chapter-3

(22-23) marks

Single Phase AC Circuit Analysis

Comparison of AC and DC

imp for short note

1) A.C
- → Voltage and current change periodically.
- → Low cost production.
- → A.C voltage can be lowered or raised by using capacitor or choke.
- → AC can be converted into D.C. by using converter.
- → Generally frequency 50Hz.
- → more dangerous than DC.

2) D.C.
- → Voltage and current remain constant.
- → Higher cost production.
- → D.C. current can be decreased by using the resistance.
- → DC can be converted into AC by inverter
- → frequency is zero '0'.
- → It is less dangerous than AC.

1) # AC Voltage
- An alternating voltage is any voltage that varies both in magnitude and polarity with respect to time.

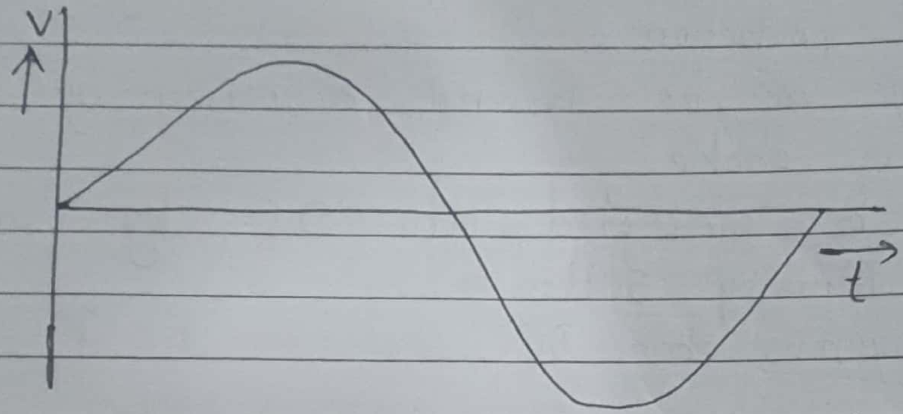
2) AC Current

Any current that varies both in magnitude & direction w.r.t time.

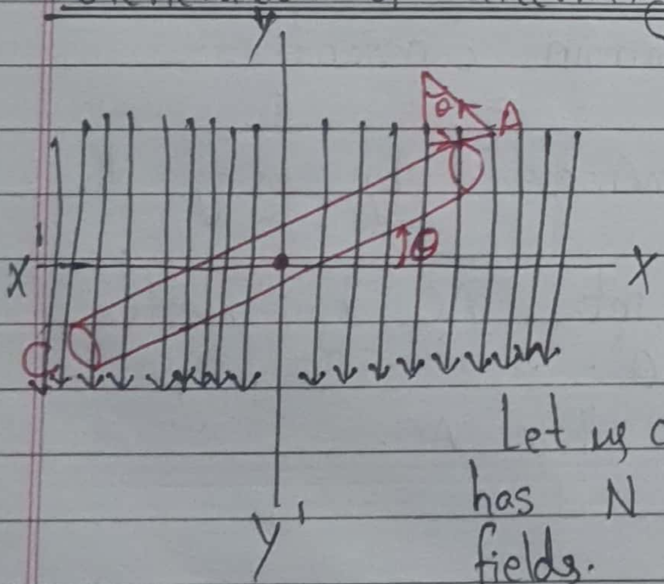
Eqn of alternating voltage & current

Sinusoidal Waveform:

The waveform traced out in graph of V versus t has a shape termed sinusoidal so, the wave is referred as sine - wave.



Generation of alternating voltage and current :-



Let us consider rectangular coil AC which has N turn's and rotating in magnetic fields.

Let the angle be measured from Horizontal position.

where,

B = flux density (Wb/m^2)

l = length of the coil (m)

v = linear velocity (m/sec)

The two sides A and C move parallel to line of magnetic flux. So, no flux cut the line,

~~that~~ hence, no emf is generated in the line when the coil is turn with angle θ, then its velocity is resolved into two parts.

(i) $V \cos\theta$ component $\Rightarrow$ parallel to direction of the magnetic flux.

(ii) $v \sin\theta$ component $\Rightarrow$ perpendicular to direction of the magnetic flux.

(iii) The emf is generated ~~due~~ entirely due to the perpendicular component i.e. $v \sin\theta$.

Now,

emf generated in one side of coil which contains N conductor,

$$e = N \times Bl v \sin\theta \quad \text{—— (i)}$$

Total emf generated in both sides of coil is

$$e = 2NBlv \sin\theta \quad \text{—— (ii)}$$

$e = E_m$ [when $E_m = e$ and $\theta = 90°$]

$$E_m = 2NBl v \sin 90°$$

$$\therefore \boxed{E_m = 2NBlv \text{ (Volts)}}$$

From eq^n (ii),

$$\boxed{e = E_m \sin\theta}$$

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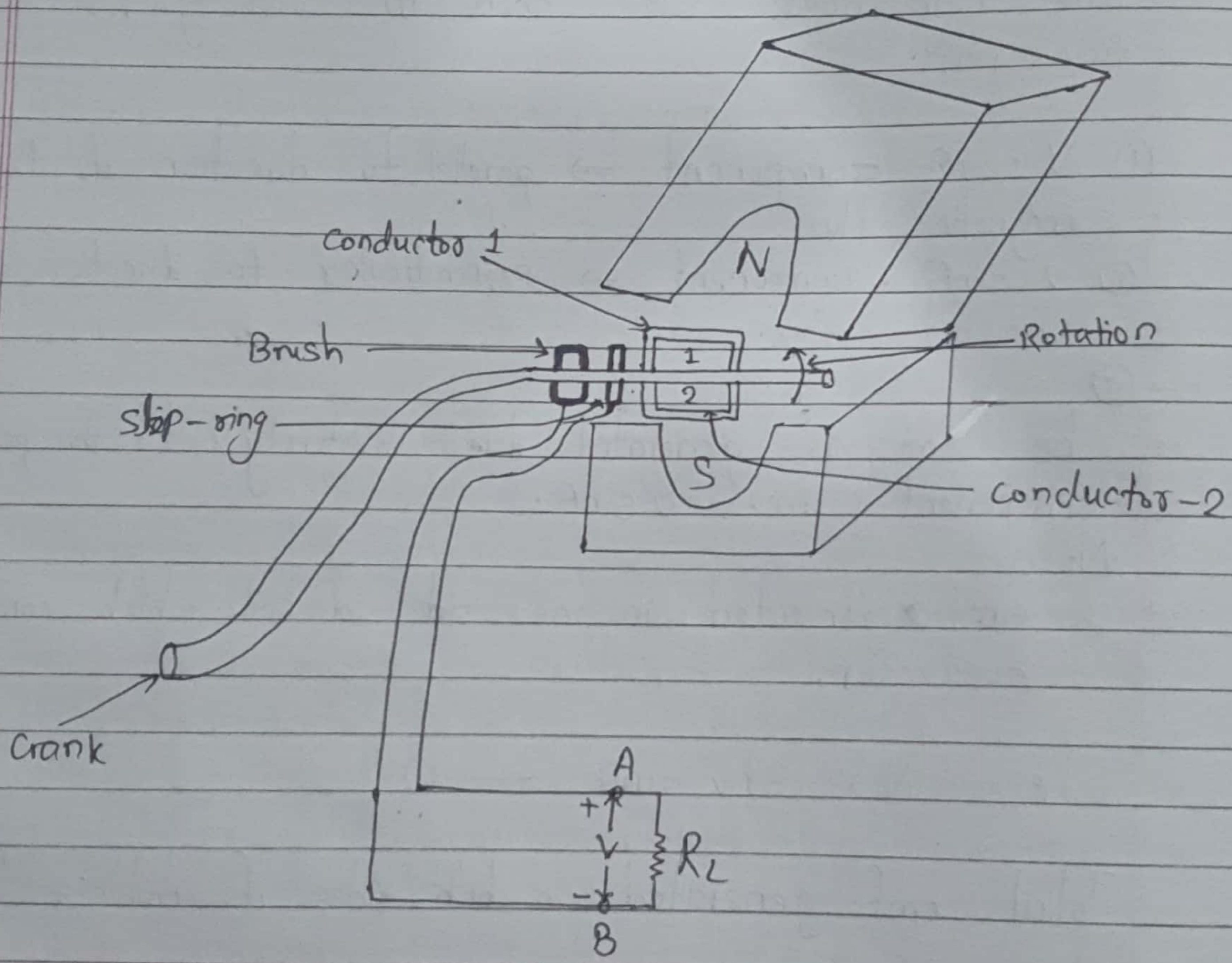
Generation of AC Voltage :



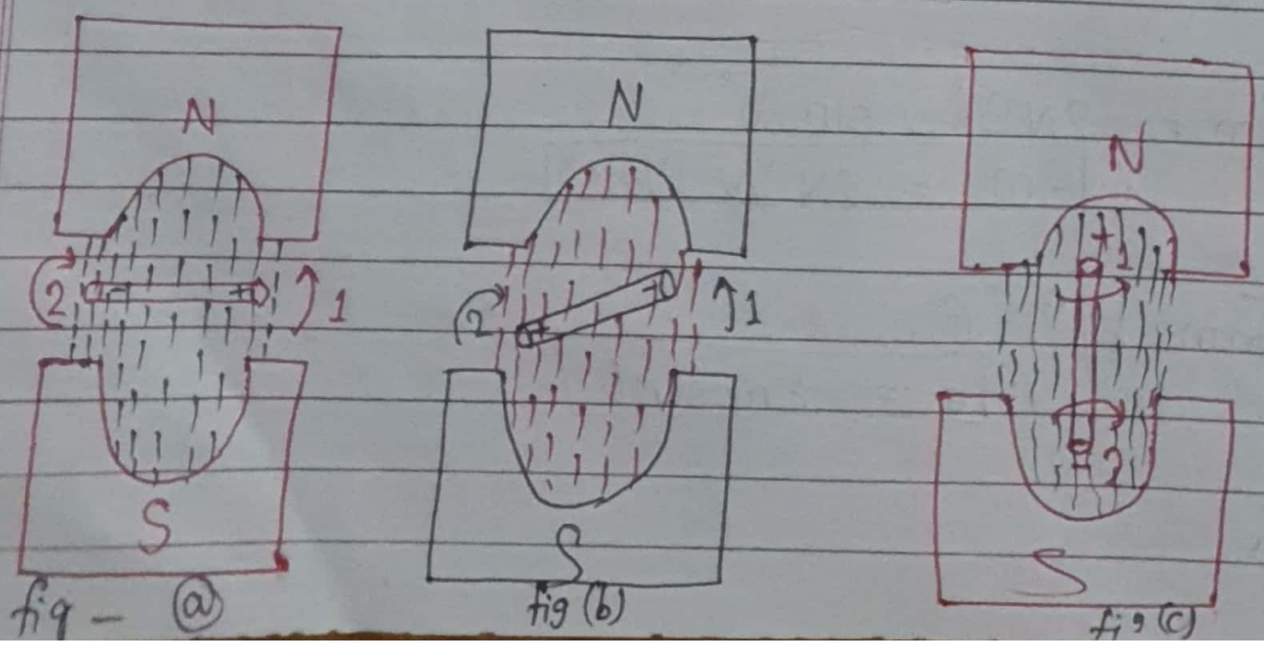
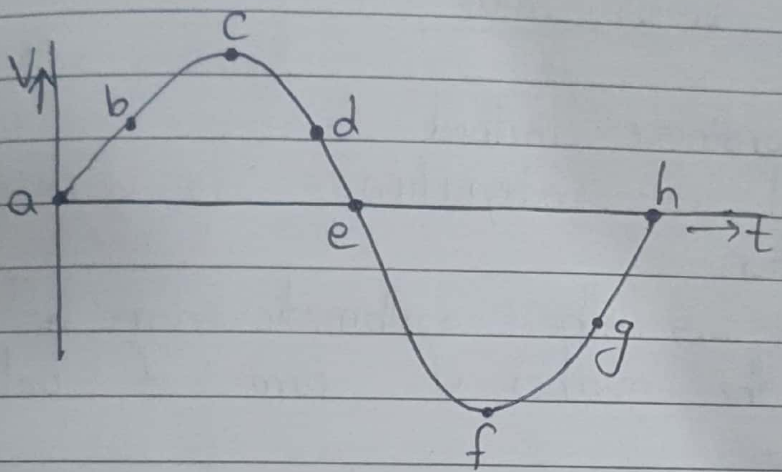


fig (d)



2014 PU

Sinusoidal function termonology:

√√ Imp 3 marks

1) Wave form.
2) Instantaneous value
3) Amplitude or peak value
4) Peak to peak value.
5) frequency
6) Phase ~~angle~~
7) Phase angle

Waveform

→ path traced by a quantity.

- Average value or mean value
- R.m.s. value or effective value of virtual value
  ↳ root mean square value

1) Average value or mean value

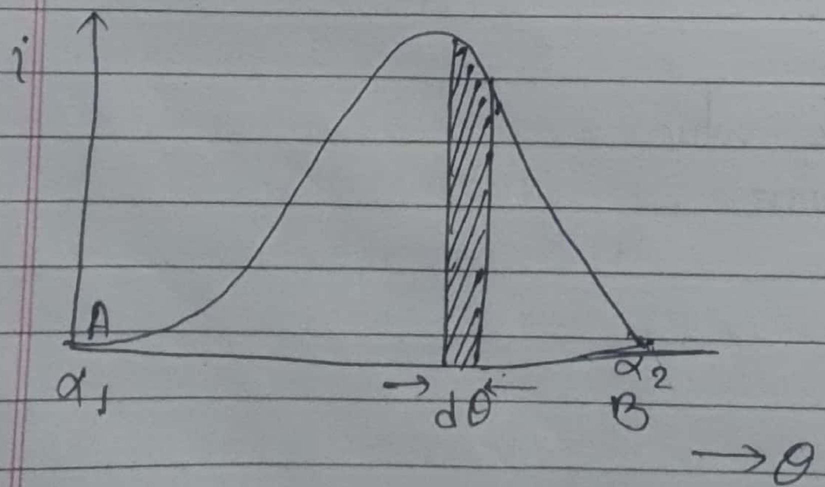
formula is req^d

a) Mid-ordinate method
b) Method of Integration

* Method of Integration

Average value is the arithmetic sum of all the value divided by the number of sum of values.

The average value of any cycle of waveform is area under the wave divided by the time period (Length of cycle).



Area of the strips $= i\,d\theta$

The area under the curve and horizontal axis will be the sum of all such strip.

Curve limits from

$\theta = \alpha_1$ to $\theta = \alpha_2$

The length of base of curve $= \alpha_2 - \alpha_1$

Total area of curve $= \int_{\alpha_1}^{\alpha_2} i\,d\theta$

The average height of the curve $= \dfrac{\text{Area under the curve}}{\text{Base length of the curve}}$

$$= \frac{1}{\alpha_2 - \alpha_1} \int_{\alpha_1}^{\alpha_2} i\,d\theta$$

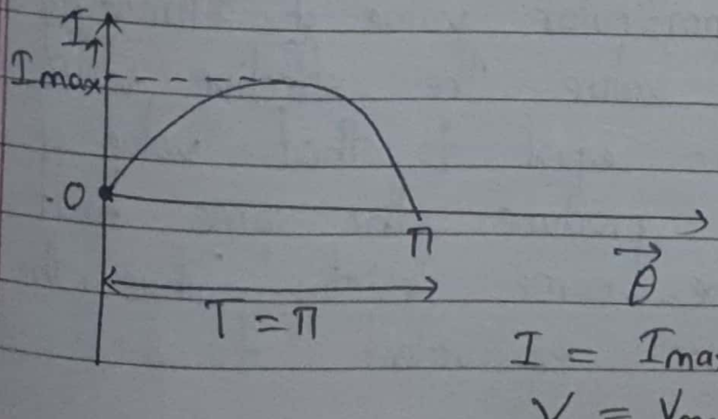
The average value of alternating current i over period T is given by

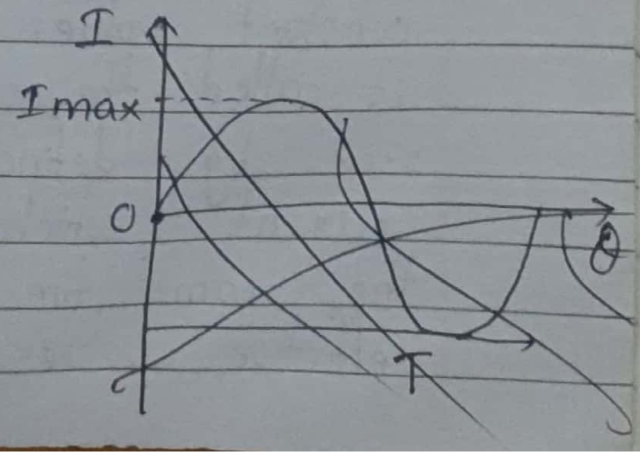
$$I_{av} = \frac{1}{T}\int_0^T i\,d\theta \quad \rightarrow \text{in terms of current}$$

$$V_{av} = \frac{1}{T}\int_0^T v\,d\theta \quad \rightarrow \text{in terms of voltage.}$$

$$P_{av} = \frac{1}{T}\int_0^T p\,d\theta \quad \rightarrow \text{in terms of power}$$

- Average Value of half-wave Sinusoidal wave form:-





$I = I_{max} \cdot \sin\theta$

$V = V_{max} \cdot \sin\theta$

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_{max} \sin\theta\, d\theta$$

$$= \frac{I_{max}}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{I_{max}}{\pi}\Big[-\cos\theta\Big]_0^{\pi}$$

$$= \frac{I_{max}}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{I_{max}}{\pi}\left[-(-1) + 1\right] = \frac{I_{max}}{\pi}\left[1+1\right]$$

$$= \frac{2\, I_{max}}{\pi}$$

$$\therefore \boxed{I_{avg} = 0.637\, I_{max}}$$

Effective or R.M.S. Value or Virtual Value:

formula

Root Mean Square Value

An alternating current varies from instant to instant if, it is necessary to determine an equivalent direct current that will be produced the 'same heat in the same interval of time, in the same resistor as a alternating currents. In other words, we measure the a.c. in terms of d.c. which has constant value. This particular value of alternating current is called the effective value. The effective value of a.c. by defination "is equal to that value of direct current which will produce the same heat in the same time in the same resistor." Thus, the effective value is d.c. equivalent to a.c.

Two methods:

(i) Mid - ordinate method
(ii) By Integration method

(ii) By Integration method

Heat produced by an alternating current of Instantaneous value i in resistor R in time dt is $= i^2 R\,dt$.

Total heat produced in one cycle [i.e. in time T] is given by

$$H_{a.c.} = \int_0^T i^2 R\,dt$$

Heat produced by an equivalent direct - current I in resistor R in time T is given:

$$H_{d.c.} = H_{a.c.}$$

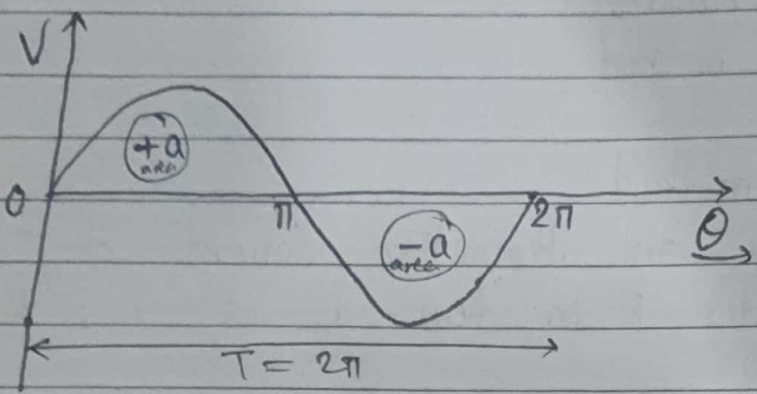
or, $$I^2 RT = \int_0^T i^2 R\,dt$$

$$\therefore I^2 = \frac{1}{T}\int_0^T i^2\,dt$$

$$\boxed{\therefore I = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}}$$

$$\boxed{V = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}}$$ in terms of voltage.

* Average Value of a Sinusoidal wave form:



$$\text{Avg. value} = \frac{+a + (-a)}{2\pi} = 0$$

By Integration Method:

$$\text{Avg. value} = \frac{1}{T}\int_0^{T} v\,d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_{max}\sin\theta\,d\theta \qquad [\because V = V_{max}\sin\theta]$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_{max}\cdot\sin\theta\,d\theta$$

$$= \frac{V_{max}}{2\pi}\left[\cancel{\sin\theta}\ -\cos\theta\right]_0^{2\pi}$$

$$= \frac{V_{max}}{2\pi} \times 0 \qquad [\because \cos 2\pi = 0]$$

$\therefore$ Avg. value $= 0$

R.M.S. value of Sinusoidal Wave form:-

$$\cancel{V}^2 = \frac{1}{T}\int_0^T v^2\, d\theta$$

$$\Rightarrow \cancel{V} = \sqrt{\frac{1}{T}\int_0^T v^2\, d\theta}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_{max}^2 \cdot \sin^2\theta\, d\theta}$$

$$V_{rms} = \sqrt{\frac{V_{max}^2}{4\pi}\int_0^{2\pi} (1-\cos 2\theta)\, d\theta}$$

$$V_{rms} = \sqrt{\frac{V_{max}^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$V_{rms} = \frac{V_{max}}{2}\sqrt{\frac{1}{\pi}\left[2\pi - \frac{\sin 4\pi}{2}\right]}$$

$$V_{rms} = \frac{V_{max}}{2}\sqrt{\frac{1}{\cancel{\pi}} \times 2\cancel{\pi}}$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$\therefore$ $V_{rms} = 0.707\, V_{max}$ (in terms of voltage)

Also,

$I_{rms} = 0.707\, I_{max}$ (in terms of current)

Form factor (K_f)

The ratio of R.M.S. value to average value.

$$K_f = \frac{\text{R.M.S. value}}{\text{Avg. value}}$$

$$= \frac{0.707\ \cancel{V_{max}}}{0.637\ \cancel{V_{max}}}$$

$$\therefore K_f = 1.11$$

Peak factor (K_p) / Amplitude factor / Crest factor:

It is defined as the ratio of maximum value to R.M.S. value.

