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CONCEPT AND DEFINITIONS

Concept and Definitions

1

1.1 Chapter Highlights

- *Definition of engineering thermodynamics:* It deals with the study of energy, its forms and transformations and its interactions with matters. It basically entails four laws namely Zeroth, First, Second and Third law of thermodynamics.
- *Applications of engineering thermodynamics:* It has wide range of applications in diverse areas of different engineering disciplines. It is important and indispensable tools in the innovation, design and development of engineering processes, equipment and devices. Its utilities are in power producing devices (heat engine, turbine etc) and consuming devices (refrigerator, heat pump etc) as well as flow devices (heat exchanger, nozzle etc).
- Thermodynamic systems can be studied by adopting either a *macroscopic (classical)* or a *microscopic (statistical)* approach. The differences between these approaches are:

Microscopic Approach	Macroscopic Approach
<ol style="list-style-type: none">1. System is analyzed with reference to its molecular behaviors.2. It needs large numbers of properties to describe a system.3. Properties that describe the behavior of the molecules can neither be felt by our senses nor easily measured by instruments.4. It requires advanced statistical mathematics for analysis of the system and calculation is time consuming and tedious.5. It is usually used in the study of statistical thermodynamics and preferred by pure scientists.6. For example: Volume of air inside a room = number of air molecules inside room \times volume of single air molecule.	<ol style="list-style-type: none">1. System is analyzed with reference to certain measurable or observable bulk properties.2. It needs few numbers of properties to describe a system.3. Properties needed to describe the system can be felt by our senses and easily measured by instruments.4. It requires simple mathematics for analysis of the system and calculation is relatively faster and simpler.5. It is used in the study of classical thermodynamics and preferred by engineers.6. For example: Volume of air inside a room = Volume of room = length \times width \times height.

- **Thermodynamic System:** It is defined as a quantity of matter or a region in space that is under study with respect to thermodynamic parameters. According to the type of interaction a system may be closed, open or isolated.
- **Surroundings:** Everything (mass or region) outside the system is called its surroundings.
- **Boundary:** Any medium that separates system from its surroundings is called boundary. The boundary may change its shape or size. It may be real or imaginary, fixed or movable, rigid or deformable and diathermic or adiabatic.
- **Universe:** A system and its surroundings together is called universe.
- **Closed system:** System in which energy (heat and work) transfer can take place but no mass transfer across the boundary is called closed system, e.g. piston cylinder device. A closed system can be completely described by the fixed amount of mass inside the system. Therefore, a closed system is also known as *control mass (CM)*.
- **Open system:** System in which both mass and energy (heat and work) transfer can take place across the boundary is called open system, e.g. turbine, compressor. Any open system can be analyzed with reference to a certain specified region in space. Therefore, an open system is also known as *control volume (CV)*.
- Differences between control mass and control volume are:

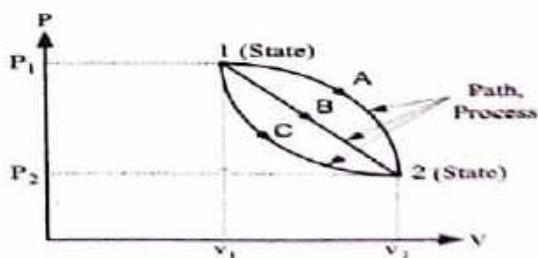
Control mass	Control volume
<ol style="list-style-type: none"> 1. Mass of the system remains constant. 2. Only energy transfer but no mass transfer takes place between system and surroundings. 3. It is also called closed system. 4. For examples: piston cylinder device, a pressure cooker, a battery etc. 	<ol style="list-style-type: none"> 1. Volume of the system remains constant. 2. Both the energy and mass transfers take place between system and surroundings. 3. It is also called open system. 4. For examples: Turbine, pump, nozzle, diffuser etc.

- **Isolated system:** System in which neither mass nor energy (heat ad work) transfer can take place is called isolated system, e.g. a closed rigid insulated container, thermo flask, universe.
- **Thermodynamic property:** Any variable that is used to define the state or condition of a system, e.g. pressure, temperature, volume, internal energy, enthalpy, entropy. According to its nature a property may be extensive or intensive.
- Differences between *extensive* and *intensive properties* are:

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Extensive property	Intensive property
1. It depends on mass or extent of the system.	1. It does not depend on mass or extent of the system.
2. Its value depends on how large a portion of the system is being considered.	2. Its value remains the same whether one considers the whole system or only a part of it.
3. They are additive when a system is divided into a number of parts, i.e., $P = P_1 + P_2 + P_3 + \dots$	3. They are not additive, i.e., $P = P_1 = P_2 = P_3 = \dots$ Here, P means any property.
4. For examples: Mass, volume, energy, enthalpy, entropy etc.	4. For examples: pressure, temperature, density etc.

- **Specific property:** Extensive property per unit mass is called specific property and is an intensive property.
- **Thermodynamic state:** It is a unique condition at which a thermodynamic system can exist.
- A **thermodynamic property** has two main features: a variable is a thermodynamic property, if and only if,
 - (i) it has a single value at each equilibrium state
 - (ii) a change in its value between any two prescribed states is independent of path.
- **Thermodynamic process:** It is the path followed by a system when it undergoes from one equilibrium state to another.
- **Why heat and work are path functions:**
 Any variable or quantity which depends not only on the end states but also on the path is known as a *path function*. Such quantities cannot be located on a property diagram by a point but are given by the area under the process curve. Work and heat are the examples of path functions and their magnitude can be represented by the area under the process curve on property diagrams, e.g. the area under the process curve on $P-V$ diagram gives the amount of the work ($W_{1-2} = \int_1^2 \delta W = \int_1^2 P dV$) and the area under the process curve on $T-S$ diagram gives the quantity of the heat ($Q_{1-2} = \int_1^2 \delta Q = \int_1^2 T dS$). Since the area under the process curve is different for the different paths (A, B, C) undertaken by the processes for the fixed initial state 1 and final state 2 as shown in figure below (e.g. work), both the heat and work are therefore the path functions.



- Differences between state or point function and path function are:

State/point function	Path function
<ol style="list-style-type: none"> 1. Variable which only depends on the end states is called state or point function. 2. It is thermodynamic property. 3. It has exact differential. 4. Its cyclic integral is zero. 5. For example: all thermodynamic properties such as pressure, temperature, volume etc. 	<ol style="list-style-type: none"> 1. Variable which depends not only on the end states but also on the path is called path function. 2. It is not thermodynamic property. 3. It has inexact differential. 4. Its cyclic integral is non-zero. 5. For example: heat and work.

- With reference to intermediate states, thermodynamic processes can be classified as *reversible* and *irreversible process*.
- Differences between reversible and irreversible processes are:

Reversible process	Irreversible process
<ol style="list-style-type: none"> 1. It is a process which passes through a series of equilibrium states such that each intermediate state can be located on a property diagram. 2. It is denoted by continuous line. 3. It occurs at an infinitely slow rate. 4. The system is in equilibrium state or in quasi-equilibrium state at any instant. 5. It can be carried out in both directions and initial state can be restored. 6. It results in higher efficiency than that of irreversible process. 7. For example: Frictionless motion, elastic deformation, restricted expansion and compression, heat transfer with infinitely small temperature difference etc. 	<ol style="list-style-type: none"> 1. It is a process which passes through a series of intermediate states which cannot be defined or located on a property diagram. 2. It is denoted by broken line indicating that intermediate states are indeterminate. 3. It occurs at a finite rate (fast). 4. The system is never in the equilibrium state at any instant. 5. It can be carried out in one direction only and initial state cannot be restored. 6. It results in lower efficiency than that of reversible process. 7. For example: Motion with friction, inelastic deformation, free expansion, heat transfer with finite temperature difference etc.

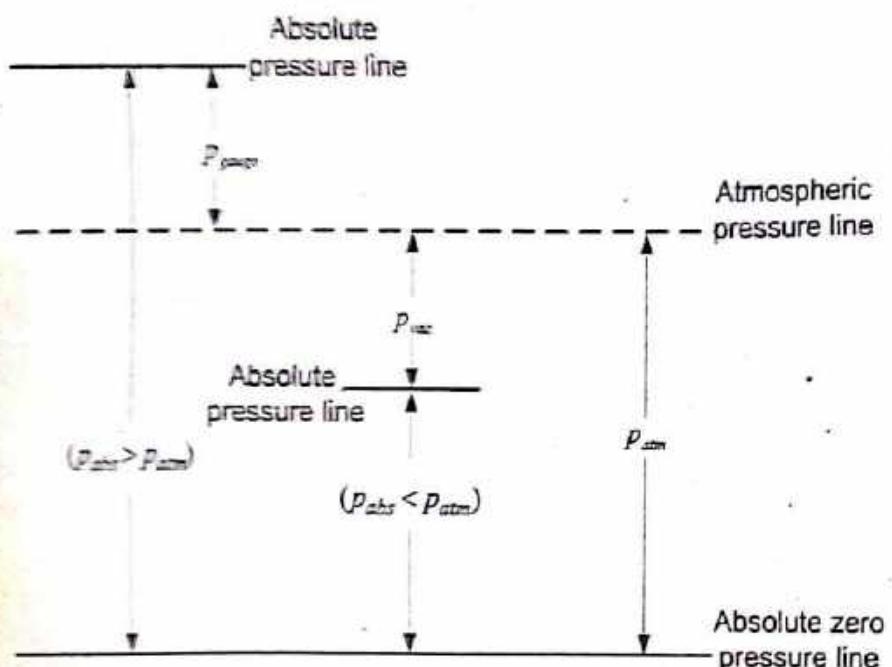
- **Cyclic process:** It is defined as a series of individual processes whose final and initial states are identical. Cyclic integral of a thermodynamic property for a cyclic process is zero.
- **Quasi-static process:** It is a succession of equilibrium states and infinite slowness is its characteristic feature. A reversible process is a quasi-static process, but a quasi-static process need not be a complete reversible process.
- **Thermodynamic equilibrium:** A system is said to be in thermodynamic equilibrium, if it is in the state of thermal, mechanical, and chemical equilibrium at a time.
- **Thermal equilibrium:** There should not be any temperature difference within the system for a system to be in a thermal equilibrium.
- **Mechanical equilibrium:** There should not be any pressure difference or unbalanced forces within the system for a system to be in a mechanical equilibrium.
- **Chemical equilibrium:** There should not be any chemical reaction during the observation period for a system to be in a chemical equilibrium.
- **Specific volume:** Volume per unit mass of the substance is called specific volume and is reciprocal of density ($v = V/m = 1/\rho$).
- **Pressure:** Force per unit area is called pressure. It is generally expressed in Pascal ($P_a = N/m^2$) and sometimes in bar (1 bar = 100 kPa) or in atmosphere (1 atm = 101.325 kPa).
- **Atmospheric pressure:** It is defined as pressure exerted by ambient (atmospheric) air on any surface and is measured by barometer. Atmospheric pressure measured by barometer with height of mercury Z_{baro} is evaluated as

$$P_{atm} = \rho g Z_{baro}$$

- **Gauge pressure:** It is the pressure of a system measured with reference to atmospheric pressure by using pressure gauge or manometer. Gauge pressure measured by manometer with height of manometric fluid Z_{mano} is determined as $P_{gauge} = \rho g Z_{mano}$.
- **Absolute pressure:** Pressure of a system relative to perfect vacuum is known as absolute pressure and it is calculated as

$$P_{abs} = P_{gauge} + P_{atm}$$

- **Vacuum gauge:** When pressure of a system is less than atmospheric pressure, gauge pressure becomes negative and this negative gauge pressure is also called vacuum gauge.
- The relationship between absolute, gauge, atmospheric and negative or vacuum gauge pressures can be represented on a pressure scale as shown in figure below:



- Differences between gauge pressure and absolute pressure are:

Gauge pressure	Absolute pressure
<ol style="list-style-type: none"> It is the pressure of a system measured with reference to atmospheric pressure or surroundings pressure. It is measured by using pressure gauge or manometer. It may be negative and is called vacuum gauge when pressure of a system is less than atmospheric pressure. Gauge pressure measured by manometer is determined as $P_{gauge} = \rho g Z_{man}$ whereas pressure gauge gives direct pressure reading. 	<ol style="list-style-type: none"> It is the pressure of a system expressed relative to perfect vacuum. It is calculated as the sum of gauge pressure and atmospheric pressure. It is positive and may be zero if the system is perfect vacuum. It is determined as $P_{abs} = P_{gauge} + P_{atm}$

- *Temperature*: It is defined as a property of a system which defines the direction and magnitude of heat transfer. It is also known as a degree of hotness or coldness of a system and is measured by an instrument called thermometer.

CONCEPT AND DEFINITIONS

- Temperature is measured in different *temperature scales* namely in *degree Celsius*, *degree Fahrenheit* and *Kelvin* (absolute temperature) scales and they have relation as:

$$\frac{N - FP}{BP - FP} = \frac{C}{100} = \frac{F - 32}{180} = \frac{K - 273}{100}$$

where N is any new scale, FP is *freezing point* of ice and BP is *boiling point* of water.

- *Equality of temperature*: If there is no change in any properties of two systems when they are brought in physical contact, then both the system are said to have equal temperatures.
- *Zeroth law of thermodynamics*: It states that two systems which are separately in thermal equilibrium with a third system are also in thermal equilibrium with each other. *Applications of zeroth law* are: measurement of the temperature of a system and comparison of the temperatures of multiple systems etc.

1.2 Solved Numerical Problems

Problem 1.1.

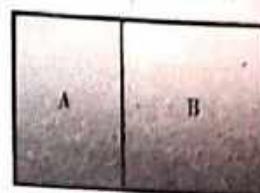
A container has two compartments separated by a membrane as shown in figure. Compartment A has 2 kg of air and a volume of 1 m³; compartment B has 1.5 m³ of air with a specific volume of 2.5 m³/kg. If the membrane is broken, determine the resultant specific volume.

Solution: Given,

$$\text{Mass of air in compartment A, } m_A = 2 \text{ kg}$$

$$\text{Volume of air in compartment A, } V_A = 1 \text{ m}^3$$

$$\text{Volume of air in compartment B, } V_B = 1.5 \text{ m}^3$$



$$\text{Specific volume of air in compartment B, } v_B = 2.5 \text{ m}^3/\text{kg}$$

$$\text{Mass of air in compartment B, } m_B = \frac{V_B}{v_B} = \frac{1.5}{2.5} = 0.6 \text{ kg}$$

$$\text{Total volume of air in compartments A \& B, } V_{AB} = V_A + V_B = 1 + 1.5 = 2.5 \text{ m}^3$$

$$\text{Total mass of air in compartments A \& B, } m_{AB} = m_A + m_B = 2 + 0.6 = 2.6 \text{ kg}$$

$$\text{Then, resultant specific volume of air, } v_{AB} = \frac{V_{AB}}{m_{AB}} = \frac{2.5}{2.6} = 0.9615 \text{ m}^3/\text{kg.}$$

Problem 1.2.

A cylinder with a total volume of 1 m³ has a movable piston as shown in figure. When the piston is at one fourth of the length, both sides have same specific volume of 4 m³/kg. Determine the specific volumes of both sides when the piston is at middle of the cylinder.

Solution: Given,

$$\text{Total volume of cylinder, } V_{AB} = 1 \text{ m}^3$$

$$\text{At piston position, } x = L/4 \text{ (at one fourth of the length)}$$

$$\text{Specific volume of side A} = \text{Specific volume of side B}$$

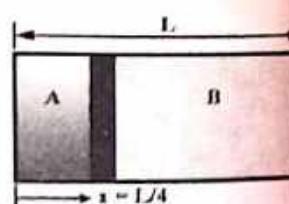
$$\Rightarrow v_A = v_B = 4 \text{ m}^3/\text{kg}$$

$$\text{At piston position, } x = L/4 \text{ (at one fourth of the length)}$$

$$\text{Volume of side A, } V_A = \frac{1}{4} V_{AB} = \frac{1}{4} \times 1 = 0.25 \text{ m}^3 \text{ (As area is constant)}$$

$$\text{Volume of side B, } V_B = \frac{3}{4} V_{AB} = \frac{3}{4} \times 1 = 0.75 \text{ m}^3$$

$$\text{Mass of side A, } m_A = \frac{V_A}{v_A} = \frac{0.25}{4} = 0.0625 \text{ kg}$$



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Mass of side B, $m_B = \frac{V_B}{v_B} = \frac{0.75}{4} = 0.1875 \text{ kg}$

At piston position, $x = L/2$ (at middle of the cylinder)

Volume of side A, $V'_A = \frac{1}{2} V_{AB} = \frac{1}{2} \times 1 = 0.5 \text{ m}^3$ (As area is constant)

Volume of side B, $V'_B = \frac{1}{2} V_{AB} = \frac{1}{2} \times 1 = 0.5 \text{ m}^3$

Although the volume of the sides A and B changes with the piston position, their masses remain constant. Therefore,

Specific volume of side A, $v'_A = \frac{V'_A}{m_A} = \frac{0.5}{0.625} = 0.8 \text{ m}^3/\text{kg}$ and

Specific volume of side B, $v'_B = \frac{V'_B}{m_B} = \frac{0.5}{0.1875} = 2.667 \text{ m}^3/\text{kg}$.

Problem 1.3.

A large chamber is separated into two compartments X and Y which are maintained at different pressures as shown in figure. Pressure gauge A reads 180 kPa and pressure gauge B reads 120 kPa. If the barometric pressure is 100 kPa, determine the absolute pressure existing in the compartments X and Y as well as the reading of gauge C.

Solution: Given,

Gauge pressure reading of gauge A, $P_{\text{gauge},A} = 180 \text{ kPa}$

Gauge pressure reading of gauge B, $P_{\text{gauge},B} = 120 \text{ kPa}$

Atmospheric (barometric) pressure, $P_{\text{atm}} = 100 \text{ kPa}$

As pressure gauge B measures the pressure of compartment X relative to the atmospheric pressure, absolute pressure in compartment X, $P_{\text{abs},X}$ is given by

$$P_{\text{abs},X} = P_{\text{atm}} + P_{\text{gauge},B} = 100 + 120 = 220 \text{ kPa}$$

Since pressure gauge A measures the pressure of compartment X relative to the pressure of compartment Y, Then $P_{\text{abs},X}$ can also be determined as

$$P_{\text{abs},X} = P_{\text{atm},Y} + P_{\text{gauge},A}$$

Then, absolute pressure in compartment Y is calculated as,

$$P_{\text{abs},Y} = P_{\text{abs},X} - P_{\text{gauge},A} = 220 - 180 = 40 \text{ kPa}$$

As pressure gauge C measures the pressure of compartment Y relative to the atmospheric pressure, $P_{\text{abs},Y}$ is also calculated as

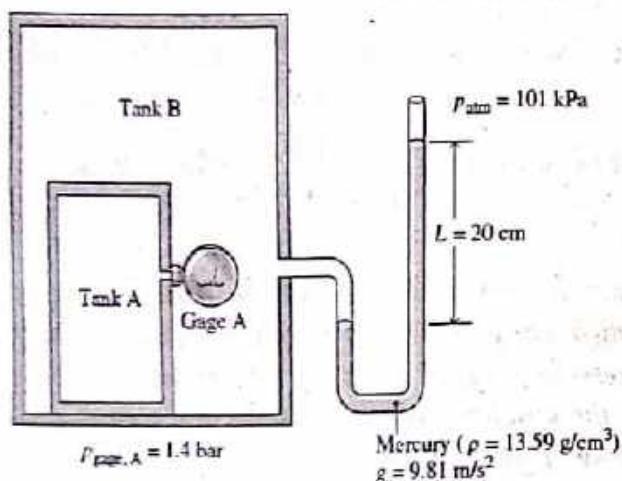
$$P_{\text{abs},Y} = P_{\text{atm}} + P_{\text{gauge},C}$$

Then, reading of gauge C, $P_{\text{gauge},C} = P_{\text{abs},Y} - P_{\text{atm}} = 40 - 100 = -60 \text{ kPa}$

This negative sign indicates that compartment Y has a pressure less than atmospheric pressure which is also called vacuum gauge.

Problem 1.4.

Figure given below shows a tank within a tank, each containing air. Pressure gauge A is located inside the tank B and reads 140 kPa. The U-tube manometer connected to tank B contains mercury. Using data given on the diagram, determine the absolute pressures inside the tank A and tank B.



Solution: Given,

Gauge pressure reading of gauge A, $P_{\text{gauge},A} = 1.4 \text{ bar} = 140 \text{ kPa}$

Atmospheric pressure, $P_{\text{atm}} = 101 \text{ kPa}$

Manometer reading, $z_{\text{mano}} = 20 \text{ cm of Hg} = 20 \times 10^{-2} \text{ m of Hg}$

Density of mercury, $\rho_{\text{Hg}} = 13.59 \text{ g/cm}^3 = 13590 \text{ kg/m}^3$ and

Acceleration of gravity, $g = 9.81 \text{ m/s}^2$

Gauge pressure reading of manometer,

$$P_{\text{gauge,mano}} = \rho_{\text{Hg}} g z_{\text{mano}} = 13590 \times 9.81 \times 20 \times 10^{-2} = 26.664 \text{ kPa}$$

As the manometer measures the pressure of tank B relative to the atmospheric pressure, absolute pressure inside tank B, $P_{\text{abs},B}$ is given by

$$P_{\text{abs},B} = P_{\text{atm}} + P_{\text{gauge,mano}} = 101 + 26.664 = 127.664 \text{ kPa}$$

Since pressure gauge A measures the pressure of tank A relative to the pressure of tank B, Then, absolute pressure inside tank A, $P_{\text{abs},A}$ can be determined as

$$P_{\text{abs},A} = P_{\text{abs},B} + P_{\text{gauge},A}$$

$$\Rightarrow P_{\text{abs},A} = 127.664 + 140 = 267.664 \text{ kPa}$$

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Problem 1.5.

A U-tube manometer containing mercury with one arm open to atmosphere is used to measure the pressure of steam flowing through a pipe as shown in figure. The level of mercury in the open ended arm stands 95 mm higher than that in the arm connected to steam pipe. Some of the steam condenses in the arm in communication with steam pipe. If height of this column is 32 mm, calculate the absolute pressure of steam. Take atmospheric pressure equal to 760 mm of mercury.

Solution: Given,

$$\begin{aligned}\text{Height of mercury in manometer, } z_{\text{mano}} &= 95 \text{ mm of Hg} \\ &= 95 \times 10^{-3} \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Height of condensed steam, } z_{\text{water}} &= 32 \text{ mm} \\ &= 32 \times 10^{-3} \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Height of mercury in Barometer, } z_{\text{baro}} &= 760 \text{ mm of Hg} \\ &= 760 \times 10^{-3} \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Atmospheric pressure, } P_{\text{atm}} &= \rho_{\text{Hg}} g z_{\text{baro}} = 13600 \times 9.81 \times 760 \times 10^{-3} \\ &= 101396 \text{ Pa} = 101.396 \text{ kPa}\end{aligned}$$

Referring to figure P1.7 under equilibrium condition, the pressure along the horizontal line 0 – 0 is same for either arm of the manometer, i.e.

$$\begin{aligned}P_{\text{abs,steam}} + \rho_{\text{water}} g z_{\text{water}} &= P_{\text{atm}} + \rho_{\text{mano}} g z_{\text{mano}} \\ \Rightarrow P_{\text{abs,steam}} + 1000 \times 9.81 \times 32 \times 10^{-3} &= 101396 + 13600 \times 9.81 \times 95 \times 10^{-3}\end{aligned}$$

∴ Absolute pressure of steam,

$$P_{\text{abs,steam}} = 101396 + 12674.5 - 313.92 = 113757 \text{ Pa} = 113.757 \text{ kPa}$$

Problem 1.6.

A piston cylinder arrangement shown in figure has a cross sectional area of 0.01 m^2 and a piston mass of 80 kg. If atmospheric pressure is 100 kPa, what should be the gas pressure to lift the piston? [Take $g = 9.81 \text{ m/s}^2$]

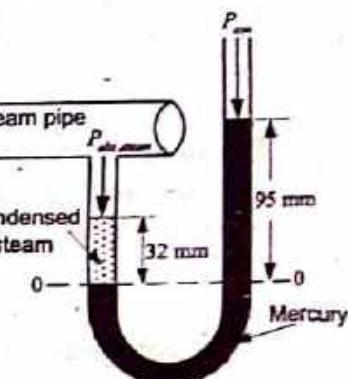
Solution: Given,

$$\text{Cross sectional area of piston, } A_p = 0.01 \text{ m}^2$$

$$\text{Mass of piston, } m_p = 80 \text{ kg}$$

$$\text{Atmospheric pressure, } P_{\text{atm}} = 100 \text{ kPa}$$

$$\text{Acceleration of gravity, } g = 9.81 \text{ m/s}^2$$



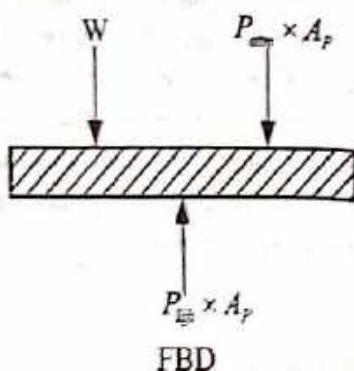
CONCEPT AND DEFINITIONS

Referring to the free body diagram (FBD) of piston, balancing the forces acting on Y-direction in equilibrium condition gives

$$P_{\text{ext}} \times A_p = P_{\text{int}} \times A_p + W = P_{\text{ext}} \times A_p + m_p \times g$$

\therefore Lifting pressure of piston.

$$\begin{aligned} P_{\text{lift}} &= P_{\text{ext}} + \frac{m_p g}{A_p} = 100 \times 10^3 + \frac{80 \times 9.81}{0.01} \\ &= 178480 \text{ Pa} = 178.48 \text{ kPa} \end{aligned}$$



Problem 1.7.

For the piston cylinder device shown in figure, determine the absolute pressure inside the device. [Take $P_{\text{atm}} = 101.3 \text{ kPa}$]

Solution: Given,

$$\text{Atmospheric pressure, } P_{\text{atm}} = 101.3 \text{ kPa}$$

$$\text{Cross sectional area of piston, } A_p = 0.004 \text{ m}^2$$

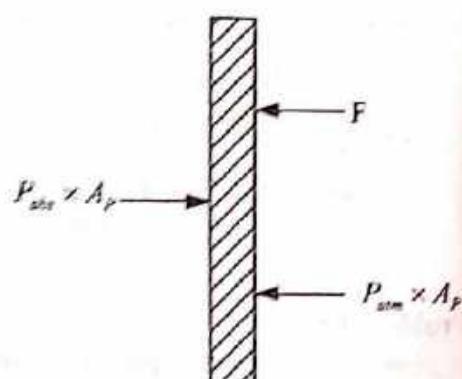
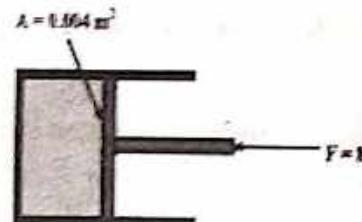
$$\text{External force applied on piston, } F = 800 \text{ N}$$

Referring to the free body diagram (FBD) of piston, balancing the forces acting on X-direction in equilibrium condition gives

$$P_{\text{ext}} \times A_p = P_{\text{int}} \times A_p + F$$

\Rightarrow Absolute pressure inside the cylinder,

$$\begin{aligned} P_{\text{int}} &= P_{\text{atm}} + \frac{F}{A_p} \\ &= 101.3 \times 10^3 + \frac{800}{0.004} \\ &= 301300 \text{ Pa} = 301.3 \text{ kPa} \end{aligned}$$

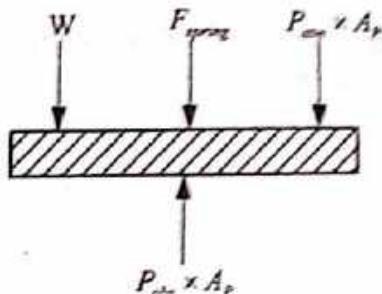
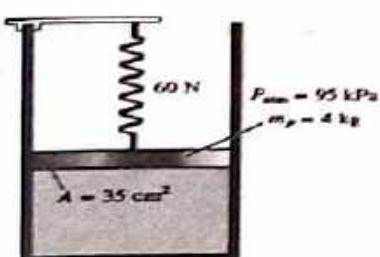


FBD

Problem 1.8.

A gas is contained in a vertical, frictionless piston-cylinder device shown in figure. The piston has a mass of 4 kg and a cross-sectional area of 35 cm^2 . A compressed spring above the piston exerts a force of 60 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.

CONCEPT AND DEFINITIONS



FBD

Solution: Given,

$$\text{Cross sectional area of piston, } A_p = 35 \text{ cm}^2 = 35 \times 10^{-4} \text{ m}^2$$

$$\text{Mass of piston, } m_p = 4 \text{ kg}$$

$$\text{Atmospheric pressure, } P_{atm} = 95 \text{ kPa}$$

$$\text{Spring force applied on piston, } F_{spring} = 60 \text{ N}$$

Referring to the free body diagram (FBD) of piston, balancing the forces acting on Y - direction in equilibrium condition gives

$$P_{int} \times A_p = P_{atm} \times A_p + F_{spring} + W$$

⇒ Absolute pressure inside the cylinder,

$$\begin{aligned} P_{int} &= P_{atm} + \frac{F_{spring}}{A_p} + \frac{m_p g}{A_p} = 95 \times 10^3 + \frac{60}{35 \times 10^{-4}} + \frac{4 \times 9.81}{35 \times 10^{-4}} \\ &= 95000 + 17143 + 11211 = 123354 \text{ Pa} = 123.354 \text{ kPa} \end{aligned}$$

Problem 1.9.

A piston cylinder device loaded with a linear spring with a spring constant of $k = 100 \text{ kN/m}$ contains a gas initially at a pressure of 100 kPa and a volume of 0.05 m^3 , as shown in figure. The cross sectional area of the piston is 0.1 m^2 . Initially spring touches the piston but exerts no force on it. Heat is supplied to the system until its volume doubles, determine the final pressure.

Solution: Given,

$$\text{Spring constant, } k = 100 \text{ kN/m}$$

$$\text{Cross sectional area of piston, } A_p = 0.1 \text{ m}^2$$

At initial state:

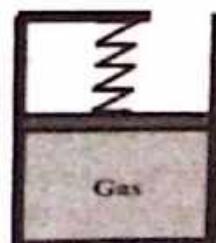
$$\text{Gas pressure, } P_1 = 100 \text{ kPa}$$

$$\text{Volume of gas, } V_1 = 0.05 \text{ m}^3$$

$$\text{Spring force, } F_{spring} = 0$$

At final state:

$$\text{Volume of gas, } V_2 = 2 \times V_1 = 2 \times 0.05 = 0.1 \text{ m}^3$$

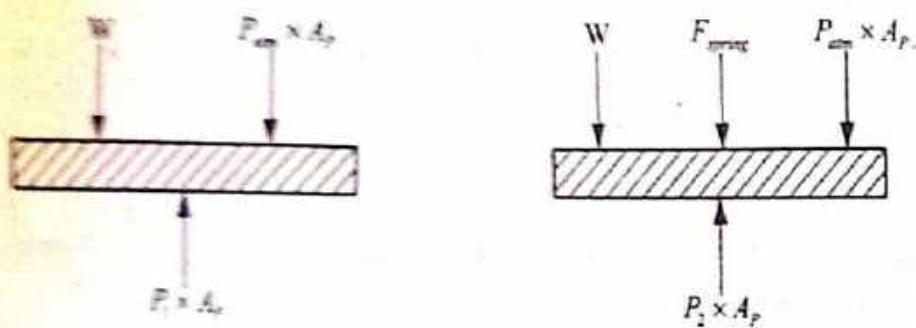


Referring to the FBD of the piston at initial state, we can write the equation for the gas pressure inside the cylinder as

$$\begin{aligned} P_1 \times A_p &= P_{\text{ext}} \times A_p + W \\ \Rightarrow P_1 &= P_{\text{ext}} + \frac{W}{A_p} = 100 \text{ kPa} \end{aligned}$$

Displacement of the spring at final state is calculated as

$$x_2 = \frac{V_2 - V_1}{A_p} = \frac{0.1 - 0.05}{0.1} = 0.5 \text{ m}$$



Similarly, referring to the FBD of the piston at final state, the pressure equation is given as

$$\begin{aligned} P_1 \times A_p &= P_{\text{ext}} \times A_p + W + F_{\text{spring}} \\ \Rightarrow \text{Final pressure, } P_2 &= P_{\text{ext}} + \frac{W}{A_p} + \frac{F_{\text{spring}}}{A_p} = P_1 + \frac{kx_2}{A_p} \\ &= 100 + \frac{100 \times 0.5}{0.1} = 600 \text{ kPa} \end{aligned}$$

Problem 1.14.

Air contained within a vertical piston-cylinder assembly is shown in figure below. On its top, the 10 kg piston is attached to a spring and exposed to an atmospheric pressure of 1 bar. Initially, the bottom of the piston is at $x = 0$, and the spring exerts a negligible force on the piston. The valve is opened and air enters the cylinder from the supply line, causing the volume of the air within the cylinder to increase by $5.9 \times 10^{-4} \text{ m}^3$. The force exerted by the spring as the air expands within the cylinder varies linearly with x according to $F_{\text{spring}} = kx$, where $k = 10,000 \text{ N/m}$. The piston face area is $7.8 \times 10^{-3} \text{ m}^2$. Ignoring friction between the piston and the cylinder wall, determine the pressure of the air within the cylinder, when the piston is in its initial position. Repeat when the piston is in its final position. The atmospheric pressure is 100 kPa and the local acceleration of gravity is 9.81 m/s^2 .

Solution: Given,

Mass of piston, $m_p = 10 \text{ kg}$

Area of piston, $A_p = 7.8 \times 10^{-3} \text{ m}^2$

Atmospheric pressure, $P_{\text{atm}} = 1 \text{ bar} = 100 \text{ kPa}$

Spring constant, $k = 10000 \text{ N/m}$

At initial state:

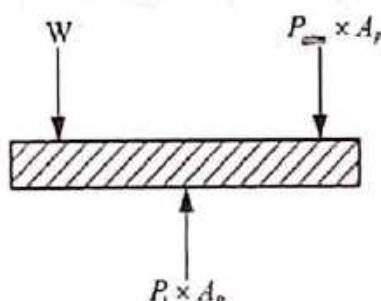
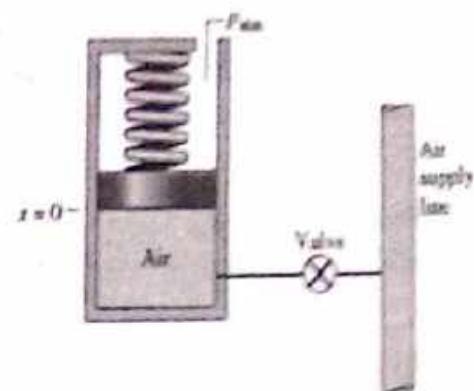
Displacement of spring, $x_1 = 0$ as $F_{\text{spring}} = 0$

At final state (when the valve is opened):

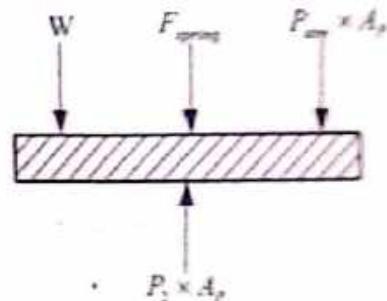
Change of volume of air,

$$(V_2 - V_1) = 3.9 \times 10^{-4} \text{ m}^3$$

$$\text{Displacement of spring, } x_2 = \frac{V_2 - V_1}{A_p} = \frac{3.9 \times 10^{-4}}{7.8 \times 10^{-3}} = 0.05 \text{ m}$$



FBD at initial state



FBD at final state

Referring to the FBD of the piston at initial state, we can write the equation for the air pressure inside the cylinder as

$$P_1 \times A_p = P_{\text{atm}} \times A_p + W$$

\Rightarrow Initial air pressure,

$$P_1 = P_{\text{atm}} + \frac{W}{A_p} = 100 \times 10^3 + \frac{10 \times 9.81}{7.8 \times 10^{-3}} = 112576.92 \text{ Pa} = 112.58 \text{ kPa}$$

Similarly, referring to the FBD of the piston at final state, the pressure equation is given as

$$P_2 \times A_p = P_{\text{atm}} \times A_p + W + F_{\text{spring}}$$

\Rightarrow Final air pressure,

$$\begin{aligned} P_2 &= P_{\text{atm}} + \frac{m_p g}{A_p} + \frac{kx_2}{A_p} \\ &= 100 \times 10^3 + \frac{10 \times 9.81}{7.8 \times 10^{-3}} + \frac{10000 \times 0.05}{7.8 \times 10^{-3}} \\ &= 176679.49 \text{ Pa} = 176.68 \text{ kPa} \end{aligned}$$

Problem 1.11.

Determine mass of the pet cock of a pressure cooker shown in figure below, whose operation pressure is gauge pressure with 100 kPa and has an opening cross sectional area of 4 mm^2 . Take an atmospheric pressure of 101 kPa and $g = 9.81 \text{ m/s}^2$.

Solution: Given,

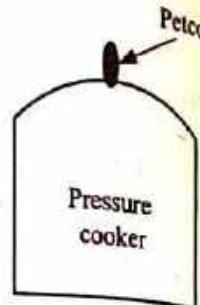
$$\text{Operation pressure of pet cock, } P_{\text{gauge}} = 100 \text{ kPa}$$

$$\begin{aligned}\text{Cross sectional area of opening, } A &= 4 \text{ mm}^2 \\ &= 4 \times 10^{-6} \text{ m}^2\end{aligned}$$

$$\text{Atmospheric pressure, } P_{\infty} = 101 \text{ kPa}$$

$$\text{Acceleration of gravity, } g = 9.81 \text{ m/s}^2$$

At equilibrium condition, the absolute pressure applied on the pet cock is equal to the sum of the atmospheric pressure and weight of the pet cock per unit area, i.e.



$$P_{\infty} = P_{\infty} + \frac{mg}{A}$$

Since absolute pressure is sum of atmospheric pressure and gauge pressure, i.e.

$$P_{\infty} = P_{\infty} + P_{\text{gauge}}$$

Equating above two equations,

$$P_{\infty} + \frac{mg}{A} = P_{\infty} + P_{\text{gauge}} \Rightarrow P_{\text{gauge}} = \frac{mg}{A}$$

= Mass of pet cock,

$$m = \frac{P_{\text{gauge}} \times A}{g} = \frac{100 \times 10^3 \times 4 \times 10^{-6}}{9.81} = 0.040775 \text{ kg} = 40.775 \text{ gm}$$

Problem 1.12.

A multi fluid container is connected to U-tube as shown in figure. The standard densities of water and mercury are 1000 kg/m^3 and 13600 kg/m^3 respectively. In the figure SG stands for specific gravity. For the given specific gravities and fluid column heights, determine the gauge pressure at A. Also determine height of the mercury column that would create same pressure at A.

Solution: Given,

$$\text{Height of oil column, } z_{\text{oil}} = 70 \text{ cm} = 0.7 \text{ m}$$

$$\text{Height of water column, } z_w = 30 \text{ cm} = 0.3 \text{ m}$$

$$\text{Height of glycerin column at left arm, } z_{\text{glyc}} = 20 + 15 \text{ cm} = 0.35 \text{ m}$$

$$\text{Height of glycerin column at right arm, } z_{\text{glyc}} = 90 + 15 \text{ cm} = 1.05 \text{ m}$$

CONCEPT AND DEFINITIONS

Specific gravity of oil, $(SG)_{oil} = 0.90$

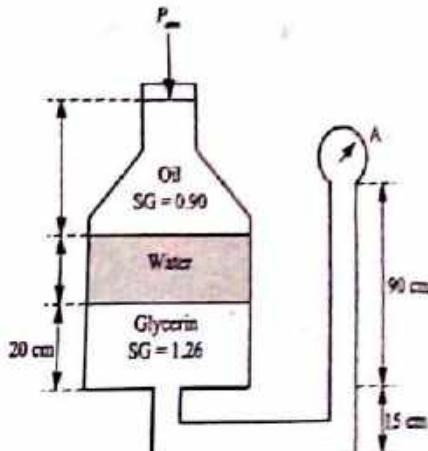
Specific gravity of glycerin, $(SG)_{gly} = 1.26$

Density of water, $\rho_w = 1000 \text{ kg/m}^3$

Density of mercury, $\rho_{Hg} = 13600 \text{ kg/m}^3$

$$\begin{aligned}\text{Density of oil, } \rho_{oil} &= \rho_w \times (SG)_{oil} \\ &= 1000 \times 0.90 = 900 \text{ kg/m}^3\end{aligned}$$

$$\begin{aligned}\text{Density of glycerin, } \rho_{gly} &= \rho_w \times (SG)_{gly} \\ &= 1000 \times 1.26 = 1260 \text{ kg/m}^3\end{aligned}$$



Here, the pressure at left arm is equal to the pressure at right arm of the U-tube, i.e.

$$P_{atm} + P_{oil} + P_w + P_{gly,l} = P_{gly,r} + P_{abs,A}$$

$$P_{atm} + P_{oil} + P_w + P_{gly,l} = P_{gly,r} + P_{atm} + P_{gauge,A}$$

$$\rho_{oil} \times g \times z_{oil} + \rho_w \times g \times z_w + \rho_{gly} \times g \times z_{gly,l} = \rho_{gly} \times g \times z_{gly,r} + P_{gauge,A}$$

$$\Rightarrow 900 \times 9.81 \times 0.7 + 1000 \times 9.81 \times 0.3 + 1260 \times 9.81 \times 0.35 = 1260 \times 9.81 \times 1.05 + P_{gauge,A}$$

∴ gauge pressure at A,

$$P_{gauge,A} = 6180.3 + 2943 + 4326.21 - 12978.63 = 470.88 \text{ Pa} = 0.471 \text{ kPa}$$

Again, the height of the mercury column that would create same pressure at A is determined as

$$P_{gauge,A} = \rho_{Hg} \times g \times z_{Hg}$$

Therefore, height of mercury column for same pressure at A,

$$z_{Hg} = \frac{P_{gauge,A}}{\rho_{Hg} \times g} = \frac{470.88}{13600 \times 9.81} = 0.003529 \text{ m} = 0.353 \text{ cm}$$

Problem 1.13.

A new absolute temperature scale is proposed. On this scale the ice point of water is $150^\circ S$ and the steam point is $300^\circ S$. Determine the temperatures in ${}^\circ C$ that correspond to $100^\circ S$ and $400^\circ S$, respectively. At what temperature, both the Celsius and new temperature scale reading would be the same?

Solution: Given,

For the new absolute temperature scale S,

Ice point of water on S, F.P. = $150^\circ S$

Steam point on S, B.P. = $300^\circ S$

We know the relationship between different temperature scales as

$$\frac{S - F.P.}{B.P. - F.P.} = \frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{K - 273}{373 - 273}$$

Using first two expressions,

$$\frac{S-150}{300-150} = \frac{C-0}{100-0}$$

For $S = 100^{\circ}\text{S}$,

$$\frac{100-150}{150} = \frac{C}{100} \Rightarrow C = -33.33^{\circ}\text{C}$$

For $S = 400^{\circ}\text{S}$,

$$\frac{400-150}{150} = \frac{C}{100} \Rightarrow C = 166.66^{\circ}\text{C}$$

Again, using the expression,

$$\frac{S-150}{300-150} = \frac{C-0}{100-0}$$

For ${}^{\circ}\text{S} = {}^{\circ}\text{C} = X$,

$$\frac{X-150}{300-150} = \frac{X-0}{100-0} \Rightarrow X = -300$$

- \therefore Reading on new temperature scale, $S = -300^{\circ}\text{S}$ and
Reading on Celsius temperature scale, $C = -300^{\circ}\text{C}$

Problem: 1.14.

Air (0.01 kg) is contained in a piston cylinder device restrained by a linear spring ($k = 500 \text{ kN/m}$) as shown in figure. Spring initially touches the piston but exerts no force on it. Heat is added to the system until the piston is displaced upward by 80 mm. Determine:

- (a) the temperature at which piston leaves the stops and
(b) the final pressure.

[Take $R = 287 \text{ J/kgK}$, $P_{\text{atm}} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$].

Solution: Given,

Mass of air, $m = 0.01 \text{ kg}$

Spring constant, $k = 500 \text{ kN/m}$

Displacement of spring, $x = 80 \text{ mm} = 0.08 \text{ m}$

Mass of piston, $m_p = 400 \text{ kg}$

Diameter of piston, $D_p = 250 \text{ mm} = 0.25 \text{ m}$

Height of piston position, $h = 200 \text{ mm} = 0.2 \text{ m}$

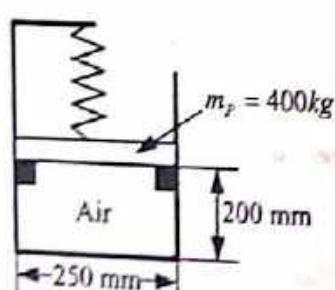
Atmospheric pressure, $P_{\text{atm}} = 100 \text{ kPa}$

Gas constant, $R = 287 \text{ J/kgK}$

Acceleration of gravity, $g = 9.81 \text{ m/s}^2$

$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi (0.25)^2}{4} = 0.0491 \text{ m}^2$$

$$\text{Initial volume of air, } V_i = A_p \times h = 0.0491 \times 0.2 = 0.0098175 \text{ m}^3$$

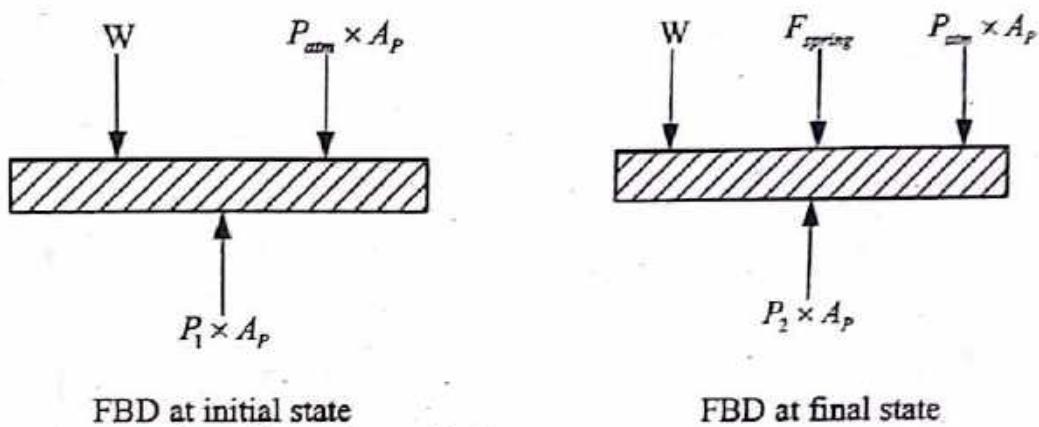


CONCEPT AND DEFINITIONS

Referring to the FBD of the piston at initial state when the piston just leaves the stops, the pressure inside the cylinder is

$$P_1 \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_1 = P_{atm} + \frac{m_p g}{A_p} = 100 \times 10^3 + \frac{400 \times 9.81}{0.0491} = 179918.53 \text{ Pa} = 179.92 \text{ kPa}$$



Now, using equation of state for an ideal gas

$$P_1 V_1 = m R T_1$$

\Rightarrow Temperature at which piston leaves stops,

$$T_1 = \frac{P_1 V_1}{m R} = \frac{179918.53 \times 0.0098175}{0.01 \times 287} = 615.45 \text{ K} = 342.45^\circ\text{C}$$

Similarly, referring to the FBD of the piston at final state, the pressure equation is given as

$$P_2 \times A_p = P_{atm} \times A_p + W + F_{spring}$$

\Rightarrow Final pressure,

$$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = P_1 + \frac{kx_2}{A_p} = 179.92 + \frac{500 \times 0.08}{0.0491} = 994.58 \text{ kPa}$$

Problem 1.15.

A 15 kg piston in a cylinder with diameter of 0.15 m is loaded with a linear spring and the outside atmospheric pressure is 100 kPa, as shown in figure. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 300 kPa with volume of 0.02 m³. The valve is opened to let some air in, causing the piston to raise 5 cm. Find the new pressure. (Take g = 9.81 m/s²).

Solution: Given,

Mass of piston, $m_p = 15 \text{ kg}$

Diameter of piston, $D_p = 0.15 \text{ m}$

Atmospheric pressure, $P_{\text{atm}} = 100 \text{ kPa}$

At initial state:

Air pressure, $P_1 = 300 \text{ kPa}$

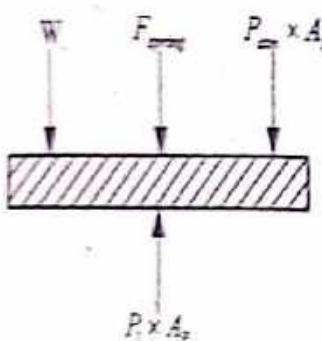
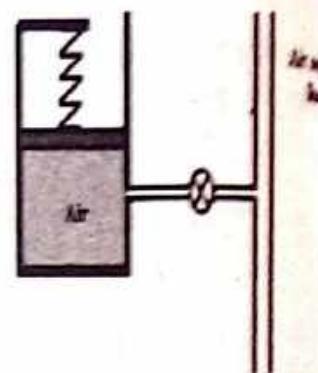
Volume of air, $V_1 = 0.02 \text{ m}^3$

At final state (when the valve is opened):

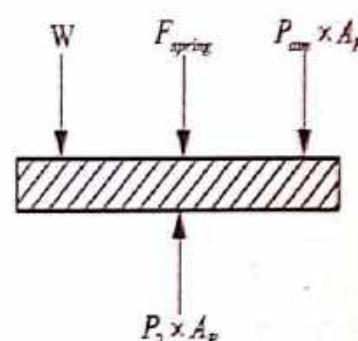
Displacement of spring, $x_2 = 5 \text{ cm} = 0.05 \text{ m}$

$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi(0.15)^2}{4} = 0.01767 \text{ m}^2$$

$$\text{Displacement of spring, } x_1 = \frac{V_1}{A_p} = \frac{0.02}{0.01767} = 1.13186 \text{ m}$$



FBD at initial state



FBD at final state

Referring to the FBD of the piston at initial state, we can write the equation for the air pressure inside the cylinder as

$$\begin{aligned} P_1 \times A_p &= P_{\text{atm}} \times A_p + W + F_{\text{spring}} \\ \Rightarrow P_1 &= P_{\text{atm}} + \frac{W}{A_p} + \frac{F_{\text{spring}}}{A_p} = P_{\text{atm}} + \frac{m_p g}{A_p} + \frac{kx_1}{A_p} \\ \Rightarrow 300 \times 10^3 &= 100 \times 10^3 + \frac{15 \times 9.81}{0.01767} + \frac{k \times 1.13186}{0.01767} \end{aligned}$$

$$\therefore k = 2992.3 \text{ N/m} = 2.9923 \text{ kN/m}$$

Total displacement of the spring at final state is given by

$$x = x_1 + x_2 = 1.13186 + 0.05 = 1.18186 \text{ m}$$

Similarly, referring to the FBD of the piston at final state, the pressure equation is given as

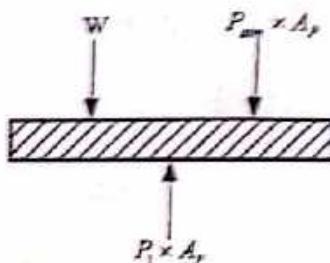
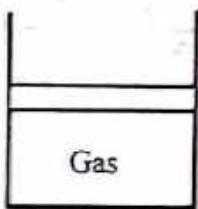
$$\begin{aligned} P_2 \times A_p &= P_{\text{atm}} \times A_p + W + F_{\text{spring}} \\ \Rightarrow \text{New air pressure, } P_2 &= P_{\text{atm}} + \frac{m_p g}{A_p} + \frac{kx}{A_p} \end{aligned}$$

CONCEPT AND DEFINITIONS

$$\begin{aligned}
 &= 100 \times 10^3 + \frac{15 \times 9.81}{0.01767} + \frac{29923 \times 1.18186}{0.01767} \\
 &= 308468 \text{ Pa} = 308.468 \text{ kPa}
 \end{aligned}$$

Problem 16.

A cylinder encloses a gas with a piston as shown in figure. The area of the piston is 0.01 m^2 . Take the atmospheric pressure to be 0.101 MPa and the local gravitational acceleration as 9.81 m/s^2 . If the weight of the piston is 490.50 N , what is the gas pressure? Will the gas pressure change if the gas volume beneath the piston is double?



FBD for given state

Solution: Given,

$$\text{Area of piston, } A_p = 0.01 \text{ m}^2$$

$$\text{Atmospheric pressure, } P_{\text{atm}} = 0.101 \text{ MPa} = 101 \text{ kPa}$$

$$\text{Weight of piston, } W = 490.50 \text{ N}$$

$$\text{Local gravitational acceleration, } g = 9.81 \text{ m/s}^2$$

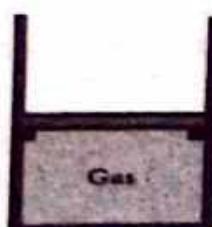
Referring to the FBD of the piston for the given state, we can write the equation for the gas pressure inside the cylinder as

$$\begin{aligned}
 P_1 \times A_p &= P_{\text{atm}} \times A_p + W \\
 \Rightarrow \text{Gas pressure, } P_1 &= P_{\text{atm}} + \frac{W}{A_p} = 101 \times 10^3 + \frac{490.50}{0.01} \\
 &= 150050 \text{ Pa} = 150.05 \text{ kPa}
 \end{aligned}$$

The gas pressure will not change even the gas volume beneath the piston is doubled. This is because as volume increases the piston moves upward balancing the pressure equal to gas pressure existing inside the cylinder (gas pressure depends on area of piston not on volume with constant area).

Problem 1.17.

A piston cylinder arrangement shown in figure below has cross sectional area of 0.01 m^2 and a piston mass of 80 kg . If the atmospheric pressure is 1 bar , what would be the gas pressure to lift the piston? If 50 kg mass is added above the piston, what would be the new pressure?



Solution: Given,

Area of piston, $A_p = 0.01 \text{ m}^2$

Mass of piston, $m_p = 80 \text{ kg}$

Atmospheric pressure, $P_{atm} = 1 \text{ bar} = 100 \text{ kPa}$

Additional mass, $m = 50 \text{ kg}$

Referring to the FBD of the piston when the piston just leaves the stop, we can write the equation for the lifting pressure of the piston as

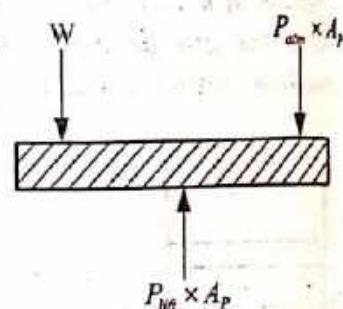
$$P_{lift} \times A_p = P_{atm} \times A_p + W$$

\Rightarrow Lifting pressure of piston,

$$\begin{aligned} P_{lift} &= P_{atm} + \frac{m_p g}{A_p} = 100 \times 10^3 + \frac{80 \times 9.81}{0.01} \\ &= 178480 \text{ Pa} \\ &= 178.48 \text{ kPa} \end{aligned}$$

If an additional mass is put above the piston,
then total mass will be

$$m_{total} = m_p + m = 80 + 50 = 130 \text{ kg.}$$



FBD of piston

Now, new gas pressure to lift the piston is determined as

$$P_{lift,N} = P_{atm} + \frac{m_{total}g}{A_p} = 100 \times 10^3 + \frac{130 \times 9.81}{0.01} = 227530 \text{ Pa} = 227.53 \text{ kPa}$$

Problem 1.18.

At the inlet and exhaust of turbine the absolute steam pressures are 6000 kPa and 4.0 cm of Hg respectively. Barometric pressure is 75 cm of Hg. Calculate the gauge pressure for the entering steam and vacuum gauge pressure for the exhaust steam. [Take $\rho_{Hg} = 13600 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$].

Solution: Given,

At the inlet of turbine:

Absolute steam pressure, $P_{abs,1} = 6000 \text{ kPa}$

Height of mercury in barometer, $z_{baro} = 75 \text{ cm} = 0.75 \text{ m}$

Density of mercury, $\rho_{Hg} = 13600 \text{ kg/m}^3$

Acceleration of gravity, $g = 9.81 \text{ m/s}^2$

At exhaust of turbine:

$$\begin{aligned} \text{Absolute steam pressure, } P_{abs,2} &= 4.0 \text{ cm of Hg} = \rho_{Hg} g z = 13600 \times 9.81 \times 4 \times 10^{-3} \\ &= 5336.64 \text{ Pa} \end{aligned}$$

$$\begin{aligned} \text{Atmospheric pressure, } P_{atm} &= \rho_{Hg} g z_{baro} = 13600 \times 9.81 \times 0.75 = 100062 \text{ Pa} \\ &= 100.062 \text{ kPa} \end{aligned}$$

Using the relation between absolute and gauge pressure at the inlet,

CONCEPT AND DEFINITIONS

$$P_{abs,1} = P_{gauge,1} + P_{atm}$$

⇒ Gauge pressure for the entering steam,

$$P_{gauge,1} = P_{abs,1} - P_{atm} = 6000 - 100.062 = 5899.938 \text{ kPa}$$

Similarly, using the relation between absolute and gauge pressure at the exhaust,

$$P_{abs,2} = P_{gauge,2} + P_{atm}$$

⇒ Vacuum gauge pressure for exhaust steam,

$$P_{gauge,2} = P_{abs,2} - P_{atm} = 5336.64 - 1000.62 = -94725.36 \text{ Pa} = -94.725 \text{ kPa}$$

The negative sign indicates that the gauge pressure for the exhaust steam is less than the atmospheric pressure and it is also known as vacuum gauge pressure.

Problem 1.19.

A mercury manometer is used to measure the pressure in a steam pipe. The level of the mercury in the manometer is 97.5 mm. Find the absolute pressure of steam inside the pipe. If the reading of the manometer drops to 80 mm, what is the new pressure of steam? [Take specific gravity of Hg = 13.6, $P_{atm} = 760 \text{ mm of Hg}$ and $g = 9.81 \text{ m/s}^2$].

Solution: Given,

Height of mercury in manometer, $z_{mano} = 97.5 \text{ mm} = 0.0975 \text{ m}$

Specific gravity of mercury, $(SG)_{Hg} = 13.6$

Atmospheric pressure, $P_{atm} = 760 \text{ mm of Hg}$

Acceleration of gravity, $g = 9.81 \text{ m/s}^2$

New height of mercury in manometer, $z_{mano,N} = 80 \text{ mm} = 0.08 \text{ m}$

Density of mercury, $\rho_{Hg} = (SG)_{Hg} \times \rho_{water} = 13.6 \times 1000 = 13600 \text{ kg/m}^3$

Atmospheric pressure, $P_{atm} = 760 \text{ mm of Hg} = \rho_{Hg} g z_{baro}$

$$= 13600 \times 9.81 \times 760 \times 10^{-3} = 101396.16 \text{ Pa}$$

Using the relation between absolute and gauge pressure for steam,

$$P_{abs} = P_{gauge} + P_{atm} = \rho_{Hg} g z_{mano} + P_{atm}$$

$$= 13600 \times 9.81 \times 0.0975 + 101396.16 = 114404.22 \text{ Pa} = 114.404 \text{ kPa}$$

When the height of mercury drops in manometer, similarly the new absolute pressure of steam is given by

$$P_{abs,N} = P_{gauge,N} + P_{atm} = \rho_{Hg} g z_{mano,N} + P_{atm}$$

$$= 13600 \times 9.81 \times 0.08 + 101396.16 = 112069.44 \text{ Pa} = 112.069 \text{ kPa}$$

Problem 1.20.

On a new scale N of temperature the freezing point of ice and boiling point of water are 100°N and 400°N respectively. Derive an expression to convert a temperature reading on N scale to °C scale. Also determine the change in N scale when the temperature of a system increases by 50°C.

Solution: Given,

For the new absolute temperature scale N

Freezing point of ice on N, F.P. = 100°N

Boiling point of water on N, B.P. = 400°N

We know the relationship between different temperature scales as

$$\frac{N - F.P.}{B.P. - F.P.} = \frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{K - 273}{373 - 273}$$

Using first two expressions,

$$\frac{N - 100}{400 - 100} = \frac{C - 0}{100 - 0}$$

$$\Rightarrow N = 100 + 3C \quad \text{which is the required expression.}$$

Let us assume N and C are new values of temperatures of the system when it is increased by 50°C. Then, using expression, $N = 100 + 3C$ to find value of N for $C = C + 50$, we get

$$N = 100 + 3(C + 50) = 250 + 3C$$

$$\text{Now, change in N scale, } N - N = 250 + 3C - (100 + 3C) = 150^\circ\text{N}$$

Problem 1.21.

During the operation of a lift, it can be subjected to a maximum gauge pressure of 500 kPa. If it is designed to lift a mass up to 900 kg, what should be diameter of the piston/cylinder? [Take $g = 9.81 \text{ m/s}^2$]

Solution: Given,

Maximum gauge pressure, $P_{\text{gauge}} = 500 \text{ kPa} = 500 \times 10^3 \text{ Pa}$

Mass to be lifted, $m = 900 \text{ kg}$

Referring to the Free Body Diagram (FBD) of piston, we can write

$$P_{\text{atm}} \times A_P = P_{\text{gauge}} \times A_P - W = P_{\text{atm}} \times A_P + mg \quad \dots \dots \dots \text{(i)}$$

where absolute pressure, $P_{\text{atm}} = P_{\text{gauge}} + P_{\text{ext}}$.

Putting this value in equation (i), we get

$$P_{\text{gauge}} \times A_P - P_{\text{atm}} \times A_P = P_{\text{atm}} \times A_P + mg$$

$$\Rightarrow P_{\text{gauge}} \times A_P = mg$$

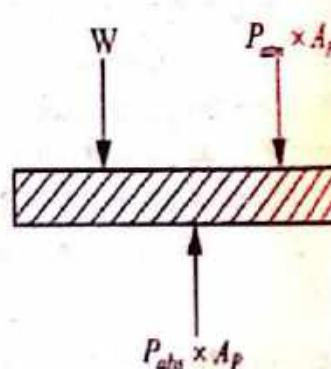
$$\therefore A_P = \frac{mg}{P_{\text{gauge}}} = \frac{900 \times 9.81}{500 \times 1000} \\ = 0.017658 \text{ m}^2$$

Area of the piston/cylinder is given by

$$A_P = \pi d_P^2 / 4$$

Hence, diameter of the piston is calculated as

$$d_P = \sqrt{\frac{4A_P}{\pi}} = \sqrt{\frac{4 \times 0.017658}{\pi}} = 0.149943 \text{ m}$$



FBD

Problem 1.22.

Air (0.1 kg) is contained in piston-cylinder assembly as shown in figure below. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is 100 kN/m . The piston weighs 30 kN and atmospheric pressure is 101 kPa . The air is initially at 300 K and 200 kPa . Heat transfer occurs until the air temperature reaches the surrounding temperature of 700 K . Find the final pressure and volume. [Take $R = 287 \text{ J/kgK}$].

Solution: Given,

Mass of air, $m = 0.1 \text{ kg}$

Spring constant, $k = 100 \text{ kN/m}$

Piston weight, $W = 30 \text{ kN}$

Atmospheric pressure, $P_{atm} = 101 \text{ kPa}$

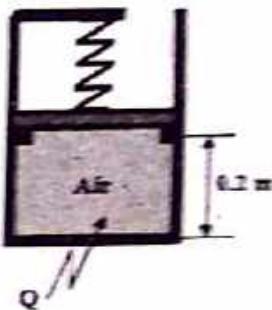
Initial temperature of air, $T_1 = 300 \text{ K}$

Initial pressure of air, $P_1 = 200 \text{ kPa}$

Final temperature of air, $T_f = 700 \text{ K}$

From the system diagram, area of piston can be determined as

$$A_p = \frac{V_1}{h} = \frac{0.04305}{0.2} = 0.21525 \text{ m}^2$$



Initial state 1:

$P_1 = 200 \text{ kPa}$, $T_1 = 300 \text{ K}$

From the equation of state, volume at state 1, V can be determined as

$$PV_1 = mRT_1 \Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{0.1 \times 287 \times 300}{200 \times 10^3} = 0.04305 \text{ m}^3$$

The heating process is shown on $P - V$ diagram in figure below. During the initial heating the piston remains stationary until the initial pressure reaches lifting pressure of the piston. Hence, process 1 – 2 is the constant volume heating and state 2 is defined as

State 2:

Volume at state 2, $V_2 = V_1 = 0.04305 \text{ m}^3$

Pressure at state 2, $P_2 = P_{lift}$

To calculate the lifting pressure of the piston, referring to the free body diagram (FBD) of piston at state 2, we can write,

$$\begin{aligned} P_{lift} \times A_p &= P_{atm} \times A_p + W \\ \Rightarrow P_{lift} &= P_{atm} + W/A_p = 101 + 30/0.21525 \\ &= 240.373 \text{ kPa} \\ \therefore P_2 &= P_{lift} = 240.373 \text{ kPa} \end{aligned}$$

Problem 1.11.

Air ($\rho = 1 \text{ kg}$) is contained in piston-cylinder assembly as shown in figure below. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is 500 kN/m . The piston weighs 30 kN and atmospheric pressure is 101 kPa . The air is initially at 300 K and 200 kPa . Heat transfer occurs until the air temperature reaches the surrounding temperature of 700 K . Find the final pressure and volume. (Take $R = 287 \text{ J/kgK}$).

Solution: Given,

Mass of air, $m = 0.1 \text{ kg}$

Spring constant, $k = 500 \text{ kN/m}$

Piston weight, $W = 30 \text{ kN}$

Atmospheric pressure, $P_{atm} = 101 \text{ kPa}$

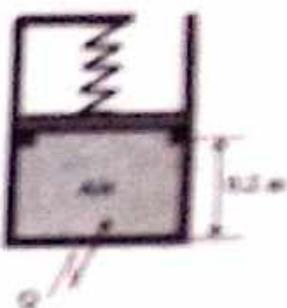
Initial temperature of air, $T_1 = 300 \text{ K}$

Initial pressure of air, $P_1 = 200 \text{ kPa}$

Final temperature of air, $T_2 = 700 \text{ K}$

From the system diagram, area of piston can be determined as

$$A_p = \frac{V_1}{h} = \frac{0.04305}{0.2} = 0.21525 \text{ m}^2$$



Initial state 1:

$P_1 = 200 \text{ kPa}$, $T_1 = 300 \text{ K}$

From the equation of state, volume at state 1, V_1 can be determined as

$$P_1 V_1 = m R T_1 \Rightarrow V_1 = \frac{m R T_1}{P_1} = \frac{0.1 \times 287 \times 300}{200 \times 10^3} = 0.04305 \text{ m}^3$$

The heating process is shown on $P - V$ diagram in figure below. During the initial heating the piston remains stationary until the initial pressure reaches lifting pressure of the piston. Hence, process 1 - 2 is the constant volume heating and state 2 is defined as

State 2:

Volume at state 2, $V_2 = V_1 = 0.04305 \text{ m}^3$

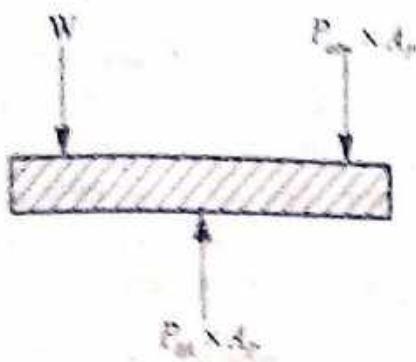
Pressure at state 2, $P_2 = P_{lift}$

To calculate the lifting pressure of the piston, referring to the free body diagram (FBD) of piston at state 2, we can write,

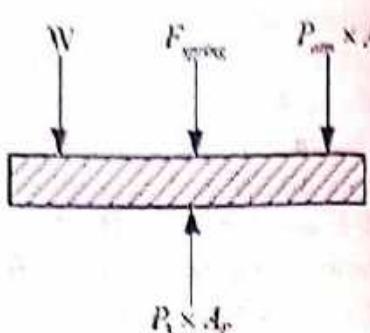
$$P_{lift} \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_{lift} = P_{atm} + W / A_p = 101 + 30 / 0.21525 \\ = 240.373 \text{ kPa}$$

$$P_2 = P_{lift} = 240.373 \text{ kPa}$$



FBD at state 2



FBD at final state

From equation of state, temperature at state 2, T_2 can be calculated as

$$P_2 V_2 = mRT_2 \Rightarrow T_2 = \frac{P_2 V_2}{mR} = \frac{240.373 \times 10^3 \times 0.04305}{0.1 \times 287} = 360.56 \text{ K}$$

Since the final temperature is $T_f = 700 \text{ K}$, the system has still to be further heated to attain this final temperature. The further heating process 2 - 3 is linear due to presence of the spring. Hence, state 3 is defined as

Final state 3:

Temperature at state 3, $T_3 = T_f = 700 \text{ K}$, volume at state 3,

$$V_3 = V_2 + x \times A_p = 0.04305 + x(0.21525) = 0.21525(0.2 + x) \quad \dots \dots \dots (i)$$

Pressure at state 3, P_3 , can be determined by referring to the free body diagram (FBD) of piston at final state which will be the final pressure,

$$\begin{aligned} P_3 \times A_p &= P_{\text{atm}} \times A_p + W + F_{\text{spring}} \\ \Rightarrow P_3 &= P_{\text{atm}} + W/A_p + kx/A_p = 240.373 + 100x/0.21525 \text{ kPa} \quad \dots \dots \dots (ii) \end{aligned}$$

where x is the compression of the spring. Pressure at state 3, P_3 , can also be calculated by using equation of state,

$$P_3 V_3 = mRT_3 \Rightarrow P_3 = \frac{mRT_3}{V_3} = \frac{0.1 \times 287 \times 700}{0.21525(0.2 + x)} = \frac{93.333}{0.2 + x} \text{ kPa} \quad \dots \dots \dots (iii)$$

Equating equations (ii) and (iii) yields,

$$\begin{aligned} 240.373 + \frac{100x}{0.21525} &= \frac{93.333}{0.2 + x} \\ \Rightarrow 51.74 + 100x &= \frac{20.09}{0.2 + x} \\ \Rightarrow 100x^2 + 71.74x - 9.74 &= 0 \\ \Rightarrow x &= \frac{-71.74 \pm \sqrt{(71.74)^2 - 4 \times 100 \times (-9.74)}}{2 \times 100} = 0.11676 \text{ m} \end{aligned}$$

(taking positive value)

Putting this value of x in equations (i) and (iii), we get

$$V_3 = 0.21525(0.2 + 0.11676) = 0.06818 \text{ m}^3$$

$$\Rightarrow \text{Final volume, } V_f = 0.06818 \text{ m}^3$$

CONCEPT AND DEFINITIONS

Similarly, $P_1 = \frac{93.333}{0.2 + x} = 294.646 \text{ kPa}$

\Rightarrow Final pressure, $P_f = 294.646 \text{ kPa}$

1.3 Solved Numerical Problems from PoU Examinations

Problem 1.23.

Identify which of the followings are extensive properties and which are intensive properties.

- i) 10 m^3 of volume
- ii) 30 J of kinetic energy
- iii) Pressure of 90 kPa
- iv) Mass of 75 kg
- v) Velocity of 60 m/s

Convert all extensive properties to intensive properties assuming $m = 75 \text{ kg}$.

(PoU, 2016 spring).

Solution:

The (i) volume, (ii) kinetic energy and (iv) mass are extensive properties as they are dependent of mass and size of the system.

The (iii) pressure and (v) velocity are intensive properties because these properties are independent of mass and size of the system.

Conversion of extensive property into intensive property can be done by dividing it by the mass of the system which is also called specific property. Hence

- (i) Extensive property: Volume, $V = 10 \text{ m}^3$
Intensive property: Specific volume, $v = V/m = 10/75 = 0.1333 \text{ m}^3/\text{kg}$
- (ii) Extensive property: Kinetic energy, $KE = 30 \text{ J}$
Intensive property: Specific kinetic energy, $ke = KE/m = 30/75 = 0.4 \text{ J/kg}$
- (iv) Extensive property: Mass, $m = 75 \text{ kg}$
Mass can be converted into intensive property by dividing it by volume and it is called density of the system. So,
Intensive property: Density, $\rho = m/V = 75/10 = 7.5 \text{ kg/m}^3$

1.4 Review Theoretical Questions

1. Define engineering thermodynamics and state its scope in the field of energy technology.
2. Discuss the value of energy to society.
3. Differentiate between classical thermodynamics and statistical thermodynamics.
4. Differentiate between the microscopic and macroscopic view point in thermodynamics with example.
5. Define thermodynamic system, surroundings, boundary and universe with examples.
6. Explain different types of thermodynamic system with examples.
7. Differentiate between the control mass and control volume with examples.
8. Define a thermodynamic property and thermodynamic state. List two important features of a thermodynamic property.
9. Difference between the intensive and extensive properties. Derive the expression for work done during adiabatic process.
10. Derive a general expression for the displacement work (Quasi-static work) transfer for a piston cylinder device. Also reduce it for an ideal gas undergoing constant temperature process.
11. Mention the characteristics of thermodynamic properties. Differentiate between extensive property and extensive property of thermodynamics with examples.
12. Explain the difference between state function and path function with example.
13. Define state function with example and mention main characteristic of a thermodynamic property in relation to a cyclic process.
14. Define a thermodynamic process. Sketch $P - V$, $T - V$ and $P - T$ diagrams for an ideal gas undergoing isothermal and isobaric expansion.
15. Sketch $P - V$, $T - V$ and $P - T$ diagrams for an ideal gas undergoing
 - (i) constant volume cooling process
 - (ii) constant temperature heat rejection process
16. Define thermodynamic equilibrium. Explain reversible and irreversible processes with reference to nature of intermediate states.

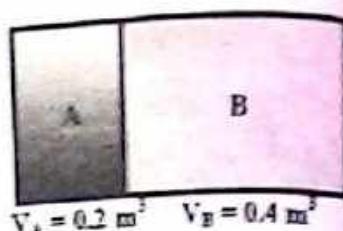
CONCEPT AND DEFINITIONS

17. Define a quasi - static or quasi - equilibrium process and state its salient characteristics.
18. Define these terms: (a) Quasi - equilibrium process (b) An open system (c) Thermodynamic equilibrium (d) Intensive property.
19. Define specific volume and temperature. Derive a relationship between Celsius, Fahrenheit and Kelvin temperature scales.
20. Differentiate between the gauge pressure and absolute pressure.
21. Define vacuum gauge. Show atmospheric pressure, gauge pressure and absolute pressure on a common pressure scale.
22. State and explain equality of temperature. Also state zeroth law of thermodynamics.
23. State and explain Zeroth law of thermodynamics. Also write down its application.

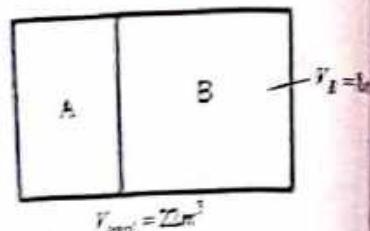
CONCEPT AND DEFINITIONS

1.5 Unsolved Numerical Problems

1. A container has two compartments separated by a membrane as shown in figure below. Specific volume of steam in compartment A and compartment B are $5 \text{ m}^3/\text{kg}$ and $10 \text{ m}^3/\text{kg}$ respectively. If the membrane breaks and steam comes to a uniform state, determine the resulting specific volume. (Ans: $7.5 \text{ m}^3/\text{kg}$)



2. Steam in a container is separated by a membrane as shown in figure below. The specific volume of steam in B is $0.5951 \text{ m}^3/\text{kg}$. The membrane breaks and the resulting specific volume is $0.7964 \text{ m}^3/\text{kg}$. Find the original specific volume in A. (Ans: $0.9873 \text{ m}^3/\text{kg}$)

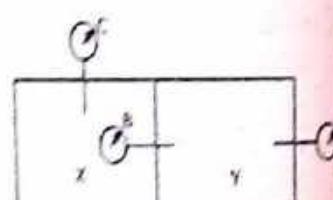


3. A fan delivers 4 m^3 of air having a mass equal to 5 kg . Calculate specific weight, density and specific volume of the air being delivered. (Ans: 12.26 N/m^3 , 1.25 kg/m^3 , $0.8 \text{ m}^3/\text{kg}$)

4. A tube contains an oil of specific gravity 0.9 to a depth of 120 cm. Find the gauge pressure at this depth. (Ans: 10.595 kPa)

5. A vessel of cylindrical shape, 40 cm diameter and 60 cm height, contains 3 kg gas. The pressure gauge mounted on the vessel indicates 600 mm of mercury. If the barometer reading is 750 mm of mercury, calculate the absolute pressure of gas. Also determine the density and specific volume of the gas. (Ans: 180.112 kPa , 39.81 kg/m^3 , $0.0251 \text{ m}^3/\text{kg}$)

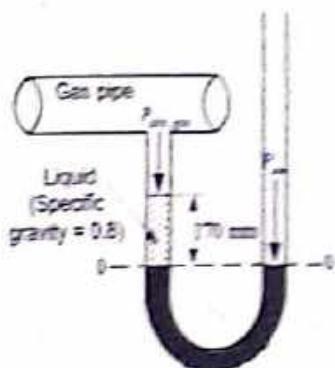
6. A vessel shown in figure below in which there are two compartments at different temperatures. The pressure gauge A reads 4 bar gauge and B reads 2 bar gauge. The barometer reads 760 mm of Hg. Calculate the absolute pressure existing in the compartments and the reading of gauge C. (Ans: 3.813 bar, 5.813 bar, 2 bar)



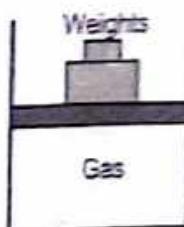
7. A turbine is supplied with steam at a pressure of 20 bar gauge. After expansion in the turbine the steam passes to a condenser which is maintained

at a vacuum of 250 mm of mercury by means of pumps. Find the inlet and exhaust steam pressures if the barometer reads 750 mm of mercury.
 (Ans: 2100 kPa, 66.708 kPa)

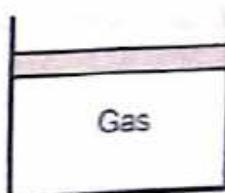
8. A U-tube manometer is connected to a gas pipe as shown in figure below. The level of the liquid in the manometer arm open to the atmosphere is 170 mm lower than the level of the liquid in the arm connected to the gas pipe. The liquid in the manometer has specific gravity of 0.8. Find the absolute pressure of the gas if the barometer reads 760 mm of Hg.
 (Ans: 99.99 kPa)



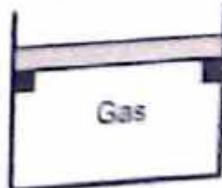
9. A vertical piston - cylinder device shown in figure below contains a gas at a pressure of 100 kPa. The piston has a mass of 5 kg and a diameter of 12 cm. Pressure of the gas is to be increased by placing some weights on the piston. Determine the local atmospheric pressure and the mass of the weights that will double the pressure of the gas inside the cylinder. (Ans: 95.663 kPa, 115.29 kg)



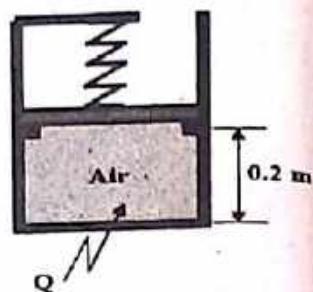
10. The piston of a vertical piston cylinder device containing a gas has a mass of 50 kg and a cross sectional area of 0.02 m^2 , as shown in figure below. (a) Determine the pressure inside the cylinder. (b) During some process heat is lost by the gas to the surroundings and its volume decreases to $\frac{1}{4}$ of the initial volume, determine its final pressure. [Take $P_{\text{ext}} = 100 \text{ kPa}$, $g = 9.81 \text{ m/s}^2$] (Ans: 124.525 kPa, Pressure remains constant)



11. A piston cylinder assembly shown in figure contains a gas at a pressure of 200 kPa. If the area of cross section of the piston 0.01 m^2 and the mass of the piston is 50 kg. What external atmospheric pressure is required to just keep the piston resting on the stops? [Take $g = 9.81 \text{ m/s}^2$]. (Ans: 150.95 kPa)



12. Air ($m = 0.1 \text{ kg}$) is contained in piston/cylinder assembly as shown in figure below. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is 100 kN/m . The piston weighs 30 kN and atmospheric pressure is 101 kPa . The air is initially at 300 K and 200 kPa . Heat transfer occurs until the air temperature reaches the surrounding temperature of 700 K . Find the final pressure and volume. [Take $R = 287 \text{ J/kgK}$]
 (Ans: 194.84 kPa , 0.06829 m^3)



13. A piston cylinder has a diameter of 0.1 m . With an outside atmospheric pressure of 100 kPa , determine the piston mass that will create an inside pressure of 500 kPa . [Take $g = 9.81 \text{ m/s}^2$] (Ans: 320.24 kg)
14. The temperature of a system drops by 36°F during a process. Express this drop in temperatures in, ${}^\circ\text{C}$ and K . (Ans: 20°C , 20 K)
15. Two thermometers, one centigrade and other Fahrenheit immersed in a fluid read the same numerical value. Find the identical numerical value shown by the thermometer. Also find their temperature of fluid expressed in K .
 (Ans: -40°C , -40°F , 233 K)

Properties Of Pure Working Substances

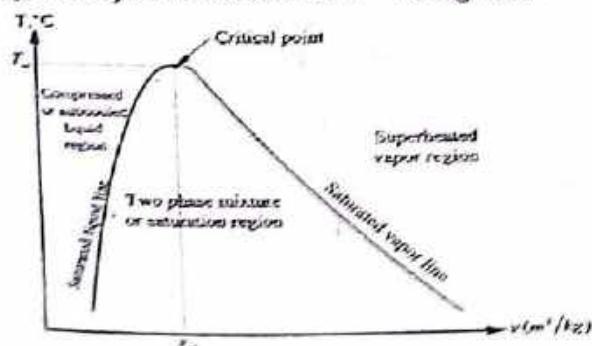
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2.1 Chapter Highlights

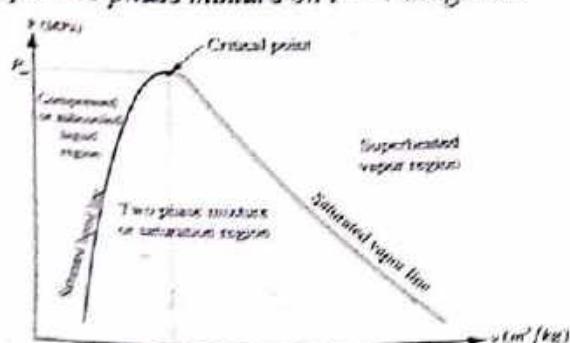
- ***Working substance:*** A substance used in any thermodynamic system (device) with the help of which energy is transported between the system and its surroundings is called a working substance. For examples, water, air, refrigerants etc.
- ***Pure substance:*** A pure substance is homogeneous in composition, homogenous in chemical aggregation and invariable in chemical aggregation and is also called simple compressible substance.
- ***Two property rule (state postulate):*** It states that “Any two independent intensive properties are sufficient to specify a thermodynamic equilibrium state of a system composed of a pure substance”.
- ***Ideal gas:*** An ideal gas has very negligible or no force of intermolecular attraction. It follows ideal gas equation or gas laws at all ranges of pressure and temperature. Most of real gases tend to behave an ideal gas at low pressure and high temperature.
- ***Boyle's law:*** It states that “At a constant temperature, the volume of a given mass of a gas is inversely proportional to the pressure, i.e. $PV = \text{constant}$ ”.
- ***Charle's law or Gay Lussac's law:*** (i) It states that “At constant pressure, the volume of a given mass of a gas is directly proportional to the absolute temperature”, i.e., $V/T = \text{constant}$.
(ii) It can also be stated as “At constant volume, the pressure of a given mass of a gas is directly proportional to the absolute temperature”, i.e., $P/T = \text{constant}$.
- ***Equation of state:*** It is the combined gas equation and given by $PV = mRT$ or, $Pv = RT$.
- ***Saturation pressure:*** For a given temperature, the pressure at which phase change takes place is called saturation pressure, e.g. saturation pressure for 100°C is 1 atm.
- ***Saturation temperature:*** For a given pressure, the temperature at which phase change takes place is called saturation temperature, e.g., saturation temperature for 1 atm is 100°C.

PROPERTIES OF PURE WORKING SUBSTANCES

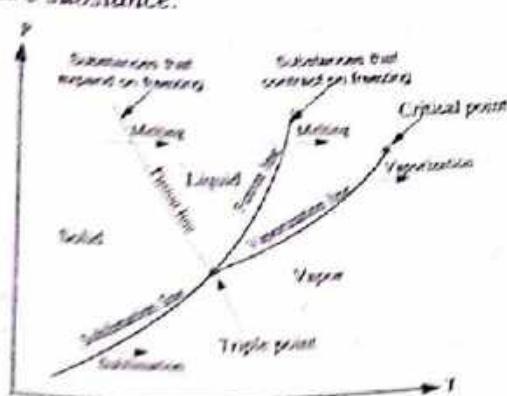
- **Saturated liquid:** It is the state of a substance at which evaporation just starts during heating or condensation just completes during cooling.
- **Saturated vapor:** It is the state of a substance at which evaporation just completes during heating and condensation just starts during cooling.
- **Sub-cooled or compressed liquid:** When saturated liquid is further cooled below its saturation temperature or compressed above its saturation pressure, it is called sub-cooled or compressed liquid.
- **Superheated vapor:** When saturated vapor is further heated or expanded, it is called a superheated vapor.
- **Two phase mixture:** It is the mixture of saturated liquid and saturated vapor.
- **Saturation curve for two phase mixture on $T - v$ diagram:**



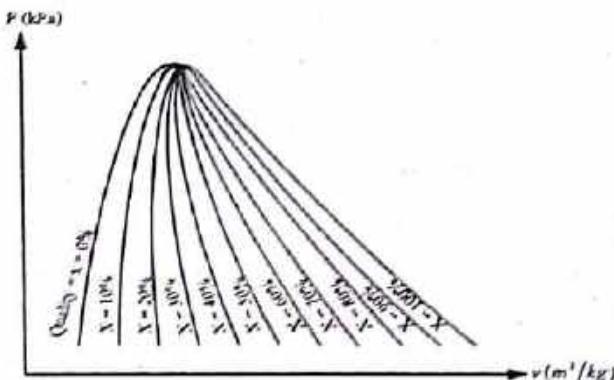
- **Saturation curve for two phase mixture on $P - v$ diagram:**



- **$P - T$ diagram of pure substance:**



- *Degree of superheat:* It is the difference in temperature of superheated vapor and its corresponding saturation temperature.
- *Degree of sub-cooling:* It is the difference in the corresponding saturation temperature and the temperature of a sub-cooled or compressed liquid.
- *Critical point:* It is the meeting point of saturated liquid line and saturated vapor line at which saturated liquid and vapor have identical properties. The pressure, temperature and volume corresponding to critical point are called critical pressure, critical temperature and critical volume respectively.
- *Tripole point:* It is the state of a substance at which the solid, liquid and vapor phase co-exist.
- *Quality or dryness fraction:* It is defined as the ratio of mass of saturated vapor and the total mass of the two phase mixture, i.e. $x = m_g/m$. It has a value ranges from 0 to 1.
- *Distribution of quality lines on P – v diagram:*



- *Moisture content or liquid fraction:* It is defined as the ratio of mass of saturated liquid and the total mass of two phase mixture, i.e.

$$y = m_l/m = (m - m_g)/m = 1 - x.$$

- *Necessity of quality:*
Both pressure and temperature are independent for sub-cooled or compressed liquid states and superheated vapor states. Hence, states can be fixed with given pressure and temperature. But for two phase mixture or saturation region, pressure and temperature are no more independent and states cannot be fixed with dependent temperature and pressure as per state postulate. Therefore, quality is necessary to fix the states within the saturation region.

- *Specific properties of a two phase mixture:*

- Specific volume, $v = v_l + xv_{lg}$ where $v_{lg} = v_g - v_l$
- Specific internal energy, $u = u_l + xu_{lg}$
- Specific enthalpy, $h = h_l + xh_{lg}$

- In the absence of sub-cooled or compressed liquid data, a general approximation is to treat a sub-cooled or compressed liquid as a saturated liquid at the given *temperature*, hence

$$z \cong z_l$$

where z stands for v , u , s , or h .

- *Specific heats of an ideal gas:*

- *Specific heat at constant volume:* It is defined as the change in specific internal energy per degree change in temperature during a constant volume process. Mathematically,

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

- *Specific heat at constant pressure:* It is defined as the change in specific internal enthalpy per degree change in temperature during a constant pressure process. Mathematically,

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

2.2 Solved Numerical Problems

Problem 2.1.

A rigid vessel contains 8 kg of water at 120°C. If 5 kg of the water is in the liquid form and the rest in the vapor form, determine

- (a) the pressure in the vessel,
- (b) the volume of the vessel,
- (c) the volume of saturated liquid and saturated vapor respectively, and
- (d) the specific enthalpy of H₂O.

Solution: Given,

Volume, $V = \text{constant}$

(As it is rigid vessel)

Total mass of water, $m = 8 \text{ kg}$

Temperature of water, $T = 120^\circ\text{C}$

Mass of liquid water, $m_l = 5 \text{ kg}$

Mass of vapor, $m_g = m - m_l = 8 - 5 = 3 \text{ kg}$

- Specific entropy, $s = s_i + xs_{fg}$
- **Enthalpy:** It is the summation of internal energy and the product of pressure and volume and mathematically, $H = U + PV$ or, specific enthalpy $h = u + Pv$.
- **Sensible heat:** It is defined as the quantity of heat required to raise the temperature of certain mass of a substance through certain rise in temperature.
- **Latent heat:** The amount of energy absorbed or released during a phase-change process is called the *latent heat*. More specifically, the amount of energy absorbed during melting is called the *latent heat of fusion* and is equivalent to the amount of energy released during freezing. Similarly, the amount of energy absorbed during vaporization is called the *latent heat of vaporization* and is equivalent to the energy released during condensation.
- **Identification of states:**
 - For a given specific volume, if specific volume is than specific volume of saturated liquid, its state will be sub-cooled or compressed liquid and if specific volume is higher than specific volume of saturated vapor, the state will be superheated vapor and if it lies between these values, the state will be two phase mixture, i.e.,
 - $v < v_i$ implies the state is sub-cooled or compressed liquid.
 - $v > v_f$ implies the state is superheated vapor.
 - $v_i < v < v_f$ implies the state is two phase mixture.

The same rule is applied for specific internal energy (u), enthalpy (h) and entropy (s).

 - For a given pressure, if temperature of a state is less than the saturation temperature, its state will be a sub-cooled or compressed liquid and if temperature is higher than the saturation temperature, the state will be a superheated vapor, i.e. $T < T_{sat}$ implies the state is sub-cooled or compressed liquid.
 $T > T_{sat}$ implies the state is superheated vapor.
 - For a given temperature, if pressure of a state is higher than the saturation pressure, its state will be a sub-cooled or compressed liquid and if pressure is less than the saturation pressure, the state will be superheated vapor, i.e.,
 $P > P_{sat}$ implies the state is sub-cooled or compressed liquid.
 $P < P_{sat}$ implies the state is superheated vapor.

- In the absence of sub-cooled or compressed liquid data, a general approximation is to treat a sub-cooled or compressed liquid as a saturated liquid at the given *temperature*, hence

$$z \cong z_l$$

where z stands for v , u , s , or h .

- *Specific heats of an ideal gas:*

- *Specific heat at constant volume:* It is defined as the change in specific internal energy per degree change in temperature during a constant volume process. Mathematically,

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

- *Specific heat at constant pressure:* It is defined as the change in specific internal enthalpy per degree change in temperature during a constant pressure process. Mathematically,

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

2.2 Solved Numerical Problems

Problem 2.1.

A rigid vessel contains 8 kg of water at 120°C. If 5 kg of the water is in the liquid form and the rest in the vapor form, determine

- (a) the pressure in the vessel,
- (b) the volume of the vessel,
- (c) the volume of saturated liquid and saturated vapor respectively, and
- (d) the specific enthalpy of H₂O.

Solution: Given,

Volume, $V = \text{constant}$

(As it is rigid vessel)

Total mass of water, $m = 8 \text{ kg}$

Temperature of water, $T = 120^\circ\text{C}$

Mass of liquid water, $m_l = 5 \text{ kg}$

Mass of vapor, $m_g = m - m_l = 8 - 5 = 3 \text{ kg}$

PROPERTIES OF PURE WORKING SUBSTANCES

Here, water is in two phase mixture region, so referring to the saturated water temperature table A2.2 for temperature, $T = 120^\circ\text{C}$,

$$\text{Pressure, } P = P_{\infty}(T = 120^\circ\text{C}) = 198.48 \text{ kPa.}$$

$$\text{Quality, } x = \frac{m_s}{m} = \frac{3}{8} = 0.375$$

Again, referring to the saturated water temperature table A2.2 for temperature, $T = 120^\circ\text{C}$, $v_f = 0.001060 \text{ m}^3/\text{kg}$, $v_{fg} = 0.8911 \text{ m}^3/\text{kg}$, $v_g = 0.8922 \text{ m}^3/\text{kg}$.

$$\begin{aligned}\text{Hence, specific volume, } v &= v_f + x v_{fg} = 0.001060 + 0.375 \times 0.8911 \\ &= 0.3352 \text{ m}^3/\text{kg.}\end{aligned}$$

$$\text{Now, volume of vessel, } V = m \times v = 8 \times 0.3352 = 2.682 \text{ m}^3$$

$$\text{Volume of saturated liquid, } V_f = m_f \times v_f = 5 \times 0.001060 = 0.0053 \text{ m}^3 \text{ and}$$

$$\text{Volume of saturated vapor, } V_g = m_g \times v_g = 3 \times 0.8922 = 2.6766 \text{ m}^3$$

Again, referring to the saturated water temperature table A2.2 for temperature, $T = 120^\circ\text{C}$, $h_f = 503.78 \text{ kJ/kg}$, $h_{fg} = 2202.4 \text{ kJ/kg}$, $h_g = 2706.2 \text{ kJ/kg}$.

$$\begin{aligned}\text{Hence, specific enthalpy of water, } h &= h_f + x h_{fg} = 503.78 + 0.375 \times 2202.4 = 1329.685 \text{ kJ/kg.}\end{aligned}$$

Problem 2.2.

Determine the pressure for water at 250°C with specific volume of $0.25 \text{ m}^3/\text{kg}$.

Solution: Given,

Temperature of water, $T = 250^\circ\text{C}$

Specific volume of water, $v = 0.25 \text{ m}^3/\text{kg}$

Referring to saturated water temperature table A2.2 for temperature, $T = 250^\circ\text{C}$, $v_f = 0.001251 \text{ m}^3/\text{kg}$, $v_{fg} = 0.04886 \text{ m}^3/\text{kg}$, $v_g = 0.05011 \text{ m}^3/\text{kg}$.

Here, $v > v_f$. Hence, it is a superheated vapor.

Now, pressure of superheated vapor can be found by referring to superheated vapor table A2.4 for temperature $T = 250^\circ\text{C}$ and specific volume, $v = 0.25 \text{ m}^3/\text{kg}$. As there is no value listed for given value of specific volume, the value of pressure is determined by linear interpolation method in an interval which includes the given specific volume. Then, from table A2.4 for temperature, $T = 250^\circ\text{C}$

$$P_1 = 800 \text{ kPa}$$

$$v_1 = 0.2931 \text{ m}^3/\text{kg}$$

$$P = ?$$

$$v = 0.25 \text{ m}^3/\text{kg}$$

$$P_2 = 1000 \text{ kPa}$$

$$v_2 = 0.2326 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$\begin{aligned}P &= P_1 + \frac{v - v_1}{v_2 - v_1} (P_2 - P_1) = 800 + \frac{0.25 - 0.2931}{0.2326 - 0.2931} (1000 - 800) \\ &= 942.479 \text{ kPa.}\end{aligned}$$

Problem 2.3.

Determine the temperature and quality (if needed) for water at a pressure of 200 kPa and having a specific volume of

- (a) $0.8 \text{ m}^3/\text{kg}$
- (b) $1.25 \text{ m}^3/\text{kg}$.

Solution: Given,

Pressure of water, $P = 200 \text{ kPa}$

- (a) Specific volume, $v_a = 0.8 \text{ m}^3/\text{kg}$
- (b) Specific volume, $v_b = 1.25 \text{ m}^3/\text{kg}$

Referring to the saturated water pressure table A2.1 for pressure, $P = 200 \text{ kPa}$, temperature for two phase mixture, $T(P = 200 \text{ kPa}) = T_{sat} = 120.24^\circ\text{C}$.

$v_l = 0.001060 \text{ m}^3/\text{kg}$, $v_{lg} = 0.8848 \text{ m}^3/\text{kg}$, $v_g = 0.8859 \text{ m}^3/\text{kg}$.

- (a) Here, $v_l < v_a < v_g$. Hence, it is a two phase mixture.

$$\text{Quality, } x = \frac{v_a - v_l}{v_{lg}} = \frac{0.8 - 0.001060}{0.8848} = 0.90296 = 90.3\%$$

- (b) Here, $v_b > v_g$. Hence, it is a superheated vapor.

There is no meaning of quality in superheated vapor region.

The temperature of superheated vapor can be read from the superheated vapor table A2.4 for pressure, $P = 200 \text{ kPa}$ and specific volume, $v_b = 1.25 \text{ m}^3/\text{kg}$. As there is no value listed for given values of the pressure and specific volume, the value of temperature is determined by linear interpolation method in an interval which includes the given specific volume. Then, from table A2.4 for pressure, $P = 200 \text{ kPa}$

$$T_1 = 250^\circ\text{C} \quad v_1 = 1.1988 \text{ m}^3/\text{kg}$$

$$T_b = ? \quad v_b = 1.25 \text{ m}^3/\text{kg}$$

$$T_2 = 300^\circ\text{C} \quad v_2 = 1.3162 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$T_b = T_1 + \frac{v_b - v_1}{v_2 - v_1} (T_2 - T_1)$$

$$\Rightarrow T_b = 250 + \frac{1.25 - 1.1988}{1.3162 - 1.1988} (300 - 250) = 271.896^\circ\text{C}$$

Problem 2.4.

Water is contained in a rigid vessel of 5 m^3 at a quality of 0.8 and a pressure of 2 MPa. If it is cooled to a pressure of 400 kPa, determine the mass of saturated liquid and saturated vapor at the final state.

Solution: Given,

Volume of water, $V = 5 \text{ m}^3 = \text{constant}$

(As it is rigid vessel)

Initial quality, $x_i = 0.8$

Initial pressure of water, $P_1 = 2 \text{ MPa} = 2000 \text{ kPa}$

Final pressure of water, $P_2 = 400 \text{ kPa}$

Initial state 1:

$$V_1 = 5 \text{ m}^3, P_1 = 2000 \text{ kPa}, x_1 = 0.8$$

$P-v$ diagram of cooling process is shown in figure below. As quality is given for pressure, $P = 2000 \text{ kPa}$, it is a two phase mixture. Referring to saturated water pressure table for A2.1 for pressure, $P_1 = 2000 \text{ kPa}$,

$$v_f = 0.001177 \text{ m}^3/\text{kg}, v_{fg} = 0.09841 \text{ m}^3/\text{kg}, v_g = 0.09959 \text{ m}^3/\text{kg},$$

$$\text{Specific volume at state 1, } v_1 = v_l + x_1 v_{fg} = 0.001177 + 0.8 \times 0.09841 \\ = 0.079905 \text{ m}^3/\text{kg}$$

$$\text{Mass of water, } m = \frac{V_1}{v_1} = \frac{5}{0.079905} = 62.5743 \text{ kg}$$

As the vessel is rigid, the process 1-2 is a constant volume cooling process. Hence, final state 2 is defined as

Final state 2:

$$P_2 = 400 \text{ kPa}, V_2 = V_1 = 5 \text{ m}^3$$

$$= \text{specific volume at state 2, } v_2 = v_1 = 0.079905 \text{ m}^3/\text{kg}.$$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$, $v_f = 0.001084 \text{ m}^3/\text{kg}, v_{fg} = 0.4614 \text{ m}^3/\text{kg}, v_g = 0.4625 \text{ m}^3/\text{kg}$.

Here, $v_f < v_2 < v_g$. Hence, it is still a two phase mixture. Then the quality at final state 2,

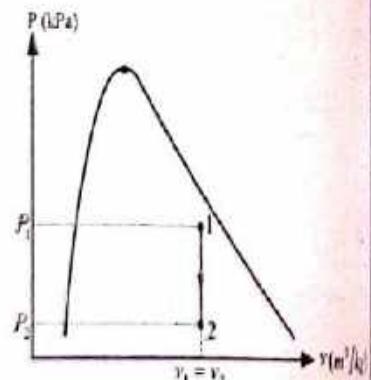
$$x_{21} = \frac{v_2 - v_f}{v_{fg}} = \frac{0.079905 - 0.001084}{0.4614} \\ = 0.17083$$

Now, mass of saturated vapor at final state 2,

$$m_{21} = x_{21} m = 0.17083 \times 62.5743 \\ = 10.6896 \text{ kg}$$

Mass of saturated liquid at final state 2,

$$m_{12} = m - m_{21} = 62.5743 - 10.6896 \\ = 51.8847 \text{ kg}$$



Problem 1.5.

Saturated contained in a closed container initially at a pressure of 2 MPa and temperature of 250°C . The temperature drops as a result of heat transfer to the surroundings until the temperature reaches 80°C . Determine:

- the pressure at which the condensation first occurs,
- the pressure and quality at final state, and
- the percentage of volume occupied by the saturated liquid at final state.

Solution: Given,

Initial pressure of steam, $P_1 = 2 \text{ MPa} = 2000 \text{ kPa}$

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Initial temperature of steam, $T_i = 250^\circ\text{C}$

Final temperature, $T_f = 80^\circ\text{C}$

Initial state 1:

$P_i = 2000 \text{ kPa}, T_i = 250^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_i = 2000 \text{ kPa}$, saturation temperature $T_{sat}(P = 2000 \text{ kPa}) = 212.42^\circ\text{C}$.

Here, $T_i > T_{sat}$. Hence, it is a superheated vapor. Now, referring to the superheated vapor table A2.4 for pressure, $P_i = 2000 \text{ kPa}$ and temperature, $T_i = 250^\circ\text{C}$, specific volume at state 1, $v_i = 0.1114 \text{ m}^3/\text{kg}$.

The $P-v$ diagram of the cooling process is shown in figure below. As the container is closed (rigid), the process 1 – 2 is a constant volume cooling. Hence, state 2 is defined as

State 2:

Volume at state 2, $V_2 = V_i \Rightarrow$ Specific volume, $v_2 = v_i = 0.1114 \text{ m}^3/\text{kg}$.

Since superheated vapor starts to condense when it reaches saturated vapor line, its specific volume is equal to specific volume of saturated vapor at state 2, i.e.

$$v_2 = v_g = 0.1114 \text{ m}^3/\text{kg}$$

Referring to the saturated water pressure table A2.1 for specific volume of saturated vapor, $v_g = 0.1114 \text{ m}^3/\text{kg}$ to determine the pressure at which first condensation starts, $P_2 = P_{sat}$. As there is no value listed for $v_g = 0.1114 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required pressure as

$$P_a = 1700 \text{ kPa} \quad v_{ga} = 0.1167 \text{ m}^3/\text{kg}$$

$$P_2 = ? \quad v_g = 0.1114 \text{ m}^3/\text{kg}$$

$$P_b = 1800 \text{ kPa} \quad v_{gb} = 0.1104 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$P_2 = P_a + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (P_b - P_a) = 1700 + \frac{0.1114 - 0.1167}{0.1104 - 0.1167} (1800 - 1700)$$
$$= 1784.127 \text{ kPa}$$

Further cooling the system causes the conversion of the saturated vapor into two phase mixture. As the container is closed, the further cooling process 2 – 3 also occurs at constant volume. Hence, final state 3 is defined as

Final state 3:

Temperature at state 3, $T_3 = T_f = 80^\circ\text{C}$, Specific volume, $v_3 = v_2 = 0.1114 \text{ m}^3/\text{kg}$.

Referring to the saturated water temperature table A2.2, for temperature $T_3 = 80^\circ\text{C}$, pressure at final state 3, $P_3 = P_f = P_{sat}(T = 80^\circ\text{C}) = 47.373 \text{ kPa}$

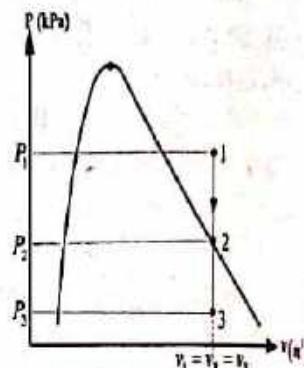
$v_i = 0.001029 \text{ m}^3/\text{kg}$, $v_{lg} = 3.4078 \text{ m}^3/\text{kg}$, $v_g = 3.4088 \text{ m}^3/\text{kg}$.

Now, quality at final state 3,

$$x_3 = x_f = \frac{v_3 - v_i}{v_{lg}} = \frac{0.1114 - 0.001029}{3.4078} = 0.03239 = 3.24\%$$

Then, the percentage of volume of saturated liquid at final state 3,

$$\begin{aligned} \left(\frac{V_l}{V} \right)_3 &= \frac{m_l v_l}{m v_3} = \frac{m_l}{m} \frac{v_l}{v_3} = (1 - x_3) \frac{v_l}{v_3} \\ &= (1 - 0.03239) \frac{0.001029}{0.1114} \\ &= 0.0089378 = 0.894\% \end{aligned}$$



Problem 2.6.

A piston cylinder device shown in figure contains 0.2 kg of a mixture of saturated liquid water and saturated water vapor at a temperature of 50°C and a volume of 0.03 m³. The mass of the piston resting on the stops is 50 kg and the cross sectional area of the piston is 12.2625 cm². The atmospheric pressure is 100 kPa. Heat is transferred until it becomes saturated vapor. Sketch the process on P-v and T-v diagrams and determine:

- (a) the temperature at which the piston just leaves the stops,
- (b) the final pressure, and
- (c) the total work transfer. [Take g = 9.81 ms⁻²]



Solution: Given,

Total mass of two phase mixture, $m = 0.2 \text{ kg}$

Initial temperature, $T_1 = 50^\circ\text{C}$

Initial volume, $V_1 = 0.03 \text{ m}^3$

Mass of piston, $m_p = 50 \text{ kg}$

Area of piston, $A_p = 12.2625 \text{ cm}^2 = 12.2625 \times 10^{-4} \text{ m}^2$

Atmospheric pressure, $P_{atm} = 100 \text{ kPa}$

Final state is saturated vapor. $\Rightarrow v_f = v_g$

Initial state 1:

$T_1 = 50^\circ\text{C}$, $V_1 = 0.03 \text{ m}^3$

Specific volume at initial state 1, $v_1 = \frac{V_1}{m} = \frac{0.03}{0.2} = 0.15 \text{ m}^3/\text{kg}$

Referring to the saturated water temperature table A2.2 for temperature $T_1 = 50^\circ\text{C}$, pressure at initial state 1, $P_1 = P_{sat}(T_1 = 50^\circ\text{C}) = 12.344 \text{ kPa}$.

The $P - v$ and $T - v$ diagrams are shown in figures below. During the initial heating process 1 – 2, the process is constant volume until the system pressure reaches lifting pressure of the piston. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_{lift}$, Volume, $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.15 \text{ m}^3/\text{kg}$

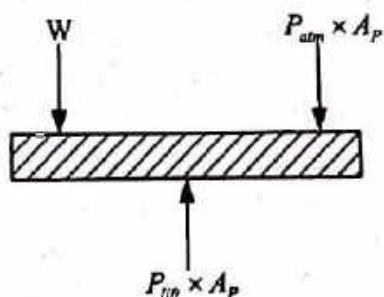
PROPERTIES OF PURE WORKING SUBSTANCES

To determine the lifting pressure of the piston, P_{lift} , referring to the free body diagram (FBD) of the piston, we can write

$$P_{lift} \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_{lift} = P_{atm} + \frac{m_p \times g}{A_p} = 100 \times 10^3 + \frac{50 \times 9.81}{12.2625 \times 10^{-4}}$$

$$= 500 \text{ kPa}$$



FBD of piston

Now, referring to the saturated water pressure table A2.1 for pressure, $P_2 = P_{lift} = 500 \text{ kPa}$, $v_l = 0.001093 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3738 \text{ m}^3/\text{kg}$, $v_g = 0.3749 \text{ m}^3/\text{kg}$.

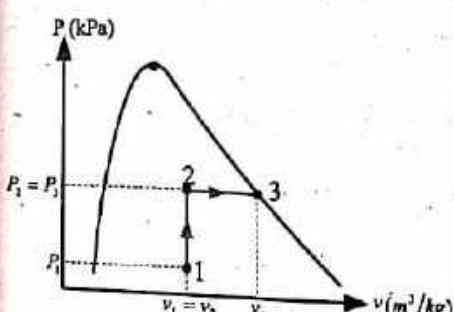
Here, $v_l < v_2 < v_g$. Hence, it is still a two phase mixture. Then, the temperature at which the piston just leaves the stops is: $T_2 = T_{sat} (P_2 = 500 \text{ kPa}) = 151.87^\circ\text{C}$.

According to the question, heat is given till the mixture becomes saturated vapor. This further heating process 2 - 3 occurs with constant pressure. So, state 3 is defined as

State 3:

Pressure at state 3, $P_3 = P_2 = 500 \text{ kPa}$, $v_3 = v_g = 0.3749 \text{ m}^3/\text{kg}$

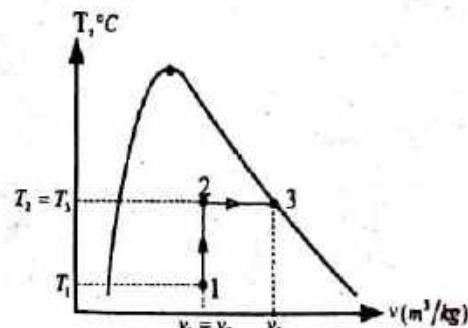
Hence, the final pressure, $P_f = P_3 = 500 \text{ kPa}$



Total work transfer,

$$W_t = W_{1-2} + W_{2-3} = 0 + P_2 (V_3 - V_2) = P_2 \times m (v_3 - v_2)$$

$$= 500 \times 10^3 \times 0.2 (0.3749 - 0.15) = 22.49 \text{ kJ}$$



Problem 2.7.

A piston cylinder device shown in the figure contains 2 kg of H_2O with an initial temperature and volume of 80°C and 0.05 m^3 respectively. It requires a pressure of 400 kPa to lift the piston from the stops. The system is heated until its temperature reaches 250°C . Sketch the process on P-v and T-v diagrams and determine the total work transfer.

Solution: Given,
Mass of water, $m = 2 \text{ kg}$

Initial temperature, $T_1 = 80^\circ\text{C}$

Initial volume, $V_1 = 0.05 \text{ m}^3$

Lifting pressure of piston, $P_{lift} = 400 \text{ kPa}$

Final temperature, $T_f = 250^\circ\text{C}$

Initial state 1: $T_1 = 80^\circ\text{C}$, $V_1 = 0.05 \text{ m}^3$

$$\text{Specific volume of water, } v_1 = \frac{V_1}{m} = \frac{0.05}{2} = 0.025 \text{ m}^3/\text{kg}$$

Referring to the saturated water temperature table A2.2 for temperature $T_1 = 80^\circ\text{C}$, $v_1 = 0.001093 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3738 \text{ m}^3/\text{kg}$, $v_g = 0.3749 \text{ m}^3/\text{kg}$.

Here, $v_1 < v_1 < v_g$. Hence, it is a two phase mixture. Then,

the pressure $P_1 = P_{sat}(T_1 = 80^\circ\text{C}) = 47.373 \text{ kPa}$.

The $P - v$ and $T - v$ diagrams are shown in figures below. During the initial heating process 1 – 2, the process is constant volume until the system initial pressure reaches the lifting pressure of the piston. Hence, state 2 is defined as

State 2: Pressure at state 2, $P_2 = P_{lift} = 400 \text{ kPa}$, Volume, $V_2 = V_1$

$$\Rightarrow v_2 = v_1 = 0.025 \text{ m}^3/\text{kg}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$, $v_1 = 0.001084 \text{ m}^3/\text{kg}$, $v_{lg} = 0.4614 \text{ m}^3/\text{kg}$, $v_g = 0.4625 \text{ m}^3/\text{kg}$.

Here, $v_1 < v_2 < v_g$. Hence, it is still a two phase mixture. Then, the temperature,

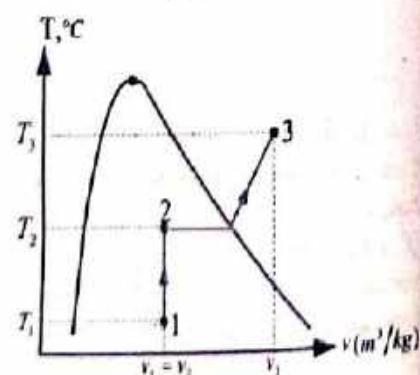
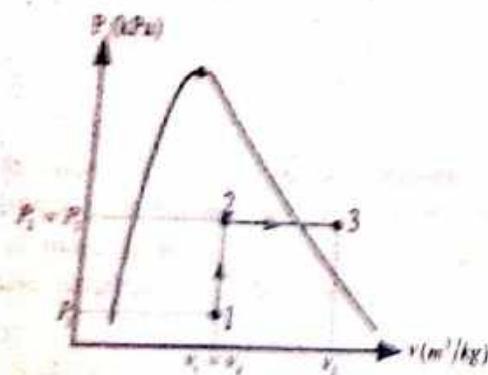
$T_2 = T_{sat}(P_2 = 400 \text{ kPa}) = 143.64^\circ\text{C}$. But the given final temperature, $T_f = 250^\circ\text{C}$. Therefore, the system should be further heated. This further heating process 2 – 3 occurs with constant pressure thereby increasing the volume and temperature. So, state 3 is defined as

State 3: Pressure at state 3, $P_3 = P_2 = 400 \text{ kPa}$, $T_3 = T_f = 250^\circ\text{C}$.

Referring to the saturated water pressure table A2.1 for pressure $P_3 = 400 \text{ kPa}$, saturation temperature, $T_{sat}(P_3 = 400 \text{ kPa}) = 143.64^\circ\text{C}$.

Here, $T_3 > T_{sat}$. Hence, it is superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 400 \text{ kPa}$, and temperature, $T_3 = 250^\circ\text{C}$, specific volume, $v_3 = 0.5951 \text{ m}^3/\text{kg}$.



Total work transfer,

$$W_t = W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = P_2 \times m(v_3 - v_2)$$

$$= 400 \times 10^3 \times 2 (0.5951 - 0.025) = 456.08 \text{ kJ}$$

Problem 2.8.

A piston cylinder device shown in figure below contains water initially at a pressure of 125 kPa with a quality of 50%. Heat is added to the system until it reaches to a final temperature of 800°C. It takes a pressure of 600 kPa to lift the piston from the stops. Sketch the process on P-v and T-v diagrams and determine:

- (a) the mass of H₂O in the system, and
- (b) the total work transfer.

Solution: Given,

Initial pressure, $P_1 = 125 \text{ kPa}$

Initial quality, $x_1 = 50\% = 0.5$

Final Temperature, $T_f = 800^\circ\text{C}$

Lifting pressure of piston, $P_{lift} = 600 \text{ kPa}$

Radius of piston, $r_p = 0.07 \text{ m}$

Height of piston position, $h = 0.3 \text{ m}$

Area of piston, $A_p = \pi r^2 = \pi(0.07)^2 = 0.015394 \text{ m}^2$

Initial state 1: $P_1 = 125 \text{ kPa}, x_1 = 0.5$

As the quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 125 \text{ kPa}$, initial temperature, $T_1 = T_{sat}(P_1 = 125 \text{ kPa}) = 105.99^\circ\text{C}$,

$$v_1 = 0.001048 \text{ m}^3/\text{kg}, v_{lg} = 1.3742 \text{ m}^3/\text{kg}, v_g = 1.3752 \text{ m}^3/\text{kg}.$$

The specific volume at state 1,

$$v_1 = v_l + x_1 v_{lg} = 0.001048 + 0.5 \times 1.3742 = 0.688148 \text{ m}^3/\text{kg}$$

Initial volume of water, $V_1 = A_p \times h = 0.015394 \times 0.3 = 0.004618 \text{ m}^3$

$$\text{Mass of water, } m = \frac{V_1}{v_1} = \frac{0.004618}{0.688148} = 0.006711 \text{ kg.}$$

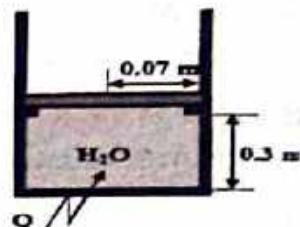
The $P - v$ and $T - v$ diagrams are shown in figures below. During the initial heating process 1 – 2, the process is constant volume until the system initial pressure reaches lifting pressure of the piston. Hence, state 2 is defined as

State 2: Pressure at state 2, $P_2 = P_{lift} = 600 \text{ kPa}$, and volume, $V_2 = V_1$

$$\Rightarrow v_2 = v_1 = 0.688148 \text{ m}^3/\text{kg.}$$

Referring to the saturated water pressure table A2.1 for pressure $P_2 = 600 \text{ kPa}$, $v_1 = 0.001101 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3145 \text{ m}^3/\text{kg}$, $v_g = 0.3156 \text{ m}^3/\text{kg}$.

