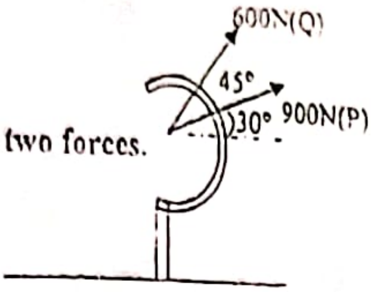
3.1 Composition of Forces

1. Determine the magnitude and direction of the resultant of two forces.



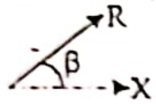
Sol^n-Here,

$R = \sqrt{600^2 + 900^2 + 2 \times 600 \times 900 \times \cos 45°}$

$= 1390.567N$ Ans

$\beta = \tan^{-1}\left(\frac{600 \sin 45°}{900 + 600 \cos 45°}\right) + 30°$ (angle made by base 900N with horizon)

$= 47.764°$ Ans



2. Find the angle between two equal forces P, when their resultant is equal forces P, when their resultant is equal to (i) P and (iii) p/2.

Sol^n - Here,

i) $p^2 = p^2 + p^2 + 2p^2 \cos\theta$

or, $p^2 = 2p^2(1+\cos\theta)$ or, $p^2 = 2p^2(2\cos^2\frac{\theta}{2})$

or, $p^2 = 4p^2 \cos^2\frac{\theta}{2}$ or, $\cos\frac{\theta}{2} = \frac{1}{2}$

or, $\theta = 120°$ Ans

ii) $\left(\frac{p}{2}\right)^2 = 4p^2\cos^2\frac{\theta}{2}$ or, $\frac{1}{16} = \cos^2\frac{\theta}{2}$

or, $\frac{\theta}{2} = 75.522$ or, $\theta = 151.045°$ Ans

3. The resultant of two forces, one is double of other, is 260N. If direction of a force is reversed, the other unchanged, then the resultant reduces to 180N. Determine the magnitude of the forces and the angle between the forces.

Sol^n- Here,

$260^2 = p^2 + 4p^2 + 4p^2\cos\theta$ or, $260^2 = p^2(5+4\cos\theta)$

$180^2 = p^2 + 4p^2 + 4p^2\cos(180° - \theta)$ or, $180^2 = p^2(5 - 4\cos\theta)$

Equating, we get

$\frac{260^2}{5 + 4\cos\theta} = \frac{180^2}{5 - 4\cos\theta}$ or, $845 - 676\cos\theta = 405 + 324\cos\theta$

or, $\theta = 63.896°$ and $p = 100N, 200N$ Ans

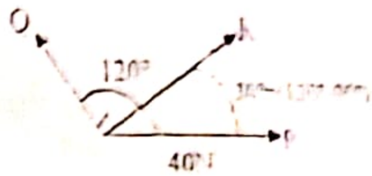
4. Two forces [illegible]

the smaller one. Find the smaller force

Sol^n- Here,

Let P > Q

$$\tan(120^\circ-90^\circ) = \frac{Q\sin 120^\circ}{P + Q\cos 120^\circ}$$



$$\text{or, } 0.577 = \frac{0.866Q}{40 - Q/2}$$

or, Q = 19.991N Ans

5. Find the magnitude of two forces, if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N. (Bihar U, 1986)

Sol^n- Here

If $\theta = 90^\circ$, $R^2=P^2+Q^2$ or, $10 = P^2+Q^2$

If $\theta = 60^\circ$, $R^2 = P^2+Q^2+PQ$ or, $13=10+PQ$

Solving,

$$10 = \left(\frac{13-10}{Q}\right)^2 + Q^2$$

$$\text{or, } 10 = \frac{9+Q^4}{Q^2}$$

or, $Q^4-10Q^2+9=0$ or, $Q^2 = \dfrac{10\pm\sqrt{10^2-4\times1\times9}}{2\times1}$

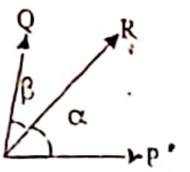
or, $Q = \pm 3, \pm 1$N, So $P = \pm 1, \pm 3$N Ans

6. The 1500-N force is to be resolved in to components along lines a-a' and b-b'. (a) Determine α if the component along line a-a' is 1200N. (b) What is the corresponding value along b-b'?

Sol^n- Here

Way - 1

We can directly use resolution of a force in required directions by a formula below



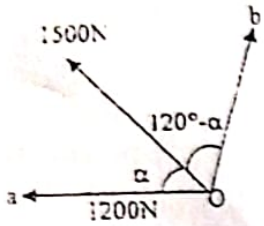
$$P = \frac{R\sin\beta}{\sin(\alpha+\beta)}, \quad Q = \frac{R\sin\alpha}{\sin(\alpha+\beta)}$$

If α + β = 90° (case of rectangular resolution),

$P = R\cos\alpha,\ Q = R\sin\alpha$

Problem:

Let us resolve the force along Oa and Ob, then



a) $$1200 = \frac{1500\sin(120^\circ - \alpha)}{\sin 120^\circ}$$

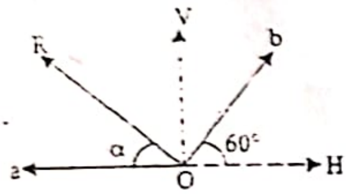
or, $\alpha = 76.146^\circ$ Ans

b) $$F_{ob} = \frac{1500\sin\alpha}{\sin 120^\circ}$$

$= 1681.665\text{N}$ Ans

Way-2

Using rectangular resolution in horizontal and vertical direction



$\Sigma H = \text{Ob}\cos60^\circ - \text{oa}$

$\Sigma V = \text{Ob}\sin60^\circ$

These ΣH, ΣV are components of Oa and Ob forces; the resolved parts of resultant R.

$$\Sigma H = \text{Ob}\cos60^\circ - 1200$$

$$\Sigma V = \text{Ob}\sin60^\circ$$

But $\Sigma F = R = 1500\text{N}$

So, $1500^2 = (\text{Ob}\cos60^\circ - 1200)^2 + (\text{b}\sin60^\circ)^2$

or, $1500^2 = \text{Ob}^2 - 2400\,\text{Ob}\cos60^\circ + 1440000$

or, $$\text{Ob} = \frac{1200 \pm \sqrt{4680000}}{2}$$

$= 1681.66\text{N}$ or -481.66N

but we are given Oa = 1200N i.e. ← in this direction so, Ob = 1681.66N ↗ is only permittable for getting resultant ↖ So,

Ob = 1681.66N Ans

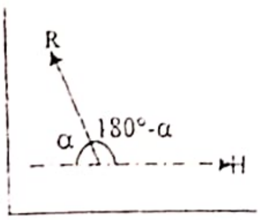
For α: $\Sigma H = R\cos(180^\circ - \alpha)$

or, $\text{Ob}\cos60^\circ - 1200 = 1500\cos(180^\circ - \alpha)$

or, $1681.66\cos60^\circ - 1200 = 1500\cos(180^\circ - \alpha)$

or, $180^\circ - \alpha = 103.854^\circ$

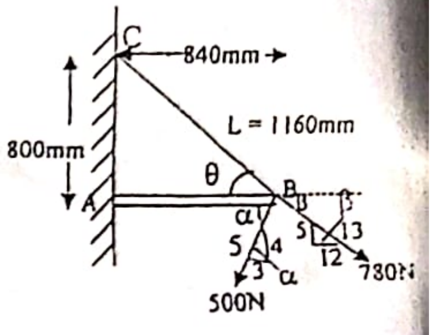
or, $\alpha = 76.146^\circ$ Ans



3.2 Resolution of forces and finding resultant

3.2.1 In 2D plane

7. Knowing that the tension in cable BC is 725N, determine the resultant of the three forces exerted at the point B of beam AB.

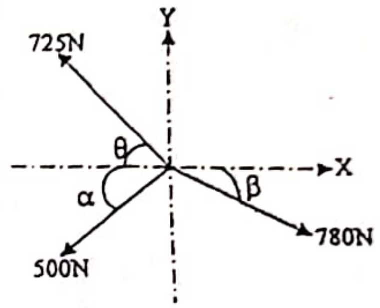


Sol^n-Here,

$$\theta = \sin^{-1}\left(\frac{800}{1160}\right) = 43.603°$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.130°$$

$$\beta = \tan^{-1}\left(\frac{5}{12}\right) = 22.620°$$



$\theta = 43.603°$

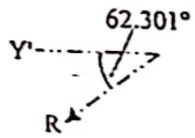
$\alpha = 53.130°$

$\beta = 22.620°$

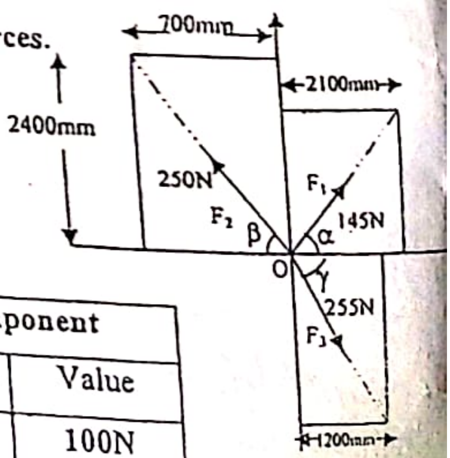
Magnitude (in N)	X-component (N)	Y-component (N)
725	-725cos 43.603	725 sin 43.603
500	-500 cos 53.130	-500sin 53.130
780	780 cos 22.620	-780 sin 22.620
	ΣFx= -105.000	ΣFy = -200

So, $R = \sqrt{(-105)^2 + (-200)^2} = 225.887N$ **Ans**

$\theta = \tan^{-1}\left(\frac{200}{105}\right) = 62.301°$ Ans



8. Determine x, y components of the given three forces.

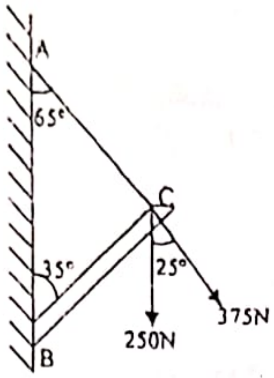


Sol^n- Here,

$\alpha = 43.603°$, $\beta = 73.740°$, $\gamma = 61.928°$

Force	X-component		Y-component	
	Force	Value	Force	Value
145N	145 cosα	105N	145 sinα	100N
250N	-250 cosβ	-69.999N	250 sinβ	240N
255N	255 cosγ	119.998N	-255 sinγ	-225.001N

Determine (a) the tension in cable AC, knowing that the resultant of the three forces exerted at C of boom BC must be along CB, (b) the corresponding magnitude of the resultant.

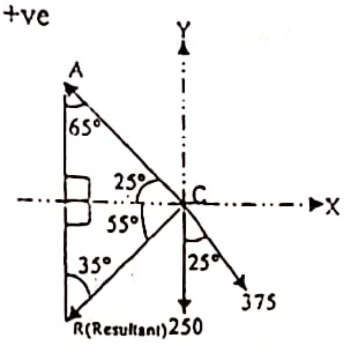


- Here,

horizontal and vertical axes through C to decompose each force.

Take →+ve
and Take ↑ +ve

Component	X-component	Y-component
T	-Tcos25°	Tsin 25°
250	0	-250
375	375 sin 25°	-375 cos25°



x = - Tcos25° + 158.482

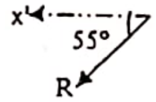
y = Tsin25° - 589.865

ΣFx = Rx or, -Tcos25° + 158.482 = -Rcos55°

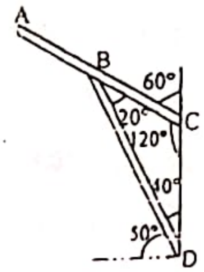
y = Ry or, Tsin25° -589.865 = -Rsin 55°

ving these two equations, we get

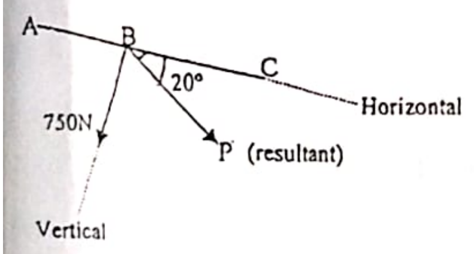
= 475.376N Ans and R = 474.836N Ans and θ=55° Ans



The hydraulic cylinder BD exerts on member ABC a force P directed along line BD. Knowing the P must have a 750 N component perpendicular to member ABC, determine (a) magnitude of P, (b) its component parallel to ABC.



- Here,



We see,

a) P sin20° = 750

or, P =2192.853N#

b) Component along BC

= P cos20°

= 2060.608N Ans

: *Rajan Gautam, Prajwal Giri, Devendra Man Palikhe*

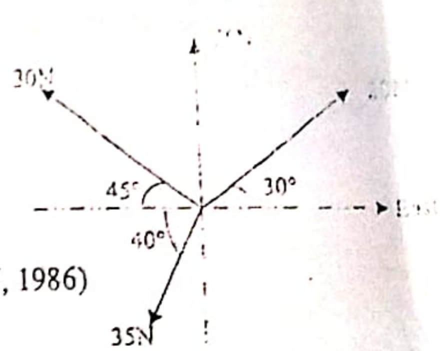
11. The following forces act at a point.

i) 20N inclined at 30° towards N of E,

ii) 25N towards N,

iii) 30N towards NW and

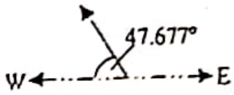
iv) 35N inclined at 40° towards S to W



Find the magnitude and direction of the resultant force (Jiwaji U, 1986)

Sol^n - Here,

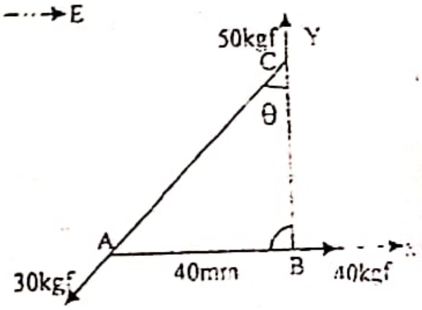
Component	X-component	Y-component
20N	20 cos 30°	20 sin 30
25N	0	25
30N	-30 cos 45°	30 sin 45°
35N	-35 cos 40°	-35 sin 40°



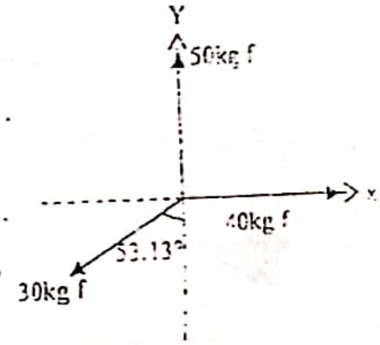
ΣFx = -30.704N ΣFy = 33.716N

So, R = 45.602N Ans θ = 47.677° Ans

12. **A triangle ABC has its sides AB = 40mm along positive x-axis and sides BC=30mm along positive y-axis. Three forces of 40kgf, 50kgf and 30kgf act along AB, BC and CA respectively. Determine the resultant of such a system of forces. (Osmania U, 1985).**



Sol^n - Here, $\tan\theta = \frac{40}{30} = \theta = 53.130°$



ΣFx = 40-30 sin53.13°

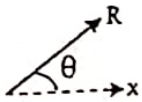
= 16kgf

ΣFy = 50-30 cos 53.13°

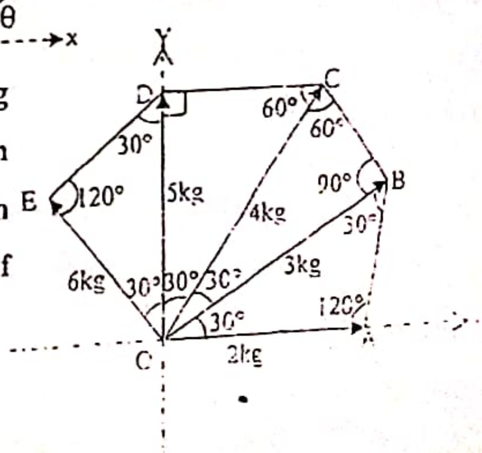
= 32kgf

R = 35.777kgf

θ = 63.435° Ans



13. **The force of 2kg, 3kg, 4kg, 5kg and 6kg are acting on one of the angular points of a regular hexagon towards the other five angular points taken in order. Find the magnitude and direction of resultant force. (Cambridge U)**



Sol^n - Here,

Using, one interior angle of a regular polygon,

$$a = \frac{(2n-4)\times 90}{n}$$

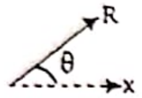
We get $a = 120°$

$\Sigma F_x = 2 + 3\cos 30° - 4\cos 60° - 6\sin 30°$

$= 3.598\text{kg}$

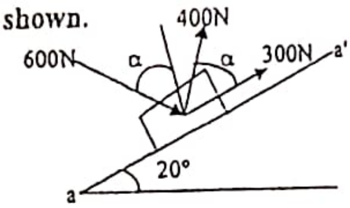
$\Sigma F_y = 3\sin 30° + 4\sin 60° + 5 + 6\cos 30°$

$= 15.160\text{kg}$



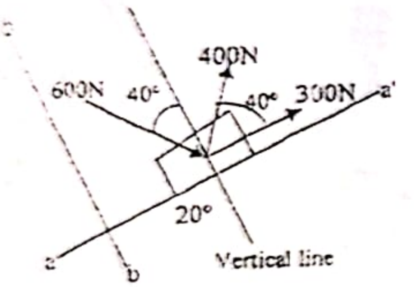
So. $R = 15.581\text{kg}$ Ans $\theta = 76.649°$ Ans

14. **Knowing that $\alpha = 40°$, determine the resultant of the forces shown.**



Sol^n - Here,

Let aa' and bb' be new X & Y axes.



$\Sigma H = 400\cos 40° + 600$

($\swarrow$ +ve) $\sin 40° + 300$

$= 992.090\text{N}$

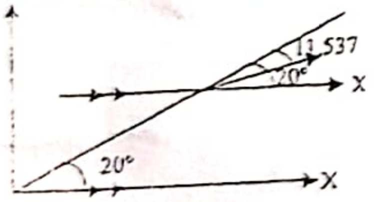
$\Sigma V = 400\sin 40° - 600\cos 40°$

($\nwarrow$+ve) $= -202.512\text{N}$

$\theta = 11.537°$

$R = 1012.548\text{N}$

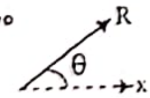
But with respect to original X & Y axes.



So, $R = 1012.548\text{N}$ Ans

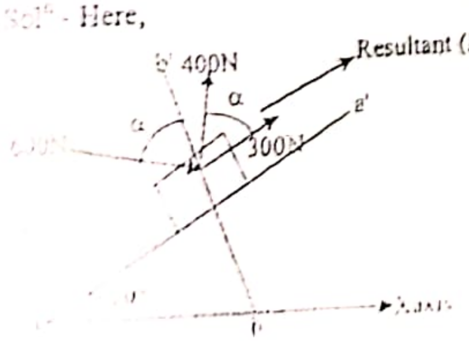
$\theta = 20° - 11.537°$

$= 8.463°$ Ans



15. **For the above Q^n. 14, determine (a) the required value of α if the resultant of the three forces shown is to be parallel to the incline, (b) the corresponding magnitude of the resultant.**

Sol^n - Here,



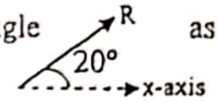
Here resultant is along horizontal direction so sum of forces in bb' must be zero that means resultant is only due to aa'.

a) $\sum F_{bb'} = 0$ (+ve) $= 400 \sin\alpha - 600 \cos\alpha = 0$

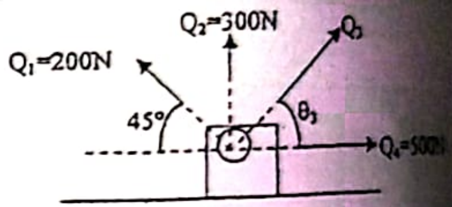
$$\text{or, } \alpha = \tan^{-1}\left(\frac{600}{400}\right) = 56.310° \text{ Ans}$$

b) $R = \sqrt{\left(\sum F_{aa'}\right)^2} = 300 + 400\cos\alpha + 600\sin\alpha$

$= 1021.11\text{N Ans}$

The resultant makes an angle  as it is parallel to incline but incline makes 20° with X-axis.

16. **The resultant of the four forces acting on the anchor shown is known to be $\vec{R} = 559\vec{i} + 788\vec{j}$ N, determine the force $\vec{Q}_3$.**



Sol^n - Here,

$\Sigma F_x = 500 + Q_3 \cos\theta_3 - 200 \cos 45°$

$= 358.579 + Q_3\cos\theta_3$

$\Sigma F_Y = Q_3 \sin\theta_3 + 300 + 200 \sin 45°$

$= 441.421 + Q_3 \sin\theta_3$

But We have, $R_x = 559$, $R_Y = 788$

So, $559 = 358.579 + Q_3\cos\theta_3$

$788 = 441.421 + Q_3\sin\theta_3$

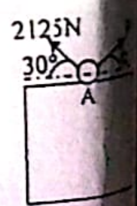
Solving, $788 = 441.421 + \dfrac{200.421}{\cos\theta_3}\sin\theta_3$

or, $\theta_3 = 59.960°$, $Q_3 = 400.358$N

$Q_{3\text{-}x} = Q_3 \cos\theta_3 = 200.421$, $Q_{3\text{-}Y} = 346.580$

So, $\vec{Q}_3 = 200.421\vec{i} + 346.580\vec{j}$ Ans

17. **A steel tank is to be positioned in an excavation. If α=20°, determine**



a) **magnitude of P if R (resultant) is to be vertical,**

b) **the value of R.**

Sol^n- Here,

a) For the resultant to be vertical,

$\Sigma H = 0$ (→+ve)

or, $P\cos 20° - 2125 \cos 30° = 0$ or, $P = 1958.411$N Ans

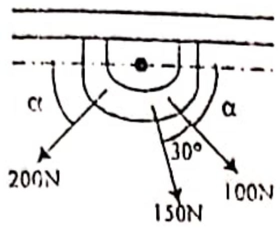
b) $R = \Sigma V = P\sin 20° + 2125 \sin 30° = 1732.316$N Ans

$\theta = \tan^{-1}\left(\dfrac{1732.316}{0}\right) = 90°$ 

18. For the collar, determine

a) the value of α if the resultant of the three forces is to be vertical,

b) the corresponding magnitude of resultant



Sol^n- Here,

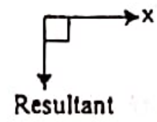
a) For the resultant to be vertical,

$\Sigma H = 0$ (+ve→)

or, $100 \cos\alpha + 150 \cos(30°+\alpha) - 200\cos\alpha = 0$

or, $100 \cos\alpha + 150 \cos30°\cos\alpha - 150 \sin30° \sin\alpha - 200\cos\alpha = 0$

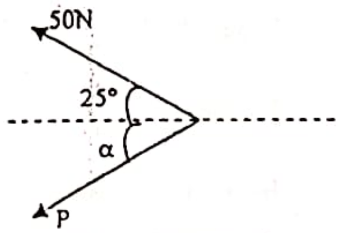
or, $29.90\cos\alpha = 75\sin\alpha$ or, $\tan\alpha = \dfrac{29.90}{75}$



or, $\alpha = 21.736°$ **Ans**

b) $R = \Sigma V = -100\sin\alpha - 150 \sin(30+\alpha) - 200 \sin\alpha =$ **-228.874N Ans**

19. Find the angle α made by P(35N) if the resultant is horizontal.



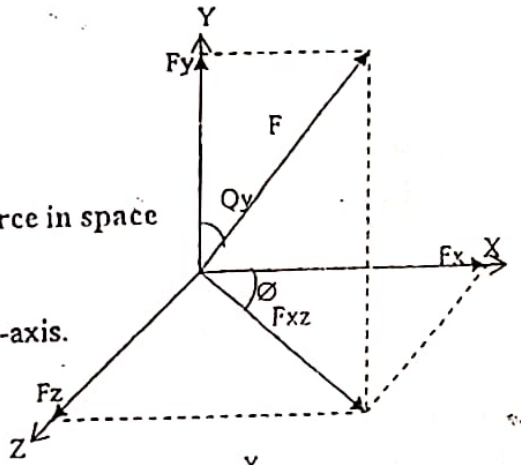
Sol^n-Here,

If the resultant of two or more forces is horizontal then the sum of their vertical components is zero and vice-versa.

So,

$P\sin\alpha = 50 \sin25°$

$\Rightarrow \alpha = 37.138°$ Ans



3.2.2 In 3D-space; Rectangular components of a force in space

Rectangular Components of a force in space

Here F is a force in space which makes θy angle with Y-axis.

