the fluid at any instant,

$(p - dp)\, A$ = Restraining *force exerted* by the surroundings on the piston, and

dl = The distance moved by the piston under the action of the *force exerted*.

Then work done by the fluid on the piston is given by force times the distance moved,

i.e., Work done by the fluid

$$= (pA) \times dl = pdV$$

(where dV = a small increase in volume)

Or considering unit mass

Work done $= pdv$ (where v = specific volume)

This is only *true* when (*a*) the *process is frictionless and (b) the difference in pressure between the fluid and its surroundings during the process is infinitely small*. Hence when a reversible process takes place between state 1 and state 2, we have

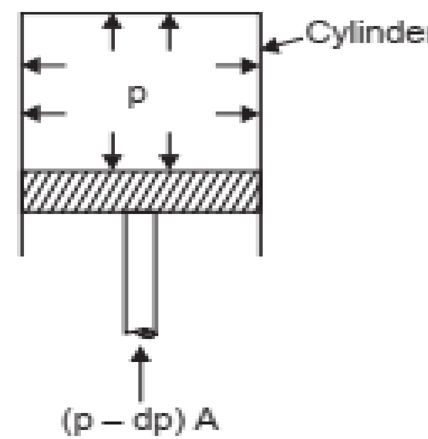


Fig. 2.32

$$\text{Work done by the unit mass of fluid} = \int_1^2 pdv \qquad \text{...(2.15)}$$

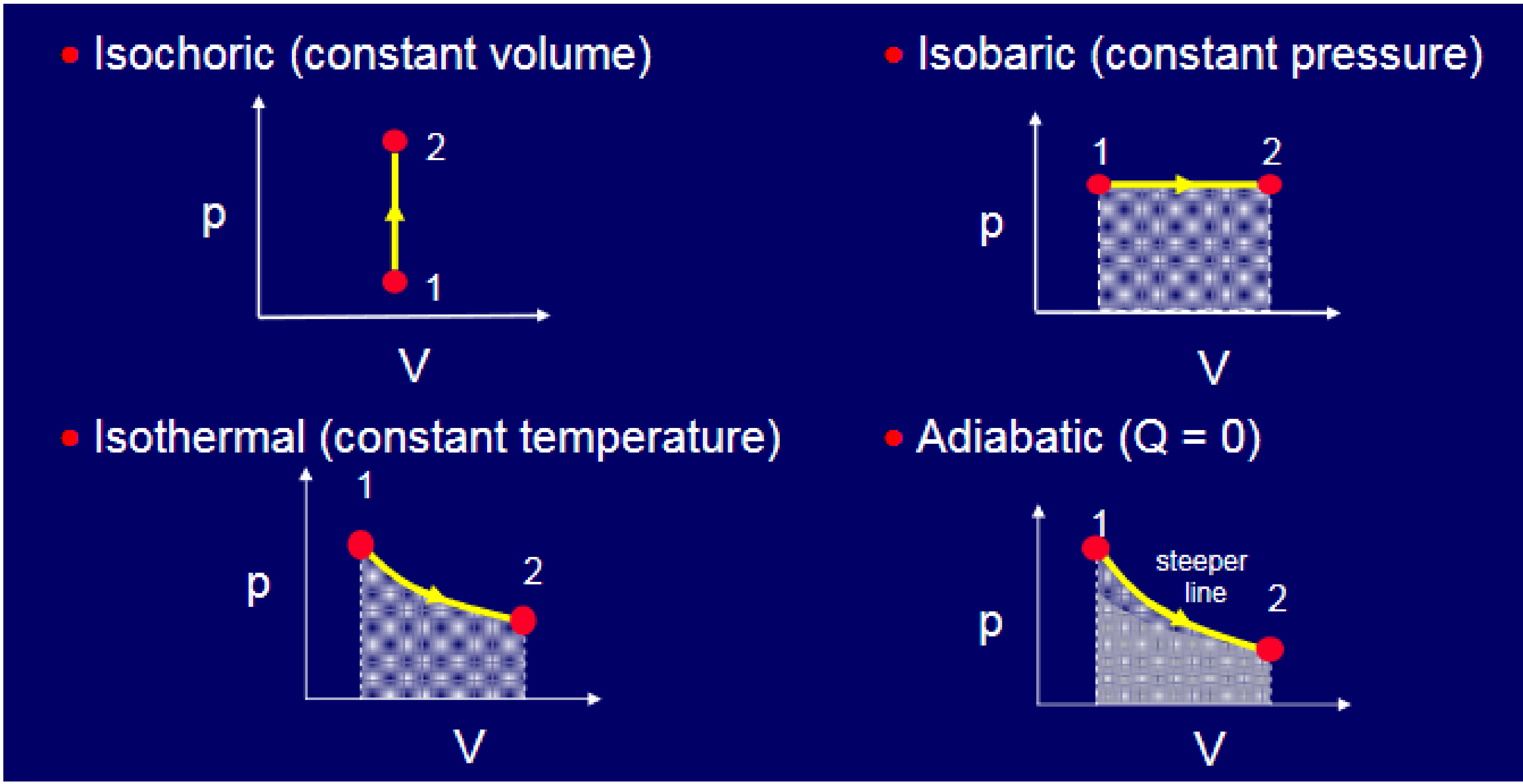
When a fluid undergoes a reversible process a series of state points can be joined up to form a line on a diagram of properties. The work done by the fluid during any reversible process is therefore given by the area under the line of process plotted on a p-v diagram (Fig. 2.32).