Here, $v_2 > v_g$. Hence, it is a superheated vapor. Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 600 \text{ kPa}$ and specific volume, $v_2 = 0.688148 \text{ m}^3/\text{kg}$ to determine the temperature at state 2. As there is no value listed



for $v_2 = 0.68815 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_a = 600^\circ\text{C}$$

$$v_a = 0.6697 \text{ m}^3/\text{kg}$$

$$T_2 = ?$$

$$v_2 = 0.68815 \text{ m}^3/\text{kg}$$

$$T_b = 650^\circ\text{C}$$

$$v_b = 0.7085 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$T_2 = T_a + \frac{v_2 - v_a}{v_b - v_a} (T_b - T_a) = 600 + \frac{0.68815 - 0.6697}{0.7085 - 0.6697} (650 - 600) = 623.776^\circ\text{C}$$

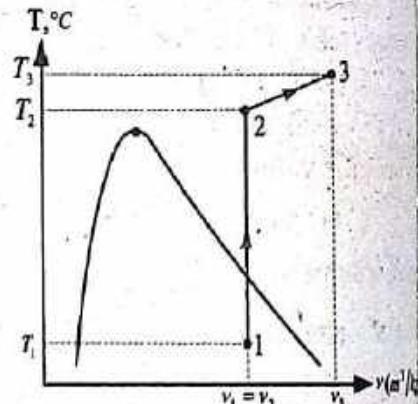
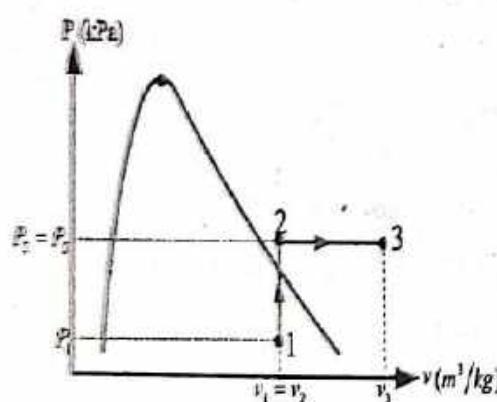
Since the given final temperature, $T_f = 800^\circ\text{C}$, the system should be further heated. This further heating process 2 - 3 occurs with constant pressure thereby increasing the volume and temperature. Hence, state 3 is defined as

State 3: Pressure at state 3, $P_3 = P_2 = 600 \text{ kPa}$, $T_3 = T_f = 800^\circ\text{C}$.

Since state 2 is already a superheated vapor, again referring to the superheated vapor table A2.4 for pressure $P_3 = 600 \text{ kPa}$, and temperature, $T_3 = 800^\circ\text{C}$, specific volume, $v_3 = 0.8246 \text{ m}^3/\text{kg}$.

Total work transfer,

$$\begin{aligned} W_t &= W_{1-2} + W_{2-3} = 0 + P_2 (V_3 - V_2) = P_2 \times m (v_3 - v_2) \\ &= 600 \times 10^3 \times 0.006711 (0.8246 - 0.68815) \\ &= 549.43 \text{ J} = 0.5494 \text{ kJ} \end{aligned}$$



Problem 2.9.

A piston cylinder device shown in figure contains 2 kg of water initially at pressure of 500 kPa with a quality of 20%. The water is heated until it becomes a saturated vapor. The volume of the system when the piston is at the upper stops is 0.4 m^3 . Sketch the process on P-v and T-v diagrams and determine:

- (a) the final pressure, and
- (b) the total work transfer.

Solution: Given,

Mass of water, $m = 2 \text{ kg}$



Initial pressure, $P_1 = 500 \text{ kPa}$

Initial quality, $x_1 = 20\% = 0.2$

Final volume, $V_f = 0.4 \text{ m}^3$

Final state is saturated vapor $\Rightarrow v_f = v_g$

$$\text{Final specific volume, } v_f = \frac{V_f}{m} = \frac{0.4}{2} = 0.2 \text{ m}^3/\text{kg}$$

Initial state 1: $P_1 = 500 \text{ kPa}, x_1 = 0.2$

As the quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 500 \text{ kPa}$, initial temperature, $T_1 = T_{sat}(P_1 = 500 \text{ kPa}) = 151.87^\circ\text{C}$,

$$v_l = 0.001093 \text{ m}^3/\text{kg}, v_{lg} = 0.3738 \text{ m}^3/\text{kg}, v_g = 0.3749 \text{ m}^3/\text{kg}.$$

The specific volume at state 1,

$$v_1 = v_l + x_1 v_{lg} = 0.001093 + 0.2 \times 0.3738 = 0.07585 \text{ m}^3/\text{kg}$$

The $P - v$ and $T - v$ diagrams are shown in figures below. During the initial heating process 1–2, the piston moves upwards with constant pressure until it just touches the stops. Hence, state 2 is defined as

State 2: Pressure, $P_2 = P_1 = 500 \text{ kPa}$, and volume, $V_2 = V_f \Rightarrow v_2 = v_f = 0.2 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for pressure $P_2 = 500 \text{ kPa}$, $v_l = 0.001093 \text{ m}^3/\text{kg}, v_{lg} = 0.3738 \text{ m}^3/\text{kg}, v_g = 0.3749 \text{ m}^3/\text{kg}$.

Here, $v_l < v_2 < v_g$. Hence, it is still a two phase mixture. As the given final state is a saturated vapor, the system should be further heated until it becomes a saturated vapor. This further heating process 2 – 3 occurs with constant volume due to presence of the stops thereby increasing the pressure and temperature. So, state 3 is defined as

State 3: Volume, $V_3 = V_2 = 0.4 \text{ m}^3 \Rightarrow$ specific volume, $v_3 = v_2 = 0.2 \text{ m}^3/\text{kg}$ and as it is a saturated vapor, $v_3 = v_g = 0.2 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for specific volume, $v_g = 0.2 \text{ m}^3/\text{kg}$ to determine the final pressure at state 3, $P_3 = P_f$. As there is no value listed for $v_g = 0.2 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required pressure as

$$P_a = 950 \text{ kPa} \quad v_{ga} = 0.2041 \text{ m}^3/\text{kg}$$

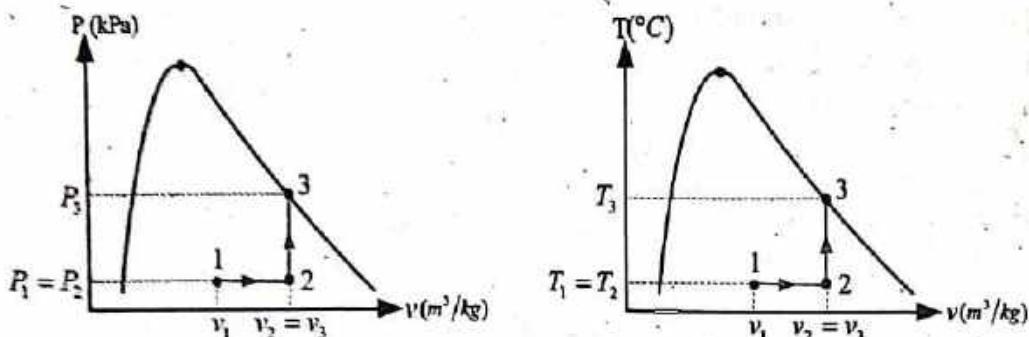
$$P_b = ? \quad v_g = 0.2 \text{ m}^3/\text{kg}$$

$$P_b = 1000 \text{ kPa} \quad v_{gb} = 0.1944 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$P_3 = P_a + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (P_b - P_a) = 950 + \frac{0.2 - 0.2041}{0.1944 - 0.2041} (1000 - 950)$$

$$= 971.134 \text{ kPa} = P_f$$



Total work transfer,

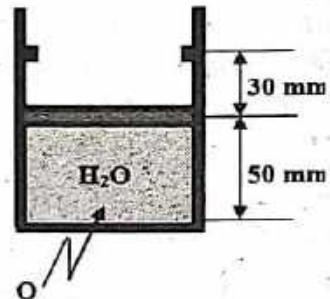
$$W_t = W_{1-2} + W_{2-3} = P_1 (V_2 - V_1) + 0 = P_1 \times m (v_2 - v_1)$$

$$= 500 \times 10^3 \times 2(0.2 - 0.07585) = 124150 \text{ J} = 124.15 \text{ kJ.}$$

Problem 2.10.

The frictionless piston shown in figure has a mass of 20 kg and a cross sectional area of 78.48 cm^2 . Heat is added until the temperature reaches 400°C . If the quality of the H_2O at the initial state is 0.2, determine:

- (a) the initial pressure,
- (b) the mass of H_2O ,
- (c) the quality of the system when the piston hits the stops,
- (d) the final pressure, and
- (e) the total work transfer.
- (f) [Take $P_{atm} = 100 \text{ kPa}$, $g = 9.81 \text{ m/s}^2$].



Solution: Given,

Mass of piston, $m_p = 20 \text{ kg}$

Area of piston, $A_p = 78.48 \text{ cm}^2 = 78.48 \times 10^{-4} \text{ m}^2$

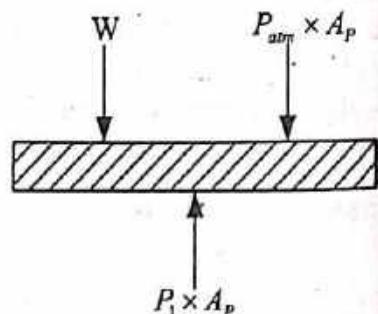
Final temperature of water, $T_f = 400^\circ\text{C}$

Initial quality of water, $x_i = 0.2$

Atmospheric pressure, $P_{atm} = 100 \text{ kPa}$

Initial state 1:

$x_i = 0.2$



FBD of piston

Initial volume of water, $V_i = A_p \times h_i = 78.48 \times 10^{-4} \times 50 \times 10^{-3} = 0.0003924 \text{ m}^3$

To calculate the pressure at initial state 1, P_i , referring to the free body diagram (FBD) of piston, we can write

$$P_i \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_i = P_{atm} + \frac{m_p \times g}{A_p} = 100 \times 10^3 + \frac{20 \times 9.81}{78.48 \times 10^{-4}} = 125 \text{ kPa}$$

As the quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 125 \text{ kPa}$,

$$v_1 = 0.001048 \text{ m}^3/\text{kg}, v_{lg} = 1.3742 \text{ m}^3/\text{kg}, v_g = 1.3752 \text{ m}^3/\text{kg}.$$

The specific volume at state 1,

$$v_1 = v_l + x_1 v_{lg} = 0.001048 + 0.2 \times 1.3742 = 0.275888 \text{ m}^3/\text{kg}$$

$$\text{Mass of water, } m = \frac{V_1}{v_1} = \frac{0.0003924}{0.275888} = 0.001422 \text{ kg.}$$

Initial temperature, $T_1 = T_{sat} (P_1 = 125 \text{ kPa}) = 105.99^\circ\text{C}$.

The $P - v$ and $T - v$ diagrams are shown in figures below. During the initial heating process 1–2, the piston moves upwards with constant pressure until it just touches the stops. Hence, state 2 is defined as

State 2: Pressure at state 2, $P_2 = P_1 = 125 \text{ kPa}$, and volume, V_2 is the final volume which can be calculated from the given parameters in system diagram as

$$V_2 = A_P \times h_t = 78.48 \times 10^{-4} \times (50 + 30) \times 10^3 = 0.00062784 \text{ m}^3$$

$$\text{Specific volume at state 2, } v_2 = \frac{V_2}{m} = \frac{0.00062784}{0.001422} = 0.44152 \text{ m}^3/\text{kg.}$$

Here, for pressure $P_2 = 125 \text{ kPa}$, $v_l < v_2 < v_g$. Hence, it is still a two phase mixture.

Quality at state 2,

$$x_2 = \frac{v_2 - v_l}{v_{lg}} = \frac{0.44152 - 0.001048}{1.3742} = 0.32053$$

Temperature at state 2, $T_2 = T_{sat} (P_2 = 125 \text{ kPa}) = 105.99^\circ\text{C} = T_1$. As the given final temperature, $T_f = 400^\circ\text{C}$, the system should be further heated. This further heating process 2 – 3 occurs with constant volume due to presence of the stops thereby increasing the pressure and temperature. So, state 3 is defined as

State 3: Temperature, $T_3 = T_f = 400^\circ\text{C}$, Volume, $V_3 = V_2 = 0.00062784 \text{ m}^3$

$$\Rightarrow \text{specific volume, } v_3 = v_2 = 0.44152 \text{ m}^3/\text{kg.}$$

Here, the temperature at state 3, $T_3 = 400^\circ\text{C}$ is greater than the critical temperature of water ($T_{cr} = 373.98^\circ\text{C}$). Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for temperature, $T_3 = 400^\circ\text{C}$ and specific volume, $v_3 = 0.44152 \text{ m}^3/\text{kg}$ to determine the pressure at state 3, P_3 .

As there is no value listed for $v_3 = 0.44152 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required pressure as

$$P_a = 600 \text{ kPa} \quad v_a = 0.5137 \text{ m}^3/\text{kg}$$

$$P_b = ? \quad v_b = 0.44152 \text{ m}^3/\text{kg}$$

$$P_b = 800 \text{ kPa} \quad v_b = 0.3843 \text{ m}^3/\text{kg}$$

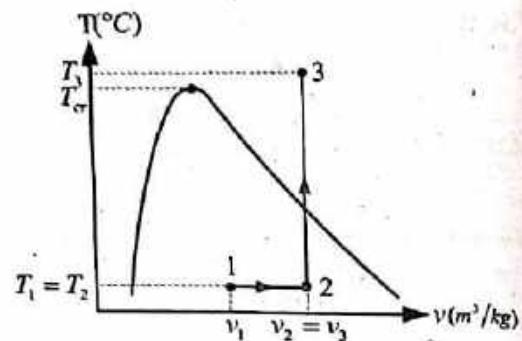
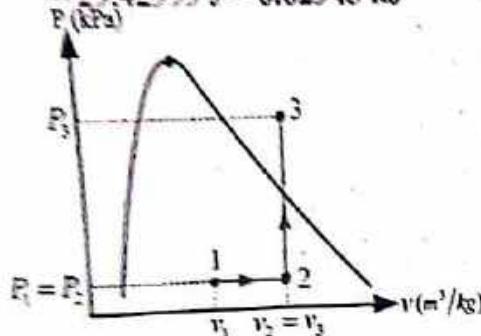
Using linear interpolation method,

$$P_3 = P_a + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (P_b - P_a) = 600 + \frac{0.44152 - 0.5137}{0.3843 - 0.5137} (800 - 600)$$

$$= 711.561 \text{ kPa} = P_f$$

Total work transfer,

$$W_t = W_{1-2} + W_{2-3} = P_1(V_2 - V_1) + 0 = 125 \times 10^3 \times (0.00062784 - 0.0003924) \\ = 29.42999 \text{ J} = 0.02943 \text{ kJ}$$



Problem 2.11.

Water (2 kg) is contained within a piston-cylinder arrangement as shown in figure. The initial temperature is 105°C , and there are equal masses of liquid and vapor initially. The system is heated to a position where the piston is locked, and then the system is cooled to the saturated vapor state at $T = 50^\circ\text{C}$. Draw the process on a P-v and T-v diagrams, and evaluate the total work done during the process.



Solution: Given,

Mass of water, $m = 2 \text{ kg}$

Initial temperature, $T_i = 105^\circ\text{C}$

Initial masses of saturated liquid (m_l) and vapor (m_g) are equal

$$\Rightarrow m_l = m_g \Rightarrow \text{Initial quality, } x_i = \frac{m_g}{m_l + m_g} = 0.5$$

Final temperature of saturated vapor, $T_f = 50^\circ\text{C}$

Initial state 1:

$T_i = 105^\circ\text{C}, x_i = 0.5$

As the quality is given, it is a two phase mixture. Referring to the saturated water temperature table A2.2 for temperature, $T_i = 105^\circ\text{C}$,

$$v_f = 0.001047 \text{ m}^3/\text{kg}, v_{fg} = 1.4190 \text{ m}^3/\text{kg}, v_g = 1.4200 \text{ m}^3/\text{kg}.$$

The specific volume at state 1,

$$v_i = v_f + x_i v_{fg} = 0.001047 + 0.5 \times 1.419 = 0.710547 \text{ m}^3/\text{kg}$$

Initial volume, $V_i = mv_i = 2 \times 0.710547 = 1.4211 \text{ m}^3$

Pressure at initial state 1, $P_i = P_{sat}(T_i = 105^\circ\text{C}) = 120.79 \text{ kPa}$.

The $P - v$ and $T - v$ diagrams are shown in figures below. During the initial heating process 1-2, the piston moves upwards with constant pressure to a position where it is locked. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_I = 120.79 \text{ kPa}$, and as the system is finally cooled to the saturated vapor state at $T_f = 50^\circ\text{C}$, it must be a superheated vapor in state 2. Since the piston is locked at state 2, the cooling process 2 - 3 occurs with constant volume until superheated vapor becomes saturated vapor thereby decreasing the pressure and temperature. So, state 3 is defined as

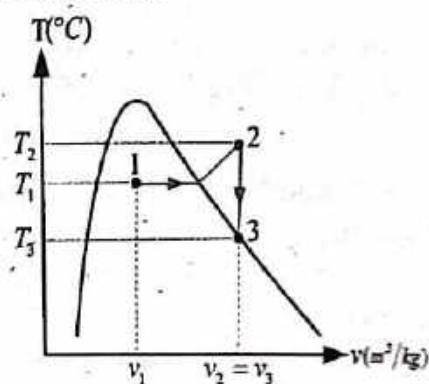
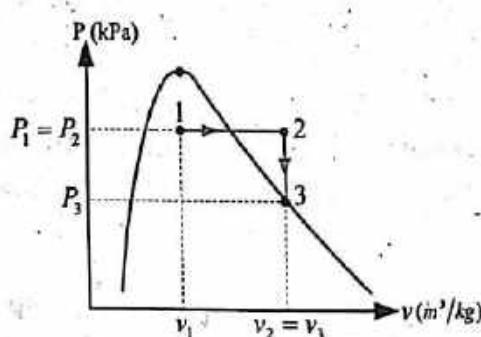
State 3:

Temperature at state 3, $T_3 = T_f = 50^\circ\text{C}$ with saturated vapor.

Volume at state 3, $V_3 = V_2 \Rightarrow v_3 = v_2$ and specific volume, $v_3 = v_g$.

Now, referring to the saturated water temperature table A2.2 for temperature, $T_3 = 50^\circ\text{C}$, $v_g = 12.037 \text{ m}^3/\text{kg} = v_3 = v_2$.

Pressure at final state 3, $P_3 = P_{sat}(T_3 = 50^\circ\text{C}) = 12.344 \text{ kPa}$.



Total work done,

$$\begin{aligned} W_t &= W_{1-2} + W_{2-3} = P_I(V_2 - V_1) + 0 = P_I \times m(v_2 - v_1) \\ &= 120.79 \times 10^3 \times 2(12.037 - 0.710547) = 2736244.5 \text{ J} \\ &= 2736.2445 \text{ kJ} \end{aligned}$$

Problem 2.12.

A piston cylinder arrangement shown in figure contains 1 kg of water initially at a pressure of 1 MPa and a temperature of 500°C . The water is cooled until it is completely converted into the saturated liquid. It requires a pressure of 400 kPa to support the piston. Sketch the process on P-v and T-v diagrams and determine the total work transfer.

Solution: Given,

Mass of water, $m = 1 \text{ kg}$

Initial pressure, $P_I = 1 \text{ MPa} = 1000 \text{ kPa}$

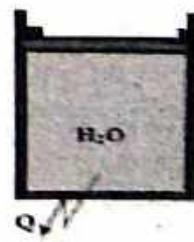
Initial temperature, $T_I = 500^\circ\text{C}$

Supporting pressure of piston, $P_{sup} = 400 \text{ kPa}$

Final state is saturated liquid.

Initial state 1:

$P_I = 1000 \text{ kPa}, T_I = 500^\circ\text{C}$



Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 1000 \text{ kPa}$, saturation temperature, $T_{sat}(P_1 = 1000 \text{ kPa}) = 179.92^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor. Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 1000 \text{ kPa}$ and temperature, $T_1 = 500^\circ\text{C}$, the specific volume at initial state 1, $v_1 = 0.3541 \text{ m}^3/\text{kg}$.

The $P - v$ and $T - v$ diagrams are shown in figures below. During initial state of cooling, piston remains stationary although heat is removed from the system. So, process 1 - 2 is constant volume cooling during which initial pressure decreases to supporting pressure of piston. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_{sup} = 400 \text{ kPa}$ and volume at state 2, $V_2 = V_1$

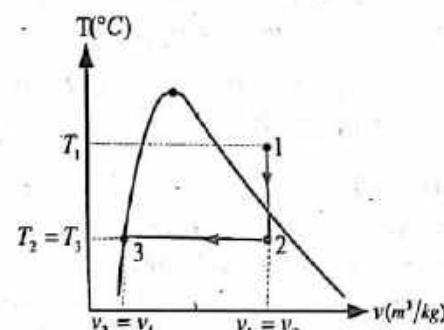
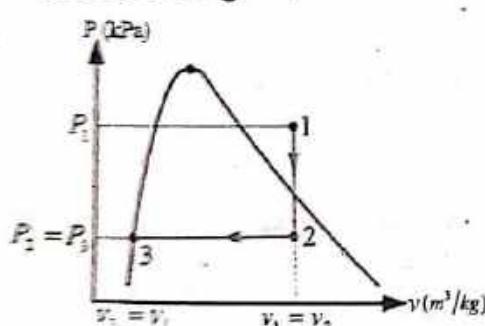
⇒ Specific volume at state 2, $v_2 = v_1 = 0.3541 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$, $v_1 = 0.001084 \text{ m}^3/\text{kg}$, $v_{lg} = 0.4614 \text{ m}^3/\text{kg}$, $v_g = 0.4625 \text{ m}^3/\text{kg}$.

Here, $v_1 < v_2 < v_g$. Hence, it is a two phase mixture. As the required final state is saturated liquid, the system should be further cooled until it becomes saturated liquid. This further cooling process 2 - 3 occurs at constant supporting pressure of piston. Then state 3 can be defined as

Final state 3:

Pressure, $P_3 = P_{sup} = 400 \text{ kPa}$ and specific volume, $v_3 = v_l$ ($P_3 = 400 \text{ kPa}$)
 $= 0.001084 \text{ m}^3/\text{kg}$.



Total work transfer,

$$\begin{aligned} W_t &= W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = P_2 \times m(v_3 - v_2) \\ &= 400 \times 10^3 \times 1 (0.001084 - 0.3541) \\ &= -141206.4 \text{ J} = -141.2064 \text{ kJ} \end{aligned}$$

The negative sign indicates the work is done on the system.

Problem 2.13.

A piston cylinder arrangement shown in figure below contains 2 kg of water initially at a pressure of 200 kPa and a temperature of 50°C . Heat is added until the piston reaches the upper stops where the total volume is 1.5 m^3 . It takes a pressure of 600 kPa to lift the piston. Sketch the process on $P-v$ and $T-v$ diagrams and determine the final temperature and the work transfer.

Solution: Given,

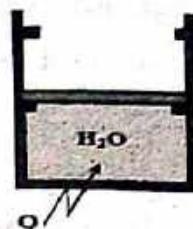
Mass of water, $m = 2 \text{ kg}$

Initial pressure, $P_1 = 200 \text{ kPa}$

Initial temperature, $T_1 = 50^\circ\text{C}$

Final total volume, $V_f = 1.5 \text{ m}^3$

Lifting pressure of piston, $P_{lift} = 600 \text{ kPa}$



Initial state 1:

$P_1 = 200 \text{ kPa}, T_1 = 50^\circ\text{C}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 200 \text{ kPa}$, saturation temperature, $T_{sat}(P_1 = 200 \text{ kPa}) = 120.24^\circ\text{C}$.

Here, $T_1 < T_{sat}$. Hence, it is a compressed or sub-cooled liquid.

Now, referring to the compressed liquid table A 2.3 for pressure, $P_1 = 200 \text{ kPa}$ and temperature, $T_1 = 50^\circ\text{C}$ to find the specific volume at initial state 1, v_1 . But this value is not available in the table for this low pressure (as liquid is incompressible at low pressure).

Then, referring to the saturated water temperature table A2.2 for temperature, $T_1 = 50^\circ\text{C}$ to take the value of specific volume of saturated liquid for the specific volume at initial state 1, $v_1 = v_l(T_1 = 50^\circ\text{C}) = 0.001012 \text{ m}^3/\text{kg}$ with slight error.

The $P - v$ and $T - v$ diagrams are shown in figures below. During initial state of heating, piston remains stationary although heat is added to the system. So, process 1–2 is constant volume heating during which initial pressure increases to the lifting pressure of piston. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_{lift} = 600 \text{ kPa}$ and volume at state 2, $V_2 = V_1$

⇒ Specific volume at state 2, $v_2 = v_1 = 0.001012 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 600 \text{ kPa}$, $v_1 = 0.001101 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3145 \text{ m}^3/\text{kg}$, $v_g = 0.3156 \text{ m}^3/\text{kg}$.

Here, $v_2 < v_l$. Hence, it is still a compressed or sub-cooled liquid. Since heat is added until the piston reaches the upper stops where the total volume is 1.5 m^3 ,

the final specific volume, $v_f = \frac{V_f}{m} = \frac{1.5}{2} = 0.75 \text{ m}^3/\text{kg}$ and this further heating

process 2 – 3 occurs with constant pressure. Hence, final state 3 is defined as

Final state 3:

$$P_3 = P_2 = 600 \text{ kPa}, v_3 = v_f = 0.75 \text{ m}^3/\text{kg}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 600 \text{ kPa}$, $v_1 = 0.001101 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3145 \text{ m}^3/\text{kg}$, $v_g = 0.3156 \text{ m}^3/\text{kg}$.

Here, $v_3 > v_g$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 600 \text{ kPa}$ and specific volume, $v_3 = 0.75 \text{ m}^3/\text{kg}$ to find the temperature at final state 3, $T_3 = T_f$

As there is no value listed for $v_3 = 0.7500 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_a = 700^\circ\text{C}$$

$$v_a = 0.7472 \text{ m}^3/\text{kg}$$

$$T_b = ?$$

$$v_b = 0.7500 \text{ m}^3/\text{kg}$$

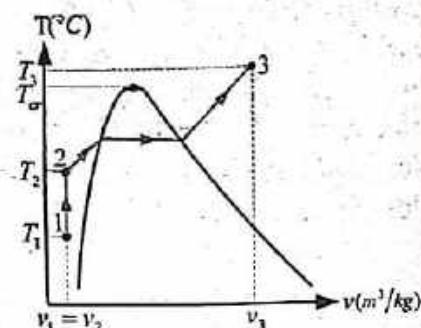
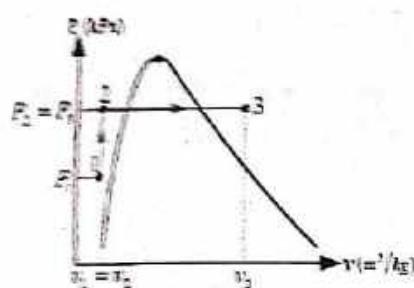
$$T_c = 750^\circ\text{C}$$

$$v_c = 0.7859 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$T_f = T_a + \frac{v_3 - v_a}{v_b - v_a} (T_b - T_a) = 700 + \frac{0.75 - 0.7472}{0.7859 - 0.7472} (750 - 700)$$

$$= 703.618^\circ\text{C} = T_f$$



Total work transfer,

$$\bar{W}_t = \bar{W}_{1-2} + \bar{W}_{2-3} = 0 + P_2 (V_3 - V_2) = P_2 \times m(v_3 - v_2)$$

$$= 600 \times 10^3 \times 2 (0.75 - 0.001012) = 898785.6 \text{ J} = 898.7856 \text{ kJ}$$

Problem 2.14.

A piston cylinder arrangement shown in figure below contains water initially at $P_1 = 100 \text{ kPa}$, $x_1 = 0.8$ and $V_1 = 0.01 \text{ m}^3$. When the system is heated, it encounters a linear spring ($k = 100 \text{ kN/m}$). At this state volume is 0.015 m^3 . The heating continues till its pressure is 200 kPa . If the diameter of the piston is 0.15 m , determine

- (a) the final temperature, and
- (b) the total work transfer.

Also sketch the process on $P-v$ diagram.

Solution: Given

Initial pressure of water, $P_1 = 100 \text{ kPa}$

Initial quality of water, $x_1 = 0.8$

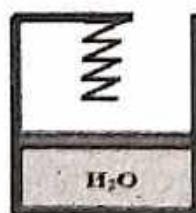
Initial volume of water, $V_1 = 0.01 \text{ m}^3$

Spring constant, $k = 100 \text{ kN/m}$

Volume when piston touches spring, $V_2 = 0.015 \text{ m}^3$

Final pressure, $P_f = 200 \text{ kPa}$

Diameter of piston, $D_p = 0.15 \text{ m}$



$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi (0.15)^2}{4} = 0.01767 \text{ m}^2$$

Initial state 1:

$$P_1 = 100 \text{ kPa}, x_1 = 0.8, V_1 = 0.01 \text{ m}^3$$

As the quality is given, it is a two phase mixture.

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 100 \text{ kPa}$,

$$v_f = 0.001043 \text{ m}^3/\text{kg}, v_{lg} = 1.6933 \text{ m}^3/\text{kg}, v_g = 1.6943 \text{ m}^3/\text{kg}.$$

The specific volume at state 1,

$$v_1 = v_f + x_1 v_{lg} = 0.001043 + 0.8 \times 1.6933 = 1.35568 \text{ m}^3/\text{kg}$$

$$\text{Mass of water, } m = \frac{V_1}{v_1} = \frac{0.01}{1.35568} = 0.007376 \text{ kg.}$$

The $P - v$ diagram is shown in figure below. During initial state of heating, piston moves upward with constant initial pressure until it encounters the spring. Hence, the process 1 - 2 is constant pressure heating during which volume increases to 0.015 m^3 . Then state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_1 = 100 \text{ kPa}$ & volume at state 2, $V_2 = 0.015 \text{ m}^3$

$$\Rightarrow \text{Specific volume at state 2, } v_2 = \frac{V_2}{m} = \frac{0.015}{0.007376} = 2.03362 \text{ m}^3/\text{kg.}$$

Here, $v_2 > v_g$ ($P_2 = 100 \text{ kPa}$). Hence, it is a superheated vapor. Since the given final pressure is $P_f = 200 \text{ kPa}$, the system should be further heated to attain this final pressure and the spring gets compressed (process 2-3). Then final state 3 can be defined as

Final state 3:

$P_3 = P_f = 200 \text{ kPa}$ and volume at state 3, $V_3 = V_2 + A_p \times x$

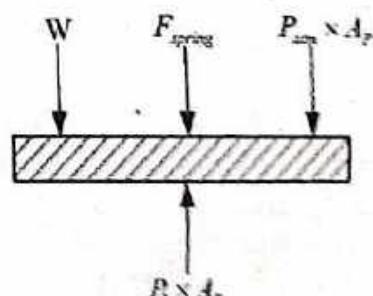
where x is the compression of the spring.

To find the value of x , referring to free body diagram (FBD) of piston at state 3, we can write

$$P_3 \times A_p = P_{atm} \times A_p + W + F_{spring}$$

$$\Rightarrow P_3 = P_{atm} + \frac{W}{A_p} + \frac{kx}{A_p} = P_2 + \frac{kx}{A_p}$$

$$\Rightarrow x = (P_3 - P_2) \frac{A_p}{k} = (200 - 100) \times 10^3 \times \frac{0.01767}{100 \times 10^3} \\ = 0.01767 \text{ m}$$



FBD at state 3

$$\text{Thus, } V_3 = V_2 + A_p \times x = 0.015 + 0.01767 \times 0.01767 = 0.015312 \text{ m}^3$$

$$\text{Specific volume at state 3, } v_3 = \frac{V_3}{m} = \frac{0.015312}{0.007376} = 2.075953 \text{ m}^3/\text{kg.}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 200 \text{ kPa}$, $v_1 = 0.001060 \text{ m}^3/\text{kg}$, $v_{lg} = 0.8848 \text{ m}^3/\text{kg}$, $v_g = 0.8859 \text{ m}^3/\text{kg}$.

Here, $v_3 > v_g$. Hence it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 200 \text{ kPa}$ and specific volume, $v_3 = 2.075953 \text{ m}^3/\text{kg}$ to find the temperature at final state 3, $T_f = T_3$. As there is no value listed for $v_3 = 2.075953 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$\begin{array}{ll} T_a = 600^\circ\text{C} & v_a = 2.0130 \text{ m}^3/\text{kg} \\ T_s = ? & v_3 = 2.075953 \text{ m}^3/\text{kg} \\ T_b = 650^\circ\text{C} & v_b = 2.1287 \text{ m}^3/\text{kg} \end{array}$$

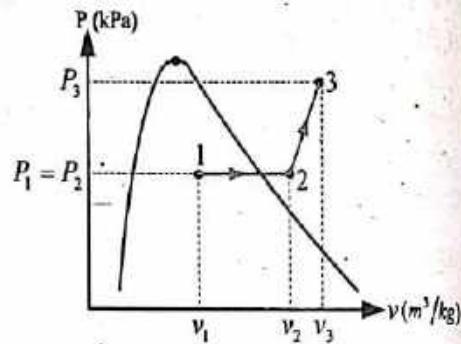
Using linear interpolation method,

$$T_s = T_a + \frac{v_3 - v_a}{v_b - v_a} (T_b - T_a) = 600 + \frac{2.075953 - 2.0130}{2.1287 - 2.0130} (650 - 600)$$

$$= 627.205^\circ\text{C} = T_f$$

Total work transfer,

$$\begin{aligned} W_t &= \overline{W}_{2-3} + \overline{W}_{3-1} \\ &= P_2 (V_2 - V_1) + \frac{1}{2} (P_2 + P_3)(V_3 - V_2) \\ &= 100 \times 10^3 (0.015 - 0.01) + \\ &\quad \frac{1}{2} (100 + 200) \times 10^3 (0.015312 - 0.015) \\ &= 546.8 \text{ J} = 0.5468 \text{ kJ} \end{aligned}$$



Problem 2.15.

A rigid container with a volume of 0.170 m^3 is initially filled with steam at 200 kPa and 300°C . It is cooled to 90°C .

- At what temperature does the phase change start to occur?
- What is the final pressure?
- What mass fraction of water is in the final state?

Solution: Given,

Volume of container, $V = 0.170 \text{ m}^3 = \text{constant}$ (As it is a rigid container)

Initial pressure of steam, $P_1 = 200 \text{ kPa}$

Initial temperature of steam, $T_1 = 300^\circ\text{C}$

Final temperature, $T_f = 90^\circ\text{C}$

Initial state 1: $P_1 = 200 \text{ kPa}$, $T_1 = 300^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 200 \text{ kPa}$, saturation temperature $T_{sat}(P = 200 \text{ kPa}) = 120.24^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 200 \text{ kPa}$ and temperature, $T_1 = 300^\circ\text{C}$, specific volume at state 1, $v_1 = 1.3162 \text{ m}^3/\text{kg}$.

The $P-v$ diagram of the cooling process is shown in figure below. As the container is rigid, the process 1 – 2 is a constant volume cooling. Hence, state 2 is defined as

State 2: Volume, $V_2 = V_1 \Rightarrow$ Specific volume, $v_2 = v_1 = 1.3162 \text{ m}^3/\text{kg}$.

Since superheated vapor starts to phase change (condense) when it reaches saturated vapor line, its specific volume is equal to specific volume of saturated vapor at state 2, i.e. $v_2 = v_g = 1.3162 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for specific volume of saturated vapor, $v_g = 1.3162 \text{ m}^3/\text{kg}$ to determine the temperature at which phase change (condensation) starts, $T_2 = T_{sat}$. As there is no value listed for $v_g = 1.3162 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_a = 105^\circ\text{C}$$

$$v_{ga} = 1.4200 \text{ m}^3/\text{kg}$$

$$T_2 = ?$$

$$v_g = 1.3162 \text{ m}^3/\text{kg}$$

$$T_b = 110^\circ\text{C}$$

$$v_{gb} = 1.2106 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$T_2 = T_a + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (T_b - T_a) = 105 + \frac{1.3162 - 1.4200}{1.2106 - 1.4200} (110 - 105) = 107.478 \text{ kPa}$$

Further cooling the system causes the conversion of the saturated vapor into two phase mixture. As the container is rigid, the further cooling process 2 - 3 also occurs at constant volume. Hence, final state 3 is defined as

Final state 3:

Temperature, $T_3 = T_f = 90^\circ\text{C}$, Specific volume, $v_3 = v_2 = 1.3162 \text{ m}^3/\text{kg}$.

Referring to the saturated water temperature table A2.2, for temperature $T_3 = 90^\circ\text{C}$, $v_l = 0.001036 \text{ m}^3/\text{kg}$, $v_{lg} = 2.3607 \text{ m}^3/\text{kg}$, $v_g = 2.3617 \text{ m}^3/\text{kg}$.

Pressure at final state 3, $P_3 = P_f = P_{sat}(T = 90^\circ\text{C}) = 70.117 \text{ kPa}$

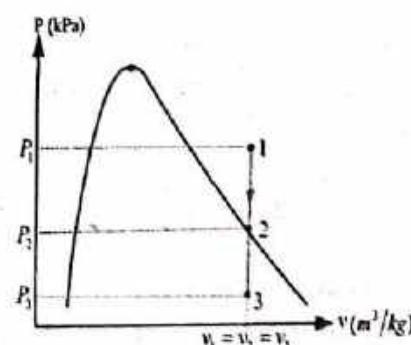
Now, quality at final state 3,

$$x_3 = x_f = \frac{v_3 - v_l}{v_{lg}} = \frac{1.3162 - 0.001036}{2.3607} \\ = 0.5571$$

Then, the mass fraction of water (saturated liquid)

at final state 3,

$$\left(\frac{m_l}{m} \right)_3 = y_3 = y_f = (1 - x_3) = (1 - 0.5571) = 0.4429 = 44.29\%$$



Problem 2.16.

A one liter closed vessel contains water at its critical condition. The vessel is cooled until its pressure drops to 1 MPa. Calculate the mass of water in the vessel, the final dryness fraction and final temperature. Also show the process on P-v and T-v diagrams.

Solution: Given,

Volume of vessel, $V = 1 \text{ Ltr.} = 0.001 \text{ m}^3 = \text{constant}$ (As it is a closed vessel)

Initially, water is at critical condition.

Final pressure, $P_2 = 1 \text{ MPa} = 1000 \text{ kPa}$

Initial state 1: $V_1 = 0.001 \text{ m}^3$

State 1 is the critical condition and for water the critical pressure, temperature and specific volume are as follows:

$P_c = P_{cr} = 22055 \text{ kPa}$, $T_c = 373.98^\circ\text{C}$ and $v_c = v_{cr} = 0.00311 \text{ m}^3/\text{kg}$

Mass of water, $m = V/v_1 = 0.001/0.00311 = 0.321543 \text{ kg}$.

The P-v and T-v diagrams of the cooling process are shown in figures below. Since the vessel is closed, the process 1-2 is the constant volume cooling process from critical state 1 to state 2 at given final pressure (1000 kPa). Hence, final state 2 is defined as

Final state 2: $P_2 = 1000 \text{ kPa}$, volume at state 2, $V_2 = V_1 = 0.001 \text{ m}^3$

\Rightarrow Specific volume at state 2, $v_2 = v_1 = 0.00311 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for $P_2 = 1000 \text{ kPa}$

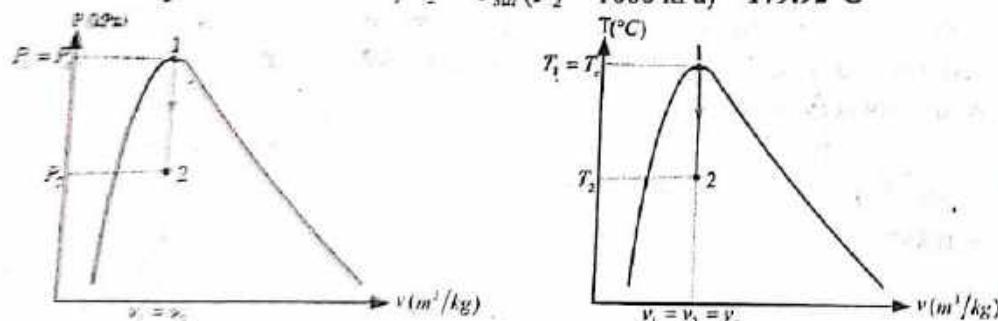
$v_f = 0.001127 \text{ m}^3/\text{kg}$, $v_{fg} = 0.1933 \text{ m}^3/\text{kg}$, $v_g = 0.1944 \text{ m}^3/\text{kg}$.

Here, $v_f < v_2 < v_g$. Hence, it is still a two phase mixture.

The dryness fraction (quality) at final state 2,

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.00311 - 0.001127}{0.1933} = 0.0103 = 1.03\%$$

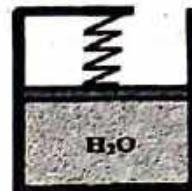
The final temperature at state 2, $T_2 = T_{sat}(P_2 = 1000 \text{ kPa}) = 179.92^\circ\text{C}$


Problem 2.17.

A piston cylinder device with a linear spring initially contains water at a pressure of 4 MPa and 500°C with the initial volume being 0.1 m^3 , as shown in figure below. The system now cools until the pressure reaches 1000 kPa. If the

piston is at the bottom, the system pressure is 300 kPa. Sketch the process on P-v diagram and determine

- the mass of water
- the final temperature and volume, and
- the total work transfer.



Solution: Given,

Initial pressure, $P_1 = 4 \text{ MPa} = 4000 \text{ kPa}$

Initial temperature, $T_1 = 500^\circ\text{C}$

Initial volume, $V_1 = 0.1 \text{ m}^3$

Final pressure, $P_2 = 1000 \text{ kPa}$

Boundary condition:

when $V_3 = 0$ (piston is at bottom), pressure, $P_3 = 300 \text{ kPa}$

Initial state 1:

$$P_1 = 4000 \text{ kPa}, T_1 = 500^\circ\text{C}, V_1 = 0.1 \text{ m}^3$$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 4000 \text{ kPa}$, the saturation temperature, $T_{sat} = 250.39^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 4000 \text{ kPa}$ and temperature, $T_1 = 500^\circ\text{C}$, the specific volume, $v_1 = 0.08642 \text{ m}^3/\text{kg}$.

$$\text{The mass of water, } m = \frac{V_1}{v_1} = \frac{0.1}{0.08642} = 1.15714 \text{ kg}$$

The $P - v$ diagram of the cooling process is shown in figure below. Since the piston is loaded with a linear spring, the process 1 – 2 is the cooling process during which initial pressure decreases linearly to given final pressure (1000 kPa) thereby decreasing temperature and volume also. By using boundary condition of state 3, final state 2 is defined as

Final state 2:

The final volume at state 2, V_2 can be determined by applying linear interpolation method. We have

$$P_1 = 4000 \text{ kPa}$$

$$V_1 = 0.1 \text{ m}^3$$

$$P_2 = 1000 \text{ kPa}$$

$$V_2 = ?$$

$$P_3 = 300 \text{ kPa}$$

$$V_3 = 0 \text{ m}^3$$

Then,

$$V_2 = V_1 + \frac{P_2 - P_1}{P_3 - P_1} (V_3 - V_1) = 0.1 + \frac{1000 - 4000}{300 - 4000} (0.0 - 0.1) = 0.01892 \text{ m}^3$$

The specific volume at state 2,

$$v_2 = \frac{V_2}{m} = \frac{0.01892}{1.15714} = 0.01635 \text{ m}^3/\text{kg}$$

Referring to the saturated water pressure table A2.1 for $P_2 = 1000 \text{ kPa}$

$$v_l = 0.001127 \text{ m}^3/\text{kg}, v_{lg} = 0.1933 \text{ m}^3/\text{kg}, v_g = 0.1944 \text{ m}^3/\text{kg}$$

Here, $v_1 < v_2 < v_g$. Hence, it is a two phase mixture. The temperature at state 2, T_2 will be then saturation temperature for pressure, P_2 , i.e.

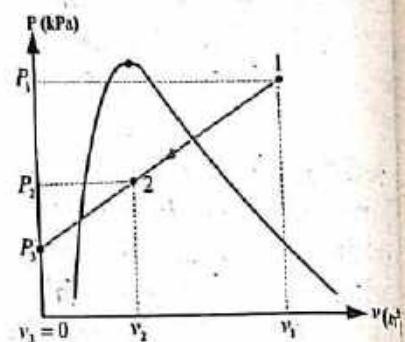
$$T_2 = T_{sat}(P_2 = 1000 \text{ kPa}) = 179.92^\circ\text{C}.$$

Total work transfer,

$$W_{1-2} = \text{Area under the process curve } 1 - 2$$

$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= \frac{1}{2}(4000 + 1000)(0.01892 - 0.1) = -202.7 \text{ kJ}$$



The negative sign indicates that the work is done on the system.

Problem 2.18.

A vessel contains 2 kg of saturated liquid water and saturated water vapor mixture at a temperature of 150°C . One third of the volume is saturated liquid and two third is saturated vapor. Determine the pressure, quality, volume, internal energy and enthalpy of the mixture.

Solution: Given,

Total mass of water, $m = 2 \text{ kg}$

Temperature of water, $T = 150^\circ\text{C}$

Volume of saturated liquid, $V_l = V/3 \text{ m}^3$

Volume of saturated vapor, $V_g = 2V/3 \text{ m}^3$, where V is volume of mixture.

Here, water is in two phase mixture region, so referring to the saturated water temperature table A2.2 for temperature, $T = 150^\circ\text{C}$,

$v_l = 0.001090 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3918 \text{ m}^3/\text{kg}$, $v_g = 0.3929 \text{ m}^3/\text{kg}$.

Pressure, $P = P_{sat}(T = 150^\circ\text{C}) = 475.72 \text{ kPa}$.

$$\text{Mass of saturated liquid, } m_l = \frac{V_l}{v_l} = \frac{V}{3 \times 0.001090} = \frac{V}{0.00327}$$

$$\text{Mass of saturated vapor, } m_g = \frac{V_g}{v_g} = \frac{2V}{3 \times 0.3929} = \frac{V}{0.58935}$$

Quality of mixture,

$$x = \frac{m_g}{m} = \frac{m_g}{m_l + m_g} = \frac{V}{0.58935[(V/0.00327) + (V/0.58935)]}$$

$$= 0.00552 = 0.552\%$$

Specific volume, $v = v_l + x v_{lg} = 0.001090 + 0.00552 \times 0.3918 = 0.003252 \text{ m}^3/\text{kg}$

$$\text{Now, } V = m \times v = 2 \times 0.003252 = 0.006504 \text{ m}^3$$

To calculate specific internal energy, again, referring to the saturated water temperature table A2.2 for temperature, $T = 150^\circ\text{C}$,

$$u_l = 631.80 \text{ kJ/kg}, u_{lg} = 1927.7 \text{ kJ/kg}, u_g = 2559.5 \text{ kJ/kg}$$

$$\text{Specific internal energy, } u = u_l + xu_{lg} = 631.80 + 0.00552 \times 1927.7$$

$$= 642.441 \text{ kJ/kg}$$

Then, $U = m \times u = 2 \times 642.441 = 1284.882 \text{ kJ}$

Again, referring to the saturated water temperature table A2.2 for temperature, $T = 150^\circ\text{C}$, $h_l = 632.32 \text{ kJ/kg}$, $h_{lg} = 2114.1 \text{ kJ/kg}$, $h_g = 2746.4 \text{ kJ/kg}$.

$$\text{Specific enthalpy, } h = h_l + xh_{lg} = 632.32 + 0.00552 \times 2114.1 = 643.9898 \text{ kJ/kg.}$$

Hence, $H = m \times h = 2 \times 643.9898 = 1287.9796 \text{ kJ}$

Problem 2.19.

A rigid container with a volume of 0.170 m^3 is initially filled with steam at 1500 kPa and 200°C . It is cooled to 100°C . Determine

- (a) total mass of the system
- (b) final pressure and
- (c) mass fraction of liquid in final state.

Also sketch the process on P-v and T-v diagrams.

Solution: Given,

Volume of container, $V = 0.170 \text{ m}^3 = \text{constant}$ (As it is a rigid container)

Initial pressure of steam, $P_1 = 1500 \text{ kPa}$

Initial temperature of steam, $T_1 = 200^\circ\text{C}$

Final temperature, $T_2 = 100^\circ\text{C}$

Initial state 1:

$$P_1 = 1500 \text{ kPa}, T_1 = 200^\circ\text{C}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 1500 \text{ kPa}$, saturation temperature $T_{sat}(P = 1500 \text{ kPa}) = 198.33^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 1500 \text{ kPa}$ and temperature, $T_1 = 200^\circ\text{C}$, specific volume at state 1, $v_1 = 0.1324 \text{ m}^3/\text{kg}$.

$$\text{Total mass of system, } m = \frac{V_1}{v_1} = \frac{0.170}{0.1324} = 1.284 \text{ kg}$$

The P-v and T-v diagrams of the cooling process are shown in figures below. As the container is rigid, the process 1 – 2 is a constant volume cooling until the system reaches final temperature, T_2 . Hence, state 2 is defined as

Final state 2:

$T_2 = 100^\circ\text{C}$ and volume, $V_2 = V_1 \Rightarrow$ Specific volume, $v_2 = v_1 = 0.1324 \text{ m}^3/\text{kg}$.

Referring to the saturated water temperature table A2.2 for temperature $T_2 = 100^\circ\text{C}$, $v_l = 0.001043 \text{ m}^3/\text{kg}$, $v_{lg} = 1.6726 \text{ m}^3/\text{kg}$, $v_g = 1.6736 \text{ m}^3/\text{kg}$.

Here, $v_l < v_2 < v_g$. Hence, it is a two phase mixture. The pressure at state 2, P_2 will be then saturation pressure for temperature, T_2 , i.e.

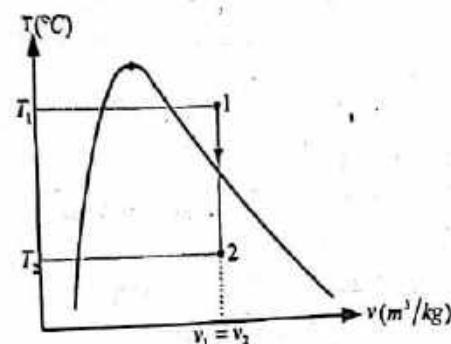
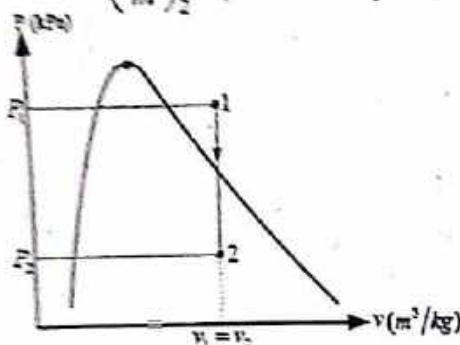
$$P_2 = P_{sat}(T_2 = 100^\circ\text{C}) = 101.32 \text{ kPa.}$$

Now, quality at final state 2,

$$x_2 = \frac{v_2 - v_f}{v_g} = \frac{0.1324 - 0.001043}{1.6726} = 0.078535$$

Then, the mass fraction of saturated liquid at final state 3,

$$y_2 = \left(\frac{m_2}{m} \right)_2 = (1 - x_2) = (1 - 0.078535) = 0.9215 = 92.15\%$$



2.3 Solved Numerical Problems from PoU Examinations

Problem 2.20.

5 kg of H_2O is contained in a closed rigid tank at initial pressure of 2000 kPa and a quality of 50%. Heat transfer occurs until the tank contains only saturated vapor. Determine

- (i) the volume of tank in m^3 ,
- (ii) the initial temperature of water and
- (iii) the final pressure is in kPa.

(PoU, 2004 spring).

Solution: Given,

Mass of H_2O , $m = 5 \text{ kg}$

Initial pressure, $P_1 = 2000 \text{ kPa}$

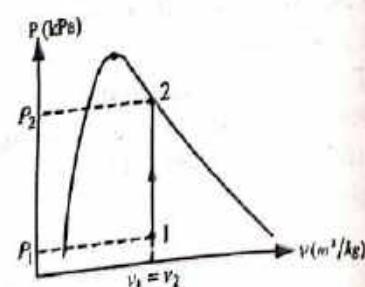
Initial quality, $x_1 = 50\% = 0.5$

Final state is saturated vapor \Rightarrow

$$v_f = v_2 = v_g$$

Initial state 1:

$$P_1 = 2000 \text{ kPa}, x_1 = 0.5$$



As the quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 2000 \text{ kPa}$, initial temperature, $T_1 = T_{\text{sat}} (P_1 = 2000 \text{ kPa}) = 212.42^\circ\text{C}$,

$v_l = 0.001177 \text{ m}^3/\text{kg}$, $v_{lg} = 0.09841 \text{ m}^3/\text{kg}$, $v_g = 0.09959 \text{ m}^3/\text{kg}$.

The specific volume at state 1,

$$v_1 = v_l + x_1 v_{lg} = 0.001177 + 0.5 \times 0.09841 = 0.050382 \text{ m}^3/\text{kg}$$

Volume at state 1, $V_1 = m \times v_1 = 5 \times 0.050382 = 0.25191 \text{ m}^3 = V$

The $P - v$ diagram is shown in figure above. As the tank is closed and rigid, the heating process 1–2 is a constant volume and the water converts to saturated vapor. Hence, the final state 2 is defined as

State 2:

Volume, $V_2 = V_1 = 0.25191 \text{ m}^3 \Rightarrow$ specific volume, $v_2 = v_1 = 0.050382 \text{ m}^3/\text{kg}$ and as it is a saturated vapor, $v_2 = v_g = 0.050382 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for specific volume, $v_g = 0.050382 \text{ m}^3/\text{kg}$ to determine the final pressure at state 2, $P_2 = P_f$. As there is no value listed for $v_g = 0.050382 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required pressure as

$$P_a = 3750 \text{ kPa} \quad v_{ga} = 0.05318 \text{ m}^3/\text{kg}$$

$$P_2 = ? \quad v_g = 0.050382 \text{ m}^3/\text{kg}$$

$$P_b = 4000 \text{ kPa} \quad v_{gb} = 0.04977 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$P_2 = P_a + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (P_b - P_a) = 3750 + \frac{0.050382 - 0.05318}{0.04977 - 0.05318} (4000 - 3750)$$

$$= 3955.132 \text{ kPa} = P_f$$

Problem 2.21.

A vessel contains 1 kg of steam which contains 1/3 liquid and 2/3 vapor by volume. The temperature of steam is 151.87°C . Find the quality, specific volume and specific enthalpy of the mixture.

(PoU, 2006 spring, 2010 spring, 2011 fall, 2015 spring).

Solution: Given,

Total mass of steam, $m = 1 \text{ kg}$

Temperature of steam, $T = 151.87^\circ\text{C}$

Volume of saturated liquid, $V_l = V/3 \text{ m}^3$

Volume of saturated vapor, $V_g = 2V/3 \text{ m}^3$, where V is volume of vessel or mixture. Here, water is in two phase mixture region, so referring to the saturated water pressure table A2.1 for temperature, $T = 151.87^\circ\text{C}$, $P_{\text{sat}} (151.87^\circ\text{C}) = 500 \text{ kPa}$, $v_l = 0.001093 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3738 \text{ m}^3/\text{kg}$, $v_g = 0.3749 \text{ m}^3/\text{kg}$, and $h_l = 640.38 \text{ kJ/kg}$, $h_{lg} = 2108.2 \text{ kJ/kg}$, $h_g = 2748.6 \text{ kJ/kg}$.

PROPERTIES OF PURE WORKING SUBSTANCES

$$\text{Mass of saturated liquid, } m_l = \frac{V_l}{v_l} = \frac{V}{3 \times 0.001093} = \frac{V}{0.003279}$$

$$\text{Mass of saturated vapor, } m_g = \frac{V_g}{v_g} = \frac{2V}{3 \times 0.3749} = \frac{V}{0.56235}$$

Quality of mixture,

$$x = \frac{m_g}{m} = \frac{m_g}{m_l + m_g} = \frac{V}{0.56235[(V/0.003279) + (V/0.56235)]} \\ = 0.005797 = 0.558\%$$

$$\text{Specific volume of mixture, } v = v_l + xv_{lg} = 0.001093 + 0.005797 \times 0.3738 \\ = 0.0032599 \text{ m}^3/\text{kg.}$$

Similarly, specific enthalpy of mixture,

$$h = h_l + xh_{lg} = 640.38 + 0.005797 \times 2108.2 = 652.6012 \text{ kJ/kg}$$

Problem 2.22.

A vessel having a volume of 0.6 m^3 contains 3 kg of liquid water and water vapor mixture in equilibrium at a pressure of 0.5 MPa. Calculate:

- (a) the volume and mass of liquid
- (b) the volume and mass of steam

(PoU, 2007 spring/fall).

Solution: Given,

Volume of vessel, $V = 0.6 \text{ m}^3$

Total mass of mixture, $m = 3 \text{ kg}$

pressure of mixture, $P = 0.5 \text{ MPa} = 500 \text{ kPa}$

$$\text{Specific volume, } v = \frac{V}{m} = \frac{0.6}{3} = 0.2 \text{ m}^3/\text{kg}$$

Here, water is in two phase mixture region, so referring to the saturated water pressure table A2.1 for pressure, $P = 500 \text{ kPa}$,

$$v_l = 0.001093 \text{ m}^3/\text{kg}, v_{lg} = 0.3738 \text{ m}^3/\text{kg}, v_g = 0.3749 \text{ m}^3/\text{kg.}$$

$$\text{Quality, } x = \frac{v - v_l}{v_{lg}} = \frac{0.2 - 0.001093}{0.3738} = 0.532121$$

$$\text{Mass of saturated vapor, } m_g = x \times m = 0.532121 \times 3 = 1.59636 \text{ kg} = 1.6 \text{ kg}$$

$$\text{Volume of saturated vapor, } V_g = m_g \times v_g = 1.59636 \times 0.3749 = 0.598477 \text{ m}^3$$

$$\text{Mass of saturated liquid, } m_l = m - m_g = 3 - 1.59636 = 1.4036 \text{ kg} = 1.4 \text{ kg}$$

$$\text{Volume of saturated liquid, } V_l = m_l \times v_l = 1.4036 \times 0.001093 = 0.0015342 \text{ m}^3$$

Problem 2.23.

A vessel having a volume of 0.4 m^3 contains 2 kg of liquid water and water vapor mixture in equilibrium at a pressure of 0.6 MPa. Calculate

- (a) the volume and mass of liquid
- (b) the volume and mass of vapor

(PoU, 2009 spring).

Solution: Refer problem 2.22 (very similar).

Problem 2.24.

A two phase liquid vapor mixture of H_2O is at 30 bar. If on heating a fixed volume, critical point is attained. Determine the quality at initial state.

(PoU, 2010 fall).

Solution: Given,

Initial pressure of mixture, $P_1 = 30 \text{ bar} = 3000 \text{ kPa}$

Final state is critical point, hence specific volume at final state 2,

$$v_2 = v_{cr} = 0.00311 \text{ m}^3/\text{kg}$$

The $P - v$ diagram is given below. The heating process 1 – 2 is at fixed volume which means it is an isochoric process.

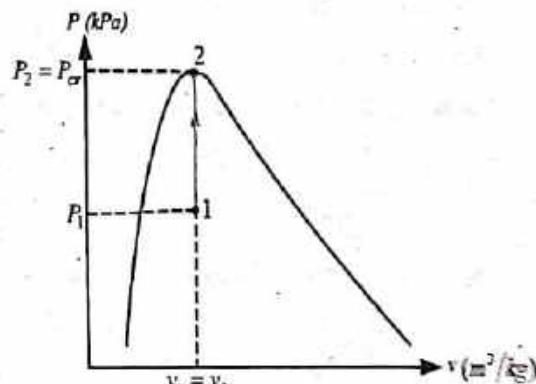
So, specific volume at initial state 1, $v_1 = v_2 = 0.00311 \text{ m}^3/\text{kg}$

Here, water is in two phase mixture region, so referring to the saturated water pressure table A2.1 for pressure, $P = 3000 \text{ kPa}$,

$$v_l = 0.001217 \text{ m}^3/\text{kg}, v_{lg} = 0.06544 \text{ m}^3/\text{kg}, v_g = 0.06666 \text{ m}^3/\text{kg}$$

Now, quality at initial state 1,

$$x_1 = \frac{v_1 - v_l}{v_{lg}} = \frac{0.00311 - 0.001217}{0.06544} \\ = 0.028927 = 2.8927\%$$



Problem 2.25.

A piston cylinder arrangement shown in figure below contains 2 kg of water initially at pressure of 600 kPa and temperature of 350 °C. Heat is rejected from the system until it is completely converted into saturated liquid. It requires a pressure of 400 kPa to support the piston. Sketch the process on $P - v$ and $T - v$ diagrams and determine the temperature when piston just starts to move and the total work transfer.

(PoU 2013 spring).

Solution: Given,

Mass of water, $m = 2 \text{ kg}$

Initial pressure, $P_1 = 600 \text{ kPa}$

Initial temperature, $T_1 = 350^\circ\text{C}$

Supporting pressure of piston, $P_{sup} = 400 \text{ kPa}$

Final state is saturated liquid.

Initial state 1:

$$P_1 = 600 \text{ kPa}, T_1 = 350^\circ\text{C}$$



Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 600 \text{ kPa}$, saturation temperature, $T_{sat}(P_1 = 600 \text{ kPa}) = 158.86^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 600 \text{ kPa}$ and temperature, $T_1 = 350^\circ\text{C}$, the specific volume at initial state 1, $v_1 = 0.4742 \text{ m}^3/\text{kg}$.

The $P - v$ and $T - v$ diagrams are shown in figures below. During initial state of heat rejection, piston remains stationary although heat is removed from the system. So, process 1 - 2 is constant volume cooling during which initial pressure decreases to supporting pressure of piston. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_{sup} = 400 \text{ kPa}$ and volume at state 2, $V_2 = V_1 \Rightarrow$ Specific volume at state 2, $v_2 = v_1 = 0.4742 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$, $v_1 = 0.001084 \text{ m}^3/\text{kg}$, $v_{fg} = 0.4614 \text{ m}^3/\text{kg}$, $v_g = 0.4625 \text{ m}^3/\text{kg}$.

Here, $v_2 > v_g$. Hence it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 400 \text{ kPa}$ and specific volume, $v_2 = 0.4742 \text{ m}^3/\text{kg}$ to find the temperature at state 2, T_2 . As there is no value listed for $v_2 = 0.4742 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_a = 150^\circ\text{C}$$

$$v_a = 0.4708 \text{ m}^3/\text{kg}$$

$$T_b = ?$$

$$v_b = 0.4742 \text{ m}^3/\text{kg}$$

$$T_b = 200^\circ\text{C}$$

$$v_b = 0.5342 \text{ m}^3/\text{kg}$$

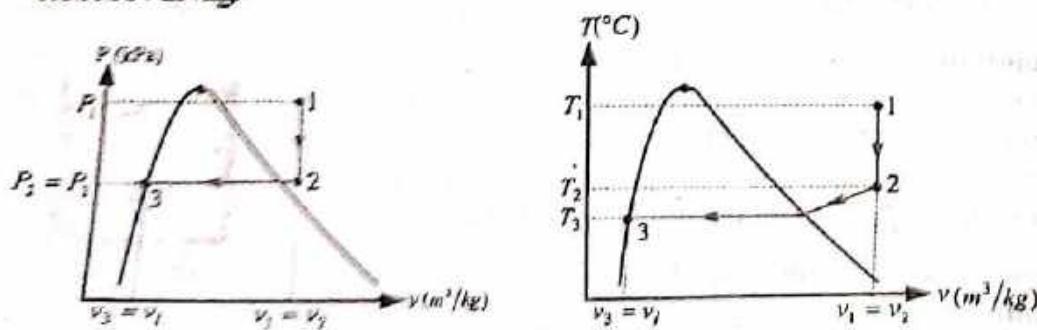
Using linear interpolation method,

$$T_2 = T_a + \frac{v_2 - v_a}{v_b - v_a} (T_b - T_a) = 150 + \frac{0.4742 - 0.4708}{0.5342 - 0.4708} (200 - 150) = 152.68^\circ\text{C}.$$

As the required final state is saturated liquid, the system should be further cooled until it becomes saturated liquid. This further cooling process 2 - 3 occurs at constant supporting pressure of piston. Then state 3 can be defined as

Final state 3:

Pressure at state 3, $P_3 = P_{sup} = 400 \text{ kPa}$ and specific volume, $v_3 = v_l$ ($P_3 = 400 \text{ kPa}$) $= 0.001084 \text{ m}^3/\text{kg}$.



Total work transfer,

$$W_t = W_{1-2} + W_{2-3} = 0 + P_2 (V_3 - V_2) = P_2 \times m(v_3 - v_2)$$

$$= 400 \times 10^3 \times 2 (0.001084 - 0.4742) = -378492.8 \text{ J} = -378.4928 \text{ kJ}$$

The negative sign indicates the work is done on the system.

Problem 2.26.

A vessel having a volume of 0.8 m^3 contains 4 kg of liquid water and water vapor mixture in equilibrium at a pressure of 7 bar. Calculate:

- (a) the volume and mass of liquid
- (b) the volume and mass of vapor

(PoU 2014 spring, PoU 2015 fall, PoU 2019 fall).

Solution: Refer problem 2.22. (Hint: 7 bar = 700 kPa, Ans: $V_l = 0.001186 \text{ m}^3$, $m_l = 1.07 \text{ kg}$, $V_g = 0.799304 \text{ m}^3$, $m_g = 2.93 \text{ kg}$). (Very similar).

Problem 2.27.

A rigid tank contains steam at 200 kPa. One third of its volume is in liquid phase and rest is in vapor phase. Determine the temperature of steam and specific volume. (PoU 2015).

Solution: Given,

Pressure of steam, $P = 200 \text{ kPa}$

Volume of saturated liquid, $V_l = V/3 \text{ m}^3$

Volume of saturated vapor, $V_g = 2V/3 \text{ m}^3$, where V is volume of tank or mixture.

Here, water is in two phase mixture region, so referring to the saturated water pressure table A2.1 for pressure, $P = 200 \text{ kPa}$, the saturation temperature of steam, T_{sat} (200 kPa) = $T = 120.24^\circ\text{C}$.

Also, $v_l = 0.001060 \text{ m}^3/\text{kg}$, $v_{lg} = 0.8848 \text{ m}^3/\text{kg}$, $v_g = 0.8859 \text{ m}^3/\text{kg}$.

$$\text{Mass of saturated liquid, } m_l = \frac{V_l}{v_l} = \frac{V}{3 \times 0.001060} = \frac{V}{0.00318}$$

$$\text{Mass of saturated steam, } m_g = \frac{V_g}{v_g} = \frac{2V}{3 \times 0.8859} = \frac{V}{1.32885}$$

Quality of mixture,

$$x = \frac{m_g}{m} = \frac{m_g}{m_l + m_g} = \frac{V}{1.32885[(V/0.00318) + (V/1.32885)]} = 0.0023873$$

$$\begin{aligned} \text{Specific volume of steam, } v &= v_l + xv_{lg} = 0.001060 + 0.0023873 \times 0.8848 \\ &= 0.0031723 \text{ m}^3/\text{kg}. \end{aligned}$$

Problem 2.28.

A piston cylinder device contains 0.1 m^3 of liquid water and 0.9 m^3 of water vapor in equilibrium at 800 kPa. Heat is transferred at constant pressure until the temperature reaches 350°C .

PROPERTIES OF PURE WORKING SUBSTANCES

- (i) What is the initial temperature of water?
- (ii) Determine total mass of water.
- (iii) Calculate the final volume.
- (iv) Show the process on $P - v$ diagram with respect to saturation lines. (PoU 2017 fall).

Solution: Given,

Volume of liquid water, $V_l = 0.1 \text{ m}^3$

Volume of water vapor, $V_g = 0.9 \text{ m}^3$

Initial pressure, $P_1 = 800 \text{ kPa}$

Final temperature, $T_2 = 350^\circ\text{C}$

Heating process is isobaric process.

As the initial state is two phase mixture, the initial temperature of the water is equal to the saturation temperature corresponding to the initial pressure.

State 1: $P_1 = 800 \text{ kPa}$, $V_l = 0.1 \text{ m}^3$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 800 \text{ kPa}$, saturation temperature, $T_{sat}(P_1 = 800 \text{ kPa}) = T_1 = 170.44^\circ\text{C}$.

Also, for pressure, $P_1 = 800 \text{ kPa}$,

$$v_l = 0.001115 \text{ m}^3/\text{kg}, v_{lg} = 0.2393 \text{ m}^3/\text{kg}, v_g = 0.2404 \text{ m}^3/\text{kg}.$$

$$\text{Now, mass of liquid water, } m_l = \frac{V_l}{v_l} = \frac{0.1}{0.001115} = 89.6861 \text{ kg}$$

$$\text{Mass of water vapor, } m_g = \frac{V_g}{v_g} = \frac{0.9}{0.2404} = 3.74376 \text{ kg}$$

$$\text{Total mass of water, } m = m_l + m_g = 89.6861 + 3.74376 = 93.42986 \text{ kg}$$

The $P - v$ diagram of the heating process is given above. Heat is transferred at constant pressure until the temperature becomes 350°C . Hence, the final state 2 is defined as

State 2: $P_2 = P_1 = 800 \text{ kPa}$, $T_2 = 350^\circ\text{C}$

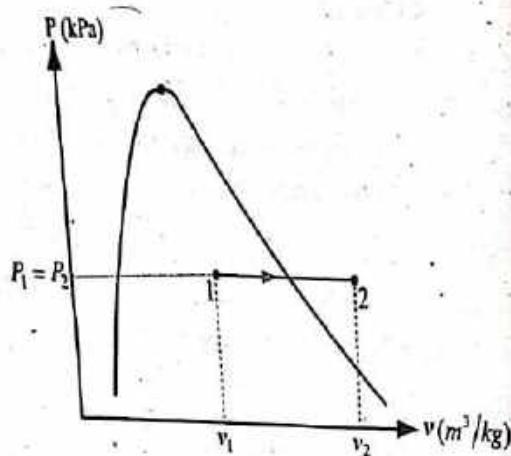
Here, $T_2 > T_{sat}(800 \text{ kPa})$. So, state 2 is a superheated vapor. Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 800 \text{ kPa}$ and temperature $T_2 = 350^\circ\text{C}$, the specific volume, $v_2 = 0.3544 \text{ m}^3/\text{kg}$.

$$\text{Then, final volume, } V_2 = m \times v_2 = 93.42986 \times 0.3544 = 33.1115 \text{ m}^3$$

Problem 2.29.

A piston cylinder device initially contains 50 L of liquid water at 40°C and 200 kPa. Heat is transferred to the water at constant pressure until entire liquid is vaporized. Determine

- (i) the mass of the water
- (ii) the final temperature
- (iii) the total enthalpy change



Also show the process on $T - v$ diagram with respect to saturation lines.

(PoU 2018 fall).

Solution: Given,

$$\begin{aligned}\text{Initial volume of water, } V_1 &= 50 \text{ L} \\ &= 50 \times 0.001 = 0.05 \text{ m}^3\end{aligned}$$

Initial temperature, $T_1 = 40^\circ\text{C}$

Initial pressure, $P_1 = 200 \text{ kPa}$

Heating process is isobaric.

Final state is saturated vapor $\Rightarrow v_f = v_g$

Initial state 1:

$$P_1 = 200 \text{ kPa}, T_1 = 40^\circ\text{C}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 200 \text{ kPa}$, saturation temperature, $T_{sat}(P_1 = 200 \text{ kPa}) = 120.24^\circ\text{C}$.

Here, $T_1 < T_{sat}$. Hence, it is a subcooled or compressed liquid.

Now, referring to the compressed liquid table A 2.3 for pressure, $P_1 = 200 \text{ kPa}$ and temperature, $T_1 = 40^\circ\text{C}$ to find the specific volume and enthalpy at initial state 1, v_1 and h_1 . But these values are not available in the table for this low pressure (as liquid is incompressible at low pressure).

Then, referring to the saturated water temperature table A2.2 for temperature, $T_1 = 40^\circ\text{C}$ to take the value of specific volume and enthalpy of saturated liquid for the specific volume and enthalpy at initial state 1, $v_1 = v_l(T_1 = 40^\circ\text{C}) = 0.001008 \text{ m}^3/\text{kg}$ and

$$h_1 = h_l(T_1 = 40^\circ\text{C}) = 167.50 \text{ kJ/kg} \text{ with slight error.}$$

$$\text{Mass of water, } m = V_1/v_1 = 0.05/0.001008 = 49.6032 \text{ kg.}$$

The $T - v$ diagram is shown in figure above. As the heating process 1 – 2 is a constant pressure and the water is completely vaporized (saturated vapor). Hence, the final state 2 is defined as

State 2:

$$P_2 = P_1 = 200 \text{ kPa} \text{ and as it is a saturated vapor, } v_2 = v_g = 0.8859 \text{ m}^3/\text{kg.}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 200 \text{ kPa}$ and specific volume, $v_2 = v_g = 0.8859 \text{ m}^3/\text{kg}$ to determine the final temperature $T_f = T_2$ at state 2, $T_2 = 120.24^\circ\text{C}$.

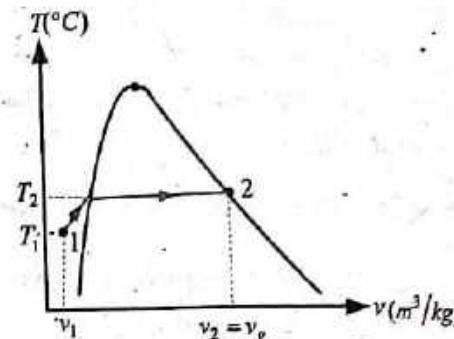
Similarly, the specific enthalpy at state 2, $h_2 = h_g(P_2 = 200 \text{ kPa}) = 2706.5 \text{ kJ/kg}$.

Now, total change in enthalpy during the process is given by

$$H_2 - H_1 = m(h_2 - h_1) = 49.6032 \times (2706.5 - 167.5) = 125942.46 \text{ kJ}$$

Problem 2.30.

A rigid tank with a volume of 2.5 m^3 contains 15 kg of saturated liquid - vapor mixture of water at 75°C . Now the water is slowly heated until the liquid in the tank is completely vaporized. Determine (i) the final temperature (ii) change in the internal energy during the process. Also, show the process on a $T - v$ diagram with respect to saturation lines. (PoU 2019 spring)



Solution : Given,

Volume of water, $V = 2.5 \text{ m}^3 = \text{constant}$

(As it is a closed rigid vessel)

Mass of mixture, $m = 15 \text{ kg}$

Initial temperature of mixture, $T_1 = 75^\circ\text{C}$

Final specific volume of water, $v_f = v_g$

(As water is completely vaporized)

Initial state 1: $V_1 = 0.2 \text{ m}^3$, $T_1 = 75^\circ\text{C}$

Specific volume of water mixture, $v_1 = \frac{V_1}{m} = \frac{2.5}{15} = 0.166666 \text{ m}^3/\text{kg}$

As water is in a two phase mixture, referring to saturated water temperature table for A2.2 for temperature, $T_1 = 75^\circ\text{C}$,

$v_f = 0.001026 \text{ m}^3/\text{kg}$, $v_g = 4.1323 \text{ m}^3/\text{kg}$, $v_{fg} = 4.1333 \text{ m}^3/\text{kg}$,

$u_f = 313.92 \text{ kJ/kg}$, $u_{fg} = 2161.3 \text{ kJ/kg}$, $u_g = 2475.2 \text{ kJ/kg}$.

$$\text{Quality at state 1, } x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.166666 - 0.001026}{4.1323} = 0.0400844$$

$T-v$ diagram of heating process is shown in figures below. As the initial state is a two phase mixture, the initial specific internal energy for the initial temperature 75°C can be calculated as

$$u_1 = u_f + x_1 u_{fg} = 313.92 + 0.0400844 \times 2161.3 = 400.5544 \text{ kJ/kg}$$

As the vessel is rigid, the process 1 – 2 is a constant volume heating process. Hence, final state 2 is defined as

Final state 2: $v_2 = v_g$ and $V_2 = V_1 = 2.5 \text{ m}^3 \Rightarrow$ specific volume at state 2,

$$v_2 = v_g = 0.166666 \text{ m}^3/\text{kg} \Rightarrow v_2 = v_g = 0.166666 \text{ m}^3/\text{kg}$$

Referring to the saturated water temperature table A2.2 for specific volume, $x_2 = x_g = 0.166666 \text{ m}^3/\text{kg}$ to find the required final temperature at final state 2, T_2 . As there is no value listed for $v_g = 0.166666 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_e = 185^\circ\text{C}$$

$$v_{g0} = 0.1741 \text{ m}^3/\text{kg}$$

$$T_2 = ?$$

$$v_{g1} = 0.166666 \text{ m}^3/\text{kg}$$

$$T_e = 190^\circ\text{C}$$

$$v_{g2} = 0.1565 \text{ m}^3/\text{kg}$$

Using linear interpolation method, pressure at which the container holds only saturated vapor,

$$T_2 = T_e + \frac{v_{g1} - v_{g0}}{v_{g0} - v_{g2}} (T_e - T_e) = 185 + \frac{0.166666 - 0.1741}{0.1565 - 0.1741} (190 - 185)$$

$$= 187.392^\circ\text{C}$$

Similarly, the specific internal energy at final state 2 is given by $u_2 = u_g$ and it can be calculated by using the linear interpolation method for the specific volume, $v_2 = v_g = 0.166666 \text{ m}^3/\text{kg}$ as

$$u_{ga} = 2586.6 \text{ kJ/kg}$$

$$v_{ga} = 0.1741 \text{ m}^3/\text{kg}$$

$$u_{g2} = ?$$

$$v_{g2} = 0.166666 \text{ m}^3/\text{kg}$$

$$u_{gb} = 2589.6 \text{ kJ/kg}$$

$$v_{gb} = 0.1565 \text{ m}^3/\text{kg}$$

Using linear interpolation method, the specific internal energy at state 2,

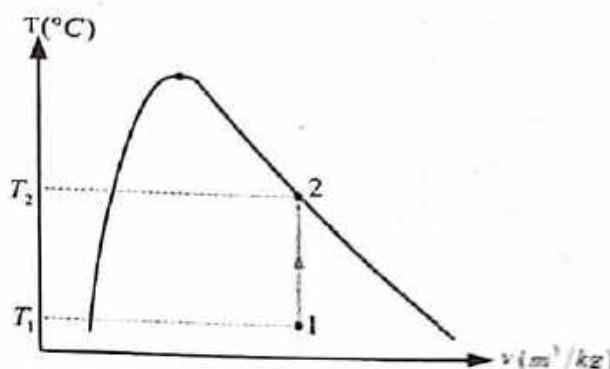
$$u_2 = u_{g2} = u_{ga} + \frac{v_{g2} - v_{ga}}{v_{gb} - v_{ga}} (u_{gb} - u_{ga})$$

$$= 2586.6 + \frac{0.166666 - 0.1741}{0.1565 - 0.1741} (2589.6 - 2586.6)$$

$$= 2587.867 \text{ kJ/kg}$$

Hence, the change in the internal energy during the process is given by

$$dU_{12} = m(u_2 - u_1) = 15(2587.867 - 400.5544) = 32809.689 \text{ kJ}$$



$$v_1 = v_2$$

Problem 2.31.

A closed, rigid container with a volume 0.2 m^3 initially contains a mixture of saturated liquid water and saturated water vapor at a pressure of 100 kPa with a quality of 50% . Heat is added, the pressure to the system until it reaches 200 kPa . Sketch the process on $P-v$ and $T-v$ diagram and determine

- (a) the temperature of each state
- (b) the mass of vapor present at each state.
- (c) If the heating is continued, determine the pressure at which the container holds only saturated vapor.

(PoU 2020 fall).

Solution : Given,

Volume of water, $V = 0.2 \text{ m}^3 = \text{constant}$

(As it is a closed rigid vessel)

Initial quality, $x_1 = 50\% = 0.5$

Initial pressure of mixture, $P_1 = 100 \text{ kPa}$

Final pressure of mixture, $P_2 = 200 \text{ kPa}$

Initial state 1: $V_1 = 0.2 \text{ m}^3$, $P_1 = 100 \text{ kPa}$, $x_1 = 0.5$

$P-v$ and $T-v$ diagrams of heating process are shown in figures below. As quality is given for pressure, $P_1 = 100 \text{ kPa}$, it is a two phase mixture.

Referring to saturated water pressure table for A2.1 for pressure, $P_1 = 100 \text{ kPa}$,
 $v_l = 0.001043 \text{ m}^3/\text{kg}$, $v_{lg} = 1.6933 \text{ m}^3/\text{kg}$, $v_g = 1.6943 \text{ m}^3/\text{kg}$.

$$\begin{aligned}\text{Specific volume at state 1, } v_1 &= v_l + x_1 v_{lg} = 0.001043 + 0.5 \times 1.6933 \\ &= 0.847693 \text{ m}^3/\text{kg}\end{aligned}$$

$$\text{Mass of water, } m = \frac{V_1}{v_1} = \frac{0.2}{0.847693} = 0.235934 \text{ kg}$$

As the initial state is a two phase mixture, the initial temperature is the saturation temperature at given pressure 100 kPa. Hence, $T_1 = T_{sat}(100 \text{ kPa}) = 99.632^\circ\text{C}$.

Initial mass of saturated vapor, $m_{g1} = x_1 \times m = 0.5 \times 0.235934 = 0.117967 \text{ kg}$

As the vessel is rigid, the process 1 – 2 is a constant-volume heating process. Hence, final state 2 is defined as

Final state 2: $P_2 = 200 \text{ kPa}$, $V_2 = V_1 = 0.2 \text{ m}^3 \Rightarrow$ specific volume at state 2,
 $v_2 = v_l = 0.847693 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 200 \text{ kPa}$,
 $v_l = 0.001060 \text{ m}^3/\text{kg}$, $v_{lg} = 0.8848 \text{ m}^3/\text{kg}$, $v_g = 0.8859 \text{ m}^3/\text{kg}$.

Here, $v_l < v_2 < v_g$. Hence, it is still a two phase mixture. Then, the final temperature at state 2 is the saturation temperature at given pressure 200 kPa. Hence, $T_2 = T_{sat}(200 \text{ kPa}) = 120.24^\circ\text{C}$.

The quality at final state 2,

$$x_2 = \frac{v_2 - v_l}{v_{lg}} = \frac{0.847693 - 0.001060}{0.8848} = 0.95686$$

Now, final mass of saturated vapor at state 2,

$$m_{g2} = x_2 m = 0.95686 \times 0.235934 = 0.225756 \text{ kg}$$

If the system is further heated, the heating process 2 – 3 is again a constant-volume process due to closed rigid container and the final state 3 with saturated vapor can be defined as

Final state 3: Volume, $V_3 = V_2 = 0.2 \text{ m}^3 \Rightarrow$ Specific volume at state 3, $v_3 = v_2 = 0.847693 \text{ m}^3/\text{kg}$ and $v_3 = v_g = 0.847693 \text{ m}^3/\text{kg}$.

Referring to the saturated water temperature table A2.2 for specific volume, $v_3 = v_g = 0.847693 \text{ m}^3/\text{kg}$ to find the required pressure at final state 3, T_3 . As there is no value listed for $v_g = 0.847693 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$P_a = 200 \text{ kPa}$$

$$v_{ga} = 0.8859 \text{ m}^3/\text{kg}$$

$$P_b = ?$$

$$v_{g3} = 0.847693 \text{ m}^3/\text{kg}$$

$$P_b = 225 \text{ kPa}$$

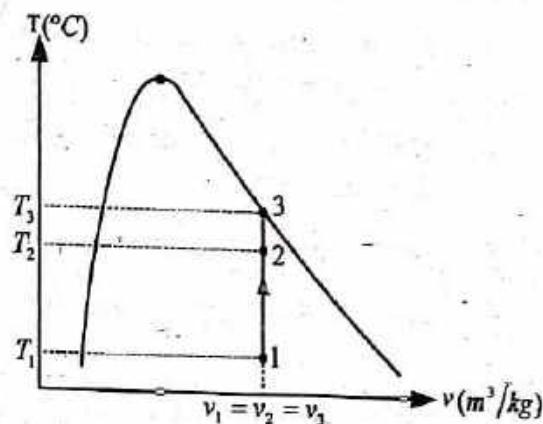
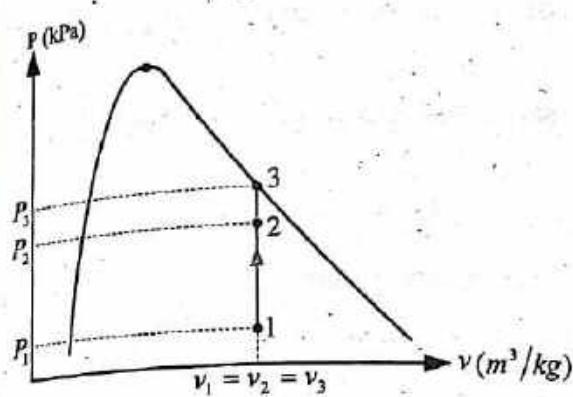
$$v_{gb} = 0.7934 \text{ m}^3/\text{kg}$$

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Using linear interpolation method, pressure at which the container holds only saturated vapor,

$$P_3 = P_a + \frac{v_{g3} - v_{ga}}{v_{gb} - v_{ga}} (P_b - P_a) = 200 + \frac{0.847693 - 0.8859}{0.7934 - 0.8859} (225 - 200)$$

$$= 210.326 \text{ kPa}$$



2.4 Review Theoretical Questions

1. Define pure substance. Derive an expression for specific volume of a two phase mixture in terms of quality.
2. Define pure substance. State and explain 'state postulate' or 'two property rule' with examples.
3. Define pure substance. Explain why quality is necessary to define state of a two phase mixture.
4. Define pure substance. Explain how saturation curve is formed on $T-v$ diagram.
5. Define pure substance. Explain why property tables and charts are necessary.
6. State 'two property rule' for a state. Explain the importance of graphical and tabular data presentation.
7. Define an ideal gas. Explain ideal gas equations.
8. Derive an expression for the specific volume of a two phase mixture in term of quality. Which takes more energy to vaporize 1 kg of saturated liquid water at 100°C or 120°C ? Why?
9. Define the following terms: saturation temperature, saturated vapor, quality, subcooled liquid, and critical point.
10. Define compressed liquid line, saturation temperature and quality. Derive the relation $v = v_l + x v_{fg}$ for the two phase mixture.
11. Define the terms: saturated vapor, superheated vapor and critical point. Also derive an expression for specific volume of a two phase mixture in terms of quality.
12. Define moisture content and critical point. Derive an expression for specific volume of a two phase (liquid and vapor) mixture in terms of quality.
13. Define quality and moisture content. Derive an expression for specific volume of a two phase mixture in terms of quality.
14. Define the following terms: critical point, quality, enthalpy, specific heat of an ideal gas at constant volume.
15. Define the following terms: saturation temperature, superheated vapor, moisture content, critical point and specific heat at constant pressure.
16. Define saturation pressure, saturation temperature and critical point. Write down the effect of pressure on
 - (a) Specific volume of a saturated liquid (v_l)

PROPERTIES OF PURE WORKING SUBSTANCES

- (b) Specific volume of a saturated vapor (v_g)
 - (c) Change in specific volume due to evaporation (v_{fg})
17. Define quality and write why it is necessary. Also derive an expression for specific volume of a two phase mixture.
18. Define the terms saturation curve for two phase mixture, quality and moisture content.
19. Define quality and explain why it is necessary to define the state of two phase mixture. Sketch saturation curve on $P - v$ diagram and also show constant quality lines.
20. Explain how saturation curve is formed on $T - v$ diagram.
21. Sketch the saturation curve on $P - v$ and $T - v$ diagrams. Also label all important points, lines and regions.
22. Sketch the following processes on $P - v$ and $T - v$ diagrams. Show both initial and final states properly relative to saturation curves.
(a) A saturated vapor in a rigid tank is heated.
(b) A superheated vapor is condensed isobarically to the saturated liquid state.
(c) A compressed liquid is heated isobarically to the saturated vapor.
(d) A two phase mixture in a rigid tank is heated such that it passes through a critical point.
23. Define enthalpy. Derive an expression for specific heat at constant volume for an ideal gas.

2.5 Unsolved Numerical Problems

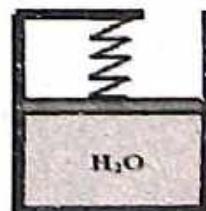
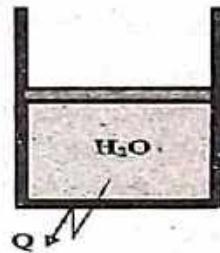
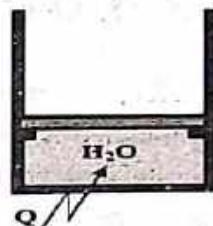
1. A 0.08 m^3 vessel contains liquid water and water vapor at 235°C . Find the mass of each if their volumes are equal. (Ans: 33.76 kg , 0.63 kg)
2. A 1.8 m^3 rigid tank contains steam at 220°C . One third of the volume is in the liquid phase and the rest is in the vapor phase. Determine
 - (a) the pressure of the steam
 - (b) the quality of the saturated mixture and
 - (c) the specific volume. (Ans: 2317.8 kPa , $0.02686 \text{ m}^3/\text{kg}$)
3. A 0.3 m^3 rigid vessel contains 5 kg of water at 150 kPa . Determine
 - (a) the temperature,
 - (b) the mass of each phase, and
 - (c) the specific enthalpy.

(Ans: 111.38°C , 0.2544 kg , 4.7456 kg , 580.45 kJ/kg)
4. Five kg of steam at 200 kPa occupies a volume of (a) 2.06 m^3 (b) 5.20 m^3 . Determine the temperature and quality in each case. (Ans: (a) 120.24°C , 0.4644 (b) 183.3°C , steam is superheated i.e. quality \Rightarrow meaningless)
5. A one liter closed vessel contains water at critical conditions. The vessel is cooled until its pressure drops to 1 MPa . Calculate the mass of water in the vessel and final dryness fraction.
 (Ans: Hint: $1 \text{ ltr} = 0.001 \text{ m}^3$, 0.3215 kg , 0.01026)
6. A rigid vessel contains 8 kg of water at 120°C . If 5 kg of the water is in the liquid form and the rest is in the vapor form. Determine
 - (a) the pressure in the vessel
 - (b) the volume of the vessel
 - (c) the volume of saturated liquid and the saturated vapor and
 - (d) the specific enthalpy of water.

(Ans: 198.48 kPa , 2.6819 m^3 , 0.0053 m^3 , 2.6766 m^3 , 1329.68 kJ/kg)
7. Two kg of water is contained in a rigid vessel of volume 0.5 m^3 . Heat is added until the temperature is 150°C . Determine:
 - (a) the final pressure,
 - (b) the mass of the vapor at the final state, and
 - (c) the volume of the vapor at the final state.

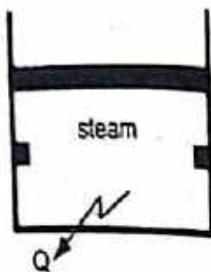
(Ans: 475.72 kPa , 1.271 kg , 0.499 m^3)
8. Five kg of H_2O is contained in a closed rigid container with an initial pressure and quality of 1000 kPa and 40% respectively. Heat is added to the system until the container holds only saturated vapor. Sketch the process on $P-v$ and $T-v$ diagrams and determine:

- (a) the volume of the container, and
 (b) the final pressure. (Ans: 0.3922 m^3 , 2551.87 kPa)
9. Steam is contained in a closed rigid container. Initially, the pressure and temperature of steam are 1500 kPa and 250°C respectively. The temperature drops as a result of heat transfer to surroundings. Determine the pressure at which condensation first occurs in bar and the fraction of total mass that has been condensed when the temperature reaches 100°C . What percentage of volume is occupied by saturated liquid at the final state?
 (Ans: 12.9339 bar , 0.91 , 0.6244%)
10. Ten kg of water in a piston cylinder device as shown in figure exists as saturated liquid vapor mixture at 100 kPa with the quality of 50%. It is now heated so that the volume triples. The mass of the piston is such that cylinder pressure of 200 kPa will float it.
- (a) Find the final temperature and volume of water.
 - (b) Find the work given out by water.
 - (c) Plot the process on $P - v$ and $T - v$ diagrams.
 (Ans: 829.23°C , 25.43 m^3 , 3390.69 kJ)
11. A piston cylinder device shown in figure contains 0.5 kg of water initially at a pressure of 1 MPa and 250°C . Heat is lost by the system until it becomes saturated liquid. Sketch the process on $P - v$ and $T - v$ diagrams and determine the work transfer.
 (Ans: -115.7365 kJ)
12. A piston cylinder device shown in figure below loaded with a linear spring initially contains steam at 200 kPa , 200°C and 0.5 m^3 . At this state linear spring is touching the piston but exerts no force on it. Heat is now slowly transferred to the steam causing the pressure and volume to rise to 400 kPa and 0.5785 m^3 respectively. Show the process on $P - v$ diagram with respect to saturation lines and determine
- (a) the final temperature and
 - (b) the work done by the steam.
 (Ans: 810.41°C , 23.5 kJ)

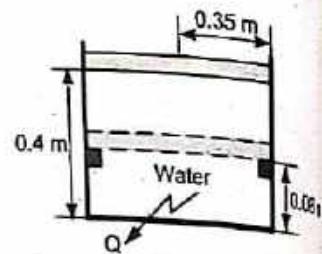


PROPERTIES OF PURE WORKING SUBSTANCES

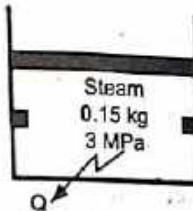
13. A piston cylinder device shown in figure below initially contains 0.15 kg steam at 3.0 MPa, 250°C. Now the steam loses heat to surroundings and piston moves down, hitting the stops at which point the cylinder contains saturated liquid water. The cooling continues until the cylinder contains water at 200°C. Sketch the process on $P - v$ and $T - v$ diagrams and determine (a) the final pressure and quality (b) the boundary work.
 (Ans: 1533.6 kPa, 0.0004, -35.96 kJ)



14. Water is contained in the frictionless piston cylinder device shown in figure. The water is cooled from an initial temperature of 300°C and pressure of 0.8 MPa to a final pressure of 0.6 MPa. Plot and label the process on $P - v$ and $T - v$ diagrams. Determine:
 (a) the mass of water
 (b) the total work done.
 (Ans: 0.475 kg, -98.4 kJ)



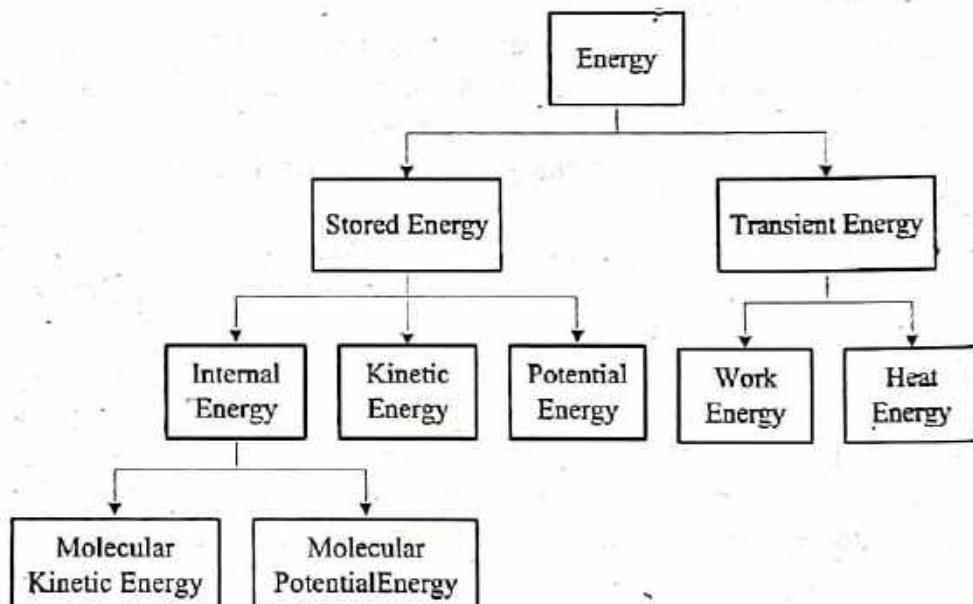
15. A piston cylinder device initially contains 0.15 kg steam at 3.0 MPa, superheated by 5°C. Now the steam loses heat to surroundings and piston moves down, hitting a set of stops at which point the cylinder contains saturated water vapor. The cooling continues until the cylinder contains water at 200°C. Determine (a) the final pressure and quality (if mixture) (b) boundary work.
 (Ans: 1533.6 kPa, 0.52, -0.5445 kJ)



Work and Heat

3.1 Chapter Highlights

- *Energy*: Energy of a system is defined as its capacity to perform work and it has different forms.
- The different forms of energy in thermodynamics are shown in chart below:



- Differences between *stored energy* and *transient energy* are:

Stored Energy	Transient Energy
1. Energy that remains within the system boundary as an inherent property of the system. 2. It can be defined for a state. 3. It is thermodynamic a property. 4. It is called a point or state function. 5. It is not a boundary phenomenon. 6. Its cyclic integral is zero. 7. It has single value for each equilibrium state. 8. For example: internal energy, kinetic energy and potential energy.	1. Energy that can cross the boundary of the system during a thermodynamic process. 2. It can be defined only for a process. 3. It is not a thermodynamic property. 4. It is called a path function. 5. It is a boundary phenomenon. 6. Its cyclic integral is non-zero. 7. It does not have single value for each equilibrium state. 8. For example: work energy, heat energy and electrical energy.

WORK AND HEAT

- **Internal energy:** It is defined as the energy of a system due to its molecular activities (motion and position). It is the summation of *molecular potential energy* and *molecular kinetic energy* of the molecules of the system. As the molecular activities increase with temperature, internal energy of the system increases with increase in temperature. The change in internal energy denoted by dU depends upon the change in temperature of the system dT as

$$dU = m c_v dT$$

where m is the mass and c_v is the specific heat capacity at constant volume.

- **Molecular potential energy:** It is the internal or microscopic form of energy that molecules possess as a result of their position in relative to one another. Latent energy, chemical energy and nuclear energy are the examples of the molecular potential energy.
- **Molecular kinetic energy:** It is the internal or microscopic form of energy due to the motion or velocity of the molecules of the system and it has different forms due to different types of motions of the molecules e.g. translational kinetic energy, rotational kinetic energy, and vibrational kinetic energy of the molecules.
- **Potential Energy:** It is defined as the external or macroscopic form of energy of a system for doing work by virtue of its position or elevation in a gravitational field and is expressed as

$$PE = mgz$$

where m is the mass of the system, g is the acceleration due to gravity and z is the elevation of the system.

- **Kinetic Energy:** It is defined as the external or macroscopic form of energy of a system for doing work by virtue of its velocity and is expressed as

$$KE = \frac{1}{2} m \bar{V}^2$$

where m is the mass of the system and \bar{V} is the velocity of the system.

- **Total energy:** It is the summation of all internal as well as external forms of energy of a system namely internal energy, potential energy and kinetic energy. Mathematically,

$$E = U + PE + KE = U + mgz + \frac{1}{2} m \bar{V}^2$$

- Energy can be transferred from one system to another or to surroundings in the form of work and heat. Hence, work and heat are energy interactions which bring about changes in the properties of the system.
- **Similarities** of work and heat are:
1. Both are forms of energy and are transient type.
 2. Both are path functions.
 3. Both have inexact differentials.
 4. Both are boundary phenomenon.
 5. Both are not thermodynamic properties.

WORK AND HEAT

- 6. Both are equal for a cyclic process and their cyclic integral is non-zero.
- 7. Both are physically equivalent and their unit is Joule.

➤ Differences between work transfer and heat transfer are:

Work Transfer	Heat Transfer
<ol style="list-style-type: none"> 1. It is the transfer of energy without transfer of mass because of any property difference (pressure, electrical and gravitational potential) other than temperature between the system and surroundings. 2. Work is high grade energy. 3. Work is ordered motion of molecules in one direction. 4. Work can be totally converted into heat. 5. Entropy of the system does not change during work transfer. 6. For the displacement work transfer displacement of the system boundary is necessary. 7. Work transfer to a system is taken as negative and work transfer from a system is taken as positive. 	<ol style="list-style-type: none"> 1. It is the transfer of energy without transfer of mass because of temperature difference between the system and surroundings. 2. Heat is low grade energy. 3. Heat is random motion of molecules. 4. Heat cannot be totally converted into work. 5. Entropy of the system increases during heat transfer. 6. Displacement of the system boundary is not necessary for heat transfer. 7. Heat transfer to a system is taken as positive and heat transfer from a system is taken as negative.

- *Sign convention:* Heat supplied to the system from the surroundings is taken as positive and heat lost from the system to the surroundings is taken as negative. Whereas, work done by the system is taken as positive and work done on the system is taken as negative.
- *Mechanical definition of work:* Work is said to be done if a system is displaced to certain distance (s) by applying a force (F) in the direction of the force. Mathematically,

$$W_{1-2} = \int_1^2 F \cdot ds$$

➤ *Displacement work:* The magnitude of work transfer during any quasi-equilibrium process is given by the area under the process curve on $P-V$ diagram and calculated by the expression

$$W_{1-2} = \int_1^2 P \cdot dV$$

➤ Expressions for work transfer for different processes:

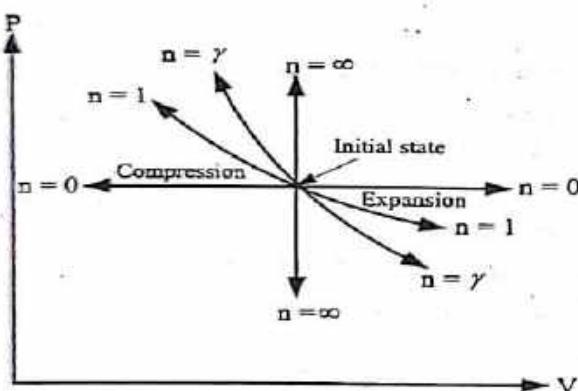
1. For an *isochoric process*: $W_{1-2} = 0$
2. For an *isobaric process*: $W_{1-2} = P(V_2 - V_1)$

WORK AND HEAT

3. For isothermal process: $W_{1-2} = P_1 V_1 \ln(V_2/V_1)$
 4. For an adiabatic process: $W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-\gamma}$ where γ is the specific heats ratio.
 5. For a polytropic process: $W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n}$ where n is the polytropic index.
- A polytropic process represents common thermodynamic processes for different values of polytropic index n as given below:

Polytropic index	Relation	Process
(i) When $n = 0$	$P = \text{constant}$	Constant pressure or isobaric
(ii) When $n = 1$	$PV = \text{constant}$	Constant temperature or isothermal
(iii) When $n = \infty$	$V = \text{constant}$	Constant volume or isochoric
(iv) When $n = \gamma$	$PV^\gamma = \text{constant}$	Adiabatic

This is illustrated on a P - V diagram in figure below.



- *Shaft work*: Work energy interaction with a thermodynamic system through a rotating shaft is known as shaft work and it is given by

$$W_{\text{shaft}} = 2\pi n t$$
 where t is the torque.
- *Spring force*: It is the force applied to a linear elastic spring to cause the certain displacement and is expressed as

$$F = kx$$
- *Spring work*: It is the work done to stretch the spring or an elastic object and is given by

$$W_{\text{spring}} = \frac{1}{2} kx^2$$

where k is the spring constant and x is the displacement of the spring.

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- *Electrical work:* It is the work done on a charged particle by an electric field and is calculated as

$$W_{elec} = \int_1^2 VIdt$$

where I is the current and V is the potential difference.

- *Paddle wheel work:* It is the work done on the fluid system due to the rotation of a paddle wheel and is given by

$$W_{paddle} = \int_1^2 \tau d\theta = \int_1^2 mgdz = mg(z_2 - z_1)$$

where τ is the torque and θ is the angular displacement of the paddle.

- *Flow work:* It is the work done by a fluid when it flows in or out of any passage, e.g. through inlet or outlet of an open system and is derived as

$$W_{flow} = PV$$

- *Power:* It is defined as a rate of energy transfer and can be classified into thermal power and mechanical power. *Mechanical power* is defined as the rate of work transfer. Mathematically,

$$\dot{W} = \lim_{\Delta t \rightarrow 0} \frac{\delta W}{\Delta t}$$

Thermal power is defined as the rate of heat transfer. Mathematically,

$$\dot{Q} = \lim_{\Delta t \rightarrow 0} \frac{\delta Q}{\Delta t}$$

- *Equation of state* for an ideal gas is given by $PV = mRT$ where R is the characteristic gas constant.

3.2 Solved Numerical Problems

Problem 3.1.

A gas is contained in a piston cylinder device initially at a pressure of 150 kPa and a volume of 0.04 m³. Calculate the work done by the gas when it undergoes the following processes to a final volume of 0.1 m³.

- (a) constant pressure
- (b) constant temperature
- (c) $PV^{1.35} = \text{constant}$

Solution: Given.

Initial gas pressure, $P_1 = 150 \text{ kPa}$

Initial gas volume, $V_1 = 0.04 \text{ m}^3$

Final gas volume, $V_2 = 0.1 \text{ m}^3$

- (a) For constant pressure process

Final gas pressure, $P_2 = P_1 = P = 150 \text{ kPa}$

Work done by the gas, $W_{1-2} = P(V_2 - V_1) = 150(0.1 - 0.04) = 9 \text{ kJ}$

- (b) For constant temperature process

Work done by the gas, $W_{1-2} = P_1 V_1 \ln(V_2/V_1) = 150 \times 0.04 \ln(0.1/0.04) = 5.4977 \text{ kJ}$

- (c) For $PV^{1.35} = \text{constant}$ process

It is a polytropic process with polytropic index, $n = 1.35$.

Using pressure – volume relationship to find the final gas pressure, P_2

$$P_1 V_1^{1.35} = P_2 V_2^{1.35}$$

$$\Rightarrow P_2 = P_1 (V_1/V_2)^{1.35} = 150(0.04/0.1)^{1.35} = 43.538 \text{ kPa}$$

$$\text{Work done, } W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{43.538 \times 0.1 - 150 \times 0.04}{1-1.35} = 4.703 \text{ kJ}$$

Problem 3.2.

A gas undergoes a polytropic process from an initial state of 500 kPa and 0.02 m³ to a final state of 100 kPa and 0.05 m³. Determine the work transfer.

Solution: Given,

Initial gas pressure, $P_1 = 500 \text{ kPa}$

Initial gas volume, $V_1 = 0.02 \text{ m}^3$

Final gas pressure, $P_2 = 100 \text{ kPa}$

Final gas volume, $V_2 = 0.05 \text{ m}^3$

Pressure – volume relationship for a polytropic process is given by

$$PV^n = \text{constant} \quad \text{where } n \text{ is the polytropic index.}$$

Using pressure – volume relationship to find the value of n ,

$$P_1 V_1^n = P_2 V_2^n \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^n$$

Taking logarithmic of both sides, we get

$$n = \frac{\ln(P_2/P_1)}{\ln(V_1/V_2)} = \frac{\ln(100/500)}{\ln(0.02/0.05)} = 1.7565$$

Now, work transfer,

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{100 \times 0.05 - 500 \times 0.02}{1-1.7565} = 6.609 \text{ kJ}$$

Problem 2.3.

Air undergoes three processes in series to form a cycle:

Process 1-2: compression with $PV^{1.3} = \text{constant}$ from $P_1 = 100 \text{ kPa}$, $V_1 = 0.04 \text{ m}^3$ to $V_2 = 0.02 \text{ m}^3$

Process 2-3: constant pressure process to $V_3 = V_1$

Process 3-1: constant volume

Sketch the process on P-V diagram and determine the total work transfer.

Solution: Given,

Process 1 – 2: pressure – volume relationship is $PV^{1.3} = \text{constant}$ with polytropic index, $n = 1.3$.

Initial pressure, $P_1 = 100 \text{ kPa}$

Initial volume, $V_1 = 0.04 \text{ m}^3$

Final volume, $V_2 = 0.02 \text{ m}^3$

Process 2 – 3: Constant pressure process

⇒ Initial pressure, $P_2 = \text{final pressure, } P_3$

Final volume, $V_3 = V_1 = 0.04 \text{ m}^3$

Process 3 – 1: Constant volume process ⇒ Initial volume = final volume
⇒ $V_3 = V_1 = 0.04 \text{ m}^3$

Process 1 – 2:

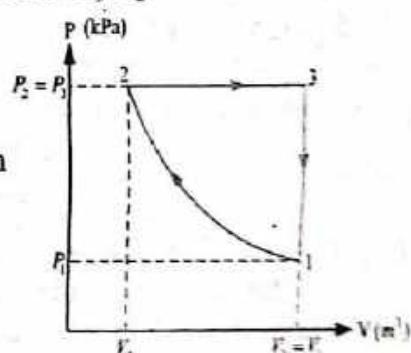
Using pressure – volume relationship to find final pressure, P_2

$$P_1 V_1^{1.3} = P_2 V_2^{1.3}$$

$$\Rightarrow P_2 = P_1 (V_1/V_2)^{1.3} = 100 (0.04/0.02)^{1.3} = 246.23 \text{ kPa}$$

Work transfer in a polytropic process 1 - 2 is given by

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{246.23 \times 0.02 - 100 \times 0.04}{1-1.3} = -3.082 \text{ kJ}$$



The negative sign indicates that the work is done on the system due to compression.

Process 2 – 3: constant pressure process

$$P_3 = P_2 = P = 246.23 \text{ kPa}$$

$$V_3 = V_1 = 0.04 \text{ m}^3$$

Work transfer in a constant pressure process 2 - 3 is given by

$$W_{2-3} = P(V_3 - V_2) = 246.23(0.04 - 0.02) = 4.9246 \text{ kJ}$$

Process 3 - 1: constant volume process

$$V_3 = V_1 = 0.04 \text{ m}^3 \Rightarrow dV = 0$$

Work transfer in a constant volume process 2 - 3 is

$$W_{3-1} = \int_1^2 P dV = 0$$

Hence, total work transfer of the cycle,

$$W_t = W_{1-2} + W_{2-3} + W_{3-1} = -3.082 + 4.9246 + 0 = 1.8426 \text{ kJ}$$

Problem 3.4.

Argon (1 kg) contained in a piston cylinder device shown in figure is initially at a pressure of 500 kPa and a temperature of 70°C. Heat is added until the final temperature is 600°C and a pressure of 1 MPa is required to lift the piston from the stops. Sketch the process on P-V and T-V diagrams and determine the total work transfer.

[Take R=208 J/kgK]



Solution: Given,

Mass of Argon, $m = 1 \text{ kg}$

Initial pressure, $P_1 = 500 \text{ kPa} = 500000 \text{ Pa}$

Initial temperature, $T_1 = 70^\circ\text{C} + 273 = 343 \text{ K}$

Final temperature, $T_f = 600^\circ\text{C}$

Lifting pressure of piston, $P_{lift} = 1 \text{ MPa} = 1000 \text{ kPa}$

Characteristic gas constant, $R = 208 \text{ J/kgK}$

State 1:

$P_1 = 500 \text{ kPa} = 500000 \text{ Pa}$

$T_1 = 70^\circ\text{C} + 273 = 343 \text{ K}$

Initial volume of argon, V_1 is determined by using equation of state for an ideal gas as:

$$V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 208 \times 343}{500000} = 0.142688 \text{ m}^3$$

$P - V$ and $T - V$ diagrams are shown in figures below. During initial state of heating, the piston remains stationary till the system pressure reaches the lifting pressure of the piston. So, process 1 - 2 is constant volume (isochoric) heating. Hence, state 2 is defined as

State 2:

$V_2 = V_1 = 0.142688 \text{ m}^3$ and $P_2 = P_{lift} = 1 \text{ MPa} = 1000 \text{ kPa}$

Using equation of state, temperature at state 2, T_2 is calculated as

$$T_2 = \frac{P_2 V_2}{mR} = \frac{1000 \times 10^3 \times 0.142688}{1 \times 208} = 686 \text{ K} = 413^\circ\text{C}$$

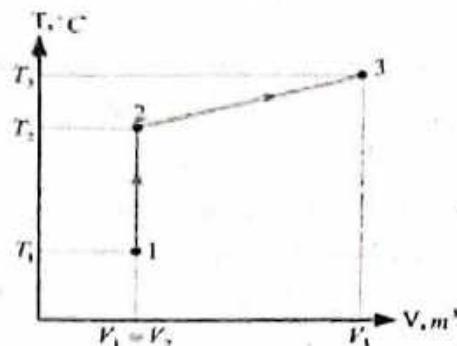
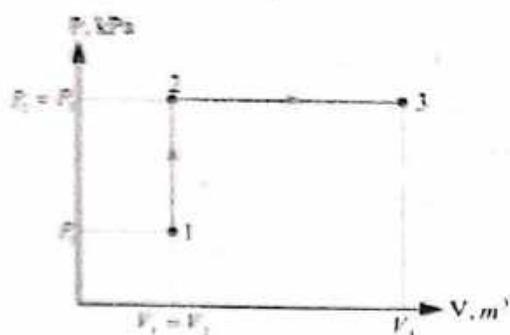
Here T_1 is less than the final temperature T_3 , hence the system should be further heated. After reaching the lifting pressure, further heating causes displacement of the piston to upward direction with a constant lifting pressure (isobaric process 2 – 3) until the system reaches final temperature. Hence, final state 3 is defined as

State 3:

$$P_1 = P_2 = 1000 \text{ kPa}, T_1 = 600^\circ\text{C} = 600 + 273 = 873 \text{ K}$$

Using equation of state, volume at final state 3, V_3 is determined as

$$V_3 = \frac{mRT_3}{P_3} = \frac{1 \times 208 \times 873}{1000 \times 10^3} = 0.181584 \text{ m}^3$$



Total work transfer during the process is given by

$$W_t = W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = 1000(0.181584 - 0.142688) = 38.896 \text{ kJ}$$

Problem 3.5.

A piston cylinder arrangement shown in figure contains gas initially at $P_1 = P_{atm} = 100 \text{ kPa}$ and $T_1 = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$. Piston with a cross sectional area of 0.01 m^2 has a mass of 50 kg and is initially resting on the bottom stops. Heat is added to the system until it touches the upper stops.

- (a) Sketch the process on P-V and T-V diagrams.
- (b) Determine the total work transfer.
- (c) Determine the final temperature of the gas. [Take $g = 9.81 \text{ m/s}^2$]

Solution: Given,

Initial pressure, $P_1 = P_{atm} = 100 \text{ kPa}$

Initial temperature, $T_1 = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$

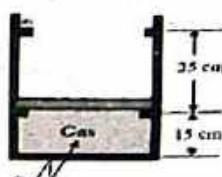
Area of piston, $A_P = 0.01 \text{ m}^2$

Mass of piston, $m_P = 50 \text{ kg}$

State 1:

$$P_1 = 100 \text{ kPa}, T_1 = 293 \text{ K}$$

$$\text{Initial volume, } V_1 = A_P \times h = 0.01 \times 15 \times 10^{-2} = 0.0015 \text{ m}^3$$



P – V and T – V diagrams are shown in figures below. During initial state of heating, the piston remains stationary till the system pressure reaches the lifting pressure of the piston. So, process 1 – 2 is constant volume (isochoric) heating. Hence, state 2 is defined as

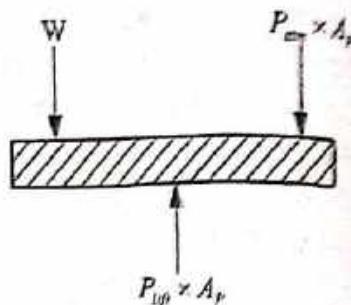
State 2:

$$V_2 = V_1 = 0.0015 \text{ m}^3 \text{ and } P_2 = P_{lift}$$

To calculate lifting pressure, referring to the free body diagram (FBD) of the piston, we can write equation for the pressure inside the cylinder as

$$P_{lift} \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_{lift} = P_{atm} + \frac{m_p \times g}{A_p} = 100 \times 10^3 + \frac{50 \times 9.81}{0.01} \\ = 149.05 \text{ kPa} = P_2$$



FBD of Piston

Using Charle's law for constant volume, temperature at state 2, T_2 is calculated as

$$T_2 = \frac{P_2}{P_1} \times T_1 = \frac{149.05 \times 293}{100} = 436.716 \text{ K} = 163.776^\circ\text{C}$$

Heat is added to the system until the piston touches the upper stops which causes displacement of the piston to upward direction with a constant lifting pressure (isobaric process 2 – 3) till it just touches the upper stops. Hence, final state 3 is defined as

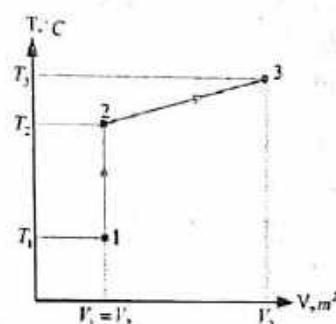
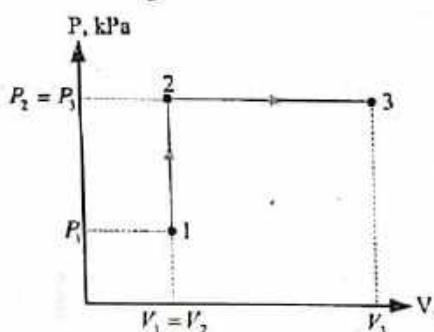
State 3:

$$P_3 = P_2 = 149.05 \text{ kPa}$$

Volume at state 3 is equal to final volume, so from the system diagram, $V_3 = A_p \times h_i = 0.01(0.15 + 0.25) = 0.004 \text{ m}^3$

Temperature at state 3 is the final temperature which can be determined by using Charle's law for constant pressure, so

$$T_f = T_3 = \frac{V_3}{V_2} \times T_2 = \frac{0.004}{0.0015} \times 436.716 = 1164.576 \text{ K} = 891.576^\circ\text{C}$$



Total work transfer,

$$W_t = W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = 149.05 \times 1000 (0.004 - 0.0015) \\ = 372.625 \text{ J}$$

Problem 3.6.

