Superposition Theorem

Statement:

The superposition theorem states that, " In any linear networks containing two or more sources, the responce in any element is equal to the algebric sum of the responce causes by individual source acting along, while the other source are non-operative;" ~~i.e.~~

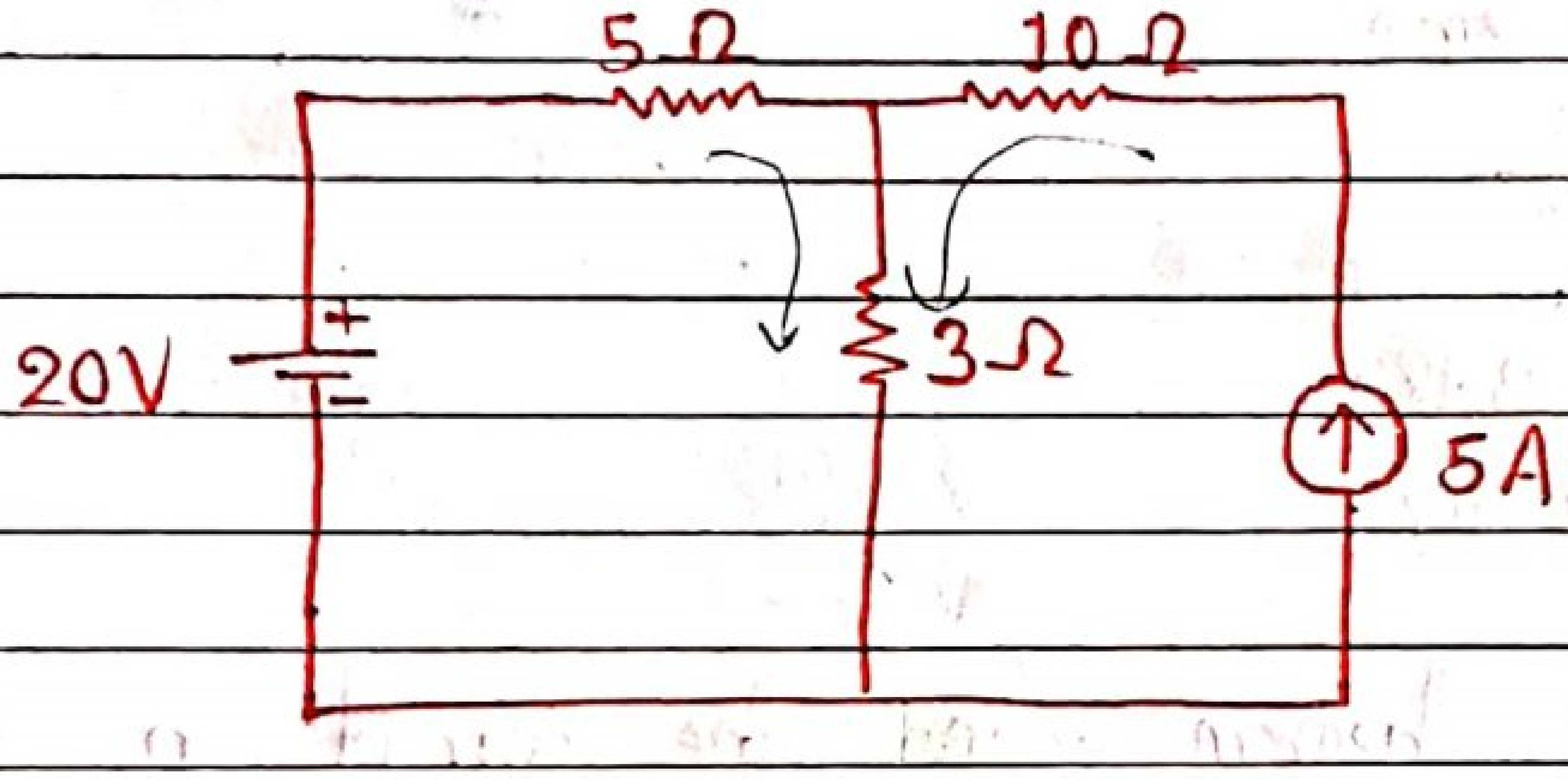
i.e. while considering the effect of individual source, other ideal voltage source & other ideal current source in the network are replaced by short ckt & open ckt across their terminals.

This theorem is valid only for linear system.

Procedure

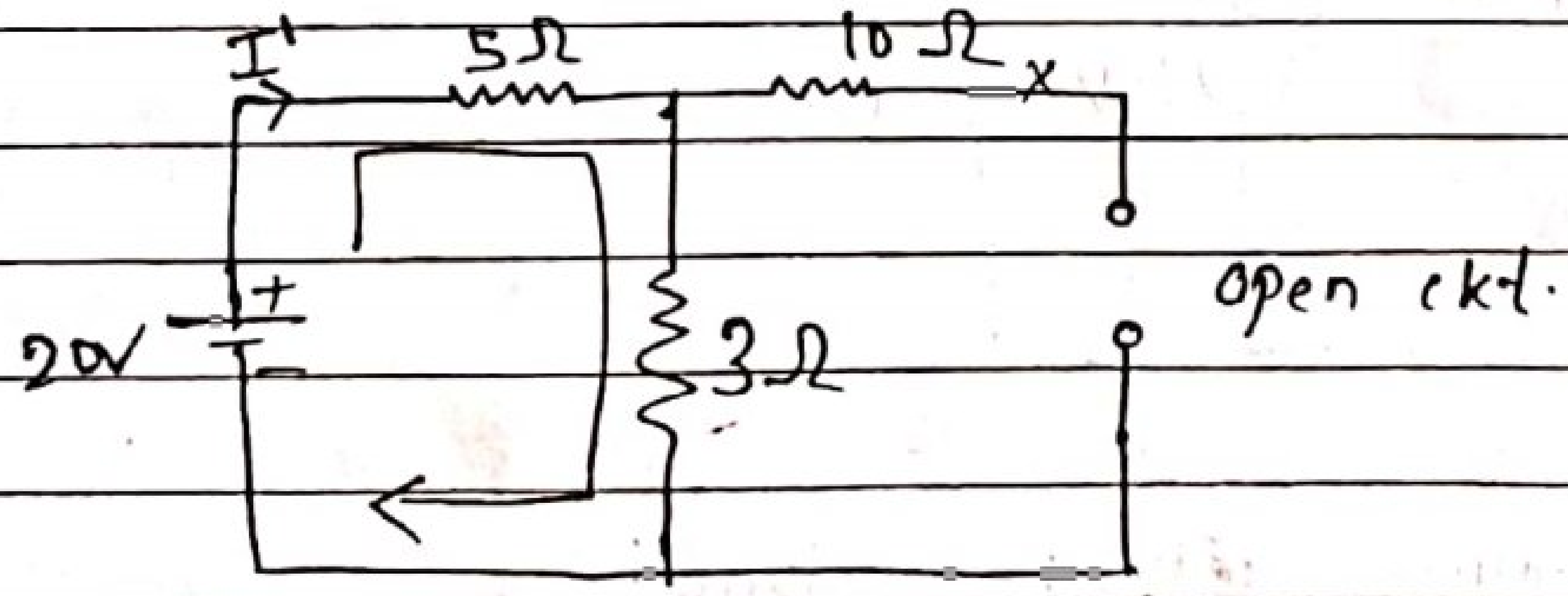
1) ~~Remove the source~~ Pick one source,
2) Remove the source,
3) Voltage – short ckt (s.c.)
   Current – open ckt (o.c.)
4) Determine the individual responce,
5) Sum of the responce of reaction.

Q.1) Use the superposition theorem, find the current passing through 3-Ω resistor.



solution:-

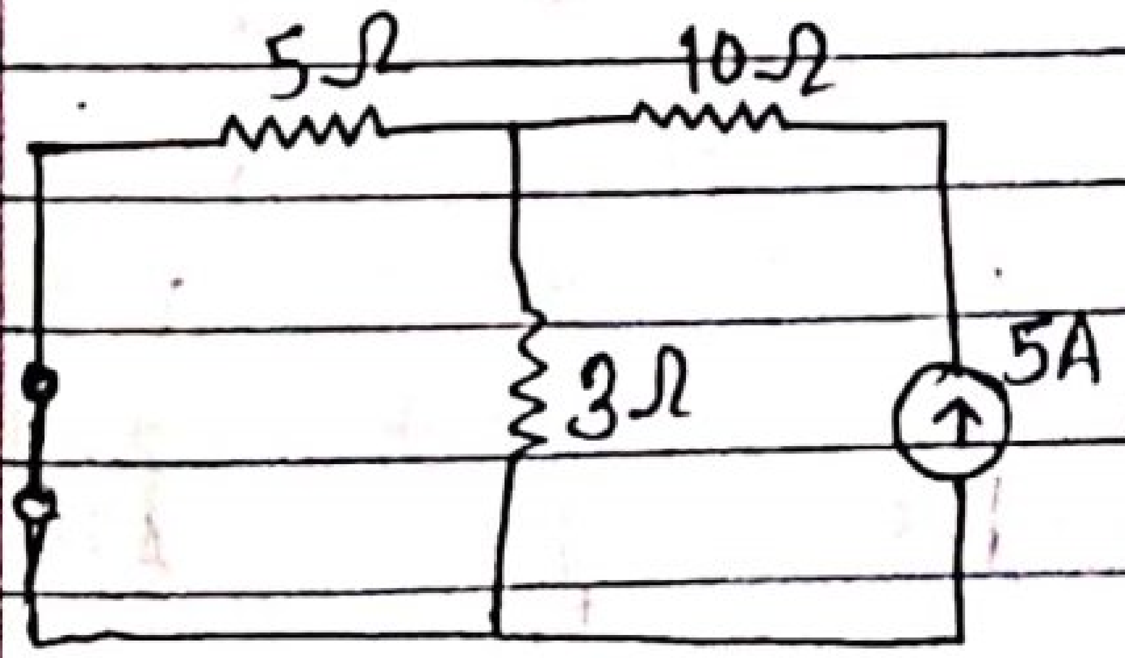
Case I: Consider the voltage source (20V):



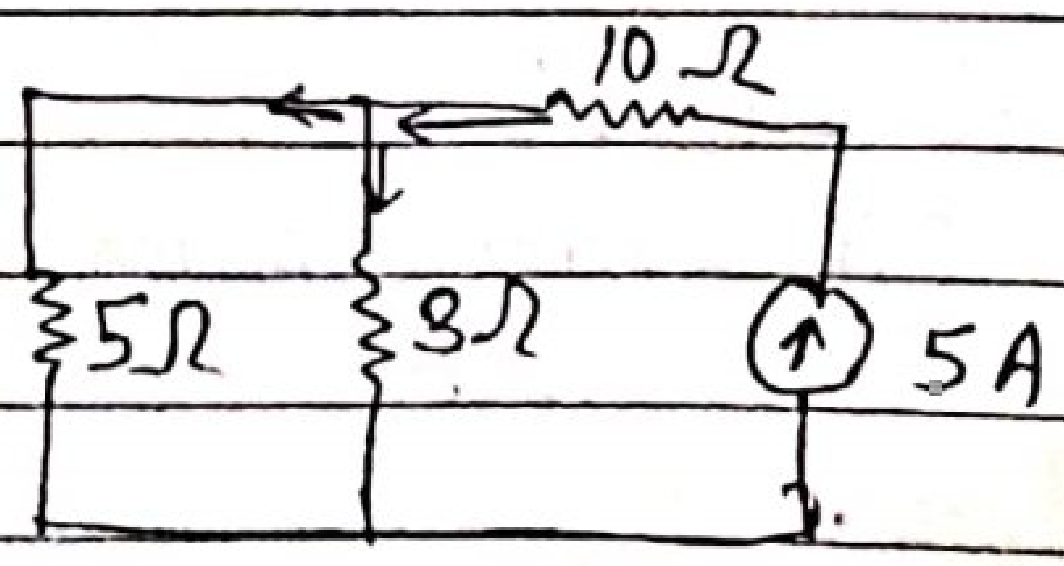
$$I' = \frac{20}{5+3} = 2.5\,A$$

$$\Rightarrow \boxed{I' = 2.5\ A}$$

Case II: Consider the current source (5A):-



$\Rightarrow$



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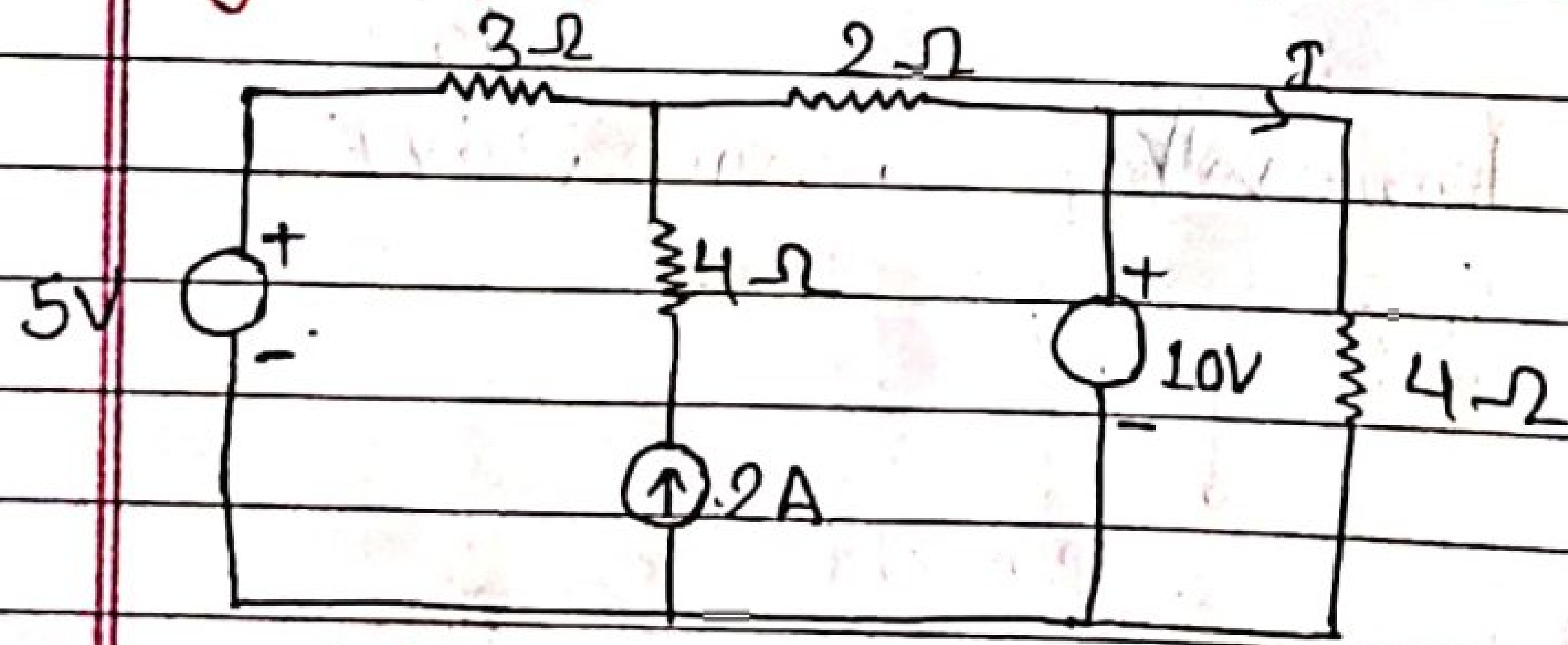
$$I''_2 = 5 * \frac{5}{3+5} = 3.125 \text{ amp}$$ [current division formula is used]

$$I'' = 3.25 \text{ amp} \qquad I_2 = I_T \times \frac{R_1}{R_1+R_2}$$

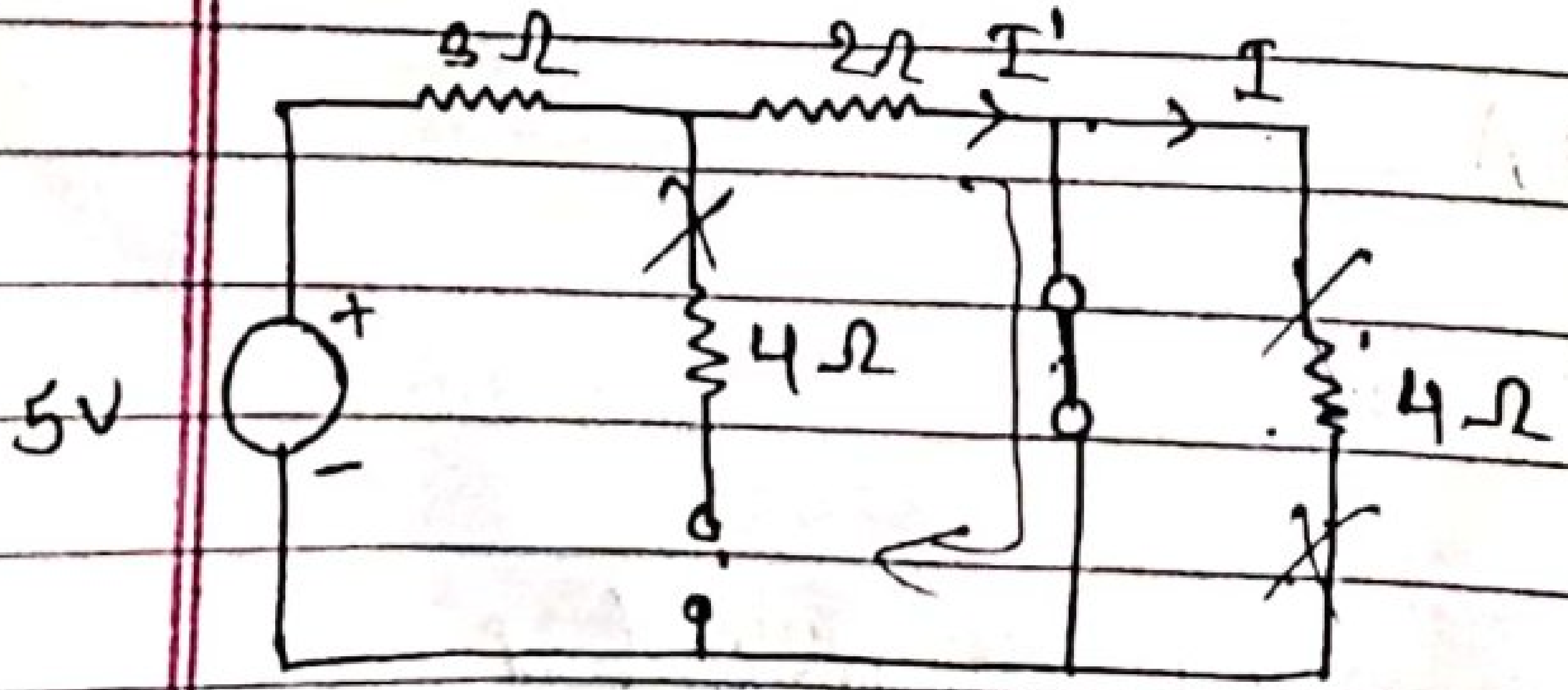
$$\therefore I = I' + I''$$
$$= 2.5 + 3.125$$
$$= 5.625 \text{ amp}$$

Q.2) By superposition theorem, find the current in 4-Ω resis



solution:

Step-1: 5V source acting alone:



$$~~I' = \frac{5}{3+2} = 1 \text{ amp}~~$$

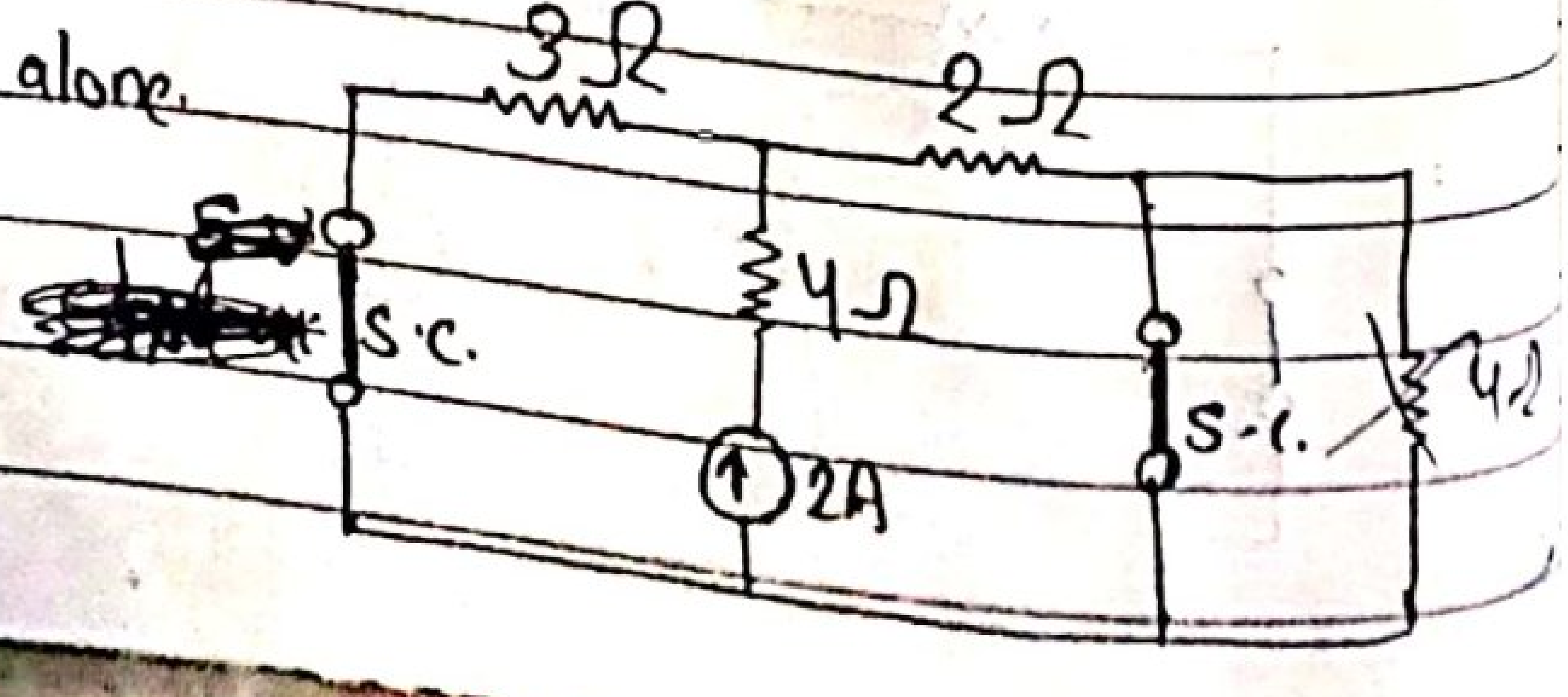
$$~~\boxed{\therefore I' = 1 \text{ amp}}~~$$

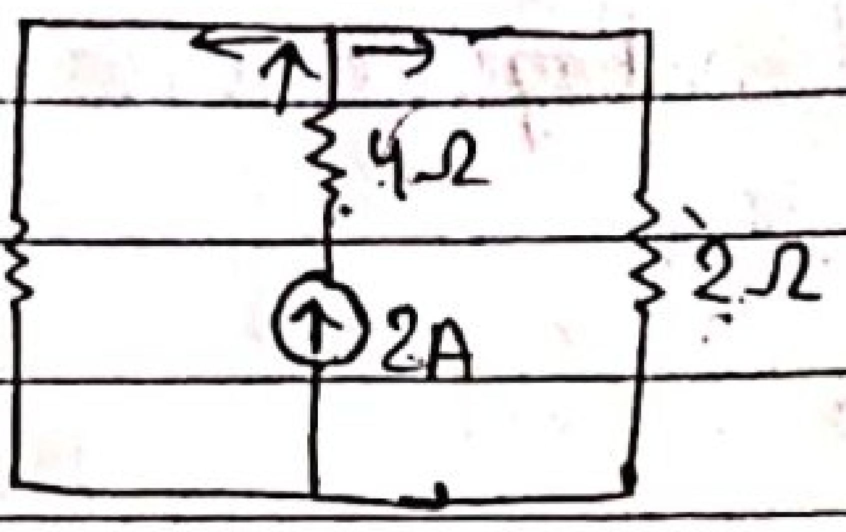
no current flows through 4Ω resistor

$I' = 0$

Step-2:

2A current source acting alone.



$\Rightarrow$ 

~~$I'' = 2 \times \frac{3}{2+3}$~~

~~$= 1.2$ amp~~

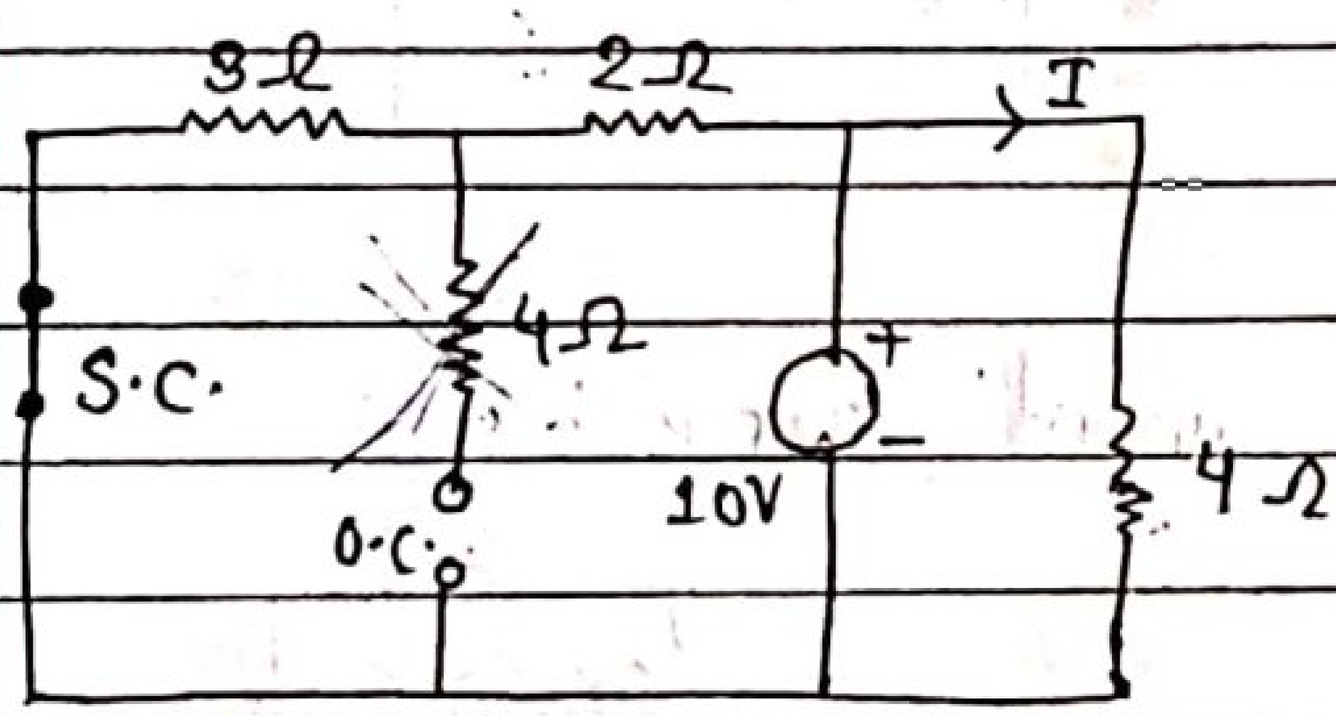
~~$\therefore \boxed{I'' = 1.2 \text{ amp}}$~~

no current flows through 4Ω resistor.

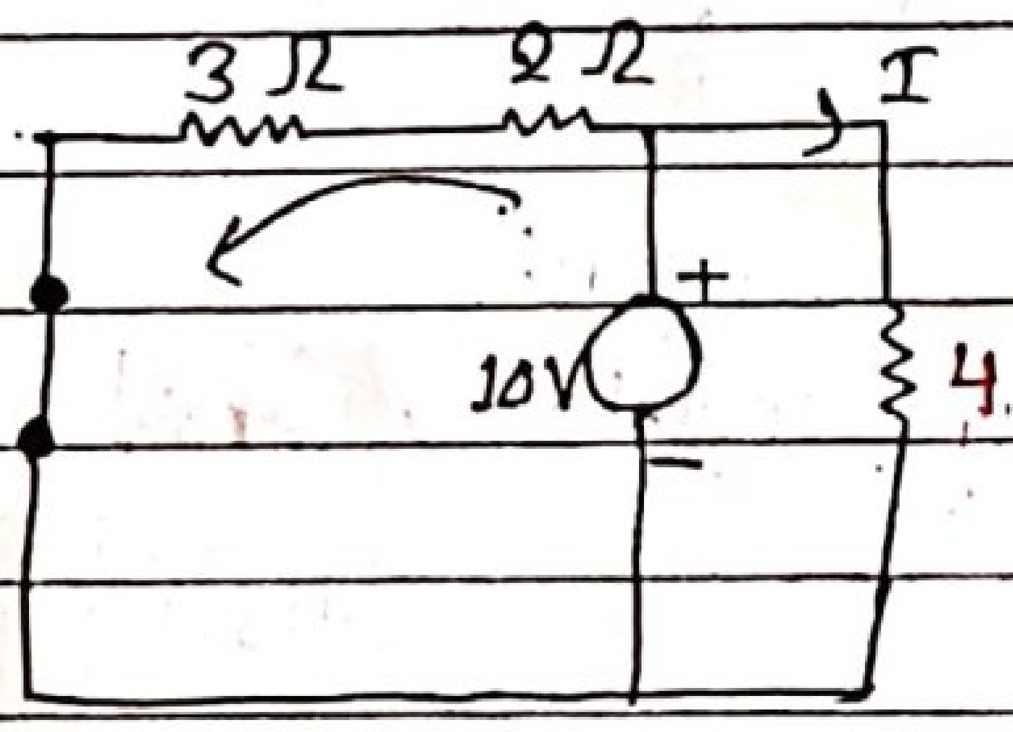
$I'' = 0$

Step - 3:

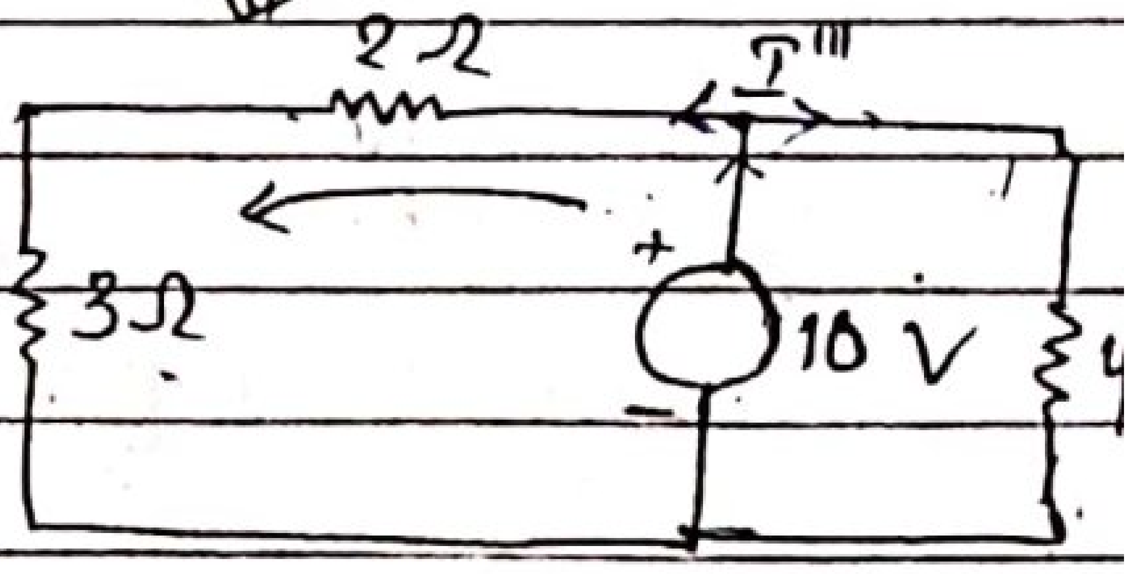
10V source acting along:



$\Rightarrow$



$\Downarrow$



~~$I''' = \frac{10}{3+2} = 2$ amp~~

~~$\therefore \boxed{I''' = 2 \text{ amp}}$~~

~~$\therefore I = I' + I'' + I'''$~~

~~$= 1 + 1.2 - 2$~~

~~$= 0.2$ amp~~

$I_T = \frac{V}{R}$

$= \frac{10}{2.22}$

$= 4.5$

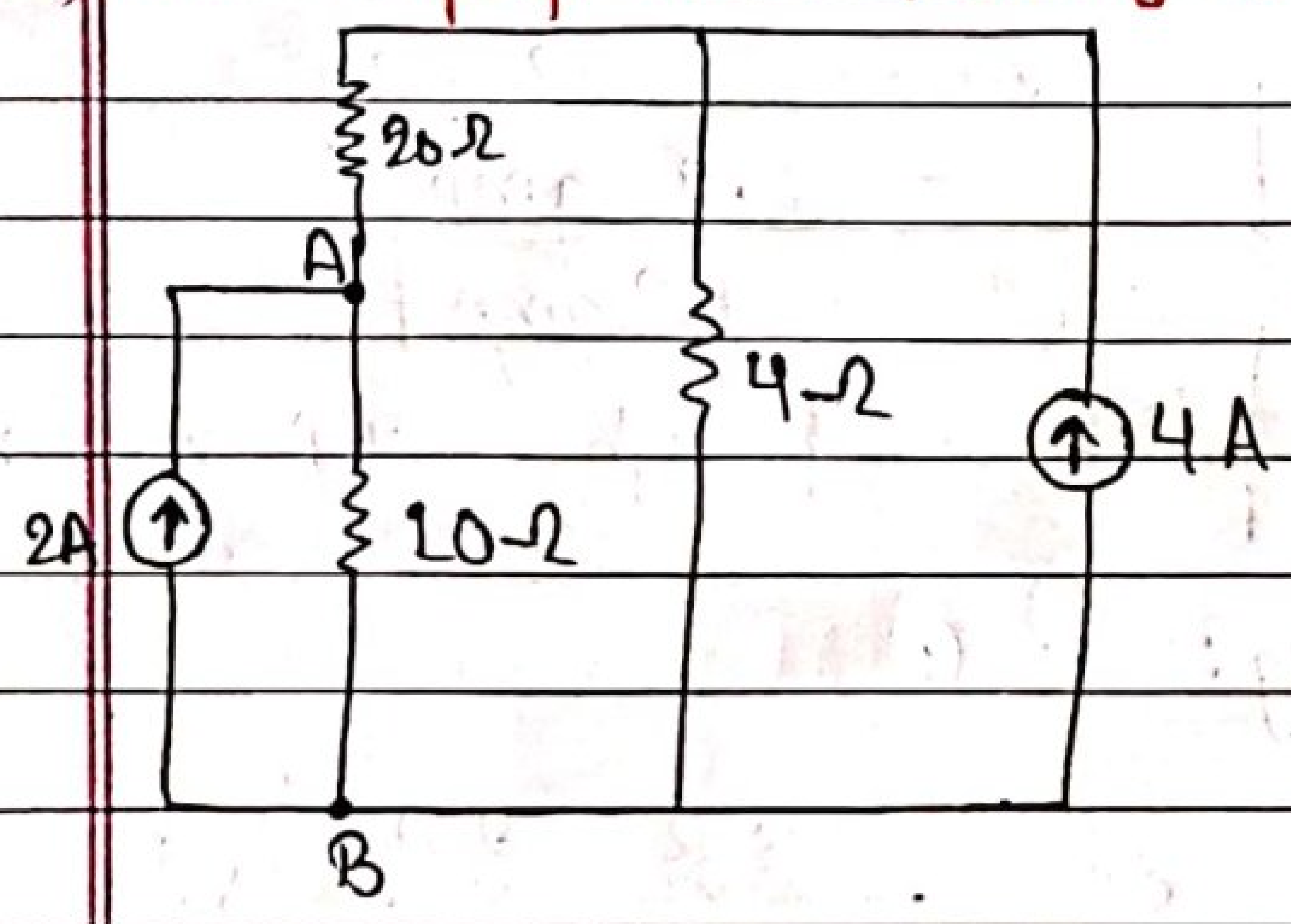
$I''' = 4.5 \times \frac{5}{9}$

$= 2.5$ amp

$I = I' + I'' + I'''$

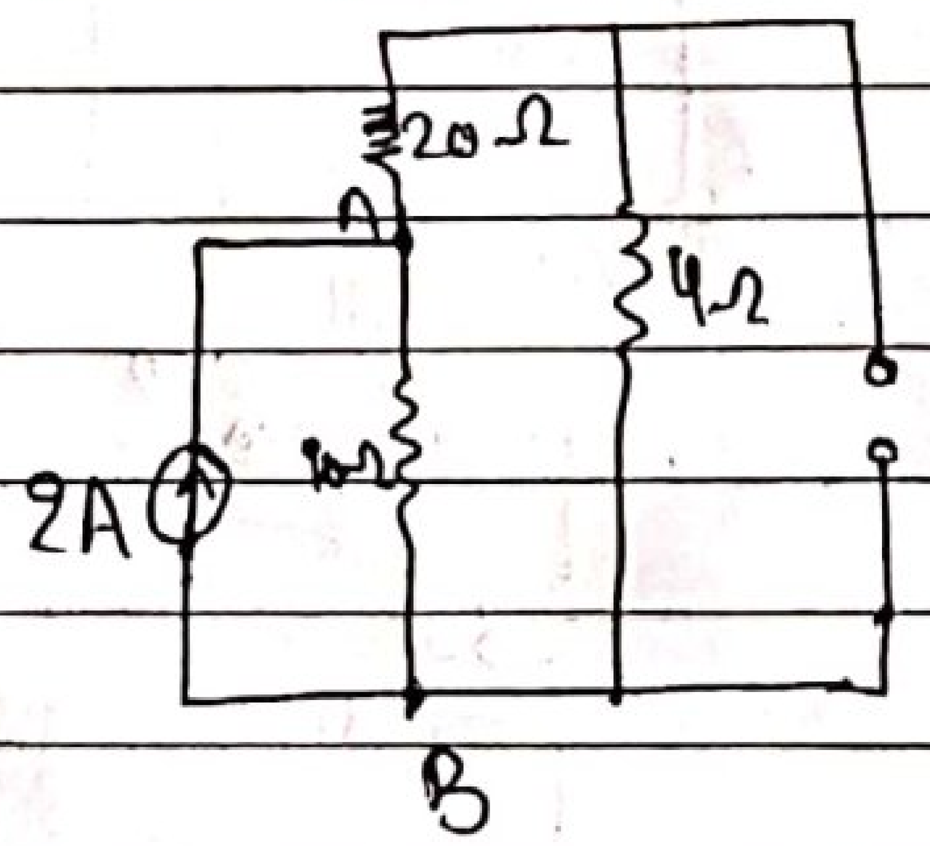
$= 0 + 0 + 2.5 = 2.5$ amp

Q:3) Use superposition theorem to find voltage across 10 Ω resistor.

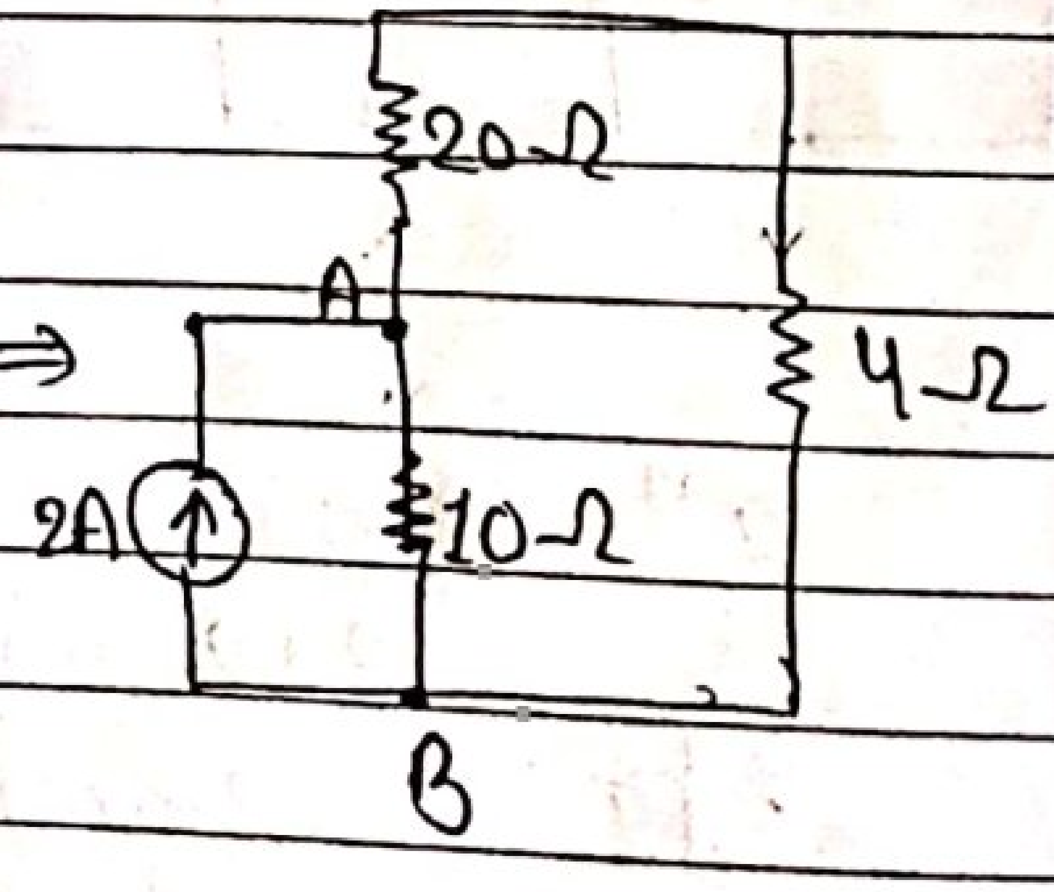


solution:

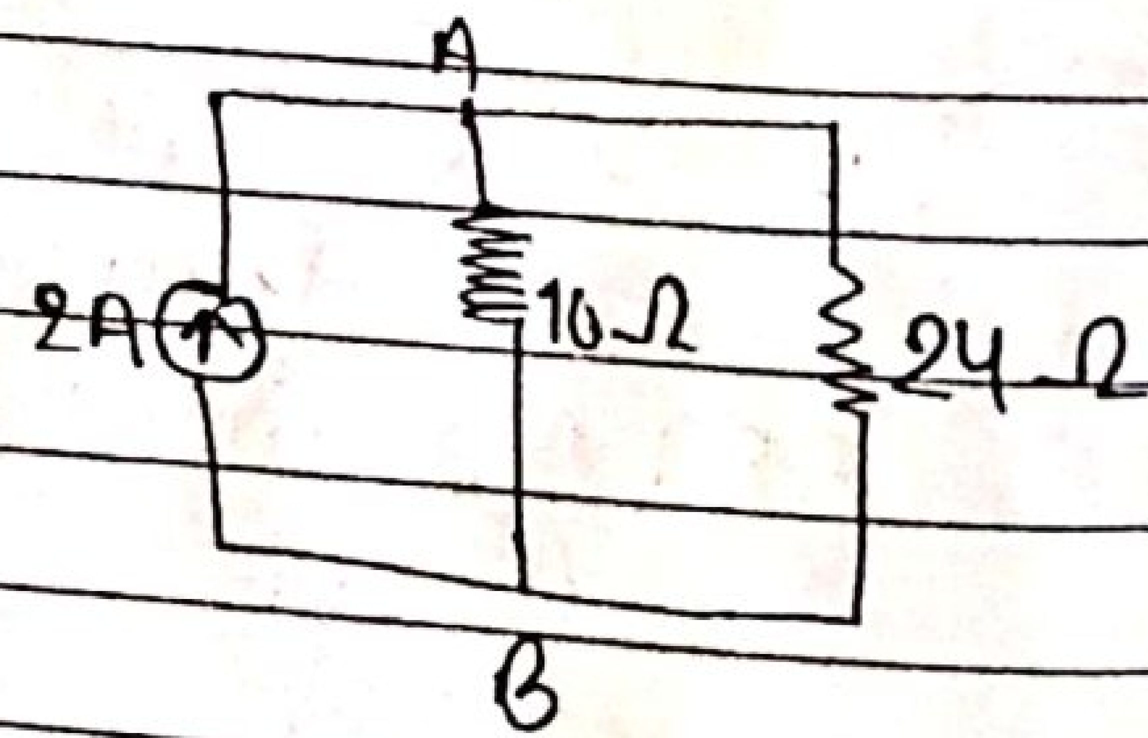
Step -1: Consider 2A current source:



⇒



⇓



$$I_{AB} = 2 \times \frac{24}{24+10} = 1.41 \text{ amp}$$

$\Rightarrow \boxed{I_{AB}' = 1.41 \text{ amp}}$

$$V_{AB}' = 1.41 \times 10 = 14.1 \text{ V}$$

$\Rightarrow \boxed{V_{AB} = 14.1V}$

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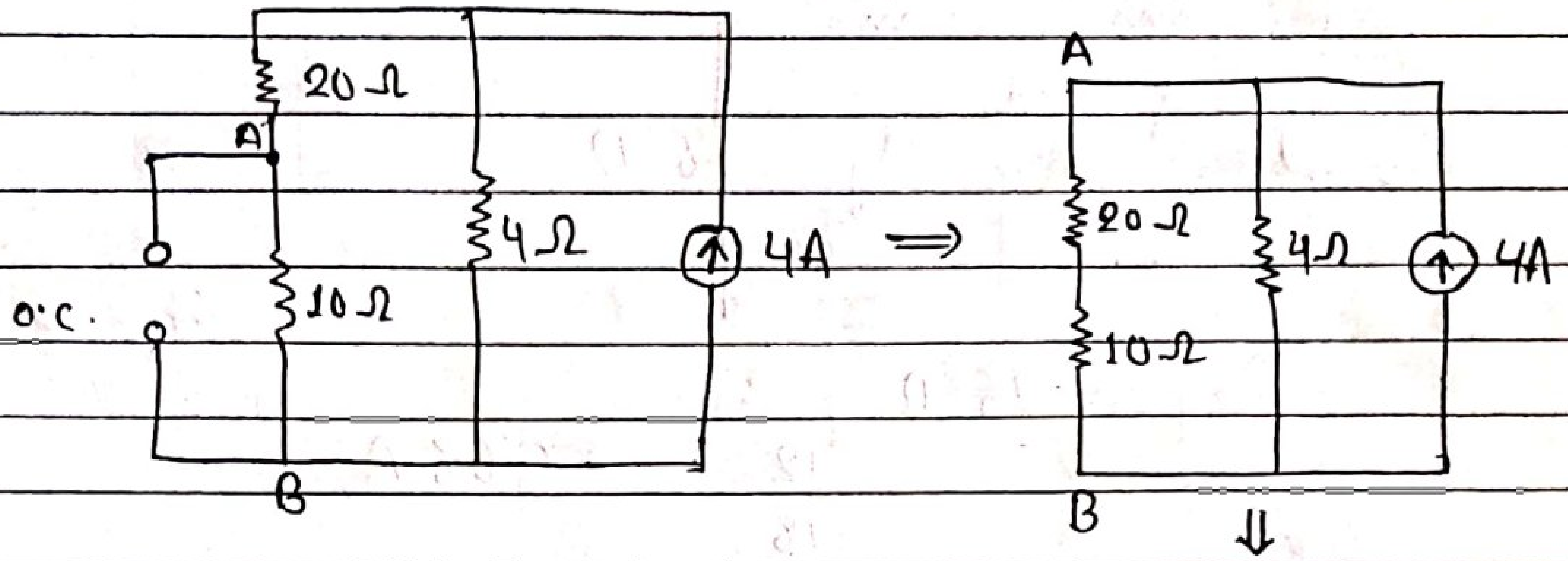
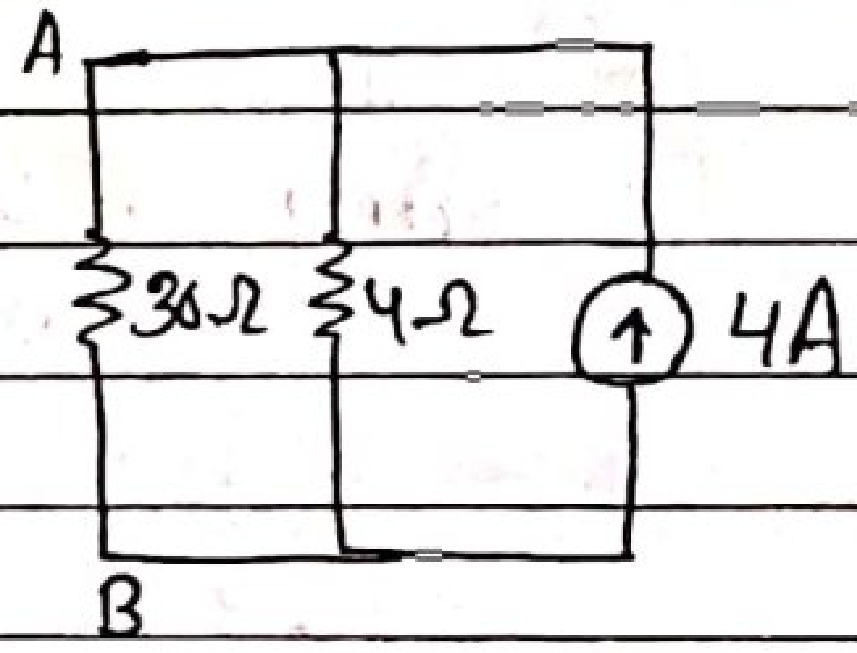
Step-2: Consider 4A current source.



$$I_{AB}'' = 4 \times \frac{4}{30+4} = 0.470 \text{ amp}$$



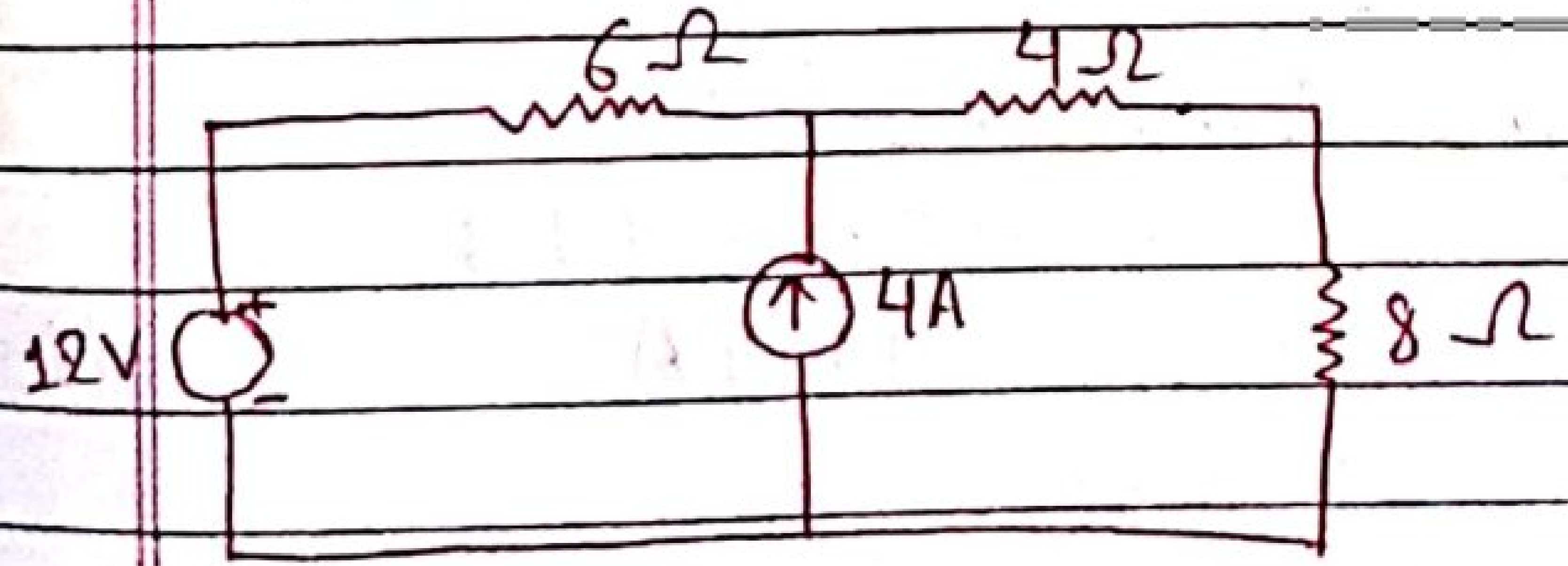
$$V_{AB}'' = 0.470 \times 10$$
$$= 4.70 \text{ V}$$

So,

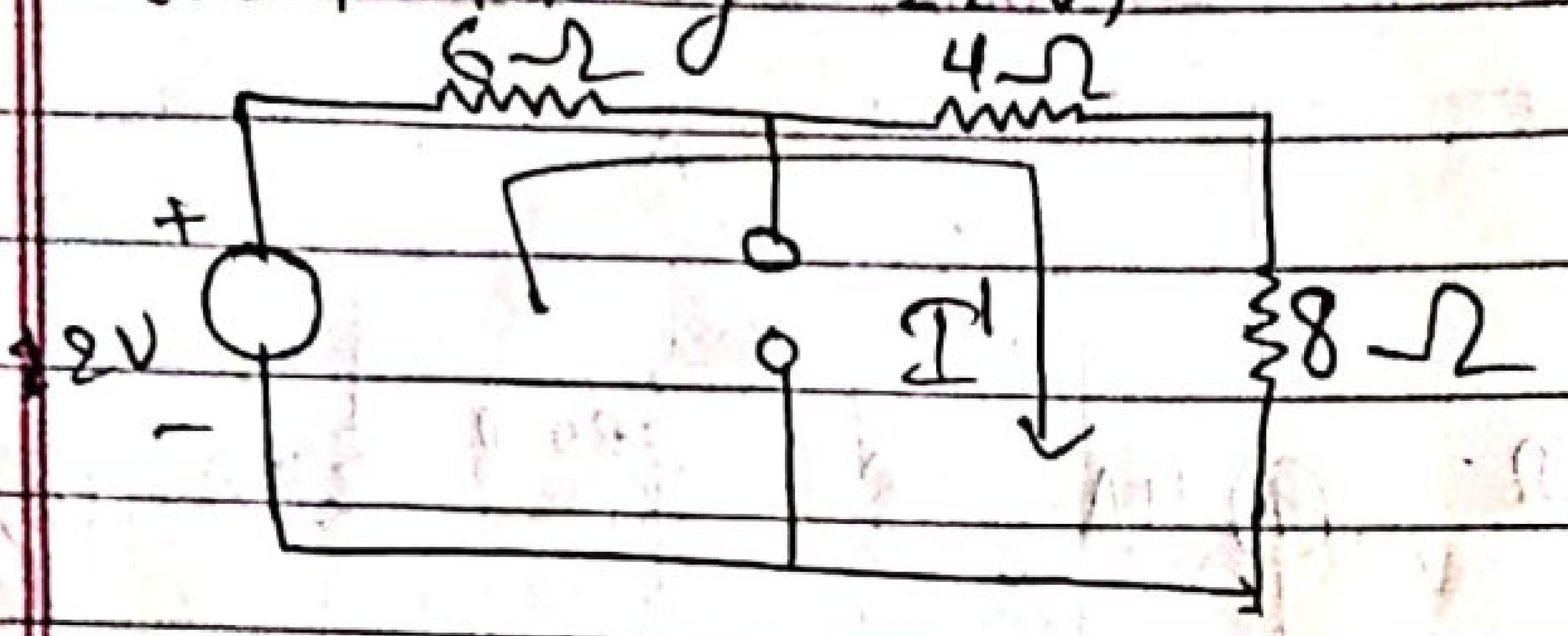
$$V_{AB} = 14.1 \text{ V} + 4.7 \text{ V}$$
$$= 18.8 \text{ V}$$

Q. 4) Determine the current flowing in 8 Ω ~~theo~~ resistor by superposition theorem:

Ans: 1.98 Amp.