We use rectangular resolution to get

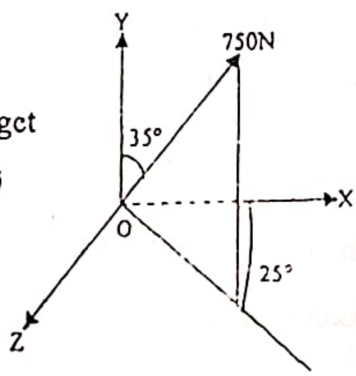
$F_y = F\cos\theta_y$

$F_{xz} = F\sin\theta_y$

Here Fxz lies in x-z plane. Now resolving Fxz using Ø angle, we get

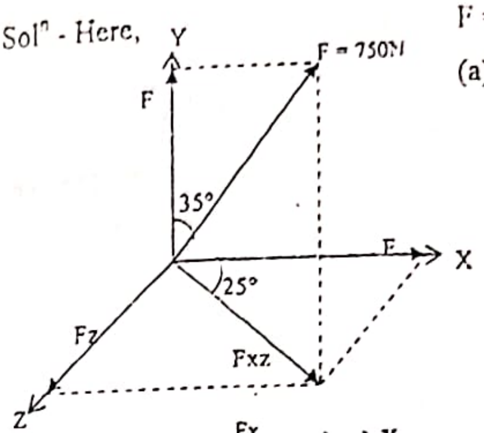
$F_x = F_{xz} \cos\varnothing = F \sin\theta_y \cos\varnothing$ and $F_z = F_{xz} \sin\varnothing = F \sin\theta_y \sin\varnothing$

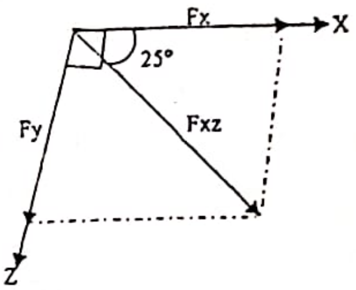
Remember $F^2 = F_x^2 + F_y^2 + F_z^2$



20. Determine (a) x, y, z components of the 750 N force (b) the angles θx, θy and θz that the force forms with the coordinate axes.

Sol^n - Here,





F = 750N

(a) Fy = 750cos 35°

= 614.364N Ans

Fxz = 750sin 35°

Now, for xz- plane

Fx = Fxz cos25°

= 389.878N

Fy = Fxz sin25°

= 181.803N Ans

b) We know,

Fx = F cosα, Fy = Fcosβ, Fz = Fcosγ

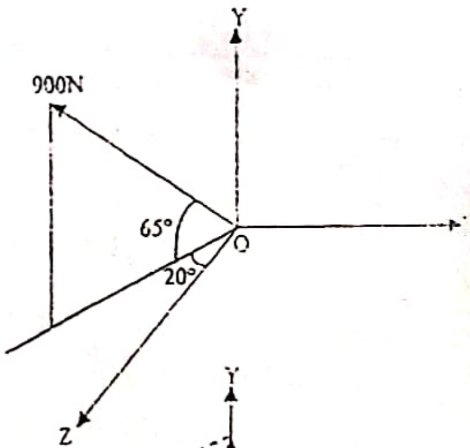
where α, β, γ are angles made by F with X, Y and Z-axis respectively.

So, $\theta x = \cos^{-1}\left(\frac{Fx}{F}\right) = 58.68°$ Ans

$\theta y = \cos^{-1}\left(\frac{Fy}{F}\right) = 35°$ Ans

$\theta z = \cos^{-1}\left(\frac{Fz}{F}\right) = 75.97°$ Ans

21. Determine (a) x, y, z components of the 900N force (b) the angles θx, θy and θz that the force forms with the coordinate axes.

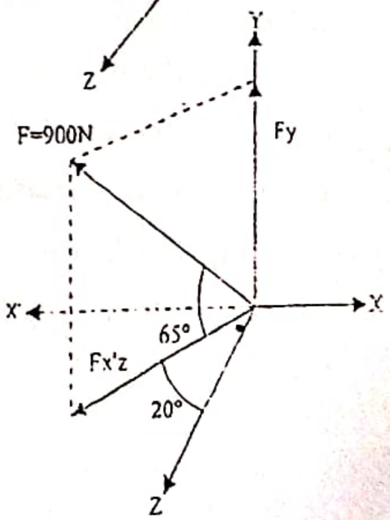


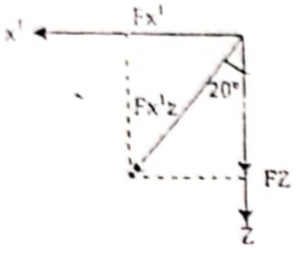
Sol^n - Here,

a) Fy = 900 sin65°

= 815.68N Ans

Fx'z = 900cos 65°





Now, for x'z- plane

$Fx' = Fx'z \sin 20°$

$= 130.09N$ Ans

But $F_x = -130.09N$ Ans

and $Fz = Fx'z \cos 20° = 357.42N$ Ans

b) $\theta x = \cos^{-1}\left(\frac{Fx}{F}\right) = 98.31°, \theta y = 25°, \theta z = 66.6°$ Ans

22. Find the resultant of forces of Q^n.20 and Q^n.21.

Sol^n- Here,

We use the tabulation for our simplicity where each row gives information of each force where our calculation begins with first finding vertical component and then the horizontal component, then we again decompose the horizontal component into its parts. Please look figures of above questions and give proper direction of force components like (+) X or (-) X similarly give proper notations for plane like XY' or XY or X'Y or X'Z ....

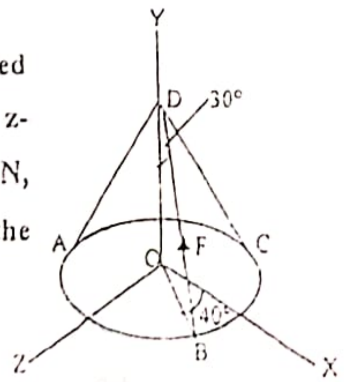
	Vertical Compo.		Horizontal Compo.		Compo of horizontal Compo.			
Force	Axis	Magnitude	Plane	Magnitude	Axis	Magnitude	Axis	Magnitude
750N	(+) Y	750 cos35°	XZ	750 sin35°	(+)X	750sin35° cos 25°	(+)Z	750 sin35° sin25°
900N	(+)Y	900 sin65°	X'Z	900cos65°	(-)X	900 cos65° sin20°	(+)Z	900 cos65° cos20°
	$\sum Y = 1430.04$				$\sum X = 259.79$		$\sum Z = 539.22$	

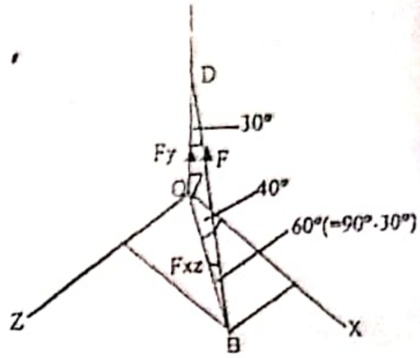
Now, $R = \sqrt{(\sum X)^2 + (\sum Y)^2 + (\sum Z)^2} = 1550.25N$ Ans

So, $\theta x = \cos^{-1}\left(\frac{\sum X}{R}\right) = 80.35°, \theta z = 69.65°$ Ans

Now our procedure will begin with some how directly but figure shows clearly its direction so plz read figures carefully.

23. A horizontal circular plate is suspended from three wires attached to a support D and form 30° with the vertical knowing that the z-component of the force exerted by wire BD on the plate is -32.14N, determine (a) tension in wire BD, (b) the angles θx, θy and θz that the force exerted at B forms with the coordinate axes.





Sol^n- Here,

We see that $\vec{F} = \vec{F}xz + \vec{F}y$

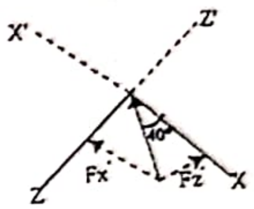
But $Fy = F\sin 60°$ (1)

$Fxz = F\cos 60°$ (2)

We see decomposition of horizontal force of XZ - plane:

$Fx^{l} = Fxz \cos 40°$ (3)

$Fz^{l} = Fxz \sin 40°$ (4)



But Fz = -32.14N (given)

$= Fz^{l} = 32.14N$

So, from (4), Fxz = 50N

From (2), F = 100N Ans

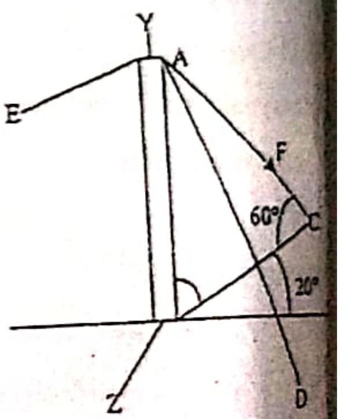
From (1), Fy = 86.603N

From (3), Fx' = 38.30N = Fx = -38.30N

So, $\theta x = \cos^{-1}\left(\frac{-38.30}{100}\right) = 112.520°$ Ans

$\theta y = 29.999°$ and $\theta z = 108.748°$ Ans

24. The end of the coaxial cable AE is attached to the pole AB, strengthened by guy wires AC and AD. Knowing that the tension in wire AC is 600N, determine (a) the components of the force exerted by this wire on the pole, (b) angles θx, θy and θz that the force forms with the coordinate axes.



Sol^n- Here,

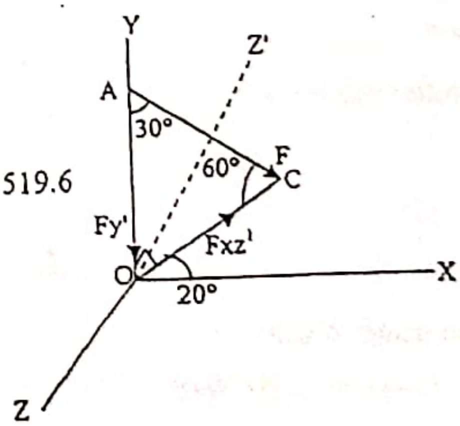
F = 600N

$Fy' = F\cos 30° = 519.6$

$Fy = -519.6N$#

$Fxz' = F\sin 30°$

= 300N



Now,

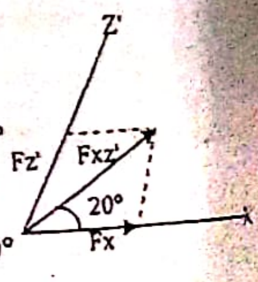
$Fx = Fxz' \cos 20°$

= 281.91N#

$FZ' = Fxz' \sin 20°$

= 102.61N

or, Fz = -102.61N#



So, $\theta x = 61.98°$, $\theta y = 150°$, $\theta z = 99.83°$ Ans

Remember: $\theta x+\theta y+\theta z < 360°$ & $\theta x+\theta y+\theta z \geq 180°$

A force acts at the origin of a coordinate system in a direction defined by the angles θx 70.9° and θy =144.9°. Knowing that the Z component of the force is -260N, determine (a) θz (b) the other components and magnitude of F.

- Here,

We know

$\cos^2\theta x+\cos^2\theta y+\cos^2\theta z=1$

or, $\cos^2\theta z = 0.224$ or, $\cos\theta z = \pm 0.473$

e know,

$$\cos\theta z = \frac{Fz}{F} \quad or, \cos\theta z = \frac{-260}{F}$$

ere F being square root of summation of square, can't be negative.

, $\cos\theta z$ must be -ve.

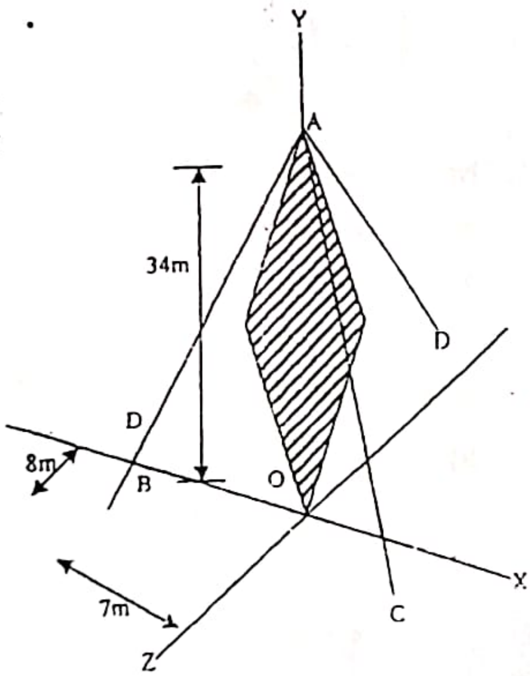
. $\cos\theta z$ = -0.473 or, θz = 118.229°Ans

$F\cos\theta z = Fz$

or, F = 549.687N#

$Fx = F\cos\theta x$ = 179.867NAns Fz = -449.726NAns

6. A transmission tower is held by three guy wires anchored by bolts at B, C, D. If the tension in wire AB is 2625N, determine the components of the force exerted by the wire on the bolt at B.



ol^n- Here,

Ve need to find force exerted by wire on the bolt at B which is directed from B to A.

(-7, 0, 8) and A(0, 34, 0)

$$\text{o, } \vec{BA} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$= (0 + 7)\vec{i} + (34 - 0)\vec{j} + (0 - 8)\vec{k}$$

$$= 7\vec{i} + 34\vec{j} - 8\vec{k}$$

$$\hat{BA} = \frac{7\vec{i} + 34\vec{j} - 8\vec{k}}{\sqrt{7^2 + 34^2 + 8^2}} = \frac{7\vec{i} + 34\vec{j} - 8\vec{k}}{35.62}$$

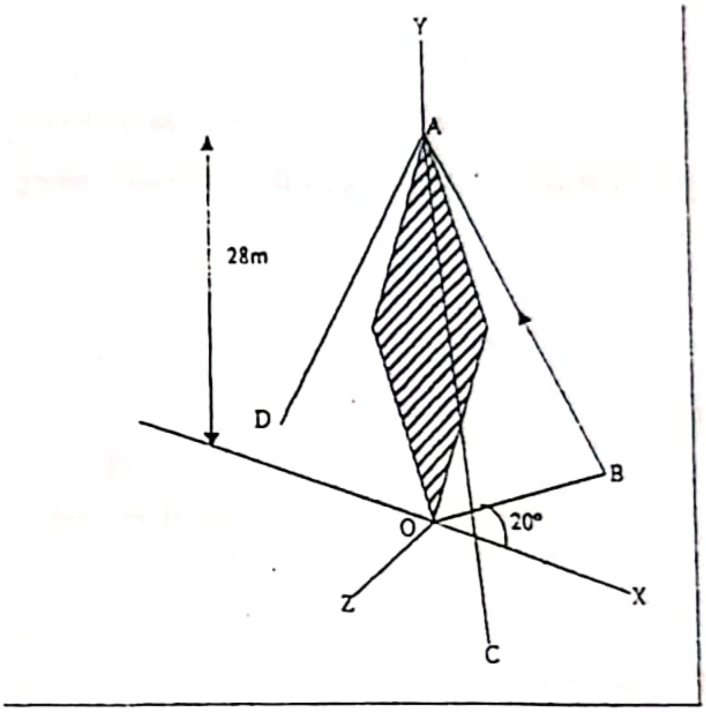
$$\text{So, } \vec{F_{BA}} = F \times \hat{BA}$$

$= 2625 \times \frac{7\vec{i} + 34\vec{j} - 8\vec{k}}{35.62}$

$= 515.86\vec{i} + 2506.62\vec{j} - 589.56\vec{k}$

So, Fx = 515.86NAns Fy = 2506.62NAns Fz = -589.56NAns

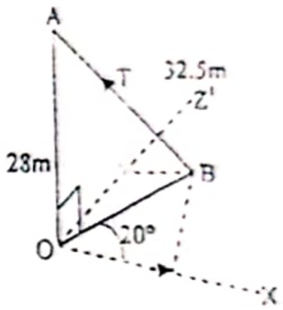
27. Cable AB is 32.5m long, and the tension in that cable is 20KN. Determine (a) x, y, z components of the force exerted by the cable on the anchor B. (b) the angles θx, θy and θz defining the direction of that force.



Sol^n- Here,

AB = 32.5m

T_{BA} = 20kN



$OB = \sqrt{32.5^2 - 28^2} = 16.5$

$$\hat{T}_{BA} = \frac{-OB\cos 20°\,\vec{i} + 28\,\vec{j} + OB\sin 20°\,\vec{k}}{\sqrt{(-OB\cos 20°)^2 + 28^2 + (OB\sin 20°)^2}} = -0.477\vec{i} + 0.862\vec{j} + 0.174\vec{k}$$

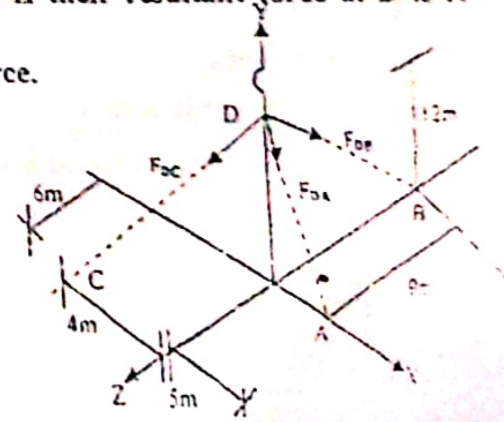
$T_{BA} = -9.54\vec{i} + 17.24\vec{j} + 3.48\vec{k}$

a) Fx = -9.54kN, Fy = 17.24kN, Fz = 3.48kN Ans

b) θx = 118.490° θy = 30.458° θz=79.98° Ans

28. Three forces act on a hook at D as shown. If their resultant force at D is R = -370 $\vec{j}$ N, determine the magnitude of each force.

Sol^n- Here,



$$\vec{F}_{DA} = F_{DA} \times \frac{5\vec{i} - 12\vec{j}}{\sqrt{5^2 + 12^2}} = \frac{5\vec{i} - 12\vec{j}}{13} F_{DA}$$

$$\vec{F}_{DA} = F_{DA} \times \frac{-12\vec{j} - 9\vec{k}}{15}$$

$$\vec{F}_{DC} = F_{DC} \times \frac{-4\vec{i} - 12\vec{j} + 6\vec{k}}{14}$$

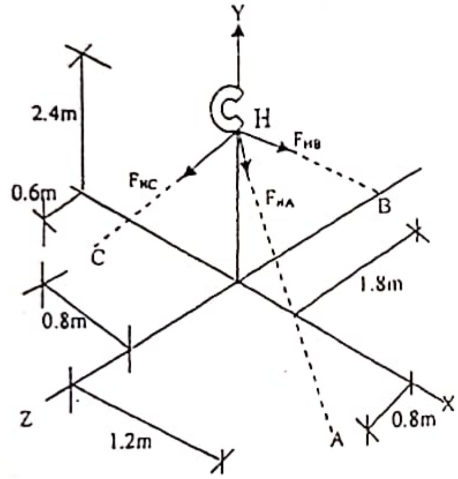
Adding, we get

$$\left(\frac{5}{13}F_{DA} - \frac{4}{14}F_{DC}\right)\vec{i} + \left(-\frac{12}{13}F_{DA} - \frac{12}{15}F_{DB} - \frac{12}{14}F_{DC}\right)\vec{j} + \left(-\frac{9}{15}F_{DB} + \frac{6}{14}F_{DC}\right) = -370\vec{j}$$

Solving,

F_{DA} = 130NAns $\quad$ F_{DB} = 125NAns $\quad$ F_{DC} = 175NAns

29. Three forces $\vec{F}_{HA}, \vec{F}_{HB}$ *and* $\vec{F}_{HC}$ act on hook at H. If the magnitude of these forces are F_{HA} = 420N, F_{HB} = 500N and F_{HC} = 390N, determine the resultant force R acting on the hook.



Sol^n- Here,

H(0, 2.4, 0)

A (1.2, 0, 0.8), B(0, 0, -1.8), C(-0.8, 0, 0.6)

$$\vec{F}_{HA} = 420 \times \frac{1.2\vec{i} - 2.4\vec{j} + 0.8\vec{k}}{2.8}$$

$$\vec{F}_{HB} = 500 \times \frac{-2.4\vec{j} - 1.8\vec{k}}{3}$$

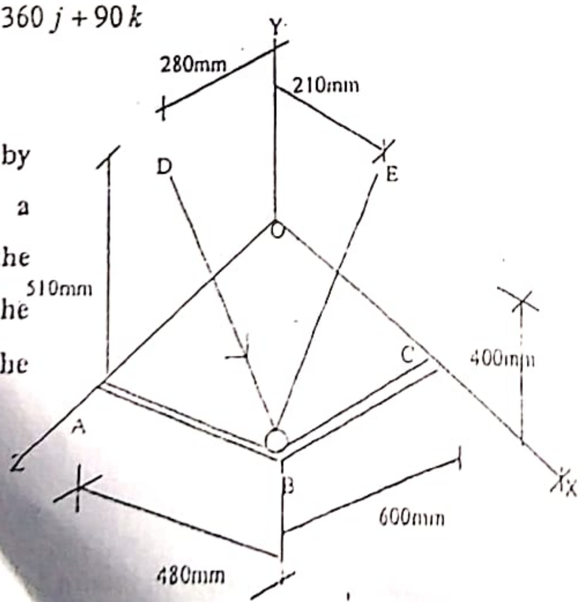
$$\vec{F}_{HC} = 390 \times \frac{-0.8\vec{i} - 2.4\vec{j} + 0.6\vec{k}}{2.6}$$

Now,

$$\vec{R} = 180\vec{i} - 360\vec{j} + 120\vec{k} - 400\vec{j} - 300\vec{k} - 120\vec{i} - 360\vec{j} + 90\vec{k}$$

$$= 60\vec{i} - 1120\vec{j} - 90\vec{k} \text{ Ans}$$

30. A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B. Knowing that the tension in the cable is 385N, determine the components of the force exerted by the cable on the support at D.



Sol^n- Here,

T_{DB} = 385N

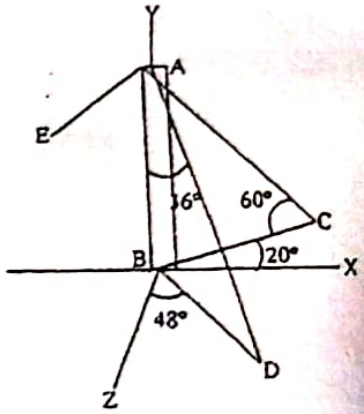
$$\vec{DB} = \frac{480\vec{i} - 510\vec{j} + (600-280)\vec{k}}{770} \times 385$$

$$= 240\vec{i} - 255\vec{j} - 160\vec{k}$$

So, $F_x = 240N$, $F_y = -255N$, $F_z = 160N$ Ans

Note:- Study that the dimensions given in parallel to an axis shows distance in that axis.

31. **The end of the coaxial cable is AE attached to the pole AB, which is strengthened by guy wires AC and AD. Knowing that the tension in AC is 750N and that the resultant of the forces exerted at A by the wires AC and AD must be contained in the xy plane, determine (a) the tension in AD, (b) the magnitude and direction of the resultant of the two forces.**



Sol^n- Here,

Force	X-compo.	Y-Compo.	Z-Compo.
$\vec{T}_{AC}$	$T_{AC}\cos60°\cos20°$	$-T_{AC}\sin60°$	$-T_{AC}\cos60°\sin20°$
$\vec{T}_{AD}$	$T_{AD}\sin36°\sin48°$	$-T_{AD}\cos36°$	$T_{AD}\sin36°\cos48°$

Now, for the resultant to be in XY plane i.e. the condition that all the sum of tensions has zero horizontal component so pole is then vertical,

$\sum Z$- component = 0

or, $-750\cos60°\cos20° + T_{AD}\sin36°\cos48° = 0$

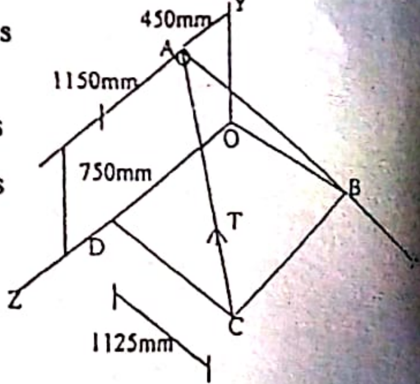
or, $T_{AD} = 326.102N$ Ans

b) $F = \sqrt{\sum X^2 + \sum Y^2}$

$= \sqrt{(494.78)^2 + (-913.26)^2} = 1038.67N$ Ans

$\theta x = \cos^{-1}\left(\frac{\sum x}{F}\right) = 61.55°$, $\theta y = 151.55°$, $\theta z = 90°$ Ans

32. Knowing that the tension in the cable AC is 2000N, determine the component of the forces exerted on the plane at C. (PU-2012-Spring)



$$\frac{1125\,\vec{i} + 750\,\vec{j} - (1600 - 450)\,\vec{k}}{1775}$$

$$= 2000 \times \frac{-1125\,\vec{i} + 750\,\vec{j} - 1150\,\vec{k}}{1775}$$

$7.606\,\vec{i} + 845.070\,\vec{j} - 1295.775\,\vec{k}$

$= -1267.606N$, $F_y = 845.070N$, $F_z = -1295N$ **Ans**

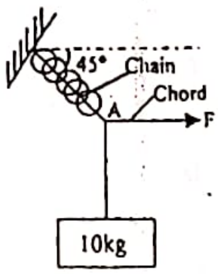
$s^{-1}\left(\frac{-1267.606}{2000}\right) = 129.331°$, $\theta_y = 65.005°$, $\theta_z = 130°$ **Ans**

ou see the ring or hook at A, so tension T_{AC} i.e. 2000N must be equal to T_{AB} only in ude of force but not necessarily in components.

Equilibrium of a particle

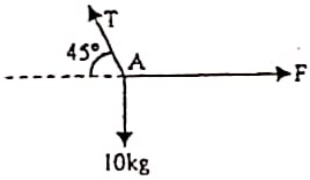
In 2D plane

A 10kg weigh is suspended from ceiling by a chain as in figure. The chain makes an angle of 45° with the ceiling. When it is pulled by a chord horizontally, find the tension in the chain and the pulling force in the chord.