A piston cylinder device shown in figure contains 1 kg of Nitrogen initially at a pressure of 250 kPa and a temperature of 500°C. Heat is lost from the system till its temperature reaches 25°C. Sketch the process on P-V and T-V diagrams and determine the final pressure and the total work transfer. [Take R = 297 J/kg K]

Solution: Given,

Mass of gas, $m = 1 \text{ kg}$

Initial pressure, $P_i = 250 \text{ kPa}$

$$\begin{aligned}\text{Initial temperature, } T_i &= 500^\circ\text{C} = 500 + 273 \\ &= 773 \text{ K}\end{aligned}$$

$$\text{Final temperature, } T_f = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$$

State 1:

Initial volume is determined by using equation of state for ideal gas as

$$V_i = \frac{mRT_i}{P_i} = \frac{1 \times 297 \times 773}{250 \times 1000} = 0.918324 \text{ m}^3$$

$P - V$ and $T - V$ diagrams are shown in figures below. During initial state of heat loss, the piston moves downward with constant initial pressure until it just reaches stops located at the half of the initial volume. So, process 1 - 2 is constant pressure (isobaric) cooling. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_i = 250 \text{ kPa}$

$$\text{Volume at state 2, } V_2 = \frac{1}{2} V_i = \frac{0.918324}{2} = 0.459162 \text{ m}^3$$

Temperature at state 2, T_2 is determined by using equation of state for ideal gas as

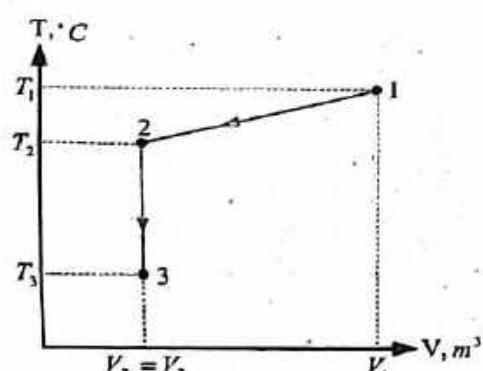
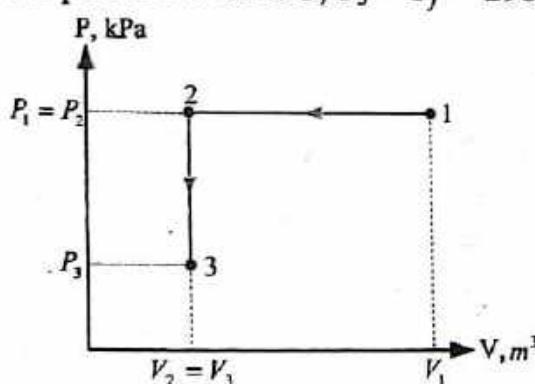
$$T_2 = \frac{P_2 V_2}{mR} = \frac{250 \times 1000 \times 0.459162}{1 \times 297} = 386.5 \text{ K} = 113.5^\circ\text{C}$$

Here T_2 is greater than the final temperature T_f hence the system should be further cooled. However, further cooling causes no displacement of the piston due to presence of stops. Therefore, cooling process 2 - 3 occurs with a constant volume (isochoric) until the system cooled up to given final temperature. Hence, final state 3 is defined as

State 3:

Volume at state 3, $V_3 = V_2 = 0.459162 \text{ m}^3$

Temperature at state 3, $T_3 = T_f = 298 \text{ K}$



Pressure at state 3, P_3 is determined by using equation of state for an ideal gas as

$$P_1 = \frac{mRT_1}{V_1} = \frac{1 \times 297 \times 298}{0.459162} = 192755.5 \text{ Pa} = 192.755 \text{ kPa}$$

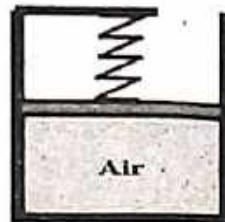
Total work transfer,

$$\begin{aligned} W_1 &= W_{1,2} + W_{2,3} = P_1(V_1 - V_f) + 0 = 250 \times 1000 (0.459162 - 0.918324) \\ &= 114790.5 \text{ Pa} = 114.79 \text{ kPa} \end{aligned}$$

Problem 3.7.

A piston cylinder arrangement loaded with a linear spring ($k = 20 \text{ kN/m}$) as shown in figure contains air. Spring is initially unstretched and undergoes a compression of 40 mm during a process. If the mass of the piston is 80 kg and piston diameter is 0.1 m, determine the total work transfer.

[Take $P_{atm} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$]



Solution: Given,

Spring constant, $k = 20 \text{ kN/m}$

Displacement of spring, $x = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$

Mass of piston, $m = 80 \text{ kg}$

Diameter of piston, $D_p = 0.1 \text{ m}$

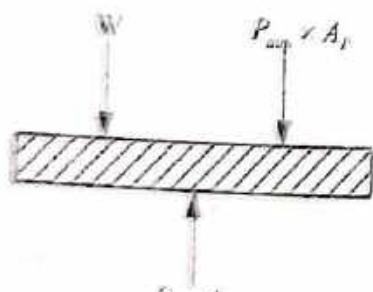
Atmospheric pressure, $P_{atm} = 100 \text{ kPa}$

$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi (0.1)^2}{4} = 0.007854 \text{ m}^2$$

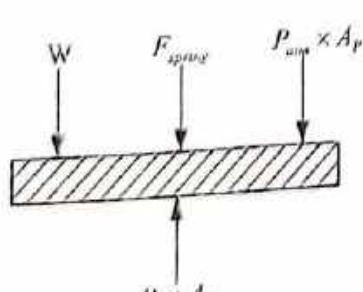
State 1:

To calculate pressure P_1 at initial state 1, referring to the free body diagram (FBD) of the piston at initial state, we can write equation for the pressure inside the cylinder as

$$\begin{aligned} P_1 \times A_p &= P_{atm} \times A_p + W \\ \Rightarrow P_1 &= P_{atm} + \frac{m_p \times g}{A_p} = 100 \times 10^3 + \frac{80 \times 9.81}{0.007854} = 199923.6 \text{ Pa} = 199.924 \text{ kPa} \end{aligned}$$



FBD at initial state 1



FBD at final state 2

State 2:

To calculate pressure P_2 at state 2 after compression of the spring, referring to the free body diagram (FBD) of the piston at final state, we can write equation for the pressure inside the cylinder as

$$P_2 \times A_p = P_{atm} \times A_p + W + F_{spring}$$

$$\Rightarrow P_2 = P_{atm} + \frac{m_p \times g}{A_p} + \frac{kx}{A_p} = P_1 + \frac{kx}{A_p}$$

$$= 199923.6 + \frac{20 \times 10^3 \times 40 \times 10^{-3}}{0.007854} = 301782.525 \text{ Pa}$$

$$= 301.782 \text{ kPa}$$

Work transfer,

$W_i = W_{1-2}$ = Area under process curve 1 - 2 on P - V diagram

$$= P_1(V_2 - V_1) + \frac{1}{2}(P_2 - P_1)(V_2 - V_1)$$

$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)(A_p \times x)$$

$$= \frac{1}{2}(199.924 + 301.782) \times 10^3 \times (0.007854 \times 40 \times 10^{-3})$$

$$= 78.81 \text{ J}$$

Problem 3.8

A piston cylinder arrangement with two set of stops is restrained by a linear spring ($k = 12 \text{ kN/m}$) as shown in figure. The cross sectional area of the piston is 0.05 m^2 . The initial pressure of the gas is 500 kPa and the pressure required to lift the piston is 1000 kPa . Heat is supplied to the gas until its pressure reaches 6000 kPa . Sketch the process on P - V diagram and determine the total work transfer.

Solution: Given,

Spring constant, $k = 12 \text{ kN/m}$

Area of piston, $A_p = 0.05 \text{ m}^2$

Initial pressure, $P_1 = 500 \text{ kPa}$

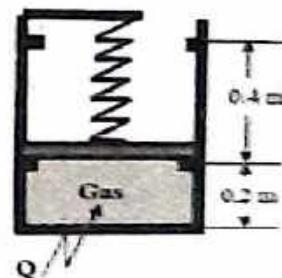
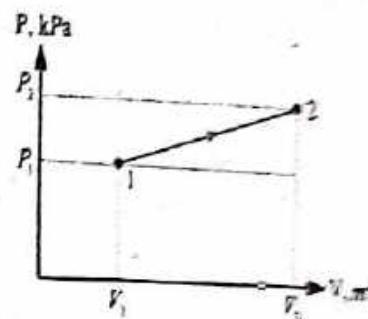
Lifting pressure of piston, $P_{lift} = 1000 \text{ kPa}$

Final pressure, $P_f = 6000 \text{ kPa}$

Initial State 1:

$P_1 = 500 \text{ kPa}$

Initial volume, $V_1 = A_p \times h = 0.05 \times 0.2 = 0.01 \text{ m}^3$



$P-V$ diagram is shown in figure below. During initial state of heating, the piston remains stationary till the system pressure reaches the lifting pressure of the piston. So, process 1 - 2 is constant volume (isochoric) heating. Hence, state 2 is defined as

State 2:

Volume at state 2, $V_2 = V_1 = 0.01 \text{ m}^3$

Pressure at state 2, $P_2 = P_{atm} = 1000 \text{ kPa}$

After reaching the lifting pressure, further heating causes the piston to move upwards until the piston reaches the upper stops (process 2 - 3) thereby pressure, temperature and volume increase. Hence, state 3 is defined as

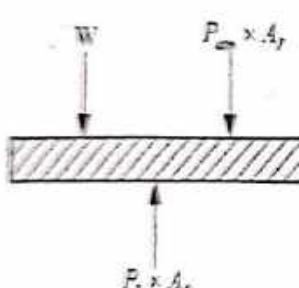
State 3:

Volume of state 3, $V_3 = V_1 = A_p \times h_t = 0.05(0.2 + 0.4) = 0.03 \text{ m}^3$

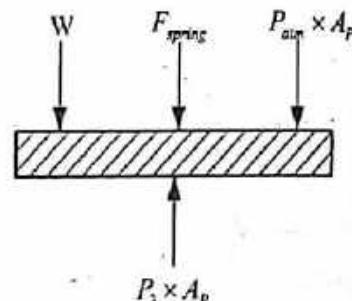
Pressure at state 3, $P_3 = ?$

Referring to the free body diagram (FBD) of the piston at state 2, we can write equation for the pressure inside the cylinder as

$$P_2 \times A_p = P_{atm} \times A_p + W \Rightarrow P_2 = P_{atm} + W/A_p$$



FBD at state 2

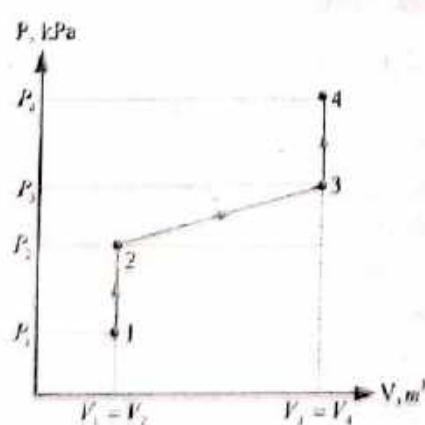


FBD at state 3

Again, referring to the free body diagram (FBD) of the piston at state 3, equation for the pressure inside the cylinder is

$$\begin{aligned} P_3 \times A_p &= P_{atm} \times A_p + W + F_{spring} \\ &\Rightarrow P_3 = P_{atm} + W/A_p + F_{spring}/A_p = P_2 + kx/A_p \\ &= 1000 \times 10^3 + \frac{12 \times 10^3 \times 0.4}{0.05} = 1096000 \text{ Pa} = 1096 \text{ kPa} \end{aligned}$$

Here P_3 is less than the final pressure P_4 , hence the system should be further heated. However, further heating causes no displacement of the piston due to presence of stops. Therefore, heating process 3 - 4 occurs with a constant volume (isochoric) until the system heated up to given final pressure. Hence, final state 4 is defined as



State 4:

 Pressure at state 4, $P_4 = 6000 \text{ kPa}$

 Volume at state 4, $V_4 = V_f = 0.03 \text{ m}^3$

Total work transfer,

$$W_t = W_{1,2} + W_{2,3} + W_{3,4} \quad (W_{2,3} \text{ is area under process curve } 2-3)$$

$$= 0 + \frac{1}{2}(P_2 + P_3)(V_3 - V_2) + 0 = \frac{1}{2}(1000 + 1096)(0.03 - 0.01) = 20.96 \text{ kJ}$$

Problem 3.9.

Air (0.01 kg) is contained in a piston cylinder device restrained by a linear spring ($k = 500 \text{ kN/m}$) as shown in figure below. Spring initially touches the piston but exerts no force on it. Heat is added to the system until the piston is displaced upward by 80 mm. Determine:

- (a) the temperature at which piston leaves the stops
- (b) work done by the air

[Take $R = 287 \text{ J/kg.K}$, $K, P_{atm} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$]

Solution: Given,

 Mass of air, $m = 0.01 \text{ kg}$

 Spring constant, $k = 500 \text{ kN/m}$

 Displacement of spring, $x = 80 \text{ mm} = 0.08 \text{ m}$

 Mass of piston, $m_p = 400 \text{ kg}$

 Diameter of piston, $D_p = 250 \text{ mm} = 0.25 \text{ m}$

 Height of position of stops, $h = 200 \text{ mm} = 0.2 \text{ m}$

 Atmospheric pressure, $P_{atm} = 100 \text{ kPa}$

$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi (0.25)^2}{4} = 0.049087 \text{ m}^2$$

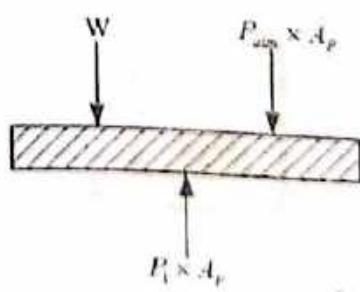
State 1: (When the piston just leaves the stops)

$$\text{Volume at state 1, } V_1 = A_p \times h = 0.049087 \times 0.2 = 0.009817 \text{ m}^3$$

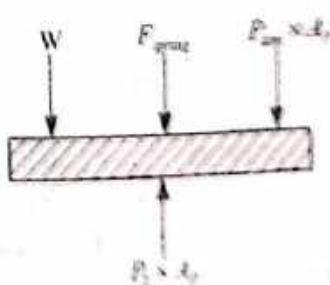
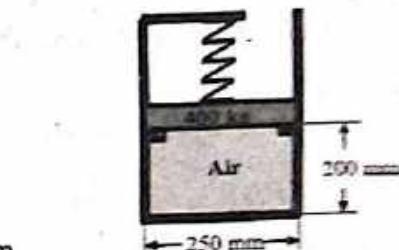
 To calculate pressure P_1 at state 1, referring to the free body diagram (FBD) of the piston at state 1, we can write equation for the pressure inside the cylinder as

$$P_1 \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_1 = P_{atm} + \frac{m_p g}{A_p} = 100 \times 10^3 + \frac{400 \times 9.81}{0.049087} = 179939.7 \text{ Pa} = 179.94 \text{ kPa}$$



FBD at state 1



FBD at state 2

Temperature T_1 at which the piston leaves the stops can be calculated by using equation of state as

$$T_1 = \frac{PV_1}{mR} = \frac{179.94 \times 10^3 \times 0.009817}{0.01 \times 287} = 615.5 \text{ K} = 342.5^\circ\text{C}$$

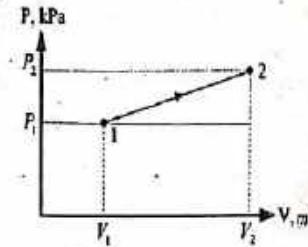
Heating causes the piston to move upwards until the piston is displaced upward by 80 mm (process 1 - 2) thereby pressure, temperature and volume increase. Hence, state 2 is defined as

State 2:

Volume at state 2, $V_2 = A_p \times h_2 = 0.049087 (0.2 + 0.08) = 0.013744 \text{ m}^3$

To calculate pressure P_2 at state 2, referring to the free body diagram (FBD) of the piston at state 2, equation for the pressure inside the cylinder is

$$\begin{aligned} P_2 \times A_p &= P_{atm} \times A_p + W + F_{spring} \\ \Rightarrow P_2 &= P_{atm} + W/A_p + F_{spring}/A_p = P_1 + kx/A_p \\ &= 179.94 \times 10^3 + \frac{500 \times 10^3 \times 0.08}{0.049087} \\ &= 994819.7 \text{ Pa} = 994.82 \text{ kPa} \end{aligned}$$



Total work transfer,

$$\bar{W}_1 = \bar{W}_{1,2} \quad (\bar{W}_{1,2} \text{ is area under process curve } 1 - 2)$$

$$\begin{aligned} &= \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(179.94 + 994.82)(0.013744 - 0.009817) \\ &= 2.307 \text{ kJ} \end{aligned}$$

Problem 3.10.

A gas enclosed by a piston shown in figure below starts to expand due to heating. The initial movement of 0.2 m is restrained by a fixed mass of 30 kg and the final 0.05 m is restrained both by the mass and a spring of stiffness 10 kN/m. The cross sectional area of the piston is 0.15 m^2 and the atmospheric pressure is 100 kPa.

(a) Neglecting the mass of the spring and the piston, sketch a P-V diagram of the process.

(b) Calculate the work during the initial 0.2 m movement.

(c) Calculate the total work done.

Solution: Given,

Stiffness of spring, $k = 10 \text{ kN/m}$

Area of piston, $A_p = 0.15 \text{ m}^2$

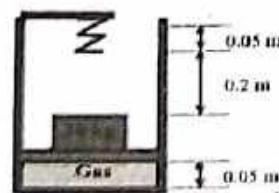
Atmospheric pressure, $P_{atm} = 100 \text{ kPa}$

Additional mass on piston, $m = 30 \text{ kg}$

P - V diagram is shown in figure below.

Initial state 1:

$$\text{Volume at state 1, } V_1 = A_p \times h = 0.15 \times 0.05 = 0.0075 \text{ m}^3$$



WORK AND HEAT

To calculate pressure P_1 at state 1, referring to the free body diagram (FBD) of the piston at state 1, we can write equation for the pressure inside the cylinder as

$$P_1 \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_1 = P_{atm} + \frac{mg}{A_p} = 100 \times 10^3 + \frac{30 \times 9.81}{0.15} = 101962 \text{ Pa} = 101.96 \text{ kPa}$$

Heating of gas causes the piston to move upwards with constant pressure until it touches the spring (isobaric process 1 – 2) while both temperature and volume increase. Hence, state 2 is defined as

State 2:

Pressure at state 2, $P_2 = P_1 = 101.96 \text{ kPa}$

Volume at state 2, $V_2 = A_p \times h_t = 0.15 \times (0.05 + 0.2) = 0.0375 \text{ m}^3$

(After moving 0.2 m from system diagram)

Now, work done during initial 0.2 m movement with constant pressure,

$$W_{1-2} = P_1(V_2 - V_1) = 101.96 (0.0375 - 0.0075) = 3.0588 \text{ kJ}$$

When the system is further heated, the spring gets compressed with the displacement 0.05 m (process 2 - 3). This increases the pressure of the system as well. Hence, state 3 is defined as

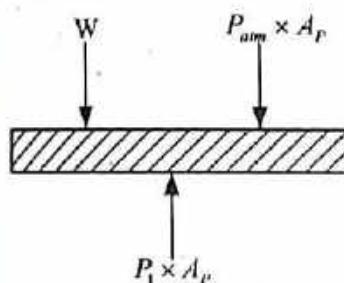
State 3:

Volume at state 3, $V_3 = A_p \times h_t = 0.15 (0.05 + 0.2 + 0.05) = 0.045 \text{ m}^3$

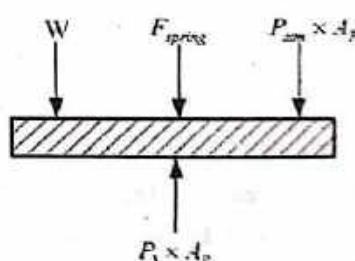
To calculate pressure P_3 at state 3, referring to the free body diagram (FBD) of the piston at state 3, equation for the pressure inside the cylinder is

$$P_3 \times A_p = P_{atm} \times A_p + W + F_{spring}$$

$$\Rightarrow P_3 = P_{atm} + W/A_p + F_{spring}/A_p = P_1 + kx/A_p \\ = 101.96 \times 10^3 + \frac{10 \times 10^3 \times 0.05}{0.15} = 105293.33 \text{ Pa} = 105.293 \text{ kPa}$$



FBD at state 1



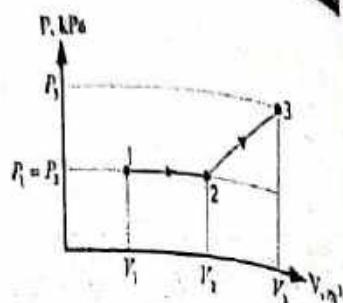
FBD at state 3

Then, work done during final 0.05 m movement is found out by evaluating the area under the process curve 2 – 3 on P – V diagram,

$$W_{2-3} = \frac{1}{2} (P_2 + P_3)(V_3 - V_2) \\ = 0.5 \times (101.96 + 105.293) (0.045 - 0.0375) \\ = 777.2 \text{ J}$$

Total work done by the gas is given by

$$W_1 = W_{1,2} + W_{2,3} = 3.0588 \times 10^3 + 777.2 \\ = 3836 \text{ J} = 3.836 \text{ kJ}$$



Problem 3.11.

A piston cylinder device containing a fluid has a stirrer as shown in figure below. The piston is frictionless and it is held down against the fluid due to atmospheric pressure of 101.3 kPa. The stirrer is rotated 9500 revolutions with an average torque against the fluid of 1.25 Nm. Meanwhile the piston of 0.65 m diameter moves out 0.6 m. Find the net-work transfer for the system.

Solution: Given,

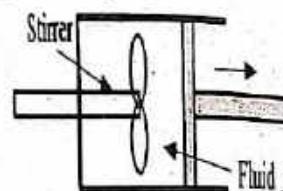
Atmospheric pressure, $P_{atm} = 101.3 \text{ kPa}$

No of rotations, $n = 9500$ revolutions

Torque, $\tau = 1.25 \text{ Nm}$

Diameter of piston, $D_p = 0.65 \text{ m}$

Displacement of piston, $L = 0.6 \text{ m}$



$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi (0.65)^2}{4} = 0.33183 \text{ m}^2$$

Here, it should be noted that there are two works, namely work done by the stirrer on the fluid system and work done by the fluid system on the atmosphere due to the piston displacement. Hence, the net work transfer for the system is the difference between these two works.

Work done by stirrer, $W_{stirr} = 2\pi n \tau = 2\pi \times 9500 \times 1.25 = 74612.825 \text{ J} = 74.613 \text{ kJ}$

Work done by piston, $W_p = F \times L = P_{atm} \times A_p \times L = 101.3 \times 0.33183 \times 0.6$

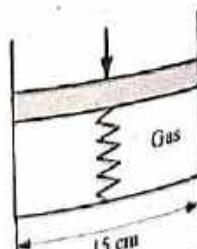
$$= 20.1687 \text{ kJ}$$

Now, net work transfer, $W_{net} = W_p - W_{stirr} = 20.1687 - 74.613 = -54.444 \text{ kJ}$

The negative sign indicates that the net work is done on the system.

Problem 3.12.

A weightless and frictionless piston moves 10 cm against an ideal gas contained in a cylinder that has an inside diameter of 15 cm. A spring is also within the cylinder positioned as shown in the figure. The spring exerts no force on the piston in the initial position, but is touching the piston. The spring has a spring constant of $2 \times 10^4 \text{ N/m}$. During the motion, the gas in the cylinder is maintained at 1 atm.



- (a) How much work has done by the piston?
- (b) What fraction of the work is done in compressing the gas?

- (c) What fraction of total work done by the piston is done in first 5 cm of piston travel?

Solution: Given,

Displacement of piston, $x = 10 \text{ cm} = 0.1 \text{ m}$

Diameter of piston, $D_p = 15 \text{ cm} = 0.15 \text{ m}$

Spring constant, $k = 2 \times 10^4 \text{ N/m}$

Gas Pressure, $P = 1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$

$$\text{Area of piston, } A_p = \frac{\pi D_p^2}{4} = \frac{\pi (0.15)^2}{4} = 0.01767 \text{ m}^2$$

Displaced volume, $V_2 - V_1 = A_p \times x = 0.01767 \times 0.1 = 0.001767 \text{ m}^3$

Here, it should be noted that the work done by the piston consists of two works, namely work done against compressing the gas and work done against compressing the spring.

$$\begin{aligned}\text{The work done against compressing the gas, } W_C &= \int_1^2 P dV = P(V_2 - V_1) \\ &= 1.01325 \times 10^5 \times 0.001767 \\ &= 179.041 \text{ J}\end{aligned}$$

The work done against compressing the spring,

$$W_{\text{spring}} = \int_1^2 F_{\text{spring}} dx = \int_1^2 kx dx = \frac{1}{2} kx^2 = 0.5 \times 2 \times 10^4 \times (0.1)^2 = 100 \text{ J}$$

(As $x_1 = 0$ when $F_{\text{spring}} = 0$)

Hence, the work done by the piston,

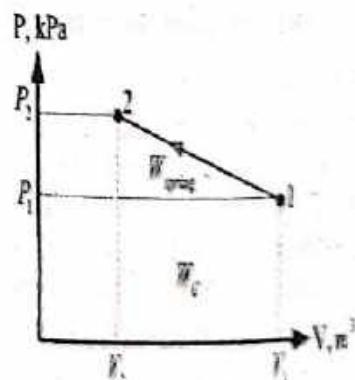
$$W = W_C + W_{\text{spring}} = 179.041 + 100 = 279.041 \text{ J}$$

Fraction of work done in compressing the gas,

$$W_1 = \frac{W_C}{W} = \frac{179.041}{279.041} = 0.64163 = 64.163\%$$

Fraction of work done by piston during first $x = 5 \text{ cm} = 0.05 \text{ m}$ travel,

$$\begin{aligned}W_2 &= \frac{PdV + \frac{1}{2}kx^2}{W} = \frac{PA_p x + \frac{1}{2}kx^2}{W} \\ &= \frac{1.01325 \times 10^5 \times 0.01767 \times 0.05 + 0.5 \times 2 \times 10^4 \times (0.05)^2}{279.041} \\ &= \frac{114.52}{279.041} = 0.4104 = 41.04\%\end{aligned}$$



Problem 3.13.

A cylinder with a frictionless piston contains 0.1 m^3 of gas at 200 kPa pressure is connected to a coil spring, which exerts a force proportional to displacement from its equilibrium position. The gas is heated until the volume is doubled at which pressure is 500 kPa . Determine the work done by the gas. Take atmospheric pressure equal to 100 kPa .

Solution: Given.

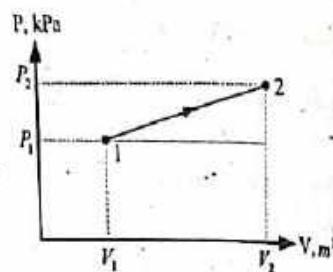
Initial volume, $V_1 = 0.1 \text{ m}^3$

Initial pressure, $P_1 = 200 \text{ kPa}$

Final pressure, $P_2 = 500 \text{ kPa}$

Atmospheric pressure, $P_{atm} = 100 \text{ kPa}$

Final volume, $V_2 = 2V_1 = 2 \times 0.1 = 0.2 \text{ m}^3$



The $P - V$ diagram of the process 1-2 is shown in figure above. The work done by the gas can be calculated by determining the area under the process curve 1-2 on $P - V$ diagram.

Hence,

$$W_{1-2} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(200 + 500)(0.2 - 0.1) = 35 \text{ kJ}$$

Problem 3.14.

4 kg of air contained in a piston cylinder arrangement at initial pressure of 2500 kPa and initial volume of 0.25 m^3 is allowed to expand according to pressure - volume relation of $PV^3 = \text{constant}$ until its volume equals to 0.65 m^3 . The air is then cooled at constant pressure until the piston comes to its initial position. Then heat is supplied to air as the piston is locked with a screw until the pressure rises to its initial pressure. Determine the total work transfer.

Solution: Given.

Mass of air, $m = 4 \text{ kg}$

Initial pressure, $P_1 = 2500 \text{ kPa}$

Initial volume, $V_1 = 0.25 \text{ m}^3$

Volume at state 2, $V_2 = 0.65 \text{ m}^3$

Pressure - volume relation for process 1-2 is: $PV^3 = C$

Air is cooled at constant pressure (process 2-3) $\Rightarrow P_2 = P_3$

Piston comes to its initial position \Rightarrow Volume at state 3, $V_3 = V_1 = 0.25 \text{ m}^3$

Heat is supplied to air at constant volume (process 3-1) until pressure rises to initial pressure \Rightarrow cyclic process.

The $P - V$ diagram is shown in figure below.

During process 1 - 2, pressure volume relationship for states 1 and 2 is given by

$$P_1 V_1^3 = P_2 V_2^3 \Rightarrow P_2 = \frac{P_1 V_1^3}{V_2^3} = \frac{2500 \times (0.25)^3}{(0.65)^3} = 142.24 \text{ kPa}$$

Work transfer during process 1-2 can be calculated by using relation for polytropic process with index $n = 3$ as

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{142.24 \times 0.65 - 2500 \times 0.25}{1-3} = 266.272 \text{ kJ}$$

Work transfer during constant pressure process 2-3,

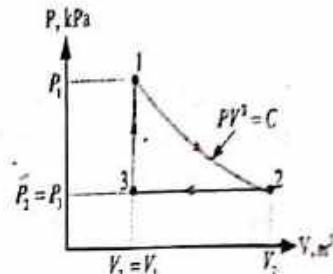
$$W_{2-3} = P_2 (V_3 - V_2) = 142.24 (0.25 - 0.65) = -56.896 \text{ kJ}$$

Work transfer during constant volume process 3-1

is zero due to no change of volume, i.e. $W_{3-1} = 0$.

Now, total work transfer will be

$$\begin{aligned} W_t &= W_{1-2} + W_{2-3} + W_{3-1} \\ &= 266.272 + (-56.896) + 0 \\ &= 209.376 \text{ kJ} \end{aligned}$$



3.3 Solved Numerical Problems from PoU Examinations

Problem 3.15.

A gas compresses from initial state where $P_1 = 550 \text{ kPa}$ and $V_1 = 0.1 \text{ m}^3$ to final state where $P_2 = 1000 \text{ kPa}$. The relationship between pressure and volume during the process is $PV^2 = \text{constant}$. Determine the work done in kJ.

(PoU, 2004 spring, 2008 fall).

Solution: Given,

Initial pressure, $P_1 = 550 \text{ kPa}$

Initial volume, $V_1 = 0.1 \text{ m}^3$

Final pressure, $P_2 = 1000 \text{ kPa}$

Pressure - volume relationship during process is $PV^2 = \text{constant}$ which means it is a polytropic process with polytropic index, $n = 2$.

The final volume can be calculated as

$$P_1 V_1^2 = P_2 V_2^2 \Rightarrow V_2 = \sqrt{\frac{P_1 V_1^2}{P_2}} = \sqrt{\frac{550 \times 0.1^2}{1000}} = 0.07416 \text{ m}^3$$

Work done during the polytropic process 1 – 2 is determined as

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{1000 \times 0.07416 - 550 \times 0.1}{1-2} = -19.16 \text{ kJ}$$

Negative sign indicates that the work is done on the system.

Problem 3.16.

A gas expands from initial state where $P_1 = 500 \text{ kPa}$ and $V_1 = 0.1 \text{ m}^3$ to final state where $P_2 = 100 \text{ kPa}$. The relationship between pressure and volume during the process is $PV = \text{constant}$. Determine the work done in kJ.

(PoU, 2006 spring).

Solution: Given,

Initial pressure, $P_1 = 500 \text{ kPa}$

Initial volume, $V_1 = 0.1 \text{ m}^3$

Final pressure, $P_2 = 100 \text{ kPa}$

Pressure - volume relationship during process is $PV = \text{constant}$ which means it is an isothermal process.

The final volume can be calculated as

$$P_1 V_1 = P_2 V_2 \Rightarrow V_2 = \frac{P_1 V_1}{P_2} = \frac{500 \times 0.1}{100} = 0.5 \text{ m}^3$$

Work done during the isothermal process 1 - 2 is determined as

$$W_{1-2} = P_1 V_1 \ln(V_2/V_1) = 500 \times 0.1 \ln(0.5/0.1) = 80.472 \text{ kJ}$$

Problem 3.17.

A mass of gas is compressed in a quasi-static process from 72 kPa , 0.1 m^3 to 0.3 MPa , 0.03 m^3 . Assuming that the pressure and volume are related by $PV^n = \text{constant}$. Find the work done by the gas system. (PoU 2006 fall).

Solution: Given,

Initial pressure, $P_1 = 72 \text{ kPa}$

Initial volume, $V_1 = 0.1 \text{ m}^3$

Final pressure, $P_2 = 0.3 \text{ MPa} = 300 \text{ kPa}$

Final volume, $V_2 = 0.03 \text{ m}^3$

Pressure - volume relationship during process is $PV^n = \text{constant}$ which means it is a polytropic process with polytropic index, n .

The polytropic index (n) can be calculated as

$$\begin{aligned} P_1 V_1^n = P_2 V_2^n &\Rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \Rightarrow \ln\left(\frac{P_1}{P_2}\right) = n \ln\left(\frac{V_2}{V_1}\right) \\ &\Rightarrow n = \ln\left(\frac{P_1}{P_2}\right) / \ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{72}{300}\right) / \ln\left(\frac{0.03}{0.1}\right) = 1.18534 \end{aligned}$$

Work done during the polytropic process 1 - 2 is determined as

$$W_{1-2} = \frac{P_1 V_2 - P_1 V_1}{1-n} = \frac{300 \times 0.03 - 72 \times 0.1}{1-1.185} = -9.712 \text{ kJ}$$

Negative sign indicates that the work is done on the system.

Problem 3.18.

A mass of gas is compressed in a quasi - static process from $80 \text{ kPa}, 0.1 \text{ m}^3$ to $0.4 \text{ MPa}, 0.03 \text{ m}^3$. Assuming that the pressure and volume are related by $PV^n = \text{constant}$. Find the work done by the gas system. (PoU 2007 spring, 2013 fall).

Solution: Refer problem 3.17 (very similar) (Ans: -11.87 kJ)

Problem 3.19.

For the cycle shown in figure, determine the work output and heat transfer. (PoU, 2007 fall, 2015 spring).

Solution: Given,

At state 1:

Pressure, $P_1 = 100 \text{ kPa}$

Volume, $V_1 = 0.08 \text{ m}^3$

At state 2:

Pressure, $P_2 = 800 \text{ kPa}$

Volume, $V_2 = 0.08 \text{ m}^3$

At state 3:

Pressure, $P_3 = P_1 = 100 \text{ kPa}$

As the process $1 - 2$ is a constant volume process, work done is zero. Hence

$$W_{1-2} = 0$$

Since the process $2 - 3$ is an isothermal process, volume at state 3 can be calculated as

$$P_2 V_2 = P_3 V_3 \Rightarrow V_3 = \frac{P_2 V_2}{P_3} = \frac{800 \times 0.08}{100} = 0.64 \text{ m}^3$$

Work done during the isothermal process $2 - 3$ is determined as

$$W_{2-3} = P_2 V_2 \ln(V_3/V_2) = 800 \times 0.08 \ln(0.64/0.08) = 133.084 \text{ kJ}$$

As the process $3 - 1$ is a constant pressure process, work done is calculated as

$$W_{3-1} = P_3 (V_1 - V_3) = 100 \times (0.08 - 0.64) = -56 \text{ kJ}$$

Then, the net work done is determined as

$$W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-1} = 0 + 133.084 + (-56) = 77.084 \text{ kJ}$$

For a cyclic process the net work transfer is equal to the net heat transfer, so

$$Q_{\text{net}} = W_{\text{net}} = 77.084 \text{ kJ}$$

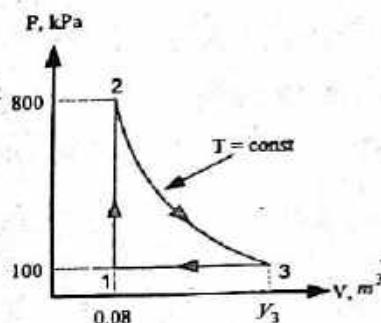
Problem 3.20.

A gas is compressed from initial state from $V_1 = 0.09 \text{ m}^3$, $P_1 = 1 \text{ bar}$ and to $V_2 = 0.03 \text{ m}^3$, $P_2 = 3 \text{ bar}$. Pressure and volume are related linearly during the process. For the gas, find the work done in kJ. (PoU, 2009 spring).

Solution: Given,

Initial volume, $V_1 = 0.09 \text{ m}^3$

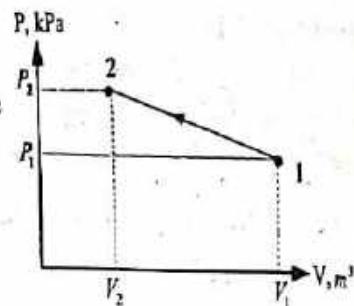
Initial pressure, $P_1 = 1 \text{ bar} = 100 \text{ kPa}$



Final volume, $V_2 = 0.03 \text{ m}^3$

Final pressure, $P_2 = 3 \text{ bar} = 300 \text{ kPa}$

Pressure - volume are related linearly during the process means the process curve is as shown in $P - V$ diagram below.



Work done during the given process 1 – 2 is calculated as the area under the process curve in $P - V$ diagram, i.e., area of triangle plus area of rectangle below the process curve.

$$W_{1-2} = \frac{1}{2}(V_2 - V_1)(P_2 - P_1) + (V_2 - V_1)P_1 = \frac{1}{2}(0.03 - 0.09)(300 - 100) + (0.03 - 0.09)100 \\ = -12 \text{ kJ}$$

Negative sign indicates the work is done on the gas system.

Problem 3.21.

In a non-flow process, a gas expands from volume 0.5 m^3 to a volume of 1 m^3 according to the law $P = 3V^2 + 1/V$, where P is the pressure in bar and V is the volume in m^3 . Determine (i) the pressure at the end of expansion and (ii) the work done by the gas in the expansion process in kJ. (PoU, 2010 fall).

Solution: Given,

Initial gas volume, $V_1 = 0.5 \text{ m}^3$

Final gas volume, $V_2 = 1 \text{ m}^3$

Pressure – volume relationship for the given process is

$$P = 3V^2 + \frac{1}{V}$$

where P is in bar.

The final pressure at the end of expansion can be determined as

$$P_2 = 3V_2^2 + \frac{1}{V_2} = 3 \times 1^2 + \frac{1}{1} = 4 \text{ bar} = 400 \text{ kPa}$$

Work done by the gas during the expansion process is calculated as

$$W_{1-2} = \int_1^2 P dV = \int_1^2 \left(3V^2 + \frac{1}{V} \right) dV = \left[\frac{3V^3}{3} + \ln(V) \right]_1^2 = (V_2^3 - V_1^3) + (\ln V_2 - \ln V_1) \\ = (1^3 - 0.5^3) + (\ln 1 - \ln 0.5) = 1.56815 \text{ bar m}^3 \\ = 156.815 \text{ kJ} \quad (\text{As } 1 \text{ bar} = 100 \text{ kPa})$$

Problem 3.22.

Calculate the work done in a piston cylinder arrangement during an expansion process where the volume changes from 1 m^3 to 4 m^3 and the process is given by the equation $P = (V^2 + 6V)$ bar. (PoU, 2010 spring, 2011 fall).

Solution: Given,

Initial volume, $V_1 = 1 \text{ m}^3$

Final volume, $V_2 = 4 \text{ m}^3$

Pressure – volume relationship for the given process is

$$P = (V^2 + 6V) \quad \text{where } P \text{ is in bar.}$$

$$\begin{aligned} \text{Work done, } W_{1-2} &= \int_1^2 P dV = \int_1^2 (V^2 + 6V) dV = \left[\frac{V^3}{3} + 6 \times \frac{V^2}{2} \right]_1^2 \\ &= \frac{(V_2^3 - V_1^3)}{3} + 3(V_2^2 - V_1^2) = \frac{4^3 - 1^3}{3} + 3(4^2 - 1^2) \\ &= 66 \text{ bar m}^3 = 6600 \text{ kJ} \quad (\text{As } 1 \text{ bar} = 100 \text{ kPa}) \end{aligned}$$

Problem 3.23.

If during an expansion process the volume of gas changes from 0.2 m^3 to 0.5 m^3 and pressure changes according to equation $P = 1500 \left(\frac{V}{100} + 1 \right)$, where P is in Pa and V is in m^3 . What is the work done by the gas in kJ? (PoU, 2012 fall).

Solution: Given,

Initial gas volume, $V_1 = 0.2 \text{ m}^3$

Final gas volume, $V_2 = 0.5 \text{ m}^3$

Pressure – volume relationship for the given process is

$$P = 1500 \left(\frac{V}{100} + 1 \right) \quad \text{where } P \text{ is in Pascal.}$$

Work done by the gas is calculated as

$$\begin{aligned} W_{1-2} &= \int_1^2 P dV = \int_1^2 1500 \left(\frac{V}{100} + 1 \right) dV = 1500 \left[\frac{V^2}{2 \times 100} + V \right]_1^2 \\ &= 7.5(V_2^2 - V_1^2) + 1500(V_2 - V_1) = 7.5(0.5^2 - 0.2^2) + 1500(0.5 - 0.2) \\ &= 451.575 \text{ J} = 0.4516 \text{ kJ} \end{aligned}$$

Problem 3.24.

Consider two part process with an expansion from 0.1 m^3 to 0.2 m^3 at constant pressure of 150 kPa followed by an expansion from 0.2 m^3 to 0.4 m^3 with a linearly rising pressure from 150 kPa ending at 300 kPa . Show the process in a P-V diagram and find the boundary work. (PoU, 2013 spring).

Solution: Given,

At state 1:

Volume, $V_1 = 0.1 \text{ m}^3$

Pressure, $P_1 = 150 \text{ kPa}$

At state 2:

Volume, $V_2 = 0.2 \text{ m}^3$

Pressure, $P_2 = P_1 = 150 \text{ kPa}$

At state 3:

Volume, $V_3 = 0.4 \text{ m}^3$

Pressure, $P_3 = 300 \text{ kPa}$

The $P - V$ diagram of the process is given above, where process 1 - 2 is constant pressure process (horizontal line) and process 2 - 3 in which both pressure and volume increases linearly (straight and upwards inclined line).

The boundary work during the process is equal to the area under the process curve in $P - V$ diagram. Hence, the areas under 1 - 2 and 2 - 3 lines gives the boundary works or total work and it can be calculated as

$$\begin{aligned} W_{\text{ext}} &= W_{1-2} + W_{2-3} = P_1(V_2 - V_1) + \frac{1}{2}(P_2 + P_3)(V_3 - V_2) \\ &= 150(0.2 - 0.1) + \frac{1}{2}(150 + 300)(0.4 - 0.2) = 60 \text{ kJ} \end{aligned}$$

Problem 3.25.

0.2 m^3 of an ideal gas at a pressure of 2 MPa and 600 K is expanded isothermally to 5 times the initial volume. It is then cooled to 300 K at constant volume and then compressed back polytropically to its initial state. Show the process on a $P - V$ diagram and determine the work done. (PoU, 2014 fall).

Solution: Given,

At state 1:

Volume, $V_1 = 0.2 \text{ m}^3$

Pressure, $P_1 = 2 \text{ MPa} = 2000 \text{ kPa}$

Temperature, $T_1 = 600 \text{ K}$

At state 2:

Volume, $V_2 = 5 V_1 = 5 \times 0.2 \text{ m}^3 = 1 \text{ m}^3$

At state 3:

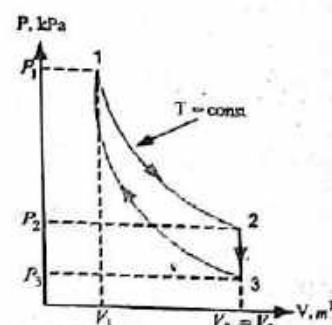
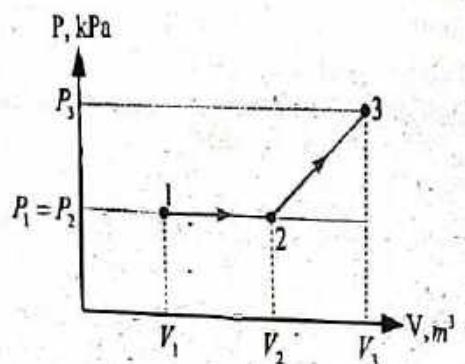
Volume, $V_3 = V_2 = 1 \text{ m}^3$

Temperature, $T_3 = 300 \text{ K}$

It is a cyclic process because the initial and final states are identical. The $P - V$ diagram of this cyclic process is given below, where process 1 - 2 is constant temperature or isothermal process (downwards inclined line), process 2 - 3 is the constant volume process (vertical line) and process 3 - 1 is the polytropic process (upwards inclined line).

The net work done during the cyclic process is equal to the sum of the individual work done of different processes and it can be calculated as

$$W_{\text{ext}} = W_{1-2} + W_{2-3} + W_{3-1}$$



Using Boyle's law for isothermal process 1 - 2,

$$P_1 V_1 = P_2 V_2 \Rightarrow P_2 = \frac{P_1 V_1}{V_2} = \frac{2000 \times 0.2}{1} = 400 \text{ kPa}$$

Work done during isothermal process is determined as

$$W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1} = 2000 \times 0.2 \ln \frac{1}{0.2} = 643.775 \text{ kJ}$$

Using Charle's law for isochoric process 2 - 3,

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow P_3 = T_3 \times \frac{P_2}{T_2} = P_3 = 300 \times \frac{400}{600} = 200 \text{ kPa}$$

Work done during the isochoric process is zero. Hence, $W_{2-3} = 0$

For polytropic process 3 - 1, we know

$$P_3 V_3^n = P_1 V_1^n \Rightarrow n = \frac{\ln\left(\frac{P_3}{P_1}\right)}{\ln\left(\frac{V_1}{V_3}\right)} = \frac{\ln\left(\frac{200}{2000}\right)}{\ln\left(\frac{0.2}{1}\right)} = 1.431$$

Work done during the polytropic process is given by

$$W_{3-1} = \frac{P_1 V_1 - P_3 V_3}{1-n} = \frac{2000 \times 0.2 - 200 \times 1}{1-1.431} = -464.037 \text{ kJ}$$

Now, net work done during the cyclic process is

$$W_{net} = 643.775 + 0 + (-464.037) = 179.738 \text{ kJ}$$

Positive work done means the cycle is power cycle.

Problem 3.26.

A fluid at a pressure of 3 bar and with specific volume $0.18 \text{ m}^3/\text{kg}$ contained in a cylinder behind a piston that expands reversibly to a pressure of 0.6 bar according to a law $P = C/v^2$, where C is a constant. Show the expansion process on a P - v diagram and calculate the net work done by the fluid on the piston.

(PoU, 2014 spring, 2016 fall).

Solution: Given,

Initial pressure, $P_1 = 3 \text{ bar} = 300 \text{ kPa}$

Initial specific volume, $v_1 = 0.18 \text{ m}^3/\text{kg}$

Final pressure, $P_2 = 0.6 \text{ bar} = 60 \text{ kPa}$

Pressure - volume relationship is given by

$$P = C/v^2 \Rightarrow Pv^2 = C,$$

which is a polytropic process with polytropic index is 2.

The P - v diagram for the polytropic process is given here.

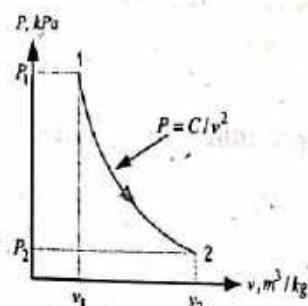
For the polytropic process, we know

$$P_1 v_1^2 = P_2 v_2^2$$

$$\Rightarrow v_2 = v_1 \times \sqrt{\frac{P_1}{P_2}} = 0.18 \times \sqrt{\frac{300}{60}} = 0.4025 \text{ m}^3/\text{kg}$$

Work done during the polytropic process is calculated as

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{60 \times 0.4025 - 300 \times 0.18}{1-2} = 29.85 \text{ kJ/kg}$$



Problem 3.27.

A gas expands from an initial state where $P_1 = 550 \text{ kPa}$ and volume $V_1 = 0.1 \text{ m}^3$ to the final state where $P_2 = 100 \text{ kPa}$. The relationship between pressure and volume during the process is $PV^2 = \text{constant}$. Determine the work done in kJ.

(PoU, 2015 fall).

Solution: Given,

Initial pressure, $P_1 = 550 \text{ kPa}$

Initial volume, $V_1 = 0.1 \text{ m}^3$

Final pressure, $P_2 = 100 \text{ kPa}$

Pressure - volume relationship during the polytropic process is given by $PV^2 = \text{constant}$ where polytropic index is 2.

For the polytropic process, we know

$$P_1 V_1^2 = P_2 V_2^2 \Rightarrow V_2 = V_1 \times \sqrt{\frac{P_1}{P_2}} = 0.1 \times \sqrt{\frac{550}{100}} = 0.2345 \text{ m}^3$$

Work done during the polytropic process is calculated as

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{100 \times 0.2345 - 550 \times 0.1}{1-2} = 31.55 \text{ kJ}$$

Problem 3.28.

A piston cylinder device contains air (0.15 kg) initially at 2 MPa and 350°C . The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle. Assume data if necessary.

(PoU, 2017 spring).

Solution: Given,

Mass of air, $m = 0.15 \text{ kg}$

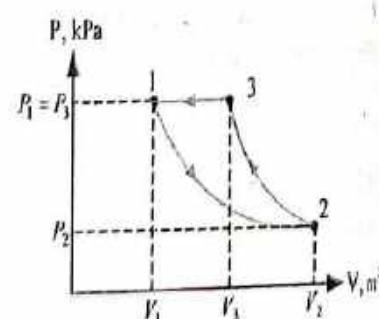
At state 1: Pressure, $P_1 = 2 \text{ MPa} = 2000 \text{ kPa}$

Temperature, $T_1 = 350^\circ\text{C} = 350 + 273 = 623 \text{ K}$

At state 2: Pressure, $P_2 = 500 \text{ kPa}$

At state 3: Pressure, $P_3 = P_1 = 2000 \text{ kPa}$

Polytropic index, $n = 1.2$



The $P - V$ diagram for the cyclic process is given here, where downwards curve line 1 – 2 is an isothermal expansion process, upwards curve line 2 – 3 is a polytropic compression process and horizontal straight line 3 – 1 is a isobaric compression process.

Using equation of state for an ideal gas at state 1, we have

$$P_1 V_1 = mRT_1 \Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{0.15 \times 287 \times 623}{2000 \times 1000} = 0.01341 \text{ m}^3$$

For the isothermal process 1 – 2, we know

$$P_1 V_1 = P_2 V_2 \Rightarrow V_2 = \frac{P_1 V_1}{P_2} = \frac{2000 \times 0.01341}{500} = 0.05364 \text{ m}^3$$

And boundary work is given by

$$W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1} = 2000 \times 0.01341 \times \ln \frac{0.05364}{0.01341} = 37.18 \text{ kJ}$$

For the polytropic process 2 – 3, we know

$$P_2 V_2^n = P_3 V_3^n \Rightarrow V_3 = V_2 \times \left(\frac{P_2}{P_3} \right)^{1/n} = 0.05364 \times \left(\frac{500}{2000} \right)^{1/1.2} = 0.01689 \text{ m}^3$$

And boundary work is calculated as

$$W_{2-3} = \frac{P_3 V_3 - P_2 V_2}{1-n} = \frac{2000 \times 0.01689 - 500 \times 0.05364}{1-1.2} = -34.8 \text{ kJ}$$

For the isobaric process 3 – 1, boundary work is determined as

$$W_{3-1} = P_3 (V_1 - V_3) = 2000 \times (0.01341 - 0.01689) = -6.96 \text{ kJ}$$

Now, net work of the cycle is given by

$$W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = 37.18 + (-34.8) + (-6.96) = -4.58 \text{ kJ}$$

The negative sign indicates that the net work is done on the system and such a cycle is known as a refrigeration cycle.

Problem 3.29.

Air enters a compressor at 10^5 Pa and 25°C having volume of $1.8 \text{ m}^3/\text{kg}$ and is compressed to $5 \times 10^5 \text{ Pa}$ isothermally. Determine

- (i) work done
 - (ii) change in internal energy and
 - (iii) heat transferred.
- (PoU 2018 spring).

Solution: Given,

Initial pressure, $P_1 = 10^5 \text{ Pa} = 100 \text{ kPa}$

Initial temperature, $T_1 = 25^\circ\text{C}$

Initial volume, $v_1 = 1.8 \text{ m}^3/\text{kg}$

Final pressure, $P_2 = 5 \times 10^5 \text{ Pa} = 500 \text{ kPa}$

For an isothermal process 1 – 2, we have



WORK AND HEAT

$$P_1 v_1 = P_2 v_2 \Rightarrow v_2 = \frac{P_1 v_1}{P_2} = \frac{100 \times 1.8}{500} = 0.36 \text{ m}^3/\text{kg}$$

For an isothermal process; work done per kg of air is given by

$$w_{1-2} = P_1 v_1 \ln \frac{v_2}{v_1} = 100 \times 1.8 \times \ln \frac{0.36}{1.8} = -289.7 \text{ kJ/kg}$$

Negative sign indicates that the work is done on the system.

For an isothermal process ($T_1 = T_2$), the change in internal energy is given by

$$\Delta u_{1-2} = c_v (T_2 - T_1) = 0$$

Now, the heat transferred per kg of air during isothermal process 1 - 2 is given by

$$q_{1-2} = \Delta u_{1-2} + w_{1-2} = 0 + (-289.7) = -289.7 \text{ kJ/kg}$$

Negative sign indicates that the heat is lost by the system.

Problem 3.30.

A non flow reversible process occurs for which pressure and volume are correlated by the expression, $P = V^2 + \frac{6}{V}$ where P is in bar and V is in m^3 . What amount of work will be done when volume changes from 2 to 4 m^3 ?

(PoU 2019 Fall).

Solution: Given,

Initial gas volume, $V_1 = 2 \text{ m}^3$

Final gas volume, $V_2 = 4 \text{ m}^3$

Pressure – volume relationship for the given process is

$$P = V^2 + \frac{6}{V}$$

where P is in bar.

Work done,

$$\begin{aligned} W_{1-2} &= \int_1^2 P dV = \int_1^2 \left(V^2 + \frac{6}{V} \right) dV = \left[\frac{V^3}{3} + 6 \ln(V) \right]_1^2 \\ &= \frac{(V_2^3 - V_1^3)}{3} + 6(\ln V_2 - \ln V_1) \\ &= \frac{4^3 - 2^3}{3} + 6(\ln 4 - \ln 2) \\ &= 22.8255 \text{ bar m}^3 \\ &= 2282.55 \text{ kJ} \quad (\text{As } 1 \text{ bar} = 100 \text{ kPa}) \end{aligned}$$

Problem 3.31.

1 kg of a fluid is compressed reversibly according to the law $Pv = 0.25$ where P is in bar and v is in m^3/kg . The final volume is 1/4 of the initial volume. Calculate the work done on the fluid and sketch the process on a $P - v$ diagram.

(PoU 2019 spring).

Solution: Given,

Mass of fluid, $m = 1 \text{ kg}$

$$\text{Final volume, } V_2 = V_1/4 \Rightarrow V_2/V_1 = 1/4$$

pressure - volume relationship of the process is given as

$$Pv = 0.25$$

where P is in bar.

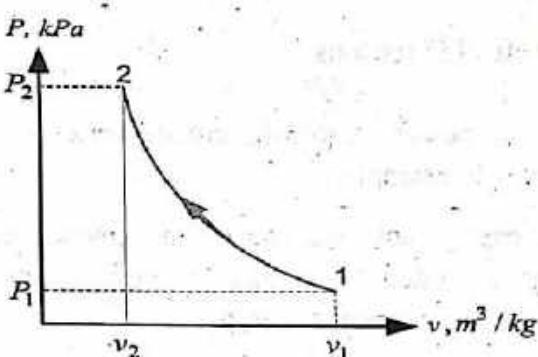
Here the product of pressure and volume is constant. This means the process is an isothermal or constant temperature process. So, we can also write

$$P_1V_1 = 0.25 \Rightarrow P_1V_1/m = 0.25 \Rightarrow P_1V_1 = 0.25 \quad (\text{As } m = 1 \text{ kg})$$

For an isothermal process the work transfer is calculated as

$$W_{1-2} = P_1V_1 \ln(V_2/V_1) = 0.25 \times \ln(1/4) = -0.34657 \text{ bar m}^3 \\ = -34.657 \text{ kJ} \quad (\text{As } 1 \text{ bar} = 100 \text{ kPa})$$

Negative sign indicates that the work is done on the system fluid.



Problem 3.32.

A gas undergoes a polytropic process from an initial state of 450 kPa and 0.03 m^3 to a final state of 150 kPa and 0.06 m^3 . Determine the work transfer.

(PoU 2020 fall)

Solution: Given,

Initial pressure, $P_1 = 450 \text{ kPa}$

Initial volume, $V_1 = 0.03 \text{ m}^3$

Final pressure, $P_2 = 150 \text{ kPa}$

Final volume, $V_2 = 0.06 \text{ m}^3$

Pressure - volume relationship during a polytropic process is given by

$PV^n = \text{constant}$ where n is the polytropic index.

The polytropic index (n) can be calculated as

$$P_1V_1^n = P_2V_2^n \Rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \Rightarrow \ln\left(\frac{P_1}{P_2}\right) = n \ln\left(\frac{V_2}{V_1}\right) \\ \Rightarrow n = \ln\left(\frac{P_1}{P_2}\right) / \ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{450}{150}\right) / \ln\left(\frac{0.06}{0.03}\right) = 1.58496$$

Work done during the polytropic process 1 – 2 is determined as

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{150 \times 0.06 - 450 \times 0.03}{1-1.58496} = 7.6928 \text{ kJ}$$

Positive sign indicates that the work is done by the system.

3.4 Review Theoretical Questions

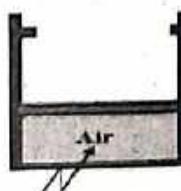
1. Define energy and power. Also differentiate between the stored energy and transient energy with examples.
2. Define internal energy, potential energy and kinetic energy and total energy of a thermodynamic system. Also differentiate between microscopic potential energy and macroscopic potential energy.
3. Define total energy of a system. Also differentiate between the stored energy and transient energy with examples.
4. Define stored energy and transient energy. Also list their features.
5. Differentiate between the stored energy and transient energy with examples. Sketch polytropic processes on a common $P - V$ diagram for different values of n .
6. Define a polytropic process. Sketch polytropic process with $n = 0$, $n = 1$, $n = 1.4$, $n = \infty$ on a common $P - V$ diagram.
7. Define work transfer and heat transfer. Also mention their sign conventions used in the analysis of thermodynamic problems.
8. Compare heat and work with suitable examples. Prove that work is a path function.
9. In what respect the heat and work interactions are (i) similar and (ii) dissimilar?
10. Write down the similarities and differences between heat transfer and work transfer.

11. Differentiate between heat transfer and work transfer. Derive the mathematical expression for work transfer for an ideal gas undergoing an isothermal/isobaric process.
12. Define heat and work thermodynamically. Why heat and work are denoted by inexact differential?
13. Define work transfer. Derive an expression for work transfer for polytropic process.
14. Define internal energy. Derive an expression for work transfer for polytropic process.
15. Derive an expression for displacement work transfer.
16. Derive a general expression for the displacement work (Quasi-static work) transfer for a piston cylinder device. Also reduce it for an ideal gas undergoing constant temperature process.
17. Derive an expression for work transfer for any process on a piston cylinder device. Also reduce it to get the expression for work transfer during a polytropic process.
18. Derive expressions for displacement work transfer for the following processes:
 - (a) Constant pressure process
 - (b) Constant temperature process and
 - (c) Polytropic process.
19. Define a polytropic process. Sketch polytropic processes on a common graph for different values of n . Also derive an expression for work transfer for a polytropic process.

3.5 Unsolved Numerical Problems

- A gas is contained in a piston cylinder device initially at a pressure of 800 kPa and a volume of 0.1 m^3 . Calculate the work done by the gas when it undergoes the following processes to a final volume of 0.5 m^3 .
 - constant pressure.
 - according to linear law to a final pressure of 200 kPa.
 - according to law $PV = \text{constant}$.
 - according to law $PV^2 = \text{constant}$.
 - according to the relation $P = a + b/V$ to a final pressure of 200 kPa; where a and b are constants.

(Ans: 320 kJ, 200 kJ, 128.755 kJ, 64 kJ, 140.708 kJ)
- The pressure volume correlation for a non-flow reversible process is given by $P = (8 - 4V) \text{ bar}$ where V is in m^3 . If 150 kJ of work is supplied to the system, determine the final pressure and volume of the system taking initial volume equal to 0.6 m^3 . (Ans: 517.2 kPa)
- One kg of certain fluid is contained in piston cylinder arrangement at 10 bar. The fluid expands reversibly to its two times of initial volume according to relation $PV^2 = \text{constant}$. The fluid is then cooled at constant pressure to its initial volume. Finally heat is supplied to the fluid with the piston firmly locked and the final pressure rises to initial value of 10 bar. If the fluid has initial volume of 0.05 m^3 , determine the net work done by the fluid.
(Ans: 12.5 kJ)
- An ideal gas undergoes an isobaric expansion from $V_1 = 0.1 \text{ m}^3$ and $V_2 = 0.2 \text{ m}^3$ at a constant pressure of 200 kPa. It is further expanded until its volume rises to 0.4 m^3 with linear rising pressure from 200 kPa to 400 kPa. Determine the total work transfer for the process. (Ans: 80 kJ)
- A piston cylinder device contains air initially at 2 MPa and 350°C . The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net-work of the cycle. Assume data if necessary.
(Ans: 37.18 kJ, -34.86 kJ, -6.97 kJ, -4.65 kJ)
- A piston cylinder device shown in figure contains 0.1 kg of air initially at a pressure of 4 MPa and temperature of 200°C . Heat is added to the system until the pressure is 8 MPa and the temperature is 800°C . Sketch the process on $P-V$ and $T-V$ diagrams and determine total work transfer.
[Take $R = 287 \text{ J/kg K}$] (Ans: 1.82 kJ)



7. A piston cylinder device shown in figure contains 2 kg of air initially at a pressure of 200 kPa and a temperature of 50°C. Heat is added until the piston reaches the upper stops where the total volume is 1.5 m³. There is further heat transfer to air until its temperature reaches 1200°C. It takes a pressure of 500 kPa to lift the piston from the bottom stops. Sketch the process on $P - V$ and $T - V$ diagrams and determine the final pressure and the total work transfer. [Take $R = 287 \text{ J/kgK}$].

(Ans: 563.667 kPa, 286.495 kJ)



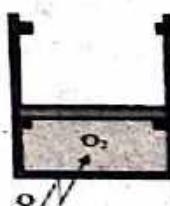
8. Air (0.5 kg) in the piston cylinder device shown in figure has an initial pressure and temperature of 1 MPa and 500°C respectively. The system is cooled until the temperature reaches 50°C. It takes a pressure of 0.5 MPa to support the piston. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take $R = 287 \text{ J/kg K}$]

(Ans: -9.115 kJ)

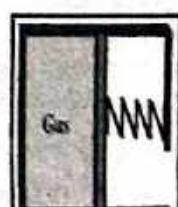


9. Oxygen (3.6 kg) contained in a piston cylinder device shown in figure is initially at a pressure of 200 kPa and a temperature of 50°C. Heat is added until the piston just reaches the upper stops where the total volume is 3 m³. It requires a pressure of 500 kPa to lift the piston. Sketch the process on $P-V$ and $T-V$ diagrams and determine the total work transfer.

[Take $R = 260 \text{ J/kg K}$] (Ans: 744.18 kJ)



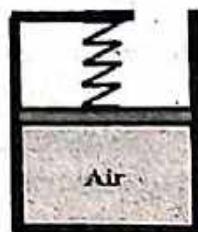
10. An unstretched spring ($k = 1 \text{ kN/m}$) is attached to a piston cylinder device as shown in figure. Heat is added until the gas pressure inside the cylinder is 400 kPa. If the diameter of the piston is 50 mm, determine the work done by the gas on the piston. [Take $P_{atm} = 100 \text{ kPa}$] (Ans: 0.289 kJ)



11. A piston cylinder arrangement loaded with a linear spring ($k = 20 \text{ kN/m}$) as shown in figure contains air. Spring is initially unstretched and undergoes a

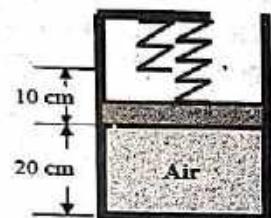
compression of 40 mm during a process. If the mass of the piston is 80 kg and piston diameter is 0.1 m, determine the total work transfer.

[Take $P_{atm} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$] (Ans: 78.81 J)

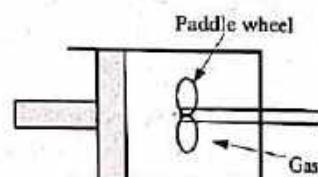


12. A piston cylinder device restrained by a linear spring ($k = 200 \text{ kN/m}$) shown in figure (see figure of problem 12) contains air initially at a pressure of 400 kPa and a volume of 0.5 m^3 . Initially spring touches the piston but exerts no force on it. Heat is transferred to the system until its volume doubles. If the cross sectional area of the piston is 0.4 m^2 , determine
 (a) The final pressure inside the cylinder,
 (b) The total work done by the air, and
 (c) The fraction of total work done consumed for the compression of spring.
 (Ans: 1025 kPa, 356.25 kJ, 43.86%)

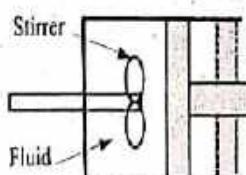
13. A piston cylinder arrangement shown in figure below is restrained by two linear springs as shown in figure. The system contains air initially at a pressure of 150 kPa and a volume of 0.002 m^3 . Heat is added to the system until its volume doubles; determine the total work transfer. Also sketch the process on $P-V$ diagram. Both springs have spring constant of 100 kN/m (Ans: 2.8 kJ)



14. A gas in the cylinder and piston arrangement comprises the system. It expands from 1.5 m^3 to 2.0 m^3 while receiving 200 kJ of work from a paddle wheel. The pressure on the gas remains constant at 600 kPa. Determine net work done by the system. (Ans: 100 kJ)



15. A piston and cylinder machine containing a fluid system has a stirring device as shown in figure. The piston is frictionless and it is held down against the fluid due to atmospheric pressure of 101.3 kPa. The stirring device is turned 9500 revolutions with an average torque against the fluid of 1.25 mN. Meanwhile the piston of 0.65 m diameter moves out 0.6 m. Find the net work transfer for the system. (Ans: - 54.44 kJ).



FIRST LAW OF THERMODYNAMICS

4

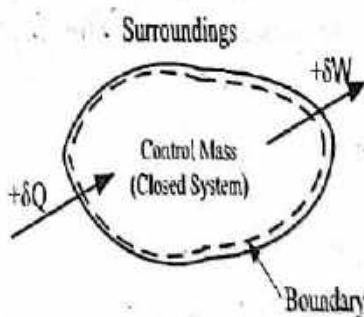
4.1 Chapter Highlights

- *First law of thermodynamics:* It is based on the mass and energy conservation principles. It provides the effect of the interactions between the system and surroundings on the total energy of the system.
- Since in a *closed system (control mass)* interactions can take place only in terms of energy transfer, first law of thermodynamics for a closed system is explained with reference to conservation of energy alone.
- Since in an *open system (control volume)* interactions can take place in terms of both mass and energy transfer, first law of thermodynamics for an open system is explained with reference to both mass and energy conservation principles.
- *Conservation of mass for a control mass:* "The total mass of a control mass always remains constant". Mathematically, $dm = 0$.
- *Conservation of energy for a control mass:* "The change in total energy of a control mass is equal to the heat supplied to the control mass minus the work produced by the control mass".
Mathematically, $dE = \delta Q - \delta W$

On the rate basis, $\frac{dE}{dt} = \dot{Q} - \dot{W}$

For any process from state 1 to state 2: $E_2 - E_1 = Q_{12} - W_{12}$

where Q_{12} and W_{12} are the total heat transferred to the control mass and total work transferred by the control mass respectively.



Then, heat transferred to the control mass

$$Q = \Delta E + W = \Delta U + W, \text{ when neglecting kinetic energy and potential energy.}$$

- First law of thermodynamics for a *control mass undergoing cyclic process* ($\oint \delta E = 0$) can be expressed as: $\oint \delta Q = \oint \delta W$ or, $\sum Q = \sum W$.

➤ First law of thermodynamics for a *power cycle*: "Whenever a control mass is undergoing cyclic process, then net heat supplied to the control mass is equal to net work produced by the control mass."

➤ First law of thermodynamics for a *refrigeration cycle*: "Whenever a control mass is undergoing a cyclic process, then net work supplied to the control mass is equal to the net heat rejected by the control mass."

➤ *Applications of first law of thermodynamics for non-flow processes:*

- *Isochoric process*: For a control mass, heat transfer during a constant volume process is equal to the increase in internal energy of the system. Mathematically,

$$\text{As } W = 0, Q = \Delta U = U_2 - U_1 = mc_v(T_2 - T_1).$$

- *Isobaric process*: For a control mass, heat transfer during a constant pressure process is equal to the increase in enthalpy of the system. Mathematically,

$$\begin{aligned} \text{As } W = P(V_2 - V_1), Q &= \Delta U + W = U_2 - U_1 + P(V_2 - V_1) \\ &= H_2 - H_1 = mc_p(T_2 - T_1). \end{aligned}$$

- *Isothermal Process*: For a control mass, heat transfer during a constant temperature process is equal to the work done by the system. Mathematically,

$$\text{As } \Delta U = 0, Q = W = P_1 V_1 \ln(V_2/V_1)$$

- *Adiabatic process*: For a control mass, heat transfer during an adiabatic process is equal to zero. Mathematically,

$$\text{As } Q = 0, W = -\Delta U$$

This means work is done by the system at the expense of the reduction in the internal energy during an adiabatic process.

- *Polytropic process*: For a control mass, heat transfer during an polytropic process with polytropic index n is equal to

$$Q = \left(\frac{\gamma - n}{\gamma - 1} \right) \times W \text{ where } W = \frac{mR(T_2 - T_1)}{1 - n} \text{ and } \gamma = c_p/c_v.$$

➤ *Conservation of mass for control volume*: "The change in mass within a control volume is equal to mass entering the control volume minus mass leaving the control volume".

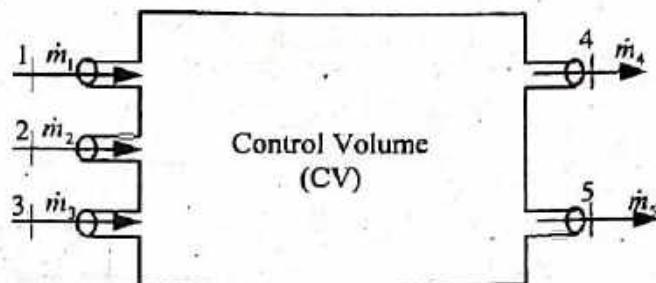
➤ *Mathematically*,

$$dm_{cv} = \sum m_{in} - \sum m_{out}$$

$$\text{On rate basis, } \frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

where $\dot{m}_{in} = \dot{m}_1 + \dot{m}_2 + \dot{m}_3$ is the sum of mass flow rate at each inlet and $\dot{m}_{out} = \dot{m}_4 + \dot{m}_5$ is the sum of mass flow rate at each outlet.

FIRST LAW OF THERMODYNAMICS



- Mass flow rate is given by: $\dot{m} = \frac{A\bar{V}}{v} = \rho A \bar{V}$

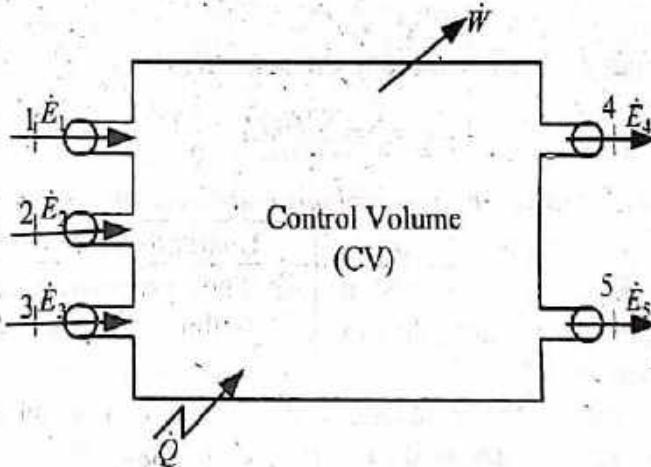
where A = Area of inlet or outlet, \bar{V} = velocity of fluid, v = specific volume of fluid and ρ = density of fluid

- Conservation of energy for a control volume: "The change in total energy of a control volume is equal to net energy transported by the fluid into the control volume plus the heat supplied to the control volume minus the work produced by the control volume". Mathematically,

$$dE_{cv} = E_{net} + \delta Q - \delta W$$

On rate basis, $\frac{dE_{cv}}{dt} = \dot{E}_{net} + \dot{Q} - \dot{W}$

where $\dot{E}_{net} = \sum \dot{E}_{in} - \sum \dot{E}_{out}$ and $\dot{E} = \dot{m}(u + \frac{1}{2}\bar{V}^2 + gz)$.

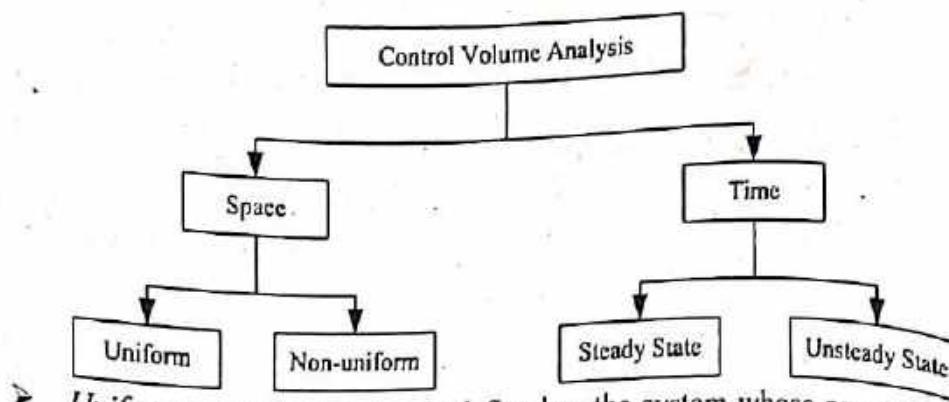


- The general energy equation for control volume is

$$\frac{dE_{cv}}{dt} = \sum [\dot{m}(h + \frac{1}{2}\bar{V}^2 + gz)]_{in} - \sum [\dot{m}(h + \frac{1}{2}\bar{V}^2 + gz)]_{out} + \dot{Q}_{cv} - \dot{W}_{cv}$$

- Analysis of a control volume: Any control volume can be analyzed with reference to either space or time.

FIRST LAW OF THERMODYNAMICS



- **Uniform control volume:** It is defined as the system whose properties do not vary with space at a particular instant of time.
- **Non-uniform control volume:** It is defined as the system whose properties vary with space at a particular instant of time.
- **Steady state control volume:** It is the system whose properties at a particular point do not vary with time.
- **Unsteady state control volume:** It is the system whose properties at a particular point vary with time.
- For steady state control volume it reveals:

$$\frac{dm_{cv}}{dt} = 0 \quad \text{and} \quad \frac{dE_{cv}}{dt} = 0$$

- The steady state mass equation for control volume is:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

- The steady state energy equation for control volume is:

$$\sum [\dot{m}(h + \frac{1}{2}\bar{V}^2 + gz)]_{in} + \dot{Q}_{cv} = \sum [\dot{m}(h + \frac{1}{2}\bar{V}^2 + gz)]_{out} + \dot{W}_{cv}$$

- Differences between steady state and unsteady control volume:

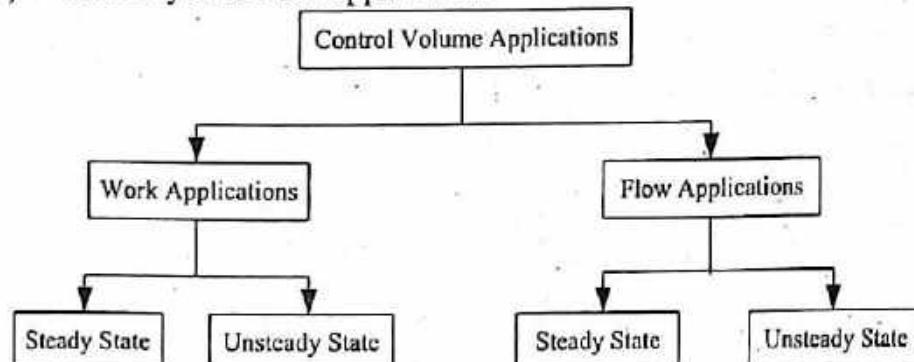
Steady state control volume	Unsteady state control volume
<ol style="list-style-type: none"> 1. The properties of the system within control volume do not change with time. 2. The properties at the boundaries of the control volume do not change with time. 3. The mass and energy interactions (heat and work) between the system and surroundings do not change with time. 4. The system continues indefinitely. 	<ol style="list-style-type: none"> 1. The properties of the system within control volume change with time. 2. The properties at the boundaries of the control volume change with time. 3. The mass and energy interactions (heat and work) between the system and surroundings do not change with time. 4. The system starts and stops over some finite time.

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- | | |
|--|---|
| 5. For examples: Common devices such as Turbine, compressor, pump, fan, nozzle, diffuser, heat exchanger etc. at their normal operation. | 5. For examples: Discharging and filling of a gas cylinder, Balloons, during start up and shut down period of devices such as turbine, compressor, pump, fan etc. |
|--|---|

➤ Depending upon the operation and functions of devices, *control volume applications* can be classified into four groups:

- (i) Steady state work applications
- (ii) Steady state flow applications
- (iii) Unsteady state work applications
- (iv) Unsteady state flow applications



➤ Differences between steady state work applications and steady state flow applications:

Steady state work applications	Steady state flow applications
<ol style="list-style-type: none"> 1. They are any devices which operate under steady state conditions and either produce or consume work. 2. For examples: Devices such as Turbine, compressor, pump, fan etc. 	<ol style="list-style-type: none"> 1. They are any devices which operate under steady state condition and neither produce nor consume work. 2. For examples: Devices such as nozzle, diffuser, heat exchanger, boiler, condenser, throttling valve etc.

➤ Differences between steady state work applications and steady state flow applications:

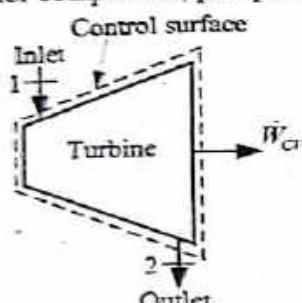
Unsteady state work applications	Unsteady state flow applications
<ol style="list-style-type: none"> 1. They are any devices which operate under unsteady state conditions and either produce or consume work. 2. For examples: During start up and shut down period of devices such as turbine, compressor, pump, fan etc. 	<ol style="list-style-type: none"> 1. They are any devices which operate under unsteady state condition and neither produce nor consume work. 2. For examples: Discharging a gas cylinder e.g. during cooking and filling of cylinder by gas etc.

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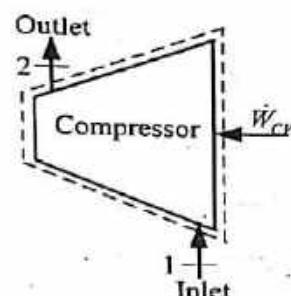
- **Turbine:** It is a device which produces the shaft work (mechanical work) at the expense of the energy of the working fluid.
- **Compressor, pump and fan:** They are the devices which increase the fluid energy by consuming mechanical work. Compressor increases the pressure of a gas, pump increases the pressure of a liquid and fan increases the kinetic energy or velocity of the fluid.
- For the common steady state work devices such as turbine, compressor, pump and fan having single inlet "1" and single outlet "2", steady state energy equation becomes

$$\dot{Q}_o - \dot{W}_o = \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2) + (z_2 - z_1)]$$

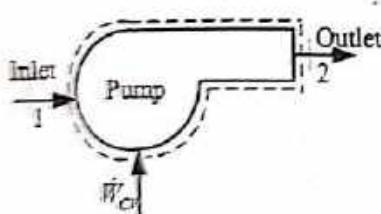
where $\dot{Q}_o = 0$, if devices are adiabatic, \dot{W}_o is positive for turbine and \dot{W}_o is negative for compressor, pump and fan.



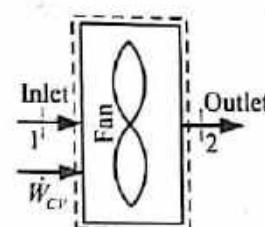
(a) Turbine



(b) Compressor



(c) Pump



(d) Fan

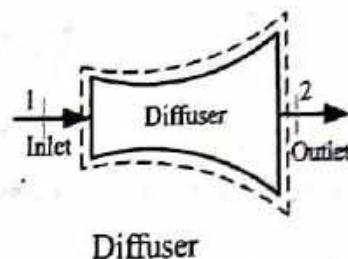
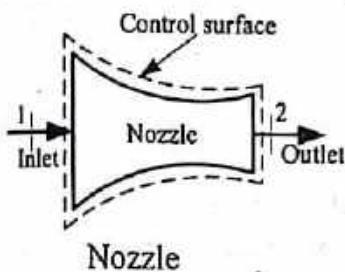
- **Nozzle and diffuser:** A nozzle is a device with decreasing cross-sectional area which increases the velocity of a fluid at the expense of pressure. A diffuser is a device with increasing cross-sectional area which decreases the velocity of a fluid thereby increases the pressure of a fluid.
- For the steady state flow devices ($\dot{W}_o = 0$) such as nozzle and diffuser having single inlet "1" and single outlet "2", steady state energy equation becomes

$$\dot{Q}_o = \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2) + (z_2 - z_1)]$$

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For these devices change in potential energy between inlet and outlet is negligible as compare to the change in kinetic energy and enthalpy, then for adiabatic ($\dot{Q}_{cv} = 0$) nozzle and diffuser above equation reduces to

$$h_1 + \frac{1}{2} \bar{V}_1^2 = h_2 + \frac{1}{2} \bar{V}_2^2$$



- **Heat exchanger:** It is a flow device in which two moving hot and cold fluid streams having different temperatures exchange heat without mixing. In this device, the change in kinetic and potential energy between inlet and outlet are negligible in comparison to the change in enthalpy. Hence, the steady state energy equation for the hot fluid is given by

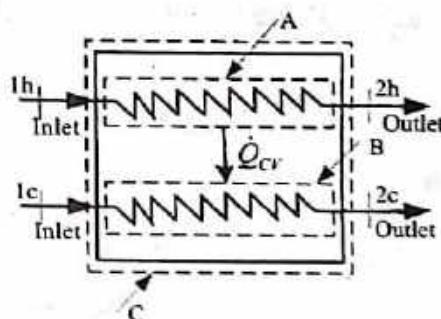
$$\dot{Q}_{cv} = \dot{m}_h (h_{2h} - h_{1h})$$

Similarly, the steady state energy equation for the cold fluid is given by

$$\dot{Q}_{cv} = \dot{m}_c (h_{2c} - h_{1c})$$

If both hot and cold fluids together are taken as a system, then the steady state energy equation for the combined system is given by

$$\dot{m}_h h_{1h} + \dot{m}_c h_{1c} = \dot{m}_h h_{2h} + \dot{m}_c h_{2c}$$



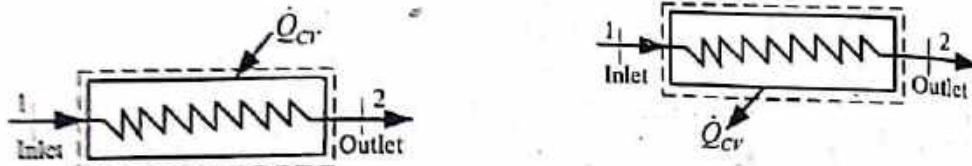
Heat exchanger

where subscripts $1h$ and $1c$ denote inlet of hot and cold fluids respectively. Likewise, subscripts $2h$ and $2c$ denote outlet of hot and cold fluids respectively. \dot{m}_h and \dot{m}_c are mass flow rates of hot and cold fluids respectively.

- **Evaporator and condenser:** Evaporator is a special type of heat exchanger which converts liquid into vapor by absorbing heat from the surroundings, whereas condenser is also a special type of heat exchanger which converts

vapor into liquid by rejecting heat to the surroundings. The steady state energy equation for these two devices is expressed as

$$\dot{Q}_c = \dot{m}(h_2 - h_1)$$



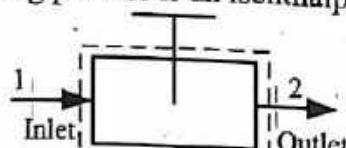
Evaporator

Condenser

- > *Throttling valve:* It is a flow device which reduces pressure of a fluid without performing work. For this device, change in kinetic and potential energy between inlet and outlet as well as heat transfer are assumed to be negligible. Hence, the steady state energy equation for the throttling valve reduces to

$$h_1 = h_2$$

which indicates the throttling process is an isenthalpic process.



Throttling valve

- > *Pipe:* In a pipe the change in potential energy between the inlet and outlet is important and there is no work. Assuming that the control surface of the pipe is adiabatic, the equation of energy conservation reduces to

$$(h_1 - h_2) + \frac{1}{2}(\bar{V}_1^2 - \bar{V}_2^2) + g(z_1 - z_2) = 0$$



Pipe

- > *Mass conservation equation for any unsteady process between two states 1 and 2 is*

$$m_{cv2} - m_{cv1} = m_{in} - m_{out}$$

where m_{cv1} and m_{cv2} are the masses of the control volume at state 1 and state 2 respectively. m_{in} is the total mass entering the control volume and

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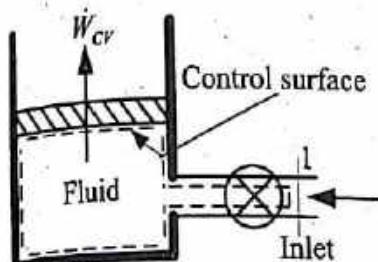
\dot{m}_{out} is the total mass exiting from the control volume during time interval t_1 to t_2 .

Energy conservation equation for any unsteady process between two states 1 and 2 is

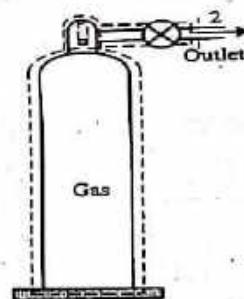
$$E_{cv2} - E_{cv1} = [\dot{m}(h + \frac{1}{2}\bar{V}^2 + gz)]_{in} - [\dot{m}(h + \frac{1}{2}\bar{V}^2 + gz)]_{out} + Q_{12} - W_{12}$$

where E_{cv1} and E_{cv2} are the total energy of the control volume at state 1 and state 2 respectively. Q_{12} is the total heat transfer to the control volume and W_{12} is the total work done by the control volume during time interval t_1 to t_2 .

The above energy equation is used for unsteady state work applications and it is also applied for the flow applications like discharging of a gas cylinder by putting $W_{12} = 0$.



Unsteady state work application



Unsteady state flow application

First law of thermodynamics for an isolated system: Total energy of a control mass remains constant when it is isolated from its surroundings, i.e.

$$dE = 0 \text{ as } \delta Q = 0 \text{ and } \delta W = 0.$$

First law of thermodynamics for a control mass undergoing an adiabatic process: The increase in total energy of a control mass during an adiabatic process is equal to the work done on the control mass, i.e.

$$dE = -\delta W \text{ as } \delta Q = 0.$$

Perpetual motion (continuously running) machine of the first kind (PMM-I) is impossible due to friction.

4.2 Solved Numerical Problems

Problem 4.1.

A piston cylinder arrangement contains 0.01 m^3 air at 150 kPa and 27°C . The air is now compressed in a process for which pressure and volume relationship is given by $PV^{1.25} = \text{constant}$ to a final pressure 600 kPa . Determine the work transfer and heat transfer for the process.

Solution: Given,

Initial volume of air, $V_1 = 0.01 \text{ m}^3$

Initial pressure of air, $P_1 = 150 \text{ kPa}$

Initial temperature of air, $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Pressure-volume relation: $PV^{1.25} = \text{const} \Rightarrow \text{Index, } n = 1.25$

Final pressure, $P_2 = 600 \text{ kPa}$

For the compression process from state 1 to 2, we can write

$$PV_1^{1.25} = PV_2^{1.25} \Rightarrow V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{1.25}} = 0.01 \left(\frac{150}{600} \right)^{\frac{1}{1.25}} = 0.00329877 \text{ m}^3$$

Work transfer for a polytropic process,

$$W_{1 \rightarrow 2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{600 \times 10^3 \times 0.00329877 - 150 \times 10^3 \times 0.01}{1-1.25} \\ = -1917.05 \text{ J} = -1.917 \text{ kJ}$$

The negative sign indicates that the work is done on the system.

$$\text{Mass of air, } m = \frac{PV_1}{RT_1} = \frac{150 \times 10^3 \times 0.01}{287 \times 300} = 0.0174216 \text{ kg}$$

$$\text{Final temperature, } T_2 = \frac{P_2 V_2}{mR} = \frac{600 \times 10^3 \times 0.00329877}{0.0174216 \times 287} = 395.85 \text{ K}$$

Change in internal energy,

$$(\Delta U)_{12} = U_2 - U_1 = m c_v (T_2 - T_1) = 0.0174216 \times 718 (395.85 - 300) \\ = 1198.99 \text{ J} = 1.199 \text{ kJ}$$

From first law of thermodynamics for control mass, the heat transfer,

$$Q_{1 \rightarrow 2} = (\Delta U)_{12} + W_{1 \rightarrow 2} = 1.199 - 1.917 = -0.718 \text{ kJ}$$

The negative sign indicates that the heat is lost by the system.

Problem 4.2.

A piston cylinder device contains 0.2 kg of a gas initially at $P_1 = 1000 \text{ kPa}$ and $V_1 = 0.02 \text{ m}^3$. It undergoes polytropic expansion to a final pressure of 200 kPa during which the relation between pressure and volume is $PV^3 = \text{constant}$. If the specific internal energy of the gas decreases by 160 kJ/kg during the process, determine the heat transfer for the process.

Solution: Given,

Mass of gas, $m = 0.2 \text{ kg}$

Initial pressure of gas, $P_1 = 1000 \text{ kPa}$

Initial volume of gas, $V_1 = 0.02 \text{ m}^3$

Final pressure of gas, $P_2 = 200 \text{ kPa}$

Pressure-volume relation: $PV^n = \text{const} \Rightarrow \text{Polytropic index, } n = 3$

Decrease in specific internal energy, $du = -160 \text{ kJ/kg}$

For the expansion process from state 1 to 2, we can write:

$$PV_1^n = P_2 V_2^n \Rightarrow V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}} = 0.02 \left(\frac{1000}{200} \right)^{\frac{1}{3}} = 0.0342 \text{ m}^3$$

Work transfer for a polytropic process,

$$W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{200 \times 10^3 \times 0.0342 - 1000 \times 10^3 \times 0.02}{1-3} = 6580 \text{ J} = 6.58 \text{ kJ}$$

From first law of thermodynamics for control mass, the heat transfer.

$$Q_{1-2} = (\Delta U)_{12} + W_{1-2} = m \times du + W_{1-2} = -0.2 \times 160 + 6.58 = -25.42 \text{ kJ}$$

The negative sign indicates that the heat is lost by the system.

Problem 4.3.

A rigid vessel initially contains 4 kg of a saturated water vapor. It is cooled to a final state where the temperature is 150°C and quality is 0.1. Determine the initial temperature and the heat transfer from the system.

Solution: Given,

Mass of saturated water vapor, $m_s = 4 \text{ kg}$

Final temperature, $T_2 = 150^\circ\text{C}$

Final quality, $x_2 = 0.1$

As the final state is two phase mixture, referring to the saturated water temperature table A2.2 for temperature, $T_2 = 150^\circ\text{C}$,

$$v_t = 0.001090 \text{ m}^3/\text{kg}, v_{fg} = 0.3918 \text{ m}^3/\text{kg} \text{ and } v_g = 0.3929 \text{ m}^3/\text{kg}$$

Specific volume at final state,

$$v_2 = v_t + x_2 v_{fg} = 0.001090 + 0.1 \times 0.3918 = 0.04027 \text{ m}^3/\text{kg}$$

As vessel contains 4 kg of saturated vapor only, total mass of water,

$$m = m_t + m_s = 0 + 4 = 4 \text{ kg}$$

Volume of water, $V_2 = m \times v_2 = 4 \times 0.04027 = 0.16108 \text{ m}^3$

Since the vessel is rigid, the process is constant volume cooling process. Hence,

$$V_1 = V_2 \Rightarrow \text{specific volume at initial state, } v_1 (= v_t) = v_2 = 0.04027 \text{ m}^3/\text{kg}$$

Referring to the saturated water temperature table A2.2 for specific volume $v_1 = 0.04027 \text{ m}^3/\text{kg}$ to find the initial temperature, T_1 . As there is no value listed

for $v_g = 0.04027 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_a = 260^\circ\text{C}$$

$$v_{ga} = 0.04219 \text{ m}^3/\text{kg}$$

$$T_i = ?$$

$$v_g = 0.04027 \text{ m}^3/\text{kg}$$

$$T_b = 265^\circ\text{C}$$

$$v_{gb} = 0.03876 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$T_i = T_a + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (T_b - T_a) = 260 + \frac{0.04027 - 0.04219}{0.03876 - 0.04219} (265 - 260)$$

$$= 262.8^\circ\text{C}$$

Similarly, to find specific internal energy at initial state 1 by interpolation method

$$u_{ga} = 2598.4 \text{ kJ/kg}$$

$$v_{ga} = 0.04219 \text{ m}^3/\text{kg}$$

$$u_i = ?$$

$$v_g = 0.04027 \text{ m}^3/\text{kg}$$

$$u_{gb} = 2596.0 \text{ kJ/kg}$$

$$v_{gb} = 0.03876 \text{ m}^3/\text{kg}$$

Then,

$$u_i = u_{ga} + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (u_{gb} - u_{ga}) = 2598.4 + \frac{0.04027 - 0.04219}{0.03876 - 0.04219} (2596.0 - 2598.4)$$

$$= 2597.056 \text{ kJ/kg}$$

Referring to the saturated water temperature table A2.2 for temperature,

$$T_2 = 150^\circ\text{C}, u_i = 631.80 \text{ kJ/kg}, u_{lg} = 1927.7 \text{ kJ/kg} \text{ and } u_g = 2559.5 \text{ kJ/kg}$$

Specific internal energy at final state,

$$u_2 = u_i + x_2 u_{lg} = 631.8 + 0.1 \times 1927.7 = 824.57 \text{ kJ/kg}$$

Change in total internal energy,

$$(\Delta U)_{12} = U_2 - U_1 = m(u_2 - u_1) = 4(824.57 - 2597.056) = -7089.944 \text{ kJ}$$

Work transfer, $W_{1-2} = P(V_2 - V_1) = 0$ (As constant volume process)

From the first law of thermodynamics for control mass, the heat transfer

$$Q_{1-2} = (\Delta U)_{12} + W_{1-2} = -7089.944 + 0 = -7089.944 \text{ kJ}$$

The negative sign indicates that the heat is lost by the system.

Problem 4.4.

A rigid vessel with a volume of 0.1 m^3 contains water initially at 500 kPa with a quality of 60% . A heater is turned on heating the water at a rate of 2 kW . Determine the time required to vaporize all the liquid.

Solution: Given,

Volume of rigid vessel, $V = 0.1 \text{ m}^3$

Initial pressure, $P_1 = 500 \text{ kPa}$

Initial quality, $x_1 = 0.6$

Thermal power, $\dot{Q} = 2 \text{ kW} = 2000 \text{ W}$

At initial state 1:

As the quality is given, initial state is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 500 \text{ kPa}$,

$$v_1 = 0.001093 \text{ m}^3/\text{kg}, v_{lg} = 0.3738 \text{ m}^3/\text{kg} \text{ and } v_g = 0.3749 \text{ m}^3/\text{kg}$$

$$u_1 = 639.84 \text{ kJ/kg}, u_{lg} = 1921.4 \text{ kJ/kg} \text{ and } u_g = 2561.2 \text{ kJ/kg}$$

Specific volume at initial state,

$$v_1 = v_l + x_1 v_{lg} = 0.001093 + 0.6 \times 0.3738 = 0.22537 \text{ m}^3/\text{kg}$$

Specific internal energy at initial state,

$$u_1 = u_l + x_1 u_{lg} = 639.84 + 0.6 \times 1921.4 = 1792.68 \text{ kJ/kg}$$

At final state 2:

Since the vessel is rigid, the vaporization process is constant volume heating and all the water is in saturated vapor state for complete vaporization. Hence, $V_2 = V_1 = V \Rightarrow$ specific volume at final state, $v_2 (\approx v_g) = v_1 = 0.22537 \text{ m}^3/\text{kg}$.

Mass of water, $m = V/v_2 = V/v_1 = 0.1/0.22537 = 0.4437 \text{ kg}$

Referring to the saturated water pressure table A2.1 for specific volume $v_2 = v_g = 0.22537 \text{ m}^3/\text{kg}$ to find the final specific internal energy, u_2 . As there is no value listed for $v_g = 0.22537 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find the required specific internal energy as

$$u_{ga} = 2578.5 \text{ kJ/kg}$$

$$v_{ga} = 0.2269 \text{ m}^3/\text{kg}$$

$$u_2 = ?$$

$$v_g = 0.22537 \text{ m}^3/\text{kg}$$

$$u_{gb} = 2580.2 \text{ kJ/kg}$$

$$v_{gb} = 0.2149 \text{ m}^3/\text{kg}$$

Now,

$$u_2 = u_{ga} + \frac{v_g - v_{ga}}{v_{gb} - v_{ga}} (u_{gb} - u_{ga}) = 2578.5 + \frac{0.22537 - 0.2269}{0.2149 - 0.2269} (2580.2 - 2578.5)$$

$$= 2578.7167 \text{ kJ/kg}$$

Change in internal energy,

$$(\Delta U)_{12} = U_2 - U_1 = m (u_2 - u_1) = 0.4437 (2578.7167 - 1792.68) = 348.776 \text{ kJ}$$

Work transfer, $W_{1-2} = 0$ (As the process is constant volume)

From first law of thermodynamics for control mass, heat transfer,

$$Q_{1-2} = (\Delta U)_{12} + W_{1-2} = 348.776 + 0 = 348.776 \text{ kJ}$$

We know, thermal power,

$$\dot{Q} = \frac{Q_{1-2}}{t} \Rightarrow \text{Time of vaporization, } t = \frac{Q_{1-2}}{\dot{Q}} = \frac{348.776 \times 10^3}{2000} \\ = 174.388 \text{ sec} \\ = 2 \text{ min } 54.388 \text{ sec}$$

Problem 4.5.

A piston cylinder device shown in figure loaded with a linear spring ($k = 20 \text{ kN/m}$) contains 0.5 kg of H_2O initially at a pressure of 200 kPa and

a volume of 0.4 m^3 . Heat is transferred to the H_2O until a final pressure of 400 kPa is reached. If the cross sectional area of the piston is 0.05 m^2 , determine the final temperature and the heat transfer for the process.

Solution: Given,

Spring constant, $k = 20 \text{ kN/m}$

Mass of water, $m = 0.5 \text{ kg}$

Initial pressure, $P_1 = 200 \text{ kPa}$

Initial volume, $V_1 = 0.4 \text{ m}^3$

Final pressure, $P_2 = 400 \text{ kPa}$

Area of piston, $A_p = 0.05 \text{ m}^2$

Initial state 1: $P_1 = 200 \text{ kPa}, V_1 = 0.4 \text{ m}^3$

Specific volume of state 1, $v_1 = V_1/m = 0.4/0.5 = 0.8 \text{ m}^3/\text{kg}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 200 \text{ kPa}$,

$$v_1 = 0.001060 \text{ m}^3/\text{kg}, v_{lg} = 0.8848 \text{ m}^3/\text{kg} \text{ and } v_g = 0.8859 \text{ m}^3/\text{kg}.$$

$$u_1 = 504.59 \text{ kJ/kg}, u_{lg} = 2024.8 \text{ kJ/kg} \text{ and } u_g = 2529.4 \text{ kJ/kg}.$$

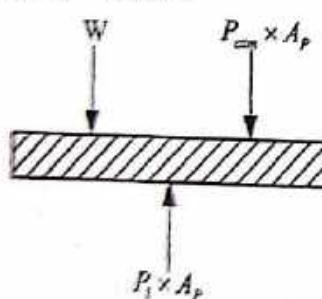
Here, $v_l < v_1 < v_g$. Hence it is a two phase mixture. Then, quality of two phase mixture at state 1,

$$x_1 = \frac{v_1 - v_l}{v_{lg}} = \frac{0.8 - 0.001060}{0.8848} = 0.90296$$

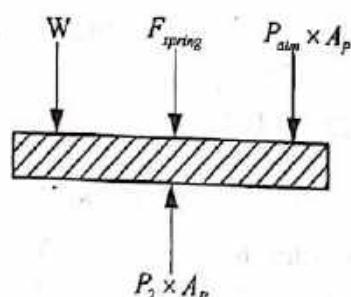
Specific internal energy at initial state,

$$u_1 = u_l + x_1 u_{lg} = 504.59 + 0.90296 \times 2024.8 = 2332.9034 \text{ kJ/kg}$$

State 2: $P_2 = 400 \text{ kPa}$



FBD at initial state 1



FBD at final state 2

Referring to the free body diagram (FBD) of piston at initial state 1, we can write,
 $P_1 \times A_p = P_{atm} \times A_p + W \quad \dots \dots \dots \text{(i)}$

Similarly, referring to the free body diagram (FBD) of piston at final state 2, we can write,

$$P_2 \times A_p = P_{atm} \times A_p + W + F_{spring} \quad \dots \dots \dots \text{(ii)}$$

Using equation (i) in (ii),

$$\Rightarrow P_2 \times A_p = P_1 \times A_p + kx_2$$

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$$\Rightarrow x_2 = (P_2 - P_1) \frac{A_p}{k} = (400 - 200) \times 10^3 \frac{0.05}{20 \times 10^3} = 0.5 \text{ m}$$

where x_2 is the compression of the spring after the piston moved from state 1 to state 2.

Now, volume at state 2, $V_2 = V_1 + A_p \times x_2 = 0.4 + 0.05 \times 0.5 = 0.425 \text{ m}^3$

Specific volume at state 2, $v_2 = V_2/m = 0.425/0.5 = 0.85 \text{ m}^3/\text{kg}$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$,

$v_l = 0.001084 \text{ m}^3/\text{kg}$, $v_{lg} = 0.4614 \text{ m}^3/\text{kg}$ and $v_g = 0.4625 \text{ m}^3/\text{kg}$.

Here, $v_2 > v_g$. Hence it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 400 \text{ kPa}$ and specific volume, $v_2 = 0.85 \text{ m}^3/\text{kg}$ to find temperature at state 2, T_2 . As there is no value listed for $v_2 = 0.85 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_a = 450^\circ\text{C} \quad v_{ga} = 0.8311 \text{ m}^3/\text{kg}$$

$$T_2 = ? \quad v_g = 0.85 \text{ m}^3/\text{kg}$$

$$T_b = 500^\circ\text{C} \quad v_{gb} = 0.8894 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$T_2 = T_a + \frac{v_2 - v_{ga}}{v_{gb} - v_{ga}} (T_b - T_a) = 450 + \frac{0.85 - 0.8311}{0.8894 - 0.8311} (500 - 450)$$

$$= 466.209^\circ\text{C}$$

Similarly, to find specific internal energy at initial state 2 (u_2) by interpolation method as

$$u_{ga} = 3046 \text{ kJ/kg} \quad v_{ga} = 0.8311 \text{ m}^3/\text{kg}$$

$$u_2 = ? \quad v_2 = 0.85 \text{ m}^3/\text{kg}$$

$$u_{gb} = 3129.3 \text{ kJ/kg} \quad v_{gb} = 0.8894 \text{ m}^3/\text{kg}$$

Then,

$$u_2 = u_{ga} + \frac{v_2 - v_{ga}}{v_{gb} - v_{ga}} (u_{gb} - u_{ga}) = 3046 + \frac{0.85 - 0.8311}{0.8894 - 0.8311} (3129.3 - 3046)$$

$$= 3073.0046 \text{ kJ/kg}$$

Change in internal energy,

$$(\Delta U)_{12} = U_2 - U_1 = m (u_2 - u_1) = 0.5 (3073.0046 - 2332.9034) = 370.0506 \text{ kJ}$$

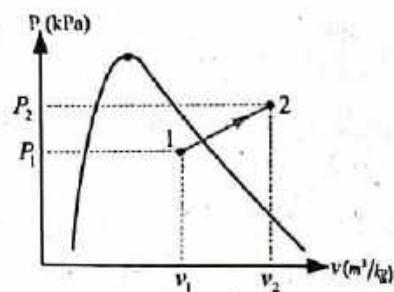
The process curve is linear due to the spring and is shown on $P - v$ diagram in figure below. The total work transfer, W_{1-2} can be calculated as the area covered by the process curve 1 - 2 as

$$W_{1-2} = \frac{1}{2} (P_1 + P_2)(V_2 - V_1) = \frac{1}{2} (200 + 400)(0.425 - 0.4)$$

$$= 7.5 \text{ kJ}$$

From first law of thermodynamics for control mass, heat transfer,

$$\begin{aligned} Q_{1-2} &= (\Delta U)_{12} + W_{1-2} \\ &= 370.0504 + 7.5 \\ &= 377.5504 \text{ kJ} \end{aligned}$$



Problem 4.6.

Nitrogen (5 kg) is contained in a piston cylinder device shown in figure initially at a pressure of 800 kPa and a temperature of 127°C. There is a heat transfer to the system until the temperature reaches to 527°C. It takes a pressure of 1500 kPa to lift the piston. Sketch the process on P-V and T-V diagrams and determine the total work and heat transfer in the process.



[Take R = 297 J/kgK and c_v = 743 J/kgK].

Solution: Given,

Mass of nitrogen, $m = 5 \text{ kg}$

Initial pressure, $P_1 = 800 \text{ kPa}$

Initial temperature, $T_1 = 127^\circ\text{C} = 127 + 273 = 400 \text{ K}$

Final temperature, $T_f = 527^\circ\text{C} = 527 + 273 = 800 \text{ K}$

Lifting pressure of piston, $P_{lift} = 1500 \text{ kPa}$

Initial state 1: $P_1 = 800 \text{ kPa}$, $T_1 = 400 \text{ K}$

Volume at state 1, V_1 can be determined by using equation of state for ideal gas as

$$P_1 V_1 = m R T_1 \Rightarrow V_1 = \frac{m R T_1}{P_1} = \frac{5 \times 297 \times 400}{800 \times 10^3} = 0.7425 \text{ m}^3$$

The heating process is shown on P-V and T-V diagrams in figures below. During the initial heating the piston remains stationary until the initial pressure reaches the given lifting pressure of the piston. Hence, process 1 - 2 is the constant volume heating and state 2 is defined as

State 2: Volume, $V_2 = V_1 = 0.7425 \text{ m}^3$ and pressure, $P_2 = P_{lift} = 1500 \text{ kPa}$

From equation of state, temperature at state 2,

$$P_2 V_2 = m R T_2 \Rightarrow T_2 = \frac{P_2 V_2}{m R} = \frac{1500 \times 10^3 \times 0.7425}{5 \times 297} = 750 \text{ K}$$

But the given final temperature is $T_f = 800 \text{ K}$. So, the system has to be further heated to reach the final temperature. This further heating process 2 - 3 is a constant pressure process. Hence, state 3 is defined as

Final state 3: Pressure, $P_3 = P_2 = 1500 \text{ kPa}$, and temperature, $T_3 = T_f = 800 \text{ K}$.

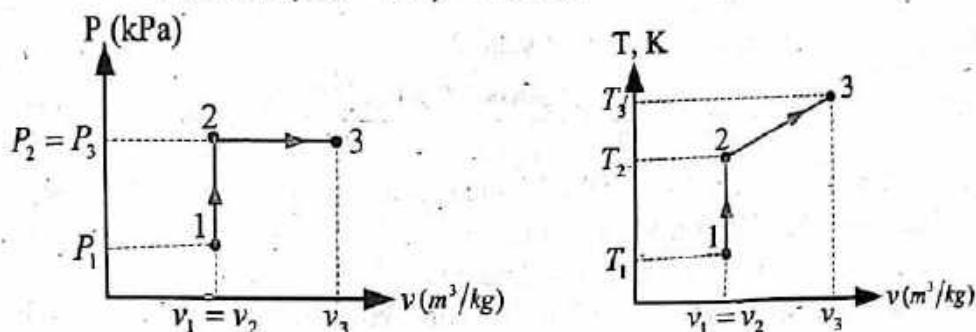
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Volume at state 3, V_3 can be determined by using equation of state for an ideal gas as

$$P_3 V_3 = m R T_3 \Rightarrow V_3 = \frac{m R T_3}{P_3} = \frac{5 \times 297 \times 800}{1500 \times 10^3} = 0.792 \text{ m}^3$$

Total change in internal energy during the process 1-3,

$$\begin{aligned} (\Delta U)_{13} &= U_3 - U_1 = m c_v (T_3 - T_1) \\ &= 5 \times 743 \times (800 - 400) = 1486 \text{ kJ} \end{aligned}$$



$$\begin{aligned} \text{Total work transfer, } W_t &= W_{1-2} + W_{2-3} = 0 + P_2 (V_3 - V_2) \\ &= 1500 \times 10^3 (0.792 - 0.7425) = 74.25 \text{ kJ} \end{aligned}$$

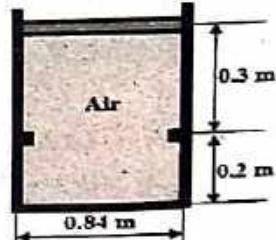
From first law of thermodynamics for control mass, total heat transfer,

$$Q_t = (\Delta U)_{13} + W_t = 1486 + 74.25 = 1560.25 \text{ kJ}$$

Problem 4.7.

Air is contained in a piston cylinder device shown in figure initially at a pressure and temperature of 1000 kPa and 800°C . Heat is lost by the system until its pressure drops to 750 kPa. Sketch the process on $P-V$ and $T-V$ diagrams and determine the total work and heat transfer.

[Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$]



Solution: Given,

Initial pressure, $P_i = 1000 \text{ kPa}$

Initial temperature, $T_i = 800^\circ\text{C} = 800 + 273 = 1073 \text{ K}$

Final pressure, $P_f = 750 \text{ kPa}$

Initial state 1: $P_i = 1000 \text{ kPa}$, $T_i = 1073 \text{ K}$

Volume of air at state 1, V_i can be determined from the system diagram,

$$V_i = \frac{\pi D_p^2}{4} \times h_i = \frac{\pi (0.84)^2}{4} \times (0.2 + 0.3) = 0.2771 \text{ m}^3$$

From the equation of state, mass of air

$$P_i V_i = m R T_i \Rightarrow m = \frac{P_i V_i}{R T_i} = \frac{1000 \times 10^3 \times 0.2771}{287 \times 1073} = 0.89978 \text{ kg}$$

The cooling process is shown on $P - V$ and $T - V$ diagrams in figures below. When heat is lost by the system, piston moves downwards until it reaches the stops with constant pressure. Hence, the process 1 – 2 is constant pressure cooling. Then, state 2 is defined as

State 2: Pressure, $P_2 = P_f = 1000 \text{ kPa}$

Volume of air at state 2, V_2 can be determined from the system diagram,

$$V_2 = \frac{\pi D^2}{4} \times h = \frac{\pi (0.84)^2}{4} \times (0.2) = 0.110835 \text{ m}^3$$

From equation of state, temperature at state 2,

$$PV_2 = mRT_2 \Rightarrow T_2 = \frac{P_2 V_2}{mR} = \frac{1000 \times 10^3 \times 0.110835}{0.89978 \times 287} = 429.2 \text{ K}$$

Since the final pressure is $P_f = 750 \text{ kPa}$, the system has still to be further cooled to attain final pressure. The further cooling process 2 – 3 is constant volume cooling due to presence of stops. So, state 3 is defined as

Final state 3: Pressure, $P_3 = P_f = 750 \text{ kPa}$, and volume, $V_3 = V_2 = 0.110835 \text{ m}^3$.

From equation of state, temperature at state 3,

$$PV_3 = mRT_3 \Rightarrow T_3 = \frac{P_3 V_3}{mR} = \frac{750 \times 10^3 \times 0.110835}{0.89978 \times 287} = 321.899 \text{ K}$$

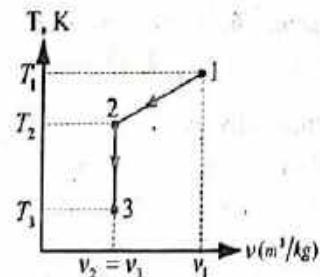
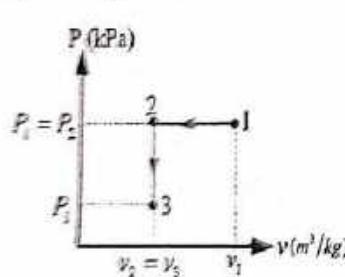
Total change in internal energy during the process 1 – 3,

$$\begin{aligned} (\Delta U)_{13} &= U_3 - U_1 = m c_v (T_3 - T_1) \\ &= 0.89978 \times 718 \times (321.899 - 1073) = -485.243 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{Total work transfer, } W_t &= W_{1-2} + W_{2-3} = P_1 (V_2 - V_1) + 0 \\ &= 1000 \times 10^3 (0.110835 - 0.2771) = -166.265 \text{ kJ} \end{aligned}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_t = (\Delta U)_{13} + W_t = -485.243 - 166.265 = -651.508 \text{ kJ}$$



Problem 4.8.

Air (0.4 kg) is contained in a piston cylinder device shown in figure initially at a pressure of 1500 kPa and 800 K . The cylinder has stops such that the minimum volume of the system is 0.04 m^3 . The air in the cylinder is cooled to 300 K . Sketch the process on $P - V$ and $T - V$ diagrams and determine

- (a) the final volume and pressure of the air, and
- (b) the total work and heat transfer in the process.

(Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$).

Solution: Given,

Mass of air, $m = 0.4 \text{ kg}$

Initial pressure, $P_i = 1500 \text{ kPa}$

Initial temperature, $T_i = 800 \text{ K}$

Minimum volume, $V_{\min} = 0.04 \text{ m}^3$

Final temperature, $T_f = 300 \text{ K}$

Initial state 1: $P_i = 1500 \text{ kPa}$, $T_i = 800 \text{ K}$

From the equation of state, volume at state 1, V_i can be determined as

$$P_i V_i = m R T_i \Rightarrow V_i = \frac{m R T_i}{P_i} = \frac{0.4 \times 287 \times 800}{1500 \times 10^3} = 0.06123 \text{ m}^3$$



The cooling process is shown on $P - V$ and $T - V$ diagrams in figures below. When heat is lost by the system, piston moves downwards until it reaches the stops with constant pressure. Hence, the process 1 – 2 is constant pressure cooling. Then, state 2 is defined as

State 2: Pressure, $P_2 = P_i = 1500 \text{ kPa}$

Volume of air at state 2, V_2 can be determined as

$$V_2 = V_{\min} = 0.04 \text{ m}^3$$

From equation of state, temperature at state 2, T_2 can be calculated as

$$P_2 V_2 = m R T_2 \Rightarrow T_2 = \frac{P_2 V_2}{m R} = \frac{1500 \times 10^3 \times 0.04}{0.4 \times 287} = 522.65 \text{ K}$$

Since the final temperature is $T_f = 300 \text{ K}$, the system has still to be further cooled to attain final temperature. The further cooling process 2 – 3 is constant volume cooling due to presence of stops. So, state 3 is defined as

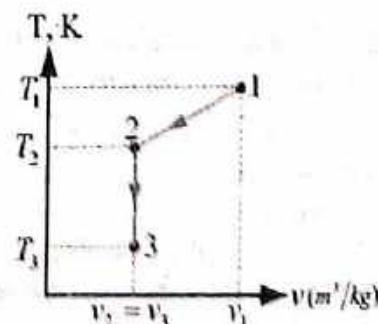
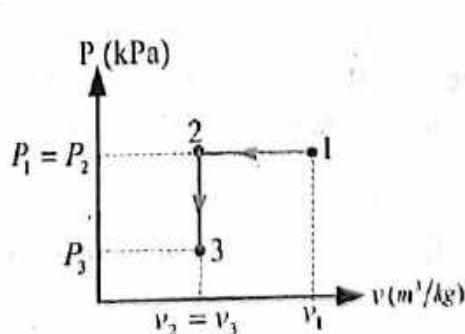
Final state 3: Temperature, $T_3 = T_f = 300 \text{ K}$, and volume, $V_3 = V_2 = 0.04 \text{ m}^3$.