i.e., Work done = Shaded area on Fig. 2.33

$$= \int_1^2 pdv.$$

When p can be expressed in terms of v then the integral, $\int_1^2 pdv$, can be evaluated.

Work done for various quasi-static processes



3.2.2 *pdV*-Work in Various Quasi-Static Processes

1. Constant pressure process (Fig. 3.7) (isobaric or isopiestic process)

$$W_{1-2} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) \tag{3.4}$$

2. Constant volume process (Fig. 3.8) (isochoric process)

$$W_{1-2} = \int p\,dV = 0 \tag{3.5}$$

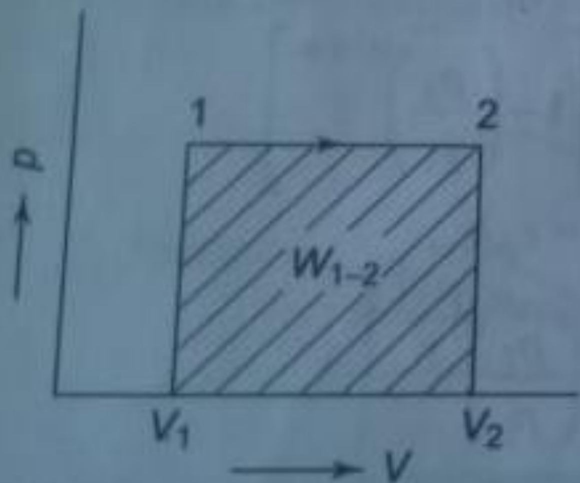


Fig. 3.7 *Constant pressure process*

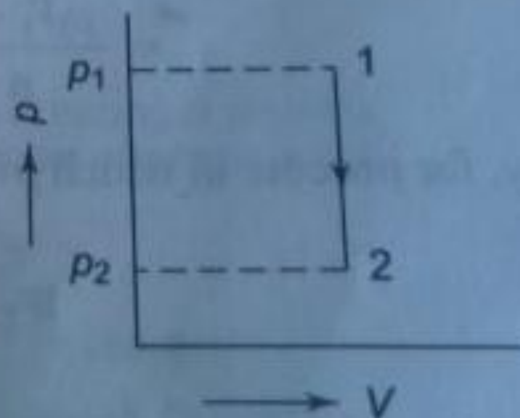


Fig. 3.8 *Constant volume process*

Work done for isothermal process

3. Process in which $pV = C$ (Fig. 3.9)

$$\therefore \quad W_{1-2} = \int_{V_1}^{V_2} p\,dV \qquad pV = p_1 V_1 = C$$

$$p = \frac{(p_1 V_1)}{V}$$

$$W_{1-2} = p_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} = p_1 V_1 \ln \frac{V_2}{V_1}$$

$$= p_1 V_1 \ln \frac{p_1}{\;}$$

Work Done for Polytropic Process

$${}_1W_2 = \int_1^2 p\,dV$$

$$= \int_1^2 \frac{C}{V^n}\,dV \quad \text{where } C = pV^n$$

$$= C\int_1^2 V^{-n}\,dV$$

$$= C\left[\frac{V^{-n+1}}{-n+1}\right]_1^2$$

$$\boxed{= \left[\frac{CV_2^{-n+1} - CV_1^{-n+1}}{-n+1}\right]_1^2}$$

$$= \left[\frac{p_2V_2^{\cancel{n}}V_2^{\cancel{-n}+1} - p_1V_1^{\cancel{n}}V_1^{\cancel{-n}+1}}{-n+1}\right] \quad \text{since } C = p_1V_1^n = p_2V_2^n$$

$${}_1W_2 = \left[\frac{p_2V_2 - p_1V_1}{-n+1}\right] = \frac{P_1V_1 - P_2V_2}{n-1}$$