Here,

ody diagram at 'A'

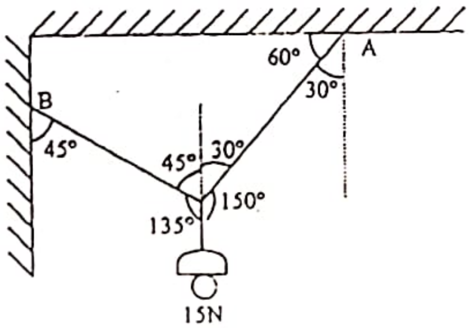


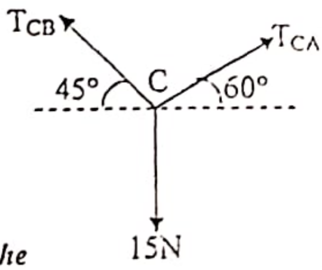
equilibrium equation at 'A',

) $(+ve\rightarrow) \Rightarrow F - T\cos 45° = 0$

) $(+ve\uparrow) \Rightarrow T\sin 45° = 10g \Rightarrow T = 138.593N$ **Ans** $F = 98N$ **Ans**

An electric light fixture weighing 15N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the vertical as shown in figure. Determine the forces in the strings AC and BC. (Kanpur U, 1986)





Here,

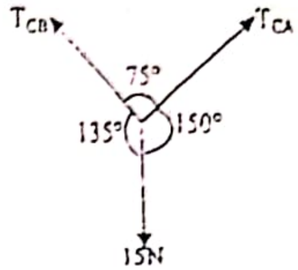
1

equilibrium eq^n,

$T_{CA}\sin 60^\circ + T_{CB}\sin 45^\circ = 15$

Solving, $T_{CA} = 10.981N$, $T_{CB} = 7.765N$ Ans

WAY-2

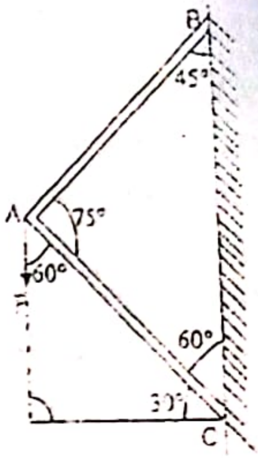


By sine law,

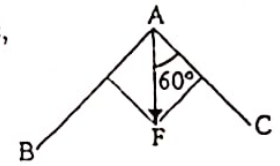
$$\frac{15}{\sin 75^\circ} = \frac{T_{CA}}{\sin 135^\circ} = \frac{T_{CB}}{\sin 150^\circ}$$

or, $T_{CA} = 10.981N$ & $T_{CB} = 7.765N$ Ans

35. **The vertical force F = 500N acts downward at A on the two membered frame. Determine the magnitudes of the two components of F directed along the axes of AB and AC.**



Sol^n- Here,

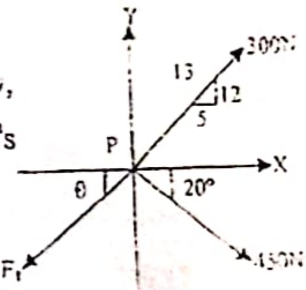


$F_{AC} = 500\cos 60^\circ =$ **250N Ans**

$F_{AB} = 500\sin 60^\circ =$ **433.013N Ans**

36. **Determine the magnitude and angle of F_1 so that particle P is in equilibrium.**

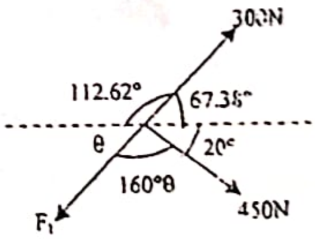
Sol^n- Here,



We can use sine law as well as equilibrium eq^n in 2d-pane. As we know, 2doplane equilibrium eq^n can solve for at most two unknowns as two eq^{n}s can only solve at most two unknowns.

WAY-1

By sine law,



$$\frac{450}{\sin(112.62+\theta)} = \frac{F_1}{\sin 87.38^\circ} = \frac{300}{\sin(160-\theta)}$$

Equating values of F_1,

$$F_1 = \frac{450\sin 87.38^\circ}{\sin(112.62+\theta)} = \frac{300\sin 87.38^\circ}{\sin(160-\theta)}$$

or, $449.530\sin(160-\theta) = 299.686\sin(112.62+\theta)$

or, $449.530(\sin 160\cos\theta - \cos 160\sin\theta) = 299.686(\sin 112.62\cos\theta$ ...

or, $537.68\sin\theta = 122.885\cos\theta$ or, $\theta = \tan^{-1}\left(\frac{122.885}{537...}\right)$

WAY -2

$\sum Fx(+ve\rightarrow) = 0 \Rightarrow 450\cos20° + 300\cos67.38° = F_1\cos\theta$

$\sum Fy(\uparrow+ve) = 0 \Rightarrow 450\cos20° + 300\cos67.38° = F_1\sin\theta$

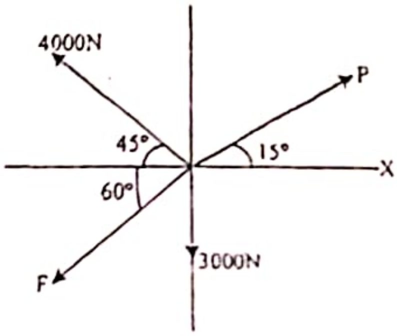
Squaring and adding,

$(23.014)^2 + (538.247)^2 = F_1^2(\sin^2\theta + \cos^2\theta)$

or, $F_1 = 552.125N$ $\theta = 12.874°$ Ans

37. The forces on the gusset plate of a joint in a bridge trusses act as shown. Determine the values of P and F to maintain the equilibrium of the joint.

Sol^n- Here,



$\sum Fx(\rightarrow+ve) = 0$

$\Rightarrow P\cos15° - 4000\cos45° - F\cos60° = 0$

$\sum Fy(\uparrow+ve) = 0$

$\Rightarrow P\sin15° + 4000\sin45° - F\sin60° - 3000 = 0$

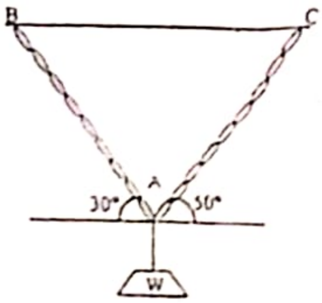
Solving, $P = 3342.781N$ Ans

$F = 800.903N$ Ans

For Q^n.38, $\sum Fx=0$ $\Rightarrow T_{AB}\cos30° = T_{AC}\cos50°$

$$\Rightarrow \frac{TAB}{TAC} = 0.7422 \text{ so } TAC > TAB \text{ SO } TAC \text{ limits tension.}$$

38. Determine the largest weight W that can be supported by two wires. The stress in either wire is not to exceed 30N/cm^2. The cross-sectional areas of wires AB and AC are 0.4cm^2 and 0.5cm^2 respectively. (Refer Q^{n}44 for details)



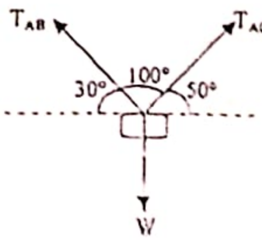
Sol^n- Here,

The question provides no limitation in arrangement like length of wire so we can directly use maximum tension for each wire.

$T_{AB} = 12N$ $(=0.4\times30)$

$T_{AC} = 15N$

By sine law,

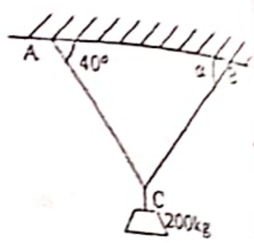


$$\frac{W}{\sin100°} = \frac{12}{\sin140°} = \frac{15}{\sin120°}$$

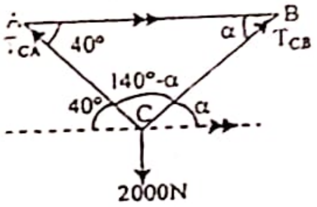
[illegible] 17.057N Ans

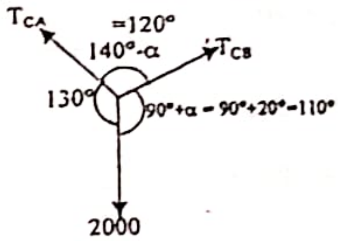
W [illegible] when T_{AC} = 15N [illegible]

39. Two cables are tied at C and are loaded as shown Knowing $\alpha=20^\circ$, determine the tension (a) in cable AC (b) in cable BC.



Sol^n - Here,





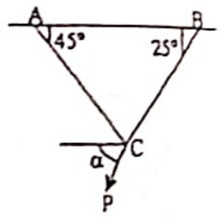
By sine law,

$$\frac{T_{CB}}{\sin 130^\circ}=\frac{T_{CA}}{\sin 110^\circ}=\frac{2000}{\sin 120^\circ}$$

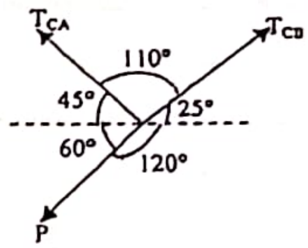
$\Rightarrow T_{CB} = 1769.104N$ & $T_{CA} = 2170.127N$ Ans

40. Two cables are tied together at C and are loaded as shown. Knowing that P=500N and $\alpha=60^\circ$, determine the tension.

a) in cable AC, b) in BC.



Sol^n- Here,

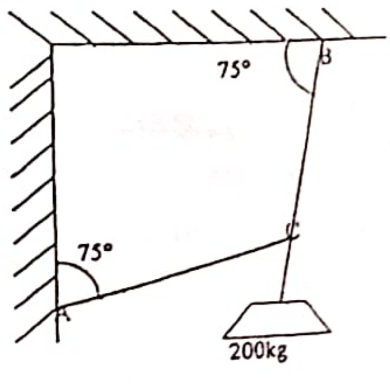


By sine law,

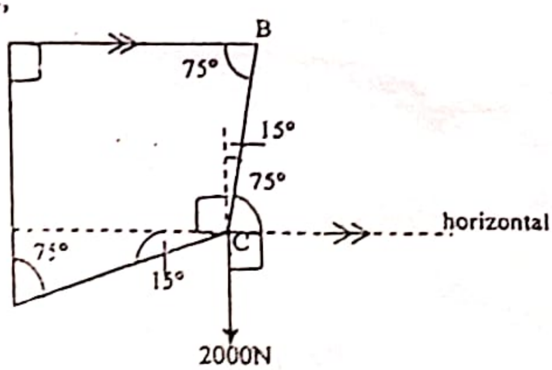
$$\frac{500}{\sin 110^\circ}=\frac{T_{CB}}{\sin 105^\circ}=\frac{T_{CA}}{\sin 145^\circ}$$

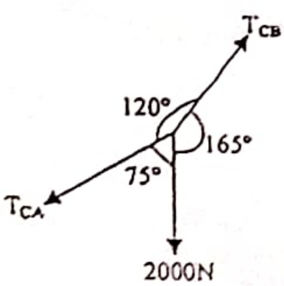
$\Rightarrow T_{CB} = 513.958N$ and $T_{CA} = 305.194N$ Ans

41. Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC (b) in cable BC.



Sol^n- Here,



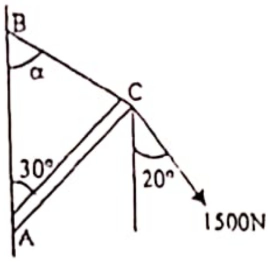


By sine law,

So drawing FBD, we get $\dfrac{2000}{\sin 120°} = \dfrac{T_{CB}}{\sin 75°} = \dfrac{T_{CA}}{\sin 165°}$

∴ T_{CB} = 2230.710N and T_{CA} = 597.717N Ans

42. Knowing that α = 55° and that boom AC exerts on pin C a force directed along line AC, determine (a) the magnitude of that force (b) the tension in BC.

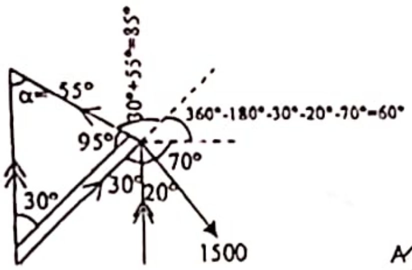


Sol^n- Here,

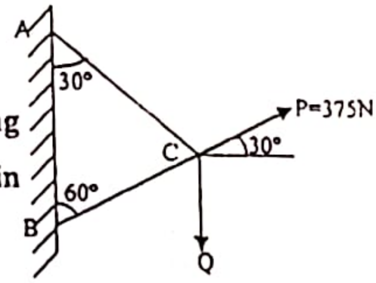
By sine law,

$$\frac{1500}{\sin 85°} = \frac{T_{AC}}{\sin 145°} = \frac{T_{CB}}{\sin 130°}$$

= F_{AC} = 863.651N, T_{CB}=1153.456N Ans



43. Two cables tied together at C are loaded as shown. Knowing that Q=300N, determine the tension (a) in cable AC, (b) in cable BC.



Sol^n-Here,

We have four forces in equilibrium and we can't use sine law as sine law is applicable for only three forces however we can combine two forces to one to use sine law. The rectangular resolution of forces and solving by equilibrium eq^n is applicable for any no. of forces 2, 3, 4, 5 or 10 but there is limitation that two equilibrium eq^{n}s of 2d plane can solve at most two unknowns only.

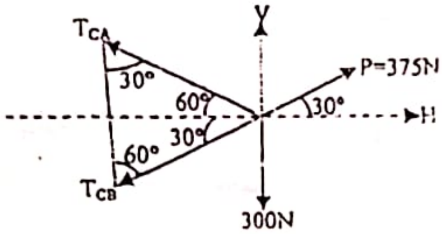
$\Sigma H=0$

$\Rightarrow 375\cos 30° - T_{CA}\cos 60° - T_{CB}\cos 30° = 0$ (a)

$\Sigma V = 0$

$\Rightarrow -300 + 375\sin 30° + T_{CA}\sin 60° - T_{CB}\sin 30° = 0$ (b)

Solving (a) & (b), T_{CB} = 225N, T_{CA}=260N Ans



44. For the above Q^n.43, Determine the range of values of Q for which the tension will not exceed 300N in either cable.

Sol^n- Here,

$\Sigma H=0$ (+ve→)

$\Rightarrow T_{CA}\cos 60° + T_{CB}\cos 20° = 375\cos 30°$ (a)

and $\Sigma V = 0 \Rightarrow 375\sin 30° = Q - T_{CA}\sin 60° + T_{CB}\sin 30°$ (b)

From (a) we get T_{CA} in terms of T_{CB} as $T_{CA} = \dfrac{375\cos 30° - T_{CB}\cos 30°}{\cos 60°}$ .....

From (b), $375\sin 30° = Q - \sin 60° \dfrac{375\cos 30° - T_{CB}\cos 30°}{\cos 60°} + T_{CB}\sin 30°$

$\Rightarrow Q = 750 - 2\,T_{CB}$ .... (**)

When $T_{CB}=0 \Rightarrow Q = 750,\ T_{CA} = 649.519$.............................. (1)

$T_{CB} = 300 \Rightarrow Q = 150,\ T_{CA} = 129.904$......................... (2)

When $T_{CA} = 0 \Rightarrow Q = 0,\ T_{CB} = 375$ .... (3)

$T_{CA} = 300 \Rightarrow Q = 346.410,\ T_{CB} = 201.795$.............................. (4)

We get (2), (4) applicable so, $150 \le Q \le 346.410$N

The reason for such direct inspection by looking some limiting values is that the relation of T_{CA}, T_{CB}, Q is linear by (*), (**) but not quadratic or other.

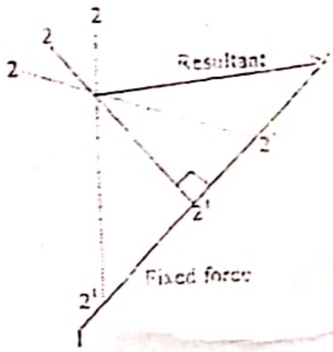
Let us look back Q^n (38),

$\Sigma F_x = 0 \Rightarrow T_{AC}\cos 50° - T_{BA}\cos 30° = 0$

At $T_{AC} = 15$N, $T_{BA} = 11.131$N (a)

At $T_{BA} = 12$N, $T_{AC} = 16.168$ (b)

So, (a) is applicable & $T_{AC} = 15$ is used where $T_{BA} = 11.131$N not 12N..



If two forces give a resultant then the condition for a force to be minimum is that it makes an angle 90° with the other. Let R be resultant. 11' be a fixed force. 22' the other force is shown for three possible directions. We see that the minimum value of 22' occurs when it is ⊥ to 11' as the shortest distance bet^n a point and a line is 90°. So the solid line 22' is minimum.

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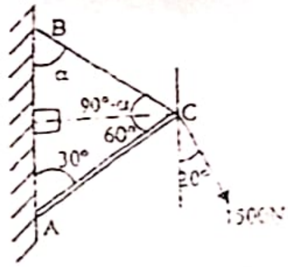
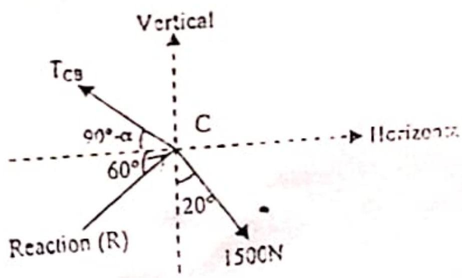
45. For the structure and loading, determine (a) the value of α for which the tension in cable BC is as small as possible, (b) the corresponding value of the tension.



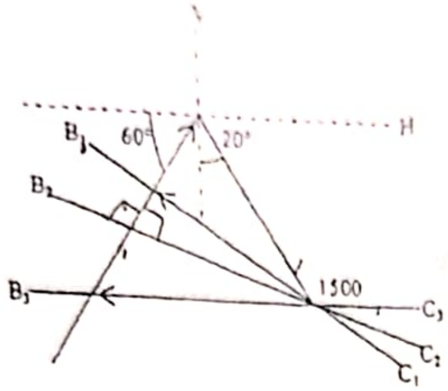
Sol^n- Here,

The equilibrium has generally two types of solutions-one is graphical and next is trigonometrical calculations.

WAY-1 Graphical Way

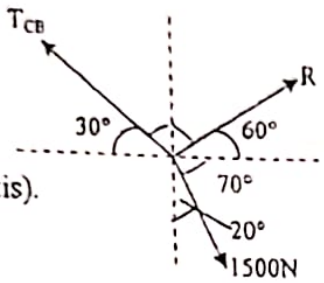


Look the above FBD, we see that 1500N has fixed direction 20° with the vertical, reaction is also fixed at 60° to horizontal but T_{CB} is not fixed as α is unknown to be determined so the figure shows various possible lines of action of T_{CB}. As we know for equilibrium, the forces form a triangle (for three forces) is according to def^n of vector or triangle law of vector addition.



We see that C_2B_2 has minimum value out of three possible values of T_{CB} because this line makes 90° with reaction (R). So,

By sine law, $\dfrac{1500}{\sin 90°} = \dfrac{T_{CB}}{\sin(60° + 70°)}$



$\Rightarrow T_{CB} = 1149.067N$ Ans

From FBD, 90°-α is angle made by T_{CB} with horizontal (-ve axis).

So, 90°-α=30°

$\Rightarrow$ α=60° Ans

WAY-2 Trigonometrical calculation

$\sum H=0 \Rightarrow 1500 \sin 20° - T_{CB} \cos (90°-\alpha) + R \cos 60°=0$

$\sum V=0 \Rightarrow -1500 \cos 20° + T_{CB} \sin(90°-\alpha) + R \sin 60°=0$

Equating R of two eq^{n}s, we get

$$\frac{1500 \sin 20° - T_{CB} \sin \alpha}{\cos 60°} = -\frac{1500 \cos 20° + T_{CB} \cos \alpha}{\sin 60°}$$

$$\Rightarrow T_{CB} = \frac{1500(\sin 20° \sin 60° + \cos 20° \cos 60°)}{\cos 60° \cos \alpha + \sin 60° \sin \alpha} \quad \text{..........} (*)$$

For T_{CB} be minimum, cos60°cosα+sin60° sinα should be maximum.

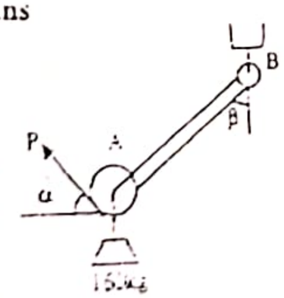
d(maximum)=0 [At maximum point, slope=0]

$$= \frac{d(\cos 60° \cos \alpha + \sin 60° \sin \alpha)}{d\alpha} = 0$$

$\Rightarrow -\cos 60° \sin \alpha + \sin 60° \cos \alpha = 0 \quad \Rightarrow \tan \alpha = \dfrac{\sin 60°}{\cos 60°} \Rightarrow \alpha=60°$ Ans

From (*), $T_{CB} = 1149.07N$ Ans

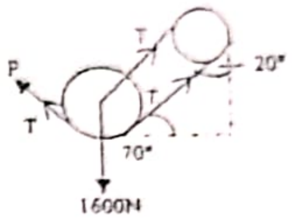
46. A 160kg load is supported by rope-pulley. Knowing β=20°, determine the magnitude and direction of P to maintain equilibrium.



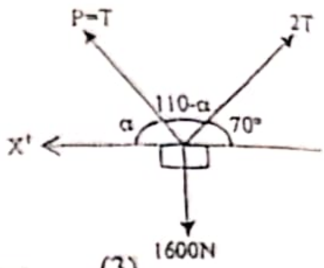
Sol^n- Here,

Tension in the rope is same on each side of a simple pulley be T (say).

By sine law,



FBD at A



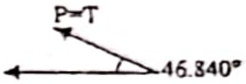
$$\frac{1600}{\sin(110-\alpha)} \ldots(1) \quad = \frac{T}{\sin 160^\circ} \ldots..(2) \quad = \frac{2T}{\sin(90^\circ+\alpha)} \ldots\ldots(3)$$

From (2) (3), 2 sin 160° = sin (90° + α)

2sin 160° = cos α

$\Rightarrow \alpha = 46.840^\circ$ **Ans**

$\Rightarrow T = 613.303N$ **Ans**,

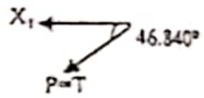


2 sin 160° = sin (90°+α)

sin (sin^{-1}(2sin160°)) = sin(90°+α)

$\Rightarrow$ sin (43.160) = sin(90°+α)

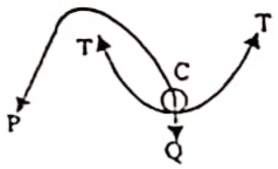
$\Rightarrow \alpha = -46.840^\circ$ **Ans**

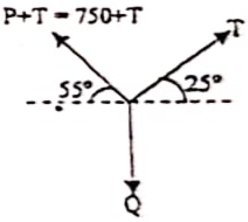


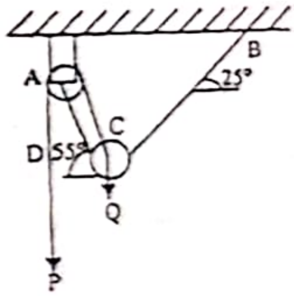
T = 1391.386N **Ans**

47. A load Q is applied to the pulley C, which can roll on the cable ACB. The pull held by cable CAD which supports a load P(750N), determine (a) tension in c ACB, (b) magnitude of load Q.

Sol^n-Here,



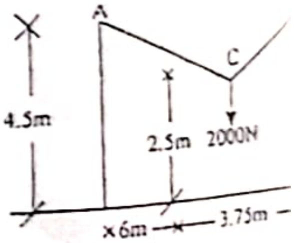




By sine law, $\frac{T}{\sin 145^\circ} = \frac{750+T}{\sin 115^\circ} = \frac{Q}{\sin 100^\circ}$

$\Rightarrow$ T = 1292.88N Q=2219.83N Ans

48. Two cables are tied together at C and loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

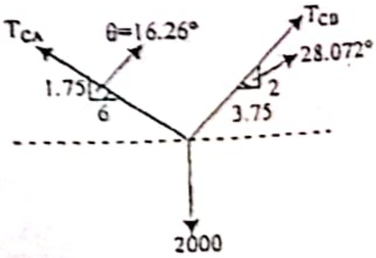


Sol^n- Here,

By sine law,

$$\frac{2000}{\sin 135.668} = \frac{T_{CA}}{\sin 118.072^\circ} = \frac{T_{CB}}{\sin 106.260}$$

$\Rightarrow T_{CA} = 2.525kN$, $T_{CB} = 2.748kN$ Ans

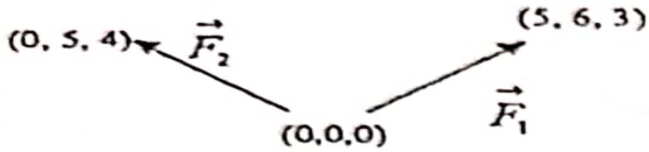


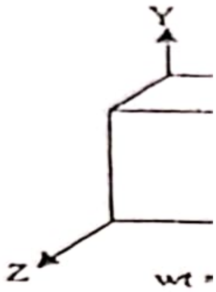
.2 In 3D space

Find the magnitude and direction of $\vec{F_3}$, that is required for ec

ven: $F_1 = 60N$

$F_2 = 50N$





1n - Here,

$$= 60 \times \frac{5\vec{i} + 6\vec{j} + 3\vec{k}}{\sqrt{5^2 + 6^2 + 3^2}} = 35.855\vec{i} + 43.026\vec{j} + 21.513\vec{k}$$

$$= 50 \times \frac{5\vec{j} + 4\vec{k}}{\sqrt{5^2 + 4^2}} = 39.044\vec{j} + 31.235\vec{k}$$

$$= -200\vec{j}$$

$$= x\vec{i} + y\vec{j} + z\vec{k}\ (say)$$

en, By equilibrium,

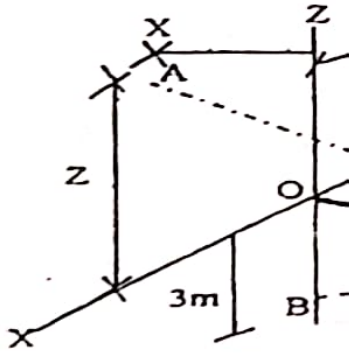
$$F = \vec{F_1} + \vec{F_2} + \vec{wt} + \vec{F_3} = 0$$

$$(35.855+x)\ \vec{i} + (43.026+39.044-200+y)\ \vec{j} + (21.513+31.235+z)\ \vec{k} =$$

lving, $x = -35.855$, $y = 117.93$, $z = -52.748$

$$\vec{F_3} = -35.855\vec{i} + 117.93\vec{j} - 52.748\vec{k}\ \text{Ans}$$

The pole is held in place by three cables determine the posi cable DA so that the resultant force exerted on the pole is from D towards O.