From equation of state, pressure at state 3, P_3 can be found out which will be the final pressure,

$$P_3 V_3 = m R T_3 \Rightarrow P_3 = \frac{m R T_3}{V_3} = \frac{0.4 \times 287 \times 300}{0.04} = 861 \text{ kPa} = P_f$$

Total change in internal energy during the process 1 – 3,

$$\begin{aligned} (\Delta U)_{13} &= U_3 - U_1 = m c_v (T_3 - T_i) = 0.4 \times 718 \times (300 - 800) \\ &= -143600 \text{ J} = -143.6 \text{ kJ} \end{aligned}$$



$$\text{Total work transfer, } W_i = W_{1,2} + W_{2,3} = P_i(V_2 - V_1) + 0$$

$$= 1500 \times 10^3 (0.04 - 0.06123) = -31845 \text{ J} = -31.845 \text{ kJ}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_i = (\Delta U)_{1,3} + W_i = -143.6 - 31.845 = -175.445 \text{ kJ}$$

Problem 4.9.

Air (0.1 kg) is contained in piston-cylinder assembly as shown in figure below. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is 100 kN/m . The piston weighs 30 kN and atmospheric pressure is 101 kPa . The air is initially at 300 K and 200 kPa . Heat transfer occurs until the air temperature reaches the surrounding temperature of 700 K .

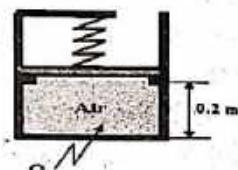
(a) Find the final pressure and volume.

(b) Find the process work.

(c) Find the heat transfer.

(d) Draw the P-V diagram of the process.

[Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$].



Solution: Given.

Mass of air, $m = 0.1 \text{ kg}$

Spring constant, $k = 100 \text{ kN/m}$

Piston weight, $W = 30 \text{ kN}$

Atmospheric pressure, $P_{atm} = 101 \text{ kPa}$

Initial temperature of air, $T_i = 300 \text{ K}$

Initial pressure of air, $P_i = 200 \text{ kPa}$

Final temperature of air, $T_f = 700 \text{ K}$

From the system diagram, area of piston can be determined as

$$A_p = \frac{V_i}{h} = \frac{0.04305}{0.2} = 0.21525 \text{ m}^2$$

Initial state 1: $P_i = 200 \text{ kPa}$, $T_i = 300 \text{ K}$

From the equation of state, volume at state 1, V_i can be determined as

$$P_i V_i = m R T_i \Rightarrow V_i = \frac{m R T_i}{P_i} = \frac{0.1 \times 287 \times 300}{200 \times 10^3} = 0.04305 \text{ m}^3$$

The heating process is shown on $P - V$ diagram in figure below. During the initial heating the piston remains stationary until the initial pressure reaches lifting pressure of the piston. Hence, process 1 – 2 is the constant volume heating and state 2 is defined as

State 2: Volume, $V_2 = V_i = 0.04305 \text{ m}^3$ and pressure, $P_2 = P_{lift}$

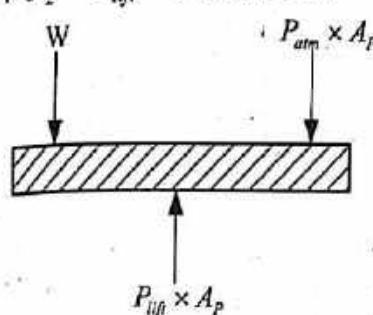
To calculate the lifting pressure of the piston, referring to the free body diagram (FBD) of piston at state 2, we can write,

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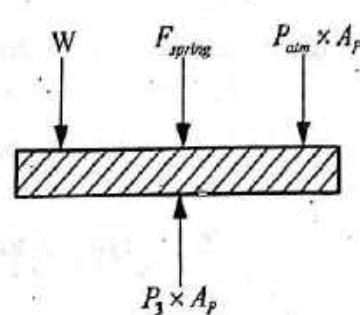
$$P_{lift} \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_{lift} = P_{atm} + W / A_p = 101 + 30 / 0.21525 = 240.373 \text{ kPa}$$

$$\therefore P_2 = P_{lift} = 240.373 \text{ kPa}$$



FBD at state 2



FBD at final state

From equation of state, temperature at state 2, T_2 can be calculated as

$$P_2 V_2 = m R T_2 \Rightarrow T_2 = \frac{P_2 V_2}{m R} = \frac{240.373 \times 10^3 \times 0.04305}{0.1 \times 287} = 360.56 \text{ K}$$

Since the final temperature is $T_f = 700 \text{ K}$, the system has still to be further heated to attain this final temperature. The further heating process 2 - 3 is linear due to presence of the spring. Hence, state 3 is defined as

Final state 3: Temperature, $T_3 = T_f = 700 \text{ K}$, and volume at state 3,

$$V_3 = V_2 + x \times A_p = 0.04305 + x (0.21525) = 0.21525 (0.2 + x) \quad \dots \dots \dots \text{(i)}$$

Pressure at state 3, P_3 can be determined by referring to the free body diagram (FBD) of piston at final state which will be the final pressure,

$$P_3 \times A_p = P_{atm} \times A_p + W + F_{spring}$$

$$\Rightarrow P_3 = P_{atm} + W / A_p + kx / A_p = 240.373 + 100x / 0.21525 \text{ kPa} \quad \dots \dots \text{(ii)}$$

where x is the compression of the spring. Pressure at state 3, P_3 can also be calculated by using equation of state,

$$P_3 V_3 = m R T_3 \Rightarrow P_3 = \frac{m R T_3}{V_3} = \frac{0.1 \times 287 \times 700}{0.21525(0.2 + x)} = \frac{93.333}{0.2 + x} \text{ kPa} \quad \dots \dots \text{(iii)}$$

Equating equations (ii) and (iii) yields,

$$240.373 + \frac{100x}{0.21525} = \frac{93.333}{0.2 + x}$$

$$\Rightarrow 51.74 + 100x = \frac{20.09}{0.2 + x}$$

$$\Rightarrow 100x^2 + 71.74x - 9.74 = 0$$

$$\Rightarrow x = \frac{-71.74 \pm \sqrt{(71.74)^2 - 4 \times 100 \times (-9.74)}}{2 \times 100}$$

$$= 0.11676 \text{ m} \quad (\text{taking positive value})$$

Putting this value of x in equations (i) and (iii), we get

$$V_3 = 0.21525(0.2 + 1.11676) = 0.06818 \text{ m}^3 \Rightarrow V_f = 0.06818 \text{ m}^3$$

$$\text{Similarly, } P_3 = \frac{93.333}{0.2 + x} = 294.646 \text{ kPa} \Rightarrow P_f = 294.646 \text{ kPa}$$

Total change in internal energy during the process 1 - 3,

$$(\Delta U)_{13} = U_3 - U_1 = m c_v (T_3 - T_1) \\ = 0.1 \times 718 \times (700 - 300) = 28720 \text{ J} = 28.72 \text{ kJ}$$

Total work transfer,

$$W_t = W_{12} + W_{23} = 0 + \frac{1}{2} (P_2 + P_3)(V_3 - V_2)$$

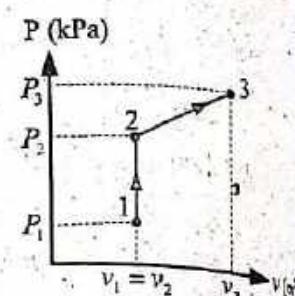
$$\approx 0.5(240.373 + 294.646) \times 10^3 \times$$

$$(0.06818 - 0.04305) = 6722.51 \text{ J} = 6.72 \text{ kJ}$$

From first law of thermodynamics for control mass,

total heat transfer,

$$Q_i = (\Delta U)_{13} + W_t = 28.72 + 6.72 = 35.44 \text{ kJ}$$



Problem 4.10.

Water (1.5 kg) is contained in a piston cylinder device shown in figure initially at a pressure of 400 kPa with a quality of 50 %. There is a heat transfer to the system until it reaches a final temperature of 500°C. It takes a pressure of 800 kPa to lift the piston. Sketch the process on P - v and T - v diagrams and determine the total work and heat transfer.

Solution: Given,

Mass of water, $m = 1.5 \text{ kg}$

Initial pressure, $P_i = 400 \text{ kPa}$

Initial quality, $x_i = 50\% = 0.5$

Final temperature, $T_f = 500^\circ\text{C}$

Lifting pressure of piston, $P_{lift} = 800 \text{ kPa}$

Initial state 1: $P_i = 400 \text{ kPa}$, $x_i = 0.5$

As the quality is given, initial state is a two phase mixture.

Referring to the saturated water pressure table A2.1 for pressure, $P_i = 400 \text{ kPa}$,

$$v_f = 0.001084 \text{ m}^3/\text{kg}, v_{lg} = 0.4614 \text{ m}^3/\text{kg} \text{ and } v_g = 0.4625 \text{ m}^3/\text{kg}.$$

$$u_f = 604.47 \text{ kJ/kg}, u_{lg} = 1949.0 \text{ kJ/kg} \text{ and } u_g = 2553.5 \text{ kJ/kg}.$$

Specific volume at initial state 1,

$$v_i = v_f + x_i v_{lg} = 0.001084 + 0.5 \times 0.4614 = 0.23178 \text{ m}^3/\text{kg}$$

Specific internal energy at initial state 1,

$$u_i = u_f + x_i u_{lg} = 604.47 + 0.5 \times 1949.0 = 1578.97 \text{ kJ/kg}$$

The heating process is shown on $P - v$ and $T - v$ diagrams in figures below. During the initial heating the piston remains stationary until the initial pressure

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reaches the given lifting pressure of the piston. Hence, process 1 - 2 is the constant volume heating and state 2 is defined as

State 2: Volume, $V_2 = V_1 \Rightarrow$ Specific volume, $v_2 = v_1 = 0.23178 \text{ m}^3/\text{kg}$

And pressure, $P_2 = P_{lift} = 800 \text{ kPa}$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 800 \text{ kPa}$,

$$v_1 = 0.001115 \text{ m}^3/\text{kg}, v_{lg} = 0.2393 \text{ m}^3/\text{kg} \text{ and } v_g = 0.2404 \text{ m}^3/\text{kg}.$$

Here, $v_1 < v_2 < v_g$. Hence, it is still a two phase mixture. Then, temperature at state 2 is the saturation temperature at given pressure. This means $T_2 = T_{sat} (P_2 = 800 \text{ kPa}) = 170.44^\circ\text{C}$ which is less than given final temperature, $T_f = 500^\circ\text{C}$. Therefore, the system has to be heated further to attain final temperature. This further heating process 2 - 3 occurs at the constant pressure and final state 3 is defined as

Final state 3: Pressure, $P_3 = P_2 = 800 \text{ kPa}$ and Temperature, $T_3 = T_f = 500^\circ\text{C}$,

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 800 \text{ kPa}$, the saturation temperature, $T_{sat} (P_3 = 800 \text{ kPa}) = 170.44^\circ\text{C}$. Here, $T_3 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 800 \text{ kPa}$ and temperature, $T_3 = 500^\circ\text{C}$, specific volume at state 3, $v_3 = 0.4433 \text{ m}^3/\text{kg}$ and specific internal energy at state 3,

$$u_3 = 3126.1 \text{ kJ/kg.}$$

Total change in internal energy during the process 1 - 3,

$$(\Delta U)_{13} = U_3 - U_1 = m(u_3 - u_1) = 1.5(3126.1 - 1578.97) = 2320.695 \text{ kJ}$$

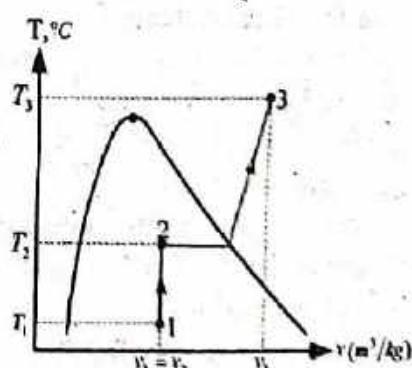
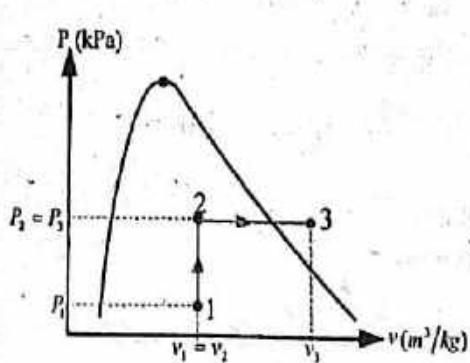
Total work transfer, $W_t = W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = P_2 \times m(v_3 - v_2)$

$$= 800 \times 10^3 \times 1.5(0.4433 - 0.23178)$$

$$= 253824 \text{ J} = 253.824 \text{ kJ}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_t = (\Delta U)_{13} + W_t = 2320.695 + 253.824 = 2574.519 \text{ kJ}$$

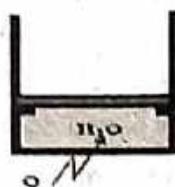


Problem 4.11.

Water (4 kg) is contained in a piston cylinder device shown in figure below initially at a pressure of 100 kPa with a quality of 10 %. The piston has a mass of 100 kg and a cross sectional area of 24.525 cm². Heat is now added until

H_2O reaches a saturated vapor state. Sketch the process on $P-v$ and $T-v$ diagrams and determine

- the initial volume
- the final pressure
- the total work transfer,
and
- the total heat transfer.



Solution: Given,

Mass of water, $m = 4 \text{ kg}$

Initial pressure, $P_1 = 100 \text{ kPa}$

Initial quality, $x_1 = 10\% = 0.1$

Mass of piston, $m_p = 100 \text{ kg}$

Area of piston, $A_p = 24.525 \text{ cm}^2 = 24.525 \times 10^{-4} \text{ m}^2$

Final state is saturated vapor.

Initial state 1: $P_1 = 100 \text{ kPa}, x_1 = 0.1$

As the quality is given, initial state is a two phase mixture.

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 100 \text{ kPa}$,

$$v_1 = 0.001043 \text{ m}^3/\text{kg}, v_{lg} = 1.6933 \text{ m}^3/\text{kg} \text{ and } v_g = 1.6943 \text{ m}^3/\text{kg}.$$

$$u_1 = 417.41 \text{ kJ/kg}, u_{lg} = 2088.3 \text{ kJ/kg} \text{ and } u_g = 2505.7 \text{ kJ/kg}.$$

Specific volume at initial state 1,

$$v_1 = v_l + x_1 v_{lg} = 0.001043 + 0.1 \times 1.6933 = 0.170373 \text{ m}^3/\text{kg}$$

Initial volume at state 1, $V_1 = m \times v_1 = 4 \times 0.170373 = 0.681492 \text{ m}^3$

Specific internal energy at initial state,

$$u_1 = u_l + x_1 u_{lg} = 417.41 + 0.1 \times 2088.3 = 626.24 \text{ kJ/kg}$$

The heating process is shown on $P-v$ and $T-v$ diagrams in figures below.

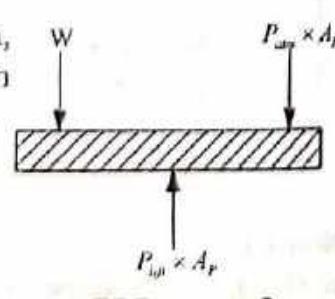
During the initial heating the piston remains stationary until the initial pressure reaches lifting pressure of the piston. Hence, process 1 - 2 is the constant volume heating and state 2 is defined as

State 2: Volume, $V_2 = V_1 \Rightarrow$ Specific volume, $v_2 = v_1 = 0.170373 \text{ m}^3/\text{kg}$
and pressure, $P_2 = P_{lift}$

To calculate the lifting pressure of the piston, referring to the free body diagram (FBD) of piston at state 2, we can write,

$$P_{lift} \times A_p = P_{atm} \times A_p + W$$

$$\Rightarrow P_{lift} = P_{atm} + \frac{m_p \times g}{A_p} = 100 + \frac{100 \times 9.81}{24.525 \times 10^{-4}} \\ = 500 \text{ kPa} = P_2$$



FBD at state 2

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 500 \text{ kPa}$,

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$$v_l = 0.001093 \text{ m}^3/\text{kg}, v_{lg} = 0.3738 \text{ m}^3/\text{kg} \text{ and } v_g = 0.3749 \text{ m}^3/\text{kg}$$

$$u_l = 639.84 \text{ kJ/kg}, u_{lg} = 1921.4 \text{ kJ/kg} \text{ and } u_g = 2561.2 \text{ kJ/kg.}$$

Here, $v_l < v_2 < v_g$. Hence, it is still a two phase mixture. But the required final state is saturated vapor. Therefore, the system has to be heated further so that the system becomes saturated vapor. This further heating process 2 - 3 occurs at constant pressure. Then state 3 is defined as

Final state 3: Pressure, $P_3 = P_2 = 500 \text{ kPa}$

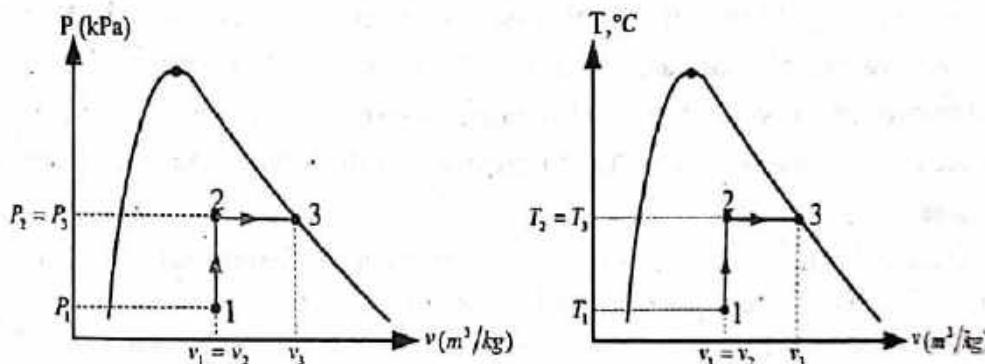
Now, the final pressure, $P_f = P_3 = 500 \text{ kPa}$

Specific volume at state 3, $v_3 = v_g (P_3 = 500 \text{ kPa}) = 0.3749 \text{ m}^3/\text{kg}$ and

specific internal energy at state 3, $u_3 = u_g (P_3 = 500 \text{ kPa}) = 2561.2 \text{ kJ/kg}$ as the system is saturated vapor.

Total change in internal energy during the process 1 - 3,

$$(\Delta U)_{13} = U_3 - U_1 = m (u_3 - u_1) = 4 (2561.2 - 626.24) = 7739.84 \text{ kJ}$$



$$\begin{aligned}\text{Total work transfer, } W_t &= W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = P_2 \times m(v_3 - v_2) \\ &= 500 \times 10^3 \times 4(0.3749 - 0.170373) \\ &= 409054 \text{ J} = 409.054 \text{ kJ}\end{aligned}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_t = (\Delta U)_{13} + W_t = 7739.84 + 409.054 = 8148.894 \text{ kJ}$$

Problem 4.12.

A piston cylinder device shown in figure below contains 4 kg of water initially at saturated liquid state at 5 MPa. There is a heat transfer to the system until it hits the stops at which time its volume is 0.08 m³. There is further heat transfer to the device until water is completely vaporized. Sketch the process on P - v and T - v diagrams and determine the total work and heat transfer.

Solution: Given,

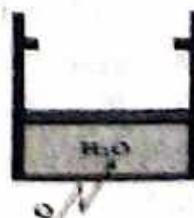
Mass of water in saturated liquid form, $m = 4 \text{ kg}$

Initial pressure, $P_i = 5 \text{ MPa} = 5000 \text{ kPa}$

Final volume, $V_f = 0.08 \text{ m}^3$

Final state is completely vaporized.

Initial state 1: $P_i = 5000 \text{ kPa}$, saturated liquid



Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 5000 \text{ kPa}$, $v_1 = 0.001286 \text{ m}^3/\text{kg}$ and $u_1 = 1147.8 \text{ kJ/kg}$.

As the initial state is a saturated liquid, the specific volume of water at state 1, $v_1 = v_f = 0.001286 \text{ m}^3/\text{kg}$ and the specific internal energy of water at state 1, $u_1 = u_f = 1147.8 \text{ kJ/kg}$.

The heating process is shown on $P - v$ and $T - v$ diagrams in figures below. When the system is heated, the piston moves upwards with constant pressure until it reaches the stops. So, process 1 – 2 is the constant pressure heating and state 2 is defined as

State 2: Pressure, $P_2 = P_1 = 5000 \text{ kPa}$ and volume, $V_2 = V_f = 0.08 \text{ m}^3$

Specific volume at state 2, $v_2 = V_2/m = 0.08/4 = 0.02 \text{ m}^3/\text{kg}$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 5000 \text{ kPa}$,

$$v_1 = 0.001286 \text{ m}^3/\text{kg}, v_{lg} = 0.03815 \text{ m}^3/\text{kg} \text{ and } v_g = 0.03944 \text{ m}^3/\text{kg}$$

Hence, $v_1 < v_2 < v_g$. Hence, it is still a two phase mixture. But the final state is completely vaporized (saturated vapor). Hence, the system has to be further heated to convert the system into the saturated vapor. This further heating process 2 – 3 occurs at constant volume due to presence of the stops. Then final state 3 is defined as

Final state 3: Volume, $V_3 = V_2 = 0.08 \text{ m}^3 \Rightarrow$ Specific volume at state 3, $v_3 = v_g = 0.02 \text{ m}^3/\text{kg}$. As the state 3 is saturated vapor, $v_3 = v_g = 0.02 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for specific volume of saturated vapor, $v_g = 0.02 \text{ m}^3/\text{kg}$ to find the specific internal energy at state 3, $u_3 = u_g$. As there is no value listed for $v_g = 0.02 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required specific internal energy as

$$u_{gc} = 2557.6 \text{ kJ/kg} \quad v_{go} = 0.02048 \text{ m}^3/\text{kg}$$

$$u_g = ? \quad v_g = 0.02 \text{ m}^3/\text{kg}$$

$$u_{gb} = 2544.2 \text{ kJ/kg} \quad v_{gb} = 0.01803 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$u_g = u_{gc} + \frac{v_g - v_{go}}{v_{gb} - v_{go}} (u_{gb} - u_{gc}) = 2557.6 + \frac{0.02 - 0.02048}{0.01803 - 0.02048} (2544.2 - 2557.6) \\ = 2554.9747 \text{ kJ/kg} = u_g$$

Total change in internal energy during the process 1 – 3,

$$(\Delta U)_{13} = U_3 - U_1 = m(u_3 - u_1) = 4(2554.9747 - 1147.8) = 5628.6988 \text{ kJ}$$

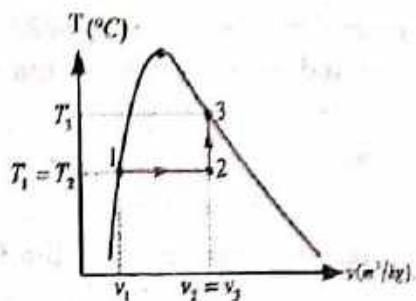
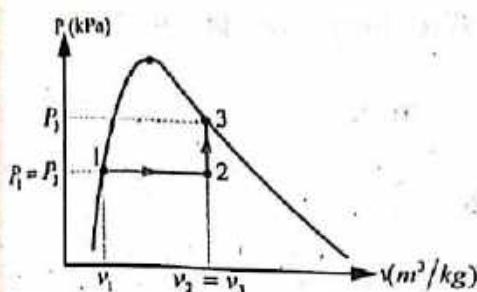
Total work transfer,

$$W_1 = W_{1,2} + W_{2,3} = P_1(V_2 - V_1) + 0 = P_1 \times m(v_2 - v_1)$$

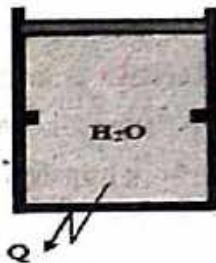
$$= 5000 \times 10^3 \times 4(0.02 - 0.001286) = 374280 \text{ J} = 374.28 \text{ kJ}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_1 = (\Delta U)_{13} + W_1 = 5628.6988 + 374.28 = 6002.9788 \text{ kJ.}$$


Problem P4.13.

Water (2 kg) is contained in a piston cylinder device shown in figure. The mass of the piston is such that the H_2O exists at a pressure of 10 MPa and a temperature of $800^{\circ}C$. There is a heat transfer from the device until the piston just rests on stops at which time the volume inside the cylinder is $2.45 \times 10^{-3} m^3$. Sketch the process on $P-v$ and $T-v$ diagrams and determine the total work transfer and total heat transfer.



Solution: Given,

Mass of water, $m = 2 \text{ kg}$

Initial pressure, $P_1 = 10 \text{ MPa} = 10000 \text{ kPa}$

Initial temperature, $T_1 = 800^{\circ}C$

Final volume, $V_f = 2.45 \times 10^{-3} m^3$

Initial state 1: $P_1 = 10000 \text{ kPa}$, $T_1 = 800^{\circ}C$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 10000 \text{ kPa}$, saturation temperature, $T_{sat}(P_1 = 10000 \text{ kPa}) = 311.03^{\circ}C$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 10000 \text{ kPa}$ and temperature, $T_1 = 800^{\circ}C$, the specific volume at state 1, $v_1 = 0.04863 \text{ m}^3/\text{kg}$ and the specific internal energy at state 1, $u_1 = 3627.2 \text{ kJ/kg}$.

The cooling process is shown on $P-v$ and $T-v$ diagrams in figures below. When heat is lost by the system, the piston moves downwards with constant pressure until it just rests on stops. So, process 1-2 is the constant pressure cooling and final state 2 is defined as

Final state 2: Pressure, $P_2 = P_1 = 10000 \text{ kPa}$; volume, $V_2 = V_f = 2.45 \times 10^{-3} m^3$

Specific volume at state 2, $v_2 = V_2/m = (2.45 \times 10^{-3})/2 = 0.001225 \text{ m}^3/\text{kg}$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 10000 \text{ kPa}$,

$$v_l = 0.001452 \text{ m}^3/\text{kg}, v_{lg} = 0.01658 \text{ m}^3/\text{kg} \text{ and } v_g = 0.01803 \text{ m}^3/\text{kg}$$

Here, $v_2 < v_l$. Hence, it is a sub-cooled or compressed liquid. Now, referring to the compressed liquid table A2.3 for pressure, $P_2 = 10000 \text{ kPa}$ and specific volume, $v_l = 0.001225 \text{ m}^3/\text{kg}$ to find the specific internal energy at state 2, u_2 . As there is

no value listed for $v_2 = 0.001225 \text{ m}^3/\text{kg}$ in the table, linear interpolation method is used to find required specific internal energy as

$$u_e = 979.55 \text{ kJ/kg} \quad v_e = 0.001199 \text{ m}^3/\text{kg}$$

$$u_2 = ? \quad v_2 = 0.001225 \text{ m}^3/\text{kg}$$

$$u_b = 1073.0 \text{ kJ/kg} \quad v_b = 0.001241 \text{ m}^3/\text{kg}$$

Using linear interpolation method,

$$u_2 = u_e + \frac{v_2 - v_e}{v_b - v_e} (u_b - u_e) = 979.55 + \frac{0.001225 - 0.001199}{0.001241 - 0.001199} (1073.0 - 979.55)$$

$$= 1037.4 \text{ kJ/kg}$$

Total change in internal energy during the process 1-2,

$$(\Delta U)_{12} = U_2 - U_1 = m(u_2 - u_1) = 2(1037.4 - 3627.2) = -5179.6 \text{ kJ}$$

Total work transfer,

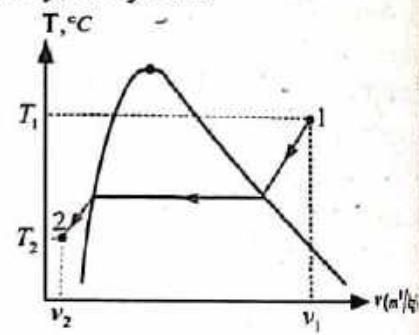
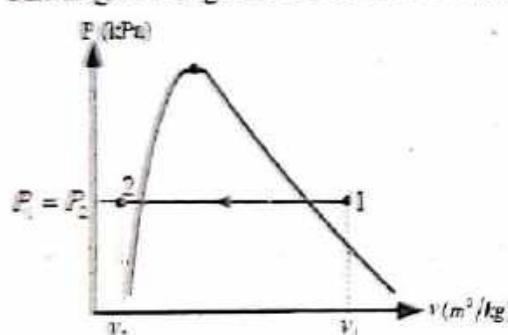
$$W_i = W_{12} = P_i(V_2 - V_1) = P_i \times m(v_2 - v_1) = 10000 \times 10^3 \times 2(0.001225 - 0.04863)$$

$$= -948100 \text{ J} = -948.1 \text{ kJ}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_i = (\Delta U)_{12} + W_i = -5179.6 - 948.1 = -6127.7 \text{ kJ}$$

The negative sign indicates that the heat is lost by the system.



Problem 4.14.

A piston cylinder device shown in figure below contains water initially at $P_1 = 1 \text{ MPa}$ and $T_1 = 500^\circ\text{C}$. A pressure of 500 kPa is required to support the piston. There is a heat transfer from the device until its temperature drops to 30°C . Sketch the process on $P-v$ and $T-v$ diagrams and determine the total work and heat transfer per kg of water.

Solution: Given,

Initial pressure, $P_1 = 1 \text{ MPa} = 1000 \text{ kPa}$

Initial temperature, $T_1 = 500^\circ\text{C}$

Supporting pressure of piston, $P_{sup} = 500 \text{ kPa}$

Final temperature, $T_f = 30^\circ\text{C}$

Initial state 1: $P_1 = 1000 \text{ kPa}$, $T_1 = 500^\circ\text{C}$



Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 1000 \text{ kPa}$, saturation temperature $T_{sat}(P_1 = 1000 \text{ kPa}) = 179.92^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 1000 \text{ kPa}$ and temperature, $T_1 = 500^\circ\text{C}$,

the specific volume at state 1, $v_1 = 0.3541 \text{ m}^3/\text{kg}$ and

the specific internal energy at state 1, $u_1 = 3124.5 \text{ kJ/kg}$.

The cooling process is shown on $P - v$ and $T - v$ diagrams in figures below. During the initial stage of cooling of the system, the piston remains stationary till the initial system pressure decreases up to the given supporting pressure of the piston. So, process 1 - 2 is the constant volume cooling and state 2 is defined as

Final state 2: Pressure, $P_2 = P_{sup} = 500 \text{ kPa}$ and volume, $V_2 = V_1 \Rightarrow$

Specific volume at state 2, $v_2 = v_1 = 0.3541 \text{ m}^3/\text{kg}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 500 \text{ kPa}$,

$v_f = 0.001093 \text{ m}^3/\text{kg}$, $v_{lg} = 0.3738 \text{ m}^3/\text{kg}$ and $v_g = 0.3749 \text{ m}^3/\text{kg}$

Here, $v_f < v_2 < v_g$. Hence, it is a two phase mixture. So, temperature at state 2 is the saturation temperature at given pressure, i.e. $T_2 = T_{sat}(P_2 = 500 \text{ kPa}) = 151.87^\circ\text{C}$. But the final temperature is given as $T_f = 30^\circ\text{C}$. Therefore, the system has to be further cooled to attain the final temperature. This further cooling process 2 - 3 occurs at constant pressure until the piston just touches the stops. Then final state 3 is defined as

Final state 3: Pressure, $P_3 = P_2 = 500 \text{ kPa}$, and temperature, $T_3 = T_f = 30^\circ\text{C}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 500 \text{ kPa}$, saturation temperature, $T_{sat}(P_3 = 500 \text{ kPa}) = 151.87^\circ\text{C}$.

Here, $T_3 < T_{sat}$. Hence, it is a sub-cooled or compressed liquid.

Now, referring to the compressed liquid table A2.3 for pressure, $P_3 = 500 \text{ kPa}$ and temperature, $T_3 = 30^\circ\text{C}$ to find the specific volume at state 3, v_3 . But there is no property data available for compressed liquid at pressure, $P_3 = 500 \text{ kPa}$ in the table. As the compression of the liquid is very less with the change of pressure, we can take the specific volume of saturated liquid (v_f) at temperature, $T_3 = 30^\circ\text{C}$ for v_3 with less error, i.e.

$$v_3 = v_f(T_3 = 30^\circ\text{C}) = 0.001004 \text{ m}^3/\text{kg} \quad (\text{from temperature table A2.2})$$

Similarly, specific internal energy at state 3, $u_3 = u_f(T_3 = 30^\circ\text{C}) = 125.67 \text{ kJ/kg}$.

Total change in specific internal energy during the process 1 - 3,

$$(\Delta u)_{13} = (u_3 - u_1) = 125.67 - 3124.5 = -2998.83 \text{ kJ/kg}$$

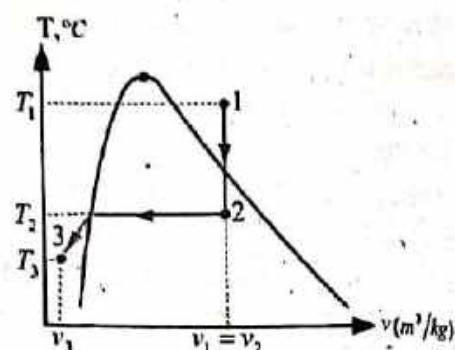
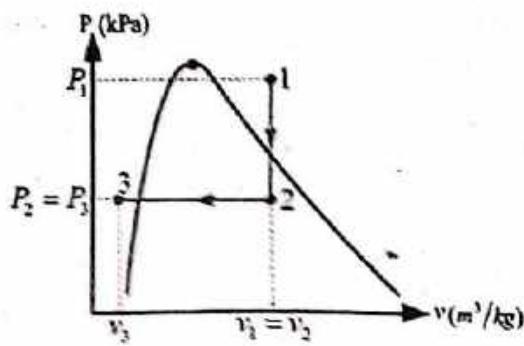
Total specific work transfer,

$$w_t = w_{1-2} + w_{2-3} = 0 + P_2(v_3 - v_2)$$

$$= 500 \times 10^3 \times (0.001004 - 0.3541) = -176548 \text{ J/kg} = -176.548 \text{ kJ/kg}$$

From first law of thermodynamics for control mass, total specific heat transfer,

$$q_t = (\Delta u)_{13} + w_t = -2998.83 - 176.548 = -3175.378 \text{ kJ/kg}$$


Problem 4.15.

A closed system undergoes a process A from state 1 to state 2 as shown in figure below which requires a heat input of $Q_A = 18.6 \text{ kJ}$. The system returns adiabatically from state 2 to state 1 through process B. Determine the work transfer for process B. [Take $\gamma = 1.4$]

Solution: Given,

Heat input for process A (state 1 - 2),

$$Q_A = Q_{1-2} = 18.6 \text{ kJ}$$

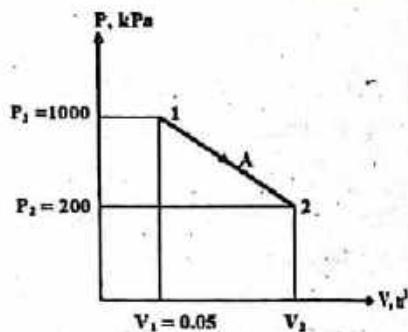
Pressure at state 1, $P_1 = 1000 \text{ kPa}$

Volume at state 1, $V_1 = 0.05 \text{ m}^3$

Pressure at state 2, $P_2 = 200 \text{ kPa}$

Process B (state 2 - 1) is adiabatic

$$\Rightarrow Q_B = Q_{2-1} = 0$$



Since process B (state 2 - 1) is adiabatic, the pressure - volume relation for an adiabatic process can be written as

$$P_2 V_2^\gamma = P_1 V_1^\gamma \Rightarrow V_2 = V_1 (P_1/P_2)^{1/\gamma} = 0.05 (1000/200)^{1/1.4} = 0.157846 \text{ m}^3$$

Work transfer for process A (state 1 - 2) can be calculated as the area under the process curve as

$$\begin{aligned} W_{1-2} &= W_A = \frac{1}{2} (P_1 + P_2)(V_2 - V_1) = 0.5(1000 + 200)(0.157846 - 0.05) \\ &= 64.7076 \text{ kJ} \end{aligned}$$

From the first law of thermodynamics for process A, change in internal energy is determined as

$$\Delta U_{1-2} = U_2 - U_1 = Q_{1-2} - W_{1-2} = 18.6 - 64.7076 = -46.1076 \text{ kJ}$$

Now, from first law of thermodynamics for process B, change in internal energy is determined as

$$\begin{aligned} \Delta U_{2-1} &= U_1 - U_2 = Q_{2-1} - W_{2-1} = 0 - W_{2-1} = -W_{2-1} \\ \Rightarrow W_{2-1} &= -\Delta U_{2-1} = \Delta U_{1-2} = U_2 - U_1 = -46.1076 \text{ kJ} \end{aligned}$$

Alternate method:

Using work transfer relation for adiabatic (polytropic) process ($n = \gamma$),

$$W_{2-1} = \frac{P_1 V_1 - P_2 V_2}{1-n} = \frac{P_1 V_1 - P_2 V_2}{1-\gamma} = \frac{1000 \times 0.05 - 200 \times 0.157846}{1-1.4} = -46.077 \text{ kJ}$$

Problem 4.16.

Air enters a compressor operating at steady state at 100 kPa, 300 K and leaves at 1000 kPa, 400 K, with a volumetric flow rate of 1.5 m³/min. The work consumed by the compressor is 250 kJ per kg of air. Neglecting the effects of potential and kinetic energy, determine the heat transfer rate, in kW.

[Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$].

Solution: Given,

Pressure at inlet, $P_1 = 100 \text{ kPa}$

Temperature at inlet, $T_1 = 300 \text{ K}$

Pressure at outlet, $P_2 = 1000 \text{ kPa}$

Temperature at outlet, $T_2 = 400 \text{ K}$

Volumetric flow rate at outlet, $\dot{V}_2 = 1.5 \text{ m}^3/\text{min} = 1.5/60 = 0.025 \text{ m}^3/\text{s}$

Specific work consumed by compressor, $w_{cv} = -250 \text{ kJ/kg}$

Using equation of state, volume at outlet is given by

$$P_2 V_2 = m R T_2$$

Dividing both sides of above equation by time, we get

$$\dot{P}_2 \dot{V}_2 = \dot{m} R T_2$$

$$\text{Then, mass flow rate, } \dot{m} = \frac{\dot{P}_2 \dot{V}_2}{R T_2} = \frac{1000 \times 10^3 \times 0.025}{287 \times 400} = 0.21777 \text{ kg/s}$$

$$\text{Power consumed by compressor, } \dot{W}_{cv} = \dot{m} w_{cv} = 0.21777 (-250)$$

$$= -54.4425 \text{ kW} = -54442.5 \text{ W}$$

From the energy equation for control volume at steady state condition, we have

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_2 - h_1) \quad (\text{when neglecting KE & PE change})$$

$$\Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m} c_p (T_2 - T_1) = -54442.5 + 0.21777 \times 1005 (400 - 300)$$

$$= -32556.615 \text{ W} = -32.5566 \text{ kW}$$

The negative sign indicates that heat is lost by the system (compressor).

Problem 4.17.

An adiabatic turbine operating under steady state condition develops 12 MW of power output for a steam mass flow rate of 15 kg/s. The steam enters at 4 MPa with a velocity of 20 m/s and exits at 60 kPa with a quality of 85% and velocity of 100 m/s. Determine the inlet temperature of steam.

Solution: Given,

Rate of heat loss, $\dot{Q}_{cv} = 0$

(As turbine is adiabatic)

Power output, $\dot{W}_o = 12 \text{ MW} = 12 \times 10^6 \text{ W}$

Mass flow rate, $\dot{m} = 15 \text{ kg/s}$

(As $\dot{m}_1 = \dot{m}_2 = \dot{m}$)

Pressure at inlet, $P_1 = 4 \text{ MPa} = 4000 \text{ kPa}$

Velocity at inlet, $\bar{V}_1 = 20 \text{ m/s}$

Pressure at outlet, $P_2 = 60 \text{ kPa}$

Velocity at outlet, $\bar{V}_2 = 100 \text{ m/s}$

Quality at outlet, $x_2 = 85\% = 0.85$

Here, two properties namely pressure and quality are given for outlet of the turbine. Hence, only the turbine outlet or state 2 is clearly defined as

At outlet: $P_2 = 60 \text{ kPa}$, $x_2 = 0.85$

Since quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 60 \text{ kPa}$,

$$h_2 = 359.9 \text{ kJ/kg}, \quad h_{fg} = 2293.1 \text{ kJ/kg}, \quad h_g = 2653.0 \text{ kJ/kg}$$

Using enthalpy relation for two phase mixture, specific enthalpy at outlet of turbine,

$$h_2 = h_1 + x_2 h_{fg} = 359.9 + 0.85 \times 2293.1 = 2309.035 \text{ kJ/kg}$$

From the energy equation for control volume system at steady state condition, we can determine specific enthalpy at inlet of turbine, h_1 as

$$\dot{Q}_o - \dot{W}_o = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{when neglecting PE change})$$

$$= h_2 - \frac{\dot{W}_o}{\dot{m}} + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \quad (\text{As } \dot{Q}_o = 0)$$

$$= 2309.035 \times 10^3 - \frac{12 \times 10^6}{15} + \frac{1}{2} (100^2 - 20^2)$$

$$= 3113.835 \text{ J/kg} = 3113.835 \text{ kJ/kg}$$

At inlet: $P_1 = 4000 \text{ kPa}$, $h_1 = 3113.835 \text{ kJ/kg}$

Referring to the saturated water pressure table A2.1 for pressure $P_1 = 4000 \text{ kPa}$,

$h_1 = 1067.2 \text{ kJ/kg}$, $h_{fg} = 1713.4 \text{ kJ/kg}$, $h_g = 2800.6 \text{ kJ/kg}$

Here, $h_1 > h_g$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 4000 \text{ kPa}$ and specific enthalpy, $h_1 = 3113.835 \text{ kJ/kg}$ to find the temperature at inlet, T_1 . As there is no value listed for $h_1 = 3113.835 \text{ kJ/kg}$ in the table, linear interpolation method is used to find required temperature as

$$T_s = 350^\circ\text{C}$$

$$h_s = 3091.8 \text{ kJ/kg}$$

$$T_i = ?$$

$$h_i = 3113.835 \text{ kJ/kg}$$

$$T_e = 400^\circ\text{C}$$

$$h_e = 3213.4 \text{ kJ/kg}$$

Using linear interpolation method,

$$T_i = T_s + \frac{h_i - h_s}{h_e - h_s} (T_e - T_s) = 350 + \frac{3113.835 - 3091.8}{3213.4 - 3091.8} (400 - 350) = 359.06^\circ\text{C}$$

Problem 4.18.

A steam turbine develops 60 MW of power output. Mass flow rate of steam is found to be 80 kg/s. Properties of steam at inlet and exit of the turbine are as follows:

Property	Inlet	Exit
Pressure	8 MPa	0.4 MPa
Temperature	500°C	
Quality		80 %
Velocity	50 m/s	150 m/s
Elevation above the reference level	10 m	5 m

- (a) Determine the rate at which heat is lost from the turbine surface.
- (b) Determine the inlet and outlet areas.

Solution: Given,

$$\text{Power output, } \dot{W}_o = 60 \text{ MW} = 60 \times 10^6 \text{ W}$$

$$\text{Mass flow rate, } \dot{m} = 80 \text{ kg/s}$$

$$(\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

$$\text{Pressure at inlet, } P_1 = 8 \text{ MPa} = 8000 \text{ kPa}$$

$$\text{Temperature at inlet, } T_1 = 500^\circ\text{C}$$

$$\text{Velocity at inlet, } \bar{V}_1 = 50 \text{ m/s}$$

$$\text{Elevation at inlet, } z_1 = 10 \text{ m}$$

$$\text{Pressure at outlet, } P_2 = 0.4 \text{ MPa} = 400 \text{ kPa}$$

$$\text{Velocity at outlet, } \bar{V}_2 = 150 \text{ m/s}$$

$$\text{Quality at outlet, } x_2 = 80\% = 0.8$$

$$\text{Elevation at outlet, } z_2 = 5 \text{ m}$$

$$\text{At inlet: } P_1 = 8000 \text{ kPa}, T_1 = 500^\circ\text{C}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 8000 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 8000 \text{ kPa}) = 295.04^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 8000 \text{ kPa}$ and temperature, $T_1 = 500^\circ\text{C}$, specific volume at inlet, $v_1 = 0.04174 \text{ m}^3/\text{kg}$ and specific enthalpy at inlet, $h_1 = 3398.5 \text{ kJ/kg}$.

$$\text{At outlet: } P_2 = 400 \text{ kPa}, x_2 = 0.8$$

As quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$,

$$v_t = 0.001084 \text{ m}^3/\text{kg}, v_{lg} = 0.4614 \text{ m}^3/\text{kg} \text{ and } v_g = 0.4625 \text{ m}^3/\text{kg}$$

$$h_t = 604.91 \text{ kJ/kg}, h_{lg} = 2133.6 \text{ kJ/kg}, \text{ and } h_g = 2738.5 \text{ kJ/kg}$$

Specific volume at outlet,

$$v_2 = v_1 + x_2 v_{lg} = 0.001084 + 0.8 \times 0.4614 = 0.3702 \text{ m}^3/\text{kg}$$

Specific enthalpy at outlet,

$$h_2 = h_1 + x_2 h_{lg} = 604.91 + 0.8 \times 2133.6 = 2311.79 \text{ kJ/kg}$$

From the energy equation for control volume at steady state condition, we have

$$\begin{aligned}\dot{Q}_o - \dot{W}_o &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \\ \Rightarrow \dot{Q}_o &= \dot{W}_o + \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \\ \therefore \dot{Q}_o &= 60 \times 10^6 + 80 \left[(2311.79 - 3398.5) \times 10^3 + \frac{1}{2} (150^2 - 50^2) + 9.81(5 - 10) \right] \\ &= -26140724 \text{ W} = -26.141 \text{ MW}\end{aligned}$$

The negative sign indicates that the heat is lost by the system (turbine).

As $\dot{m}_1 = \dot{m}_2 = \dot{m}$ for steady state condition and using expression for rate of mass flow, the inlet area, A_1 , and outlet area, A_2 are given by

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{\bar{V}_1} = \frac{80 \times 0.04174}{50} = 0.066784 \text{ m}^2$$

$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} \Rightarrow A_2 = \frac{\dot{m} v_2}{\bar{V}_2} = \frac{80 \times 0.3702}{150} = 0.19744 \text{ m}^2$$

Problem 4.19.

Air enters a nozzle steadily at 300 kPa, 127°C and with a velocity of 40 m/s and leaves at 100 kPa and with a velocity of 300 m/s. The heat loss from the nozzle surface is 20 kJ/kg of the air. The inlet area of the nozzle is 100 cm². Determine:

- (a) the exit temperature of the air, and
- (b) the exit area of the nozzle. Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$

Solution: Given,

Pressure at inlet, $P_1 = 300 \text{ kPa}$

Temperature at inlet, $T_1 = 127^\circ\text{C} = 127 + 273 = 400 \text{ K}$

Velocity at inlet, $\bar{V}_1 = 40 \text{ m/s}$

Pressure at outlet, $P_2 = 100 \text{ kPa}$

Velocity at outlet, $\bar{V}_2 = 300 \text{ m/s}$

Specific heat loss, $q_{ex} = -20 \text{ kJ/kg}$

Inlet area, $A_1 = 100 \text{ cm}^2 = 0.01 \text{ m}^2$

From the energy equation for control volume system like flow device nozzle ($\dot{W}_o = 0$) at steady state condition, we have

$$\dot{Q}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{Neglecting PE change})$$

$$\Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2)$$

$$\Rightarrow q_{cv} = c_p (T_2 - T_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2)$$

$$\Rightarrow T_2 = T_1 + \frac{1}{c_p} \left[q_{cv} - \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] = 400 + \frac{1}{1005} \left[-20 \times 10^3 - \frac{1}{2} (300^2 - 40^2) \right]$$

$$= 336.12 \text{ K} = 63.12^\circ\text{C}$$

Using equation of state, specific volume at inlet, v_1 is given by

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 400}{300 \times 10^3} = 0.382666 \text{ m}^3/\text{kg}$$

Similarly, specific volume at outlet, v_2 is given by

$$P_2 v_2 = RT_2 \Rightarrow v_2 = \frac{RT_2}{P_2} = \frac{287 \times 336.12}{100 \times 10^3} = 0.964664 \text{ m}^3/\text{kg}$$

Now, rate of mass flow of the air,

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2} \quad (\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

$$\Rightarrow A_2 = A_1 \frac{v_2}{v_1} \frac{\bar{V}_1}{\bar{V}_2} = 0.01 \times \frac{0.964664}{0.382666} \times \frac{40}{300} = 0.0033612 \text{ m}^2 = 33.612 \text{ cm}^2$$

Problem 4.20.

Steam enters into a nozzle at $P_1 = 1000 \text{ kPa}$, $T_1 = 300^\circ\text{C}$ and with a velocity of 75 m/s and leaves the nozzle at $P_2 = 80 \text{ kPa}$, $T_2 = 200^\circ\text{C}$ and with a velocity of 350 m/s . Determine the heat loss per unit mass of the steam from the nozzle surface.

Solution: Given,

Pressure at inlet, $P_1 = 1000 \text{ kPa}$

Temperature at inlet, $T_1 = 300^\circ\text{C}$

Velocity at inlet, $\bar{V}_1 = 75 \text{ m/s}$

Pressure at outlet, $P_2 = 80 \text{ kPa}$

Temperature at outlet, $T_2 = 200^\circ\text{C}$

Velocity at outlet, $\bar{V}_2 = 350 \text{ m/s}$

From the energy equation for control volume system like flow device nozzle ($\dot{W}_n = 0$) at steady state condition, we have

$$\dot{Q}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{Neglecting PE change})$$

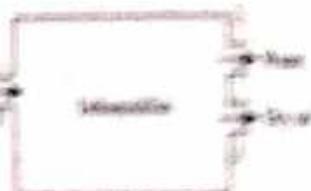
Mass flow rate of air with water vapor at inlet 1, $\dot{m}_1 = 210 \text{ kg/h}$

Tenthalpy of water at outlet 2, $h_2 = 34 \text{ kJ/kg}$

Mass flow rate of water at outlet 2, $\dot{m}_2 = 4 \text{ kg/h}$

Tenthalpy of dry air at outlet 3, $h_3 = 23.8 \text{ kJ/kg}$

As shown in figure, there are two outlets and one inlet of the control volume (dehumidifier). Let us assume that the mass flow rate of dry air at outlet 3 is \dot{m}_3 . Using principle of conservation of mass for control volume at steady state condition,



$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}$$

$$\Rightarrow \dot{m}_1 = \dot{m}_2 + \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 210 - 4 = 206 \text{ kg/h}$$

Similarly, using principle of conservation of energy for control volume like flow device dehumidifier ($\dot{W}_{cv} = 0$) at steady state condition,

$$\dot{Q}_{cv} + \sum_{\text{in}} m h_i = \sum_{\text{out}} m h_f \quad (\text{Neglecting KE & PE change})$$

$$\Rightarrow \dot{Q}_{cv} + \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_2 h_2 + \dot{m}_3 h_3 - \dot{m}_1 h_1 = 4 \times 34 + 206 \times 23.8 - 210 \times 90 \\ = -13861.2 \text{ kJ/h} = -3.85 \text{ kJ/s}$$

Problem 4.22.

In a water heater operating under steady state condition, water at 50°C flowing with a mass flow rate of 5 kg/s is mixed with the saturated water vapor at 120°C . The mixture leaves the heater as a saturated liquid water at 100°C . Determine the rate at which saturated water vapor must be supplied to the heater.

Solution: Given,

Temperature of water at inlet 1, $T_1 = 50^\circ\text{C}$

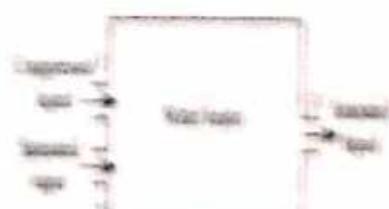
Mass flow rate of water at inlet 1, $\dot{m}_1 = 5 \text{ kg/s}$

Temperature of saturated vapor at inlet 2,

$T_2 = 120^\circ\text{C}$

Temperature of saturated liquid at outlet 3,

$T_3 = 100^\circ\text{C}$



As shown in above figure, there are two inlets and one outlet of the control volume (water heater). Let us assume that the water heater is adiabatic ($\dot{Q}_{cv} = 0$) and mass flow rate of saturated liquid at its outlet 3 is \dot{m}_3 . Using principle of conservation of mass for control volume at steady state condition,

$$\Rightarrow \frac{\dot{Q}_o}{\dot{m}} = (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2)$$

$$\Rightarrow q_o = (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2)$$

At inlet: $P_1 = 1000 \text{ kPa}$, $T_1 = 300^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 1000 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 1000 \text{ kPa}) = 179.92^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 1000 \text{ kPa}$ and temperature, $T_1 = 300^\circ\text{C}$, specific enthalpy at inlet, $h_1 = 3050.6 \text{ kJ/kg}$.

At outlet: $P_2 = 80 \text{ kPa}$, $T_2 = 200^\circ\text{C}$

Similarly, referring to the saturated water pressure table A2.1 for pressure, $P_2 = 80 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_2 = 80 \text{ kPa}) = 93.511^\circ\text{C}$. Here, $T_2 > T_{sat}$. Hence, it is also a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 80 \text{ kPa}$ and temperature, $T_2 = 200^\circ\text{C}$, to find the specific enthalpy at outlet, h_2 . As there is no value listed for $P_2 = 80 \text{ kPa}$ in the table, linear interpolation method is used to find the required specific enthalpy as

$$P_a = 50 \text{ kPa}$$

$$h_a = 2877.2 \text{ kJ/kg}$$

$$P_2 = 80 \text{ kPa}$$

$$h_2 = ?$$

$$P_b = 100 \text{ kPa}$$

$$h_b = 2874.8 \text{ kJ/kg}$$

Using linear interpolation method,

$$h_2 = h_a + \frac{P_2 - P_a}{P_b - P_a} (h_b - h_a) = 2877.2 + \frac{80 - 50}{100 - 50} (2874.8 - 2877.2) \\ = 2875.76 \text{ kJ/kg}$$

$$\text{Now, } q_o = (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) = (2875.76 - 3050.6) \times 10^3 + 0.5 (350^2 - 75^2) \\ = -116402.5 \text{ J/kg} = -116.4025 \text{ kJ/kg}$$

Problem 4.21.

Warm atmospheric air containing water vapor enters the dehumidifier with an enthalpy of 90 kJ/kg at a rate of 210 kg/h . Heat is removed from the air while it passes over a bank of tubes through which cold water flows. Atmospheric moisture that condenses on the tube drains from the dehumidifier with an enthalpy of 34 kJ/kg at a rate of 4 kg/h . Leaving air has an enthalpy of 23.8 kJ/kg . Velocities through the dehumidifier are quite low. Determine the rate of heat removal from the air stream through the dehumidifier.

Solution: Given,

Enthalpy of air with water vapor at inlet 1, $h_1 = 90 \text{ kJ/kg}$

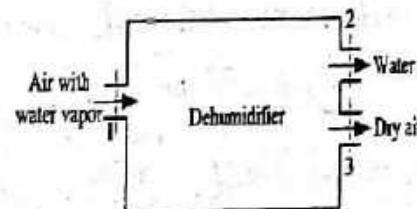
Mass flow rate of air with water vapor at inlet 1, $\dot{m}_1 = 210 \text{ kg/h}$

Enthalpy of water at outlet 2, $h_2 = 34 \text{ kJ/kg}$

Mass flow rate of water at outlet 2, $\dot{m}_2 = 4 \text{ kg/h}$

Enthalpy of dry air at outlet 3, $h_3 = 23.8 \text{ kJ/kg}$

As shown in figure, there are two outlets and one inlet of the control volume (dehumidifier). Let us assume that the mass flow rate of dry air at outlet 3 is \dot{m}_3 . Using principle of conservation of mass for control volume at steady state condition,



$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\Rightarrow \dot{m}_1 = \dot{m}_2 + \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 210 - 4 = 206 \text{ kg/h}$$

Similarly, using principle of conservation of energy for control volume like flow device dehumidifier ($\dot{W}_{cv} = 0$) at steady state condition,

$$\dot{Q}_{cv} + \sum_{in} \dot{m}h = \sum_{out} \dot{m}h \quad (\text{Neglecting KE & PE change})$$

$$\Rightarrow \dot{Q}_{cv} + \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_2 h_2 + \dot{m}_3 h_3 - \dot{m}_1 h_1 = 4 \times 34 + 206 \times 23.8 - 210 \times 90 \\ = -13861.2 \text{ kJ/h} = -3.85 \text{ kJ/s}$$

Problem 4.22.

In a water heater operating under steady state condition, water at 50°C flowing with a mass flow rate of 5 kg/s is mixed with the saturated water vapor at 120°C . The mixture leaves the heater as a saturated liquid water at 100°C . Determine the rate at which saturated water vapor must be supplied to the heater.

Solution: Given,

Temperature of water at inlet 1, $T_1 = 50^\circ\text{C}$

Mass flow rate of water at inlet 1, $\dot{m}_1 = 5 \text{ kg/s}$

Temperature of saturated vapor at inlet 2,

$T_2 = 120^\circ\text{C}$

Temperature of saturated liquid at outlet 3,

$T_3 = 100^\circ\text{C}$

As shown in above figure, there are two inlets and one outlet of the control volume (water heater). Let us assume that the water heater is adiabatic ($\dot{Q}_{cv} = 0$) and mass flow rate of saturated liquid at its outlet 3 is \dot{m}_3 . Using principle of conservation of mass for control volume, at steady state condition,



$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{S} + \dot{m}_2 \quad \dots \dots \dots \text{(i)}$$

Similarly, using principle of conservation of energy for control volume like flow device dehumidifier ($\dot{W}_c = 0$) at steady state condition,

$$\sum_{in} \dot{m}h = \sum_{out} \dot{m}h \quad (\text{Neglecting KE & PE change with } \dot{Q}_c = 0)$$

$$\Rightarrow \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Using equation (i), we get

$$\Rightarrow \dot{S}h_1 + \dot{m}_2 h_2 = (\dot{S} + \dot{m}_2)h_3 \quad \dots \dots \dots \text{(ii)}$$

Referring to the saturated water temperature table A2.2 for temperature, $T_1 = 50^\circ\text{C}$, specific enthalpy of water is taken as of saturated liquid, i.e.

$$h_1 = h_f (T_1 = 50^\circ\text{C}) = 209.33 \text{ kJ/kg.}$$

Similarly, specific enthalpy of saturated vapor for temperature, $T_2 = 120^\circ\text{C}$, i.e.

$$h_2 = h_g (T_2 = 120^\circ\text{C}) = 2706.2 \text{ kJ/kg}$$

and specific enthalpy of saturated liquid for temperature $T_3 = 100^\circ\text{C}$, i.e.

$$h_3 = h_f (T_3 = 100^\circ\text{C}) = 419.06 \text{ kJ/kg.}$$

Putting these specific enthalpy values in above equation (ii), we get

$$5 \times 209.33 + \dot{m}_2 \times 2706.2 = (5 + \dot{m}_2) \times 419.06$$

$$\Rightarrow 1046.65 + 2706.2 \times \dot{m}_2 = 2095.3 + 419.06 \times \dot{m}_2$$

$$\Rightarrow 2287.14 \times \dot{m}_2 = 1048.65$$

$$\therefore \dot{m}_2 = 0.4585 \text{ kg/s}$$

Problem 4.23.

An adiabatic diffuser has air entering at 100 kPa, 300 K with a velocity of 200 m/s. The inlet cross sectional area of the diffuser is 100 mm². At the exit the area is 860 mm², and the exit velocity is 20 m/s. Determine the exit temperature and pressure of the air. [Take $C_p = 1005 \text{ J/kgK}$, $R = 287 \text{ J/kgK}$].

Solution: Given,

Pressure at inlet, $P_1 = 100 \text{ kPa}$

Temperature at inlet, $T_1 = 300 \text{ K}$

Velocity at inlet, $\bar{V}_1 = 200 \text{ m/s}$

Inlet area, $A_1 = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$

Exit area, $A_2 = 860 \text{ mm}^2 = 860 \times 10^{-6} \text{ m}^2$

Velocity at exit, $\bar{V}_2 = 20 \text{ m/s}$

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From the energy equation for control volume system like flow device diffuser ($\dot{W}_{cv} = 0$) at steady state condition, neglecting PE change and for adiabatic diffuser ($\dot{Q}_{cv} = 0$), we have

$$0 = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right]$$

$$\Rightarrow c_p (T_2 - T_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) = 0$$

$$\Rightarrow T_2 = T_1 + \frac{1}{2c_p} (\bar{V}_1^2 - \bar{V}_2^2) = 300 + \frac{1}{2 \times 1005} (200^2 - 20^2) = 319.7 \text{ K}$$

Using equation of state, specific volume at inlet, v_1 is given by

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{100 \times 10^3} = 0.861 \text{ m}^3/\text{kg}$$

Now, rate of mass flow of the air,

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2} \quad (\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

\Rightarrow Specific volume at exit,

$$v_2 = v_1 \frac{A_2 \bar{V}_2}{A_1 \bar{V}_1} = 0.861 \times \frac{860 \times 10^{-6}}{100 \times 10^{-6}} \times \frac{20}{200} = 0.74046 \text{ m}^3$$

Using equation of state, pressure at exit, P_2 is given by

$$P_2 v_2 = RT_2 \Rightarrow P_2 = \frac{RT_2}{v_2} = \frac{287 \times 319.7}{0.74046} = 123914.73 \text{ Pa} = 123.915 \text{ kPa}$$

Problem 4.24.

Steam enters into a well insulated throttling valve at 10 MPa, 600 °C and exits at 5 MPa. Determine the final temperature of the steam.

Solution: Given,

Pressure at inlet, $P_1 = 10 \text{ MPa} = 10000 \text{ kPa}$

Temperature at inlet, $T_1 = 600^\circ\text{C}$

Pressure at outlet, $P_2 = 5 \text{ MPa} = 5000 \text{ kPa}$

From the principle of conservation of energy for control volume like flow device throttling valve ($\dot{W}_{cv} = 0$) at steady state condition, (Neglecting KE & PE change with $\dot{Q}_{cv} = 0$ for insulated throttling valve)

$$h_1 = h_2$$

At Inlet: $P_1 = 10000 \text{ kPa}$, $T_1 = 600^\circ\text{C}$

Referring to the saturated water pressure table A2.1, for pressure, $P_1 = 10000 \text{ kPa}$, the saturation temperature for given pressure, i.e. T_{sat} ($P_1 = 10000 \text{ kPa}$) = 311.03°C. Here, $T_1 > T_{sat}$. Hence it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 10000 \text{ kPa}$ and temperature, $T_1 = 600^\circ\text{C}$, the specific enthalpy at inlet, $h_1 = 3624.7 \text{ kJ/kg}$ which is equal to specific enthalpy at outlet, $h_2 = h_1 = 3624.7 \text{ kJ/kg}$.

At outlet: $P_2 = 5000 \text{ kPa}$, $h_2 = 3624.7 \text{ kJ/kg}$

Referring to the saturated water pressure table A2.1, for pressure, $P_2 = 5000 \text{ kPa}$, $h_f = 1154.2 \text{ kJ/kg}$, $h_{fg} = 1639.5 \text{ kJ/kg}$, $h_g = 2793.7 \text{ kJ/kg}$.

Here, $h_2 > h_g$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 5000 \text{ kPa}$ and specific enthalpy, $h_2 = 3624.7 \text{ kJ/kg}$ to find the temperature at outlet, T_2 . As there is no value listed for $h_2 = 3624.7 \text{ kJ/kg}$ in the table, linear interpolation method is used to find the required specific enthalpy as

$$T_a = 550^\circ\text{C} \quad h_a = 3550.2 \text{ kJ/kg}$$

$$T_2 = ? \quad h_2 = 3624.7 \text{ kJ/kg}$$

$$T_b = 600^\circ\text{C} \quad h_b = 3666.2 \text{ kJ/kg}$$

Using linear interpolation method,

$$T_2 = T_a + \frac{h_2 - h_a}{h_b - h_a} (T_b - T_a) = 550 + \frac{3624.7 - 3550.2}{3666.2 - 3550.2} (600 - 550)$$

$$= 582.112^\circ\text{C}$$

4.3 Solved Numerical Problems from PoU Examinations

Problem 4.25.

Steam enters a steam turbine at a pressure of 1 MPa, temperature of 300°C and velocity of 50 m/s. The steam leaves the turbine at a pressure of 150 kPa and velocity of 200 m/s. Determine the work per kg of steam flowing through the turbine, assuming the process to be reversible and adiabatic. (PoU 2004 spring)

Solution: Given,

Pressure at inlet, $P_1 = 1 \text{ MPa} = 1000 \text{ kPa}$

Temperature at inlet, $T_1 = 300^\circ\text{C}$

Velocity at inlet, $\bar{V}_1 = 50 \text{ m/s}$

Pressure at outlet, $P_2 = 150 \text{ kPa}$

Velocity at outlet, $\bar{V}_2 = 200 \text{ m/s}$

Process is reversible and adiabatic, i.e. isentropic process.

At inlet: $P_1 = 1000 \text{ kPa}$, $T_1 = 300^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 1000 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 1000 \text{ kPa}) = 179.92^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 1000 \text{ kPa}$ and temperature, $T_1 = 300^\circ\text{C}$, specific enthalpy at inlet, $h_1 = 3050.6 \text{ kJ/kg}$ and specific entropy at inlet, $s_1 = 7.1219 \text{ kJ/kgK}$.

At outlet: $P_2 = 150 \text{ kPa}$, and $s_2 = s_1 = 7.1219$ (As the process is isentropic)

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 150 \text{ kPa}$,

$$h_1 = 467.18 \text{ kJ/kg}, \quad h_{lg} = 2226.2 \text{ kJ/kg}, \quad h_g = 2693.4 \text{ kJ/kg}$$

$$s_1 = 1.4338 \text{ m}^3/\text{kg}, \quad s_{lg} = 5.7894 \text{ m}^3/\text{kg} \text{ and } s_g = 7.2232 \text{ m}^3/\text{kg}$$

Here, $s_1 < s_2 < s_g$. Hence, it is a two phase mixture. Now, the quality of steam at outlet of the turbine is given by

$$x_2 = \frac{s_2 - s_1}{s_{lg}} = \frac{7.1219 - 1.4338}{5.7894} = 0.9825$$

$$\begin{aligned} \text{Then, enthalpy of steam at outlet, } h_2 &= h_1 + x_2 h_{lg} = 467.18 + 0.9825 \times 2226.2 \\ &= 2654.4215 \text{ kJ/kg} \end{aligned}$$

From the energy equation for control volume at steady state condition, we have

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right]$$

$$\Rightarrow 0 - \dot{W}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right]$$

(As $\dot{Q}_{cv} = 0$ for adiabatic turbine and neglecting PE change)

$$\begin{aligned} \frac{\dot{W}}{\dot{m}} &= w = \left[(h_1 - h_2) + \frac{1}{2} (\bar{V}_1^2 - \bar{V}_2^2) \right] = (3050.6 - 2654.4215) \times 10^3 + \frac{1}{2} (50^2 - 200^2) \\ &= 377428.5 \text{ J/kg} = 377.4285 \text{ kJ/kg} \end{aligned}$$

Problem 4.26.

Steam enters a convergent divergent nozzle with a velocity of 60 m/s, 800 kPa and 300 °C. The steam leaves the nozzle at 160 kPa and 0.96 dryness. The cross sectional area at exit is 12 cm² and considering the flow as adiabatic, determine the steam velocity at the exit and steam flow rate. (PoU 2005 fall).

Solution: Given,

Velocity at inlet, $\bar{V}_1 = 60 \text{ m/s}$

Pressure at inlet, $P_1 = 800 \text{ kPa}$

Temperature at inlet, $T_1 = 300^\circ\text{C}$

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Pressure at exit, $P_2 = 160 \text{ kPa}$

Quality of steam at exit, $x_2 = 0.96$

Outlet area of nozzle, $A_1 = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$

Heat loss from nozzle surface, $\dot{Q}_\text{ov} = 0$ (As flow is adiabatic)

At inlet: $P_1 = 800 \text{ kPa}$, $T_1 = 300^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 800 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{\text{sat}}(P_1 = 800 \text{ kPa}) = 170.44^\circ\text{C}$. Here, $T_1 > T_{\text{sat}}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 800 \text{ kPa}$ and temperature, $T_1 = 300^\circ\text{C}$,

specific volume at inlet, $v_1 = 0.3241 \text{ m}^3/\text{kg}$ and

specific enthalpy at inlet, $h_1 = 3055.9 \text{ kJ/kg}$.

Now, mass flow rate of steam,

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{80 \times 10^{-4} \times 50}{0.08002} = 4.99875 \text{ kg} \quad (\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

At exit: $P_2 = 160 \text{ kPa}$, $x_2 = 0.96$

As the quality is given, it is a two phase mixture. Now, referring to the saturated water pressure table A2.1 for pressure, $P_2 = 160 \text{ kPa}$ to find the values of specific volume and specific enthalpy. As there is no value listed for $P_2 = 160 \text{ kPa}$ in the table, linear interpolation method is used to find the required specific volume at exit as

$$\begin{array}{lll} P_a = 150 \text{ kPa} & v_{la} = 0.001053 \text{ m}^3/\text{kg} & v_{lg a} = 1.1584 \text{ m}^3/\text{kg} \\ P_b = 175 \text{ kPa} & v_{lb} = ? & v_{lg 2} = ? \\ P_2 = 160 \text{ kPa} & v_{lb} = 0.001057 \text{ m}^3/\text{kg} & v_{lg b} = 1.0027 \text{ m}^3/\text{kg} \end{array}$$

Using linear interpolation method,

$$v_{l2} = v_{la} + \frac{P_2 - P_a}{P_b - P_a} (v_{lb} - v_{la}) = 0.001053 + \frac{160 - 150}{175 - 150} (0.001057 - 0.001053) \\ = 0.0010546 \text{ m}^3/\text{kg}$$

$$v_{lg 2} = v_{lg a} + \frac{P_2 - P_a}{P_b - P_a} (v_{lg b} - v_{lg a}) = 1.1584 + \frac{160 - 150}{175 - 150} (1.0027 - 1.1584) \\ = 1.09612 \text{ m}^3/\text{kg}$$

Then, $v_2 = v_{l2} + x_2 v_{lg 2} = 0.0010546 + 0.96 \times 1.09612 = 1.05333 \text{ m}^3/\text{kg}$

Similarly, specific enthalpy at exit h_2 can be found out by interpolation as

$$\begin{array}{lll} P_a = 150^\circ\text{C} & h_{la} = 467.18 \text{ kJ/kg} & h_{lg a} = 2226.2 \text{ kJ/kg} \\ P_b = 175^\circ\text{C} & h_{lb} = ? & h_{lg 2} = ? \\ P_2 = 160^\circ\text{C} & h_{l2} = ? & h_{lg b} = 2213.3 \text{ kJ/kg} \\ P_2 = 160 \text{ kPa} & h_{lb} = 487.08 \text{ kJ/kg} & \end{array}$$

Now,

$$h_{l2} = h_{la} + \frac{P_2 - P_a}{P_b - P_a} (h_{lb} - h_{la}) = 467.18 + \frac{160 - 150}{175 - 150} (487.08 - 467.18) = 475.14 \text{ kJ/kg}$$

$$h_{lg2} = h_{lg a} + \frac{P_2 - P_a}{P_b - P_a} (h_{lg b} - h_{lg a}) = 2226.2 + \frac{160 - 150}{175 - 150} (2213.3 - 2226.2) \\ = 2221.04 \text{ kJ/kg}$$

$$h_2 = h_{l2} + x_2 h_{lg2} = 475.14 + 0.96 \times 2221.04 = 2607.3384 \text{ kJ/kg}$$

From the energy equation for control volume system like flow device nozzle ($\dot{W}_v = 0$) at steady state condition, we have

$$\dot{Q}_v = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{Neglecting PE change})$$

$$\Rightarrow 0 = (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \quad (\text{As flow is adiabatic})$$

$$\Rightarrow \bar{V}_2 = [\bar{V}_1^2 - 2(h_2 - h_1)]^{1/2} = [60^2 - 2(2607.3384 - 3055.9)10^3]^{1/2} = 949.064 \text{ m/s}$$

Again, mass flow rate of steam can be expressed as

$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} = \frac{12 \times 10^{-4} \times 949.064}{1.05333} = 1.0812 \text{ kg/s}$$

Problem 5.27.

Steam at 0.6 MPa, 200 °C enters an insulated nozzle with a velocity of 50 m/s and it leaves at a pressure of 0.2 MPa and velocity of 60 m/s. Determine the final temperature if the steam is superheated. (PoU 2006 spring).

Solution: Given,

Pressure at inlet, $P_1 = 0.6 \text{ MPa} = 600 \text{ kPa}$

Temperature at inlet, $T_1 = 200^\circ\text{C}$

Velocity at inlet, $\bar{V}_1 = 50 \text{ m/s}$

Pressure at exit, $P_2 = 0.2 \text{ MPa} = 200 \text{ kPa}$

Velocity at exit, $\bar{V}_2 = 60 \text{ m/s}$

Steam at exit is superheated.

At inlet: $P_1 = 600 \text{ kPa}$, $T_1 = 200^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 600 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 600 \text{ kPa}) = 158.86^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 600 \text{ kPa}$ and temperature, $T_1 = 200^\circ\text{C}$,

specific enthalpy, $h_1 = 2849.7 \text{ kJ/kg}$ and specific volume, $v_1 = 0.3520 \text{ m}^3/\text{kg}$.

From the energy equation for control volume system like flow device nozzle ($\dot{W}_v = 0$) at steady state condition, we have

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$$\dot{Q}_v = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{Neglecting PE change})$$

$$\Rightarrow 0 = (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \quad (\text{As nozzle is insulated, } \dot{Q}_v = 0)$$

$$\Rightarrow h_2 = h_1 - \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) = 2849.7 \times 10^3 - 0.5(60^2 - 50^2) = 2849150 \text{ J/kg}$$

$$= 2849.15 \text{ kJ/kg}$$

At exit: $P_2 = 200 \text{ kPa}$, $h_2 = 2849.15 \text{ kJ/kg}$

Similarly, referring to the saturated water pressure table A2.1 for pressure, $P_2 = 200 \text{ kPa}$, $h_f = 504.80 \text{ kJ/kg}$, $h_{fg} = 2201.7 \text{ kJ/kg}$, $h_g = 2706.5 \text{ kJ/kg}$. Here, $h_2 > h_g$. Hence, it is also a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 200 \text{ kPa}$ and specific enthalpy, $h_2 = 2849.15 \text{ kJ/kg}$, to find the temperature at exit, T_2 . As there is no value listed for $h_2 = 2849.15 \text{ kJ/kg}$ in the table, linear interpolation method is used to find the required exit temperature as

$$\begin{array}{ll} T_a = 150^\circ\text{C} & h_a = 2768.6 \text{ kJ/kg} \\ T_b = ? & h_b = 2849.15 \text{ kJ/kg} \\ T_c = 200^\circ\text{C} & h_c = 2870.0 \text{ kJ/kg} \end{array}$$

Using linear interpolation method,

$$T_2 = T_a + \frac{h_2 - h_a}{h_b - h_a} (T_b - T_a) = 150 + \frac{2849.15 - 2768.6}{2870.0 - 2768.6} (200 - 150) = 189.7189^\circ\text{C}$$

Problem 4.28.

A certain water heater operates under steady conditions receiving 4.2 kg/s of water at 75°C temperature, and enthalpy 313.93 kJ/kg . The water is heated by mixing with steam which is supplied to the heater at temperature 100.2°C and enthalpy 2676 kJ/kg . The mixer leaves the heater as liquid water at temperature 100°C and enthalpy 419 kJ/kg . How much steam must be supplied to the heater
(PoU 2006 fall).

Solution: Given,

Temperature of water at inlet 1, $T_1 = 75^\circ\text{C}$

Mass flow rate of water at inlet 1, $\dot{m}_1 = 4.2 \text{ kg/s}$

Enthalpy of water at inlet 1, $h_1 = 313.93 \text{ kJ/kg}$

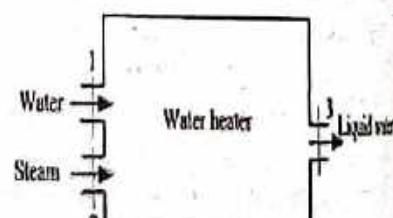
Temperature of steam at inlet 2, $T_2 = 100.2^\circ\text{C}$

Enthalpy of water at inlet 2, $h_2 = 2676 \text{ kJ/kg}$

Temperature of liquid water at outlet 3, $T_3 = 100^\circ\text{C}$

Enthalpy of liquid water at outlet 3, $h_3 = 419 \text{ kJ/kg}$

As shown in above figure, there are two inlets and one outlet of the control volume (water heater). Let us assume that the water heater is adiabatic ($\dot{Q}_v = 0$)



and mass flow rate of liquid water (mixture) at its outlet 3 is \dot{m}_3 . Using principle of conservation of mass for control volume at steady state condition,

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \dot{m}_3 = 4.2 + \dot{m}_2 \quad \dots \dots \dots (i)$$

Similarly, using principle of conservation of energy for control volume like flow device dehumidifier ($\dot{W}_{cv} = 0$) at steady state condition,

$$\sum_{in} \dot{m} h = \sum_{out} \dot{m} h \quad (\text{Neglecting KE & PE change with } \dot{Q}_o = 0)$$

$$\Rightarrow \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Using equation (i), we get

$$\Rightarrow 4.2 h_1 + \dot{m}_2 h_2 = (4.2 + \dot{m}_2) h_3$$

$$\Rightarrow 4.2 \times 313.93 + \dot{m}_2 \times 2676 = (4.2 + \dot{m}_2) \times 419$$

$$\Rightarrow 1318.506 + 2676 \times \dot{m}_2 = 1759.8 + 419 \times \dot{m}_2$$

$$\Rightarrow 2257 \times \dot{m}_2 = 441.294$$

$$\therefore \dot{m}_2 = 0.19552237 \text{ kg/s} = 703.88055 \text{ kg/hr}$$

Problem 4.29.

Steam enters a turbine operating a steady state with a mass flow rate of 4600 kg/h and the turbine develops a power of 1000 kW. At the inlet, pressure is 60 bar, the temperature is 400°C and velocity is 10 m/s. At the exit, the pressure is 1 bar, the quality is 0.9 and the velocity is 50 m/s. Calculate the rate of heat transfer between the turbine and surroundings in kW. (PoU 2007 fall).

Solution: Given,

$$\text{Mass flow rate, } \dot{m} = 4600 \text{ kg/h} = 4600/3600 = 1.28 \text{ kg/s}$$

$$\text{Power output, } \dot{W}_{cv} = 1000 \text{ kW} = 1000 \times 10^3 \text{ W}$$

$$\text{Pressure at inlet, } P_1 = 60 \text{ bar} = 6000 \text{ kPa}$$

$$\text{Temperature at inlet, } T_1 = 400^\circ\text{C}$$

$$\text{Velocity at inlet, } \bar{V}_1 = 10 \text{ m/s}$$

$$\text{Pressure at outlet, } P_2 = 1 \text{ bar} = 100 \text{ kPa}$$

$$\text{Quality at outlet, } x_2 = 0.9$$

$$\text{Velocity at outlet, } \bar{V}_2 = 50 \text{ m/s}$$

$$\text{At inlet: } P_1 = 6000 \text{ kPa}, T_1 = 400^\circ\text{C}$$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 6000 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 6000 \text{ kPa}) = 275.62^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 6000 \text{ kPa}$ and temperature, $T_1 = 400^\circ\text{C}$, the specific enthalpy at inlet, $h_1 = 3177.0 \text{ kJ/kg}$.

At outlet: $P_2 = 100 \text{ kPa}$, $x_2 = 0.9$

As quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 100 \text{ kPa}$,

$$h_f = 417.51 \text{ kJ/kg}, h_{fg} = 2257.6 \text{ kJ/kg}, h_g = 2675.1 \text{ kJ/kg}$$

Specific enthalpy at outlet,

$$h_2 = h_f + x_2 h_{fg} = 417.51 + 0.9 \times 2257.6 = 2449.35 \text{ kJ/kg}$$

From the energy equation for control volume at steady state condition, we have

$$\dot{Q}_s - \dot{W}_s = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{Neglecting PE change})$$

$$\begin{aligned} \Rightarrow \dot{Q}_s &= \dot{W}_s + \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \\ &= 1000 \times 10^3 + 1.28 [(2449.35 - 3177.0) \times 10^3 + 0.5 (50^2 - 10^2)] \\ &= 70144 \text{ W} = 70.144 \text{ kW} \end{aligned}$$

Problem 4.30.

A gas is held in a cylinder behind a piston undergoes a cycle composed of three processes:

Process 1 – 2: an adiabatic compression process. For this process $W_{1,2} = -58 \text{ kJ}$.

Process 2 – 3: an expansion during which heat is added and work is done on piston, $Q_{2,3} = 253 \text{ kJ}$, $W_{2,3} = 95 \text{ kJ}$.

Process 3 – 1: a cooling of gas while piston is stationary to bring the gas to its initial state. Determine the net heat transfer and heat transfer for the process 3–1. (PoU 2008 spring)

Solution: Given,

For process 1 – 2: compression with $Q_{1,2} = 0$, $W_{1,2} = -58 \text{ kJ}$

For process 2 – 3: expansion with $Q_{2,3} = 253 \text{ kJ}$, $W_{2,3} = 95 \text{ kJ}$.

For process 3 – 1: cooling with constant volume, $V_3 = V_1 \Rightarrow W_{3,1} = 0$

Using first law of thermodynamics for process 1 – 2, we have

$$Q_{1,2} = dU_{1,2} + W_{1,2} \Rightarrow dU_{1,2} = Q_{1,2} - W_{1,2} = 0 - (-58) = 58 \text{ kJ}$$

Similarly, applying first law for process 2 – 3, we get

$$Q_{2,3} = dU_{2,3} + W_{2,3} \Rightarrow dU_{2,3} = Q_{2,3} - W_{2,3} = 253 - 95 = 158 \text{ kJ}$$

We know for a cyclic process, net change in internal energy is zero, i.e.

$$dU_{1,2} + dU_{2,3} + dU_{3,1} = 0 \Rightarrow dU_{3,1} = -dU_{1,2} - dU_{2,3} = -58 - 158 = -216 \text{ kJ}$$

Now, applying first law for process 3 – 1, we get

$$Q_{3,1} = dU_{3,1} + W_{3,1} = -216 - 0 = -216 \text{ kJ}$$

Then, net heat transfer is calculated as

$$Q_{\text{net}} = Q_{1,2} + Q_{2,3} + Q_{3,1} = 0 + 253 + (-216) = 37 \text{ kJ}$$

Alternately,

For a cyclic process, net heat transfer is equal to net work transfer, so

$$Q_{net} = W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = -58 + 95 + 0 = 37 \text{ kJ}$$

Problem 4.31.

The property of a system during a reversible constant pressure non-flow process at $P = 1.6 \text{ bar}$ changes from $v_1 = 0.3 \text{ m}^3/\text{kg}$, $T_1 = 20^\circ\text{C}$ to $v_2 = 0.55 \text{ m}^3/\text{kg}$, $T_2 = 260^\circ\text{C}$. The specific heat of the fluid is given by $c_p = 1.5 + \frac{75}{T+45} \text{ kJ/kg}^\circ\text{C}$.

where T is in $^\circ\text{C}$. Determine

- (a) heat added per kg
- (b) change in internal energy per kg
- (c) change in enthalpy per kg.

(PoU 2009 fall, PoU 2010 spring, PoU 2014 fall).

Solution: Given,

Constant pressure, $P_1 = P_2 = 1.6 \text{ bar} = 160 \text{ kPa}$

Specific volume at state 1, $v_1 = 0.3 \text{ m}^3/\text{kg}$

Temperature at state 1, $T_1 = 20^\circ\text{C}$

Specific volume at state 2, $v_2 = 0.55 \text{ m}^3/\text{kg}$

Temperature at state 2, $T_2 = 260^\circ\text{C}$

Specific heat and temperature relationship is given by $c_p = 1.5 + \frac{75}{T+45}$

The heat added per kg during the process 1–2 is given by

$$\begin{aligned} q_{1-2} &= \int_1^2 c_p dT = \int_1^2 \left(1.5 + \frac{75}{T+45} \right) dT = [1.5T + 75 \ln(T+45)] \\ &= 1.5(T_2 - T_1) + 75 \{\ln(T_2+45) - \ln(T_1+45)\} \\ &= 1.5(260 - 20) + 75 \{\ln(260+45) - \ln(20+45)\} \\ &= 475.944 \text{ kJ/kg} \end{aligned}$$

The work done per kg during an isobaric process 1–2 is determined as

$$w_{1-2} = P_1(v_2 - v_1) = 160(0.55 - 0.3) = 40 \text{ kJ/kg}$$

Using first law of thermodynamic for a control mass system, we have change in internal energy per kg as

$$du_{1-2} = q_{1-2} - w_{1-2} = 475.944 - 40 = 435.944 \text{ kJ/kg}$$

From the definition of specific enthalpy, we have

$$\begin{aligned} h &= u + Pv \Rightarrow dh = du + pdv + vdP \\ &= du + w + 0 \quad (\text{As pressure is constant}) \end{aligned}$$

⇒ Change in enthalpy per kg ,

$$dh_{1-2} = du_{1-2} + w_{1-2} = 435.944 + 40 = 475.944 \text{ kJ/kg}$$

Problem 4.32.

Steam enters a horizontally aligned well insulated nozzle of inlet area 0.15 m^2 at the velocity of 65 m/s . At the inlet, the specific enthalpy of steam is 3000 kJ/kg , specific volume $0.187 \text{ m}^3/\text{kg}$ and at the outlet, specific enthalpy is 2762 kJ/kg , specific volume $0.498 \text{ m}^3/\text{kg}$. Determine

- (i) velocity of steam at the outlet
- (ii) mass flow rate of steam
- (iii) exit area of nozzle.

(PoU 2010 fall).

Solution: Given,

Inlet area of nozzle, $A_1 = 0.15 \text{ m}^2$

Velocity at inlet, $\bar{V}_1 = 65 \text{ m/s}$

Specific enthalpy at inlet, $h_1 = 3000 \text{ kJ/kg}$

Specific volume at inlet, $v_1 = 0.187 \text{ m}^3/\text{kg}$

Specific enthalpy at outlet, $h_2 = 2762 \text{ kJ/kg}$

Specific volume at outlet, $v_2 = 0.498 \text{ m}^3/\text{kg}$

From the energy equation for control volume system like flow device nozzle ($\dot{W}_o = 0$) at steady state condition, we have

$$\begin{aligned}\dot{Q}_o &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] && \text{(Neglecting PE change)} \\ \Rightarrow 0 &= (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) && (\dot{Q}_o = 0, \text{ as nozzle is well insulated}) \\ \Rightarrow \bar{V}_2 &= [\bar{V}_1^2 - 2(h_2 - h_1)]^{1/2} = [65^2 - 2(2762 - 3000)10^3]^{1/2} = 692.983 \text{ m/s.}\end{aligned}$$

Now, mass flow rate of steam,

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{0.15 \times 65}{0.187} = 52.139 \text{ kg} \quad (\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

Again, mass flow rate of steam can be expressed as

$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} \Rightarrow A_2 = \frac{\dot{m} v_2}{\bar{V}_2} = \frac{52.139 \times 0.498}{692.983} = 0.0374688 \text{ m}^2$$

Problem 4.33.

A system undergoes a cycle consisting of four processes. Complete the missing table entries. Is this power cycle or refrigeration cycle? (PoU 2012 fall).

Process	$\Delta U, \text{kJ}$	W, kJ	Q, kJ
1 - 2	-500		0
2 - 3	0	-100	
3 - 4			400
4 - 1		300	500

Solution: Given,

Process 1 – 2: Change in internal energy, $\Delta U_{1-2} = -500 \text{ kJ}$

Heat transfer, $Q_{1-2} = 0$

Process 2 – 3: Change in internal energy, $\Delta U_{2-3} = 0$

Work transfer, $W_{2-3} = -100 \text{ kJ}$

Process 3 – 4: Heat transfer, $Q_{3-4} = 400 \text{ kJ}$

Process 4 – 1: Work transfer, $W_{4-1} = 300 \text{ kJ}$

Heat transfer, $Q_{4-1} = 500 \text{ kJ}$

Using first law of thermodynamics for process 1 – 2, work transfer

$$W_{1-2} = Q_{1-2} - \Delta U_{1-2} = 0 - (-500) = 500 \text{ kJ}$$

For process 2 – 3: Heat transfer, $Q_{2-3} = \Delta U_{2-3} + W_{2-3} = 0 + (-100) = -100 \text{ kJ}$

Negative sign indicates that heat is lost by the system.

For process 4 – 1: Change in internal energy is given by

$$\Delta U_{4-1} = Q_{4-1} - W_{4-1} = 500 - 300 = 200 \text{ kJ}$$

We know that the total change in internal energy in a cyclic process is zero, i.e.

$$\Delta U_{1-2} + \Delta U_{2-3} + \Delta U_{3-4} + \Delta U_{4-1} = 0 \Rightarrow \Delta U_{3-4} = 0 - \Delta U_{1-2} - \Delta U_{2-3} - \Delta U_{4-1}$$

⇒ Change in internal energy for process 3 – 4,

$$\Delta U_{3-4} = -(-500) - 0 - 200 = 300 \text{ kJ}$$

For process 3 – 4: Work transfer, $W_{3-4} = Q_{3-4} - \Delta U_{3-4} = 400 - 300 = 100 \text{ kJ}$

Alternately, use the relation, $\sum W = \sum Q$ for a cycle to find the value of W_{3-4} .

Problem 4.34.

A steam turbine receives steam from two boilers. One flow is 5 kg/s at 3 MPa, 700°C and the other flow is 15 kg/s at 800 kPa, 500°C. The exit state is 10 kPa with a quality of 96%. Find the total power output of the adiabatic turbine.

(PoU 2013 fall).

Solution: Given,

Let A and B are two boilers.

Mass flow rate at inlet for boiler A, $\dot{m}_{1A} = 5 \text{ kg/s}$

Pressure at inlet for boiler A, $P_{1A} = 3 \text{ MPa} = 3000 \text{ kPa}$

Temperature at inlet for boiler A, $T_{1A} = 700^\circ\text{C}$

Mass flow rate at inlet for boiler B, $\dot{m}_{1B} = 15 \text{ kg/s}$

Pressure at inlet for boiler B, $P_{1B} = 800 \text{ kPa}$

Temperature at inlet for boiler B, $T_{1B} = 500^\circ\text{C}$

Pressure at outlet, $P_2 = 10 \text{ kPa}$

Quality at outlet, $x_2 = 96\% = 0.96$

Rate of heat transfer, $\dot{Q}_{cv} = 0$ (As the turbine is adiabatic)

At Inlet for boiler A: $P_{1A} = 3000 \text{ kPa}$, $T_{1A} = 700^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_{1A} = 3000 \text{ kPa}$, the saturation temperature for given pressure, i.e.

$T_{sat}(P_{1A} = 3000 \text{ kPa}) = 233.89^\circ\text{C}$,

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_{1A} = 3000 \text{ kPa}$ and temperature, $T_{1A} = 700^\circ\text{C}$, specific enthalpy at inlet for boiler A, $h_{1A} = 3911.7 \text{ kJ/kg}$.

At inlet for boiler B: $P_{1B} = 800 \text{ kPa}$, $T_{1B} = 500^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_{1B} = 800 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_{1B} = 800 \text{ kPa}) = 170.44^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is also a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_{1B} = 800 \text{ kPa}$ and temperature, $T_{1B} = 500^\circ\text{C}$, specific enthalpy at inlet, for boiler B, $h_{1B} = 3480.7 \text{ kJ/kg}$.

Now, total enthalpy at inlet can be calculated as

$$H_1 = \dot{m}_{1A} h_{1A} + \dot{m}_{1B} h_{1B} = 5 \times 3911.7 + 15 \times 3480.7 = 77169 \text{ kW}$$

At outlet: $P_2 = 10 \text{ kPa}$, $x_2 = 0.96$

As the quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 10 \text{ kPa}$,

$h_f = 191.83 \text{ kJ/kg}$, $h_{fg} = 2392.0 \text{ kJ/kg}$, $h_g = 2583.8 \text{ kJ/kg}$

Now, the specific enthalpy of steam at outlet of the turbine is given by

$$h_2 = h_f + x_2 h_{fg} = 191.83 + 0.96 \times 2392.0 = 2488.15 \text{ kJ/kg}$$

From the conservation principle of mass for control volume, we have

$$\dot{m}_2 = \dot{m}_{1A} + \dot{m}_{1B} = 5 + 15 = 20 \text{ kg/s}$$

Then, enthalpy of steam at outlet, $H_2 = \dot{m}_2 h_2 = 20 \times 2488.15 = 49763.0 \text{ kW}$

From the energy equation for control volume system at steady state condition, we have

$$\begin{aligned} \dot{Q}_{cv} - \dot{W}_{cv} &= \dot{m}_2 h_2 - (\dot{m}_{1A} h_{1A} + \dot{m}_{1B} h_{1B}) && \text{(Neglecting KE \& PE)} \\ \Rightarrow \dot{Q}_{cv} - \dot{W}_{cv} &= H_2 - H_1 && \text{(As } \dot{Q}_{cv} = 0) \\ \therefore \dot{W}_{cv} &= H_1 - H_2 = 77169 - 49763 = 22006 \text{ kW} = 22.006 \text{ MW} \end{aligned}$$

Problem 4.35.

0.3 kg of nitrogen gas at 100 kPa and 40 °C is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 2 MPa and temperature becomes 160 °C. The work done during the process is 60 kJ. Calculate the heat transferred from the nitrogen to the surroundings. Take for nitrogen, $c_v = 0.75 \text{ kJ/kgK}$. (PoU 2014 spring, PoU 2015 fall).

Solution: Given,

Mass of nitrogen, $m = 0.3 \text{ kg}$

Initial pressure, $P_1 = 100 \text{ kPa}$

Initial Temperature, $T_1 = 40^\circ\text{C} = 40 + 273 = 313 \text{ K}$

Final pressure, $P_2 = 2 \text{ MPa} = 2000 \text{ kPa}$

Final temperature, $T_2 = 160^\circ\text{C} = 160 + 273 = 433 \text{ K}$

Work done on system during compression, $W_{1,2} = -60 \text{ kJ}$

Specific heat of nitrogen, $c_v = 0.75 \text{ kJ/kgK}$

For ideal gas, change in internal energy is determined as

$$\Delta U_{1,2} = mc_v(T_2 - T_1) = 0.3 \times 0.75 \times (433 - 313) = 27 \text{ kJ}$$

Now, using first law of thermodynamics, heat transfer can be calculated as

$$Q_{1,2} = \Delta U_{1,2} + W_{1,2} = 27 + (-60) = -33 \text{ kJ}$$

Negative sign indicates that heat is transferred from system to surroundings.

Problem 4.36.

Air enters turbine at 1 MPa and 327°C with a velocity of 100 m/s and exits at 100 kPa and 27°C with low velocity. Heat transfer (loss) from the turbine surface is 1200 kJ/min and power output of turbine is 240 kW . Determine the mass flow rate of air through the turbine. (PoU 2016 fall).

Solution: Given,

Pressure at inlet, $P_1 = 1 \text{ MPa} = 1000 \text{ kPa}$

Temperature at inlet, $T_1 = 327^\circ\text{C} = 327 + 273 = 600 \text{ K}$

Velocity at inlet, $\bar{V}_1 = 100 \text{ m/s}$

Pressure at exit, $P_2 = 100 \text{ kPa}$

Temperature at exit, $T_2 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Velocity at exit, $\bar{V}_2 = 0 \text{ m/s}$ (As exit velocity is low)

Heat loss by turbine, $\dot{Q}_{cv} = -1200 \text{ kJ/min} = -1200/60 = -20 \text{ kJ/s} = -20 \text{ kW}$

Power output of turbine, $\dot{W}_{cv} = 240 \text{ kW}$

Change in enthalpy, $h_2 - h_1 = c_p(T_2 - T_1) = 1005(300 - 600)$
 $= -301500 \text{ J/kg} = -301.5 \text{ kJ/kg}$

From the energy equation for control volume at steady state condition, we have

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{Neglecting PE change})$$

$$\Rightarrow \dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2)} = \frac{-20 - 240}{-301.5 + \frac{1}{2 \times 1000} (0^2 - 100^2)} = 0.848287 \text{ kg/s}$$

Problem 4.37.

Steam enters a nozzle at 400°C and 800 kPa with a velocity of 10 m/s and leaves at 300°C and 200 kPa while losing heat at a rate of 25 kW . For an inlet area of 800 cm^2 , determine the velocity and volume flow rate of steam at the nozzle exit. (PoU 2016 spring, PoU 2017 spring).

Solution: Given,

Temperature at inlet, $T_1 = 400^\circ\text{C}$

Pressure at inlet, $P_1 = 800 \text{ kPa}$

Velocity at inlet, $\bar{V}_1 = 10 \text{ m/s}$

Inlet area of nozzle, $A_1 = 800 \text{ cm}^2 = 800 \times 10^{-4} \text{ m}^2$

Temperature at exit, $T_2 = 300^\circ\text{C}$

Pressure at exit, $P_2 = 200 \text{ kPa}$

Heat loss from nozzle surface, $\dot{Q}_{\infty} = -25 \text{ W} = -25 \times 10^3 \text{ W}$

At inlet: $P_1 = 800 \text{ kPa}$, $T_1 = 400^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 800 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 800 \text{ kPa}) = 170.44^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 800 \text{ kPa}$ and temperature, $T_1 = 400^\circ\text{C}$,

specific volume at inlet, $v_1 = 0.3843 \text{ m}^3/\text{kg}$ and

specific enthalpy at inlet, $h_1 = 3267.0 \text{ kJ/kg}$.

Now, mass flow rate of steam,

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{800 \times 10^{-4} \times 10}{0.3843} = 2.0817 \text{ kg} \quad (\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m})$$

At exit: $P_2 = 200 \text{ kPa}$, $T_2 = 300^\circ\text{C}$

Similarly, referring to the saturated water pressure table A2.1 for pressure, $P_2 = 200 \text{ kPa}$, the saturation temperature for given pressure, i.e.

$T_{sat}(P_2 = 200 \text{ kPa}) = 120.24^\circ\text{C}$.

Here, $T_2 > T_{sat}$. Hence, it is also a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 200 \text{ kPa}$ and temperature, $T_2 = 300^\circ\text{C}$,

specific volume, $v_2 = 1.3162 \text{ m}^3/\text{kg}$ and specific volume, $h_2 = 3071.4 \text{ kJ/kg}$.

From the energy equation for control volume system like flow device nozzle ($\dot{W}_{\infty} = 0$) at steady state condition, we have

$$\begin{aligned} \dot{Q}_{\infty} &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] && \text{(Neglecting PE change)} \\ \Rightarrow \frac{\dot{Q}_{\infty}}{\dot{m}} &= (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \\ \Rightarrow \bar{V}_2 &= [\bar{V}_1^2 + \frac{2\dot{Q}_{\infty}}{\dot{m}} - 2(h_2 - h_1)]^{1/2} = [10^2 + \frac{2 \times -25 \times 10^3}{2.0817} - 2(3071.4 - 3267.0)10^3]^{1/2} \\ &= 606.037 \text{ m/s} \end{aligned}$$

Again, mass flow rate of steam can be expressed as

$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} \Rightarrow \text{Exit area, } A_2 = \frac{\dot{m} v_2}{\bar{V}_2} = \frac{2.0817 \times 1.3162}{606.037} = 0.004521 \text{ m}^2$$

Now, volume flow rate of steam at exit can be calculated as

$$\dot{V}_2 = A_2 \bar{V}_2 = 0.004521 \times 606.037 = 2.74 \text{ m}^3/\text{s}$$

Problem 4.38.

Air flows through an open system at the steady mass flow rate of 5 kg/s. At inlet the air velocity is negligible, the pressure is 1 bar and the temperature is 15°C. The air flow is compressed isentropically and leaves the system at a pressure of 5 bar through a pipe with an internal diameter of 80 mm. Calculate:

- the temperature of the air at it leaves the system.
- the velocity in the exit pipe.
- the power input to the air.

(Assume for air, $R = 287 \text{ J/kgK}$ and $\gamma = 1.4$). (PoU 2018 fall).

Solution: Given,

Mass flow rate of air, $\dot{m} = 5 \text{ kg/s}$

Velocity at inlet, $\bar{V}_1 = 0 \text{ m/s}$ (negligible)

Pressure at inlet, $P_1 = 1 \text{ bar} = 100 \text{ kPa}$

Temperature at inlet, $T_1 = 15^\circ\text{C} = 15 + 273 = 288 \text{ K}$

Pressure at outlet, $P_2 = 5 \text{ bar} = 500 \text{ kPa}$

Rate of heat loss, $\dot{Q}_{cv} = 0 \text{ kW}$ (Isentropic compression)

Outlet diameter, $D_2 = 80 \text{ mm} = 0.08 \text{ m}$

Outlet area of the pipe is given by

$$A_2 = \frac{\pi \times D_2^2}{4} = \frac{\pi (0.08)^2}{4} = 0.00502655 \text{ m}^2$$

For isentropic process, we have

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 288 \times \left(\frac{500}{100} \right)^{\frac{1.4-1}{1.4}} = 456.14 \text{ K}$$

Using equation of state, specific volume at outlet, v_2 is given by

$$P_2 v_2 = RT_2 \Rightarrow v_2 = \frac{RT_2}{P_2} = \frac{287 \times 456.14}{500 \times 10^3} = 0.261824 \text{ m}^3/\text{kg}$$

$$\text{Mass flow rate of air is: } \dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2}$$

Then, the velocity of the exit pipe is calculated as

$$\bar{V}_2 = \frac{\dot{m} \times v_2}{A_2} = \frac{5 \times 0.261824}{0.00502655} = 260.441 \text{ m/s}$$

From the energy equation for an open system at steady state condition, we have

$$\begin{aligned} \dot{Q}_{cv} - \dot{W}_{cv} &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \\ \Rightarrow \quad \dot{Q}_{cv} - \dot{W}_{cv} &= \dot{m}(h_2 - h_1) \quad (\text{Neglecting KE \& PE}) \\ \Rightarrow \quad 0 - \dot{W}_{cv} &= \dot{m}(h_2 - h_1) \quad (\text{As } \dot{Q}_{cv} = 0) \\ \therefore \quad \dot{W}_{cv} &= \dot{m} \times (h_1 - h_2) = \dot{m} \times c_p \times (T_1 - T_2) \end{aligned}$$

$$= 5 \times 1005 (288 - 456.14) = - 844903.5 \text{ W} = - 844.9035 \text{ kW}$$

Problem 4.39.

Air at 100 kPa and 127°C enters an adiabatic diffuser at a rate of 1.5 kg/s and leaves at a pressure of 150 kPa. The velocity of the air is decreased from 250 m/s to 50 m/s as it passes through the diffuser. Determine the exit temperature of the air and the exit area of the diffuser. [For air, take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$. Assume suitable data if necessary.] (PoU 2019 fall).

Solution: Given,

Pressure at inlet, $P_1 = 100 \text{ kPa}$

Temperature at inlet, $T_1 = 127^\circ\text{C} = 127 + 273 = 400 \text{ K}$

Mass flow rate of air, $\dot{m} = 1.5 \text{ kg/s}$

Pressure at exit, $P_2 = 150 \text{ kPa}$

Velocity at inlet, $\bar{V}_1 = 250 \text{ m/s}$

Velocity at exit, $\bar{V}_2 = 50 \text{ m/s}$

From the energy equation for control volume system like flow device diffuser ($\dot{W}_c = 0$) at steady state condition, neglecting PE change and for adiabatic diffuser ($\dot{Q}_c = 0$), we have

$$\begin{aligned} 0 &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \\ &\Rightarrow c_p (T_2 - T_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) = 0 \\ &\Rightarrow T_2 = T_1 + \frac{1}{2c_p} (\bar{V}_1^2 - \bar{V}_2^2) = 400 + \frac{1}{2 \times 1005} (250^2 - 50^2) = 429.851 \text{ K} \end{aligned}$$

Using equation of state, the specific volume at the exit is given by

$$P_2 v_2 = RT_2 \Rightarrow v_2 = \frac{RT_2}{P_2} = \frac{287 \times 429.851}{150 \times 10^3} = 0.82245 \text{ m}^3/\text{kg}$$

Now, rate of mass flow of the air,

$$\begin{aligned} \dot{m} &= \frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2} \quad (\text{As } \dot{m}_1 = \dot{m}_2 = \dot{m}) \\ \Rightarrow \text{Exit area, } A_2 &= \frac{\dot{m} \times v_2}{\bar{V}_2} = \frac{1.5 \times 0.82245}{50} = 0.0246735 \text{ m}^2 \end{aligned}$$

Problem 4.40.

Air flow at a rate of 1.2 kg/s through a compressor, entering at 100 kPa, 25°C with a velocity of 60 m/s and leaving at 500 kPa, 150°C with a velocity of 120 m/s. Heat lost by compressor to surroundings is estimated to be 20 kJ/kg.

Calculate the power required to drive the compressor and diameters of inlet and exhaust pipes.
Take $R = 287 \text{ J/kgK}$ and $C_p = 1005 \text{ J/kgK}$. (PoU 2019 spring).

Solution: Given,

Rate of mass flow, $\dot{m} = 1.2 \text{ kg/s}$

Pressure at inlet, $P_1 = 100 \text{ kPa}$

Temperature at inlet, $T_1 = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$

Velocity at inlet, $\bar{V}_1 = 60 \text{ m/s}$

Pressure at outlet, $P_2 = 500 \text{ kPa}$

Temperature at outlet, $T_2 = 150^\circ\text{C} = 150 + 273 = 423 \text{ K}$

Velocity at outlet, $\bar{V}_2 = 120 \text{ m/s}$

Specific heat lost by compressor, $q_{cv} = -20 \text{ kJ/kg}$

Rate of heat lost by compressor, $\dot{Q}_{cv} = \dot{m} \times q_{cv} = 1.2 \times -20 \text{ kJ/kg}$

$$= -24 \text{ kW} = -24000 \text{ W}$$

Change in enthalpy, $h_2 - h_1 = c_p(T_2 - T_1) = 1005(423 - 298) = 125625 \text{ J/kg}$

From the energy equation for control volume at steady state condition, we have

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] \quad (\text{when neglecting PE change})$$

\Rightarrow Power required to drive the compressor,

$$\dot{W}_{cv} = \dot{Q}_{cv} - \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right]$$

$$= -24000 - 1.2 [125625 + 0.5 (120^2 - 60^2)] = -181230 \text{ W} = -181.23 \text{ kW}$$

The negative sign indicates that the power is supplied to the system (compressor).

Using equation of state, specific volumes at inlet and outlet are given by

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 298}{100 \times 10^3} = 0.85526 \text{ m}^3/\text{kg}$$

$$P_2 v_2 = RT_2 \Rightarrow v_2 = \frac{RT_2}{P_2} = \frac{287 \times 423}{500 \times 10^3} = 0.242802 \text{ m}^3/\text{kg}$$

As $\dot{m}_1 = \dot{m}_2 = \dot{m}$ for steady state condition and using expression for rate of mass flow, the inlet area and outlet area of pipes are given by

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{\bar{V}_1} = \frac{1.2 \times 0.85526}{60} = 0.0171052 \text{ m}^2$$

$$\dot{m} = \frac{A_2 \bar{V}_2}{v_2} \Rightarrow A_2 = \frac{\dot{m} v_2}{\bar{V}_2} = \frac{1.2 \times 0.242802}{120} = 0.00242802 \text{ m}^2$$

Now, using area calculation formula, the inlet and outlet diameters are determined as

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$$A_1 = \frac{\pi D_1^2}{4} \Rightarrow D_1 = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4 \times 0.0171052}{\pi}} = 0.1476 \text{ m}$$

$$A_2 = \frac{\pi D_2^2}{4} \Rightarrow D_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 \times 0.00242802}{\pi}} = 0.0556 \text{ m}$$

Problem 4.41.

A system undergoes a cycle consisting of three processes listed in the table. Compute the missing values. All quantities are in kJ. (PoU 2020 fall).

Process	Q	W	ΔE
1 - 2	a	100	100
2 - 3	b	-50	c
3 - 1	100	d	-200

Solution: Given.

Process 1 - 2: Heat transfer, $Q_{1-2} = a$

Work transfer, $W_{1-2} = 100 \text{ kJ}$

Change in total energy, $\Delta E_{1-2} = 100 \text{ kJ}$

Process 2 - 3: Heat transfer, $Q_{2-3} = b$

Work transfer, $W_{2-3} = -50 \text{ kJ}$

Change in total energy, $\Delta E_{2-3} = c$

Process 3 - 1: Heat transfer, $Q_{3-1} = 100 \text{ kJ}$

Work transfer, $W_{3-1} = d$

Change in total energy, $Q_{3-1} = -200 \text{ kJ}$

Using first law of thermodynamics for process 1 - 2, heat transfer

$$Q_{1-2} = \Delta E_{1-2} + W_{1-2} \Rightarrow a = 100 + 100 = 200 \text{ kJ}$$

For process 3 - 1: Work transfer is given by

$$W_{3-1} = Q_{3-1} - \Delta E_{3-1} \Rightarrow d = 100 - (-200) = 300 \text{ kJ}$$

We know that the sum of change in total energy in a cyclic process is zero, i.e.

$$\begin{aligned} \Delta E_{1-2} + \Delta E_{2-3} + \Delta E_{3-1} &= 0 \Rightarrow \Delta E_{2-3} = 0 - \Delta E_{1-2} - \Delta E_{3-1} \\ &\Rightarrow c = 0 - 100 - (-200) = 100 \text{ kJ} \end{aligned}$$

For process 2 - 3: Heat transfer, $Q_{2-3} = \Delta E_{2-3} + W_{2-3}$

$$\Rightarrow b = c + (-50) = 100 - 50 = 50 \text{ kJ}$$

Alternately, use the relation, $\sum W = \sum Q$ for a cycle to find the value of Q_{2-3} .

Hence, the complete table will be

Process	Q	W	ΔE
1 - 2	200	100	100
2 - 3	50	-50	100
3 - 1	100	300	-200

4.4 Review Theoretical Questions

1. Explain first law of thermodynamics for a control mass with reference to conservation of mass and conservation of energy.
2. Define a cycle. Explain first law of thermodynamics for control mass undergoing cyclic process. Write down statements of first law for power cycle and refrigeration cycle.
3. Derive the mass and energy equations for a control volume and reduce them for a steady state process.
4. State and explain conservation of mass for a control volume. Also derive an expression for mass flow rate through a uniform section.
5. Write down general steady state energy equation for a control volume. Reduce it for an adiabatic turbine, an adiabatic diffuser and throttling valve. Also mention relevant assumptions for each device.
6. Write down general energy equation for a heat exchanger, condenser, compressor, evaporator and throttling valve.
7. Derive the general expression for conservation of energy for control volume. Modify it for turbine and nozzle.
8. Write down general mass conservation and energy conservation equations for a control volume. Also derive mass and energy conservation equations for a gas filling process in a gas station.
9. Derive a general expression for energy conservation for a control volume and reduce it for steady state condition.
10. Write down general mass conservation and energy conservation equations for a control volume. Also reduce them for a control volume operating under unsteady state condition.
11. State and explain conservation of energy for a control volume.
12. Derive unsteady state energy equation for an open volume. Apply this equation to derive governing equations for the discharge of a gas from a cylinder.
13. Differentiate between steady state and unsteady state control volume. Derive mass and energy conservation equations for a process in which gas is being supplied to a rigid cylinder.
14. Derive an expression for flow work for any inlet or outlet section of a control volume.

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15. Write down general mass conservation and energy conservation equations for a steady state process and reduce them for an adiabatic turbine.
16. Write down general mass and energy conservation equations for a control volume under steady state. Reduce them for heat exchanger and adiabatic pump.
17. Define steady state flow application. Write down general energy equation for a nozzle and diffuser and reduce it for an adiabatic nozzle and diffuser.
18. Define steady and unsteady state system. Derive the expressions of conservation of mass and conservation of energy for a control volume having steady and unsteady flow.
19. Derive general expressions for mass and energy conservation for a control volume operating under unsteady state condition.
20. Define steady state work application. Write down general energy equation for a turbine and compressor and reduce it for an adiabatic turbine or compressor.
21. Differentiate between steady state work applications and steady state flow applications. Write down the functions of (an adiabatic) turbine and nozzle. Also derive governing equations for them when they operate under steady state condition.
22. Differentiate between steady state work applications and steady state flow applications. Write down mass and energy conservation equations for steady state process. Reduce them for a nozzle and a condenser.
23. Write down general energy equation for a heat exchanger, condenser/evaporator and throttling valve. Also mention the relevant assumptions for each.
24. Derive mass and energy conservation equations for a gas filling process in a gas station.
25. State first law of thermodynamics for an isolated system and a control mass undergoing an adiabatic process.
26. Define PMM I. Explain why it is not possible.

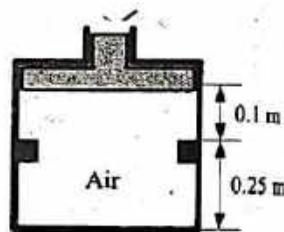
4.5 Unsolved Numerical Problems

1. Air undergoes a cyclic process in a cylinder and piston arrangement. First the atmospheric air is at 1 bar and 27°C is compressed adiabatically to 10 bar, then expanded isothermally up to initial pressure and then brought to initial condition under constant pressure. Find
 - (a) the change in specific internal energy
 - (b) change in specific enthalpy
 - (c) the specific heat transfer and
 - (d) the specific work transfer for each process and also for the cycle.

(Ans: 0 kJ/kg, 0 kJ/kg (for cyclic process), 102.3 kJ/kg, -200.3325 kJ/kg, 382.7656 kJ/kg, -80.133 kJ/kg, 102.3 kJ/kg)
2. A fluid system undergoes a non-flow frictionless process, following the pressure volume relation as $P = \frac{5}{V} + 1.5$, where P is in bar and V is in m^3 . During the process the volume changes from $0.15 m^3$ to $0.05 m^3$ and system rejects 45 kJ of heat. Calculate
 - (a) change in internal energy
 - (b) change in enthalpy. (Ans: 519 kJ, 504.05 kJ)
3. A gas contained in a piston cylinder device undergoes a polytropic process for which pressure volume relationship is given by $PV^{2.5} = \text{constant}$. The initial pressure is 400 kPa, the initial volume is $0.2 m^3$ and the final volume is $0.4 m^3$. The internal energy of the gas decreases by 20 kJ during the process. Determine the work transfer and heat transfer for the process.
 (Ans: 34.48 kJ, 14.48 kJ)
4. Air undergoes a three process cycle. Find the net work done for 2 kg of air if the processes are constant pressure expansion (1 – 2), constant volume (2 – 3) and constant temperature compression (3 – 1). The necessary information are $T_1 = 100^\circ\text{C}$, $T_2 = 600^\circ\text{C}$ and $P_1 = 200 \text{ kPa}$. Sketch the cycle on $P - v$ diagram.
 (Ans: 214.34 kJ)
5. A rigid vessel initially contains 4 kg of a saturated water vapor. It is cooled to a final state where the temperature is 150°C and the quality is 0.1. Determine the initial temperature and the heat transfer from the system.
 (Ans: 262.799°C, -7089.948 kJ)
6. A rigid vessel having a volume of $0.4 m^3$ initially contains a two-phase mixture at a pressure of 100 kPa with 2 % of its volume occupied by saturated liquid and the remaining by the saturated vapor. Heat is supplied to the vessel until it holds only saturated vapor. Determine the total heat transfer for the process. (Ans: 16775.98 kJ)

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7. A rigid vessel of volume 0.2 L contains water at its critical state. It is cooled down to room temperature of 25°C . Determine the heat loss from the water.
 (Ans: -122.969 kJ)
8. A piston cylinder device as shown in figure restrained by a linear spring ($k = 20 \text{ kN/m}$) contains 0.5 kg of water initially at a pressure of 200 kPa and a volume of 0.4 m^3 . It is now heated until a final pressure of 400 kPa is reached. If the cross sectional area of the piston is 0.05 m^2 , determine the final temperature and the heat transfer in the process.
 (Ans: 466.21°C , 377.5 kJ)
9. A 25 kg piston is required to travel a specific distance within the piston cylinder arrangement shown in figure. The piston initially rests on the bottom stops. The air within the cylinder is heated, until pressure reaches at 0.3 MPa . The initial pressure and temperature are $P_i = P_{atm} = 0.101 \text{ MPa}$ and $T_i = 20^\circ\text{C}$. Cross sectional area of the piston is 0.005 m^2 .
- Draw $P-V$ diagram for the process
 - Find the temperature of the air inside the cylinder when the piston reaches the upper stops
 - Calculate the heat transfer during the process. [Take $R = 287 \text{ J/kgK}$, $c_p = 1004 \text{ J/kgK}$, $c_v = 717 \text{ J/kgK}$]
 (Ans: 609.96 K , 1.071485 kJ)
10. Steam enters a turbine operating at steady state with a mass flow rate of 1.2 kg/s . Properties of the steam at the inlet are $P_1 = 5 \text{ MPa}$, $T_1 = 450^\circ\text{C}$, $\bar{V}_1 = 10 \text{ m/s}$ and at the exit are $P_2 = 100 \text{ kPa}$, $x_2 = 80\%$, $\bar{V}_2 = 50 \text{ m/s}$. If the power output of the turbine is 1200 kW , determine the rate of heat transfer from the turbine. (Ans: -109.812 kW)
11. An adiabatic turbine operating under steady state condition develops 12 MW of power output for a steam mass flow rate of 15 kg/s . The steam enters at 4 MPa with a velocity of 20 m/s and exits at 60 kPa with a quality of 85% and velocity of 100 m/s . Determine the inlet temperature of steam.
 (Ans: 359.1°C)
12. Steam enters a turbine operating at steady state with mass flow rate of 4600 kg/h and the turbine develops a power output of 1000 kW . At the inlet, the pressure is 60 bar , the temperature is 400°C and velocity is 10 m/s . At the exit, the pressure is 0.1 bar , the quality is 0.9 and the velocity is 50 m/s .