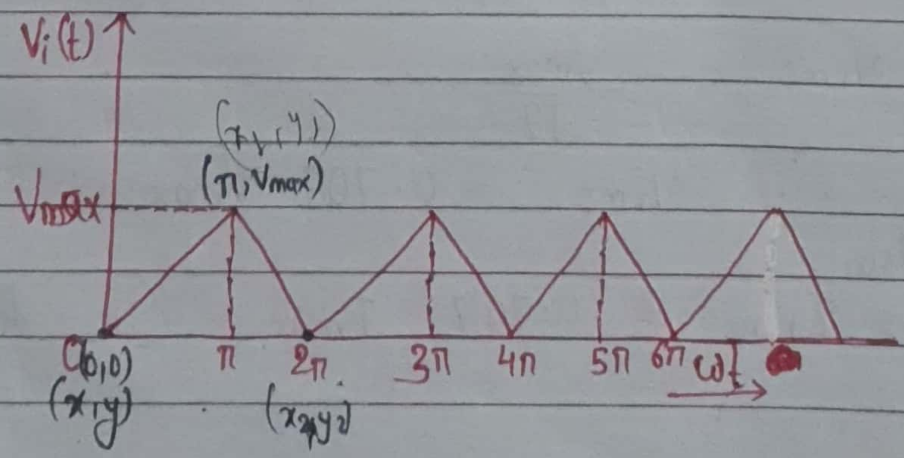
$$K_p = \frac{\text{max}^m \text{ value (Peak value)}}{\text{R.M.S. value}}$$

$$= \frac{\cancel{\text{max}^m \text{ value}}}{0.707 \cdot \cancel{\text{max}^m \text{ value}}}$$

$$\therefore K_p = 1.414$$

Problems

1) Q. Find the avg. value or R.M.S. value of given waveform.



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$$\boxed{\begin{aligned} V_{avg} &= \frac{1}{T}\int_0^T v\, d(\omega t) \\ V_{rms} &= \sqrt{\frac{1}{T}\int_0^T v^2\, d(\omega t)} \end{aligned}}$$

Solution :-

$$V_{avg} = \frac{1}{T}\int_0^T v \cdot d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} \frac{V_{max}(\omega t)}{\pi} \cdot d(\omega t) + \int_{\pi}^{2\pi}\left(\frac{-V_{max}\cdot \omega t}{\pi} + 2V_{max}\right) d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[\frac{V_{max}\cdot \omega^2}{\pi}\int_0^{\pi} t\,dt - \frac{V_{max}\cdot \omega^2}{\pi}\int_{\pi}^{2\pi} t\,dt + 2V_{max}\cdot \omega \int_{\pi}^{2\pi} dt\right]$$

$$= \frac{1}{2\pi} \cdot V_{max}$$

Rough

$x_1 = 0$, $y_1 = 0$

$x_2 = \pi$, $y_2 = V_{max}$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

on $y = \frac{V_{max}}{\pi} x$

$$\Rightarrow \boxed{V_i(t) = \frac{V_{max}}{\pi}(\omega t)}$$

Again,

$x_1 = \pi$, $y_1 = V_{max}$

$x_2 = 2\pi$, $y_2 = 0$

So,

$$y - V_{max} = -\frac{V_{max}}{\pi}(x - \pi)$$

$$\Rightarrow V_i(t) - V_{max} = -\frac{V_{max}}{\pi}(\omega t - \pi)$$

$$\Rightarrow V_i(t) = -\frac{V_{max}\cdot \omega t}{\pi} + \frac{V_{max}\cancel{\pi}}{\cancel{\pi}} + V_{max}$$

$$\Rightarrow \boxed{V_i(t) = -\frac{V_{max}\cdot \omega t}{\pi} + 2V_{max}}$$

$$= V_{max}\left[\frac{-\omega t}{\pi} + 2\right]$$

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$$V_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} \frac{V_{max}(\omega t)}{\pi} \cdot d(\omega t) + \int_{\pi}^{2\pi} \frac{-V_{max}(\omega t - 2\pi)}{\pi} d(\omega t)\right]$$

$$= \frac{V_{max}}{2\pi^2}\left[\int_0^{\pi} \omega t \cdot d(\omega t) - \int_{\pi}^{2\pi} (\omega t - 2\pi)\, d(\omega t)\right]$$

$$= \frac{V_{max}}{2\pi^2}\left[\left[\frac{(\omega t)^2}{2}\right]_0^{\pi} - \left[\frac{(\omega t)^2}{2}\right]_{\pi}^{2\pi} + 2\pi\left[\omega t\right]_{\pi}^{2\pi}\right]$$

$$= \frac{V_{max}}{2\pi^2}\left[\left[\frac{\pi^2}{2} - \frac{0}{2}\right] - \left[\frac{4\pi^2}{2} + \frac{\pi^2}{2} + 2\pi(2\pi - \pi)\right]\right]$$

$$= \frac{V_{max}}{2\pi^2}\left[-\frac{2\pi^2}{2} + 2\pi^2\right]$$

$$= \frac{V_{max}}{2}$$

Avg. value of the a.c. voltage is given $= \frac{V_{max}}{2}$

Now,

RMS Value for the given a.c. waveform:

$$V_{rms} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_i^2(t) \cdot d(\omega t) + \int_{\pi}^{2\pi} V_i(t)^2 \cdot d(\omega t)\right.}$$

$$= \sqrt{\frac{1}{2\pi}\left[\frac{V_{max}^2}{\pi^2}\left\{\left[\frac{\omega t}{3}\right]^3\right\}_0^{\pi} + \frac{1}{2\pi} \cdot \frac{-V_{max}^2}{\pi^2}\int_{\pi}^{2\pi} (\omega t)^2 - 2\omega t \cdot 2\pi + 4\pi^2\right]}$$

$$= \frac{V_{max}}{\pi}\sqrt{\frac{1}{2\pi}\left\{\frac{\pi^3}{3} - 0 - \left[\frac{(\omega t)^3}{3} - \frac{4\pi(\omega t)^2}{2} + 4\pi^2\omega t\right]_{\pi}^{2\pi}\right\}}$$

$$= \frac{V_{max}}{\pi}\sqrt{\frac{\pi^2}{3}} = \frac{V_{max}}{\sqrt{3}}$$

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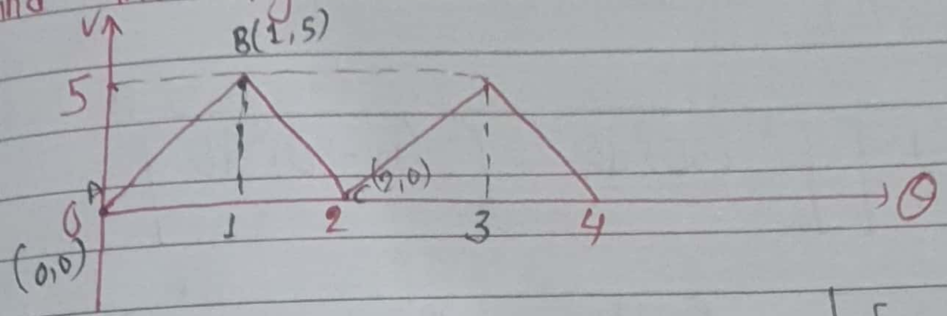
2. Q. Find the Avg. value and RMS. Value :



So,

$$V_{avg} = \frac{1}{T}\int_0^T v\, d\theta$$

$$= \frac{1}{2}\left[\int_0^1 5\theta\, d\theta + \int_1^2 (-5\theta+10)\, d\theta\right]$$

$$= \frac{1}{2}\left[5\left[\frac{\theta^2}{2}\right]_0^1 - 5\left(\frac{\theta^2}{2}\right)_1^2 + 10[\theta]_1^2\right]$$

$$= \frac{1}{2}\left[\frac{5}{2} - \frac{15}{2} + 10\right]$$

$$= \frac{1}{2} \times \left[\frac{5-15+20}{2}\right]$$

$$= \frac{1}{2} \times \frac{10}{2}$$

$$= \frac{1}{2} \times 5$$

$$= 2.5V$$

For AB,

$x_1 = 0$, $x_2 = 1$

$y_1 = 0$, $y_2 = 5$

So,

$$y - 0 = \frac{5}{1}(x - 0)$$

$\Rightarrow y = 5x$

$\Rightarrow V = 5\theta$

From BC,

$x_1 = 1$ $y_1 = 5$

$x_2 = 2$ $y_2 = 0$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or, $$y - 5 = \frac{-5}{2-1}(x-1)$$

$\Rightarrow y - 5 = -5x + 5$

$\Rightarrow y = -5x + 10$

$\Rightarrow V = -5\theta + 10$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, d\theta}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 (5\theta)^2 d\theta + \int_1^2 (10-5\theta)^2 d\theta\right]}$$

$$= \sqrt{\frac{1}{2}\left[25\left[\frac{\theta^3}{3}\right]_0^1 + \int_1^2 (100 - 100\theta + 25\theta^2)\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2}\left[25 \times \frac{1}{3} + 100\left[\frac{\theta}{1}\right]_1^2 - 100\cdot\left[\frac{\theta^2}{2}\right]_1^2 + 25\left[\frac{\theta^3}{3}\right]_1^2\right]}$$

$$= \sqrt{\frac{1}{2}\left[\frac{25}{3} + 100 - 150 + \frac{175}{3}\right]}$$

$$= \sqrt{\frac{1}{2} \times \frac{50}{3}}$$

$$= \sqrt{\frac{25}{3}}$$

$$= \frac{5}{\sqrt{3}}$$

3) Find the avg. value, r.m.s. value and form factor of given wave form shown below:
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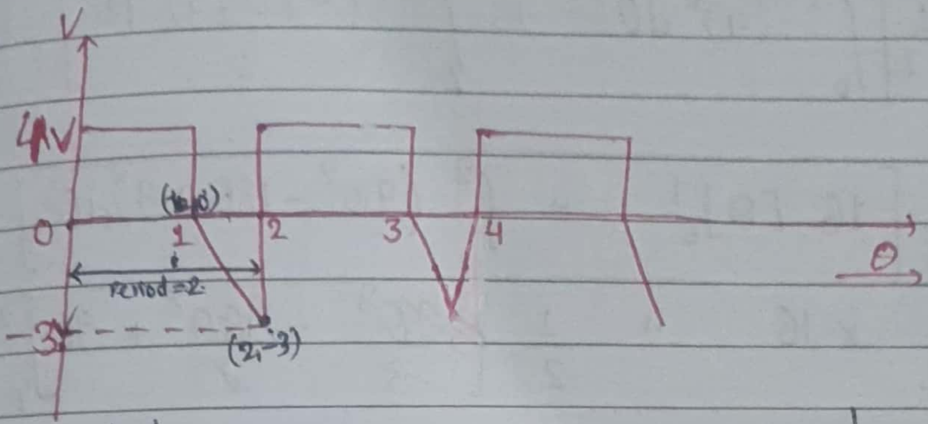

solution :-

Here, 0 to 1 , voltage = 4V
1 to 2 , voltage = -3V

$x_1 = 1$, $y_1 = 0$
$x_2 = 2$, $y_2 = -3$

Now,

$$\text{Avg. value} = \frac{1}{T}\int_0^T v \cdot d\theta$$

$$= \frac{1}{2}\left[\int_0^1 4\,d\theta + \int_1^2 (-3\theta+3)\,d\theta\right]$$

$$= \frac{1}{2}\left[4\cdot[\theta]_0^1 - 3\left[\frac{\theta^2}{2}\right]_1^2 + 3[\theta]_1^2\right]$$

$$= \frac{1}{2}\left[4\times 1 - 3\times\frac{2^2}{2} + 3\times\frac{1^2}{2} + 3\times 2 - 3\times 1\right]$$

$$= \frac{1}{2}\left[4 \cancel{-6} + \frac{3}{2} \cancel{+6} - 3\right]$$

$$= \frac{1}{2}\left[\frac{8+3-6}{2}\right]$$

$$= \frac{5}{4} = 1.25\ V$$

Rough

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{-3-0}{2-1}(x-1)$$

$$\Rightarrow y = -3x + 3$$

$$\Rightarrow V = -3\theta + 3$$

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RMS. value $= \sqrt{\frac{1}{T}\int_0^T v^2\, d\theta}$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 (4)^2\, d\theta + \int_1^2 (-3\theta+3)^2\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2}\left[16\,[\theta]_0^1 + \int_1^2 (9\theta^2 - 18\theta + 9)\,d\theta\right]}$$

$$= \sqrt{\frac{1}{2} \times 16 + \frac{1}{2}\left[\frac{\cancel{3}9\theta^3}{\cancel{3}} - \frac{\cancel{9}18\theta^2}{\cancel{2}} + 9\theta\right]_1^2}$$

$$= \sqrt{8 + \frac{1}{2}\left[3\times 2^3 - 3\times 1^3 - 9\times 2^2 + 9\times 1^2 + 9\times 2 - 9\times 1\right]}$$

$$= \sqrt{8 + \frac{1}{2}\left[24 - 3 - 36 \cancel{+9} + 18 \cancel{-9}\right]}$$

$$= \sqrt{8 + \frac{1}{2} \times 3}$$

$$= \sqrt{\frac{19}{2}} = 3.08$$

Form factor (K_f) $= \frac{\text{rms value}}{\text{avg. value}}$

$$= \frac{3.08}{1.25}$$

$$= 2.464$$

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4) Find the avg. value, rms value & form factor of the given triangular waveform.
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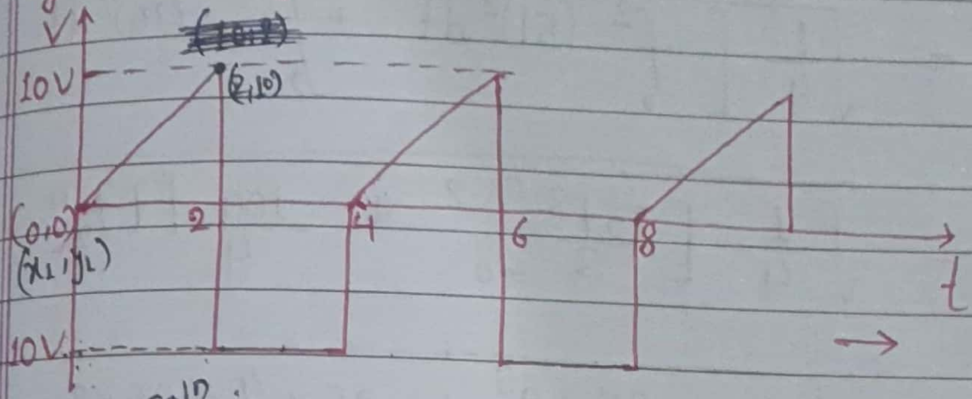

$0 \leq t \leq 2$
$2 \leq t \leq 4$

Sol^n :-

$x_1 = 0, y_1 = 0$

$x_2 = 2, y_2 = 10$

Now,

$$\text{Avg. value} = \frac{1}{T}\int_0^{4} v\, d\theta$$

$$= \frac{1}{4}\left[\int_0^2 5t \cdot dt + \int_2^4 10\, dt\right]$$

$$= \frac{1}{4}\left[\left[\frac{5t^2}{2}\right]_0^2 + 10[t]_2^4\right]$$

$$= \frac{1}{4}\left[\frac{5 \times 2^2}{2} + 10 \times 4 - 10 \times 2\right]$$

$$= \frac{1}{4}\left[10 + 40 - 20\right]$$

$$= \frac{30}{4}$$

$$= 7.5\,V$$

Rough

~~$y - 0 = \frac{2}{10\,5}(x - 0)$~~

~~$\Rightarrow 5y = x$~~

$y - 0 = \frac{10}{2}(x - 0)$

$\Rightarrow y = 5x$

$\Rightarrow v = 5t$ — (i)

$v = 10$ — (ii)

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R.M.S. value $= \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$

$= \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (10)^2\,dt\right]}$

$= \sqrt{\frac{1}{4}\left[\frac{25t^3}{3}\right]_0^2 + \frac{100}{4}[t]_2^4}$

$= \sqrt{\frac{1}{4} \times \frac{25 \times 2^3}{3} + 25 \times 4 - 25 \times 2}$

$\cancel{= \sqrt{\frac{50}{3} + 100 - 50}}$

$= \cancel{7.67}\ 8.16$

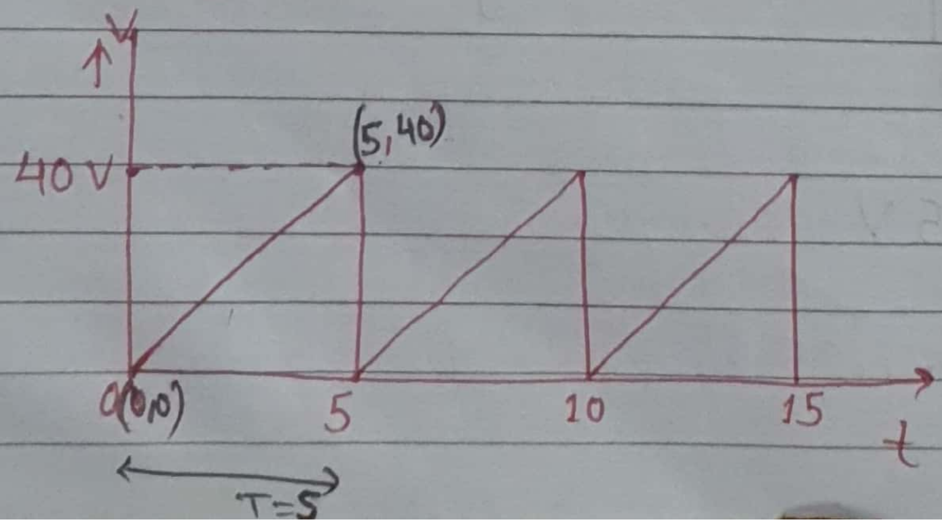
Form factor $(k_f) = \frac{\text{rms value}}{\text{avg. value}}$

$= \frac{8.16}{7.5}$

$= 1.08$

5) Compute the form factor of a given sawtooth wave of (triangular wave).

PU 2019
v.Imp



Solⁿ :-

First method:

$x_1 = 0, \; y_1 = 0$

$x_2 = 5, \; y_2 = 40$

$y - 0 = \frac{\cancel{8}\,40}{5}\, x - 0$

$\Rightarrow y = 8x$

$\Rightarrow V = 8t \quad \text{— ①}$

Now,

$$V_{Avg}\text{ value} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{5}\int_0^5 8t\cdot dt$$

$$= \frac{1}{5}\left[\frac{8t^2}{2}\right]_0^5$$

$$= \frac{1}{\cancel{5}} \times 4 \times 5^{\cancel{2}}$$

$$= 20\,V$$

Now,

$$V_{RMS}\text{ value} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

$$= \sqrt{\frac{1}{5}\int_0^5 64t^2\,dt}$$

$$= \sqrt{\frac{1}{5}\left[\frac{64\times t^3}{3}\right]_0^5} = \sqrt{\frac{1}{5}\left[\frac{64\times 5^3}{3}\right]}$$

$$\cancel{= \sqrt{\frac{1}{5}\times 3250}}$$

$$\cancel{= \sqrt{50}}$$

$$= 23.09\,V$$

$$\text{Form factor }(k_f) = \frac{\text{rms value}}{\text{avg. value}}$$

$$= \frac{23.09}{20}$$

$$= 1.73$$

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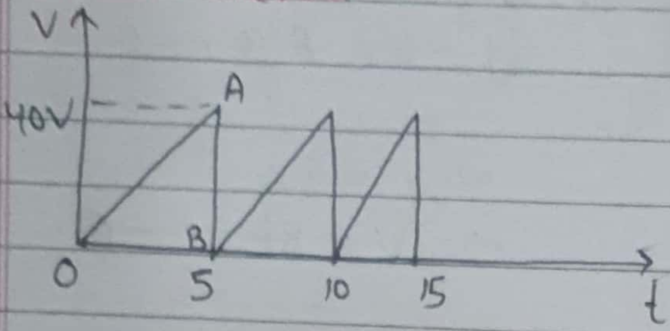
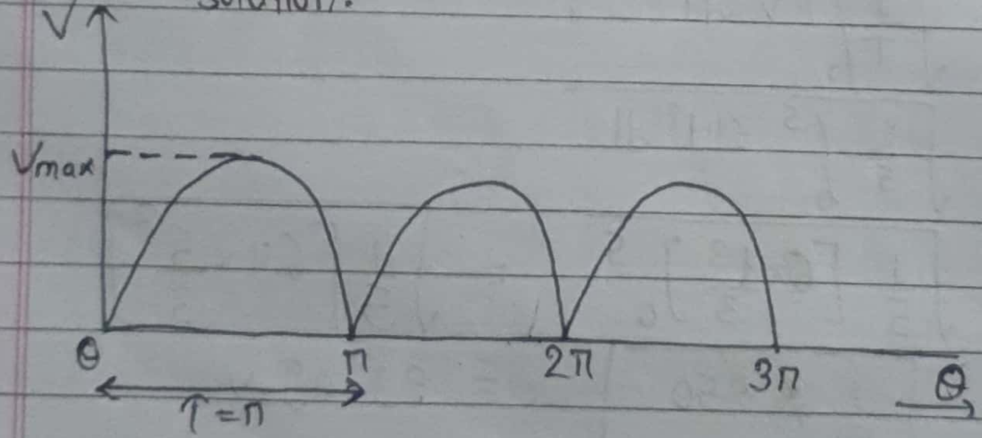
Second method:



$$y = mx + c$$
$$= \frac{40}{5}x + c \qquad \left[m = \frac{\text{height}}{\text{base length}} \right]$$
$$\therefore y = 8x$$
$$\Rightarrow V = 8t$$