1n - Here,

$$A = 300 \times \frac{-4\vec{j} + z\vec{k} + x\vec{i}}{\sqrt{4^2 + z^2 + x^2}}$$

$$B = 350 \times \frac{-4\vec{j} - 3\vec{k}}{5} = -280\vec{j} - 210\vec{k}$$

$$C = 350 \times \frac{-4\vec{j} + 2\vec{k} - 3\vec{i}}{5.385} = -259.981\vec{j} + 129.991\vec{k} - 194.986\vec{i}$$

w,

$$A + \vec{BD} + \vec{DC} = \left(\frac{300x}{\sqrt{4^2 + z^2 + x^2}} - 194.986\right)\vec{i} + \left(\frac{-1200}{\sqrt{4^2 + z^2 + x^2}} - 280 -\right.$$

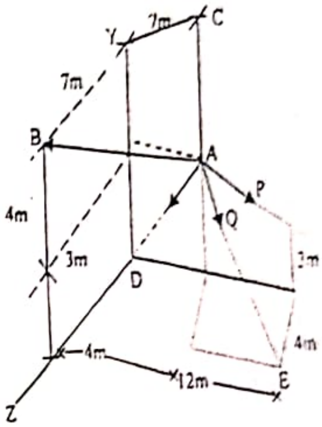
$$+\left(\frac{300z}{\sqrt{4^2+z^2+x^2}}-210+129.991\right)\vec{k}$$

But by question, $\sum F_x=0$, $\sum F_z=0$ as the pole is directed along Y-axis.

So, solving, $\left(\frac{300x}{\sqrt{16+z^2+x^2}}\right)^2=38019.54$ and $\left(-\frac{300z}{\sqrt{16+z^2+x^2}}\right)^2=6401.440$

we get, $x = 3.653$, $z = 1.499$ Ans

51. Three cables are connected at A, where the forces P and Q are applied. Knowing that Q=3.6kN and that the tension in cable AD=O, find (a) magnitude & sense of P (b) tensions in AB, AC.



Sol^n, Here,

$$\vec{T}_{AB}=T_{AB}\times\frac{-4\vec{i}+4\vec{j}+7\vec{k}}{9}$$

$$\vec{T}_{AC}=T_{AC}\times\frac{-4\vec{i}+4\vec{j}-7\vec{k}}{9}$$

$$\vec{Q}=3.6\times\frac{12\vec{i}-3\vec{j}+4\vec{k}}{13}$$

$$\vec{P}=P\vec{i}$$

We know, by equilibrium,

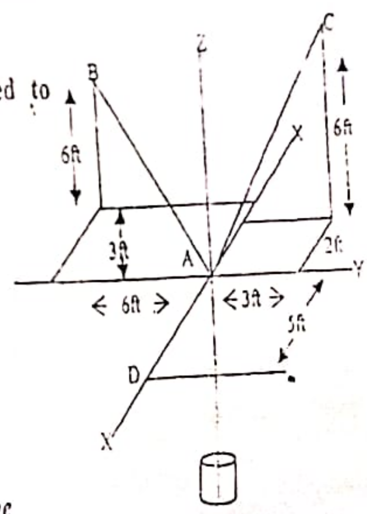
$$\vec{T}_{AB}+\vec{T}_{AC}+\vec{P}+\vec{Q}=0$$

$$\Rightarrow\left(-\frac{4}{9}T_{AB}-\frac{4}{9}T_{AC}+\frac{43.2}{13}+P\right)\vec{i}+\left(\frac{4}{9}T_{AB}+\frac{4}{9}T_{AC}-\frac{10.8}{13}\right)\vec{j}+$$

$$\left(\frac{7}{9}T_{AB}-\frac{7}{9}T_{AC}+\frac{14.4}{13}\right)\vec{k}=0$$

Solving, $T_{AB} = 0.223$kN, $T_{AC} = 1.647$kN, P =0.250kN Ans

52. Determine the force in each cable needed to support the 500lb bucket.



Sol^n- Here,

$$\vec{T}_{AC}=T_{AC}\times\frac{3\vec{j}+6\vec{k}-2\vec{i}}{7}$$

$$\vec{T}_{AB}=T_{AB}\times\frac{6\vec{j}+6\vec{k}-3\vec{i}}{9}$$

$\vec{T}_{AE} = T_{AE} \times \frac{5\vec{i}}{5}$

$\vec{W} = -500\vec{k}$

By equilibrium, we have

$\Sigma Fx = 0 \Rightarrow \frac{-2}{7}T_{AC} - \frac{3}{9}T_{AB} + T_{AD} = 0$

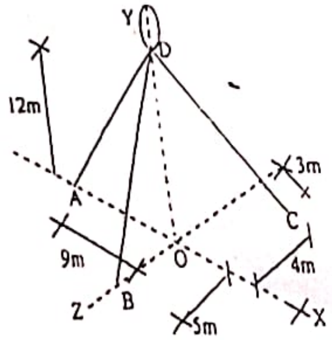
$\Sigma Fy = 0 \Rightarrow \frac{3}{7}T_{AC} + \frac{6}{9}T_{AB} = 0$

$\Sigma Fz = 0 \Rightarrow \frac{6}{7}T_{AC} + \frac{6}{9}T_{AB} - 500 = 0$

Solving, T_{AC} = 1166.667 lb, T_{AB} = -750 lb, T_{AD} = 83.333 lb Ans

53. Three cables DA, DB, and DC are used to tie down a balloon at D as shown. Knowing that the balloon exerts as 640N forces at D, determine the tension in each cable.

Sol^n- Here,



$\vec{T}_{DB} = T_{DB} \times \frac{-12\vec{j} + 5\vec{k}}{13}$

$\vec{T}_{DA} = T_{DA} \times \frac{-9\vec{i} - 12\vec{j}}{15}$

$\vec{T}_{DC} = T_{DC} \times \frac{3\vec{i} - 12\vec{j} - 4\vec{k}}{13}$

$\Sigma F + 640\vec{j} = 0$

So, $\Sigma Fy = -640$

$\Rightarrow \frac{-12}{13}T_{DB} - \frac{12}{15}T_{DA} - \frac{12}{13}T_{C} = -640$

$\Sigma Fx = 0$

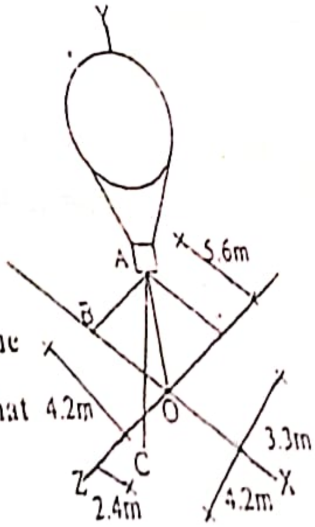
$\Rightarrow \frac{-9}{15}T_{DA} + \frac{3}{13}T_{DC} = 0$

$\Sigma Fz = 0$

$\Rightarrow \frac{5}{13}T_{DB} - \frac{4}{13}T_{DC} = 0$

Solving, T_{DA} = + 125N, T_{DB} = 260N, T_{DC} = 325N Ans

54. Three cables are used to tether a balloon as shown. Determine the vertical force P exerted by the balloon at A knowing that the tension in cable AD is 481N.



Sol^n- Here,

Force	Vector Showing line of action	Ma
TAD	$-3.3\vec{k} - 5.6\vec{j}$	
TAC	$2.4\vec{i} - 5.6\vec{j} + 4.2\vec{k}$	
TAB	$-4.2\vec{i} - 5.6\vec{j}$	
P	$\vec{j}$	

For equilibrium,

$\Sigma Fx = 0 \Rightarrow 0.324TAC$

$\Sigma Fy = 0 \Rightarrow -414.622$

$\Sigma Fz = 0 \Rightarrow -244.348$

From (3), TAC =

From (1), TAB =

From (2), P = 9

55. A cra

show

knc

27

Sol^n- He

...gnitude	Unit vector showing line of action	Vector representation of force
6.5	$-0.508\vec{k} -0.862\vec{j}$	$-244.348\vec{k} -414.622\vec{j}$
7.4	$0.324\vec{i} - 0.757\vec{j} +0.568\vec{k}$	$0.324TAC\vec{i} - 0.757TAC\vec{j} +0.568TAC\vec{k}$
7	$-0.6\vec{i} -0.8\vec{j}$	$-0.6TAB\vec{i} -0.8TAB\vec{j}$
1	$\vec{j}$	$P\vec{j}$

...$\vec{i}$ $- 0.6TAB\vec{i} = 0 \Rightarrow 0.324TAC-0.6TAB=0$ (1)

...0.757 TAC -0.8 TAB + P=0 (2)

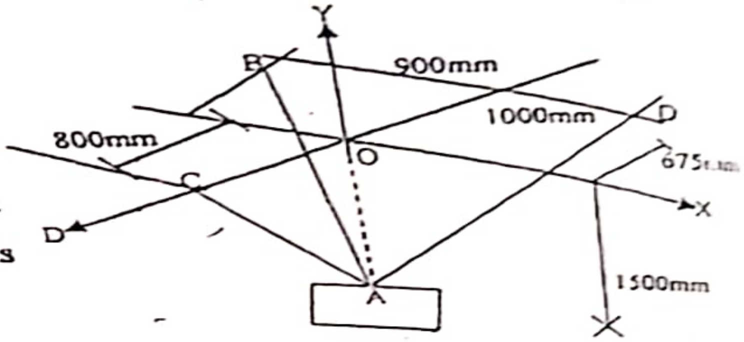
...3 + 0.568 TAC =0 (3)

...430.19N

...= 232.303N

...926.12N Ans

...ate is supported by three cables as ...wn. Determine the weight of the crate ...owing that the tension in cable AC is ...120N.



...ere,

Force	Unit vector of line of action	Vector representation of force
AD	$\frac{1000\vec{i} + 1500\vec{j} - 675\vec{k}}{1925}$	$\frac{1000}{1925}T_{AD}\vec{i} + \frac{1000}{1925}T_{AD}\vec{j} - \frac{675}{1500}T_{AD}\vec{k}$
AB	$\frac{-900\vec{i} + 1500\vec{j} - 675\vec{k}}{1875}$	$\frac{-900}{1875}T_{AB}\vec{i} + \frac{1500}{1875}T_{AB}\vec{j} - \frac{675}{1875}T_{AB}\vec{k}$
AC	$\frac{0\vec{i} + 1500\vec{j} + 800\vec{k}}{1700}$	$2400\vec{j} + 1280\vec{k}$
Weight	$-\vec{j}$	$-w\vec{j}$

Now, for equilibrium

$\Sigma Fx = 0 \Rightarrow \frac{1000}{1925}TAD - \frac{900}{1875}TAB = 0$ (1)

$\Sigma Fy = 0 \Rightarrow \frac{1500}{1925}TAD + \frac{1500}{1875}TAB + 2400 - w = 0$ (2)

$$\frac{-675}{1925}TAD + \frac{-675}{1875}TAB + 1280 = 0 \qquad (3)$$

Solving, we get TAB = 1871.32N, TAD = 1730.70N and w = 5245.65

6. A beamway carries a load $\vec{P}$ and is supported by two cables as in figure. Knowing that the tension in the cable AB is 732N and that the resultant of the load $\vec{P}$ and of the forces exerted at A by the two cables must be directed along OA. Determine the tension in cable AC, resultant and load P.

Soln- Here,

or $\vec{AB}, \vec{TAB} = TAB \times \frac{-960\vec{i} + 580\vec{j} + 480\vec{k}}{1220}$

or $\vec{AC}, \vec{TAC} = TAC \times \frac{-960\vec{i} + 500\vec{j} - 720\vec{k}}{1300}$

Now, for equilibrium,

$\Sigma Fx = 0 \Rightarrow$ R - 0.8 TAB - 0.7385TAC = 0 ....

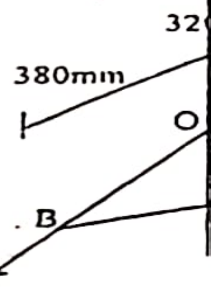
$\Sigma Fz = 0 \Rightarrow$ R - 0.3934 TAB - 0.5538TAC = 0 ....

$\Sigma Fy = 0 \Rightarrow$ -P + 0.4833TAB + 0.3846TAC = 0 ....

Given TAB = 732N

We get, TAC = 519.99N, R = 969.61N, P = 553.76N Ans

7. Three cables are connected at A, where the forces P and Q are applied as shown. Knowing that P =1200N, determine the values of Q for which cable AD is taut.



Soln- Here,

We implement values at slag (≠taut) i.e. tension = zero so that we ... which there is slag but below that the wire will be taut, which is our req...

S.N.	Force	Unit vector along live of action	Vector repres...
1	TAC	$\frac{-960\vec{i} - 240\vec{j} - 320\vec{k}}{1040}$	$\frac{-960}{1040}TAC\vec{i}\ \frac{-24}{104}$
2	TAB	$\frac{-960\vec{i} - 240\vec{j} + 380\vec{k}}{1060}$	$\frac{-960}{1060}TAB\vec{i}\ \frac{-24}{1060}$

3	TAD	$\frac{960\vec{i} + (960-240)\vec{j} - 220\vec{k}}{1220}$	[illegible]
4	P	$\vec{i}$	$1200\vec{i}$
5	Q	$\vec{j}$	$Q\vec{j}$

At equilibrium, $\sum Fx = 0 \Rightarrow \frac{-960}{1040}TAC - \frac{960}{1060}TAB + \frac{960}{1220}TAD + 1200 = 0$

$\sum Fy = 0 \Rightarrow \frac{-240}{1040}TAC \frac{-240}{1060}TAB + \frac{720}{1220}TAD + Q = 0$

$\sum Fz = 0 \Rightarrow \frac{-320}{1040}TAC \frac{+380}{1060}TAB \frac{-220}{1220}TAD = 0$

When TAD = 0 (the condition when AD is slag $\neq$ taut)

-0.923TAC - 0.906 TAB +1200=0

+0.308 TAC - 0.380 TAB = 0

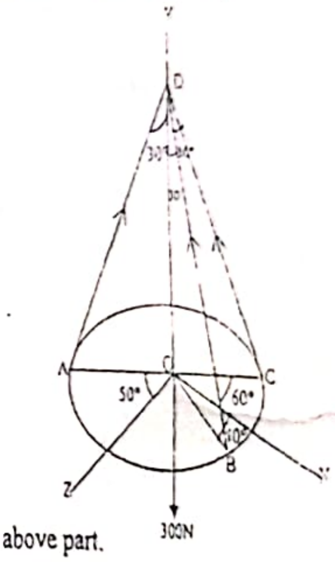
+TAB(0.630) = 1200×0.308 $\Rightarrow$ TAB = 586.667N and TAC = 724.25N

So, Q = 299.97N Ans

Hence, Q must be less than 299.97N so that $\vec{TAD}$ does have some value (i.e. it is taut)

So, 0≤Q<299.97N.

58. A horizontal circular plate weighing 300N is suspended as in figure from three wires that are attached to a support at D and form 30° with the vertical. Determine the tension in each wire.



Soln- Here,

The below procedure is the continued part from forces in space- above part.

Force	X-compo.	Y-compo.	Z-compo
CD	-TCD sin30°cos60°	TCD cos30°	TCD sin30° sin60°
BD	-TBD sin30°cos40°	TBD cos30°	-TBD sin30° sin40°
AD	TAD sin30° sin50°	TAD cos30°	-TAD sin30° cos50°
Weight	0	-300	0

For equilibrium in 3d-space,

$\Sigma F_x = 0 \Rightarrow$ -TCD sin30°cos60°-TBD sin30°cos40°+TADsin30° sin50°=0

$\Sigma F_y = 0 \Rightarrow$ TCD cos30° TBD cos30°+TADcos30°-300=0

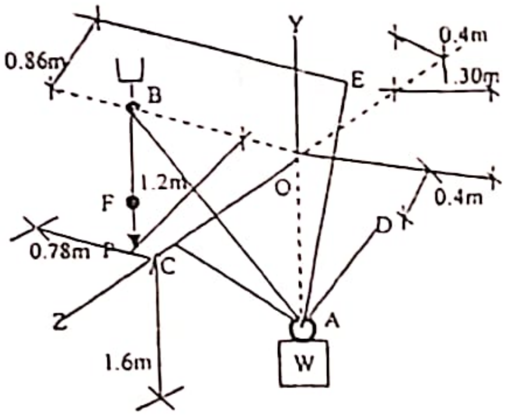
$\Sigma F_z = 0 \Rightarrow$ TCD sin30° sin60°-TBDsin30° sin40°-TADsin30° cos50°=0

Solving by cross- multiply, we get

TCD = 147.6N, TBD = 51.3N and TAD =147.6N Ans

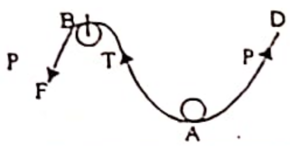
Note:- Three unknowns can be determined by using three equilibrium equations of 3d-equilibrium.

59. A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force $\vec{P}$ is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that W=1000N, determine the magnitude of $\vec{P}$.



Solⁿ- Here,

The tension is the same in all over portions of cable FBAD.



Force	Unit vector Representation	Complete Vector Representation
AE	$\dfrac{-0.4\vec{i}+1.6\vec{j}-0.86\vec{k}}{1.86}$	$\dfrac{-0.4}{1.86}TAE\,\vec{i}+\dfrac{1.6}{1.86}TAE\,\vec{j}-\dfrac{0.86}{1.86}TAE\,\vec{k}$
AC	$\dfrac{1.6\vec{j}-1.2\vec{k}}{2}$	$\dfrac{1.6}{2}TAC\,\vec{j}+\dfrac{1.2}{2}TAC\,\vec{k}$
AB	$\dfrac{-0.78\vec{i}+1.6\vec{j}}{1.78}$	$\dfrac{-0.78}{1.78}P\,\vec{i}+\dfrac{1.6}{1.78}P\,\vec{j}$
AD	$\dfrac{1.3\vec{i}+1.6\vec{j}-0.4\vec{k}}{2.1}$	$\dfrac{1.3}{2.1}P\,\vec{i}+\dfrac{1.6}{2.1}P\,\vec{j}+\dfrac{0.4}{2.1}P\,\vec{k}$
Weight	$\dfrac{-\vec{j}}{1}$	$-1000\,\vec{j}$

[illegible]

$\Sigma F_x = 0$ [illegible] $\dfrac{0.78}{1.78}P$ [illegible]

$$\Sigma F_y = 0 \Rightarrow \frac{1.6}{1.86}TAE + [illegible] + \frac{1.6}{1.78}P + \frac{1.6}{2.1}P - 1000 = 0 \qquad (2)$$

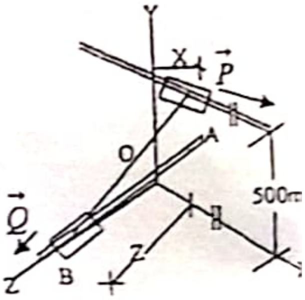
$$\Sigma F_z = 0 \Rightarrow \frac{-0.86}{1.86}TAE + \frac{1}{2}TAC + \frac{0.4}{2.1}P = 0 \qquad (3)$$

$$\text{From (1), } P = \frac{0.4\,TAE}{1.86\left(\frac{1.3}{2.1} - \frac{0.78}{1.78}\right)} = 1.189\,TAE$$

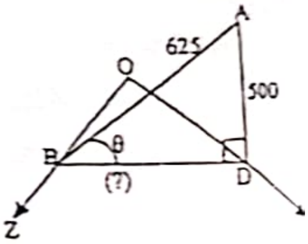
From (2), 0.8 TAC + TAE (2.335) = 1000

From (3), -0.6TAC - TAE (-0.236) = 0

Solving, TAE = 317.460N and P = 377.46N Ans



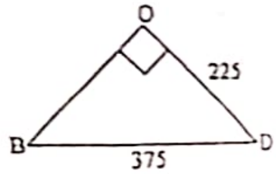
60. **Collars A and B are connected by a 625mm long wire and can slide freely on frictionless rods. If a 300N force $\vec{Q}$ is applied to collar B as shown, determine (a) tension in the wire when x = 225mm, (b) the corresponding magnitude of $\vec{P}$ to maintain equilibrium.**



Sol^n- Here,

Let D be projection point of A on x-axis,

$$\theta = \sin^{-1}\left(\frac{500}{625}\right) = 53.13°$$



BD = 375mm

$$OB = \sqrt{375^2 - 225^2} = 300mm$$

x = 225, y = 500, z = 300

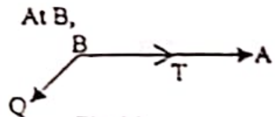


Fig-(a)

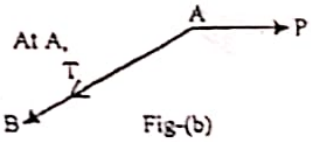


Fig-(b)

For	x	y	z	d	x	y	z
AB	-225	-500	300	625	-0.36	-0.8	0.48
BA	225	500	-300	625	0.36	0.8	-0.48

From fig (a),

$(0.36T)\vec{i} + (0.8T)\vec{j} + (-0.48T + Q)\vec{k}$ ....(*) is force directed BA.

From fig (b),

$(-0.36T+P)\vec{i} + (-0.8T)\vec{j} + (0.48T)\vec{k}$ ....(**) is directed A to B.

Adding we get, $P\vec{i} + Q\vec{j} = 0$

Which is req^d condition to equilibrium (*) is the net force obtained by opposition with $\vec{Q}$ and there is equilibrium so Q-component is zero $\Rightarrow -0.48T + Q = 0 \Rightarrow T = 625N$ Ans

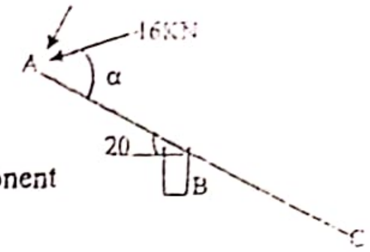
Similarly, from (**)

$-0.36T + P = 0$

$\Rightarrow P = 225N$ Ans

3.4 Moment I

1) A foot value for a pneumatic system is hinged at B. Knowing that α = 28° determine the moment of the 16KN force about point B.



⇒ Sol^n

AB = 170mm BC = 80mm

Resolving the 16KN force into horizontal and vertical component

horizontal component (along AC) = 16 * cosα= 14.127KN

Vertical component ($\perp^{re}$ to AC) = 16 * sinα =7.5115.

Since, the horizontal component has the same line of action of force. Hence, moment at point B is given by the vertical component only.

∴ M = 7.5115* ($\perp^{re}$ distance) = 7.5115* 0.170 = **1.277Nm (anticlockwise)//**

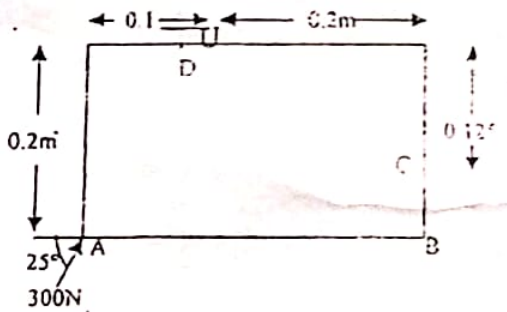
2. **A 300N force is applied at point. A as shown. Determine (a) the moment of the 300N force about D, (b) the magnitude and sense of the force applied at B that creates the same moment at D (c) the smallest force applied at c that creates the same moment about D.**

⇒ Sol^n

a) Moment of 300N force about D.