Calculate the rate of heat transfer between the turbine and surroundings in kW. (Ans: 2065.3 kW)

13. A steam turbine receives steam from the boiler at the rate of 5 kg/s and at 3.5 MPa pressure in dry saturated condition. The velocity of steam entering the turbine is 75 m/s and the elevation at the entry is 5 m. It leaves the turbine at a pressure of 7 kPa and with a dryness fraction of 0.8. The velocity of steam at the exit of turbine is 160 m/s and elevation at the exit is 1 m. The loss of heat from the turbine to surroundings is 18 MJ/hr. Calculate the power developed by the turbine. (Ans: 3.71431 MW)
14. A water pump delivers 2750 kg of water per minute from initial pressure of 0.8 bar to a final pressure of 2.8 bar. The suction is 2 m below and the delivery is 5 m above the center of pump. If the suction and delivery pipes are of 15 cm and 10 cm diameter respectively, calculate the power required to run the pump. (Ans: -12.942 kW)
15. Air at 1 bar pressure, 290 K temperature flows steadily at the rate of 120 m³/hr into a compressor where its pressure and temperature are respectively raised to 15 bar and 390 K. During the compression process, the heat loss from the compressor is 10% of work input to the compressor. Neglecting changes in kinetic energy and potential energy, determine the work input and heat loss. Assume that air behaves as a perfect gas.
(Ans: -4.47 kW, -0.447 kW)
16. Steam enters a nozzle operating at steady state with $P_1 = 10$ bar, $T_1 = 400^\circ\text{C}$ and velocity of 10 m/s. The steam flows through the horizontal adiabatic nozzle. At the exit $P_2 = 1.5$ bar and the velocity of 1068.13 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle. (Ans: 0.002171 m²)
17. Steam at 6 bar, 200°C enters an insulated nozzle with a velocity of 50 m/s, and it leaves at a pressure of 1.37 bar and velocity of 500 m/s. Determine final enthalpy of steam if inlet area of nozzle and exit specific volume are 900 cm² and 0.498 m³/kg respectively. Find the mass flow rate and exit area of nozzle. (Ans: 2725.95 kJ/kg, 12.78 kg/s, 0.01273 m²)
18. Air at 80 kPa, 27°C and 220 m/s enters a diffuser at a rate of 2.5 kg/s and leaves at 40°C. The exit area of diffuser is 400 cm². The air is estimated to lose heat at a rate of 18 kJ/s during this process. Determine the exit velocity and pressure of air. (Ans: 62.048 m/s, 91.06 kPa)
19. A well insulated valve is used to throttle steam from 8 MPa, 500°C to 6 MPa. Determine the final temperature of steam. Determine the final temperature of the steam. (Ans: 490.12°C)

20. A water heating arrangement operates at steady state with liquid water entering at inlet 1 with $P_1 = 500 \text{ kPa}$ and $T_1 = 50^\circ\text{C}$. Steam enters at inlet 2 with $P_2 = 500 \text{ kPa}$ and $T_2 = 200^\circ\text{C}$. Saturated liquid water exits with a pressure of $P_3 = 500 \text{ kPa}$ from the outlet 3. Determine the ratio of mass flow rates \dot{m}_1/\dot{m}_2 . (Ans: 5.1375)

Second Law of Thermodynamics

5

5.1 Chapter Highlights

➤ **Heat engine:** It is a device which operates on a cyclic process and converts heat energy into mechanical work.

➤ **Efficiency:** It is defined as the ratio of desired output to the required input, i.e.

$$\eta = \frac{\text{Desired output}}{\text{Required input}}$$

➤ For heat engine (reversible or irreversible), the efficiency is the ratio of work output to the heat supplied and it can be derived for both reversible and irreversible heat engines as

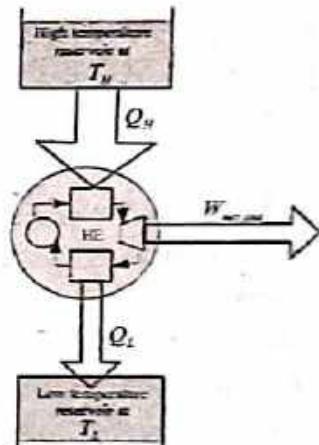
$$\eta_{HE} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Using second law of thermodynamics for reversible heat engine,

$$\frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

The efficiency of a reversible heat engine which is also called a *Carnot efficiency* is given by

$$\eta_{HE,rev} = \eta_{Carnot} = 1 - \frac{T_L}{T_H}$$



As the work output from the real engine decreases with increase in irreversibility, efficiency of a real engine (or irreversible engine) is always less than that of reversible engine, i.e.

$$\eta_{HE,irrev} < \left(1 - \frac{T_L}{T_H}\right)$$

- **Heat pump:** It is a device operating on a cyclic process which transfers heat from a low temperature reservoir (surroundings) to a high temperature reservoir (desired space) with the aid of external work. It maintains the temperature of a desired space higher than that of the surroundings thereby providing the heating effect.
- **Coefficient of performance (COP):** It is defined as the ratio of desired effect (heating or cooling) to the work supplied, i.e.

$$COP = \frac{\text{Desired effect}}{\text{Work input}}$$

- For heat pump (reversible or irreversible), desired effect is the amount of heat supplied to desired space (Q_H) and therefore COP is given by

$$(COP)_{HP} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L}$$

Using second law of thermodynamics for reversible heat pump,

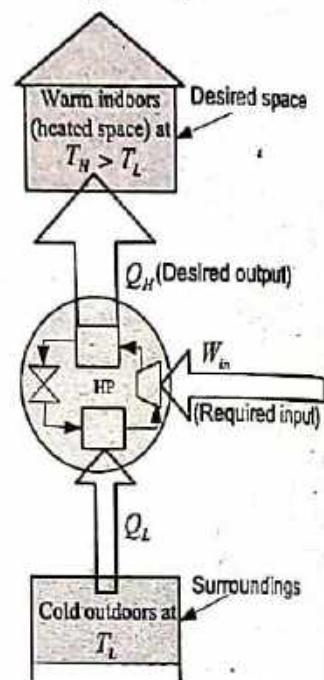
$$\frac{Q_L}{T_L} - \frac{Q_H}{T_H} = 0$$

the COP of a reversible heat pump is given by

$$(COP)_{HP,rev} = \frac{T_H}{T_H - T_L}$$

As the work input to the real heat pump should be increased with increase in irreversibility for the same heating effect, COP of a real heat pump (or irreversible heat pump) is always less than that of reversible heat pump, i.e.

$$(COP)_{HP,irrev} < \frac{T_H}{T_H - T_L}$$



- **Refrigerator:** It is a device operating on a cyclic process which transfers heat from a low temperature reservoir (desired space) to a high temperature reservoir (surroundings) with the aid of external work. It maintains the temperature of a desired space lower than that of the surroundings thereby providing the cooling effect.
- For refrigerator (reversible or irreversible), desired effect is the amount of heat taken out from desired space (Q_L) and therefore COP is given by

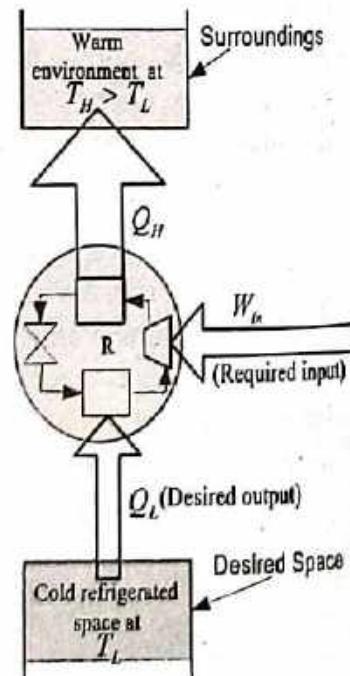
$$(COP)_R = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

Using second law of thermodynamics for reversible refrigerator,

$$\frac{Q_L}{T_L} - \frac{Q_H}{T_H} = 0$$

The COP of a reversible refrigerator is given by

$$(COP)_{R,rev} = \frac{T_L}{T_H - T_L}$$



SECOND LAW OF THERMODYNAMICS

As the work input to the real refrigerator should be increased with increase in irreversibility for the same cooling effect, COP of a real refrigerator (or irreversible refrigerator) is always less than that of reversible refrigerator, i.e.

$$(COP)_{R,irrev} < \frac{T_L}{T_H - T_L}$$

For the same machine, it reveals: $(COP)_{HP} - (COP)_R = 1$.

- *Differences between heat pump and refrigerator are:*

Heat Pump	Refrigerator
<p>1. Its main purpose is to heat the desired space (room during winter season).</p> <p>2. It absorbs heat from the atmosphere which is at low temperature and throws it to desired space that is to be heated and is at high temperature.</p> <p>3. In heat pump the evaporator is located outside the room (desired space) which is to be heated.</p> <p>4. The condenser in heat pump is located inside the room (desired space) and it acts as heating device.</p> <p>5. In heat pump, condenser performs main function of heating the room while the evaporator absorbs heat from the atmosphere.</p> <p>6. COP of heat pump is higher than that of refrigerator and is given by</p> $(COP)_{HP} = \frac{Q_H}{W} = 1 + (COP)_R$	<p>1. Its purpose is to cool or freeze a substance in desired space (compartment of kitchen refrigerator or freezer).</p> <p>2. It absorbs heat from the desired space which is to be cooled and is at low temperature and throws it to the atmosphere at high temperature.</p> <p>3. In refrigerator the evaporator is located inside the freezer (desired space) and it acts as cooling device.</p> <p>4. The condenser in the refrigerator is located outside the freezer compartment and it is exposed to the atmosphere.</p> <p>5. In refrigerator, evaporator performs main function of cooling or freezing the desired space while the condenser delivers heat to atmosphere.</p> <p>6. COP of refrigerator is lower than that of heat pump and given by</p> $(COP)_R = \frac{Q_L}{W} = (COP)_{HP} - 1$

- *Similarities between heat pump and refrigerator are:*

1. Cycle execution direction of both devices is same and it is opposite to that of heat engine.
2. Both devices work on the vapor compression refrigeration cycle and have same cycle components, e.g. compressor, condenser, expansion valve (throttling valve) and evaporator.

3. Both devices need external work for the execution of cycle.
4. Net work of cycle is negative as both are refrigeration cycles.
5. Working substance is a refrigerant in both devices.
6. Both devices transfer heat from low temperature reservoir (sink) to the high temperature reservoir (source).
7. Performance of both devices is measured by their COP which is greater than unity.

➤ *Limitations (drawbacks) of first law of thermodynamics which lead to necessity of second law of thermodynamics are:*

- It deals with the quantitative aspect of the energy and gives only the condition that any process is possible provided that the total energy remains constant. But some processes in nature cannot occur although the total energy is conserved, because most of the processes in nature proceed in only one direction. Hence, it does not focus any restriction towards the direction of energy flow.
 - It states that for a closed system undergoing a cyclic process the net heat supplied to the system is equal to the net work output of the system. However, any device even operating on a cycle cannot convert heat supplied to it completely into work. Hence, it also does not focus any restriction towards what portion of heat supplied is converted into mechanical work.
- *Second law of thermodynamics* deals with quality or nature of energy and defines the direction of the process in which the system can proceed. It defines the direction of the process with reference to the system property called entropy. Hence, it focus towards the direction of flow of energy and is also named as directional law.
- *According to the second law of thermodynamics*, only a portion of heat input to an engine can be converted into work and remaining portion is rejected to a sink at lower temperature. Thus, energy gets degraded during conversion of heat into work and therefore it is also known as law of degradation of energy.
- *Entropy*: It is a measure of molecular disorder, randomness, disorganization or uncertainty existing in a system. It is an extensive property.
- *Classical statements of second law of thermodynamics*: Those statements with the help of which a direction or possibility of any process can be defined without using the property entropy are called classical statements of second law.
- *Kelvin – Planck statement*: “It is impossible to construct a heat engine operating on a cycle which converts all the heat energy supplied to it into mechanical work”.
 - *Claude's statement*: “It is impossible to construct a device operating on a cycle which transfers heat from a body at a lower temperature to a body at a higher temperature without aid of external work”.

SECOND LAW OF THERMODYNAMICS

- *Perpetual motion machine of the second kind (PMM II):* Without violating the first law of thermodynamics a machine can be imagined which could continuously absorbs heat from a source and would converts it continuously into mechanical work. This machine is known as PMM II. It is impossible to construct such a machine and it will violate the second law of thermodynamics as it would have 100% efficiency.
- *Equivalence of Kelvin-Planck and Clausius Statements:*
The Kelvin-Planck and the Clausius statements are equivalent in their consequences, and either statement can be used as the expression of the second law of thermodynamics. Any device that violates the Kelvin-Planck statement also violates the Clausius statement, and vice versa.
- *The major factors causing irreversibility in any process are:*
 - Process involving friction.
 - Process involving non-linear or inelastic spring.
 - Free expansion process.
 - Heat transfer with finite temperature difference
 - Mixing of fluids
- *Carnot Theorem:*
"No heat engine operating between two given constant temperature source and sink can have a higher efficiency than a reversible engine operating between the same source and sink".
- *Thermodynamic Temperature Scale:*
A temperature scale which is independent of the thermodynamic property of thermometric substance is called a *thermodynamic temperature scale*. The SI unit of thermodynamic temperature is the Kelvin (K). The relation between the thermodynamic temperature and Celsius scale which is in common use is

$$T(K) = T(^{\circ}\text{C}) + 273.15^{\circ}$$

5.2 Solved Numerical Problems

Problem 5.1.

An inventor makes the following claims. Determine whether the claims are valid or not and explain why or why not.

- A heat engine receives 400 kJ from a source at a temperature of 1000 K. It rejects 150 kJ of heat to sink at a temperature of 300 K.
- A heat pump can maintain a building at 20°C when the surroundings is at 0°C. The heat loss from the room occurs at a rate of 1000 kJ/min and the heat pump requires 1 kW of power input.
- During an experiment conducted in a room at 27°C, it is found that a refrigerator consumes 2 kW of power and removes 36000 kJ of heat from the desired space at -23°C. The running time for the refrigerator during the experiment was 30 min.

Solution:

(a) Given,

Source temperature, $T_H = 1000\text{ K}$

Sink temperature, $T_L = 300\text{ K}$

Heat rejection, $Q_L = 150\text{ kJ}$

Heat addition, $Q_H = 400\text{ kJ}$

Here, from the first law of thermodynamics for a cyclic process, work output of a heat engine,

$$W_{out} = Q_H - Q_L = 400 - 150 = 250\text{ kJ}$$

Maximum possible efficiency (Carnot efficiency) of the heat engine,

$$\eta_{Carnot} = \eta_{rev, HE} = 1 - \frac{T_L}{T_H} = 1 - 300/1000 = 0.7 = 70\%$$

Efficiency of the heat engine as per the claim of inventor,

$$\eta_{inventor} = \frac{W_{out}}{Q_H} = 250/400 = 0.625 = 62.5\%.$$

Here, $\eta_{inventor} < \eta_{Carnot}$. Hence, inventor's claim is valid and the heat engine is running under the irreversible condition.

(b) Given,

Higher temperature, $T_H = 20^\circ\text{C} = 20 + 273 = 293\text{ K}$

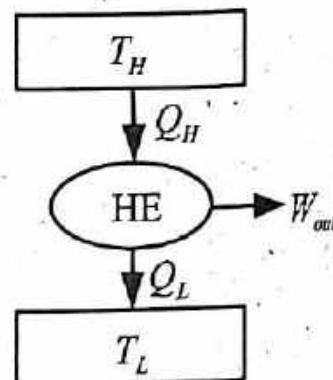
Lower temperature, $T_L = 0^\circ\text{C} = 0 + 273 = 273\text{ K}$

Rate of heat loss, $\dot{Q}_L = 1000\text{ kJ/min} = 1000/60 = 16.666\text{ kW}$

Power input, $\dot{W}_{in} = 1\text{ kW}$

Maximum possible coefficient of performance, $(COP)_{rev}$ of a heat pump,

$$(COP)_{rev, HP} = \frac{T_H}{T_H - T_L} = \frac{293}{293 - 273} = 14.65$$



Coefficient of performance of the heat pump as per inventor's claim,

$$(COP)_{inventor} = \frac{\dot{Q}_H}{\dot{W}_{in}} = 16.666/1 = 16.666.$$

Here, $(COP)_{inventor} > (COP)_{rev,HP}$. Hence, the inventor's claim is not valid. Because no practical heat pump can have more COP than that of reversible heat pump.

(c) Given,

Higher temperature, $T_H = 27^\circ C = 27 + 273 = 300 K$

Lower temperature, $T_L = -23^\circ C = -23 + 273 = 250 K$

Running time for refrigerator, $t = 30 \text{ min}$

Power input, $\dot{W}_{in} = 2 \text{ kW}$

Heat rejection, $\dot{Q}_L = 36000 \text{ kJ}$

$$\Rightarrow \text{Rate of heat rejection, } \dot{Q}_L = \dot{Q}_L/t \\ = 36000/(30 \times 60) \approx 20 \text{ kW}$$

Maximum possible coefficient of performance, $(COP)_{rev}$ of a refrigerator,

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L} = \frac{250}{300 - 250} = 5$$

Coefficient of performance of the refrigerator as per inventor's claim,

$$(COP)_{inventor} = \frac{\dot{Q}_L}{\dot{W}_{in}} = 20/2 = 10$$

Here, $(COP)_{inventor} > (COP)_{rev,R}$. Hence, the inventor's claim is not valid. Because no practical refrigerator can have more COP than that of reversible refrigerator.

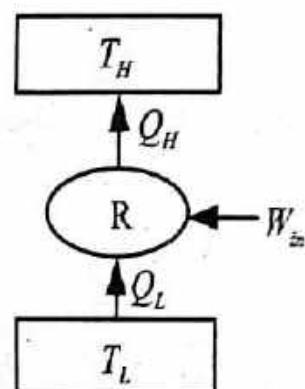
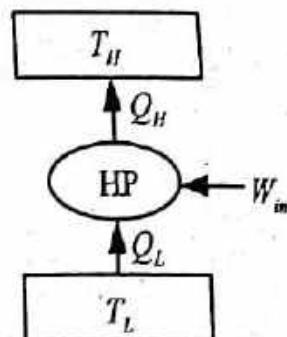
Problem 5.2.

A power cycle operating between two reservoirs receives Q_H from a high temperature source at $T_H = 1000 K$ and rejects energy Q_L to a low temperature sink at $T_L = 300 K$. For each of the following cases, determine whether the cycle operates reversibly, irreversibly or is impossible.

- (a) $Q_H = 800 \text{ kJ}$, $W = 600 \text{ kJ}$
- (b) $Q_H = 800 \text{ kJ}$, $Q_L = 240 \text{ kJ}$
- (c) $W = 960 \text{ kJ}$, $Q_L = 640 \text{ kJ}$
- (d) $\eta = 50\%$

Solution: Given,

Source temperature, $T_H = 1000 K$



Sink temperature, $T_L = 300 \text{ K}$

Maximum possible efficiency (Carnot efficiency) of the power cycle,

$$\eta_{\text{carnot}} = \eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - 300/1000 = 0.7 = 70\%$$

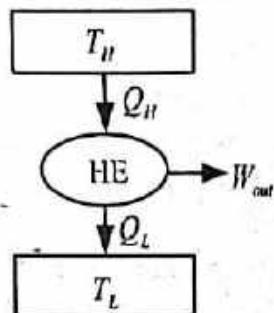
(a) Given,

Heat addition, $Q_H = 800 \text{ kJ}$

Work input, $W = 600 \text{ kJ}$

Efficiency of the power cycle as per the given condition,

$$\eta = \frac{W_{\text{out}}}{Q_H} = 600/800 = 0.75 = 75\%$$



Here, $\eta > \eta_{\text{carnot}}$. Hence, the given power cycle is not possible.

(b) Given,

Heat addition, $Q_H = 800 \text{ kJ}$

Heat rejection, $Q_L = 240 \text{ kJ}$

Here, from the first law of thermodynamics for a cyclic process, work output of a power cycle,

$$W_{\text{out}} = Q_H - Q_L = 800 - 240 = 560 \text{ kJ}$$

Efficiency of the power cycle as per the given condition,

$$\eta = \frac{W_{\text{out}}}{Q_H} = 560/800 = 0.7 = 70\%$$

Here, $\eta = \eta_{\text{carnot}}$. Hence, the given power cycle operates reversibly.

(c) Given,

Work output, $W_{\text{out}} = 960 \text{ kJ}$

Heat rejection, $Q_L = 640 \text{ kJ}$

Here, from the first law of thermodynamics for a cyclic process, work output of a power cycle,

$$W_{\text{out}} = Q_H - Q_L \Rightarrow Q_H = W_{\text{out}} + Q_L = 960 + 640 = 1600 \text{ kJ}$$

Efficiency of the power cycle as per the given condition,

$$\eta = \frac{W_{\text{out}}}{Q_H} = 960/1600 = 0.6 = 60\%$$

Here, $\eta < \eta_{\text{carnot}}$. Hence, the given power cycle operates irreversibly.

(d) $\eta = 50\%$

Here, $\eta < \eta_{\text{carnot}}$. Hence, the given power cycle operates irreversibly.

Problem 5.3.

An inventor claims that his engine absorbs 300 kJ of energy from a thermal reservoir at 325 K and delivers 75 kJ of work. The inventor also states that his engine has two rejections: 125 kJ to reservoir at 300 K and 100 kJ to reservoir at 275 K. Check the validity of his claim.

Solution: Given,

Source temperature, $T_H = 325 \text{ K}$

Absorbed heat, $Q_H = 300 \text{ kJ}$

Work output, $W_{out} = 75 \text{ kJ}$

Rejected heat to reservoir 1, $Q_{L1} = -125 \text{ kJ}$

Sink temperature of reservoir 1, $T_{L1} = 300 \text{ K}$

Rejected heat to reservoir 2, $Q_{L2} = -100 \text{ kJ}$

Sink temperature of reservoir 2, $T_{L2} = 275 \text{ K}$

The validity of the proposed engine is checked on the basis of fact that the engine satisfies the both first and second laws of thermodynamics. If it satisfies them then engine is theoretically feasible and if it violates any of two laws, then the engine is impossible.

From the first law of thermodynamics for a cyclic process,

$$\oint \delta Q = \oint \delta W$$

Here, $\oint \delta Q = \sum Q = Q_H + Q_{L1} + Q_{L2} = 300 - 125 - 100 = 75 \text{ kJ}$ and

$$\oint \delta W = \sum W = W_{out} = 75 \text{ kJ}$$

This shows that the first law of thermodynamics is satisfied.

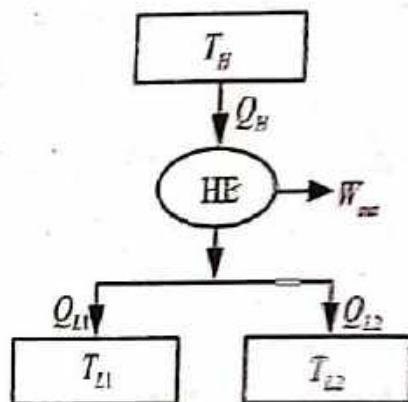
From the second law of thermodynamics for a cyclic process,

$$\oint dS_{CM} = 0 \geq \oint \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM} \Rightarrow \oint \frac{\delta Q}{T} \leq 0$$

$$\text{Here, } \oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} + \frac{-Q_{L1}}{T_{L1}} + \frac{-Q_{L2}}{T_{L2}} = \frac{300}{325} - \frac{125}{300} - \frac{100}{275} = 0.14277 > 0$$

This shows that the second law of thermodynamics is not satisfied.

Since the data does not satisfy the second law of thermodynamics, the claim of inventor is not valid.



Problem 5.4.

An engine burns 0.5 kg of a fuel at 1800 K and rejects energy at an average temperature of 600 K. If the calorific value of the fuel is 42000 kJ/kg, determine the amount of work output that the engine can provide.

Solution: Given,

Source temperature, $T_H = 1800 \text{ K}$

Sink temperature, $T_L = 600 \text{ K}$

Mass of fuel, $m_f = 0.5 \text{ kg}$

Calorific value, $CV = 42000 \text{ kJ/kg}$

Work output, $\dot{W}_{out} = ?$

Heat addition, $Q_H = m_f \times CV = 0.5 \times 42000 = 21000 \text{ kJ}$

Maximum possible efficiency (Carnot efficiency) of the engine,

$$\eta_{\text{Carnot}} = \eta_{\text{cv}} = 1 - \frac{T_L}{T_H} = 1 - 600/1800 = 0.6666 = 66.66\%$$

Then, maximum work output of the engine,

$$\eta_{\text{cv}} = \frac{\dot{W}_{out,\text{max}}}{Q_H} \Rightarrow \dot{W}_{out,\text{max}} = \eta_{\text{cv}} \times Q_H = 0.6666 \times 21000 = 14000 \text{ kJ} = 14 \text{ MJ}$$

Problem 5.5.

A heat pump having a COP of 5 maintains a building at a temperature of 24°C by supplying heat at a rate of 72000 kJ/h when the surroundings is at 0°C . The heat pump runs 12 hour in a day and the electricity costs Rs 10/kWh.

(a) Determine the actual and minimum theoretical cost per day.

(b) Compare the actual operating cost with the cost of direct electric resistance heating.

Solution: Given,

COP of heat pump, $(COP)_{HP} = 5$

Higher temperature, $T_H = 24^\circ\text{C} = 24 + 273 = 297 \text{ K}$

Lower temperature, $T_L = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}$

Running time, $t = 12 \text{ h/day}$

Rate of heating, $\dot{Q}_H = 72000 \text{ kJ/h} = 72000/3600 = 20 \text{ kJ/s} = 20 \text{ kW}$

Cost of electricity, $R = \text{Rs } 10/\text{kWh}$

(a) Actual power required to drive the heat pump is given by

$$\dot{W}_n = \frac{\dot{Q}_H}{(COP)_{HP}} = 20/5 = 4 \text{ kW}$$

Then, the actual cost per day is given by

$$C_{\text{actual}} = \dot{W}_n \times t \times R = 4 \times 12 \times 10 = \text{Rs } 480/\text{day}$$

Maximum theoretical COP of the heat pump is calculated as

$$(COP)_{rev,HP} = \frac{T_H}{T_H - T_L} = \frac{297}{297 - 273} = 12.375$$

Hence, theoretical power required to drive the heat pump is determined as

$$\dot{W}_{th} = \frac{\dot{Q}_H}{(COP)_{rev,HP}} = \frac{20}{12.375} = 1.6162 \text{ kW}$$

Now, the minimum theoretical cost per day is calculated as

$$C_{th} = \dot{W}_{th} \times t \times R = 1.6162 \times 12 \times 10 = \text{Rs } 193.94/\text{day}$$

(b) The cost of direct electric resistance heating per day is given by

$$C_{elec} = \dot{Q}_H \times t \times R = 20 \times 12 \times 10 = \text{Rs } 2400/\text{day}$$

Problem 5.6.

An air conditioning unit having a COP of 4 maintains a hall at 20°C on a day when the outside temperature is 35°C . The thermal load consists of heat energy entering through the walls at a rate of 600 kJ/min and from the occupants, computers and lighting at a rate of 120 kJ/min . Determine the power required to drive the unit and compare it with the minimum theoretical power required.

Solution: Given,

COP of air conditioning unit, $(COP)_R = 4$

Higher temperature, $T_H = 35^\circ\text{C} = 35 + 273 = 308 \text{ K}$

Lower temperature, $T_L = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$

$$\begin{aligned} \text{Rate of total heat taken out from hall, } \dot{Q}_L &= (600 + 120) \text{ kJ/min} = 720/60 \\ &= 12 \text{ kJ/s} = 12 \text{ kW} \end{aligned}$$

Here, the air conditioning unit is working as a refrigerator; its COP is therefore given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

$$\Rightarrow \text{Power required to drive the unit, } \dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = 12/4 = 3 \text{ kW}$$

The maximum COP of an air conditioning unit working as a reversible refrigerator is given by

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L} = \frac{293}{308 - 293} = 19.5333$$

Then, the minimum theoretical power required is calculated as

$$\dot{W}_{th} = \frac{\dot{Q}_L}{(COP)_{rev,R}} = 12/19.5333 = 0.6143 \text{ kW}$$

Problem 5.7.

A heat engine operates between a high temperature source T_H and a low temperature sink at 300 K. The engine develops 60 kW of power and rejects heat to the sink at the rate of 72 MJ/h. Determine the minimum theoretical value for T_H in K.

Solution: Given,

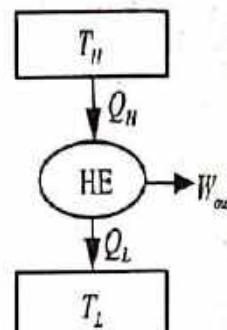
Lower temperature, $T_L = 300 \text{ K}$

Power output, $\dot{W}_{out} = 60 \text{ kW}$

Rate of heat rejection, $\dot{Q}_L = 72 \text{ MJ/h} = 72000 / 3600 = 20 \text{ kW}$

From the first law of thermodynamics for a cyclic process, we have

$$\begin{aligned}\sum \dot{Q}_i &= \sum \dot{W}_i \Rightarrow \dot{Q}_H - (\dot{Q}_L) = \dot{W}_{out} \\ \Rightarrow \dot{Q}_H &= \dot{Q}_L + \dot{W}_{out} = 20 + 60 = 80 \text{ kW}\end{aligned}$$



For minimum theoretical value for T_H , the heat engine should be reversible. Hence, efficiency of the reversible heat engine,

$$\begin{aligned}\eta_{rev, HE} &= 1 - \frac{T_L}{T_H} = \frac{\dot{W}_{out}}{\dot{Q}_H} = \frac{60}{80} = 0.75 \\ \Rightarrow T_H &= T_L / (1 - 0.75) = 300 / 0.25 = 1200 \text{ K}\end{aligned}$$

Problem 5.8.

A heat engine has a solar collector receiving 0.25 kW/m^2 , and provides a high temperature source at 400 K. The heat engine rejects heat to the ambient at 30°C. If the required power output is 2 kW, what is the minimum size of the solar collector?

Solution: Given,

Rate of heat addition per unit area, $\dot{Q}_H / A = 0.25 \text{ kW/m}^2$

Higher temperature, $T_H = 400 \text{ K}$

Lower temperature, $T_L = 30^\circ\text{C} = 30 + 273 = 303 \text{ K}$

Power output, $\dot{W}_{out} = 2 \text{ kW}$

For minimum size of solar collector, the heat engine should be reversible. Hence, efficiency of the reversible heat engine,

$$\begin{aligned}\eta_{rev, HE} &= 1 - \frac{T_L}{T_H} = \frac{\dot{W}_{out}}{\dot{Q}_H} = \frac{2}{0.25 \times A} \Rightarrow 1 - \frac{303}{400} = \frac{2}{0.25 \times A} \\ \therefore \text{Area of solar collector, } A &= 32.9897 \text{ m}^2\end{aligned}$$

Problem 5.9.

An ideal engine can develop 27 kW power output while rejecting 15 kJ heat per cycle. The engine operates between $T_H = 1200\text{ K}$ and $T_L = 300\text{ K}$. Determine the minimum theoretical number of cycles per minute.

Solution: Given,

$$\text{Power output of ideal engine, } \dot{W}_{out,ideal} = 27\text{ kW}$$

$$\text{Heat rejection per cycle, } Q_L = 15\text{ kJ/cycle}$$

$$\text{Higher temperature, } T_H = 1200\text{ K}$$

$$\text{Lower temperature, } T_L = 300\text{ K}$$

From the first law of thermodynamics for a cyclic process, we have

$$\begin{aligned} \sum Q_i &= \sum W_i \Rightarrow Q_H - (Q_L) = W_{out} \\ &\Rightarrow Q_H = W_{out} + Q_L \end{aligned} \quad \dots \dots \dots \text{(i)}$$

The efficiency of the ideal (reversible) heat engine,

$$\eta_{rev,HE} = 1 - \frac{T_L}{T_H} = \frac{W_{out}}{Q_H}$$

Using equation (i), we get

$$\Rightarrow W_{out} = Q_H \left(1 - \frac{T_L}{T_H}\right) = (W_{out} + Q_L) \left(1 - \frac{300}{1200}\right) = \frac{3}{4}(W_{out} + Q_L)$$

$$\Rightarrow W_{out} = 3 \times Q_L = 3 \times 15 = 45\text{ kJ/cycle}$$

Now, minimum number of cycle per minute,

$$\begin{aligned} N &= \frac{\dot{W}_{out,ideal}}{W_{out}} = \frac{27}{45} = 0.6 \text{ cycles per second} \\ &= 0.6 \times 60 = 36 \text{ cycles per minute} \end{aligned}$$

Problem 5.10.

A heat pump having a coefficient of performance 50% of the theoretical maximum maintains a house at a temperature of 20°C . The heat leakage from the house occurs at a rate of 0.8 kW per degree temperature difference. For a maximum power input of 1.5 kW, determine the minimum surroundings temperature for which the heat pump will be sufficient?

Solution: Given,

$$\text{Coefficient of performance of heat pump, } (COP)_{HP}$$

$$= 50\% \times (COP)_{rev,HP}$$

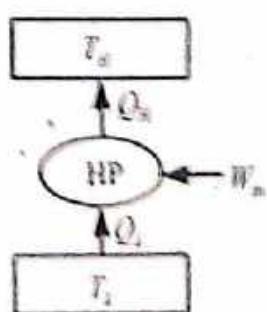
$$\text{Higher temperature, } T_H = 20^\circ\text{C} = 20 + 273 = 293\text{ K}$$

Rate of heat leakage = rate of heat addition,

$$\Rightarrow \dot{Q}_H = 0.8(T_H - T_L)\text{ kW}$$

$$\text{Power input, } \dot{W}_{in} = 1.5\text{ kW}$$

Theoretical maximum (reversible) COP of a heat pump is given by



$$(COP)_{rev,HP} = \frac{T_H}{T_H - T_L} = \frac{293}{293 - T_L} \quad \dots \dots \dots \text{(i)}$$

COP of the heat pump.

$$(COP)_{SP} = \frac{\dot{Q}_S}{\dot{W}_{in}} = \frac{0.8(T_H - T_L)}{1.5} = \frac{0.8(293 - T_L)}{1.5} \quad \dots \dots \dots \text{(ii)}$$

According to the question, we have

$$(COP)_{SP} = 50\% \times (COP)_{rev,HP}$$

Using equations (i) and (ii), we get

$$\begin{aligned} \frac{0.8(293 - T_L)}{1.5} &= 0.5 \times \frac{293}{293 - T_L} \\ \Rightarrow (293 - T_L)^2 &= 274.6875 \\ \Rightarrow 293 - T_L &= \pm 16.5737 \end{aligned}$$

Taking positive value for minimum surroundings temperature,

$$\begin{aligned} \therefore \text{Lower temperature, } T_L &= 293 - 16.5737 \\ &= 276.4263 \text{ K} = 3.4263^\circ\text{C} \end{aligned}$$

Problem 5.11.

An air condition unit having COP 50% of the theoretical maximum maintains a house at a temperature of 20°C by cooling it against the surroundings temperature. The house gains energy at a rate of 0.8 kW per degree temperature difference. For a maximum rate of work input of 1.8 kW , determine the maximum surroundings temperature for which it provides sufficient cooling.

Solution: Given,

Coefficient of performance of air condition,

$$(COP)_R = 50\% \times (COP)_{rev,R}$$

$$\text{Lower temperature, } T_L = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$$

$$\text{Rate of energy gained} = \text{Rate of energy removed} \Rightarrow \dot{Q}_L = 0.8(T_H - T_L) \text{ kW}$$

$$\text{Rate of work input, } \dot{W}_{in} = 1.8 \text{ kW}$$

Theoretical maximum (reversible) COP of an air condition unit working as a refrigerator is given by

$$(COP)_{rest} = \frac{T_L}{T_H - T_L} = \frac{293}{T_H - 293} \quad \dots \dots \dots \text{(i)}$$

COP of the actual air condition unit working as a refrigerator,

$$(COP)_x = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{0.8(T_H - T_L)}{1.8} = \frac{0.8(T_H - 293)}{1.8} \quad \dots \dots \dots \text{(ii)}$$

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According to the question, we have

$$(COP)_R = 50\% \times (COP)_{rev,R}$$

Using equations (i) and (ii), we get

$$\begin{aligned} \frac{0.8(T_H - 293)}{1.8} &= 0.5 \times \frac{293}{T_H - 293} \\ \Rightarrow (T_H - 293)^2 &= 329.625 \\ \Rightarrow T_H - 293 &= \pm 18.1556 \end{aligned}$$

Taking positive value for maximum surroundings temperature,

$$\therefore T_H = 293 + 18.1556 = 311.1556 \text{ K} = 38.1556^\circ\text{C}$$

Problem 5.12.

A house is to be maintained at 25°C in summer as well as winter. For this it is proposed to use a reversible device as a refrigerator in summer and a heat pump in winter. The ambient temperature is 40°C in summer and 3°C in winter. The energy losses as heat from the roof and walls are estimated as 5 kW per degree Celsius temperature difference between the room and ambient conditions. Calculate the power required to operate the device in summer and winter.

Solution: Given,

In summer:

Higher temperature (ambient temperature), $T_H = 40^\circ\text{C} = 40 + 273 = 313 \text{ K}$

Lower temperature (temperature of house), $T_L = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$

Rate of heat loss = Rate of heat removed from house

$$\Rightarrow \dot{Q}_L / (T_H - T_L) = 5 \text{ kW} \Rightarrow \dot{Q}_L = 5 (T_H - T_L) \text{ kW} = 5 (313 - 298) = 75 \text{ kW}$$

In winter:

Higher temperature (temperature of house), $T_H = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$

Lower temperature (ambient temperature), $T_L = 3^\circ\text{C} = 3 + 273 = 276 \text{ K}$

Rate of heat loss = Rate of heat added to house $\Rightarrow \dot{Q}_H / (T_H - T_L) = 5 \text{ kW}$

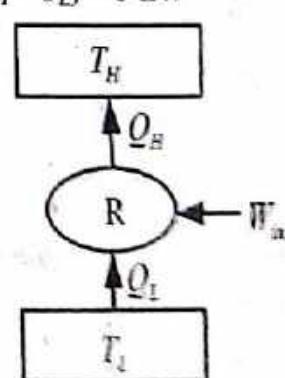
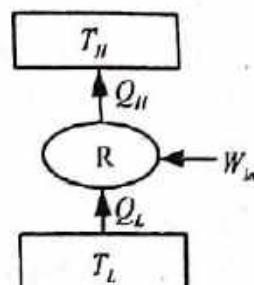
$$\begin{aligned} \Rightarrow \dot{Q}_H &= 5 (T_H - T_L) \text{ kW} \\ &= 5 (298 - 276) = 110 \text{ kW} \end{aligned}$$

In summer, a refrigerator is used to cool the house.

Hence, coefficient of performance of a reversible refrigerator,

$$\begin{aligned} (COP)_{rev,R} &= \frac{T_L}{T_H - T_L} = \frac{\dot{Q}_L}{\dot{W}_{in}} \\ \Rightarrow \frac{298}{313 - 298} &= \frac{75}{\dot{W}_{in}} \\ \Rightarrow \text{Power input, } \dot{W}_{in} &= 3.775 \text{ kW} \end{aligned}$$

In winter, a heat pump is used to heat the house. Hence, coefficient of performance of a reversible heat pump,



$$(COP)_{rev,HP} = \frac{T_H}{T_H - T_L} = \frac{\dot{Q}_H}{\dot{W}_{in}} \Rightarrow \frac{298}{298 - 276} = \frac{110}{\dot{W}_{in}}$$

$$\Rightarrow \text{Power input, } \dot{W}_{in} = 8.121 \text{ kW}$$

Problem 5.13.

An air conditioning unit with power input of 1.5 kW. It has COP of 3 while working as a cooling unit in summer and 4 while working as heating unit in winter. It maintains a hall at 22°C year around, which exchanges heat of rate 0.8 kW per degree temperature difference with the surroundings. Determine the maximum and the minimum outside temperature for which this unit is sufficient.

Solution: Given,

$$\text{Power input, } \dot{W}_{in} = 1.5 \text{ kW}$$

In summer:

COP of an air conditioning unit working as a cooling unit (refrigerator),
 $(COP)_R = 3$

$$\text{Lower temperature (temperature of hall), } T_L = 22^\circ\text{C} = 22 + 273 = 295 \text{ K}$$

$$\text{Rate of heat removed from hall, } \dot{Q}_L / (T_H - T_L) = 0.8 \text{ kW}$$

$$\Rightarrow \dot{Q}_L = 0.8 (T_H - T_L) \text{ kW} = 0.8 (T_H - 295) \text{ kW}$$

In winter:

COP of an air conditioning unit working as a heating unit (heat pump), $(COP)_{HP} = 4$

$$\text{Higher temperature (temperature of hall), } T_H = 22^\circ\text{C} = 22 + 273 = 295 \text{ K}$$

$$\text{Rate of heat added to hall, } \dot{Q}_H / (T_H - T_L) = 0.8 \text{ kW}$$

$$\Rightarrow \dot{Q}_H = 0.8 (T_H - T_L) \text{ kW} = 0.8 (295 - T_L) \text{ kW}$$

In summer, a refrigerator is used to cool the hall. Hence, coefficient of performance of a refrigerator,

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}_{in}} \Rightarrow 3 = \frac{0.8(T_H - 295)}{1.5}$$

∴ Higher temperature (maximum outside temperature),

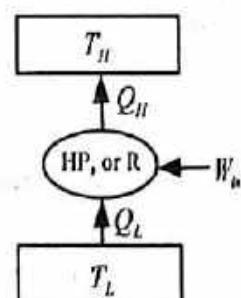
$$T_H = 300.625 \text{ K} = 27.625^\circ\text{C}$$

In winter, a heat pump is used to heat the hall. Hence, coefficient of performance of a heat pump,

$$(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} \Rightarrow 4 = \frac{0.8(295 - T_L)}{1.5}$$

∴ Lower temperature (minimum outside temperature),

$$T_L = 287.5 \text{ K} = 14.5^\circ\text{C}$$



Problem 5.14.

A heat pump heats a house in the winter and then reverses to cool in summer. The room temperature should be 22 °C in the winter and 26 °C in the summer. Heat transfer through the walls and ceilings is estimated to be 3000 kJ/h per degree temperature difference between the inside and outside.

- Determine the power required to run it in winter when the outside temperature decreases to 0 °C.
- If unit is run by same power as calculated in (a), throughout the year, determine the maximum outside summer temperature for which the house can be maintained at 26 °C.

Solution: Given,

In winter:

A heat pump is used to heat the house in winter.

Higher temperature (room temperature), $T_H = 22^\circ\text{C} = 22 + 273 = 295 \text{ K}$

Lower temperature (outside temperature), $T_L = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}$

Rate of heat leakage from house = Rate of heat added to house,

$$\dot{Q}_H / (T_H - T_L) = 3000 \text{ kJ/h} = 3000/3600 \text{ kJ/s} = 5/6 \text{ kW}$$

$$\Rightarrow \dot{Q}_H = 5(T_H - T_L)/6 = 5(295 - 273)/6 = 18.3333 \text{ kW}$$

In summer:

A heat pump reverses to cool the house in summer means it works as a refrigerator. Power input is same as in the case of heat pump.

Lower temperature (room temperature), $T_L = 26^\circ\text{C} = 26 + 273 = 299 \text{ K}$

Rate of heat leakage from house = Rate of heat removed from house,

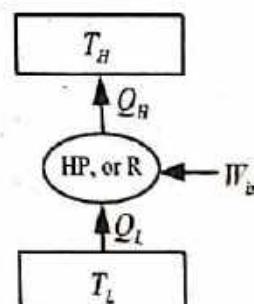
$$\dot{Q}_L / (T_H - T_L) = 3000 \text{ kJ/h} = 3000/3600 \text{ kJ/s} = 5/6 \text{ kW}$$

$$\Rightarrow \dot{Q}_L = 5(T_H - T_L)/6 \text{ kW} = 5(T_H - 299)/6 \text{ kW}$$

Let the heat pump be reversible, then coefficient of performance of heat pump, $(COP)_{rev,HP}$ is given by

$$(COP)_{rev,HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L}$$

$$\Rightarrow \text{Power input, } \dot{W}_{in} = \frac{18.3333(295 - 273)}{295} \\ = 1.367 \text{ kW}$$



Let the refrigerator used to cool the house in summer be reversible, then coefficient of performance of the refrigerator, $(COP)_{rev,R}$ with same power input as that of the heat pump is given as

$$(COP)_{rev,R} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{T_L}{T_H - T_L}$$

$$\Rightarrow \frac{5(T_H - 299)}{6 \times 1.367} = \frac{299}{T_H - 299}$$

$$\Rightarrow (T_H - 299)^2 = 490.48 \Rightarrow T_H - 299 = \pm 22.147$$

Taking positive value for higher temperature (maximum outside temperature),

$$T_H = 321.147 \text{ K} = 48.147^\circ\text{C}$$

Problem 5.15.

A Carnot engine operates between two reservoirs at temperatures T_L and T_H . The work output of the engine is 0.6 times the heat rejected. The difference in temperature between the source and sink is 200°C . Calculate the thermal efficiency, source temperature and sink temperature.

Solution: Given,

Here, source and sink temperatures are given as T_H and T_L respectively. Let work output of heat engine be W_{out} and heat rejection be Q_L . Then, according to the question, we have

$$W_{out} = 0.6 Q_L \quad \dots \dots \dots \text{(i)}$$

$$T_H - T_L = 200^\circ\text{C} \quad \dots \dots \dots \text{(ii)}$$

From the first law of thermodynamics for a cyclic process, work output of Carnot engine,

$$W_{out} = Q_H - Q_L \Rightarrow Q_H = W_{out} + Q_L$$

Using equation (i), we get

$$Q_H = 0.6 Q_L + Q_L = 1.6 Q_L \quad \dots \dots \dots \text{(iii)}$$

Then, efficiency of the Carnot engine is given by

$$\eta_{Carnot} = \frac{W_{out}}{Q_H} = \frac{0.6 Q_L}{1.6 Q_L} = 0.375 = 37.5\%$$

Efficiency of a Carnot engine or a reversible heat engine can be determined as,

$$\begin{aligned} \eta_{Carnot} &= 1 - \frac{T_L}{T_H} \Rightarrow 0.375 = \frac{T_H - T_L}{T_H} \\ &\Rightarrow 0.375 T_H = T_H - T_L \end{aligned}$$

Using equation (ii), we get

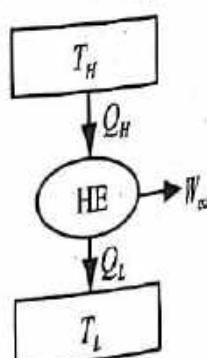
$$0.375 T_H = 200$$

\therefore Source temperature, $T_H = 533.33 \text{ K}$

Putting this value of T_H in equation (ii) gives

$$533.33 - T_L = 200$$

\therefore Sink temperature, $T_L = 533.33 - 200 = 333.33 \text{ K}$



Problem 5.16.

Work output of an ideal engine is 4 times the heat rejected by it. Determine its efficiency. If the sink temperature increases by 300 °C, its efficiency reduces to 60%. Determine its source and sink temperature.

Solution: Given,

$$\text{Work output} = 4 \text{ times of heat rejected}, \Rightarrow W_{out} = 4 Q_L \quad \dots \text{(i)}$$

Let initial sink temperature be T_L and new sink temperature be T'_L , then

$$T'_L = T_L + 300 \quad \dots \text{(ii)}$$

Let new efficiency be $\eta'_{rev,HE}$, then

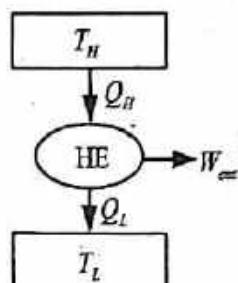
$$\eta'_{rev,HE} = 60\% = 0.6 \quad \dots \text{(iii)}$$

From the first law of thermodynamics for a cyclic process, work output of an ideal heat engine is given by

$$W_{out} = Q_H - Q_L \Rightarrow Q_H = W_{out} + Q_L$$

Using equation (i), we get

$$Q_H = 4 Q_L + Q_L = 5 Q_L$$



For the first case, the efficiency of the ideal (reversible) heat engine is given by

$$\eta_{rev,HE} = 1 - \frac{T_L}{T_H} = \frac{W_{out}}{Q_H} = \frac{4Q_L}{5Q_H} = 4/5 = 0.8 \quad \dots \text{(iv)}$$

For second case, similarly the new efficiency of the ideal heat engine,

$$\eta'_{rev,HE} = 1 - \frac{T'_L}{T_H}$$

Using equations (ii) and (iii), it becomes

$$0.6 = 1 - \frac{T_L + 300}{T_H}$$

From equation (iv),

$$1 - \frac{T_L}{T_H} = 0.8 \quad \dots \text{(v)}$$

Solving equations (iv) and (v) for temperatures T_H and T_L , we get

Source temperature, $T_H = 1500 \text{ K}$

Sink temperature, $T_L = 300 \text{ K}$

Problem 5.17.

A refrigerator having a COP of 4 maintains the freezer compartment at -3 °C by removing heat at a rate of 10800 kJ/h and rejects heat to the surroundings at 27 °C. Determine the power input to the refrigerator and compare it with minimum theoretical power input. If the electricity cost is Rs. 10/kWh, determine the actual and minimum theoretical cost per day for effective operation of 12 h/day.

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Solution: Given,

COP of refrigerator, $(COP)_R = 4$

Higher temperature (surroundings temperature), $T_H = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Lower temperature (freezer temperature), $T_L = -3^\circ\text{C} = -3 + 273 = 270 \text{ K}$

Rate of cooling, $\dot{Q}_L = 10800 \text{ kJ/h} = 10800/3600 = 3 \text{ kJ/s} = 3 \text{ kW}$

Cost of electricity, $R = \text{Rs } 10/\text{kWh}$

Running time, $t = 12 \text{ h/day}$

Actual power input to the refrigerator is given by

$$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{3}{4} = 0.75 \text{ kW}$$

Maximum theoretical COP of the refrigerator is calculated as

$$(COP)_{theor.R} = \frac{T_L}{T_H - T_L} = \frac{270}{300 - 270} = 9$$

Hence, theoretical power input to the refrigerator is determined as

$$\dot{W}_{th} = \frac{\dot{Q}_L}{(COP)_{theor.R}} = \frac{3}{9} = \frac{1}{3} \text{ kW} = 0.33333 \text{ kW}$$

Then, the actual cost per day is given by

$$C_{actual} = \dot{W}_{in} \times t \times R = 0.75 \times 12 \times 10 = \text{Rs } 90/\text{day}$$

Now, the minimum theoretical cost per day is calculated as

$$C_{th} = \dot{W}_{th} \times t \times R = \frac{1}{3} \times 12 \times 10 = \text{Rs } 40/\text{day}$$

5.3 Review Theoretical Questions

1. Define heat engine, heat pump and refrigerator. Explain how first and second laws are applied to determine the performance of a heat engine.
2. Define heat engine and heat pump. Explain how first law and second law of thermodynamics can be applied to analyze the performance of a heat pump.
3. Define heat pump and COP. Explain how performance of reversible and irreversible heat pump can be evaluated by applying first law and second law of thermodynamics.
4. Define refrigerator and its COP. Explain how first law and second law of thermodynamics can be applied to analyze the performance of the refrigerator.
5. Differentiate between heat engine and refrigerator. Also explain the factors used for their performance evaluation.
6. Write down the similarities and differences between heat pump and refrigerator.
7. Explain the limitations of the first law of thermodynamics with examples.
8. What is the key feature of the second law of thermodynamics?
9. Explain why heat transfer is an irreversible process. How it can be approximated as a reversible?
10. Write down classical statements of second law of thermodynamics.
11. Show the equivalence between Kelvin-Planck's and Clausius's statement of second law of thermodynamics.
12. Write down the factors causing irreversibility.
13. State and explain Carnot theorem.
14. Describe about the thermodynamic temperature scale.

5.4 Unsolved Numerical Problems

- An inventor claims to have developed a power cycle capable of delivering a net work output of 450 kJ for an energy input by heat transfer of 1000 kJ. The system undergoing cycle receives the heat transfer from hot gases at temperature of 500 K and discharges to the atmosphere at 300 K. Evaluate this claim. (Ans: Inventor's claim is invalid, because $\eta_{\text{inventor}} > \eta_{\text{rev}}$)
- During an experiment a student claims that based on his measurements, a heat engine receives 300 kJ from a source at 500 K converts 160 kJ of it into work and rejects heat to the sink at 300 K. Are these data reasonable? (Ans: Not reasonable, because $\eta_{\text{student}} > \eta_{\text{rev}}$)
- An engine is to be designed to produce 0.735 kW by taking continuously 6000 kJ/hr from heat source which is at 505 K, and the heat sink is at atmospheric temperature. The atmospheric temperature varies from 38°C in summer to 0°C in winter. Comment on the performance of the engine. (Ans: Engine works in summer but not in winter, so it has to be redesigned for winter, because $\eta_{\text{summer}} < \eta_{\text{carnot}}$ and $\eta_{\text{winter}} > \eta_{\text{carnot}}$)
- During an experiment conducted in a room at 25°C, a laboratory assistance measures that a refrigerator consumes 2 kW of power and removes 30000 kJ of heat from the desired (refrigerated) space which is maintained at -3°C. The running time of the refrigerator during experiment was 20 min. Determine if these measurements are reasonable? (Ans: Not reasonable, because $(COP)_{\text{act}} > (COP)_{\text{rev}}$)
- Find the efficiency, power output and heat rejection rate for a heat engine operating on a Carnot cycle which receives heat at a rate of 6 kW at 327°C and rejects heat to 27°C. (Ans: 50%, 3 kW, 3 kW)
- A car engine consumes fuel at a rate of 30 L/h and delivers 80 kW of power output. If the calorific value of the fuel is 42000 kJ/kg and a density of 0.8 g/cm³, determine the efficiency of the engine. (Ans: 28.57%)
- An ideal engine have a efficiency of 25 %. If the sink temperature is reduced by 100°C, its efficiency gets doubled, determine its source and sink temperatures. (Ans: 400 K, 300 K)
- A building is maintained at a temperature of 25°C by a heat pump having a coefficient of performance of 2.5. It loses heat at a rate of 1 kW per degree temperature difference between the inside and the outside. If the outside temperature is -10°C, determine the power required to drive the heat pump. (Ans: 14 kW)

SECOND LAW OF THERMODYNAMICS

9. A refrigerator operates in a room at 22°C . Heat must be taken from the desired space at a rate of 2.5 kW to maintain its temperature at -20°C . What is the minimum power required to drive the refrigerator? (Ans: 0.415 kW)
10. A refrigerator takes heat from a desired space maintained at -5°C at a rate of 100 kJ/min and rejects heat to the surroundings at 20°C . If the coefficient of performance of the refrigerator is 50 % of that of a reversible refrigerator cycle operating between the same temperature limits. Determine the power required to drive the cycle. (Ans: 0.3109 kW)

Entropy**6.1 Chapter Highlights**

- **Inequality of Clausius:** The expression $\oint \left(\frac{\delta Q}{T} \right) \leq 0$ for a cyclic process is known as *Clausius inequality* and it not only gives mathematical expression to the second law of thermodynamics, but also throws light on irreversibility of the system. Hence, for any cycle if
 - $\oint \left(\frac{\delta Q}{T} \right) < 0$, then the cycle is irreversible.
 - $\oint \left(\frac{\delta Q}{T} \right) = 0$, then the cycle is reversible.
 - $\oint \left(\frac{\delta Q}{T} \right) > 0$, then the cycle is impossible.
- **Entropy as a property of a system:** Second law of thermodynamics defines the direction of the process with reference to the system property called entropy. Entropy of a system is a measure of molecular disorder, randomness, disorganization or uncertainty existing in a system. It is an extensive property and denoted by S .
- For any process between state 1 and state 2, it reveals $S_2 \geq S_1$.
 - If $S_1 = S_2$, the process is *reversible* and
 - If $S_2 > S_1$, the process is *irreversible*.
- **Entropy of a pure substance** is calculated by using *Gibb's equation*:

$$dU = TdS - PdV$$
- **Change in entropy for any pure substance** is given by

$$dS = \frac{dU}{T} + \frac{P}{T} dV \quad \text{and} \quad dS = \frac{dH}{T} - \frac{V}{T} dP$$
- **Entropy change of an ideal gas:**

$$S_2 - S_1 = mc_v \ln(T_2/T_1) + mR \ln(V_2/V_1)$$

$$S_2 - S_1 = mc_p \ln(T_2/T_1) - mR \ln(P_2/P_1)$$
- **Entropy change of an incompressible substance:**

$$S_2 - S_1 = mc \ln(T_2/T_1)$$

where c is specific heat of incompressible substance.

ENTROPY

- *Principle of increase of entropy:* "Entropy of an isolated system always increases or may remain constant for an ideal process". Mathematically,

$$(dS)_{\text{isolated}} \geq 0.$$

- Entropy of an isolated system at final state is always greater than that of initial state during any process.
- The difference between the entropy at the final state and initial state during any process is called *entropy generation* or *entropy production* (δS_{gen}). It can be expressed as:

$$(dS - \delta S_{\text{gen}})_{\text{isolated}} = 0 \quad \text{where } \delta S_{\text{gen}} \geq 0.$$

In terms of rate basis,

$$\left(\frac{dS}{dt}\right)_{\text{isolated}} \geq 0 \quad \text{or, } \left(\frac{dS}{dt}\right)_{\text{isolated}} - \dot{S}_{\text{gen}} = 0 \quad \text{where } \dot{S}_{\text{gen}} \geq 0$$

- *Reversible heat transfer reservoir:* It is an idealized system having infinite heat capacity (i.e., no change of its temperature by the heat transfer) which can interact with its surroundings only by heat transfer but not by work transfer.
- *Reversible work transfer reservoir:* It is an idealized system having infinite work capacity (i.e., no change of its pressure by the work transfer) which can interact with its surroundings only by work transfer but not by heat transfer.
- *Change in entropy for a reversible heat transfer reservoir* is given by

$$dS = \frac{\delta Q}{T}.$$

- *Change in entropy for a reversible work transfer reservoir* is given by
 $dS = 0.$

This means that work transfer does not have any contribution on the entropy of the system.

- *Balance of entropy for a closed system or control mass:* "The change in entropy of a closed system is greater than or equal to the sum of heat transfers divided by the corresponding boundary absolute temperatures". It is expressed as:

$$(dS)_{CM} \geq \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM}$$

$$\text{Or, } (dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM} - \delta S_{\text{gen}} = 0$$

$$\text{where } \delta S_{\text{gen}} = (dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM} \geq 0$$

On rate basis:

$$\left(\frac{dS}{dt}\right)_{CM} \geq \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM}$$

$$\text{Or, } \left(\frac{dS}{dt}\right)_{CM} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM} - \dot{S}_{gen} = 0$$

$$\text{where } \dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CM} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM} \geq 0.$$

- For a single thermal reservoir undergoing reversible process, the change in entropy of a control mass is given by

$$(dS)_{CM} = \frac{\delta Q}{T} \Rightarrow \delta Q = T.dS$$

This means the area under the $T - S$ diagram gives the magnitude of heat transfer.

- Balance of entropy for an open system or a control volume: "The change in entropy of an open system is greater than or equal to the net entropy transported into the system due to mass transfer plus the sum of heat transfers divided by the corresponding boundary absolute temperatures". It can be expressed as:

$$(dS)_{CV} \geq S_{net} + \sum \left(\frac{\delta Q_i}{T_i}\right)_{CV}$$

$$\text{where } S_{net} = S_{in} - S_{out} = m_{in}S_{in} - m_{out}S_{out}.$$

$$\text{Or, } (dS)_{CV} - S_{net} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CV} - \delta S_{gen} = 0$$

$$\text{where } \delta S_{gen} = (dS)_{CV} - S_{net} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CV} \geq 0$$

On rate basis,

$$\left(\frac{dS}{dt}\right)_{CV} \geq \dot{S}_{net} + \sum \left(\frac{\delta \dot{Q}_i}{T_i}\right)_{CV}$$

$$\text{where } \dot{S}_{net} = \dot{S}_{in} - \dot{S}_{out} = \dot{m}_{in}S_{in} - \dot{m}_{out}S_{out}$$

$$\text{Or, } \left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{net} - \sum \left(\frac{\delta \dot{Q}_i}{T_i}\right)_{CV} - \dot{S}_{gen} = 0$$

$$\text{where } \dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{net} - \sum \left(\frac{\delta \dot{Q}_i}{T_i}\right)_{CV} \geq 0$$

➤ **Isentropic process:** The process during which the entropy remains constant is known as isentropic process. A process must be a reversible as well as adiabatic to be an isentropic process.

➤ **Isentropic relations for an ideal gas:**

- Pressure-volume relation:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^r$$

- Temperature-volume relation:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

- Temperature-pressure relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

➤ **Isentropic relation for an incompressible substance:**

$$h_2 - h_1 = v(P_2 - P_1)$$

➤ **High grade energy:** It is the energy which can be completely converted into shaft work without any loss, e.g. mechanical work, electrical work, water and wind power, manual or animal power etc.

➤ **Low grade energy:** It is the energy which cannot be completely converted into mechanical work, i.e. only a portion of it can be converted into mechanical work, e.g. heat energy, heat from nuclear fission and fusion, heat from combustion of fuels such as wood, gas, oil, coal etc.

➤ **Reversible process:** It is the process in which the initial conditions of both system and surroundings can be restored by the reverse action such that net change in entropy is zero for both forward and reverse processes.

➤ **Irreversible process:** It is the process in which the initial conditions of both the system and surroundings cannot be restored by the reverse action. During this process, entropy of the system increases in forward direction and the reverse direction is not possible.

➤ **Internally reversible process:** In this process the initial condition of only the system is restored when the system is taken through reverse direction.

➤ **Externally reversible process:** In this process the initial condition of only the surroundings is restored when the system is taken through the reverse direction.

➤ **Completely reversible process:** A process is said to be a completely reversible, if it is internally as well as externally reversible.

➤ **Available energy:** It is the portion of heat energy that can be converted into mechanical work in a cyclic engine. It is also known as exergy.

- *Unavailable energy:* It is the portion of heat energy that cannot be converted into mechanical work and is rejected to the surroundings (sink). It is also known as *anergy*.
- *Lost Work:* When transfer of heat takes place through the finite temperature difference, there is a decrease in the available energy so transferred which is also known as *loss of available energy or lost work*.
- *Irreversibility and Degradation of energy:* *Irreversibility* denotes the loss of available work due to the fact that all real processes are irreversible and represents an increase in unavailable energy (*anergy*). This loss or decrease of available energy is called *degradation of energy*.
- *Mechanical irreversibility:* It is due to friction within the system as well as between the system and surroundings.
- *Thermal irreversibility:* It is due to heat transfer within the system as well as between the system and surroundings.
- *Internal irreversibility:* It is due to friction as well as heat transfer within the system.
- *External irreversibility:* It is due to friction as well as heat transfer between the system and surroundings.
- *Availability:* The available work from a system becomes a maximum when its state is brought in equilibrium with the surroundings at a maximum possible lower temperature by means of work and heat transfer. This maximum work is called *availability*. Thus, availability is a composite property depending on the state of both the system and surroundings.

6.2 Solved Numerical Problems

Problem 6.1.

2 kg of water at 100°C is mixed with 4 kg of water at 20°C in an isolated system. Calculate the net change in entropy due to the mixing process.

[Take specific heat of water $c = 4.18 \text{ kJ/kgK}$].

Solution: Given,

Mass of hot water, $m_1 = 2 \text{ kg}$

Temperature of hot water, $T_1 = 100^{\circ}\text{C} = 100 + 273 = 373 \text{ K}$

Mass of cold water, $m_2 = 4 \text{ kg}$

Temperature of cold water, $T_2 = 20^{\circ}\text{C} = 20 + 273 = 293 \text{ K}$

Specific heat of water, $c = 4.18 \text{ kJ/kgK}$

Let T_3 be the final equilibrium temperature after mixing process, then heat lost by the hot water is equal to heat absorbed by the cold water, i.e.

$$m_1 c (T_1 - T_3) = m_2 c (T_3 - T_2)$$

$$\Rightarrow 2 \times 4.18 (373 - T_3) = 4 \times 4.18 (T_3 - 293)$$

$$\Rightarrow T_3 = 319.666 \text{ K}$$

The change in entropy of the hot water during mixing is given by

$$(\Delta S)_{W1} = m_1 c \ln(T_3/T_1) = 2 \times 4.18 \times \ln(319.666/373) = -1.28996 \text{ kJ/K}$$

The change in entropy of the cold water during mixing is given by

$$(\Delta S)_{W2} = m_2 c \ln(T_3/T_2) = 4 \times 4.18 \times \ln(319.666/293) = 1.45638 \text{ kJ/K}$$

The net change in entropy due to the mixing process is given by

$$(\Delta S)_{net} = (\Delta S)_{W1} + (\Delta S)_{W2} = -1.28996 + 1.45638 = 0.1664 \text{ kJ/K}$$

Problem 6.2.

Block A ($m_A = 0.5 \text{ kg}$, $c_A = 1 \text{ kJ/kgK}$) and block B ($m_B = 1 \text{ kg}$, $c_B = 0.5 \text{ kJ/kgK}$) which are initially at 100°C and 500°C respectively are brought in contact inside an isolated system. Determine the change in entropy when they reach to a final state of thermal equilibrium.

Solution: Given,

Mass of block A, $m_A = 0.5 \text{ kg}$

Specific heat of block A, $c_A = 1 \text{ kJ/kgK}$

Temperature of block A, $T_A = 100^{\circ}\text{C} = 100 + 273 = 373 \text{ K}$

Mass of block B, $m_B = 1 \text{ kg}$

Specific heat of block B, $c_B = 0.5 \text{ kJ/kgK}$

Temperature of block B, $T_B = 500^{\circ}\text{C} = 500 + 273 = 773 \text{ K}$

Let T be the final equilibrium temperature, then heat lost by the hot block B is equal to heat absorbed by the cold block A, i.e.

$$m_B c_B (T_B - T) = m_A c_A (T - T_A)$$

$$\Rightarrow 1 \times 0.5 (773 - T) = 0.5 \times 1 (T - 373)$$

$$\Rightarrow T = 573 \text{ K}$$

The change in entropy of the hot block B due to the decrease in its temperature is given by

$$(\Delta S)_B = m_B c_B \ln(T/T_B) = 1 \times 0.5 \times \ln(573/773) = -0.149697 \text{ kJ/K}$$

The change in entropy of the cold block A due to the increase in its temperature is given by

$$(\Delta S)_A = m_A c_A \ln(T/T_A) = 0.5 \times 1 \times \ln(573/373) = 0.214654 \text{ kJ/K}$$

The net change in entropy after thermal equilibrium is given by

$$(\Delta S)_{\text{net}} = (\Delta S)_A + (\Delta S)_B = 0.214654 - 0.149697 = 0.064957 \text{ kJ/K}$$

Problem 6.3.

1 kg of air enclosed in an isolated box with volume V_1 , pressure P_1 and temperature T_1 is allowed to expand freely until its volume increases to $V_2 = 2V_1$. Determine the change in entropy. [Take $R = 287 \text{ J/kgK}$].

Solution: Given,

Mass of air, $m = 1 \text{ kg}$

Let initial volume of air = V_1

Initial pressure of air = P_1

Final volume of air, $V_2 = 2V_1 \Rightarrow V_2/V_1 = 2$

As the air is enclosed in an isolated box, there is no interaction between system and surroundings in terms of mass and energy transfer namely heat and work transfer ($\therefore Q = 0 \text{ & } W = 0$).

From the first law of thermodynamics for an isolated system,

$$\Delta U = Q - W = 0$$

Thus, the change in internal energy of the system is zero.

For an ideal gas, the change in internal energy is solely function of temperature difference, i.e.

$$\Delta U = mc_v \Delta T = 0 \Rightarrow \Delta T = 0 \Rightarrow T_2 - T_1 = 0$$

$$\therefore \text{Temperature at final state, } T_2 = T_1 \Rightarrow T_2/T_1 = 1$$

Hence, the free expansion of air in an isolated box is an isothermal process.

For an ideal gas, change in entropy during the process 1 – 2 is given by

$$\begin{aligned} \Delta S &= mc_v \ln(T_2/T_1) + mR \ln(V_2/V_1) = mc_v \ln(1) + mR \ln(2) \\ &= 0 + 1 \times 287 \times \ln(2) = 198.933 \text{ J/K} \end{aligned}$$

Problem 6.4.

Five kg of water at 30°C is mixed with 1 kg of ice at 0°C . Assuming the process of mixing is adiabatic, find the change in entropy. Latent heat of ice, $L = 336 \text{ kJ/kg}$, c for water = 4.2 kJ/kgK .

Solution: Given,

Mass of hot water, $m_1 = 5 \text{ kg}$

Temperature of water, $T_1 = 30^\circ\text{C} = 30 + 273 = 303 \text{ K}$

Mass of ice, $m_2 = 1 \text{ kg}$

Temperature of ice, $T_2 = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}$

Latent heat of ice, $L = 336 \text{ kJ/kg}$

Specific heat of water, $c = 4.2 \text{ kJ/kgK}$

Let T_3 be the final equilibrium temperature after mixing process, then heat lost by the water is equal to latent heat absorbed by the ice during melting at 0°C plus heat absorbed by the water when its temperature increases from 0°C (273 K) to T_3 , i.e.

$$\begin{aligned} m_1 c(T_1 - T_3) &= m_2 L + m_2 c(T_3 - T_2) \\ \Rightarrow 5 \times 4.2 (303 - T_3) &= 1 \times 336 + 1 \times 4.2 (T_3 - 273) \\ \Rightarrow T_3 &= 284.666 \text{ K} \end{aligned}$$

The change in entropy of the water during mixing is given by

$$\begin{aligned} (\Delta S)_{W1} &= m_1 c \ln(T_3/T_1) = 5 \times 4.2 \times \ln(284.666/303) \\ &= -1.31074 \text{ kJ/K} \end{aligned}$$

The change in entropy of the ice during mixing is given by the summation of change in entropy of the ice during melting at 0°C (273 K) and the change in entropy of water when its temperature increases from 0°C (273 K) to T_3 , i.e.

$$\begin{aligned} (\Delta S)_{I2} &= \frac{m_2 \times L}{T_2} + m_2 c \ln(T_3/T_2) = \frac{1 \times 336}{273} + 1 \times 4.2 \times \ln(284.666/273) \\ &= 1.40652 \text{ kJ/K} \end{aligned}$$

The net change in entropy due to the mixing process is given by

$$(\Delta S)_{net} = (\Delta S)_{W1} + (\Delta S)_{I2} = -1.31074 + 1.40652 = 0.095777 \text{ kJ/K}$$

Problem 6.5.

Steam at 1 MPa and 300°C is flowing with a velocity of 50 m/s reversibly and adiabatically through a nozzle and leaves the nozzle at 150 kPa . Determine the exit velocity of steam from the nozzle.

Solution: Given,

Pressure at inlet, $P_1 = 1 \text{ MPa} = 1000 \text{ kPa}$

Temperature at inlet, $T_1 = 300^\circ\text{C}$

Velocity at inlet, $\bar{V}_1 = 50 \text{ m/s}$

Exit pressure, $P_2 = 150 \text{ kPa}$

Steam is flowing reversibly and adiabatically through nozzle

\Rightarrow Isentropic flow ($dS = 0$) \Rightarrow Entropy of steam at inlet (S_1) is equal to entropy at exit (S_2) of nozzle $\Rightarrow S_1 = S_2 \Rightarrow$ specific entropy, $s_1 = s_2$.

From the energy equation for control volume system like flow device nozzle ($\dot{W}_c = 0$) at steady state condition, we have

$$\begin{aligned}\dot{Q}_c &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \right] && \text{(Neglecting PE change)} \\ \Rightarrow \frac{\dot{Q}_c}{\dot{m}} &= (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) \\ \Rightarrow 0 &= (h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) && \text{(As } \dot{Q}_c = 0, \text{ for adiabatic nozzle)} \\ \Rightarrow \bar{V}_2 &= \sqrt{2(h_1 - h_2) + \bar{V}_1^2} && \dots \dots \dots \text{(i)}\end{aligned}$$

At inlet: $P_1 = 1000 \text{ kPa}$, $T_1 = 300^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 1000 \text{ kPa}$, the saturation temperature for given pressure, i.e.

$T_{sat}(P_1 = 1000 \text{ kPa}) = 179.92^\circ\text{C}$.

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 1000 \text{ kPa}$ and temperature, $T_1 = 300^\circ\text{C}$,

specific enthalpy at inlet, $h_1 = 3050.6 \text{ kJ/kg}$

specific entropy at inlet, $s_1 = 7.1219 \text{ kJ/kgK}$

At exit: Pressure, $P_2 = 150 \text{ kPa}$, and specific entropy, $s_2 = s_1 = 7.1219 \text{ kJ/kgK}$

(As the flow is isentropic)

Similarly, referring to the saturated water pressure table A2.1 for pressure, $P_2 = 150 \text{ kPa}$,

$$h_f = 467.18 \text{ kJ/kg}, \quad h_{fg} = 2226.2 \text{ kJ/kg}, \quad h_g = 2693.4 \text{ kJ/kg}$$

$$s_f = 1.4338 \text{ kJ/kgK}, \quad s_{fg} = 5.7894 \text{ kJ/kgK}, \quad s_g = 7.2232 \text{ kJ/kgK}$$

Here, $s_f < s_1 < s_g$. Hence, it is a two phase mixture.

Now, referring to the saturated water vapor table A2.1 for pressure, $P_2 = 150 \text{ kPa}$ and specific entropy, $s_2 = 7.1219 \text{ kJ/kgK}$, to find the specific enthalpy at exit, h_2

$$\text{Quality at exit, } x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{7.1219 - 1.4338}{5.7894} = 0.9825$$

$$\text{Then, } h_2 = h_f + x_2 h_{fg} = 467.18 + 0.9825 \times 2226.2 = 2654.427 \text{ kJ/kgK.}$$

Putting the values of h_1 and h_2 in equation (i), we get exit velocity,

$$\bar{V}_2 = \sqrt{2(3050.6 - 2654.427) / 10^3 + 50^2} = 891.54 \text{ m/s}$$

Problem 6.6.

A compressor receives air at 100 kPa and 27°C and requires a power input of 60 kW . If the mass flow rate of the air is 0.1 kg/s , determine the maximum exit pressure of the compressor.

Solution: Given,

Pressure at inlet of compressor, $P_1 = 100 \text{ kPa}$

Temperature at inlet, $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Power input, $\dot{W}_{cv} = -60 \text{ kW}$

Mass flow rate of air, $\dot{m} = 0.1 \text{ kg/s}$

Here, it should be noted that for the maximum exit pressure of the compressor, the compressor is to be adiabatic ($\dot{Q}_{cv} = 0$) and the compression process to be reversible (i.e., isentropic process).

From the energy equation for control volume system like work device compressor at steady state condition, we have

$$\begin{aligned}\dot{Q}_{cv} - \dot{W}_{cv} &= \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1)] \\ \Rightarrow \dot{Q}_{cv} - \dot{W}_{cv} &= \dot{m}(h_2 - h_1) \quad (\text{Neglecting KE & PE change})\end{aligned}$$

As $\dot{Q}_{cv} = 0$, for adiabatic compressor

$$\therefore \dot{W}_{cv} = \dot{m}(h_1 - h_2) \quad \dots \dots \dots \text{(i)}$$

For an ideal gas, we know

$$h_1 - h_2 = c_p(T_1 - T_2) \quad \dots \dots \dots \text{(ii)}$$

Substituting equation (ii) into equation (i), we get

$$\begin{aligned}\dot{W}_{cv} &= \dot{m}c_p(T_1 - T_2) \\ \therefore T_2 &= T_1 - \frac{\dot{W}_{cv}}{\dot{m}c_p} = 300 - \frac{-60 \times 10^3}{0.1 \times 1005} = 897.015 \text{ K}\end{aligned}$$

Now, for an adiabatic and reversible (isentropic) process, the pressure of air at exit of the compressor is given by

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{r}{r-1}} = 100 \left(\frac{897.015}{300} \right)^{\frac{1.4}{1.4-1}} = 4622.475 \text{ kPa}$$

Problem 6.7.

Steam enters into a well insulated throttling valve at 10 MPa and 600°C and exits at 5 MPa . Determine the change in entropy per unit mass of the steam.

Solution: Given,

Pressure at inlet, $P_1 = 10 \text{ MPa} = 10000 \text{ kPa}$

Temperature at inlet, $T_1 = 600^\circ\text{C}$

Pressure at exit, $P_2 = 5 \text{ MPa} = 5000 \text{ kPa}$

From the energy equation for control volume system like flow device throttling valve ($\dot{W}_{cv} = 0$) at steady state condition, we have

$$\dot{Q}_{\alpha} = \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1)]$$

Neglecting change in KE and PE and as the throttling valve is well insulated ($\dot{Q}_{\alpha} = 0$), the above energy equation reduces to

$$h_1 = h_2 \quad \dots \dots \dots \text{(i)}$$

At inlet: $P_1 = 10000 \text{ kPa}$, $T_1 = 600^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 10000 \text{ kPa}$, the saturation temperature for given pressure, i.e.

$$T_{sat}(P_1 = 10000 \text{ kPa}) = 311.03^\circ\text{C}$$

Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 10000 \text{ kPa}$ and temperature, $T_1 = 600^\circ\text{C}$,

specific enthalpy at inlet, $h_1 = 3624.7 \text{ kJ/kg}$ and

specific entropy at inlet, $s_1 = 6.9022 \text{ kJ/kgK}$.

From equation (i), the specific enthalpy at exit of the throttling valve,

$$h_2 = h_1 = 3624.7 \text{ kJ/kg}$$

At exit: $P_2 = 5000 \text{ kPa}$, $h_2 = 3624.7 \text{ kJ/kg}$

Similarly, referring to the saturated water pressure table A2.1 for pressure, $P_2 = 5000 \text{ kPa}$,

$$h_1 = 1154.2 \text{ kJ/kg}, \quad h_{fg} = 1639.5 \text{ kJ/kg}, \quad h_g = 2793.7 \text{ kJ/kg}$$

Here, $h_2 > h_g$. Hence, it is also a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_2 = 5000 \text{ kPa}$ and specific enthalpy, $h_2 = 3624.7 \text{ kJ/kg}$, to find the specific entropy at exit, s_2 . As there is no value listed for $h_2 = 3624.7 \text{ kJ/kg}$ in the table, linear interpolation method is used to find the required specific entropy as

$$s_a = 7.1218 \text{ kJ/kgK} \quad h_a = 3550.2 \text{ kJ/kg}$$

$$s_b = ? \quad h_b = 3624.7 \text{ kJ/kg}$$

$$s_c = 7.2586 \text{ kJ/kgK} \quad h_c = 3666.2 \text{ kJ/kg}$$

Using linear interpolation method,

$$s_2 = s_a + \frac{h_2 - h_a}{h_b - h_a} (s_b - s_a) = 7.1218 + \frac{3624.7 - 3550.2}{3666.2 - 3550.2} (7.2586 - 7.1218)$$

$$= 7.209658 \text{ kJ/kgK}$$

Now, the change in entropy per unit mass of the steam,

$$\Delta S/m = \Delta s = s_2 - s_1 = 7.209658 - 6.9022 = 0.307458 \text{ kJ/kgK}$$

Problem 6.8.

Steam enters into a turbine at 2 MPa , 400°C and with a velocity of 200 m/s and saturated vapor exits from the turbine at 100 kPa with a velocity of 80 m/s . The power output of the turbine is 800 kW when the mass flow rate of steam is 1.5 kg/s . Turbine rejects heat to the surroundings at 300 K . Determine the rate at which the entropy is generated within the turbine.

Solution: Given,

Pressure at inlet, $P_1 = 2 \text{ MPa} = 2000 \text{ kPa}$

Temperature at inlet, $T_1 = 400^\circ\text{C}$

Velocity at inlet, $\bar{V}_1 = 200 \text{ m/s}$

Pressure at exit, $P_2 = 100 \text{ kPa}$

Velocity at exit, $\bar{V}_2 = 80 \text{ m/s}$

Power output of turbine, $\dot{W}_{cv} = 800 \text{ kW} = 800 \times 10^3 \text{ W}$

Surroundings temperature, $T_{sur} = 300 \text{ K}$

Mass flow rate of steam, $\dot{m} = 1.5 \text{ kg/s}$

At inlet: $P_1 = 2000 \text{ kPa}$, $T_1 = 400^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 2000 \text{ kPa}$, the saturation temperature for given pressure, i.e. $T_{sat}(P_1 = 2000 \text{ kPa}) = 212.42^\circ\text{C}$. Here, $T_1 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_1 = 2000 \text{ kPa}$ and temperature, $T_1 = 400^\circ\text{C}$,

the specific enthalpy at inlet, $h_1 = 3247.5 \text{ kJ/kg}$

the specific entropy at inlet, $s_1 = 7.1269 \text{ kJ/kgK}$.

At exit: $P_2 = 100 \text{ kPa}$, and exit steam is saturated vapor \Rightarrow exit specific enthalpy, $h_2 = h_g$ and exit specific entropy, $s_2 = s_g$.

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 100 \text{ kPa}$, $h_g = 2675.1 \text{ kJ/kg}$ and $s_g = 7.3589 \text{ kJ/kgK}$.

Then, the exit specific enthalpy, $h_2 = h_g = 2675.1 \text{ kJ/kg}$ and

the exit specific entropy, $s_2 = s_g = 7.3589 \text{ kJ/kgK}$.

From the energy equation for control volume system at steady state condition, we have

$$\begin{aligned}\dot{Q}_{cv} - \dot{W}_{cv} &= \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1)] \\ \Rightarrow \quad \dot{Q}_{cv} &= \dot{W}_{cv} + \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2)] \quad (\text{Neglecting PE change}) \\ \Rightarrow \quad \dot{Q}_{cv} &= 800 \times 10^3 + 1.5[(2675.1 - 3247.5) \times 10^3 + \frac{1}{2}(80^2 - 200^2)] \\ &= -83800 \text{ W} = -83.8 \text{ kW}\end{aligned}$$

Now, the rate of entropy generation of any control volume is given by

$$\dot{S}_{gen} = \left(\frac{dS}{dt} \right)_{cv} - (\dot{S}_{in} - \dot{S}_{out}) - \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{cv}$$

Assuming that the turbine operates in steady state condition, then change of entropy per unit time within the control volume is zero, i.e. $(dS/dt)_{cv} = 0$. Hence,

$$\dot{S}_{gen} = \dot{S}_{out} - \dot{S}_{in} - \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{cv} = \dot{m}(s_2 - s_1) - \frac{\dot{Q}_{cr}}{T_{sur}}$$

$$= 1.5 (7.3589 - 7.1269) - (-83.8)/300 = 0.62733 \text{ kW/K}$$

Problem 6.9.

The conditions of steam at entrance and exit of a turbine are: $h_1 = 3456.5 \text{ kJ/kg}$, $s_1 = 7.2338 \text{ kJ/kgK}$, and velocity of 150 m/s; $h_2 = 2792.8 \text{ kJ/kg}$, $s_2 = 7.4665 \text{ kJ/kgK}$, velocity of 100 m/s respectively. The work output per kg of the steam flow is 600 kJ/kg. Heat is transferred to the surroundings at 500 K. Determine the entropy generation per kg steam flow.

Solution: Given,

Specific enthalpy at inlet, $h_1 = 3456.5 \text{ kJ/kg}$

Specific entropy at inlet, $s_1 = 7.2338 \text{ kJ/kgK}$

Velocity at inlet, $\bar{V}_1 = 150 \text{ m/s}$

Specific enthalpy at exit, $h_2 = 2792.8 \text{ kJ/kg}$

Specific entropy at exit, $s_2 = 7.4665 \text{ kJ/kgK}$

Velocity at exit, $\bar{V}_2 = 100 \text{ m/s}$

Work output of turbine per kg of steam, $w_{cr} = 600 \text{ kJ/kg}$

Surroundings temperature, $T_{sur} = 500 \text{ K}$

From the energy equation for control volume at steady state condition, we have

$$\begin{aligned} \dot{Q}_{cr} - \dot{W}_{cr} &= \dot{m}[(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1)] \\ &= \frac{\dot{Q}_{cr}}{\dot{m}} - \frac{\dot{W}_{cr}}{\dot{m}} = [(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2)] \quad (\text{Neglecting PE change}) \\ \Rightarrow q_{cr} - w_{cr} &= [(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2)] \\ \therefore q_{cr} &= w_{cr} + [(h_2 - h_1) + \frac{1}{2}(\bar{V}_2^2 - \bar{V}_1^2)] \\ &= 600 \times 10^3 + [(2792.8 - 3456.5) \times 10^3 + \frac{1}{2} (100^2 - 150^2)] \\ &= 69949.99 \text{ J/kg} = -69.95 \text{ kJ/kg} \end{aligned}$$

Now, entropy generation per kg of steam for a control volume like turbine in steady state condition [$(ds/dt)_{cv} = 0$] is given by

$$\begin{aligned} s_{gen} &= (s_{sur} - s_{in}) - \sum \left(\frac{q_i}{T_i} \right)_{cv} \\ \Rightarrow s_{gen} &= (s_2 - s_1) - \frac{q_{cr}}{T_{sur}} = (7.4665 - 7.2338) - \frac{(-69.95)}{500} = 0.3726 \text{ kJ/kgK} \end{aligned}$$

Problem 6.10.

A rigid vessel consists of 0.4 kg of hydrogen initially at 200 kPa and 27 °C. Heat is transferred to hydrogen from a reservoir at 600 K until its temperature reaches 450 K. Determine the heat transfer, the change in entropy of hydrogen and amount of entropy produced. [Take $c_v = 10.183 \text{ J/kgK}$].

Solution: Given,

Mass of hydrogen, $m = 0.4 \text{ kg}$

Initial pressure, $P_1 = 200 \text{ kPa}$

Initial temperature, $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Temperature of reservoir, $T_{sur} = 600 \text{ K}$

Final temperature, $T_2 = 450 \text{ K}$

Since the vessel is rigid, the heating process occurs at constant volume from initial state 1 to final state 2. Hence, work transfer

$$W_{1-2} = \int_1^2 P dV = 0$$

Change in internal energy of hydrogen is given by

$$(\Delta U)_{12} = U_2 - U_1 = mc_v(T_2 - T_1) = 0.4 \times 10.183 \times (450 - 300) = 610.98 \text{ J}$$

Now, from the first law of thermodynamics, the heat transfer is given by

$$Q_{1-2} = (\Delta U)_{12} + W_{1-2} = 610.98 + 0 = 610.98 \text{ J}$$

The change in entropy for an incompressible substance or for a constant volume process 1 – 2,

$$dS = S_2 - S_1 = mc_v \ln\left(\frac{T_2}{T_1}\right) = 0.4 \times 10.183 \times \ln\left(\frac{450}{300}\right) = 1.65154 \text{ J/K}$$

Then, entropy production for a control mass,

$$S_{gen} = (dS)_{CM} - \sum \left(\frac{Q_i}{T_i} \right)_{CM} = S_2 - S_1 - \frac{Q_{1-2}}{T_{sur}} = 1.65154 - \frac{610.98}{600} = 0.63324 \text{ J/K}$$

Problem 6.11.

A piston cylinder device shown in figure contains 1.5 kg of water initially at 100 kPa with 10 % of quality. The mass of the piston is such that a pressure of 400 kPa is required to lift the piston. Heat is added to the system from a source at 500°C until its temperature reaches 400°C. Determine the total entropy generation during the process.



Solution: Given,

Mass of water, $m = 1.5 \text{ kg}$

Initial pressure, $P_1 = 100 \text{ kPa}$

Initial quality, $x_1 = 10\% = 0.1$

Final temperature, $T_f = 400^\circ\text{C}$

Surroundings source temperature, $T_{\text{sur}} = 500^\circ\text{C} = 500 + 273 = 773 \text{ K}$

Lifting pressure of piston, $P_{\text{lif}} = 400 \text{ kPa}$

Initial state 1: $P_1 = 100 \text{ kPa}$, $x_1 = 0.1$

As the quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_1 = 100 \text{ kPa}$

$$v_i = 0.001043 \text{ m}^3/\text{kg}, \quad v_{lg} = 1.6933 \text{ m}^3/\text{kg}, \quad v_g = 1.6943 \text{ m}^3/\text{kg}$$

$$u_i = 417.41 \text{ kJ/kg}, \quad u_{lg} = 2088.3 \text{ kJ/kg}, \quad u_g = 2505.7 \text{ kJ/kg}$$

$$s_i = 1.3027 \text{ kJ/kgK}, \quad s_{lg} = 6.0562 \text{ kJ/kgK}, \quad s_g = 7.3589 \text{ kJ/kgK}$$

Specific volume at state 1,

$$v_1 = v_i + x_1 v_{lg} = 0.001043 + 0.1 \times 1.6933 = 0.170373 \text{ m}^3/\text{kg}$$

Specific internal energy at state 1,

$$u_1 = u_i + x_1 u_{lg} = 417.41 + 0.1 \times 2088.3 = 626.24 \text{ kJ/kg}$$

Specific entropy at state 1,

$$s_1 = s_i + x_1 s_{lg} = 1.3027 + 0.1 \times 6.0562 = 1.90832 \text{ kJ/kgK}$$

The process on $P - v$ and $T - v$ diagrams are given below. During initial stage of heating, the piston remains stationary until the initial pressure reaches the lifting pressure of the piston. Therefore, the process 1 – 2 is a constant volume heating thereby increasing pressure and temperature. Hence, state 2 is defined as

State 2: Pressure, $P_2 = P_{\text{lif}} = 400 \text{ kPa}$ and volume, $V_2 = V_1 \Rightarrow$

specific volume, $v_2 = v_1 = 0.170373 \text{ m}^3/\text{kg}$

Referring to the saturated water pressure table A2.1 for pressure, $P_2 = 400 \text{ kPa}$

$$v_i = 0.001084 \text{ m}^3/\text{kg}, \quad v_{lg} = 0.4614 \text{ m}^3/\text{kg}, \quad v_g = 0.4625 \text{ m}^3/\text{kg}$$

Here, $v_i < v_2 < v_g$. Hence, it is still a two phase mixture. So, temperature at state 2, T_2 is the saturation temperature at given pressure, i.e.

$$T_2 = T_{\text{sat}}(P_2 = 400 \text{ kPa}) = 143.64^\circ\text{C}$$

Since the given final temperature of the system is $T_f = 400^\circ\text{C}$, the system has to be heated further to attain this final temperature. This further heating process 2 – 3 occurs with constant pressure and thus final state 3 is defined as

Final state 3: Pressure, $P_3 = P_2 = 400 \text{ kPa}$ and temperature, $T_3 = T_f = 400^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 400 \text{ kPa}$, the saturation temperature at given pressure, i.e. $T_{\text{sat}}(P_3 = 400 \text{ kPa}) = 143.64^\circ\text{C}$.

Here, $T_3 > T_{\text{sat}}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 400 \text{ kPa}$, and temperature, $T_3 = 400^\circ\text{C}$, to determine specific volume (v_3), specific internal energy (u_3) and specific entropy (s_3) at state 3 as

ENTROPY

$$v_3 = 0.7726 \text{ m}^3/\text{kg}$$

$$u_3 = 2964.3 \text{ kJ/kg}$$

$$s_3 = 7.8982 \text{ kJ/kgK}$$

Change in total internal energy during the process 1 - 3,

$$(\Delta U)_{13} = U_3 - U_1 = m(u_3 - u_1) = 1.5(2964.3 - 626.24) = 3507.09 \text{ kJ/kg}$$

Total work transfer during the process 1 - 3,

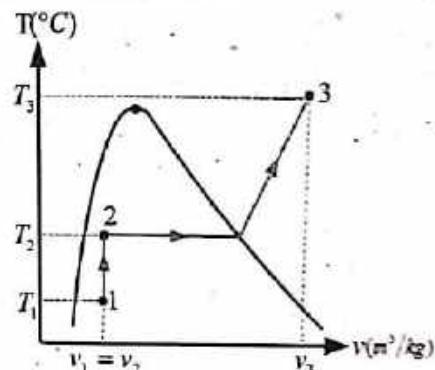
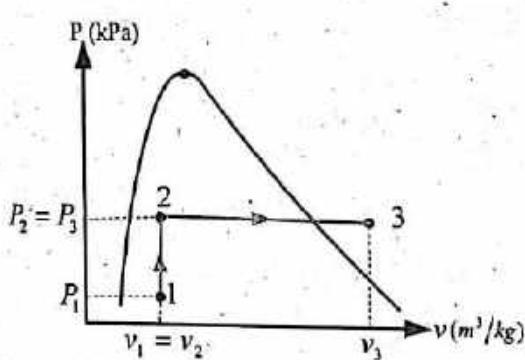
$$\begin{aligned} W_t &= W_{1,3} = W_{1,2} + W_{2,3} = 0 + P_2(V_3 - V_2) = P_2 \times m(v_3 - v_2) \\ &= 400 \times 1.5(0.7726 - 0.170373) = 361.3362 \text{ kJ} \end{aligned}$$

Then, from the first law of thermodynamics for control mass, total heat transfer during the process 1 - 3 is given by

$$Q_t = Q_{1,3} = (\Delta U)_{13} + W_{1,3} = 3507.09 + 361.3362 = 3868.4262 \text{ kJ}$$

Therefore, total entropy generated for control mass is given by

$$\begin{aligned} S_{gen} &= (\Delta S)_{CM} - \sum (Q_i/T_i)_{CM} = m(s_3 - s_1) - Q_t/T_{sur} \\ &= 1.5(7.8982 - 1.90832) - 3868.4262/773 = 3.9804 \text{ kJ/K} \end{aligned}$$



Problem 6.12.

A piston cylinder device shown in figure below contains 1 kg of Nitrogen initially at a pressure of 250 kPa and temperature of 500 °C. Heat is lost from the system till its temperature reaches 40 °C. Sketch the process on P - v and T - v diagrams and determine the entropy generation. Assume that surroundings temperature is at 20 °C. [Take R = 297 J/kgK and c_v = 743 J/kgK]

Solution: Given,

Mass of nitrogen, $m = 1 \text{ kg}$

Initial pressure, $P_1 = 250 \text{ kPa}$

Initial temperature, $T_1 = 500^\circ\text{C} = 500 + 273 = 773 \text{ K}$

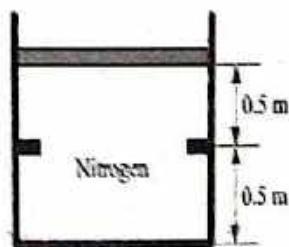
Final temperature, $T_f = 40^\circ\text{C} = 40 + 273 = 313 \text{ K}$

Surroundings temperature, $T_{sur} = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$

Initial state 1: $P_1 = 250 \text{ kPa}$, $T_1 = 773 \text{ K}$

From the equation of state, volume of nitrogen at state 1, V_1 can be determined as

$$PV_1 = mRT_1 \Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 297 \times 773}{250 \times 10^3} = 0.918324 \text{ m}^3$$



ENTROPY

The cooling process is shown on $P - v$ and $T - v$ diagrams in figures below. When heat is lost by the system, piston moves downwards until it reaches the stops with constant pressure. Hence, the process 1 – 2 is constant pressure cooling. Then, state 2 is defined as

State 2: Pressure, $P_2 = P_1 = 250 \text{ kPa}$

Volume of nitrogen at state 2, V_2 can be determined from the system diagram,

$$V_2 = \frac{V_1}{2} = 0.459162 \text{ m}^3 \quad (\text{As stops are located in middle})$$

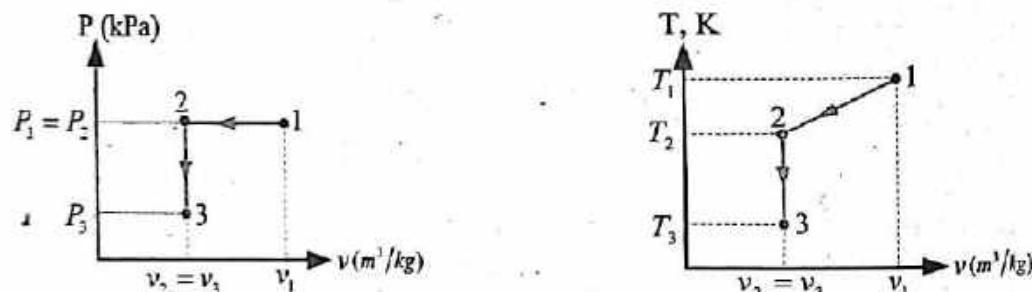
From equation of state, temperature at state 2,

$$P_2 V_2 = mRT_2 \Rightarrow T_2 = \frac{P_2 V_2}{mR} = \frac{250 \times 10^3 \times 0.459162}{1 \times 297} = 386.5 \text{ K}$$

Since the final temperature is $T_f = 40^\circ\text{C} = 313 \text{ K}$, the system has still to be further cooled to attain final temperature. The further cooling process 2 – 3 is constant volume cooling due to presence of stops. So, state 3 is defined as

Final state 3: Temperature, $T_3 = T_f = 313 \text{ K}$, and volume, $V_3 = V_2 = 0.459162 \text{ m}^3$. From equation of state, pressure at state 3,

$$P_3 V_3 = mRT_3 \Rightarrow P_3 = \frac{mRT_3}{V_3} = \frac{1 \times 297 \times 313}{0.459162} = 202.458 \text{ kPa}$$



Total change in internal energy during the process 1 – 3,

$$(\Delta U)_{13} = U_3 - U_1 = m c_v (T_3 - T_1) = 1 \times 743 \times (313 - 773) = -341.78 \text{ kJ}$$

Total work transfer,

$$\begin{aligned} W_{1-3} &= W_{1-2} + W_{2-3} = P_1 (V_2 - V_1) + 0 \\ &= 250 \times 10^3 (0.459162 - 0.918324) = -114.79 \text{ kJ} \end{aligned}$$

From first law of thermodynamics for control mass, total heat transfer,

$$Q_{1-3} = (\Delta U)_{13} + W_{1-3} = -341.78 - 114.79 = -456.57 \text{ kJ}$$

Now, total change in entropy during the process 1 – 3 for an ideal gas,

$$\begin{aligned} dS_{1-3} &= S_3 - S_1 = mc_v \ln\left(\frac{T_3}{T_1}\right) + mR \ln\left(\frac{V_3}{V_1}\right) \\ &= 1 \times 743 \times \ln(313/773) + 1 \times 297 \times \ln(0.459162/0.918324) \\ &= -877.593 \text{ J/K} = -0.877593 \text{ kJ/K} \end{aligned}$$

Then, entropy generation for a control mass is given by

$$S_{gen} = (dS)_{CM} - \sum \left(\frac{Q_i}{T_i} \right)_{CM} = S_3 - S_1 - \frac{Q_{1-3}}{T_{sur}}$$

$$= -0.877593 - \frac{(-456.57)}{293} = 0.680666 \text{ kJ/K}$$

6.3 Solved Numerical Problems from PoU Examinations

Problem 6.13.

Two blocks A ($c_p = 0.9 \text{ kJ/kgK}$) and B ($c_p = 0.38 \text{ kJ/kgK}$) which are initially at 95°C and 540°C respectively are brought together into contact and isolated from the surroundings. They are allowed to reach a final state of thermal equilibrium. Determine the entropy change of each block and of isolated system. Mass of block A and B are 0.45 kg and 0.9 kg respectively. (PoU 2013 spring).

Solution: Given,

Specific heat of block A, $c_A = 0.9 \text{ kJ/kgK}$

Temperature of block A, $T_A = 95^\circ\text{C} = 95 + 273 = 368 \text{ K}$

Mass of block A, $m_A = 0.45 \text{ kg}$

Specific heat of block B, $c_B = 0.38 \text{ kJ/kgK}$

Temperature of block B, $T_B = 540^\circ\text{C} = 540 + 273 = 813 \text{ K}$

Mass of block B, $m_B = 0.9 \text{ kg}$

Net change in entropy after thermal equilibrium, $(\Delta S)_{net} = ?$

Let T be the final equilibrium temperature, then heat lost by the hot block B is equal to heat absorbed by the cold block A, i.e.

$$m_B c_B (T_B - T) = m_A c_A (T - T_A)$$

$$\Rightarrow 0.9 \times 0.38 (813 - T) = 0.45 \times 0.9 (T - 368)$$

$$\Rightarrow T = 571.735 \text{ K}$$

The change in entropy of the hot block B due to the decrease in its temperature is given by

$$(\Delta S)_B = m_B c_B \ln(T/T_B) = 0.9 \times 0.38 \times \ln(571.735/813) = -0.120403 \text{ kJ/K}$$

The change in entropy of the cold block A due to the increase in its temperature is given by

$$(\Delta S)_A = m_A c_A \ln(T/T_A) = 0.45 \times 0.9 \times \ln(571.735/368) = 0.17844 \text{ kJ/K}$$

The net change in entropy of isolated system after thermal equilibrium is given by

$$(\Delta S)_{\text{net}} = (\Delta S)_A + (\Delta S)_B = 0.17844 + (-0.120403) = 0.058037 \text{ kJ/K}$$

Problem 6.14.

A frictionless piston cylinder device contains a saturated liquid vapor mixture of water at 100°C. During a constant pressure process, 600 kJ of heat is transferred to the surroundings air at 25°C. As a result, a part of the water vapor contained in the cylinder condenses. Determine

- (i) entropy change of water
- (ii) total entropy generation during this heat transfer process.

(PoU 2016 spring).

Solution: Given,

Initial temperature of mixture, $T_1 = 100^\circ\text{C} = 100 + 273 = 373 \text{ K}$

Heat transfer (loss) to surroundings, $Q_{1-2} = -600 \text{ kJ}$

Surroundings temperature, $T_{\text{surr}} = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$

Initially, saturated liquid vapor mixture is at 100°C. Finally, a part of the water contained in the cylinder gets condensed at same pressure and same temperature (as two phase mixture) due to heat transfer.

So, the change of entropy of saturated water at 100°C due to heat loss is given by

$$dS = \frac{dH}{T} - \frac{V}{T} dP = \frac{Q_{1-2}}{T_1} = \frac{-600}{373} = -1.60858 \text{ kJ/K} \quad (\text{As pressure is constant})$$

Now, the total entropy generation during this heat transfer process,

$$\dot{S}_{\text{gen}} = dS - \frac{Q_{1-2}}{T_{\text{surr}}} = -1.60858 - \frac{-600}{298} = 0.404843 \text{ kJ/K}$$

6.4 Review Theoretical Questions

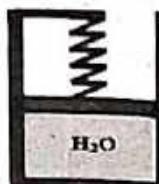
1. State and explain Inequality of Clausius.
2. Define entropy and relate available and unavailable energy with it.
3. Differentiate between the reversible and irreversible processes with reference to entropy.
4. Explain why most of the real processes are irreversible.
5. Derive the relations for entropy for an ideal gas and an incompressible substance.
6. State and explain principle of increase of entropy for an isolated system. Also explain entropy generation.
7. Define reversible heat transfer reservoir and reversible work transfer reservoir. Derive expressions for change in entropy for reversible heat transfer reservoir and reversible work transfer reservoir.
8. Derive an expression for the balance of entropy of a closed system.
9. State the entropy change statement for an open system and derive an expression for its entropy generation.
10. Write down the Gibb's equation and use it to derive an expression for change in entropy for an incompressible substance. Also derive isentropic relation for an incompressible substance.
11. Define isentropic process. Derive isentropic relations for an ideal gas and an incompressible substance.
12. Define the low grade energy and high grade energy.
13. Define internally and externally reversible process.
14. Differentiate between thermal and mechanical irreversibilities.
15. Define and describe the term lost work.
16. Explain the terms irreversibility and availability.

6.5 Unsolved Numerical Problems

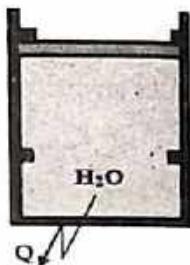
1. A piston cylinder device shown in figure contains 2 kg of water at 2 MPa and 300°C. Heat is added from the source at 800°C to the water until its temperature reaches 800°C. Determine the total entropy generated during the process. (Ans: 0.7211 kJ/K)



2. A piston cylinder device loaded with a linear spring as shown in figure contains 0.5 kg of water at 100 kPa and 25°C. Heat is transferred from a source at 750°C until water reaches to a final state of at 1000 kPa and 600°C. Determine the total entropy generated during the process. (Ans: 2.1633 kJ/K)



3. Water is contained in a piston cylinder device with two set of stops as shown in figure is initially at 1 MPa and 400°C. The limiting volumes are $V_{min} = 1 m^3$ and $V_{max} = 2 m^3$. The weight of the piston is such that a pressure of 400 kPa is required to support the piston. The system is cooled to 100°C by allowing system to reject heat to the surroundings at 25°C. Sketch the process on $P-v$ and $T-v$ diagrams and determine the total entropy generated during the process. (Ans: 16.1838 kJ/K)



4. 4 kg of water at 25°C is mixed with 1 kg of ice 0°C in an isolated system. Calculate the change in entropy due to mixing process. [Take latent heat of ice, L = 336 kJ/kgK and specific heat of water, c = 4.18 kJ/kgK] (Ans: 4.063964 kJ/K)
5. An isolated system of total mass m is formed by mixing two equal masses of the same liquid initially at the temperatures T_1 and T_2 . Eventually, the system attains an equilibrium state. Each mass is incompressible with constant specific heat c . Show that the net change of entropy due to the mixing process is

$$m \times c \times \ln \left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right]$$

6. A rigid cylinder contains nitrogen initially at 100 kPa , 300 K and 0.005 m^3 . It is heated reversibly until its temperature reaches 400 K . Determine the entropy change of the nitrogen during the process.
[Take $R = 297 \text{ J/kgK}$, $c_v = 743 \text{ J/kgK}$] (Ans: 1.1995 kJ/K)
7. 1 kg of air initially at 500 kPa and 400 K expand polytropically ($PV^{1.2} = \text{constant}$) until its pressure reduces to 100 kPa . Determine the entropy change of air during the process. [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$] (Ans: 0.1923 kJ/K)
8. Air at a pressure of 100 kPa and a temperature of 25°C enters into a diffuser with a velocity of 150 m/s and exits with a velocity of 40 m/s . Assuming the process to be reversible and adiabatic, determine the exit pressure and temperature of the air. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$] (Ans: 112.75 kPa , 308.398 K)
9. Steam enters a turbine at a pressure of 1.5 MPa and a temperature of 300°C with a velocity of 60 m/s expands in a reversible adiabatic process and exits at a pressure of 200 kPa with a velocity of 150 m/s . Determine the specific work output. (Ans: 403.712 kJ/kg)
10. Steam enters a nozzle at a pressure of 1.5 MPa and a temperature of 300°C with a velocity of 50 m/s undergoes a reversible adiabatic process and exits at a pressure of 200 kPa . Determine the exit velocity. (Ans: 910.398 m/s)
11. Steam enters an adiabatic nozzle at a pressure of 4 MPa and a temperature of 400°C with a velocity of 50 m/s and exits at a pressure of 2 MPa with a velocity of 300 m/s . If the nozzle has an inlet area of 8 cm^2 , determine
 (a) the exit temperature of steam from the nozzle, and
 (b) the rate of entropy generation for the process.
 (Ans: 364.9°C , 0.1296 kW/K)
12. Air enters a compressor operating steadily at a pressure of 100 kPa and a temperature of 27°C with a volumetric flow rate of $1.2 \text{ m}^3/\text{min}$ and exits at a pressure of 400 kPa and a temperature of 177°C . The power required to drive the compressor is 3.6 kW . Determine
 (a) the heat transfer rate from the compressor surface and
 (b) the rate of entropy generation if heat is transferred to the surroundings at a temperature of 20°C . [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$] (Ans: -98.1 W , 0.55841 W/K)
13. Air enters a nozzle operating steadily at a pressure of 500 kPa and a temperature of 327°C with a velocity of 50 m/s and exits at a pressure of 100 kPa and a temperature of 27°C with a velocity of 500 m/s . Determine
 (a) the heat loss per kg of air from the nozzle surface and

ENTROPY

- (b) the rate of entropy generation per kg of air if heat is transferred to surroundings at a temperature 20°C . [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$] (Ans: -177.75 kJ/kg , 0.37195 kJ/kgK)
14. Air at a pressure of 1 MPa and a temperature of 327°C is throttled down to a pressure of 100 kPa . Determine the change in entropy per unit mass of air. [Take $R = 287 \text{ J/kgK}$] (Ans: 0.6608 kJ/kgK)
15. Steam enters an adiabatic turbine at a pressure of 5 MPa and a temperature of 500°C with a velocity of 50 m/s and exits at a pressure of 50 kPa and a temperature of 100°C with a velocity of 150 m/s . If the power output of the turbine is 5 MW , determine the mass flow rate of steam flowing through the turbine. (Ans: 6.7404 kg/s)

Some Power Cycles

7

7.1 Chapter Highlights

- ***Thermodynamic Cycle:*** It is defined as a repeated series of thermodynamic processes occurring in a way that initial and final states are identical. Most of the devices which are used for continuous energy conversion operate on cyclic processes. These devices are used either to produce work output by supplying heat from the combustion of fuel (e.g. heat engine) or to provide heating or cooling effect with the aid of work input usually through electrical power (e.g. heat pump, refrigerator).
- ***Classification of practical cycles:***
- According to the working principle: Closed and open cycle.
 - According to the power: Power and refrigeration cycle.
 - According to the working substance: Gas and vapor cycle.
 - According to the combustion location: Internal combustion and external combustion cycle.
- ***Differences between closed cycle and open cycle:***

Closed cycle	Open cycle
<ol style="list-style-type: none"> 1. In a closed cycle the working substance is continuously circulated in the cycle. 2. The working substance does not need the replenishment for next cycle. 3. For example, most of the steam power cycles and refrigeration cycles are closed type. 	<ol style="list-style-type: none"> 1. In an open cycle the working substance may be thrown into environment after completion of each cycle. 2. A fresh charge of the working substance enters the cycle again for next cycle. 3. For example, most of the internal combustion engines and gas turbines work on open cycle.

- ***Differences between power cycle and refrigeration cycle:***

Power cycle	Refrigeration cycle
<ol style="list-style-type: none"> 1. It produces power or delivers work to the surroundings when the cycle is executed. 2. It operates the heat engine. 3. Net work is always positive. 4. For examples: Otto, Diesel, Brayton, Rankine cycles. 	<ol style="list-style-type: none"> 1. It needs power which should be supplied from surroundings to execute the cycle. 2. Heat operates the heat pump and refrigerator. 3. Net work is always negative. 4. Examples: Vapor compression refrigeration cycle.

SOME POWER CYCLES

➤ *Differences between gas cycle and vapor cycle:*

Gas cycle	Vapor cycle
<ol style="list-style-type: none"> 1. In this cycle, the working substance remains in gaseous state throughout the cycle. 2. For examples: Otto, Diesel, Brayton cycles etc. 	<ol style="list-style-type: none"> 1. Working substance can undergo phase change (from liquid to vapor or from vapor to liquid) throughout the cycle. 2. For examples: Rankine cycle, vapor compression refrigeration cycle.

➤ *Differences between internal and external combustion cycle:*

Internal combustion cycle	External combustion cycle
<ol style="list-style-type: none"> 1. The combustion takes place inside the power producing unit of the cycle. 2. For examples: Otto cycle and Diesel cycle. 3. The combustion takes place inside the power producing unit of the cycle. 4. For examples: Otto cycle and Diesel cycle 	<ol style="list-style-type: none"> 1. The combustion takes place outside the power producing unit of the cycle. 2. For examples: Brayton and Rankine cycle. 3. The combustion takes place outside the power producing unit of the cycle. 4. For examples: Brayton and Rankine cycle.

➤ *Comparison between different power cycles:*

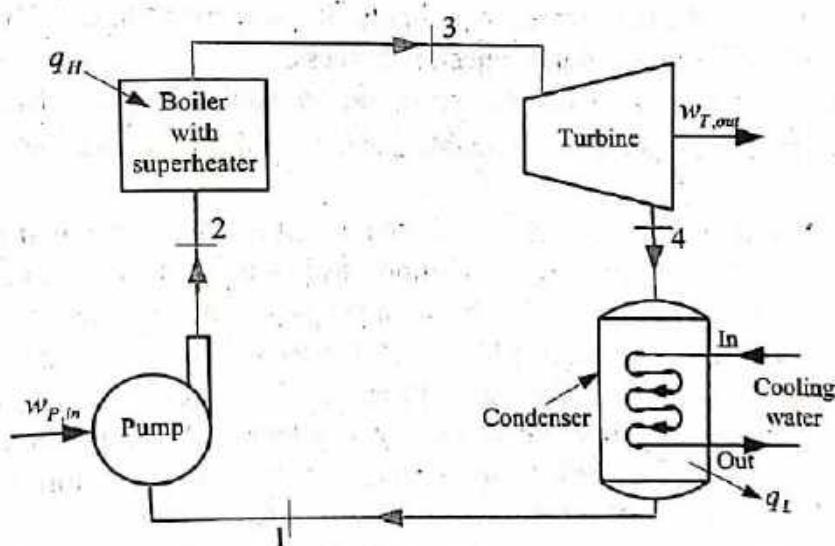
Processes	Power cycles				
	Carnot	Otto	Diesel	Brayton	Rankine
Compression	Isentropic	Isentropic	Isentropic	Isentropic	Isentropic
Heat Addition	Isothermal	Isochoric	Isobaric	Isobaric	Isobaric
Expansion	Isentropic	Isentropic	Isentropic	Isentropic	Isentropic
Heat Rejection	Isothermal	Isochoric	Isochoric	Isobaric	Isobaric

➤ *Air standard analysis:* An idealized analysis of a cycle considering only air as the work substance is called an *air standard analysis*. Such a simplified analysis can be carried out in Otto or petrol cycle and Diesel cycle by considering only air as its working substance. Although this analysis differs from the actual operation, it gives basic idea about the variation of efficiency of the cycle with different parameters.

- **Air standard efficiency:** The efficiency of the engine using air as the working substance is known as an *air standard efficiency*.
- **Assumptions of air standard analysis:**
 - (i) The cycle consists of fixed mass of air.
 - (ii) The expansion and compression processes are isentropic.
 - (iii) The combustion process is replaced by an equivalent heat addition process from an external source.
 - (iv) The exhaust process is replaced by an equivalent heat rejection process to an external sink.
 - (v) The properties of air such as c_p , c_v , R etc. remain constant.