6) The output voltage (full wave rectifier) shown below. find the average value and rms value (effective & virtual value).

solution:



solution:

$$\text{Avg. value} = \frac{1}{T}\int_0^T v\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_{max}\sin\theta\,d\theta \qquad \left[\because \text{sinusoidal wave form } V = V_{max}\sin\theta \right]$$

$$= \frac{V_{max}}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{V_{max}}{\pi}\left[-\cos\pi + \cos 0\right]$$

So, Avg. value $= \frac{V_{max}}{\pi} \times 2$

$= 0.636 \; V_{max}$

Now,

RMS value $= \sqrt{\frac{1}{T}\int_0^T v^2 \, d\theta}$

$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_{max}^2 \cdot \sin^2\theta \, d\theta}$

$= \sqrt{\frac{V_{max}^2}{2\pi} \int_0^{\pi} 2\sin^2\theta \cdot d\theta}$

$= \sqrt{\frac{V_{max}^2}{2\pi} \int_0^{\pi} (1 + \cos 2\theta) d\theta}$

$= \sqrt{\frac{V_{max}^2}{2\pi} \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi}}$

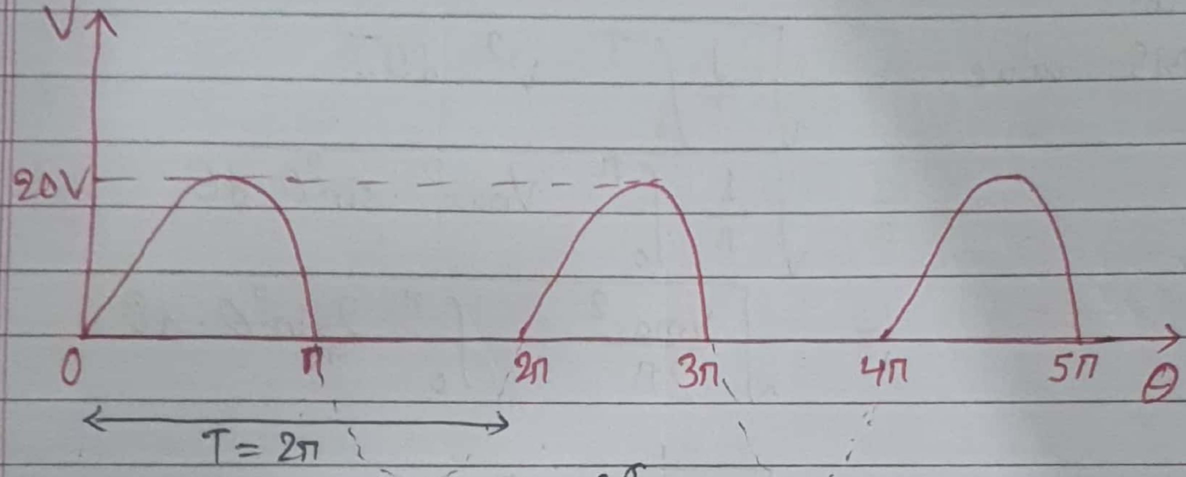
$= \sqrt{\frac{V_{max}^2}{2\pi} \left[\pi + \frac{\sin 2\pi}{2} - \frac{\sin 0}{2}\right]}$

$= \sqrt{\frac{V_{max}^2}{2 \cancel{\pi}} \times \cancel{\pi}}$

$= \frac{V_{max}}{\sqrt{2}}$

$= \cancel{0.56} \; 0.707 \; V_{max}$

7) The output voltage (half wave rectifier) whose maximum value is 20V. Find the avg. value, rms value (virtual or effective value), peak factor and form factor.



Avg. value $= \frac{1}{T}\int_0^T v \cdot d\theta$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 20\sin\theta \, d\theta + \int_{\pi}^{2\pi} 0 \cdot \sin\theta \, d\theta\right]$$

$$= \frac{1}{2\pi}\left[20\left[-\cos\theta\right]_0^{\pi} + 0\right]$$

$$= \frac{20}{2\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{20}{\cancel{2}\pi} \cancel{\times 2}$$

$$= \frac{20}{\pi} = 6.37\,V$$

rms value $= \sqrt{\frac{1}{2\pi}\int_0^{\pi} 400 \sin^2\theta \, d\theta}$

$$= \sqrt{\frac{400}{4\pi}\int_0^{\pi} (1+\cos 2\theta)\, d\theta}$$

$$= \frac{1}{\cancel{2\pi}} \sqrt{\frac{200}{2\pi}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{\cancel{1}}{\cancel{2\pi}} \sqrt{\frac{200}{2\pi}\left[\pi + 0 - 0 - 0\right]}$$

$$= \frac{\cancel{1}}{\cancel{2\pi}} \sqrt{\frac{200\cancel{\pi}}{2\cancel{\pi}}} = \sqrt{100}$$

$$\cancel{= \sqrt{\frac{200\pi}{4\pi^2}}} = 10V$$

$$\cancel{= 3.99}$$

$$\text{Peak factor}(k_p) = \frac{\text{max}^m \text{ value}}{\text{rms}}$$

$$= \frac{20}{10}$$

$$= 2$$

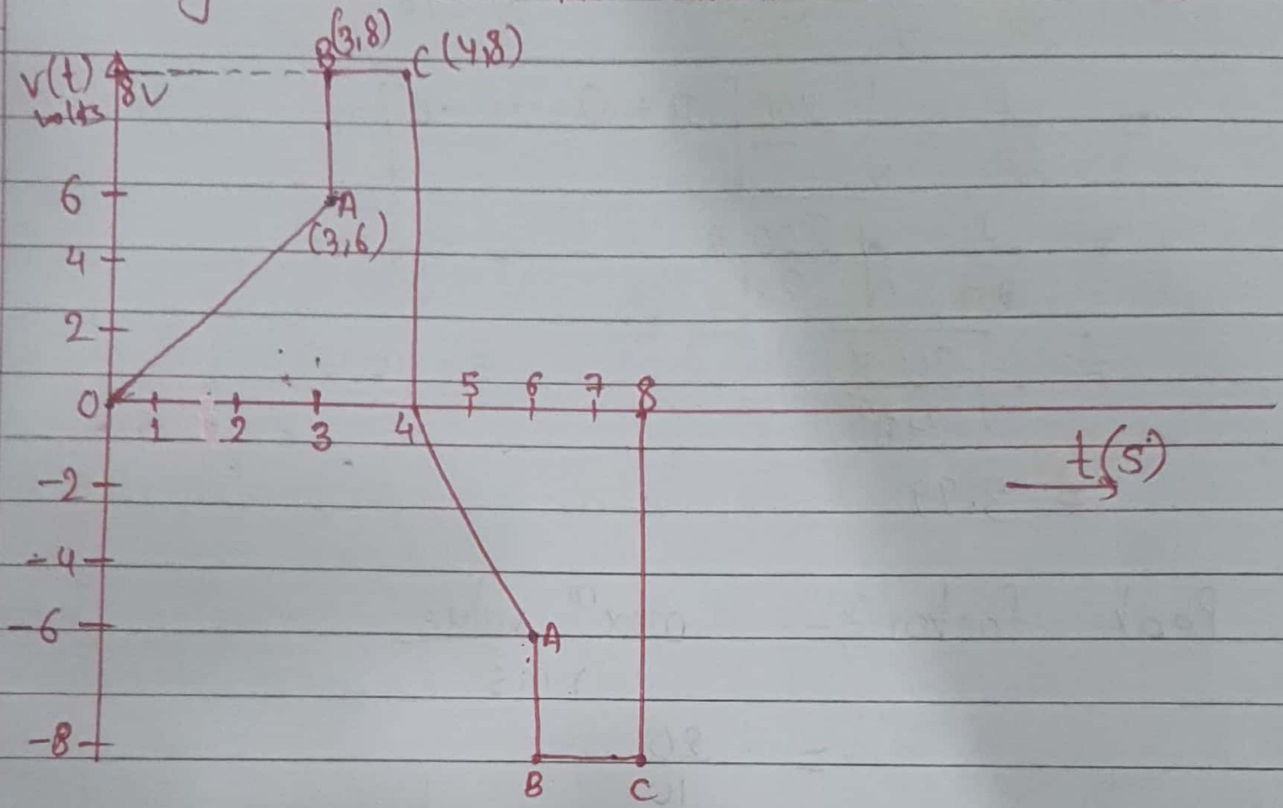
$$\text{form factor}(k_f) = \frac{\text{rms value}}{\text{avg. value}}$$

$$= \frac{10}{6.36}$$

$$= 1.572$$

8) A voltage wave has the variation as shown below:

2016 fall PU



solution: Case 1 - ~~(AB)~~ (OA)

$x_1 = 0, y_1 = 0$

$x_2 = 3, y_2 = 6$

$y = \frac{6}{3}x$

$\Rightarrow y = 2x$

$\Rightarrow V = 2t$ —— (1)

Case - 2 (AB)

$x_1 = 3, y_1 = 6$

$x_2 = 3, y_2 = 8$

X

$\Rightarrow y - 6 = \frac{2}{0}(x-3)$

$\Rightarrow 0$

~~$\Rightarrow 0 = 2x - 6$~~

Case - 3 (BC)

$x_1 = 3, y_1 = 8$

$x_2 = 4, y_2 = 8$

So,

$y - 8 = \frac{0}{1}(x-3)$

$\Rightarrow y = 8$

$\Rightarrow V = 8$

$$\text{Avg. value} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{4}\left[\int_0^3 2t\cdot dt + \int_3^4 8\,dt\right]$$

$$= \frac{1}{4}\left[\left[\frac{2t^2}{2}\right]_0^3 + 8[t]_3^4\right]$$

$$= \frac{1}{4}\left[\frac{\cancel{2}\times 9}{\cancel{2}} + 8\times 4 - 8\times 3\right]$$

$$= \frac{1}{4}\times 17$$

$$= 4.25V$$

$$\text{RMS value} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

$$= \sqrt{\frac{1}{4}\left[\int_0^3 4t^2\cdot dt + \int_3^4 64\,dt\right]}$$

$$= \sqrt{\frac{1}{\cancel{4}}\cdot\cancel{4}\left[\frac{t^3}{3}\right]_0^3 + \frac{1}{4}\times 64\,[t]_3^4}$$

$$= \sqrt{\frac{\cancel{3^3}}{\cancel{3}} + 16\times 4 - 16\times 3}$$

$$= \sqrt{9+64-48} = \sqrt{25}$$

$$= \cancel{4.36}\ 5V$$

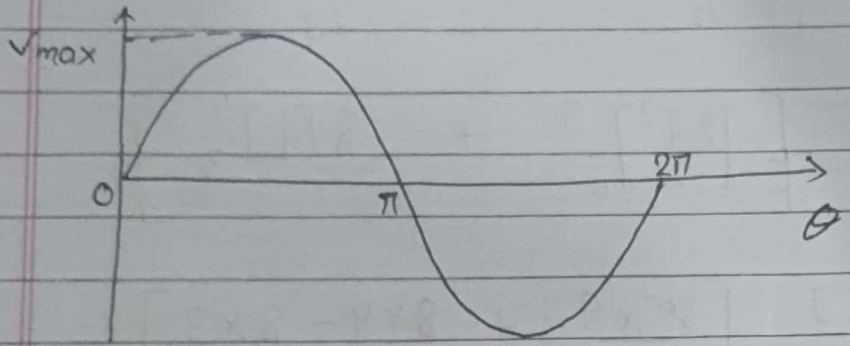
$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{5^2}{50}$$ [given in ques]

$$= 0.5\text{ watt}$$

2019

9) Show that the peak factor of sine value is equal to 1.414.

solution:



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_{max}^2 \sin^2\theta\, d\theta}$$

$$= 0.707\, V_{max}$$

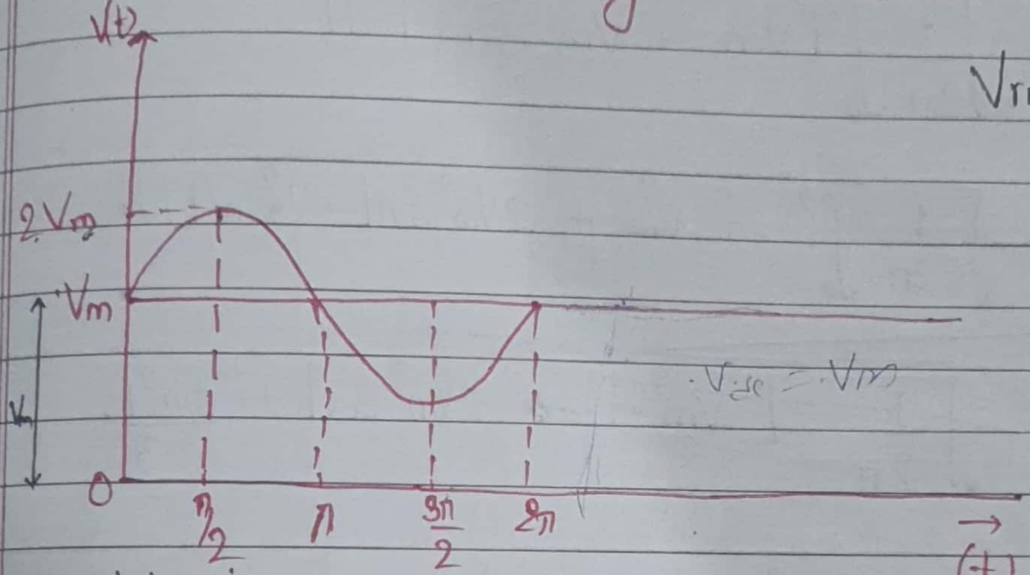
$$\text{Peak value} = \frac{V_{max}}{rms}$$

$$= \frac{V_{max}}{0.707\, V_{max}}$$

$$= 1.414$$

10) Determine the average value and rms value in which the ac waveform is ~~receiving~~ residing on dc (with a magnitude Vm.)

$$V_{rms} = \sqrt{\frac{3}{2}}\, V_m$$



solution :-

Here the waveform eq^n for a given waveform is

For, dc component:

$V_{d.c.} = V_m$

~~$v = V_{max} \cdot \sin t$~~

For, ac component:

$V_{ac} = V_m \sin t$

So, that, total wave form eq^n are

$$V = V_{dc} + V_{ac}$$
$$= V_m + V_m \sin t \qquad 0 \le t \le 2\pi$$

Now, avg. value for one complete cycle is

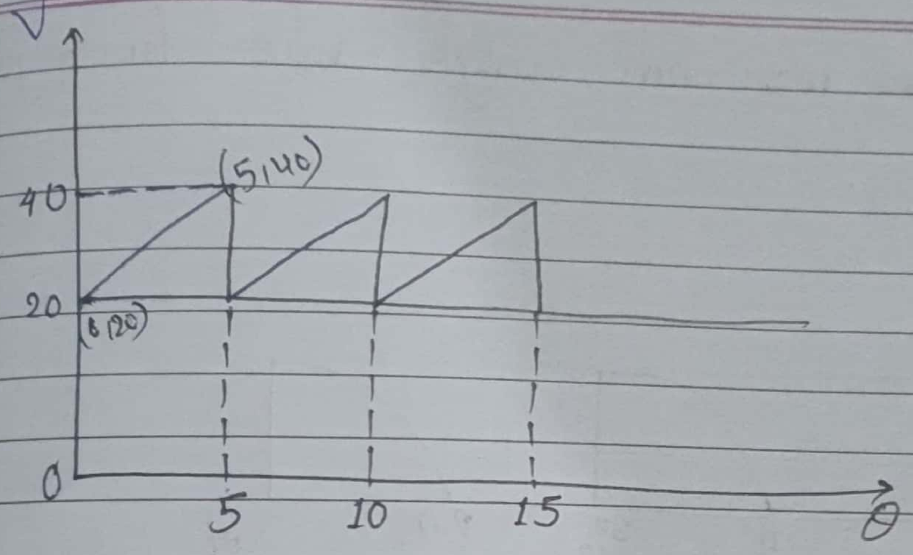
$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} (V_m + V_m \sin t)\, dt$$

$$= \frac{1}{2\pi} \cancel{(V_m)} [t \cdot V_m]_0^{2\pi} + \frac{1}{2\pi} [V_m \cdot -\cos t]_0^{2\pi}$$

$$= \frac{1}{2\pi} \times 2\pi \cdot V_m - \frac{1}{2\pi} \times V_m \cdot [-1 + 1] = V_m - 0$$

$$= V_m$$

Q.



d.c. component = 20 V

ac component = ~~8~~ 4θ $[0 \le t \le 5]$

$y = mx + c$

$V = \frac{\text{total ht.}}{\text{base l.}}\, x + c$

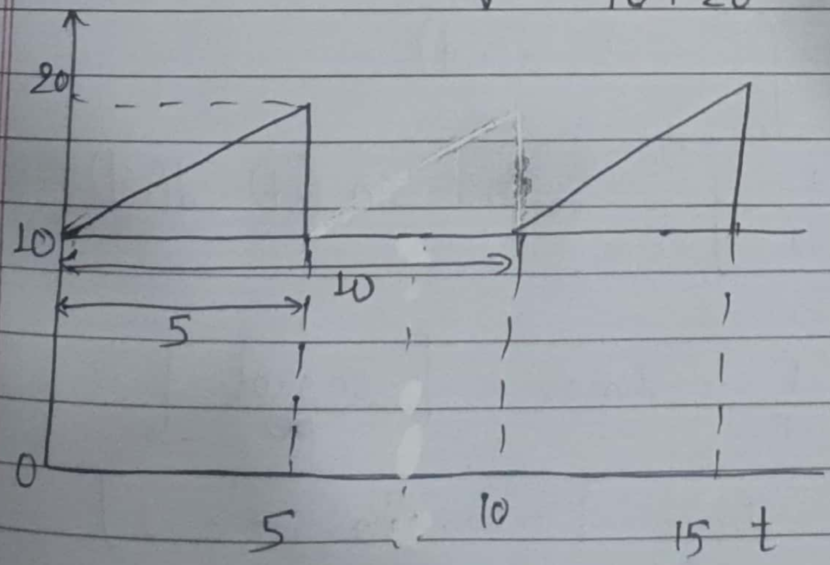
$V = \frac{20x}{5} + 20 = 4x + 20$ ~~4t~~

$y - 20 = \frac{40-20}{5}(x - 0)$

$y = 4x + 20$

$V = 4\theta + 20$

Q.



~~m = 10/10~~

~~dc = 10~~

~~ac = 2t~~

$V_{max} = \cancel{\frac{1}{T}\int_0^T v\,dt}$

$= \cancel{\frac{1}{10}\int_0^5 (10+2t) + \int_5^{10} 0\,dt}$

ac = $2t + 10$

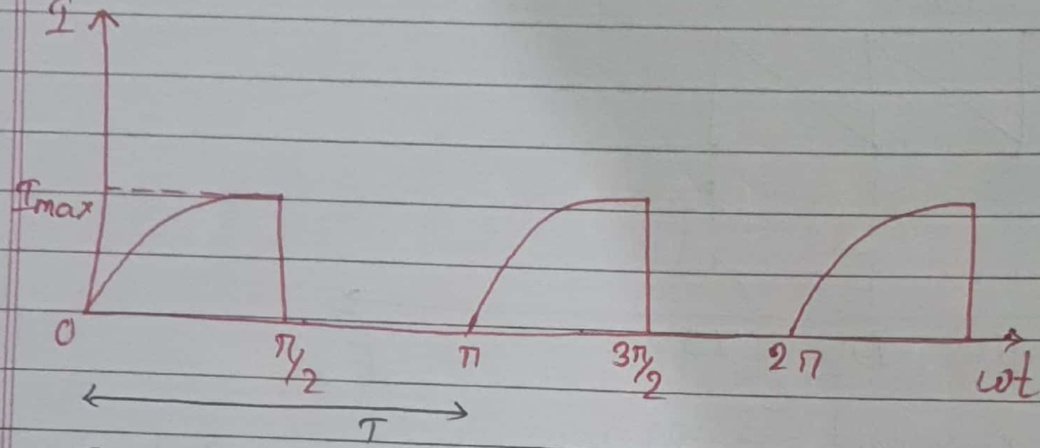
dc = 10V. $(0 \le t \le 10)$

~~$y = mx + c$~~

~~$= \frac{10}{5}x = \frac{\text{height}}{\text{base length}}$~~

~~$= 2x$~~

Q. Determine the rms and average value for a given waveform:



$I = I_{max} \cdot \sin \omega t$ ——— (i) $(0 \leq \omega t \leq \pi/2)$

$\underset{\Downarrow\,0}{I} = I_{max} \cdot \sin \omega t$ ——— (ii) $(\pi/2 \leq \omega t \leq \pi)$

So,

$$\cancel{I_{max}}\ I_{avg} = \frac{1}{T}\int_0^T I \, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi/2} I_{max} \sin(\omega t)\, d(\omega t)$$

$$= \frac{1}{\pi} \cdot I_{max}\left[-\cos \omega t\right]_0^{\pi/2}$$

$$= \frac{I_{max}}{\pi}\left[-\cos \pi/2 + \cos 0\right]$$

$$= \frac{I_{max}}{\pi}[0+1]$$

$$I_{avg} = \frac{I_{max}}{\pi}$$

$\frac{I_{max}}{2}$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi/2} (I_{max} \cdot \sin \omega t)^2 \, d(\omega t)}$$

$$= \sqrt{\frac{1}{\pi} \cdot I_{max}^2 \cdot \frac{1}{2}\int_0^{\pi/2} (1 - \cos 2(\omega t)) \, d(\omega t)}$$

$$= \sqrt{\frac{I_{max}^2}{2\pi}\left[\omega t - \frac{\sin 2(\omega t)}{2}\right]_0^{\pi/2}}$$

$$= \sqrt{\frac{I_{max}^2}{2\pi}\left(\frac{\pi}{2} - \frac{0}{2}\right)}$$

$$= \sqrt{\frac{I_{max}^2}{2\cancel{\pi}} \times \frac{\cancel{\pi}}{2}}$$

$$I_{rms} = \frac{I_{max}}{2}$$

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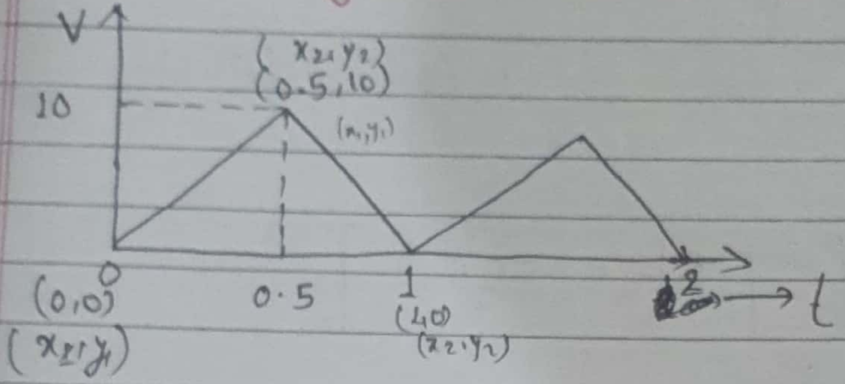
2014 Fall