Resolving 300N force into horizontal and vertical components

horizontal = 300* cos25°



Vertical = 300 * sin 25

Taking moment equilibrium at point D

$\sum M_D = 0$ (+ve clock wise)

300* cos25°* 0.2 + 300 * sin25° * 0.1

-54.38+12.68 = -41.70

41.70Nm (anticlockwise)//

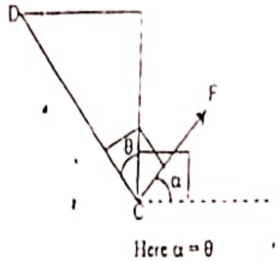
b) Let F be the smallest force applied at point B such that the moment is 41.70 (anticlockwise) for the same moment with the smallest force the $\perp^{re}$ distance should be greater and the maximum distance possible is DB

$DB = \sqrt{0.2^2 + 0.2^2}$ ⇒ DB = 0.28m

∴ F * 0.283 = 41.70 ⇒ F = 147.43N (←)

$\therefore \tan\theta = \dfrac{0.2}{0.2}$ ⇒ θ= 45°//

The smallest force applied at C that creates the same moment at D



$DC = \sqrt{0.2^2 + 0.125^2} = 0.236m$

$M = 0.236 \cdot F$

$41.70 = 0.236 \cdot F \Rightarrow F = 176.80N\ (\leftarrow)$

$\tan\theta = \dfrac{200}{125}$ $\quad \theta = 58°$// so

3 **Determine with respect to x, y, z axes the moment of a 600KN force whose line of action has the d.c.s $\left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)$ and acts at the point (1, -1, 0). All units are KN and meters.**

$\Rightarrow$ Sol^n

The unit vector along the force 600KN $\quad \hat{F} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{K}$

Force in vector form $\quad \vec{F} = F\,\hat{F} \quad = 600\left(\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{K}\right) = \left(400\hat{i} + 400\hat{j} - 200\hat{k}\right)KN$

The position vector of point of action of force $\vec{F}$ w. r. to origin is

$\vec{r} = 1\hat{i} + (-1-0)\hat{j} + (0-0)\hat{k} \quad \Rightarrow \quad \vec{r} = 1\hat{i} - \hat{j}$

Now, the moment of the force about the point (0, 0, 0) $\quad \vec{m} = \vec{r} \times \vec{f}$

$(\hat{i} - \hat{j}) \times (400\hat{i} + 400\hat{j} - 200\hat{k}) = 400\hat{k} + 200\hat{j} + 400\hat{k} + 200\hat{i} = (200\hat{i} + 200\hat{j} + 800\hat{k})$ kNm

Hence, moment about the axes are

$m_x = \vec{m} \cdot \hat{j} = 200$kNm//

$m_y = \vec{m} \cdot \hat{j} = 200$kNm//

$m_z = \vec{m} \cdot \hat{k} = 800$kNm//

4 **Determine the resultant of the system of concurrent forces having following magnitude and passing through the origin and the indicated points. P = 25N (3, 4, 5) m, Q = 30N (4, 5, 6) m and R = 35N (6, 7, 8)m. What is the moment of this resultant force about a point (1, -1, 1)?**

$\Rightarrow$ Sol^n

Let $\vec{F}_P$, $\vec{F}_Q$ and $\vec{F}_R$ be the force vector from the points P, Q, R respectively.

$\vec{F}_P = \frac{3\hat{i}+4\hat{j}+5\hat{k}}{\sqrt{9+16+25}} \qquad = 0.42\hat{i}+0.57\hat{j}+0.71\hat{k}$

$0.57\hat{j}+0.71\hat{k}) \qquad = 10.5\hat{i}+14.25\hat{j}+17.75\hat{k}$

e Q $= \frac{4\hat{i}+5\hat{j}+6\hat{k}}{\sqrt{16+25+36}} = \frac{4\hat{i}+5\hat{j}+6\hat{k}}{\sqrt{77}} = 0.456\hat{i}+0.57\hat{j}+0.687\hat{k}$

$30(0.456\hat{i}+0.57\hat{j}+0687\hat{k}) \qquad \Rightarrow \vec{F}_Q = 13.67\hat{i}+17.09\hat{j}+20.51\hat{k})$

$= \frac{6\hat{i}+7\hat{j}+8\hat{k}}{\sqrt{36+49+64}} = \frac{6\hat{i}+7\hat{j}+8\hat{k}}{\sqrt{149}}$

orm $\vec{F}_R = 41.37\vec{i}+53.41\vec{j}+61.2\vec{k}$

rs of different points i.e P, Q, R . be $\vec{r}_P, \vec{r}_Q, \vec{r}_R$ w.r.t point (1,-1, 1) then,

$)\hat{j} + (5-1)\hat{k} = 2\hat{i}+5\vec{j}+4\vec{k}$

$)\hat{j} + (6-1)\hat{k} = 3\hat{i}+6\vec{j}+5\vec{k}$

$\hat{j} + (8-1)\hat{k} = 5\hat{i}+8\vec{j}+7\vec{k}$

librium at point (1, -1, 1)

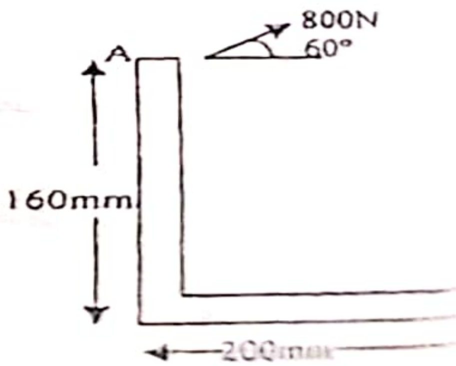
$- \vec{r}_R \times \vec{f}_R$

$$+\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & 5 \\ 26.67 & 17.09 & 20.51 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 8 & 7 \\ 17.2 & 20.07 & 22.34 \end{vmatrix}$$

$\hat{k} + 37.6\hat{i} + 6.82\hat{j} - 30.75\hat{k} + 43.03\hat{i} + 5.7\hat{j} - 37.25\hat{k}$

$92\hat{k}$ //

0N acts on a bracket as shown. Determine the moment of the



ce 800N about point B is represented by M_B.

$M_B = r_{A/B} \times F$

$6\hat{j}$ m.

$(800\cos 60°)\ \hat{i} + (800 \sin 60°)\ \hat{j} \quad = 400\hat{i} + 693\hat{j}$

Now, $M_B = [0.2\hat{i} + 0.16\hat{j}] * [400\hat{i} + 693\hat{j}]$

$= -138.6\hat{k} - 64.0\hat{k} \qquad = -202.6\hat{k}$ //

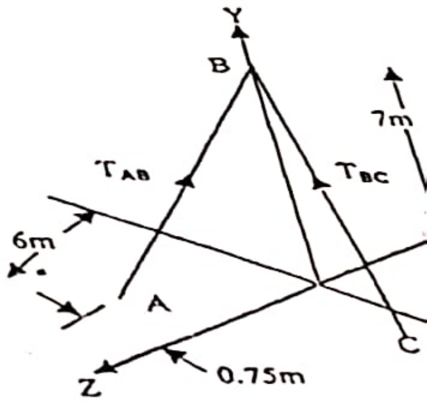
M_B = 202.6kNm (anticlockwise)

The moment M_B is a vector $\perp^{re}$ to the plane of the figure and pointing into the paper.

We can also solve this problem by resolving 800N force into two components and ta moment equilibrium at point B.

6. **Before the electric pole dismantled cables AB and BC attached as shown tension in the cables are 555N and 660N for AB and BC, determine the m about O of the resultant force exerted on the pole by the cables at B.**

⇒ Sol^n



We have

$M_o = r_{B/O} \times R_B$

Where, $r_{B/O} = 7\vec{j}$

R_B = Resultant of the forces due to cables.

$R_B = T_{AB} + T_{BC}$

$$T_{AB} = \left(\frac{-0.75\hat{i} - 7\hat{j} + 6\hat{k}}{\sqrt{0.75^2 + 7^2 + 6^2}}\right) 555$$

$= (-0.75\hat{i} - 7\hat{j} + 6\hat{k})\ 60$

$= -45\hat{i} - 420\hat{j} + 360\hat{k}$

Unit vector along BC.

$$\frac{4.25\hat{i} - 7\hat{j} + 1\hat{k}}{\sqrt{4.25^2 + 49 + 1}} = \frac{1}{8.25}(4.25\hat{i} - 7\hat{j} + 1\hat{k}$$

Now,

$$\vec{T}_{BC} = \frac{1}{8.25}(4.25\hat{i} - 7\hat{j} + 1\hat{k})\ 660 \quad = (4.25\hat{i} - 7\hat{j} + 1\hat{k})\ 80 \quad = 340\hat{i} - 560$$

∴ Resultant forces at point B $\vec{F}_B = \vec{T}_{AB} + \vec{T}_{BC}$

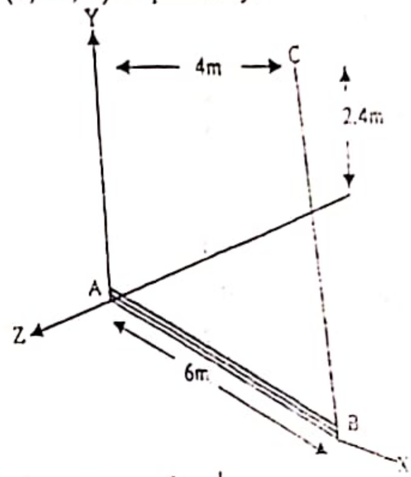
$$\vec{F}_B = [-45\hat{i} - 420\hat{j} + 360\hat{k}] + [340\hat{i} - 560\hat{j} + 80\hat{k}] = 295\hat{i} - 980\hat{j}$$

Now moment at O $\quad Mo = \begin{vmatrix} i & j & k \\ 0 & 7 & 0 \\ 295 & -980 & 440 \end{vmatrix} \Rightarrow (3080\hat{i} - 2070\hat{k})$ Nm.

7. The 6m boom AB has a fixed end A. A cable holds the boom at point B, attached at point C. If the tension in the cable is 2.5kN. Determine the moment about A of the force exerted by the cable at B.

$\Rightarrow$ Sol^n

Coordinates of the point A, B, C are A(0,0,0), B(6,0,0), C(0,2.4,-4) respectively.



moment at O Mo $= r_{A/B} \times \vec{T}_{BC}$

Now,

$r_{A/B} = 6\hat{i} + 0\hat{j} + 0\hat{k}$

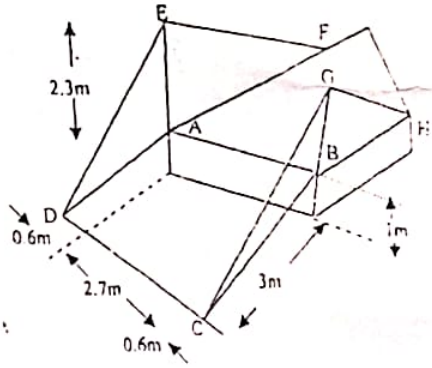
$$\vec{T}_{BC} = \hat{T}_{BC}\cdot|T_{BC}| = \frac{-6\hat{i}+2.4\hat{j}-4\hat{k}}{\sqrt{(2.4)^2+6^2+4^2}} * 2.5$$

$= 0.789\hat{j} - 1.31\hat{k} - 1.97\hat{i}$

$\therefore$ Moment at point A $\quad M_A = \vec{r}_{A/B} \times \vec{F}_{BC} \quad \begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ -1.97 & 0.789 & -1.31 \end{vmatrix}$

$\Rightarrow (7.86\hat{j} + 4.734\hat{k})$Nm. //

3.5 Moment is space



A ramp ABCD is supported by cables at corner C and D. The tension in each of the cable is 810N. Determine the moment about A of the force exerted by (a) The cable at D (b) The cable at C.

$\Rightarrow$Sol^n

Coordinates of point A(0,0,0), B(2.7,0,0), C(3.3,-1,3), D(-0.6,-1,3), G(2.7,2.3,0), E(0,2.3,0)

$\vec{F}_{CE} = \hat{F}_{DE}\,|\vec{F}_{DE}|$

$$= (0.6\hat{i} + 3.3\hat{j} - 3\hat{k}) * \frac{810}{\sqrt{0.6^2 + 3.3^2 + 9}}$$

$= 108\hat{i} + 594\hat{j} - 540\hat{k}$

$\vec{r}_{\cdot c} = -0.6\hat{i} - \hat{j} + 3\hat{k}$

Moment about A $M_A = \begin{vmatrix} i & j & k \\ -0.6 & -1 & +3 \\ 108 & +594 & -540 \end{vmatrix}$ $\Rightarrow M_{A/D} = -1242$

ii) The cable at C $\vec{r}_{A/C} = 3.3\hat{i} + 3\hat{k} - \hat{j}$

Now, tension in wire in vector form.

$$\vec{F}_{CC} = 810 \times \frac{(2.7-3.3)\vec{i} + (2.3+1)\vec{j} + (0.3)\vec{k}}{\sqrt{(-0.6)^2 + 3.3^2 + (-3)^2}} = -108\vec{i} + 594\vec{j} -$$

Now, moment at point A $M_A = \begin{vmatrix} i & j & k \\ 3.3 & -1 & 3 \\ -108 & 594 & -540 \end{vmatrix}$

$M_{A/C} = -12.42\hat{i} + 1458\hat{j} + 1852\hat{k}$ //

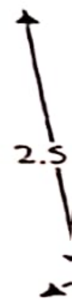
9 **A small boat swings from two davits one of which i the line AB-AD is 410N. Determine the moment a exerted on the davit at A.**

$\Rightarrow$ Sol^n

Coordinates of A(0,2.5,1) D(2,0,0) C(0,0,0).

Due to the double pulley system.

$R_A = 2F_{AB} + F_{AD}$



$\vec{F}_{AB} = -410\hat{j}\,N$

$\vec{F}_{AD} = \lambda_{AD}|F_{AD}|$

$$= \frac{2\hat{i} - 2.5\hat{j} - 1\hat{k}}{\sqrt{4 + (2.5)^2 + 1}} \cdot 410 \qquad = 244.47\hat{i} - 305.59\hat{j} - 122.$$

$R_A = -2 \cdot 410\hat{j} + 244.47\hat{i} - 305.59\hat{j} - 122.24\hat{k}$

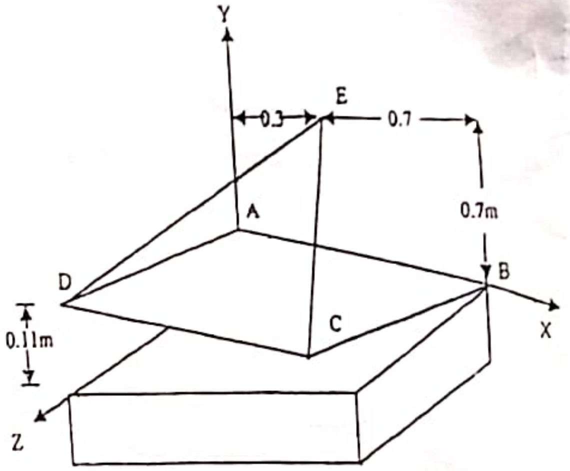
$= \ldots 244.47\hat{i} - 305.59\hat{j} - 122.24\hat{k} \quad = 244.47\hat{i} - 11$

$\therefore$ Moment at point C due to resultant force R_A. $M_C = \begin{vmatrix} i & j & k \\ 0 & 2.5 & 1 \\ 244.47 & -1125.59 & -122.24 \end{vmatrix}$

$\therefore M_C = (-305.6+1125.59)\,\hat{i} + (244.47)\,\hat{j} + (-2.5*244.47)\,\hat{k}$

$M_C = 819.99\,\hat{i} + 244.47\,\hat{j} - 611.175\,\hat{k}$ Nm//

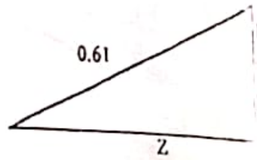
10. **The 0.61×1m lid ABCD of storage bin is hinged along side AB and is held open by looping cord DEC over a frictionless hook at E. If the tension in the cord is 66N determine the moment about each of the coordinate axes of the force exerted by the cord at D.**



$\Rightarrow$ Sol^n

Coordinates of A(0,0,0), B(1,0,0), E(0.3,0.7, 0), D(0,0.11, 0.6) and C(1,0.11,0.6)

$Z = \sqrt{0.61^2 - 0.11^2} = 0.6$m.



[$\therefore$ The lid is always greater than the box which it covers.]

$\vec{T}_{DE} = \lambda_{D/E} \left|\vec{T}_{DE}\right| = \dfrac{(-0.3\hat{i} - 0.6\hat{j} + 0.6\hat{k})}{\sqrt{0.3^2 + 0.6^2 + 0.6^2}} * 66 \Rightarrow -22\hat{i} - 44\hat{j} + 44\hat{k}$

Now, moment at A $M_A = r_{A/D} \times T_{DE}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -0.11 & -0.6 \\ -22 & -44 & 44 \end{vmatrix} = -31.24\,\hat{i} + 13.2\,\hat{j} - 2.42\,\hat{k}$ Nm.

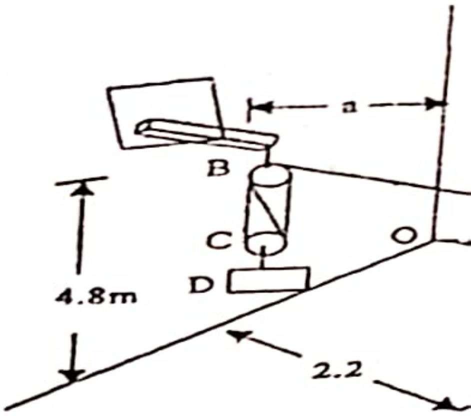
Now, moment about the axes, moment about x-axis $M_x = M.\,\hat{i}$ = -31.24Nm//

moment about y-axis $My = M.\,\hat{j}$ = -13.2Nm//

moment about 2 axis $Mz = M.\,\hat{k}$ = -2.42Nm//

To lift a heavy crate, a man uses a block and tackle to t hook B. Knowing that the moment about the y and z axes by portion AB of the rope are, resp. 120 Nm and 460Nm.

⇒ Sol^n



Coordinates of points A and B

A(2.2.1.6,0) & B(0,4.8,a)

$$\vec{F}_{AB} = \lambda |F_{AB}| = \left| \frac{-2.2\hat{i}+3.2\hat{j}+a\hat{k}}{\sqrt{15.08+a^2}} \right| * F_{AB}$$

moment at point B $M_B = r_{A/O} * F_{AB}$

$$(2.2\hat{i}+1.6\hat{j}) \times \frac{F_{AB}}{d_{AB}}[-2.2\hat{i}+3.2\hat{j}+a\hat{k}]$$

$$= \frac{F_{AB}}{d_{AB}} \begin{vmatrix} i & j & k \\ 2.2 & 1.6 & 0 \\ 2.2 & 3.2 & a \end{vmatrix} = \frac{F_{AB}}{d_{AB}}[1.6a\hat{i}+2.2a\hat{j}+10.56\hat{k}]$$

Now, moment about y-axis. $M_y = 2.2a \cdot \frac{F_{AB}}{d_{AB}}$

$$\Rightarrow 120 = 2.2a * \frac{F_{AB}}{d_{AB}} \qquad \text{(i)}$$

moment about z - axis

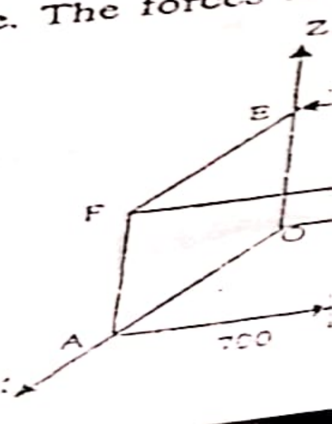
$$M_z = 10.56 \cdot \frac{F_{AB}}{d_{AB}} \Rightarrow 460 = 10.56 \cdot \frac{F_{AB}}{d_{AB}} \quad \ldots\ldots\ldots\ldots \quad \text{(ii)}$$

Dividing (i) by (ii) $\frac{120}{460} = \frac{2.2a}{10.56}$

a = 1.252m //

12. Four forces are acting along the edges of the 0.8m cube as sh the resultant of these forces by a force $\vec{R}$ through point A an same point A.

⇒ Sol^n

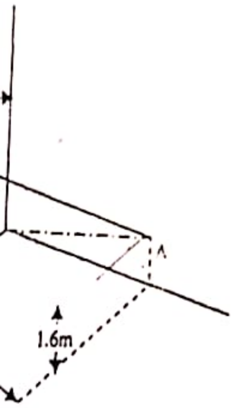


Let A, B, C, D, E, F, G be the points as shown in figure. The forces are direction.

F_{AB} = 700N , F_{BG} = 600N , F_{DE} = 500N , F_{DC} = 400N

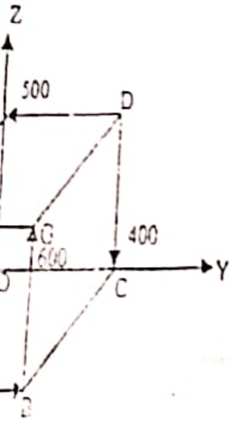
The coordinates A(0.8,0,0), B(0.8, 0.8, 0), C(0,0.8,0), D(0,0.8, 0.8), E(0,0,0.8), f(0.8,0,0.8), G(0.8, 0.8, 0.8)

the bottom of an I-beam at
xes of the forces exerted at B
n. Determine a.



shown in fig. Represent
and a couple about the

are represented by their



$\vec{F}_{AB} = 700\hat{j}$, $\vec{F}_{BG} = 600\hat{k}$, $\vec{F}_{DE} = -500\hat{j}$ & $\vec{F}_{DC} = -400\hat{k}$

Now,

$\vec{r}_{AB} = 0.8\hat{j}$ $\quad\quad$ $\vec{r}_{AC} = -0.8\hat{i} + 0.8\hat{j}$

$\vec{r}_{AE} = 0.8\hat{j} + 0.8\hat{k}$, $\quad$ $\vec{r}_{AD} = 0.8\hat{i} + 0.8\hat{j} + 0.8\hat{k}$

Taking the moment about point A.

$\vec{m} = \vec{r}_{AB} \times \vec{F}_{AB} + \vec{r}_{AE} \times \vec{F}_{BG} + \vec{r}_{AD} \times \vec{F}_{DC} + \vec{r}_{AD} \times \vec{F}_{DE}$

$= 0.8\hat{j} \times 700\hat{j} + 0.8\hat{j} \times 600\hat{k} + (-0.8\hat{j} + 0.8\hat{j} + 0.8\hat{k}) \times (-400\hat{k}) + (-0.8\hat{i} + 0.8\hat{j} + 0.8\hat{k}) \times (-500\hat{j})$

$= 0 + 480\hat{i} - 320\hat{j} - 320\hat{j} + 400\hat{k} + 400\hat{i}$

$= 560\hat{i} - 320\hat{j} + 400\hat{k}$ Nm//

Resultant force is $\vec{R} = \vec{F}_{AB} + \vec{F}_{BG} + \vec{F}_{DE} + \vec{F}_{DC}$

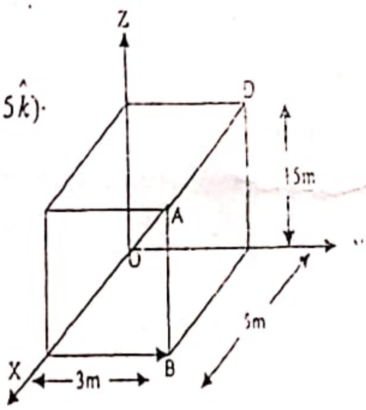
$= 700\hat{j} + 600\hat{k} - 500\hat{j} - 400\hat{k} \quad \Rightarrow \vec{R} = 200\hat{j} + 200\hat{k}$ //

13. **What are the force component of 500kN force acting as shown in figure. What are the d.c.s associated with the force?**

⇒ Sol^n

Let A be the point of application of the force 500kN coordinate of A (5,3,15)

The unit vector along the direction of the force



$$\vec{F}_{AO} = \frac{(0-5)\hat{i} + (0-3)\hat{j} + (0-15)\hat{k}}{\sqrt{(-5)^2 + 9 + 15^2}} = \frac{1}{\sqrt{259}}(-5\hat{i} - 3\hat{j} - 15\hat{k})$$

Now,

$$\vec{F}_{AO} = \hat{F}_{AO}\,|F_{AO}| = \frac{500}{\sqrt{259}}(-5\hat{i} - 3\hat{j} - 15\hat{k})$$

$= -155.34\hat{i} - 93.21\hat{j} - 466.03\hat{k}$

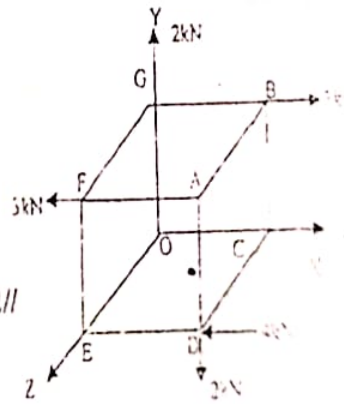
Hence, components of the force along x, y, z axis are

$F_x = -155.34$N, $F_y = -93.21$N, $F_z = 466.03$N

The unit vector along the direction of force is

$$\frac{1}{\sqrt{259}}(-5\hat{i} - 3\hat{j} - 15\hat{k}) = -0.311\hat{i} - 0.186\hat{j} - 0.932\hat{k}$$

Hence, d.c.s are $\cos\theta_x = -0.311$, $\cos\theta_y = -0.186$, $\cos\theta_z = -0.932$//



Find the resultant of the given system of forces as shown in figure.