➤ **Vapour Power Cycles:**

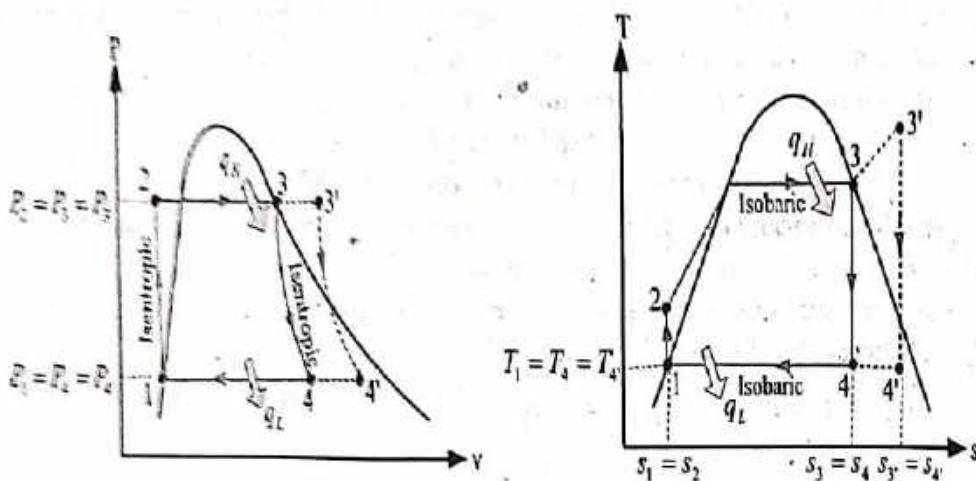
- **Rankine cycle:** It is a power cycle which consists of a steam turbine. It comprises of
 - two isentropic processes, namely an isentropic pumping process and an isentropic expansion process and
 - two isobaric processes, namely an isobaric heat addition process and an isobaric heat rejection process.
- **Different components of an ideal Rankine cycle:**



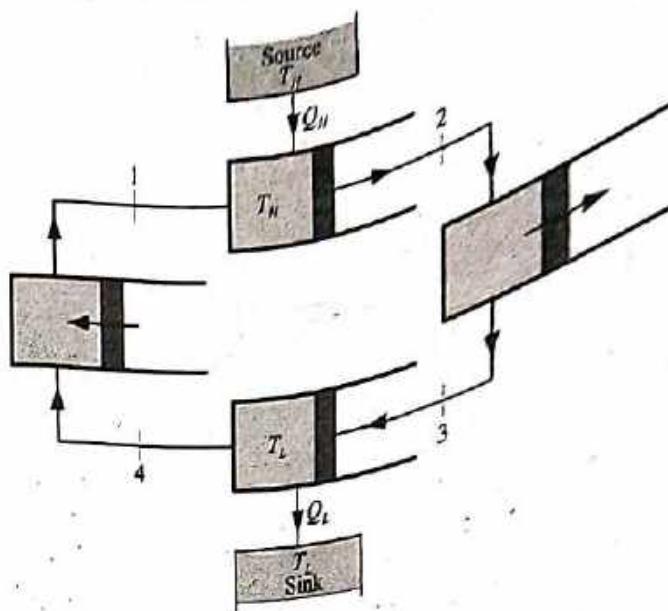
- Efficiency of a Rankine cycle is given by

$$\eta_R = \frac{w_{net}}{q_H} = \frac{w_T - w_P}{q_H} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

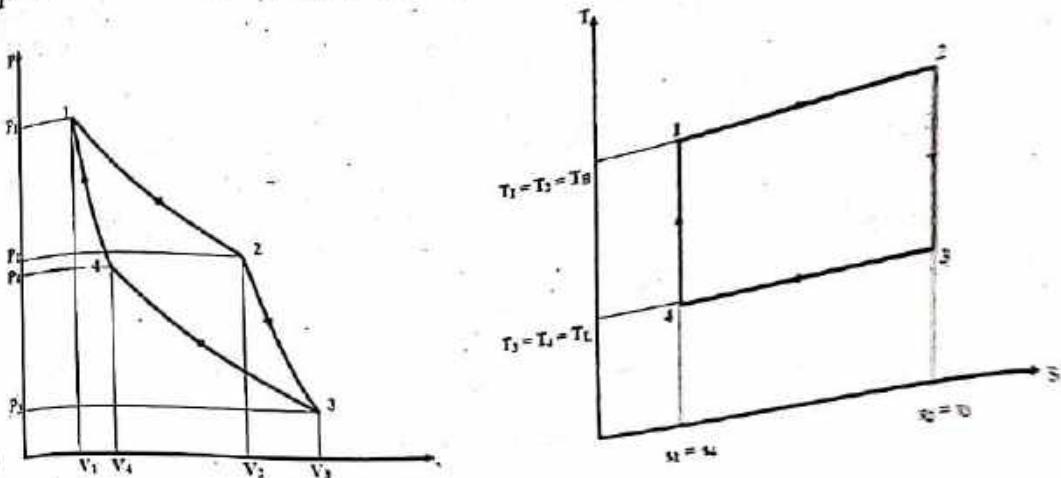
- *P-V and T-S diagrams of an ideal Rankine cycle:*



SOME POWER CYCLES



➤ *P-V and T-S diagrams of a Carnot cycle:*



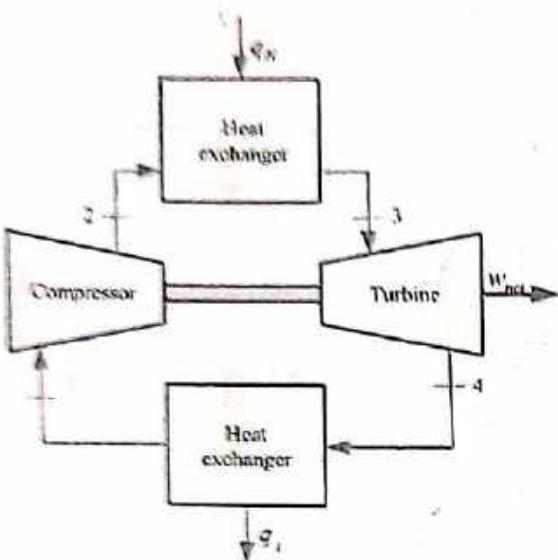
➤ Efficiency of a Carnot cycle is given by

$$\eta_{\text{Carnot}} = \frac{W_{\text{net}}}{q_H} = \frac{q_H - q_L}{q_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_4}{T_2}$$

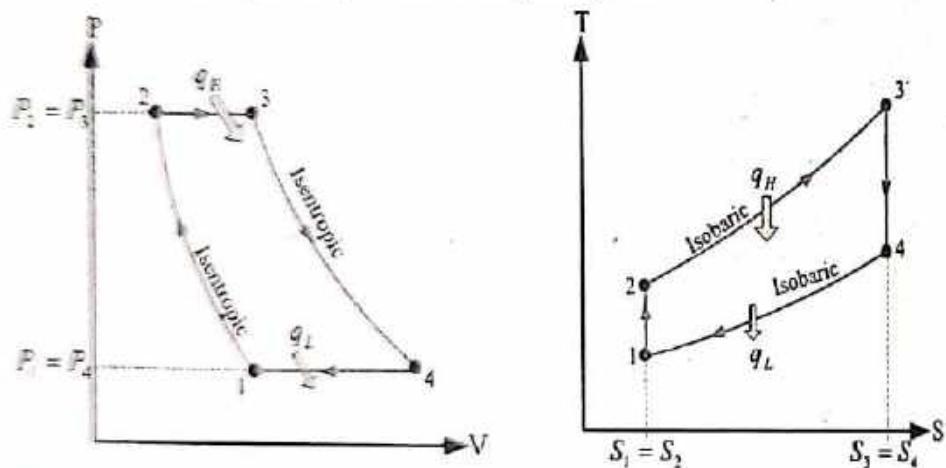
➤ *Brayton cycle:* It is a power cycle which consists of a gas turbine. It comprises of:

- two isentropic processes, namely an isentropic compression process and isentropic expansion process and
- two isobaric processes, namely an isobaric heat addition process and an isobaric heat rejection process,

➤ *Different components of an ideal Brayton cycle:*



➤ *P – V and T – S diagrams of an ideal Brayton cycle:*



➤ *Efficiency of a Brayton cycle is given by*

$$\eta_L = \frac{W_{net}}{q_H} = \frac{W_T - W_c}{q_H} = 1 - \left(\frac{1}{r_p} \right)^{\frac{1}{r}}$$

where $r_p = \frac{P_2}{P_1}$ is called pressure ratio. The pressure ratio for maximum net work developed by the cycle can be derived as:

$$r_p = \left(\frac{T_2}{T_1} \right)^{\frac{1}{2(r-1)}}$$

Efficiency of Brayton cycle increases with increase in pressure ratio. But high pressure ratio causes high temperature and there is limitation to which an

actual turbine blade can withstand maximum temperature and pressure. Hence, the pressure ratio is usually between 10 and 16.

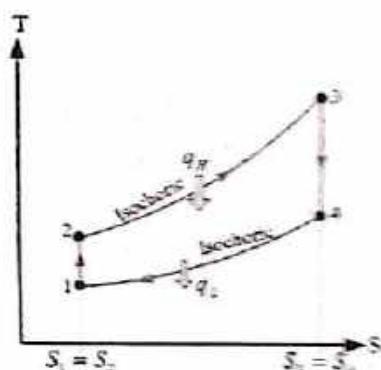
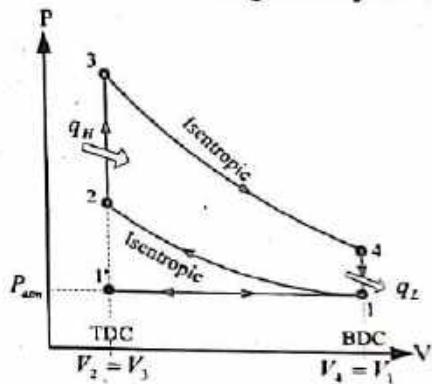
- **Back work:** It is the amount of work consumed by the compressor that is produced by the turbine.
- **Back work ratio:** It is defined as the ratio of compressor work to the turbine work.

➤ **Internal Combustion Engines:**

- **Otto cycle:** It is an idealized model for the operation of petrol engines. It consists of

- two isentropic processes, namely an isentropic compression process and an isentropic expansion process and
- two isochoric processes, namely an isochoric heat addition process and an isochoric heat rejection process.

➤ **P – V and T – S diagrams of an ideal Otto cycle:**



- Efficiency of an ideal Otto cycle is given by

$$\eta_O = \frac{W_{net}}{q_H} = \frac{q_H - q_L}{q_H} = 1 - \frac{q_L}{q_H} = 1 - \left(\frac{1}{r_c} \right)^{\gamma-1}$$

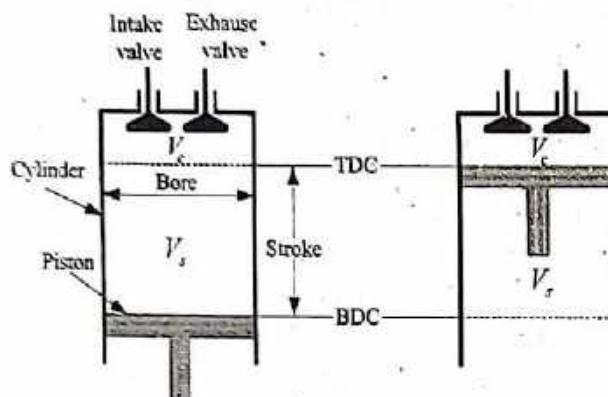
where $r_c = \frac{V_1}{V_2}$ is the compression ratio. The compression ratio for maximum net work developed by the cycle can be derived as

$$r_c = \left(\frac{T_3}{T_1} \right)^{\frac{1}{2(\gamma-1)}}$$

Efficiency of Otto cycle increases with increase in compression ratio, but high compression ratio causes high pressure and temperature and there is a limitation to which an actual engine cylinder can withstand maximum pressure or temperature. Hence, the compression ratio is usually from 8 to 12.

- Compression ratio can be expressed as the function of cylinder dimensions as:

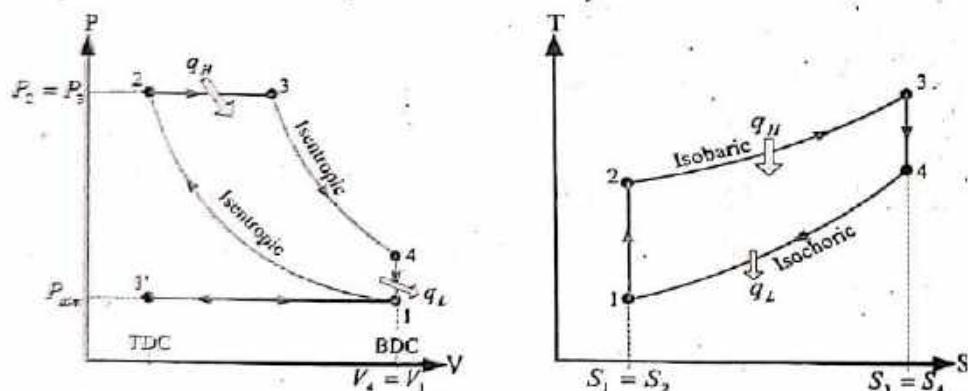
$$r_c = \frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = 1 + \frac{V_s}{V_c} = 1 + \frac{\pi D_p^2 L_s}{4V_c}$$



- Diesel cycle: It is an idealized model for the operation of diesel engines. It consists of

- Two isentropic processes, namely an isentropic compression process and an isentropic expansion process,
- one isobaric heat addition process and
- one isochoric heat rejection process.

- P – V and T – S diagrams of an ideal Diesel cycle:



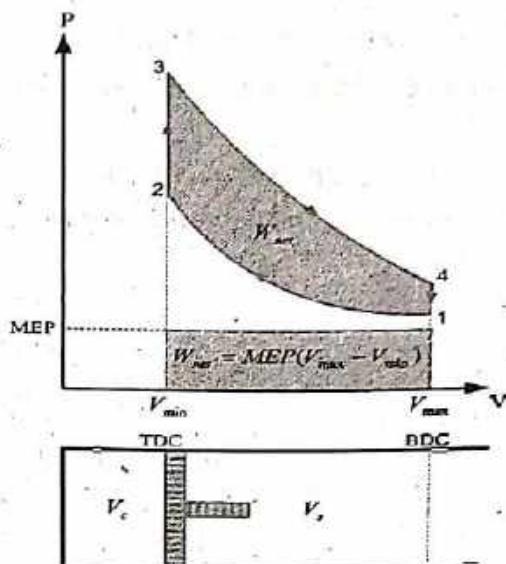
- Efficiency of an ideal Diesel cycle is derived as

$$\eta_D = \frac{w_{net}}{q_H} = \frac{q_H - q_L}{q_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left(\frac{\alpha^\gamma - 1}{\alpha - 1} \right)$$

where $\alpha = \frac{V_3}{V_2}$ is the cut-off ratio.

- Mean effective pressure (MEP): It is the constant pressure which acting on piston through one stroke would do the same amount of work as it done by the varying pressure during the cycle. It is one of the parameters used for the comparison of different internal combustion cycles and it is given by

$$MEP = \frac{\text{Work done}}{\text{Stroke volume}} = \frac{W}{V_1 - V_2} = \frac{W}{v_1 - v_2}$$



➤ Comparison between Otto cycle and Diesel cycle:

Otto cycle	Diesel cycle
<ol style="list-style-type: none"> It is also called spark ignition (SI) engine cycle. A mixture of air and fuel is compressed during isentropic compression process. Fuel is ignited by the electric spark produced by spark plug at the end of compression stroke. It is also known as constant volume cycle. Compression ratio is usually between 8 and 12. Generally, it is less efficient than Diesel cycle but for the same compression ratio and heat supplied its efficiency is higher than that of Diesel cycle. 	<ol style="list-style-type: none"> It is also called compression ignition (CI) engine cycle. Only air is compressed during the isentropic compression process. Fuel is injected to a high temperature air at the end of compression stroke and gets self burnt. It is also known as constant pressure cycle. Compression ratio is usually between 16 and 22. Generally, it is more efficient than Otto cycle but for the same compression ratio and heat supplied its efficiency is less than that of Otto cycle.

7.2 Solved Numerical Problems

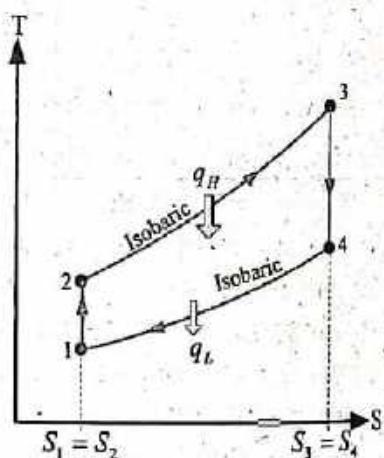
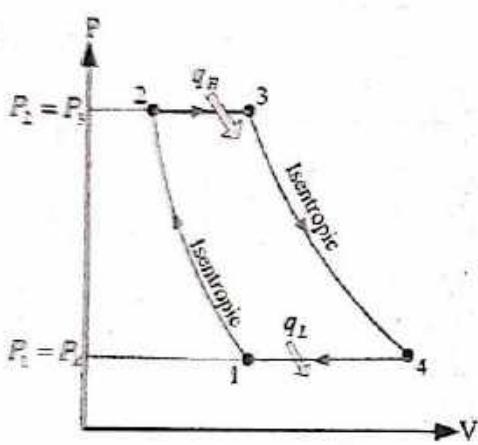
Problem 7.1.

Air at 100 kPa and 25°C enters into a compressor of an ideal Brayton cycle and exits at 1000 kPa. The maximum temperature during the cycle is 1127°C. Determine

- the pressure and temperature at each states of the cycle.
- the compressor work, turbine work and net work per kg of air, and
- the cycle efficiency.

Solution: Given,

It is an ideal Brayton cycle or gas turbine cycle. The $P - V$ and $T - S$ diagrams are given below.



At inlet of compressor of the cycle,
pressure, $P_1 = 100$ kPa

temperature, $T_1 = 25^\circ\text{C} = 25 + 273 = 298$ K

At outlet of compressor,

Pressure, $P_2 = 1000$ kPa

Maximum temperature during the cycle can be attained after heat addition process at state 3, hence maximum temperature, $T_3 = 1127^\circ\text{C} \approx 1127 + 273 = 1400$ K

Process 1 – 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = 298 \times \left(\frac{1000}{100}\right)^{\frac{1.4-1}{1.4}} \\ = 575.33 \text{ K} = 302.33^\circ\text{C}$$

Process 2 – 3 is an isobaric heat addition process, hence $P_3 = P_2 = 1000$ kPa

Process 4 – 1 is an isobaric heat rejection process, then $P_4 = P_1 = 100$ kPa

Process 3 – 4 is an isentropic expansion process, so

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{r-1}{r}} \Rightarrow T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}} = 1400 \times \left(\frac{100}{1000}\right)^{\frac{1.4-1}{1.4}} \\ = 725.15 \text{ K} = 452.15^\circ\text{C}$$

SOME POWER CYCLES

Compressor work per kg of air (specific work),

$$w_C = h_2 - h_1 = c_p(T_2 - T_1) = 1005(575.33 - 298) = 278.717 \text{ kJ/kg}$$

Turbine work per kg of air (specific work),

$$w_T = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1400 - 725.15) = 678.224 \text{ kJ/kg}$$

Net work per kg of air (net specific work),

$$w_{net} = w_T - w_C = 678.224 - 278.717 = 399.507 \text{ kJ/kg}$$

Efficiency of an ideal Brayton cycle is given by

$$\eta_B = 1 - \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} = 1 - \left(\frac{100}{1000} \right)^{\frac{1.4-1}{1.4}} = 0.482 = 48.2\%$$

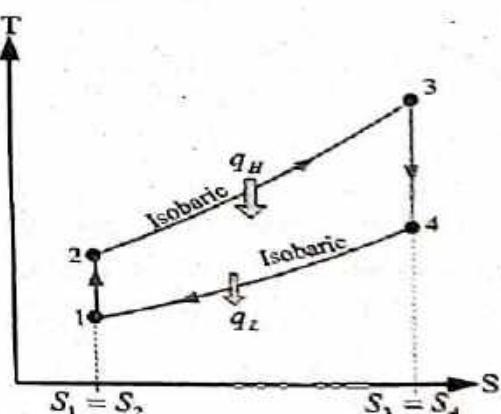
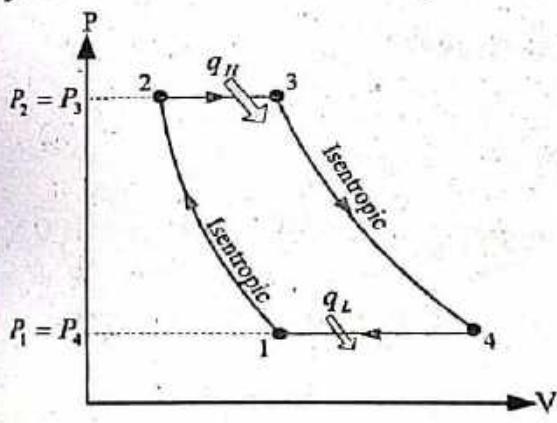
Problem 7.2.

Air at the compressor inlet of an ideal gas turbine cycle is at 100 kPa and 20°C. The heat added to the cycle per kg of air is 800 kJ/kg. The maximum temperature during the cycle is limited to 1400 K. Determine

- (a) the pressure ratio,
- (b) the net work output per kg of air, and
- (c) the cycle efficiency.

Solution: Given,

It is an ideal Brayton cycle or gas turbine cycle. The P – V and T – S diagrams are given below.



At inlet of compressor of the cycle,

pressure, $P_1 = 100 \text{ kPa}$

temperature, $T_1 = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$

Heat added to cycle per kg of air, $q_H = 800 \text{ kJ/kg}$

Maximum temperature during the cycle can be attained after heat addition process at state 3, hence maximum temperature, $T_3 = 1400 \text{ K}$

From process 2 – 3 heat added per kg of air is given by

$$q_H = q_{23} = h_3 - h_2 = c_p(T_3 - T_2)$$

$$\Rightarrow 800 \times 10^3 = 1005(1400 - T_2)$$

$$\therefore T_2 = 603.98 \text{ K}$$

Process 1 – 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} \Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{r}{r-1}} = \left(\frac{603.98}{293}\right)^{\frac{1.4}{1.4-1}} \therefore r_p = 12.576$$

Process 2 – 3 is an isobaric heat addition process, hence $P_3 = P_2$

Process 4 – 1 is an isobaric heat rejection process, then $P_4 = P_1$

Process 3 – 4 is an isentropic expansion process, so

$$\begin{aligned} \frac{T_4}{T_3} &= \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}} = \left(\frac{P_1}{P_2}\right)^{\frac{r-1}{r}} = \left(\frac{1}{r_p}\right)^{\frac{r-1}{r}} \\ \Rightarrow T_4 &= T_3 \left(\frac{1}{r_p}\right)^{\frac{r-1}{r}} = 1400 \times \left(\frac{1}{12.576}\right)^{\frac{1.4-1}{1.4}} = 679.14 \text{ K} \end{aligned}$$

Compressor work per kg of air (specific work),

$$w_C = h_2 - h_1 = c_p(T_2 - T_1) = 1005(603.98 - 293) = 312.535 \text{ kJ/kg}$$

Turbine work per kg of air (specific work),

$$w_T = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1400 - 679.14) = 724.464 \text{ kJ/kg}$$

Net work per kg of air (net specific work),

$$w_{net} = w_T - w_C = 724.464 - 312.535 = 411.929 \text{ kJ/kg}$$

Efficiency of an ideal Brayton cycle is given by

$$\eta_B = 1 - \left(\frac{1}{r_p}\right)^{\frac{r-1}{r}} = 1 - \left(\frac{1}{12.576}\right)^{\frac{1.4-1}{1.4}} = 0.5149 = 51.49\%$$

Problem 7.3.

In an ideal Brayton cycle, air enters the compressor at 100 kPa and 300 K and the turbine at 1000 kPa and 1200 K. Heat is transferred to the air at a rate of 30 MW. Determine the efficiency and the power output of the plant.

Solution: Given,

At inlet of compressor of the cycle,

pressure, $P_1 = 100 \text{ kPa}$ and temperature, $T_1 = 300 \text{ K}$

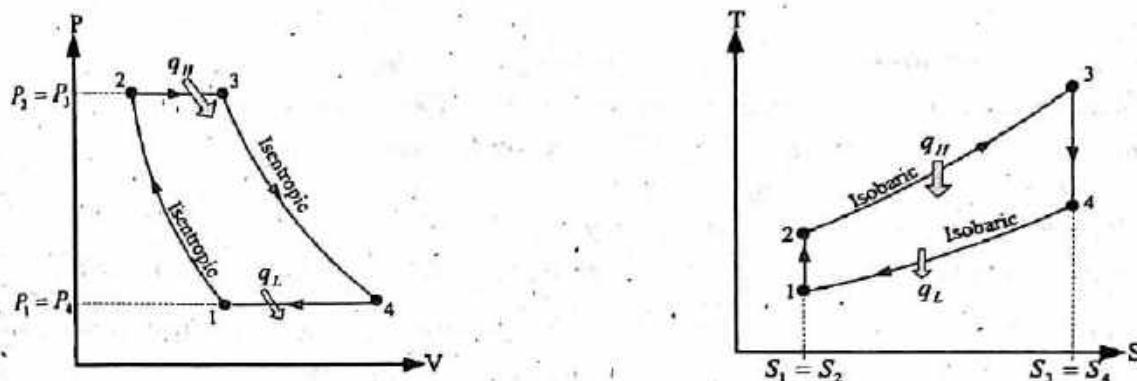
Rate of heat transferred to air, $\dot{Q}_H = 30 \text{ MW} = 30 \times 10^6 \text{ W}$

At inlet of turbine of the cycle,

pressure, $P_3 = 1000 \text{ kPa}$, and temperature, $T_3 = 1200 \text{ K}$

It is an ideal Brayton cycle or gas turbine cycle. The $P - V$ and $T - S$ diagrams are given below.

SOME POWER CYCLES



Process 2 – 3 is an isobaric heat addition process, so $P_2 = P_3 = 1000 \text{ kPa}$

Process 4 – 1 is an isobaric heat rejection process, so $P_4 = P_1 = 100 \text{ kPa}$

Efficiency of an ideal Brayton cycle is given by

$$\eta_B = 1 - \left(\frac{1}{r_p} \right)^{\frac{r-1}{r}} = 1 - \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} = 1 - \left(\frac{100}{1000} \right)^{\frac{1.4-1}{1.4}} = 0.482 = 48.2\%$$

Process 1 – 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = 300 \times \left(\frac{1000}{100} \right)^{\frac{1.4-1}{1.4}} = 579.21 \text{ K}$$

Process 3 – 4 is an isentropic expansion process, so

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} = \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} \Rightarrow T_4 = T_3 \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} = 1200 \times \left(\frac{100}{1000} \right)^{\frac{1.4-1}{1.4}} = 621.557 \text{ K}$$

Compressor work per kg of air (specific work),

$$w_C = h_2 - h_1 = c_p(T_2 - T_1) = 1005(579.21 - 300) = 280.606 \text{ kJ/kg}$$

Turbine work per kg of air (specific work),

$$w_T = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1200 - 621.557) = 581.335 \text{ kJ/kg}$$

Net work per kg of air (net specific work),

$$w_{net} = w_T - w_C = 581.335 - 280.606 = 300.729 \text{ kJ/kg}$$

From process 2 – 3 the rate of heat transferred to air is given by

$$\dot{Q}_H = \dot{m}q_H = \dot{m}q_{23} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2)$$

Then, the mass flow rate of air,

$$\Rightarrow \dot{m} = \frac{\dot{Q}_H}{c_p(T_3 - T_2)} = \frac{30 \times 10^6}{1005(1200 - 579.21)} = 48.085 \text{ kg/s}$$

Now, power output of the plant,

$$\dot{W}_{net} = \dot{m}w_{net} = 48.085 \times 300.729 = 14.46 \text{ MW}$$

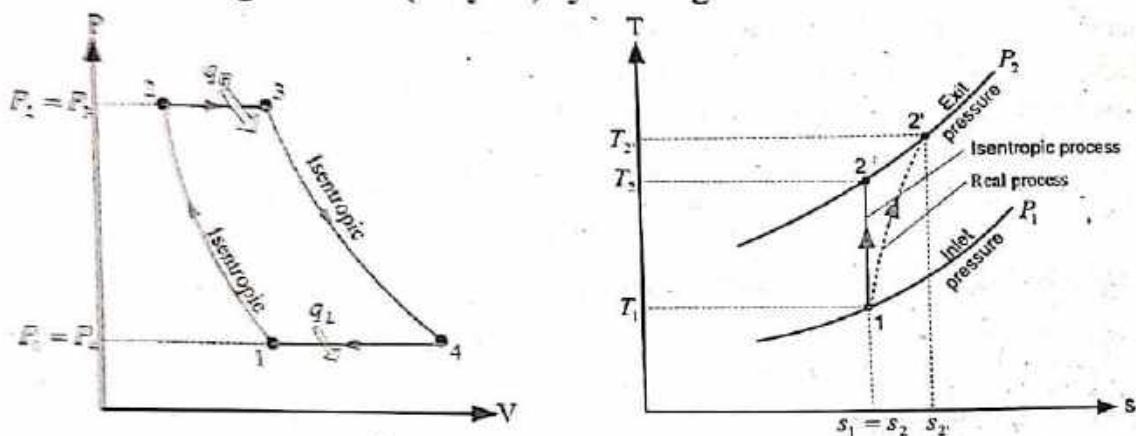
Alternate method: $\eta_B = \frac{\dot{W}_{net}}{\dot{Q}_H} \Rightarrow \dot{W}_{net} = \eta_B \dot{Q}_H = 0.482 \times 30 = 14.46 \text{ MW}$

Problem 7.4.

The compressor and turbine of an ideal gas turbine each have isentropic efficiencies of 80 %. The pressure ratio is 10. The minimum and maximum temperatures are 300 K and 1200 K respectively. Determine:

- the net work per kg of air,
- the thermal efficiency of the cycle, and
- Compare both of these for a cycle with ideal compressor and turbine.

Solution: The $P - V$ diagram and $T - S$ diagram with the real and isentropic compressors of a gas turbine (Brayton) cycle are given below.



Given:

Isentropic efficiency of compressor, $\eta_{isen,C} = 80\% = 0.8$

Isentropic efficiency of turbine, $\eta_{isen,T} = 80\% = 0.8$

$$\text{Pressure ratio, } r_p = \frac{P_2}{P_1} = 10$$

Minimum temperature of cycle is at inlet of compressor, so
minimum temperature at state 1, $T_1 = 300 \text{ K}$

Maximum temperature of cycle is attained after heat addition, so
maximum temperature at state 3, $T_3 = 1200 \text{ K}$

Process 1-2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = 300 \times (10)^{\frac{1.4-1}{1.4}} = 579.19 \text{ K}$$

Process 3-4 is an isentropic expansion process, so

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} = \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} \Rightarrow T_4 = T_3 \left(\frac{1}{r_p} \right)^{\frac{r-1}{r}} = 1200 \times \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 621.557 \text{ K}$$

Isentropic (ideal) compressor work per kg of air (specific work),

$$w_{c,isen} = h_2 - h_1 = c_p(T_2 - T_1) = 1005 (579.19 - 300) = 280.586 \text{ kJ/kg}$$

Isentropic (ideal) turbine work per kg of air (specific work)

$$w_{T,isen} = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1200 - 621.557) = 581.335 \text{ kJ/kg}$$

Isentropic net work per kg of air (net specific work),

$$w_{net,isen} = w_{T,isen} - w_{C,isen} = 581.335 - 280.586 = 300.749 \text{ kJ/kg}$$

Heat added to the ideal (isentropic) cycle per kg of air (specific heat),

$$q_H = q_{23} = h_3 - h_2 = c_p(T_3 - T_2) = 1005(1200 - 579.19) = 623.914 \text{ kJ/kg}$$

Isentropic thermal efficiency of the cycle is given by

$$\eta_{B,isen} = \frac{w_{net,isen}}{q_{H,isen}} = \frac{300.749}{623.914} = 0.482 = 48.2\%$$

Alternate method:

Isentropic thermal efficiency of the cycle is also given by

$$\eta_{B,isen} = 1 - \left(\frac{1}{r_p} \right)^{\frac{r-1}{r}} = 1 - \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 0.482 = 48.2\%$$

Isentropic thermal efficiency of the compressor can be defined as

$$\eta_{C,isen} = \frac{w_{C,isen}}{w_{C,real}}$$

Hence, real compressor work per kg of air (specific work),

$$w_{C,real} = \frac{w_{C,isen}}{\eta_{C,isen}} = \frac{280.586}{0.8} = 350.7325 \text{ kJ/kg}$$

Similarly, isentropic thermal efficiency of the turbine can be defined as

$$\eta_{T,isen} = \frac{w_{T,real}}{w_{T,isen}}$$

Hence, real turbine work per kg of air (specific work),

$$w_{T,real} = \eta_{T,isen} \times w_{T,isen} = 0.8 \times 581.335 = 465.068 \text{ kJ/kg}$$

Real net work per kg of air (net specific work),

$$w_{net,real} = w_{T,real} - w_{C,real} = 465.068 - 350.7325 = 114.3355 \text{ kJ/kg}$$

Heat added to the real cycle per kg of air (specific heat),

$$q_{H,real} = q_{23} = h_3 - h_2 = c_p(T_3 - T_2)$$

where T_2' is the temperature at state 2' when the compressor works with frictional losses, heat losses etc. Hence, real compressor work per kg of air is also calculated as the function of T_2' as

$$w_{C,real} = h_2' - h_1 = c_p(T_2' - T_1)$$

$$T_2' = T_1 + \frac{w_{C,real}}{c_p} = 300 + \frac{350.7325 \times 10^3}{1005} = 648.987 K$$

Then,

$$q_{H,\text{real}} = c_p(T_3 - T_2) = 1005(1200 - 648.987) = 553.768 \text{ kJ/kg}$$

Now, real thermal efficiency of the cycle is given by

$$\eta_{B,\text{real}} = \frac{W_{net,\text{real}}}{q_{H,\text{real}}} = \frac{114.3355}{553.768} = 0.2065 = 20.65\%$$

Problem 7.5.

In an ideal Brayton cycle, the ambient conditions determine the minimum temperature while the maximum temperature is determined by the design conditions and metallurgical considerations of the turbine blades. For fixed value of minimum temperature T_1 and maximum temperature T_3 , show that for maximum work output, the pressure ratio should have the value

$$r_p = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(y-1)}}$$

Solution:

Work output, W_{out} = Heat supplied – Heat rejected = $Q_H - Q_L$

$$\Rightarrow W_{out} = C_p(T_3 - T_2) - C_p(T_4 - T_1) \quad \dots \dots \dots \text{(i)}$$

Processes 1 – 2 and 3 – 4 are isentropic compression and expansion respectively, therefore temperature – pressure relationships are given by

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{r}} \quad \text{and} \quad \frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{1}{r}} \quad \dots \dots \dots \text{(ii)}$$

Processes 2 – 3 and 4 – 1 are isobaric heat addition and rejection respectively, hence $P_2 = P_3$ and $P_1 = P_4$.

$$\text{The pressure ratio is defined as: } r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

Then, from equation (i), we get

$$T_2 = T_1(r_p)^{\frac{1}{r}} \quad \text{and} \quad T_4 = T_3\left(\frac{1}{r_p}\right)^{\frac{1}{r}} \quad \dots \dots \dots \text{(iii)}$$

Using equation (iii) in (i),

$$\begin{aligned} W_{out} &= C_p[T_3 - T_1(r_p)^{\frac{1}{r}}] - C_p[T_3\left(\frac{1}{r_p}\right)^{\frac{1}{r}} - T_1] \\ \Rightarrow W_{out} &= C_p T_3 [1 - (r_p)^{\frac{1}{r}}] - C_p T_1 [(r_p)^{\frac{1}{r}} - 1] \end{aligned}$$

SOME POWER CYCLES

The pressure ratio for maximum work output can be obtained by differentiating the above expression with respect to r_p and setting the derivative equal to zero, i.e.

$$\frac{dW_{out}}{dr_p} = -C_p T_3 \frac{1-\gamma}{\gamma} (r_p)^{\frac{1-2\gamma}{\gamma}} - C_p T_1 \frac{\gamma-1}{\gamma} (r_p)^{\frac{-1}{\gamma}} = 0$$

Or, $C_p T_3 \frac{\gamma-1}{\gamma} (r_p)^{\frac{1-2\gamma}{\gamma}} = C_p T_1 \frac{\gamma-1}{\gamma} (r_p)^{\frac{-1}{\gamma}}$

Or, $\frac{T_3}{T_1} = (r_p)^{\frac{2(\gamma-1)}{\gamma}} \Rightarrow r_p = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}}$

Problem 7.6.

The compression ratio of an ideal Otto cycle is 8.5. At the beginning of the compression stroke, air is at 100 kPa and 27°C. The pressure is doubled during the constant volume heat addition process. Determine:

- (a) the heat added per kg of air,
- (b) the net work output per kg of air,
- (c) the thermal efficiency, and
- (d) the mean effective pressure.

Solution: Given,

Compression ratio, $r_c = V_1/V_2 = 8.5$

Pressure at beginning of compression stroke, $P_1 = 100 \text{ kPa}$

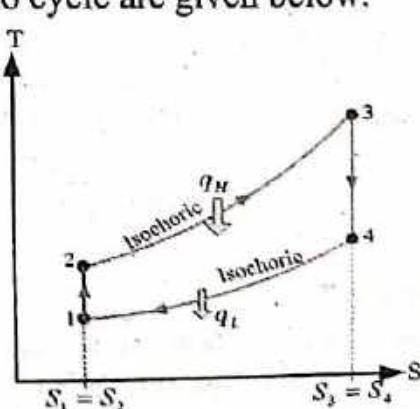
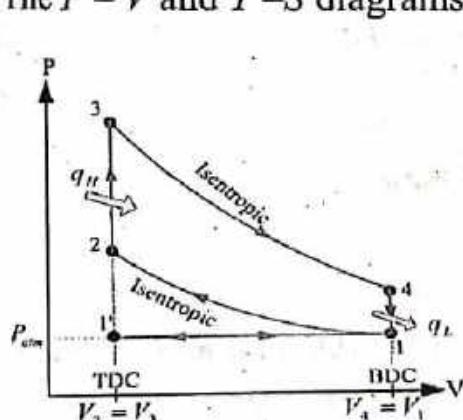
Temperature at beginning of compression stroke, $T_1 = 27^\circ\text{C} = 27 + 273 \approx 300 \text{ K}$

The pressure is doubled during the constant volume heat addition process 2 – 3 means the pressure at state 3, $P_3 = 2 P_2 \Rightarrow P_3/P_2 = 2$.

Process 1 – 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow T_2 = T_1 (r_c)^{\gamma-1} = 300 \times (8.5)^{1.4-1} = 706.1367 \text{ K}$$

The P – V and T – S diagrams of an ideal Otto cycle are given below.



SOME POWER CYCLES

Process 2 – 3 is an isochoric process, so using Charles law for ideal gas,

$$\frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow T_3 = T_2 \frac{P_3}{P_2} = 706.1367 \times 2 = 1412.2734 \text{ K}$$

As the process 2 – 3 is an isochoric process, i.e., $V_2 = V_3$ or $dV = 0$, there is no work done ($w_{2-3} = 0$) and from first law of thermodynamics, heat added per kg of air is given by

$$q_H = q_{2-3} = (\Delta u)_{23} + w_{2-3} = (u_3 - u_2) + 0 = c_v(T_3 - T_2) \\ = 718(1412.2734 - 706.1367) = 507.00615 \text{ kJ/kg}$$

Process 4 – 1 is also an isochoric process, i.e. $V_4 = V_1$.

Process 3 – 4 is an isentropic expansion process, so

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{r_c}\right)^{\gamma-1} \\ \Rightarrow T_4 = T_3 \left(\frac{1}{r_c}\right)^{\gamma-1} = 1412.2734 \times \left(\frac{1}{8.5}\right)^{1.4-1} = 600 \text{ K}$$

Heat loss per kg of air during the isochoric process 4 – 1 is given by

$$q_L = q_{4-1} = (\Delta u)_{41} + w_{4-1} = (u_4 - u_1) + 0 = c_v(T_4 - T_1) \\ = 718(600 - 300) = 215.4 \text{ kJ/kg}$$

Then, the net work output per kg of air,

$$w_{net} = q_H - q_L = 507.00615 - 215.4 = 291.606 \text{ kJ/kg}$$

The thermal efficiency,

$$\eta_O = \frac{w_{net}}{q_H} = \frac{291.606}{507.00615} = 0.57515 = 57.515\%$$

Now, the mean effective pressure is determined as

$$MEP = \frac{w_{net}}{V_{stroke}} = \frac{w_{net}}{v_1 - v_2}$$

$$\text{Using equation of state, } v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{100 \times 10^3} = 0.861 \text{ m}^3/\text{Kg}$$

From compression ratio,

$$r_c = \frac{V_1}{V_2} = \frac{v_1}{v_2} \Rightarrow v_2 = \frac{v_1}{r_c} = \frac{0.861}{8.5} = 0.101294 \text{ m}^3/\text{Kg}$$

Hence, the mean effective pressure,

$$MEP = \frac{w_{net}}{v_1 - v_2} = \frac{291.606}{0.861 - 0.101294} = 383.844 \text{ kPa}$$

Problem 7.7.

An ideal Otto cycle has a compression ratio of 8. The minimum and maximum temperatures during the cycle are 300 K and 1500 K respectively. Determine:

- (a) the heat added per kg of air,

- (b) the thermal efficiency, and
 (c) the efficiency of a Carnot cycle operating between the same temperature limits.

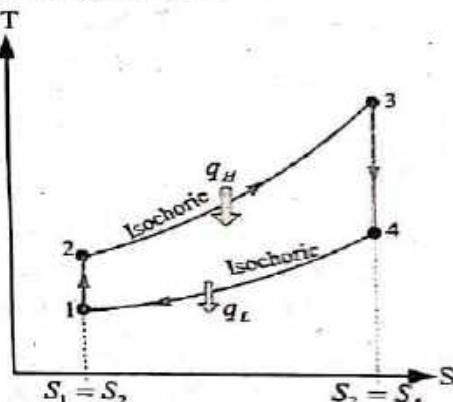
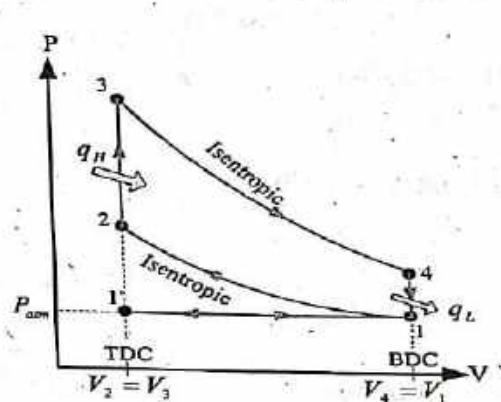
Solution: Given,

$$\text{Compression ratio, } r_c = V_1/V_2 = 8$$

The minimum temperature is at the beginning of compression stroke, so minimum temperature, $T_1 = 300 \text{ K}$

The maximum temperature is attained after the heat addition, so maximum temperature, $T_3 = 1500 \text{ K}$

The $P - V$ and $T - S$ diagrams of an ideal Otto cycle are given below.



Process 1 – 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = T_1 (r_c)^{\gamma-1} = 300 \times (8)^{1.4-1} = 689.219 \text{ K}$$

As the process 2 – 3 is an isochoric process, i.e., $V_2 = V_3$ or $dV = 0$, there is no work done ($w_{2-3} = 0$) and from first law of thermodynamics, heat added per kg of air is given by

$$q_H = q_{2-3} = (\Delta u)_{23} + w_{2-3} = (u_3 - u_2) + 0 = c_v(T_3 - T_2)$$

$$= 718 (1500 - 689.219) = 582.141 \text{ kJ/kg}$$

Then, the thermal efficiency,

$$\eta_O = 1 - \left(\frac{1}{r_c}\right)^{\gamma-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 0.5647 = 56.47\%$$

Now, the Carnot efficiency,

$$\eta_{\text{carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_1}{T_3} = 1 - \frac{300}{1500} = 0.8 = 80\%$$

Problem 7.8.

In an ideal Otto cycle, heat added to the system due to combustion is twice the heat rejected through the exhaust gas. Determine the thermal efficiency and compression ratio of the engine.

Solution: Given,

Heat added to the system is twice the heat rejected means: $q_H = 2q_L$

The thermal efficiency of an Otto cycle can be determined as

$$\eta_O = \frac{w_{net}}{q_H} = \frac{q_H - q_L}{q_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{q_L}{2q_L} = 1 - \frac{1}{2} = 0.5 = 50\%$$

Again, the thermal efficiency of an Otto cycle can be expressed as

$$\eta_O = 1 - \left(\frac{1}{r_c} \right)^{\gamma-1}$$

$$\text{Then, the compression ratio, } r_c = \frac{1}{(1-\eta_O)^{\frac{1}{\gamma-1}}} = \frac{1}{(1-0.5)^{\frac{1}{1.4-1}}} = 5.6568$$

Problem 7.9.

The following data are obtained for a four stroke petrol engine:

Cylinder bore = 14 cm

Stroke length = 15 cm

Clearance volume = 231 cm³

Determine:

(a) *the ratio of clearance volume and swept volume,*

(b) *the compression ratio, and*

(c) *the thermal efficiency.*

Solution: Given,

Cylinder bore = Piston diameter, $D_p = 14 \text{ cm} = 14 \times 10^{-2} \text{ m}$

Stroke length, $L_s = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$

Clearance volume, $V_C = 231 \text{ cm}^3 = 231 \times 10^{-6} \text{ m}^3$

The stroke volume of an Otto or a petrol engine is calculated

$$V_s = \frac{\pi D_p^2}{4} L_s = \frac{\pi (14 \times 10^{-2})^2}{4} \times 15 \times 10^{-2} = 2309.0706 \times 10^{-6} \text{ m}^3$$

$$\text{Then, } \frac{V_C}{V_s} = \frac{231 \times 10^{-6}}{2309.0706 \times 10^{-6}} = 0.1$$

The compression ratio of a petrol engine can be expressed as

$$r_c = \frac{V_1}{V_2} = \frac{V_1 + V_s}{V_2} = \frac{V_C + V_s}{V_C} = 1 + \frac{V_s}{V_C} = 1 + \frac{2309.0706 \times 10^{-6}}{231 \times 10^{-6}} \\ = 11 \quad (\text{As } V_2 = V_C)$$

Now, the thermal efficiency of the petrol engine,

$$\eta_O = 1 - \left(\frac{1}{r_c} \right)^{\gamma-1} = 1 - \left(\frac{1}{11} \right)^{1.4-1} = 0.61678 = 61.678\%$$

Problem 7.10.

An engine with bore of 8 cm and stroke of 12 cm has a compression ratio of 6. To increase the compression ratio, 1.5 mm is machined off the cylinder head face to increase stroke volume while the clearance volume remains same. Determine the new compression ratio.

Solution: Given,

$$\text{Cylinder bore} = \text{Piston diameter}, D_p = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$$

$$\text{Stroke length}, L_s = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$$

$$\text{Compression ratio}, r_c = 6$$

$$\text{Machined off length}, L_m = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

Clearance volume remains same means

$$\text{New clearance volume}, V_{Cn} \approx V_C$$

$$\text{Stroke volume}, V_s = \frac{\pi D_p^2}{4} L_s = \frac{\pi (8 \times 10^{-2})^2}{4} \times 12 \times 10^{-2} = 6.032 \times 10^{-4} \text{ m}^3$$

The compression ratio of a petrol engine can be expressed as

$$r_c = \frac{V_1}{V_2} = \frac{V_2 + V_s}{V_2} = \frac{V_C + V_s}{V_C} = 1 + \frac{V_s}{V_C} \quad \dots \dots \dots \text{(i)} \quad (\text{As } V_2 = V_C)$$

$$\Rightarrow \text{The clearance volume}, V_C = \frac{V_s}{r_c - 1} = \frac{6.032 \times 10^{-4}}{6 - 1} = 1.2064 \times 10^{-4} \text{ m}^3$$

After cylinder head face is machined off, the new stroke length will be

$$L_{Sn} = L_s + L_m = 12 \times 10^{-2} + 1.5 \times 10^{-3} = 12.15 \times 10^{-2} \text{ m}$$

Then, new stroke volume will be

$$V_{Sn} = \frac{\pi D_p^2}{4} L_{Sn} = \frac{\pi (8 \times 10^{-2})^2}{4} \times 12.15 \times 10^{-2} = 6.1073 \times 10^{-4} \text{ m}^3$$

Using equation (i) for new compression ratio,

$$\begin{aligned} r_{cn} &= 1 + \frac{V_{Sn}}{V_{Cn}} = 1 + \frac{V_{Sn}}{V_C} = 1 + \frac{6.1073 \times 10^{-4}}{1.2064 \times 10^{-4}} \\ &= 6.06242 \quad (\text{As } V_{Cn} = V_C) \end{aligned}$$

Problem 7.11.

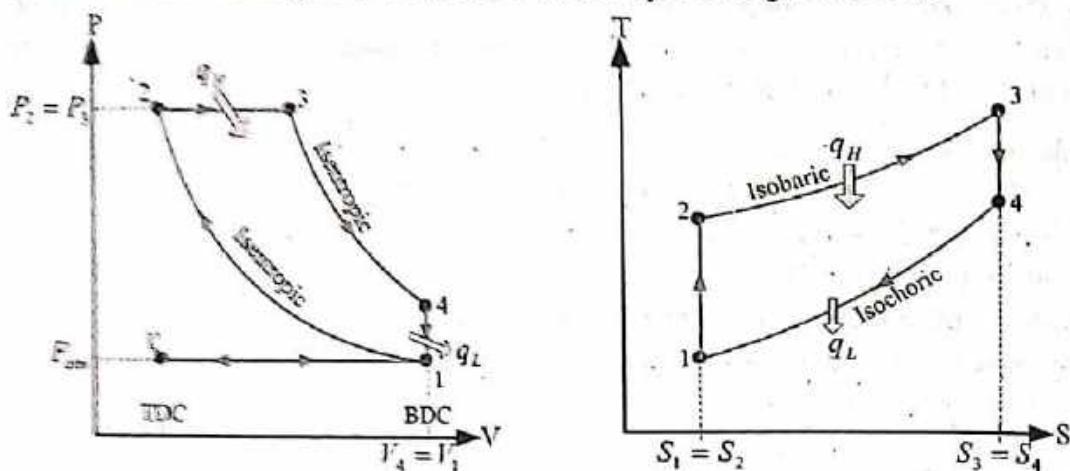
The pressure and temperature at the beginning of the compression stroke of an air standard Diesel cycle are 100 kPa and 300 K. The peak pressure and temperature during the cycle are 8000 kPa and 3000 K respectively. Determine the compression ratio, the cycle efficiency and the mean effective pressure.

Solution: Given,

$$\text{Pressure at beginning of compression stroke}, P_i = 100 \text{ kPa}$$

$$\text{Temperature at beginning of compression stroke}, T_i = 300 \text{ K}$$

Peak pressure and temperature are attained after the heat addition, so peak pressure, $P_3 = 8000 \text{ kPa}$ and peak temperature, $T_3 = 3000 \text{ K}$
 The $P - V$ and $T - S$ diagrams of an ideal Diesel cycle are given below.



Process 1 – 2 is an isentropic compression process, so pressure – volume relationship is given by

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{\gamma} = (r_c)^{\gamma} \quad (\text{As } r_c = \frac{V_1}{V_2})$$

Hence, the compression ratio, $r_c = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} = \left(\frac{8000}{100} \right)^{\frac{1}{1.4}} = 22.8744$

Again, for the isentropic process 1 – 2, temperature – volume relationship is given by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r_c)^{\gamma-1} \Rightarrow T_2 = T_1 (r_c)^{\gamma-1} = 300 (22.8744)^{1.4-1} \\ = 1049.2 \text{ K} \quad (\text{As } r_c = \frac{V_1}{V_2})$$

Process 2 – 3 is an isobaric heat addition process, so using Charles law for ideal gas,

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} \Rightarrow \alpha = \frac{T_3}{T_2} = \frac{3000}{1049.2} = 2.859$$

where $\alpha = V_3/V_2$ is the cut-off ratio.

Then, efficiency of the Diesel cycle,

$$\eta_D = 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^{\gamma} - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{22.8744} \right)^{1.4-1} \left[\frac{2.859^{1.4} - 1}{2.859 - 1} \right] \\ = 0.6317 = 63.17\%$$

$$\text{Mean effective pressure, } MEP = \frac{W_{net}}{v_1 - v_2}$$

Process 2 – 3 is an isobaric heat addition process, so

$$q_H = q_{23} = c_p(T_3 - T_2) = 1005(3000 - 1049.2) = 1960.554 \text{ kJ/kg}$$

The net work output per kg of air of the cycle can be determined as

$$w_{net} = \eta_D \times q_H = 0.6317 \times 1960.554 = 1238.482 \text{ kJ/kg}$$

Now, using equation of state,

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{100 \times 10^3} = 0.861 \text{ m}^3/\text{kg} \quad \text{and}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 1049.2}{8000 \times 10^3} = 0.03764 \text{ m}^3/\text{kg}$$

$$\text{Then, } MEP = \frac{w_{net}}{v_1 - v_2} = \frac{1238.482}{0.861 - 0.03764} = 1504.18 \text{ kPa}$$

Problem 7.12.

The properties of air at the beginning of compression stroke in an air standard Diesel cycle are 100 kPa and 300 K. The air at the beginning of the expansion stroke is at 6500 kPa and 2000 K. Determine:

- (a) *the compression ratio,*
- (b) *the thermal efficiency, and*
- (c) *the mean effective pressure.*

Solution: Given,

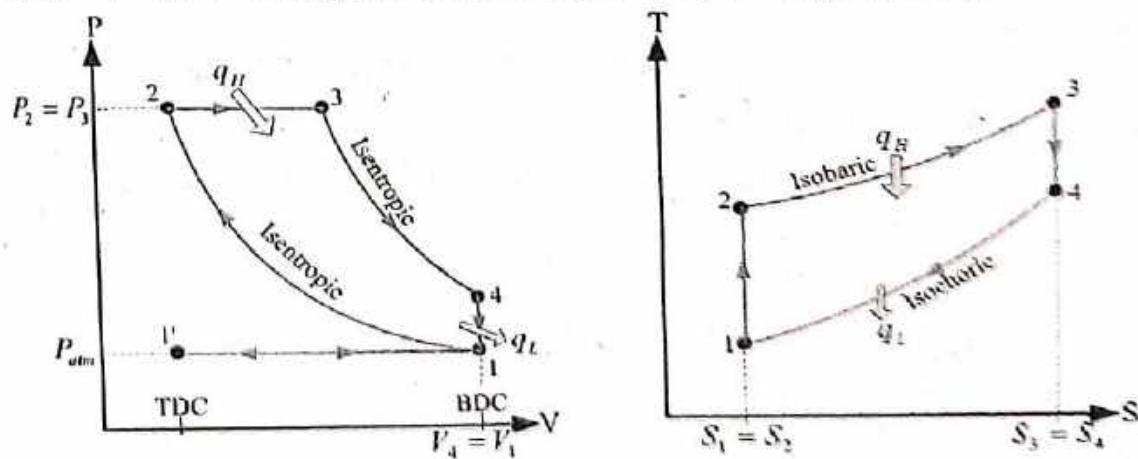
Pressure at beginning of compression stroke, $P_1 = 100 \text{ kPa}$

Temperature at beginning of compression stroke, $T_1 = 300 \text{ K}$

Pressure at beginning of expansion stroke, $P_3 = 6500 \text{ kPa}$

Temperature at beginning of expansion stroke, $T_3 = 2000 \text{ K}$

The P – V and T – S diagrams of an ideal Diesel cycle are given below.



SOME POWER CYCLES

Process 1 - 2 is an isentropic compression process, so pressure - volume relationship is given by

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{\gamma} = (r_c)^{\gamma} \quad (\text{As } r_c = \frac{V_1}{V_2})$$

Process 2 - 3 is an isobaric process, so $P_2 = P_3 = 6500 \text{ kPa}$.

Hence, $r_c = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} = \left(\frac{6500}{100} \right)^{\frac{1}{1.4}} = 19.72$

Again, for isentropic process 1 - 2, temperature - volume relationship is given by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r_c)^{\gamma-1} \Rightarrow T_2 = T_1(r_c)^{\gamma-1} = 300(19.72)^{1.4-1} = 988.744 \text{ K}$$

(As $r_c = \frac{V_1}{V_2}$)

Process 2 - 3 is an isobaric process, so using Charles law for ideal gas,

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$\Rightarrow \alpha = \frac{T_3}{T_2} = \frac{2000}{988.744} = 2.023$$

where $\alpha = V_3/V_2$ is the cut-off ratio.

Then thermal efficiency of the Diesel cycle,

$$\eta_D = 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^{\gamma} - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{19.72} \right)^{1.4-1} \left[\frac{2.023^{1.4} - 1}{2.023 - 1} \right]$$

$$= 0.64375 = 64.375\%$$

Mean effective pressure, $MEP = \frac{W_{net}}{v_1 - v_2}$

Process 2 - 3 is an isobaric heat addition process, so

$$q_H = q_{23} = c_p(T_3 - T_2) = 1005(2000 - 988.744) = 1016.312 \text{ kJ/kg}$$

Thermal efficiency of a cycle can be determined as

$$\eta_D = \frac{W_{net}}{q_H} = w_{net} = \eta_D \times q_H = 0.64375 \times 1016.312 = 654.251 \text{ kJ/kg}$$

$\eta_D = \frac{W_{net}}{q_H}$ using equation of state,

$$w_{net} = \frac{RT_1}{P_1} = \frac{287 \times 300}{100 \times 10^3} = 0.861 \text{ m}^3/\text{kg} \text{ and}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 988.744}{6500 \times 10^3} = 0.043657 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 300}{6500 \times 10^3}$$

$$\text{Then, } MEP = \frac{W_{net}}{v_1 - v_2} = \frac{654.251}{0.861 - 0.043657} = 800.461 \text{ kPa}$$

Problem 7.13.

Air at the beginning of compression stroke in an ideal Diesel cycle is at 100 kPa and 295 K and the compression ratio is 20. Determine the maximum temperature during the cycle to have an efficiency of 65 %.

Solution: Given,

Pressure at beginning of compression stroke, $P_1 = 100 \text{ kPa}$

Temperature at beginning of compression stroke, $T_1 = 295 \text{ K}$

Compression ratio, $r_c = V_1/V_2 = 20$

Efficiency of cycle, $\eta_D = 65\% = 0.65$

For the isentropic process 1 – 2, temperature – volume relationship is given by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r_c)^{\gamma-1} \Rightarrow T_2 = T_1(r_c)^{\gamma-1} = 295(20)^{1.4-1} = 977.764 \text{ K}$$

(As $r_c = \frac{V_1}{V_2}$)

The efficiency of the Diesel cycle,

$$\eta_D = 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^{\gamma} - 1}{\alpha - 1} \right] \quad \text{where } \alpha = V_3/V_2 \text{ is the cut-off ratio.}$$

$$\Rightarrow 0.65 = 1 - \frac{1}{1.4} \left(\frac{1}{20} \right)^{1.4-1} \left[\frac{\alpha^{1.4} - 1}{\alpha - 1} \right]$$

$$\Rightarrow \alpha^{1.4} - 1.624 \alpha + 0.624 = 0$$

$$\therefore \alpha = 1.9289$$

Process 2 – 3 is an isobaric process, so using Charles law for ideal gas,

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

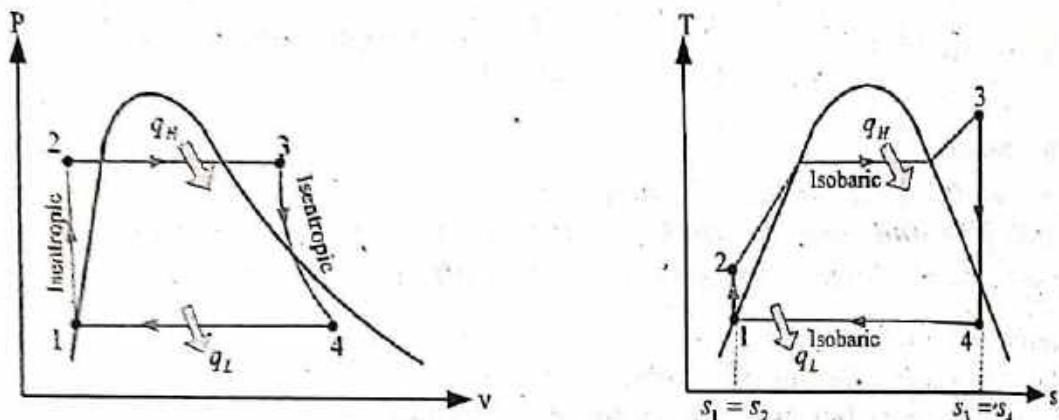
⇒ Maximum temperature during the cycle,

$$T_3 = T_2 \times \alpha = 977.764 \times 1.9289 = 1886.01 \text{ K}$$

Problem 7.14.

A Rankine cycle has a boiler working at a pressure of 2 MPa. The maximum and minimum temperatures during the cycle are 400°C and 50°C respectively. Determine the efficiency of the cycle and compare it with that of the Carnot cycle operating between the same temperature limits.

Solution: The $P-v$ and $T-s$ diagrams of an ideal Rankine cycle are given below.



Given.

As process 2 – 3 is an isobaric heat addition in boiler, boiler working pressure, $P_2 = P_3 = 2 \text{ MPa} = 2000 \text{ kPa}$

Maximum temperature is attained after heat addition at boiler exit, so

$$T_s = 400^\circ\text{C} = 400 + 273 = 673 \text{ K}$$

Minimum temperature is maintained before heat addition at boiler inlet or after heat rejection at condenser exit, so

$$T_1 = T_4 = 50^\circ\text{C} = 50 + 273 = 323 \text{ K}$$

The efficiency of Rankine cycle is: $\eta_R = \frac{w_{net}}{q_H}$

where $w_{net} = w_T - w_P = (h_3 - h_4) - (h_2 - h_1)$ and $q_H = h_3 - h_2$

Determination of values of specific enthalpy at all states of the cycle:

Initial state 1: Temperature, $T_1 = 50^\circ\text{C}$ and water is in saturated liquid state.

Referring to the saturated water temperature table A 2.2 for temperature,

$T_1 = 50^\circ\text{C}$, pressure, $P_1 = P_{sat}(T_1 = 50^\circ\text{C}) = 12.344 \text{ kPa}$,

specific enthalpy, $h_1 = h_l(T_1 = 50^\circ\text{C}) = 209.33 \text{ kJ/kg}$ and

specific volume, $v_1 = v_l(T_1 = 50^\circ\text{C}) = 0.001012 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 2000 \text{ kPa}$ and water is in subcooled or compressed liquid.

Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for an incompressible substance (liquid water),

$$h_2 - h_1 = v_1(P_2 - P_1)$$

$$\Rightarrow h_2 = h_1 + v_1(P_2 - P_1) = 209.33 + 0.001012(2000 - 12.344) \\ = 211.3415 \text{ kJ/kg}$$

State 3: Pressure, $P_3 = 2000 \text{ kPa}$, and temperature, $T_3 = 400^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 2000 \text{ kPa}$, saturation temperature, $T_{sat}(P_3 = 2000 \text{ kPa}) = 212.42^\circ\text{C}$.

Here, $T_3 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 2000 \text{ kPa}$, and temperature, $T_3 = 400^\circ\text{C}$,

specific enthalpy, $h_3 = 3247.5 \text{ kJ/kg}$ and specific entropy, $s_3 = 7.1269 \text{ kJ/kgK}$

State 4: Temperature, $T_4 = 50^\circ\text{C}$ and as the process $3 - 4$ is the isentropic expansion, the specific entropy at state 4, $s_4 = s_3 = 7.1269 \text{ kJ/kgK}$.

Referring to the saturated water temperature table A2.2 for temperature, $T_4 = 50^\circ\text{C}$,

$$s_l = 0.7037 \text{ kJ/kgK}, s_{lg} = 7.3708 \text{ kJ/kgK}, s_g = 8.0745 \text{ kJ/kgK}.$$

$$h_l = 209.33 \text{ kJ/kg}, h_{lg} = 2381.9 \text{ kJ/kg}, h_g = 2591.2 \text{ kJ/kg}$$

Here, $s_l < s_4 < s_g$. Hence, it is a two phase mixture. Then,

$$\text{the quality at state 4, } x_4 = \frac{s_4 - s_l}{s_{lg}} = \frac{7.1269 - 0.7037}{7.3708} = 0.8714$$

and the specific enthalpy at state 4,

$$h_4 = h_l + x_4 h_{lg} = 209.33 + 0.8714 \times 2381.9 = 2284.917 \text{ kJ/kg}$$

Now, work output from the turbine,

$$w_T = h_3 - h_4 = 3247.5 - 2284.917 = 962.583 \text{ kJ/kg}$$

Work input to the pump,

$$w_P = h_2 - h_1 = 211.3415 - 209.33 = 2.0115 \text{ kJ/kg}$$

Net work delivered to the surroundings,

$$w_{net} = w_T - w_P = 962.583 - 2.0115 = 960.5715 \text{ kJ/kg}$$

Heat addition to the boiler during the process $2 - 3$,

$$q_H = h_3 - h_2 = 3247.5 - 211.3415 = 3036.16 \text{ kJ/kg}$$

Efficiency of the Rankine cycle,

$$\eta_R = \frac{w_{net}}{q_H} = \frac{960.5715}{3036.16} = 0.31637 = 31.637\%$$

Efficiency of the Carnot cycle,

$$\eta_C = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{T_1}{T_3} = 1 - \frac{323}{673} = 0.52006 = 52.006\%$$

Problem 7.15.

Determine the efficiency of an ideal Rankine cycle operating between the boiler pressure of 1.5 MPa and a condenser pressure of 8 kPa . The steam leaves the boiler as saturated vapor.

Solution: Given,

As the process $2 - 3$ is an isobaric heat addition in the boiler,

boiler pressure, $P_2 = P_3 = 1.5 \text{ MPa} = 1500 \text{ kPa}$

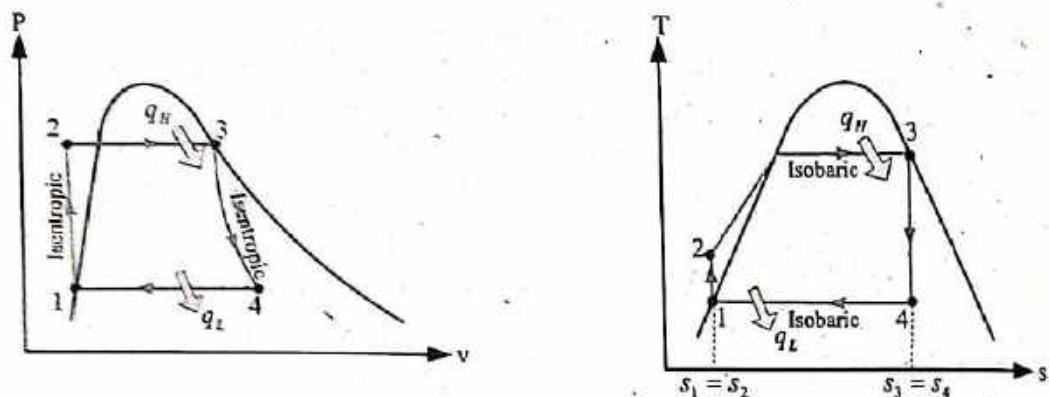
Since the process $4 - 1$ is an isobaric heat rejection in the condenser,

condenser pressure, $P_1 = P_4 = 8 \text{ kPa}$

Steam leaves the boiler as saturated vapor means the state 3 is saturated vapor state.

The $P - v$ and $T - s$ diagrams of an ideal Rankine cycle are given below.

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The efficiency of Rankine cycle is given by

$$\eta_R = \frac{w_{net}}{q_H} = \frac{w_T - w_P}{q_{23}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

Determination of values of specific enthalpy at all states of the cycle:

Initial state 1: Pressure, $P_1 = 8 \text{ kPa}$ and water is in saturated liquid state.

Referring to the saturated water pressure table A 2.1 for pressure, $P_1 = 8 \text{ kPa}$, specific enthalpy, $h_1 = h_l(P_1 = 8 \text{ kPa}) = 173.85 \text{ kJ/kg}$ and specific volume, $v_1 = v_l(P_1 = 8 \text{ kPa}) = 0.001008 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 1500 \text{ kPa}$ and water is in subcooled or compressed liquid. Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for an incompressible substance (liquid water),

$$h_2 - h_1 = v_1(P_2 - P_1)$$

$$\Rightarrow h_2 = h_1 + v_1(P_2 - P_1) = 173.85 + 0.001008(1500 - 8) = 175.354 \text{ kJ/kg}$$

State 3: Pressure, $P_3 = 1500 \text{ kPa}$, and it is a saturated vapor.

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 1500 \text{ kPa}$, specific enthalpy, $h_3 = h_g(P_3 = 1500 \text{ kPa}) = 2791.5 \text{ kJ/kg}$ and specific entropy, $s_3 = s_g(P_3 = 1500 \text{ kPa}) = 6.4438 \text{ kJ/kgK}$

State 4: Pressure, $P_4 = 8 \text{ kPa}$ and as the process 3 – 4 is the isentropic expansion, the specific entropy at state 4, $s_4 = s_3 = 6.4438 \text{ kJ/kgK}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_4 = 8 \text{ kPa}$,

$$s_1 = 0.5925 \text{ kJ/kgK}, s_{lg} = 7.6342 \text{ kJ/kgK}, s_g = 8.2267 \text{ kJ/kgK}$$

$$h_l = 173.85 \text{ kJ/kg}, h_{lg} = 2402.3 \text{ kJ/kg}, h_g = 2576.1 \text{ kJ/kg}$$

Here, $s_1 < s_4 < s_g$. Hence, it is a two phase mixture. Then,

$$\text{the quality at state 4, } x_4 = \frac{s_4 - s_1}{s_{lg}} = \frac{6.4438 - 0.5925}{7.6342} = 0.76646$$

and the specific enthalpy at state 4,

$$h_4 = h_l + x_4 h_{lg} = 173.85 + 0.76646 \times 2402.3 = 2015.117 \text{ kJ/kg}$$

Then, the efficiency of Rankine cycle is given by

$$\eta_R = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = \frac{(2791.5 - 2015.117) - (175.354 - 173.85)}{2791.5 - 175.354}$$

$$= 0.2962 = 29.62\%$$

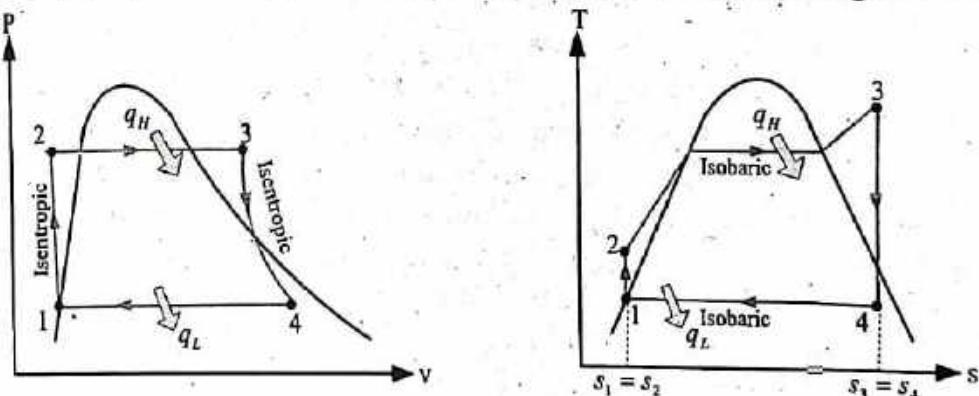
Problem 7.16.

An ideal Rankine cycle operates between a boiler pressure of 4 MPa and a condenser pressure of 10 kPa. The exit steam from the turbine should have a quality of 96 % and the power output of the turbine should be 80 MW. Determine

- (a) the minimum boiler exit temperature,
- (b) the efficiency of the cycle, and
- (c) the mass flow rate of steam.

Solution:

The $P-v$ and $T-s$ diagrams of an ideal Rankine cycle are given below.



Given,

As the process 2 – 3 is an isobaric heat addition in the boiler,
boiler pressure, $P_2 = P_3 = 4 \text{ MPa} = 4000 \text{ kPa}$

Since the process 4 – 1 is an isobaric heat rejection in the condenser,
condenser pressure, $P_1 = P_4 = 10 \text{ kPa}$

Quality of exit steam from turbine, $x_4 = 96\% = 0.96$

Power output of turbine, $\dot{W}_{out,T} = 80 \text{ MW} = 80 \times 10^3 \text{ kW}$

Initial state 1: Pressure, $P_1 = 10 \text{ kPa}$ and water is in saturated liquid state.

Referring to the saturated water pressure table A 2.1 for pressure, $P_1 = 10 \text{ kPa}$,
specific enthalpy, $h_1 = h_l (P_1 = 10 \text{ kPa}) = 191.83 \text{ kJ/kg}$ and
specific volume, $v_1 = v_l (P_1 = 10 \text{ kPa}) = 0.001010 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 4000 \text{ kPa}$ and water is in subcooled or compressed liquid.

Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for
an incompressible substance (liquid water),

$$h_2 - h_1 = v_1(P_2 - P_1)$$

Now, the power output of the turbine is given by

$$\dot{W}_{out,T} = \dot{m} \times w_T$$

Hence, flow rate of steam,

$$\dot{m} = \frac{\dot{W}_{out,T}}{w_T} = \frac{\dot{W}_{out,T}}{w_T} = \frac{\dot{W}_{out,T}}{(h_3 - h_4)} = \frac{80 \times 10^3}{(4139.544 - 2488.15)} = 48.444 \text{ kg/s}$$

Problem 7.17.

Saturated vapor enters into a turbine of an ideal Rankine cycle at 10 MPa and saturated liquid exits the condenser at 10 kPa. The power output of the cycle is 120 MW. Determine:

- (a) the thermal efficiency of the cycle,
- (b) the back work ratio,
- (c) the mass flow rate of steam,
- (d) the rate at which heat is supplied to the boiler,
- (e) the rate at which heat is rejected from the condenser, and
- (f) the mass flow rate of condenser cooling water, if the cooling water enters at 20°C and exits at 35°C .

[Take specific heat of water as 4.18 kJ/kgK].

Solution: Given,

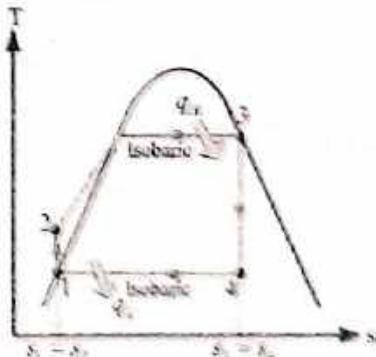
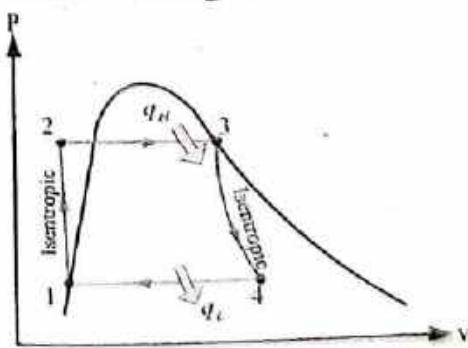
The boiler exit pressure is same the turbine inlet pressure and process 2 – 3 is an isobaric heat addition in the boiler, i.e. $P_2 = P_3 = 10 \text{ MPa} = 10000 \text{ kPa}$ (State 3 is given as saturated vapor)

As process 4 – 1 is an isobaric heat rejection in condenser, condenser exit pressure, $P_1 = P_4 = 10 \text{ kPa}$ (State 1 is given as saturated liquid)

Power output of cycle, $\dot{W}_{out} = 120 \text{ MW} = 120 \times 10^3 \text{ kW}$

Cooling water enters at $T_{in} = 20^\circ\text{C}$ and exits at $T_{out} = 35^\circ\text{C}$.

The $P-v$ and $T-s$ diagrams of an ideal Rankine cycle are given below.



Initial state 1: Pressure, $P_1 = 10 \text{ kPa}$ and water is in saturated liquid state.

SOME POWER CYCLES

$\Rightarrow h_2 = h_1 + v_1(P_2 - P_1) = 191.83 + 0.001010(4000 - 10) = 195.86 \text{ kJ/kg}$
 As the process 3 – 4 is the isentropic expansion, the specific entropy at state 3, $s_3 = s_4$. To find the specific entropy, we deal first with the state 4 since it is clearly defined.

State 4: Pressure, $P_4 = 10 \text{ kPa}$ and quality, $x_4 = 0.96$

As quality is given, it is a two phase mixture. Referring to the saturated water pressure table A2.1 for pressure, $P_4 = 10 \text{ kPa}$,

$$s_l = 0.6493 \text{ kJ/kgK}, s_{lg} = 7.4989 \text{ kJ/kgK}, s_g = 8.1482 \text{ kJ/kgK}.$$

$$h_l = 191.83 \text{ kJ/kg}, h_{lg} = 2392.0 \text{ kJ/kg}, h_g = 2583.8 \text{ kJ/kg}$$

the specific entropy at state 4,

$$s_4 = s_l + x_4 s_{lg} = 0.6493 + 0.96 \times 7.4989 = 7.848244 \text{ kJ/kgK}$$

and the specific enthalpy at state 4,

$$h_4 = h_l + x_4 h_{lg} = 191.83 + 0.96 \times 2392.0 = 2488.15 \text{ kJ/kg}$$

State 3: Pressure, $P_3 = 4000 \text{ kPa}$ and the specific entropy at state 3,

$$s_3 = s_4 = 7.848244 \text{ kJ/kgK}.$$

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 4000 \text{ kPa}$,

$$s_l = 2.7962 \text{ kJ/kgK}, s_{lg} = 3.2727 \text{ kJ/kgK}, s_g = 6.0689 \text{ kJ/kgK}.$$

$$h_l = 1087.2 \text{ kJ/kg}, h_{lg} = 1713.4 \text{ kJ/kg}, h_g = 2800.6 \text{ kJ/kg}$$

Here, $s_3 > s_g$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 4000 \text{ kPa}$ and specific entropy, $s_3 = 7.848244 \text{ kJ/kgK}$ to find the temperature, T_3 and specific enthalpy h_3 . As there is no value listed for $s_3 = 7.848244 \text{ kJ/kgK}$ in the table, linear interpolation method is used to find the temperature as

$$T_a = 750^\circ\text{C} \quad h_a = 4023.0 \text{ kJ/kg} \quad s_a = 7.7371 \text{ kJ/kgK}$$

$$T_b = ? \quad h_b = ? \quad s_b = 7.848244 \text{ kJ/kgK}$$

$$T_t = 800^\circ\text{C} \quad h_t = 4141.7 \text{ kJ/kg} \quad s_t = 7.8503 \text{ kJ/kgK}$$

Using linear interpolation method to find the minimum boiler exit temperature ,

$$T_3 = T_a + \frac{s_3 - s_a}{s_b - s_a} (T_b - T_a) = 750 + \frac{7.848244 - 7.7371}{7.8503 - 7.7371} (800 - 750)$$

$$= 799.092^\circ\text{C}$$

Similarly, again using interpolation method to find specific enthalpy as

$$h_3 = h_a + \frac{s_3 - s_a}{s_b - s_a} (h_b - h_a) = 4023.0 + \frac{7.848244 - 7.7371}{7.8503 - 7.7371} (4141.7 - 4023.0)$$

$$= 4139.544 \text{ kJ/kg}$$

Then, the efficiency of Rankine cycle is given by

$$\eta_r = \frac{(h_3 - h_t) - (h_2 - h_1)}{h_3 - h_2} = \frac{(4139.544 - 2488.15) - (195.86 - 191.83)}{4139.544 - 195.86}$$

$$= 0.4177 = 41.77\%$$

Now, the power output of the turbine is given by

$$\dot{W}_{out,T} = \dot{m} \times w_T$$

Hence, flow rate of steam,

$$\dot{m} = \frac{\dot{W}_{out,T}}{w_T} = \frac{\dot{W}_{out,T}}{\dot{w}_T} = \frac{\dot{W}_{out,T}}{(h_3 - h_4)} = \frac{80 \times 10^3}{(4139.544 - 2488.15)} = 48.444 \text{ kg/s}$$

Problem 7.17.

Saturated vapor enters into a turbine of an ideal Rankine cycle at 10 MPa and saturated liquid exits the condenser at 10 kPa. The power output of the cycle is 120 MW. Determine:

- (a) the thermal efficiency of the cycle,
- (b) the back work ratio,
- (c) the mass flow rate of steam,
- (d) the rate at which heat is supplied to the boiler,
- (e) the rate at which heat is rejected from the condenser, and
- (f) the mass flow rate of condenser cooling water, if the cooling water enters at 20°C and exits at 35°C .

[Take specific heat of water as 4.18 kJ/kgK].

Solution: Given,

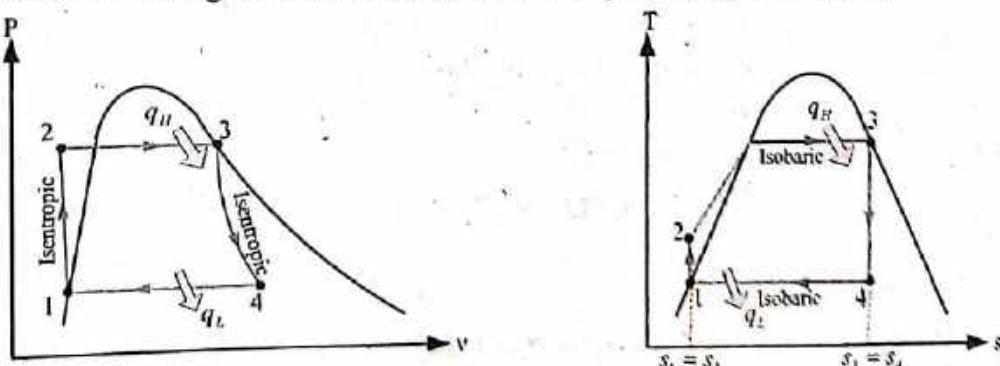
The boiler exit pressure is same the turbine inlet pressure and process 2 – 3 is an isobaric heat addition in the boiler, i.e. $P_2 = P_3 = 10 \text{ MPa} = 10000 \text{ kPa}$ (State 3 is given as saturated vapor)

As process 4 – 1 is an isobaric heat rejection in condenser, condenser exit pressure, $P_1 = P_4 = 10 \text{ kPa}$ (State 1 is given as saturated liquid)

Power output of cycle, $\dot{W}_{out} = 120 \text{ MW} = 120 \times 10^3 \text{ kW}$

Cooling water enters at $T_{in} = 20^\circ\text{C}$ and exits at $T_{out} = 35^\circ\text{C}$,

The $P-v$ and $T-s$ diagrams of an ideal Rankine cycle are given below.



Initial state 1: Pressure, $P_1 = 10 \text{ kPa}$ and water is in saturated liquid state.

Referring to the saturated water pressure table A 2.1 for pressure, $P_1 = 10 \text{ kPa}$, specific enthalpy, $h_1 = h_l (P_1 = 10 \text{ kPa}) = 191.83 \text{ kJ/kg}$ and specific volume, $v_1 = v_l (P_1 = 10 \text{ kPa}) = 0.001010 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 10000 \text{ kPa}$ and water is in subcooled or compressed liquid. Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for an incompressible substance (liquid water),

$$h_2 - h_1 = v_1(P_2 - P_1)$$

$$\Rightarrow h_2 = h_1 + v_1(P_2 - P_1) = 191.83 + 0.001010 (10000 - 10) = 201.92 \text{ kJ/kg}$$

State 3: Pressure, $P_3 = 10000 \text{ kPa}$, and saturated vapor state.

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 10000 \text{ kPa}$, specific enthalpy, $h_3 = h_g = 2724.5 \text{ kJ/kg}$ and specific entropy, $s_3 = s_g = 5.6139 \text{ kJ/kgK}$

State 4: Pressure, $P_4 = 10 \text{ kPa}$ and as the process 3 – 4 is the isentropic expansion, the specific entropy at state 4, $s_4 = s_3 = 5.6139 \text{ kJ/kgK}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_4 = 10 \text{ kPa}$,

$$s_l = 0.6493 \text{ kJ/kgK}, s_{lg} = 7.4989 \text{ kJ/kgK}, s_g = 8.1482 \text{ kJ/kgK}$$

$$h_l = 191.83 \text{ kJ/kg}, h_{lg} = 2392.0 \text{ kJ/kg}, h_g = 2583.8 \text{ kJ/kg}$$

Here, $s_l < s_4 < s_g$. Hence, it is a two phase mixture. Then,

$$\text{the quality at state 4, } x_4 = \frac{s_4 - s_l}{s_{lg}} = \frac{5.6193 - 0.6493}{7.4989} = 0.662$$

and the specific enthalpy at state 4,

$$h_4 = h_l + x_4 h_{lg} = 191.83 + 0.662 \times 2392.0 = 1775.334 \text{ kJ/kg}$$

Now, work output from the turbine,

$$w_T = h_3 - h_4 = 2724.5 - 1775.334 = 949.166 \text{ kJ/kg}$$

Work input to the pump,

$$w_P = h_2 - h_1 = 201.92 - 191.83 = 10.0899 \text{ kJ/kg}$$

Net work delivered to the surroundings,

$$w_{net} = w_T - w_P = 949.166 - 10.0899 = 939.076 \text{ kJ/kg}$$

Heat addition to the boiler during the process 2 – 3,

$$q_H = q_{23} = h_3 - h_2 = 2724.5 - 201.92 = 2522.58 \text{ kJ/kg}$$

Heat rejection from the condenser during the process 4 – 1,

$$q_L = q_{41} = h_4 - h_1 = 1775.334 - 191.83 = 1583.504 \text{ kJ/kg}$$

Thermal efficiency of Rankine cycle,

$$\eta_R = \frac{w_{net}}{q_H} = \frac{939.076}{2522.58} = 0.3723 = 37.237\%$$

Back work ratio is given by

$$\frac{W_P}{W_T} = \frac{w_P}{w_T} = \frac{10.0899}{949.166} = 0.01063 = 1.063\%$$

Power output of the cycle is defined as

$$\dot{W}_{\text{out}} = \dot{m} \times w_{\text{net}}$$

Hence, the mass flow rate of steam,

$$\dot{m} = \frac{\dot{W}_{\text{out}}}{w_{\text{net}}} = \frac{120 \times 10^3}{939.076} = 127.7852 \text{ kg/s}$$

Rate of heat supplied to the boiler,

$$\dot{Q}_B = \dot{m} \times q_B = 127.7852 \times 2522.58 = 322348.39 \text{ kW} = 322.3484 \text{ MW}$$

Rate of heat rejected from the condenser,

$$\dot{Q}_L = \dot{m} \times q_L = 127.7852 \times 1583.504 = 202348.37 \text{ kW} = 202.3484 \text{ MW}$$

Heat absorbed by the condenser cooling water is given by

$$q_{cw} = h_{in} - h_{out} = c(T_{in} - T_{out}) = 4.18 (35 - 20) = 62.7 \text{ kJ/kg}$$

Rate of heat absorbed by condenser cooling water is then calculated as

$$\dot{Q}_{cw} = \dot{m}_{cw} \times q_{cw}$$

which is equal to the rate of heat rejected by the condenser, i.e.

$$\dot{Q}_{cw} = \dot{m}_{cw} \times q_{cw} = \dot{Q}_L$$

Hence, the mass flow rate of the condenser cooling water,

$$\dot{m}_{cw} = \frac{\dot{Q}_L}{q_{cw}} = \frac{202.3484 \times 10^3}{62.7} = 3227.247 \text{ kg/s}$$

Problem 7.18.

Superheated steam at 8 MPa, 500°C enters into turbine of a steam power plant working on a Rankine cycle. The steam leaves the condenser as saturated liquid at 8 kPa. The turbine and pump have isentropic efficiencies of 90% and 80% respectively. For the cycle, determine:

- (a) the net work per kg of steam,
- (b) the heat supplied into the boiler per kg of steam, and
- (c) the thermal efficiency.

Solution: Given,

The boiler exit pressure is same the turbine inlet pressure and process 2 – 3 is an isobaric heat addition in the boiler, i.e. $P_2 = P_3 = 8 \text{ MPa} = 8000 \text{ kPa}$ (State 3 is given as superheated steam)

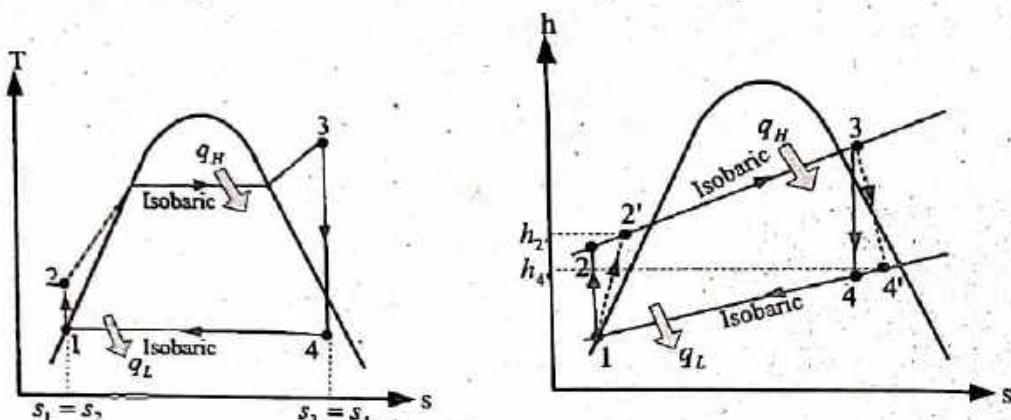
As process 4 – 1 is an isobaric heat rejection in condenser, condenser exit pressure, $P_1 = P_4 = 8 \text{ kPa}$ (State 1 is given as saturated liquid)

Temperature of steam at inlet of turbine, $T_3 = 500^\circ\text{C}$

Isentropic efficiency of turbine, $\eta_{isen,T} = 90\% = 0.9$

Isentropic efficiency of pump, $\eta_{isen,P} = 80\% = 0.8$

The $T-s$ and $h-s$ diagrams of an ideal Rankine cycle are given below.



Initial state 1: Pressure, $P_1 = 8 \text{ kPa}$ and water is in saturated liquid state.

Referring to the saturated water pressure table A 2.1 for pressure, $P_1 = 8 \text{ kPa}$, specific enthalpy, $h_1 = h_f (P_1 = 8 \text{ kPa}) = 173.85 \text{ kJ/kg}$ and specific volume, $v_1 = v_f (P_1 = 8 \text{ kPa}) = 0.001008 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 8000 \text{ kPa}$ and water is in subcooled or compressed liquid. Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for an incompressible substance (liquid water),

$$h_2 - h_1 = v_1(P_2 - P_1)$$

$$\Rightarrow h_2 = h_1 + v_1(P_2 - P_1) = 173.85 + 0.001008(8000 - 8) = 181.906 \text{ kJ/kg}$$

State 3: Pressure, $P_3 = 8000 \text{ kPa}$, temperature, $T_3 = 500^\circ\text{C}$ and superheated vapor state.

Referring to the superheated vapor table A2.4 for pressure, $P_3 = 8000 \text{ kPa}$ and temperature, $T_3 = 500^\circ\text{C}$, specific enthalpy, $h_3 = 3398.5 \text{ kJ/kg}$ and specific entropy, $s_3 = 6.7243 \text{ kJ/kgK}$.

State 4: Pressure, $P_4 = 8 \text{ kPa}$ and as the process 3 – 4 is the isentropic expansion, the specific entropy at state 4, $s_4 = s_3 = 6.7243 \text{ kJ/kgK}$.

Referring to saturated water pressure table A2.1 for pressure, $P_4 = 8 \text{ kPa}$,

$$s_1 = 0.5925 \text{ kJ/kgK}, s_{lg} = 7.6342 \text{ kJ/kgK}, s_g = 8.2267 \text{ kJ/kgK}$$

$$h_1 = 173.85 \text{ kJ/kg}, h_{lg} = 2402.3 \text{ kJ/kg}, h_g = 2576.1 \text{ kJ/kg}$$

Here, $s_1 < s_4 < s_g$. Hence, it is a two phase mixture. Then,

$$\text{the quality at state 4, } x_4 = \frac{s_4 - s_1}{s_{lg}} = \frac{6.7243 - 0.5925}{7.6342} = 0.8032$$

and the specific enthalpy at state 4,

$$h_4 = h_1 + x_4 h_{lg} = 173.85 + 0.8032 \times 2402.3 = 2103.377 \text{ kJ/kg}$$

Now, isentropic work output from the turbine,

$$W_{isen,T} = h_3 - h_4 = 3398.5 - 2103.377 = 1295.123 \text{ kJ/kg}$$

Isentropic work input to the pump,

$$W_{isen,P} = h_2 - h_1 = 181.906 - 173.85 = 8.056 \text{ kJ/kg}$$

Isentropic efficiency of the turbine is defined as

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$$\eta_{isen,T} = \frac{W_{real,T}}{W_{isen,T}} = \frac{w_{real,T}}{w_{isen,T}}$$

Hence, real work output from the turbine,

$$w_{real,T} = \eta_{real,T} \times w_{isen,T} = 0.9 \times 1295.123 = 1165.611 \text{ kJ/kg}$$

Similarly, isentropic efficiency of the pump is defined as

$$\eta_{isen,P} = \frac{W_{isen,P}}{W_{real,P}} = \frac{w_{isen,P}}{w_{real,P}}$$

Hence, real work input to the pump,

$$w_{real,P} = \frac{w_{isen,P}}{\eta_{isen,P}} = \frac{8.056}{0.8} = 10.07 \text{ kJ/kg}$$

Real net work per kg of steam,

$$w_{net,real} = w_{real,T} - w_{real,P} = 1165.611 - 10.07 = 1155.541 \text{ kJ/kg}$$

Real work input to the pump can also be determined as

$$w_{real,P} = h_2 - h_1$$

where h_2 is the real specific enthalpy of the steam at the pump exit or at the boiler inlet when the pumping process is not isentropic (or real).

Then, real specific enthalpy at the inlet of the boiler,

$$h_2 = h_1 + w_{real,P} = 173.85 + 10.07 = 183.92 \text{ kJ/kg}$$

So, the real heat supplied into the boiler per kg of steam is calculated as

$$q_{H,real} = h_3 - h_2 = 3398.5 - 183.92 = 3214.58 \text{ kJ/kg}$$

Now, the real thermal efficiency of Rankine cycle is given by

$$\eta_{R,real} = \frac{w_{real,net}}{q_{H,real}} = \frac{1155.541}{3214.58} = 0.3595 = 35.95\%$$

7.3 Solved Numerical Problems from PoU Examinations

Problem 7.19.

The compression ratio in an air standard Otto cycle is 8. At the beginning of the compression stroke, the pressure is 0.1 MPa and the temperature is 15°C. The heat transfer to the air per cycle is 1800 kJ/kg of air. Determine:

- the pressure and temperature at the end of each process of the cycle,
- the thermal efficiency, and
- the mean effective pressure. [$R = 287 \text{ J/kgK}$, $c_v = 718 \text{ J/kgK}$]

(PoU 2004 spring, 2008 fall, 2010 fall, 2011 fall, 2013 fall).

Solution: Given,

Compression ratio, $r_c = V_1/V_2 = 8$

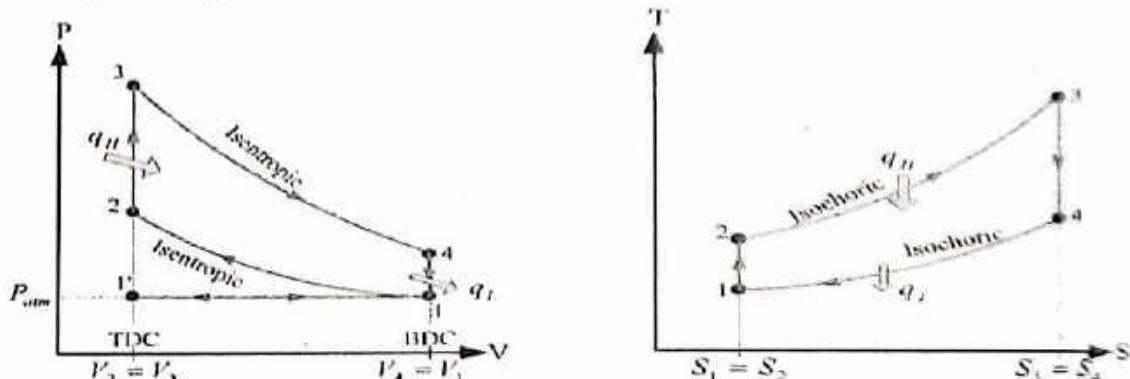
At the beginning of compression stroke:

pressure, $P_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$

temperature, $T_1 = 15^\circ\text{C} = 15 + 273 = 288 \text{ K}$

Heat transfer to air per cycle, $q_H = 1800 \text{ kJ/kg}$

For the calculation reference, the $P - V$ and $T - S$ diagrams for an air standard Otto cycle are given in below.



Process 1 – 2 is the isentropic compression, hence applying pressure – volume and temperature – volume relations for an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^r \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^r = 100(8)^{1.4} = 1837.917 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{r-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1} = 288(8)^{1.4-1} = 661.65 \text{ K}$$

Heat added during the process 2 - 3 is given by

$$q_H = q_{23} = c_v(T_3 - T_2)$$

$$\Rightarrow T_3 = T_2 + \frac{q_H}{c_v} = 661.65 + \frac{1800 \times 10^3}{718} = 3168.614 \text{ K}$$

Process 2 – 3 is an isochoric heat addition process, so using Charles law for an ideal gas,

$$\frac{T_3}{P_2} = \frac{T_1}{P_1} \Rightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{3168.614}{661.65} \times 1837.917 = 8801.707 \text{ kPa}$$

Similarly, process 3 – 4 is the isentropic expansion, hence applying pressure – volume and temperature – volume relations for an isentropic process,

$$\frac{P_3}{P_4} = \left(\frac{V_3}{V_4} \right)^{\gamma-1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow P_4 = P_3 \left(\frac{1}{r_e} \right)^{\gamma-1} = 8801.707 \left(\frac{1}{8} \right)^{1.4-1} = 478.896 \text{ kPa}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma-1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow T_4 = T_3 \left(\frac{1}{r_e} \right)^{\gamma-1} = 3168.614 \left(\frac{1}{8} \right)^{1.4-1} = 1379.219 \text{ K}$$

The efficiency of the cycle is given by

$$\eta_{\text{c}} = 1 - \left(\frac{1}{r_e} \right)^{\gamma-1} = 1 - \left(\frac{1}{8} \right)^{1.4-1} = 0.5647 = 56.47\%$$

Work output per kg of air is determined as

$$w_{\text{out}} = \eta_c \times q_{\text{in}} = 0.5647 \times 1800 = 1016.46 \text{ kJ/kg}$$

Applying equation of state to find the specific volume of air at state 1,

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 288}{100 \times 10^3} = 0.82656 \text{ m}^3/\text{kg}$$

Applying equation of state to find the specific volume of air at state 2,

$$P_2 v_2 = RT_2 \Rightarrow v_2 = \frac{RT_2}{P_2} = \frac{287 \times 661.65}{1837.917 \times 10^3} = 0.10332 \text{ m}^3/\text{kg}$$

Now, the mean effective pressure,

$$MEP = \frac{W_{\text{out}}}{V_{\text{mean}}} = \frac{W_{\text{out}}}{v_1 - v_2} = \frac{1016.46}{0.82656 - 0.10332} = 1405.426 \text{ kPa}$$

Problem 7.20.

Calculate the efficiency and work output of a simple gas turbine plant operating on Brayton cycle. The minimum and maximum temperatures of the cycle are 288 K and 1000 K respectively. The pressure ratio is 6. The isentropic efficiencies of compressor and turbine are 85% and 95% respectively.

(PoU 2006 spring).

Solution: Given,

Minimum temperature of cycle is at inlet of compressor, so

minimum temperature at state 1, $T_1 = 288 \text{ K}$

Maximum temperature of cycle is attained after heat addition, so

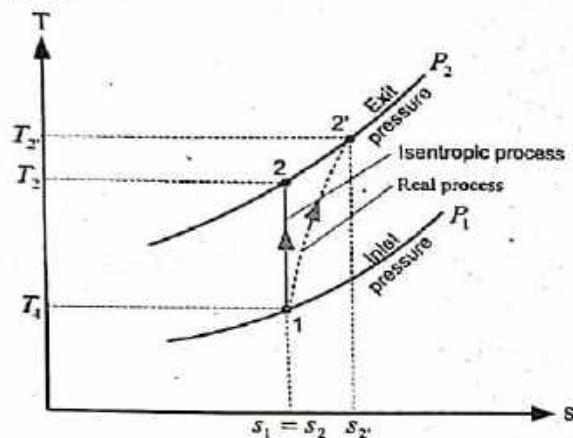
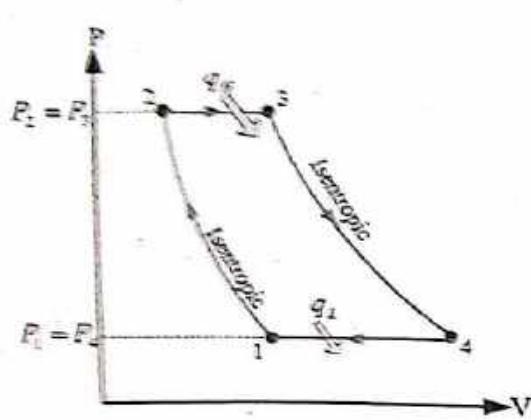
maximum temperature at state 3, $T_3 = 1000 \text{ K}$

$$\text{Pressure ratio, } r_p = \frac{P_2}{P_1} = 6$$

Isentropic efficiency of compressor, $\eta_{\text{isen}, C} = 85\% = 0.85$

Isentropic efficiency of turbine, $\eta_{\text{isen}, T} = 95\% = 0.95$

The $P - V$ diagram and $T - S$ diagram with the real and isentropic compressors of a gas turbine (Brayton) cycle are given below.



Process 1 – 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 288 \times (6)^{\frac{1.4-1}{1.4}} = 480.531 \text{ K}$$

Process 3 – 4 is an isentropic expansion process, so

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = T_3 \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1000 \times \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = 599.337 \text{ K}$$

Isentropic (ideal) compressor work per kg of air (specific work),

$$w_{C,\text{isen}} = h_2 - h_1 = c_p(T_2 - T_1) = 1005(480.531 - 288) = 193.494 \text{ kJ/kg}$$

Isentropic (ideal) turbine work per kg of air (specific work)

$$w_{T,\text{isen}} = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1000 - 599.337) = 402.666 \text{ kJ/kg}$$

Isentropic thermal efficiency of the compressor can be defined as

$$\eta_{C,\text{real}} = \frac{w_{C,\text{real}}}{w_{C,\text{isen}}}$$

Hence, real compressor work per kg of air (specific work),

$$w_{C,\text{real}} = \frac{w_{C,\text{isen}}}{\eta_{C,\text{real}}} = \frac{193.494}{0.85} = 227.64 \text{ kJ/kg}$$

Similarly, isentropic thermal efficiency of the turbine can be defined as

$$\eta_{T,\text{real}} = \frac{w_{T,\text{real}}}{w_{T,\text{isen}}}$$

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Hence, real turbine work per kg of air (specific work),

$$w_{T,real} = \eta_{T,isen} \times w_{T,isen} = 0.95 \times 402.666 = 382.533 \text{ kJ/kg}$$

Real net work per kg of air (net specific work),

$$w_{net,real} = w_{T,real} - w_{C,real} = 382.533 - 227.64 = 154.893 \text{ kJ/kg}$$

Heat added to the real cycle per kg of air (specific heat),

$$q_{H,real} = q_{2'3} = h_3 - h_2 = c_p(T_3 - T_2')$$

where T_2' is the temperature at state 2 when the compressor works with frictional loses, heat loses etc. Hence, real compressor work per kg of air is also calculated as the function of T_2' as

$$w_{C,real} = h_2' - h_1 = c_p(T_2' - T_1)$$

$$T_2' = T_1 + \frac{w_{C,real}}{c_p} = 288 + \frac{227.64 \times 10^3}{1005} = 514.507 \text{ K}$$

$$\text{Then, } q_{H,real} = c_p(T_3 - T_2') = 1005(1000 - 514.507) = 487.92 \text{ kJ/kg}$$

Now, real thermal efficiency of the cycle is given by

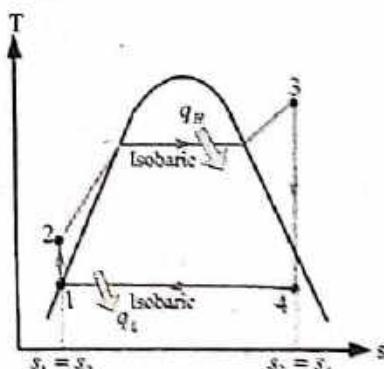
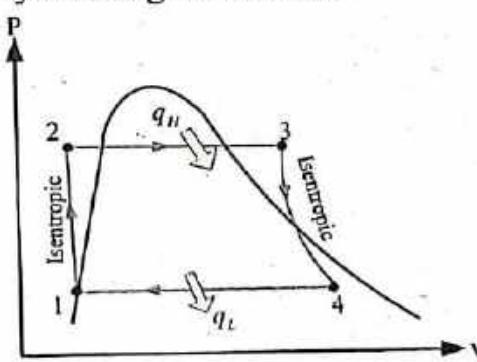
$$\eta_{B,real} = \frac{w_{net,real}}{q_{H,real}} = \frac{154.893}{487.92} = 0.31745 = 31.745\%$$

Problem 7.21.

Consider a steam power plant operating on the simple ideal Rankine cycle. The steam enters the turbine at 3 MPa and 350 °C and is condensed in a condenser at a pressure 80 kPa. Determine the thermal efficiency of the cycle.

(PoU 2007 spring).

Solution: For the calculation reference, the $P-v$ and $T-s$ diagrams of an ideal Rankine cycle are given below.



Given,

Turbine inlet pressure of steam, $P_3 = P_2 = 3 \text{ MPa} = 3000 \text{ kPa}$

(As process 2 – 3 is an isobaric heat addition in boiler)

Turbine exit pressure of steam, $P_4 = P_1 = 80 \text{ kPa}$

(As process 4 – 1 is an isobaric heat rejection from condenser)

Temperature of steam at turbine inlet, $T_3 = 350^\circ\text{C} = 350 + 273 = 623 \text{ K}$
 The thermal efficiency of Rankine cycle is:

$$\eta_R = \frac{W_{net}}{q_H}$$

where $w_{net} = w_T - w_P = (h_3 - h_4) - (h_2 - h_1)$ and $q_H = h_3 - h_2$

Determination of values of specific enthalpy at all states of the cycle:

Initial state 1: Pressure, $P_1 = 80 \text{ kPa}$ and water is in saturated liquid state.

Referring to the saturated water pressure table A 2.1 for pressure, $P_1 = 80 \text{ kPa}$, specific enthalpy, $h_1 = h_f(P_1 = 80 \text{ kPa}) = 391.71 \text{ kJ/kg}$ and specific volume, $v_1 = v_f(P_1 = 80 \text{ kPa}) = 0.001038 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 3000 \text{ kPa}$ and water is in sub-cooled or compressed liquid. Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for an incompressible substance (liquid water),

$$h_2 - h_1 = v_1(P_2 - P_1) \\ \Rightarrow h_2 = h_1 + v_1(P_2 - P_1) = 391.71 + 0.001038(3000 - 80) = 394.741 \text{ kJ/kg}$$

State 3: Pressure, $P_3 = 3000 \text{ kPa}$, and temperature, $T_3 = 350^\circ\text{C}$

Referring to the saturated water pressure table A2.1 for pressure, $P_3 = 3000 \text{ kPa}$, saturation temperature, $T_{sat}(P_3 = 3000 \text{ kPa}) = 233.89^\circ\text{C}$.

Here, $T_3 > T_{sat}$. Hence, it is a superheated vapor.

Now, referring to the superheated vapor table A2.4 for pressure, $P_3 = 3000 \text{ kPa}$, and temperature, $T_3 = 350^\circ\text{C}$,

specific enthalpy, $h_3 = 3114.8 \text{ kJ/kg}$ and specific entropy, $s_3 = 6.7420 \text{ kJ/kgK}$

State 4: Pressure, $P_4 = 80 \text{ kPa}$ and as the process 3 – 4 is isentropic expansion, the specific entropy at state 4, $s_4 = s_3 = 6.7420 \text{ kJ/kgK}$.

Referring to the saturated water pressure table A2.1 for pressure, $P_4 = 80 \text{ kPa}$,

$$s_l = 1.2330 \text{ kJ/kgK}, s_{lg} = 6.2009 \text{ kJ/kgK}, s_g = 7.4339 \text{ kJ/kgK}$$

$$h_l = 391.71 \text{ kJ/kg}, h_{lg} = 2273.6 \text{ kJ/kg}, h_g = 2665.3 \text{ kJ/kg}$$

Here, $s_l < s_4 < s_g$. Hence, it is a two phase mixture. Then,

$$\text{the quality at state 4, } x_4 = \frac{s_4 - s_l}{s_{lg}} = \frac{6.7420 - 1.2330}{6.2009} = 0.88842 \text{ and}$$

the specific enthalpy at state 4,

$$h_4 = h_l + x_4 h_{lg} = 391.71 + 0.88842 \times 2273.6 = 2411.62 \text{ kJ/kg}$$

Now, work output per kg of steam from the turbine,

$$w_T = h_3 - h_4 = 3114.8 - 2411.62 = 703.18 \text{ kJ/kg}$$

Work input per kg of steam to the pump,

$$w_P = h_2 - h_1 = 394.741 - 391.71 = 3.031 \text{ kJ/kg}$$

Net work per kg of steam delivered to the surroundings,

$$w_{net} = w_T - w_P = 703.18 - 3.031 = 700.15 \text{ kJ/kg}$$

Heat addition to the boiler during the process 2 – 3,

$$q_H = q_{23} = h_3 - h_2 = 3114.8 - 394.741 = 2720.06 \text{ kJ/kg}$$

Then, the thermal efficiency of the Rankine cycle,

$$\eta_R = \frac{w_{net}}{q_H} = \frac{700.15}{2720.06} = 0.2574 = 25.74\%$$

Problem 7.22.

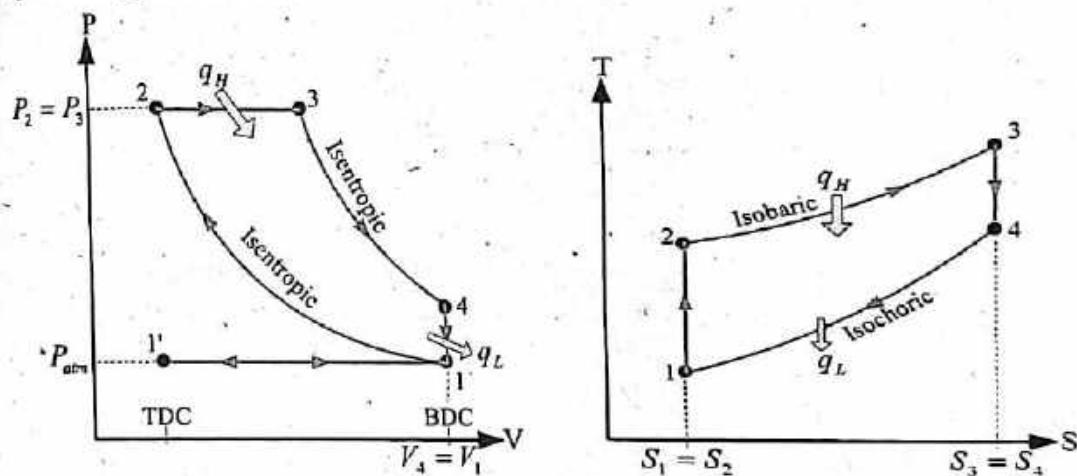
An air standard Diesel cycle has compression ratio of 18, and the heat transfer to the working fluid is 1800 kJ/kg. At the beginning of the compression process, the pressure is 0.1 MPa and temperature is 15 °C. Determine

- (i) the thermal efficiency
- (ii) the mean effective pressure.

(PoU 2007 fall, PoU 2010 spring, PU 2013).

Solution:

For the calculation reference, the $P = V$ and $T - S$ diagrams of an ideal Diesel cycle are given below.



Given,

Compression ratio, $r_c = V_1/V_2 = 18$

Heat transferred to working fluid, $q_H = 1800 \text{ kJ/kg}$

Pressure at the beginning of compression stroke, $P_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$

Temperature at the beginning of compression stroke, $T_1 = 15^\circ\text{C}$

$$= 15 + 273 = 288 \text{ K}$$

For the isentropic compression process 1 – 2, temperature – volume relation is given by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{r-1} = (r_c)^{r-1} \Rightarrow T_2 = T_1(r_c)^{r-1} = 288(18)^{1.4-1} = 915.17 \text{ K}$$

From the heat transferred to the fluid during the process 2 – 3, maximum temperature at state 3, T_3 is calculated as

$$q_H = c_p(T_3 - T_2)$$

$$\Rightarrow T_3 = T_2 + \frac{q_H}{c_p} = 915.17 + \frac{1800 \times 1000}{1005} = 2706.215 \text{ K}$$

Using Charles law for an ideal gas during an isobaric process 2 – 3 to find the value of cut-off ratio (α), we have

$$\frac{T_2}{V_2} = \frac{T_3}{V_3} \Rightarrow \alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2706.215}{915.17} = 2.957$$

Then, the thermal efficiency of the Diesel cycle,

$$\eta_D = 1 - \frac{1}{r} \left(\frac{1}{r_c} \right)^{r-1} \left[\frac{\alpha^r - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{18} \right)^{1.4-1} \left[\frac{2.957^{1.4} - 1}{2.957 - 1} \right] = 0.5908 = 59.08\%$$

Net work output per kg of air during the cycle,

$$w_{net} = \eta_D \times q_H = 0.5908 \times 1800 = 1063.44 \text{ kJ/kg}$$

Applying equation of state to find the specific volume of air at state 1,

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 288}{100 \times 10^3} = 0.82656 \text{ m}^3/\text{kg}$$

Using compression ratio to find the specific volume of air at state 2,

$$r_c = \frac{V_1}{V_2} = \frac{v_1}{v_2} \Rightarrow v_2 = \frac{v_1}{r_c} = \frac{0.82656}{18} = 0.04592 \text{ m}^3/\text{kg}$$

Now, the mean effective pressure,

$$MEP = \frac{W_{net}}{V_{stroke}} = \frac{W_{net}}{V_1 - V_2} = \frac{w_{net}}{v_1 - v_2} = \frac{1063.44}{0.82656 - 0.04592} = 1362.267 \text{ kPa}$$

Problem 7.23.

The compression ratio in air standard Otto cycle is 7. At the beginning of the compression stroke, pressure is 0.1 MPa and temperature is 20 °C. The heat transferred to the air per cycle is 1920 kJ/kg. Determine.

- (i) the pressure and temperature at the end of each process of cycle
 - (ii) the thermal efficiency.
- (PoU 2008 spring).

Solution: Given,

Compression ratio, $r_c = V_1/V_2 = 7$

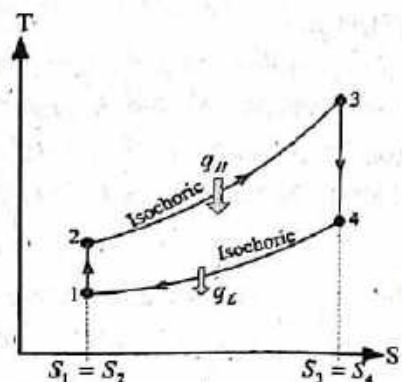
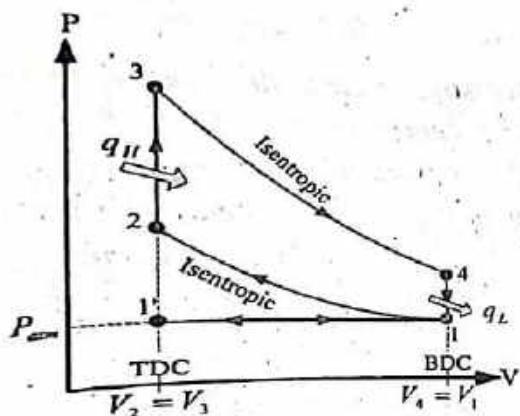
At the beginning of compression stroke:

pressure, $P_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$

temperature, $T_1 = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$

Heat transfer to air per cycle, $q_H = 1920 \text{ kJ/kg}$

For the calculation reference, the $P - V$ and $T - S$ diagrams for an air standard Otto cycle are given in below.