Q. Find the avg. value & rms value:



avg. = 5.5

rms = 5.77

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y = \frac{10}{0.5}x$$

$$= V = 20t$$

~~Now~~ Also,

$$y - 10 = \frac{-10}{0.5}(x - 0.5)$$

or $y - 10 = -20(x - 0.5)$

or $y = -20x + 10 + 10$

$$\Rightarrow y = -20x + 20$$

$$\Rightarrow V = -20t + 20$$

Now,

$$\text{Avg. value} = \frac{1}{T}\int_0^T V\,dt$$

$$= \frac{1}{1}\left[\int_0^{0.5} 20t \cdot dt + \int_{0.5}^{1} (-20t + 20)dt\right]$$

$$= \left[\frac{20t^2}{2}\right]_0^{0.5} + \left[\frac{-20t^2}{2} + 20t\right]_{0.5}^{1}$$

$$= [10t^2]_0^{0.5} + [-10t^2 + 20t]_{0.5}^{1}$$

$$= (10\times 0.5^2) + (-10\times 1^2 + 20\times 1 + 10\times 0.5^2 - 20\times 0.5)$$

$$= 2.5 - 10 + 20 + 2.5 - 10$$

$$= 5$$

Now,

RMS Value $= \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$

$$= \sqrt{\frac{1}{1}\left[\int_0^{0.5} 400t^2\,dt + \int_{0.5}^{1} (20-20t)^2\,dt\right]}$$

$$= \sqrt{\int_0^{0.5} 400t^2\,dt + \int_{0.5}^{1} (400 - 800t + 400t^2)\,dt}$$

$$= \sqrt{400\left[\frac{t^3}{3}\right]_0^{0.5} + 400\left[t - \frac{2t^2}{2} + \frac{t^3}{3}\right]_{0.5}^{1}}$$

$$= \sqrt{400\left[\frac{0.5^3}{3} - \frac{0^3}{3}\right] + 400\left[1 - 0.5 - \frac{\cancel{2}\times 1^2}{\cancel{2}} + \frac{\cancel{2}\times 0.5^2}{\cancel{2}} + \frac{1^3}{3} - \frac{0.5^3}{3}\right]}$$

$$= 20\sqrt{\cancel{\frac{1}{24}} + 0.5 - 1 + 0.25 + \frac{1}{3} - \cancel{\frac{1}{24}}}$$

$$= \cancel{20 \times \frac{1}{\sqrt{12}}} \quad 5.77$$

$$\cancel{= 1.67}$$

Page No.:
Date : / /

A.C.

Civil low
In C, I leads V.
~~In~~ V leads I in L.

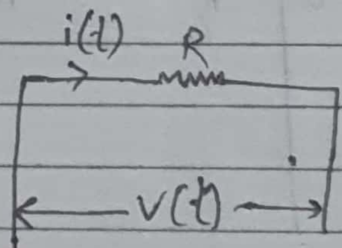
Steady state responce of pure ckt:

1) phase relation in a pure resistor
2) " " " " " inductor
3) " " " " " capacitor

① Phase relation in a pure resistor

When a sinusoidal voltage of certain magnitude is applying to a resistor certain amount of sine-wave current passing through it. So, we know the relation of $V(t)$ & $i(t)$

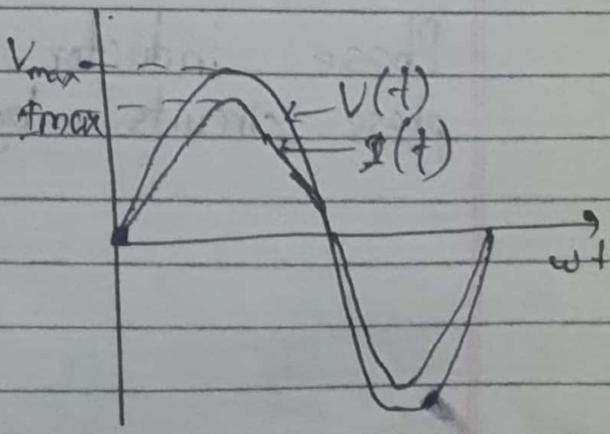
$$\boxed{V(t) = i(t) \cdot R}$$



Consider the function,

$$i(t) = I_{max} \cdot \sin\omega t$$

Both voltage & current in the phase but amplitude is different according to the value of resistor.

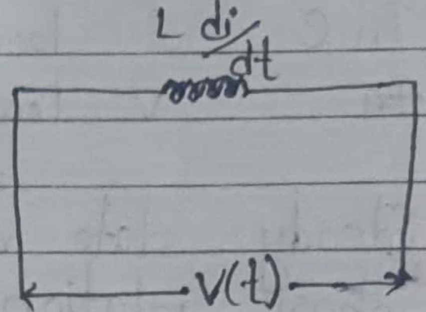


Voltage & currents said to phase since phase ~~is~~ difference is zero.

$$\boxed{Z = R}$$

2) Phase relation in a pure ~~resistor~~ inductor:

$$v(t) = L \cdot \frac{di}{dt}$$

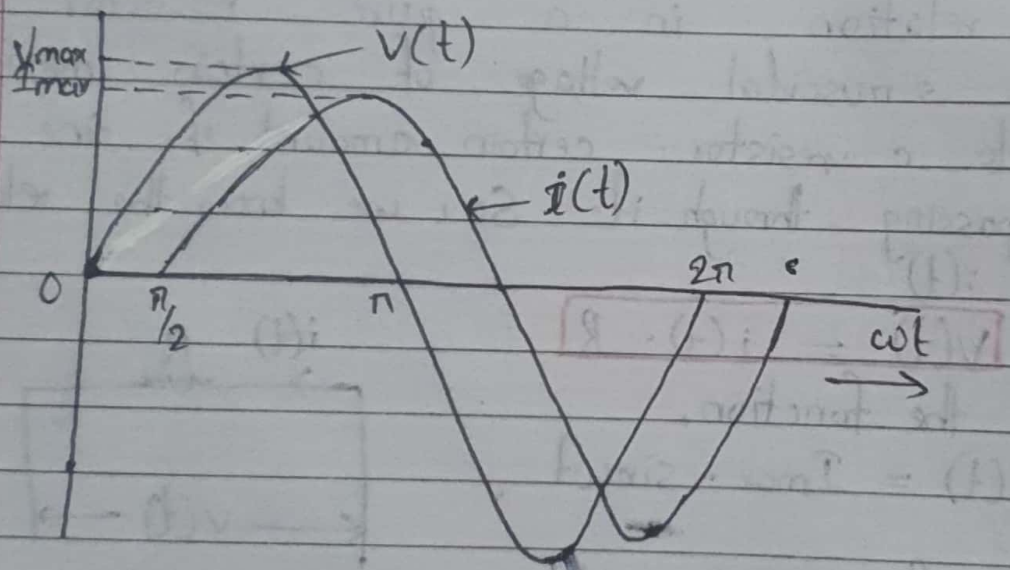


$$i(t) = I_{max} \cdot \sin(\omega t)$$

$$v(t) = L \cdot \frac{d(I_{max} \sin \omega t)}{dt}$$

$$V(t) = \omega L \cdot I_{max} \omega t$$

$$\therefore V(t) = V_{max} \cdot \cos \omega t$$



Phase inductor voltage and current are out of phase circuits lags behind the voltage by 90° or (π/2)

3> Phase resistor in a pure capacitor:

$$V(t) = \frac{1}{C} \int i \, dt \quad \text{--- ①}$$ [The relation betⁿ voltage & current]

$$i(t) = I_{max} \cdot \sin\omega t$$

$$V(t) = \frac{1}{C} \int I_{max} \cdot \sin\omega t \, d(\omega t)$$

$$= \frac{1}{\omega C} I_{max} (-\cos\omega t)$$

$$= \frac{I_{max} \sin(\omega t - \pi/2)}{\omega C}$$

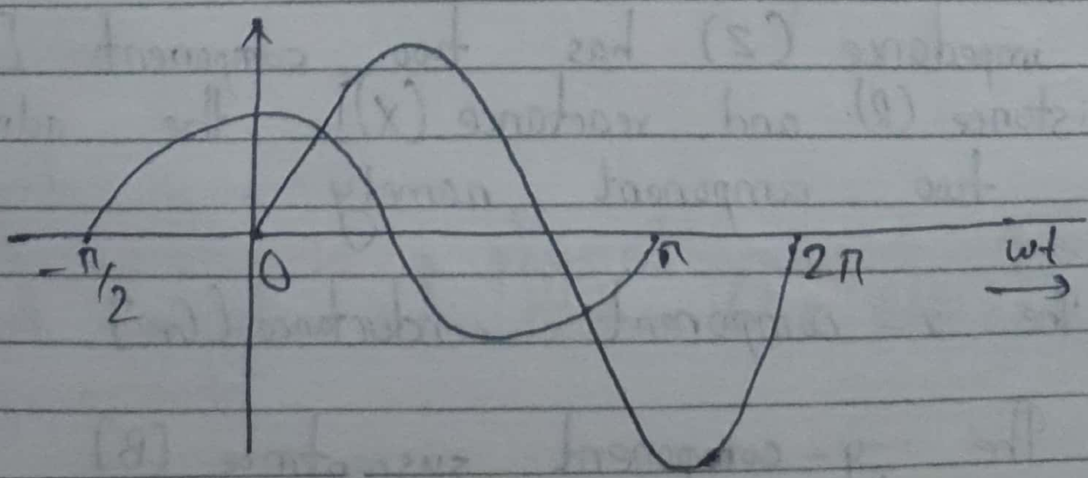
$$\therefore V(t) = V_{max} \cdot \sin(\omega t - \pi/2)$$

where, $V_{max} = \dfrac{I_{max}}{\omega C}$

$$\Rightarrow \frac{V_{max}}{I_{max}} = Z$$

$$\therefore Z = \frac{1}{\omega C}$$

Impedance $(Z) = X_C = \dfrac{1}{\omega C}$



1) Polar form $\longrightarrow e^{j\theta}$

2) Rectangular form $\longrightarrow$ $x + jy$

$4+j5$ } rectangular form

$r = 6.40$
$\theta = 51.34$ } polar form

Polar to rectangular form :

- Rec (6.40, 51.34) $\Rightarrow x = 4, y = 5$

$\Rightarrow 4+j5$

$$\frac{\cancel{6.4}^{2.13} \angle 51.34}{\cancel{3} \angle 5^\circ} = 2.13 \angle 51.34 - 5^\circ = 2.13 \angle 46.34^\circ.$$

$$\frac{5+j6}{8+j10} = \frac{7.81 \angle 50.19^\circ}{12.81 \angle 51.34^\circ}$$

Admittance (Y)

It is the reciprocal of impedance (Z) of an a.c. circuit. So,

$$Y = \frac{1}{Z}$$

Thus, As impedance (Z) has two components [via resistance (R) and reactance (X)], the admittance has ~~also~~ two component namely :

1) The x- component conductance [G]

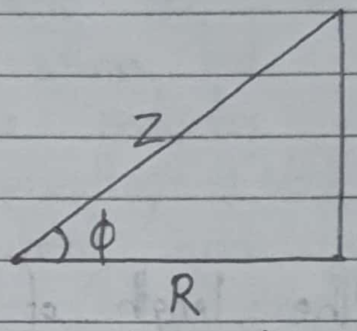
(ii) The y- component susceptance [B]

Conductance $(G) = Y\cos\phi = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2}$

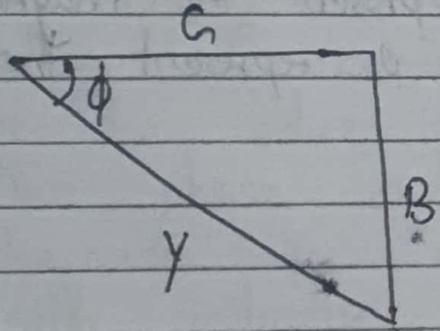
& Susceptance $(B) = Y\sin\phi = \frac{1}{Z} \cdot \frac{X}{Z} = \frac{X}{Z^2}$

$\therefore$ Admittance $(Y) = \sqrt{G^2 + B^2}$

The unit's of
G = mho
$B = \text{ohm}^{-1}$
Y = simon's



impedance triangle



Admittance triangle

Phasor diagram

Imp

A phasor diagram can be used to represent a sine-wave in terms of magnitude and its angular position.

Vector diagram

A vector is a physical quantity that have the both magnitude and direction (X, Y, Z) or polar co-ordinate

Phasor diagram:
E.g.

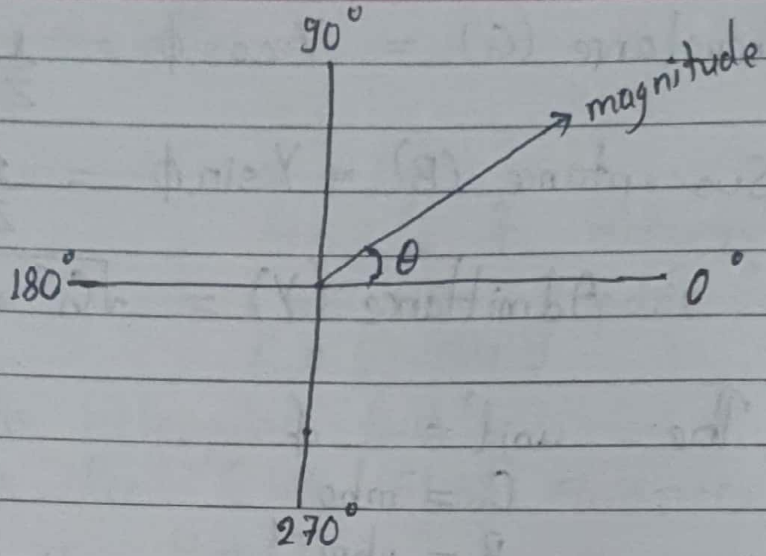


fig (a)

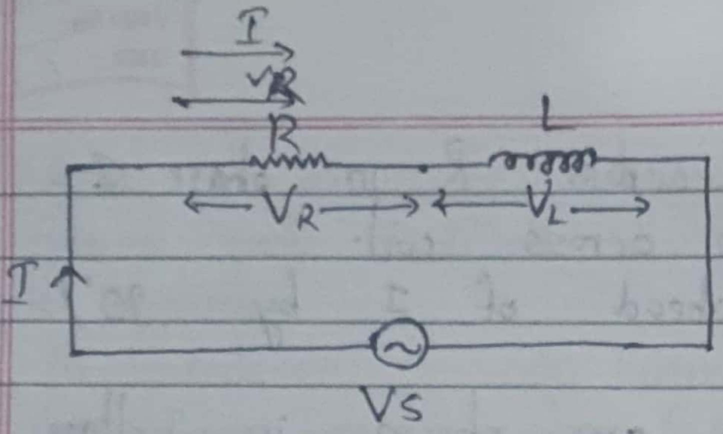
The length of arrow represent the magnitude of sine wave & the angle θ represent the angular position of sine wave.

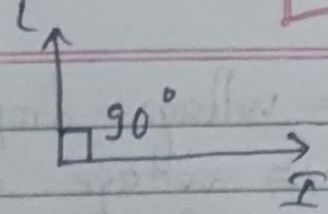
Series Circuit

1) Series RL circuit:-

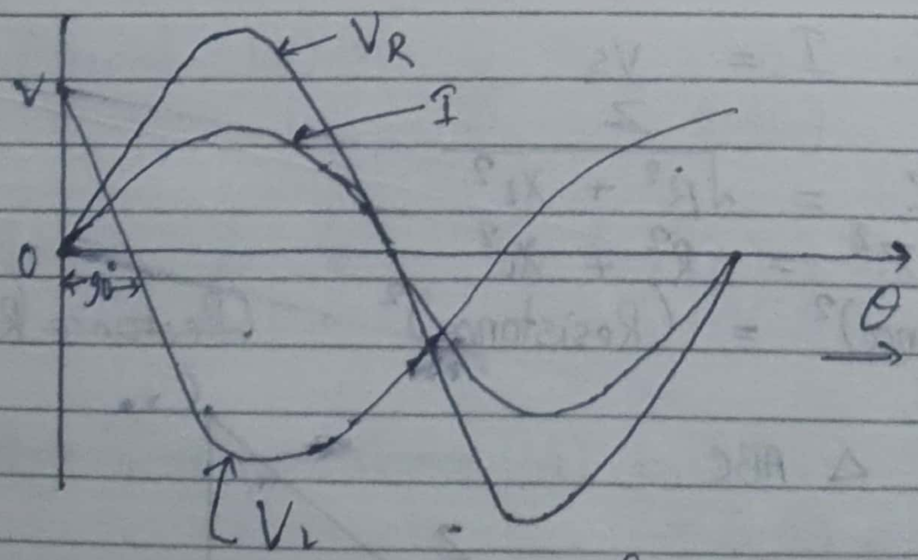
If we apply the sinusoidal input in the RL ckt, all the current in the elements & all the voltage across the element are sinusoidal. In the analysis of RL ckt we find

i) impedance
ii) current
iii) phase angle
iv) Voltage drops





The resistor voltage (V_R) and the current (I) are in same phase with each other but lag behind the source voltage (V_s). The inductor voltage V_L leads the source ~~sou~~ voltage (V_s). The phase angle between current and voltage in the inductor is always $90°$. The amplitude of voltage & current in the complete ckt depend on the value of resistor and inductive reactance. The phase angle is somewhere betn 0 to $90°$ (because the series combination of resistance with inductive rectance).



V_s = RMS value of the applied voltage
I = " " " " resultant current.

V_R = voltage drops across R. in phase I.
V_L = voltage drop across coil.
$V_L = I \cdot X_L$ (ahead of I by 90°).