$\Rightarrow$ Sol^n

Side of the cube = 3m

The coordinates O(0,0,0), A(3,3,3), B(3,3,0) C(3,0,0), D(3,0,3), E(0,0,3), F(0,3,3), C

Let $\vec{F}_{AF}, \vec{F}_{CB}, \vec{F}_{OC}, \vec{F}_{DE}, \vec{F}_{AD}, \vec{F}_{OG}$ be the forces along respective direction.

Then,

$$\vec{F}_{AF} = 3\left[\frac{(0-3)\hat{i}+(3-3)\hat{j}+(3-3)\hat{k}}{\sqrt{(0-3)^2+(3-3)^2+(3-3)^2}}\right] = -3\hat{i}$$

Similarly,

$\vec{F}_{CB} = 3\hat{i}$ $\quad \vec{F}_{AD} = -2\hat{j}$ $\quad$ And $\vec{r}_{OA} = 3\hat{i}+3\hat{j}+3\hat{k}$ $\quad \vec{r}_{OB} = 3$

$\vec{F}_{OG} = 2\hat{j}$ $\quad \vec{F}_{OC} = 4\hat{i}$ $\quad \vec{r}_{OC} = 3\hat{i}$ $\quad \vec{r}_{OD} = 3\hat{i} + 3\hat{k}$

$\vec{F}_{DE} = -4\hat{i}$ $\quad \vec{r}_{OG} = 3\hat{j}$

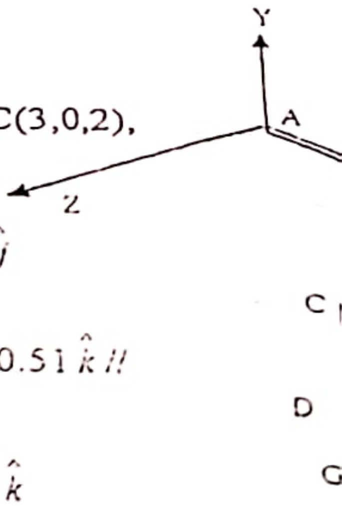
Now, taking moments about point 0.

$$\vec{M} = \vec{r}_{OA} \times \vec{F}_{AF} - \vec{r}_{OB} \times \vec{F}_{CB} + \vec{r}_{OC} \times \vec{F}_{OC} + \vec{r}_{OG} \times \vec{F}_{OG} + \vec{r}_{OD} \times \vec{F}_{DE} + \vec{r}_{OD} \times \vec{F}_{AD}$$

$$= (3\hat{i}+3\hat{j}+3\hat{k})\times(-3\hat{i}) + (3\hat{i}+3\hat{j})\times 3\hat{i} + (3\hat{i}+3\hat{k})\times(-4\hat{i}) + (3\hat{i}+3\hat{k})\times(-2\hat{j})$$

$$= 9\hat{k} - 9\hat{j} - 9\hat{k} - 12\hat{j} - 6\hat{k} + 6\hat{i} \quad = 6\hat{i} - 21\hat{j} - 6\hat{k} \,//$$

Find the moment of force F=500N as shown in figure about the p passes through the point D and E. The length are AB =3m E DG=1.5m, GH=2m HE=1.5m.



$\Rightarrow$ Sol^n

The coordinates of different point are A(0,0,0), B(3,0,0), C(3,0,2), D(3,-1,2), G(3,-2.5,2), H(5,-2.5,2) E (5, -2.5, 0.5)

Now, Unit vectors along $\vec{DE}$ $\quad \hat{F} = (5-3)\hat{i} + 9 - 2.5 + 1)\hat{j}$

$$\hat{F} = \frac{(5-3)\hat{i}+(-2.5+1)\hat{j}+(0.5-2)\hat{k}}{\sqrt{(5-3)^2+(-2.5+1)^2+(0.5-2)^2}} = 0.69\hat{i} - 0.51\hat{j} - 0.51\hat{k}\,//$$

$$\vec{F} = 500(0.69\hat{i} - 0.51\hat{j} - 0.51\hat{k}) = 345\hat{i} - 255\hat{j} - 255\hat{k}$$

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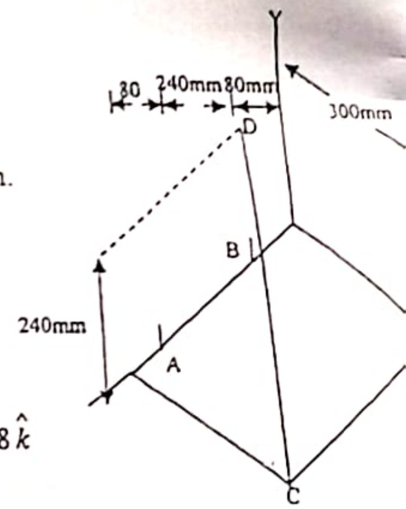
Now, Position vector of point D w.r.t A is $r_{A/D} = 3\hat{i} + 2\hat{k} - \hat{j}$

$\therefore$ moment of the force $\vec{F}$ about A is $(3\hat{i} - \hat{j} + 2\hat{k}) \times (345\hat{i} - 255\hat{j} - 255\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 345 & -255 & -255 \end{vmatrix} \Rightarrow 765\hat{i} + 1455\hat{j} - 420\hat{k} \,//$$

16. **A rectangular plate is supported by bracket at A and B and by a wire CD. Know that tension in cable is 200N. Determine the moment about A of the force exerted the wire on point C.**



$\Rightarrow$ Sol^n

Coordinates of the point

A(0, 0, 0.32), B(0,0,0.08), C(0.3, 0,0.4), D(0,0.24, 0.08)m.

The force exerted by the cable

$$\vec{F}_{DC} = \hat{F}_{CD} \cdot \left|\vec{F}_{CD}\right| = \left(\frac{-0.3\hat{i} + 0.24\hat{j} - 0.32\hat{k}}{\sqrt{0.3^2 + 0.24^2 + 0.32^2}} \right) \times 200$$

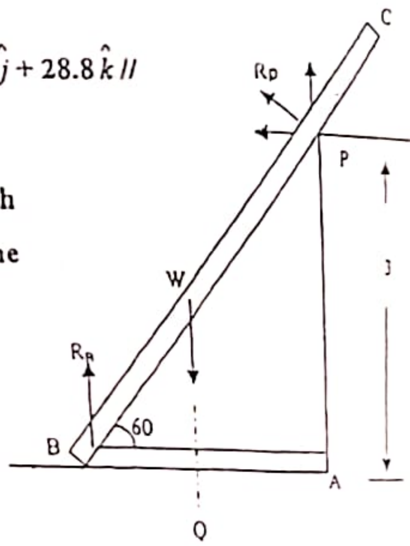
$$= (-0.3\hat{i} + 0.24\hat{j} - 0.32\hat{k}) \times \frac{200}{0.5} = -120\hat{i} + 96\hat{j} - 128\hat{k}$$

Position vector of AC = $r_{C/A} = \vec{AC} = 0.3\hat{i} + 0.08\hat{k}$

Now, moment about A of the force exerted by the wire on point C.

$$M_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix} \Rightarrow -7.68\hat{i} + 28.8\hat{j} + 28.8\hat{k} \,//$$

17. **Determine the tension in the cable AB which hold a post of 4m length from sliding. The post has mass of 9kg.**



$\Rightarrow$ Sol^n

For the equilibrium of the post

$$\vec{R}_B + \vec{R}_P + \vec{W} + \vec{R}_{AB} = 0 \ldots\ldots\ldots\ldots (i)$$

Here, $\vec{R}_B = R_B\,\hat{j}$

$\vec{R}_P = R_P(-\cos 30\,\hat{i} + \sin 30\,\hat{j}) = -0.866R_P\,\hat{i} + 0.5R_P\,\hat{j}$ $\qquad \vec{W} = (-9 * 9.81)\,\hat{j} = -88.29\,\hat{j}$

$T_{BA}\hat{i}$ So, from eqn ---------(i)

$R_B\hat{j} - 0.866R_P\hat{i} + 0.5R_P\hat{j} - 88.29\hat{j} + T_{BA}\hat{i} = 0$

Taking similar components together.

$\hat{i}(T_{BA} - 0.866R_P) + \hat{j}(R_B + 0.5R_P - 88.29) = 0$

Equating the coefficient.

$T_{BA} = 0.866R_P$ ------------------ (ii)

$R_B + 0.5R_P = 88.29$ ------------------------ (iii)

Taking moment about B. $\Sigma \vec{M}_B = 0$

$(\vec{B}, \times \vec{R}_p) + \vec{BQ}\times(-88.29\vec{j}) = (3\vec{j}\times R_P\cos 30\vec{j}) + (\vec{BA}\,\hat{i}\times R_P\sin 30°) + (\vec{BQ}\,\hat{i}\times -88.29\hat{j})$

In triangle BWQ $\cos 60° = \frac{BQ}{BW} \Rightarrow BQ = 2\times\cos 60° = 1$

In ΔBPA $\tan 60° = \frac{AP}{BA} \Rightarrow BA = 1.732$

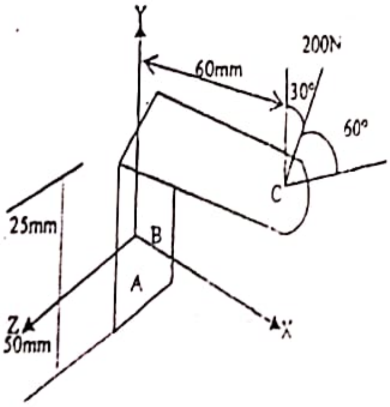
So, $\vec{m}_B = (3\vec{j}\times -Rp\cos 30°\hat{i}) - (1.732\hat{i}\times -Rp\sin 30\hat{j}) + (\hat{i}\times -88.29\hat{j})$

$= 2.598Rp\hat{k} + 0.866Rp\hat{k} - 88.29\hat{k} \Rightarrow \hat{k}(2.598Rp + 0.866Rp) = 88.29\hat{k}$

$\therefore Rp = \frac{88.29}{(2.598+0.866)} \Rightarrow Rp = 25.48N$

and from eqn ------------------(ii) $T_{BA} = 0.866Rp \Rightarrow T_{BA} = 22.07N$//

18. A 200N force is applied as shown to the bracket ABC. Determine the moment of the force about A.



⇒ Sol^n - Here,

The components of 200N force are

$200\cos 60°\vec{k}$ and $-200\cos 30°\vec{j}$

So, $\vec{F} = (-200\cos 30°\vec{j} + 200\cos 60°\vec{k})$

and $\vec{AC} = +75\vec{j} + 60\vec{i}$

So, $M = \vec{AC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 60 & 75 & 0 \\ 0 & -173.2 & 100 \end{vmatrix} = \vec{i}(7500) - \vec{j}(6000) + \vec{k}(-1039.2)$

$= (7.5\vec{i} - 6\vec{j} - 10.392\vec{k})$ k Nm//

19. Find the moment of the for

⇒ Sol^n, Here,

$\vec{r} = 1.2\vec{i} - 0.6\vec{j} + 0.5\vec{k}$

$\vec{F} = 300\times\frac{-1.2\vec{i} + (0.6+0.6)\vec{j} + (1.1}{\sqrt{3.24}}$

$= -200\vec{i} + 200\vec{j} + 100\vec{k}$

So, $M = \vec{r}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.2 & -0.6 & 0.5 \\ -200 & 200 & 100 \end{vmatrix}$

20. A Force $\vec{F} = -5\vec{i} + 3\vec{j} - 4\vec{k}$ l

origin. If the forces acts at

and z coordinates.

⇒ Sol^n-Here, $Mo = \vec{r}\times\vec{F}$

Or, $-17\vec{i} - 7\vec{j} + 16\vec{k} = \begin{vmatrix} i & j & k \\ x & 2 & z \\ -5 & 3 & -4 \end{vmatrix}$

or, $-17\vec{i} - 7\vec{j} + 16\vec{k} = (-8-3z)\,\vec{i} - \vec{j}$ (

Solving Z=3m, x =2m//

21. A guyed pole is shown. The t

= 28kN. Determine the rang

which the magnitude of the

not exceed 200kNm.

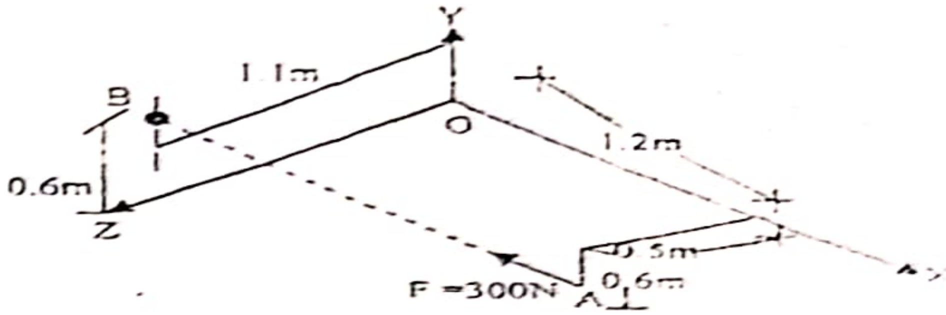
⇒ Sol^n - Here,

$\vec{T}_{AB} = 27\times\frac{12\vec{i} - 12\vec{j} - 6\vec{k}}{18} = 18\vec{i} - 18$

$\vec{T}_{AC} = 28\times\frac{-4\vec{i} - 12\vec{j} - 6\vec{k}}{14} = -8\vec{i} - 2$

orce F shown about the origin.

$1.1-0.5)\vec{k}$



$= -160\text{Nm}\,\vec{i}\ +220\text{Nm}\,\vec{j} + 120\text{Nm}\,\vec{k}$ //

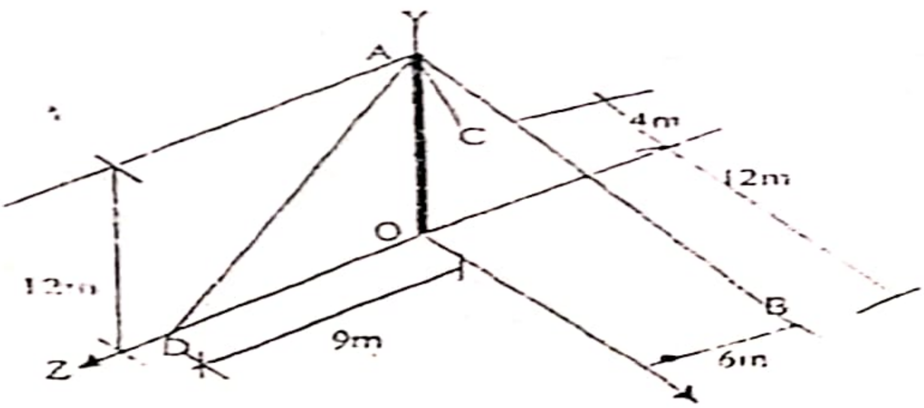
kN Produces a moment $\vec{Mo} = -17\,\vec{i}\ -7\,\vec{j}\ +16\,\vec{k}$ kN m about
t a point p having a y coordinate of 2m. Determine the

$\vec{j}\ (-4x+5z) + \vec{k}\ (3x+10)$

e tensions in the cables AB & AC are T_{AB} =27kN and T_{AC}
nge of the values of the tension TAD in the cable AD for
he resultant moment developed at base O of the pole will

$8\,\vec{j} - 9\,\vec{k}$

$24\,\vec{j} - 12\,\vec{k}$



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∴ $T_{BA}\hat{i}$ So, from eqn ---------(i)

$R_B\hat{j} - 0.866R_P\hat{i} + 0.5R_P\hat{j} - 88.29\hat{j} + T_{BA}\hat{i} = 0$

Taking similar components together.

$\hat{i}\,(T_{BA} - 0.866R_P) + \hat{j}\,(R_B + 0.5R_P - 88.29) = 0$

Equating the coefficient.

$T_{BA} = 0.866R_P$ -------------------- (ii)

$R_B + 0.5R_P = 88.29$ -------------------------- (iii)

Taking moment about B. $\Sigma \vec{M}_B = 0$

$(\vec{B}_, \times \vec{R}_p) + \vec{BQ}\times(-88.29\vec{j}) = (3\vec{j}\times R_P\cos30\vec{j}) + (\vec{BA}\,\hat{i}\times Rp\sin30°) + (\vec{BQ}\,\hat{i}\times -88.29\hat{j})$

In triangle BWQ $\cos60° = \frac{BQ}{BW} \Rightarrow BQ = 2\times\cos60° = 1$

In ΔBPA $\tan 60° = \frac{AP}{BA} \Rightarrow BA = 1.732$

So. $\vec{m}_B = (3\hat{j}\times -Rp\cos30°\hat{i}) - (1.732\hat{i}\times -Rp\sin30\hat{j}) + (\hat{i}\times -88.29\hat{j})$

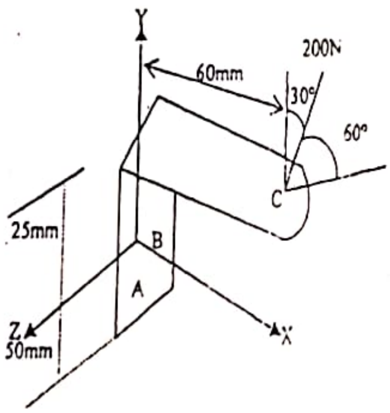
$= 2.598Rp\hat{k} + 0.866Rp\hat{k} - 88.29\hat{k} \Rightarrow \hat{k}(2.598Rp + 0.866Rp) = 88.29\hat{k}$

$\therefore Rp = \frac{88.29}{(2.598 + 0.866)} \Rightarrow Rp = 25.48N$

and from eqn -----------------(ii) $T_{BA} = 0.866Rp \Rightarrow T_{BA} = 22.07N$//

18. A 200N force is applied as shown to the bracket ABC. Determine the moment of the force about A.

⇒ Sol^n - Here,



The components of 200N force are

$200\cos 60°\vec{k}$ and $-200\cos30°\vec{j}$

So, $\vec{F} = (-200\cos30°\vec{j} + 200\cos60°\vec{k})$

and $\vec{AC} = +75\vec{j} + 60\vec{i}$

$$\text{So, } M = \vec{AC}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 60 & 75 & 0 \\ 0 & -173.2 & 100 \end{vmatrix} = \vec{i}(7500) - \vec{j}(6000) + \vec{k}(-1039.2)$$

$= (7.5\vec{i} - 6\vec{j} - 10.392\vec{k})$ k Nm//

19. Find the moment of the for

⇒ Sol^n, Here,

$\vec{r} = 1.2\vec{i} - 0.6\vec{j} + 0.5\vec{k}$

$\vec{F} = 300\times\frac{-1.2\vec{i} + (0.6+0.6)\vec{j} + (1.}{\sqrt{3.24}}$

$= -200\vec{i} + 200\vec{j} + 100\vec{k}$

$$\text{So, } M = \vec{r}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.2 & -0.6 & 0.5 \\ -200 & 200 & 100 \end{vmatrix}$$

20. A Force $\vec{F} = -5\vec{i} + 3\vec{j} - 4\vec{k}$ l

origin. If the forces acts at

and z coordinates.

⇒ Sol^n-Here, $Mo = \vec{r}\times\vec{F}$

$$\text{Or, } -17\vec{i} - 7\vec{j} + 16\vec{k} = \begin{vmatrix} i & j & k \\ x & 2 & z \\ -5 & 3 & -4 \end{vmatrix}$$

or, $-17\vec{i} - 7\vec{j} + 16\vec{k} = (-8-3z)\vec{i} - \vec{j}$ (

Solving Z=3m, x =2m//

21. A guyed pole is shown. The t = 28kN. Determine the rang which the magnitude of the not exceed 200kNm.

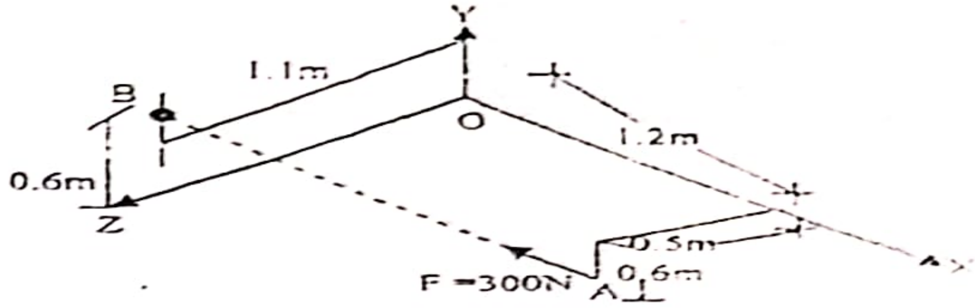
⇒ Sol^n - Here,

$\vec{T}_{AB} = 27\times\frac{12\vec{i} - 12\vec{j} - 6\vec{k}}{18} = 18\vec{i} - 18\vec{j}$

$\vec{T}_{AC} = 28\times\frac{-4\vec{i} - 12\vec{j} - 6\vec{k}}{14} = -8\vec{i} - 2$

orce F shown about the origin.

$(1.1-0.5)\vec{k}$



$= -160Nm\ \vec{i}\ +220Nm\ \vec{j} + 120Nm\ \vec{k}$ //

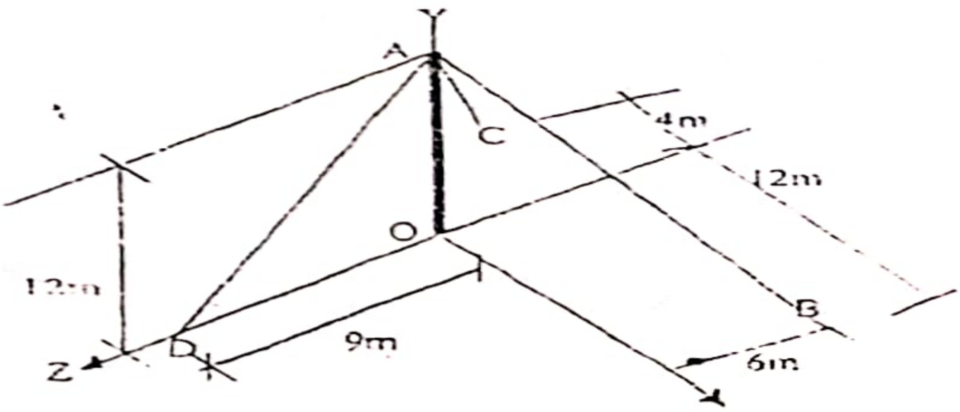
kN Produces a moment $\vec{Mo} = -17\vec{i} - 7\vec{j} + 16\vec{k}$ kN m about
t a point p having a y coordinate of 2m. Determine the x

$(-4x+5z) + \vec{k}\,(3x+10)$

e tensions in the cables AB & AC are T_{AB} =27kN and T_{AC}
nge of the values of the tension TAD in the cable AD for
e resultant moment developed at base O of the pole will

$3\vec{j} - 9\vec{k}$

$24\vec{j} - 12\vec{k}$



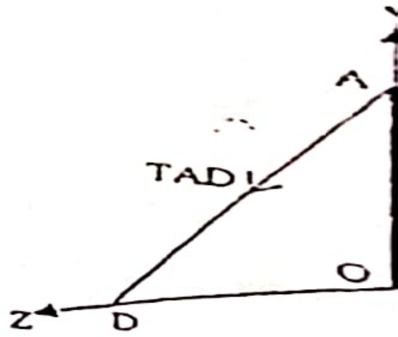
$$\vec{T}_{AD} = T \times \frac{-12\vec{j} + 9\vec{k}}{15} = -0.8T\,\vec{j} + 0.6T\,\vec{k}$$

By question,

$$\vec{OA} \times \vec{T}_{AB} + \vec{OA} \times \vec{T}_{AC} + \vec{OA} \times \vec{T}_{AD}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ 18 & -18 & -9 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ -8 & -24 & -12 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 12 & 0 \\ 0 & -0.8T & 0.6T \end{vmatrix}$$

$-108\,\vec{i} - 216\,\vec{k} - 144\,\vec{i} + 96\,\vec{k} + 7.2T\,\vec{i}$



$(-252+7.2T)\,\vec{i} - 120\,\vec{k}$

So, $\sqrt{(-252+7.2T)^2 + (-120)^2} = 200$

or, $200^2 - 120^2 = (-252+7.2T)^2$

or, $25600 = 63504 - 3628.8T + 51.84T^2$

or, $51.84T^2 - 3628.8T + 37904 = 0$

We get T = 57.22kN and 12.78kN

So, 12.78kN < T < 57.22kN//

3.6 Scalar product

22 **Determine the projection of a vector $\vec{F}$ along a line jo (3, -4, 4). The magnitude of force F is 150 and parallel**

=>Sol^n- The unit vector of $4\hat{i} - 12\hat{j} + 6\hat{k}$ i.e. $\frac{4\hat{i} - 12\hat{j} + 6\hat{k}}{\sqrt{196}}$

gives direction of $\vec{F}$

So, $\vec{F} = 150 \times \left(\frac{4}{14}\vec{i} - \frac{12}{14}\vec{j} + \frac{6}{14}\vec{k}\right)$

The projection of $\vec{F}$ on BA is F cosθ = $\frac{\vec{F}.\vec{BA}}{BA}$

[illegible] $\frac{2\vec{i} - 2\vec{j} + 5\vec{k}}{\sqrt{33}}$

projection [illegible] $\frac{-900}{7}\Big) + \left(\frac{5}{\sqrt{33}} \times \frac{450}{7}\right) =$

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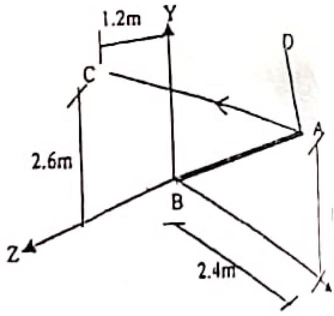
23. Find angle between the vectors $\vec{a} = 2\vec{i} + 5\vec{j} + 6\vec{k}$ and $\vec{b} = 1\vec{i} - 2\vec{j} - 8\vec{k}$

$\Rightarrow$ Sol^n - Here,

We know $\vec{P}.\vec{Q} = PQ\cos\theta \quad \Rightarrow \theta = \cos^{-1}\dfrac{\vec{P}\,\vec{Q}}{PQ}$

So, $\theta = \cos^{-1}\left(\dfrac{2\times1-10-48}{\sqrt{65}\ \sqrt{69}}\right) = 146.740°//$

24. Knowing that the tension in cable AC is 1260-N, determine (a) the angle betw cable AC and the boom AB, (b) the projection on AB of the force exerted by cable AC at A.