P

$PV^n = C$ — (1) $= \dfrac{C}{V^n}$

$P_1V_1^n = C$ — 2

$P_2V_2^n = C$ — 3

$\dfrac{1}{V^n} = V^{-n}$

$= p_1V_1 \ln \frac{p_1}{p_2}$

4. Process in which $pV^n = C$, where n is a constant (Fig. 3.10).

$$pV^n = p_1V_1^n = p_2V_2^n = C$$

$$p = \frac{(p_1V_1^n)}{V^n}$$

∴ $$W_{1-2} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{p_1V_1^n}{V^n}\,dV = (p_1V_1^n)\left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

∴

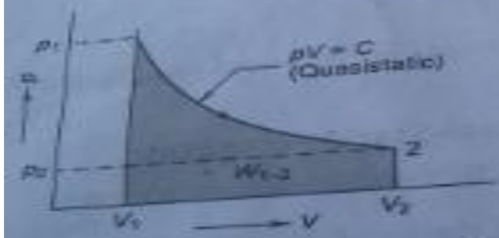


Fig. 3.9 Process in which pV = Constant

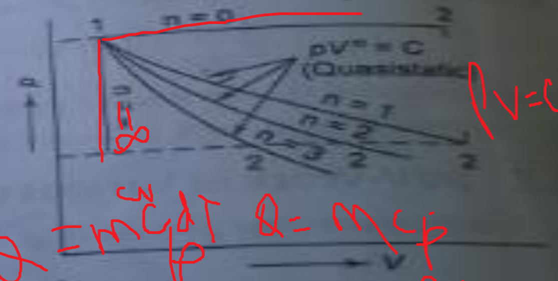


Fig. 3.10 Process in which pV^n = Constant

$Q = mc_v dT$ $\quad Q = mc_p$

$\gamma = \frac{c_p}{c_v}$

PV^n

$$= \frac{p_1V_1^n}{1-n}(V_2^{1-n} - V_1^{1-n}) = \frac{p_2V_2^n \times V_2^{1-n} - p_1V_1^n \times V_1^{1-n}}{1-n}$$

$$= \frac{p_1V_1 - p_2V_2}{n-1} = \frac{p_1V_1}{n-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{n-1/n}\right]$$

y, for process in which $pv^\gamma = c$, where $\gamma = c_p/c_v$

γ

$$W_{1-2} = \frac{p_1V_1}{n-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