Process 1 - 2 is the isentropic compression, hence applying pressure - volume and temperature - volume relations for an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^r \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^r = 100(7)^{1.4} = 1524.53 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{r-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1} = 293(7)^{1.4-1} = 638.126 \text{ K}$$

Heat added during the process 2 - 3 is given by

$$q_H = q_{23} = c_v(T_3 - T_2)$$

$$\Rightarrow T_3 = T_2 + \frac{q_H}{c_v} = 638.126 + \frac{1920 \times 10^3}{718} = 3312.221 \text{ K}$$

Process 2 - 3 is an isochoric heat addition process, so using Charles law for an ideal gas,

$$\frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow P_3 = \frac{T_3}{T_2} P_2 = \frac{3312.221}{638.126} \times 1524.53 = 7913.14 \text{ kPa}$$

Similarly, process 3 - 4 is the isentropic expansion, hence applying pressure - volume and temperature - volume relations for an isentropic process,

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4} \right)^r = \left(\frac{V_2}{V_1} \right)^r \Rightarrow P_4 = P_3 \left(\frac{1}{r_c} \right)^r = 7913.14 \left(\frac{1}{7} \right)^{1.4} = 519.053 \text{ kPa}$$

$$\frac{T_4}{P_4} = \left(\frac{V_3}{V_4} \right)^{r-1} = \left(\frac{V_2}{V_1} \right)^{r-1} \Rightarrow T_4 = T_3 \left(\frac{1}{r_c} \right)^{r-1} = 3312.221 \left(\frac{1}{7} \right)^{1.4-1} = 1520.828 \text{ K}$$

The efficiency of the cycle is given by

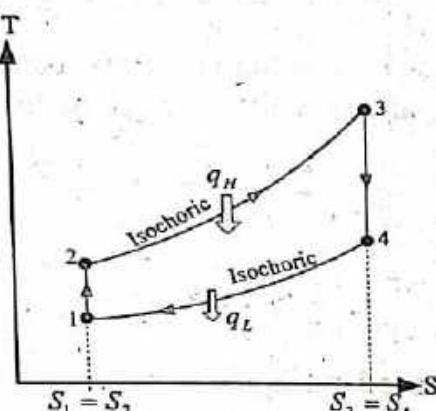
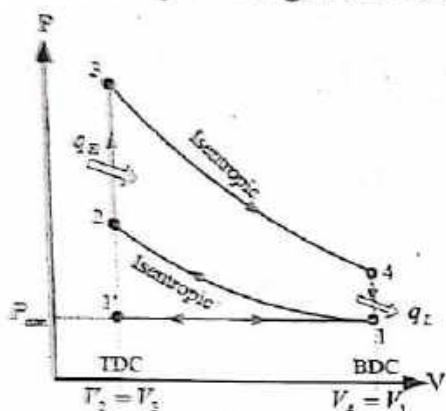
$$\begin{aligned} \eta_O &= 1 - \left(\frac{1}{r_c} \right)^{r-1} = 1 - \left(\frac{1}{7} \right)^{1.4-1} \\ &= 0.540843 = 54.08\% \end{aligned}$$

Problem 7.24.

In an Otto cycle engine, the temperature and pressure of air at the beginning of the compression are 300 K and 101 kPa. The compression ratio is 8. The maximum pressure in the cycle is 8000 kPa. Determine the air standard efficiency and heat transferred to the air during constant volume heat addition.

(PoU 2009 spring).

Solution: For the calculation reference, the $P - V$ and $T - S$ diagrams for an air standard Otto cycle are given below.



Given,

At the beginning of compression stroke:

pressure, $P_1 = 101 \text{ kPa}$

temperature, $T_1 = 300 \text{ K}$

Compression ratio, $r_c = V_1/V_2 = 8$

Maximum pressure during the cycle, $P_3 = 8000 \text{ kPa}$

The efficiency of the cycle is given by

$$\eta_O = 1 - \left(\frac{1}{r_c} \right)^{\gamma-1} = 1 - \left(\frac{1}{8} \right)^{1.4-1} = 0.5647 = 56.47\%$$

Process 1 – 2 is the isentropic compression, hence applying pressure – volume and temperature – volume relations for an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^\gamma \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 101(8)^{1.4} = 1856.296 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 300(8)^{1.4-1} = 689.22 \text{ K}$$

Process 2 – 3 is an isochoric heat addition process, so using Charles law for an ideal gas,

$$\frac{T_3}{P_2} = \frac{T_2}{P_1} \Rightarrow T_3 = \frac{P_3}{P_1} T_2 = \frac{8000}{1856.296} \times 689.22 = 2970.298 \text{ K}$$

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Now, the heat transferred to the air during constant volume process 2 - 3 can be calculated as

$$q_H = c_v(T_3 - T_2) = 718 (2970.298 - 689.22) = 1637.814 \text{ kJ/kg.}$$

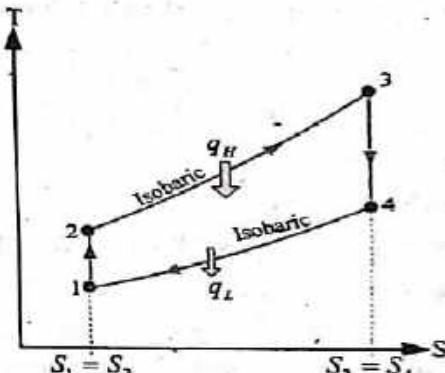
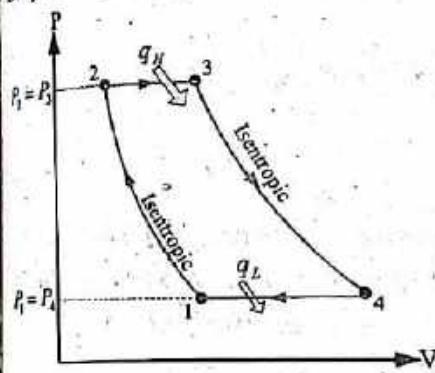
Problem 7.25.

A hot air engine works on Brayton cycle with initial and final pressure as 1 bar and 3 bar respectively. If temperature before isentropic compression and isentropic expansion are 300 K and 900 K, determine

- (i) heat supplied per kg of air
- (ii) work done per kg of air
- (iii) efficiency of the engine.

(PoU 2009 fall).

Solution: For the calculation reference, the $P - V$ and $T - S$ diagrams of an ideal Brayton cycle or gas turbine cycle are given below.



Given,

Initial pressure, $P_1 = P_4 = 1 \text{ bar} = 100 \text{ kPa}$

Final pressure, $P_2 = P_3 = 3 \text{ bar} = 300 \text{ kPa}$

Temperature before isentropic compression, $T_1 = 300 \text{ K}$

Temperature before isentropic expansion, $T_3 = 900 \text{ K}$

Process 2 - 3 is an isobaric heat addition process, so $P_2 = P_3$

Pressure ratio, $r_p = P_2/P_1 = 300/100 = 3$

Process 4 - 1 is an isobaric heat rejection process, so $P_4 = P_1$

Process 1 - 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = 300 \times (3)^{\frac{1+1}{1-1}} = 410.62 \text{ K}$$

Process 3 - 4 is an isentropic expansion process, so

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} = \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}}$$

$$\Rightarrow T_4 = T_3 \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} = 900 \times \left(\frac{1}{3} \right)^{\frac{1+1}{1-1}} = 657.54 \text{ K}$$

Heat supplied per kg of air (specific heat addition),

$$q_H = h_3 - h_2 = c_p(T_3 - T_2) = 1005 (900 - 410.62) = 491.827 \text{ kJ/kg}$$

Compressor work per kg of air (specific work),

$$w_C = h_2 - h_1 = c_p(T_2 - T_1) = 1005 (410.62 - 300) = 111.173 \text{ kJ/kg}$$

Turbine work output per kg of air (specific work output),

$$w_T = h_3 - h_4 = c_p(T_3 - T_4) = 1005 (900 - 657.54) = 243.672 \text{ kJ/kg}$$

Net work output per kg of air of the cycle (net specific work output),

$$w_{net} = w_T - w_C = 243.672 - 111.173 = 132.499 \text{ kJ/kg}$$

$$\text{Efficiency of the engine, } \eta_B = \frac{w_{net}}{q_H} = \frac{132.499}{491.827} = 0.2694 = 26.94\%$$

Alternately,

Efficiency of an ideal Brayton cycle is given by

$$\eta_B = 1 - \left(\frac{1}{r_p} \right)^{\frac{r-1}{r}} = 1 - \left(\frac{1}{3} \right)^{\frac{1.4-1}{1.4}} = 0.2694 = 26.94\%$$

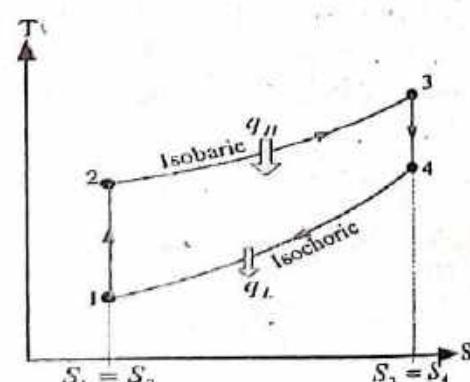
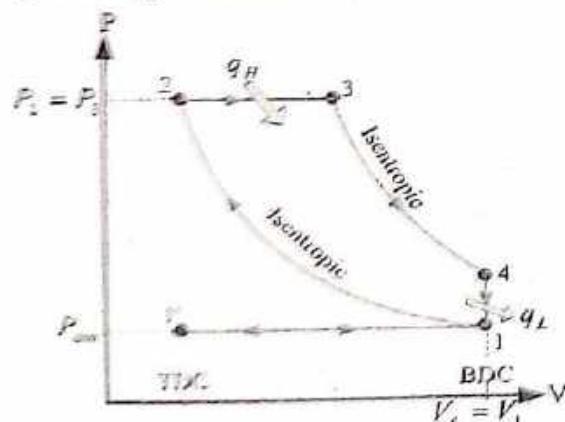
Problem 7.26.

An engine works on air standard Diesel cycle whose compression ratio is 14. The pressure and temperature at the beginning of the cycle are 1 bar and 300 K respectively. The maximum temperature of the cycle is limited to 2500°C. Determine the thermal efficiency of the cycle.

(PoU 2010 fall, 2018 spring, 2019 fall).

Solution:

For the calculation reference, the $P = V$ and $T - S$ diagrams of an ideal Diesel cycle are given below.



Given,

Compression ratio, $r_c = V_1/V_2 = 14$

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Pressure at beginning of cycle, $P_1 = 1 \text{ bar} = 100 \text{ kPa}$

Temperature at beginning of cycle, $T_1 = 300 \text{ K}$

Maximum temperature of cycle, $T_{max} = T_3 = 2500^\circ\text{C} = 2500 + 273 = 2773 \text{ K}$

For the isentropic process 1 - 2, temperature - volume relationship is given by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r_c)^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 (r_c)^{\gamma-1} = 300 (14)^{1.4-1} = 862.13 \text{ K} \quad (\text{As } r_c = \frac{V_1}{V_2})$$

Process 2 - 3 is an isobaric process, so using Charles law for ideal gas to find the value of cut-off ratio as

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} \Rightarrow \alpha = T_3/T_2 = 2773/862.13 = 3.216$$

The efficiency of the Diesel cycle,

$$\begin{aligned} \eta_D &= 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^\gamma - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{14} \right)^{1.4-1} \left[\frac{3.216^{1.4} - 1}{3.216 - 1} \right] \\ &= 0.5366 = 53.66\% \end{aligned}$$

Problem 7.27.

An air standard diesel cycle has a compression ratio of 22 and expansion ratio of 11. Determine its cut off ratio and the efficiency. (PoU 1012 fall).

Solution: Given,

Compression ratio, $r_c = V_1/V_2 = 22$

Expansion ratio, $r_e = V_4/V_3 = 11$

Process 4 - 1 is an isochoric heat rejection, so $V_4 = V_1$.

$$\text{Cut-off ratio is defined as: } \alpha = \frac{V_3}{V_2} = \frac{V_3}{V_1} \times \frac{V_1}{V_2} = \frac{V_3}{V_4} \times \frac{V_1}{V_2} = \frac{r_c}{r_e} = \frac{22}{11} = 2$$

Then, efficiency of the Diesel cycle,

$$\begin{aligned} \eta_D &= 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^\gamma - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{22} \right)^{1.4-1} \left[\frac{2^{1.4} - 1}{2 - 1} \right] \\ &= 0.6599 = 65.99\% \end{aligned}$$

Problem 7.28.

An ideal gas turbine cycle produces 15 MW of power output. The properties of air at the compressor inlet are 100 kPa and 17°C . The pressure ratio for the cycle is 15 and the heat added per kg of air per cycle is 900 kJ/kg. Determine

- (a) the efficiency of the cycle,
- (b) the maximum temperature in the cycle, and
- (c) the mass flow rate of air.

(PoU 2013 spring).

Solution: Given,

At inlet of compressor of the cycle,

pressure, $P_1 = 100 \text{ kPa}$

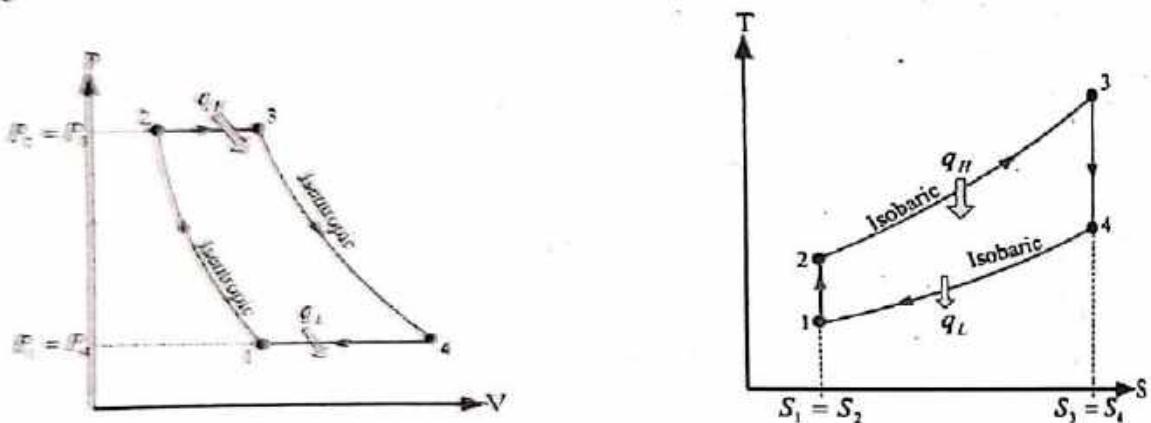
temperature, $T_1 = 17^\circ\text{C} = 17 + 273 = 290 \text{ K}$

Power output, $\dot{W}_t = 15 \text{ MW} = 15 \times 10^6 \text{ W}$

$$\text{Pressure ratio, } r_p = \frac{P_2}{P_1} = 15$$

Heat added per kg of air, $q_H = 900 \text{ kJ/kg} = 900 \times 10^3 \text{ J/kg}$

It is an ideal gas turbine cycle or Brayton cycle. The $P - V$ and $T - S$ diagrams are given below.



Efficiency of the cycle is given by,

$$\eta_t = 1 - \left(\frac{1}{r_p} \right)^{\frac{r-1}{r}} = 1 - \left(\frac{1}{15} \right)^{\frac{1.4-1}{1.4}} = 0.53869 = 53.87\%$$

Process 1 - 2 is an isentropic compression, so

$$\frac{T_2}{T_1} = \left(r_p \right)^{\frac{r-1}{r}} \Rightarrow T_2 = T_1 \left(r_p \right)^{\frac{r-1}{r}} = 290 \times (15)^{\frac{1.4-1}{1.4}} = 628.65 \text{ K} = 355.65^\circ\text{C}$$

From process 2 - 3 heat added per kg of air is given by

$$q_H = q_M = h_3 - h_2 = c_p (T_3 - T_2)$$

$$\Rightarrow 900 \times 10^3 = 1005 (T_3 - 628.65)$$

$$\therefore T_3 = 1524.17 \text{ K} = 1251.17^\circ\text{C}$$

Process 3 - 4 is an isentropic expansion, so

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} \Rightarrow T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} = 1524.17 \times \left(\frac{1}{15} \right)^{\frac{1.4-1}{1.4}}$$

$$\Rightarrow T_4 = 1524.17 \times \left(\frac{1}{15} \right)^{\frac{1.4-1}{1.4}} = 703.1114 \text{ K} = 430.1114^\circ\text{C}$$

Work output from turbine per kg of air,

$$w_T = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1524.17 - 703.11) = 825.164 \text{ kJ/kg}$$

Work input to compressor per kg of air,

$$w_C = h_2 - h_1 = c_p(T_2 - T_1) = 1005(628.65 - 290) = 340.343 \text{ kJ/kg}$$

Net work output per kg of air of the cycle,

$$w_{net} = w_T - w_C = 825.164 - 340.343 = 484.821 \text{ kJ/kg}$$

Power output of the cycle can be expressed as

$$\dot{W}_{net} = \dot{m} \times w_{net}$$

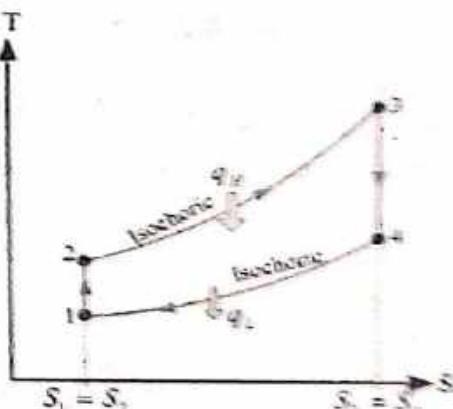
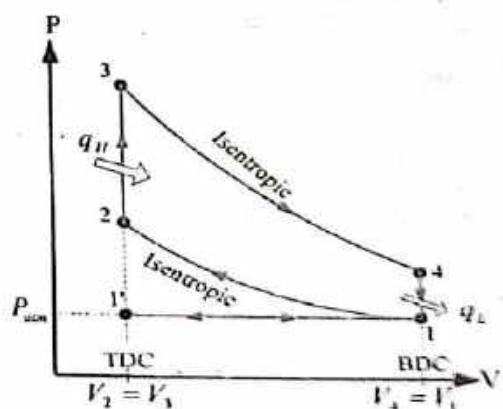
Now, the mass flow rate of air,

$$\Rightarrow \dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{15 \times 10^6}{484.821 \times 10^3} = 30.9392 \text{ kg/s}$$

Problem 7.29.

An engine working on Otto cycle is supplied with air at 0.1 MPa, 35°C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of cycle, the cycle efficiency and mean effective pressure. (PoU 2013 fall).

Solution: For the calculation reference, the $P - V$ and $T - S$ diagrams for an air standard Otto cycle are given in below:



Given,

At the beginning of compression stroke:

pressure, $P_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$

temperature, $T_1 = 35^\circ\text{C} = 35 + 273 = 308 \text{ K}$

Compression ratio, $r_c = V_1/V_2 = 8$

Heat transfer to air per cycle, $q_H = 2100 \text{ kJ/kg}$

Process 1 – 2 is the isentropic compression, hence applying pressure – volume and temperature – volume relations for an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^r \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2}\right)^r = 100(8)^{1.4} = 1837.917 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{r-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{r-1} = 308(8)^{1.4-1} = 707.6 \text{ K}$$

Heat added during the process 2 - 3 is given by

$$q_H = q_{23} = c_v(T_3 - T_2)$$

$$\Rightarrow T_3 = T_2 + \frac{q_H}{c_v} = 707.6 + \frac{2100 \times 10^3}{718} = 3632.4 \text{ K}$$

Process 2 - 3 is an isochoric heat addition process, so using Charles law for an ideal gas:

$$\frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow P_3 = \frac{T_3}{T_2} P_2 = \frac{3632.4}{707.6} \times 1837.917 = 9434.78 \text{ kPa}$$

The efficiency of the cycle is given by

$$\eta_o = 1 - \left(\frac{1}{r_c}\right)^{r-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 0.5647 = 56.47\%$$

Work output per kg of air is determined as

$$w_{out} = \eta_o \times q_H = 0.5647 \times 2100 = 1185.87 \text{ kJ/kg}$$

Applying equation of state to find the specific volume of air at state 1,

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 308}{100 \times 10^3} = 0.88396 \text{ m}^3/\text{kg}$$

Applying equation of state to find the specific volume of air at state 2,

$$P_2 v_2 = RT_2 \Rightarrow v_2 = \frac{RT_2}{P_2} = \frac{287 \times 707.6}{1837.917 \times 10^3} = 0.11049 \text{ m}^3/\text{kg}$$

Now, the mean effective pressure,

$$MEP = \frac{w_{out}}{V_{stroke}} = \frac{w_{out}}{v_1 - v_2} = \frac{1185.87}{0.88396 - 0.11049} = 1533.18 \text{ kPa}$$

Problem 7.30.

The compression ratio in an air standard Otto cycle is 8. At the beginning of the compression stroke the pressure is 0.2 MPa and the temperature is 18°C. The heat transfer to the air per cycle is 1900 kJ/kg of air. Determine

- (i) temperature and pressure at the end of each process of cycle
- (ii) thermal efficiency of cycle. (PoU 2014 fall).

Solution: Refer problem 7.23 (Very similar).

Problem 7.31.

An ideal Brayton cycle has pressure ratio of 10. The temperature of air at compressor and turbine inlets are 300 K and 1200 K respectively. Determine its thermal efficiency and mass flow rate of air required to produce net power output of 80 MW. (PoU 2014 spring).

Solution: Given,

$$\text{Pressure ratio, } r_p = \frac{P_2}{P_1} = 10$$

Temperature at compressor inlet, $T_1 = 300 \text{ K}$

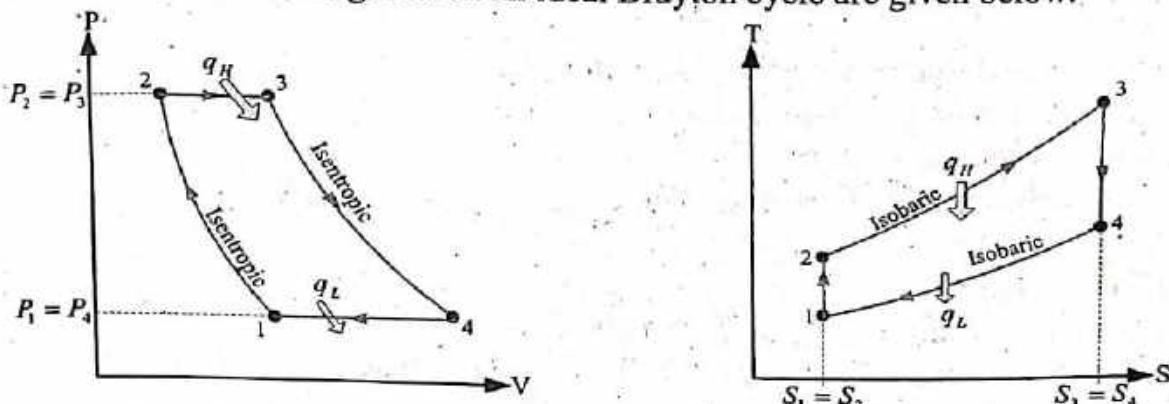
Temperature at turbine inlet, $T_3 = 1200 \text{ K}$

Net power output, $\dot{W}_{net} = 80 \text{ MW} = 80 \times 10^6 \text{ W}$

Efficiency of the cycle is given by,

$$\eta_B = 1 - \left(\frac{1}{r_p} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 0.48205 = 48.205\%$$

The $P-V$ and $T-S$ diagrams of an ideal Brayton cycle are given below.



Process 1 – 2 is an isentropic compression, so

$$\frac{T_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 300 \times (10)^{\frac{1.4-1}{1.4}} = 579.21 \text{ K}$$

From process 2 – 3 heat added per kg of air is given by

$$q_H = q_{23} = h_3 - h_2 = c_p(T_3 - T_2) = 1.005 (1200 - 579.21) = 623.894 \text{ kJ/kg}$$

Net work output per kg of air of the cycle,

$$w_{net} = \eta_B \times q_H = 0.48205 \times 623.894 = 300.748 \text{ kJ/kg}$$

Power output of the cycle can be expressed as

$$\dot{W}_{net} = \dot{m} \times w_{net}$$

Now, the mass flow rate of air,

$$\Rightarrow \dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{80 \times 10^6}{300.748 \times 10^3} = 266.003 \text{ kg/s}$$

Problem 7.32.

The compression ratio of an air standard Otto cycle is 8. At the beginning of the compression process, the pressure and temperature of air are 100 kPa and 20 °C respectively. The heat added per kg of air during the cycle is 2000 kJ/kg. Determine the pressure and temperature at the end of each process of the cycle, the thermal efficiency, and the mean effective pressure.

[Take $R = 287 \text{ J/kgK}$, and $\gamma = 1.4$] (PoU 2015 fall)

Solution: Refer problem 7.19 (Very similar). [Ans: $P_2 = 1837.917 \text{ kPa}$, $T_2 = 673.137 \text{ K}$, $T_3 = 3458.652 \text{ K}$, $P_3 = 9443.419 \text{ kPa}$, $P_4 = 513.811 \text{ kPa}$, $T_4 = 1505.465 \text{ K}$, $\eta_0 = 56.472\%$, $MEP = 1535.003 \text{ kPa}$]

Problem 7.33.

An air standard Carnot cycle is executed in a closed system between the temperature limits of 350 K and 1200 K. The pressure before and after the isothermal compression are 150 kPa and 300 kPa respectively. If net work output per cycle is 0.5 kJ, determine

- (i) the maximum pressure in the cycle
- (ii) the heat transfer to air
- (iii) the mass of air.

Assume variables, specific heats for air.

(PoU 2016 spring).

Solution: Given,

Temperature at initial state 1, $T_1 = 1200 \text{ K}$

Temperature at state 2, $T_2 = T_1 = 1200 \text{ K}$ (As process 1 – 2 is isothermal in Carnot cycle)

Temperature at state 3, $T_3 = 350 \text{ K}$

Temperature at state 4, $T_4 = T_3 = 350 \text{ K}$

Pressure at state 3, $P_3 = 150 \text{ kPa}$

Pressure at state 4, $P_4 = 300 \text{ kPa}$

Net work output of cycle, $W_{net} = 0.5 \text{ kJ}$

The $P - v$ and $T - s$ diagrams are shown below.

Using the temperature – pressure relationship for an isentropic process (reversible adiabatic process) 4 – 1,

$$\frac{T_1}{T_4} = \left(\frac{P_1}{P_4} \right)^{\frac{\gamma-1}{\gamma}}$$

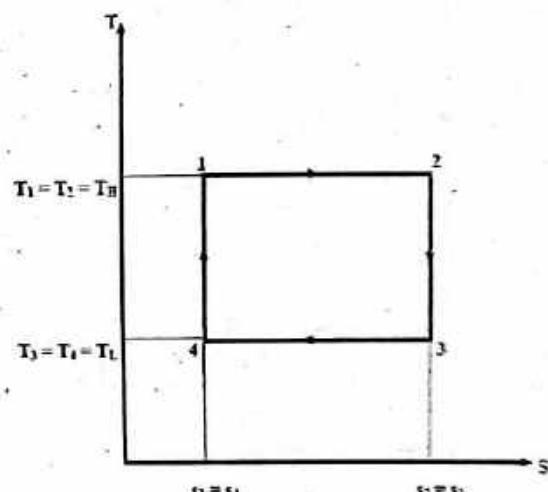
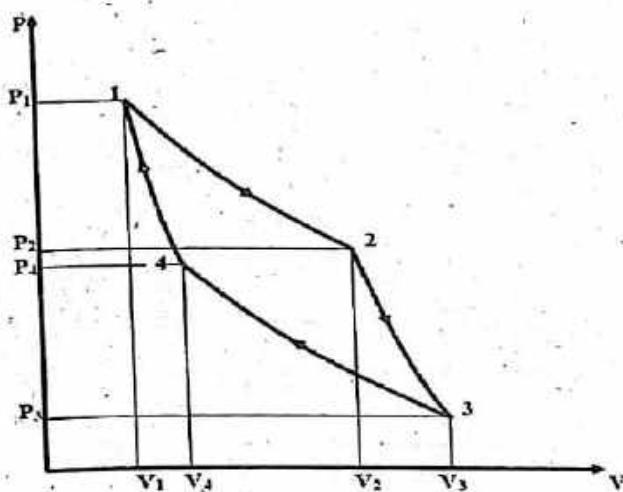
⇒ Maximum pressure in the cycle,

$$P_1 = P_4 \times \left(\frac{T_1}{T_4} \right)^{\frac{1.4}{\gamma-1}} = 300 \times \left(\frac{1200}{350} \right)^{\frac{1.4}{1.4-1}} = 22388.11 \text{ kPa}$$

Carnot efficiency is calculated as,

$$\eta_{Carnot} = \frac{W_{net}}{Q_{in}} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{T_3}{T_1}$$

⇒ Heat transfer to the air, $Q_{in} = \frac{W_{net} \times T_{max}}{T_{max} - T_{min}} = \frac{0.5 \times 1200}{1200 - 350} = 0.70588 \text{ kJ}$



Similarly, using the temperature – pressure relationship for an isentropic process (reversible adiabatic process) 2 – 3,

$$\frac{T_2}{T_3} = \left(\frac{P_2}{P_3} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow P_2 = P_3 \times \left(\frac{T_2}{T_3} \right)^{\frac{\gamma}{\gamma-1}} = 150 \times \left(\frac{1200}{350} \right)^{\frac{1.4}{1.4-1}} = 11194.0557 \text{ kPa}$$

Now, the heat transfer to air can also be calculated as

$$Q_{in} = Q_{1-2} = W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1} = m R T_1 \ln \frac{P_1}{P_2}$$

$(\Delta U_{12} = 0 \text{ as } 1-2 \text{ is isothermal process})$

$$\Rightarrow \text{Mass of air, } m = \frac{Q_{in}}{R T_1 \ln(P_1/P_2)} = \frac{0.70588 \times 1000}{287 \times 1200 \times \ln(22388.11/11194.0557)} \\ = 0.002957 \text{ kg}$$

Problem 7.34.

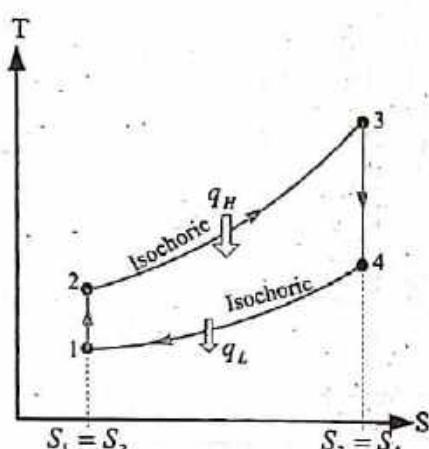
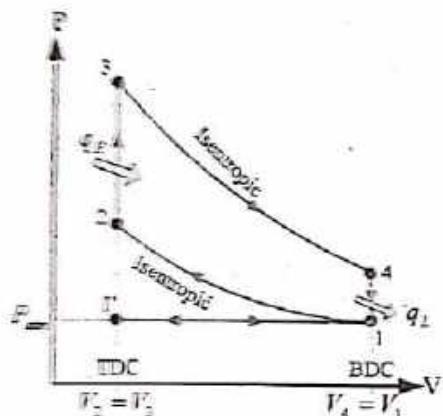
In an Otto cycle engine, the temperature and pressure of air at the beginning of the compression are 300 K and 101 kPa. The compression ratio is 8. The maximum pressure in the cycle is 8000 kPa. Determine

- the peak temperature in the cycle,
- the temperature at the end of expansion stroke,
- the cycle efficiency and
- the heat addition.

(PoU 2016 fall).

Solution:

For the calculation reference, the $P - V$ and $T - S$ diagrams for an air standard Otto cycle are given in below.



Given,

At the beginning of compression stroke:

temperature, $T_1 = 300 \text{ K}$

pressure, $P_1 = 101 \text{ kPa}$

Compression ratio, $r_c = V_1/V_2 = 8$

Maximum pressure during the cycle, $P_3 = 8000 \text{ kPa}$

The efficiency of the cycle is given by

$$\eta_O = 1 - \left(\frac{1}{r_c} \right)^{r-1} = 1 - \left(\frac{1}{8} \right)^{1.4-1} = 0.5647 = 56.47\%$$

Process 1 – 2 is the isentropic compression, hence applying pressure – volume and temperature – volume relations for an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{r-1} \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{r-1} = 101(8)^{1.4-1} = 1856.296 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{r-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1} = 300(8)^{1.4-1} = 689.22 \text{ K}$$

Process 2 – 3 is an isochoric heat addition process, so using Charles law for an ideal gas,

$$\frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow T_3 = \frac{P_3}{P_2} T_2 = \frac{8000}{1856.296} \times 689.22 = 2970.298 \text{ K}$$

Process 3 – 4 is an isentropic expansion, hence applying temperature – volume relationship for an isentropic process

$$\begin{aligned} \frac{T_4}{T_3} &= \left(\frac{V_3}{V_4} \right)^{\gamma-1} = \left(\frac{V_2}{V_1} \right)^{\gamma-1} = \left(\frac{1}{r_c} \right)^{\gamma-1} \\ \Rightarrow T_4 &= T_3 \left(\frac{1}{r_c} \right)^{\gamma-1} = 2970.298 \left(\frac{1}{8} \right)^{1.4-1} = 1292.897 \text{ K} \end{aligned}$$

Now, the heat transferred to the air during constant volume process 2 – 3 can be calculated as

$$q_H = c_v(T_3 - T_2) = 718 (2970.298 - 689.22) = 1637.814 \text{ kJ/kg.}$$

Problem 7.35.

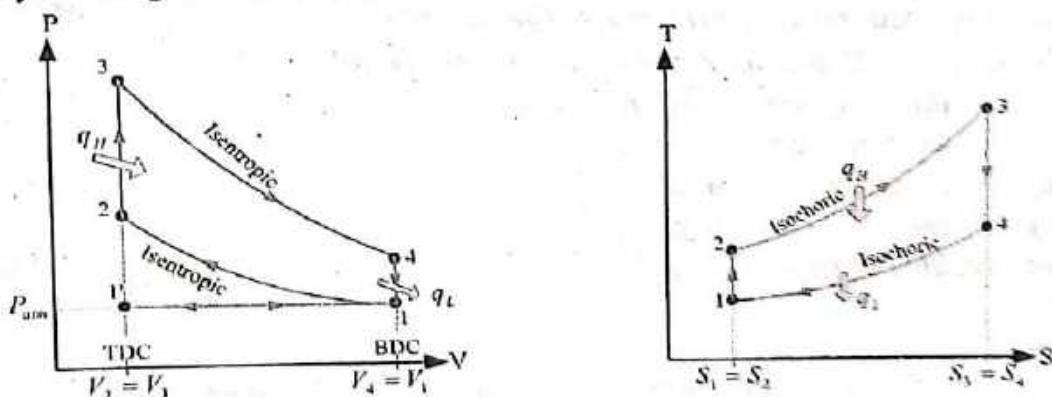
An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 95 kPa and 27°C and 750 kJ/kg of heat is transferred to air during the constant volume heat addition process. Taking into account the variation of specific heats with temperature, determine

- (a) pressure and temperature at the end of the heat addition process,
- (b) the net work output and
- (c) the thermal efficiency.

(PoU 2017 spring, KU 2012).

Solution:

For the calculation reference, the $P - V$ and $T - S$ diagrams for an air standard Otto cycle are given in below.



Given,

Compression ratio, $r_c = V_1/V_2 = 8$

At the beginning of compression process:

pressure, $P_1 = 95 \text{ kPa}$

temperature, $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Heat transfer to air per cycle, $q_H = 750 \text{ kJ/kg}$

Process 1 – 2 is the isentropic compression, hence applying pressure – volume and temperature – volume relations for an isentropic process,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^\gamma \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 95(8)^{1.4} = 1746.0215 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 300(8)^{1.4-1} = 689.219 \text{ K}$$

Heat added during the process 2 - 3 is given by

$$q_E = q_{23} = c_v(T_3 - T_2)$$

$$\Rightarrow T_3 = T_2 + \frac{q_H}{c_v} = 689.219 + \frac{750 \times 10^3}{718} = 1733.787 \text{ K}$$

Process 2 – 3 is an isochoric heat addition process, so using Charles law for an ideal gas,

$$\frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow P_3 = \frac{T_3}{T_2} P_2 = \frac{1733.787}{689.219} \times 1746.0215 = 4392.261 \text{ kPa}$$

The efficiency of the cycle is given by

$$\eta_o = 1 - \left(\frac{1}{r_e} \right)^{\gamma-1} = 1 - \left(\frac{1}{8} \right)^{1.4-1} = 0.5647 = 56.47\%$$

Net work output per kg of air is determined as

$$w_{net} = \eta_o \times q_H = 0.5647 \times 750 = 423.525 \text{ kJ/kg}$$

Problem 7.36.

In a Brayton cycle based power plant, the air at the inlet 27°C , 0.1 MPa . The pressure ratio is 6.25 and maximum temperature is 800°C . Find

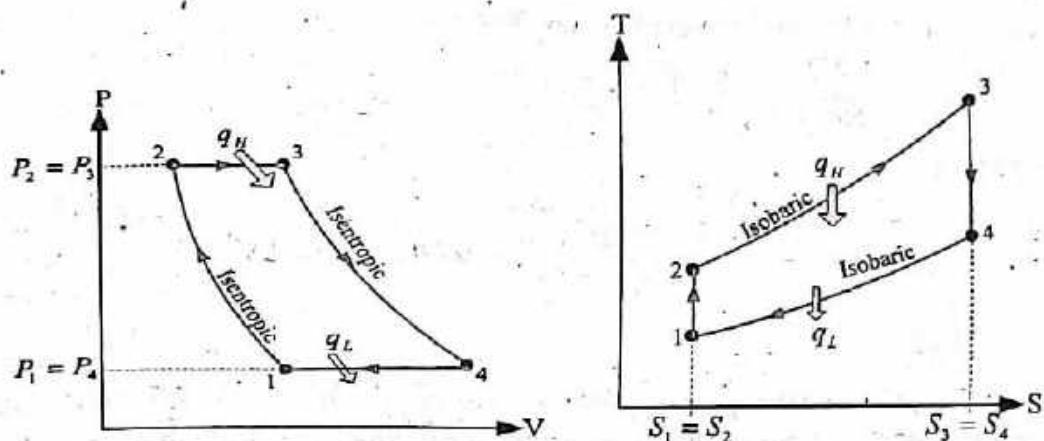
- (i) the compressor work per kg of air
- (ii) the turbine work per kg of air
- (iii) heat supplied per kg of air
- (iv) the cycle efficiency.

Assume suitable constants for air.

(PoU 2017 fall).

Solution:

The $P - V$ and $T - S$ diagrams of an ideal Brayton cycle are given below.



Given,

At inlet of compressor of the cycle,

pressure, $P_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$

temperature, $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

Pressure ratio, $r_p = P_2/P_1 = 6.25$

Maximum temperature during the cycle can be attained after heat addition process at state 3, hence

maximum temperature, $T_3 = 800^\circ\text{C} = 800 + 273 = 1073 \text{ K}$

Process 1 - 2 is an isentropic compression process, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1(r_p)^{\frac{\gamma-1}{\gamma}} = 300(6.25)^{\frac{1.4-1}{1.4}} = 506.425 \text{ K}$$

Process 2 - 3 is an isobaric heat addition process, hence $P_3 = P_2$

Process 4 - 1 is an isobaric heat rejection process, then $P_4 = P_1$

Process 3 - 4 is an isentropic expansion process, so

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow T_4 = T_3 \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1073 \times \left(\frac{1}{6.25}\right)^{\frac{1.4-1}{1.4}} = 635.63 \text{ K}$$

Compressor work per kg of air (specific work),

$$w_C = h_2 - h_1 = c_p(T_2 - T_1) = 1005(506.425 - 300) = 207.457 \text{ kJ/kg}$$

Turbine work per kg of air (specific work),

$$w_T = h_3 - h_4 = c_p(T_3 - T_4) = 1005(1073 - 635.63) = 439.557 \text{ kJ/kg}$$

Net work per kg of air (net specific work),

$$w_{net} = w_T - w_C = 439.557 - 207.457 = 232.1 \text{ kJ/kg}$$

From process 2 - 3 heat supplied per kg of air is given by

$$q_H = q_{23} = h_3 - h_2 = c_p(T_3 - T_2) = 1005(1073 - 506.425) = 569.408 \text{ kJ/kg}$$

Efficiency of an ideal Brayton cycle is given by

$$\eta_B = \frac{w_{net}}{q_H} = \frac{232.1}{569.408} = 0.4076 = 40.76\%$$

Alternately,

$$\eta_B = 1 - \left(\frac{1}{r_p} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{6.25} \right)^{\frac{1.4-1}{1.4}} = 0.4076 = 40.76\%$$

Problem 7.37.

The turbine of a steam power plant operating on a simple Rankine cycle produces 1750 kW of power when the boiler is operated at 6 MPa, the condenser at 20 kPa and the temperature at the turbine entrance is 500 °C. Find the enthalpies for state 1 and 2 if the enthalpy at state 3, $h_3 = 3423.1 \text{ kJ/kg}$ and the enthalpy at state 4, $h_4 = 2267.5 \text{ kJ/kg}$ (Refer T – s diagram below). Determine

- (i) rate of heat supply in the boiler
- (ii) rate of heat rejection in the condenser
- (iii) thermal efficiency.

(PoU 2018 fall).

Solution: Given,

Power output of turbine, $\dot{W}_{out,T} = 1750 \text{ kW}$

Boiler pressure, $P_3 = P_2 = 6 \text{ MPa} = 6000 \text{ kPa}$

(As process 2 – 3 is an isobaric heat addition in boiler)

Condenser pressure, $P_4 = P_1 = 20 \text{ kPa}$

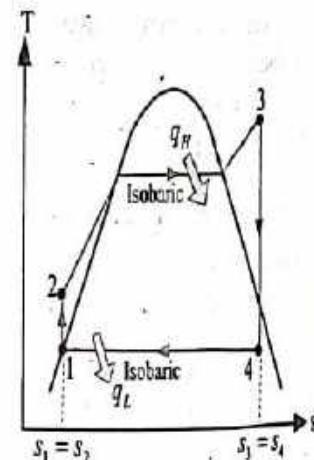
(As process 4 – 1 is an isobaric heat rejection from condenser)

Temperature at turbine entrance,

$T_3 = 500^\circ\text{C} = 500 + 273 = 773 \text{ K}$

Enthalpy at state 3, $h_3 = 3423.1 \text{ kJ/kg}$

Enthalpy at state 4, $h_4 = 2267.5 \text{ kJ/kg}$



Initial state 1: Pressure, $P_1 = 20 \text{ kPa}$ and water is in saturated liquid state.

Referring to the saturated water pressure table A.2.1 for pressure, $P_1 = 20 \text{ kPa}$, specific enthalpy, $h_1 = h_l (P_1 = 20 \text{ kPa}) = 251.46 \text{ kJ/kg}$ and specific volume, $v_1 = v_l (P_1 = 20 \text{ kPa}) = 0.001017 \text{ m}^3/\text{kg}$.

State 2: Pressure, $P_2 = 6000 \text{ kPa}$ and water is in sub-cooled or compressed liquid.

Process 1 – 2 is an isentropic pumping, hence applying the isentropic relation for an incompressible substance (liquid water),

$$h_2 - h_1 = v_1 (P_2 - P_1)$$

$$\Rightarrow h_2 = h_1 + v_1 (P_2 - P_1) = 251.46 + 0.001017 (6000 - 20) = 257.5416 \text{ kJ/kg}$$

Now, work output per kg of steam from the turbine,

$$w_T = h_3 - h_4 = 3423.1 - 2267.5 = 1155.6 \text{ kJ/kg}$$

Work input per kg of steam to the pump,

$$w_p = h_2 - h_1 = 257.5416 - 251.46 = 6.0816 \text{ kJ/kg}$$

Net work per kg of steam delivered to the surroundings,

$$w_{net} = w_T - w_p = 1155.6 - 6.0816 = 1149.5184 \text{ kJ/kg}$$

$$\text{Rate of mass flow of steam, } \dot{m} = \frac{\dot{W}_{out,T}}{w_T} = \frac{1750}{1149.5184} = 1.5224 \text{ kg/s}$$

Rate of heat supply in the boiler during the process 2 – 3,

$$\begin{aligned} \dot{Q}_H &= \dot{m} \times q_H = \dot{m} \times q_{23} = \dot{m}(h_3 - h_2) = 1.5224 (3423.1 - 257.5416) \\ &= 4819.2461 \text{ kW} \end{aligned}$$

Rate of heat rejection in the condenser during the process 4 – 1,

$$\begin{aligned} \dot{Q}_L &= \dot{m} \times q_L = \dot{m} \times q_{41} = \dot{m}(h_4 - h_1) = 1.5224 (2267.5 - 251.46) \\ &= 3069.2193 \text{ kW} \end{aligned}$$

Then, the thermal efficiency of the Rankine cycle,

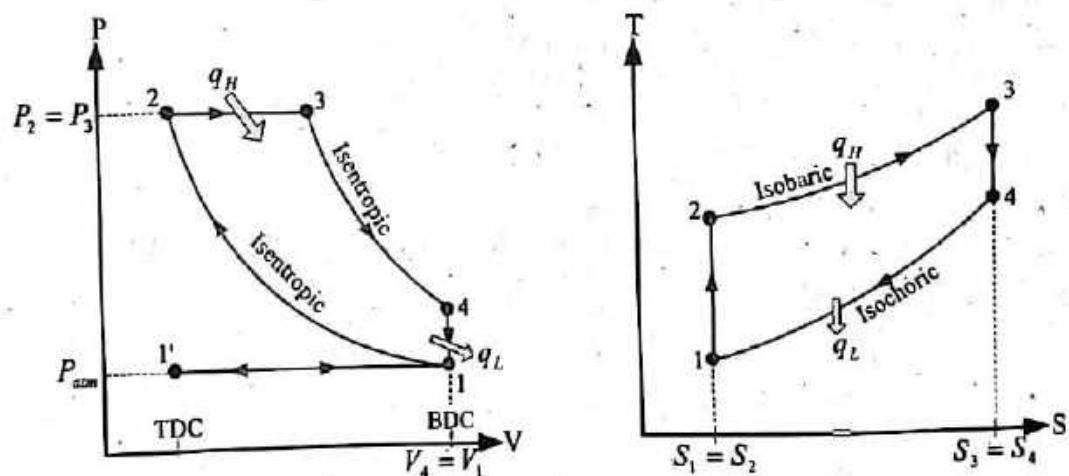
$$\eta_R = \frac{w_{net}}{q_H} = \frac{w_{net}}{h_3 - h_2} = \frac{1149.5184}{3423.1 - 257.5416} = 0.36313 = 36.313\%$$

Problem 7.38.

A diesel engine has a compression ratio of 20:1 with an inlet of 95 kPa and 290 K, state 1, with volume 0.5 L. The maximum cycle temperature is 1800 K. Find the maximum pressure, the net specific work and the thermal efficiency. [Assume necessary air constants]. (PoU 2019 spring).

Solution:

The P – V and T – S diagrams of an ideal Diesel cycle are given below.



Given,

Compression ratio, $r_c = V_1/V_2 = 20:1$

Pressure at beginning of compression stroke, $P_1 = 95 \text{ kPa}$

SOME POWER CYCLES

Temperature at beginning of compression stroke, $T_1 = 290 \text{ K}$

$$\begin{aligned} \text{Volume at beginning of compression stroke, } V_1 &= 0.5 \text{ L} = 0.5 \times 0.001 \text{ m}^3 \\ &= 5 \times 10^{-4} \text{ m}^3 \end{aligned}$$

Maximum pressure and temperature are attained after the heat addition, so maximum temperature, $T_3 = 1800 \text{ K}$

Process 1 - 2 is an isentropic compression process, so pressure - volume relationship is given by

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{-1} = (r_c)^{\gamma} \Rightarrow P_2 = P_1 (r_c)^{\gamma} = 95 (20)^{1.4} = 6297.463 \text{ kPa}$$

Process 2 - 3 is an isobaric process, so maximum pressure of the cycle is given by
 $P_3 = P_2 = 6297.463 \text{ kPa}$

Again, for the isentropic process 1 - 2, temperature - volume relationship is given by

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r_c)^{\gamma-1} \Rightarrow T_2 = T_1 (r_c)^{\gamma-1} = 290 (20)^{1.4-1} \\ &= 961.19 \text{ K} \quad (\text{As } r_c = \frac{V_1}{V_2}) \end{aligned}$$

Process 2 - 3 is an isobaric heat addition process, so using Charles law for ideal gas,

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} \Rightarrow \alpha = \frac{T_3}{T_2} = \frac{1800}{961.19} = 1.873$$

where $\alpha = V_3/V_2$ is the cut-off ratio.

Then, efficiency of the Diesel cycle,

$$\begin{aligned} \eta_D &= 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^\gamma - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{20} \right)^{1.4-1} \left[\frac{1.873^{1.4} - 1}{1.873 - 1} \right] \\ &= 0.65257 = 65.257\% \end{aligned}$$

Heat added during the process 2 - 3 is given by

$$q_H = q_{23} = c_p (T_3 - T_2) = 1.004 (1800 - 961.19) = 842.165 \text{ kJ/kg}$$

The net work output per kg of air of the cycle can be determined as

$$w_{ne} = \eta_D \times q_H = 0.65257 \times 842.165 = 549.572 \text{ kJ/kg}$$

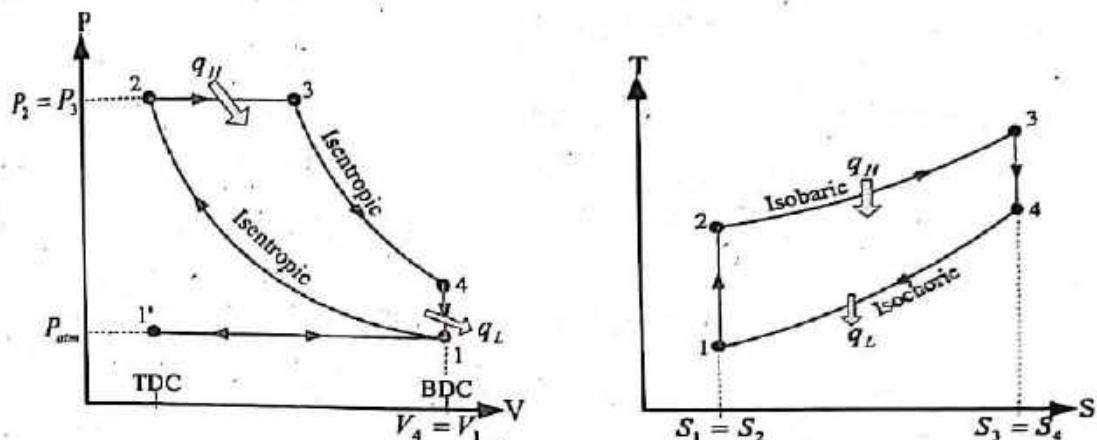
Problem 7.39.

A CI engine is proposed to have a compression ratio of 15 while operating with low temperature of 200°C and a low pressure of 200 kPa . If the work output is to be 1000 kJ/kg , calculate the maximum possible thermal efficiency. (Take $c_p = 1.004 \text{ kJ/kgK}$, $c_v = 0.717 \text{ kJ/kgK}$, $R = 0.287 \text{ kJ/kgK}$) (PoU 2020 fall).

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Solution:

For the calculation reference, the $P = V$ and $T - S$ diagrams of a CI engine or an ideal Diesel cycle are given below.



Given,

$$\text{Compression ratio, } r_c = V_1/V_2 = 15$$

$$\begin{aligned} \text{Temperature at beginning of cycle (low temperature), } T_1 &= 200^\circ\text{C} = 200 + 273 \\ &= 473 \text{ K} \end{aligned}$$

$$\text{Pressure at beginning of cycle (low pressure), } P_1 = 200 \text{ kPa}$$

$$\text{Work output, } w_{\text{out}} = 1000 \text{ kJ/kgK}$$

For the isentropic process 1 – 2, temperature – volume relationship is given by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r_c)^{\gamma-1}$$

$$\Rightarrow T_2 = T_1(r_c)^{\gamma-1} = 473(15)^{1.4-1} = 1397.3257 \text{ K} \quad (\text{As } \gamma = \frac{c_p}{c_v} \text{ and } r_c = \frac{V_1}{V_2})$$

Process 2 – 3 is an isobaric process, so using Charle's law, we get

$$\frac{T_3}{V_3} = \frac{T_2}{V_2} \Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = \alpha$$

Process 3 – 4 is an isentropic process, so using temperature – volume relationship

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma-1} = \left(\frac{V_3}{V_2} \times \frac{V_2}{V_1} \right)^{\gamma-1} = (\alpha / r_c)^{\gamma-1} \quad (\text{As } V_4 = V_1)$$

$$\text{Hence, } \frac{T_4}{T_1} = \frac{T_4}{T_3} \times \frac{T_3}{T_2} \times \frac{T_2}{T_1} = \left(\frac{\alpha}{r_c} \right)^{\gamma-1} \times \alpha \times (r_c)^{\gamma-1} = \alpha^{\gamma}$$

The specific work output for diesel cycle or CI engine can be expressed in terms of cut-off ratio (α) as follows:

$$\begin{aligned} w_{\text{out}} &= q_H - q_L = c_p(T_3 - T_2) - c_v(T_4 - T_1) \\ &= c_p T_2 (T_3/T_2 - 1) - c_v T_1 (T_4/T_1 - 1) \end{aligned}$$

SOME POWER CYCLES

$$\begin{aligned}
 &= c_p T_2 (\alpha - 1) - c_v T_1 (\alpha^\gamma - 1) \\
 \Rightarrow & 1000 = 1.004 \times 1397.3257 (\alpha - 1) - 0.717 \times 473 (\alpha^\gamma - 1) \\
 \Rightarrow & \alpha^{1.4} - 4.13667\alpha + 6.0853 = 0 \\
 \therefore & \alpha = 2.2
 \end{aligned}$$

Heat added during the process 2-3 is given by

$$\begin{aligned}
 q_H &= q_{23} = c_p (T_3 - T_2) = c_p T_2 (T_3/T_2 - 1) = c_p T_2 (\alpha - 1) \\
 &= 1.004 \times 1397.3257 (2.2 - 1) \\
 &= 1683.5 \text{ kJ/kg}
 \end{aligned}$$

The efficiency of the Diesel cycle or CI engine is determined as,

$$\eta_D = \frac{W_{out}}{q_H} = \frac{1000}{1683.5} = 0.594 = 59.4\%$$

Alternately,

$$\eta_D = 1 - \frac{1}{\gamma} \left(\frac{1}{r_c} \right)^{\gamma-1} \left[\frac{\alpha^\gamma - 1}{\alpha - 1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{15} \right)^{1.4-1} \left[\frac{2.2^{1.4} - 1}{2.2 - 1} \right] = 0.594 = 59.4\%$$

7.4 Review Theoretical Questions

1. Define a thermodynamic cycle. Explain the applications of different practical cycles.
2. Differentiate between gas and vapor cycles. Sketch $P - V$ and $T - S$ diagrams and layout for Brayton and Rankine cycles.
3. Differentiate between power cycle and refrigeration cycle. Sketch components, $P - v$ and $T - s$ diagrams for Rankine cycle.
4. Differentiate between power cycle and refrigeration cycle with the appropriate examples. Sketch the components of a steam power plant and corresponding processes on $P - v$ and $T - s$ diagrams.
5. Differentiate between power cycle and refrigeration cycle. Sketch $P - v$ and $T - s$ diagrams for an ideal Otto and ideal Diesel cycles. Also write down the expressions for their efficiencies.
6. List the common power cycles and the processes associated with them.
7. Explain the working principle of a Carnot cycle with corresponding process in $P - v$ and $T - s$ diagrams.
8. Sketch a Carnot cycle on $P - v$ and $T - s$ diagrams and explain the processes associated with it. Also write an expression for theoretical efficiency of the Carnot cycle.
9. Explain the working principle of a Brayton cycle. Sketch the cycle on $P - v$ and $T - s$ diagrams and explain the variation of its efficiency with pressure ratio.
10. Explain the working principle of Brayton cycle with corresponding processes on $P - v$ and $T - s$ diagrams.
11. Sketch an ideal Brayton cycle on $P - v$ and $T - s$ diagrams and derive an expression for its efficiency in terms of pressure ratio.
12. Describe the working principle of Rankine cycle with the help of $P - v$ and $T - s$ diagrams.
13. Explain with the help of neat diagrams the various processes of any Rankine cycle and derive an expression for its efficiency.
14. Sketch Rankine cycle on $P - v$ and $T - s$ diagrams using dry saturated steam and obtain an expression for the Rankine cycle efficiency.
15. Define air standard analysis. Also list the assumptions of an air standard analysis.

16. What is air standard cycle? Differentiate between Diesel cycle and Otto cycle.
17. Sketch an ideal Otto cycle on $P - v$ and $T - s$ diagrams. Also derive an expression for its efficiency in terms of compression ratio.
18. Sketch an ideal Otto cycle on $P - v$ and $T - s$ diagrams. Derive the expression for compression ratio in terms of cylinder dimensions.
19. Compare spark ignition and compression ignition engine. Sketch $P - v$ and $T - s$ diagrams for them.
20. In an air standard Otto cycle, the ambient conditions determine the minimum temperature while the maximum temperature is determined by the design conditions and metallurgical considerations of the piston and cylinder. For fixed value of minimum temperature T_1 and maximum temperature T_3 , show that for maximum work output, the compression ratio should have the value

$$r_c = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(y-1)}}$$

21. Sketch an ideal Diesel cycle on $P - v$ and $T - s$ diagrams and derive an expression for its efficiency in terms of compression ratio and cut-off ratio.
22. Derive an expression for compression ratio for an internal combustion engine. Discuss the effect of different parameters involved in the expression on the compression ratio.

7.5 Unsolved Numerical Problems

1. A Carnot cycle operates between source and sink temperatures of 250°C and -15°C . If the system receives 90 kJ from the source, find
 - (i) efficiency of the cycle,
 - (ii) the net work transfer and
 - (iii) heat rejected to sink. (Ans.: 50.6%, 45.54 kJ, 44.46 kJ)
2. A Brayton cycle operates with air entering the compressor at 1 bar and 25°C . The pressure ratio across the compressor is 3 to 1, and the maximum temperature in the cycle is 650°C . Determine the compressor work, turbine work, and thermal efficiency. (Ans: 112.51 kJ/kg, 250.03 kJ/kg, 26.95%)
3. A gas turbine power plant operating on an ideal Brayton cycle has a pressure ratio 8. The gas temperature is 300 K at the compressor inlet and 1300 K at turbine inlet. Utilizing the air standard assumptions, determine:
 - (a) the gas temperature at the exit of the compressor and turbine
 - (b) the back work ratio, and the thermal efficiency.
 (Ans: 543.43 K, 717.66 K, 41.8%, 44.8%)
4. The minimum and maximum temperatures during an ideal Brayton cycle are 300 K and 1200 K respectively. The pressure ratio is such that the net work developed is maximized. Determine:
 - (a) the compressor and turbine work per unit mass of air, and
 - (b) the thermal efficiency of the cycle.
 (Ans: 301.504 kJ/kg, 603.004 kJ/kg, 50%)
5. In a gas turbine plant working on Brayton cycle, the air at inlet is 27°C , 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C . The isentropic turbine and compression efficiencies are each 80%. Find the compressor work, turbine work, heat supplied, cycle efficiency and turbine exhaust temperature. Mass of air may be considered as 1 kg. Draw $T - s$ diagram. (Ans: 259.29 kJ/kg, 351.6 kJ/kg, 517.57 kJ/kg, 17.83%, 450.13°C)
6. In an Otto cycle engine the temperature and pressure of air at the beginning of compression are 300 K and 101 kPa. The compression ratio is 10. The maximum pressure in the cycle is 10000 kPa. Determine the air standard cycle efficiency and heat transferred to the air during constant volume heat addition process. [Take $R = 287 \text{ J/kgK}$, $c_v = 718 \text{ J/kgK}$].
 (Ans: 60.189%, 1553.61 kJ/kg).
7. An air standard Otto cycle has a compression ratio of 10. At the beginning of the compression stroke, the pressure and temperature are 100 kPa and 20°C respectively. The peak temperature during the cycle is 2000 K. Determine

SOME POWER CYCLES

- (a) the pressure and temperature at the end of each process of the cycle,
(b) the thermal efficiency, and
(c) the mean effective pressure. (Ans: 2511.87 kPa, 735.98 K, 6825.92 kPa, 271.75 kPa, 796.21 K, 60.19%, 721.71 kPa)
8. Air at the beginning of the compression stroke in an air standard Otto cycle is at 100 kPa and 300 K. The temperature of the air before and after the expansion stroke are 1550 K and 650 K respectively. If the air circulation rate is 3 kg/min, determine the compression ratio, air standard efficiency and the power output. (Ans: 8.78, 58.063%, 17.42 kW)
9. One kg of air is taken by through a constant volume cycle. At the commencement of the adiabatic compression, the pressure and temperature are 103 kN/m² and 100°C respectively. The adiabatic compression has a volume ratio of 6:1. The maximum pressure of cycle is 3.45 MN/m². Determine for cycle
(a) the heat transferred to air
(b) the heat rejected by air
(c) the ideal thermal efficiency
(d) the work done
(e) the mean effective pressure. (Ans: 946.683 kJ/kg, 462.32 kJ/kg, 51.18%, 484.36 kJ/kg, 559.31 kN/m²).
10. An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m³. The initial pressure and temperature are 1 bar and 50°C. The maximum pressure is limited to 25 bars, find the followings:
(a) the air standard efficiency and
(b) the mean effective pressure for cycle. (Ans: 56.47%, 59 bars)
11. A Diesel cycle has a compression ratio of 20. The air at the beginning of the compression stroke is at $P_1 = 100 \text{ kPa}$, $T_1 = 290 \text{ K}$ and $V_1 = 0.5 \times 10^{-3} \text{ m}^3$. The maximum temperature during the cycle is 2000 K. Determine:
(a) the maximum pressure during the cycle,
(b) the cycle efficiency, and
(c) the work output. (Ans: 6628.91 kPa, 64.32%, 0.4034 kJ)
12. The following data are given for a four stroke Diesel engine:
Cylinder bore = 14 cm
Stroke length = 25 cm
Clearance volume = 350 cm³
Determine the air standard efficiency, if fuel injection takes place at constant pressure for 5 % of the stroke. (Ans: 59.3%)

SOME POWER CYCLES

13. An engine is to operate on Diesel cycle with the following data:
Maximum temperature = 1500 K
Exhaust temperature at the end of expansion stroke = 700 K
The inlet condition at the start of compression = 1 bar and 300 K
Find the compression ratio, maximum pressure and cycle efficiency for the cycle. (Ans: 12.3, 3360.83 kPa, 58.04%)
14. In a steam power plant operating on the simple ideal Rankine cycle, steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of the cycle. Compare the efficiency with the Carnot cycle efficiency working with same temperature reservoirs. (Ans: 25.66%, 58.54%)
15. In a steam power cycle, the steam supply is at 15 bar and at dry and saturated state. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle neglecting pump work. (Ans: 25.9%, 23.54%)
16. In a steam turbine of a steam power plant, steam at 20 bar, 360°C is expanded to 0.08 bar. Then it enters a condenser where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Find the net work per kg of steam and the cycle efficiency assuming ideal processes.
(Ans: 969.61 kJ/kg, 32.5%)
17. In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Assuming the steam flow rate of 9.5 kg/s, Determine:
(a) the pump work
(b) the turbine work
(c) the Rankine efficiency
(d) the condenser heat flow
(e) the quality at the end of expansion.
(Ans: 33.63 kW, 7495.5 kW, 30.93%, 16734.25 kW, 0.747)

Heat Transfer**8****8.1 Chapter Highlights**

- *Heat transfer:* It is the transfer of energy without transfer of mass from one region to another region as a result of temperature difference. Heat transfer takes place from high temperature to low temperature region by three modes, namely conduction, convection and radiation.
- *Applications of heat transfer:* It is widely used in most of the engineering disciplines such as
 - Design of thermal and nuclear power plants, heat engines, boilers, condensers, heat exchangers etc.
 - Heat pump, refrigeration and air conditioning units.
 - Design of cooling systems for electric motors, generators and transformers.
 - Construction of structures and minimization of building heat losses.
 - Thermal control of space vehicles.

- *Differences between thermodynamics and heat transfer are:*

Thermodynamics	Heat transfer
<ol style="list-style-type: none"> 1. It deals with the equilibrium states of a system. 2. There should not be a temperature gradient. 3. It helps to determine the amount of work and heat transfers when a system undergoes from one equilibrium state to another. However, it cannot help to find the temperature of the system at any interval of time before the equilibrium condition is attained. 	<ol style="list-style-type: none"> 1. It occurs only when a system is in a non-equilibrium state. 2. Heat transfer takes place only when there is temperature gradient. 3. It helps to predict the distribution of temperature as a function of time and determine the heat energy transferred per unit time from one region to another due to temperature difference.

- *Conduction:* It is the mode of heat transfer due to temperature gradient from one part of a substance to other part or one substance to another in perfect physical contact without actual movement of molecules.
- In solids, heat is transferred through conduction by the following two mechanisms by movement of free electrons and lattice (crystal) vibrations. Whereas heat conduction in liquids and gases occurs comparatively in negligible amount and is based on the movements of atoms and molecules.

- *Thermal conductivity* of the good conductors decreases when temperature increases, whereas that of the insulators and alloys increases with increase in temperature.
- *Assumptions:* The following assumptions are made for calculation of rate of heat flow by using different governing laws of heat transfer:
 - Conduction of heat occurs under steady state conditions.
 - Heat flow is unidirectional.
 - No heat source within medium.
- *Fourier law:* It is the governing law of heat conduction which states that "The rate of heat flow per unit area normal to the direction of heat flow through a body is directly proportional to the temperature gradient in that direction". Mathematically,

$$\frac{\dot{Q}}{A} \propto \frac{dT}{dx} \quad \Rightarrow \quad \dot{Q} = -kA \frac{dT}{dx}$$

where k is *thermal conductivity* of the material and the negative sign indicates that heat flows in the direction of decreasing temperature. The thermal conductivity depends essentially upon the factors such as material structure, moisture content, density of the material and operating conditions such as pressure and temperature.

- *Convection:* It is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. It occurs by the actual movement of the molecules thereby causing molecular collisions and molecular diffusions.
- *Newton's law of cooling:* It is the governing law of convection heat transfer and states that "The rate of heat flow per unit surface area is directly proportional to the temperature difference between a solid surface and an adjacent fluid".

$$\frac{\dot{Q}}{A_s} \propto (T_s - T_\infty) \quad \Rightarrow \quad \dot{Q} = hA_s(T_s - T_\infty)$$

where h is the *heat transfer coefficient of convection* and it depends upon the thermodynamic and transport properties such as density, viscosity, specific heat and thermal conductivity of the fluid, the geometry of the surface, the nature of the fluid flow, and the prevailing thermal conditions.

- According to the mechanism of the fluid flow, convection heat transfer is classified into two types: free or natural convection and forced convection.
- *Differences between free convection and forced convection are:*

Free convection	Forced convection
1. It is the heat transfer process in which flow of fluid is caused by density gradient.	1. It is the heat transfer process in which flow of fluid is caused by some external devices such as pump, fan, blower etc.

- | | |
|--|---|
| 2. Mass flow rate of fluid is usually low during the free convection.
3. Rate of heat transfer is usually very low.
4. For example: Cooling of a room without a fan by natural circulation of air. | 2. Mass flow rate of fluid is comparatively higher in forced convection.
3. Rate of heat transfer is relatively higher.
4. For example: Cooling of a room by a fan. |
|--|---|

➤ *Thermal resistance:* It is the property of a substance by virtue of which it opposes the flow of heat.

➤ *Thermal resistance of a plane wall* is given by

$$R_{th,wall} = \frac{L}{kA}$$

➤ *Thermal resistance of a hollow cylinder* is given by

$$R_{th,cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

➤ *Thermal resistance of a convective layer* is given by

$$R_{th,conv} = \frac{1}{hA_s}$$

➤ *Thermal resistance of a radiation layer* is given by

$$R_{th,rad} = \frac{1}{\epsilon\sigma A_s(T_s + T_\infty)(T_s^2 + T_\infty^2)}$$

➤ *Rate of heat flow through a plane wall* is given by

$$\dot{Q} = \frac{kA}{L} (T_1 - T_2) = \frac{\Delta T_{overall}}{R_{th,wall}}$$

$$\text{where } R_{th,wall} = \frac{L}{kA}$$

➤ *Rate of heat flow through a plane wall with three layers* is given by

$$\dot{Q} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} = \frac{\Delta T_{overall}}{R_{total}}$$

$$\text{where } R_{total} = \frac{1}{A} \left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right)$$

➤ *Rate of heat flow through a hollow cylinder* is given by

$$\dot{Q} = \frac{2\pi k L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{\Delta T_{overall}}{R_{th,cyl}}$$

$$\text{where } R_{th,cyl} = \frac{\ln(r_2/r_1)}{2\pi k L}$$

- Rate of heat flow through a hollow cylinder with three layers is given by

$$\dot{Q} = \frac{2\pi L(T_1 - T_4)}{\left(\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3} \right)} = \frac{\Delta T_{overall}}{R_{total}}$$

where $R_{total} = \frac{1}{2\pi L} \left[\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3} \right]$

- Rate of heat flow through a plane wall subjected to convective medium on both sides is given by

$$\dot{Q} = \frac{A(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}} = UA(T_{\infty 1} - T_{\infty 2})$$

where $U = \frac{1}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}}$ is called overall or combined heat transfer coefficient.

- Rate of heat flow through a hollow cylinder subjected to convective medium on both sides is given by

$$\dot{Q} = \frac{A_1(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1} + \frac{A_1}{2\pi k L} \ln(r_2/r_1) + \frac{A_1}{A_2} \frac{1}{h_2}} = U_1 A_1 (T_{\infty 1} - T_{\infty 2})$$

$$\text{Or, } \dot{Q} = \frac{A_2(T_{\infty 1} - T_{\infty 2})}{\frac{A_2}{A_1} \frac{1}{h_1} + \frac{A_2}{2\pi k L} \ln(r_2/r_1) + \frac{1}{h_2}} = U_2 A_2 (T_{\infty 1} - T_{\infty 2})$$

where

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{A_1}{2\pi k L} \ln(r_2/r_1) + \frac{A_1}{A_2} \frac{1}{h_2}} \quad \text{and}$$

$$U_2 = \frac{1}{\frac{A_2}{A_1} \frac{1}{h_1} + \frac{A_2}{2\pi k L} \ln(r_2/r_1) + \frac{1}{h_2}}$$

are called *inside overall or combined heat transfer coefficient* and *outside overall or combined heat transfer coefficient* respectively. $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_2 L$ are inside curved surface area and outside curved surface area respectively.

- **Radiation:** It is the mode of heat transfer which does not require any medium, i.e. heat transfer takes place in vacuum in the form of electromagnetic wave.
- **Total emissive power:** It is defined as the total amount of radiation emitted by a body per unit area per unit time.
- **Stefan – Boltzmann law:** It states that “The maximum rate of heat radiated per unit area (heat flux) by a surface is directly proportional to the fourth power of the absolute surface temperature”. i.e.

$$\frac{\dot{Q}}{A_s} \propto T_s^4 \quad \Rightarrow \quad \dot{Q} = \sigma A_s T_s^4$$

where $\sigma = 5.67 \times 10^{-8} W/m^2 K^4$ is the Stefan – Boltzmann constant and this maximum rate of heat is radiated by a black body surface.

- **The rate of heat radiation from a real surface is given by**

$$\dot{Q} = \epsilon \sigma A_s T_s^4$$

where ϵ is the emissivity of a real surface and is defined as the ratio of the rate of heat radiated by a real body to that by a black body at the same surface temperature.

- **The net rate of heat radiated for any real body can be derived as**

$$\dot{Q}_{net} = \epsilon \sigma A_s (T_1^4 - T_2^4)$$

where T_1 and T_2 are surface temperatures of two bodies 1 and 2 which are exchanging radiation heat respectively.

- **Radiant Heat Exchange between infinite Parallel Planes-1 and 2 is given by**

$$q_{1-2} = \epsilon_{12} \sigma (T_1^4 - T_2^4)$$

$$\text{where } \epsilon_{12} = \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

is called combined emissivity of both the parallel plane surfaces.

- **Kirchhoff's law of radiation:** It states that the emissivity and the absorptivity of a surface at a given temperature and wavelength are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity.

$$\epsilon = \alpha$$

- **Wien's displacement law:** The locus of the maximum power emitted by a black body is given by the *Wien's displacement law* which relates the

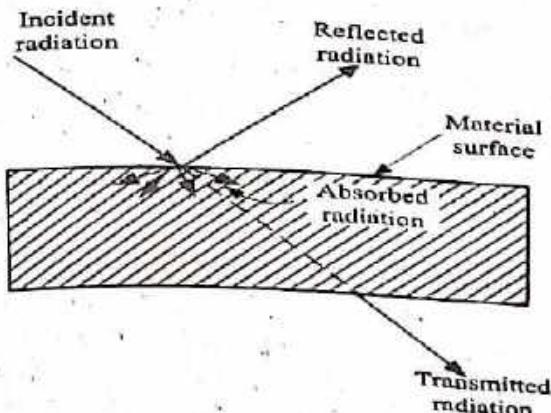
temperature T and wavelength λ at which the emissive power is maximum by the following expression,

$$\lambda T = 2898 \mu m K$$

➤ *Differences between different modes of heat transfer:*

Conduction	Convection	Radiation
<p>1. It is transfer of heat mainly in solid medium and not in vacuum.</p> <p>2. It involves free electron movement and lattice vibration and does not involve actual movement of atoms and molecules.</p> <p>3. It occurs in short range of distance.</p> <p>4. It depends on thermal conductivity k of material.</p> <p>5. Heat transfer is calculated by using Fourier's law :</p> $\dot{Q} = -kA \frac{dT}{dx}$ <p>6. For example, heat transfer in solid, plain wall, hollow cylinder etc.</p>	<p>1. It is transfer of heat mainly in liquid or gas medium and not in vacuum.</p> <p>2. It involves actual movement of atoms and molecules.</p> <p>3. It occurs in medium range of distance.</p> <p>4. It depends on heat transfer coefficient h of convective surface.</p> <p>5. Heat transfer is calculated by using Newton's law of cooling :</p> $\dot{Q} = hA_s \Delta T$ <p>6. For example, heat transfer across the room air from heater surface.</p>	<p>1. It is transfer of heat in all mediums but in vacuum it is most efficient.</p> <p>2. It involves the movement of photons as electro-magnetic wave.</p> <p>3. Wave can travel with speed of light, so it occurs in long range of distance.</p> <p>4. It depends on emissivity of radiating surface.</p> <p>5. Heat transfer is calculated by using Stefan Bolzmann law:</p> $\dot{Q}_{net} = \epsilon A_s (T_1^4 - T_2^4)$ <p>6. For example, heat transfer from the sun to the earth.</p>

➤ *Radiation surface properties:*



From conservation principle of energy,

$$Q_a + Q_r + Q_t = Q_i$$

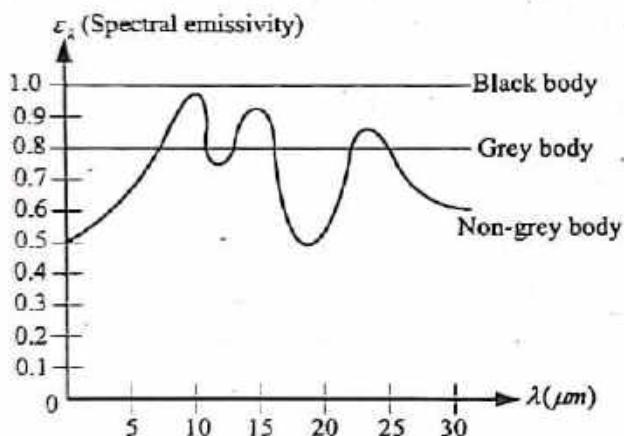
$$\Rightarrow \frac{Q_a}{Q_i} + \frac{Q_r}{Q_i} + \frac{Q_t}{Q_i} = 1$$

$$\Rightarrow \alpha + \rho + \tau = 1.$$

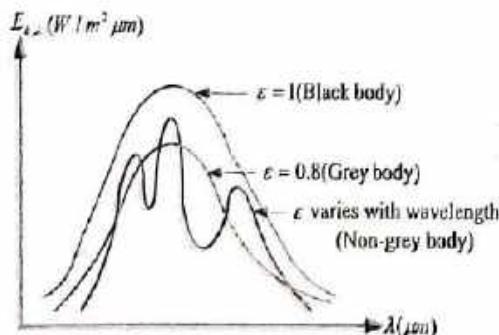
As $\alpha = \epsilon$ then $\epsilon + \rho + \tau = 1$.

where α is the *absorptivity* which is defined as the ratio of absorbed heat energy to the incident heat energy, ρ is the *reflectivity* which is defined as the ratio of reflected heat energy to the incident heat energy and τ is the *transmissivity* which is defined as the ratio of transmitted heat energy to the incident.

- **Black body:** It is the body which absorbs all the incident radiation. Hence, for a black body $\alpha = 1$, $\rho = 0$ and $\tau = 0$. As black body is a good absorber as well as good emitter, $\alpha = \epsilon = 1$.



- **Grey body:** It is defined as the body whose surface emissivity does not vary with temperature and wavelength of the radiation as in the case of the black body radiation but the value of emissivity is less than 1. Hence, for a grey body, $0 < \epsilon = \alpha < 1$.



- Differences between black body and grey body are:

Black body	Grey body
<ol style="list-style-type: none"> It is the body which absorbs all the incident radiation. It is a perfect absorber and emitter. Its surface emissivity does not vary with temperature as well as wavelength of the radiation. Its emissive power is higher than that of grey body. The value of emissivity is 1. Hence, for a black body, $\epsilon = \alpha = 1, \rho = 0, \tau = 1$. (E.g. graphite). 	<ol style="list-style-type: none"> It is the hypothetical body which cannot absorb all the incident radiation. It is not a perfect absorber and emitter. Its surface emissivity is assumed to be constant with temperature and wavelength of the radiation in contrary to the case of the non-grey body radiation. Its emissive power is less than that of black body. The value of emissivity is less than 1. Hence, for a grey body, $0 < \epsilon = \alpha < 1, \rho \neq 0, \tau \neq 0$.

- *White body*: If all the incident radiation falling on the body is reflected by its surface, then such a hypothetical body is called white body. Hence, for a white body, $\rho = 1, \alpha = \epsilon = 0$ and $\tau = 0$.
- *Opaque body*: If no incident radiation is transmitted through a body, then such a body is called opaque body. Hence, for an opaque body, $\tau = 0$ and $\alpha + \rho = 1$.
- *Transparent body*: If all the incident radiation is transmitted through a body, then such a body is called transparent body. Hence, for a transparent body, $\tau = 1$ and $\alpha + \rho = 0$.
- *Dimensionless Parameters*: In heat transfer studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless parameters or numbers* in order to reduce the number of total variables. There are many such dimensionless parameters in use in heat transfer analysis and some common dimensionless parameters are
 - *Reynolds number*: It is defined as the ratio of inertia force to dynamic viscous force in velocity boundary layer of the fluid, i.e.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho \bar{V}_{\text{avg}}^2 L^2}{\mu \bar{V}_{\text{avg}} L} = \frac{\rho \bar{V}_{\text{avg}} L}{\mu} = \frac{\bar{V}_{\text{avg}} L}{v}$$

where \bar{V}_{avg} is the average flow velocity, L is the characteristic length of the geometry, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

- *Nusselt's Number:* It is also common practice to make the heat transfer coefficient (h) dimensionless with the *Nusselt number*, and it is defined as

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h(T_2 - T_1)}{k(T_2 - T_1)/L} = \frac{hL}{k} = Nu$$

where k is the thermal conductivity of the fluid, and L is the characteristic length.

The Nusselt number can be defined as the ratio of heat transfer by convection mode to heat transfer by conduction mode across the fluid layer of thickness L . In other words, the Nusselt number therefore represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

- *Prandtl Number:* The relative thickness of the velocity boundary layer and the thermal boundary layer is best described by the dimensionless parameter *Prandtl number*, and it is defined as

$$Pr = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/(\rho c_p)} = \frac{\mu c_p}{k}$$

8.2 Solved Numerical Problems

Problem 8.1.

Determine the rate of heat loss from a brick wall ($k = 0.7 \text{ W/mK}$) of length 5 m, height 4 m and 0.25 m thick. The temperature of the inner surface is 120°C and that of outer surface is 30°C . Also calculate the temperature at the surface at 0.15 m from the inner surface.

Solution: Given,

Thermal conductivity of wall, $k = 0.7 \text{ W/mK}$

Length of wall, $l = 5 \text{ m}$

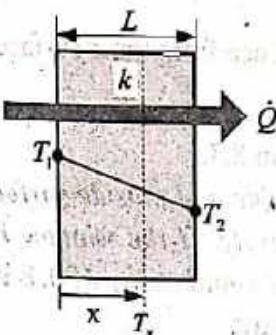
Height of wall, $H = 4 \text{ m}$

Thickness of wall, $L = 0.25 \text{ m}$

Temperature of inner surface, $T_1 = 120^\circ\text{C}$

Temperature of outer surface, $T_2 = 30^\circ\text{C}$

Applying Fourier law of conduction, we have



$$\dot{Q} = \frac{kA}{L}(T_1 - T_2)$$

where cross-sectional area of the brick wall, $A = l \times H = 5 \times 4 = 20 \text{ m}^2$

$$\Rightarrow \text{Rate of heat loss, } \dot{Q} = \frac{0.7 \times 20}{0.25} (120 - 30) = 5040 \text{ W} = 5.04 \text{ kW}$$

Similarly, as rate of heat loss is same for $L = x$, $T_2 = T_x$

$$\dot{Q} = \frac{kA}{x}(T_1 - T_x) \Rightarrow 5.04 \times 10^3 = \frac{0.7 \times 20}{0.15} (120 - T_x)$$

\therefore Temperature at $x = 0.15 \text{ m}$ from inner surface, $T_x = 66^\circ\text{C}$

Problem 8.2.

A brick wall of length 5 m, height 4 m and 0.25 m thick has the thermal conductivity, $k = 0.7 \text{ W/mK}$. The temperature of the inner surface is 120°C and that of outer surface is 30°C . Calculate the distance from the inner surface at which temperature is 90°C .

Solution: Given,

Thermal conductivity of wall, $k = 0.7 \text{ W/mK}$

Length of wall, $l = 5 \text{ m}$

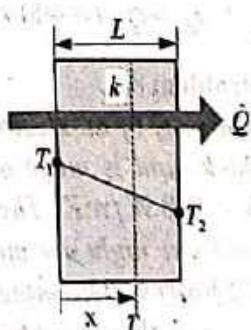
Height of wall, $H = 4 \text{ m}$

Thickness of wall, $L = 0.25 \text{ m}$

Temperature of inner surface, $T_1 = 120^\circ\text{C}$

Temperature of outer surface, $T_2 = 30^\circ\text{C}$

Applying Fourier law of conduction, we have



$$\dot{Q} = \frac{kA}{L}(T_1 - T_2)$$

where cross-sectional area of the brick wall, $A = l \times H = 5 \times 4 = 20 \text{ m}^2$

$$\Rightarrow \dot{Q} = \frac{0.7 \times 20}{0.25} (120 - 30) = 5040 \text{ W} = 5.04 \text{ kW}$$

Similarly, as rate of heat loss is same for $L = x$, $T_2 = T_x$

$$\dot{Q} = \frac{kA}{x}(T_1 - T_x) \Rightarrow 5.04 \times 10^3 = \frac{0.7 \times 20}{x} (120 - 90)$$

$$\therefore \text{Distance from inner surface at which temperature } T_x = 90^\circ\text{C}, x = \frac{0.7 \times 20}{5.04 \times 10^3} (120 - 90) = 8.3333 \text{ cm}$$

Problem 8.3.

The inside and outside surface temperatures of a window are at 20°C and 0°C , respectively. If the window is 80 cm by 50 cm and 1.6 cm thick and has a thermal conductivity of 0.8 W/mK . Determine the heat loss through the glass in 1 hour.

Solution: Given,

Inside surface temperature, $T_1 = 20^\circ\text{C}$

Outside surface temperature, $T_2 = 0^\circ\text{C}$

Length of window, $l = 80 \text{ cm} = 0.8 \text{ m}$

Height of window, $H = 50 \text{ cm} = 0.5 \text{ m}$

Thickness of window, $L = 1.6 \text{ cm} = 0.016 \text{ m}$

Thermal conductivity, $k = 0.8 \text{ W/mK}$

Applying Fourier law of conduction, heat loss per second is given by

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2)$$

where cross-sectional area of window, $A = l \times H = 0.8 \times 0.5 = 0.40 \text{ m}^2$

$$\therefore \dot{Q} = \frac{0.8 \times 0.4}{0.016} (20 - 0) = 400 \text{ W} = 400 \text{ J/s}$$

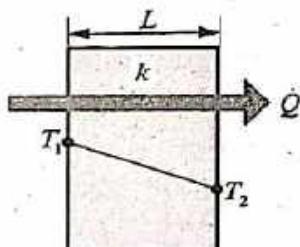
Hence, the heat loss in time, $t = 1 \text{ hour} = 60 \times 60 \text{ second}$ is calculated as

$$Q = \dot{Q} \times t = 400 \times 60 \times 60 = 1440000 \text{ J/hr} = 1440 \text{ kJ/hr}$$

Problem 8.4.

The roof of an electrically heated home is 10 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is $k = 0.8 \text{ W/mK}$. The temperatures of the inner and the outer surfaces of the roof one night are measured to be 18°C and 5°C , respectively, for a period of 12 hours. Determine:

- (a) the rate of heat loss through the roof that night and
- (b) the cost of that heat loss to the home owner if the cost of electricity is Rs 10/kWh.



Solution: Given,

Length of roof, $l = 10 \text{ m}$

Width of roof, $B = 8 \text{ m}$

Thickness of roof, $L = 0.25 \text{ m}$

Thermal conductivity, $k = 0.8 \text{ W/mK}$

Inner surface temperature, $T_1 = 18^\circ\text{C}$

Outer surface temperature, $T_2 = 5^\circ\text{C}$

If cost of electricity, $R_c = \text{Rs } 10/\text{kWh}$ and time, $t = 12 \text{ hr}$,

Applying Fourier law of conduction, heat loss per second is given by

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2)$$

where cross-sectional area of roof, $A = l \times B = 10 \times 8 = 80 \text{ m}^2$

$$\Rightarrow \dot{Q} = \frac{0.8 \times 80}{0.25}(18 - 5) = 3328 \text{ W} = 3.328 \text{ kW}$$

Hence, the cost of the heat loss in time, $t = 12 \text{ hour}$ is calculated as

$$C = \dot{Q} \times R_c \times t = 3.328 \times 10 \times 12 = \text{Rs } 399.36$$

Problem 8.5..

An electric current is passed through a wire 2 mm in diameter and 8 cm long. The wire is submerged in the liquid water. During the boiling of water temperature of water is 100°C and convection heat transfer coefficient is $4500 \text{ W/m}^2\text{K}$. Determine the power supplied to the wire to maintain the wire surface temperature at 120°C .

Solution: Given,

Diameter of wire, $D = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

Length of wire, $l = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$

Temperature of water, $T_\infty = 100^\circ\text{C}$

Wire surface temperature, $T_s = 120^\circ\text{C}$

Convective heat transfer coefficient, $h = 4500 \text{ W/m}^2\text{K}$

Electrical power supplied to the wire is equal to the rate of convection heat transfer from the wire outer surface to the liquid water. Hence,

$$P_{elec} = \dot{Q}_{conv} = h A (T_s - T_\infty)$$

where A is the peripheral surface area of wire, so

$$A = \pi D l = \pi \times 2 \times 10^{-3} \times 8 \times 10^{-2} = 5.02655 \times 10^{-4} \text{ m}^2$$

Then, power supplied to wire,

$$\begin{aligned} P_{elec} &= \dot{Q}_{conv} = h A (T_s - T_\infty) = 4500 \times 5.02655 \times 10^{-4} (120 - 100) \\ &= 45.2389 \text{ W} \end{aligned}$$

Problem 8.6.

A 2 m long, 0.35 cm diameter electrical wire extends across a room at 20°C . Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 150°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 50 V and 2 A, respectively. Neglecting the effect of heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

Solution: Given,

Length of wire, $l = 2 \text{ m}$

Diameter of wire, $D = 0.35 \text{ cm} = 0.35 \times 10^{-2} \text{ m}$

Temperature of room, $T_a = 20^{\circ}\text{C}$

Surface temperature of wire, $T_s = 150^{\circ}\text{C}$

Voltage drop across wire, $V = 50 \text{ V}$

Electric current through wire, $I = 2 \text{ A}$

The rate of convection heat transfer from the outer surface of the wire to the air in the room is equal to the electrical power supplied to the wire. Hence,

$$\dot{Q}_{conv} = hA(T_s - T_a) = P_{elec} = I \times V$$

where A is the peripheral surface area of wire, so

$$A = \pi D l = \pi \times 0.35 \times 10^{-2} \times 2 = 2.1991 \times 10^{-2} \text{ m}^2$$

Then, convective heat transfer coefficient,

$$h = \frac{I \times V}{A(T_s - T_a)} = \frac{2 \times 50}{2.1991 \times 10^{-2} (150 - 20)} = 34.979 \text{ W/m}^2\text{K}$$

Problem 8.7.

A steel pipe having an outer diameter of 4 cm is maintained at a temperature of 80°C in a room where the ambient temperature is 25°C . The emissivity of the surface is 0.8 and the convection heat transfer coefficient between the surface and air is $10 \text{ W/m}^2\text{K}$. Determine the total heat loss from the unit length of the pipe. [Take $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$]

Solution: Given,

Outer diameter of pipe, $D = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$

Surface temperature of pipe, $T_s = 80^{\circ}\text{C} = 80 + 273 = 353 \text{ K}$

Ambient temperature, $T_a = 25^{\circ}\text{C} = 25 + 273 = 298 \text{ K}$

Surface Emissivity, $\varepsilon = 0.8$

Convection heat transfer coefficient, $h = 10 \text{ W/m}^2\text{K}$

Total heat loss from the surface of the pipe to the ambient is equal to the sum of the heat loss by the convection and that by the radiation, i.e.

$$\dot{Q}_t = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_s - T_a) + \varepsilon \sigma A(T_s^4 - T_a^4)$$

where A is the peripheral surface area of the pipe, i.e. $A = \pi Dl$.

Then,

$$\dot{Q}_r = h\pi Dl(T_s - T_\infty) + \epsilon\sigma\pi Dl(T_s^4 - T_\infty^4)$$

\Rightarrow Total heat loss per unit length of pipe,

$$\frac{\dot{Q}_r}{l} = h\pi D(T_s - T_\infty) + \epsilon\sigma\pi D(T_s^4 - T_\infty^4)$$

$$\begin{aligned} \text{and } \dot{Q}_r &= 10 \times \pi \times 4 \times 10^{-2} (353 - 298) + 0.8 \times 5.67 \times 10^{-8} \times 4 \times 10^{-2} (353^4 - 298^4) \\ &= 69.115 + 43.556 = 112.671 \text{ W/m} \end{aligned}$$

Problem 8.8.

The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 5°C in winter and 26°C in summer. Determine the rate of radiation heat transfer between a person inside the house and the surrounding surfaces if the exposed surface area, the average outer surface temperature and the surface emissivity of the person are 1.4 m^2 and 30°C and 0.95 , respectively. [Take $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$]

Solution: Given,

Inner surface temperature in winter, $T_{\infty w} = 5^\circ\text{C} = 5 + 273 = 278 \text{ K}$

Inner surface temperature in summer, $T_{\infty s} = 26^\circ\text{C} = 26 + 273 = 299 \text{ K}$

Outer surface temperature of person, $T_s = 30^\circ\text{C} = 30 + 273 = 303 \text{ K}$

Exposed surface area, $A = 1.4 \text{ m}^2$

Surface emissivity, $\epsilon = 0.95$

The rate of radiation heat transfer in winter is given by

$$\dot{Q}_{rad,w} = \epsilon\sigma A(T_s^4 - T_{\infty w}^4) = 0.95 \times 5.67 \times 10^{-8} \times 1.4 (303^4 - 278^4) = 185.215 \text{ W}$$

Similarly, the rate of radiation heat transfer in summer is expressed as

$$\dot{Q}_{rad,s} = \epsilon\sigma A(T_s^4 - T_{\infty s}^4) = 0.95 \times 5.67 \times 10^{-8} \times 1.4 (303^4 - 299^4) = 32.906 \text{ W}$$

Problem 8.9.

A flat plate solar collector is insulated at the back surface and exposed to solar radiation at the front surface. The front surface absorbs solar radiation at a rate of 800 W/m^2 and losses heat by convection to the ambient air at 25°C . If the heat transfer coefficient between the plate and the air is $20 \text{ W/m}^2\text{K}$, determine the surface temperature of the plate.

Solution: Given,

Rate of heat absorption per unit area, $\dot{Q}_{abs}/A = 800 \text{ W/m}^2$

Ambient air temperature, $T_\infty = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$

Convection heat transfer coefficient, $h = 20 \text{ W/m}^2\text{K}$

Here, the rate of the heat absorption of the solar radiation by the plate surface should be equal to the rate of heat loss by the convection from the plate of solar collector to the ambient air, i.e.

$$\begin{aligned}\dot{Q}_{\text{abs}} &= \dot{Q}_{\text{conv}} = hA(T_s - T_\infty) \\ \Rightarrow \frac{\dot{Q}_{\text{abs}}}{A} &= h(T_s - T_\infty) \Rightarrow 800 = 20(T_s - 298) \\ \therefore \text{Surface temperature of plate, } T_s &= 338 \text{ K} = 65^\circ\text{C}\end{aligned}$$

Problem 8.10.

The inner surface of a 0.2 m thick wall ($k = 1 \text{ W/mK}$) is exposed to hot combustion gas and its outer surface is exposed to ambient air at 20°C . The emissivity of the wall surface is 0.8 and convection heat transfer coefficient for the wall surface and air is $25 \text{ W/m}^2\text{K}$. Under steady state condition, temperature at the outer surface of the wall is found as 75°C . Determine the wall inner surface temperature. [Take $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$]

Solution: Given,

Thickness of wall, $L = 0.2 \text{ m}$

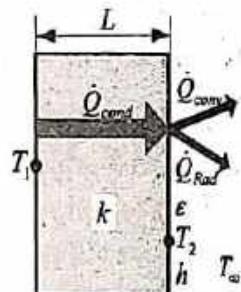
Thermal conductivity of wall, $k = 1 \text{ W/mK}$

Ambient air temperature, $T_\infty = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$

Surface emissivity of wall, $\epsilon = 0.8$

Convection heat transfer coefficient, $h = 25 \text{ W/m}^2\text{K}$

Outer surface temperature of wall, $T_2 = 75^\circ\text{C} = 75 + 273 = 348 \text{ K}$



Here, the rate of heat transfer by the conduction through the wall should be equal to the rate of heat loss by the convection plus radiation from the outer surface of the wall to the ambient air, i.e.

$$\begin{aligned}\dot{Q}_{\text{cond}} &= \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA(T_2 - T_\infty) + \epsilon\sigma A(T_2^4 - T_\infty^4) \\ &= \frac{kA}{L}(T_1 - T_2) = hA(T_2 - T_\infty) + \epsilon\sigma A(T_2^4 - T_\infty^4) \\ &\Rightarrow \frac{1}{0.2}(T_1 - 348) = 25(348 - 293) + 0.8 \times 5.67 \times 10^{-8} (348^4 - 293^4)\end{aligned}$$

\therefore Inner surface temperature of wall, $T_1 = 689.191 \text{ K} = 416.191^\circ\text{C}$

Problem 8.11.

The inner surface of a 2 cm thick $50 \text{ cm} \times 50 \text{ cm}$ plate ($k = 10 \text{ W/mK}$) is at 400°C . The outer surface dissipates heat by combined convection and radiation to the ambient air at 27°C . If the plate surface has an emissivity 0.85 and the convection heat transfer coefficient between the outer plate surface and the ambient air is $20 \text{ W/m}^2\text{K}$, determine the outer surface temperature of the plate.

Solution: Given,

Thickness of plate, $L = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

Thermal conductivity of plate, $k = 10 \text{ W/mK}$

Inner surface temperature of plate $T_1 = 400^\circ\text{C} = 400 + 273 = 673 \text{ K}$

Ambient air temperature, $T_\infty = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$

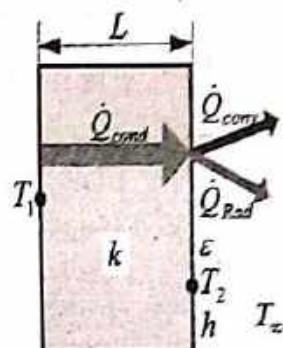
Surface emissivity of plate, $\epsilon = 0.85$

Convection heat transfer coefficient, $h = 20 \text{ W/m}^2\text{K}$

Here, the rate of heat transfer by the conduction through the plate should be equal to the rate of heat dissipation by the convection plus radiation from the outer surface of the plate to the ambient air, i.e.

$$\dot{Q}_{cond} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA(T_2 - T_\infty) + \epsilon\sigma A(T_2^4 - T_\infty^4)$$

$$\Rightarrow \frac{kA}{L}(T_1 - T_2) = hA(T_2 - T_\infty) + \epsilon\sigma A(T_2^4 - T_\infty^4)$$



$$\Rightarrow \frac{10}{2 \times 10^{-2}}(673 - T_2) = 20(T_2 - 300) + 0.85 \times 5.67 \times 10^{-8}(T_2^4 - 300^4)$$

$$\Rightarrow 4.8195 \times 10^{-8} T_2^4 + 520 T_2 - 342890.3795 = 0$$

∴ Outer surface temperature of plate, $T_2 = 643.511 \text{ K} = 370.511^\circ\text{C}$

Problem 8.12.

A furnace wall 300 mm thick is made up of an inner layer of fire brick ($k = 1 \text{ W/mK}$) covered with a layer of insulation ($k = 0.2 \text{ W/mK}$). The inner surface of the wall is at 1300°C and the outer surface is at 30°C . Under steady state condition, temperature at the interface is measured to be 1100°C .

Determine:

- (a) heat loss per unit area of the wall, and
- (b) the thickness of each layer.

Solution: Given,

Thickness of furnace wall, $L = 300 \text{ mm}$

$$= 300 \times 10^{-3} \text{ m}$$

Thermal conductivity of fire brick, $k_1 = 1 \text{ W/mK}$

Thermal conductivity of insulation, $k_2 = 0.2 \text{ W/mK}$

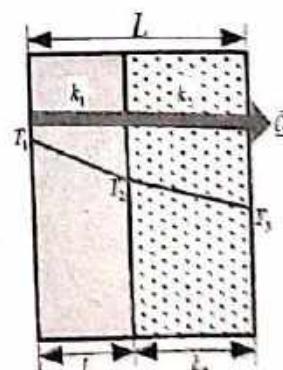
Inner surface temperature of wall, $T_1 = 1300^\circ\text{C}$

Outer surface temperature of wall, $T_3 = 30^\circ\text{C}$

Interface temperature, $T_2 = 1100^\circ\text{C}$

The rate of heat transfer through the composite structure with two plane layers is given by

$$\dot{Q} = \frac{A(T_1 - T_3)}{(L_1/k_1) + (L_2/k_2)}$$



$$\Rightarrow \frac{\dot{Q}}{A} = \frac{(T_1 - T_3)}{(L_1/k_1) + (L_2/k_2)} = \frac{1300 - 30}{(L_1/1) + (L_2/0.2)} = \frac{1270}{L_1 + 5L_2} \quad \dots \dots \dots \text{(i)}$$

Considering the first layer of fire brick only, the same rate of heat transfer is expressed as

$$\begin{aligned} \dot{Q} &= \frac{A(T_1 - T_2)}{L_1/k_1} \Rightarrow \frac{\dot{Q}}{A} = \frac{k_1(T_1 - T_2)}{L_1} \\ &\Rightarrow \frac{1270}{L_1 + 5L_2} = \frac{1(1300 - 1100)}{L_1} \end{aligned} \quad \dots \dots \dots \text{(ii)}$$

We know that the thickness of furnace wall L is equal to the thickness of the fire brick L_1 plus thickness of the insulation L_2 , i.e.

$$L = L_1 + L_2 = 300 \times 10^{-3} \quad \dots \dots \dots \text{(iii)}$$

Solving equations (ii) and (iii), we get

$$L_1 = 0.14493 \text{ m} = 144.93 \text{ mm} \text{ and } L_2 = 0.15507 \text{ m} = 155.07 \text{ mm}$$

Putting these values in equation (i), then heat transfer rate per unit area,

$$\frac{\dot{Q}}{A} = \frac{1270}{L_1 + 5L_2} = \frac{1270}{0.14493 + 5 \times 0.15507} = 1380.015 \text{ W/m}^2$$

Problem 8.13.

The walls of a furnace $4 \text{ m} \times 3 \text{ m}$ are constructed from an inner fire brick ($k = 0.4 \text{ W/mK}$) wall 30 cm thick, a layer of ceramic blanket insulation ($k = 0.2 \text{ W/mK}$) 10 cm thick and steel protective layer ($k = 50 \text{ W/mK}$) 4 mm thick. The inside temperature of the fire brick layer was measured as 500°C and the temperature of the outside of the insulation as 50°C . Determine

(a) the rate of heat loss through the wall,
(b) the temperature at the interface between fire brick layer and insulation layer,

(c) the temperature at the outside surface of the steel layer.

Solution: Given,

Area of furnace wall, $A = 4 \times 3 = 12 \text{ m}^2$

Thermal conductivity of inner fire brick,

$$k_1 = 0.4 \text{ W/mK}$$

Thickness of inner fire brick, $L_1 = 30 \text{ cm} = 0.3 \text{ m}$

Thermal conductivity of ceramic blanket insulation,

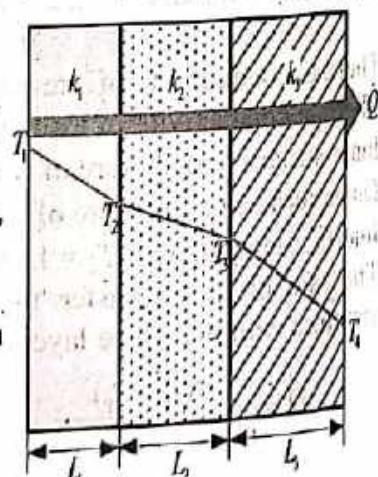
$$k_2 = 0.2 \text{ W/mK}$$

Thickness of ceramic blanket insulation, $L_2 = 10 \text{ cm} = 0.1 \text{ m}$

Thermal conductivity of steel protective layer,

$$k_3 = 50 \text{ W/mK}$$

Thickness of steel protective layer, $L_3 = 4 \text{ mm} = 0.004 \text{ m}$



Inside temperature of inner fire brick, $T_1 = 500^\circ\text{C}$

Outside temperature of ceramic blanket insulation, $T_3 = 50^\circ\text{C}$

Considering the first two layers only, the rate of heat loss through the furnace wall is given by

$$\dot{Q} = \frac{A(T_1 - T_3)}{(L_1/k_1) + (L_2/k_2)} = \frac{12(500 - 50)}{(0.3/0.4) + (0.1/0.2)} = 4320 \text{ W} = 4.32 \text{ kW}$$

Considering the first layer only, the same rate of heat loss through the furnace wall can be expressed as

$$\dot{Q} = \frac{k_1 A(T_1 - T_2)}{L_1} \Rightarrow 4320 = \frac{0.4 \times 12 \times (500 - T_2)}{0.3}$$

\therefore Interface temperature between inner fire brick and ceramic blanket insulation,

$$T_2 = 230^\circ\text{C}$$

Considering the last layer only, the same rate of heat loss through furnace wall can also be determined as

$$\dot{Q} = \frac{k_3 A(T_3 - T_4)}{L_3} \Rightarrow 4320 = \frac{50 \times 12 \times (50 - T_4)}{0.004}$$

\therefore Outside temperature of steel protective layer, $T_4 = 49.9712^\circ\text{C}$

Problem 8.14.

Find the heat transfer through the composite wall as shown in figure below. Assume one dimensional flow. The thermal conductivities of wall materials are $k_A = 150 \text{ W/mK}$, $k_B = 30 \text{ W/mK}$, $k_C = 65 \text{ W/mK}$ and $k_D = 50 \text{ W/mK}$. All dimensions are in cm.

Solution: Given,

Thermal conductivity of layer A, $k_A = 150 \text{ W/mK}$

From figure, thickness of layer A, $L_A = 30 \text{ cm}$

$$= 0.3 \text{ m}$$

Area of layer A, $A_A = 100 \text{ cm} \times (60 + 40) \text{ cm}$

$$= 10000 \text{ cm}^2 = 1 \text{ m}^2$$

Thermal conductivity of layer B, $k_B = 30 \text{ W/mK}$

From figure, thickness of layer B, $L_B = 80 \text{ cm}$

$$= 0.8 \text{ m}$$

Area of layer B, $A_B = 100 \text{ cm} \times 40 \text{ cm} = 4000 \text{ cm}^2$

$$= 0.4 \text{ m}^2$$

Thermal conductivity of layer C, $k_C = 65 \text{ W/mK}$

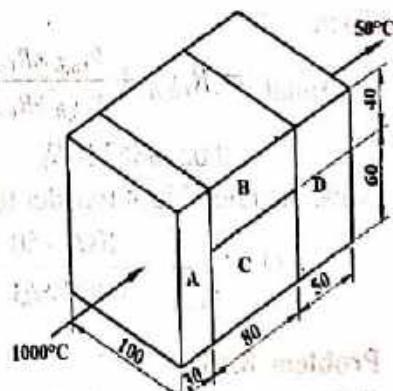
From figure, thickness of layer C, $L_C = 80 \text{ cm} = 0.8 \text{ m}$

Area of layer C, $A_C = 100 \text{ cm} \times 60 \text{ cm} = 6000 \text{ cm}^2 = 0.6 \text{ m}^2$

Thermal conductivity of layer D, $k_D = 50 \text{ W/mK}$

From figure, thickness of layer D, $L_D = 50 \text{ cm} = 0.5 \text{ m}$

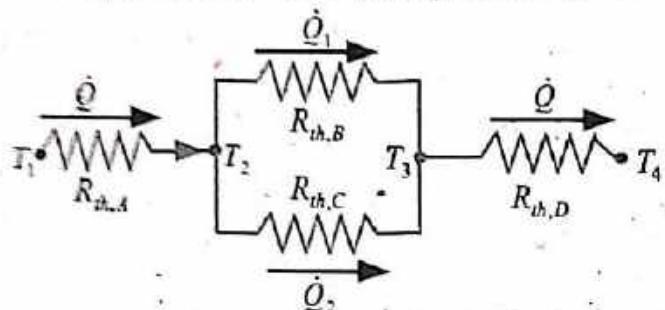
Area of layer D, $A_D = 100 \text{ cm} \times (60 + 40) \text{ cm} = 10000 \text{ cm}^2 = 1 \text{ m}^2$



Inlet temperature of composite wall, $T_1 = 1000^\circ\text{C}$

Exit temperature of composite wall, $T_4 = 50^\circ\text{C}$

Applying the electrical analogy approach for heat transfer through a composite structure, we have an equivalent circuit of given problem as



The total thermal resistance of the circuit for heat transfer is given by

$$R_{\text{total}} = R_{\text{th},A} + \frac{R_{\text{th},B} \times R_{\text{th},C}}{R_{\text{th},B} + R_{\text{th},C}} + R_{\text{th},D}$$

where thermal resistances for each layer of plane wall are calculated as

$$R_{\text{th},A} = \frac{L_A}{A_A k_A} = \frac{0.3}{1 \times 150} = 0.002 \text{ K/W}$$

$$R_{\text{th},B} = \frac{L_B}{A_B k_B} = \frac{0.8}{0.4 \times 30} = 0.06666 \text{ K/W}$$

$$R_{\text{th},C} = \frac{L_C}{A_C k_C} = \frac{0.8}{0.6 \times 65} = 0.0205128 \text{ K/W}$$

$$R_{\text{th},D} = \frac{L_D}{A_D k_D} = \frac{0.5}{1 \times 50} = 0.01 \text{ K/W}$$

Then,

$$\begin{aligned} R_{\text{total}} &= R_{\text{th},A} + \frac{R_{\text{th},B} \times R_{\text{th},C}}{R_{\text{th},B} + R_{\text{th},C}} + R_{\text{th},D} = 0.002 + \frac{0.06666 \times 0.0205128}{0.06666 + 0.0205128} + 0.01 \\ &= 0.0276863 \text{ K/W} \end{aligned}$$

Now, the rate of heat transfer through the composite wall is given by

$$\dot{Q} = \frac{T_i - T_e}{R_{\text{total}}} = \frac{1000 - 50}{0.0276863} = 34313 \text{ W} = 34.313 \text{ kW}$$

Problem 8.15.

A furnace wall is made of a layer of fire clay ($k = 0.5 \text{ W/mK}$) 12.5 cm thick and a layer of red brick ($k = 0.8 \text{ W/mK}$) 50 cm thick. If the wall temperature inside the furnace is 1200°C and that on the outside wall is 100°C . Determine the heat loss per unit area of the wall. If it is desired to reduce the thickness of the red brick layer by filling the space between the two layers by diatomite ($k = 0.1 \text{ W/mK}$) such that total thickness remains same. Determine the required thickness of the filling to ensure the amount of heat transfer for same temperature difference is reduced by 60 %.

Solution: Given,

Thermal conductivity of fire clay, $k_1 = 0.5 \text{ W/mK}$

Thickness of fire clay, $L_1 = 12.5 \text{ cm} = 0.125 \text{ m}$

Thermal conductivity of red brick, $k_2 = 0.8 \text{ W/mK}$

Thickness of red brick, $L_2 = 50 \text{ cm} = 0.5 \text{ m}$

Inside temperature of furnace wall, $T_1 = 1200^\circ\text{C}$

Outside temperature of furnace wall, $T_3 = 100^\circ\text{C}$

When the space between two layers is filled by diatomite such that total thickness remains same, i.e.

$$L_2' = L_2 - L_3 \Rightarrow L_2' = L_2 - L_3 = 0.5 - L_3$$

where L_2' is the reduced thickness of the red brick.

Rate of heat transfer is reduced by 60%, i.e. $\dot{Q}_2 = 40\% \text{ of } \dot{Q}_1$

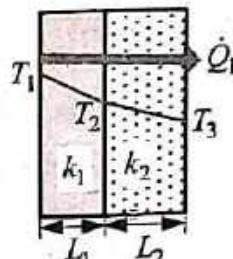
Thermal conductivity of diatomite, $k_3 = 0.1 \text{ W/mK}$

Case I: The rate of heat loss through furnace wall with two layers,

$$\dot{Q}_1 = \frac{A(T_1 - T_3)}{(L_1/k_1) + (L_2/k_2)}$$

\Rightarrow Rate of heat loss per unit area through the

$$\begin{aligned} \text{furnace wall, } \frac{\dot{Q}_1}{A} &= \frac{(T_1 - T_3)}{(L_1/k_1) + (L_2/k_2)} \\ &= \frac{1200 - 100}{(0.125/0.5) + (0.5/0.8)} \\ &= 1257.1428 \text{ W/m}^2 \end{aligned}$$



Case II: The rate of heat loss through furnace wall with three layers,

$$\dot{Q}_2 = \frac{A(T_1 - T_4)}{(L_1/k_1) + (L_3/k_3) + (L_2/k_2)}$$

\Rightarrow The rate of heat loss per unit area,

$$\frac{\dot{Q}_2}{A} = \frac{(T_1 - T_3)}{(L_1/k_1) + (L_3/k_3) + (L_2/k_2)}$$

As $(T_1 - T_3) = (T_1 - T_4)$ for same temperature difference.

$$\Rightarrow \frac{\dot{Q}_2}{A} = \frac{(1200 - 100)}{(0.125/0.5) + (L_3/0.1) + ((0.5 - L_3)/0.8)}$$

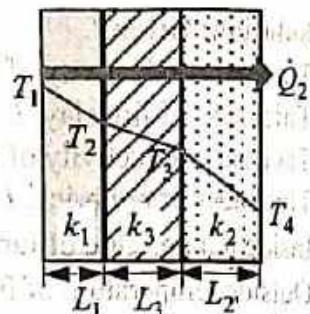
According to question, we have

$$\dot{Q}_2 = (100 - 60)\% \text{ of } \dot{Q}_1 = 40\% \text{ of } \dot{Q}_1$$

$$\begin{aligned} \Rightarrow \frac{\dot{Q}_2}{A} &= 0.4 \frac{\dot{Q}_1}{A} = 0.4 \times 1257.1428 \\ &= 502.857 \text{ W/m}^2 \end{aligned}$$

$$\Rightarrow \frac{(1200 - 100)}{(0.125/0.5) + (L_3/0.1) + ((0.5 - L_3)/0.8)} = 502.857$$

\therefore Thickness of diatomite, $L_3 = 0.15 \text{ m} = 15 \text{ cm}$



Problem 8.16.

An exterior wall of a house consists of 10 cm of common brick ($k = 0.8 \text{ W/mK}$) followed by a 4 cm layer of gypsum plaster ($k = 0.5 \text{ W/mK}$). What thickness of rock wool insulation ($k = 0.065 \text{ W/mK}$) should be added to reduce the heat transfer through the wall by 50 %?

Solution: Given,

Thermal conductivity of common brick,

$$k_1 = 0.8 \text{ W/mK}$$

Thickness of common brick, $L_1 = 10 \text{ cm} = 0.1 \text{ m}$

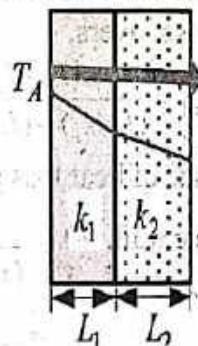
Thermal conductivity of gypsum plaster,

$$k_2 = 0.5 \text{ W/mK}$$

Thickness of gypsum plaster, $L_2 = 4 \text{ cm} = 0.04 \text{ m}$

Thermal conductivity of rock wool insulation,

$$k_3 = 0.065 \text{ W/mK}$$



Case I: Heat transfer without rock wool insulation

Let us assume that the inside and outside temperatures of the wall of the house are T_A and T_B respectively and their difference remains same after adding rock wool insulation. The rate of heat transfer through the wall is given by

$$\dot{Q}_I = \frac{A(T_A - T_B)}{(L_1/k_1) + (L_2/k_2)} = \frac{A(T_A - T_B)}{(0.1/0.8) + (0.04/0.5)}$$

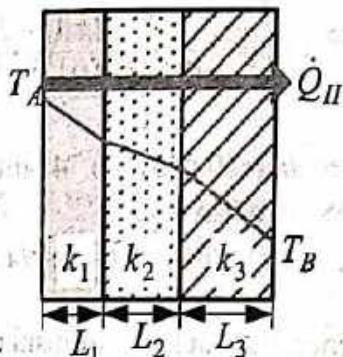
Case II: Heat transfer with rock wool insulation

The rate of heat transfer through the wall is given by

$$\begin{aligned}\dot{Q}_{II} &= \frac{A(T_A - T_B)}{(L_1/k_1) + (L_2/k_2) + (L_3/k_3)} \\ &= \frac{A(T_A - T_B)}{(0.1/0.8) + (0.04/0.5) + (L_3/0.065)}\end{aligned}$$

According to the question, we have

$$\begin{aligned}\dot{Q}_H &= (100 - 50) \% \text{ of } \dot{Q}_I \\ \Rightarrow \frac{A(T_A - T_B)}{(0.1/0.8) + (0.04/0.5) + (L_3/0.065)} \\ &= 0.5 \times \frac{A(T_A - T_B)}{(0.1/0.8) + (0.04/0.5)}\end{aligned}$$



$$\therefore \text{Thickness of rock wool insulation, } L_3 = 0.0133253 \text{ m} = 1.33253 \text{ cm}$$

Problem 8.17.

A steel pipe ($k = 45.5 \text{ W/mK}$) with outer diameter of 90 mm and thickness 3 mm is used for flow of brine at -22°C . The pipe may be insulated by any one of the two types of insulation. Insulation I has $k_1 = 0.037 \text{ W/mK}$ and insulation II has $k_2 = 0.047 \text{ W/mK}$. If one of these insulations has to be used for pipe insulation so that maximum heat transfer is to be limited to 11.6 W/m of pipe and the temperature of insulation at the outer surface could be maintained not less than 15°C , determine the required thickness of insulation for each case.