$\Rightarrow$Sol^n-Here,

$$\vec{T}_{AC} = 1260\times\frac{-2.4\vec{i}+(2.6-1.8)\vec{j}+1.2\vec{k}}{2.8}$$

$$\vec{AB} = -2.4\vec{i} - 1.8\vec{j}$$

$$\text{So, } \theta = \cos^{-1}\left(\frac{-1080\times(-2.4)-1.8\times360}{\sqrt{2.4^2+1.8^2}\ \sqrt{1080^2+360^2+540^2}}\right)$$

$= 59.050°//$

& projection $= T_{AC}\cos\theta = 1260\cos 59.050° = 648N//$

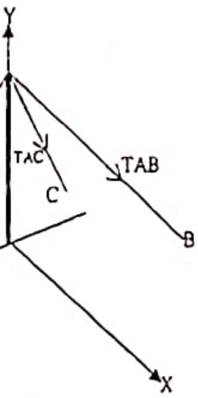
25. Determine the volume of the parallelepiped given by $\vec{P} = 4\vec{i} - 3\vec{j} + 2\vec{k}$, $\vec{Q} = -2\vec{i} - 5\vec{j} + \vec{k}$ and $\vec{S} = 7\vec{i} + \vec{j} - \vec{k}$ as respective sides.

$\Rightarrow$ Sol^n - Here, Volume $= \left[\vec{P}\ \vec{Q}\ \vec{S}\right]$

$$= \vec{P}\cdot\vec{Q}\times\vec{S} = \vec{P}\cdot\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -2 & -5 & 1\\ 7 & 1 & -1\end{vmatrix} = (4\vec{i}-3\vec{j}+2\vec{k}).(4\vec{i}+5\vec{j}+33\vec{k}) = 67//$$

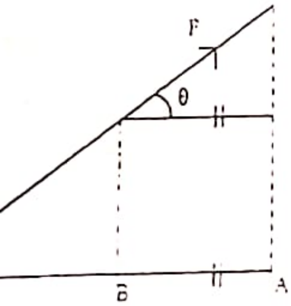
26. Given the vectors $\vec{P} = 4\vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{Q} = 2\vec{i} + 4\vec{j} - 5\vec{k}$, $S = S_x\vec{i} - \vec{j} + 2\vec{k}$, det the value of Sx for which the three vectors are coplanar.

$\Rightarrow$Sol^n - Three vectors are coplanar if their scalar triple product is zero.



oining the points (1, -2, -1) and

l to the vector $4\hat{i} - 12\hat{j} + 6\hat{k}$



$= 115.6373//$

$$\vec{P}\times\vec{Q}\cdot\vec{S} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2 & 3 \\ 2 & 4 & -5 \end{vmatrix} = (S\vec{i} - \vec{j} + 2\vec{k}) \cdot (-2\vec{i} + 26\vec{j} + 20\vec{k}$$

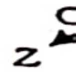
But by q^n, $\vec{S} \cdot \vec{P}\times\vec{Q} = 0 \qquad \Rightarrow -2S-26+40=0 \quad \Rightarrow S = 7$ //

3.7 Moment about an axis

27. **Compute the moment of a force $\vec{F} = 10\vec{i} + 6\vec{j}\ N$, which g about a line going through points 1 and 2 having the resp $\vec{r_1} = \left(6\vec{i} + 10\vec{j} - 3\vec{k}\right)m, \vec{r_2} = \left(-3\vec{i} - 12\vec{j} + 6\vec{k}\right)m.$**

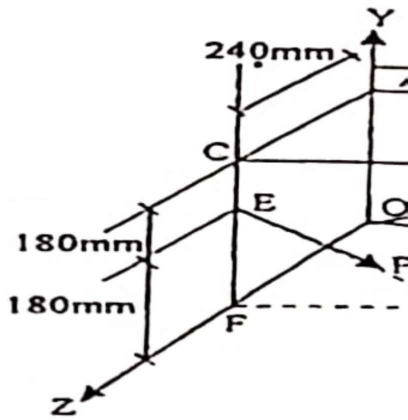
$\Rightarrow$ Sol^n So, $\vec{BC} = (-3-6)\ \vec{i}\ + (-12-10)\ \vec{j} + (6+3)\ \vec{k}$

$= -9\vec{i} - 22\vec{j} + 9\vec{k}$ //



28. **A force P of magnitude 2600N acts on the frame shown moment of P about a live joining points O and D.**

$\Rightarrow$ Sol^n - Here,



$$\hat{OD} = \frac{720\vec{i} + 360\vec{j} + 240\vec{k}}{840\times1000}\,m$$

$$\vec{P} = 2600 \times \frac{720\vec{i} - 180\vec{j} + 240\vec{k}}{7800}$$

$$\vec{OE} = 180\vec{j} + 240\vec{k}$$

So, $\vec{OE}\times\vec{P} = 288000\,\vec{i} + 576000\,\vec{j} - 432000\,\vec{k}$

$= 288Nm\,\vec{i} + 576Nm\,\vec{j} - 432Nm\,\vec{k}$

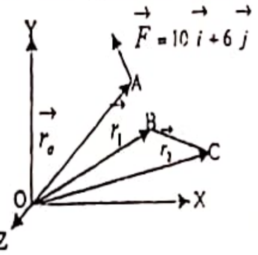
So, $\hat{OD}\cdot(\vec{OE}\times\vec{P}) = \dfrac{324}{875}$ //

29. **The 60N force P is applied at point C of the bent bar moment of P about point A. For what value of the angle point A be maximum? Determine this maximum value (M**

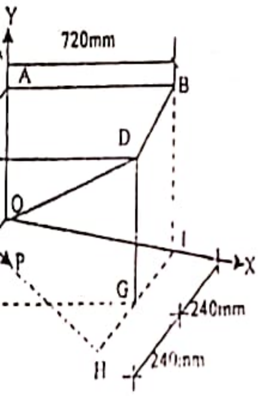
$\vec{k}) = -25 - 26 + 40$

goes through $\vec{ra} = 2\vec{i} + 6\vec{j}$ m,

espective position vectors;



wn at point E. Determine the



ar. If θ=45°, determine the

gle θ will the moment about

M_A) max.

: Sol

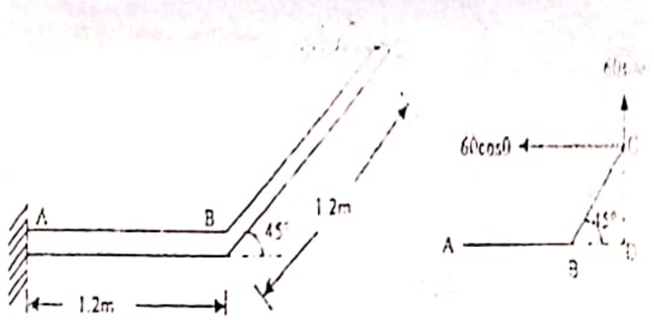
P = 60N

AB = 1.2m

BD = 1.2cos 45° = 0.8485m

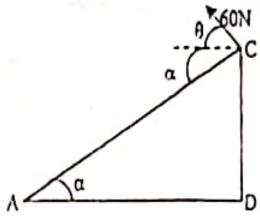
AD = 2.0485m

CD = 0.8485m



a) $M_A = 60 * \cos45° * 0.8485 + 60\sin 45° * 2.0485 = 122.909$NM

b) The moment will be maximum when the arm of the moment is ⊥re to force



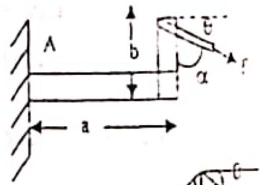
$\alpha = \tan^{-1}\left(\frac{0.8485}{2.0485}\right) = 22.499$

So, $\theta = 90° - \alpha$

$\Rightarrow \theta = 67.5$//

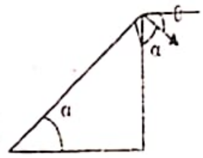
30. Determine the direction θ of the force = 40N so that it produces (a) the max. moment about point A and (b) the minimum moment about point A. Compute the moment in each case. Take a=8m, b =2m.

⇒ Sol^n



a) For max. moment about point A.

Here, $\theta + \alpha = 90°$ $\alpha = \tan^{-1}\left(\frac{2}{8}\right) = 14.036°$ $\Rightarrow \theta = 75.9637°$

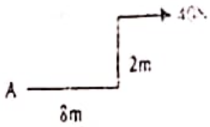


$M_A = 40 * \frac{8}{\cos\alpha} = 329.85$Nm

b) The moment will be minimum of force is parallel to the horizontal.

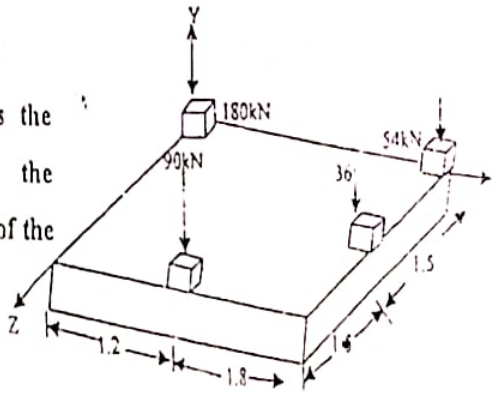
$M_A = 40 \times 2 = 80$Nm

Here, vertical gap bet^n A & B is small so, force must be horizontal to get minimum value of moment.



3.8 Equivalent of a system of forces

31) A square foundation mat supports the four column shown. Determine the magnitude and point of application of the resultant of the four loads.



⇒ Sol^n

The forces are represented into the vector form.

The position vector of the point of application of the various forces are determined and th component are arranged in tabular form.

r, m	f, KN	r×f, KNm
0	-180j	0
3i	-54j	-162k
3i+1.5k	-36j	54i-108k
1.2i+3k	-90j	270i-108k
	R =-360j	M=324i-378k

Let (x, z) be the point of application of the resultant force. Now, the position vector of the of application of the resultant force is (xi + zk) now, by varignon's theorem r×R=M

(xi+zk) (-360j) = 324i -378k

-360xk + 360zi = 324i - 378k

Equating the like vectors we get x = 1.05, z=0.9m

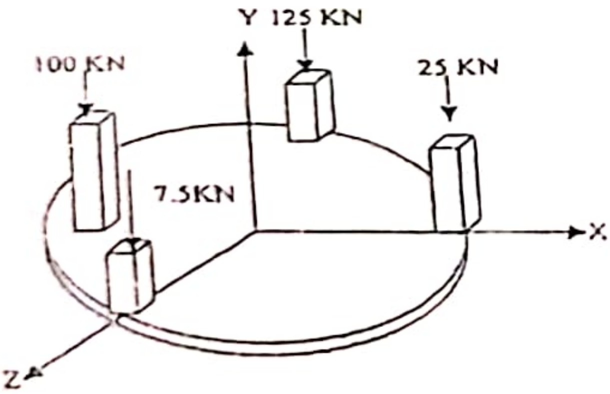
∴ **Resultant of the given system is R = 360kN↓ at x = 1.05m, z=0.9m. //**

32) **A concrete foundation mat of 5m radius supports four equally spaced colum of which is loaded 4m from the centre of the mat. Determine the magnitude point of application of the resultant of four load.**

⇒Sol^n

Let R be the resultant of the force acting due to the force given representing the forces form

$100\vec{j}, 125\vec{j}, 75\vec{j}, 25\vec{j}$



Resultant R = $125\vec{j} + 100\vec{j} + 75\vec{j} + 25\vec{j}$ - $325\vec{j}$ (Downward)

The resultant force in magnitude is $\sqrt{325^2 + 0^2 + 0^2}$ ⇒ **325kN //**

[illegible] be the coordinates at which the resultant acts. Since, the mat is locate [illegible] y=0 Now, taking mo[illegible]ilibrium at point 0

[illegible] 25 j) + (-4 [illegible] 100 j [illegible] or, $325x\vec{k} - 325z\vec{i} + 300\vec{i} - 100\vec{k} + 400\vec{k} - 500\vec{i} = 0$

or, $(300 - 325z - 500)\vec{i} + (325x - 100 + 400)\vec{k} = 0$

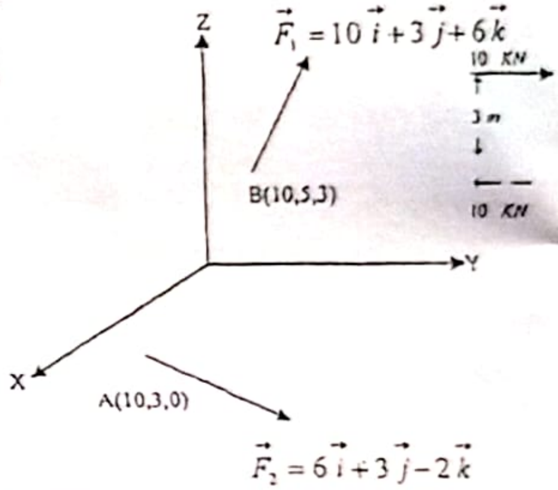
comparing the coefficient of like vectors. $\vec{i}$ $300 - 325z - 500 = 0 \quad -325z = 200 \Rightarrow z = -0.615\text{m}$

and that of $\vec{k}$ $325x - 100 + 400 = 0 \Rightarrow 325x = -300 \Rightarrow x = -0.922\text{m}//$

$\therefore$ **The point of application of the resultant is (-0.922, 0, -0.615)m**

3.9 Moment of couple

33) **Two forces and a couple are shown in fig. the couple is acting in plane z-y, find the resultant of the system at the origin o.**



$\Rightarrow$ Sol^n Resultant of forces only $\vec{F}_R = \vec{F}_1 + \vec{F}_2$

$= (10\vec{i} + 3\vec{j} + 6\vec{k}) + (6\vec{i} + 3\vec{j} - 2\vec{k})$

$\therefore F_R = 16\vec{i} + 6\vec{j} + 4\vec{k}$

Now, moment of forces about O.

$\vec{r}_1 = \vec{OB} = 10\vec{i} + 5\vec{j} + 3\vec{k}$

$\vec{r}_2 = \vec{OA} = 10\vec{i} + 3\vec{j}$

Direction of couple is $-\vec{i}$ $\quad \therefore \vec{C} = |\vec{C}|$ unit vector

$= (10*3)(-\vec{i}) \Rightarrow \vec{C} = -30\vec{i}$

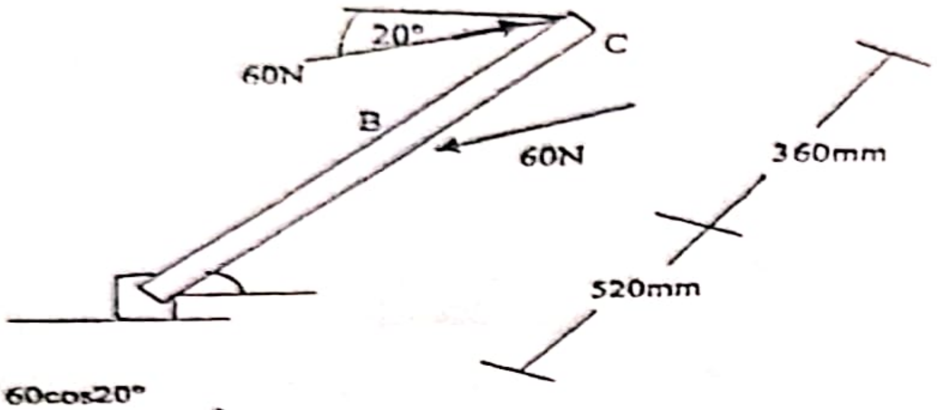
Resultant moment,

$$\vec{M}_R = \vec{C} + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 \qquad = -30\vec{i} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 5 & 3 \\ 10 & 3 & 6 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 3 & 0 \\ 6 & 3 & -2 \end{vmatrix}$$

$= -30\vec{i} + \vec{i}[30 - 9] - \vec{j}[60 - 30] + \vec{k}(30 - 50) + \vec{i}(-6 - 0) - \vec{j}(-20 - 0) + \vec{k}(30 - 18)$

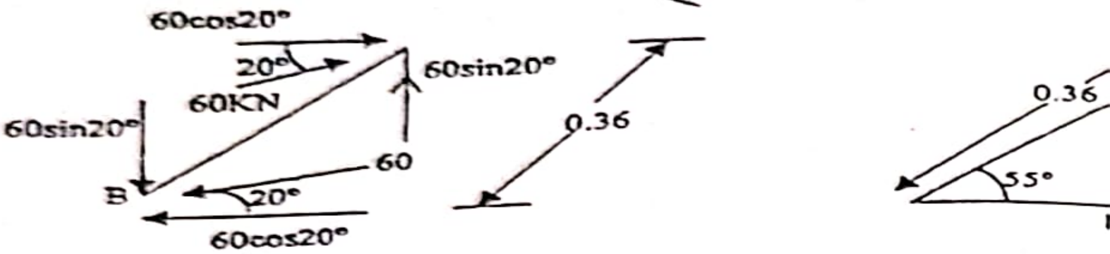
$= -30\vec{i} + 21\vec{i}\ 30\vec{j} - 20\vec{k} - 6\vec{i} + 20\vec{j} + 12\vec{k} \quad \Rightarrow M_R = -15\vec{i} - 10\vec{j} - 8\vec{k}//$

34. **Two parallel 60N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces. (a) by resolving each force into horizontal and vertical components and adding the moment of the two resulting couples. (b) using the ⊥^{r} distance bet^n the two forces, (c) by summing the moment of the forces about point A.**



⇒ Soln

Drawing the FBD of the whole system.

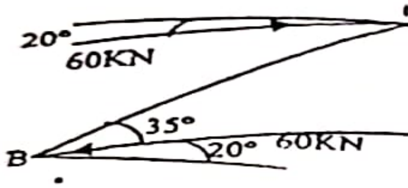


∴moment of the couple m = f×d taking clockwise positive.

60*cos20° *p - 60* sin20°*b

= 60*cos20°*0.36*sin55° - 60*sin20°×0.36* cos55°

= 12.389Nm//

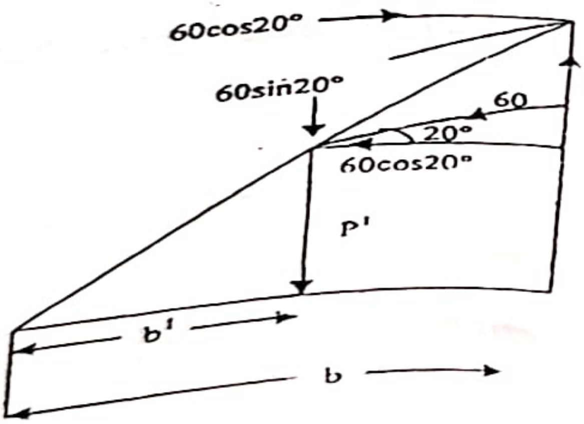


b) Moment of couple

F * ⊥r distance = 60 * d

60 * 0.36 * sin35°

= 12.39Nm



c) Using moment equilibrium at point A.

ΣMA = 0 (+ve clockwise)

60* cos20°* (0.52+0.36) sin 55°

-60 * sin20° *(0.52+0.36). cos55°

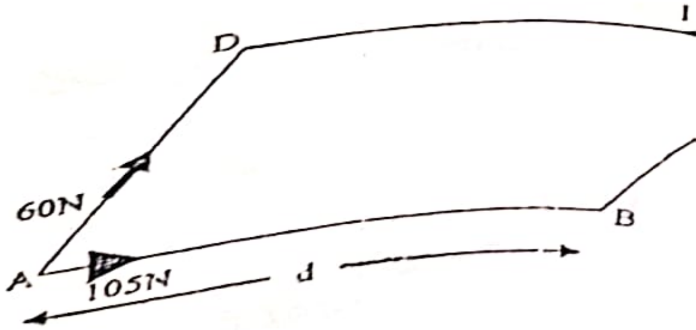
+60*sin20°*0.52cos55°

-60*cos20°*0.52.sin55°

=40.643-10.358+6.121-24.016

=12.39Nm//

35. A plate in the shape of parallelogram is acted upon by two couples. Determine (a) the moment of the couple formed by the two 105N forces. b) the ⊥r distance betn the 60N forces if



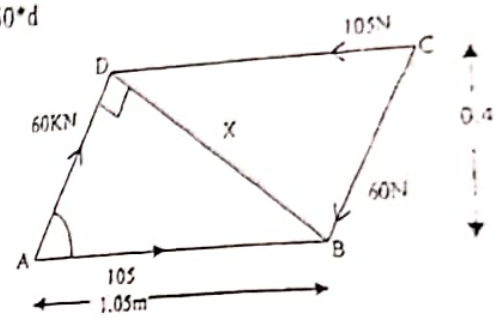
the resultant of the two couple is zero. (c) The value of α if the resultant couple is 9N.m clockwise and d is 1050mm.

⇒ Sol^n

i) Moment by the force 105N

ii) = -105 * 0.4=42 (anticlockwise)

similarly, moment by the force 60N = 60*d



ii) The sum of the two moment (ΣM_A=0)

-105*0.4 + 60*d = 0

60d = 105*0.4

d = 0.7m

iii) resultant couple = 9Nm clockwise

d = 1.05m

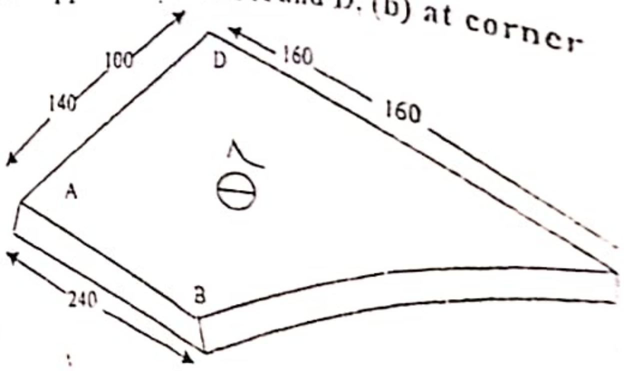
Now, moment

-105*0.4+60*x=9 ⇒ x=0.85m

from the figure, we concluded that x=0.85m and by Pythogoras theorem

$\sin\alpha = \frac{0.85}{1.05}$ ⇒ $\alpha = 54.05°$//

36. A couple M of magnitude 18Nm is applied to the handle of a screw driver to tighten a screw into a block of wood. Determine the magnitude of two smallest horizontal forces that are equivalent to M if they are applied at corners A and D. (b) at corner B and C. (c) any where.



⇒ Sol^n

Couple = 18Nm

a)When two smallest forces are acted at A & D

F * 1^{r} distance bet^n A and D = 18

F * (0.24) = 18 ⇒F = 75N

b)When two forces are applied at corner B and C.

F *(1^{r} distance bet^n BC) = 18

∴ BC = $\sqrt{240^2 + 80^2}$ = 253mm = 0.253m

Fx 0.253 =18 ⇒ F = 71.15m

By: Rajan Gautam, Prajwal Giri, Devendra Man Palikhe

c) Any where in the block

For this we find out: $AC = \sqrt{240^2 + 320^2}$

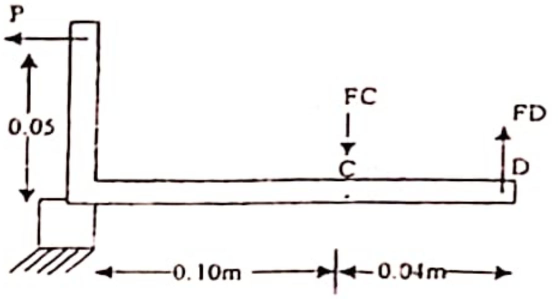
= 0.4m (longest distance)

$F * 0.4 = 18$

$F = 45N$//

37. The 80N horizontal force p acts on a bell crank as shown. (a) Replace equivalent force couple system at B. (b) find the two vertical forces at C and D equivalent to the couple found in part a.

⇒Soln The equivalent force couple system is equal to the moment about a point B.



$M_B = p*d$

$= 80 * 0.05$

$= 4Nm$

Now, $\Sigma fy = 0$ (+ve upward)

$F_D - F_C = 0$

$F_D = F_C = F$

Two vertical forces to be equivalent to moment they must be couple.