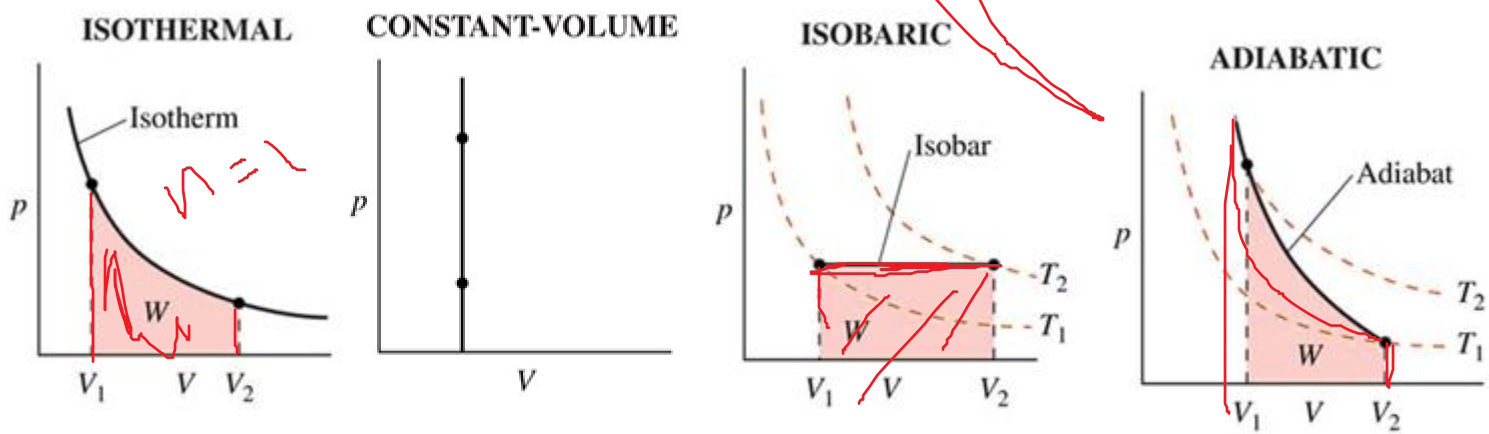
$PV^\gamma = c$

$PV^\gamma = c$ $\quad \gamma = c_p/c_v$

Ideal Gas Processes

$W = pV \ln \frac{V_2}{V_1}$

$PV = nRT$



	ISOTHERMAL	CONSTANT-VOLUME	ISOBARIC	ADIABATIC
Defining characteristic	T = constant	V = constant	p = constant	$Q = 0$
First law	$Q = W$	$Q = \Delta U$	$Q = \Delta U + W$	$\Delta U = -W$
Work done by gas	$W = nRT \ln\left(\frac{V_2}{V_1}\right)$	$W = 0$	$W = p(V_2 - V_1)$	$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$
Other relationships	pV = constant	$Q = nC_V\Delta T$	$Q = nC_p\Delta T$ $C_p = C_V + R$	pV^γ = constant $TV^{\gamma-1}$ = constant

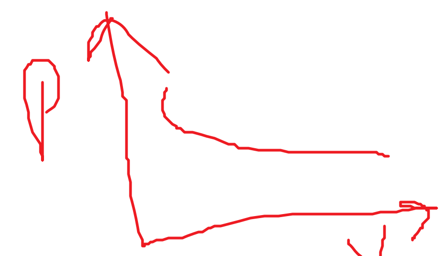
1.4

Q

$TV^{\gamma-1}$

n =

$pV^{n=1} = 2.8$



Problem 1.36. *The properties of a closed system change following the relation between pressure and volume as*

$$pv = 2.8 \text{ ; where } p \text{ is in bar and } v \text{ is in } m^3.$$

Determine the work done when the pressure increases from 1.4 bar to 7 bar.

$P_1 = 1.4$ N/m^2 P_2

$PV = 2.8$

Solution. The work done during the process is given by

$$W = \int_{v_1}^{v_2} p \, . dv$$

ln

$$v_1 = \frac{2.8}{p_1} = \frac{2.8}{1.4} = 2 \text{ ; and } v_2 = \frac{2.8}{p_2} = \frac{2.8}{7} = 0.4$$

$$W = 10^5 \int_2^{0.4} \frac{2.8}{v} \, dv \text{ Nm} = 10^5 \times 2.8 \, [\log_e v]_2^{0.4}$$

$$= - 10^5 \times 2.8 \, [\log_e 2 - \log_e 0.4] = - 2.8 \times 10^5 \log_e (5)$$

$$= - 2.8 \times 1.61 \times 10^5 \text{ Nm} = - 4.5 \times 10^5 \text{ Nm}$$

$$= - 4.5 \times 10^5 \text{ J} = \mathbf{- 450 \text{ kJ}}.$$

-450 kJ

The –ve sign indicates that the work is done on the system.

Example 18.4. Finding the Work

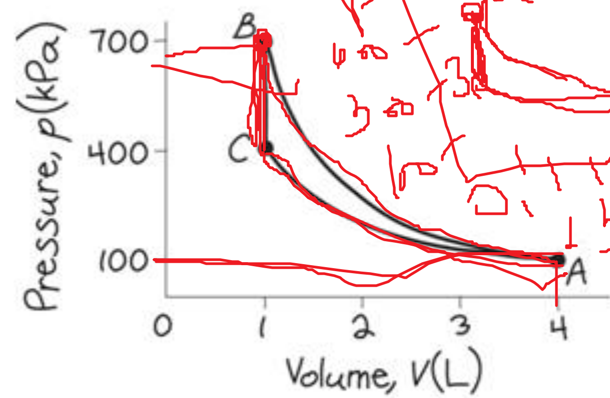
An ideal gas with γ = 1.4 occupies 4.0 L at 300 K & 100 kPa pressure.

It's compressed adiabatically to ¼ of original volume,

then cooled at constant V back to 300 K,

& finally allowed to expand isothermally to its original V.

How much work is done on the gas?



sol^n =

AB (adiabatic): $$W_{AB} = \frac{p_A V_A - p_B V_B}{\gamma - 1}$$ $$p_B = p_A \left(\frac{V_A}{V_B} \right)^{\gamma}$$

$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$W_{AB} = \frac{p_A V_A}{\gamma - 1} \left(1 - \left(\frac{V_A}{V_B} \right)^{\gamma - 1} \right) = \frac{(100\ kPa)(4.0\ L)\left(1 - 4^{1.4-1}\right)}{1.4 - 1} = -741\ J$$

$W_{1-2} = -741\ J$

BC (isometric): $$W_{BC} = 0$$

$W_{2-3} = 0$

CA (isothermal): $$W_{CA} = n\,R\,T \ln \frac{V_A}{V_C} = p_A V_A \ln 4 = 555\ J$$

$W_{3-1} = P_1 V_1 \ln \frac{V_1}{V_3}$

work done by gas: $$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = -186\ J$$

Problem 1.37. *The work supplied to a closed system is 160 kJ. The initial volume is $v_1 = 0.80\ m^3$ and the pressure of the system changes as*

$$p = 7 - 3v \text{ ; where } p \text{ is in bar and } v \text{ is in } m^3.$$

Determine the final volume and pressure of the system.