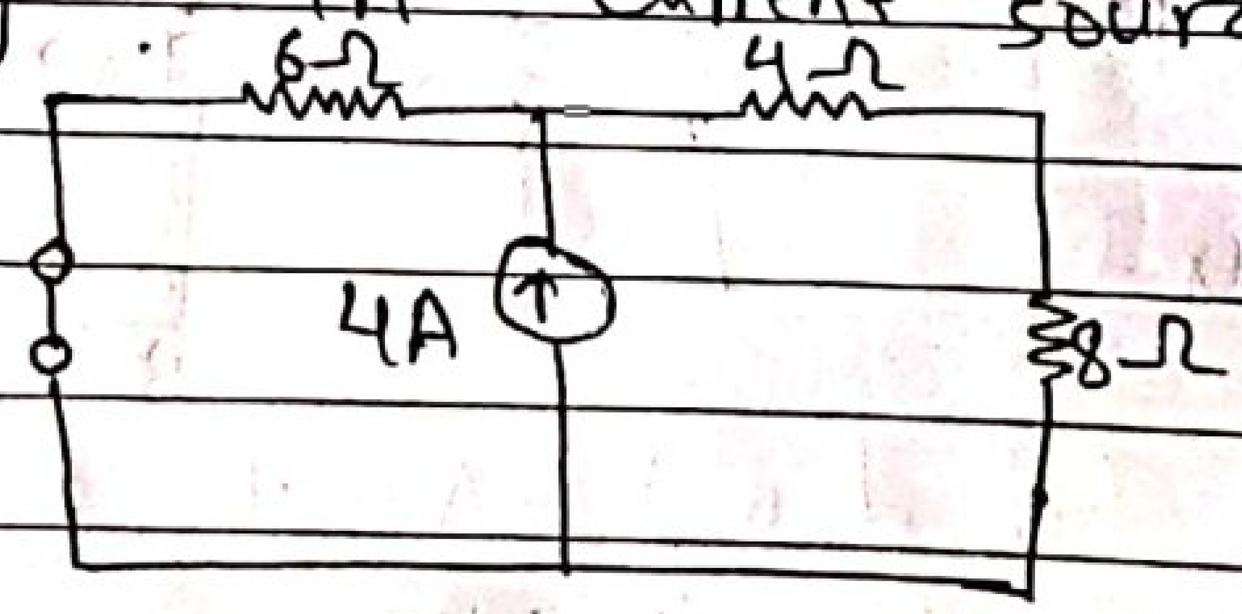
Here, taking 12 V,



$$R_{eq} = 16\,\Omega$$

$$I' = \frac{V}{R} = \frac{12}{18} = 0.66\,A$$

Now,
Taking 4A current source,



Using current division formula,

$$I'' = \frac{4 \times 6}{6+12} = 1.33\,A$$

∴ Current through $8\,\Omega = I' + I''$
$= 0.66 + 1.33$
$= 1.99\,A$

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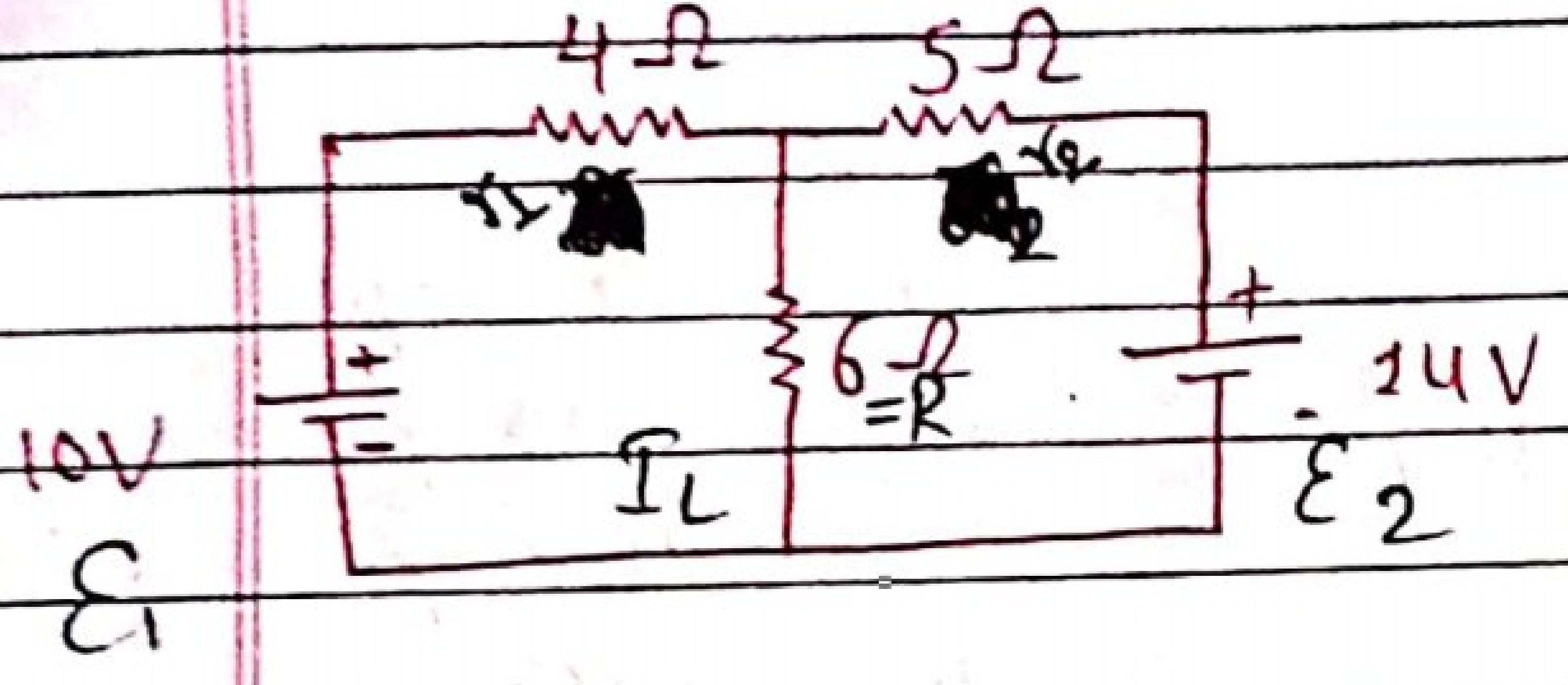
Q.5) Using superposition theorem, find the current in 6 Ω resistor

Ans: 1.42 Amp



$$I_L = \frac{\mathcal{E}_1 R_1 + \mathcal{E}_2 R_2}{R r_1 + r_1 r_2 + R r_2}$$

$$= \frac{10 \times 5 + 14 \times 4}{6 \times 4 + 4 \times 5 + 5 \times 6}$$

$$= 1.43 \text{ A}$$

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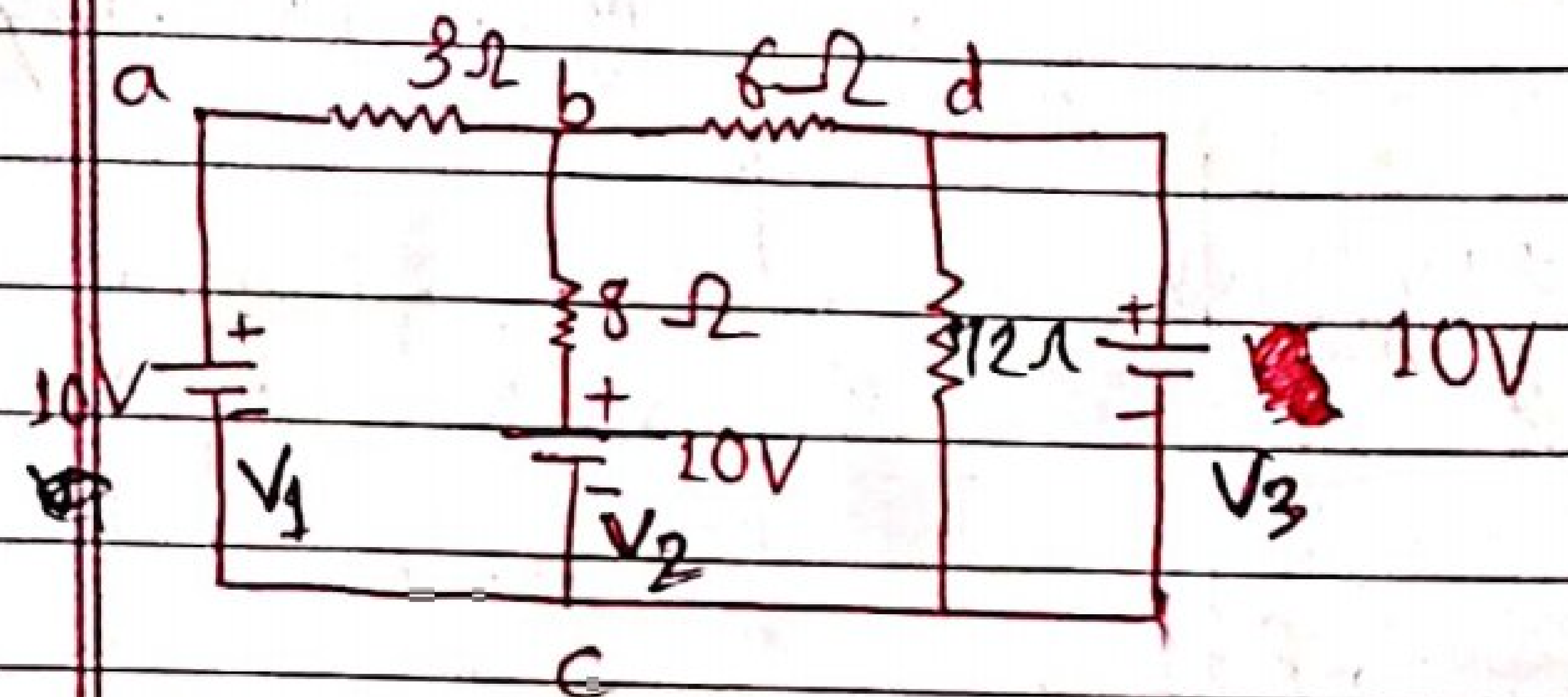
Q.6) By superposition theorem find the current in different branch of ckt.



Ans:

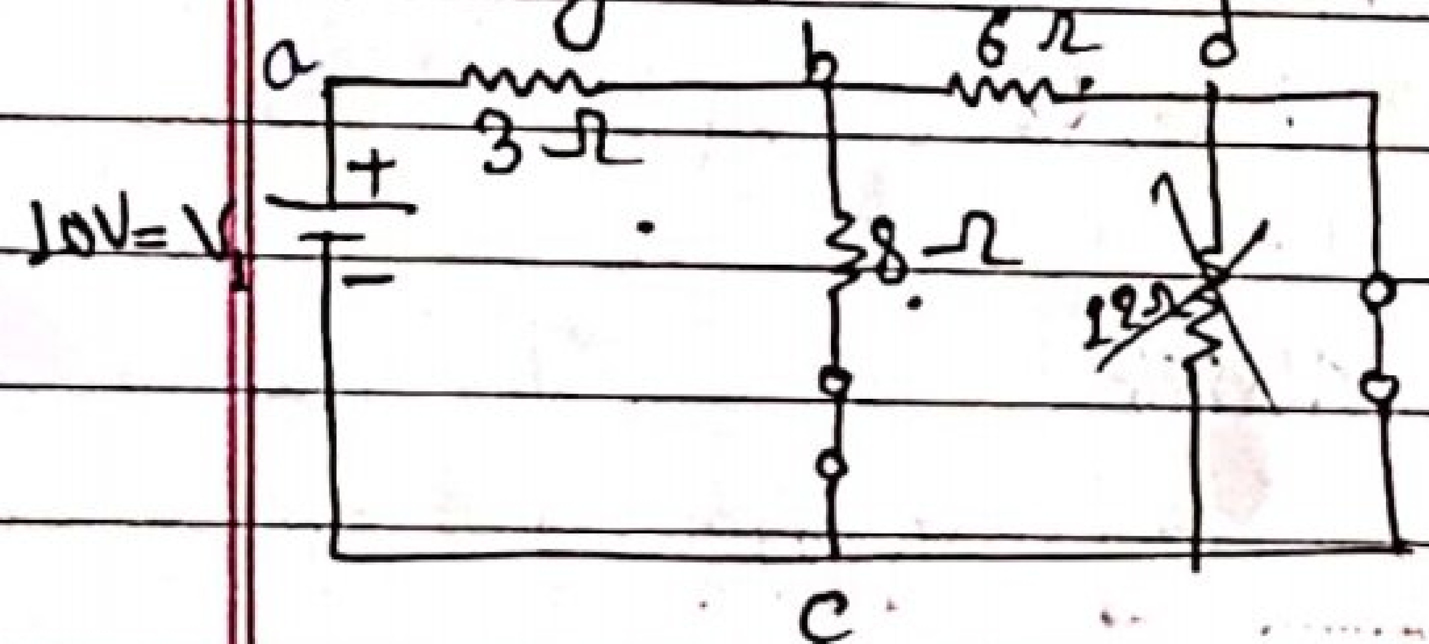
$I_{ab} = 0$ amp

$I_{bc} = 0$ amp

$I_{bd} = 0$ amp

$I_{dc} = 0.83$ amp

Taking 10V,



$$R_{eq} = 3 + \frac{8\times 6}{8+6} = 6.42\,\Omega$$

$$I_T = \frac{10}{6.42} = 1.55\,A$$

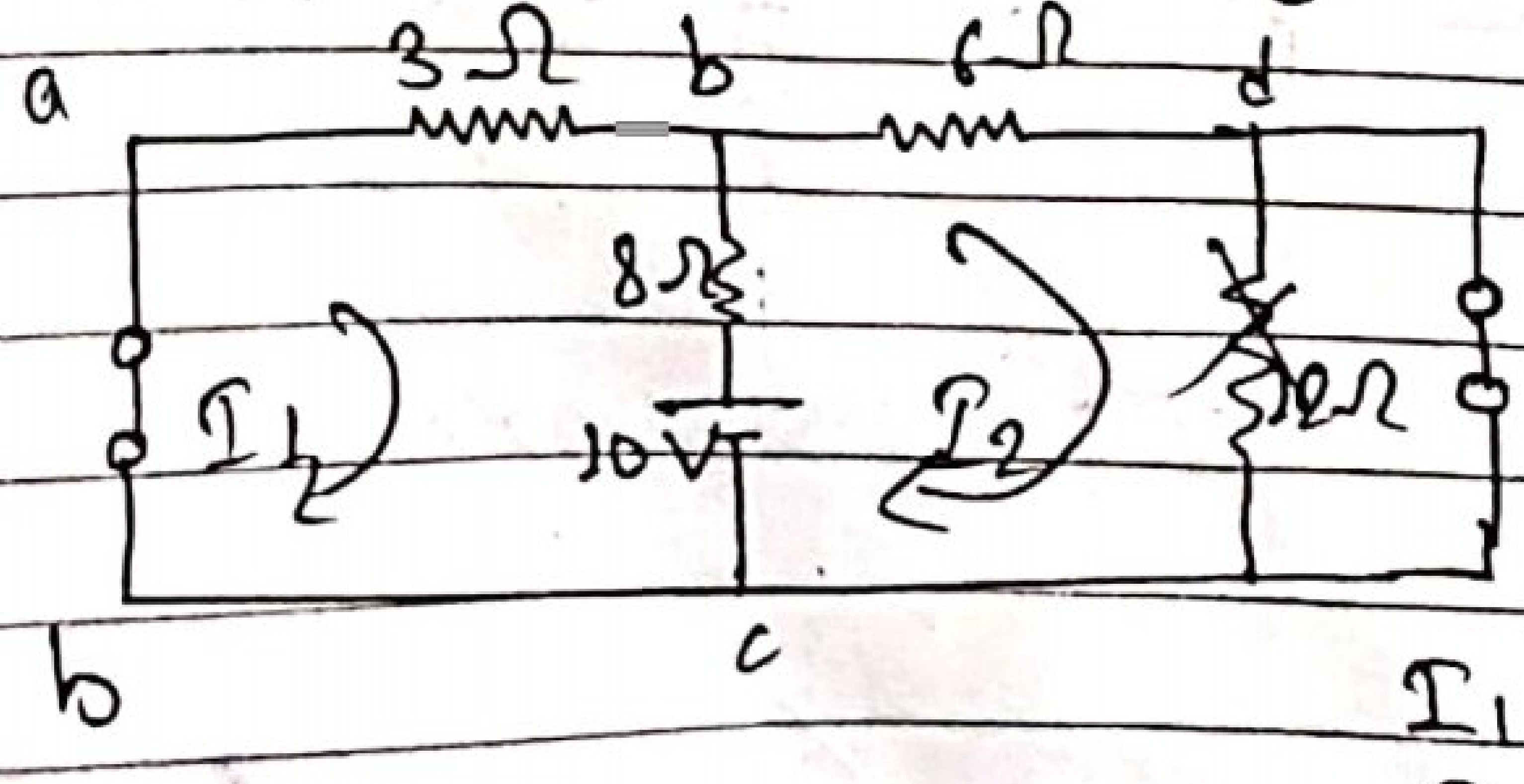
$$I'_{ab} = 1.55A \quad (a \text{ to } b)$$

$$I'_{bd} = \frac{1.55\times 8}{8+6} = 0.88 \quad (b \text{ to } d)$$

$$I'_{bc} = \frac{1.55\times 6}{8+6} = 0.66 \quad (b \text{ to } c)$$

$$I'_{cd} = 0 \ \cancel{0.88} \quad (d \text{ to } c)$$

Taking 10V (V_2) voltage source:



Loop I:

$-3I_1 - 8(I_1 - I_2) - 10 = 0$ —(I)

Loop II:

$-6I_2 + 10 - 8(I_2 - I_1) = 0$

Solving (I) & (II),

$I_1 = -0.6A$ [opposite dir^n]

$I_2 = 0.33A$

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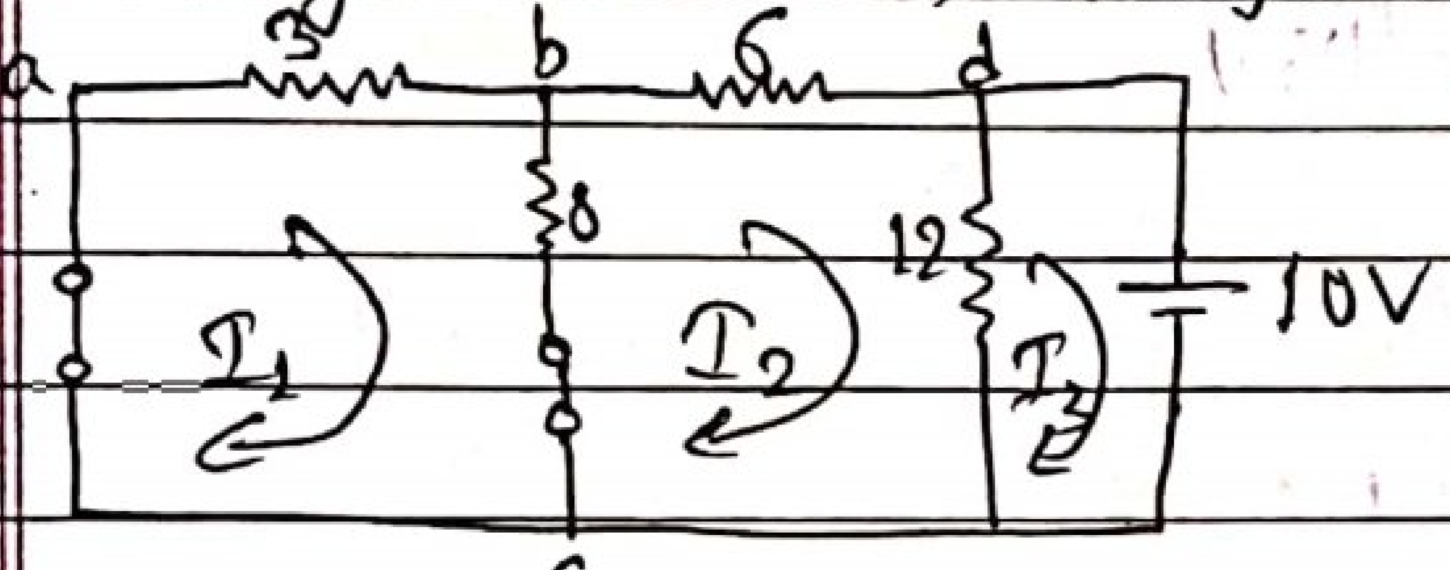
$I''_{ab} = 0.66\ A$ (b to a)
$I''_{bc} = I_1 + I_2$ (c to b)
$= 0.99\ A$

$I''_{bd} = 0.33$ (b to d)
$I''_{dc} = \cancel{0.33}\ 0$ (d to c)

Taking 10 V (V_3) voltage source :



Loop I :
$-3I_1 - 8(I_1 - I_2) = 0$ — (I)

Loop II :
$-6I_2 - 12(I_2 - I_3) - 8(I_2 - I_1) = 0$ — (II)

Loop III :
$-12(I_3 - I_2) - 10 = 0$ — (III)

Solving,
$I_1 = -0.88A$ (opposite dr^n)
$I_2 = -1.22\ A$ (" ")
$I_3 = -2.055\ A$ (" ")

$I'''_{ab} = 0.88A$ (b to a)
$I'''_{bc} = (1.22 - 0.88)\ A = 0.34$ (b to c)
$I'''_{bd} = 1.22$ (d to b)
~~I'''_{dc}~~ $I'''_{cd} = 0.83$ amp

So,
$I_{ab} = I'_{ab} + I''_{ab} + I'''_{ab}$
$= 1.55 - 0.66 - 0.88 = 0.01 \approx 0A$

$I_{bc} = I'_{bc} + I''_{bc} + I'''_{bc}$
$= 0.99 - 0.66 - 0.34 = 0.01 \approx 0\ A$

$I_{bd} = I'_{bd} + I''_{bd} + I'''_{bd}$
$= 1.22 - 0.88 - 0.33 = 0.02 \approx 0\ A$

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Old Questions

Q.7) Use the superposition theorem to find the current through the 30 Ω resistor in a given ckt.

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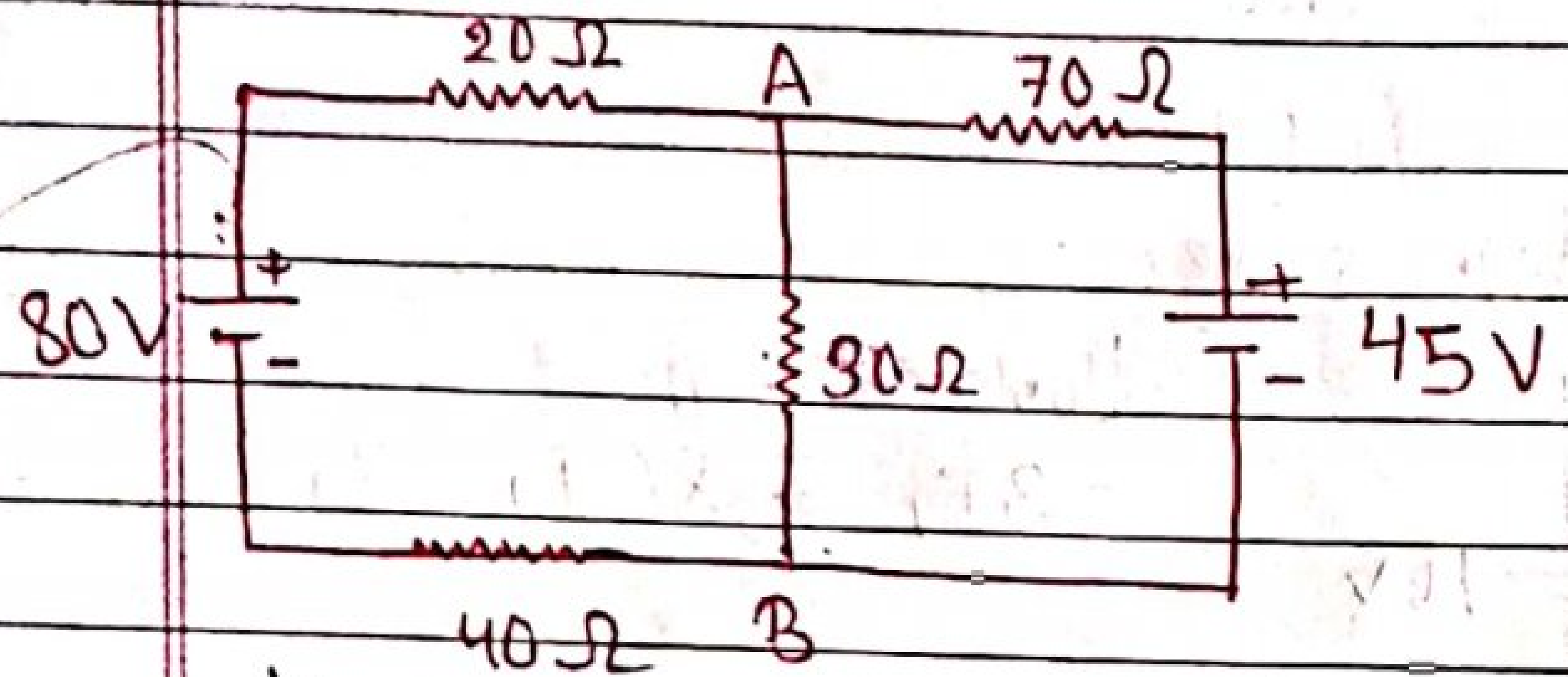
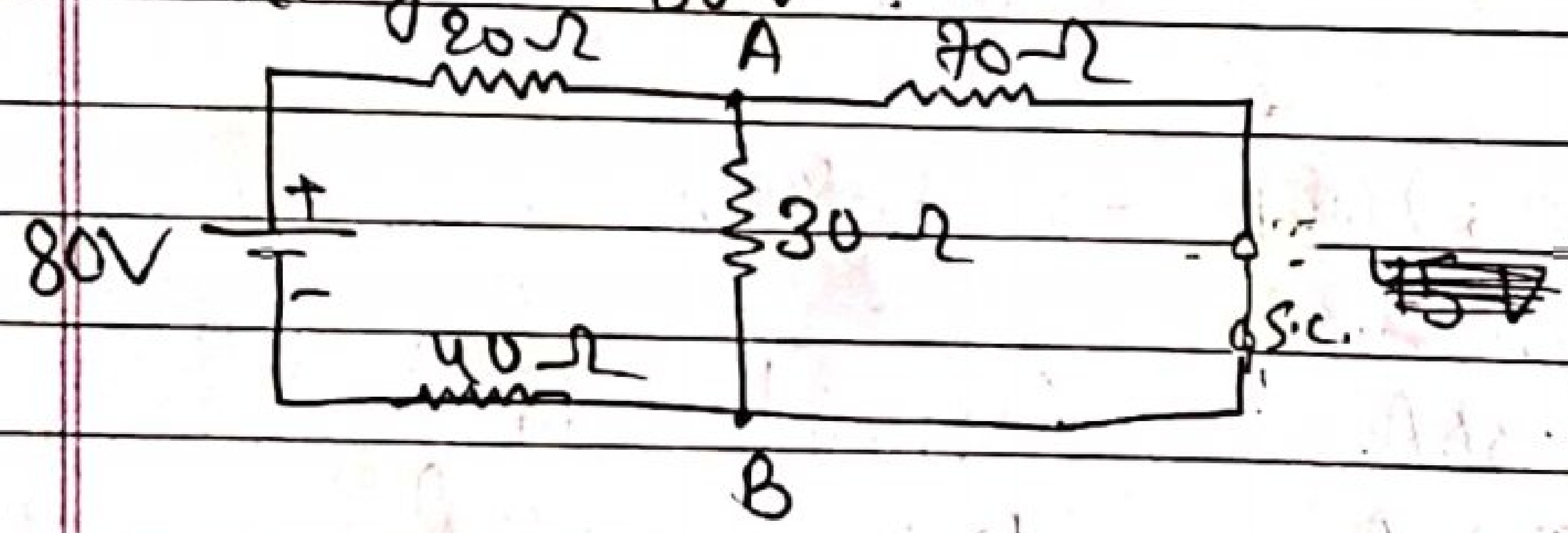


Ans: $I_{AB} = 1.024$ amp

Here,

Taking 80 V :

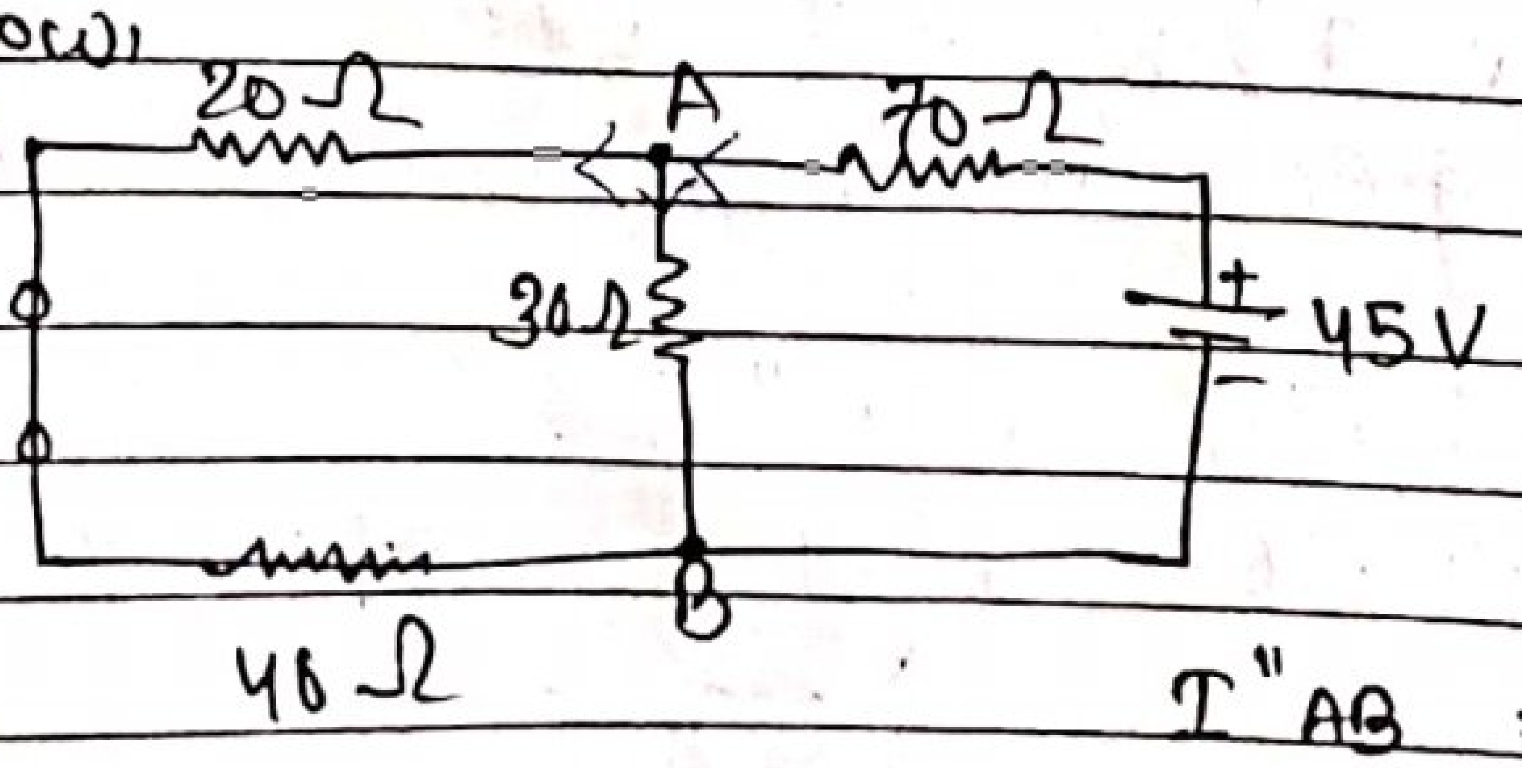


$$R_{eq} = 20 + 40 + \frac{30 \times 70}{30 + 70} = 81\ \Omega$$

$$I_T = \frac{80}{81} = 0.987\ A$$

$$I'_{AB} = \frac{I_T \times 70}{30 + 70} = 0.686 \text{ (a to b)}$$

Now,



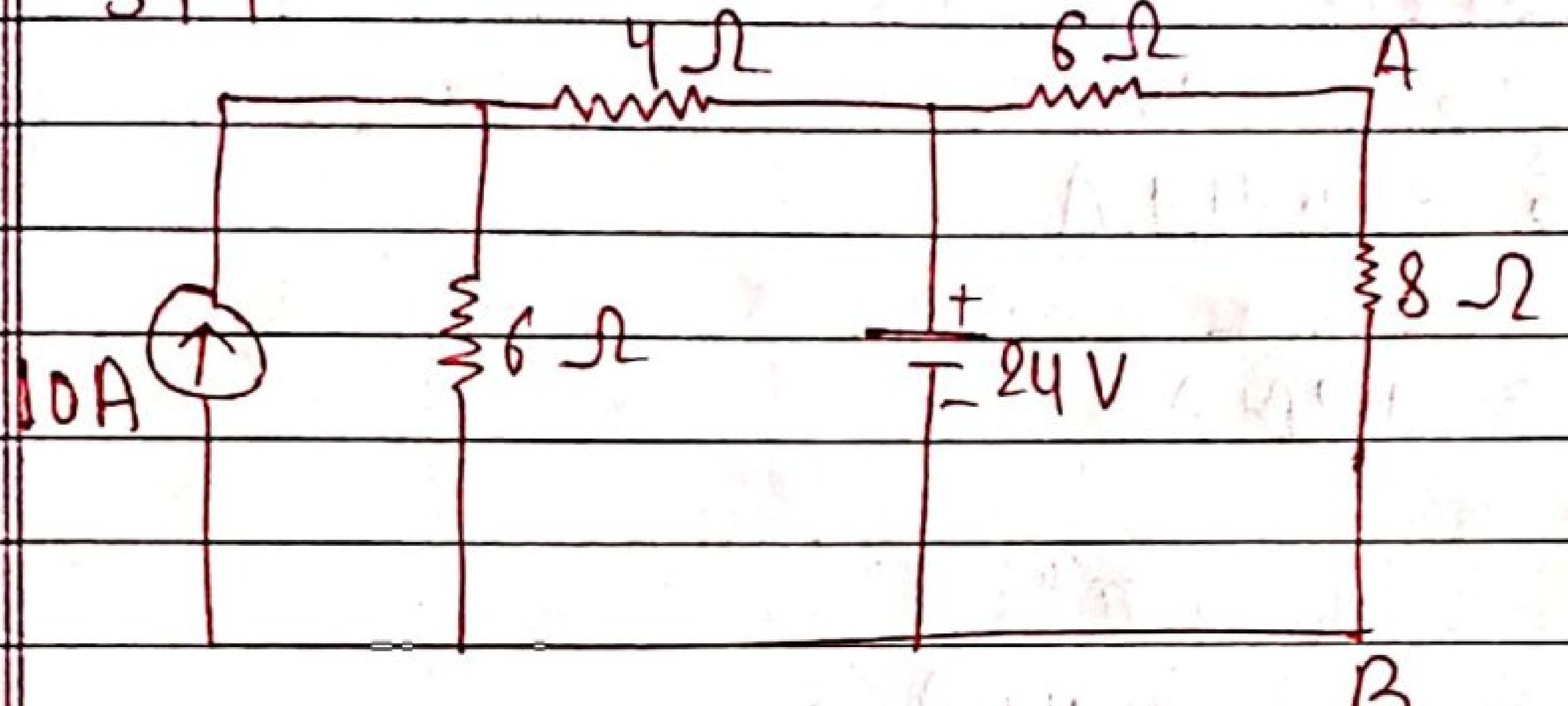
$$R_{eq} = 70 + \frac{30 \times 60}{30 + 60} = 90\ \Omega$$

$$I_T = \frac{45}{90} = 0.5A$$

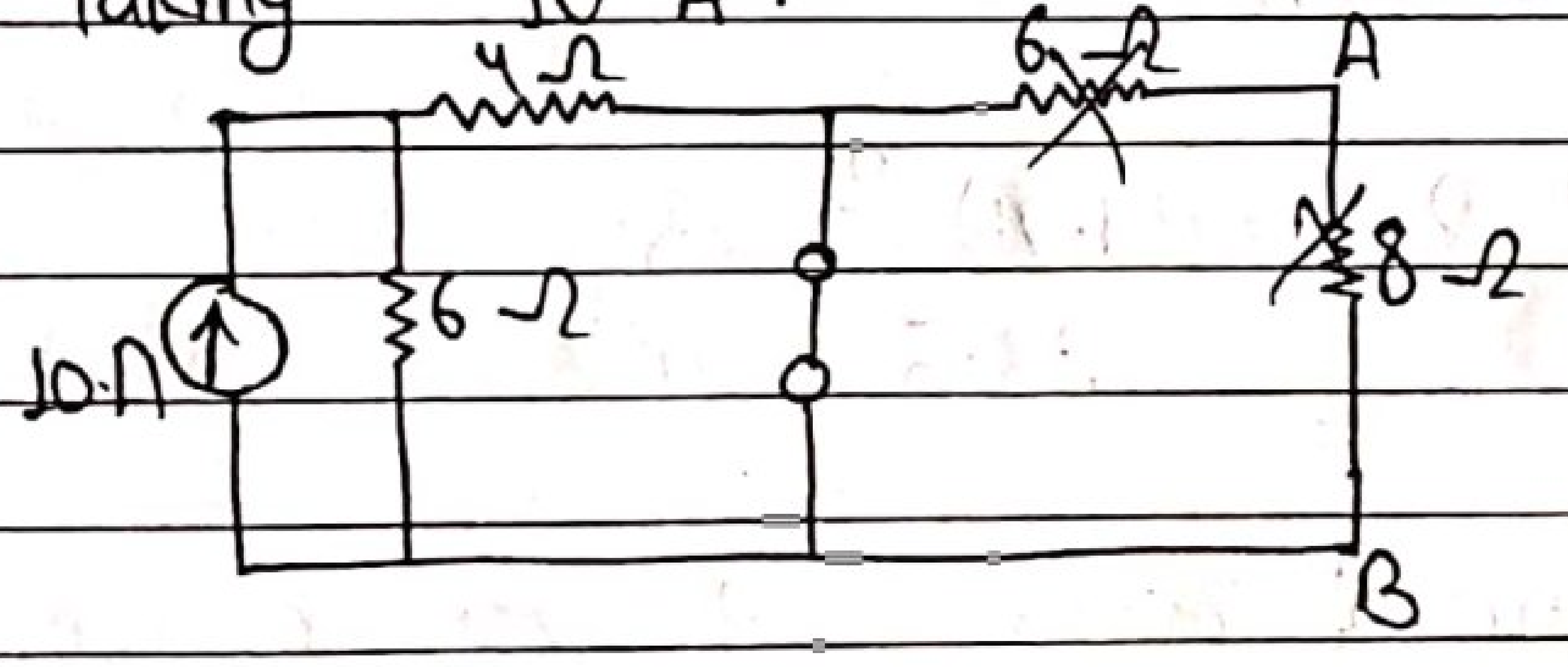
$$I''_{AB} = \frac{0.5 \times 60}{60 + 30} = 0.33A \text{ (a to b)}$$

$$\therefore I_{ab} = I'_{ab} + I''_{ab}$$
$$= 0.686 + 0.333 = 1.019 \text{ Amp}$$

Q. 8) Calculate the power consumed by 8 Ω resistor using S.P.T.

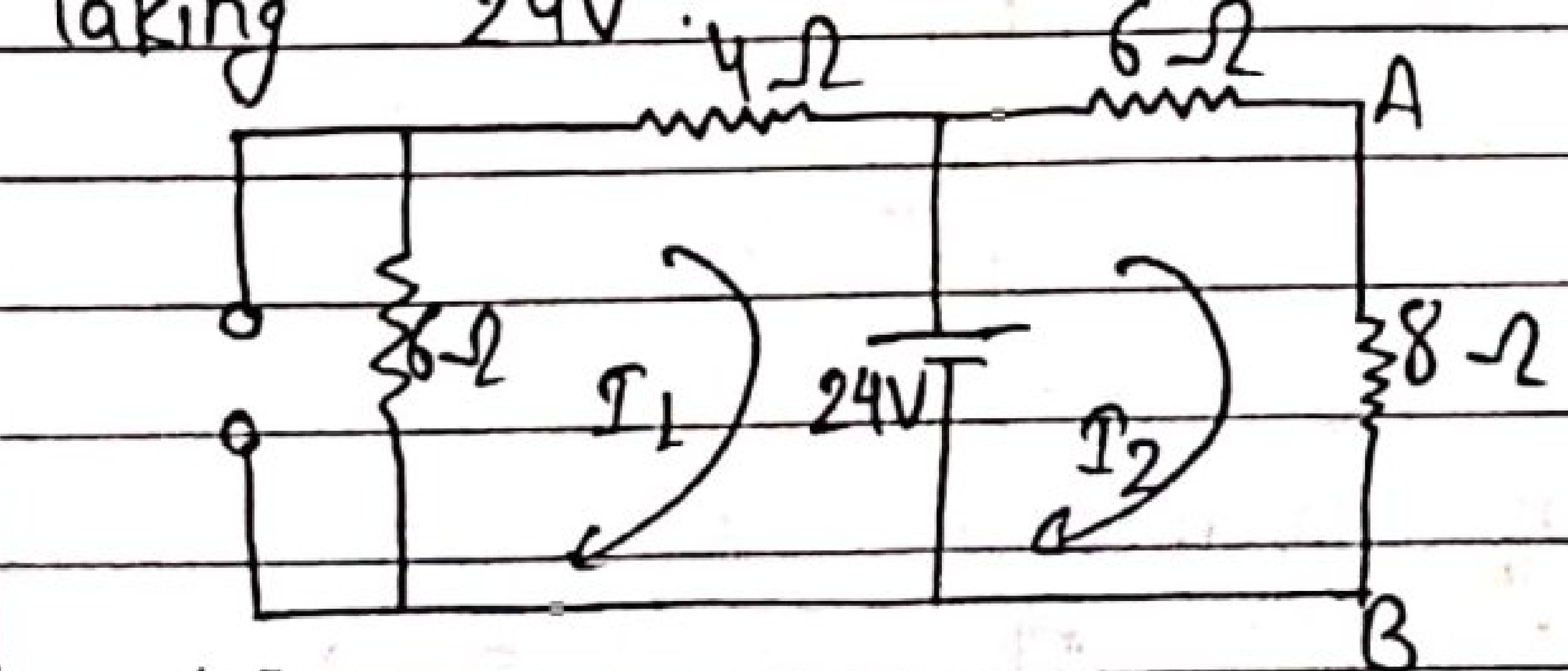


Taking 10 A:



$I'_{AB} = 0$

Taking 24V:



Loop-1:

$-6I_1 - 4I_1 - 24 = 0$

$\Rightarrow \quad -10 I_1 = 24$

$\Rightarrow \quad I_1 = -2.4$ (opp. dir^n)

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Also, loop-2:

$-6I_2 - 8I_2 + 24 = 0$

$\Rightarrow -14 I_2 = -24$

$\Rightarrow I_2 = 1.714\ A$

$\therefore I''_{ab} = 1.714\ A$

$\therefore I_{ab} = I'_{ab} + I''_{ab}$

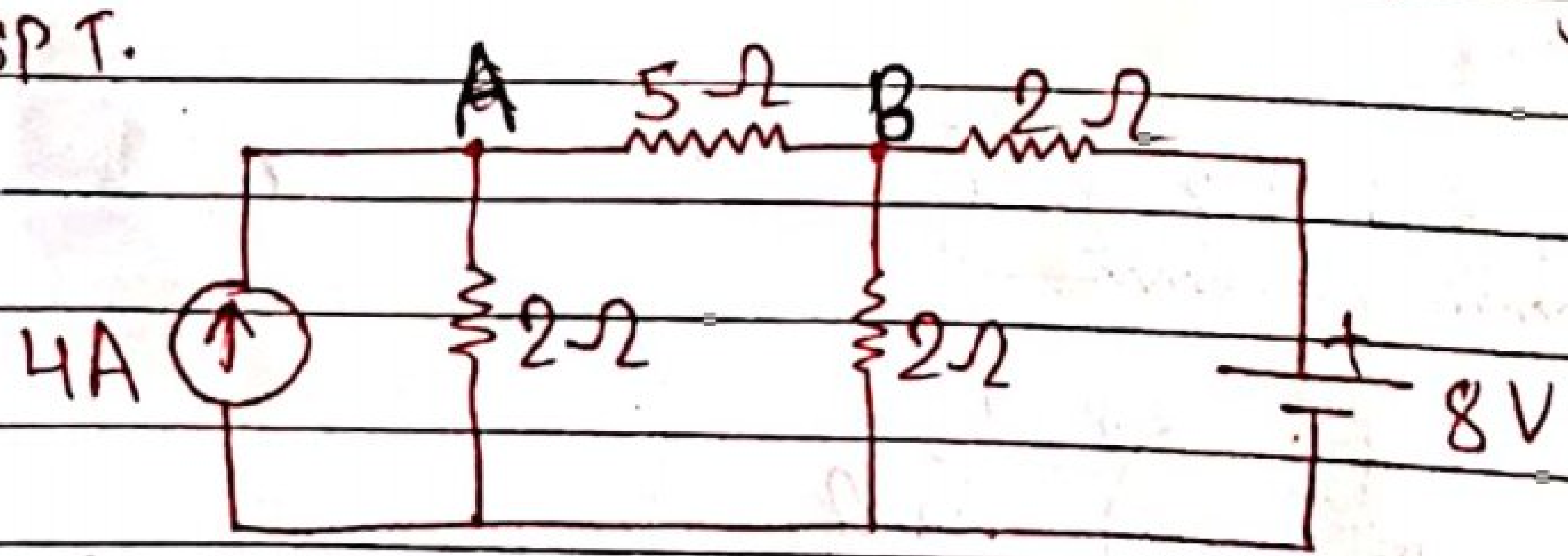
$= 0 + 1.714\ A$

$= 1.714\ A$

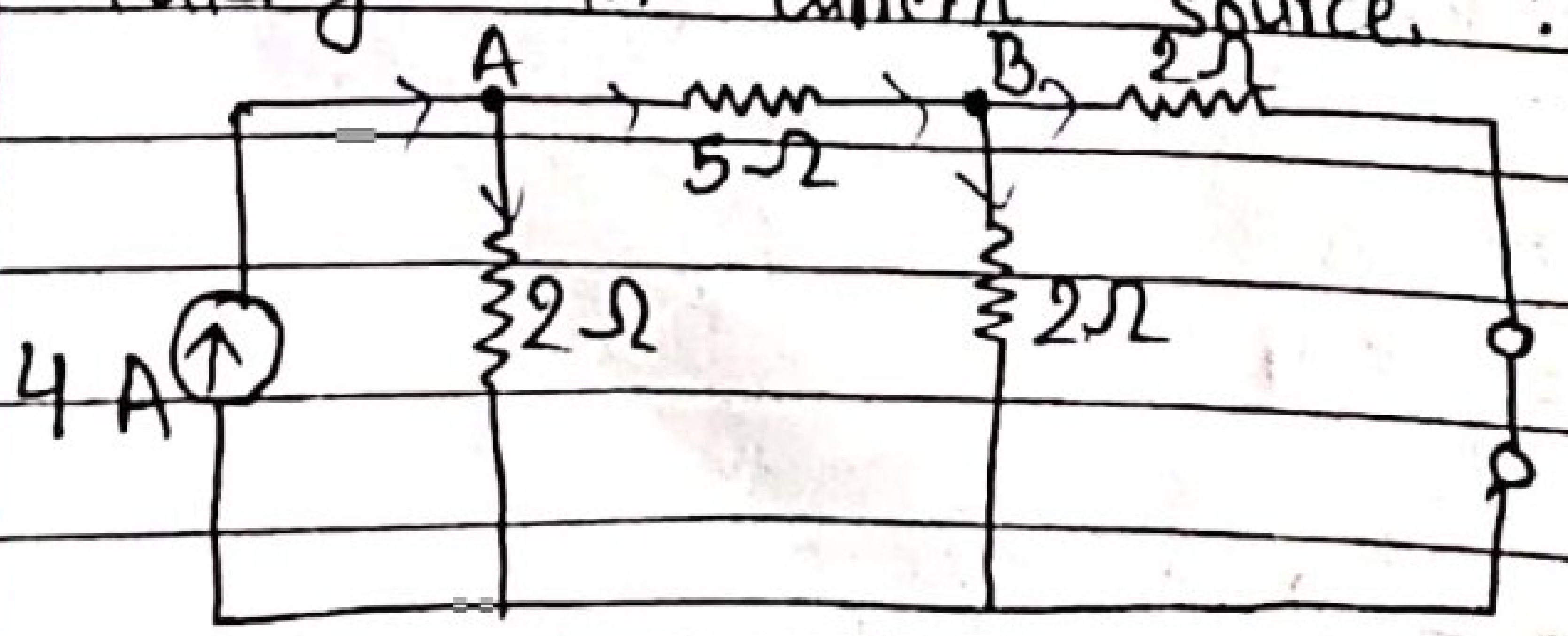
$P_{ab} = (I_{ab})^2 . R = (1.71)^2 \times 8$

$= 23.5$ watt

Q. 9) Find out the current through 5Ω resistor connected across the terminal AB in the given network. Using SPT.



Taking 4A current source :



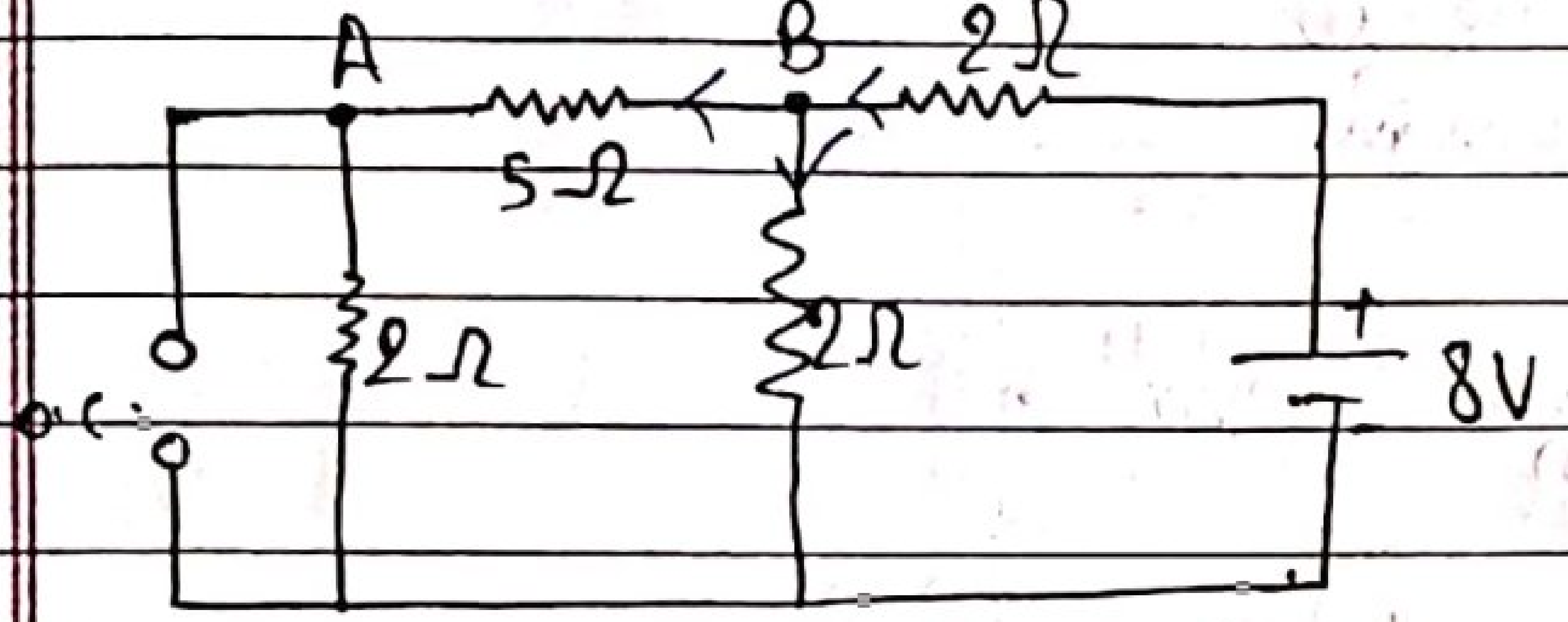
$R_1 = \frac{2 \times 2}{2+2} = 1\Omega$

$R_2 = 5\Omega + 1\Omega$
$= 6\Omega$

$I_T = 4A$

$I'_{AB} = \frac{I_T \times 2}{2+6} = \frac{4 \times 2}{2+6} = 1 Amp$ (A to

Taking 8V (Voltage source)



$R_1 = 5\,\Omega + 2\,\Omega = 7\,\Omega$

$R_2 = \frac{7 \times 2}{7+2} = 1.5\,\Omega$

$R_{eq} = (2 + 1.55)\,\Omega$

$= 3.55\,\Omega$

$I_T = \frac{8}{3.55} = 2.25\,A$

$I''_{AB} = \frac{2.25 \times 2}{7+2} = 0.5\,A$ (B to A)

$\therefore\ I_{AB} = I'_{AB} + I''_{AB}$

$= 1\,A - 0.5\,A$

$= 0.5\,A$

Q. 10) By using SPT, determine the current through 2 Ω resistor in fig. given below:
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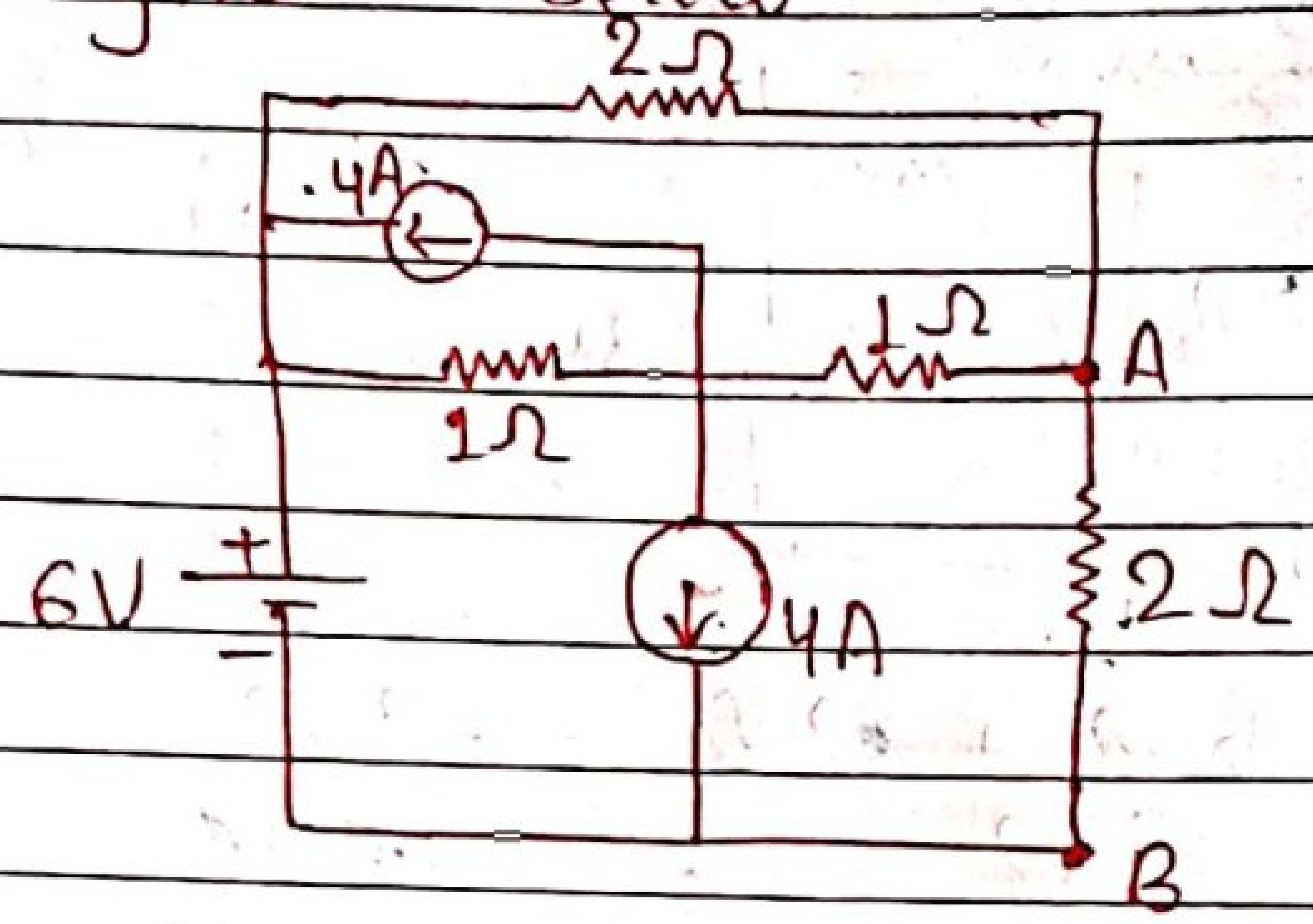
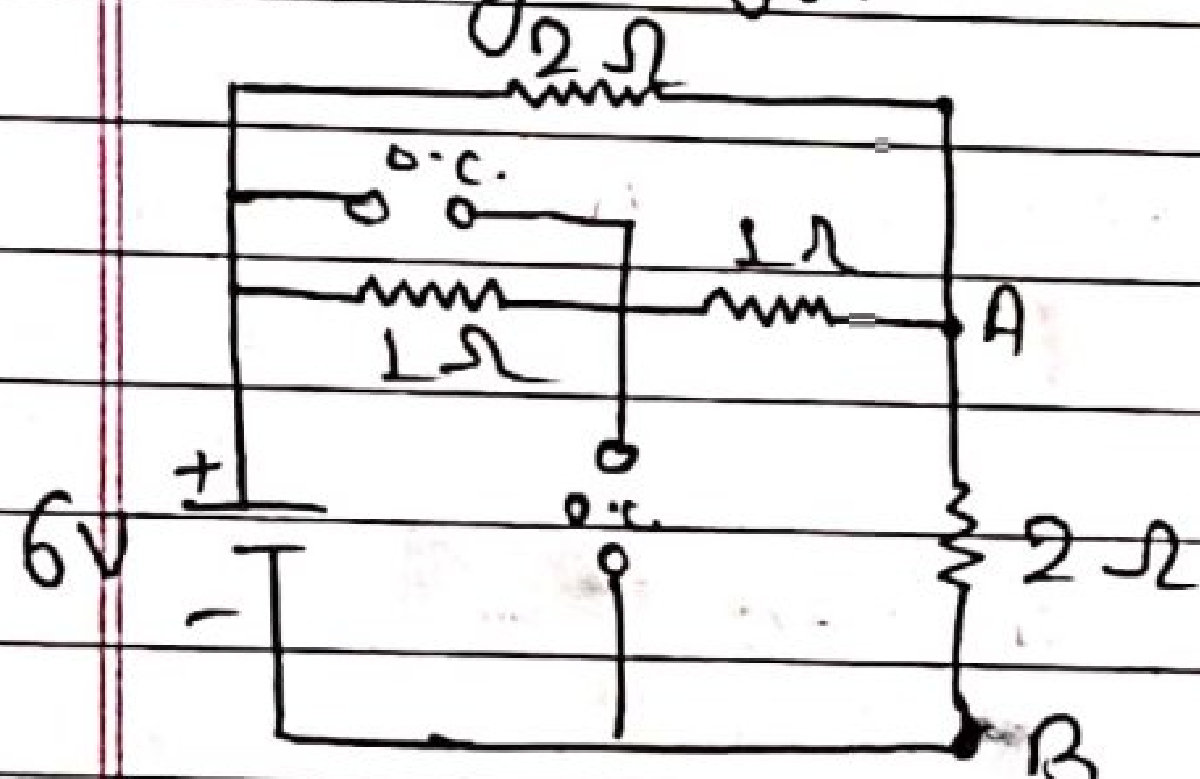


Taking 6V:



$R_1 = 1+1 = 2\Omega$

$R_2 = \frac{2\times2}{2+2} = 1\Omega$

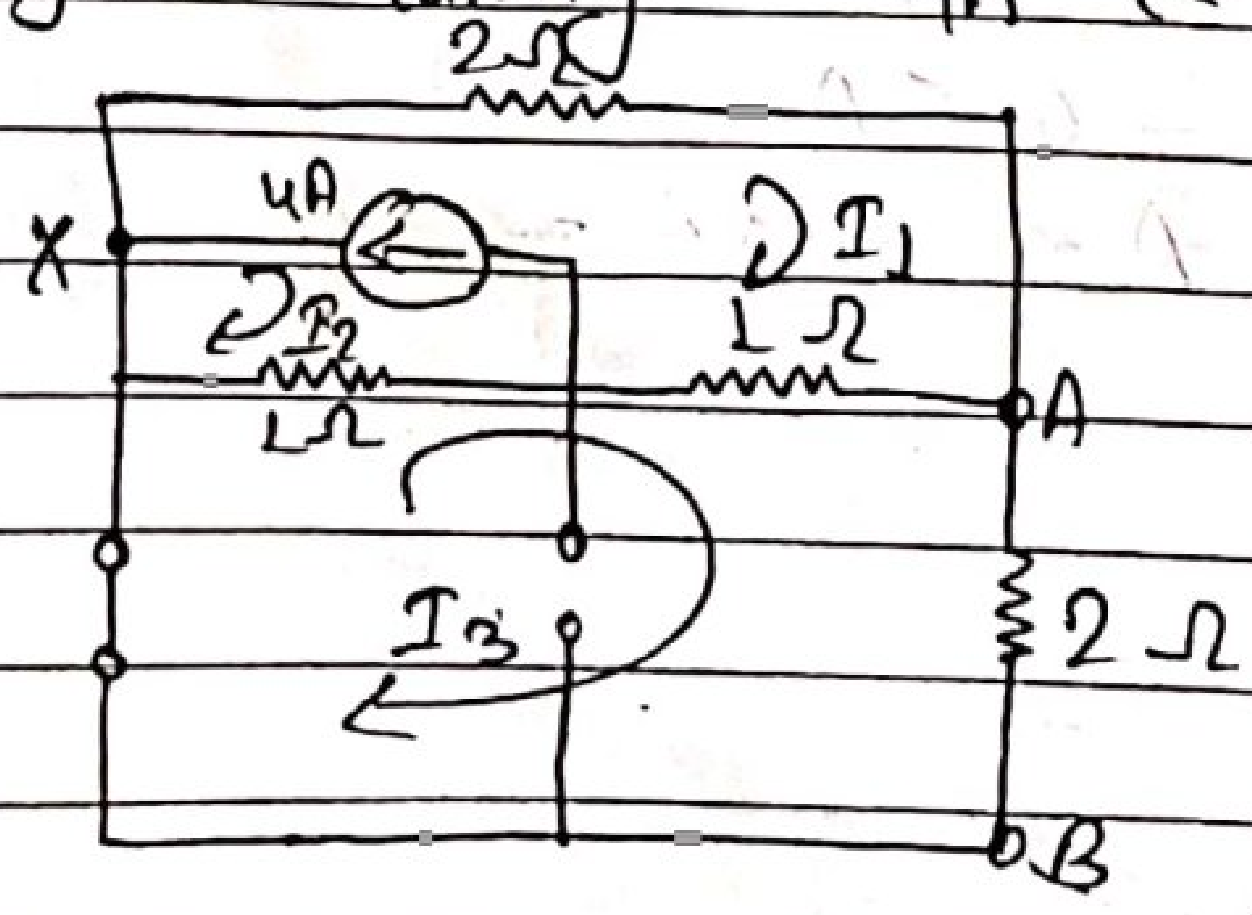
$R_{eq} = 1+2 = 3\Omega$

$I_T = \frac{V}{R_{eq}} = \frac{6}{3} = 2A$

$I'_{AB} = 2A$ [A to B]

Now,

Again taking 4A (←)



At node X:

$I_1 = I_2 + 4$

$\Rightarrow I_1 - I_2 = 4$ —(i)

Now, taking combine loop (I) & (II),

$-2I_1 - 1(I_1 - I_3) - 1(I_2 - I_3) = 0$

$\Rightarrow -2I_1 - I_1 + I_3 - I_2 + I_3 = 0$

$\Rightarrow -3I_1 - I_2 + 2I_3 = 0$ —(ii)

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Again taking loop III,

$$-1(I_3-I_2) - 1(I_3-I_1) - 2I_3 = 0$$

$$\Rightarrow I_1 + I_2 - 4I_3 = 0 \quad \text{—— (III)}$$

Equating (I), (II) & (III),

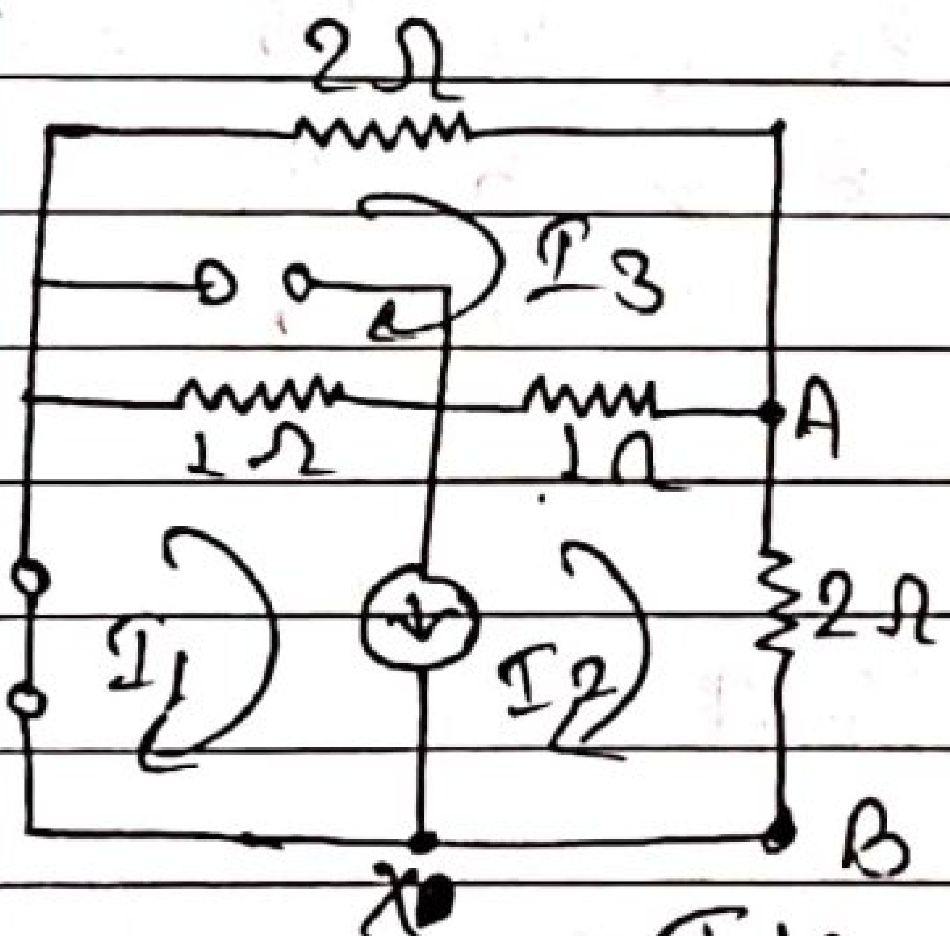
$I_1 = 0.66A$, $I_2 = -3.33A$ (opp. dirⁿ)

$I_3 = -0.66$ (opp dirⁿ)

$\therefore I''_{AB} = 0.66$ (B to A)

Now,

Taking 4A(↓)



At node x:

$$I_1 = I_2 + 4$$

$$\Rightarrow I_1 - I_2 = 4 \quad \text{—— (I)}$$

Taking combine loop (I) & (II),

$$-1(I_1-I_3) - 1(I_2-I_3) - 2I_2 = 0$$

$$\Rightarrow -I_1 - 3I_2 + 2I_3 = 0 \quad \text{—— (II)}$$

Taking loop (III),

$$-2I_3 - 1(I_3-I_2) - 1(I_3-I_1) = 0$$

$$\Rightarrow I_1 + I_2 - 4I_3 = 0 \quad \text{—— (III)}$$

Equating (I), (II) & (III),

$I_1 = 3.33$, $I_2 = -0.66A$ (opp. dirⁿ), $I_3 = 0.6$ (opp. dirⁿ)

$\therefore I'''_{AB} = 0.66$ (B to A)

$$\therefore I_{AB} = I'_{AB} + I''_{AB} + I'''_{AB}$$

$$= 2 - 0.66 - 0.66$$

$$= 0.68A$$

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Q. 11) Using SPT, to calculate the current through 10 Ω resistor of the following ckt shown below:
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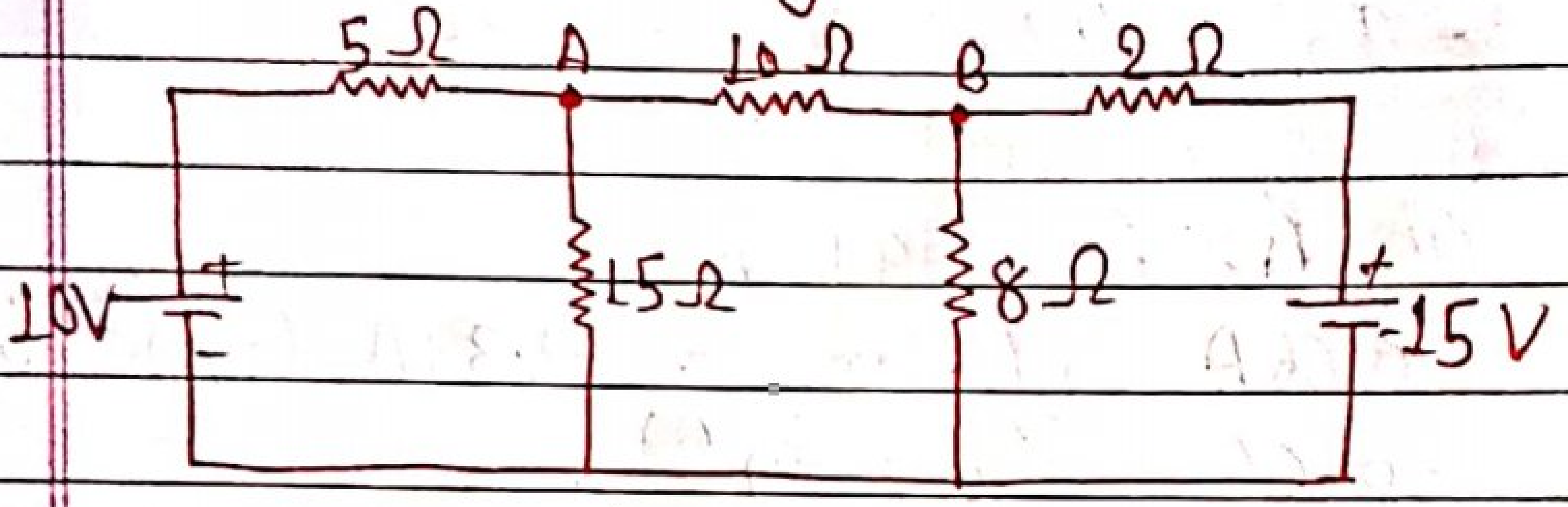
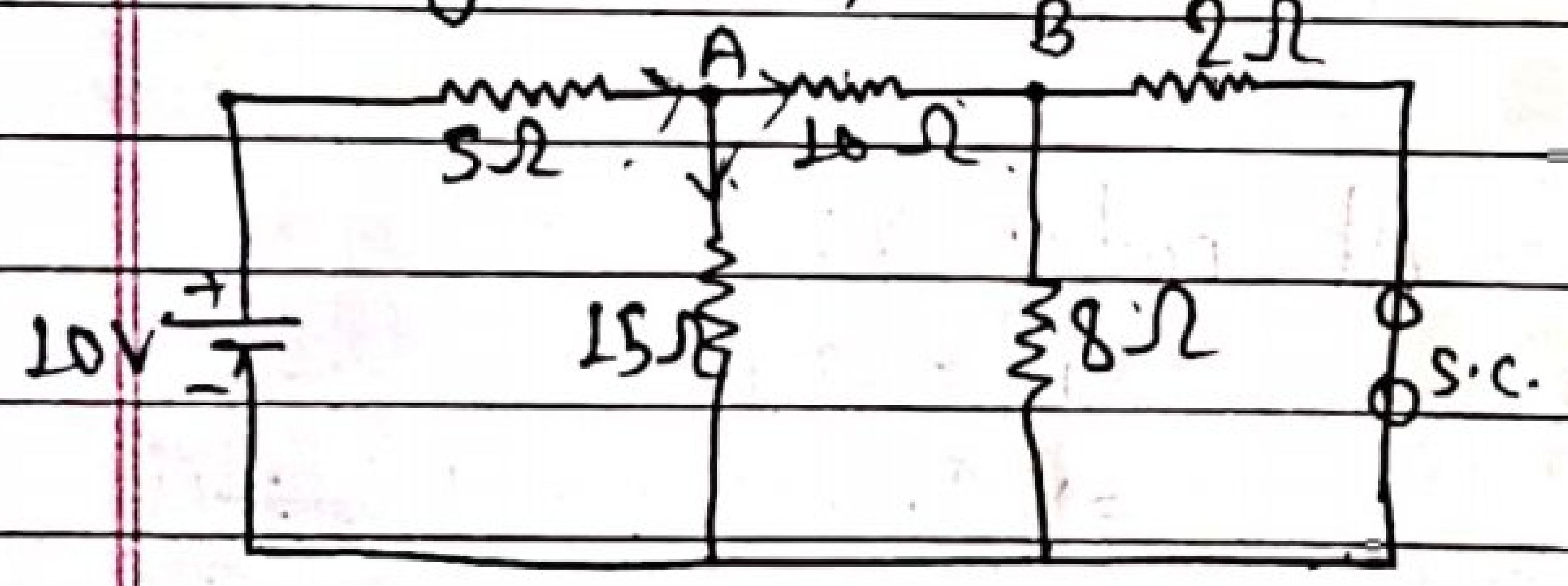


Taking 10V,



$R_1 = \frac{8 \times 2}{8+2} = 1.6\,\Omega$

$R_2 = 10 + 1.6 = 11.6\,\Omega$

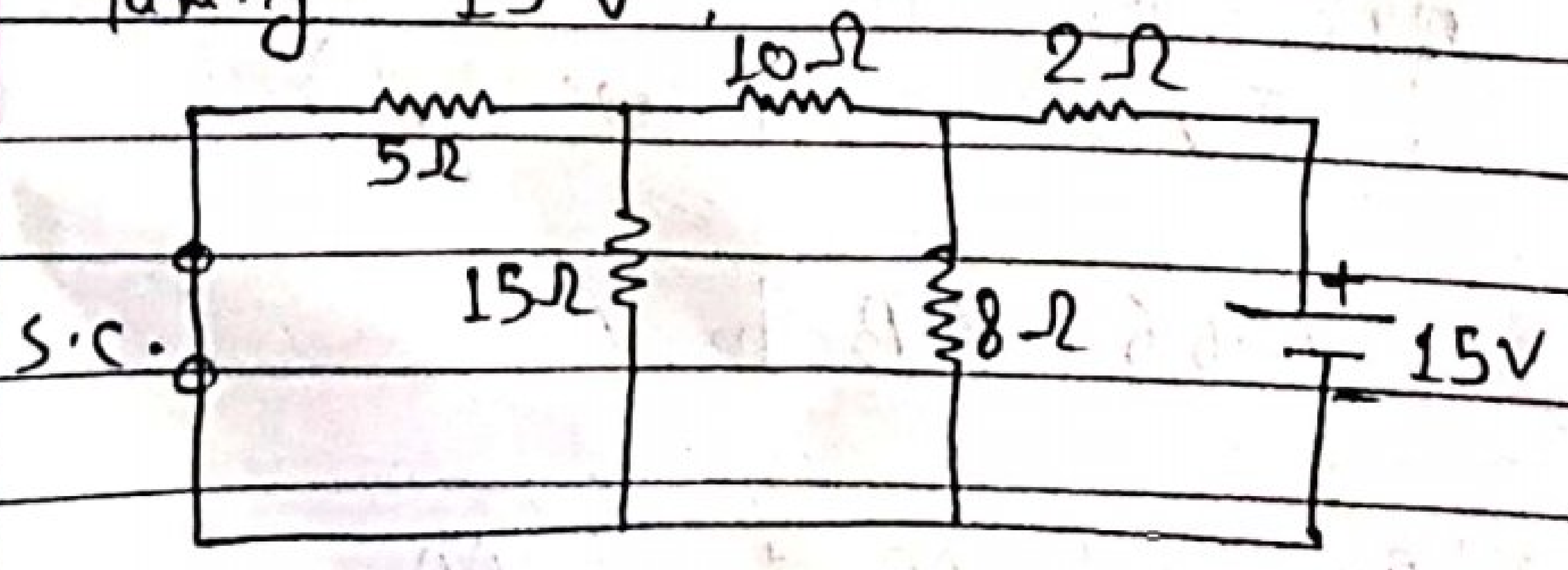
$R_3 = \frac{15 \times 11.6}{15 + 11.6} = 6.54\,\Omega$

$R_{eq} = 5 + 6.54 = 11.54\,\Omega$

$I_T = \frac{V}{R} = \frac{10}{11.54} = 0.866A$

$I'_{AB} = \frac{0.866 \times 15}{15 + 11.6} = 0.488$ Amp (A to B)

Taking 15V;



$R_1 = 3.75\,\Omega$

$R_2 = 10 + 3.75 = 13.75\,\Omega$

$R_3 = 5.05\,\Omega$

$R_{eq} = 5.05 + 2$
$= 7.05\,\Omega$

$I_T = \frac{15}{7.057} = 2.125\,\Omega$

$I''_{AB} = \frac{2.12 \times 8}{13.75 + 8} = 0.78$ Amp (B to A)

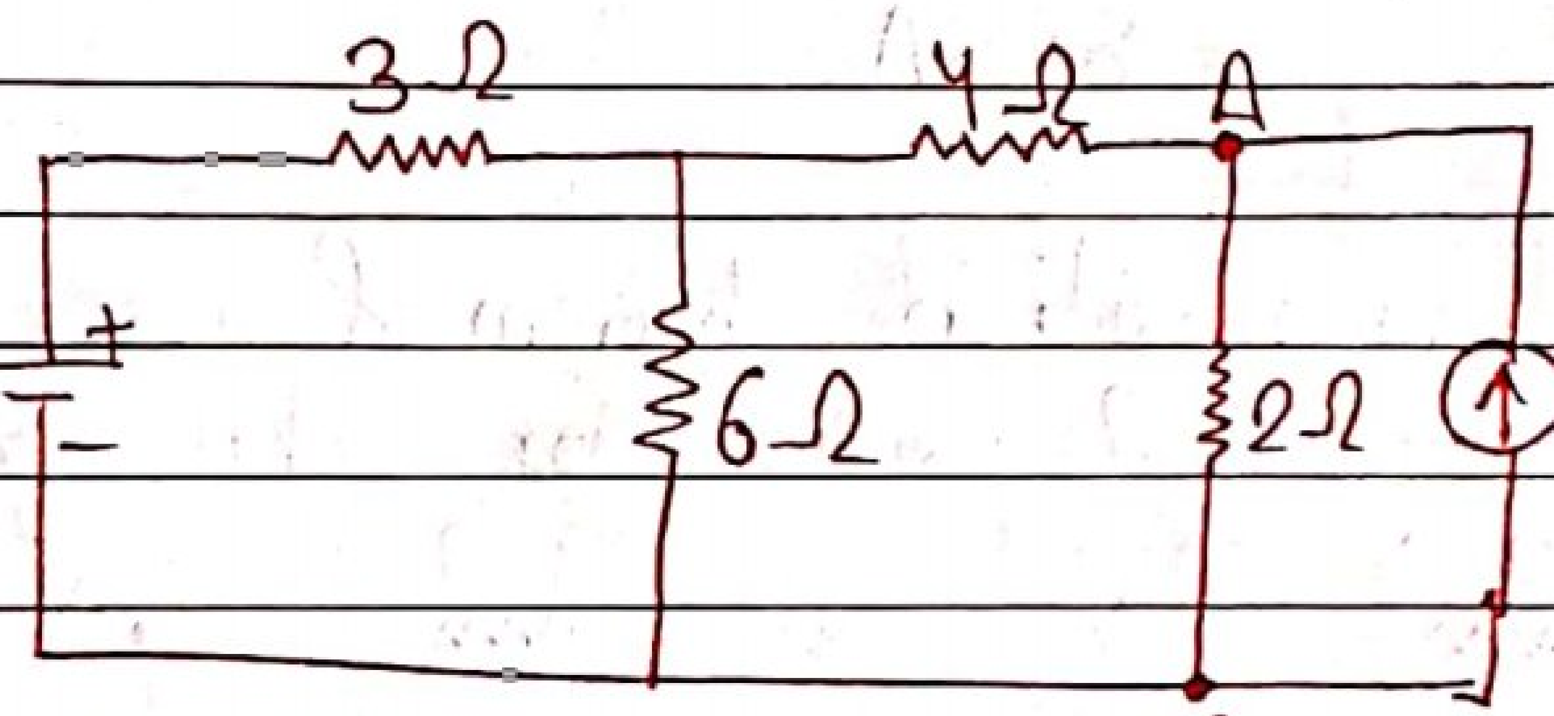
$\therefore I_{AB} = I''_{AB} - I'_{AB}$

$= 0.78 - 0.488$

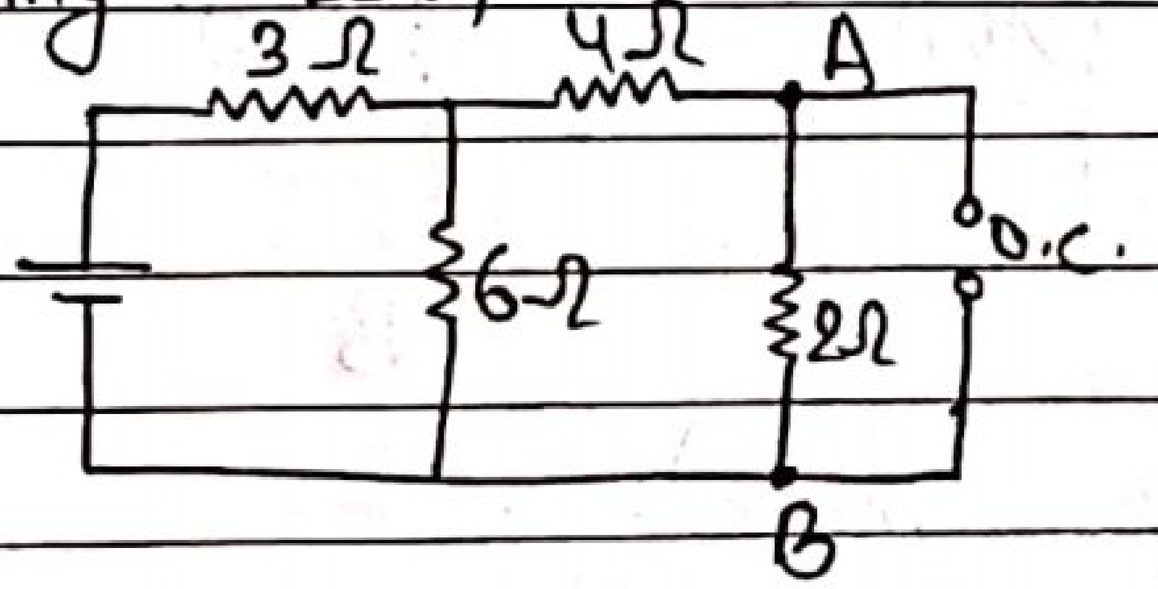
$= 0.29$ Amp

Q.12) Using SPT, calculate the current flowing through 2Ω resistor.

Imp



Taking 12V,



$R_1 = 4+2 = 6\Omega$

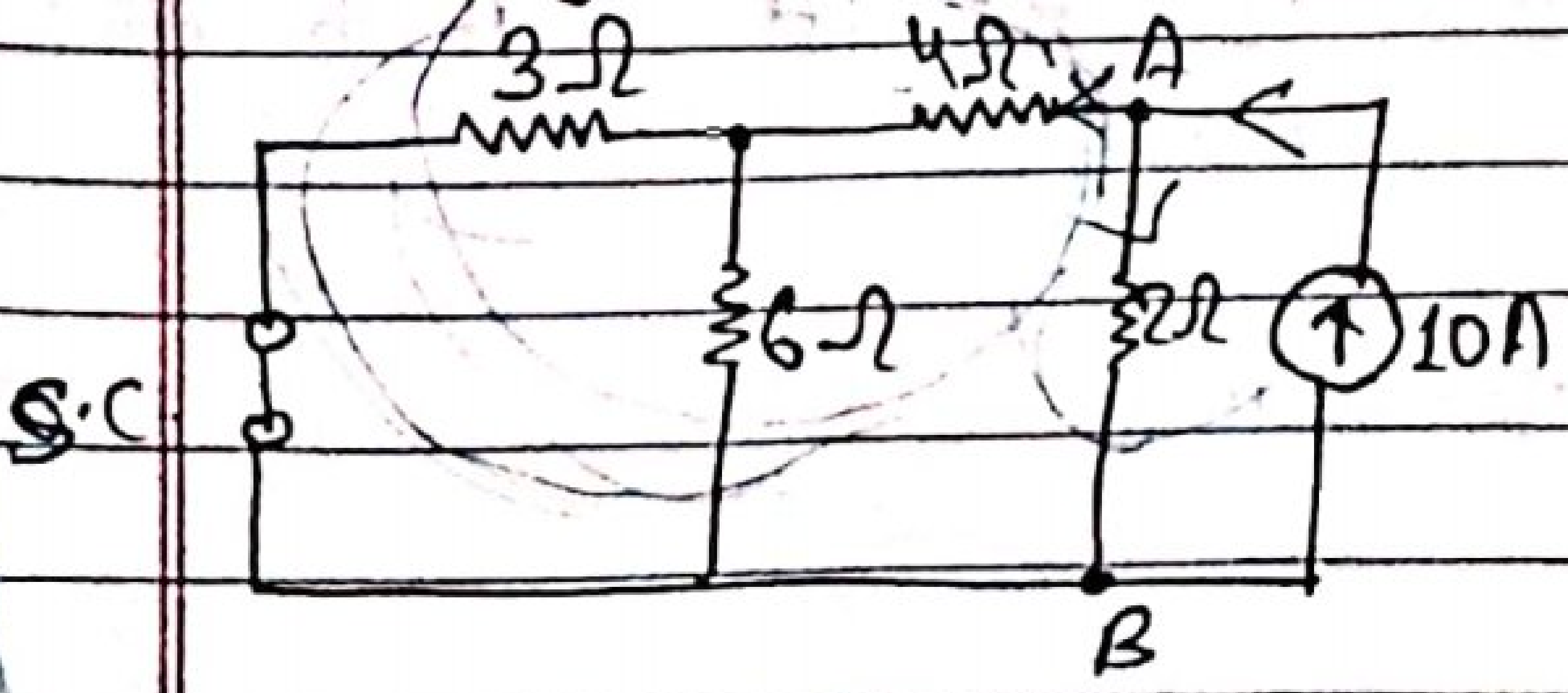
$R_2 = \frac{6 \times 6}{6+6} = 3\Omega$

$R_{eq} = 3+3 = 6\Omega$

$I_T = \frac{V}{R} = \frac{12}{6} = 2A$

$\therefore I'_{AB} = \frac{2 \times 6}{6+6} = \frac{12}{12} = 1A$ (A to B)

Taking 10A current source:



$R_1 = \frac{3 \times 6}{3+6} = 2\Omega$

$R_2 = 2+4 = 6\Omega$

$I_T = 10A$

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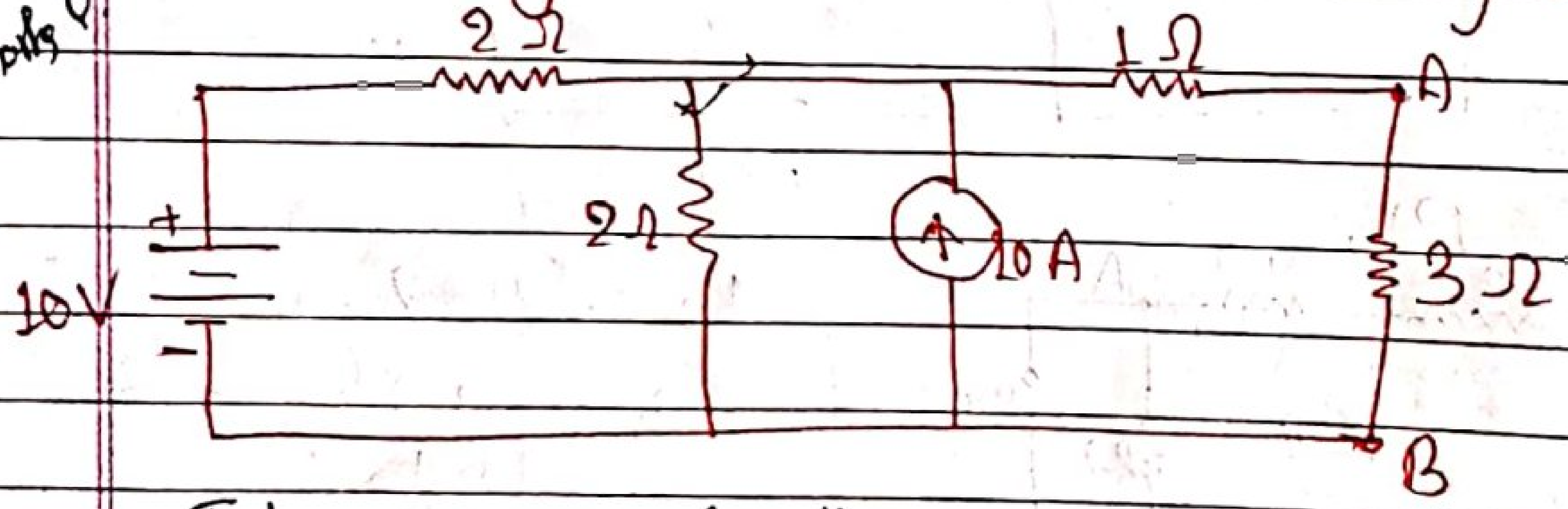
$$I''_{AB} = \frac{I_T \times R_{opp.}(R_2)}{R_2 + R_{AB}} = \frac{10 \times 6}{6+2} = 7.5A \quad (A \text{ to } B)$$

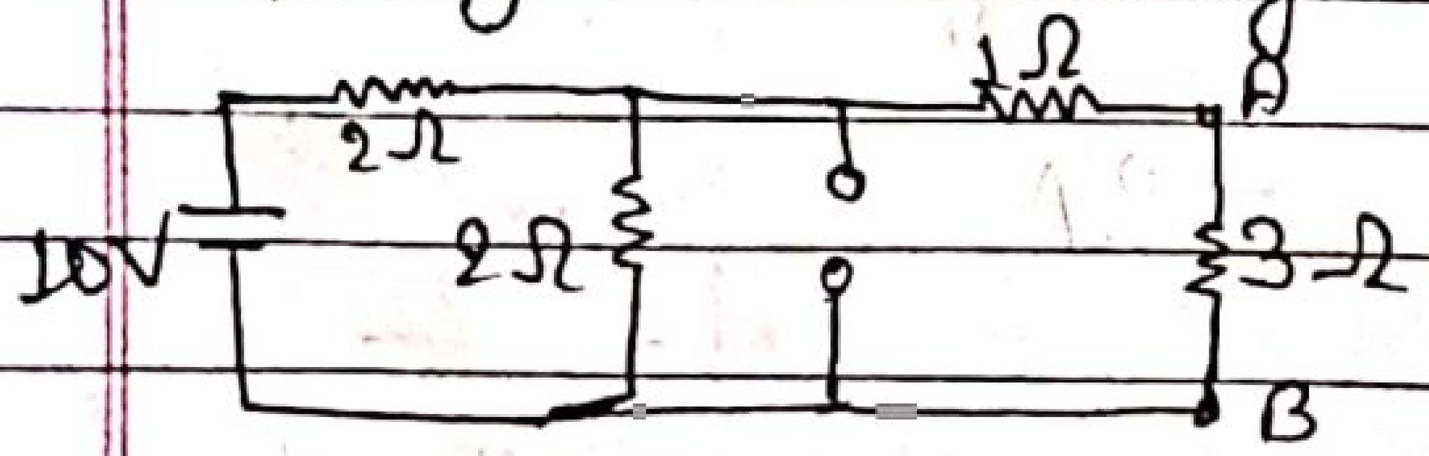
$$\therefore \quad I_{AB} = I'_{AB} + I''_{AB}$$
$$= 1A + 7.5A$$
$$= 8.5A$$

2015 spring sem 8 marks Q

13) State the superposition theorem & use it to find power consumed by 3Ω resistor for ckt given below:



Taking 10V (voltage source):



$R_1 = 1\Omega + 3\Omega = 4\Omega$

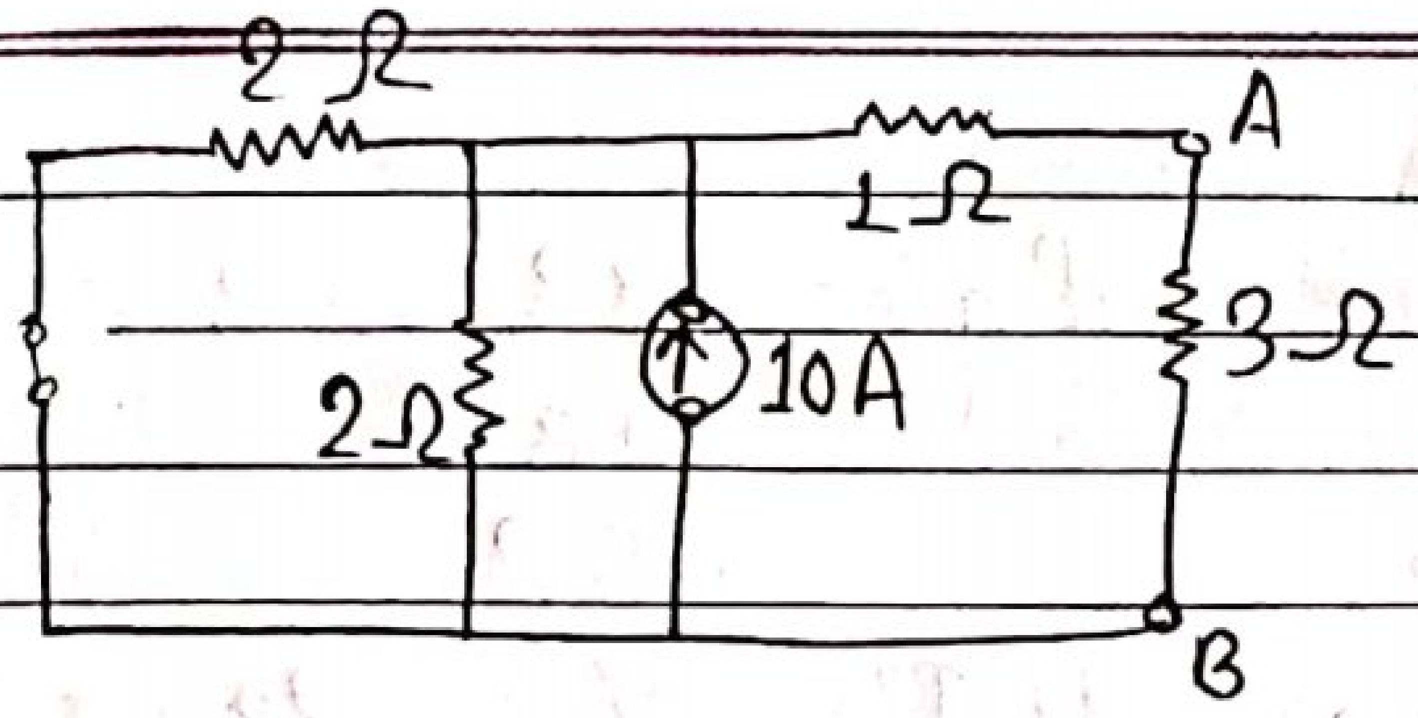
$R_2 = \frac{2 \times 4}{2+4} = 1.33 A$

$R_{eq} = 2 + 1.33 = 3.33 A$

$$I_T = \frac{V}{R_{eq}} = \frac{10}{3.33} = 3A$$

$$I'_{AB} = \frac{I_T \times 2}{2+4} = \frac{3 \times 2}{2+4} = 1A \quad (A \text{ to } B)$$

Taking 10 A current source



~~$R_1 = 1\Omega + 3\Omega$~~

$R_1 = \dfrac{2 \times 2}{2+2} = 1\,\Omega$, $R_2 = 1+3 = 4\,\Omega$.