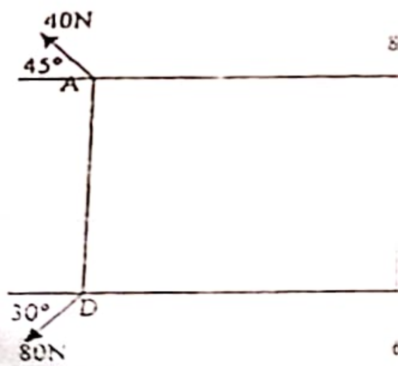
∴ $M_B = F * D$

$4 = F * 0.04$ ⇒$F = 100N$//

∴ Two forces F_C and F_D of magnitude 100N is applied at point C and D to form moment (4Nm) at B.

38 A square plate is under the action of four couple forces as shown. magnitude, direction and position of resultant of the four system.

⇒ Sol^n



Resolving the forces horizontally

$\Sigma H = 80 * \cos30° + 60\cos45° - 80\cos30° - 40\cos45°$

$= 80 * 0.86 + 60 * 0.70 - 80 * 0.86 - 40 * 0.70 = 14.14N$

Resolving the forces vertically.

$\Sigma V = 80\sin30° - 60\sin45° - 80\sin30° + 40\sin45° - 14.14N$

Resultant R $= \sqrt{\sum H^2 + \sum V^2}$

$= \sqrt{14.14^2 + 14.14^2} = 20N$

Direction of resultant, α = ?

The resultant will acts at an angle α with the horizontal so that ,

$$\tan\alpha = \frac{\sum V}{\sum H} = \frac{14.14}{14.14} = 1$$

$\alpha = 45°$

<u>Position of the resultant</u>

The moment of the resultant R can be determined by using the relation (varignon's theore

Moment of resultant about A=Algebric sum of the rectangular components of all forces a

$- R\times x = 40\sin45°\times0+40°\cos45°\times0+80\sin30°\times1.5+80\cos30°\times0+60\cos45°\times1.5$

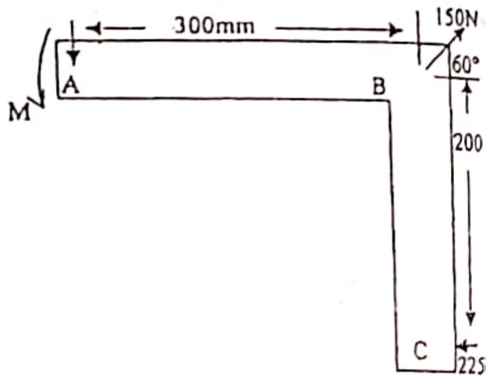
$- 60\sin60°\times1.5-80\cos30° * 1.5 +80\sin30°\times0 =0$

or, $-20*x = 60+63.63-63.63-103.92+0$

$-2x=-43.92$ $\quad x = 2.196\text{m}$ $\Rightarrow$ **x = 2.196m from A//**

39. **A couple of magnitude M =7N.m and the three forces shown are applied to a bracket. (a) find the resultant of this system of forces. (b) locate the point the line of action of the resultant interest line AB and line BC.**

$\Rightarrow$ Sol^n



Resolving the force 150N into horizontal & vertical components

Horizontal component = 150 cos60°=75N

Vertical component = $150*\sin60° = 75\sqrt{3}$

Now, representing the forces in vector form assuming that whole system is in xy plane.

$F_A = 50\vec{j}$, $F_{BH} = 75\vec{i}$, $F_{BV} = 75\sqrt{3}\,\vec{j}$ $\;F_C = -225i$

Now, the resultant force acting on the system.

$$F_R = -50\vec{j} + 75\vec{i} + 75\sqrt{3}\,\vec{j} - 225\vec{i} \quad\Rightarrow\quad F_R = -150\vec{i} + 79.9038\vec{j}$$

the resultant will form angle θ with the horizontal $\therefore \tan\theta = \dfrac{79.9038}{150}$ θ =28.04°

$R = \sqrt{150^2 + 79.9038^2}$

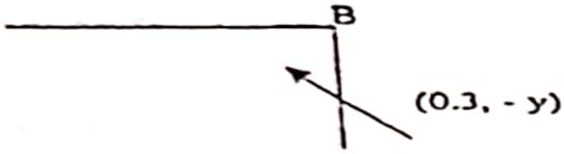
R = 169.95N//

b) Let (x, 0) and (0.3, y) be the two points on the line AB and BC respectively line of action of resultant intersect.

Taking moment about A by varignon's theorem.

$$7\vec{k} + (0.3\vec{i})\times(75\sqrt{3}\,\vec{j}) + (0.3\vec{i} - 0.2\vec{j})\times(-225\vec{i}) = (x\vec{i})\times(-150\vec{i} + 79.9\vec{j})$$

$7\vec{k} + 38.97\vec{k} - 45\vec{k} = 79.9x\,\vec{k}$



$0.01215\vec{k} = x\vec{k}$

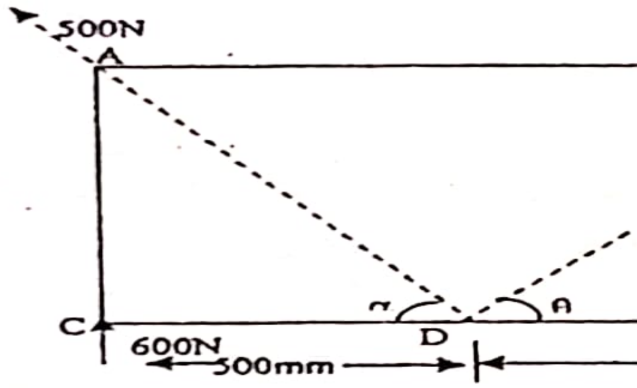
$x = 0.01215$m from A

Similarly, Moment about A.

$u + 38.97\vec{k} - 45\vec{k} = (0.3\vec{i} - y\vec{j}) \times (-150\vec{i} - 79.9\vec{j}) = 0.97\vec{k} = 23.97\vec{k}$

$y = \frac{23}{150} = 0.1533m$// 0.0367m from B to downward.

Four forces acts on a 700×375mm plate as shown. (a) Find the resultant of these forces. (b) Locate the two points where the line of action of the resultant intersects the edge of the plate.



Sol^n

Let us consider 'C' as origin and CE as x-axis and CA as y-axis and the pl... Resultant of the force system is given by.

$= 760\hat{i} + 340\cos\theta\,\hat{i} + 340\sin\theta\,\hat{j} + 500\cos\alpha\,\hat{i} + 500\sin\alpha\,\hat{j} + 600\hat{j}$

$760\hat{i} + 340\left(\frac{200}{\sqrt{200^2+375^2}}\right)\hat{i} + 340\left(\frac{375}{\sqrt{200^2+375^2}}\right)\hat{j} + 500\left(\frac{500}{\sqrt{500^2+375^2}}\right)\hat{i} + ($

$(760+160-400)\,\hat{i} + (300+300+600)\,\hat{j} \qquad = (520\hat{i} + 1200\hat{j})$N

Let us consider the resultant acts from the point (x, y). Then taking mome...

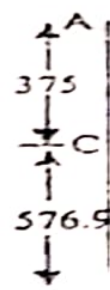
$$(x\hat{i} - y\hat{j}) \times (520\hat{j} + 1200\hat{j}) = 375\hat{j} \times \left(\frac{-500 \times 500\hat{i}}{\sqrt{500^2+375^2}}\right) + \frac{500 \times 375}{\sqrt{500^2+375^2}}\hat{j}$$

$$(700\hat{i} - 375\hat{j}) \times (340 \times \frac{200\hat{i}}{\sqrt{200^2+375^2)}}) + 340 \times \frac{375}{\sqrt{200^2+375^2)}}\hat{j}$$

$1200x\,\hat{k} - 520y\,\hat{k} = 150000\hat{k} + 210000\hat{k} - 60000\hat{k}$

$1200x - 520y = 300000$

$$\frac{x}{250} + \frac{y}{-576.9} = 1$$



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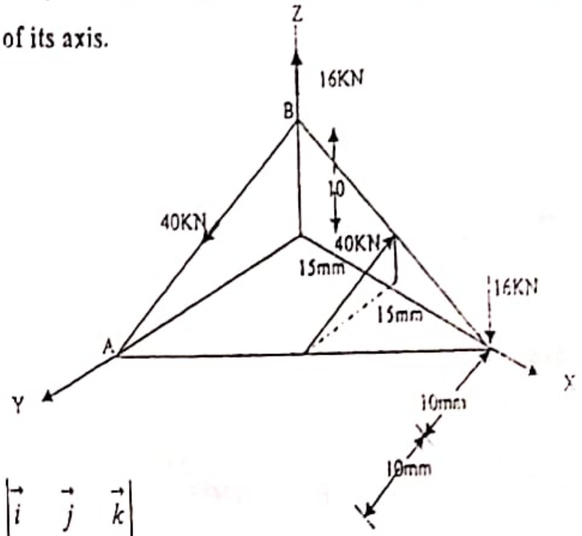
Hence, the line of action cuts the edge CE at the distance 760mm from points and cuts the side CA at a distance 576.9mm along the direction AC from the point C.

Let the resultant meets AB at a distance x. Then as shown in fig.

$$\frac{x}{(375+576.9)} = \frac{250}{576.9} \qquad x = 412.51\text{mm}//$$

Hence, the resultant Meets AB at a distance 412.51mm from point A.

41. Replace the two remaining couple with a single equivalent couple specifying its magnitude and the direction of its axis.



⇒ Sol^n

Couple of 16KN force

$C_1 = -16*30 = -480\text{KNmm}\ \vec{k}$

$\therefore \vec{C^1} = 0.48\text{KNm}\ \vec{k}$

Couple of 40KN

$\vec{AC} = 30\vec{i} - 20\vec{k}$

$\vec{AB} = 10\vec{j} - 20\vec{k} \qquad \vec{AC}\times\vec{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 30 & 0 & -20 \\ 0 & 10 & -20 \end{vmatrix}$

$= 200\vec{i} + 600\vec{j} + 300\vec{k}$

$\therefore \hat{n} = \dfrac{200\vec{i}+600\vec{j}+300\vec{k}}{\sqrt{200^2+600^2+300^2}} \qquad \Rightarrow \hat{n} = 0.286\vec{i} + 0.857\vec{j} + 0.429\vec{k}$

Perpendicular distance $P = \sqrt{15^2+5^2} = 15.811$

Couple = F ×P = 40* 15.811 = 632.456KNmm

$\therefore \vec{C_2} = 632.456(0.286\vec{i} + 0.857\vec{j} + 0.429\vec{k} \qquad \Rightarrow \vec{C_2} = 180.882\vec{i} + 542.015\vec{j} - 271.324\vec{k}$

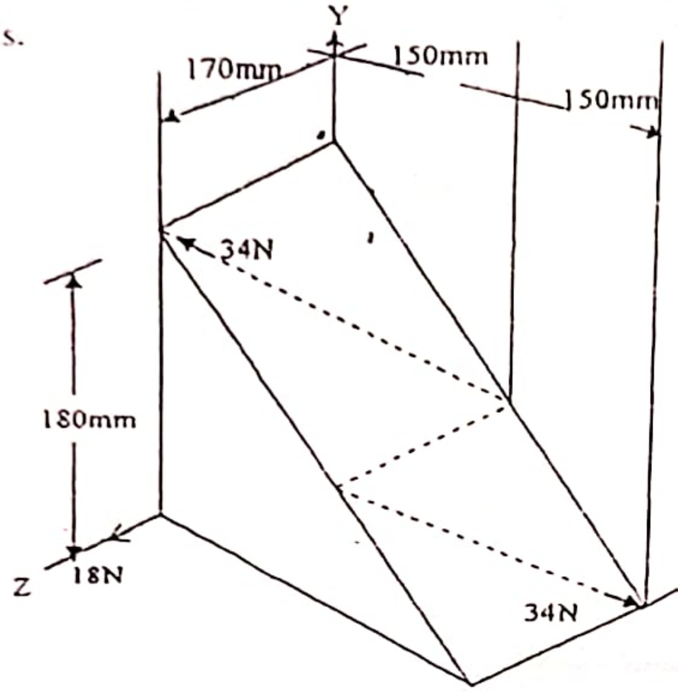
Resultant couple $C = \vec{C_1} + \vec{C_2}$

$= 180.882\vec{i} + 542.015\vec{j} + 271.324\vec{k} - 480\vec{k}$

$\therefore C = 180.882\vec{i} + 542.015\vec{j} - 208.676\vec{k}$

$\therefore$ Magnitude of equivalent couple= 608.313KNmm//

42 Replacing the two remaining couples with a single equivalent couple, specify magnitude and the direction of its axis.



⇒ Sol^n

Here, $M = \vec{r} \times \vec{p}$

Adding moment of couples.

$$(-170\vec{k} + 300\vec{i}) \times (-18\vec{k}) + (-170\vec{k} - 160\vec{j} + 300\vec{i}) \times \{34 * \frac{150\vec{i} - 80\vec{j} - 170\vec{k}}{240.42}\}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 300 & 0 & -170 \\ 0 & 0 & -18 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 300 & -160 & -170 \\ 21.21 & -1131 & -24.04 \end{vmatrix} = \vec{i}(1923.7) + \vec{j}(9006.3) + \vec{k}(0.6)$$

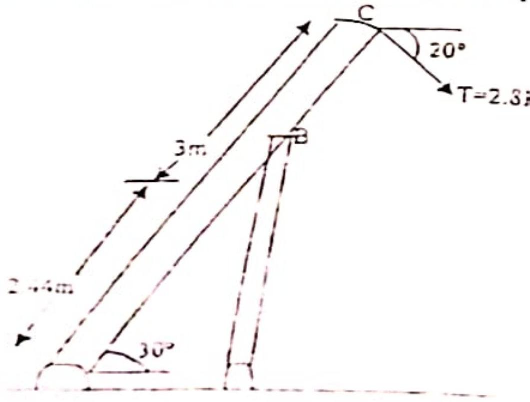
$$\text{So, } M = \sqrt{(1923.7)^2 + (9006.3)^2 + (0.6)^2}$$

$= 9209.45\text{Nmm} \quad = 9.2094\text{Nm}$

$$\text{So, } \alpha = \cos^{-1}\left(\frac{1923.7}{9209.45}\right) = 77.943° \qquad \beta = \cos^{-1}\left(\frac{9006.3}{9209.45}\right) = 12.057°$$

$$\gamma = \cos^{-1}\left(\frac{0.6}{9209.45}\right) = 89.996°\text{// Ans}$$

43 The tension in the cable attached to the end C of an adjust able boom ABC 2.8KN. Replace the force exerted by the cable at C with an equivalent force coup system (a) at A (b) at B.

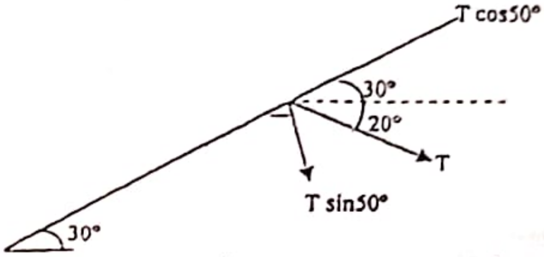


⇒ Sol^n

[illegible]

... moment at A

... Nm

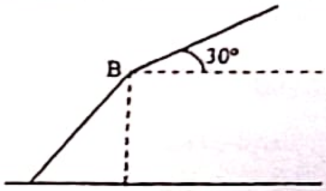


T cos50° does not give moment about A because of same line of action of forces.

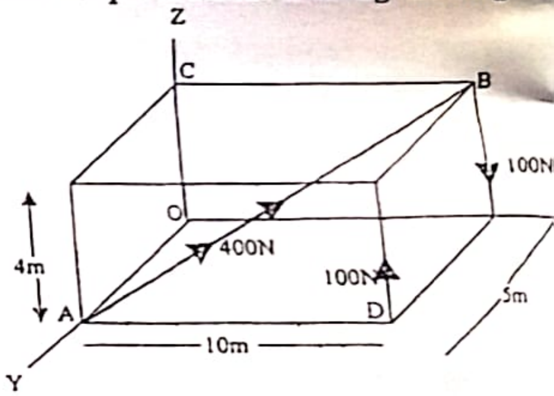
T = 2.8KN

$M_B = 2.8 * \sin 50° * 3$

= 6.435Nm (clockwise)//



44. **Find the moment about CD. The 400Nm couple moment is along the diagonal point A to point B as shown fig.**



⇒ Sol^n

Let $\vec{F} = 400\vec{i}$

$\vec{CE} = 10\vec{i} - 4\vec{k}$

Couple of 100N $\vec{C_1} = 5*100(-\vec{i}) = -500\vec{i}$

Couple of 400Nm

$\vec{AB} = 10\vec{i} - 5\vec{j} + 4\vec{k}$

$$\hat{AB} = \frac{10\vec{i} - 5\vec{j} + 4\vec{k}}{\sqrt{10^2 + 5^2 + 4^2}} = 0.842\vec{i} - 0.421\vec{j} + 0.337\vec{k}$$

$$\vec{C_2} = 400(0.842\vec{i} - 0.421\vec{j} + 0.337\vec{k}) \Rightarrow \vec{C_2} = 360\vec{i} - 168.43\vec{j} + 134.8\vec{k}$$

Adding all moment. $\vec{C_2} + \vec{C_2} + \vec{r} \times \vec{f})$

$$= -500\vec{i} + 336.861\vec{i} - 168.43\vec{j} + 134.8\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 0 & -4 \\ 400 & 0 & 0 \end{vmatrix} \quad = 836.8\vec{i} + 134.8\vec{j} - 168.4\vec{k} +$$

$$\therefore \vec{Cc} = -163.14\vec{i} + 1431.57\vec{j} + 134.74\vec{k}$$

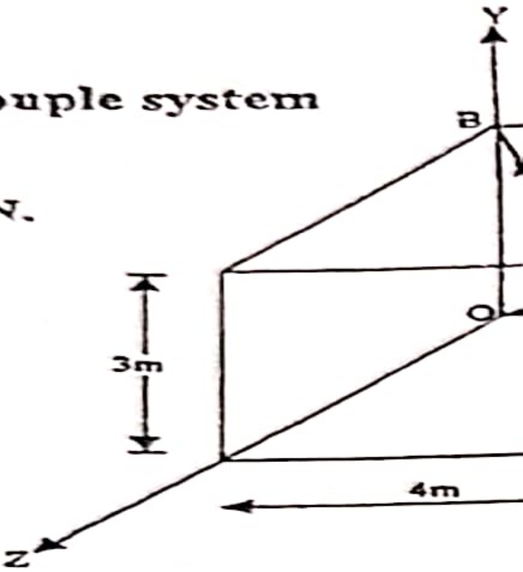
Unit vector along CD. $\vec{CD} = 10\vec{i} - 4\vec{j} + 5\vec{k}$

$$\therefore \vec{U}_{cd} = \frac{10\vec{i} - 4\vec{j} + 5\vec{k}}{\sqrt{10^2 + 4^2 + 5^2}} = 0.842\vec{i} + 0.421\vec{j} - 0.337\vec{k}$$

moment about CD = $\vec{U}_{cd}.\vec{C}_c = (0.842\vec{i} + 0.421\vec{j} - 0.337\vec{k}).$ (-

$M_{CD} = 418.488Nm$//

46. **Find the resultant of the force couple system at point as shown in fig.**
Where $F_1 = 500kN$ and $F_2 = 700kN$.



$\Rightarrow$ Sol^n

Let A, B, C be the points as shown in fig.

Then A(4,0,2), B(0,3,0), C(4,3,2)

Now, unit vectors along $F_1 = \hat{F_1}$

$$= \frac{4\vec{i} + 3\vec{j} + 2\vec{k}}{\sqrt{4^2 + 3^2 + 2^2}} = \frac{1}{\sqrt{29}}(4\vec{i} + 3\vec{j} + 2\vec{k})$$

$$\therefore \vec{F_1} = F_1.\hat{F_1} = \frac{500}{\sqrt{29}}(4\hat{i} + 3\hat{j} + 2\hat{k})kN = (371.4\hat{i} + 27854\hat{j}$$

$$\vec{AB} = 3\hat{j} - (4\hat{i} - 2\hat{k}) = -4\hat{i} + 3\hat{j} - 2\hat{k}$$

$\therefore$ Unit vector along $F_2 = \hat{F_2} = \frac{-4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{4^2 + 3^2 + 2^2}} = \frac{1}{\sqrt{29}}(-4\hat{i} +$

$$\therefore \vec{F_2} = F_2.\hat{F_2} = \frac{700}{\sqrt{29}}(-4\vec{i} + 3\vec{j} - 2\vec{k}) = -519.95\vec{i} + 389.96$$

Now, the resultant force, $\vec{R} = 371.4\vec{i} + 278.54\vec{j} + 185.7\vec{k} - 5$

$= -148.55\vec{i} + 668.5\vec{j} - 74.27\vec{k}$

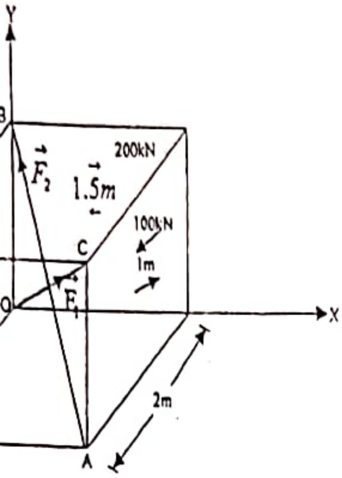
Taking moment about 0. $\vec{m} = \vec{r_2} \times \vec{f_2} + C_1 + C_2$

$= (4\vec{i} + 2\vec{k}) \times (-519.95\vec{i} + 389.96\vec{j} - 259.97\vec{k}) + 200 \times 1.5(-\vec{j}$

$= 1539.84\vec{k} + 1039.88\vec{j} - 1089.9\vec{j} - 179.92\vec{i} - 300\vec{j} + 100\vec{i}$

$= -79.92\vec{i} - 300.002\vec{j} + 1551.84\vec{k}$ //

$(-163.14\vec{i}+1431.57\vec{j}+134.74\vec{k})$



$\hat{j}+185.7\hat{k})$ kN

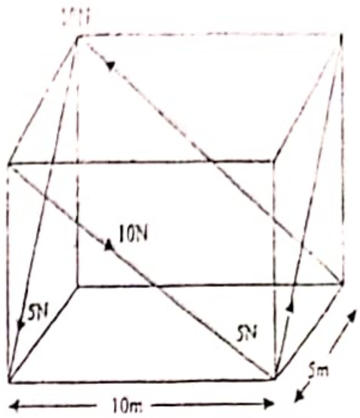
$+3\hat{j}-2\hat{k})$

$6\vec{j}-259.97\vec{k}$

$-519.95\vec{i}+389.96\vec{j}-259.97\vec{k}$

$-\vec{j})+100\times\vec{i}$

48. Add the couple whose forces acts along the diagonals of the sides of the rectangular parallelepiped.



Sol^n

$\perp^r$ distance of 10N force = 5m

$\perp^r$ distance of 5N force = 10m

Now, Moment of couple of 10N force.

$= -5\vec{k}\times 10\left|\frac{-10\vec{i}+5\vec{j}}{\sqrt{10^2+5^2}}\right|$

$= -5\vec{k}\times 10\frac{(-10\vec{i}+5\vec{j})}{10.180}$ $= -4.47\,\vec{k}\times\left[-10\vec{i}+5\vec{j}\right]$ $= 44.7\vec{j}+22.36\vec{i}$

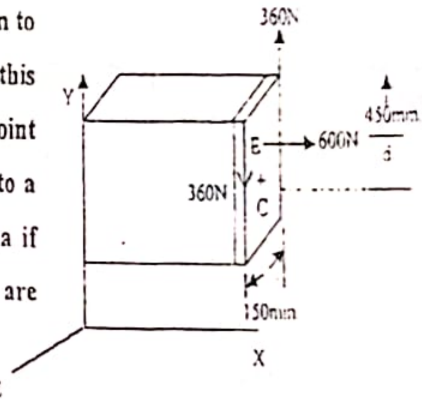
Now, moment of couple of 5N force.

$= 10\vec{i}\times 5\left|\frac{5\vec{j}-5\vec{k}}{\sqrt{5^2+5^2}}\right|$ $= 10\vec{i}\times 5\left|\frac{5\vec{j}-5\vec{k}}{\sqrt{50}}\right|$ $= 7.07\vec{i}\times[5\vec{j}-5\vec{k}]$ $= 35.35\vec{k}+35.35\vec{j}$

Adding the above equation.

$(44.7\vec{j}+22.36\vec{i})+(35.35\vec{k}+35.35\vec{j})$ $= 22.36\vec{i}+80.07\vec{j}+35.35\vec{k}$ //

49. A force and a couple are applied as shown to end of a cantilever beam. (a) Replace this system with a single force f applied at point c, and determine the distance d from C to a line drawn through D and E. (b) solve a if the directions of the two 360N force are reversed.



a) $f_c = -600\text{N}\,\vec{k}$

Taking (anticlockwise +ve)

$M_c = 360\times150-600\times d$

$0 = 54000-600d \Rightarrow d = 90\text{mm}$ below ED

b) $f_c = -600\text{N}\,\vec{k}$

$M_c = -360\times150-600\times d$

or, $0 = -54000-600d$

$d = -90\text{mm}$ i.e. 90mm above ED. //