Solution. The work done during the process is given by

$$W = \int_{v_1}^{v_2} p\,.\,dv = 10^5 \int_{0.85}^{v_2} (7 - 3v)\,.\,dv \text{ Nm (as 1 bar} = 10^5 \text{ N/m}^2)$$

$$= 10^5 \left[7v - 3\frac{v^2}{2}\right]_{0.8}^{v_2} = 10^5 \left[7(v_2 - 0.8) - 1.5\,(v_2^2 - 0.64)\right]$$

$$= 10^5 \left[7v_2 - 1.5v_2^2 - 4.64\right] \text{ Nm or J.}$$

This work should be equated to the work supplied with –ve sign as it is the work supplied to the system.

$$\therefore \quad -160 \times 10^3 = 10^5 \left[7v_2 - 1.5v_2^2 - 4.64\right] \text{ (as 1 kJ} = 10^3 \text{ J)}$$

$$\therefore \quad 1.5\,v_2^2 - 7\,v_2 + 3.04 = 0$$

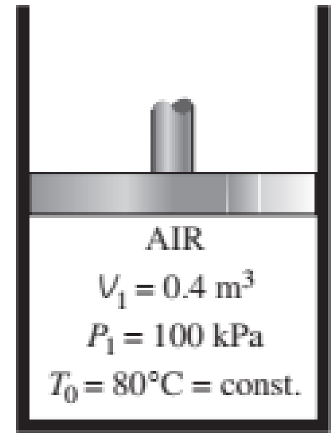
$$\therefore \quad v_2 = \frac{7 \pm \sqrt{49 - 4 \times 1.5 \times 3.04}}{2 \times 1.5} = \frac{7 \pm 5.57}{3} = \frac{7 - 5.57}{3} = 0.477 \text{ m}^3.$$

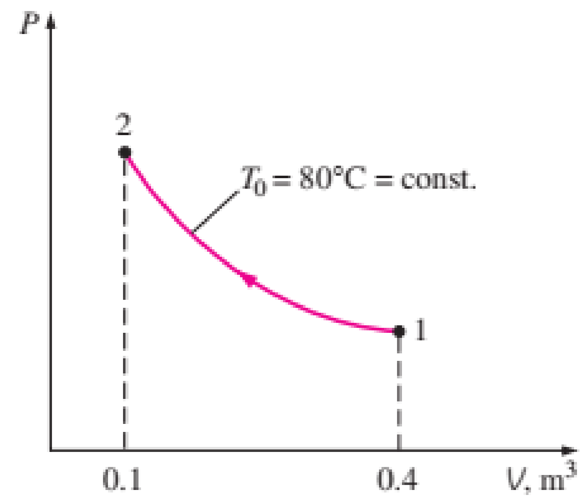
+ ve sign is incompatible with the present problem therefore it is not considered.

$$p_2 = 7 - 3 \times 0.477 = 5.56 \text{ bar} = \mathbf{5.56 \times 10^5\ N/m^2 \text{ or } Pa} = 5.56 \text{ bar}$$

EXAMPLE 4–3 Isothermal Compression of an Ideal Gas

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.





Solution Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.

Analysis A sketch of the system and the *P-V* diagram of the process are shown in Fig. 4–8.

Assumptions **1** The compression process is quasi-equilibrium. **2** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

Analysis For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

where C is a constant. Substituting this into Eq. 4–2, we have

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV = C\int_1^2 \frac{dV}{V} = C\ \ln\frac{V_2}{V_1} = P_1V_1\ \ln\frac{V_2}{V_1} \tag{4–7}$$

In Eq. 4–7, P_1V_1 can be replaced by P_2V_2 or mRT_0. Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1V_1 = P_2V_2$.

Substituting the numerical values into Eq. 4–7 yields

$$W_b = (100\ \mathrm{kPa})(0.4\ \mathrm{m}^3)\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\ \mathrm{kJ}}{1\ \mathrm{kPa\cdot m^3}}\right)$$

$$= -55.5\ \mathrm{kJ}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.