Solution: Given,

Thermal conductivity of steel pipe, $k = 45.5 \text{ W/mK}$

Outer diameter of steel pipe, $D_2 = 90 \text{ mm}$

Thickness of steel pipe, $(r_2 - r_1) = 3 \text{ mm} = 0.003 \text{ m}$

Inner surface temperature of steel pipe, $T_i = -22^\circ\text{C}$

Maximum rate of heat transfer per unit length of pipe, $\dot{Q}/L = 11.6 \text{ W/m}$

Outer surface temperature of steel pipe, $T_o = 15^\circ\text{C}$

Thermal conductivity of insulation I, $k_1 = 0.037 \text{ W/mK}$

Thermal conductivity of insulation II, $k_2 = 0.047 \text{ W/mK}$

Outer radius of steel pipe, $r_2 = D_2/2 = 90/2 = 45 \text{ mm} = 0.045 \text{ m}$

Inner radius of steel pipe, $r_1 = r_2 - 3 = 0.045 - 0.003 = 0.042 \text{ m}$

Case I: Using insulation I for pipe insulation

The rate of heat transfer through the steel pipe is given by

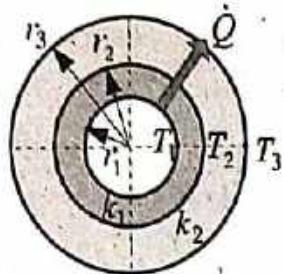
$$\begin{aligned}\dot{Q} &= \frac{2\pi L(T_3 - T_1)}{\frac{\ln(r_2/r_1)}{k} + \frac{\ln(r_3/r_2)}{k_1}} \\ \Rightarrow \frac{\dot{Q}}{L} &= \frac{2\pi \{15 - (-22)\}}{\frac{\ln(0.045/0.042)}{45.5} + \frac{\ln(r_3/0.045)}{0.037}}\end{aligned}$$

$$\Rightarrow 11.6 = \frac{2\pi \{15 + 22\}}{\ln(0.045/0.042) + \ln(r_3/0.045)} \\ 45.5 \quad \quad \quad 0.037$$

$$\Rightarrow \ln(r_3/0.045) = 0.741468$$

$$\Rightarrow r_3/0.045 = e^{0.741468}$$

$$\therefore r_3 = 0.09445566 \text{ m} = 94.45566 \text{ mm}$$

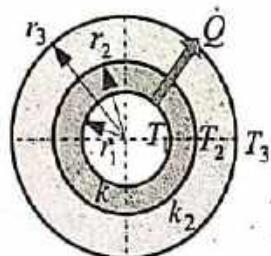


Hence, the thickness of insulation I, $(r_3 - r_2)_I = 94.45566 - 45 = 49.45566 \text{ mm}$

Case II: Using insulation II for pipe insulation

The rate of heat transfer through the steel pipe is given by

$$\begin{aligned} \dot{Q} &= \frac{2\pi L(T_3 - T_1)}{\frac{\ln(r_2/r_1)}{k} + \frac{\ln(r_3/r_2)}{k_2}} \\ \Rightarrow \frac{\dot{Q}}{L} &= \frac{2\pi \{15 - (-22)\}}{\frac{\ln(0.045/0.042)}{45.5} + \frac{\ln(r_3/0.045)}{0.047}} \\ \Rightarrow 11.6 &= \frac{2\pi \{15 + 22\}}{\frac{\ln(0.045/0.042)}{45.5} + \frac{\ln(r_3/0.045)}{0.047}} \\ \Rightarrow \ln(r_3/0.045) &= 0.941865 \\ \Rightarrow r_3/0.045 &= e^{0.941865} \\ \therefore r_3 &= 0.1154142 \text{ m} = 115.4142 \text{ mm} \end{aligned}$$



Hence, the thickness of insulation II, $(r_3 - r_2)_{II} = 115.4142 - 45 = 70.4142 \text{ mm}$

Problem 8.18.

A cast iron pipe ($k = 25 \text{ W/mK}$) with inner and outer diameters of 60 mm and 70 mm respectively is covered by an insulator ($k = 0.05 \text{ W/mK}$). Under steady state condition, temperature between the pipe and insulator interface is found to be 250°C . The allowable heat loss from the unit length of the pipe is 500 W/m and outer surface temperature of the insulator should not exceed 50°C . Determine:

- (a) minimum thickness of the insulation required, and
- (b) temperature at the inner surface of the pipe.

Solution: Given,

Thermal conductivity of pipe, $k_1 = 25 \text{ W/mK}$

Inner diameter of pipe, $D_1 = 60 \text{ mm} = 0.06 \text{ m}$

Outer diameter of pipe, $D_2 = 70 \text{ mm} = 0.07 \text{ m}$

Thermal conductivity of insulator, $k_2 = 0.05 \text{ W/mK}$

Interface temperature between pipe and insulator, $T_2 = 250^\circ\text{C}$

Rate of heat loss per unit length of pipe, $\dot{Q}/L = 500 \text{ W/m}$

Outer surface temperature of insulator, $T_3 = 50^\circ\text{C}$

Inner radius of pipe, $r_1 = D_1/2 = 0.06/2 = 0.03 \text{ m}$

Outer radius of pipe, $r_2 = D_2/2 = 0.07/2 = 0.035 \text{ m}$

Considering the insulation layer only, the rate of heat loss through the insulator is given by

$$\dot{Q} = \frac{2\pi L(T_2 - T_3)}{\ln(r_3/r_2)}$$

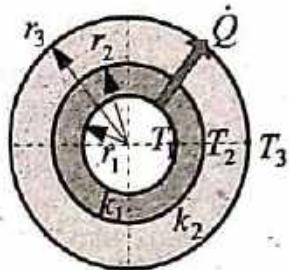
$$\Rightarrow \frac{\dot{Q}}{L} = \frac{2\pi k_2(T_2 - T_3)}{\ln(r_3/r_2)}$$

$$\Rightarrow 500 = \frac{2\pi \times 0.05(250 - 50)}{\ln(r_3/0.035)}$$

$$\Rightarrow \ln(r_3/0.035) = 0.1256637$$

$$\Rightarrow r_3/0.035 = e^{0.1256637}$$

$$\therefore r_3 = 0.0396865 \text{ m} = 39.6865 \text{ mm}$$



Hence, the minimum thickness of insulator, $r_3 - r_2 = 39.6865 - 35 = 4.6865 \text{ mm}$

Again, considering single layer of the pipe only, the same rate of heat loss through the pipe is given by

$$\dot{Q} = \frac{2\pi L(T_1 - T_2)}{\ln(r_2/r_1)}$$

$$\Rightarrow \frac{\dot{Q}}{L} = \frac{2\pi k_1(T_1 - T_2)}{\ln(r_2/r_1)} \quad \Rightarrow \quad 500 = \frac{2\pi \times 25(T_1 - 250)}{\ln(0.035/0.03)}$$

$$\therefore \text{Inner surface temperature of pipe, } T_1 = 250.491^\circ\text{C}$$

Problem 8.19.

A gas turbine blade is modeled as a flat plate. The thermal conductivity of the blade material is 15 W/mK and its thickness is 1.5 mm . The upper surface of the blade is exposed to hot gases at 1000°C and the lower surface is cooled by air bled of the compressor. The heat transfer coefficients at the upper and lower surfaces of the blade are $2500 \text{ W/m}^2\text{K}$ and $1500 \text{ W/m}^2\text{K}$ respectively. Under steady state conditions, the temperature at the upper surface of the blade is measured as 850°C , determine the temperature of the coolant air.

Solution: Given,

Thermal conductivity of turbine blade, $k = 15 \text{ W/mK}$

Thickness of turbine blade, $L = 1.5 \text{ mm} = 0.0015 \text{ m}$

Temperature of hot gases on upper surface of blade, $T_{x1} = 1000^\circ\text{C}$

Heat transfer coefficient at upper surface of blade, $h_1 = 2500 \text{ W/m}^2\text{K}$

Heat transfer coefficient at lower surface of blade, $h_2 = 1500 \text{ W/m}^2\text{K}$

Upper surface temperature of blade, $T_1 = 850^\circ\text{C}$

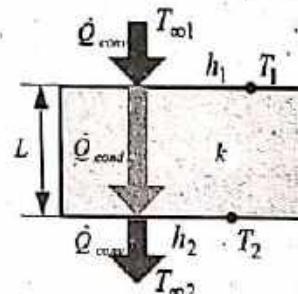
The rate of heat transfer by convection between the hot gases and the upper surface of the turbine blade is given by Newton's law of cooling,

$$\begin{aligned}\dot{Q} &= h_1 A(T_{\infty 1} - T_1) \\ \Rightarrow \frac{\dot{Q}}{A} &= h_1(T_{\infty 1} - T_1) = 2500(1000 - 850) = 375000 \text{ W/m}^2\end{aligned}$$

The same rate of heat transfer through the turbine blade is given by Fourier's law of conduction,

$$\begin{aligned}\dot{Q} &= \frac{kA}{L}(T_1 - T_2) \\ \Rightarrow \frac{\dot{Q}}{A} &= \frac{k}{L}(T_1 - T_2) \\ \Rightarrow 375000 &= \frac{15}{0.0015}(850 - T_2)\end{aligned}$$

\therefore Lower surface temperature of blade,
 $T_2 = 812.5^\circ\text{C}$



Similarly, the same rate of heat transfer by convection between the lower surface of the turbine blade and the coolant air from the compressor is given by Newton's law of cooling,

$$\begin{aligned}\dot{Q} &= h_2 A(T_2 - T_{\infty 2}) \\ \Rightarrow \frac{\dot{Q}}{A} &= h_2(T_2 - T_{\infty 2}) \\ \Rightarrow 375000 &= 1500(812.5 - T_{\infty 2})\end{aligned}$$

\therefore Temperature of coolant air on lower surface of blade, $T_{\infty 2} = 562.5^\circ\text{C}$

Problem 8.20.

The inside surface of an insulating layer is at 300°C and the outside surface is dissipating heat by convection into air at 25°C . The insulating layer has a thickness 5 cm and thermal conductivity of 0.8 W/mK . What is the minimum heat transfer coefficient at the outside surface if the outside surface temperature should not exceed 100°C ?

Solution. Given,

Inside surface temperature of insulating layer, $T_i = 300^\circ\text{C}$

Temperature of surrounding air, $T_{\infty} = 25^\circ\text{C}$

Thickness of insulating layer, $L = 5 \text{ cm} = 0.05 \text{ m}$

Thermal conductivity of insulating layer, $k = 0.8 \text{ W/mK}$

Outside surface temperature of insulating layer, $T_s = 100^\circ\text{C}$

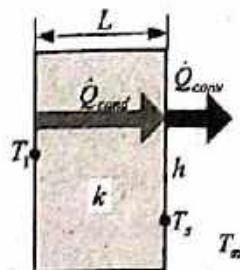
Here, the rate of heat transfer through the insulating layer by conduction is equal to the rate of heat loss from the outer surface of the insulating layer to the surrounding air by convection, i.e.

$$\dot{Q}_{\text{conduction}} = \dot{Q}_{\text{convection}}$$

$$\Rightarrow \frac{kA}{L}(T_1 - T_s) = hA(T_s - T_\infty)$$

\Rightarrow Minimum heat transfer coefficient at outside surface,

$$h = \frac{k(T_1 - T_s)}{L(T_s - T_\infty)} = \frac{0.8(300 - 100)}{0.05(100 - 25)} = 42.667 \text{ W/m}^2\text{K}$$



Problem 8.21.

A lake surface is covered by a 8 cm thick layer of ice ($k = 2.23 \text{ W/mK}$) when the ambient air temperature is -12.5°C . A thermocouple embedded on the upper surface of the layer indicates a temperature of -5°C . Assuming steady state conduction in ice and no liquid subcooling at the bottom surface of the ice layer, find the heat transfer coefficient at the upper surface. Also work out the heat loss per unit area.

Solution: Given,

Thickness of ice layer, $L = 8 \text{ cm} = 0.08 \text{ m}$

Thermal conductivity of ice, $k = 2.23 \text{ W/mK}$

Temperature of ambient air, $T_\infty = -12.5^\circ\text{C}$

Temperature of upper surface of ice layer, $T_s = -5^\circ\text{C}$

Temperature of bottom surface of ice layer, $T_1 = 0^\circ\text{C}$

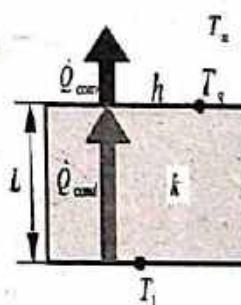
(As there is no subcooling at the bottom surface of the ice layer)

Here, the rate of heat transfer through the ice layer by conduction is equal to the rate of heat loss from the upper surface of the ice layer to the ambient air by convection, i.e.

$$\begin{aligned} \dot{Q}_{\text{conduction}} &= \dot{Q}_{\text{convection}} \\ \Rightarrow \frac{kA}{L}(T_1 - T_s) &= hA(T_s - T_\infty) \end{aligned}$$

\Rightarrow Heat transfer coefficient at upper surface of ice layer,

$$\begin{aligned} h &= \frac{k(T_1 - T_s)}{L(T_s - T_\infty)} = \frac{2.23\{0 - (-5)\}}{0.08\{-5 - (-12.5)\}} \\ &= 18.5833 \text{ W/m}^2\text{K} \end{aligned}$$



Again, considering single layer of heat transfer by convection only, the rate of heat loss from the upper surface of the ice layer to the ambient air is given by

$$\dot{Q}_{\text{convection}} = hA(T_s - T_\infty)$$

\Rightarrow Heat loss per unit area of ice layer to ambient air,

$$\frac{\dot{Q}_{\text{convection}}}{A} = h(T_s - T_\infty) = 18.5833 \{-5 - (-12.5)\} = 139.375 \text{ W/m}^2$$

Problem 8.22.

A thick walled tube of stainless steel [$k = 19 \text{ W/m}^\circ\text{C}$] with 2 cm inside diameter and 1 cm thickness is covered with a 3 cm layer of asbestos insulation [$k = 0.2 \text{ W/m}^\circ\text{C}$]. If the inside wall temperature of the pipe is maintained at 600°C and outside wall temperature of the insulation is maintained at 100°C , calculate the heat loss per meter of length. Also calculate the tube - insulation interface temperature.

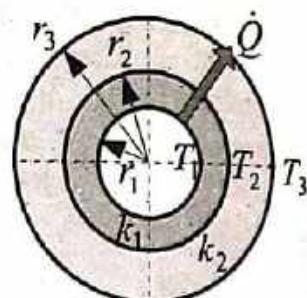
Solution: Given,

Thermal conductivity of steel pipe, $k_1 = 19 \text{ W/m}^\circ\text{C}$

Inside diameter of steel pipe, $D_1 = 2 \text{ cm} = 0.02 \text{ m}$

Thickness of steel pipe, $(r_2 - r_1) = 1 \text{ cm} = 0.01 \text{ m}$

Thickness of asbestos insulation, $(r_3 - r_2) = 3 \text{ cm}$
 $= 0.03 \text{ m}$



Thermal conductivity of asbestos insulation,

$$k_2 = 0.2 \text{ W/m}^\circ\text{C}$$

Inside wall temperature of steel pipe, $T_1 = 600^\circ\text{C}$

Outside wall temperature of asbestos insulation, $T_3 = 100^\circ\text{C}$

Inside radius of steel pipe, $r_1 = D_1/2 = 0.02/2 = 0.01 \text{ m}$

Outside radius of steel pipe, $r_2 = r_1 + 0.01 = 0.01 + 0.01 = 0.02 \text{ m}$

Outside radius of insulation, $r_3 = r_2 + 0.03 = 0.02 + 0.03 = 0.05 \text{ m}$

The rate of heat loss through the steel pipe with the layer of the asbestos insulation (composite structure) by conduction is given by

$$\dot{Q} = \frac{2\pi L(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}}$$

\Rightarrow Rate of heat loss per unit length of steel pipe,

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}} = \frac{2\pi(600 - 100)}{\frac{\ln(0.02/0.01)}{19} + \frac{\ln(0.05/0.02)}{0.2}}$$

$$= 680.3025 \text{ W/m}$$

Now, considering the single layer of the steel pipe only, the same rate of heat transfer through the steel pipe,

$$\dot{Q} = \frac{2\pi L(T_1 - T_2)}{\ln(r_2/r_1)} \quad \Rightarrow \frac{\dot{Q}}{L} = \frac{2\pi k_1(T_1 - T_2)}{\ln(r_2/r_1)}$$

$$\Rightarrow 680.3025 = \frac{2\pi \times 19(600 - T_2)}{\ln(0.02/0.01)}$$

\therefore Tube-insulation interface temperature, $T_2 = 596.05^\circ\text{C}$

Problem 8.23.

A square plate heater ($10\text{ cm} \times 10\text{ cm}$) is inserted between two slabs having same cross-sectional areas. The left slab is 100 mm thick ($k = 50\text{ W/mK}$) and the right slab is 50 mm thick ($k = 0.25\text{ W/mK}$). The heat transfer coefficients for left and right slab outer surfaces are $250\text{ W/m}^2\text{K}$ and $50\text{ W/m}^2\text{K}$ respectively. The ambient air temperature is 25°C . If the rating of the heater is 1 kW ; determine:

- (a) temperature at the heater surface, and
- (b) outer surface temperatures of each slab.

Solution: Given,

Area of heater plate, $A = 10\text{ cm} \times 10\text{ cm} = 100\text{ cm}^2 = 0.01\text{ m}^2$

Thickness of left slab, $L_1 = 100\text{ mm} = 0.1\text{ m}$

Thermal conductivity of left slab, $k_1 = 50\text{ W/mK}$

Thickness of right slab, $L_2 = 50\text{ mm} = 0.05\text{ m}$

Thermal conductivity of right slab, $k_2 = 0.25\text{ W/mK}$

Heat transfer coefficient for left slab outer surface, $h_1 = 250\text{ W/m}^2\text{K}$

Heat transfer coefficient for right slab outer surface, $h_2 = 50\text{ W/m}^2\text{K}$

Ambient air temperature, $T_{\infty 1} = T_{\infty 2} = 25^\circ\text{C}$

Rate of heat transfer from heater surfaces,

$$\dot{Q} = 1\text{ kW} = 1000\text{ W}$$

For steady state condition we know,

Total heat transfer from heater surfaces =

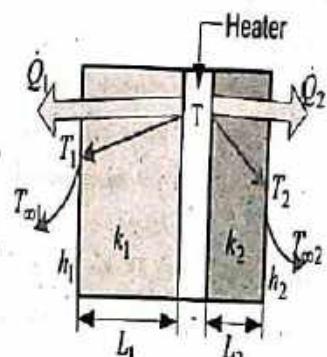
Rate of heat flow through left slab + Rate of heat flow through right slab

$$\Rightarrow \dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{A(T - T_{\infty 1})}{(L_1/k_1) + (1/h_1)} + \frac{A(T - T_{\infty 2})}{(L_2/k_2) + (1/h_2)}$$

$$\Rightarrow \frac{\dot{Q}}{A} = \frac{(T - 25)}{(0.1/50) + (1/250)} + \frac{(T - 25)}{(0.05/0.25) + (1/50)}$$

$$\Rightarrow \frac{1000}{0.01} = (T - 25) \left(\frac{1}{0.006} + \frac{1}{0.22} \right)$$

\therefore Temperature at the heater surface, $T = 609.071^\circ\text{C}$



Now, the rate of heat flow through the left slab by the conduction is equal to the rate of heat loss through the convection from its outer surface to the ambient air, i.e.

$$\begin{aligned}\dot{Q}_1 &= \frac{k_1 A(T - T_1)}{L_1} = h_1 A(T_1 - T_{\infty 1}) \\ \Rightarrow \quad &\frac{50 \times 0.01(609.071 - T_1)}{0.1} = 250 \times 0.01(T_1 - 25)\end{aligned}$$

∴ Outer surface temperature of left slab, $T_1 = 414.381^\circ\text{C}$

Similarly, the rate of heat flow through the right slab by the conduction is equal to the rate of heat loss through the convection from its outer surface to the ambient air, i.e.

$$\begin{aligned}\dot{Q}_2 &= \frac{k_2 A(T - T_2)}{L_2} = h_2 A(T_2 - T_{\infty 2}) \\ \Rightarrow \quad &\frac{0.25 \times 0.01(609.071 - T_2)}{0.05} = 50 \times 0.01(T_2 - 25)\end{aligned}$$

∴ Outer surface temperature of right slab, $T_2 = 78.097^\circ\text{C}$

8.3 Solved Numerical Problems from PoU Examinations

Problem 8.24.

A 16 cm diameter pipe carrying saturated steam is covered by a layer of insulation of thickness 40 mm ($k = 0.8 \text{ W/mK}$). Later on, an extra layer of insulation of 10 mm thick ($k = 1.2 \text{ W/mK}$) is added. If surrounding temperature remains constant and heat transfer coefficient (outside) for both insulation materials is $10 \text{ W/m}^2\text{K}$. Determine the percentage change in the rate of heat loss due to extra insulation layer. (PoU 2004 spring, PoU 2008 spring).

Solution: Given,

Outer diameter of pipe, $D_2 = 16 \text{ cm} = 0.16 \text{ m}$

Thermal conductivity of first insulation, $k_2 = 0.8 \text{ W/mK}$

Thickness of first insulation, $(r_3 - r_2) = 40 \text{ mm} = 0.04 \text{ m}$

Heat transfer coefficient between insulation and air, $h_3 = 10 \text{ W/m}^2\text{K}$

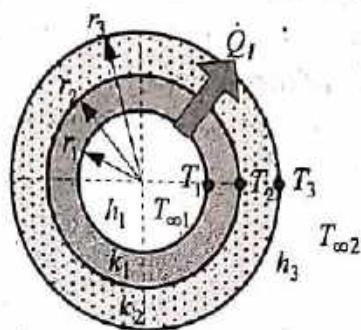
Thermal conductivity of extra insulating layer, $k_3 = 1.2 \text{ W/mK}$

Thickness of extra insulating layer, $(r_4 - r_3) = 10 \text{ mm} = 0.01 \text{ m}$

Outer radius of pipe, $r_2 = D_2/2 = 0.16/2 = 0.08 \text{ m}$

Outer radius of first insulation, $r_3 = r_2 + 0.04 = 0.08 + 0.04 = 0.12 \text{ m}$

Outer radius of extra insulation, $r_4 = r_3 + 0.01 = 0.12 + 0.01 = 0.13 \text{ m}$



Case I.

Case I: Heat transfer without extra insulating layer

Let the temperatures of inner surface of the first insulation and the ambient air are T_1 and $T_{\infty 2}$.

The rate of heat transfer through the composite hollow cylinder considering first insulation with convection to ambient air is given by

$$\dot{Q}_I = \frac{2\pi L(T_2 - T_{\infty 2})}{\frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_3 r_3}} = \frac{2\pi L(T_2 - T_{\infty 2})}{\frac{\ln(0.12/0.08)}{0.8} + \frac{1}{10 \times 0.12}} = \frac{2\pi L(T_2 - T_{\infty 2})}{1.34016}$$

Case II: Heat transfer with extra insulating layer

As the heat transfer coefficient remains the same, i.e. $h_3 = h_4$, the rate of heat transfer through the composite hollow cylinder considering both insulations with convection to ambient air is given by

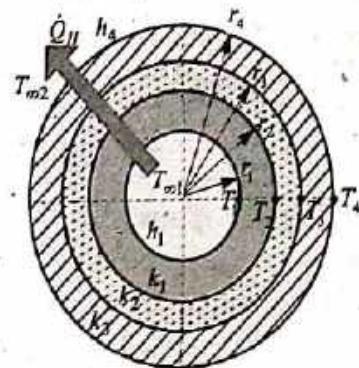
$$\begin{aligned}\dot{Q}_{II} &= \frac{2\pi L(T_2 - T_{\infty 2})}{\frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3} + \frac{1}{h_4 r_4}} = \frac{2\pi L(T_2 - T_{\infty 2})}{\frac{\ln(0.12/0.08)}{0.8} + \frac{\ln(0.13/0.12)}{1.2} + \frac{1}{10 \times 0.13}} \\ &= \frac{2\pi L(T_2 - T_{\infty 2})}{1.34276}\end{aligned}$$

Now, percentage change in rate of heat loss,

$$\frac{\dot{Q}_I - \dot{Q}_{II}}{\dot{Q}_I} = \frac{\frac{2\pi L(T_2 - T_{\infty 2})}{1.34016} - \frac{2\pi L(T_2 - T_{\infty 2})}{1.34276}}{\frac{2\pi L(T_2 - T_{\infty 2})}{1.34016}} = \frac{0.001936}{1.34016} = 0.1936\%$$

Problem 8.25.

A steam pipe 18 cm inside diameter and 20 cm outside diameter is covered with two layers of insulation. The thickness of the first and the second layers are 4 cm and 8 cm respectively and their corresponding thermal conductivities are 0.15 and 0.08 W/mK respectively. The conductivity of the pipe material is 50



Case II.

W/mK . The temperature of the inner surface of the pipe is 350°C and that of the outer surface of insulation is 30°C . Find the quantity of heat lost per meter length of the pipe. Also find out the percentage decrease in the heat loss with two layers of insulation compared to only first layer of insulation.

(PoU 2005 fall).

Solution: Given.

Inner diameter of pipe, $D_1 = 18 \text{ cm} = 0.18 \text{ m}$

Outer diameter of pipe, $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

Thermal conductivity of pipe, $k_1 = 50 \text{ W/mK}$

Thickness of first insulation, $(r_3 - r_2) = 4 \text{ cm} = 0.04 \text{ m}$

Thermal conductivity of first insulation, $k_2 = 0.15 \text{ W/mK}$

Thickness of second insulating layer, $(r_4 - r_3) = 8 \text{ cm} = 0.08 \text{ m}$

Thermal conductivity of extra insulating layer, $k_3 = 0.08 \text{ W/mK}$

Inner surface temperature of pipe, $T_1 = 350^\circ\text{C}$

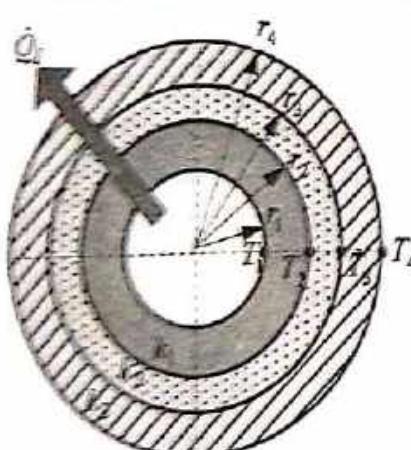
Outer surface temperature of insulation, $T_4 = 30^\circ\text{C}$

Inner radius of pipe, $r_1 = D_1/2 = 0.18/2 = 0.09 \text{ m}$

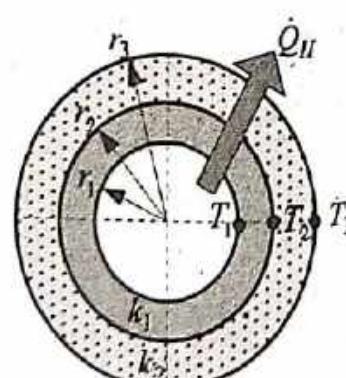
Outer radius of pipe, $r_2 = D_2/2 = 0.20/2 = 0.10 \text{ m}$

Outer radius of first insulation, $r_3 = r_2 + 0.04 = 0.10 + 0.04 = 0.14 \text{ m}$

Outer radius of second insulation, $r_4 = r_3 + 0.08 = 0.14 + 0.08 = 0.22 \text{ m}$



Case I.



Case II.

Case I: Heat transfer with both insulating layers

The rate of heat transfer through the composite hollow cylinder considering both insulations is given by

$$Q_1 = \frac{\frac{2\pi l (T_1 - T_4)}{\ln(r_2/r_1) \ln(r_4/r_2) + \ln(r_4/r_3)}}{\frac{k_1}{k_2} + \frac{k_2}{k_3}} = \frac{\frac{2\pi l (350 - 30)}{\ln(0.10/0.09) + \ln(0.22/0.14)}}{50 + 0.15 + 0.08}$$

\Rightarrow Quantity of heat loss per unit length,

$$\frac{Q_1}{L} = \frac{2\pi (350 - 30)}{7.89557} = 254.6677 \text{ W/m}$$

Case II: Heat transfer without second insulating layer

Assuming the outer surface temperature of first insulation is also $T_3 = 30^\circ\text{C}$, the rate of heat transfer through the composite hollow cylinder considering only first insulation is given by

$$\dot{Q}_{II} = \frac{2\pi L(350 - 30)}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}} = \frac{2\pi L(350 - 30)}{\frac{\ln(0.10/0.09)}{50} + \frac{\ln(0.14/0.10)}{0.15}}$$

$$\Rightarrow \frac{\dot{Q}_{II}}{L} = \frac{2\pi(350 - 30)}{2.24525} = 895.4969 \text{ W/m}$$

Now, percentage decrease in heat loss,

$$\frac{\dot{Q}_{II} - \dot{Q}_I}{\dot{Q}_{II}} = \frac{\dot{Q}_{II}/L - \dot{Q}_I/L}{\dot{Q}_{II}/L} = \frac{895.4969 - 254.6677}{895.4969} = 0.715613 = 71.5613\%$$

Problem 8.26.

A 150 mm steel pipe has inside diameter of 120 mm and outside diameter of 160 mm. It is insulated at the outside with asbestos. The steam temperature is 150°C and the air temperature is 20°C , h (steam side) = $100 \text{ W/m}^2\text{K}$, h (air side) = $30 \text{ W/m}^2\text{K}$, k (asbestos) = 0.8 W/mK , and k (steel) = 42 W/mK . How thick should the asbestos be provided in order to limit the heat loss to 2.1 kW/m^2 ?

(PoU 2006 fall).

Solution: Given,

Mean diameter of steam pipe, $D = 150 \text{ mm} = 0.15 \text{ m}$

Thermal conductivity of steam pipe, $k_1 = 42 \text{ W/mK}$

Inside diameter of steam pipe, $D_1 = 120 \text{ mm} = 0.12 \text{ m}$

Outside diameter of steam pipe, $D_2 = 160 \text{ mm} = 0.16 \text{ m}$

Thermal conductivity of asbestos, $k_2 = 0.8 \text{ W/mK}$

Steam temperature, $T_{\infty 1} = 150^\circ\text{C}$

Ambient air temperature, $T_{\infty 2} = 20^\circ\text{C}$

Heat transfer coefficient for inner surface, $h_1 = 100 \text{ W/m}^2\text{K}$

Heat transfer coefficient for outer surface, $h_3 = 30 \text{ W/m}^2\text{K}$

Rate of heat loss per unit area of pipe, $\dot{Q}/A = 2.1 \text{ kW/m}^2 = 2100 \text{ W/m}^2$

Inside radius of steam pipe, $r_1 = D_1/2 = 0.12/2 = 0.06 \text{ m}$

Outside radius of steam pipe, $r_2 = D_2/2 = 0.16/2 = 0.08 \text{ m}$

The rate of heat loss through the composite hollow cylinder subjected to convective medium on both sides is given by

$$\dot{Q} = \frac{A_1(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1} + \frac{A_1 \ln(r_2/r_1)}{2\pi k_1 L} + \frac{A_1 \ln(r_3/r_2)}{2\pi k_2 L} + \frac{A_1}{h_3 A_2}}$$

where $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_3 L$ are the inside and outside peripheral surface areas of the composite hollow cylinder (pipe) respectively.

$$\Rightarrow \dot{Q} = \frac{2\pi L(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_3 r_3}}$$

Mean area for heat transfer, $A = \pi D L = 0.15\pi L$

(where L is the length of the pipe)

Hence, rate of heat loss, $\dot{Q} = \frac{\dot{Q}}{A} \times A = 2100 \times 0.15\pi L = 989.6L \text{ W}$

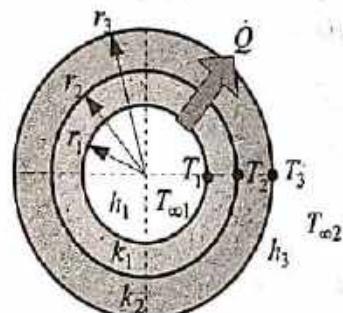
$$\Rightarrow 989.6L = \frac{2\pi L(150 - 20)}{\frac{1}{100 \times 0.06} + \frac{\ln(0.08/0.06)}{42} + \frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30 \times r_3}}$$

$$\Rightarrow 1.25 \ln(r_3) + 0.033333r_3 + 2.5053 = 0$$

$$\therefore r_3 = 0.1044 \text{ m} = 104.4 \text{ mm}$$

Then, the thickness of asbestos insulation,

$$(r_3 - r_2) = 0.1044 - 0.08 \\ = 0.0244 \text{ m} = 24.4 \text{ mm}$$



Problem 8.27.

Hot air at a temperature of 60 °C is flowing through a steel pipe of 10 cm diameter. The pipe is covered with two layers of different insulating materials of thickness 5 cm and 3 cm and their corresponding thermal conductivities are 0.23 and 0.37 W/mK. The inside and outside heat transfer coefficients are 58 and 12 W/m²K. The atmosphere is at 25 °C. Find the rate of heat loss from a 50 m long pipe. Neglect the resistance of the steel pipe. (PoU 2007 spring).

Solution: Given,

Length of pipe, $L = 50 \text{ m}$

Outer diameter of pipe, $D_2 = 10 \text{ cm} = 0.10 \text{ m}$

Thermal conductivity of first insulation, $k_2 = 0.23 \text{ W/mK}$

Thickness of first insulation, $(r_3 - r_2) = 5 \text{ cm} = 0.05 \text{ m}$

Inside heat transfer coefficient, $h_1 = 58 \text{ W/m}^2\text{K}$

Outside heat transfer coefficient, $h_4 = 12 \text{ W/m}^2\text{K}$

Thermal conductivity of extra insulating layer, $k_3 = 0.37 \text{ W/mK}$.

Thickness of second insulation, $(r_4 - r_3) = 3 \text{ cm} = 0.03 \text{ m}$

Temperature of hot air, $T_{\infty 1} = 60^\circ\text{C}$

Temperature of atmospheric air, $T_{\infty 2} = 25^\circ\text{C}$

Outer radius of pipe, $r_2 = D_2/2 = 0.10/2 = 0.05 \text{ m}$

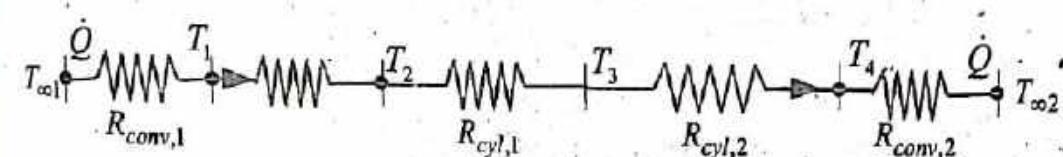
Outer radius of first insulation, $r_3 = r_2 + 0.05$

$$= 0.05 + 0.05 = 0.10 \text{ m}$$

Outer radius of extra insulation, $r_4 = r_3 + 0.03$

$$0.10 + 0.03 = 0.13 \text{ m}$$

Thermal resistance circuit is shown in figure below:



Using electrical analogy method, rate of heat transfer through composite hollow cylinder considering both insulations with both sides convection is given by

$$\dot{Q} = \frac{\Delta T_{\text{overall}}}{R_{\text{total}}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv},1} + R_{\text{cyl},1} + R_{\text{cyl},2} + R_{\text{conv},2}}$$

where the resistance of the pipe is neglected and

$$R_{\text{conv},1} = \frac{1}{h_1 A_1} = \frac{1}{h_1 2\pi r_2 L} = \frac{1}{58 \times 2 \times \pi \times 0.05 \times 50} = 0.0010976 \text{ K/W}$$

(As pipe resistance is neglected, $r_1 = r_2$)

$$R_{\text{cyl},1} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(0.10/0.05)}{2 \times \pi \times 0.23 \times 50} = 0.0095928 \text{ K/W}$$

$$R_{\text{cyl},2} = \frac{\ln(r_4/r_3)}{2\pi k_3 L} = \frac{\ln(0.13/0.10)}{2 \times \pi \times 0.37 \times 50} = 0.0022571 \text{ K/W}$$

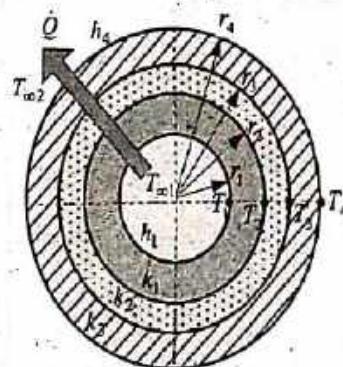
$$R_{\text{conv},2} = \frac{1}{h_4 A_2} = \frac{1}{h_4 2\pi r_4 L} = \frac{1}{12 \times 2 \times \pi \times 0.13 \times 50} = 0.0020404 \text{ K/W}$$

Now, rate of heat loss,

$$\dot{Q} = \frac{60 - 25}{0.0010976 + 0.0095928 + 0.0022571 + 0.0020404} = 2335.217 \text{ W} = 2.335217 \text{ kW}$$

Problem 8.28.

An exterior wall of a house may be approximated by a 100 mm layer of common brick ($k = 0.7 \text{ W/mK}$) followed by a 40 mm layer of gypsum plaster ($k = 0.48 \text{ W/mK}$). What thickness of loosely packed rock - wool insulation ($k = 0.065 \text{ W/mK}$) should be added to reduce the heat loss (or gain) through the wall by 80%? (PoU 2007 fall).

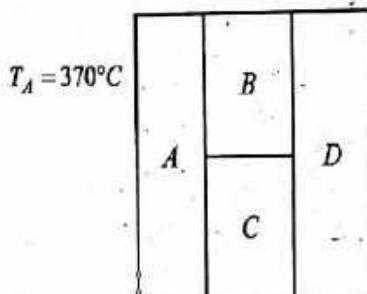


Solution: Refer problem 8.16 (very similar). (Ans: 5.77 cm)

Problem 8.29.

Find the heat transfer through the composite wall as shown in figure.

(PoU 2009 spring).



$$\begin{array}{ll} k_A = 150 \text{ W/m°C} & L_A = 2.5 \text{ cm} \\ k_B = 30 \text{ W/m°C} & L_B = 7.5 \text{ cm} \\ k_C = 150 \text{ W/m°C} & L_C = 2.5 \text{ cm} \\ k_D = 150 \text{ W/m°C} & L_D = 7.5 \text{ cm} \\ A_A = A_D = 2A_B = 2A_C = 0.2 \text{ m}^2 & \end{array}$$

Solution: Given,

Thermal conductivity of layer A, $k_A = 150 \text{ W/mK}$

From figure, thickness of layer A, $L_A = 2.5 \text{ cm} = 0.025 \text{ m}$

Thermal conductivity of layer B, $k_B = 30 \text{ W/mK}$

From figure, thickness of layer B, $L_B = 7.5 \text{ cm} = 0.075 \text{ m}$

Thermal conductivity of layer C, $k_C = 150 \text{ W/mK}$

From figure, thickness of layer C, $L_C = 2.5 \text{ cm} = 0.025 \text{ m}$

Thermal conductivity of layer D, $k_D = 150 \text{ W/mK}$

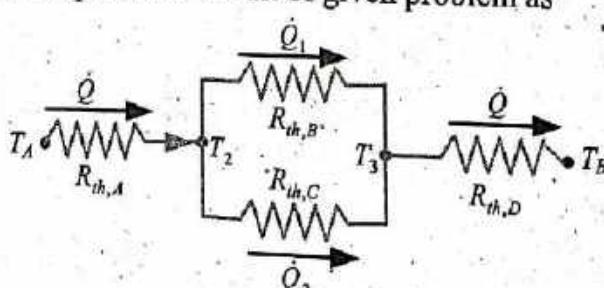
From figure, thickness of layer D, $L_D = 7.5 \text{ cm} = 0.075 \text{ m}$

Inlet temperature of composite wall, $T_A = 370^\circ\text{C}$

Exit temperature of composite wall, $T_D = 70^\circ\text{C}$

Area, $A_A = A_D = 2A_B = 2A_C = 0.2 \text{ m}^2$

Applying the electrical analogy approach for heat transfer through a composite structure, we have an equivalent circuit of given problem as



The total thermal resistance of the circuit for heat transfer is given by

$$R_{total} = R_{th,A} + \frac{R_{th,B} \times R_{th,C}}{R_{th,B} + R_{th,C}} + R_{th,D}$$

where thermal resistances for each layer of plane wall are calculated as

$$R_{th,A} = \frac{L_A}{A_A k_A} = \frac{0.025}{0.2 \times 150} = 0.0008333 \text{ K/W}$$

$$R_{th,B} = \frac{L_B}{A_B k_B} = \frac{0.075}{0.1 \times 30} = 0.024999 \text{ K/W}$$

$$R_{th,C} = \frac{L_C}{A_C k_C} = \frac{0.025}{0.1 \times 150} = 0.0016666 \text{ K/W}$$

$$R_{th,D} = \frac{L_D}{A_D k_D} = \frac{0.075}{0.2 \times 150} = 0.0025 \text{ K/W}$$

Then,

$$R_{total} = R_{th,A} + \frac{R_{th,B} \times R_{th,C}}{R_{th,B} + R_{th,C}} + R_{th,D} = 0.00083333 + \frac{0.024999 \times 0.0016666}{0.024999 + 0.0016666} + 0.0025 \\ = 0.0048957 \text{ K/W}$$

Now, the rate of heat transfer through the composite wall is given by

$$\dot{Q} = \frac{T_1 - T_4}{R_{total}} = \frac{370 - 70}{0.0048957} = 61277.42 \text{ W} = 61.277 \text{ kW}$$

Problem 8.30.

The temperatures at the inside and outside surfaces of the brick work of a furnace have been noted to be 650°C and 225°C. Calculate the percentage decrease in heat loss if the thickness of the brick work is increased by 100%. The ambient temperature is 30°C and assume that thermal conductivity of brick work and convective heat transfer coefficient remain the same before and after increase in thickness. (PoU 2009 fall, PoU 2013 fall).

Solution: Given,

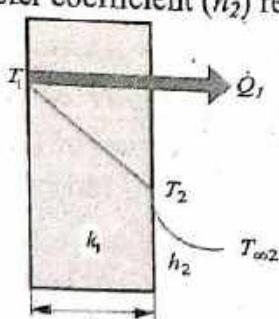
Inside temperature of furnace, $T_1 = 650^\circ\text{C}$

Outside temperature of furnace, $T_2 = 225^\circ\text{C}$

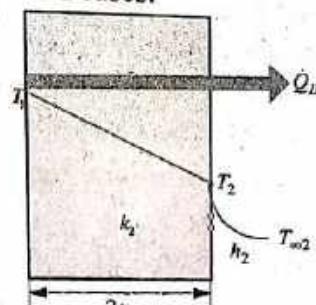
Ambient temperature, $T_{\infty 2} = 30^\circ\text{C}$

Thermal conductivity of brick work remains same, i.e., $k_1 = k_2 = k$

Heat transfer coefficient (h_2) remains same for both cases.



Case I



Case II

Case I: Heat transfer when the thickness of brick work = x .

The rate of heat transfer through the plane wall is given by

$$\dot{Q}_I = \frac{(T_1 - T_2)}{\frac{x}{kA}} = \frac{A(650 - 225)}{\frac{x}{k}} = \frac{425A}{\frac{x}{k}} \quad \dots \dots \dots \text{(i)}$$

The rate of heat transfer through the plane wall considering the convection to ambient air is given by

$$\dot{Q}_I = \frac{(T_1 - T_{\infty 2})}{\frac{x}{kA} + \frac{1}{h_2 A}} = \frac{A(650 - 30)}{\frac{x}{k} + \frac{1}{h_2}} = \frac{620A}{\frac{x}{k} + \frac{1}{h_2}} \quad \dots \dots \dots \text{(ii)}$$

Equating the equations (i) and (ii), we get

$$\frac{425A}{\frac{x}{k}} = \frac{620A}{\frac{x}{k} + \frac{1}{h_2}} \Rightarrow \frac{1}{h_2} = \frac{195x}{425k}$$

Case II: Heat transfer when thickness of brick work is increased by 100%, i.e. thickness = $2x$

As the thermal conductivity and heat transfer coefficient remain the same, the rate of heat transfer through the plane wall considering the convection to ambient air is given by

$$\dot{Q}_{II} = \frac{(T_1 - T_{\infty 2})}{\frac{2x}{kA} + \frac{1}{h_2 A}} = \frac{A(650 - 30)}{\frac{2x}{k} + \frac{1}{h_2}} = \frac{620A}{\frac{2x}{k} + \frac{195x}{425k}} = \frac{620A}{\frac{1045x}{425k}}$$

Now, Percentage decrease in heat loss,

$$\frac{\dot{Q}_I - \dot{Q}_{II}}{\dot{Q}_I} = \frac{\frac{425A}{\frac{x}{k}} - \frac{620A}{\frac{1045x}{425k}}}{\frac{425A}{\frac{x}{k}}} = 0.4067 = 40.67\%$$

Problem 8.31.

A thick walled tube of stainless steel ($k = 19 \text{ W/mK}$) with 2 cm inside diameter and 4 cm outside diameter is covered with a 3 cm layer of asbestos insulation ($k = 0.2 \text{ W/mK}$). If the inside and outside wall temperature of the pipe is maintained at 600°C and 100°C , calculate the heat loss per meter length. Also calculate the tube - insulation interface temperature. (PoU 2010 spring).

Solution: Refer problem 8.22.

Problem 8.32.

An exterior wall of a house may be approximated by a 10 cm layer of common brick ($k = 0.7 \text{ W/m } ^\circ\text{C}$) followed by a layer of a 3.8 cm layer of cement plaster ($k = 0.48 \text{ W/m } ^\circ\text{C}$). What thickness of loosely packed rock wool insulation ($k = 0.065 \text{ W/m } ^\circ\text{C}$) should be added to reduce heat loss (or gain) through the wall by 80%? (PoU 2011 fall, PoU 2015 spring).

Solution: Refer problem 8.16 (very similar) [Ans: $0.05773 \text{ m} = 5.773 \text{ cm}$]

problem 8.33.

A furnace wall is made of 20 cm of magnesite brick and 20 cm of common brick. The magnesite brick is exposed to hot gases at 1200°C and common brick outer surface is exposed to 35°C room air. The surface heat transfer coefficient of the inside wall is $40 \text{ W/m}^2\text{K}$ and that of the outer wall is $20 \text{ W/m}^2\text{K}$ respectively. Thermal conductivities of magnesite and common brick are 4 and 0.5 W/mK respectively. Determine

- heat loss per m^2 of area of the furnace wall and
- maximum temperature to which common brick is subjected.

(PoU 2012 fall).

Solution: Given,

Thickness of magnesite brick, $L_1 = 20 \text{ cm} = 0.20 \text{ m}$

Thermal conductivity of magnesite brick,

$$k_1 = 4 \text{ W/mK}$$

Thickness of common brick, $L_2 = 20 \text{ cm} = 0.20 \text{ m}$

Thermal conductivity of common brick,

$$k_2 = 0.5 \text{ W/mK}$$

Inner surface temperature of furnace wall,

$$T_{\infty 1} = 1200^\circ\text{C}$$

Outer surface temperature of furnace wall,

$$T_{\infty 2} = 35^\circ\text{C}$$

Heat transfer coefficient of inner wall, $h_1 = 40 \text{ W/m}^2\text{K}$

Heat transfer coefficient of outer wall, $h_3 = 20 \text{ W/m}^2\text{K}$

The rate of heat transfer through the composite structure with two plane layers subjected to convection on both sides is given by

$$\dot{Q} = \frac{A(T_{\infty 1} - T_{\infty 2})}{(1/h_1) + (L_1/k_1) + (L_2/k_2) + (1/h_3)}$$

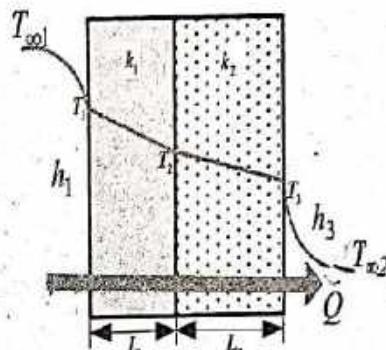
\Rightarrow Heat loss per unit area of wall,

$$\frac{\dot{Q}}{A} = \frac{(T_{\infty 1} - T_{\infty 2})}{(1/h_1) + (L_1/k_1) + (L_2/k_2) + (1/h_3)} = \frac{1200 - 35}{(1/40) + (0.2/4) + (0.2/0.5) + (1/20)} \\ = 2219.0476 \text{ W/m}^2$$

Considering the first layer of magnesite brick with inner wall convection, the same rate of heat transfer is expressed as

$$\dot{Q} = \frac{A(T_{\infty 1} - T_2)}{1/h_1 + (L_1/k_1)} \Rightarrow \frac{\dot{Q}}{A} = \frac{(T_{\infty 1} - T_2)}{(1/40) + (0.2/4)} \\ \Rightarrow 2219.0476 = \frac{(1200 - T_2)}{0.075}$$

$$\therefore \text{Interface temperature, } T_2 = 1033.571^\circ\text{C}$$



Problem 8.34.

A reactor's wall 320 mm thick is made up of an inner layer of fire brick ($k = 0.84 \text{ W/mK}$) covered with a layer of insulation ($k = 0.16 \text{ W/mK}$). The reactor operates at a temperature of 1325°C at the atmospheric temperature of 25°C . Calculate

- (i) the thickness of brick and insulation
- (ii) heat loss per unit area presuming that the insulating material has a maximum temperature of 1200°C .

(PoU 2014 spring, PoU 2015 fall).

Solution: Given,

Thickness of reactor's wall, $L = 320 \text{ mm} = 0.32 \text{ m}$

Thermal conductivity of fire brick, $k_1 = 0.84 \text{ W/mK}$

Thermal conductivity of insulation, $k_2 = 0.16 \text{ W/mK}$

Inner surface temperature of wall, $T_1 = 1325^\circ\text{C}$

Outer surface temperature of wall, $T_3 = 25^\circ\text{C}$

Interface temperature, $T_2 = 1200^\circ\text{C}$

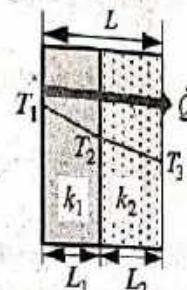
We know that the thickness of furnace wall L is equal to the thickness of the fire brick L_1 plus thickness of the insulation L_2 , i.e.

$$L = L_1 + L_2 = 0.32$$

$$\Rightarrow L_2 = 0.32 - L_1$$

The rate of heat transfer through each plane layer and the composite structure with two plane layers is same, i.e.

$$\begin{aligned} \dot{Q} &= \frac{A(T_1 - T_2)}{L_1/k_1} = \frac{A(T_2 - T_3)}{L_2/k_2} \Rightarrow \frac{(1325 - 1200)}{L_1/0.84} = \frac{1200 - 25}{L_2/0.16} \\ &\Rightarrow \frac{125}{L_1/0.84} = \frac{1175}{(0.32 - L_1)/0.16} \end{aligned}$$



∴ Thickness of brick, $L_1 = 0.1146757 \text{ m} = 114.6757 \text{ mm}$

Now, thickness of insulation, $L_2 = 0.32 - 0.1146757 = 0.205324 \text{ m} = 205.324 \text{ mm}$

Considering single layer of plane wall, heat transfer rate per unit area is given by

$$\dot{Q} = \frac{A(T_1 - T_2)}{L_1/k_1} \Rightarrow \frac{\dot{Q}}{A} = \frac{(T_1 - T_2)}{L_1/k_1} = \frac{1325 - 1200}{0.1146757/0.84} = 915.625 \text{ W/m}^2$$

Problem 8.35.

A steel pipe ($k = 45 \text{ W/mK}$) having a 5 cm OD is covered with a 4.2 cm thick layer of magnesia ($k = 0.07 \text{ W/mK}$) which in turn covered with a 2.4 cm layer of fiber glass insulation ($k = 0.048 \text{ W/mK}$). The pipe wall outside temperature is 370 K and the outside surface temperature of the fiber glass is 305 K. What is the interfacial temperature between the magnesia and the fiber glass? Assume suitable data if necessary.

(PoU 2017 spring).

Solution: Given,

Outside diameter of pipe, $D_2 = 5 \text{ cm} = 0.05 \text{ m}$

Thermal conductivity of pipe, $k_1 = 45 \text{ W/mK}$

Thickness of magnesia, $(r_3 - r_2) = 4.2 \text{ cm}$

$$= 0.042 \text{ m}$$

Thermal conductivity of magnesia, $k_2 = 0.07 \text{ W/mK}$

Thickness of fiber glass, $(r_4 - r_3) = 2.4 \text{ cm}$

$$= 0.024 \text{ m}$$

Thermal conductivity of fiber glass,

$$k_3 = 0.048 \text{ W/mK}$$

Pipe wall outside temperature, $T_2 = 370 \text{ K}$

Fiber glass outside surface temperature, $T_4 = 305 \text{ K}$

Outside radius of pipe, $r_2 = D_2/2 = 0.05/2 = 0.025 \text{ m}$

Outside radius of magnesia, $r_3 = r_2 + 0.042 = 0.025 + 0.042 = 0.067 \text{ m}$

Outside radius of fiber glass, $r_4 = r_3 + 0.024 = 0.067 + 0.024 = 0.091 \text{ m}$

Considering the layers of magnesia and fiber glass only, the rate of heat transfer per unit length of pipe, \dot{Q}/L through the composite hollow cylinder is given by.

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_2 - T_4)}{\frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3}} = \frac{2\pi(370 - 305)}{\frac{\ln(0.067/0.025)}{0.07} + \frac{\ln(0.091/0.067)}{0.048}} = 19.9597 \text{ W/m}$$

Again, the same rate of heat transfer per unit length through the single layer of magnesia is similarly given by

$$\frac{\dot{Q}}{L} = \frac{2\pi k_2(T_2 - T_3)}{\ln(r_3/r_2)} \Rightarrow 19.9597 = \frac{2\pi \times 0.07(370 - T_3)}{\ln(0.067/0.025)}$$

∴ Interface temperature between magnesia and fiber glass, $T_3 = 325.2624 \text{ K}$

Problem 8.36.

Find the heat flow rate through the composite wall as shown in figure. Assume one dimensional flow.

$k_A = 150 \text{ W/m}^\circ\text{C}$, $k_B = 30 \text{ W/m}^\circ\text{C}$,

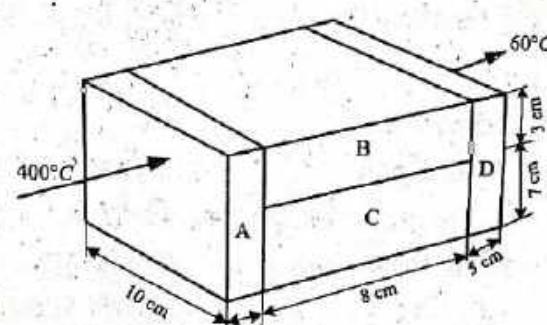
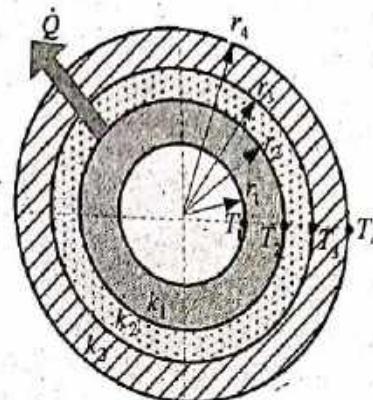
$k_C = 65 \text{ W/m}^\circ\text{C}$ and $k_D = 50 \text{ W/m}^\circ\text{C}$.

(PoU 2018 spring).

Solution: Refer problem 8.14
(very similar).

Problem 8.37.

An industrial freezer is designed to operate with an internal air temperature of $\sim 20^\circ\text{C}$ when the external air temperature is 25°C and the internal and external heat transfer coefficients are $12 \text{ W/m}^2\text{K}$ and $8 \text{ W/m}^2\text{K}$, respectively. The walls of



the freezer are composite construction, comprising of an inner layer of plastic ($k = 1 \text{ W/mK}$, and thickness of 3 mm), and an outer layer of stainless steel ($k = 16 \text{ W/mK}$, and thickness of 1 mm). Sandwiched between these two layers is a layer of insulation material with $k = 0.07 \text{ W/mK}$. Find the width of the insulation that is required to reduce the convective heat loss to 15 W/m^2 .

(PoU 2019 fall).

Solution: Given,

Internal air temperature, $T_{\infty 1} = -20^\circ\text{C}$

External air temperature, $T_{\infty 2} = 25^\circ\text{C}$

Internal heat transfer coefficient, $h_1 = 12 \text{ W/m}^2\text{K}$

External heat transfer coefficient, $h_4 = 8 \text{ W/m}^2\text{K}$

Thermal conductivity of plastic, $k_1 = 1 \text{ W/mK}$

Thickness of plastic, $L_1 = 3 \text{ mm} = 0.003 \text{ m}$

Thermal conductivity of stainless steel,

$$k_3 = 16 \text{ W/mK}$$

Thickness of stainless steel, $L_3 = 1 \text{ mm} = 0.001 \text{ m}$

Thermal conductivity of insulation, $k_2 = 0.07 \text{ W/mK}$

Convective heat loss per unit area, $\dot{Q}_{\text{conv}}/A = 15 \text{ W/m}^2$

The rate of heat transfer through the composite structure with three plane layers subjected to convection on both sides is given by

$$\begin{aligned}\dot{Q} &= \dot{Q}_{\text{conv}} = \frac{A(T_{\infty 2} - T_{\infty 1})}{(1/h_1) + (L_1/k_1) + (L_2/k_2) + (L_3/k_3) + (1/h_4)} \\ \Rightarrow \frac{\dot{Q}_{\text{conv}}}{A} &= \frac{(T_{\infty 2} - T_{\infty 1})}{(1/h_1) + (L_1/k_1) + (L_2/k_2) + (L_3/k_3) + (1/h_4)} \\ \Rightarrow 15 &= \frac{25 - (-20)}{(1/12) + (0.003/1) + (L_2/0.07) + (0.001/16) + (1/8)}\end{aligned}$$

∴ Width (thickness) of insulation, $L_2 = 0.1952 \text{ m}$

Problem 8.38.

A steam pipe 20 cm inside diameter and 22 cm outside diameter is covered with two layers of insulation. The thickness of the first and the second layers are 3 cm and 6 cm respectively and their corresponding thermal conductivities are 0.18 and 0.07 W/mK respectively. The conductivity of the pipe material is 55 W/mK. The temperature of the inner surface of the pipe is 400°C and that of the outer surface of insulation is 25°C . Find the quantity of heat lost per meter length of the pipe. Also find out the percentage decrease in the heat loss with two layers of insulation compared to only first layer of insulation.

(PoU 2019 spring).

Solution: Given,

Inner diameter of pipe, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

Outer diameter of pipe, $D_2 = 22 \text{ cm} = 0.22 \text{ m}$

Thermal conductivity of pipe, $k_1 = 55 \text{ W/mK}$

Thickness of first insulation, $(r_3 - r_2) = 3 \text{ cm} = 0.03 \text{ m}$

Thermal conductivity of first insulation, $k_2 = 0.18 \text{ W/mK}$

Thickness of second insulating layer, $(r_4 - r_3) = 6 \text{ cm} = 0.06 \text{ m}$

Thermal conductivity of second insulating layer, $k_3 = 0.07 \text{ W/mK}$

Inner surface temperature of pipe, $T_1 = 400^\circ\text{C}$

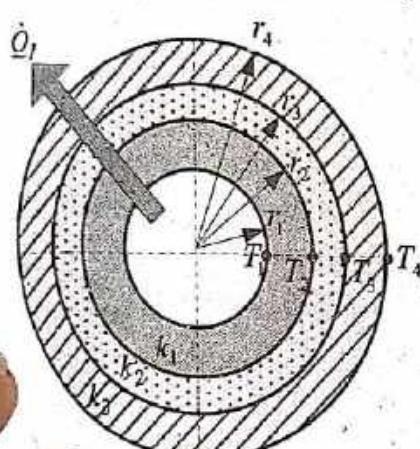
Outer surface temperature of insulation, $T_4 = 25^\circ\text{C}$

Inner radius of pipe, $r_1 = D_1/2 = 0.2/2 = 0.1 \text{ m}$

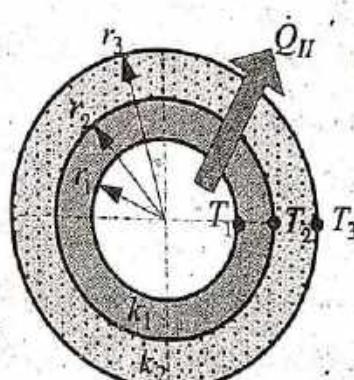
Outer radius of pipe, $r_2 = D_2/2 = 0.22/2 = 0.11 \text{ m}$

Outer radius of first insulation, $r_3 = r_2 + 0.04 = 0.11 + 0.03 = 0.14 \text{ m}$

Outer radius of second insulation, $r_4 = r_3 + 0.06 = 0.14 + 0.06 = 0.2 \text{ m}$



Case I.



Case II.

Case I: Heat transfer with both insulating layers

The rate of heat transfer through the composite hollow cylinder considering both insulations is given by

$$\dot{Q}_I = \frac{\frac{2\pi L(T_1 - T_4)}{\ln(r_2/r_1)\ln(r_3/r_2)}}{k_1} + \frac{\frac{2\pi L(T_1 - T_4)}{\ln(r_4/r_3)}}{k_2} + \frac{\frac{2\pi L(T_1 - T_4)}{\ln(r_4/r_3)}}{k_3} = \frac{2\pi L(400 - 25)}{55} + \frac{2\pi L(400 - 25)}{0.18} + \frac{2\pi L(400 - 25)}{0.07}$$

\Rightarrow Quantity of heat loss per unit length,

$$\frac{\dot{Q}_I}{L} = \frac{2\pi(400 - 25)}{6.436878} = 366.046 \text{ W/m}$$

Case II: Heat transfer without second insulating layer

Assuming the outer surface temperature of first insulation is also $T_3 = 25^\circ\text{C}$, the rate of heat transfer through the composite hollow cylinder considering only first insulation is given by

$$\dot{Q}_H = \frac{2\pi L(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}} = \frac{2\pi L(400 - 25)}{\ln(0.11/0.1) + \frac{0.18}{55}}$$

$$\Rightarrow \frac{\dot{Q}_H}{L} = \frac{2\pi(400 - 25)}{1.3415} = 1756.359 \text{ W/m}$$

Now, percentage decrease in heat loss,

$$\frac{\dot{Q}_H - \dot{Q}_I}{\dot{Q}_H} = \frac{\dot{Q}_H/L - \dot{Q}_I/L}{\dot{Q}_H/L} = \frac{1756.359 - 366.046}{1756.359} = 0.791588 = 79.1588\%$$

Problem 8.39.

An exterior wall of a house consists of 10 cm of red brick ($k = 0.9 \text{ W/mK}$) followed by a 3 cm layer of local plaster ($k = 0.4 \text{ W/mK}$). What thickness of rock wool insulation ($k = 0.07 \text{ W/mK}$) should be added to reduce the heat transfer through the wall by 85 %? (PoU 2020 fall).

Solution: Given,

Thermal conductivity of red brick, $k_1 = 0.9 \text{ W/mK}$

Thickness of red brick, $L_1 = 10 \text{ cm} = 0.1 \text{ m}$

Thermal conductivity of local plaster, $k_2 = 0.4 \text{ W/mK}$

Thickness of local plaster, $L_2 = 3 \text{ cm} = 0.03 \text{ m}$

Thermal conductivity of rock wool insulation,

$$k_3 = 0.07 \text{ W/mK}$$

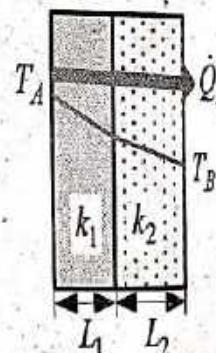
Case I: Heat transfer without rock wool insulation

Let us assume that the inside and outside temperatures of the wall of the house are T_A and T_B respectively and their difference remains same after adding rock wool insulation. The rate of heat transfer through the wall is given by

$$\dot{Q}_I = \frac{A(T_A - T_B)}{(L_1/k_1) + (L_2/k_2)} = \frac{A(T_A - T_B)}{(0.1/0.8) + (0.04/0.5)}$$

Case II. Heat transfer with rock wool insulation

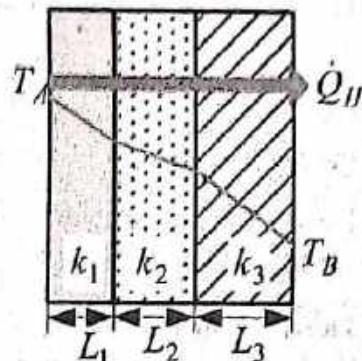
The rate of heat transfer through the wall is given by



$$\begin{aligned}\dot{Q}_n &= \frac{A(T_A - T_B)}{(L_1/k_1) + (L_2/k_2) + (L_3/k_3)} \\ &= \frac{A(T_A - T_B)}{(0.1/0.8) + (0.04/0.5) + (L_3/0.065)}\end{aligned}$$

According to the question, we have

$$\begin{aligned}\dot{Q}_n &= (100 - 85) \% \text{ of } \dot{Q}_1 \\ &\Rightarrow \frac{A(T_A - T_B)}{(0.1/0.9) + (0.03/0.4) + (L_3/0.07)} \\ &= 0.15 \times \frac{A(T_A - T_B)}{(0.1/0.9) + (0.03/0.4)}\end{aligned}$$



\therefore Thickness of rock wool insulation, $L_3 = 0.0738241 \text{ m} = 7.38241 \text{ cm}$

8.4 Review Theoretical Questions

1. Explain the differences between thermodynamics and heat transfer with examples.
2. Explain mode of heat transfer with their types and specific differences with them.
3. Differentiate between steady and unsteady state heat transfer. Derive expression for steady state heat transfer through a composite cylinder consisting of three layers.
4. Differentiate between steady and unsteady state heat transfer. Derive expression for steady state heat transfer through a composite cylinder consisting of three layers.

5. Differentiate between steady and unsteady state heat transfer. Derive expression for steady state heat transfer through a composite cylinder consisting of two different materials.
6. Derive an expression for steady state conduction heat transfer through a plane wall.
7. Derive an expression for steady state radial conduction heat transfer through a hollow cylinder.
8. Derive the heat transfer for composite plane wall. State the electrical analogy for thermal resistance.
9. Derive a heat flow equation through a composite plane wall consisting of three layers of different materials.
10. Derive an expression for conduction heat transfer through a composite cylinder (tube consisting of three layers of different materials).
11. Derive an expression for heat transfer through a mild steel pipe with a layer of insulation on the outside. Take temperature of the fluid in the pipe as t_{fluid} , temperature of air as t_{air} and length of the pipe as L . ($t_{fluid} > t_{air}$).
12. (a) Derive an expression for steady state heat transfer through a composite cylinder consisting of two different materials.
 (b) Define terms black body and gray body.
13. Define thermal resistance. Write down expressions of thermal resistance for a plane wall, hollow cylinder and convection heat transfer. Derive an expression of heat transfer for a composite plane wall consisting of three layers using thermal resistance, inside and outside wall temperature.
14. Define thermal resistance. Derive an expression for heat transfer through a composite plane wall consisting of three different layers of different materials using electric analogy approach.
15. Define thermal resistance. Write down expressions for thermal resistances for a plane wall, cylindrical layer and convective layer of a fluid.
16. Derive an expression for overall heat transfer coefficient for a composite plane wall consisting of two layers with convection on both sides.
17. Write down the expressions for thermal resistance for a plane wall and a convective fluid layer. Use them to derive overall heat transfer coefficient for a plane wall subjected to convection of both sides.
18. Derive expressions for inside and outside overall heat transfers coefficients for a hollow cylinder subjected to convection medium on both sides.

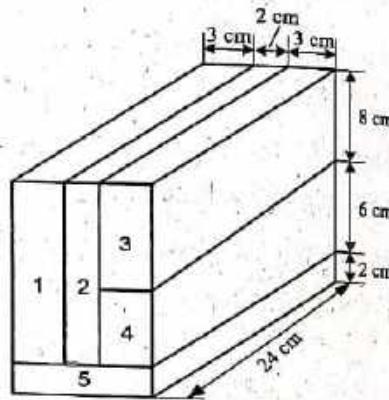
19. Write down the expressions for thermal resistance for a hollow cylinder and convective fluid layer. Use them to derive overall heat transfer coefficient for a hollow cylinder subjected to convection of both sides.
20. Using thermal resistance approach derive an expression for outside overall heat transfer coefficient for two layers of hollow cylinder subjected to convection medium on both sides.
21. Differentiate between free and forced convection with examples. Write down expression for thermal resistance for plane wall, hollow cylinder and convective layer of fluid.
22. Differentiate between black body and gray body. Derive an expression for the rate of heat flow through a composite steel pipe with a layer of insulation on the outside using electrical analogy approach.
23. Define and explain the view factor and combined emissivity.
24. State the laws of radiation: Stefan Boltzmann's Law, Kirchhoff's Law and Wien's Displacement Law.
25. Define the following terms: emissivity, reflectivity, absorptivity, transmissivity, black body, white body, and gray body.
26. Define the dimensionless parameters: Reynold Number, Nusselt Number and Prandtl Number.

8.5 Unsolved Numerical Problems

1. A brick wall 12 cm thick and 5 m^2 surface area is exposed to 250°C at one face and 50°C to another face. If the thermal conductivity of the material is 1.5 W/mK , determine the heat transfer rate. (Ans: 12.5 kW)
2. A 1.2 m long tube with outer diameter of 4 cm having outside temperature of 120°C is exposed to the ambient air at 20°C . If the heat transfer coefficient between the tube surface and the air is $20\text{ W/m}^2\text{K}$, determine the heat transfer rate from the tube to the air. (Ans: 301.6 W)
3. A flat plate collector having collection efficiency of 80% is insulated at the back surface and exposed to solar radiation at the front surface. The front surface receives solar radiation at a rate of 850 W/m^2 and dissipates heat to

the ambient air at 20°C both by convection and radiation. If the convection heat transfer coefficient between the plate and air is $16 \text{ W/m}^2\text{K}$, determine the surface temperature of the plate. (Ans: 323.031 K)

4. An oven covered with asbestos ($k = 0.2 \text{ W/mK}$) has the inner and outer dimensions of $45 \text{ cm} \times 60 \text{ cm} \times 75 \text{ cm}$ and $50 \text{ cm} \times 65 \text{ cm} \times 80 \text{ cm}$ respectively. The inside wall temperature of the oven is 250°C and the outside wall temperature is 35°C . Determine the power input required to maintain the steady state conditions. (Ans: 3637.8 W)
5. A freezer compartment consists of a cubical cavity of 1 m side. The bottom of the compartment is completely insulated. Determine the minimum thickness of the insulation ($k = 0.025 \text{ W/mK}$) that must be applied to the top and side walls to ensure a heat load of less than 400 W when the inner and outer surfaces are at -5°C and 30°C respectively. (Ans: 10.9375 mm)
6. Construct the thermal circuit model for a composite structure as shown in figure. In addition, find the total (equivalent) thermal resistance of the composite structure. The thermal conductivities are $k_1 = k_3 = 80 \text{ W/mK}$, $k_2 = 120 \text{ W/mK}$, $k_4 = 100 \text{ W/mK}$ and $k_5 = 150 \text{ W/mK}$. (Ans: 0.021213 K/W)
7. A furnace wall is made of a layer of fire clay ($k = 0.5 \text{ W/mK}$) 12.5 cm thick and a layer of red brick ($k = 0.8 \text{ W/mK}$) 50 cm thick. If the wall temperature inside the furnace is 1200°C and that on the outside wall is 100°C . Determine the heat loss per unit area of the wall. If it is desired to reduce the thickness of the red brick layer by filling the space between the two layers by diatomite ($k = 0.1 \text{ W/mK}$) such that total thickness remains same. Determine the required thickness of the filling to ensure the amount of heat transfer for the same temperature difference. (Ans: 1257.14 W/m^2 , 2.273 cm)
8. A furnace wall is made up of three layers, one of fire brick, one of insulating brick and one of red brick. The inner and outer surfaces are at 870°C and 40°C respectively. The respective coefficients of thermal conductivities of layers are 0.86 , 0.103 and $0.63 \text{ W/m}^{\circ}\text{C}$ and thicknesses are 22 cm , 7.5 cm and 11 cm . Assuming close bonding of layers at higher interfaces, find the rate of heat loss per square meter per hour and interfaces temperatures. (Ans: $2578.068 \text{ kJ/m}^2\text{hr}$, 686.67°C , 165.6°C)



9. Determine the rate of heat flow through a boiler wall made up of 20 mm thick steel ($k = 58 \text{ W/mK}$). The outer surface of boiler wall is covered with asbestos insulation ($k = 0.116 \text{ W/mK}$) 5 mm thick. The temperature of outer surface and that of fluid inside are 50°C and 300°C respectively. The inner film resistance is 0.0023 K/W .

(Hint: Inner film resistance, $R_i = 1/h_i A_i$; Ans: 5464 W/m^2)

10. A furnace has composite wall refractory material for the inside layer and insulating material on the outside to prevent the heat leakage. The total wall thickness is limited to 60 cm. The temperature of the gases in the furnace is 850°C and the atmospheric air temperature is 30°C . The temperature at the interface of two materials of the furnace wall is limited to 500°C . Find the required thickness of each material and heat loss to atmosphere in W/m^2 . Take the following data:

Conductivity of refractory material, $k_r = 1.72 \text{ W/mK}$

Conductivity of insulating material, $k_i = 0.712 \text{ W/mK}$

Heat transfer coefficient between gas and wall, $h_i = 172 \text{ W/m}^2\text{K}$

Heat transfer coefficient between wall and air, $h_o = 34.4 \text{ W/m}^2\text{K}$

(Ans: $0.533 \text{ m} = 53.3 \text{ cm}$, $0.067 \text{ m} = 6.7 \text{ cm}$, 1116.7 W/m^2)

11. The walls of a furnace $4 \text{ m} \times 3 \text{ m}$ are constructed from an inner fire brick ($k = 0.4 \text{ W/mK}$) wall 30 cm thick, a layer of ceramic blanket insulation ($k = 0.2 \text{ W/mK}$) 10 cm thick and steel protective layer ($k = 50 \text{ W/mK}$) 4 mm thick. The inside temperature of the fire brick layer was measured as 500°C and the temperature of the outside of the insulation as 50°C . Determine:

- the rate of heat loss through the wall;
- the temperature at the interface between fire brick layer and insulation layer and
- the temperature at the outside surface of the steel layer.

(Ans: 4320 W , 230°C , 49.9712°C)

12. The inner dimensions of a freezer compartments are $50 \text{ cm} \times 50 \text{ cm} \times 40 \text{ cm}$. Its wall consists of two 4 mm thick enameled steel sheet ($k = 45 \text{ W/mK}$) separated by 5 cm layer of fiber glass insulation ($k = 0.05 \text{ W/mK}$). The inside temperature is maintained at -10°C and the outside temperature on a hot summer day is 40°C . Calculate the rate at which heat should be thrown out if convective heat transfer coefficients for inner and outer surfaces are $20 \text{ W/m}^2\text{K}$ and $10 \text{ W/m}^2\text{K}$ respectively. Also calculate interface temperatures.

(Ans: 56.514 W , -7.826°C , -7.822°C , 35.65°C , 35.653°C)

13. A composite wall is made up of three layers of thicknesses 200 mm, 100 mm and 120 mm with thermal conductivities of 1.5 W/mK , 3 W/mK , $k \text{ W/mK}$ and respectively. The inside surface is exposed to hot gas at

1250°C and the outside surface is at 300°C which is exposed to ambient air at 25°C . The heat transfer coefficients for inside and outside surfaces are $20 \text{ W/m}^2\text{K}$ and $10 \text{ W/m}^2\text{K}$ respectively. Determine:

- the unknown thermal conductivity k , and
- the interface temperatures.

(Ans: $0.9318 \text{ W/m}^2\text{K}$, 1112.5°C , 745.833°C , 654.167°C)

14. In a coal-fired power plant, a furnace wall consists of a 125 mm wide refractory brick and a 125 mm wide insulating firebrick separated by an air gap. The outside wall is covered with a 12 mm thickness of plaster. The inner surface of the wall is at 1100°C , and the room temperature is 10°C . The heat transfer coefficient from the outside wall surface to the air in the room is $17 \text{ W/m}^2\text{K}$, and the resistance to heat flow of the air gap is 0.16 K/W . The thermal conductivities of the refractor brick, the insulating firebrick, and the plaster are 1.6 , 0.3 , and 0.14 W/mK , respectively. Calculate
- The rate of heat loss per unit area of wall surface.
 - The temperature at each interface throughout the wall.
 - The temperature at the outside surface of the wall.

(Ans: 1363.64 W , 993.465°C , 775.283°C , 207.09°C , 90.206°C)

15. Calculate the rate at which the heat is being lost to the surroundings per meter length of an insulated steam pipe having the following dimensions and specifications: ID of the pipe = 4 cm , thickness of the pipe = 0.2 cm and thickness of insulation = 1 cm .

Temperature of steam = 100°C ; Temperature of surroundings = 30°C

Convective heat transfer coefficient on inside surface = $5 \text{ W/m}^2\text{K}$

Convective heat transfer coefficient on outside surface = $10 \text{ W/m}^2\text{K}$

Thermal conductivity of pipe material = 25 W/mK

Thermal conductivity of insulation = 0.2 W/mK

Emissivity of outer surface of insulation = 0.8

Neglect the radiation effects from inside surface of the pipe. (Ans: 43.03 W)

16. A steel pipe with ID and OD as 80 mm and 120 mm respectively is covered with two layers of insulation, 25 mm and 40 mm thick. The thermal conductivities of insulating materials are 0.2 W/mK and 0.1 W/mK respectively while that of steel is 50 W/mK . The inner surface temperature of the pipe is 200°C while surface temperature of insulation is 40°C . Determine the heat loss from the unit length of the pipe and layer contact temperatures.

(Ans: 179.32 W/m , 199.77°C , 150.07°C)

17. A 140 mm diameter pipe carrying steam is covered by a layer of insulation ($k = 0.5 \text{ W/mK}$) of 30 mm thick. Later, an extra layer of another insulation ($k = 1 \text{ W/mK}$) having a thickness 20 mm is added. If the surrounding temperature remains constant and heat transfer coefficient for both insulating

layers is $10 \text{ W/m}^2\text{K}$, determine the percentage change in heat transfer rate due to extra insulation. (Ans: 0.905%)

8. A steam pipe ($k = 45 \text{ W/mK}$) has inside diameter of 100 mm and outside diameter of 140 mm. It is insulated at the outside with asbestos ($k = 1 \text{ W/mK}$). The steam temperature is 200°C and the air temperature is 25°C . The heat transfer coefficients for inner and outer surfaces are $120 \text{ W/m}^2\text{K}$ and $40 \text{ W/m}^2\text{K}$ respectively. Determine the required thickness of the asbestos in order to limit the heat losses to 1250 W/m ? (Ans: 43.78 mm)
9. A 200 mm diameter 50 m long pipe carrying steam is covered with 40 mm of high temperature insulation ($k = 0.1 \text{ W/m}$) and 30 mm of low temperature insulation ($k = 0.05 \text{ W/m}$). The inner and outer surfaces of the insulating layers are at 400°C and 40°C respectively. Determine:
- the rate of heat loss from the pipe,
 - the temperature at the interface of two insulating layers,
 - the rate of heat transfer from unit area of the pipe surface
 - the rate of heat transfer unit area of the outer surface of composite insulation.
- (Ans: 15604.3 W, 232.87°C , 496.7 W/m^2 , 292.177 W/m^2)

10. A 3 cm OD steam pipe is to be covered with two layers of insulation each having a thickness of 2.5 cm. The thermal conductivity of one insulation is five times that of other. Determine the percentage decrease in heat transfer if better insulating material is next to the pipe than it is at outside. Assume that outside and inside surface temperature of the composite insulation are fixed. (Ans: 36.6%)

Introduction to Refrigeration System

9.1 Chapter Highlights

- **Refrigeration:** It is defined as the technique of producing and maintaining the temperature of a space below the temperature of the surroundings atmosphere. This is possible by removing of heat from a space to be cooled.
- **Refrigerating system:** The device used to maintain the space (system) at a low temperature is known as a *refrigerating system*.
- **Refrigerated system:** The space which is kept at low temperature is called a *refrigerated system*.
- **Refrigerant:** Most of the domestic and commercial refrigeration is produced by the evaporation of a liquid so called a *refrigerant*. The refrigerant is therefore a working substance employed in a refrigeration system.
- **Coefficient of performance (COP):** The performance of a refrigeration system is expressed by a term known as the *coefficient of performance (COP)*. In general, it is defined as the ratio of desired effect to required input, i.e.

$$COP = \frac{\text{Desired effect}}{\text{Required input}}$$

- In the case of a refrigeration device, the desire effect is the refrigerating effect or heat extracted from the space being cooled (refrigerated) denoted by Q_L , and if required input or work supplied to the compressor is denoted by W_{in} , then the coefficient of performance of the refrigeration system is given by

$$(COP)_R = \frac{Q_L}{W_{in}}$$

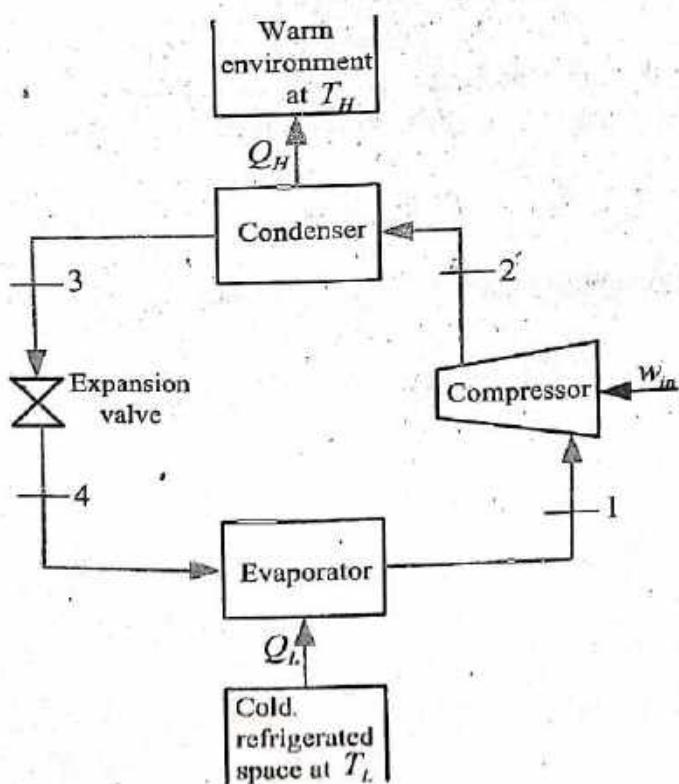
- In the case of a heat pump, the desire effect is the heating effect or heat supplied to the space being heated denoted by Q_H , and if required input or work supplied to the compressor is denoted by W_{in} , then the coefficient of performance of the heat pump is given by

$$(COP)_{HP} = \frac{Q_H}{W_{in}}$$

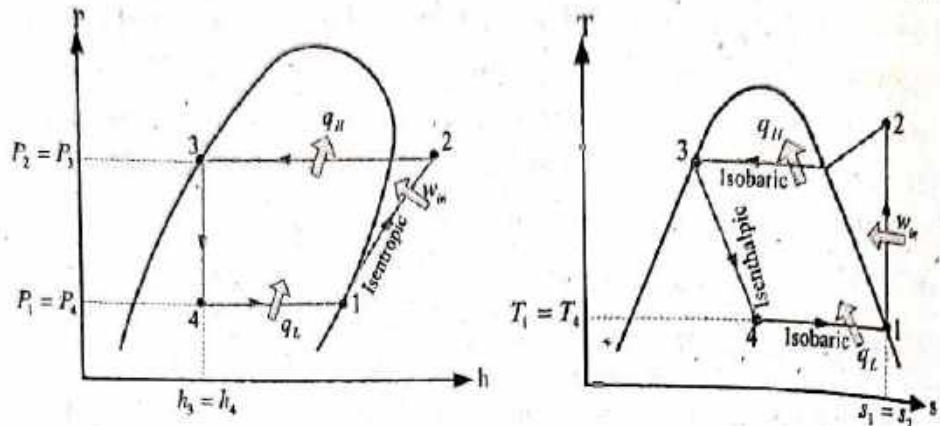
- **Ton of refrigeration:** The standard rating of a refrigeration system is given by a standard unit of refrigeration called *ton of refrigeration*, and it is defined as the refrigerating effect produced by the freezing of 1 ton (2000 pound mass = $2000/2.205 = 907$ kg) of liquid water from 0°C to ice at 0°C during a period of 24 hours. Since the latent heat (specific enthalpy) of fusion of water at 0°C is 333.43 kJ/kg, the refrigerating effect of 333.43×907 kJ in 24 hours (60×60 s) is rated as 1 ton of refrigeration. Hence,

$$1 \text{ ton of refrigeration (TR)} = \frac{333.43 \times 907}{24 \times 60 \times 60} = 3.5 \frac{\text{kJ}}{\text{s}} = 3.5 \text{ kW}$$

- **Vapor compression refrigeration cycle:** It is the most widely used cycle for heating and cooling of the desired space. The working substance used in this cycle is called refrigerant. It consists of four processes, namely
- one isentropic compression process,
 - one isenthalpic process in a throttling or expansion valve,
 - two isobaric processes, namely an isobaric heat rejection process in a condenser, and an isobaric heat addition process in an evaporator.
- **Different components of an ideal vapor compression refrigeration cycle:**



- *P-h and T-S diagrams of an ideal vapor compression refrigeration cycle:*



- When the cycle is working as a heat pump for heating of a desired space, its coefficient of performance (*COP*) is given by

$$(COP)_{HP} = \frac{Q_H}{W_{in}} = \frac{q_H}{w_{in}} = \frac{h_2 - h_3}{h_2 - h_1}$$

- When the cycle is working as a refrigerator for cooling of a desired space, its coefficient of performance (*COP*) is given by

$$(COP)_R = \frac{Q_L}{W_{in}} = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1}$$

- Since the throttling process is an isenthalpic process, i.e., $h_3 = h_4$, for fixed values of Q_H and Q_L a comparison of above equations reveals that

$$(COP)_{HP} = (COP)_R + 1$$

This relation implies that $(COP)_{HP} > 1$ since $(COP)_R$ is a positive quantity.

Solved Numerical Problems

9.1

Problem 9.1.

A refrigerating device of 1 ton rating needs 1.5 kW to maintain low temperature of evaporator. Calculate coefficient of performance of the device when it is working as a refrigerator and a heat pump.

Solution: Given,

Refrigerating effect = Rating of refrigeration, $\Rightarrow Q_L = 1 \text{ ton} = 3.5 \text{ kW}$

Work input, $W_{in} = 1.5 \text{ kW}$

The coefficient of performance of the device when it is working as a refrigerator is given by

$$(COP)_R = \frac{Q_L}{W_{in}} = \frac{3.5}{1.5} = 2.333$$

Similarly, the coefficient of performance of the device when it works as a heat pump is given by

$$(COP)_{HP} = 1 + (COP)_R = 1 + 2.333 = 3.333$$

Problem 9.2.

A refrigeration system having coefficient of performance (COP) 8.93 produces 400 kg/hr of ice at -5°C from water at 25°C . Calculate

- (i) the power needed to drive the refrigerating device and
- (ii) the heat rejected by the system.

Take latent heat of freezing = 335 kJ/kg, specific heat of ice = 2.1 kJ/kgK and specific heat of water = 4.186 kJ/kgK.

Solution: Given,

Coefficient of performance of refrigerating device, $(COP)_R = 8.93$

Rate of ice production, $\dot{m} = 400 \text{ kg/hr}$

Temperature of water, $T_1 = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$

Temperature of ice, $T_2 = -5^\circ\text{C} = -5 + 273 = 268 \text{ K}$

Latent heat of freezing, $L = 335 \text{ kJ/kg}$

Specific heat of ice, $c_i = 2.1 \text{ kJ/kgK}$

Specific heat of water, $c_w = 4.186 \text{ kJ/kgK}$

Net refrigerating effect or rate of heat absorbed, \dot{Q}_L can be calculated as

$$\begin{aligned}\dot{Q}_L &= \dot{m}[c_i(273 - T_2) + L + c_w(T_1 - 273)] \\ &= 400 \times [2.1 \times (273 - 268)] + 335 + 4.186 \times (298 - 273) \\ &= 180060.0 \text{ kJ/hr} = 50.02 \text{ kJ/s} = 50.02 \text{ kW}\end{aligned}$$

Now, coefficient of performance (COP) of refrigerating device is given by

$$(COP)_R = \frac{Q_L}{W_{in}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

From this relation, the power needed to drive the refrigerating device is calculated as

$$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{50.02}{8.93} = 5.601 \text{ kW}$$

Applying first of thermodynamics,

$$\begin{aligned} & \text{Heat rejection} = \text{heat absorbed} + \text{Work input} \\ \Rightarrow & \dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 50.02 + 5.601 = 55.62 \text{ kW} \end{aligned}$$

Problem 9.3.

A refrigerating device with COP 7.5 is used for making ice - 5°C from water at 25°C. The temperature of freezing mixture is - 10°C. Calculate the quantity of ice formed per kWh.

Take latent heat of freezing = 335 kJ/kg, specific heat of ice = 2.1 kJ/kgK and specific heat of water = 4.186 kJ/kgK.

Solution: Given,

Coefficient of performance of refrigerating device, $(COP)_R = 7.5$

Temperature of water, $T_1 = 25^\circ \text{C} = 25 + 273 = 298 \text{ K}$

Temperature of freezing mixture, $T_2 = -10^\circ \text{C} = -10 + 273 = 263 \text{ K}$

Temperature of ice, $T_3 = -5^\circ \text{C} = -5 + 273 = 268 \text{ K}$

Work input $\dot{W}_{in} = 1 \text{ kWh} = 3600 \text{ kW} \cdot \text{s} = 3600 \text{ kJ}$

Latent heat of freezing, $L = 335 \text{ kJ/kg}$

Specific heat of ice, $c_i = 2.1 \text{ kJ/kgK}$

Specific heat of water, $c_w = 4.186 \text{ kJ/kgK}$

The coefficient of performance (COP) of refrigerating device is given by

$$(COP)_R = \frac{Q_L}{W_{in}}$$

From this relation, the total heat extracted from the ice is calculated as

$$Q_L = W \times (COP)_R = 3600 \times 7.5 = 27000 \text{ kJ}$$

Removal of heat from 1 kg mass of water ($m = 1 \text{ kg}$) at 25°C so that it becomes ice at - 5°C can be determined as

$$\begin{aligned} Q_L &= m[c_w(T_1 - 273) + L + c_i(273 - T_3)] \\ \Rightarrow Q_L/m &= [4.186 \times (298 - 273) + 335 + 2.1 \times (273 - 268)] \end{aligned}$$

$$\therefore q_L = 450.15 \text{ kJ/kg}$$

Hence, the quantity of ice formation per kWh can be calculated as

$$m = \frac{Q_L}{q_L} = \frac{27000}{450.15} = 59.98 = 60 \text{ kg}$$

Problem 9.4.

A cold storage plant is required to store 20 tons of fish. The temperature of the fish when supplied is 25°C , storage temperature of fish required is -8°C , freezing point of fish is -3°C , specific heat of fish above freezing point is $2.93 \text{ kJ/kg}^\circ\text{C}$, specific heat of fish below freezing point is $1.25 \text{ kJ/kg}^\circ\text{C}$ and latent heat of fish is 232 kJ/kg .

If the cooling is achieved within 8 hours, calculate

- (a) the capacity of the refrigerating plant and
- (b) the power required to run the plant when the actual COP of the plant is 2.67.

Solution: Given,

Mass of fish to be stored, $m = 20 \text{ tons} = 20 \times 1000 = 20000 \text{ kg}$

Temperature of fish before refrigeration, $T_1 = 25^\circ\text{C}$

Freezing temperature of fish, $T_2 = -3^\circ\text{C}$

Required storage temperature of fish, $T_3 = -8^\circ\text{C}$

Specific heat of fish above freezing point, $c_{af} = 2.93 \text{ kJ/kg}^\circ\text{C}$

Specific heat of fish below freezing point, $c_{bf} = 1.25 \text{ kJ/kg}^\circ\text{C}$

Latent heat of fish, $L = 232 \text{ kJ/kg}$

Time of cooling, $t = 8 \text{ hrs}$

Actual COP of plant, $(COP)_R = 2.67$

We know for a refrigeration system,

$$1 \text{ ton of refrigeration (TR)} = \frac{333.43 \times 907}{24} = 12600.875 \text{ kJ/h} \\ = 3.5 \text{ kW}$$

Heat removed in 8 hours from 1 kg of fish can be calculated as

$$Q_L = m[c_{af}(T_1 - T_2) + L + c_{bf}(T_2 - T_3)]$$

$$\Rightarrow Q_L/m = [2.93 \times \{25 - (-3)\} + 232 + 1.25 \times \{-3 - (-8)\}]$$

$$\therefore q_L = 320.29 \text{ kJ/kg}$$

Heat removed by the cold storage plant per hour can be determined as

$$\dot{Q}_L = \frac{\dot{Q}_L}{t} = \frac{q_L \times m}{t} = \frac{320.29 \times 20000}{8} = 800725 \text{ kJ/h} = 222.423 \text{ kW}$$

Now, the capacity of the refrigerating plant is given by

$$C = \frac{\dot{Q}_L}{TR} = \frac{222.423}{3.5} = 63.55 \text{ ton}$$

The actual coefficient of performance (*COP*) of refrigerating plant is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

From this relation, the power required to run the plant is calculated as

$$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{222.423}{2.67} = 83.304 \text{ kW}$$

Problem 9.5.

*A refrigerating system operates with coefficient of performance (*COP*) 5.16. The capacity of the refrigerating system is to be 12 ton. Neglecting all losses determine*

- (a) power required in kW and
- (b) heat rejected from the system per hour.

Solution: Given,

COP of refrigerating system, $(COP)_R = 5.16$

Capacity of refrigerating system, $C = 12 \text{ ton}$

We know for a refrigeration system,

$$1 \text{ ton of refrigeration (TR)} = \frac{333.43 \times 907}{24} = 12600.875 \text{ kJ/h} \\ = 3.5 \text{ kW}$$

Now, the capacity of the refrigerating system is given by

$$C = \frac{\dot{Q}_L}{TR}$$

Then, net refrigerating effect or heat absorbed by the refrigerating system per hour can be determined as

$$\dot{Q}_L = C \times TR = 12 \times 12600.875 = 151210.5 \text{ kJ/h}$$

The coefficient of performance (*COP*) of refrigerating plant is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

From this relation, the work input required per hour to run the refrigerating system is calculated as

$$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{151210.5}{5.16} = 29304.36 \text{ kJ/h}$$

Therefore, power required in kW, $\dot{W}_{in} = \frac{\text{Work inpt per hour}}{60 \times 60}$
 $= 29304.36/3600 = 8.14 \text{ kW}$

Applying first of thermodynamics, heat rejected from the refrigerating system per hour is given by

$$\text{Heat rejection} = \text{heat absorbed} + \text{Work input}$$

$$\Rightarrow \dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 151210.5 + 29304.36 = 180514.86 \text{ kJ/h}$$

Problem 9.6.

In a vapor compression refrigeration cycle, the condenser temperature is 20 °C and the evaporator temperature is -10 °C. Saturated liquid enters the throttling (expansion) valve and saturated vapor enters the compressor. For a refrigeration effect of 3.5 kW, determine COP, mass flow rate of the refrigerant and the power input if the refrigerant is ammonia. [Take specific heat at constant pressure of ammonia vapor at 20 °C, $c_p = 2.17 \text{ kJ/kgK}$].

(Refer attached table for the properties of ammonia)

T (°C)	P (kPa)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kgK)	s_g (kJ/kgK)
-10	290.9	153.8	1450.5	0.8288	5.7564
20	857.6	294.3	1479.9	1.3308	5.3753

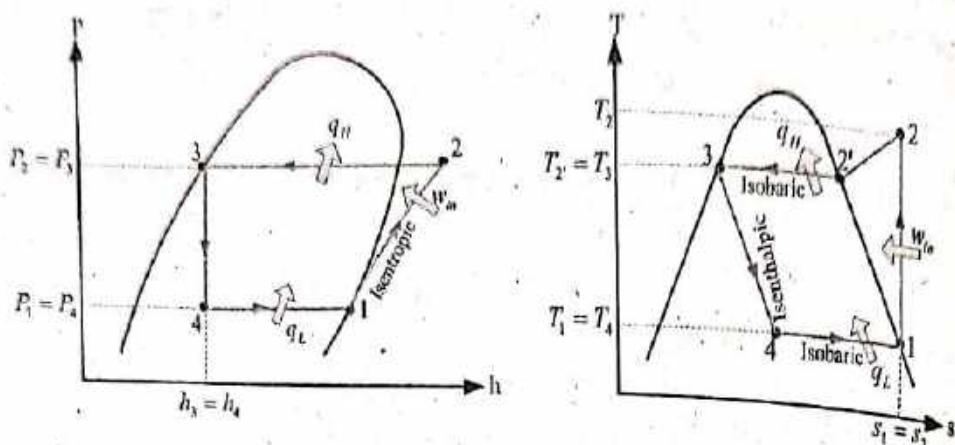
Solution: Given,

Condenser temperature, $T_3 = 20^\circ\text{C}$ and state 3 is saturated liquid.

Evaporator temperature, $T_1 = T_4 = -10^\circ\text{C}$ and state 1 is saturated vapor.

Refrigeration effect (rate of heat rejected by desired space), $\dot{Q}_L = 3.5 \text{ kW}$

With reference to $P-h$ and $T-s$ diagrams of the cycle given below, properties of refrigerant ammonia at each state are determined as follows:



Initial state 1 : Temperature, $T_1 = -10^\circ\text{C}$ and saturated vapor state.

Referring to given saturated ammonia temperature table for temperature, $T_1 = -10^\circ\text{C}$,

$$h_1 = h_g(T_1 = -10^\circ\text{C}) = 1450.5 \text{ kJ/kg}$$

$$s_1 = s_g(T_1 = -10^\circ\text{C}) = 5.7564 \text{ kJ/kgK}$$

State 2: It is a superheated vapor.

Process 1 – 2 is an isentropic compression, so specific entropy at state 2,
 $s_2 = s_1 = 5.7564 \text{ kJ/kgK}$.

State 2' in T – s diagram is the intermediate state during process 2 – 3 which lies at saturated vapor line and it is the saturated vapor state beyond which the condensation of ammonia starts.

So, referring to the given saturated ammonia temperature table for temperature, $T_2 = T_3 = 20^\circ\text{C}$ to find the specific enthalpy and specific entropy at state 2',

$$h_{2'} = h_g(T_2' = 20^\circ\text{C}) = 1479.9 \text{ kJ/kg}$$

$$s_{2'} = s_g(T_2' = 20^\circ\text{C}) = 5.3753 \text{ kJ/kgK}$$

Using the entropy relation during the isobaric process 2 – 2' ($P_2 = P_{2'}$) for an ideal gas (assuming ammonia vapor acts as an ideal gas) to find the temperature at state 2, T_2 , i.e.

$$\begin{aligned} s_2 - s_{2'} &= c_p \ln\left(\frac{T_2}{T_{2'}}\right) - R \ln\left(\frac{P_2}{P_{2'}}\right) = c_p \ln\left(\frac{T_2}{T_{2'}}\right) - 0 \\ \Rightarrow \quad \ln\left(\frac{T_2}{T_{2'}}\right) &= \frac{s_2 - s_{2'}}{c_p} = \frac{5.7564 - 5.3753}{2.17} = 0.175622 \end{aligned}$$

Taking antilog on both sides of the above equation, we get

$$T_2 / T_{2'} = e^{0.175622}$$

$$\therefore T_2 = T_{2'} \times e^{0.175622} = (20 + 273) e^{0.175622} = 349.25 \text{ K} = 76.25^\circ\text{C}$$

Now, the specific enthalpy at state 2, h_2 can be determined from the relation, $dh = c_p dT$ for process 2 – 2' as

$$\Rightarrow h_2 - h_1 = c_p (T_2 - T_1) \\ \therefore h_2 = h_1 + c_p (T_2 - T_1) = 1479.9 + 2.17(76.25 - 20) \\ = 1601.96 \text{ kJ/kg}$$

State 3: Temperature, $T_3 = 20^\circ\text{C}$ and saturated liquid state.
 Referring to the given saturated ammonia temperature table for temperature, $T_3 = 20^\circ\text{C}$ to find the specific enthalpy,

$$h_3 = h_l(T_3 = 20^\circ\text{C}) = 294.3 \text{ kJ/kg}$$

State 4: Temperature, $T_4 = -10^\circ\text{C}$ and two phase mixture.

Process 3 – 4 is an isenthalpic (constant enthalpy) expansion or throttling, so specific enthalpy at state 4, $h_4 = h_3 = 294.3 \text{ kJ/kg}$.
 Heat removed per kg of the refrigerant from the desired space,

$$q_L = h_1 - h_4 = 1450.5 - 294.3 = 1156.2 \text{ kJ/kg}$$

Work required per kg of refrigerant,

$$w_{in} = h_2 - h_1 = 1601.96 - 1450.5 = 151.46 \text{ kJ/kg}$$

Then, coefficient of performance, COP of the cycle is given by

$$COP = \frac{q_L}{w_{in}} = \frac{1156.2}{151.46} = 7.6337$$

Mass flow rate of the refrigerant is determined as

$$\dot{m} = \frac{\dot{Q}_L}{q_L} = \frac{3.5}{1156.2} = 0.003027 \text{ kg/s}$$

Power input to the compressor,

$$\dot{W}_{in} = \dot{m} \times w_{in} = 0.003027 \times 151.46 = 0.45847 \text{ kJ/s} = 458.47 \text{ W}$$

Problem 9.7.

An ice making device operates on ideal vapor compression refrigeration cycle using refrigerant R-12. The refrigerant enters the compressor as saturated vapor at -15°C and leaves the condenser as saturated liquid at 30°C . Water enters the device at 15°C and leaves as ice at -5°C . For an ice production rate of 2400 kg in a day, determine the coefficient of performance (COP) of the device and the power required to run the unit. To solve the problem, use properties of R - 12 as given below. Take the latent heat of fusion for water as $L = 335 \text{ kJ/kg}$, specific heat for water as $c_w = 4.187 \text{ kJ/kg}^\circ\text{C}$ and specific heat for ice as $c_i = 2.0935 \text{ kJ/kg}^\circ\text{C}$.

Properties of R - 12:

Specific enthalpy of saturated vapor at -15°C , $h_g = 344.927 \text{ kJ/kg}$

Specific entropy of saturated vapor at -15°C , $s_g = 1.56323 \text{ kJ/kgK}$

Specific entropy of saturated vapor at 30°C , $s_g = 1.5434 \text{ kJ/kgK}$

Specific enthalpy of saturated vapor at 30°C , $h_g = 363.575 \text{ kJ/kg}$

Specific enthalpy of saturated liquid at 30 °C, $h_l = 228.538 \text{ kJ/kg}$

Specific heat capacity at constant pressure, $c_p = 0.611 \text{ kJ/kg °C}$

Solution: Given,

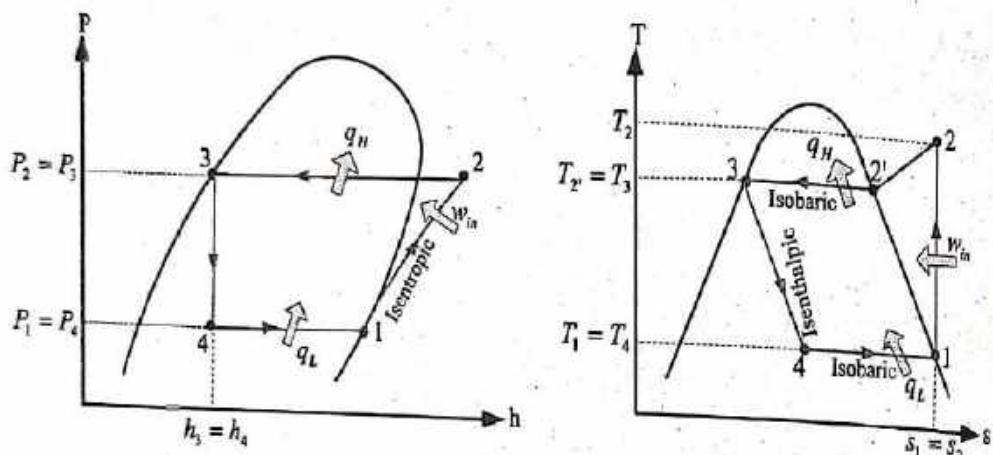
Condenser temperature, $T_3 = 30^\circ\text{C}$ and state 3 is saturated liquid.

Evaporator temperature, $T_1 = T_4 = -15^\circ\text{C}$ and state 1 is saturated vapor.

Temperature of water, $T_w = 15^\circ\text{C}$

Temperature of ice, $T_i = -5^\circ\text{C}$

Rate of ice production, $\dot{m} = 2400 \text{ kg/day} = 2400/24 = 100 \text{ kg/h} = 100/3600 = 0.028 \text{ kg/s}$



With reference to $P - h$ and $T - s$ diagrams of the cycle given above, properties of the refrigerant at each state are determined as follows:

Initial state 1 : Temperature, $T_1 = -15^\circ\text{C}$ and saturated vapor state.

Referring to given properties of the refrigerant for temperature, $T_1 = -15^\circ\text{C}$, Specific enthalpy, $h_1 = h_g(T_1 = -15^\circ\text{C}) = 344.927 \text{ kJ/kg}$

Specific entropy, $s_1 = s_g(T_1 = -15^\circ\text{C}) = 1.56323 \text{ kJ/kgK}$

State 2: It is a superheated vapor.

Process 1 – 2 is an isentropic compression, so specific entropy at state 2,
 $s_2 = s_1 = 1.56323 \text{ kJ/kgK}$.

State 2' in $T - s$ diagram is the intermediate state during process 2 – 3 which lies at saturated vapor line and it is the saturated vapor state beyond which the condensation of the refrigerant starts.

So, referring to the given properties of the refrigerant for temperature, $T_{2'} = T_3 = 30^\circ\text{C}$ to find the specific enthalpy and specific entropy at state 2',

$$h_{2'} = h_g(T_{2'} = 30^\circ\text{C}) = 363.575 \text{ kJ/kg}$$

$$s_{2'} = s_g(T_{2'} = 30^\circ\text{C}) = 1.5434 \text{ kJ/kgK}$$

Using the entropy relation during the isobaric process $2 - 2'$ ($P_2 = P_{2'}$) for an ideal gas (assuming the refrigerant vapor acts as an ideal gas) to find the temperature at state 2, T_2 , i.e.

$$s_2 - s_{2'} = c_p \ln\left(\frac{T_2}{T_{2'}}\right) - R \ln\left(\frac{P_2}{P_{2'}}\right) = c_p \ln\left(\frac{T_2}{T_{2'}}\right) - 0$$

$$\Rightarrow \ln\left(\frac{T_2}{T_{2'}}\right) = \frac{s_2 - s_{2'}}{c_p} = \frac{1.56323 - 1.5434}{0.611} = 0.032455$$

Taking antilog on both sides of the above equation, we get

$$T_2/T_{2'} = e^{0.032455}$$

$$\therefore T_2 = T_{2'} \times e^{0.032455} = (30 + 273) e^{0.032455} = 312.9952 \text{ K} = 39.995^\circ\text{C}$$

Now, the specific enthalpy at state 2, h_2 can be determined from the relation, $dh = c_p dT$

for isobaric process $2 - 2'$ as

$$\Rightarrow h_2 - h_{2'} = c_p (T_2 - T_{2'})$$

$$\therefore h_2 = h_{2'} + c_p (T_2 - T_{2'}) = 363.575 + 0.611(39.995 - 30) = 369.682 \text{ kJ/kg}$$

State 3: Temperature, $T_3 = 30^\circ\text{C}$ and saturated liquid state.

Referring to the given properties of the refrigerant for temperature, $T_3 = 30^\circ\text{C}$ to find the specific enthalpy,

$$h_3 = h_l(T_3 = 30^\circ\text{C}) = 228.538 \text{ kJ/kg}$$

State 4: Temperature, $T_4 = -15^\circ\text{C}$ and two phase mixture.

Process $3 - 4$ is an isenthalpic (constant enthalpy) expansion or throttling, so specific enthalpy at state 4, $h_4 = h_3 = 228.538 \text{ kJ/kg}$.

Heat absorbed per kg of the refrigerant,

$$q_L = h_1 - h_4 = 344.927 - 228.538 = 116.389 \text{ kJ/kg}$$

Work required per kg of refrigerant,

$$w_{in} = h_2 - h_1 = 369.682 - 344.927 = 24.755 \text{ kJ/kg}$$

Then, coefficient of performance, COP of the device is given by

$$(COP)_R = \frac{q_L}{w_{in}} = \frac{116.389}{24.755} = 4.702$$

Heat to be removed per second to produce ice can be calculated as

$$\begin{aligned} \dot{Q}_L &= \dot{m}[c_w(T_w - 0) + L + c_i(0 - T_i)] \\ &= 0.028 [4.187(15 - 0) + 335 + 2.0935\{0 - (-5)\}] \\ &= 11.432 \text{ kJ/s} = 11.432 \text{ kW} \end{aligned}$$

Power input to the compressor,

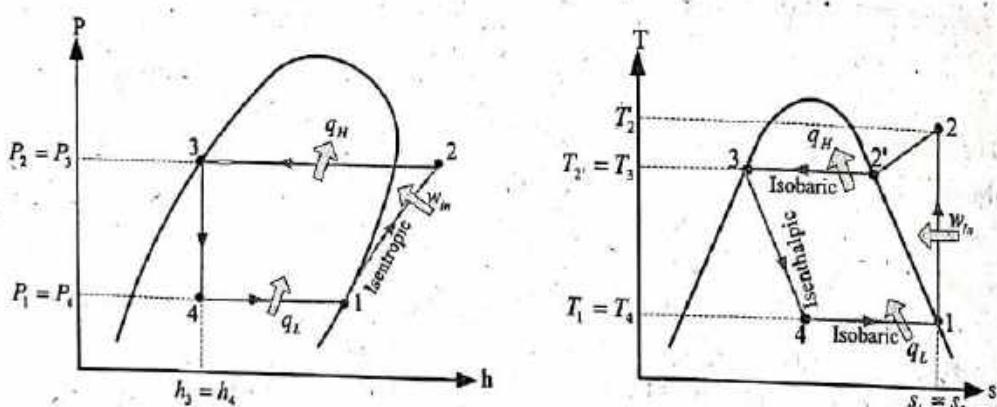
$$\dot{W}_{in} = \frac{\dot{Q}_L}{(COP)_R} = \frac{11.432}{4.702} = 2.43 \text{ kW}$$

Problem 9.8.

A refrigerator operating on an ideal vapor compression refrigeration cycle has a coefficient of performance of 6.5 and is driven by a 50 kW compressor. The enthalpies of saturated liquid and saturated vapor refrigerant at the condensing temperature of 35°C are 69.55 kJ/kg and 201.45 kJ/kg respectively. The saturated refrigerant vapor leaving evaporator has an enthalpy of 187.53 kJ/kg. Find the refrigerant temperature at compressor outlet. The specific heat (c_p) of refrigerant vapor is 0.6155 kJ/kg °C.

Solution:

For the calculation reference, $P - h$ and $T - s$ diagrams of the cycle are given below.



Given,

Coefficient of performance, $(COP)_R = 6.5$

Power input, $\dot{W}_{in} = 50 \text{ kW}$

Condenser temperature, $T_3 = T_2 = 35^\circ\text{C}$

Specific enthalpy of saturated liquid at 35°C, $h_3 = 69.55 \text{ kJ/kg}$

Specific enthalpy of saturated vapor at 35°C, $h_2 = 201.45 \text{ kJ/kg}$

Specific enthalpy of saturated vapor at evaporator exit, $h_1 = 187.53 \text{ kJ/kg}$

Specific heat of refrigerant vapor, $c_p = 0.6155 \text{ kJ/kg}^\circ\text{C}$

Net refrigerating effect or rate of heat absorbed by the refrigerator can be determined as

$$\dot{Q}_L = (COP)_R \times \dot{W}_{in} = 6.5 \times 50 = 325 \text{ kW}$$

Heat extracted or absorbed per kg of the refrigerant is given by

$$q_L = h_1 - h_4 = h_1 - h_3 = 187.53 - 69.55 = 117.98 \text{ kJ/kg}$$

(As $h_4 = h_3$)

Mass flow rate of the refrigerant is then calculated as

$$\dot{m} = \frac{\dot{Q}_L}{q_L} = \frac{325}{117.98} = 2.755 \text{ kg/s}$$

Work input to the compressor per kg of the refrigerant is given by

$$W_{in} = \frac{\dot{W}_{in}}{\dot{m}} = \frac{50}{2.755} = 18.15 \text{ kJ/kg}$$

Now, the specific enthalpy of the refrigerant vapor after compression is determined as

$$h_2 = h_1 + W_{in} = 187.53 + 18.15 = 205.68 \text{ kJ/kg}$$

Assuming the refrigerant superheated vapor to be an ideal gas, the refrigerant temperature T_2 at the compressor outlet, i.e. at state 2 can be determined from the relation, $dh = c_p dT$ for the isobaric process 2 – 2' as

$$\Rightarrow h_2 - h_{2'} = c_p (T_2 - T_{2'})$$

$$\therefore T_2 = T_{2'} + \frac{h_2 - h_{2'}}{c_p} = 35 + \frac{205.68 - 201.45}{0.6155} = 41.872^\circ\text{C}$$



9.3 Review Theoretical Questions

1. Define the terms: refrigeration, refrigerating system and refrigerated system.
2. Define the coefficient of performance (COP) of a refrigeration device.
3. What is a refrigerant?
4. What is a standard rating of a refrigeration device?
5. What do you understand by 1 ton of refrigeration?
6. Describe an ideal vapor compression refrigeration cycle with its different components giving flow diagram.
7. Draw $P - h$ and $T - s$ diagrams of an ideal vapor compression refrigeration cycle.
8. Write down the functions of the following components of an ideal vapor compression refrigeration cycle: compressor, condenser, throttling (expansion) valve and evaporator.
9. Derive the expressions for coefficient of performance (COP) for an ideal vapor compression refrigeration cycle when it is working as a refrigerator and a heat pump.
10. Prove that the coefficient of performance of heat pump is greater than that of a refrigerator by unity for fixed values of Q_H and Q_L , i.e.

$$(COP)_{HP} = (COP)_R + 1$$

Unsolved Numerical Problems

4. A refrigerator needs 1.3 kW per ton of refrigeration to maintain a space at low temperature of -38°C . Determine
 (a) coefficient of performance (*COP*) of the refrigerator,
 (b) the heat delivered and *COP* when this device is used as heat pump.
 (Ans: 2.69, 4.8 kW, 3.69)

A cold storage is to be maintained at -5°C while the surroundings are at 35°C . The heat leakage from the surroundings into the cold storage is estimated to be 29 kW. If the actual *COP* of the refrigeration plant is 2.233, determine the power needed to run the plant.
 (Ans: 12.98 kW)

3. Ice is formed at 0°C from water 20°C . The temperature of the brine is -8°C . If coefficient of performance (*COP*) of the refrigeration cycle used is 9.46, determine the mass of ice formed per kWh. Take latent heat of ice as 335 kJ/kg and specific heat of water as 4.18 kJ/kgK. (Hint: per kWh means work input is $1 \times 3600 = 3600$ kJ). (Ans: 81.36 kg)
4. A refrigerating device having coefficient of performance (*COP*) of 10.19 produces 400 kg of ice per hour at -8°C from feed water at 18°C . Determine the minimum power input to the refrigerating device assuming specific heat of water as 4.18 kJ/kgK, specific heat of ice as 2.09 kJ/kgK and latent heat of fusion 334 kJ/kg. (Ans: 4.645 kW)
5. The capacity of a refrigerator is 280 ton when operating between -10°C and 25°C . If the refrigerator has coefficient of performance (*COP*) of 7.51, calculate
 (a) quantity of ice formed within 24 hours when water is supplied at 20°C and
 (b) least power needed to drive the refrigerator in kW.
 Take specific heat of water as 4.18 kJ/kgK and latent heat of fusion as 335 kJ/kg. (Ans: 202288.3 kg = 202.29 tons, 130.5 kW)
6. In a standard vapor compression refrigeration cycle, operating between an evaporator temperature of -10°C and a condenser temperature of 40°C , the enthalpy of the refrigerant (Freon - 12) at the end of compression is 220 kJ/kg. Show the cycle on *T-s* diagram. Determine:
 (a) the *COP* of the cycle
 (b) the refrigerating capacity and the compressor power assuming a refrigerant flow rate of 1 kg/min.

Refer attached table for the properties of Freon - 12.

<i>T (°C)</i>	<i>P (MPa)</i>	<i>h_l (kJ/kg)</i>	<i>h_g (kJ/kg)</i>
-10	0.2191	26.85	183.1
40	0.9607	74.53	203.1

(Ans: 2.94, 1.81 kW, 0.615 kW)

7. A refrigerator with Freon - 12 refrigerant producing a cooling effect of 20 kW operates on a simple vapor compression refrigeration cycle, with pressure limits of 1.509 bar and 9.607 bar. The vapor leaves the evaporator as dry saturated and there is no under-cooling. Determine the power required by the machine.

Refer attached table for the properties of Freon - 12.

<i>T (°C)</i>	<i>P (bar)</i>	<i>v_g (m³/kg)</i>	<i>h_l (kJ/kg)</i>	<i>h_g (kJ/kg)</i>	<i>s_l (kJ/kgK)</i>	<i>s_g (kJ/kgK)</i>	<i>c (kJ/kgK)</i>
-20	1.509	0.1088	17.8	178.61	0.073	0.7082	-
40	9.607	-	74.53	203.05	0.2716	0.682	0.747

(Ans: 6.29 kW)

8. An ideal vapor compression refrigeration cycle uses Freon-12 as the working fluid. The temperature of the refrigerant in the evaporator is -10°C. The condensing temperature is 40°C and the cooling load is 150 W. Calculate the mass flow rate of the refrigerant.

Properties of Freon-12

<i>T (°C)</i>	<i>P (MPa)</i>	<i>h_l (kJ/kg)</i>	<i>h_g (kJ/kg)</i>
-10	0.22	26.8	183.0
40	0.96	74.5	203.1

(Ans: 0.0013825 kg/s)

9. In an ideal vapor compression refrigeration cycle, following are the properties of the refrigerant R-12 at different states:

At compressor inlet: $h_1 = 183.2 \text{ kJ/kg}$

At compressor outlet: $h_2 = 222.6 \text{ kJ/kg}$

At condenser exit: $h_3 = 84.3 \text{ kJ/kg}$

If the mass flow rate of the refrigerant is 25.03 kg/min, calculate the power rating of the compressor and the refrigerating effect in kW. (Ans: 16.436 kW, 41.0075 kW)