$I_T = 10\,A$

$I''_{AB} = \dfrac{10 \times 1}{1+4} = \dfrac{10}{5} = 2A$ (A to B) [current division]

$\therefore I_{AB} = I'_{AB} + I''_{AB}$
$= 1 + 2 = 3A$

$\therefore P_{AB} = (I_{AB})^2 \cdot R_{AB}$
$= 3^2 \cdot 3$
$= 27$ watt

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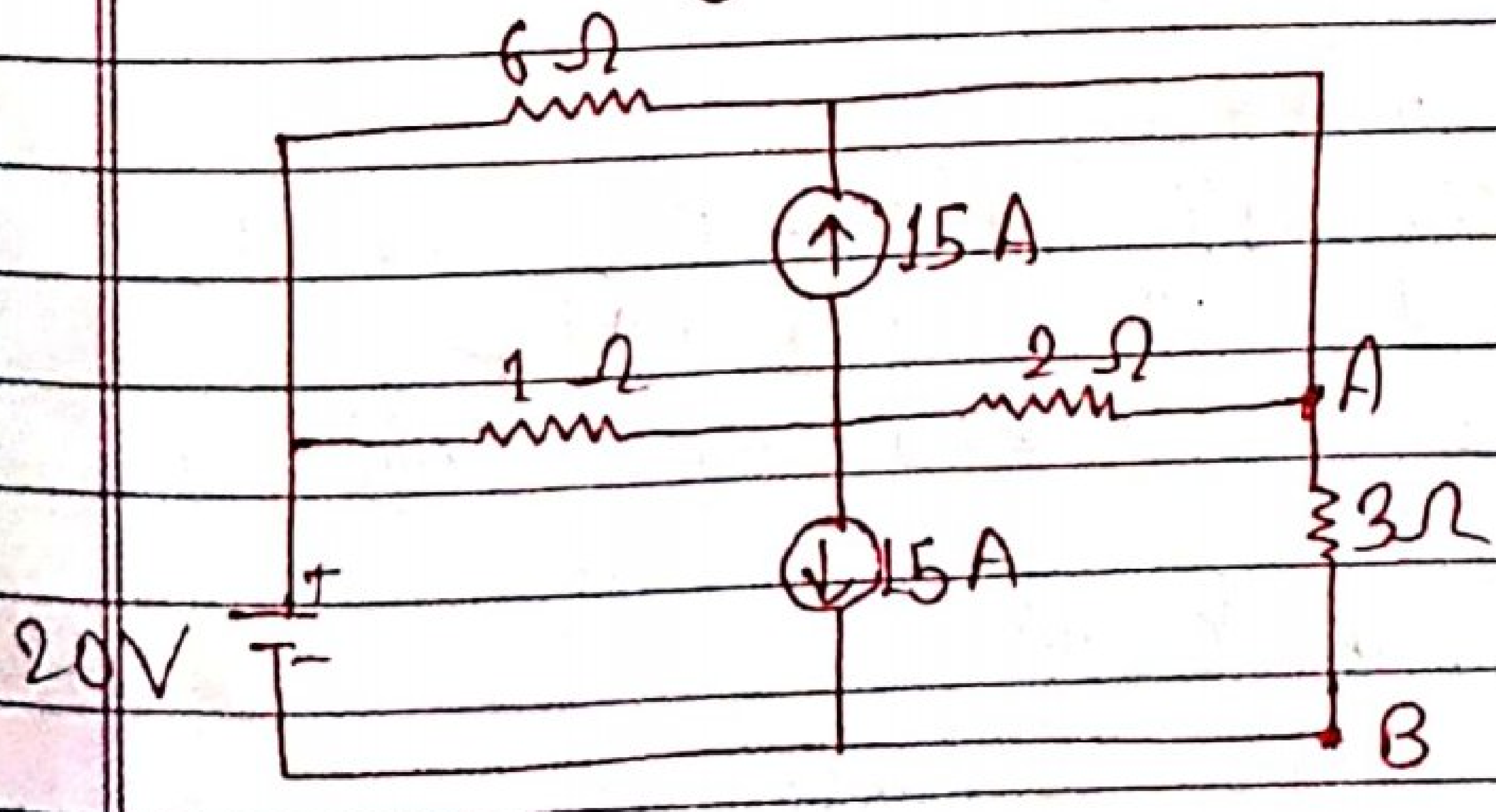
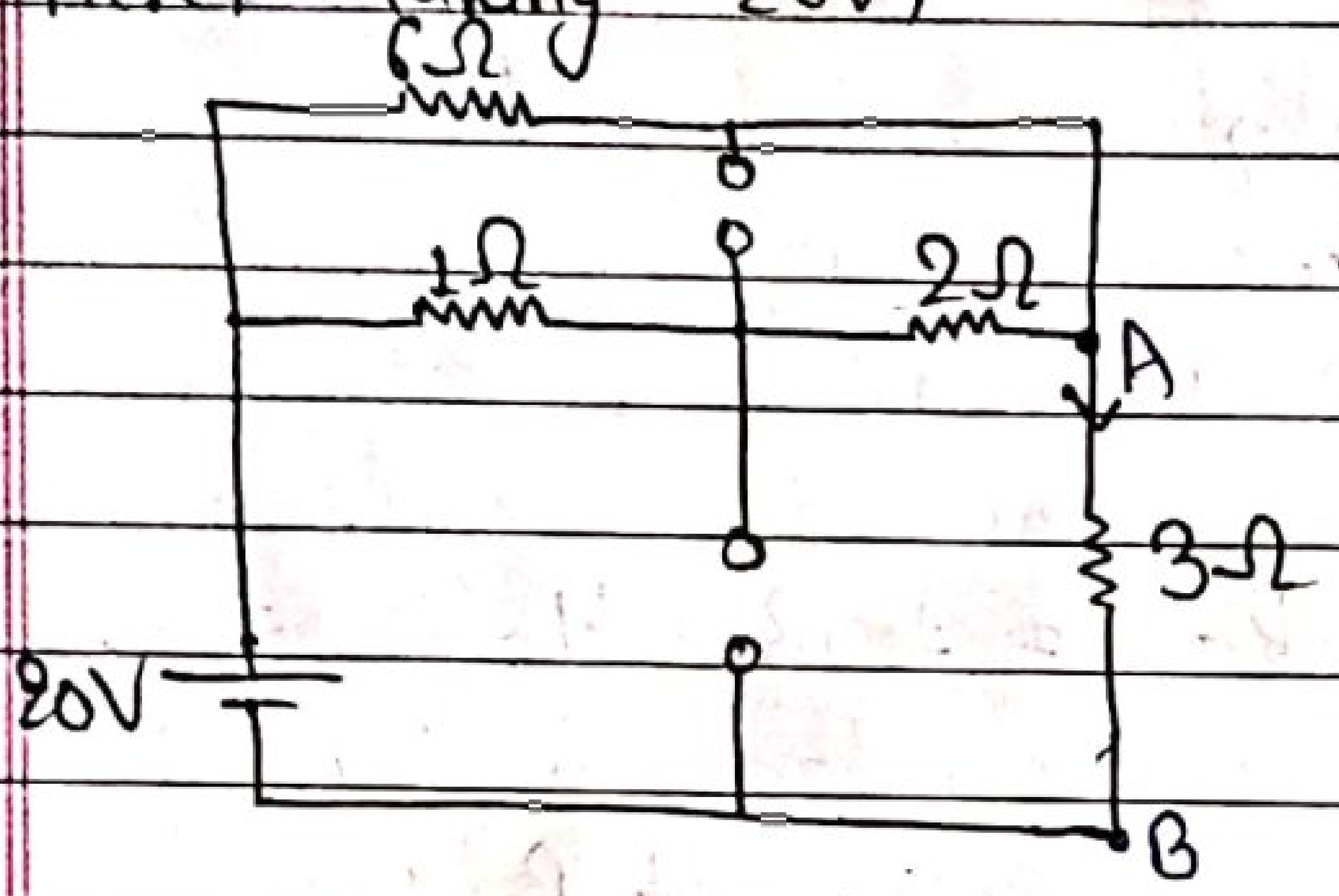
Q. Use the superposition theorem to calculate the current through 3 Ω resistor of the ckt shown in fig. below:



Here, Taking 20V,



$$R_{eq} = \frac{6\times3}{6+3} + 3 = 5\,\Omega$$

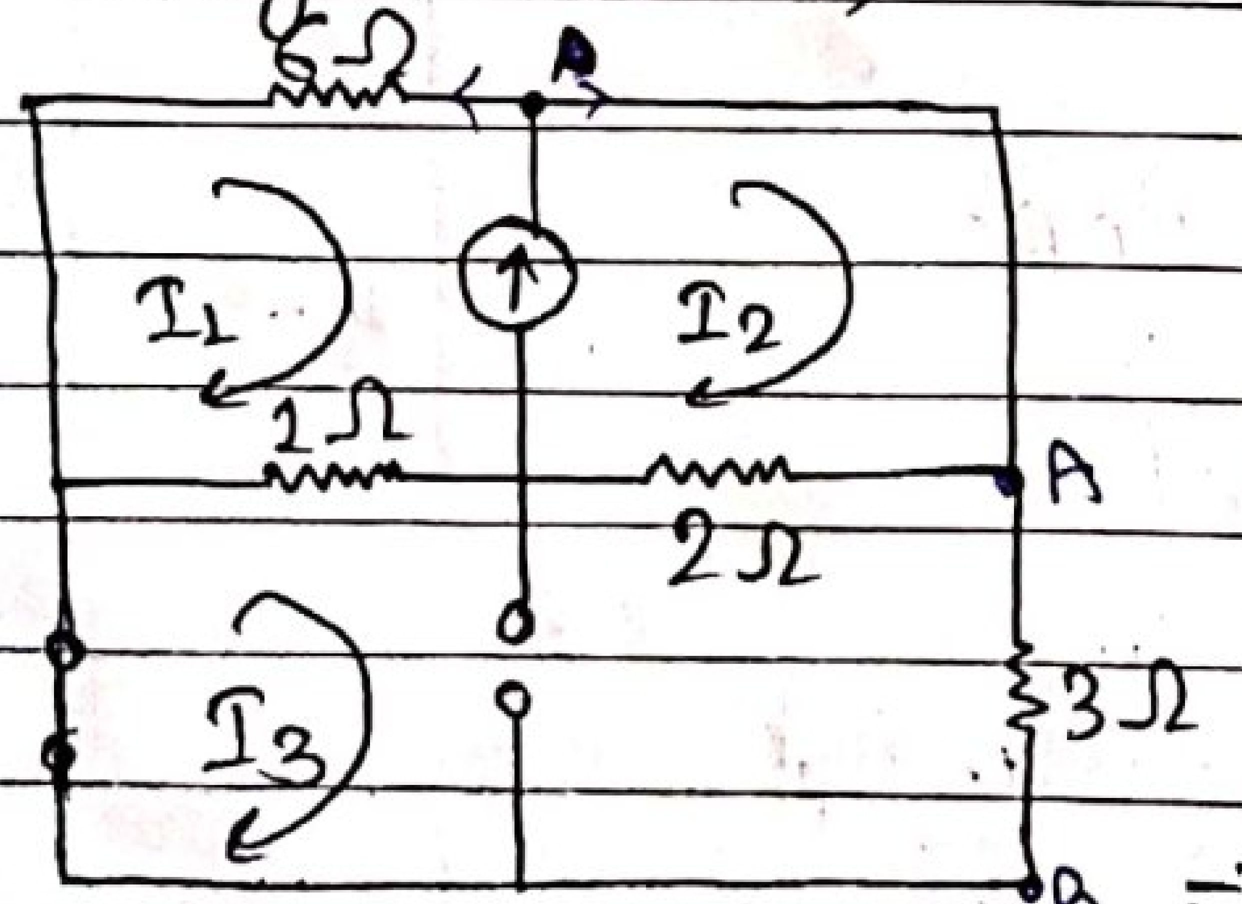
$$I = \frac{V}{5} = \frac{20}{5} = 4\,Amp$$

This I current flows through 3Ω resistor.

$\therefore I' = 4A\ (\uparrow)$

Now,

Step-2 Taking 15A (↑):



Taking node A:

$I_1 + 15 = I_2$

$\therefore I_1 - I_2 = -15$ —— (I)

Taking common loop (I) & (II)

$-6I_1 - 2(I_2 - I_3) - (I_1 - I_3) = 0$

$\Rightarrow -6I_1 - 2I_2 + 2I_3 - I_1 + I_3 = 0$

$\Rightarrow -7I_1 - 2I_2 + 3I_3 = 0$ —— (II)

Also, taking loop-3;

$-3I_3 - (I_3 - I_1) - 2(I_3 - I_2) = 0$

$\Rightarrow -3I_3 - I_3 + I_1 - 2I_3 + 2I_3 = 0$

$\Rightarrow -6I_3 + I_1 + 2I_2 = 0$ —— (III)

Solving (I), (II) & (III),

$I_1 = -2\,A$

$I_2 = 13\,A$

$I_3 = 4A$

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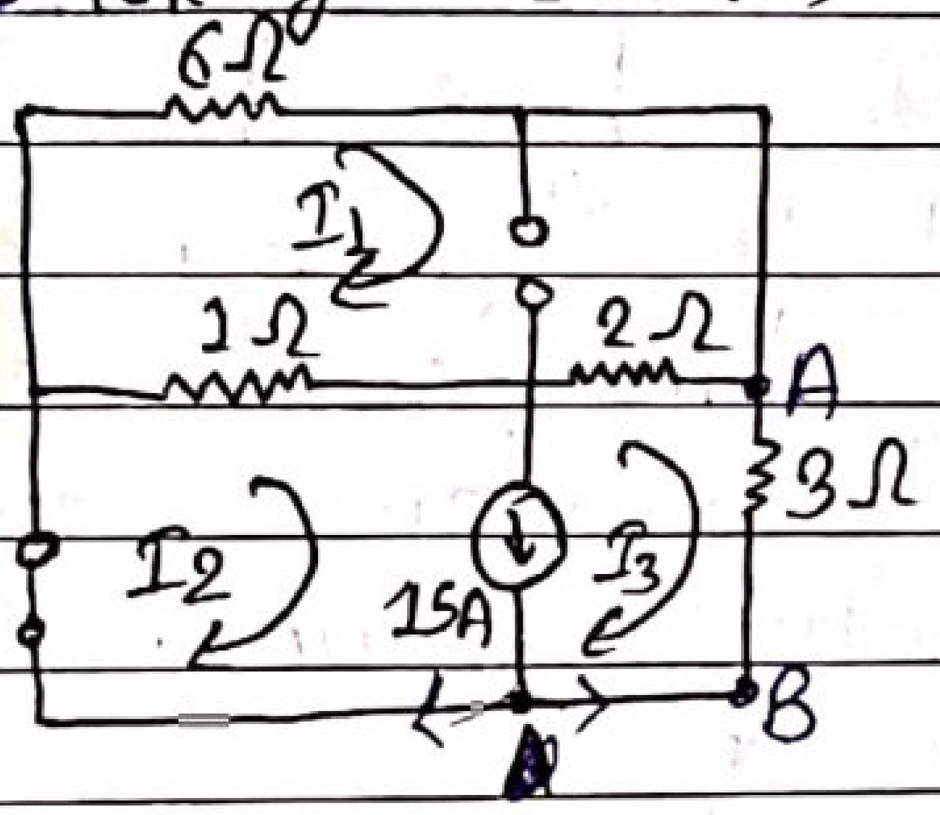
Since current flowing through 3Ω resistor is I_3.

So,

$$I'' = I_3 = 4A \ (\uparrow)$$

Now,

Step-3 Taking 15A (↓)



At node A: $I_3 + 15 = I_2$

$\Rightarrow I_2 - I_3 = 15$ —— (I)

Taking combine loop (2) & (3),

$$-3I_3 - (I_2 - I_1) - 2(I_3 - I_1) = 0$$

$$\Rightarrow -5I_3 + 3I_1 - I_2 = 0 \quad \text{—— (II)}$$

Also, taking loop (I),

$$-6I_1 - 2(I_1 - I_3) - (I_1 - I_2) = 0$$

$$\Rightarrow -9I_1 + I_2 + 2I_3 = 0 \quad \text{—— (III)}$$

Solving (I), (II) & (IV),

$I_1 = 1A$

$I_2 = 13A$

$I_3 = -2A$

Since current through 3Ω resistors I_3,

So,

$$I''' = I_3 = 2A \ (\downarrow)$$

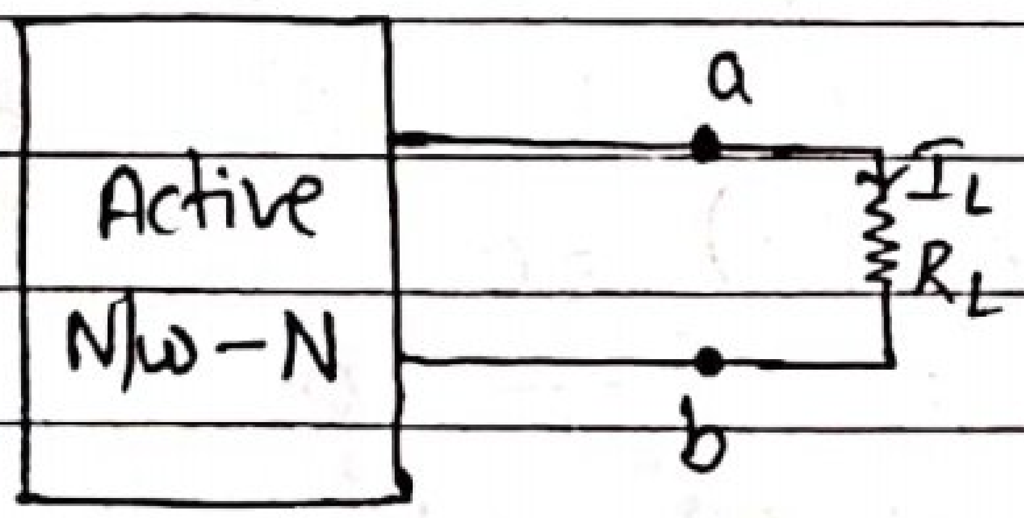
∴ Resultant current $= I' + I'' + I''' = 4 + 4 - 2$

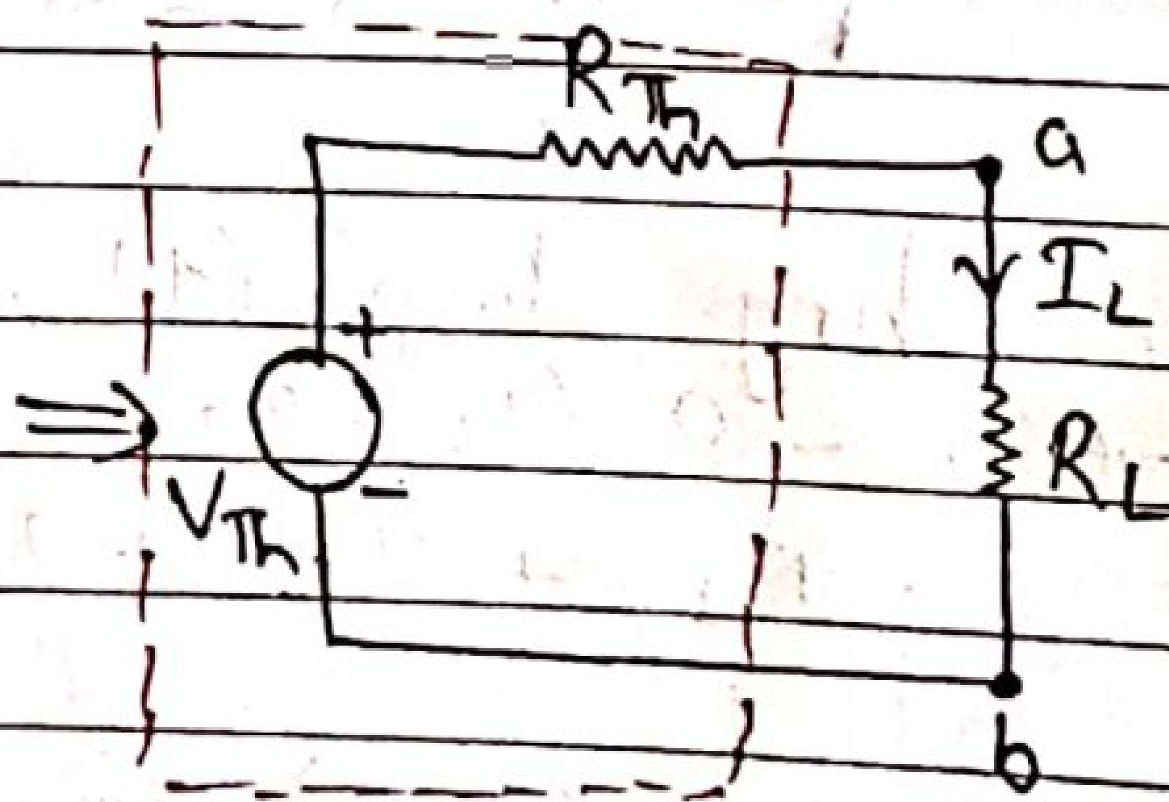
$= 6A$

∴ Voltage loop $= IR = 6 \times 3 = 18V$

Thevenin's Theorem

It states that any ckt having a number of voltage, resistance (impedance) & open output terminal can be replaced by simple equivalent ckt, consisting of single voltage source in series with resistance (impedance) where the value of voltage source is equal to the open ckt voltage across the output terminals & resistance (impedance) is equal to the resistance seen into the N/w across the output terminal.





Thevenin's equivalent N/w - N

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Procedure

Remove the load resistance.

Calculate the voltage seen by the load.

All the voltage source - short ckt (s.c.)

" " current " - open ckt (o.c.)

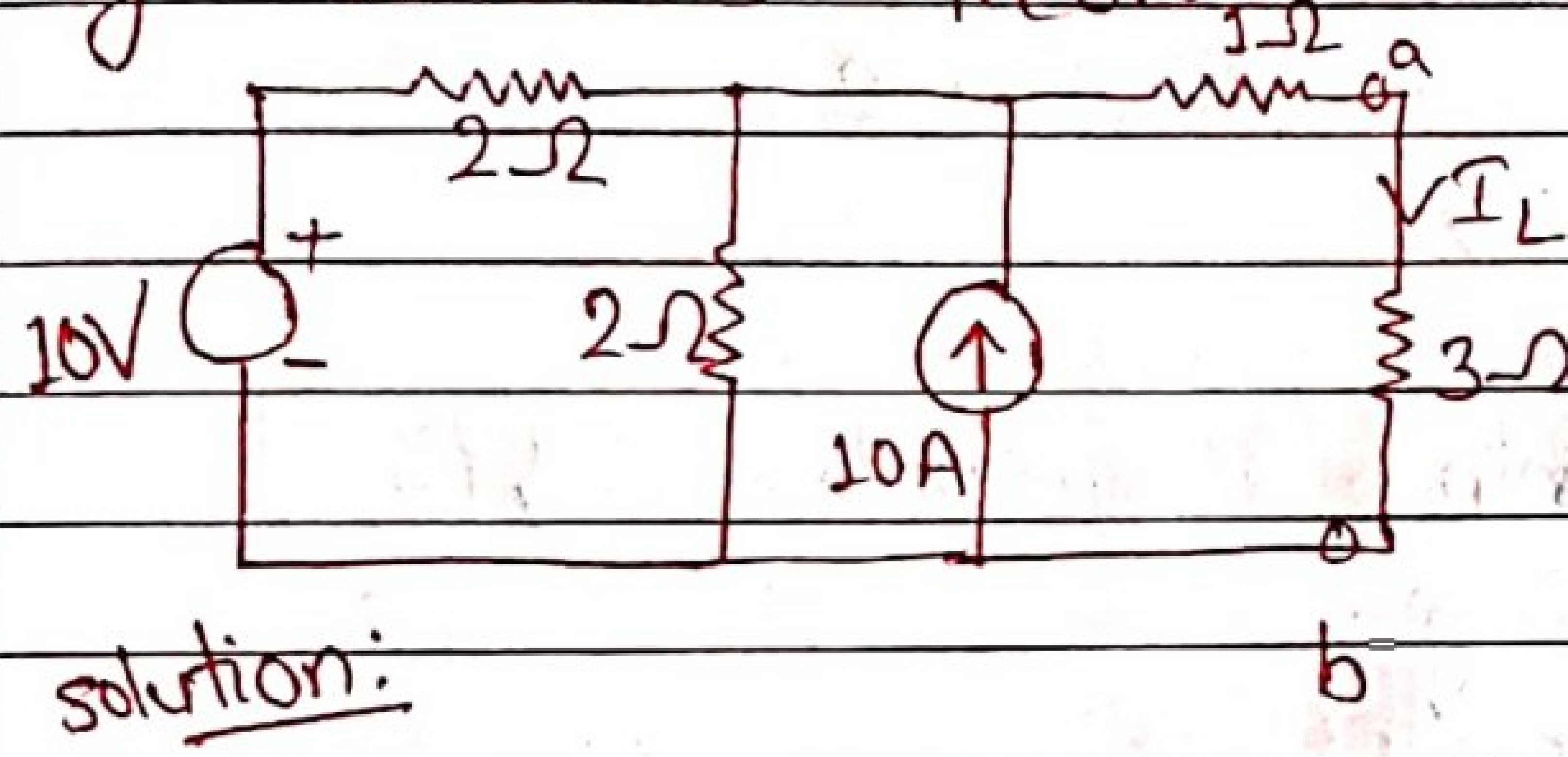
Calculate the resistance seen by the load (R_{Th}).

Thevenin's Theorem

Any ~~two~~ linear circuit containing large number of voltages and/or current sources and ~~resistances~~ resistor can be replaced by a simple equivalent ckt containing a single voltage source and a series resistor connected across the load.

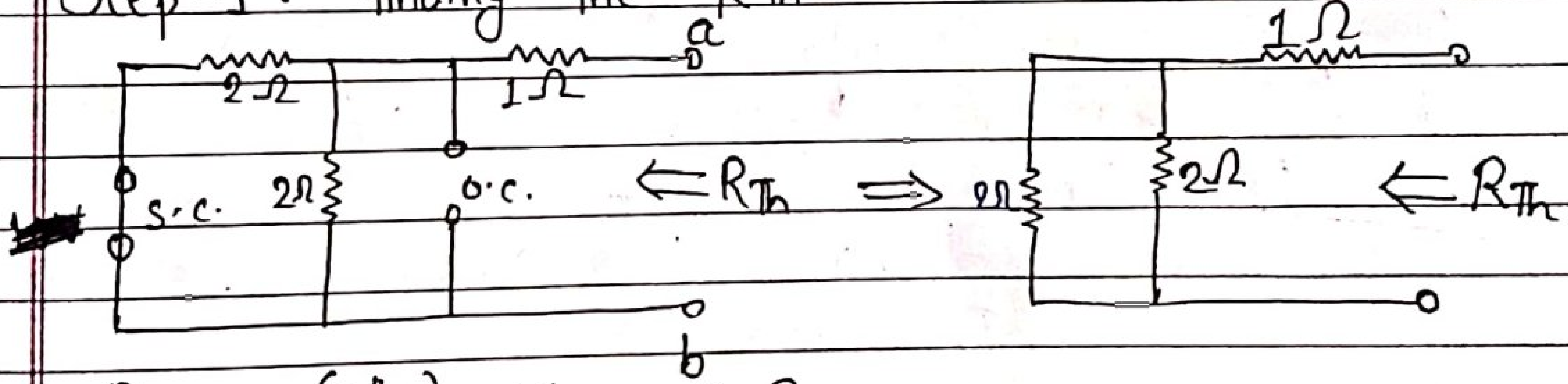
* If resistance & current are in parallel, convert current into voltage

Q.1) Determine the current through 3Ω resistor in the N/w by Thevenin's theorem.



solution:

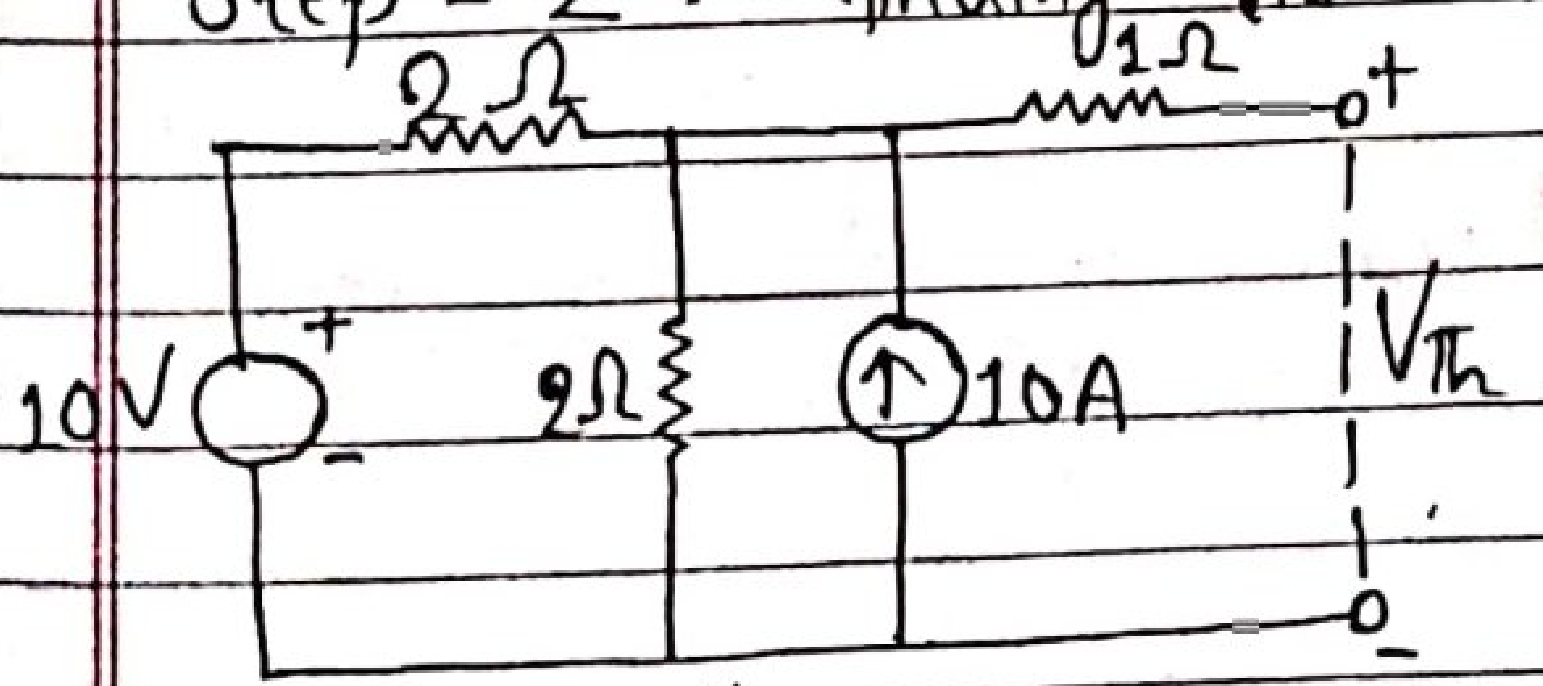
Step-1 : Finding the R_{Th} :



$R_{Th} = (2 \| 2) + 1 = 2\,\Omega$

$\therefore \boxed{R_{Th} = 2\,\Omega}$

Step - 2 : finding the V_{Th} :



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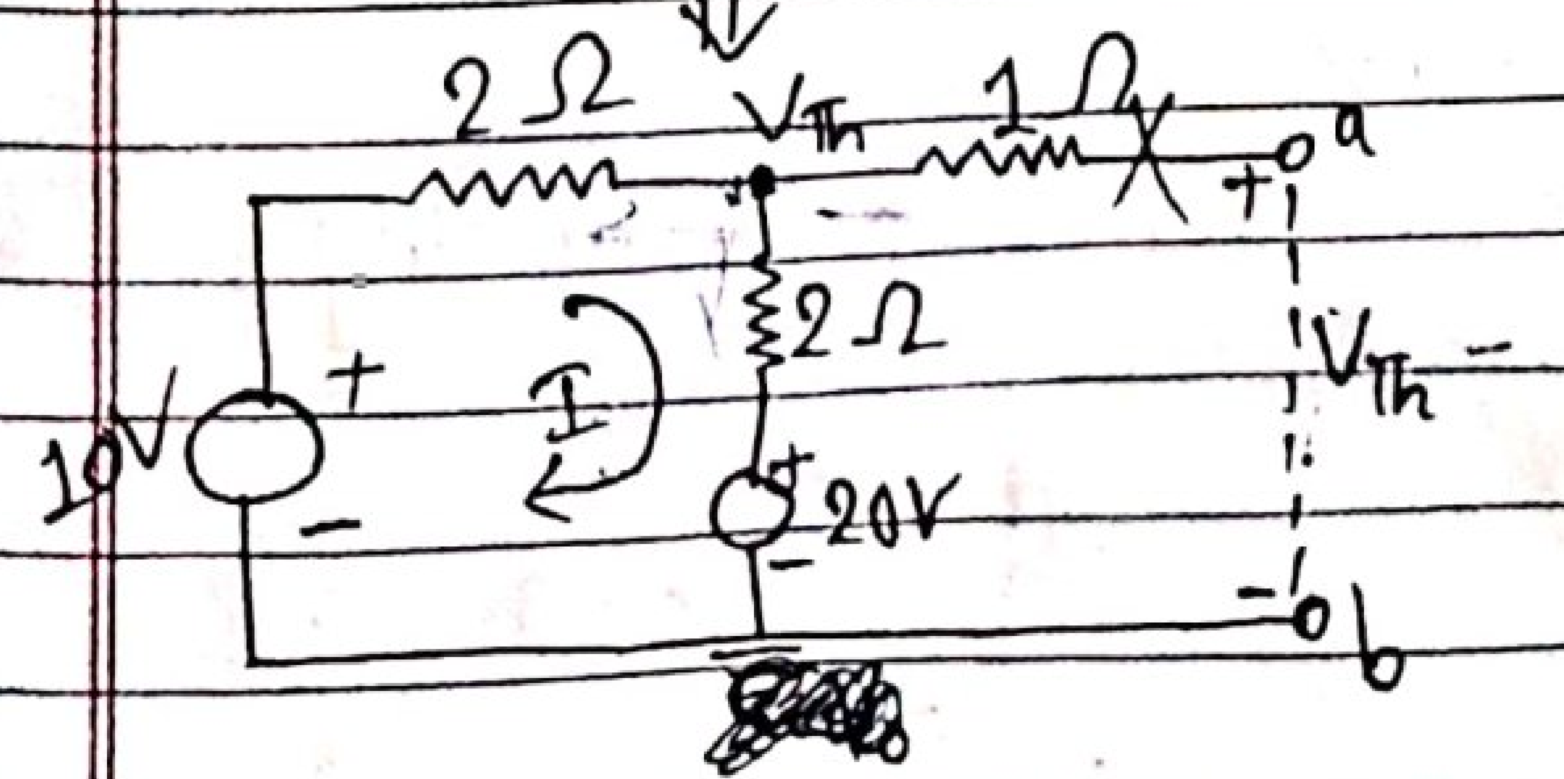


Consider mesh LHS:

$\Rightarrow 10V - 2I - 2I - 20 = 0$

$\Rightarrow I = -2.5$ amp

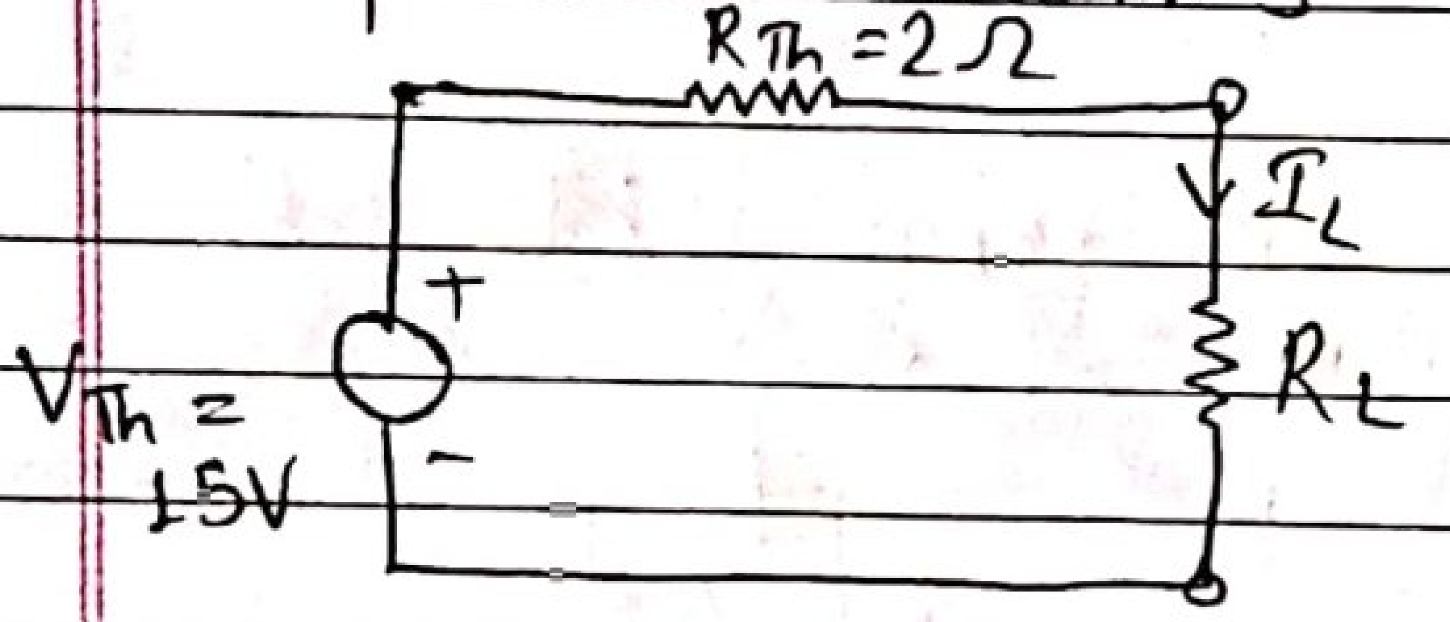
Using KVL in RHS mesh,

$$V_{Th} - 2I - 20 = 0$$

$$\therefore V_{Th} = 2 \times (-2.5) + 20$$

$$= 15\,V$$

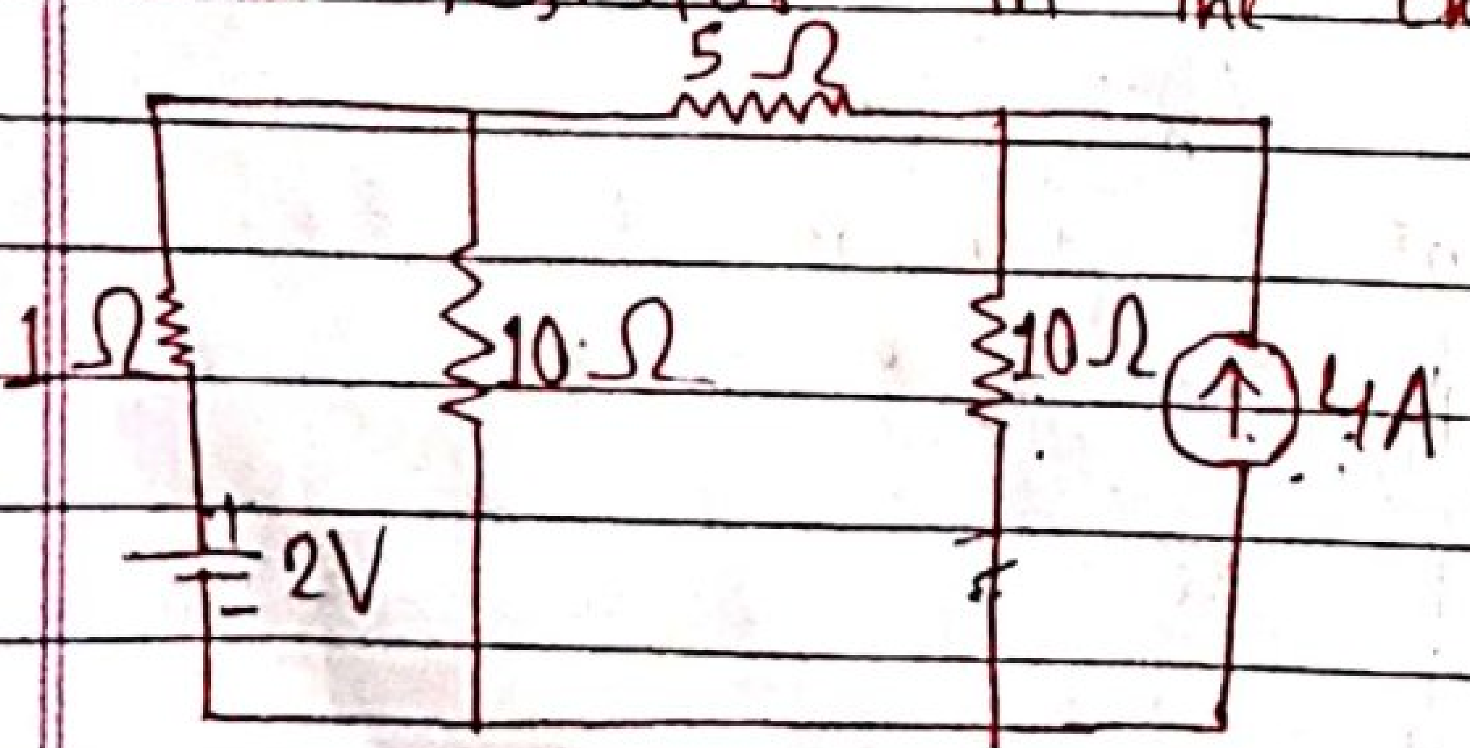
Step-3: Thevenin's equivalent ckt:



So, $I_L = \dfrac{V_{Th}}{R_{Th} + R_L} = \dfrac{15}{2+3}$

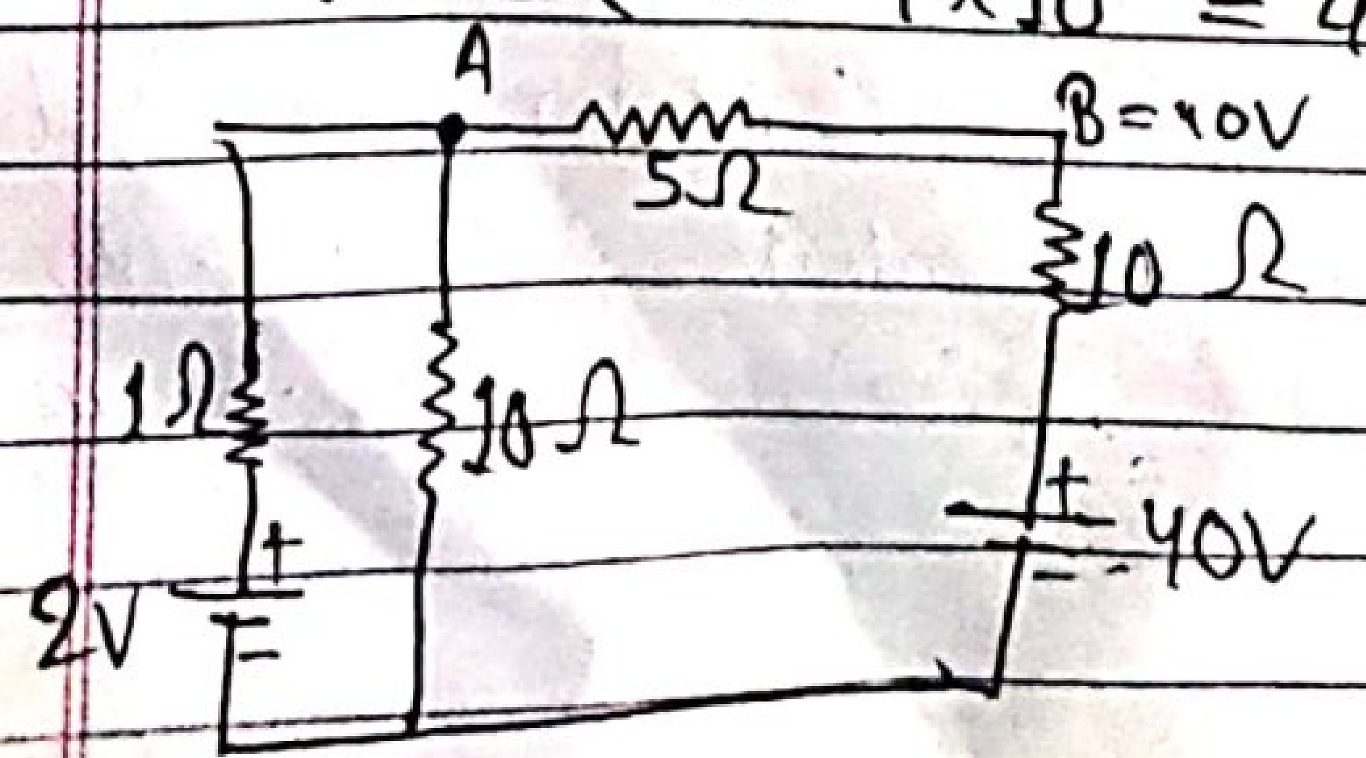
$\therefore I_L = 3$ amp

Q.2) Use the Thevenins theorem, find the current through 5Ω resistor in the ckt.