The voltage drop are shown in voltage triangle:

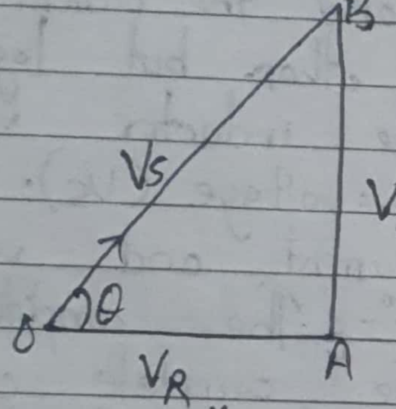


fig (a) - voltage triangle

OA = ohmic drop
AB = inductive drop
OB = vector sum of two 'V_s'

$$V_s = \sqrt{V_R^2 + V_L^2}$$
$$V_s = \sqrt{(IR)^2 + (IX_L)^2}$$
$$V_s = I\sqrt{R^2 + X_L^2}.$$

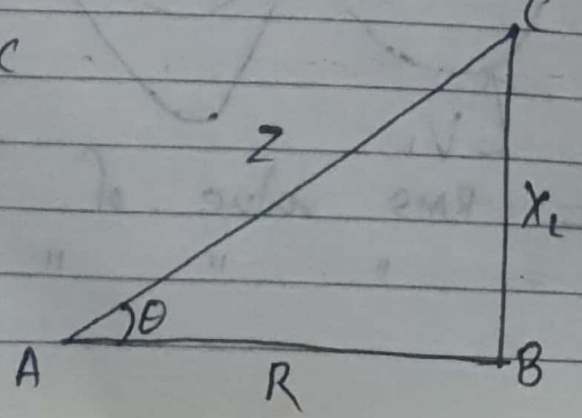
$$I = \frac{V_s}{\sqrt{R^2 + X_L^2}} \qquad \left[\because Z = \sqrt{R^2 + X_L^2}\right]$$

$$\therefore I = \frac{V_s}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$
$$Z^2 = R^2 + X_L^2$$

$(\text{Impedance})^2 = (\text{Resistance})^2 + (~~\text{Rectance}~~\ \text{Reactance})^2$

Impedance △ ABC



$$\tan\theta = \frac{X_L}{R}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

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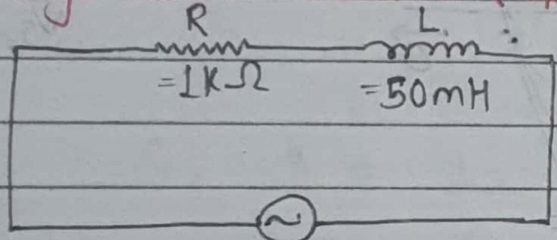
Q. 1) Circuit shown in fig. below of $1k\Omega$ connected ~~to~~ + in series with 50 mH coil, a 10 V rms, 10 kHz signal is applied

(i) Impedance (ii) Current (iii) Phase angle

(iv) Voltage across resistanc (V_R) (v) Voltage across inductor (V_L)



10 rms, 10KHz

(i) ~~inductor~~ $X_L = \omega L = 2\pi f \cdot L = 2\pi \times 10 \times 10^3 \times 50 \times 10^{-3}$

Inductance $= 3141.59\ \Omega$

So,

$$\text{Impedance } (Z) = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{(10^3)^2 + (3141.5)^2}$$
$$= 3296.9\ \Omega$$

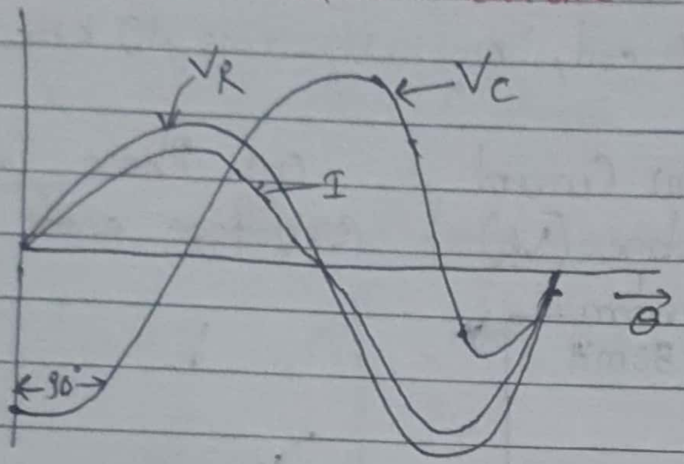
(ii) Current $(I) = \dfrac{V}{Z} = \dfrac{10}{3296.9} = 3.03 \times 10^{-3}$ amp
$= 3.03$ mA

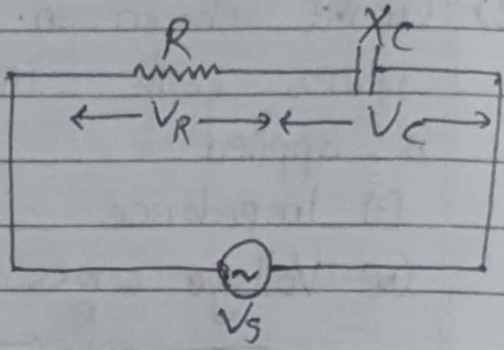
(iii) Phase angle $(\theta) = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{3141.5}{10^3}\right)$
$= 72.34°$

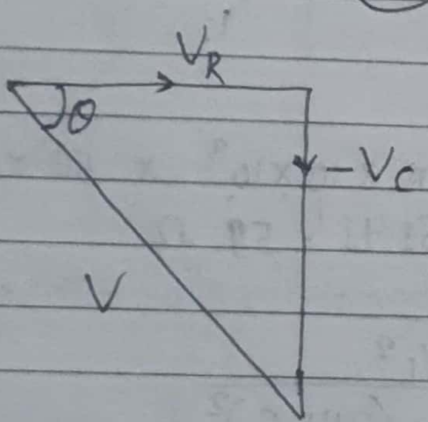
(iv) Voltage across resistance $(V_R) = IR$
$= 3.03 \times 10^{-3} \times 10^3 = 3.03$ V

(v) Voltage across inductor $(V_L) = I \cdot X_L = 3.03 \times 10^{-3} \times 3141.5$
$= 9.51$ V

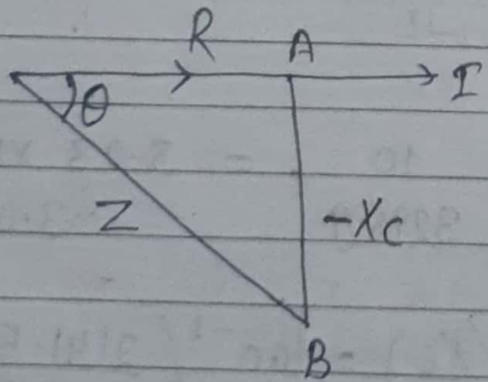
2) Series R-C Circuit







⇐ Voltage triangle



← Impedance triangle

$V_R = IR$ drop across the resistor (R)

$V_C = I \cdot X_C$ drop across the capacitor

Capacitive reactance taken as -ve.

$$V_s^2 = V_R^2 + (-V_c)^2$$

or $$V_s = \sqrt{(IR)^2 + (-IX_c)^2}$$

or $$V_s = I\sqrt{R^2 + X_c^2}$$

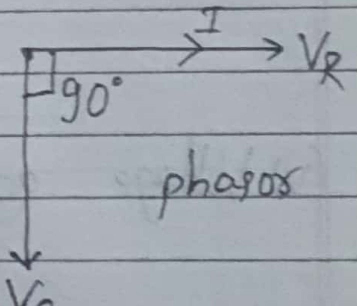
$$\therefore \quad I = \frac{V_s}{\sqrt{R^2 + X_c^2}}$$

$$\text{Impedance } (Z) = \sqrt{R^2 + X_c^2}$$

$$\text{So, } I = \frac{V_s}{Z}$$

$$\tan\theta = -\frac{X_c}{R}$$

$$\theta = \tan^{-1}\left(-\frac{X_c}{R}\right)$$



Q. The pure resistance of 50 Ω in series with pure capacitor of 100 μF. This series combination is connected across 100V, 50Hz supply. Find (i) impedance (ii) current (iii) power factor (iv) phase angle (v) voltage across resistor (vi) voltage actor capacitor (vii) draw vector diagram.

~~solution:~~ solution:

~~(i) Capacitance~~, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$= 31.83\ \Omega$

(i) $Z = \sqrt{R^2 + X_c^2}$

$= \sqrt{50^2 + (31.83)^2}$

$= 59.36\ \Omega$

(ii) Current, $I = \frac{V_s}{Z} = \frac{100}{59.36} = 1.68$ amp

(iii) power factor $= \frac{R}{Z} = \frac{50}{59.36} = 0.84$ ~~watt~~

(iv) phase angle $= \tan^{-1}\left(-\frac{X_C}{R}\right)$

$= \tan^{-1}\left(-\frac{32}{50}\right)$

$= -32.62^\circ$

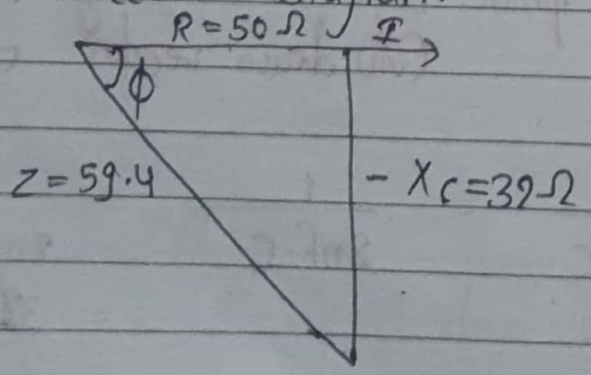
(v) Voltage across resistor $(V_R) = IR$

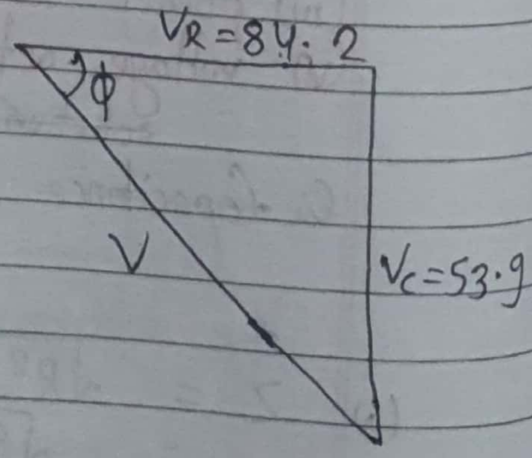
$= 1.68 \times 50 = 84\ V$

(vi) Voltage across capacitor $(V_C) = I \cdot X_C$

$= 1.68 \times 32 = 53.76\ V$

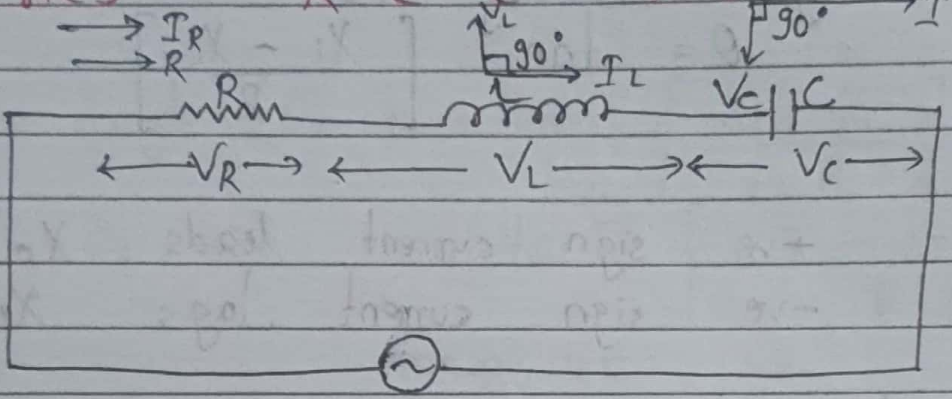
(vii) Vector diagram:

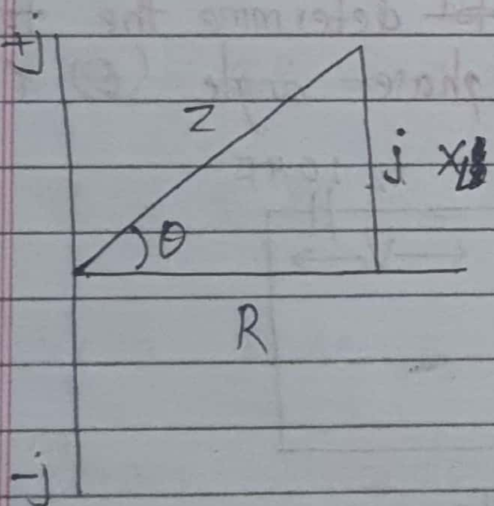


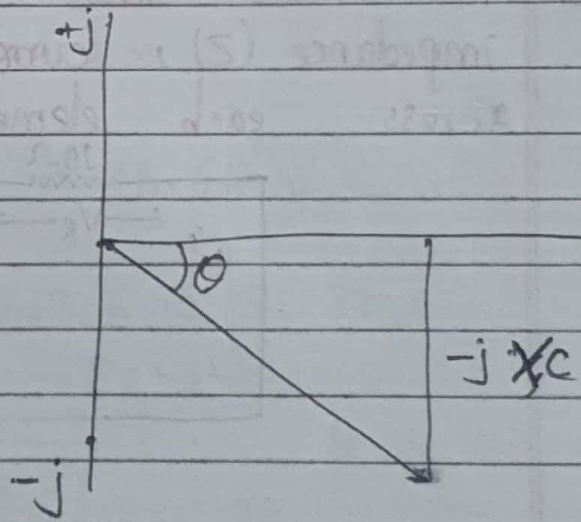


~~Series~~

3) Series R-L-C Ckt







$$V_s^2 = (IR)^2 + (IX_L - IX_C)^2$$

$$\therefore V_s = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V_s}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\therefore I = \frac{V_s}{Z} \quad ; \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

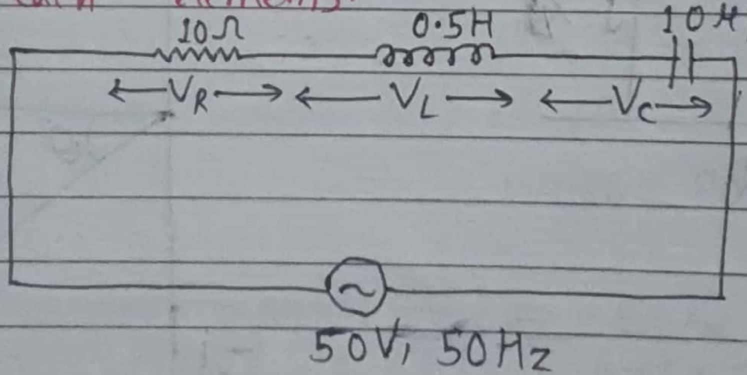
power consumed =

Similarly,
phase angle of RLC ckt:

$$\theta = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

Note:-
+ve sign current leads $X_C > X_L$
-ve sign current lags $X_L > X_C$

Q. A circuit shown in fig., ~~to~~ determine the total impedance (Z), current (I), phase angle (θ) & voltage across each elements.



$$X_L = \omega L = 2\pi f \cdot L = 314 \times 0.5 = 157\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f \times C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$
$$= 318.5\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{10^2 + (157 - 318.5)^2}$$
$$= 161.8\ \Omega$$

Current $(I) = \frac{V}{Z} = \frac{50}{161.8} = 0.31$ amp

phase angle $(\theta) = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$

$= \tan^{-1}\left[\frac{157 - 318.5}{10}\right]$

$= -86.45^\circ$

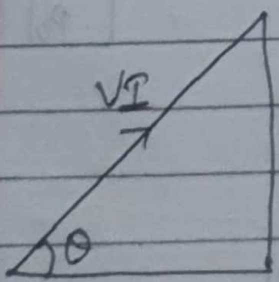
$V_R = IR = 0.31 \times 10 = 3.1\ V$

$V_L = I \cdot X_L = 0.31 \times 157 = 48.67\ V$

$V_C = I \cdot X_C = 0.31 \times 318.5 = 98.73\ V$

Power Triangle

WVI formula



$V_L I = VI\sin\theta$

$V_R I = VI\cos\theta$

(i) Active power (True or Real power) $(P) = VI\cos\phi$ — W (watt)

(ii) Reactive power $(Q) = VI\sin\phi$ — VAR (volt amp reactive)

(iii) Apparent power $(S) = VI$ — VA (volt amp)

(iv) Active component (wattful component) $= I\cos\phi$

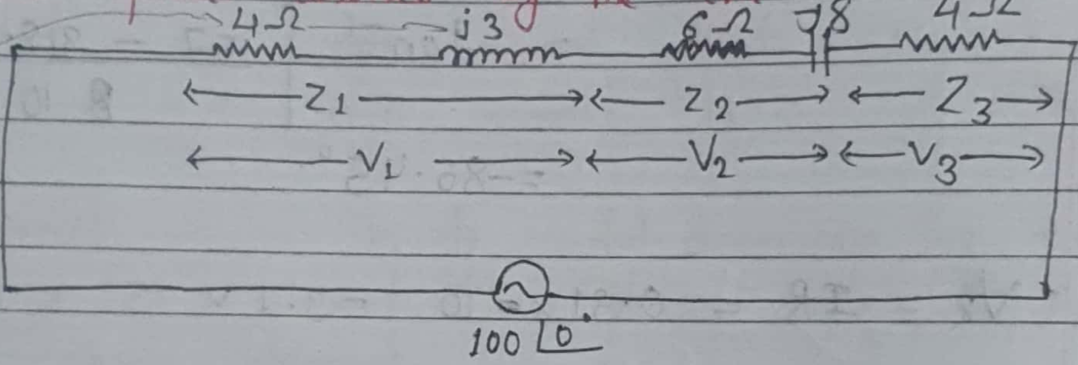
(v) Reactive comp. (wattless ") $= I\sin\phi$

$f = 50\,Hz$

~~(v) Reactive component (wattless comp.) = $I \sin\phi$~~

Q. In a figure below, calculate (i) current, (ii) voltage drops V_1, V_2 & V_3 (iii) power absorbed by each impedance & the total power absorbed by the ckt.

if L is given without 'j', find X_L then, $Z = R + jX_L$



Voltage $(V) = 100\angle 0^\circ$

$Z = Z_1 + Z_2 + Z_3$

$= (4 + j3) + (6 - j8) + (4 + j0)$

(rect form) $= (14 - 5j)\,\Omega$

(polar form) ~~Pol (14, -5)~~ $= 14.86\angle -19.65^\circ$

in calculator [Pol (14, -5)]

Now,

$I = \dfrac{V}{Z} = \dfrac{100\angle 0^\circ}{14.86\angle -19.65^\circ}$

$= \dfrac{100}{14.86}\angle 0 + 19.65^\circ$

$= 6.72\angle 19.65^\circ$

Again,

~~$V_1 = IR_1$~~ $V_1 = IZ_1 = 6.72\angle 19.65^\circ \times (4 + j3)$

$= 6.72\angle 19.65^\circ \times 5\angle 36.86^\circ$

$= 33.6\angle 56.51^\circ$

$= (18.54 + j\,28.02)\,V$

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$V_2 = IZ_2$

$= 6.72\angle 19.65^\circ \times (6 - j8)$

$= 6.72\angle 19.65^\circ \times 10\angle -53.13$

$= \cancel{6.72}\,67.2\angle -33.48$

$= (56.05 - 37.07j)V$

Hence capacitive.

Now,

$V_3 = IZ_3$

$= 6.72\angle 19.65^\circ \times 4.$

$= 26.88\angle 19.65^\circ \ V$

~~Numericals~~

* Power absorbed by each elements:

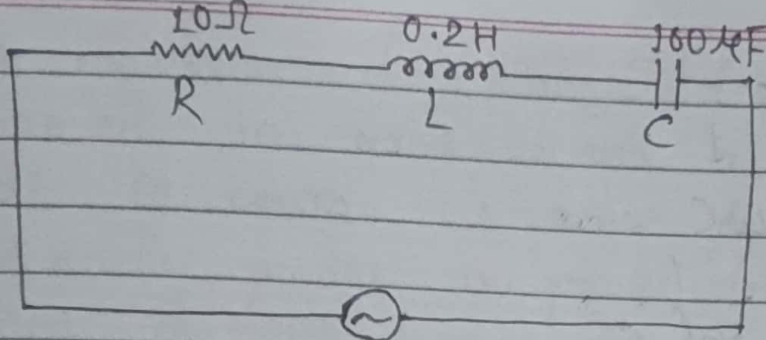
$P_1 = I^2 R_1$
$= (6.72)^2 \times 4$
$= 180.63$ watt

$P_2 = I^2 R_2$
$= (6.72)^2 \times 6$
$= 270.95$ watt

$P_3 = I^2 R_3$
$= (6.72)^2 \times 4$
$= 180.63$ watt

Now,
Power absorbed by the circuit,
$P_{total} = P_1 + P_2 + P_3$
$= (180.63 + 270.95 + 180.63)$ watt
$= 632.21$ watt

Q. A resistance of $10\,\Omega$, ~~a~~ inductance of 0.2 H and capacitor of $100\,\mu F$, are connected in series across 200V, 50 Hz supply mains. Determine
(i) impedance
(ii) current
(iii) voltage across R, L & C
(iv) power factor angle of lag (PF)
(v) power consumed in watt
(vi) VA



200V, 50 Hz

$X_L = \omega L = 2\pi f L = 314 \times 0.2 = 62.8\,\Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f \cdot C} = \dfrac{1}{314 \times 100 \times 10^{-6}} = 31.85\,\Omega$

(i) Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{10^2 + (62.8 - 31.85)^2}$

$= 32.52\,\Omega$

(ii) current, $I = \dfrac{V}{Z} = \dfrac{200}{32.52}$

$= 6.15$ amp

(iii) Voltage across ~~Resistor:~~ R, L, C:

$V_R = IR$

$= 6.15 \times 10 = 61.5\,V$

$V_L = I X_L = I \times \omega L$

$= I \times 2\pi f \cdot L$

$= 6.15 \times 2\pi \times 50 \times 0.2$

$= 386.41\,V$

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$$V_c = I \cdot X_c$$
$$= I \cdot \frac{1}{\omega C}$$
$$= I \cdot \frac{1}{2\pi f \cdot C}$$
$$= \frac{6.51}{314 \times 100 \times 10^{-6}}$$
$$= 207.32\,V$$

(iv) power factor angle of lag $= \frac{R}{Z} = \cos\phi$
$$= \frac{10}{32.52}$$
$$= 0.307$$

(v) Power consumed $= VI\cos\phi$
$$= 200 \times 6.15 \times 0.307$$
$$= 377.61$$

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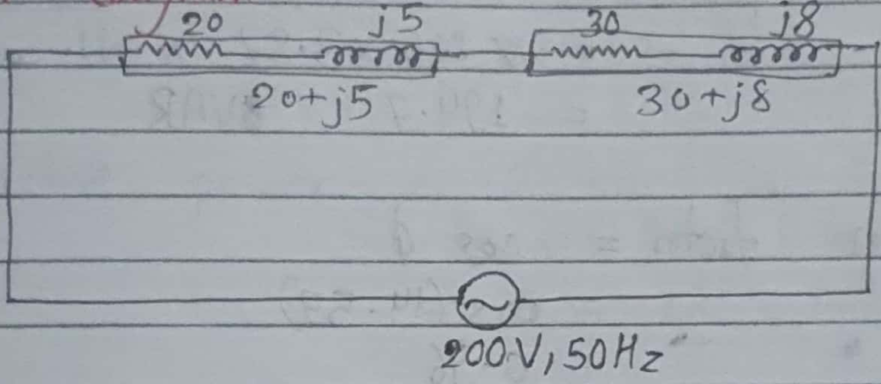
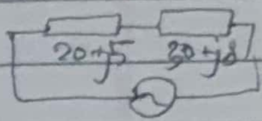
Q. Two impedance of $(20+j5)\,\Omega$ and $(30+j8)\,\Omega$ are connected in series across a 200 V, 50Hz supply. Find: (i) current (ii) active power (iii) apparent power (iv) reactive power (v) power factor & also draw a phasor diagram.

if $-ve$ j then capacitor



$\Rightarrow$ or



$$Z_T = Z_1 + Z_2$$
$$= (20+j5) + (30+j8)$$
$$= (50 + 13j)\,\Omega$$
$$= 51.66 \angle 14.57^\circ$$

$$V = 200 + j0 = 200 \angle 0^\circ$$

(i) $I = \dfrac{V}{Z} = \dfrac{200\angle 0^\circ}{51.66\angle 14.57^\circ} = 3.87 \angle -14.57^\circ$

~~Now,~~

(ii) $\cancel{\cos\phi} = \dfrac{R}{Z}$

$\cancel{\cos}\ \phi = \cos(14.57^\circ)$

active power $= VI\cos\phi$

$\cancel{= 200\angle 0^\circ \times 3.87 \angle -14.57}$

$= 200 \times 3.87 \times \cos(14.57)$

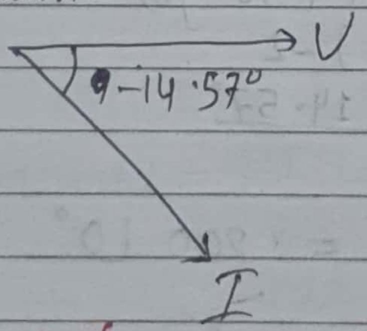
$= \cancel{11,277.18\ kW}\ 749.10\ kW$

(iii) apparent power $= VI$
$= 200 \times 3.87$
$= 774$ VA

(iv) reactive power $= VI \sin \phi$
$= 200 \times 3.87 \times \sqrt{1-\cos^2(14.57)}$
$= 194.7$ VAR

(v) power factor $= \cos \phi$
$= \cos(14.57)$
$= 0.96$

Phasor diagram:



Q. Two impedance of $(5+j10)\,\Omega$ & $(10+j5)\,\Omega$ are connected in series across 200V, 50Hz. Find (i) current (ii) active power (iii) reactive power (iv) apparent power (v) power factor & also draw phasor diagram.

solution:

$Z_1 = (5+j10)\,\Omega = 11.18 \angle 63.43°$
$Z_2 = (10+j5)\,\Omega = 11.18 \angle 26.56°$

$Z_T = Z_1 + Z_2 = (5+j10) + (10+j5)$
$= (15+j15)\,\Omega = 21.21 \angle 45°$

(i) $I = \dfrac{V}{Z_T} = \dfrac{200}{21.21\angle 45^\circ} = 9.42\angle -45^\circ$

(ii) active power $= VI\cos\phi$

$= 200 \times 9.42 \times \cos(45^\circ)$

$= 1332.18$ watt

(iii) reactive power $= VI\sin\phi$

$= 200 \times 9.42 \times \sin(45^\circ)$

$= 1332.18$ VAR

(iv) apparent power $= VI$

$= 200 \times 9.42$

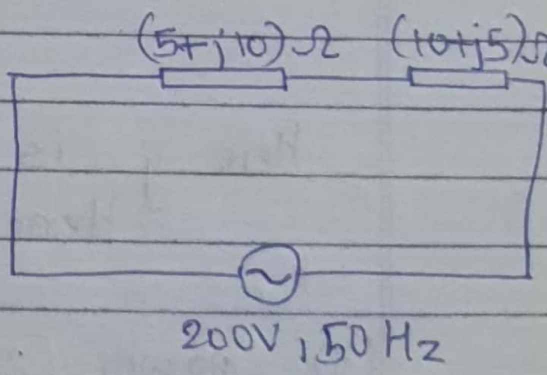
$= 1884$ VA

(v) power factor $= \cos\phi$

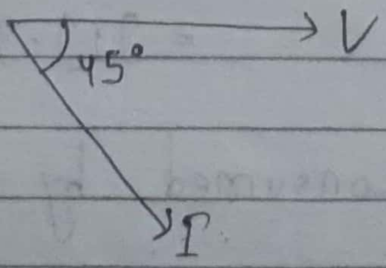
$= \cos(45^\circ)$

$= \dfrac{1}{\sqrt{2}}$

$= 0.70$



Phasor diagram:



Q. The ac voltage $V = (80 + j\,60)$ volt applied to a ckt resulting current of $(4 + j\,10)$ ampere. find
(i) impedance of ckt whether it is inductive or capacitive
(ii) power consumed
(iii) phase angle
(iv) reactive power

solution:

$V = 80 + j\,60 = 100 \angle 36.86^\circ$

$I = 4 + j\,10 = 10.77 \angle 68.19^\circ$

Now,

(i) Impedance (Z) $= \dfrac{V}{I}$

$= \dfrac{100 \angle 36.86^\circ}{10.77 \angle 68.19^\circ}$

$= 9.28 \angle -31.33$

$= 7.92 - j\,4.82$

Here j is −ve.
Hence capacitive.

(ii) power consumed $= VI \cos \phi$

$= 100 \times 10.77 \times \cos(31.33)$

$= 919.95$ watt

∴ Power consumed by given ckt $= 919.95$ watt

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(iii) phase angle (ϕ) $= 31.33°$

(iv) Reactive power:

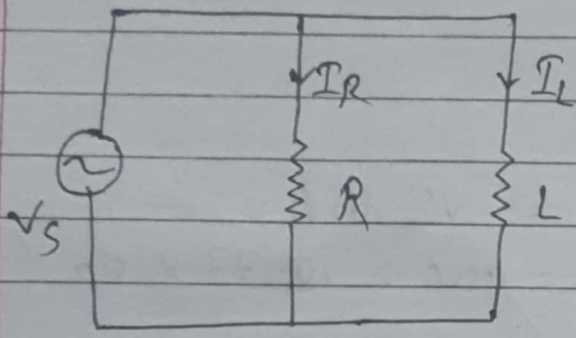
reactive power $= VI \sin\phi$

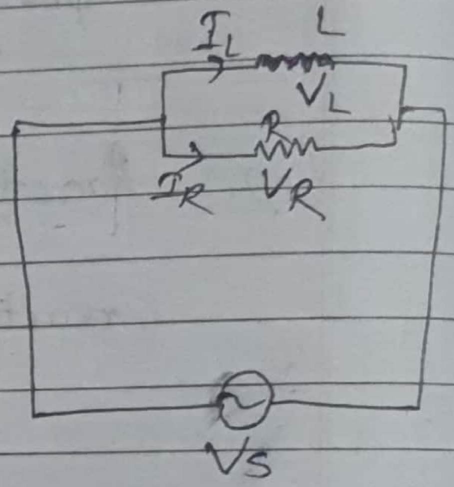
$= 100 \times 10.77 \times \sin(31.33)$

$= 560$ VAR

Parallel Circuit

1) Parallel R-L Ckt





Applied voltage $= V_s$

voltage drop across (R) $= V_R$

" " " (L) $= V_L$

Currents I_L lags behind the applied voltage.

$$I_R = \frac{V_s}{R}$$

$$I_L = \frac{V_s}{X_L}$$

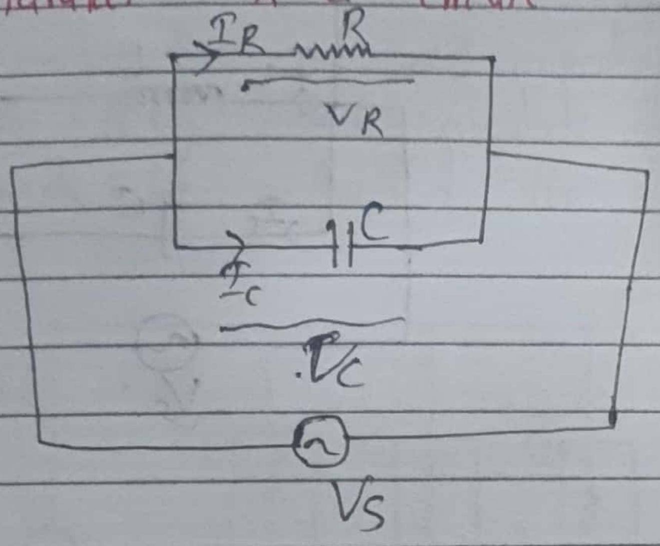
$$I = \sqrt{I_R^2 + I_L^2}$$

$$\tan\phi = \frac{I_L}{I_R} = \frac{V_s/X_L}{V_s/R} = \frac{R}{X_L}$$

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

$$\cos\phi = \frac{I_R}{\sqrt{I_R^2 + I_L^2}} = \frac{I_R}{I} = \text{power factor}$$

2) Parallel R–C circuit



$$I_R = \frac{V_s}{R} \quad \text{or,} \quad V_s = I_R \cdot R$$

$$I_c = \frac{V_s}{X_c} \quad \text{or,} \quad V_s = I_c \cdot X_c$$

$$\text{Resultant currents } (I) = \sqrt{I_R^2 + I_c^2}$$

$$\text{Power factor } (pf) = \frac{I_R}{\sqrt{I_R^2 + I_c^2}}$$

$$\tan\phi = \frac{I_c}{I_R} = \frac{V_s/X_c}{V_s/R} = \frac{R}{X_c}$$

$$\text{Phase angle } (\phi) = \tan^{-1}\left(\frac{R}{X_c}\right)$$

$$\cos\phi = \frac{I_R}{\sqrt{I_R^2 + I_c^2}}$$

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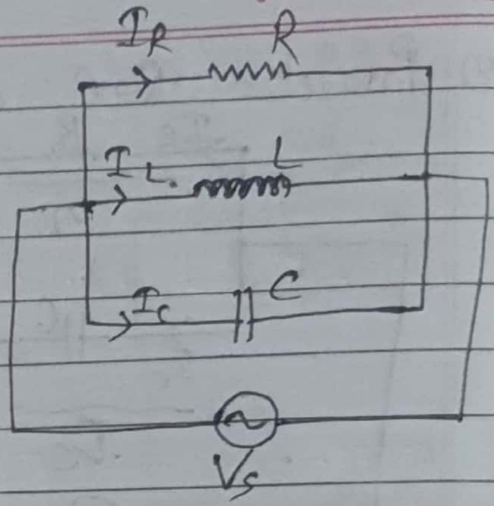
3) Parallel R-L-C ckt



$$V_S = I_R \cdot R$$
$$V_s = I_L \cdot X_L$$
$$V_s = I_C \cdot X_C$$

$$I_R = \frac{V_s}{R}$$

$$I_L = \frac{V_s}{X_L}$$

$$I_C = \frac{V_s}{X_C}$$

The resultant or total current :

$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

Power factor $= \cos\phi = \frac{I_R}{I}$

$$= \frac{I_R}{\sqrt{I_R^2 + (I_L - I_C)^2}}$$

$$\tan\phi = \frac{I_L - I_C}{I_R}$$

$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$= \tan^{-1}\left(\frac{V_s/X_L - V_s/X_C}{V_s/R}\right)$$

$$= \tan^{-1}\left[R\left(\frac{1}{X_L} - \frac{1}{X_C}\right)\right]$$

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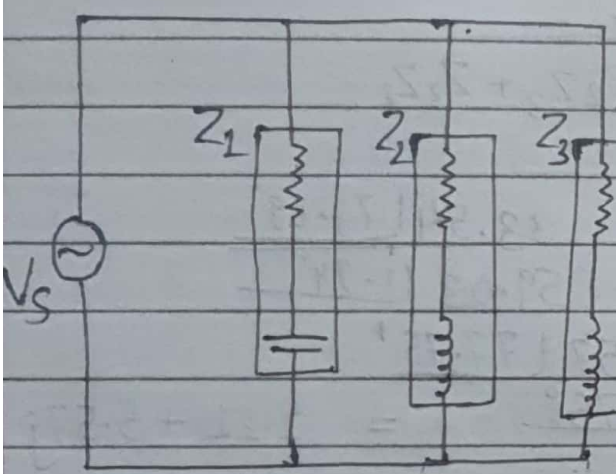
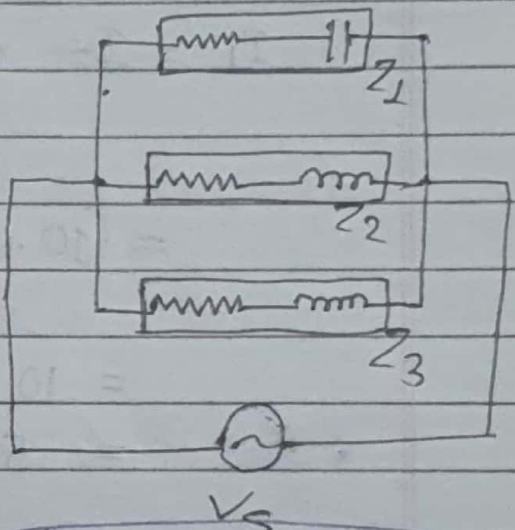
Problems of R-L-C Ckt:

Total current of 10 amp current flows through parallel combination of three impedances as $(2-j5)\Omega$, $(6+j3)\Omega$ & $(3+j4)\Omega$. Calculate the current flowing through each branch. Also find the power factor of the combination.

solution:



or,



$$Z_1 = (2-j5)\,\Omega$$
$$Z_2 = (6+j3)\,\Omega$$
$$Z_3 = (3+j4)\,\Omega$$

$$\cancel{Z_1 . Z_2 = (2-j5) \times (6+j3) =}$$
$$\cancel{Z_2 . Z_3 =}$$
$$\cancel{Z_3 . Z_1 =}$$
$$\cancel{Z_1 . Z_2 . Z_3 =}$$

$$\cancel{\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

$$(a+bj)(c+dj) = (ac-bd)+(ad+bc)j$$

$$Z_1 . Z_2 = (2-j5) \times (6+j3) = (27-j24)\,\Omega = 36.12\angle -41.63$$

$$Z_2 . Z_3 = (6+j3) \times (3+j4) = (6+j33)\,\Omega = 33.54\angle 79.69$$

$$Z_3 . Z_1 = (3+j4) \times (2-j5) = (26-j7)\,\Omega = 26.92\angle -15.06^\circ$$

$$Z_1 . Z_2 . Z_3 = (2-j5)(6+j3)(3+j4) = (177+j36)\,\Omega$$
$$= 180.62\angle 11.49^\circ$$

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$$Z_1 \cdot Z_2 + Z_2 \cdot Z_3 + Z_3 \cdot Z_1 = (27 - j24) + (6 + j33) + (26 - j7)$$

$$= (59 + j2)\ \Omega$$

$$= 59.03 \angle 1.94^\circ$$

Now,

$$I_1 = I_Z \times \frac{Z_2 \cdot Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

$$= (10 + j \cdot 0) \times \frac{33.54 \angle 79.69^\circ}{59.03 \angle 1.94^\circ}$$

$$= 10 \times 0.57 \angle 77.75^\circ$$

$$= 5.7 \angle 77.75^\circ \quad = 1.21 + 5.57j$$

$$I_2 = I_Z \times \frac{Z_3 \cdot Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

$$= 10 \times \frac{26.92 \angle -15.06}{59.03 \angle 1.94}$$

$$= 4.56 \angle -17^\circ$$

$$I_3 = I_Z \times \frac{Z_1 \cdot Z_2}{\cancel{Z_1 + Z_2}\ Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

$$= 10 \times \frac{36.12 \angle -41.63^\circ}{59.03 \angle 1.94}$$

$$= 6.1 \angle -43.57$$

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Total Z, $Z_T = \frac{Z_1 Z_2 Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}$

$= \cancel{\frac{180.62 \angle 11.49}{360.12 \angle -41.63^\circ + 33.54 \angle 79.69^\circ + 180.62 \angle 11.49^\circ}}$

$= \frac{180.62 \angle 11.49}{59.03 \angle 1.94}$

$= 3.05 \angle 9.55^\circ$

$= (3.01 + j\,0.51)\,\Omega$

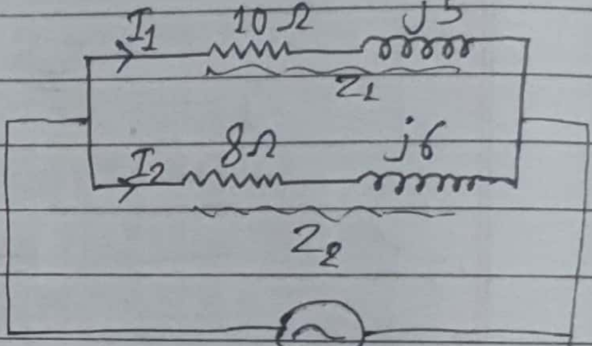
power factor $= \cos\phi$

$= \cos(9.55)$

$= 0.98$

Q. Two impedance given by $Z_1 = (10+j5)\,\Omega$ & $Z_2 = (8+j6)\,\Omega$ are joined in parallel and connected across the voltage of $V = (200+j0)$ v. Calculate the circuit current, its phase & the branch current & draw the vector diagram.

solution :-



$Z_1 = (10+j5)\,\Omega = 11.18\angle 26.56°$

$Z_2 = (8+j6)\,\Omega = 10\angle 36.86°$

$V = 200\,V$

$$Z_T = \frac{Z_1 . Z_2}{Z_1 + Z_2} = \frac{(10+j5)(8+j6)}{(10+j5)+(8+j6)}$$

$$= \frac{50+j100}{18+j11}$$

$$\cancel{= \frac{50\angle 0°}{14.86\angle 47.72}} \quad \cancel{= 3.36\angle -47.72}$$

$$= \frac{111.8\angle 63.43}{21.09\angle 31.42} = 5.30\angle 32.01°$$

Power factor $= \cos\phi = \cos(32.01) = 0.847$

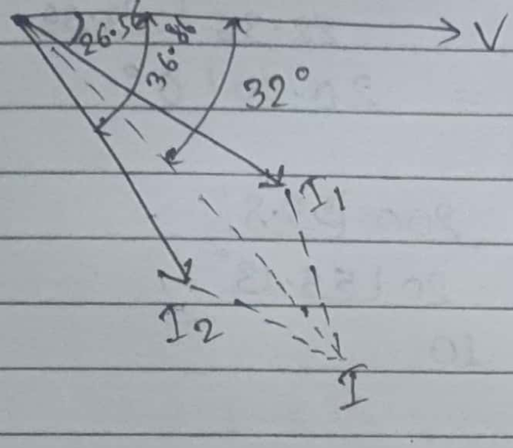
Now,

$\frac{Z \times Z_2}{Z_1+Z_2}$

$$I_1 = \frac{V_s}{Z_1} = \frac{200+j0}{11.18\angle 26.56°} = 17.88\angle -26.56°$$

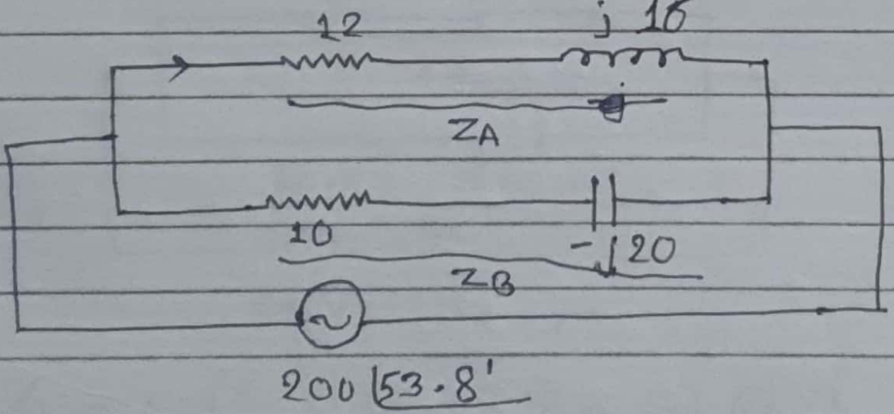
$$I_2 = \frac{V_s}{Z_2} = \frac{200}{10\angle 36.86°} = 20\angle -36.86°$$

Phasor diagram:



Q. A voltage of $200\angle 53.8°$ is applied across impedance in parallel. The value of impedance are $(12+j16)\Omega$ & $(10-j20)\Omega$. Determine the KVA, ~~KRI~~ KVAR & KW in each branch & power factor of whole ckt.

solution:



Ans

$I_A = 10 + j0$

$I_B = -4.0 + j8$

$KW = 1.2$

$KVAR = 1.6$

$KVA = 2$

$pf = \cos\phi = \frac{R}{Z}$

Here,

$V = 200\angle 53.8'$

Also,

$Z_1 = (12+j16)\Omega = 20\angle 53.13°$

$Z_2 = (10-j20)\Omega = 22.36\angle -63.43°$

$Z_1 \cdot Z_2 = (12+j16)(10-j20) = 440-80j = 447.21\angle -10.30$

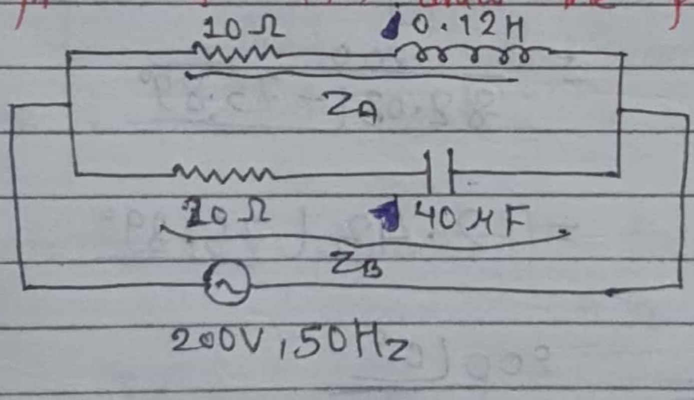
$Z_1 + Z_2 = 12+j16 + 10 - j20 = (22-4j)\Omega = 22.36\angle -10.30$

$$Z_T = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{447.21 \angle -10.30}{22.36 \angle -10.30} = 20.00 \angle 0^\circ$$

$$I_A = \frac{V}{Z_1} = \frac{200 \angle 53.8^\circ}{20 \angle 53.13^\circ} = 10$$

$$I_B = \frac{V}{Z_2} = \frac{200 \angle 53.8^\circ}{22.36 \angle -63.46} =$$

Q. Two circuits A & B are connected in parallel across 200V, 50Hz main supply. Ckt A consist of resistance of 10 Ω & inductance of 0.12H connected in series, ckt B consists of a resistance 20 Ω in series with capacitor of 40 μF. Calculate
(i) current in each branch, (ii) source current
(iii) pf & (iv) draw the phasor diagram.



$$X_L = 2\pi f L = 314 \times 0.12 = 37.67\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{314 \times 40 \times 10^{-6}} = 79.62\,\Omega$$

$$Z_A = 10 + j\,37.7\,\Omega$$

$$Z_B = 20 - j\,79.61\,\Omega$$

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$$Z_{eq} = \frac{Z_A \cdot Z_B}{Z_A + Z_B}$$

$$= 62.11 \angle 53.65^\circ$$

$$\Rightarrow Z_{eq} = (36.81 + j\,50.02)\,\Omega$$

Branch currents:

(i) $$I_A = \frac{V}{Z_A} = \frac{200 \angle 0^\circ}{10 + j\,37.7}$$

$$= \frac{200}{39 \angle 75.14^\circ}$$

$$= 5.12 \angle -75.14^\circ$$

$$I_B = \frac{V}{Z_B} = \frac{200 \angle 0^\circ}{20 - j\,79.61}$$

$$= \frac{200}{82.08 \angle -75.89^\circ}$$

$$= 2.43 \angle 75.89^\circ$$

(ii) $$I = \frac{V}{Z_{eq}} = \frac{200 \angle 0^\circ}{62.11 \angle 53.65^\circ}$$

$$= 3.22 \angle -53.65^\circ$$

(iii) power factor (pf) $= \cos\phi = \cancel{\frac{R}{Z_{eq}}}$

$$= \cos(53.65)$$

$$= 0.59$$

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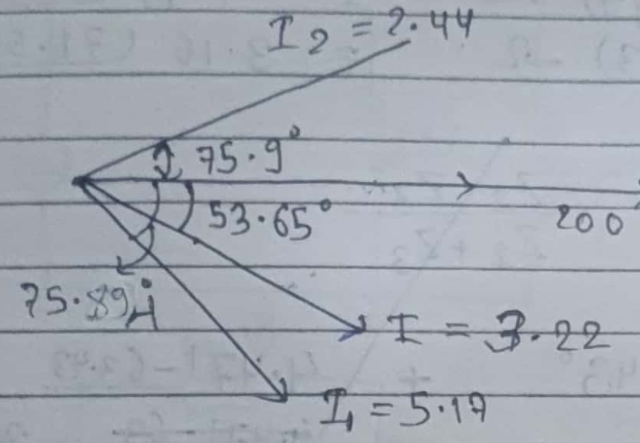
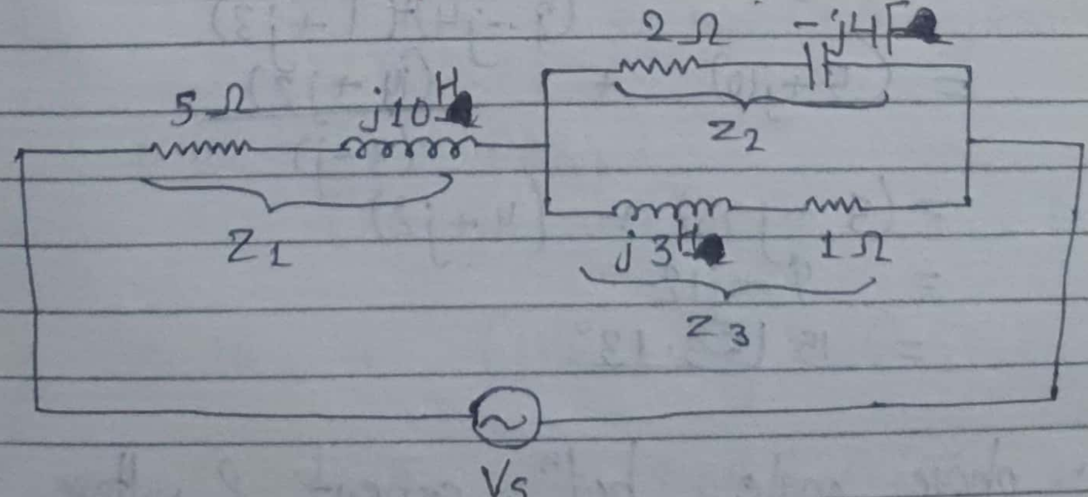
(iv) phasor diagram



Compound ckt

In many cases, the ac ckt to be analysed consist of combination of series & parallel impedance ckt. This types are knoown as series-parallel or compound ckt. Compound ckt can be simplied in the same manner as series - parallel ckt.

Q.1) Determine the equivalent Impedance of the given ckt.



$$Z_1 = (5+j10)\,\Omega = 11.18\angle 63.43^\circ$$
$$Z_2 = (2-j4)\,\Omega = 4.47\angle -63.43$$
$$Z_3 = (1+j3)\,\Omega = 3.16\angle 71.56^\circ$$

$$Z_T = Z_1 + \frac{Z_2 Z_3}{Z_2+Z_3}$$

$$= 11.18\angle 63.43^\circ + \frac{4.47\angle -63.43 \times 3.16\angle 71.56}{\cancel{4.47\angle -63}\quad 3-j1}$$

$$= 11.18\angle 63.43^\circ + \frac{14.12\angle 8.18^\circ}{3.16\angle 18.43^\circ}$$

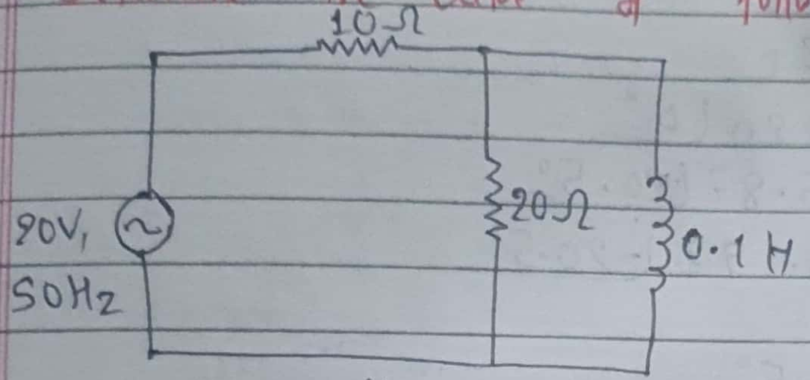
$$= 11.18\angle 63.43^\circ + 4.46\angle -10.25$$

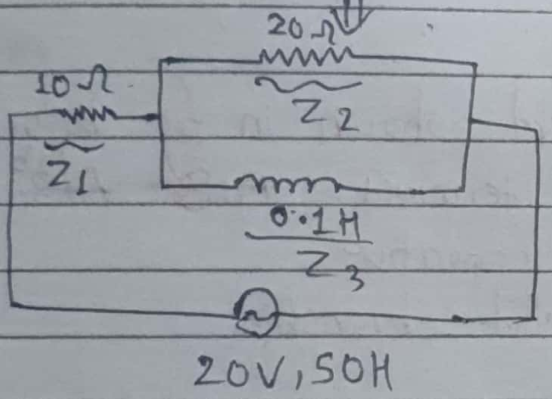
$$Z_T = Z_1 + \frac{Z_2 \cdot Z_3}{Z_2+Z_3}$$
$$= (5+j10) + \frac{(2-j4)(1+j3)}{(2-j4)+(1+j3)}$$
$$= (5+j10) + \frac{(14+j2)}{(3-j)}$$
$$= (5+j10) + (4+j2)$$
$$= 9+j12$$
$$= 15\angle 53.13^\circ$$

∴ phase angle betn current & voltage is

$$\cos\phi = \cos(53.13)$$
$$= 0.60$$

Q.2) Determine the value of following: (i) Z_T, (ii) I_T, (iii) ϕ.





$Z_1 = (10 + j0)\ \Omega$

$Z_2 = (20 + j0)\ \Omega$

$Z_3 = 0 + j \cdot X_L$

$= 0 + j \cdot 2\pi f L$

$= 0 + j \times 314 \times 0.1$

$= (0 + j \cdot 31.4)\ \Omega$

Now,

(i) $Z_T = Z_1 + \dfrac{Z_2 \cdot Z_3}{Z_2 + Z_3}$

$= 10 + j0 + \dfrac{(20 + j0)(0 + j \times 31.4)}{20 + j0 + 0 + j \times 31.4}$

$= 10 + \dfrac{(0 + 628j)}{(20 + j \cdot 31.4)}$

$= 10 + (14.22 + j\, 9.06)$

$= (24.22 + j\, 9.06)\ \Omega$

$= 25.86 \angle 20.5^\circ$

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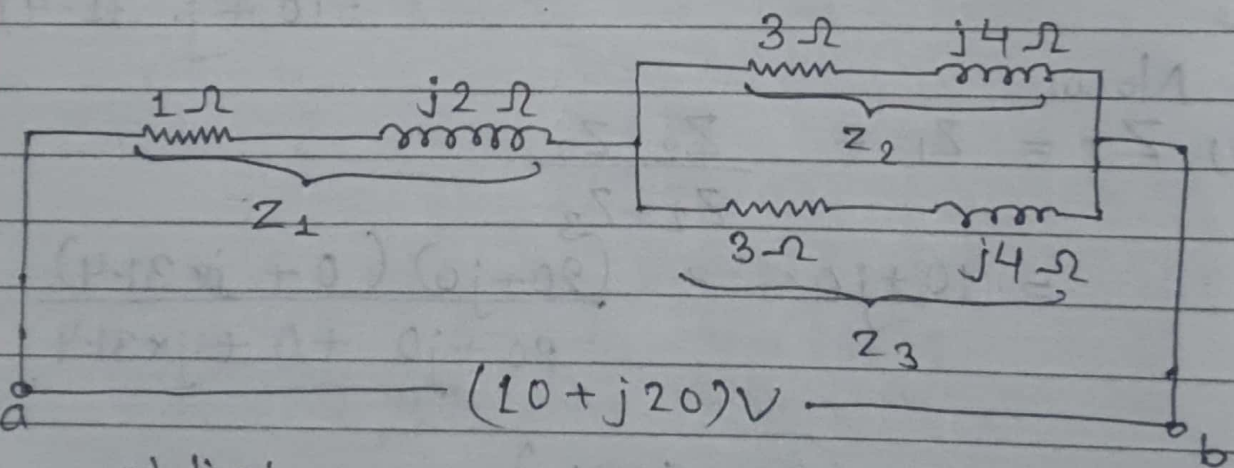
(ii) $I_T = \dfrac{V_s}{Z_T}$

$= \dfrac{20\angle 0^\circ}{25.87\angle 20.5^\circ}$

$= 0.77\angle -20.5^\circ$

(iii) $\phi = 20.5^\circ$

Q. For the series - parallel ckt shown in fig below, determine
(i) total impedance betn terminal ~~A & B~~ a, b state if it is inductive or capacitive.
(ii) voltage ~~current~~ ~~voltage~~ across the parallel branch.
(iii) phase angle.



solution:

$Z_1 = (1+j2)\,\Omega$

$Z_2 = (3+j4)\,\Omega$

$Z_3 = (3+j4)\,\Omega$

We have,

(i) $Z_T = Z_1 + \dfrac{Z_2 \cdot Z_3}{Z_2 + Z_3}$

$= (1+j2) + \dfrac{(3+j4)(3+j4)}{(3+j4)+(3+j4)}$

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$$= (1+j2) + \frac{-7+24j}{6+j8}$$

$$= 1+j2 + \frac{3}{2} + 2j$$

$$\therefore Z_T = (2.5+j4)\ \Omega$$

Hence, the given ckt is inductive.

$$I_T = \frac{V_s}{Z_T} = \frac{10+j20}{2.5+j4}$$

$$= 4.72 + j0.45$$

$$= 4.73 \angle 5.44^\circ$$

i.e. The current lags behind the voltage by 5.44°.

(ii) Voltage across parallel branch,

~~$V = I_T Z_{eq}$~~

~~$= 4.73 \angle 5.44 \times 4.5 \angle 61.18$~~

$$Z_{eq} = \frac{Z_2 \cdot Z_3}{Z_2 + Z_3} = 1.5 + j2$$

Now,

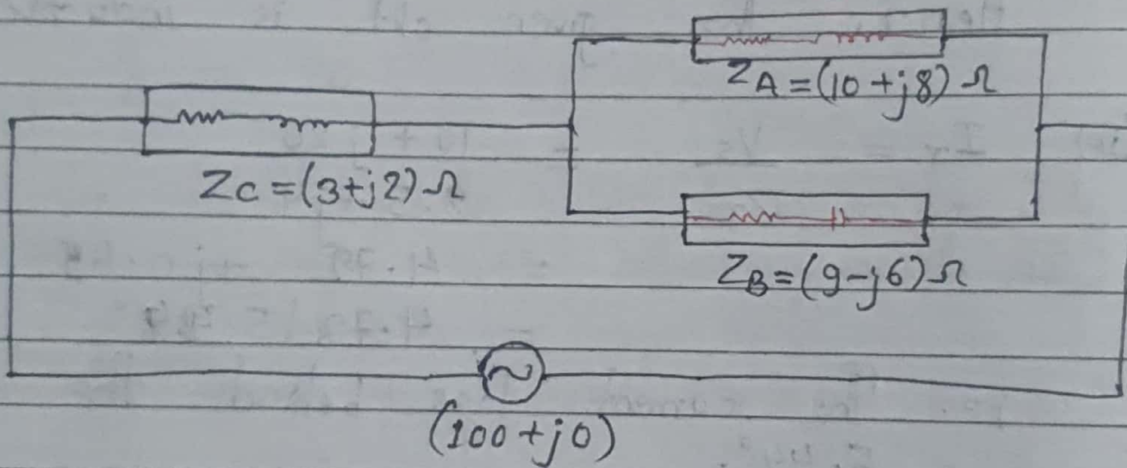
$$V = I_T \cdot Z_{eq}$$

$$= 4.74 \angle 5.44^\circ \times (1.5 + j2)$$

$$= 4.74 \angle 5.44^\circ \times 2.5 \angle 53.13$$

$$= 11.85 \angle 58.57^\circ$$

Q. In series - parallel ckt, shown in fig. below, the two parallel branches A & B are in series with C. The impedances are $Z_A = (10 + j8)\,\Omega$, $Z_B = (9 - j6)\,\Omega$ & $Z_C = (3+j2)\,\Omega$ & the voltage across the ckt is $(100 + j0)$ V. Find the current I_A, I_B & phase angle betⁿ them.



$$Z_T = Z_C + \frac{Z_A \cdot Z_B}{Z_A + Z_B}$$

$$= (3+j2) + \frac{(10+j8)(9-j6)}{(10+j8)+(9-j6)}$$

$$= (3+j2) + \frac{(138 + 12j)}{(19+j2)}$$

$$= (3+j2) + (7.24 - 0.13j)$$

$$= 10.24 + j\,1.87$$

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Total current, $I_T = \frac{V_s}{Z_T}$

$= \frac{(100 + j0)}{10.25 + j1.87}$

$= \cancel{9.5 \angle -10.34}\ 9.44 - 1.72j$

$= 9.59 \angle -10.34^\circ$

The current lags behind the voltage by 10.34°.

Now,

Current in branch 'A' is

$I_A = I_T \times \frac{Z_B}{Z_A + Z_B}$

$= 9.59 \angle -10.34 \times \frac{9 - j6}{19 + j2}$

$= 9.59 \angle -10.34 \times \frac{10.82 \angle -33.69^\circ}{19.10 \angle 6.00^\circ}$

$= 5.43 \angle -50.03^\circ$

$I_B = I_T \times \frac{Z_A}{Z_A + Z_B}$

$= 9.6 \angle -16.34^\circ \times \frac{(10 + j8)}{19 + j2}$

$= 6.42 \angle 22.32^\circ$

$\therefore$ The $\angle$ betn I_A & I_B is

$\phi = 50.04^\circ + 22.32^\circ$

$= 72.36^\circ$

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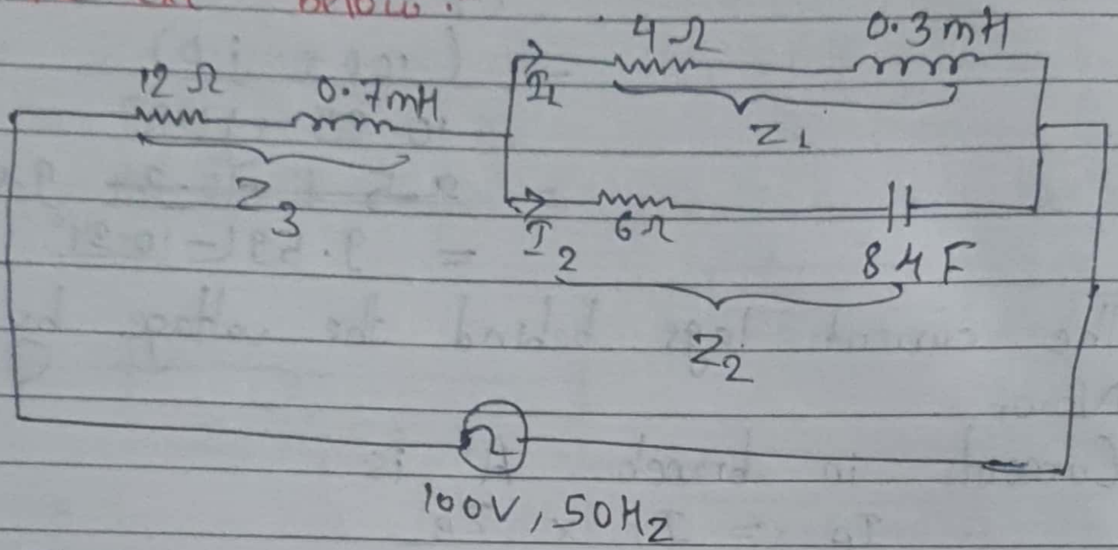
Q. Determine the total impedance & current in each branch of the ckt below:-



$$Z_3 = 12 + j \cdot X_L$$
$$= 12 + j \cdot 2\pi f \cdot L = 12 + j \cdot 314 \times 0.7 \times 10^{-3}$$
$$= (12 + j\, 0.22)\, \Omega$$

$$Z_1 = 4 + j \cdot X_L$$
$$= 4 + j \cdot 2\pi f \cdot L = 4 + j \cdot 314 \times 0.3 \times 10^{-3}$$
$$= (4 + j \cdot 0.09)\, \Omega$$

$$Z_2 = 6 - j \cdot \frac{1}{314 \times 8 \times 10^{-6}}$$
$$= 6 - j \cdot 398.09$$

Now,

$$Z_T = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$
$$= (12 + j\, 0.22) + \frac{(4 + j\, 0.09)(6 - j\, 398.09)}{(4 + j\, 0.09) + (6 - j\, 398.09)}$$
$$= (12 + j\, 0.22) + \frac{(59.83 - 1591.8i)}{10 - 398j}$$

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$= \cancel{4}\ 12 + j0.22^{\circ} + 4 + 0.05j$

$= 16 + j0.27$

$= 16 \angle 0.97^{\circ}$

Now:

~~Curre~~ $I_T = \frac{V}{Z_t} = \frac{100}{16 \angle 0.97} = 6.25 \angle -0.97^{\circ}$

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Power factor and its significance:

Power factor is cosine of angle betⁿ applied voltage & the resultant current ~~following~~ flowing in circuit, the power factor is normally called lagging when the currents lags the applied voltage, & leading when current leads the voltage. It is also the ratio of active power (P) to apparent power (S).

Importance of pf

$$pf = \frac{\text{active power}}{\text{apparent power}}$$

It is also ratio of active power to apparent power

$$pf = \frac{watt}{VA} = \frac{\cancel{VI}\cos\phi}{\cancel{VI}} = \cos\phi$$

In other words, pf is also the ratio of resistance to impedance of the ckt.

$$pf = \frac{\text{Resistance}(R)}{\text{Impedence}(Z)} = 0 \text{ to } 1$$

$$\left[\cos\phi = \frac{R}{Z}\right] \quad (0 \text{ to } 1)$$

Low pf possesses the following drawbacks:

a) The current ~~leads~~ needed ~~the~~ to obtain a given power is high, this in-turn results in the increased resistive losses, thereby resulting in decreased effeciency.

b) It causes poor voltage regulation.

c) It limits the output of generator & transformer, since higher current is drawn out from these results in

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the increase in temp.

d) It causes fall in terminal voltage.
e) It causes more power loss in line & there by reducing line efficiency.
f) It reduces KWh, since the true power is loss.
g) It reduces the torque of the consumer motor.

Use of Capacitor for improving pf.

In order to improve pf, capacitor are connected in parallel with the ckt (say an electric motor), since the capacitor draw current leading in phase by $\pi/2$ ∴ radian (90°). So, leading current compensates (reduces) the current req^d by inductive load, thereby the pf improves.

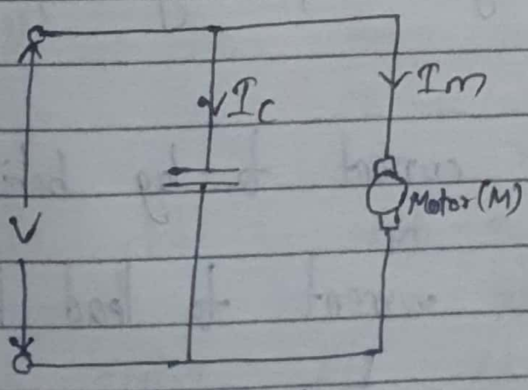

fig 'a' - use of capacitor for improving pf of a motor

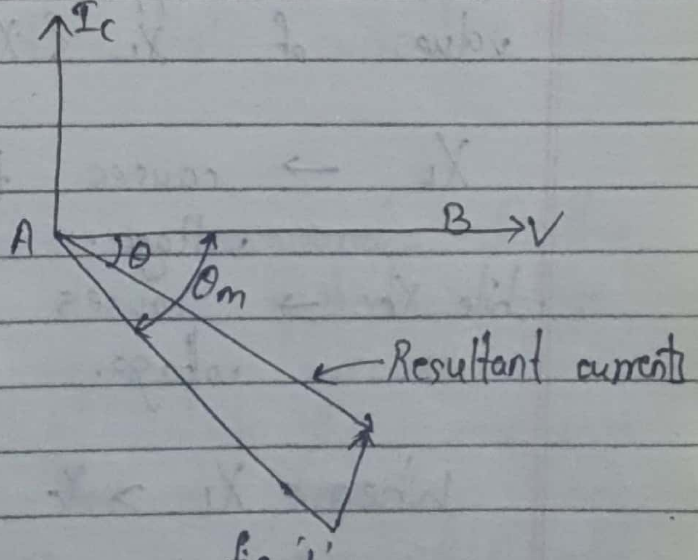

fig 'b'

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Resonance in Series and Parallel RLC ckt:

i) Series Resonance
ii) Parallel Resonance

Series Resonance

In many of electrical ckt, resonance is very important phenomenon. The study of resonance is very useful particularly in the area of communication. for example, the ability of radio receiver to select the certain frequency, transmitted by a station & eliminate the frequencies from other stations is based on the principle of resonance.