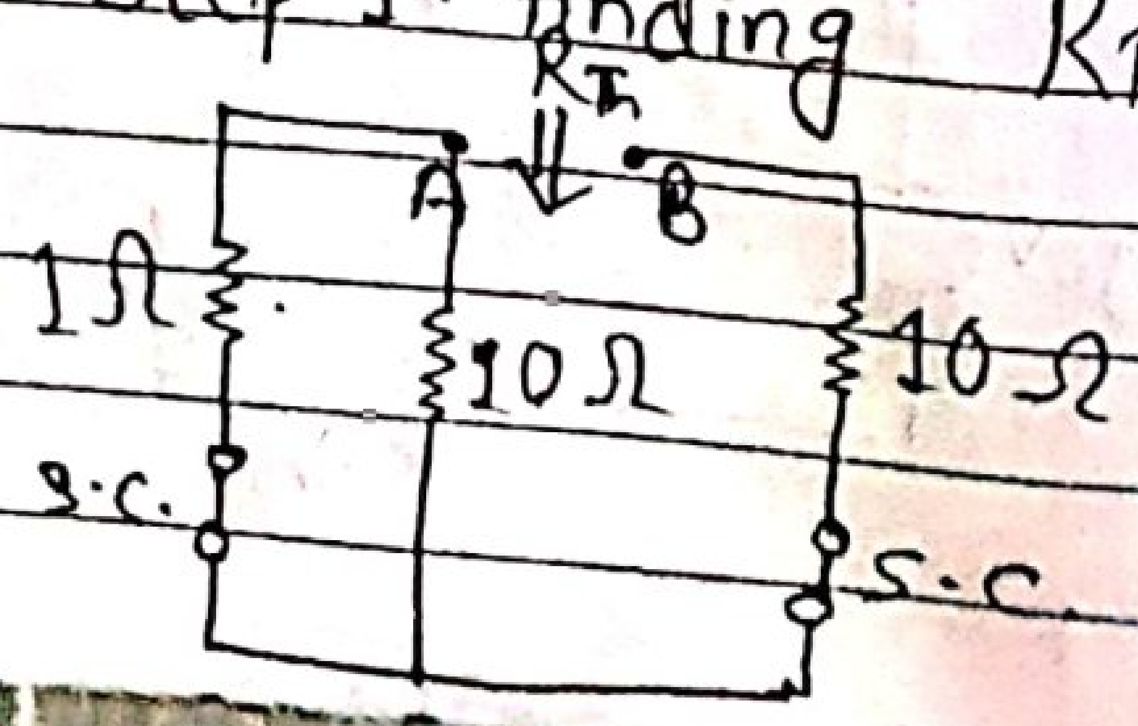


(source transformation can be done when resistor & current are in parallel)

$V = IR = 4 \times 10 = 40\,V$



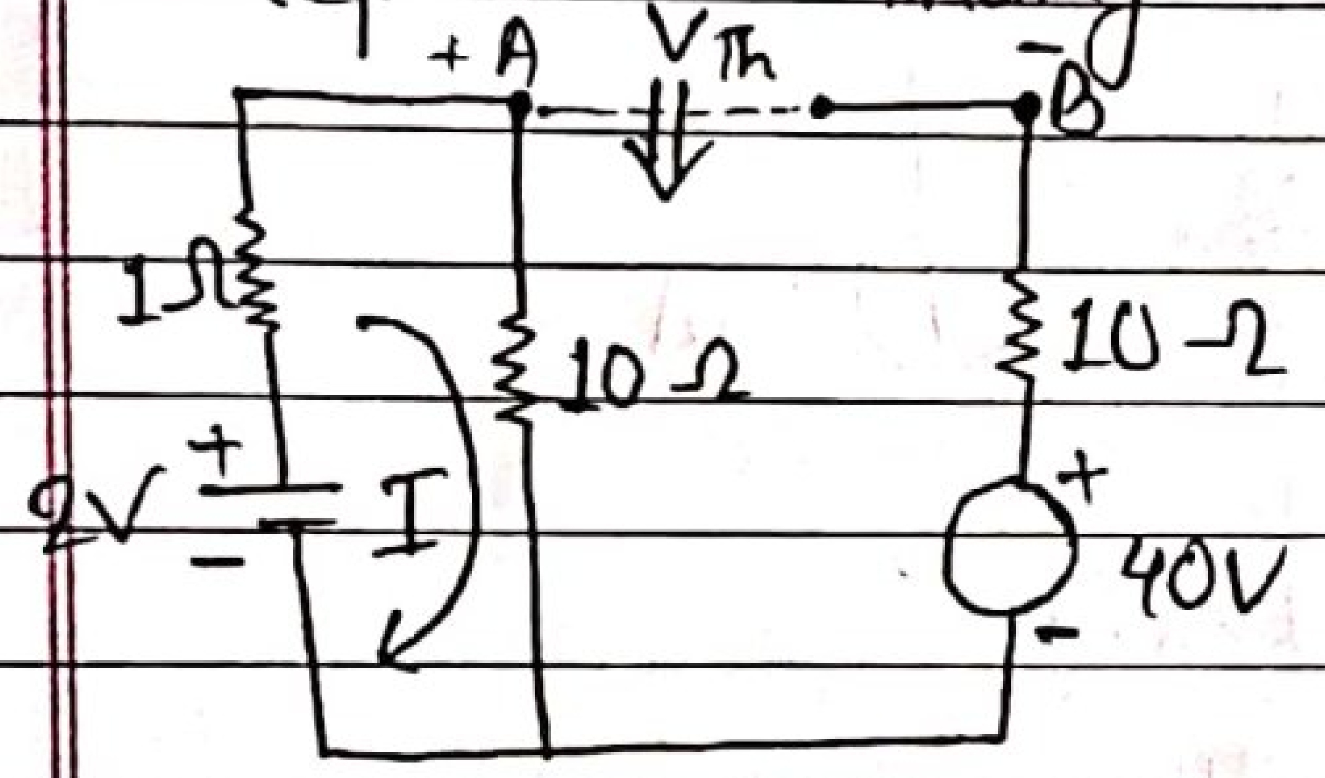
Step-1: finding R_{Th}:



$R_{Th} = 1 \parallel 10 + 10\Omega$

$= 10.90\Omega$

Step-2: finding V_{Th}:



Consider LHS mesh:

$2 - 1I - 10I = 0$

$\Rightarrow I = 0.18A$

$V_A = IR = 0.18 \times 10 = 1.8V$

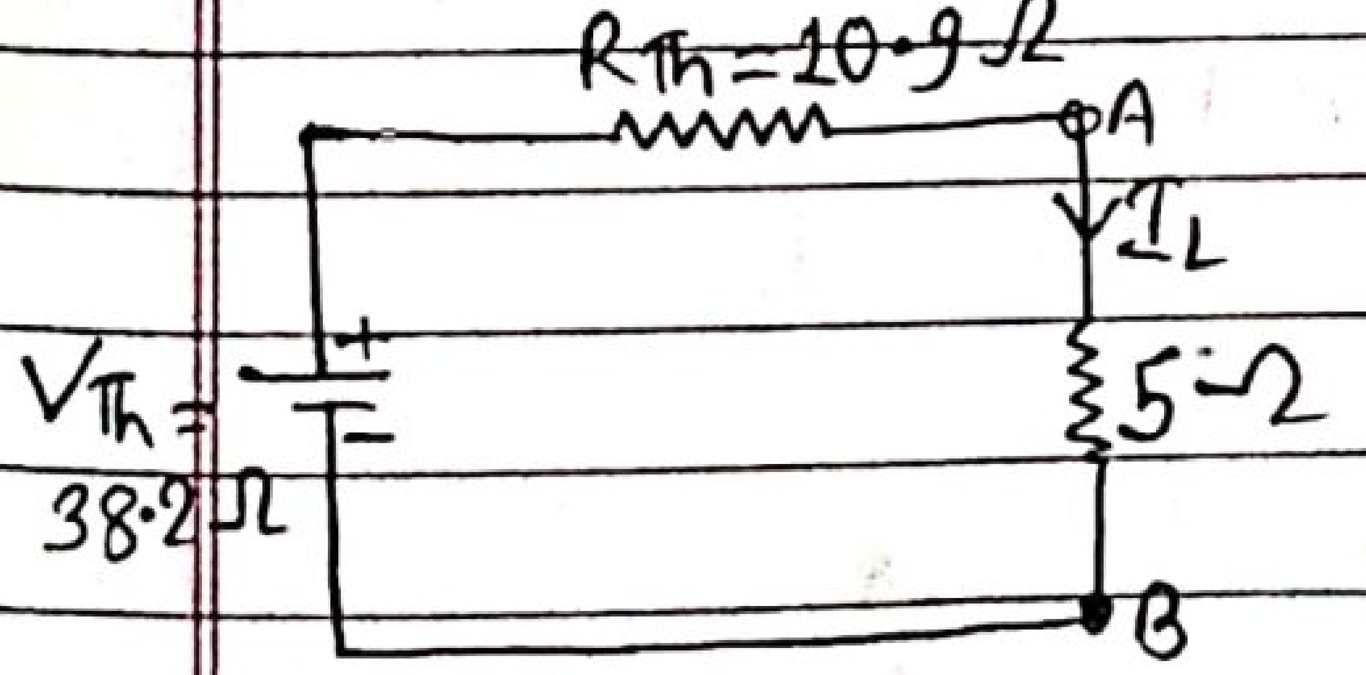
$V_{AB} = V_B - V_A$

$= 40 - 1.8$

$= 38.2V$

$\therefore V_{AB} = V_{Th} = 38.2V$

Step-3:
Find the Thevenin's equivalent ckt:



$I_L = \frac{V_{Th}}{R_{Th} + R_L}$

$= \frac{38.2}{10.9 + 5}$

$= 2.40$ amp

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Q.3) Use Thevenin's theorem to calculate the current flowing throw 4 Ω resistor.
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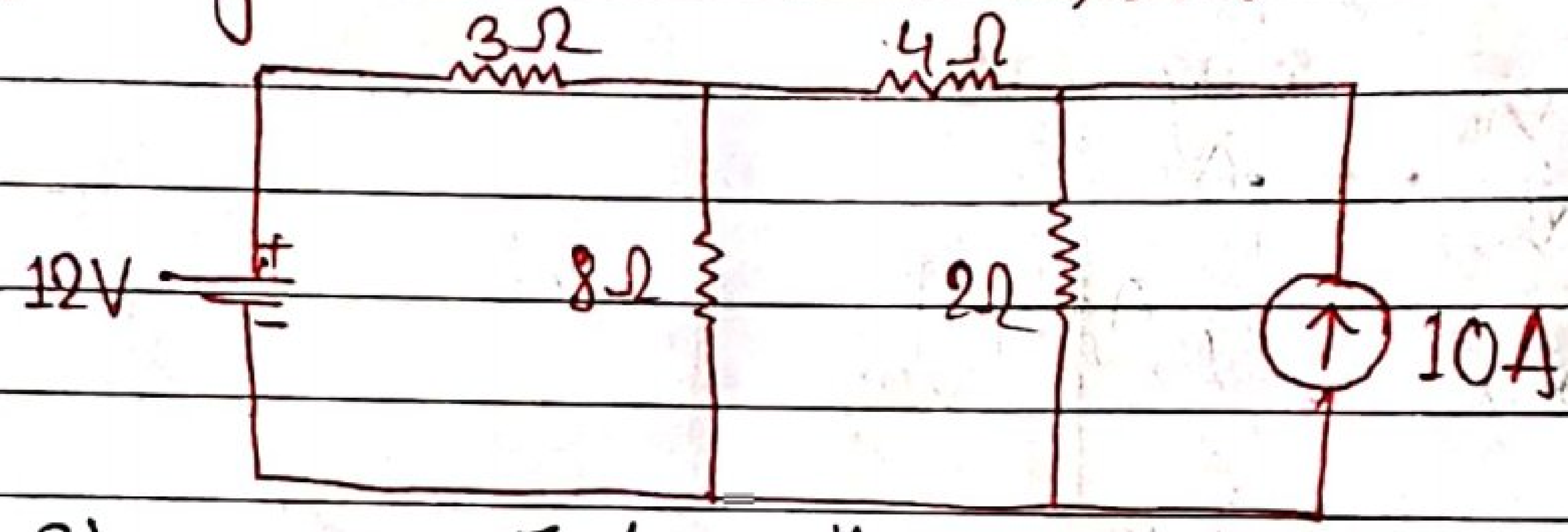
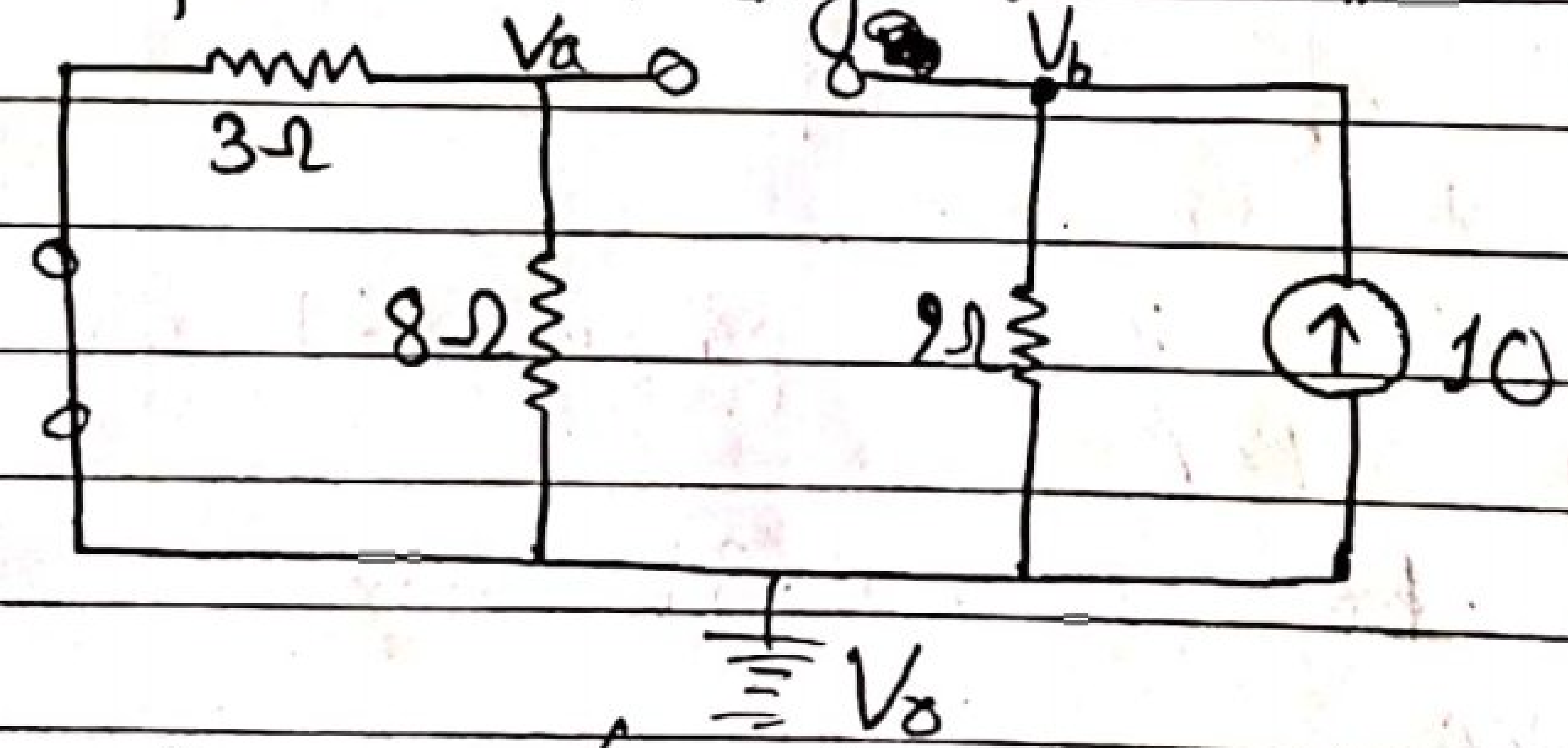
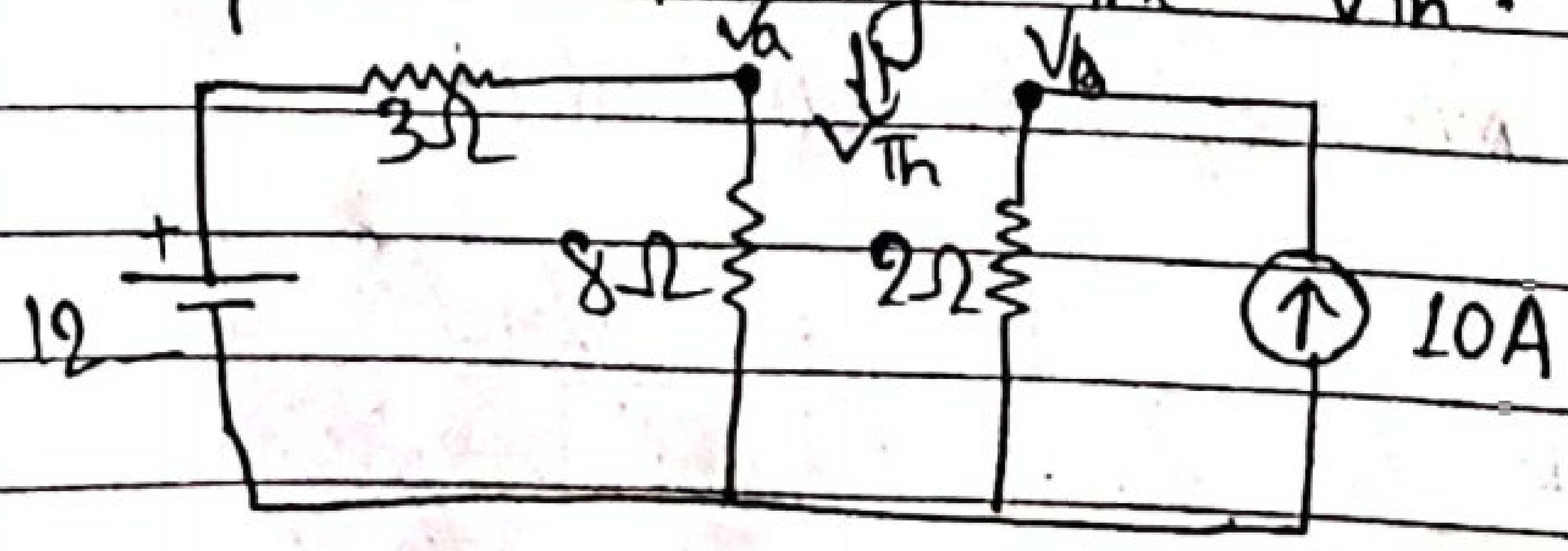


Step – 1 : Finding the R_{Th}:



$$R_{Th} = (3 \| 8) + 2$$
$$= \frac{3\times 8}{3+8} + 2$$
$$= 4.18\ \Omega$$

Step – 2 : Finding the V_{Th}:



$$\frac{V_a - 12}{3} + \frac{V_a}{8} = 0$$
$$\Rightarrow 8V_a - 96 + 3V_a = 0$$
$$\Rightarrow V_a = 8.72V$$

$$\frac{V_b}{2} = 10$$
$$\Rightarrow V_b = 20V$$

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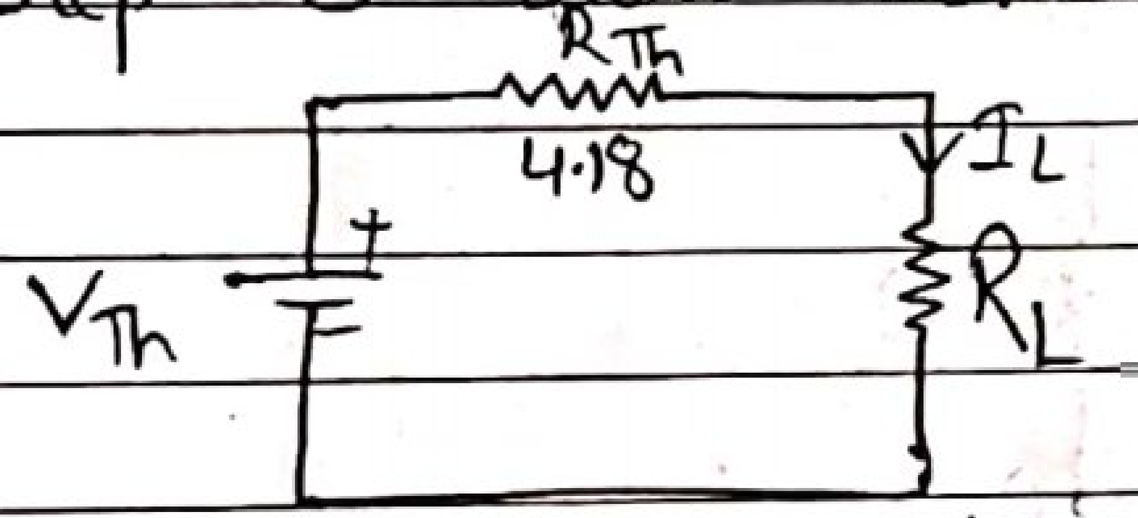
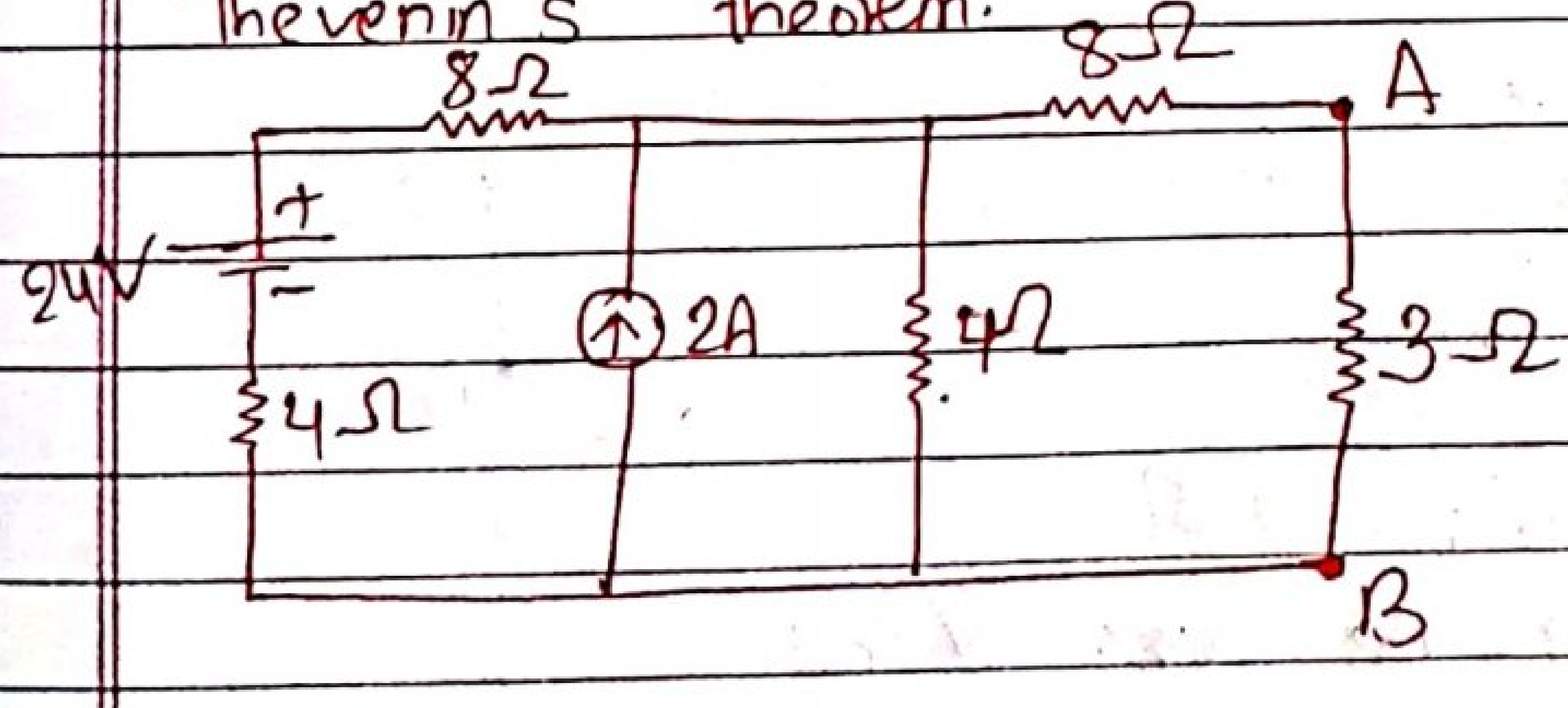
$\therefore V_{Th} = V_b - V_a$
$= 20 - 8.72$
$= 11.28\ \Omega$

Step - 3: Draw thevenin's equivalent ckt,



$$\therefore I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{11.28}{4.18 + 4} = 1.37\,A$$

Q.4) Calculate the current through $3\,\Omega$ resistor using the Thevenin's theorem.



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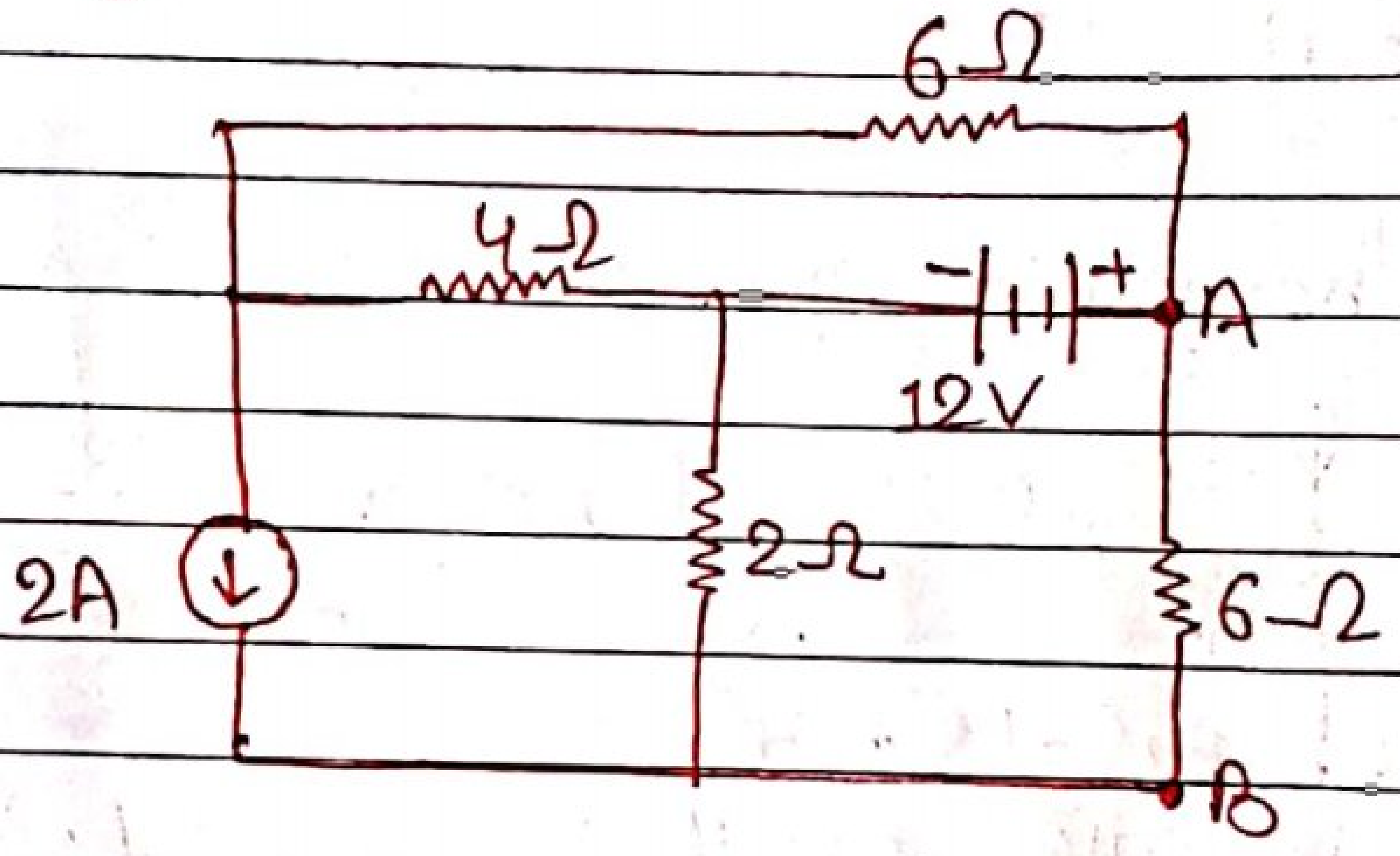
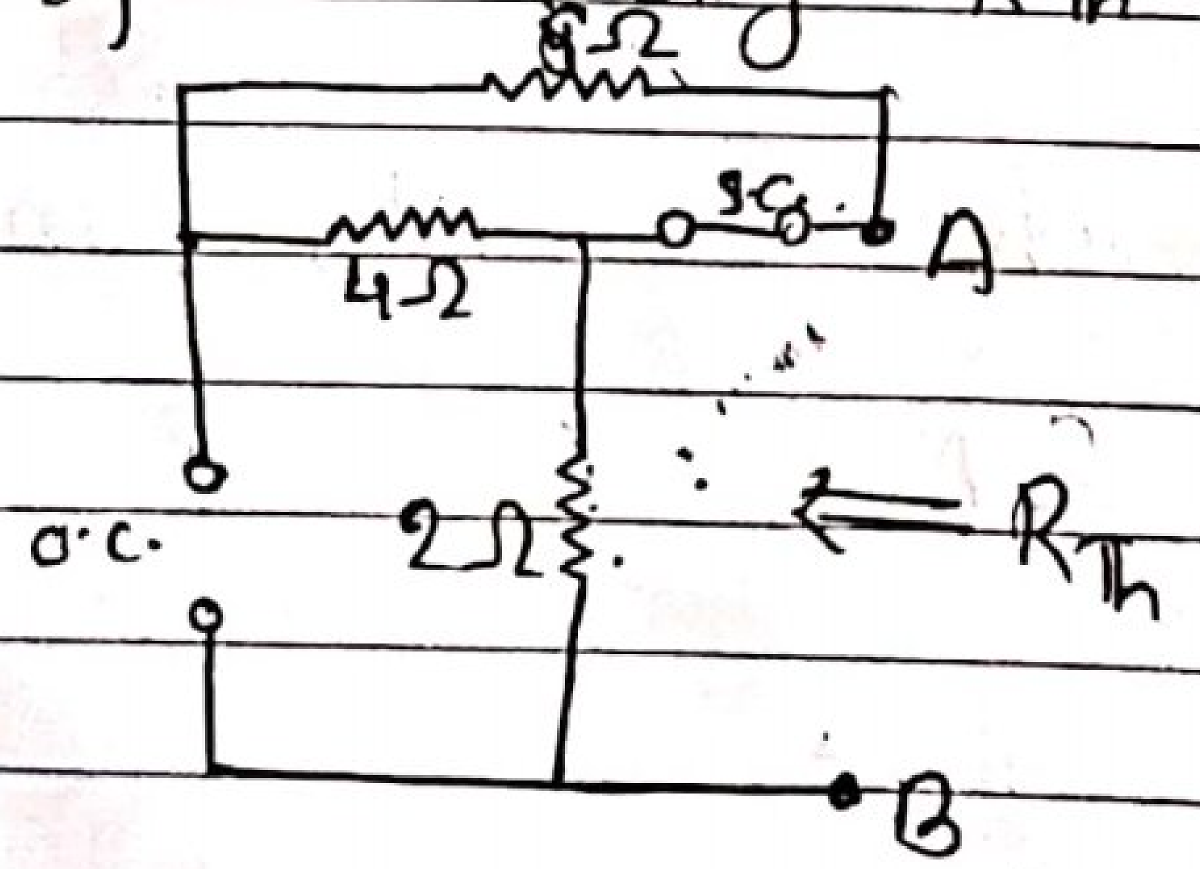
Q.5) Use Thevenin's theorem, find the current flowing through 6Ω resistor in the network:



Step-1: Finding R_{Th}:



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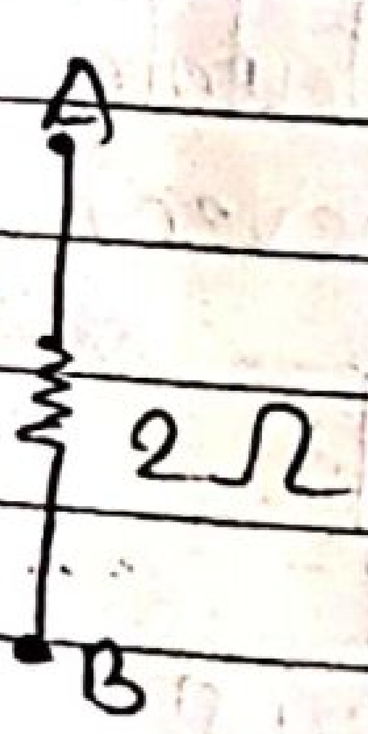
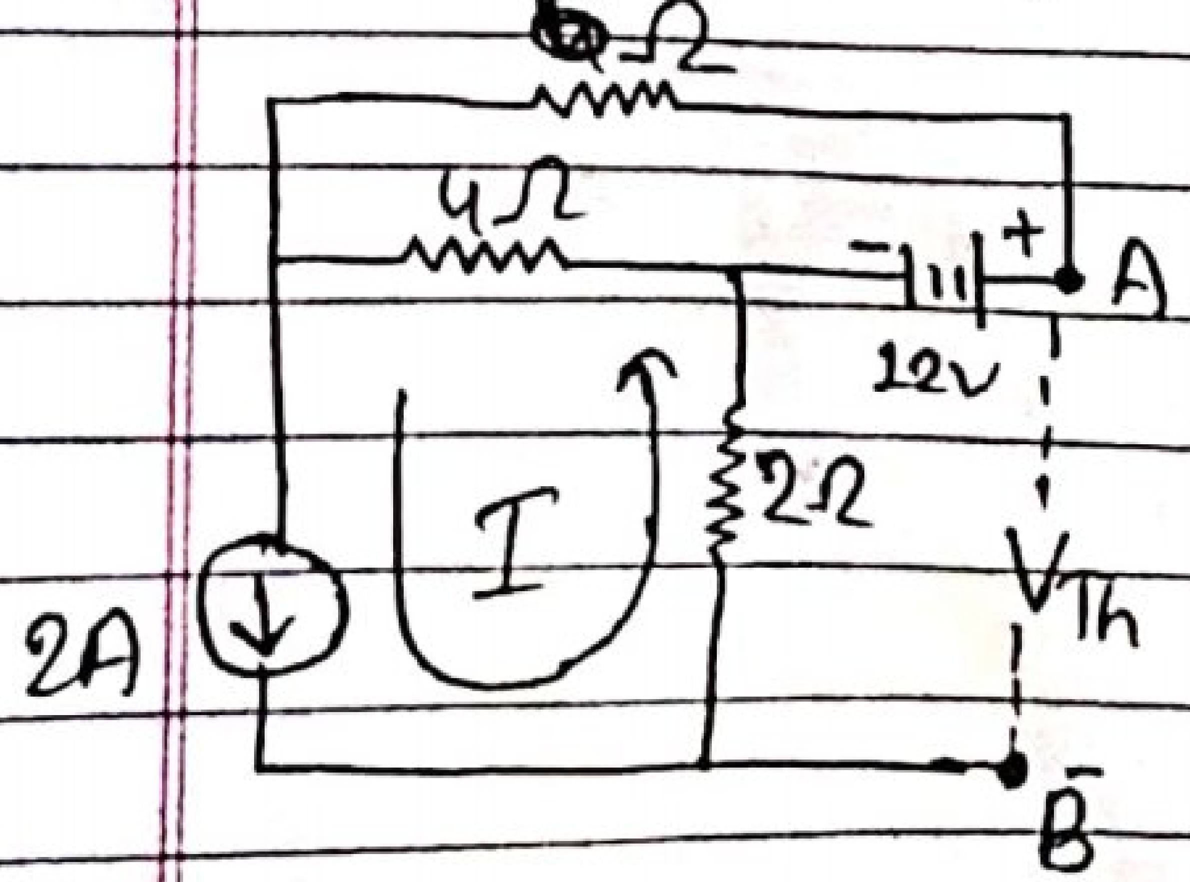


$\therefore R_{Th} = 2\Omega$

Step-2: Finding the ~~R_{Th}~~ V_{Th}:



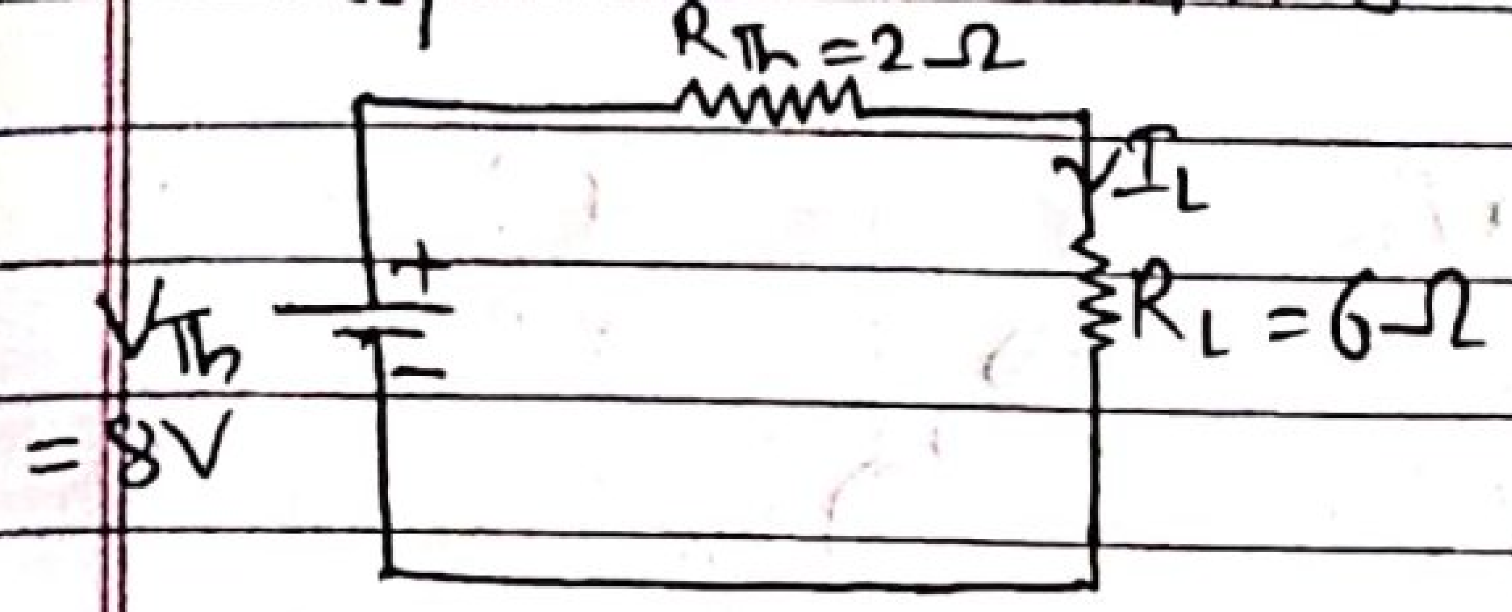
$V = IR$

$V = 2 \times 2 = 4V$

$V_{Th} - 12 + 4 = 0$

$\Rightarrow V_{Th} = 8V$

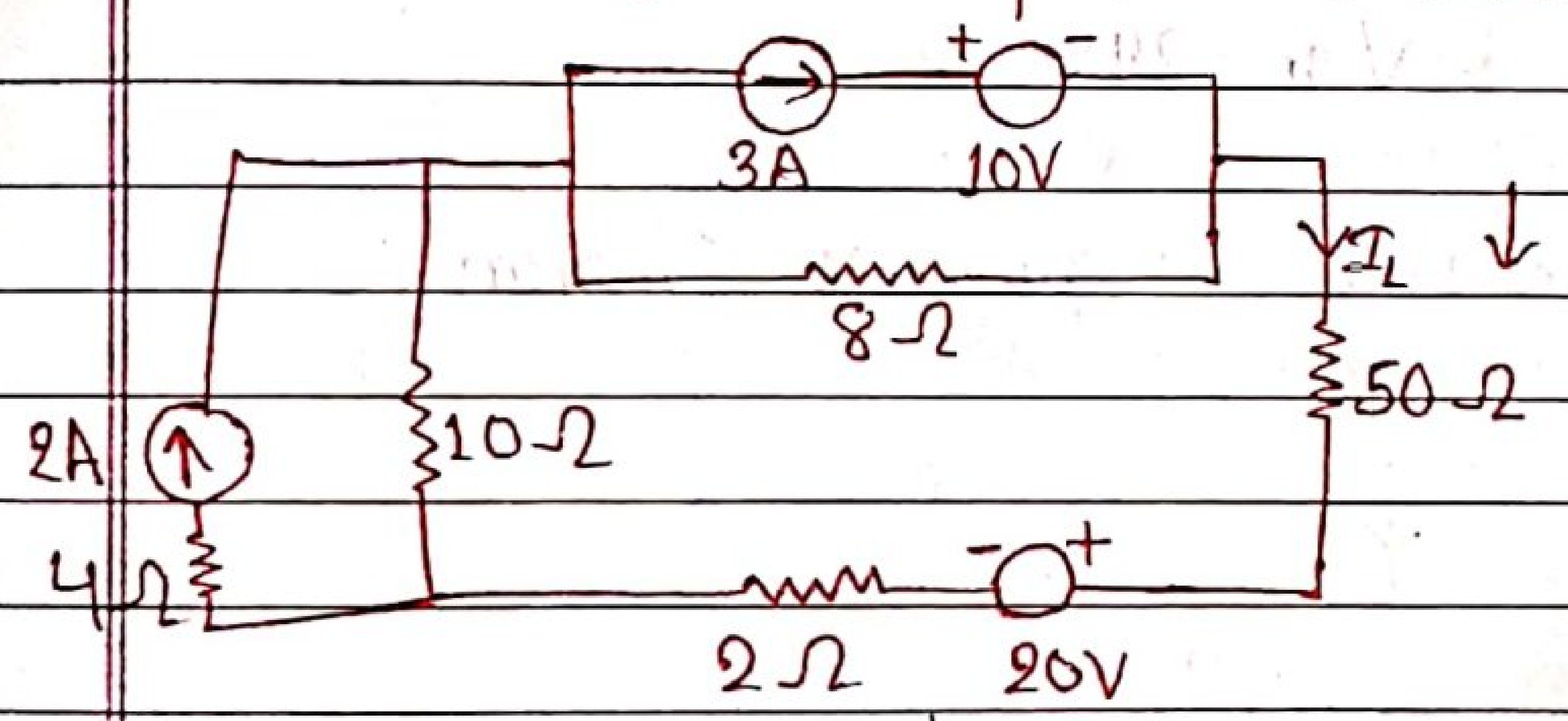
Step - 3: Thevenin's equivalent ckt:



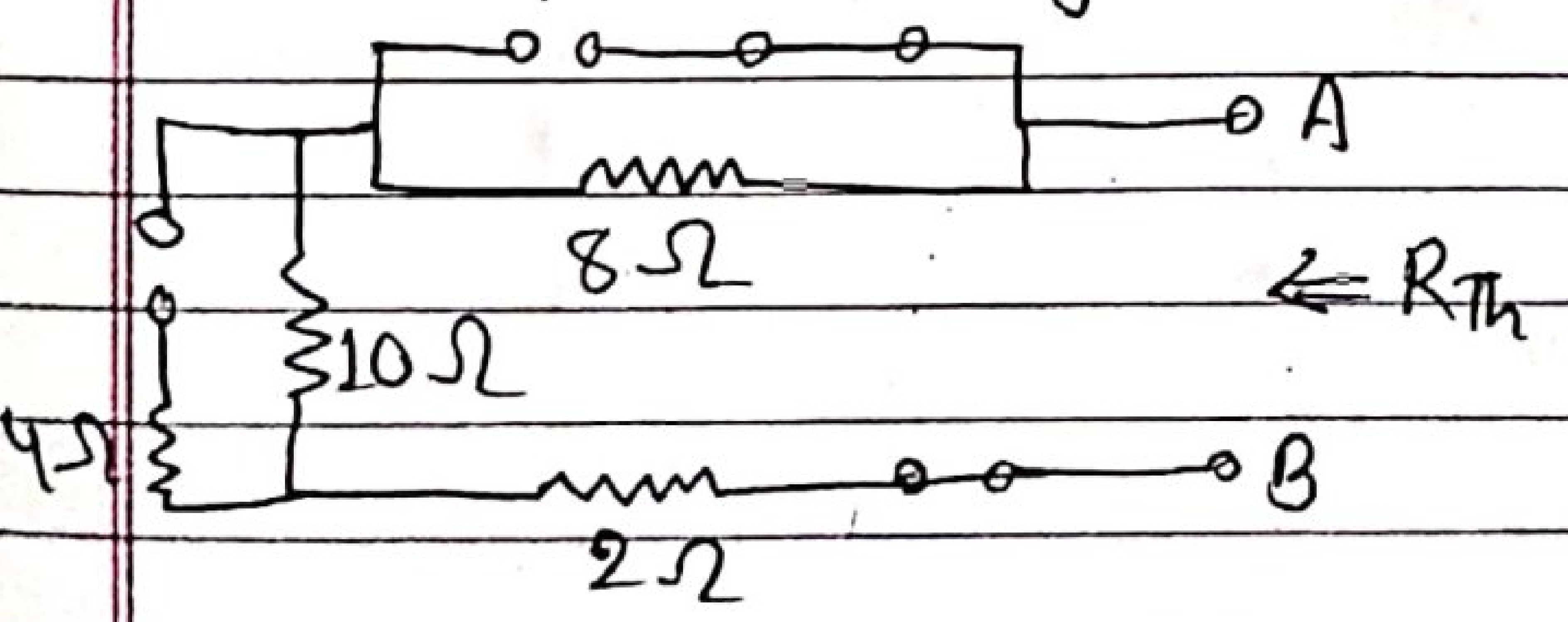
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{8}{2+6}$$

$$\therefore I_L = 1 \text{ amp}$$

Q.6) For the ckt shown in fig 'A' compute the load current (I_L) using the Thevenin's theorem and determine the value of load for which power transfer is maximum.

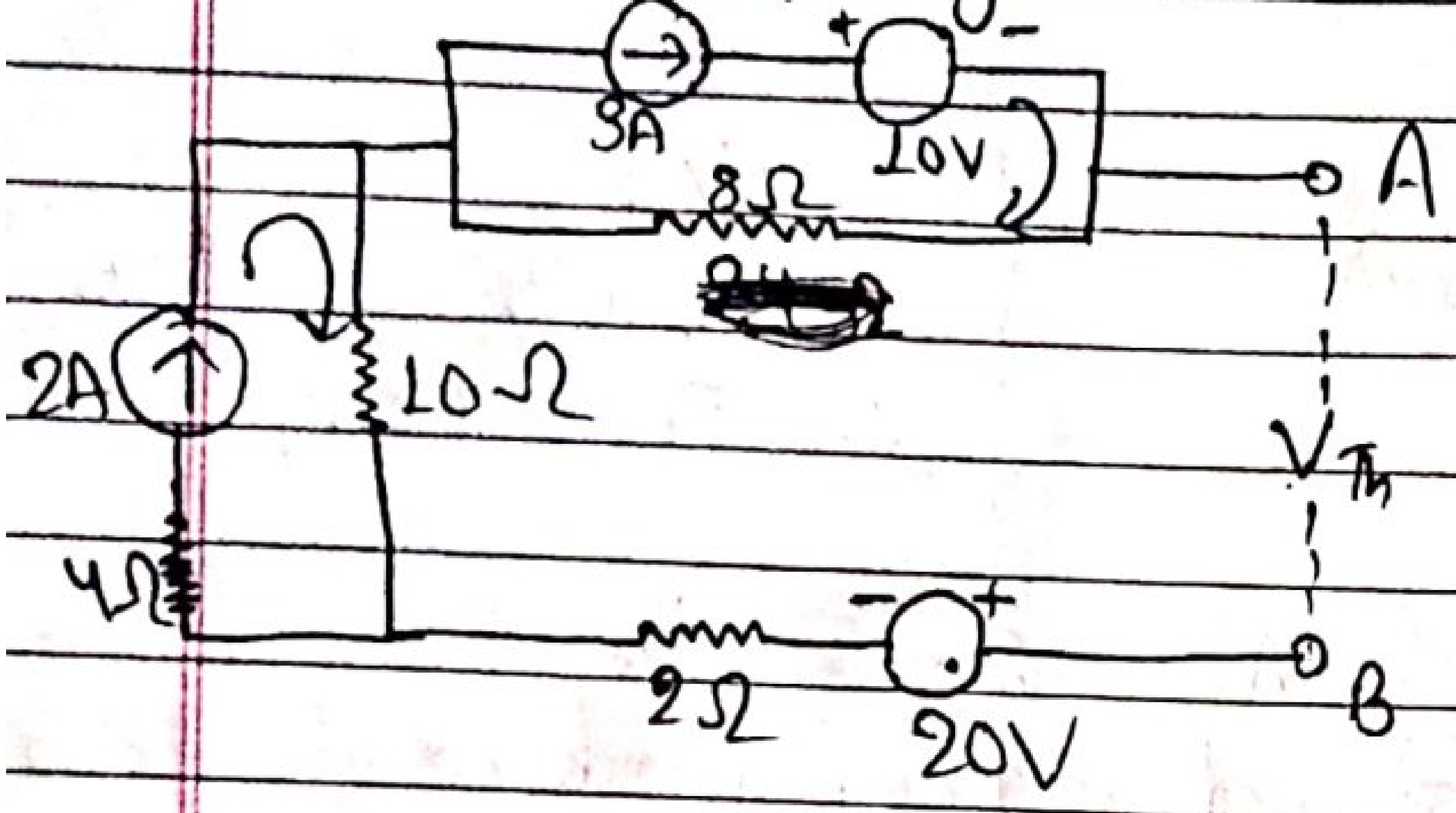


Here, Step - I :- finding R_{Th}:



$$R_{Th} = 8\Omega + 10\Omega + 2\Omega$$
$$= 20\Omega$$

Step - II : finding V_{Th}:



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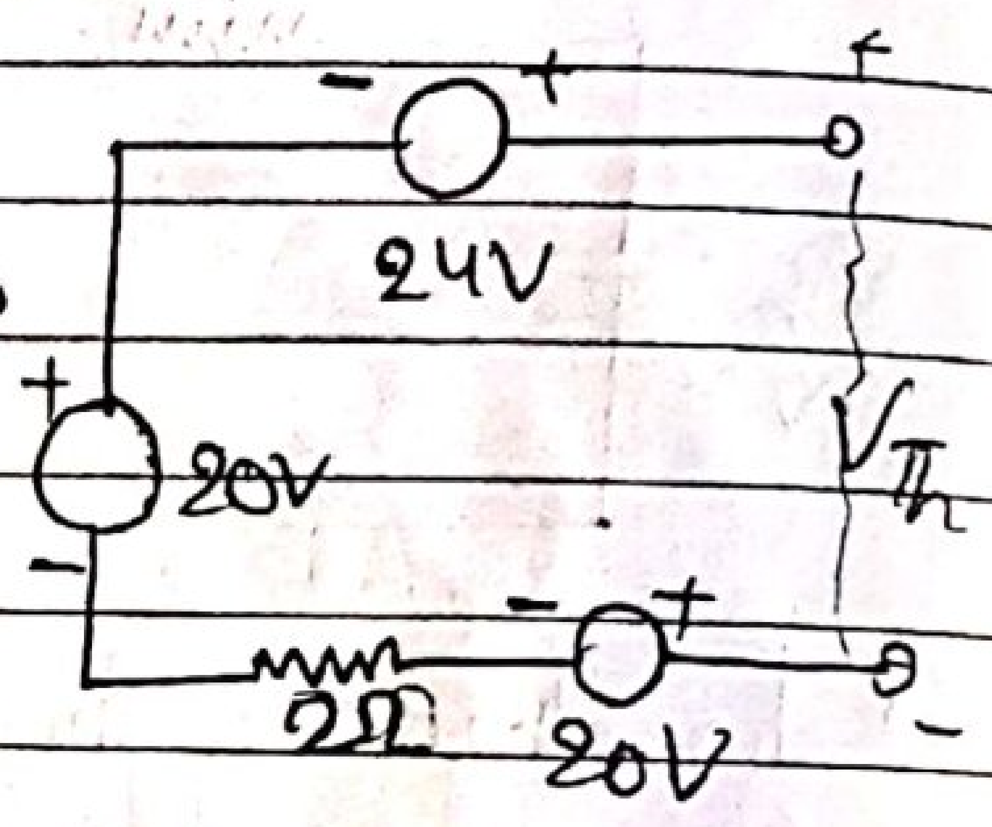
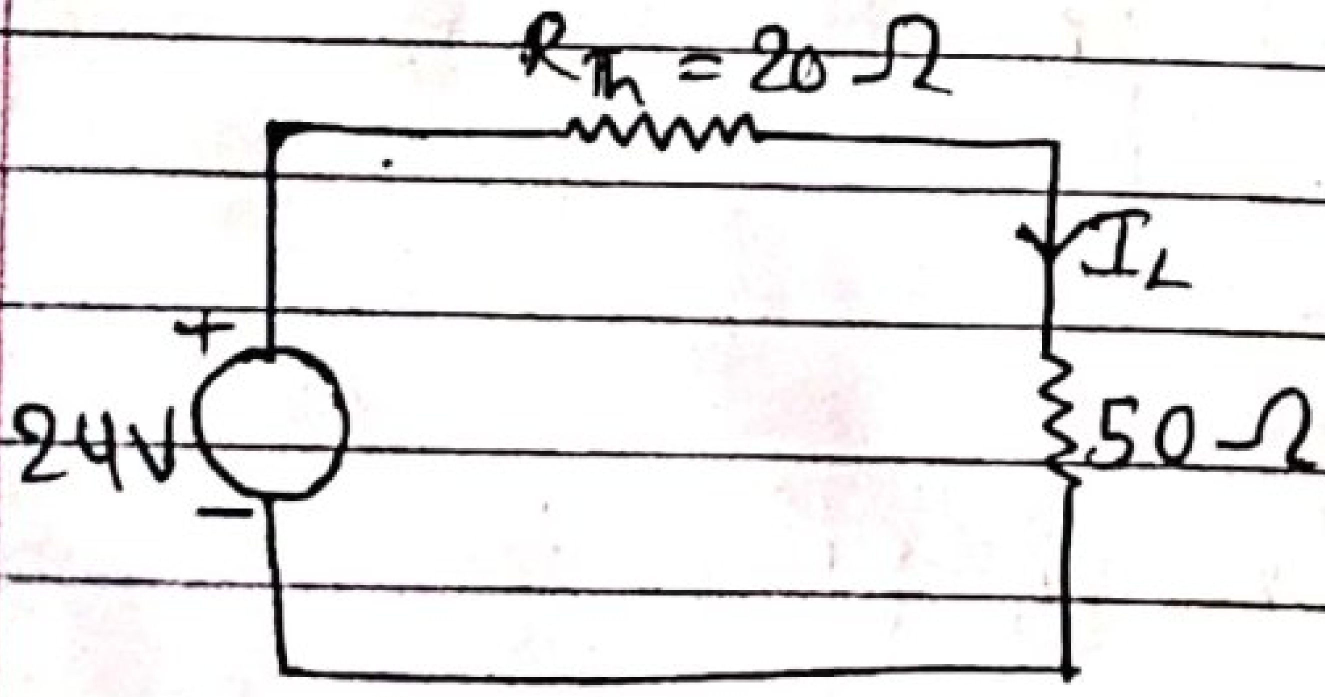


$$V_{Th} - 24 - \cancel{20} + \cancel{20} = 0$$
$$\Rightarrow V_{Th} = 24V$$

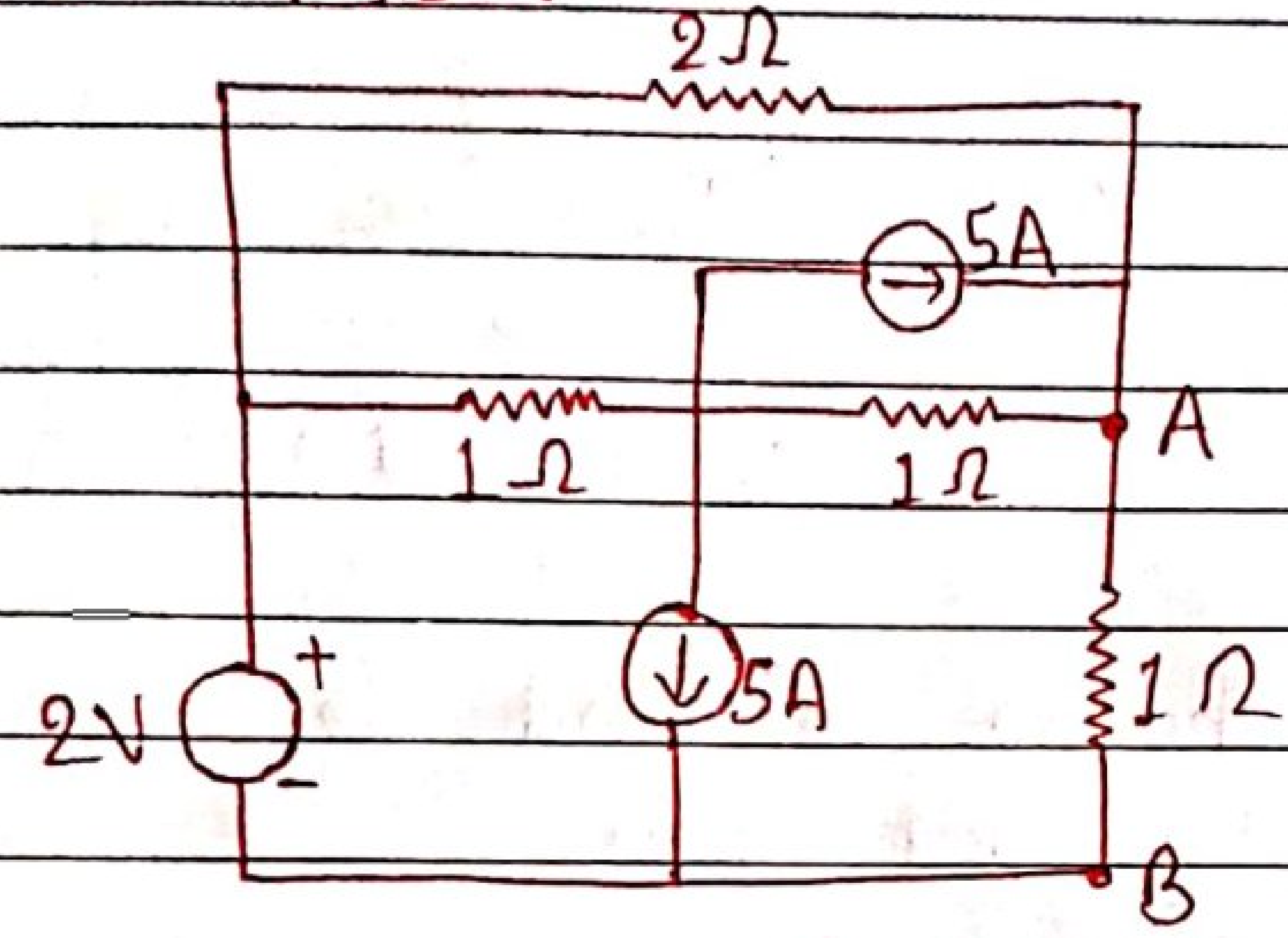
Step -3: Thevenin's equivalent Theorem



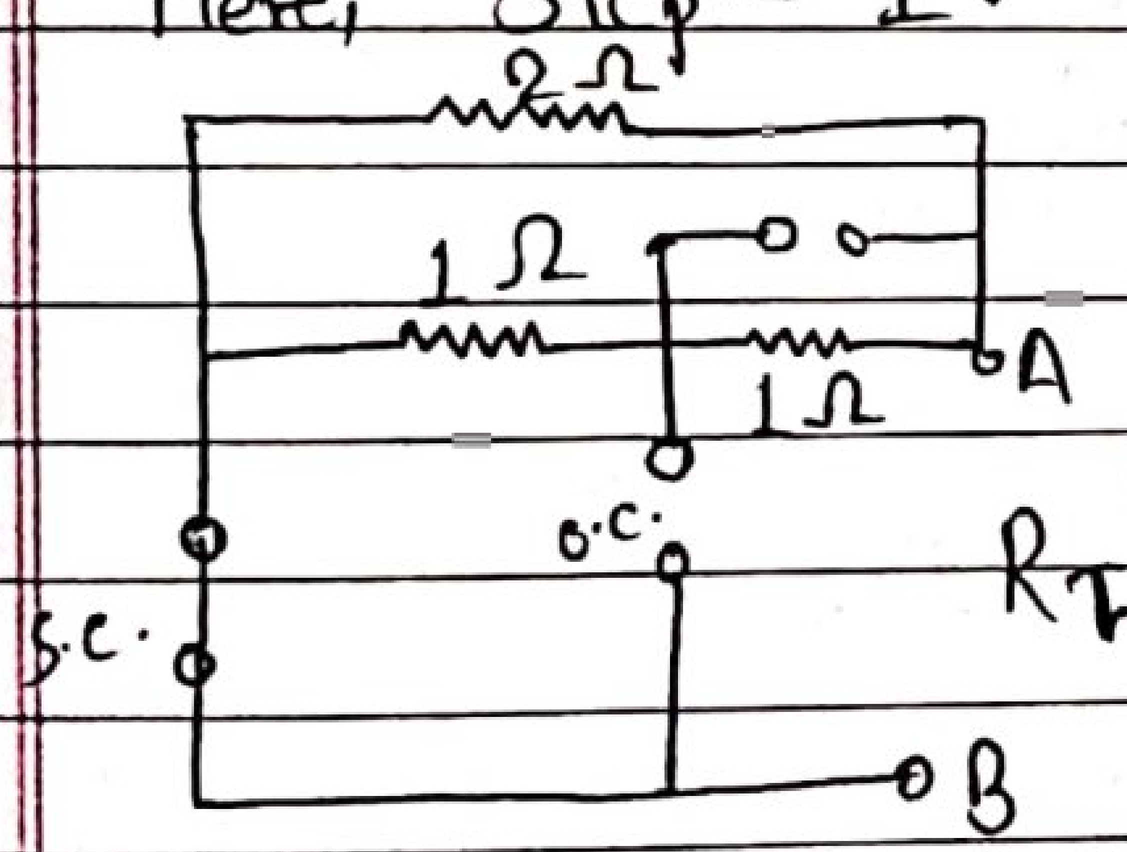
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$
$$= \frac{24}{20 + 50}$$
$$= 0.34A$$

$$P_{max} = (0.34)^2 \times 50$$
$$= 5.78 \text{ watt}$$

Q.7) By Thevenin's Theorem find the load current (I_L) through 1Ω resistor.



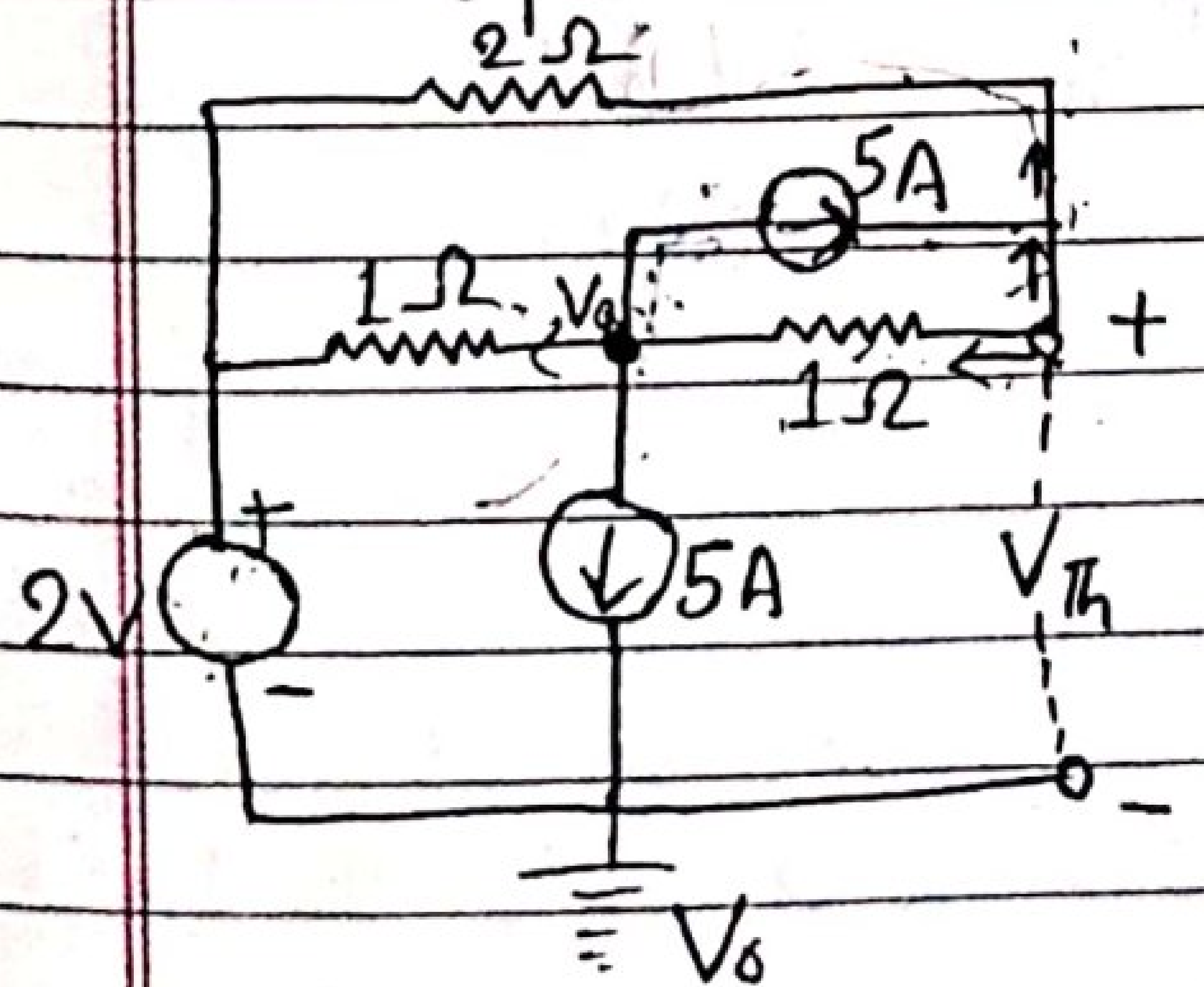
Here, Step – I:



$$R_{Th} = \frac{2\times 2}{2+2} = 1\Omega$$

R_{Th}

Step – II: finding V_{Th}:



Node – a:

$$\frac{V_a - V_{Th}}{1} + \frac{V_1 - 2}{2} + 5 + 5 = 0$$

$$\Rightarrow 2V_a - V_{Th} + 8 = 0$$

Consider node V_{Th}:

$$\frac{V_{Th} - V_a}{1} + \frac{V_{Th} - 2}{2} = 5$$

$$\Rightarrow 3V_{Th} - 2V_a = 12$$

$V_{Th} = 2V$

$V_a = -3V$

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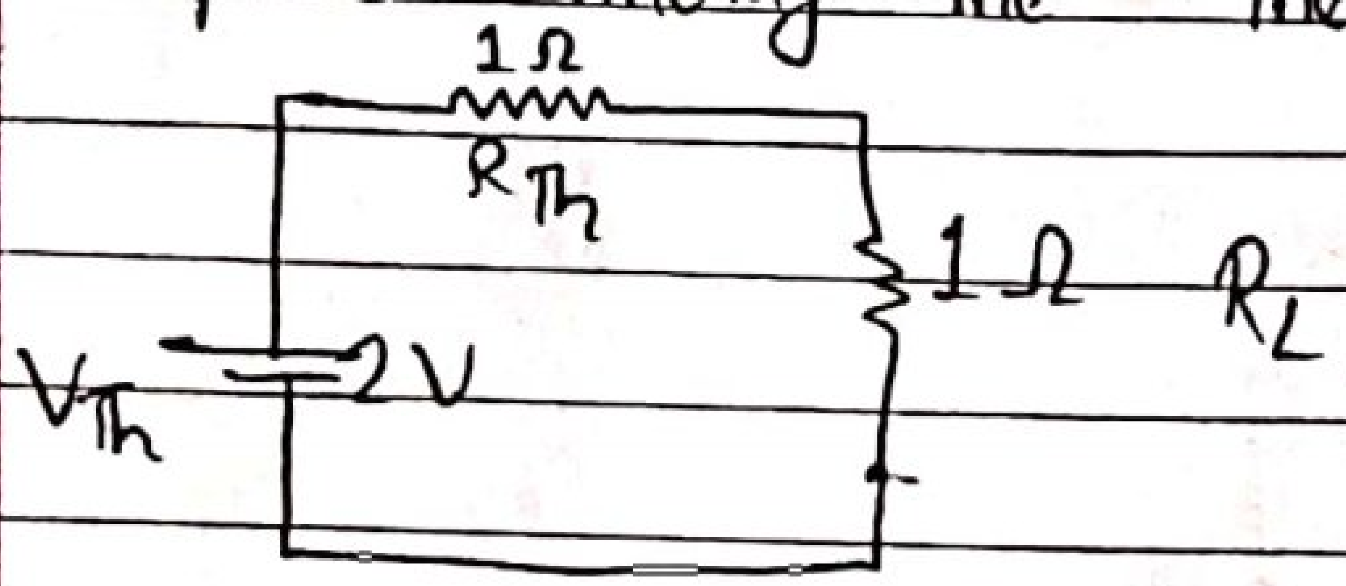
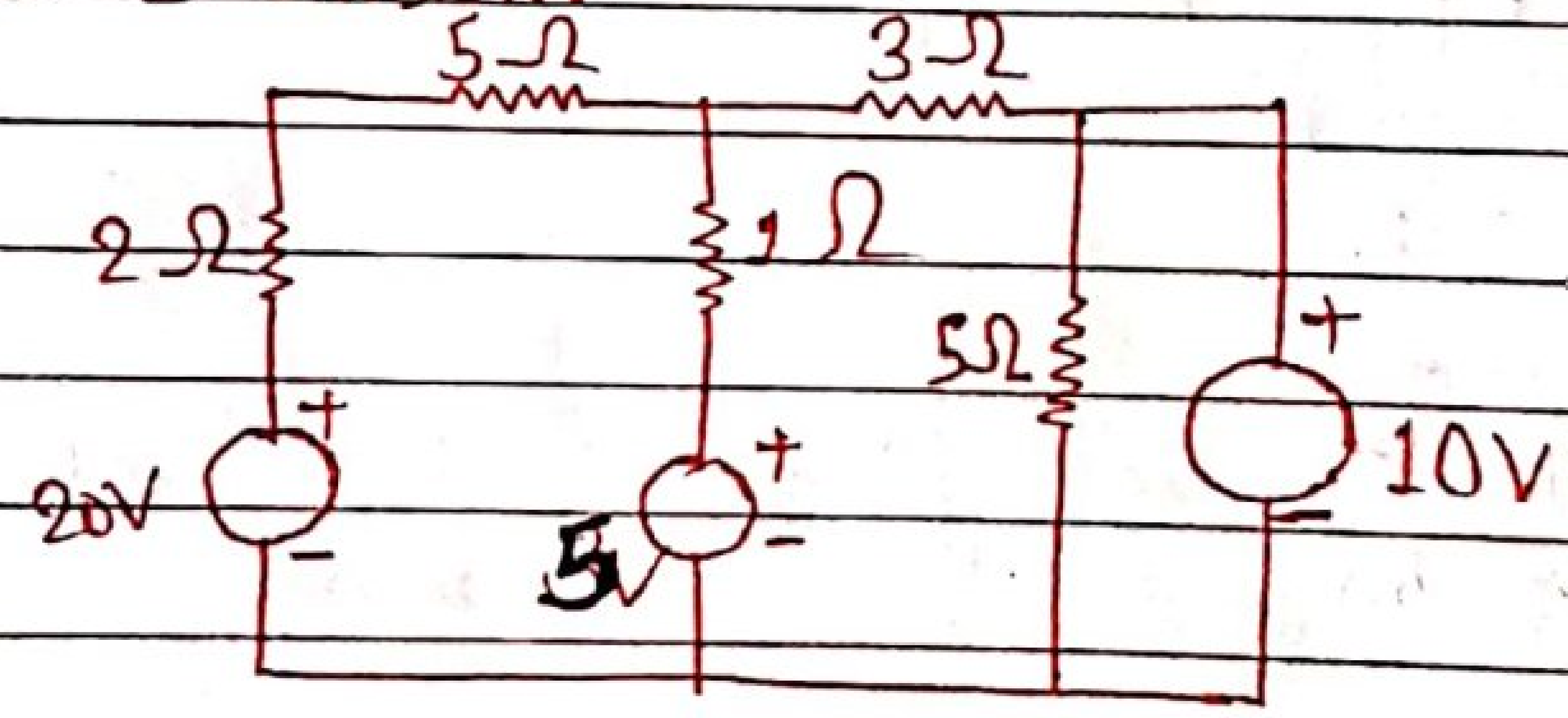
Step-3: Finding the Thevenin's eq. ckt:



$$I_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{2}{1+1}$$

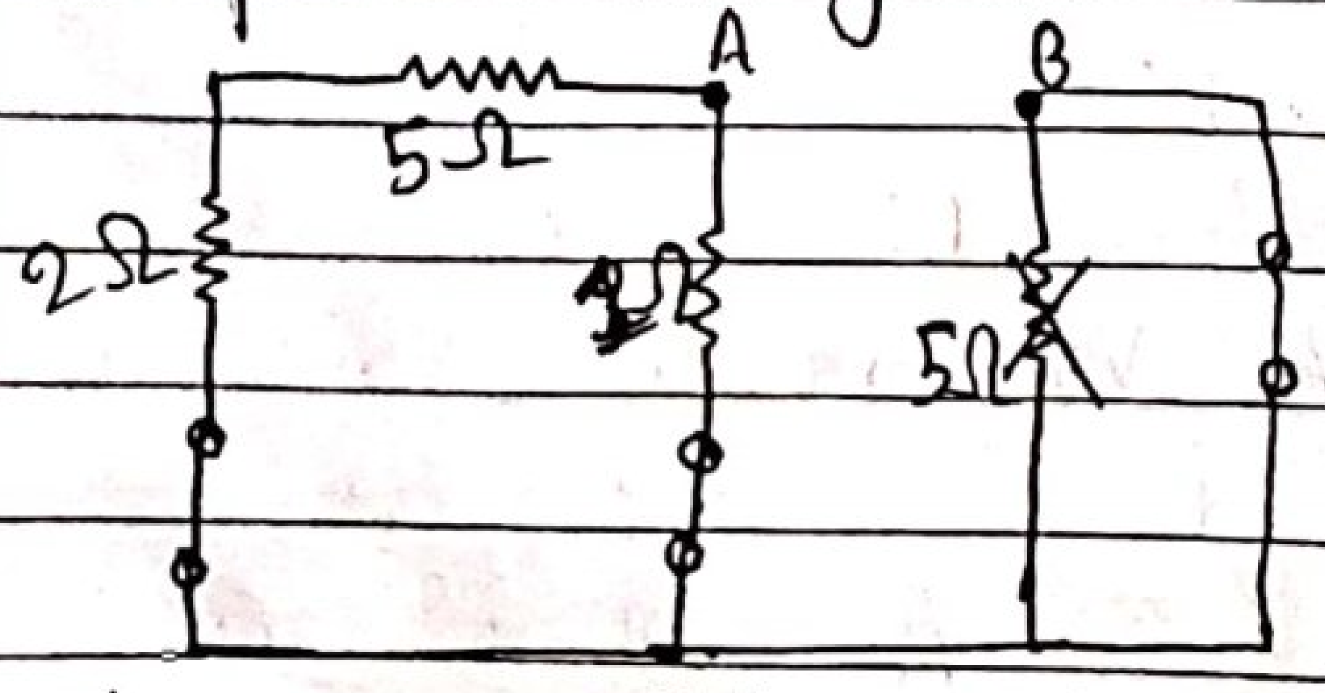
$\therefore$ $\boxed{I_L = 1A}$

(Q.8) State the Thevenin's Theorem and find the load current across 3Ω resistor.



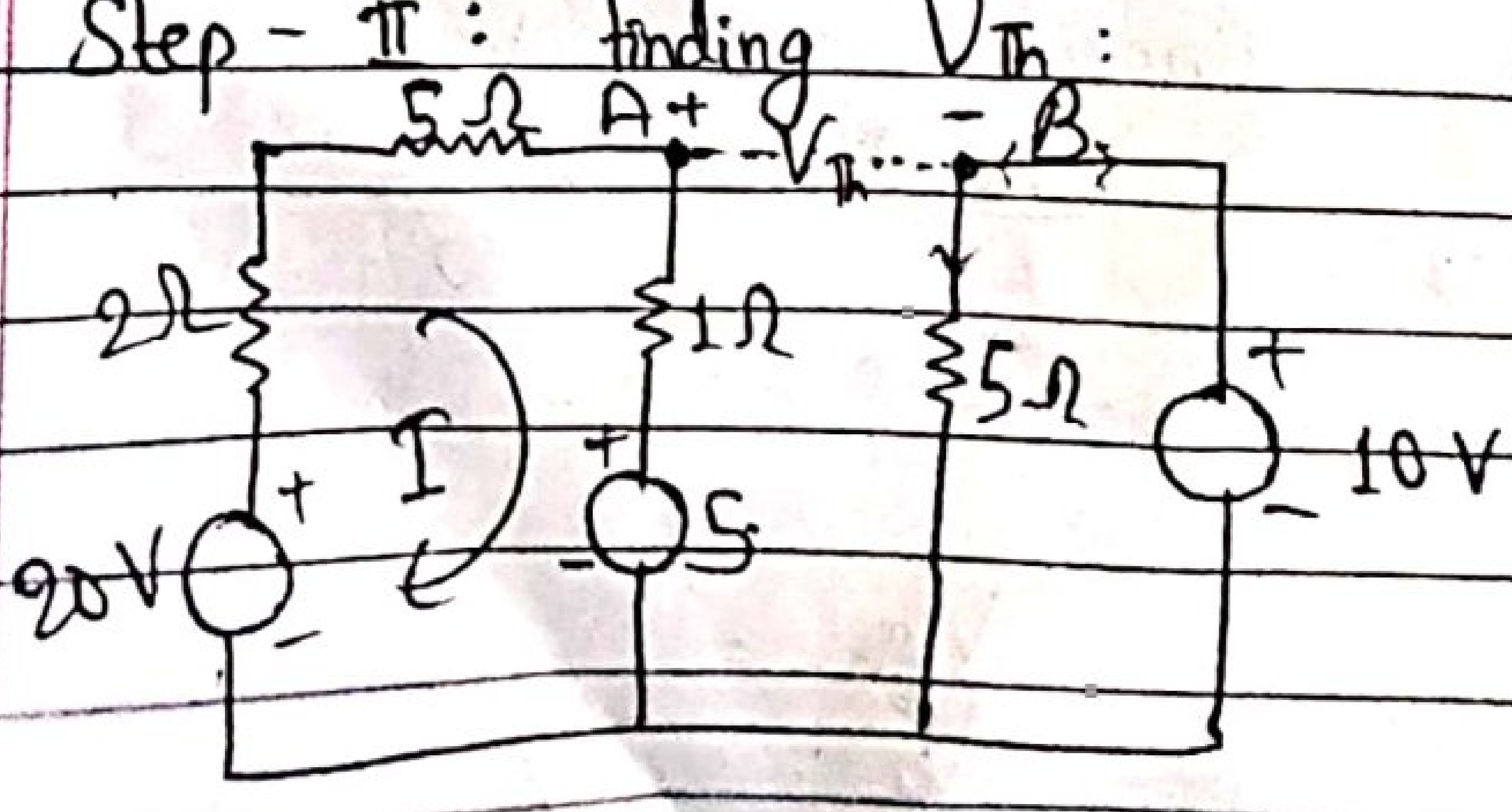
Here,

Step-I: Finding the R_{Th}:



$R_{Th} = 7 \parallel 1$
$= 0.875\ \Omega$

Step-II: Finding V_{Th}:



Consider LHS mesh:

$20 - 2I - 5I - I - 5 = 0$

$\Rightarrow 15 = 8I$

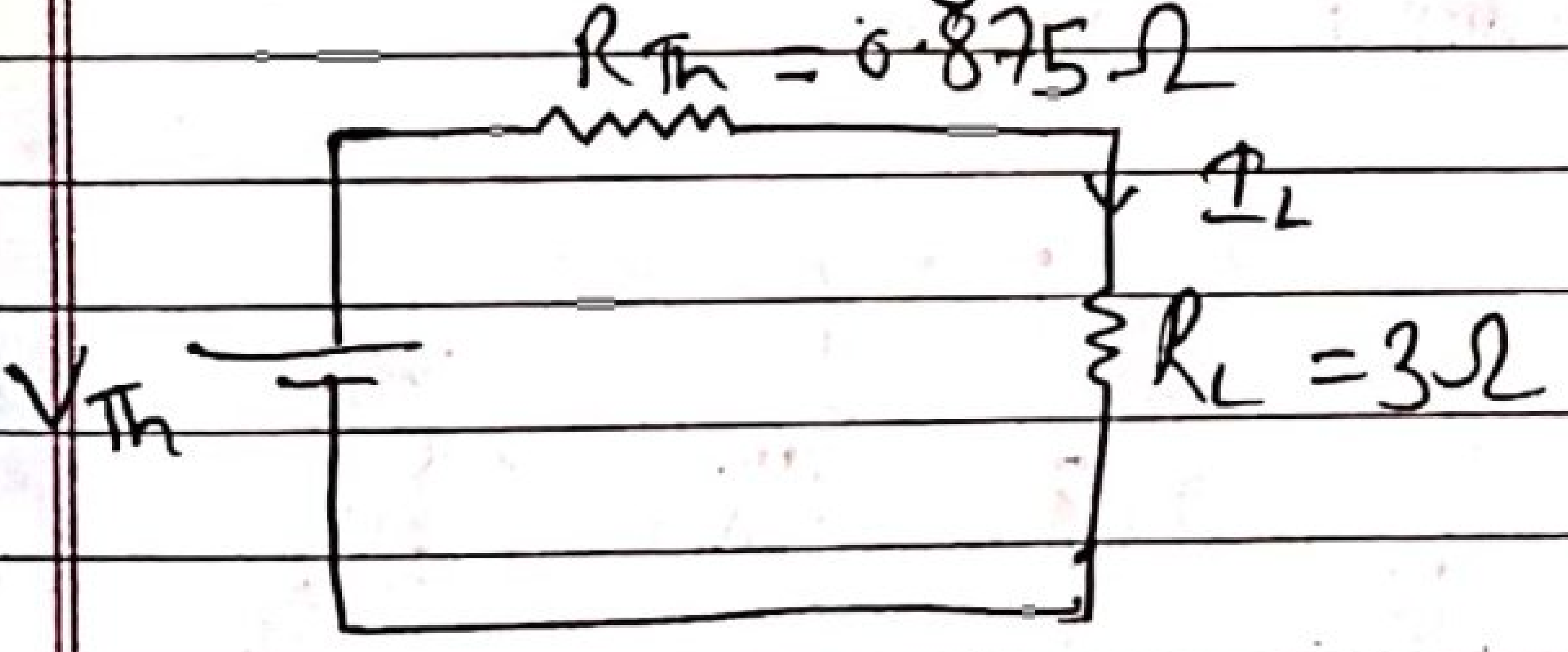
$\Rightarrow I = 1.825A$

$V_A = 20 - 7 \times 1.875$
$= 20 - 13.125$
$= 6.875V$

$V_B = 10V$

$\therefore V_{Th} = V_B - V_A$
$= 10 - 6.875$
$= 3.125V$

Step-3: Finding the Thevenin's eq. ckt:



$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{3.125}{0.875 + 3}$$

$\therefore I_L = 0.806A$

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→ Any two terminal linear circuit containing a large no. of voltage and/or current sources and resistors can be replaced by a simple equivalent ckt containing a single current source in parallel with a resistor.

Norton's Theorem

vvv Imp

It states that "Any circuit with the voltage source, resistance (impedance) and open output terminal can be replaced by single current source in parallel with single resistance, where the value of current source is equal to the current passing through the short ckt output terminals and the value of ~~current source is equal to~~ resistance (impedance) is equal to resistance seen into the output terminal.

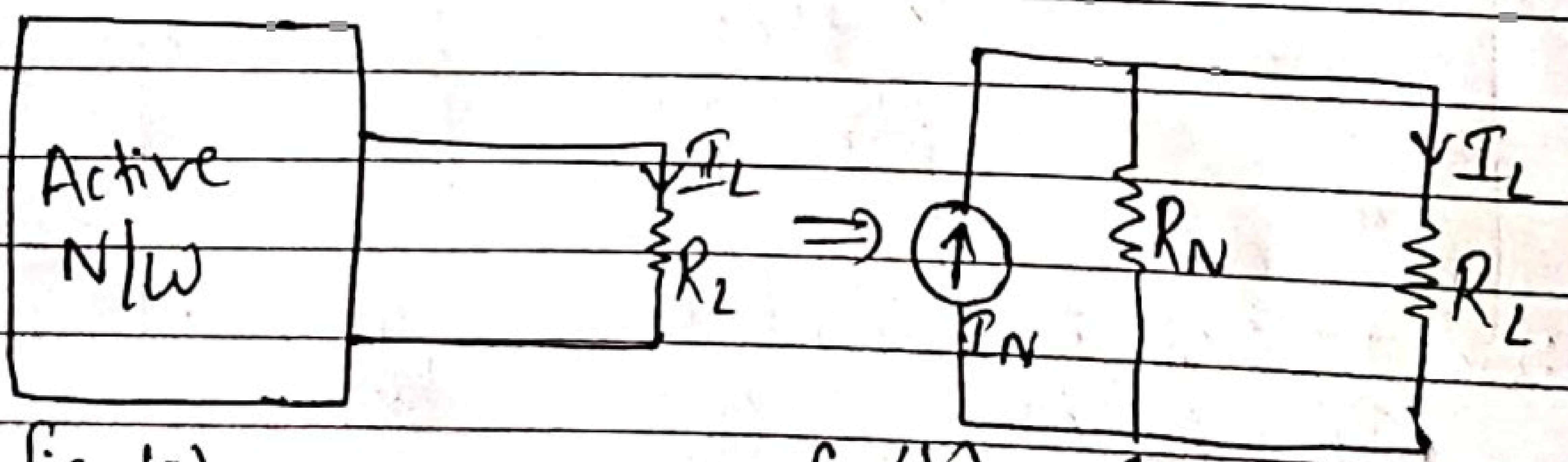

fig (a) fig (b) – Equivalent N/w or Norton's experiment

Procedure

i) Short the branch through which the current is to be measured.

ii) Obtain the current through the short ckt branch (N/w techinques).

$$\boxed{I_N = I_{s.c}}$$

iii) Calculate the R_N (eq. Norton resistance).

iv) Draw the Norton's eq. ckt.

v) Reconnect the branch resistance.

$$\boxed{I_L = I_N \times \frac{R_N}{R_N + R_L}}$$

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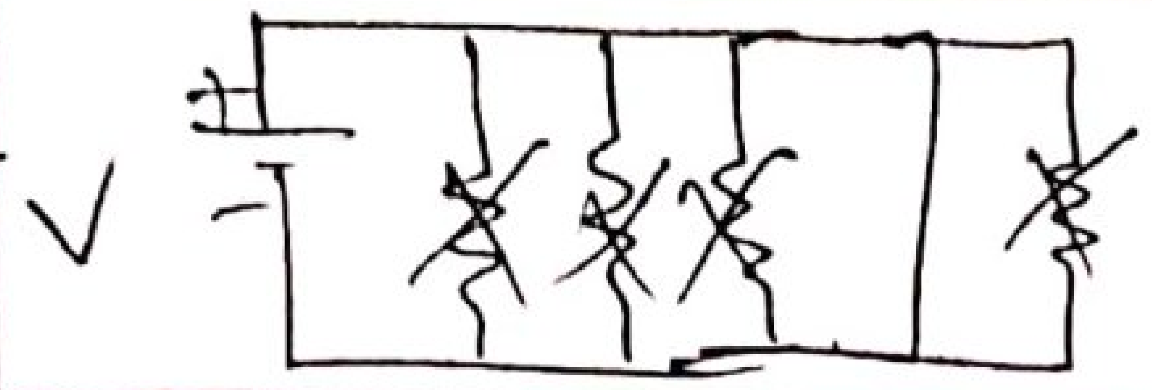
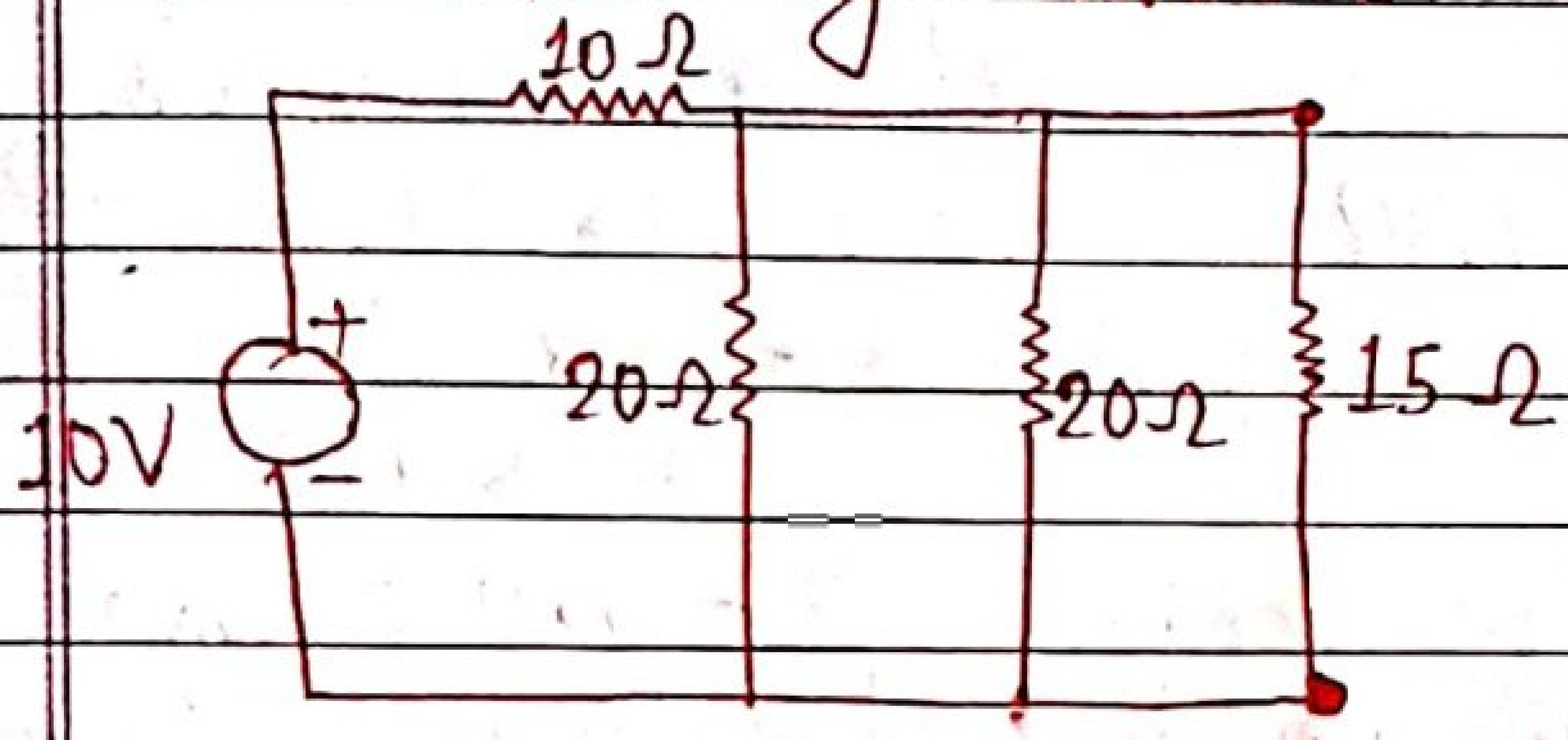
Note: ✓



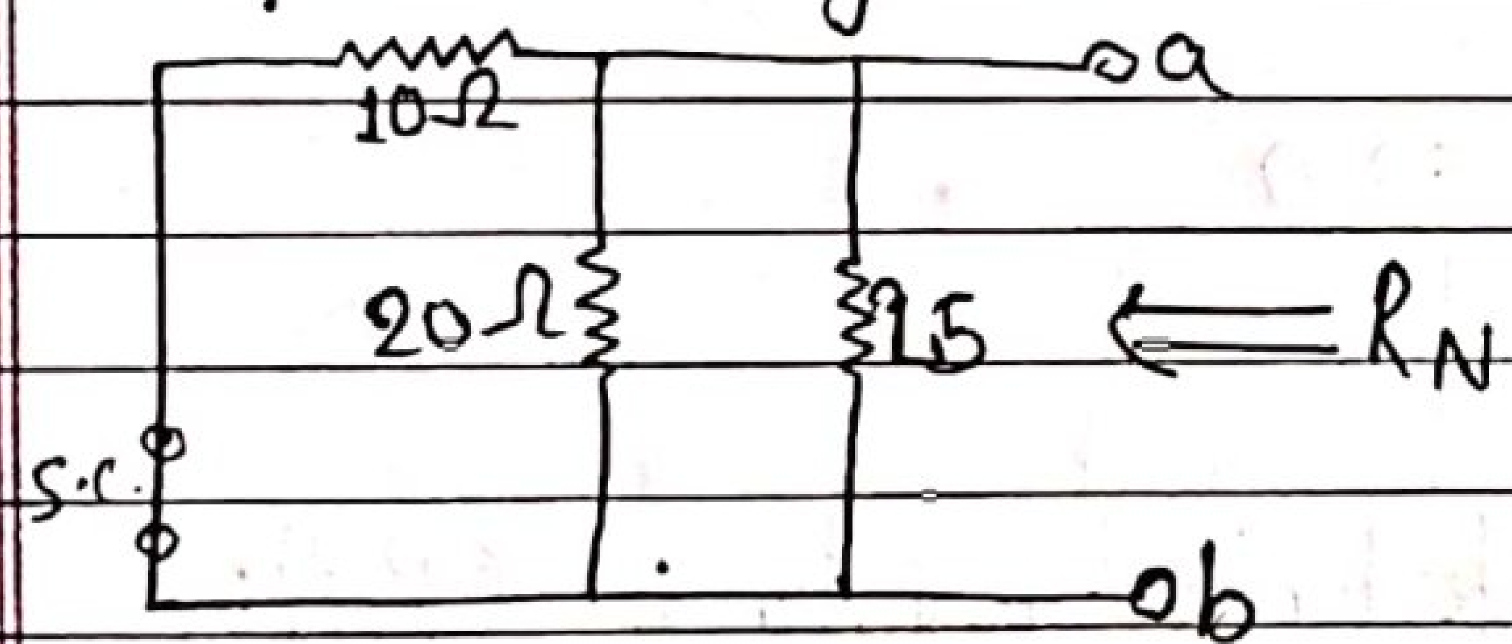
|| ckt shorts when connected with thick metallic wire.