In R-L-C series ckt:

In RLC series ckt, the current lags behind or leads the applied voltage depending upon the value of X_L & X_C.

$X_L \rightarrow$ causes total current to lag behind the applied voltage.

while $X_C \rightarrow$ causes total current to lead behind the applied voltage.

When $X_L > X_C$ [ckt is predominantly inductive]

when $X_C > X_L$ [ckt is predominantly capacitive]

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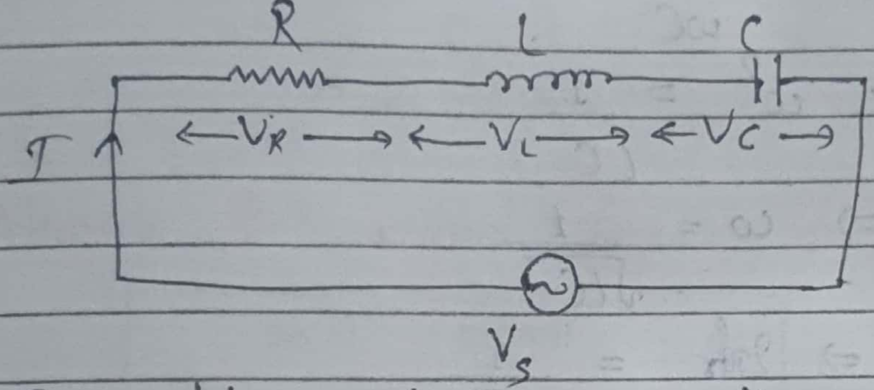
However, if one of the parameters of RLC ckt varies in such a way that current in the ckt is inphase with applied voltage. Then ckt is said to be in resonance.

Consider a series R-L-C ckt shown in fig-(c) below:



The total impedance for the series RLC ckt:

$$\boxed{\begin{aligned} Z &= R + j(X_L - X_C) \\ &= R + j\left(\omega L - \frac{1}{\omega C}\right) \end{aligned}}$$

It is clear from the ckt that current

$$\boxed{I = \frac{V_s}{Z}}$$

The ckt is said to be in resonance if the current ~~in~~ is in phase with applied voltage. In series RLC ckt, resonance occurs when $X_L = X_C$, the frequency at which the resonance occur is called the resonant frequency. Since, $X_L = X_C$; then impedance in series RLC ckt is purely resistive. At the resonant frequency (f_r), the voltage across the capacitance & inductance are equal in magnitude since they are 180° out of phase.

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with each other. Hence the zero voltage appear across the L-C combination.

At resonance,

$$X_L = X_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

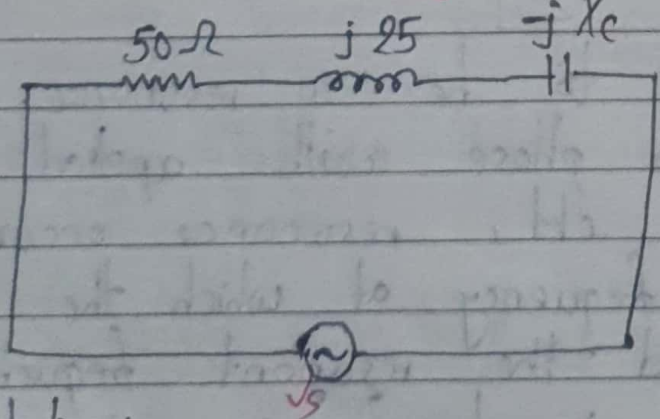
$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow 2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Q. Determine the value of a capacitive reactance & impedance at resonance in fig. below



Solution:

$X_L = X_C$

$\Rightarrow X_L = 25$

$X_C = 25$

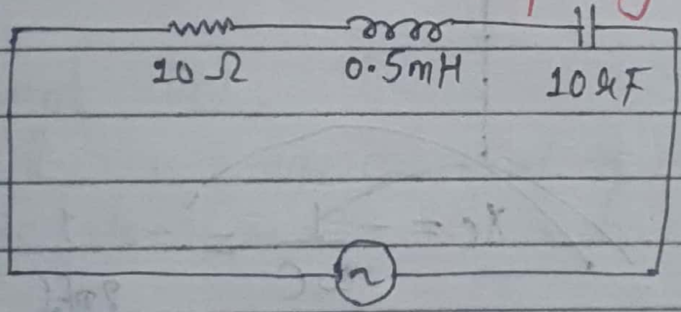
$\therefore \frac{1}{\omega C} = 25$

The value of impedance at resonance

$Z = R$

$\Rightarrow Z = 50\,\Omega$

Q. Determine the resonant frequency for the ckt shown below:



Here,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.5\times10^{-3}\times10\times10^{-6}}}$$

$$= 2250.79 \text{ Hz}$$

Difference betⁿ Series resonance & Parallel re.

Series Resonance	Parallel Resonance
(I) Series resonance occurs when the arrangement of the components creates the minimum impedance.	(I) Parallel resonance occurs when the arrangement of components creates the largest impedance.
(II) It refers to the resonance that occurs in circuits where capacitors and inductors are connected in series.	(II) It refers to the resonance that occurs in circuits where the capacitors and inductors are connected in parallel.
(III) In series resonance, a series RLC ckt contains minimum impedance at resonant frequency	(III) In parallel resonance, a parallel RLC ckt contains maximum impedance at resonant frequency.
(IV) In series resonance, maximum possible current flows through the ckt.	(IV) In parallel resonance, minimum ~~possible~~ current flows through the ckt.
(V) acceptor ckt	(V) rejector ckt

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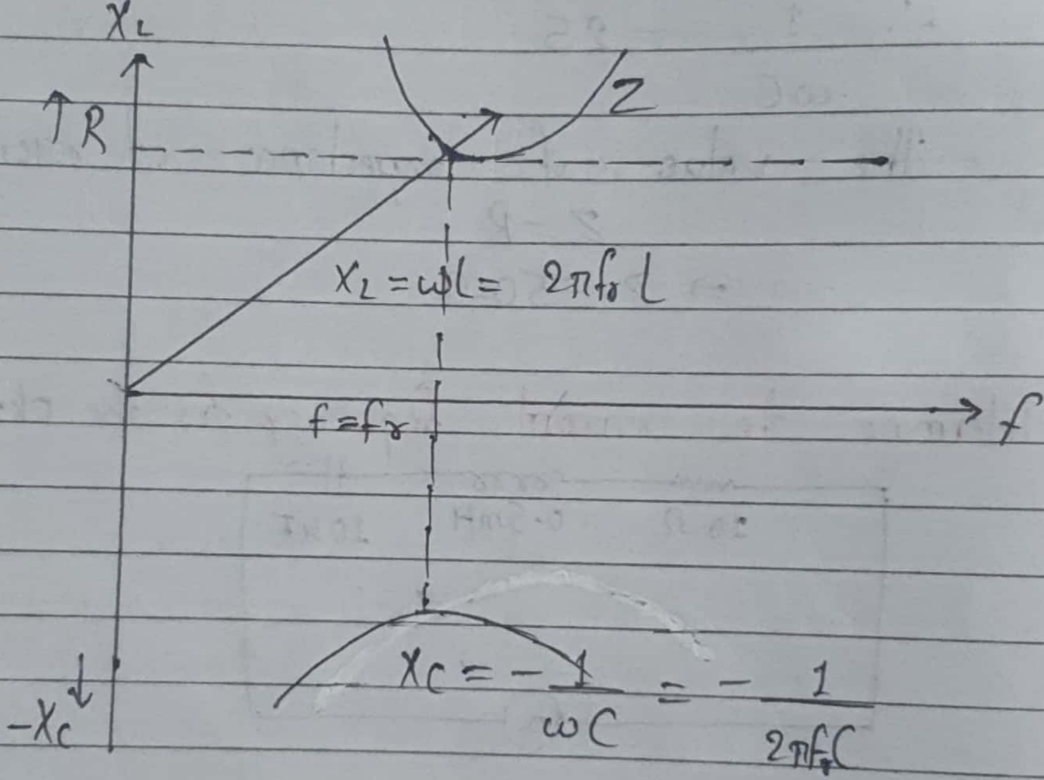
Variation of resistance, inductive reactance, capacitive reactance, impedance & current with ~~fg~~ frequency in RLC series ckt.



(a) Resistance:

It is independent of frequency & ~~st. line~~ represents st. line.

(b) Inductive reactance (X_L):

$$X_L = \omega L = 2\pi f_r L$$

X_L is directly proportional to f_r. i.e. X_L linearly increases with frequency & passes through origin.

(c) Capacitive reactance (X_C):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f_r . C}$$

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X_C is inversely prop. to f. Its graph is asymptote to horizontal axis.

(d) Impedance (Z):

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At low frequency, Z is large because X_C is large. Also, at high frequency Z is again large because X_L is large.

(e) Current (I):

It is reciprocal of ~~it~~ impedance so when Z is low, I is high & vice versa.

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Resonance Curve

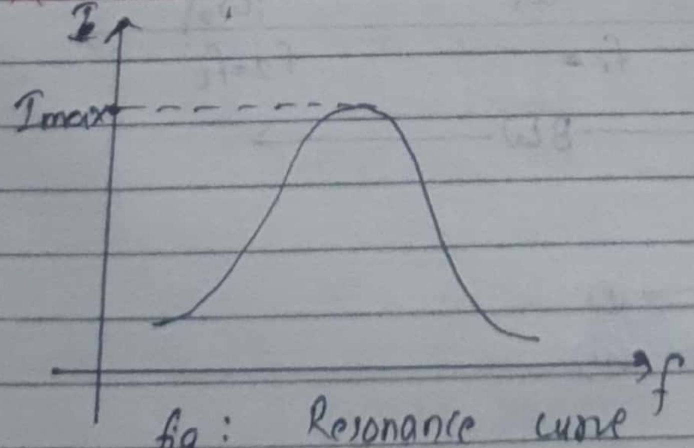

fig: Resonance curve

The graphical relation betn magnitude of current & the frequency of applied emf is called the resonance curve. It is clear that the current increases as the frequency increases. It becomes maximum at particular frequency, this frequency is called resonant frequency.

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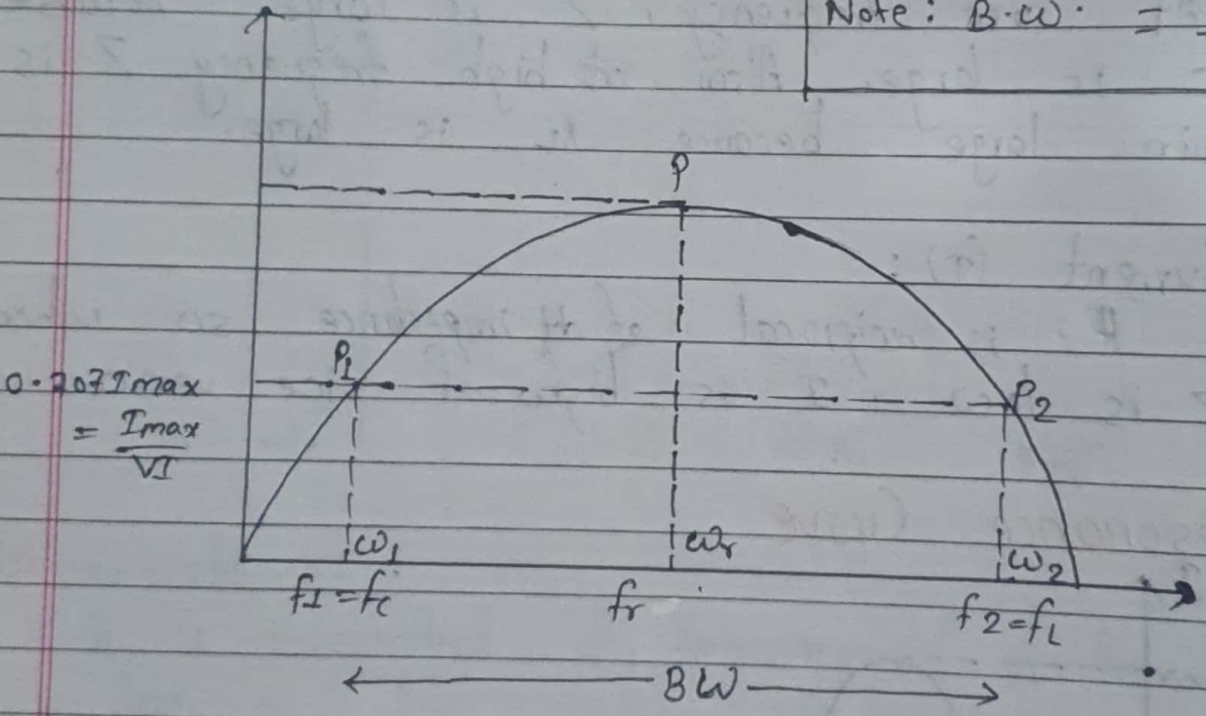
Band width of a RLC $_{series}$ ckt

The band width of any system is the range of frequencies. It is denoted by BW ~~In fi~~ shown in fig. below: The unit of band width is Hz.

Note : $B.W. = \frac{fr}{Q}$ (quality factor)



$$X_C - X_L = R$$
$$X_L - X_C = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{——— (I)}$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{——— (II)}$$

Equating (I) & (II), we get

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\Rightarrow \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = L(\omega_1 + \omega_2)$$

$$\Rightarrow \frac{1}{C}\left(\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right) = L\,\cancel{(\omega_1 + \omega_2)}$$

$$\Rightarrow \frac{1}{C}\left[\frac{1}{\omega_1 \omega_2}\right] = L \quad \text{—— (III)}$$

$$\omega_1 \omega_2 = \frac{1}{LC} \qquad [\because \cancel{\omega_1 \omega_2 = \omega_r^2}]$$

$$\cancel{\omega_r^2 = \frac{1}{LC}} \quad \text{—— (IV)}$$

If we add eq^n (I) & (II), we get

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R \quad \text{—— (V)}$$

Since, $\omega_r^2 = \frac{1}{LC}$

$$\Rightarrow C = \frac{1}{\omega_r^2} \cdot L$$

$$\Rightarrow \omega_1 \omega_2 = \omega_r^2$$

$$(\omega_2 - \omega_1)L + \frac{(\omega_2 - \omega_1)\,\omega_r^2 L}{\omega_r^2} = 2R \quad \text{—— (VI)}$$

$$(\omega_2 - \omega_1)L + (\omega_2 - \omega_1)L = 2R$$

$$\Rightarrow \cancel{2}L(\omega_2 - \omega_1) = \cancel{2}R$$

$$\Rightarrow \omega_2 - \omega_1 = \frac{R}{L} \quad \text{—— (VII)}$$

$$f_2 - f_1 = \frac{R}{2\pi L} \quad \text{--- (VIII)}$$

$$\boxed{\text{Band width} = \frac{R}{2\pi L}}$$

$$~~f_r - f_1 = \frac{R}{4\pi L}~~$$

$$f_1 = f_r - \frac{R}{4\pi L} \Rightarrow \text{The lower freq. limit} \text{ --- (}$$

$$f_2 = f_r + \frac{R}{4\pi L} \Rightarrow \text{The upper freq. limit} \text{ --- (X}$$

Imp for short notes

Quality factor(Q) of R-L-C series ckt :

Quality factor (Q) is defined as the ratio of resonant freq. to the band width of the ckt.

i.e.

$$Q = \frac{\text{resonant freq.}}{\text{band width}}$$

$$= \frac{1/2\pi\sqrt{LC}}{R/2\pi L}$$

$$\therefore \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

It is also the ratio of inductive reactance of the coil to its resistence.

So,

$$Q = \frac{X_L}{R}$$

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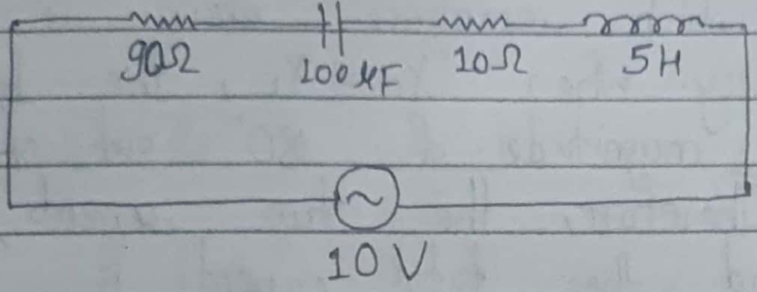
Q. In given ckt below, determine the value of Q at resonant
PU & band width of the ckt.



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5 \times 100 \times 10^{-6}}}$$

$$= 7.11 \text{ Hz}$$

Quality factor (Q) $= \frac{X_L}{R}$

$$= \frac{\omega_r L}{R}$$

$$= \frac{2\pi f_r \cdot L}{R}$$

$$= \frac{2\pi \times 7.11 \times 5}{100}$$

$$= 2.23$$

Band-width $= \frac{f_r}{Q}$

$$= \frac{7.11}{2.23}$$

$$= 3.18 \text{ Hz}$$

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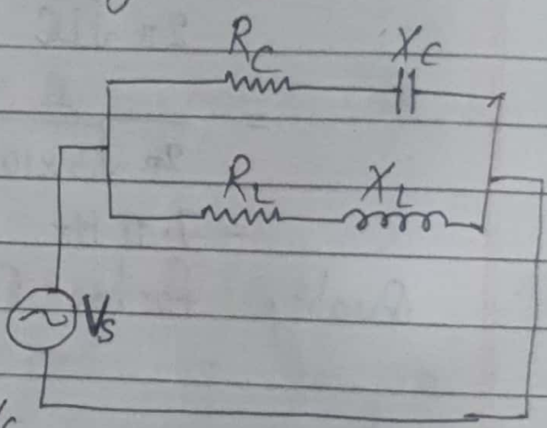
Parallel Resonance Ckt:

Basically, parallel resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the resonant frequency. When $X_C = X_L$, the branch current are equal in magnitude & 180° out of phase with each other. Therefore, the two currents cancel each other out, and the total current is 'zero'. Consider the ckt shown in fig. below:



$X_L = X_C$

For total admittance,

$$Y = \frac{1}{R_L + j \cdot X_L} + \frac{1}{R_C - j X_C}$$

$$Y = \frac{R_L - j \cdot X_L}{R_L^2 + X_L^2} + \frac{R_C + j X_C}{R_C^2 + X_C^2}$$

$$Y = \frac{R_L - j \cdot \omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j X_C}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

$$= \underbrace{\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}}_{\text{'zero'}} + j\left[\left(\frac{1/\omega C}{R_C^2 + \frac{1}{\omega^2 C^2}}\right) - \left(\frac{\omega L}{R_L^2 + \omega^2 L^2}\right)\right] \quad \text{—— (I)}$$

At resonance, the susceptance part becomes zero:

$$\frac{\frac{1}{\omega_r C}}{R_C^2 + \frac{1}{\omega_r^2 C^2}} = \frac{\omega_r \cdot L}{R_L^2 + \omega_r^2 \cdot L^2} \quad \text{—— (II)}$$

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or $\omega_r \cdot L \left[R_C^2 + \frac{1}{\omega_r^2 C^2} \right] = \frac{1}{\omega_r \cdot C} \left[R_L^2 + \omega_r^2 L^2 \right]$

or $\omega_r^2 \left[R_C^2 + \frac{1}{\omega_r^2 \cdot C^2} \right] = \frac{1}{LC} \left[R_L^2 + \omega_r^2 \cdot L^2 \right]$

or $\omega_r^2 \cdot R_C^2 - \frac{\omega_r^2 \cdot L}{C} = \frac{1}{LC} \left[R_L^2 - \frac{L}{C^2} \right]$

or $\omega_r^2 \left[R_C^2 - \frac{L}{C} \right] = \frac{1}{LC} \left[R_L^2 - \frac{L}{C} \right]$

$$\Rightarrow \boxed{\omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - L/C}}} \quad \text{—— (III)}$$

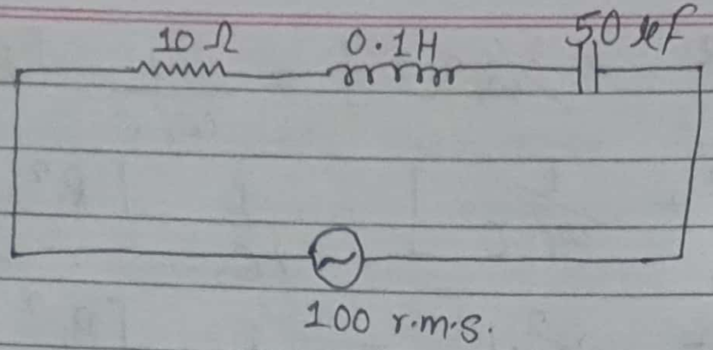
The condition for resonant frequency is given by eq^n (III). In special case,

$R_L = R_C$

So, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Q. For the ckt shown in fig. below, determine the frequency at which the ckt resonates. Also, find the voltage across the inductor as resonance & the Q factor of the ckt.



$X_L = \omega L$

$X_C = \dfrac{1}{\omega C}$

$2\pi f_r . L = \dfrac{1}{2\pi f_r . C}$

$\Rightarrow \quad f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}}$

$= 71.18 Hz$

The current passing through the ckt at resonance

$I = \dfrac{V}{R} = \dfrac{100}{10} = 10$ amp

The voltage drop across the inductor

$V_L = I X_L = I \times \omega L = I \times 2\pi f_r L$

$= 10 \times 2\pi \times 71.18 \times 0.1$

$= 447.2 V$

The quality factor ~~(Q)~~,

$Q = \dfrac{X_L}{R} = \dfrac{\omega L}{R}$

$= \dfrac{2\pi f_r . L}{R}$

$= \dfrac{2\pi \times 71.18 \times 0.1}{10}$

$= 4.47$