Q1) Determine the current I_L through $15\,\Omega$ resistor in the N/w by Norton's theorem.



solution:

Step-1: finding Norton's resistance (R_N)

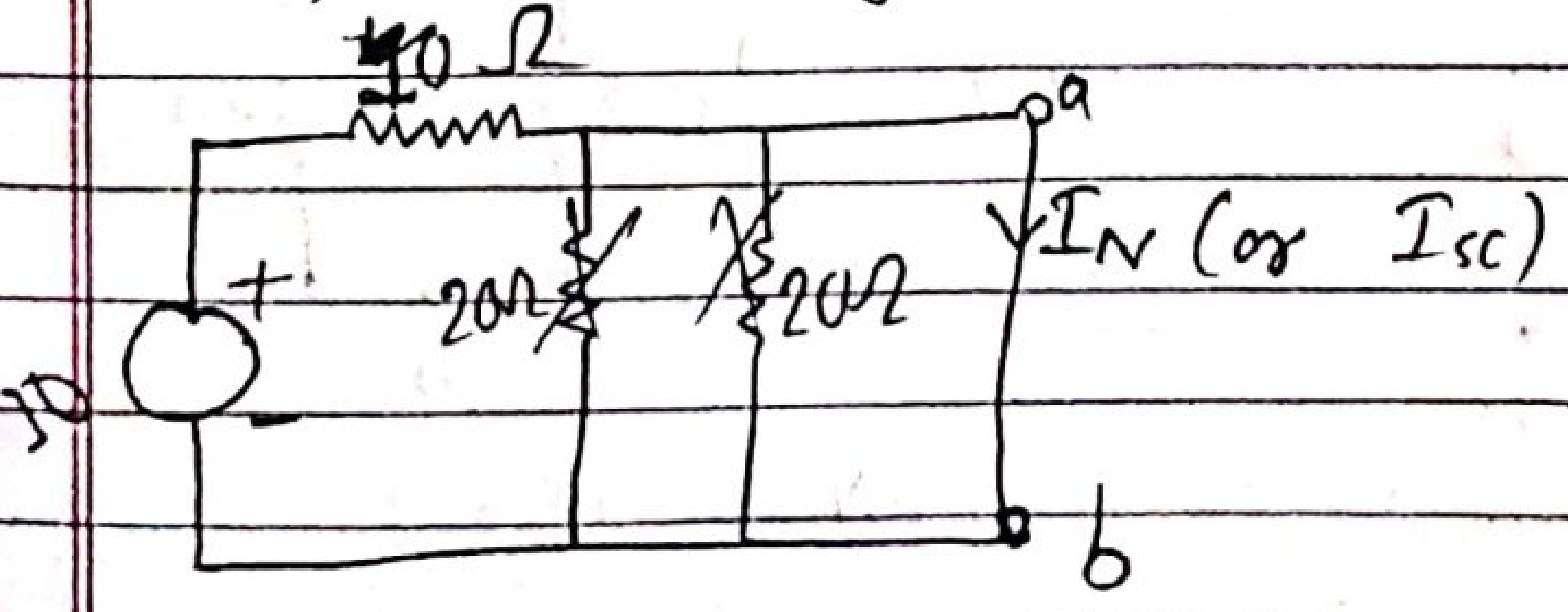


$$\frac{1}{R_N} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\Rightarrow \frac{1}{R_N} = \frac{1}{10} + \frac{1}{20} + \frac{1}{20}$$

$$\Rightarrow R_N = 5\,\Omega$$

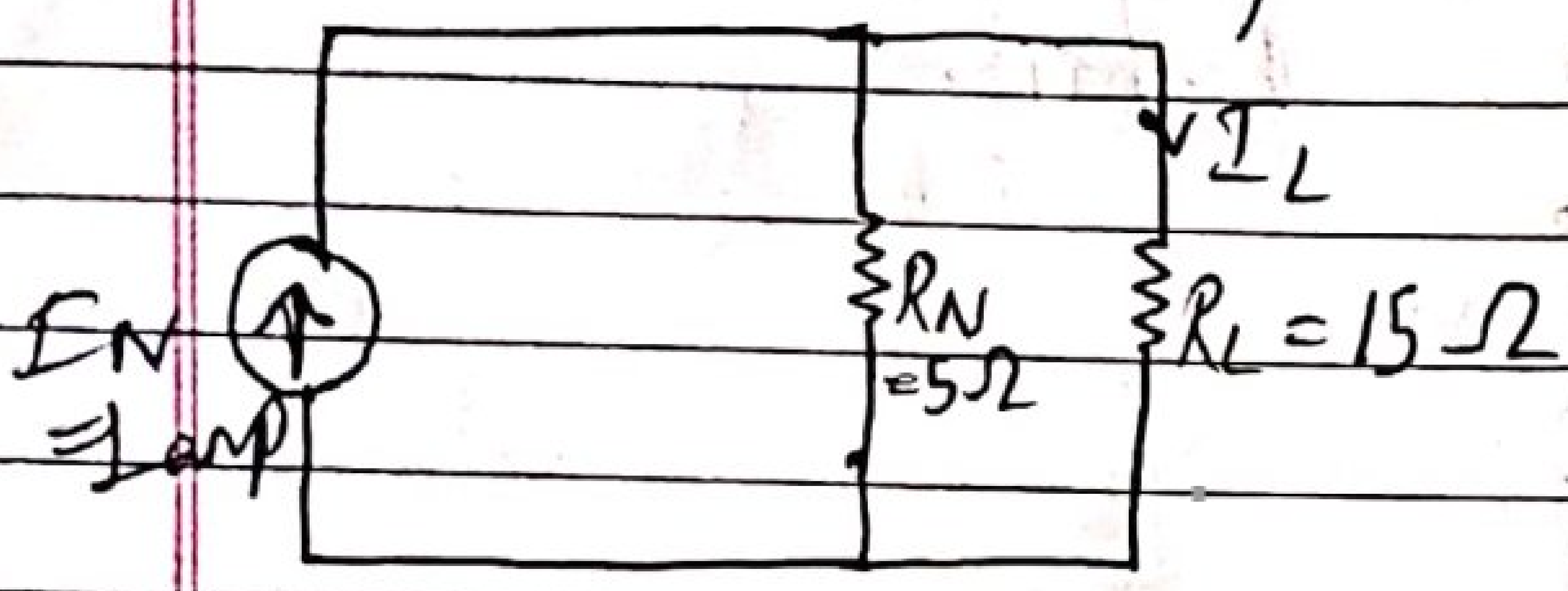
Step-2: Finding the Norton's current (I_N or I_{sc})



$$I_N = \frac{V}{R} = \frac{10}{10}$$

$$\Rightarrow I_N = 1\text{ amp}$$

Now, Norton's eq. ckt:



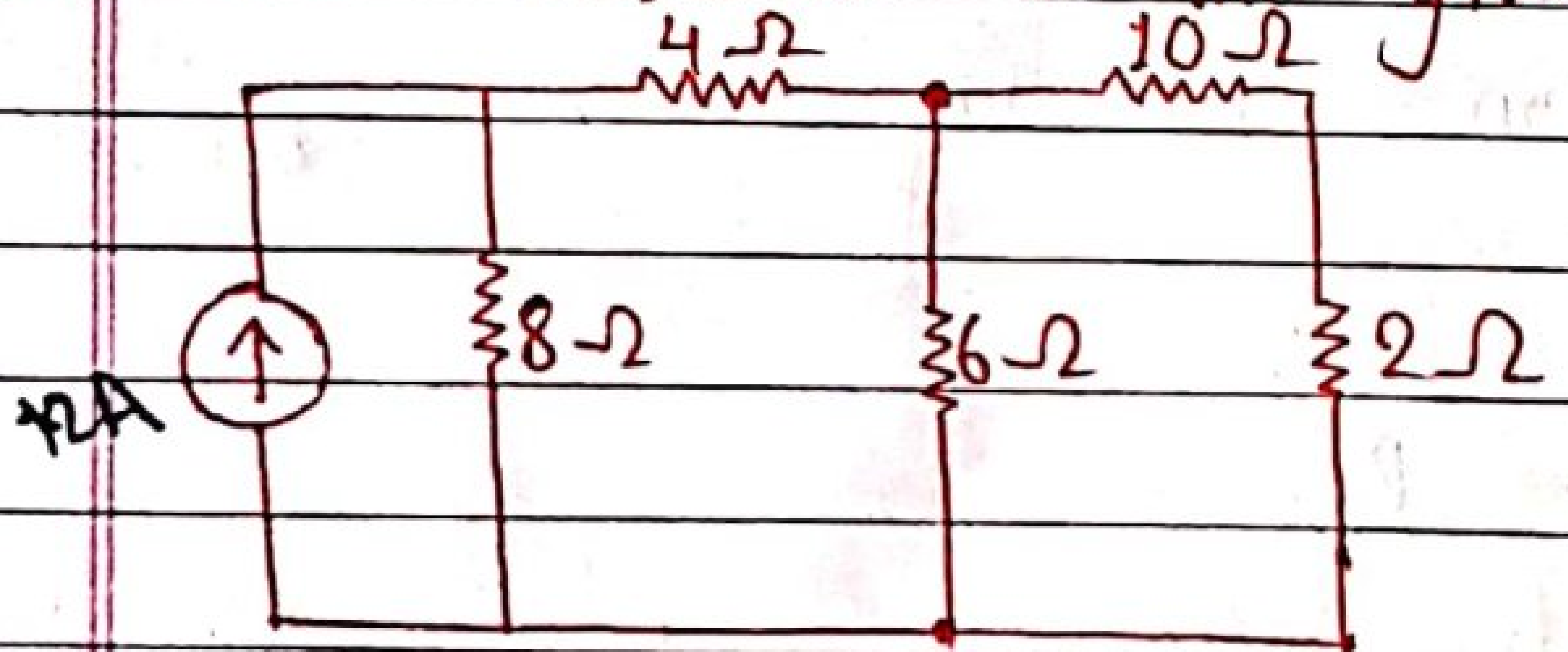
$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

$$= 1 \times \frac{5}{5+15}$$

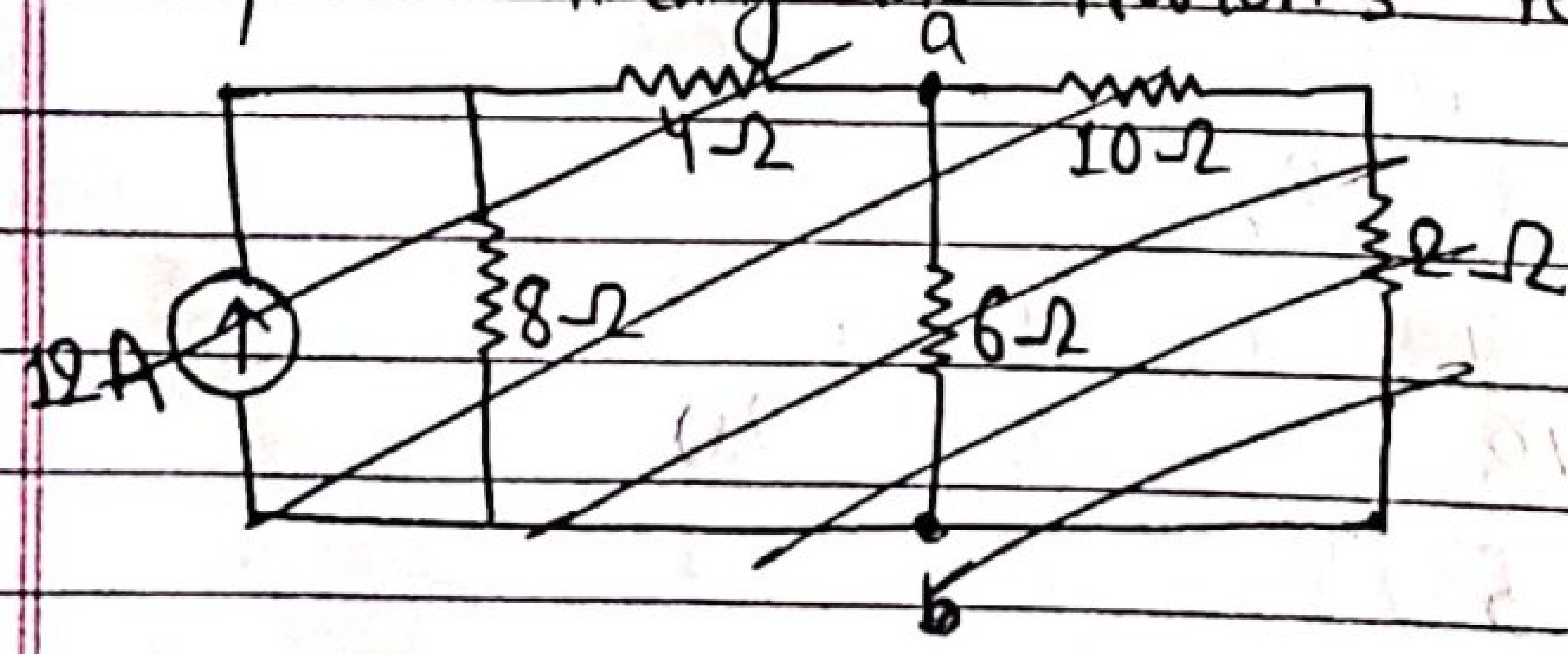
$$= 0.25 \text{ amp}$$

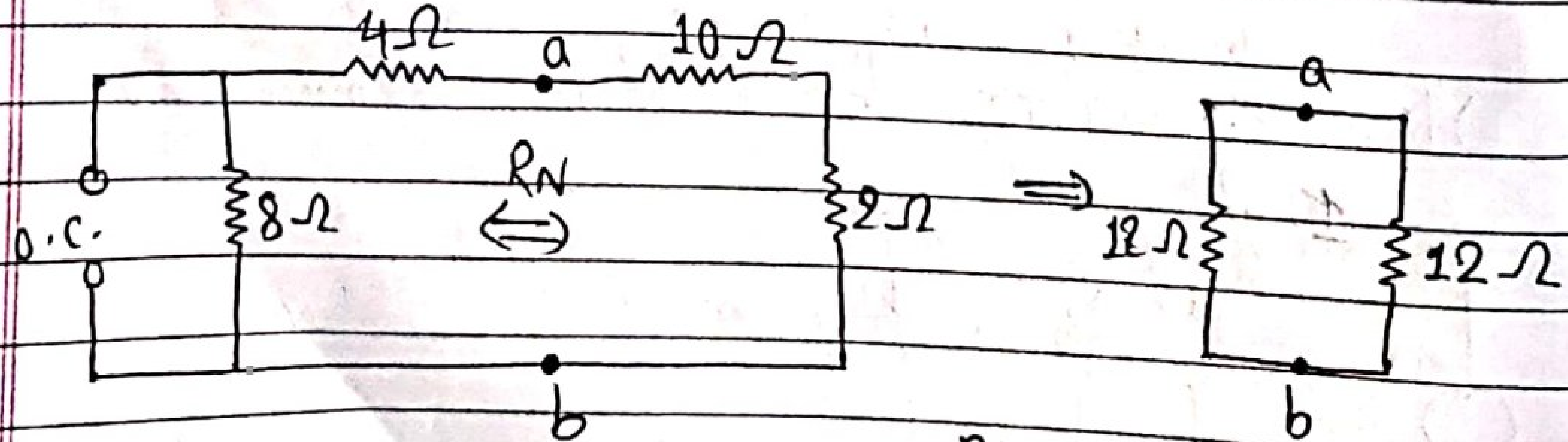
Q2) Use the Norton's theorem to calculate the current in 6 Ω resistor in the given ckt:

PU



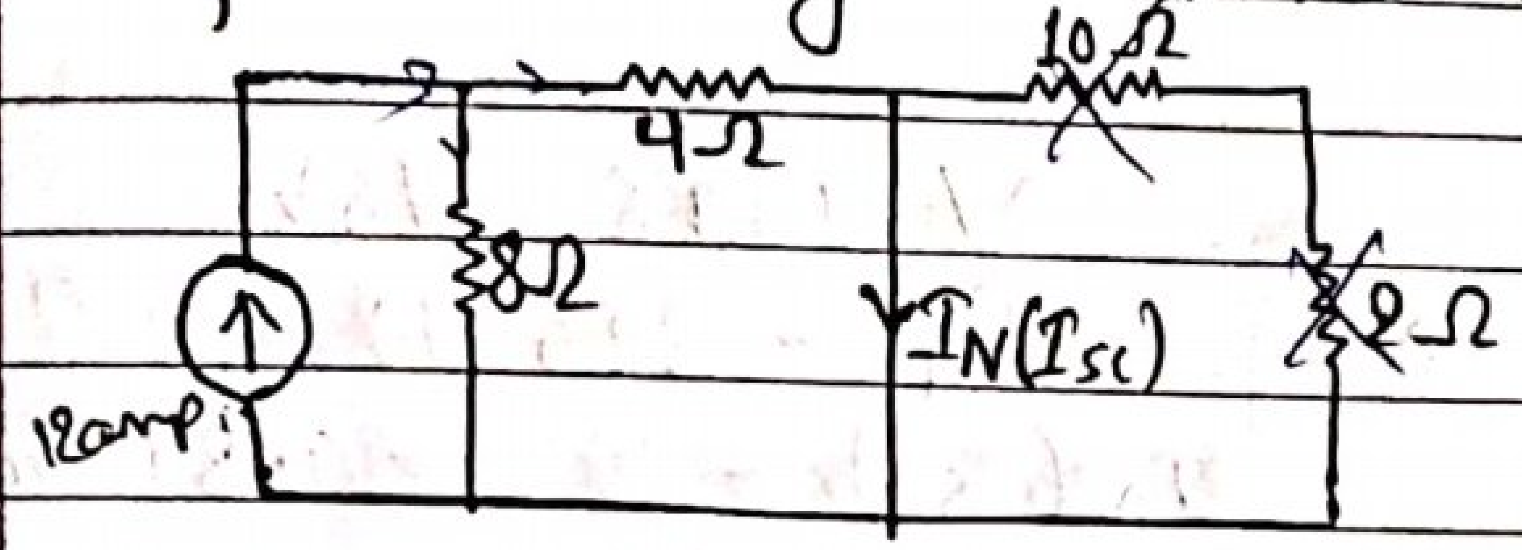
Step-1: Finding the Norton's resistance (R_N):





$$R_N = \frac{12 \times 12}{12 + 12} = 6\Omega$$

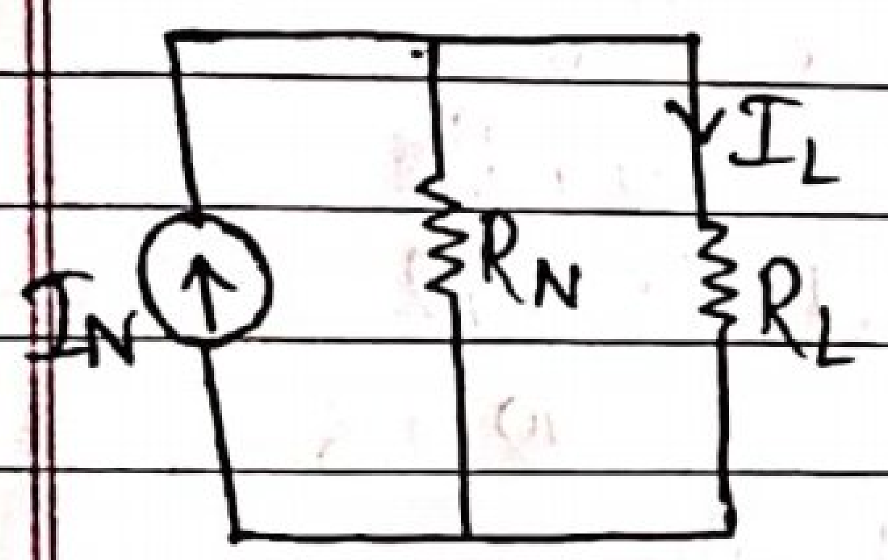
Step - 2: finding the Norton's current (I_N) or I_{sc}



$$I_N = 12 \times \frac{8}{8+4}$$

$$= 8 \text{ amp}$$

Step - 3: Norton's equivalent ckt:



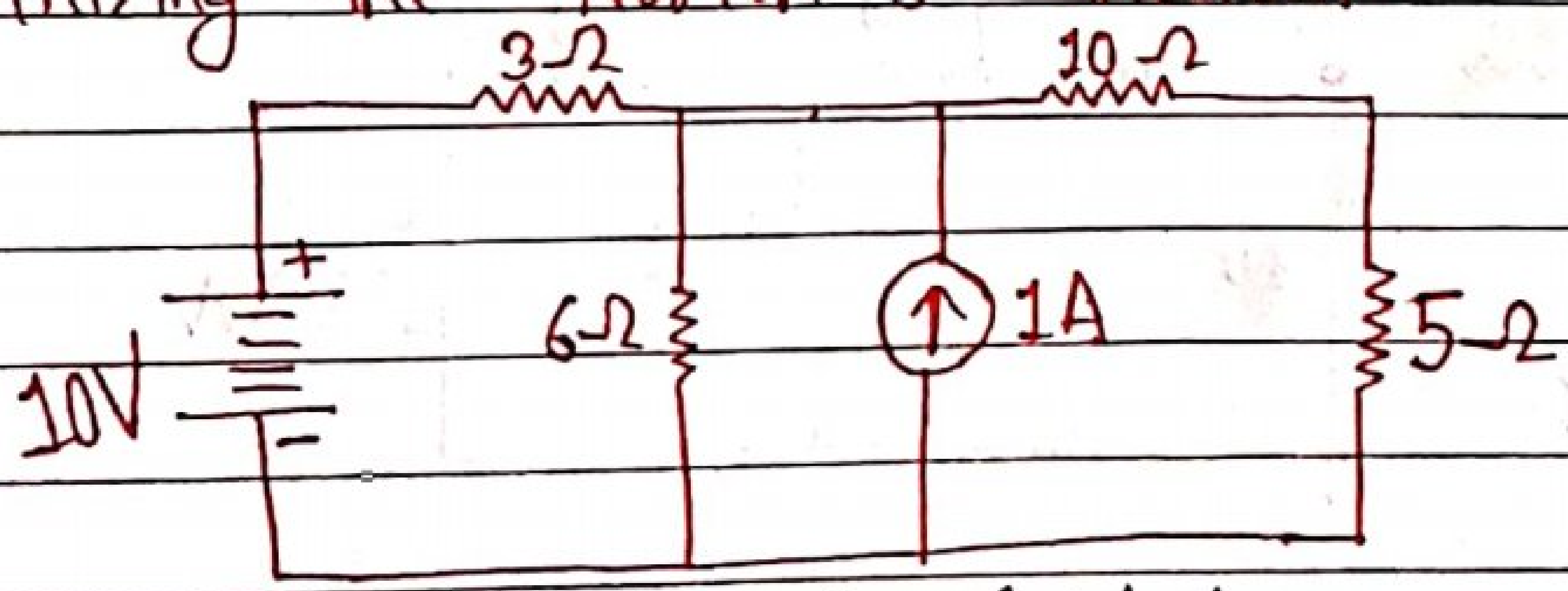
$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

$$= 8 \times \frac{6}{6+6}$$

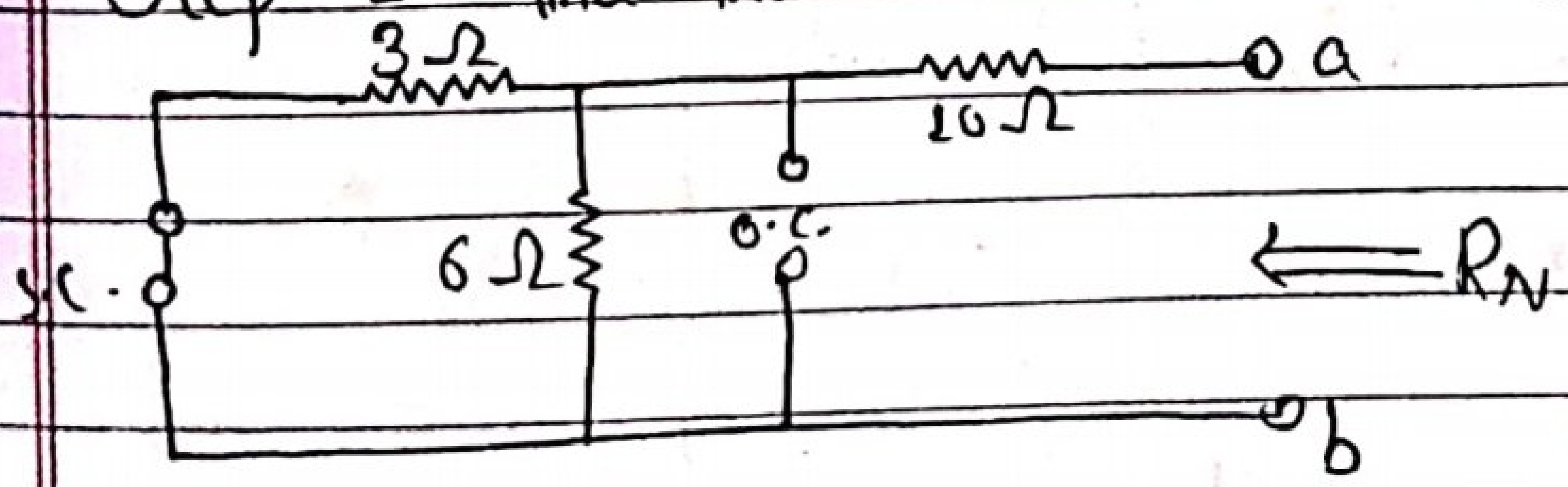
$$\therefore \boxed{I_L = 4 \text{ amp}}$$

Q.3 Find the current in 5Ω resistor, for the given ckt utilizing the Norton's Theorem:

PU



Step - 1: find the R_N (Norton resistance)



$$R_N = (3 \| 6) + 10$$

$$= 12\,\Omega$$

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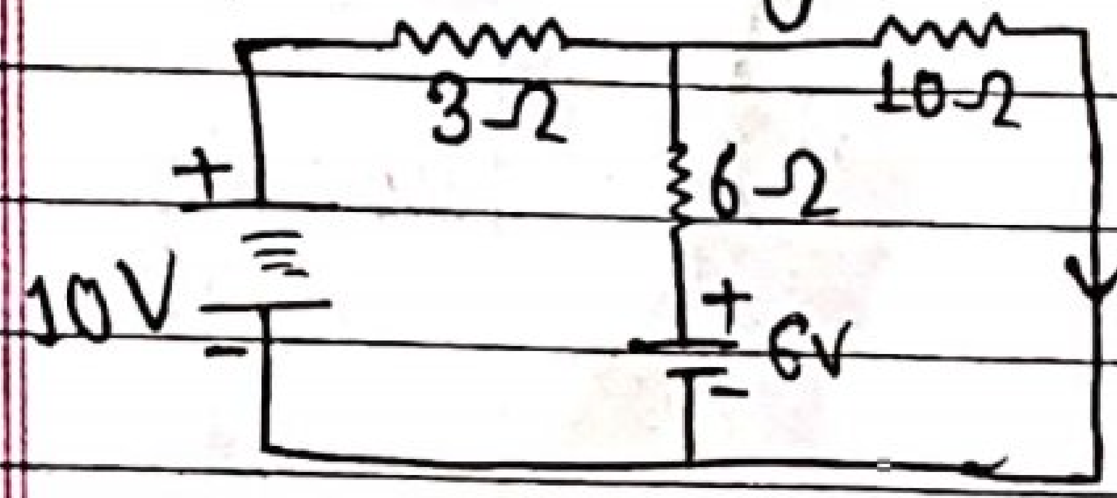
Step-2: finding the Norton's current (I_N) or I_{sc}:

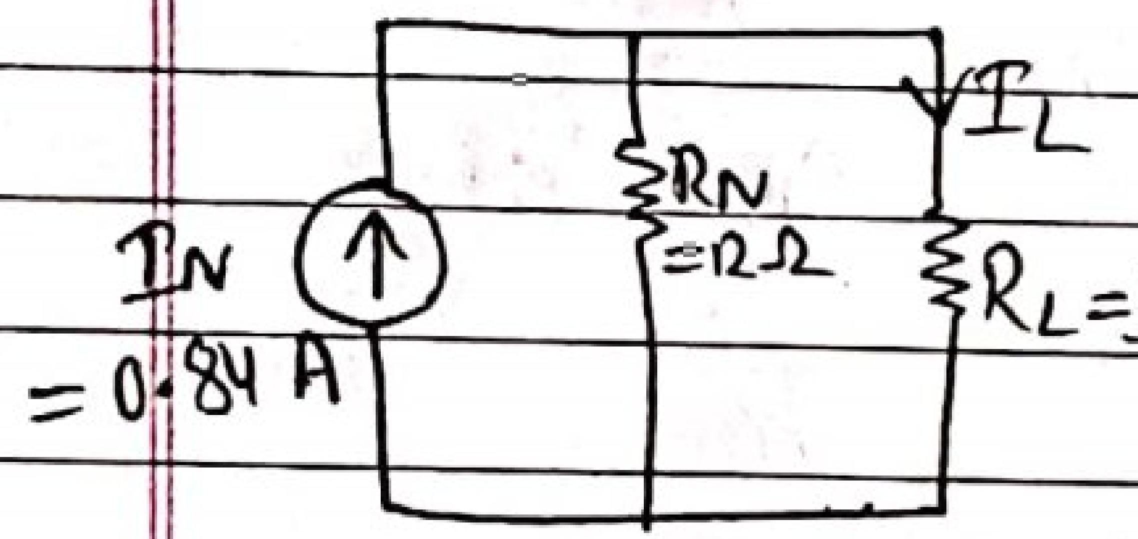


$V = 10 + 6 = 16V$

$R_T = 3 + 6 + 10 = 19\Omega$

$I_N(I_{sc}) = \frac{16}{19} = 0.84$ amp

Step-3: Norton's eq. ckt:

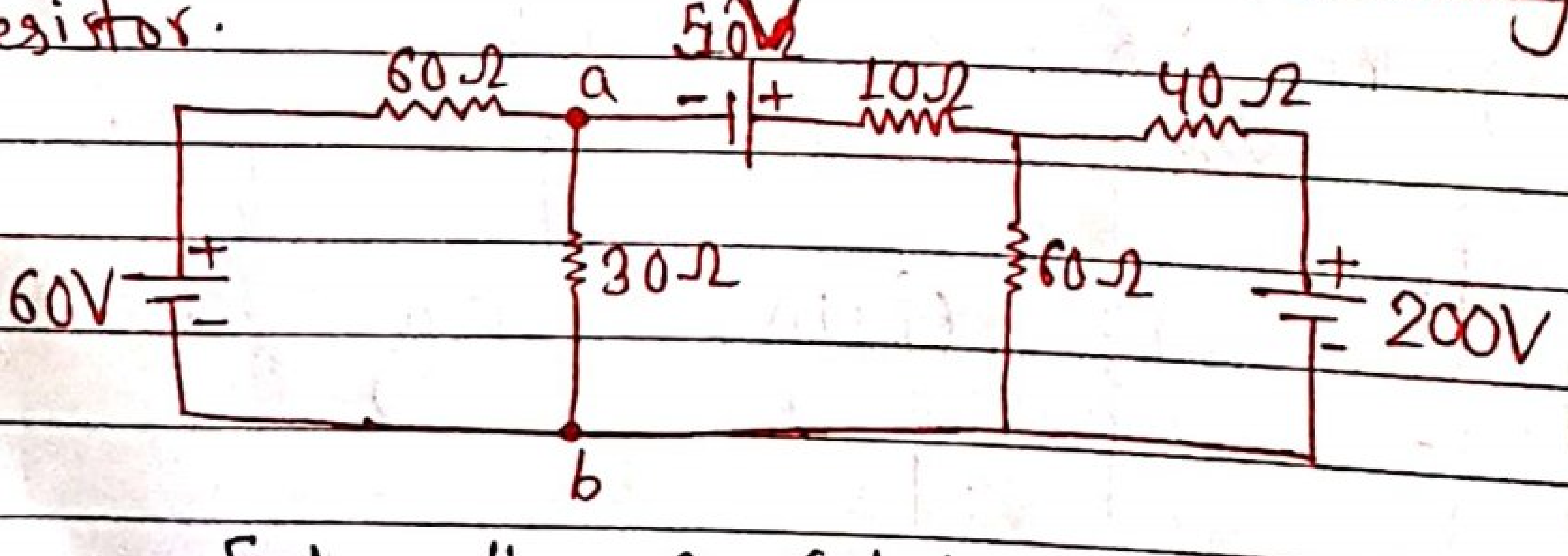


$I_L = I_N \times \frac{R_N}{R_N + R_L}$

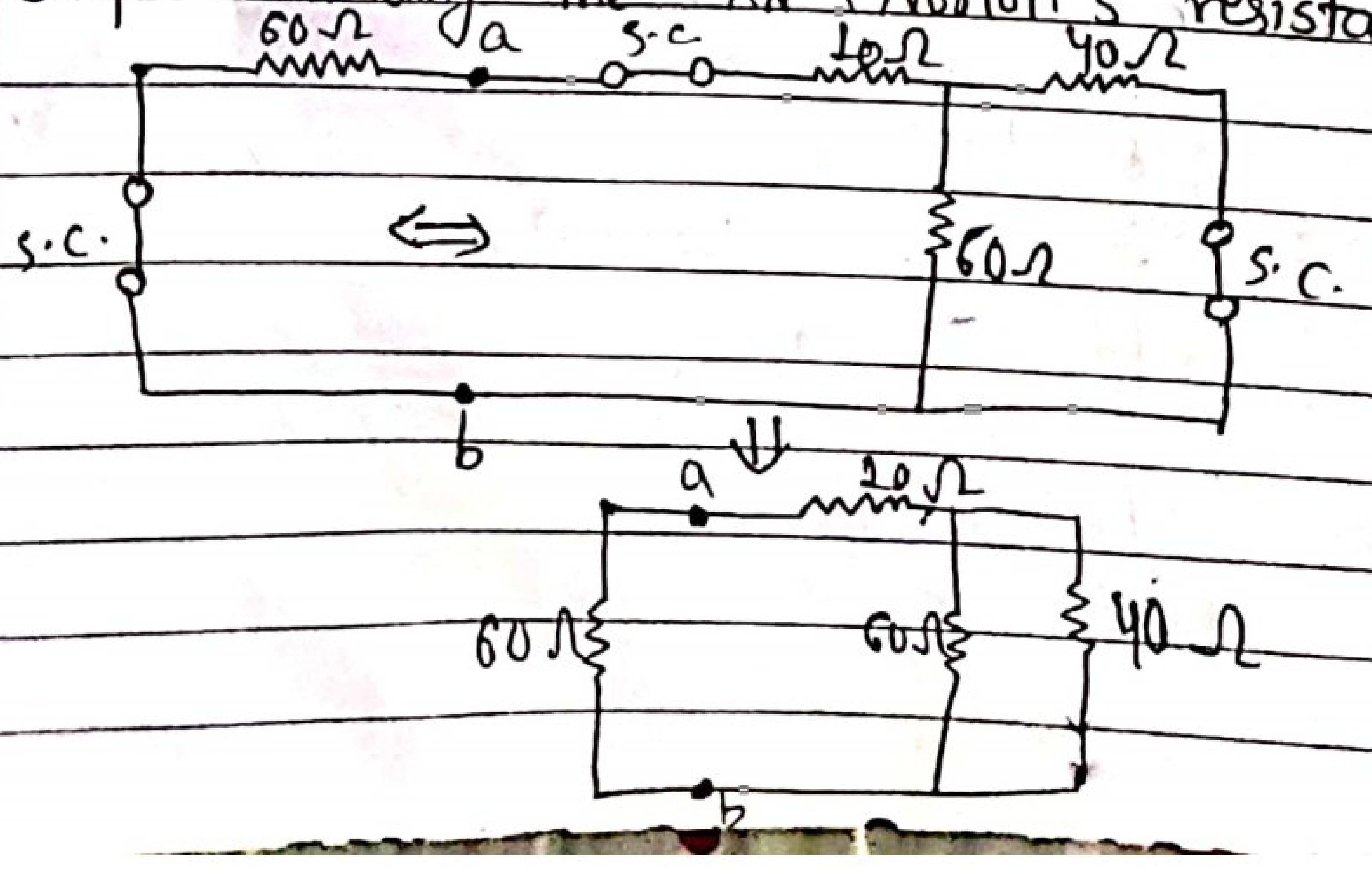
$= 0.84 \times \frac{12}{12 + 5}$

$\therefore \boxed{I_L = 0.59 \text{ amp}}$

Q.4) By Norton's Theorem, find the current through 30Ω resistor.



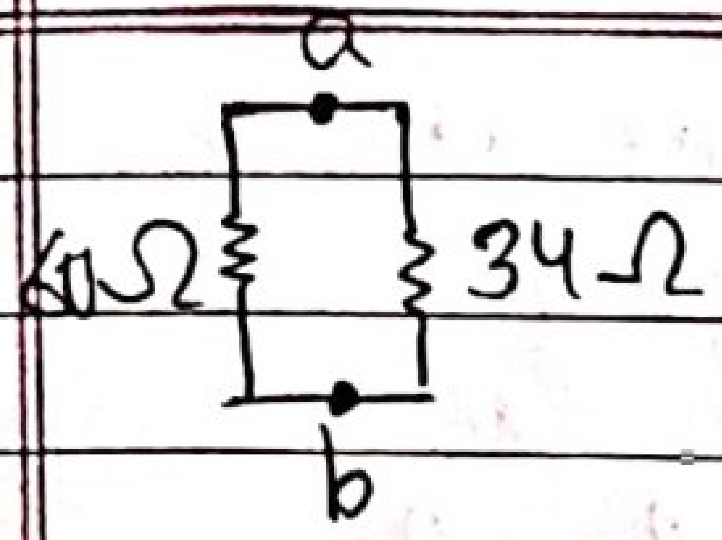
Step-1: Finding the R_N (Norton's resistance)



$R_{Th} =$

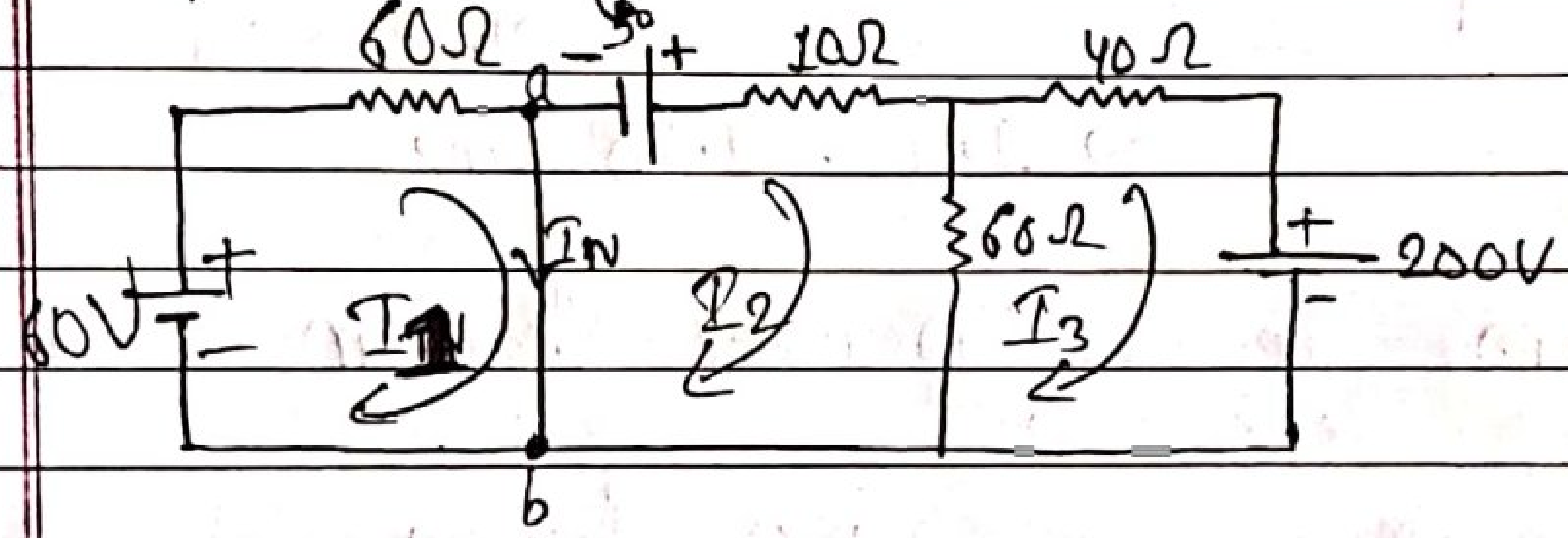
$\Rightarrow 60 \| 40 + 10$

$= 34\Omega$



$$60 \parallel 34$$
$$= 21.70\,\Omega$$

Step-2: Finding the Norton's current (I_N or I_{SC})



Loop-1 or mesh-1:

$$60 - 60I_1 = 0$$
$$\Rightarrow I_1 = 1 \text{ amp} \quad \text{—— (I)}$$

Loop-2:

$$50 - 10I_2 - 60(I_2 - I_3) = 0$$
$$\Rightarrow 50 - 70I_2 + 60I_3 = 0$$
$$\Rightarrow 70I_2 - 60I_3 = 50 \quad \text{—— (II)}$$

Loop-3:

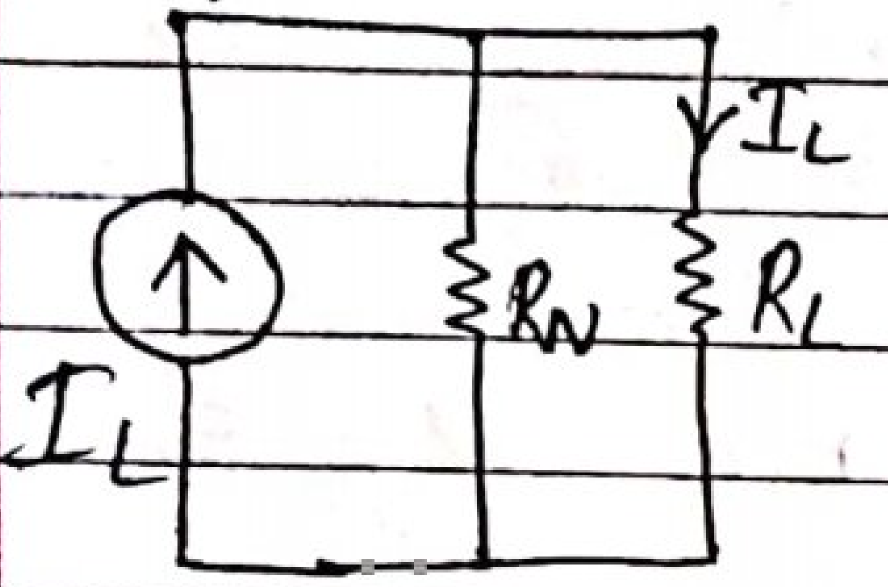
$$-200 - 60(I_3 - I_2) - 40I_3 = 0$$
$$\Rightarrow -100I_3 + 60I_2 = 200 \quad \text{—— (III)}$$

Solving,

$$I_1 = 1 \text{ amp}$$
$$I_2 = -2.05 \text{ amp}$$
$$I_3 = -3.23 \text{ amp}$$

$$\therefore I_N = I_1 - I_2$$
$$= 1 - (-2.05)$$
$$\therefore \boxed{I_N = 3.05 \text{ amp}}$$

Step-3 : Norton's eq. ckt :



$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

$$= 3.05 \times \frac{21.70}{21.70 + 30}$$

$$\Rightarrow I_L = 1.28 \text{ amp}$$

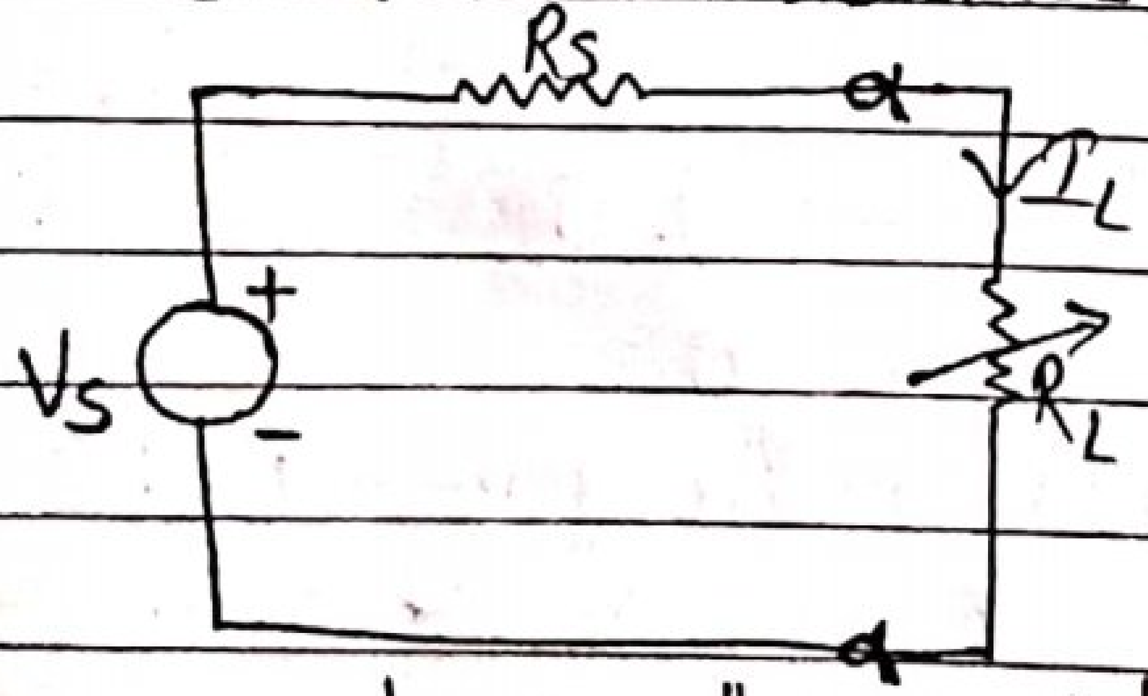
most imp

Maximum Power Transfer Theorem [MPT]:

imp derivative

Defination:

The maximum power transfer theorem states that the max^m power is delivered from source to load, when load resistance equal to source resistance. Assume that the load resistance is variable.



Load current in the ckt :

$$I_L = \frac{V_S}{R_S + R_L}$$

Power delivered to the load (R_L)

$$P_L = I_L^2 \cdot R_L = \left[\frac{V_S}{R_S + R_L}\right]^2 \cdot R_L$$

$$\therefore P_L = \frac{V_S^2 \cdot R_L}{[R_S + R_L]^2} \quad \text{——— (1)}$$

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To determine the value of R_L, the maximum power to be transfered to the load:

We have to set the first derivative of the above eq^n w.r.t. the R_L.

i.e. $\frac{dP_L}{dR_L} = 0$ [P_L is max^m]

$\therefore \frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_s^2 \cdot R_L}{(R_s+R_L)^2}\right]$

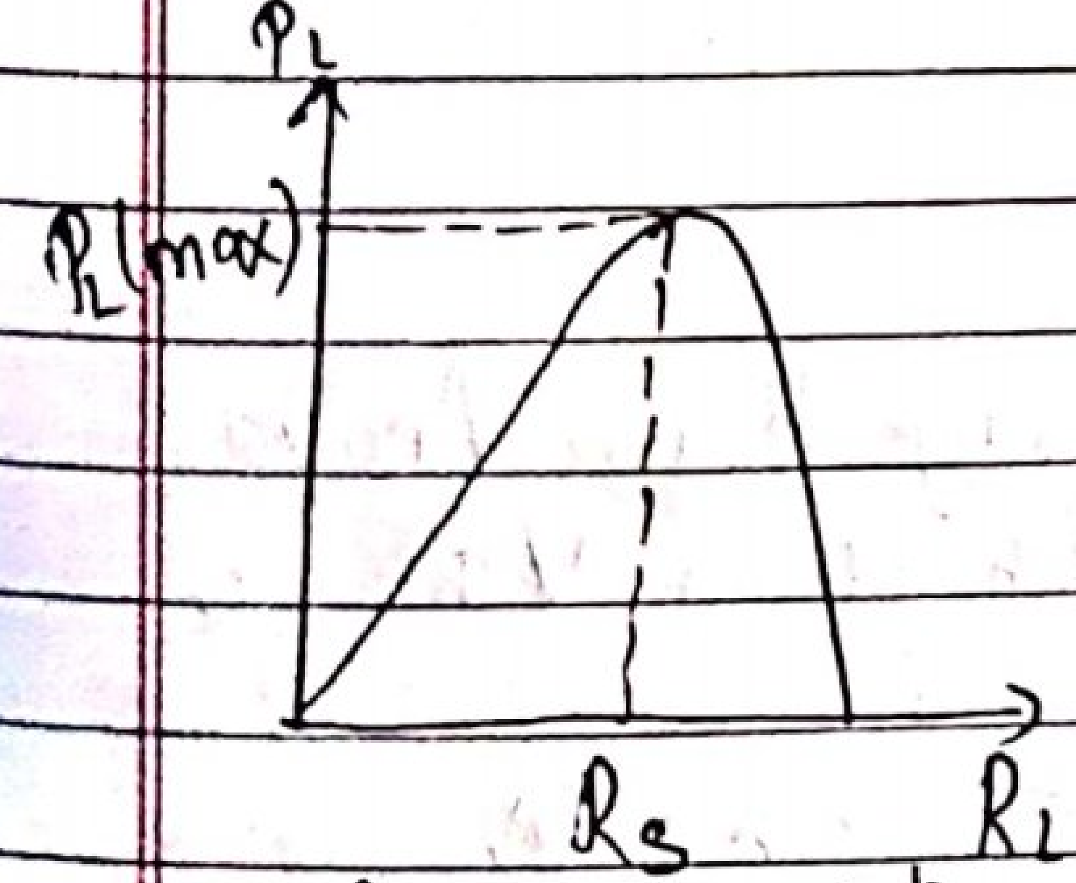
or, $0 = \frac{V_s^2\left[(R_s+R_L)^2 - 2R_L(R_s+R_L)\right]}{[R_s+R_L]^4}$

i.e. $[R_s+R_L]^2 - 2R_L[R_s+R_L] = 0$

or $R_s^2 + R_L^2 + \cancel{2R_LR_s} - \cancel{2R_LR_s} - 2R_L^2 = 0$

or $R_s^2 - R_L^2 = 0$

$\Rightarrow \boxed{R_s = R_L}$



fig(a): variation of Power with load resistance

So, the max^m power will be transfered to the load when load resistance equals to source resistance. So the max^m power

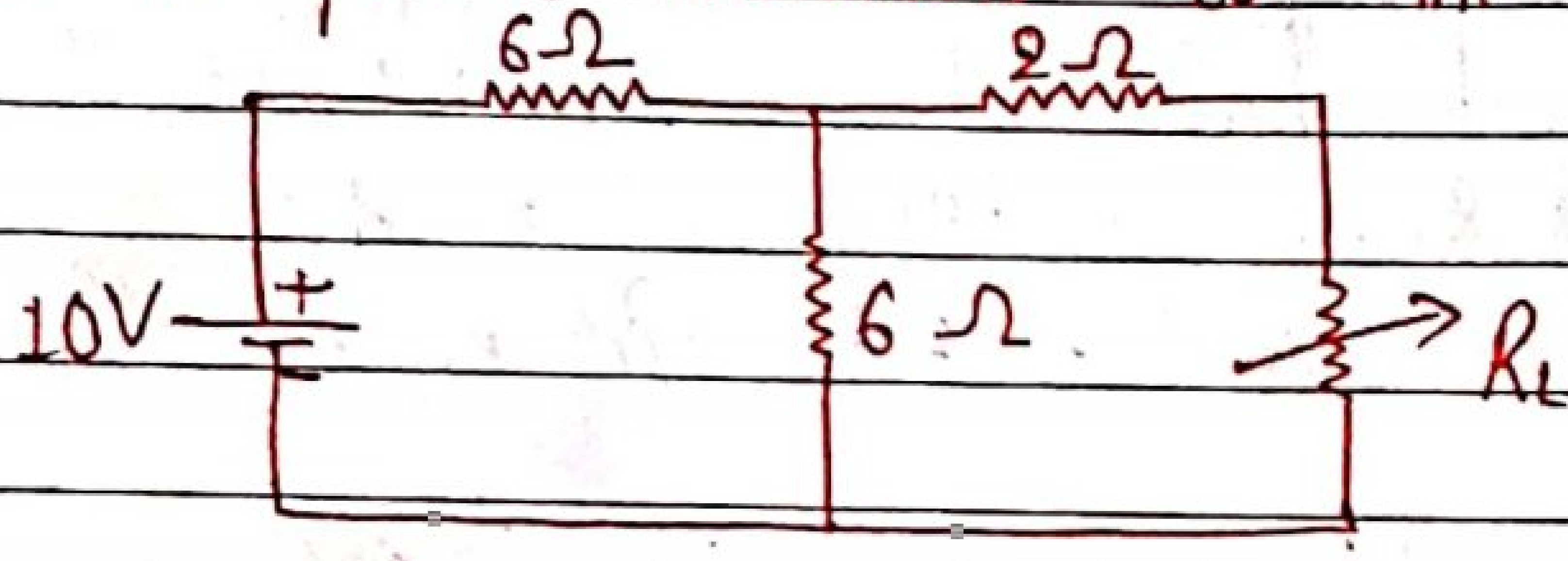
$P_L(max) = \frac{V_s^2 \cdot R_L}{(R_s+R_L)^2}$

$= \frac{V_s^2 \cdot R_L}{(R_s+R_L)^2}$

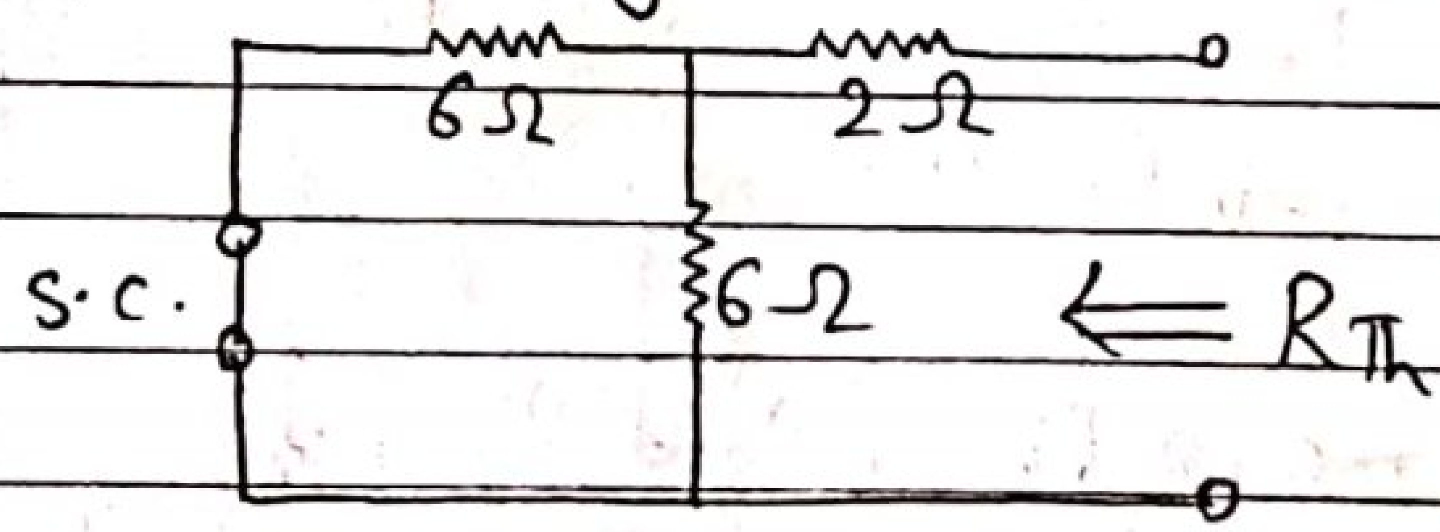
$= \frac{V_s^2 \cdot R_L}{4R_L^2}$ [$R_s = R_L$]

$\therefore$ Max^m power [P_L max] $= \frac{V_s^2}{4R_L}$

Q. In a N/w shown in fig below, determine :
(a) value of load resistance to give a max^m power transfer
(b) the power delivered to the load.



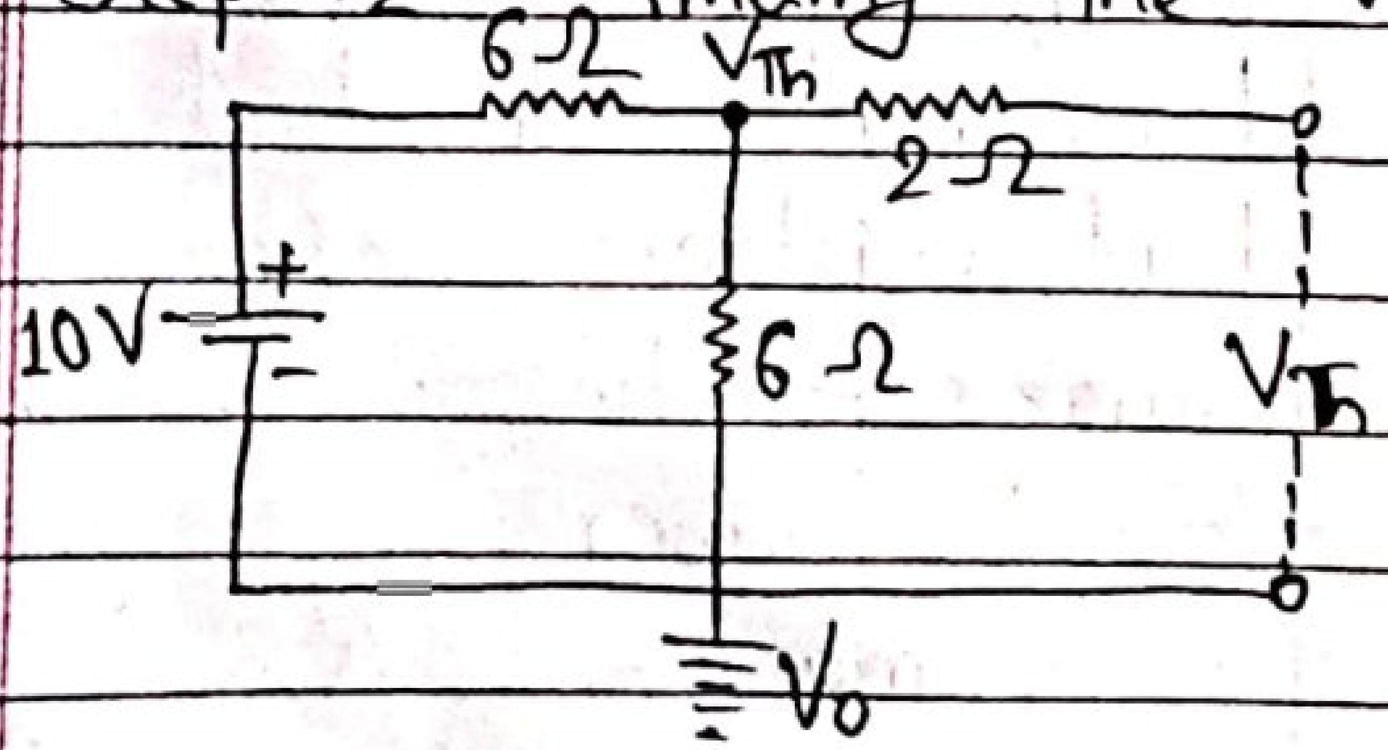
Step-1: Finding the R_{Th} :



$$R_{Th} = (6 \| 6) + 2 = \frac{36}{12} + 2$$

$$\therefore R_{Th} = 5\Omega$$

Step-2: Finding the V_{Th} :



Using Nodal Analysis:

$$\frac{V_{Th} - 10}{6} + \frac{V_{Th}}{6} = 0$$

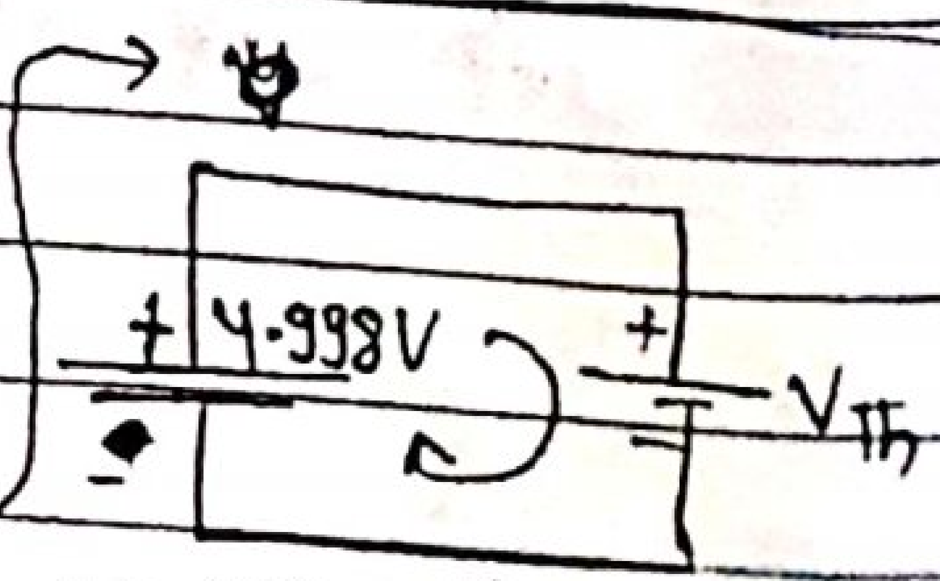
$$\Rightarrow V_{Th} = 5V$$

Taking LHS mesh,

$$10V - 6I_1 - 6I_1 = 0$$

or $$10 = 12 I_1$$

$$\Rightarrow I_1 = 0.833 \text{ amp.}$$



$$+4.998 - V_{Th} = 0$$

$$V_{Th} = 4.998V$$

Date
Page

(a) Max^m power to transfer:

$$R_L = R_{Th} = 5\Omega \quad (max^m \text{ power})$$

(b) P_L max $= I_L^2 \cdot R_L$

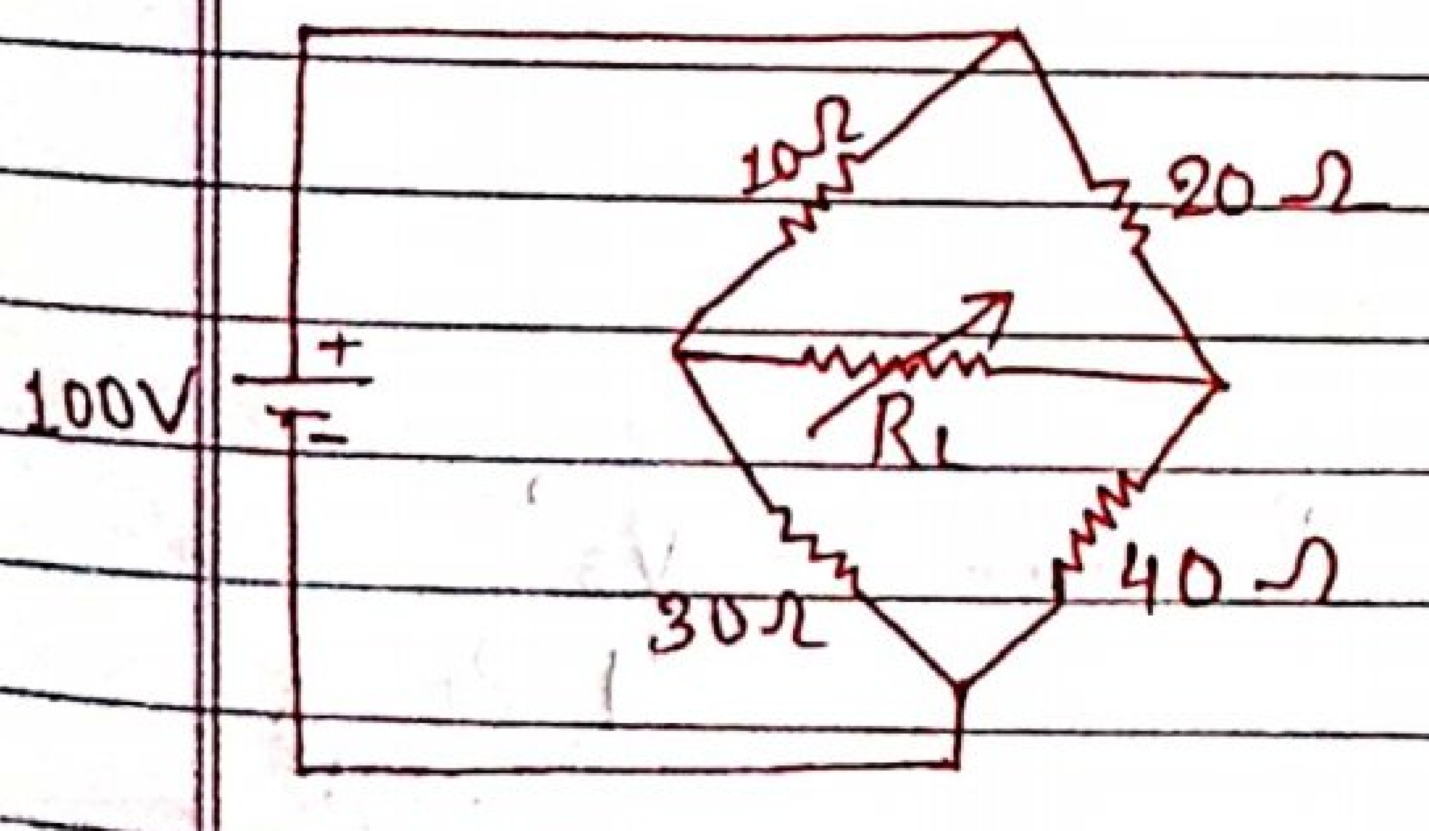
$$= \frac{V_S^2}{(R_S+R_L)^2} \cdot R_L$$

$$P_L \, max = \frac{V_S^2}{4R_L} \quad (R_S = R_L)$$

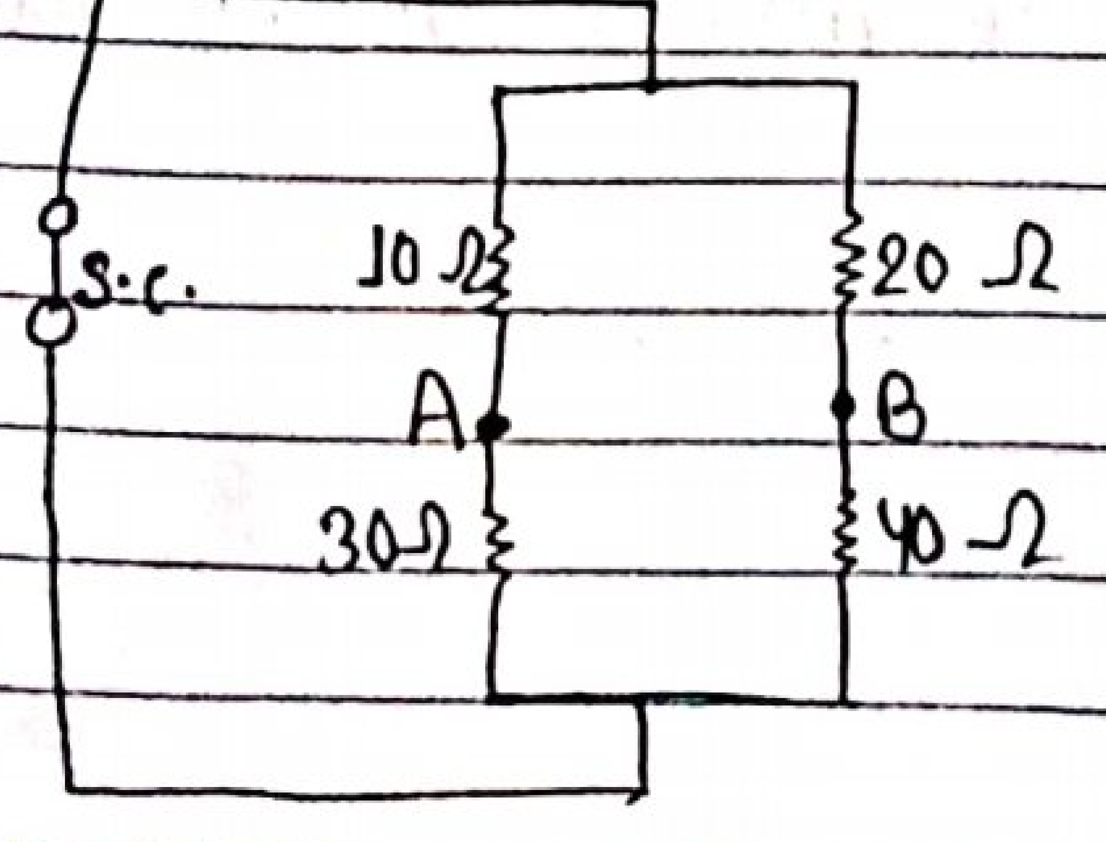
$$= \frac{5^{\cancel{2}}}{4\times\cancel{5}} \quad [\because V_S = V_{Th}]$$

$$\therefore P_L \, max = 1.25 \text{ watt}$$

Q. Determine the load resistance to receive the max^m power from the source, also find the max^m power delivered to the load in the ckt shown in fig.



Step-1: Finding the R_{Th}:



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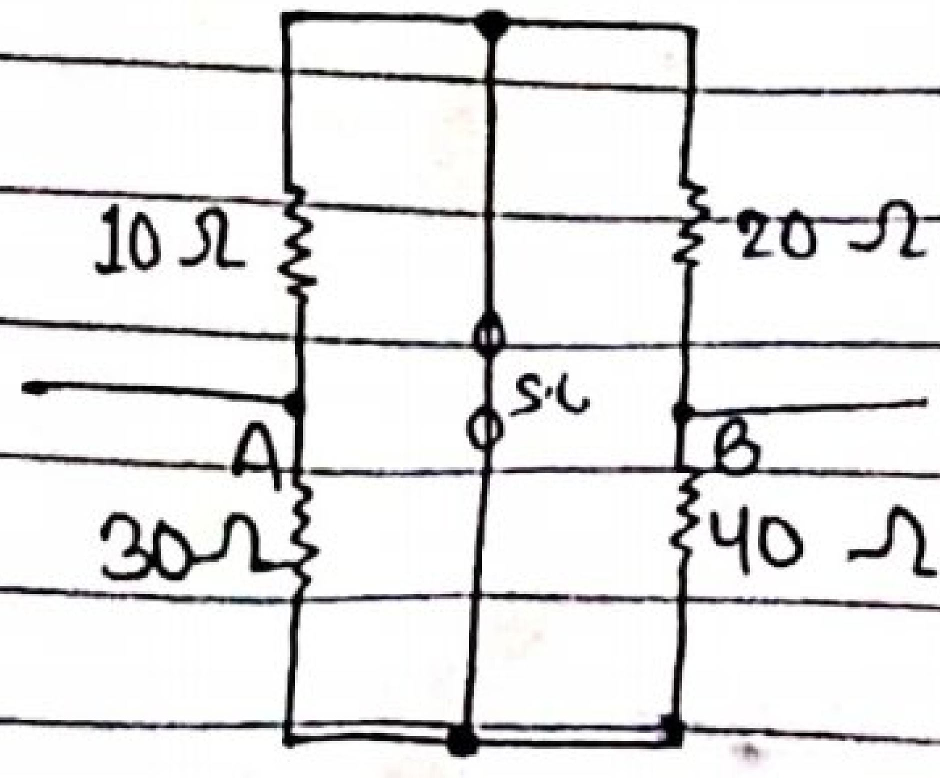
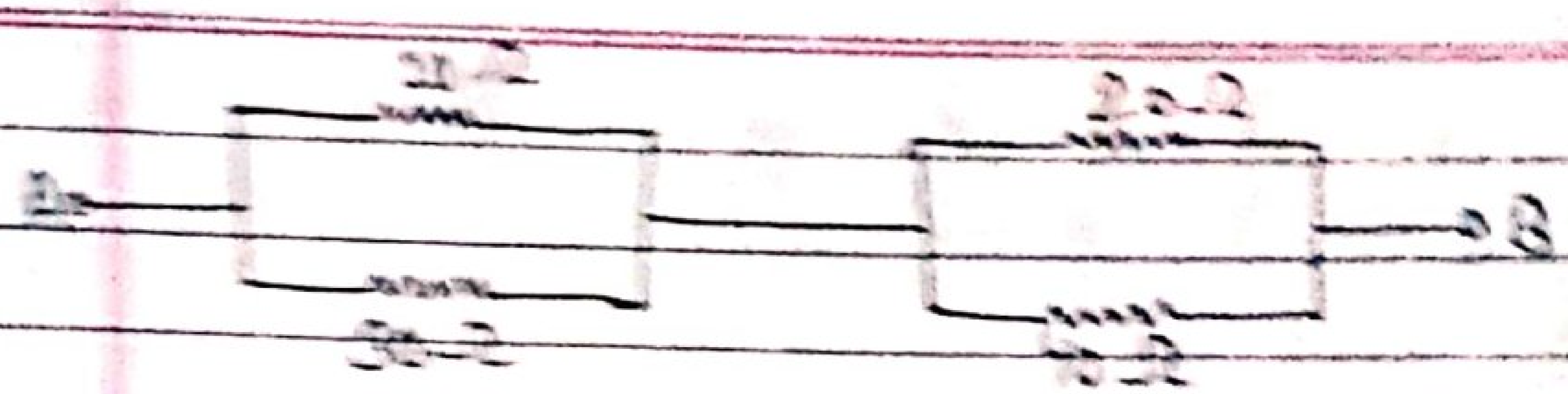
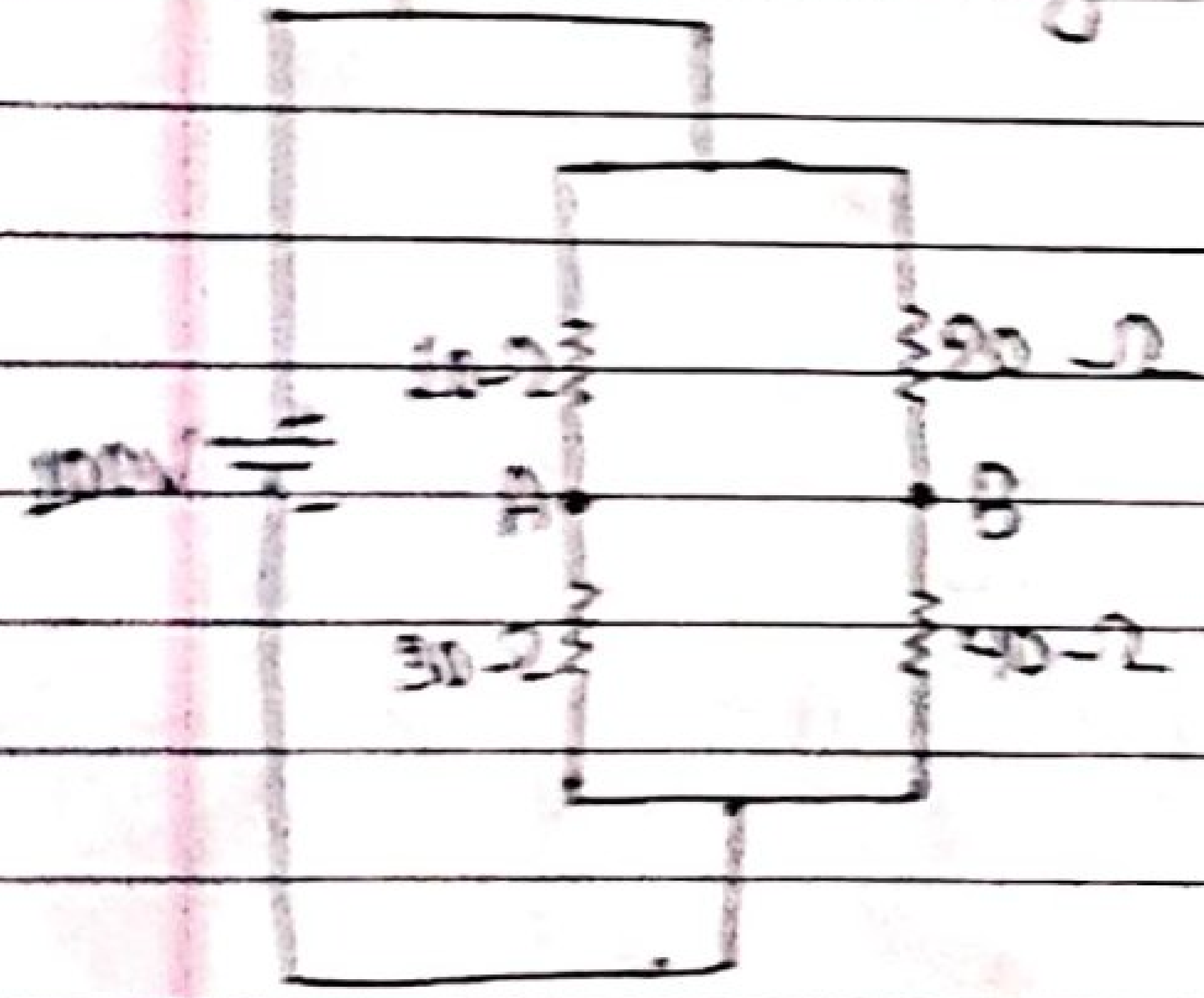




$$R_{Th} = (10 \parallel 30) + (20 \parallel 40)$$
$$= 20.83\ \Omega$$

Step-2: Finding the V_{Th}:

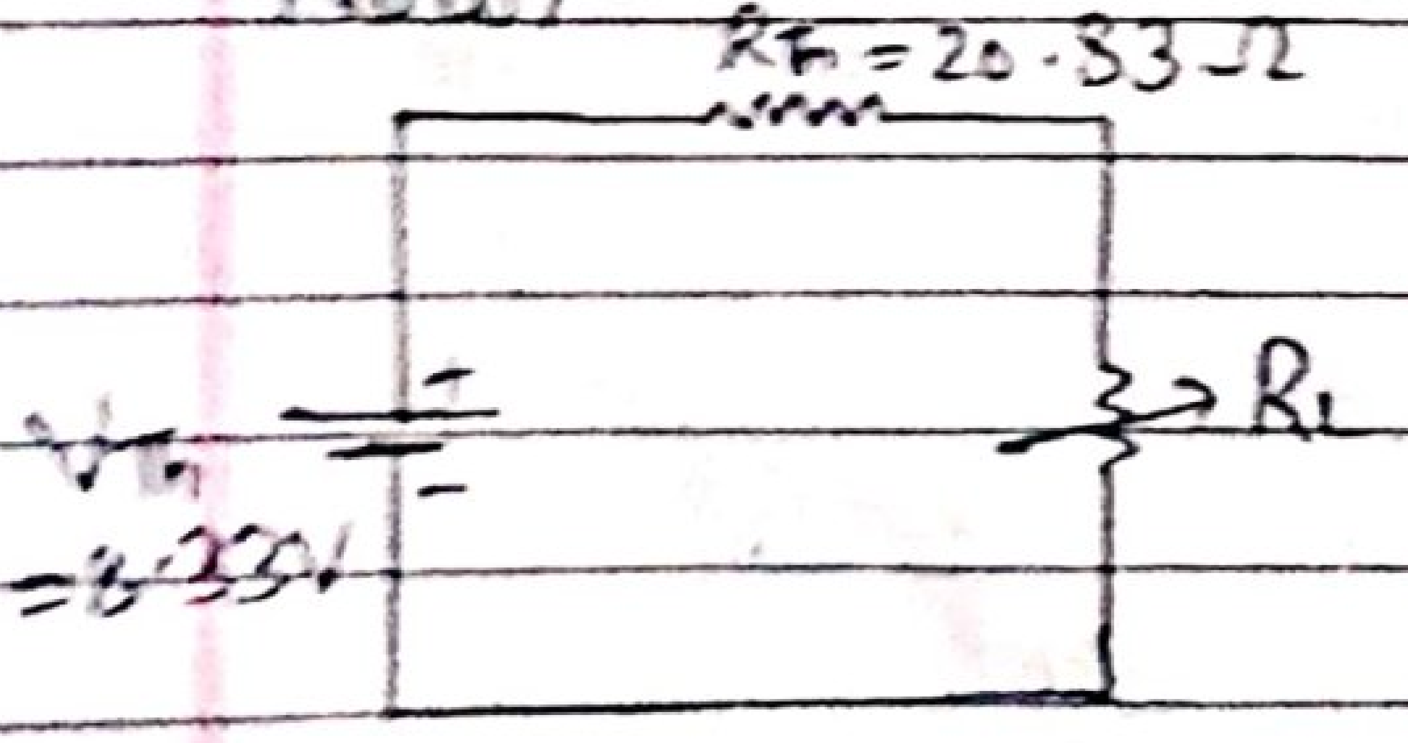


$$V_A = 100 \times \frac{10}{10+30} = 25V$$

$$V_B = 100 \times \frac{20}{20+40} = 33.33$$

$$\therefore V_{AB} = V_B - V_A$$
$$= 33.33 - 25$$
$$\Rightarrow V_{AB} = 8.33V$$

Now,



$$P_L\ max = \frac{V_s^2}{4 R_L}$$
$$= \frac{(8.33)^2}{4 \times 20.83}$$

$$\therefore P_L\ max = 0.834\ watt$$

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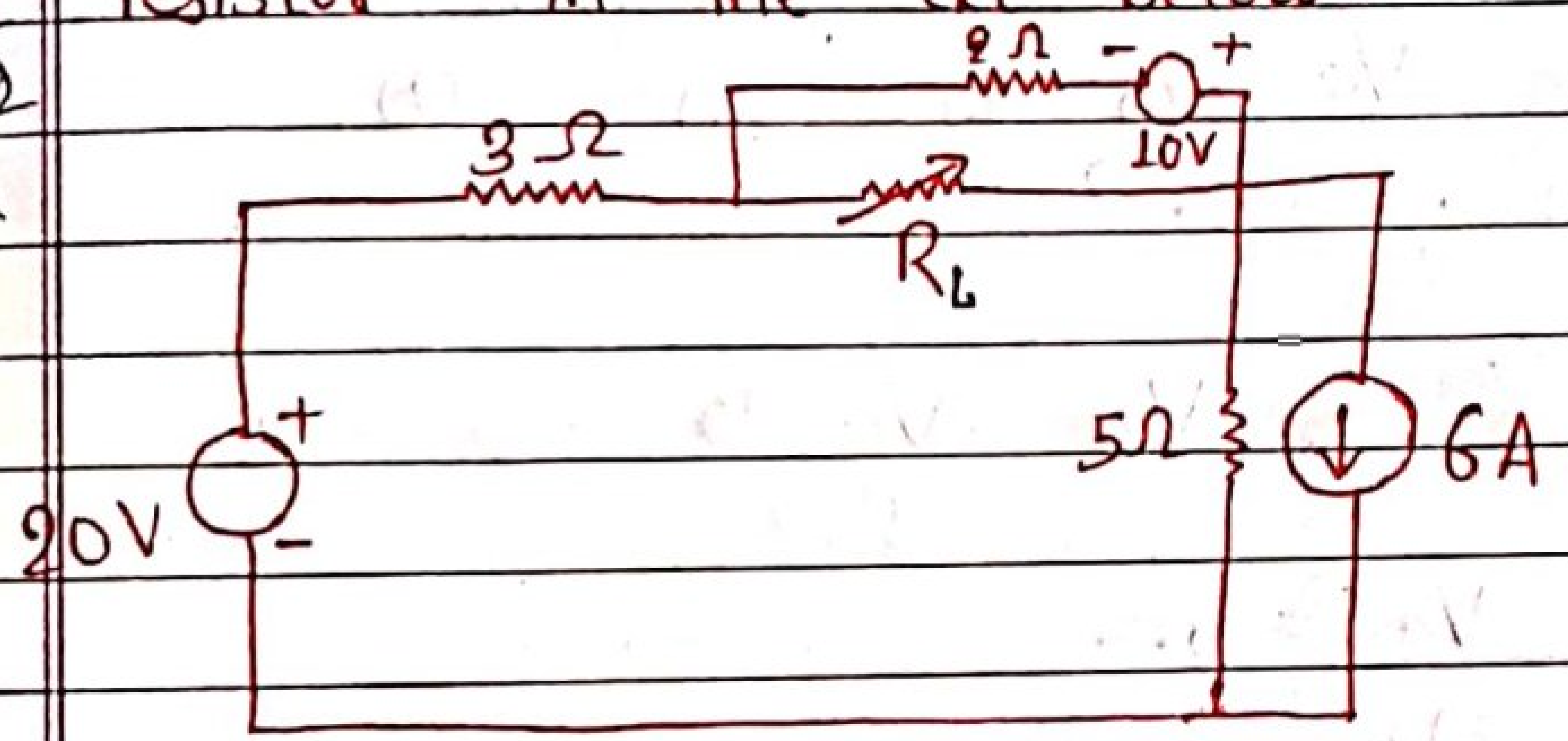
Q. Find the maximum power that can be delivered to the resistor in the ckt below:



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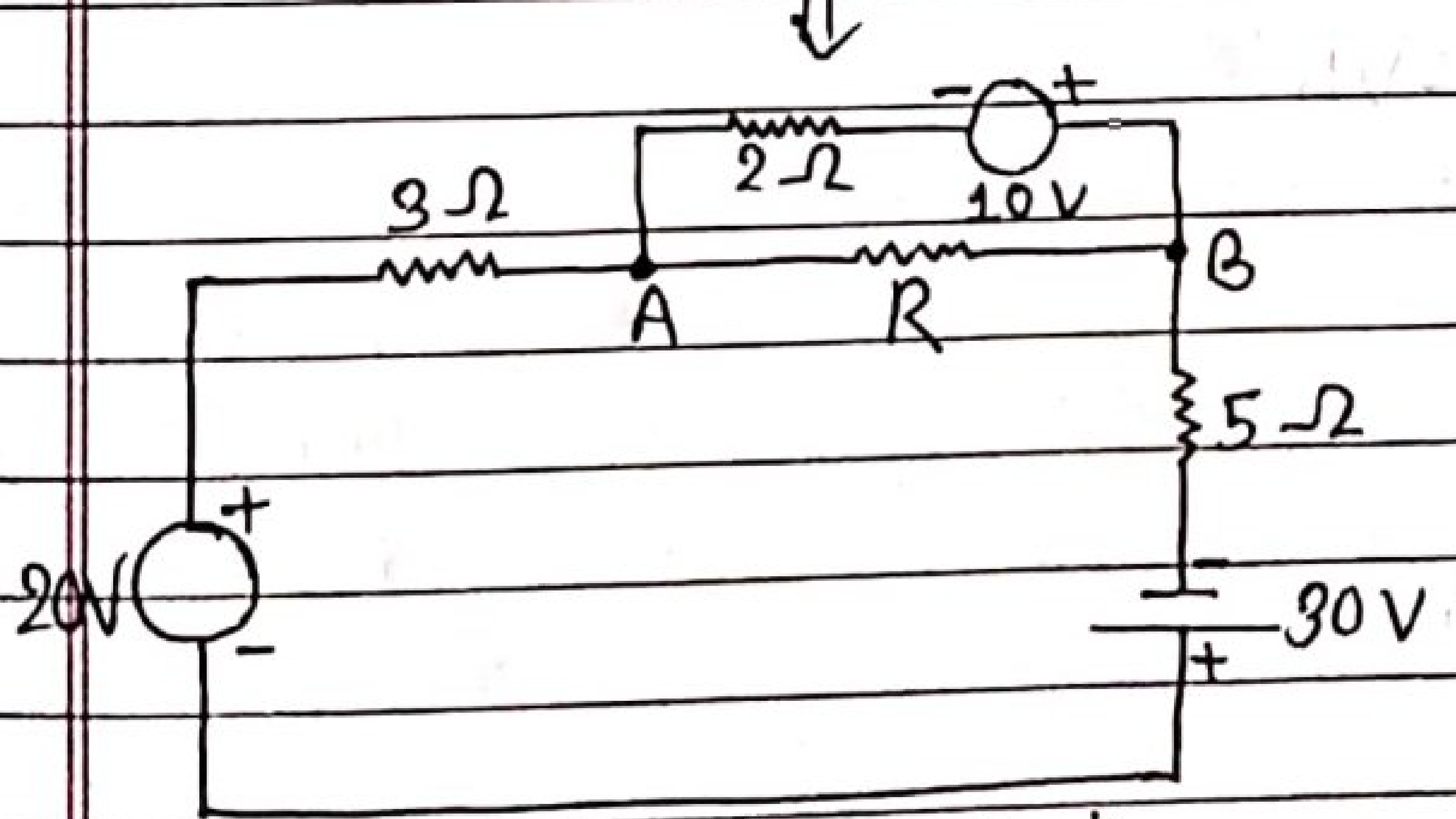
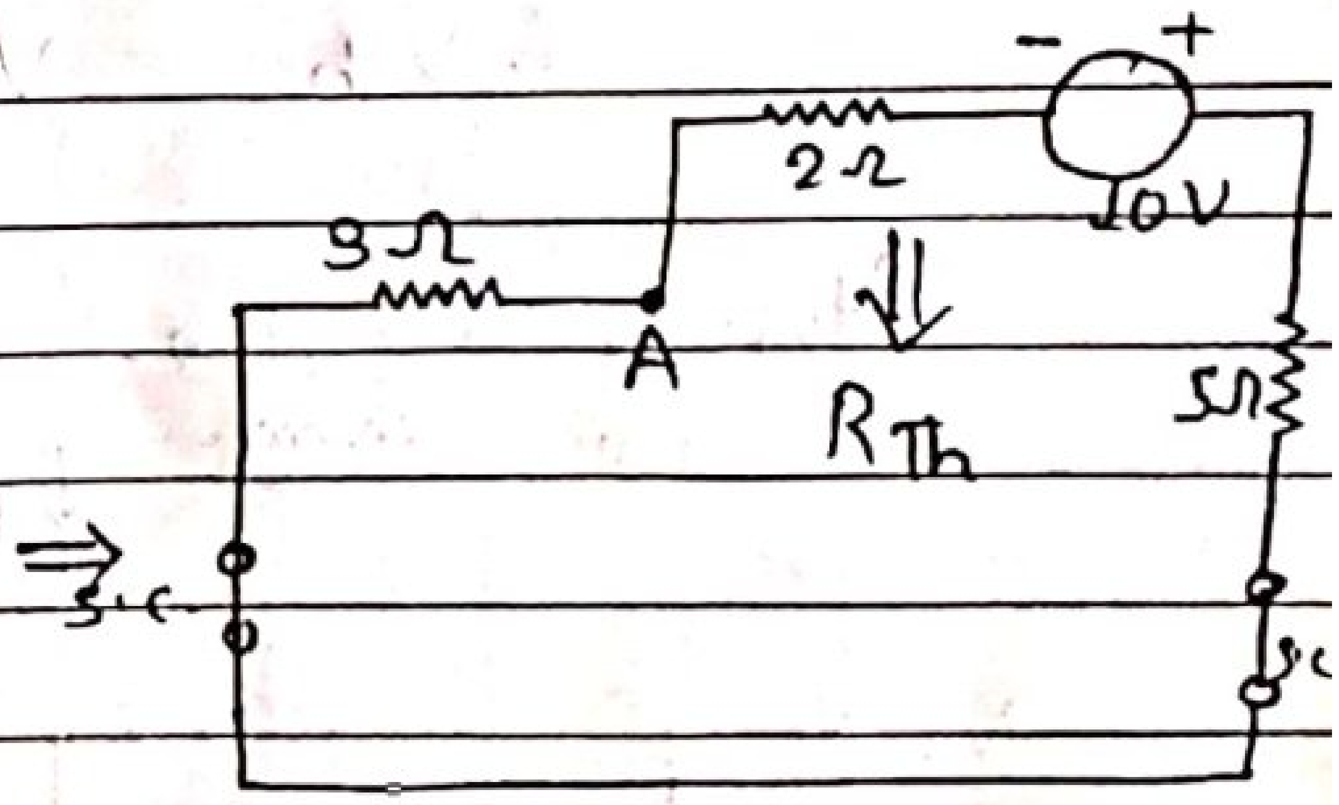
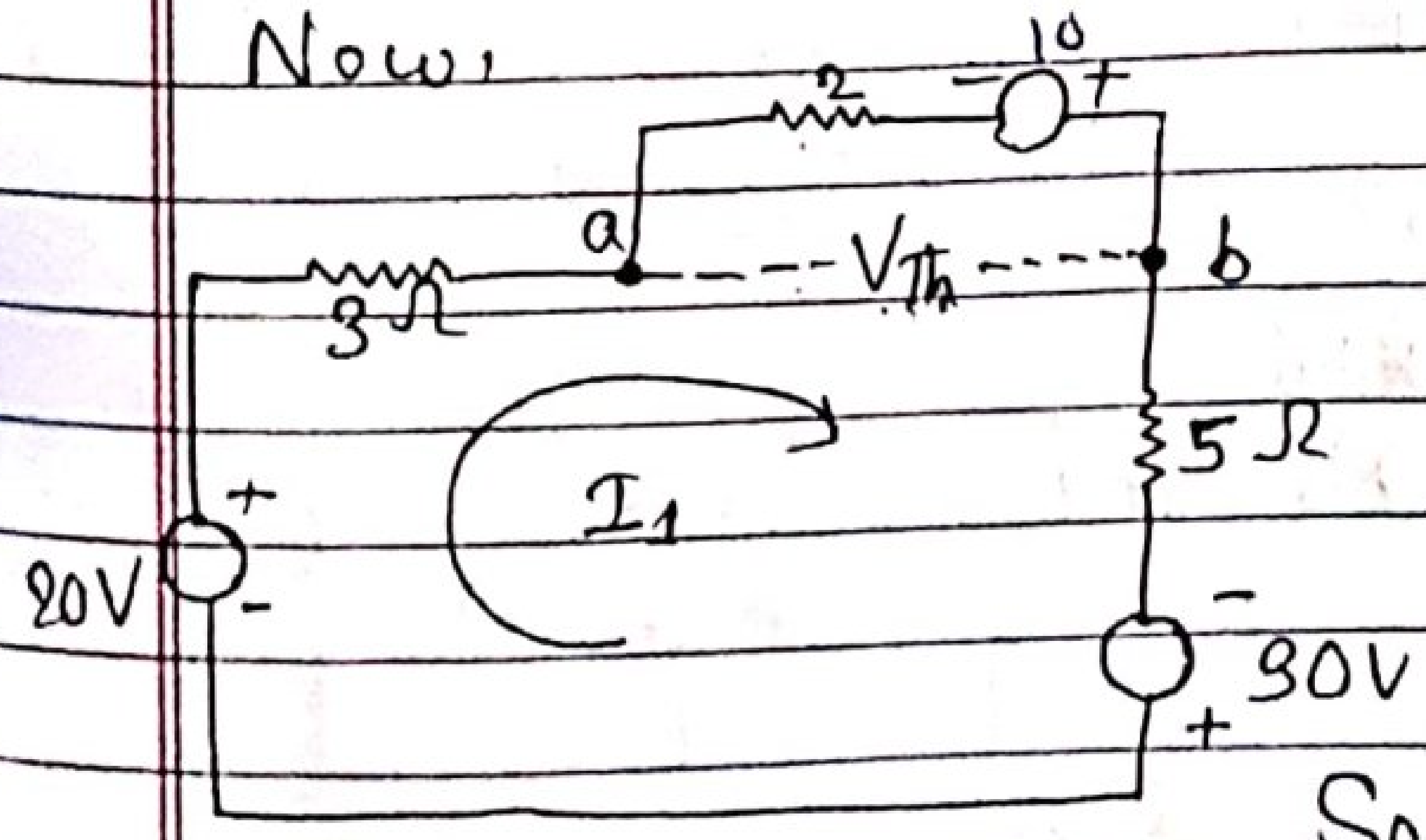




$R_{Th} = (5+3) \parallel 2$
$= 1.6\,\Omega$

Now,



Considering mesh analysis:

$20V - 3I_1 - 2I_1 + 10\cancel{V}$
$- 5I_1 + 30 = 0$

or $60\cancel{V} - 10I_1 = 0$

or $60 = 10I_1$

$\Rightarrow I_1 = 6$ Amp

So, $V_{Th} - 2I_1 + 10 = 0$

So, $V_{Th} = -10 + 2 \times 6$

$\Rightarrow V_{Th} = 2V$

or, using Nodal Analysis,

$$\frac{V_a - 20}{3} + \frac{V_a - V_b + 10}{2} = 0 \quad \text{—} (1)$$

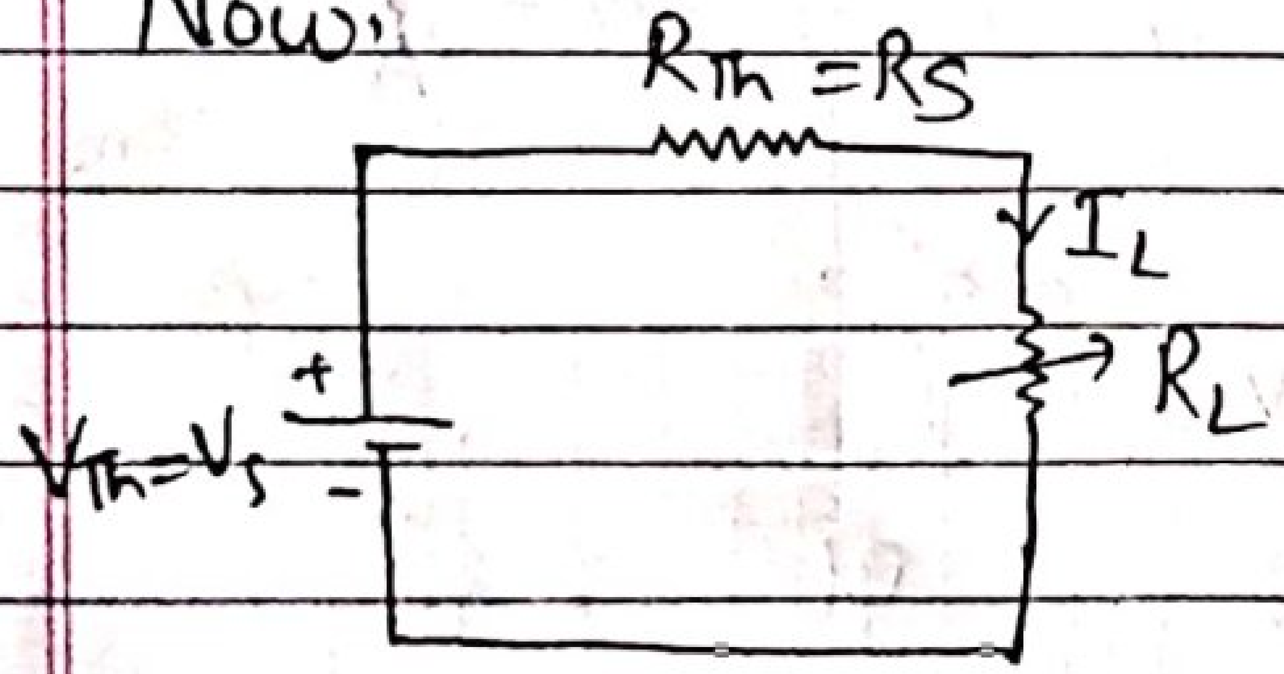
$$\frac{V_b + 30}{5} + \frac{V_b - V_a - 10}{2} = 0 \quad \text{—} (11)$$

Solving, $V_a = \ldots$

$V_b = \ldots$

$\therefore V_{Th} = V_a - V_b$

Now,



$$P_L \max = \frac{V_s^2}{4R_L}$$

$$= \frac{2^2}{4 \times 1.6}$$

$\therefore P_L \max = 0.625$ watt