given a pin-hole model

$P \longrightarrow$ ($P \in R^3$)

$P' = Rp + t$, $P_c = \begin{bmatrix} P'_x / P'_z \\ P'_y / P'_z \\ 1 \end{bmatrix}$ (R, t) $\xrightarrow{P_c}$ $u_c' = u_c(1 + k_1 r_c^2 + k_2 r_c^4)$, $v_c' = v_c(1 + k_1 r_c^2 + k_2 r_c^4)$ (k_1, k_2) $\xrightarrow{u_c', v_c'}$ $u_s = f_x u_c' + c_x$, $v_s = f_y v_c' + c_y$ (f_x, f_y, c_x, c_y) $\longrightarrow u_s, v_s$

$$\underset{\text{meas.}}{z} = h(\underset{\text{pose}}{x}, \underset{\text{landmark}}{y})$$

∴ error:

$$e = z - h(T, p)$$

∴ @ lm p_j w/ pose T_i

overall error:

$$\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\|e_{ij}\|^2$$

$$= \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\|z_{ij} - h(T_i, p_j)\|^2$$

→ we are adjusting the pose T & lm p at the same time

→ Bundle Adjustment

∴ in the final problem:

$$x = [T_1, \dots, T_m, p_1, \dots, p_n]^T$$

∴ when $x \longleftarrow x + \Delta x$

$$\frac{1}{2}\|f(x+\Delta x)\|^2$$

$$\approx \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\|e_{ij} + \underbrace{F_{ij}}_{\text{Partial Derivative w.r.t. } T_i}\Delta\xi_i + \underbrace{E_{ij}}_{\text{Partial Derivative w.r.t. } P_j}\Delta p_j\|^2$$

we then stack all variables altogether:

$$x_c = [\xi_1, \xi_2, \dots, \xi_m]^T \in R^{6m}$$

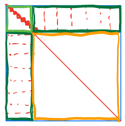
$$x_p = [p_1, p_2, \dots, p_n]^T \in R^{3n}$$

$$\rightarrow \frac{1}{2}\|f(x+\Delta x)\|^2 = \frac{1}{2}\|e + F\Delta x_c + E\Delta x_p\|^2$$

$$\rightarrow H\Delta x = g$$

- $H = J^T J$ or $J^T J + \lambda I$
- $J = [F \; E]$
- $J^T J = \begin{bmatrix} F^T F & F^T E \\ E^T F & E^T E \end{bmatrix}$

Schur Trick

$H =$ 

$$H = \begin{bmatrix} B & E \\ E^T & C \end{bmatrix}$$

$$\Rightarrow H\Delta x = g \iff \begin{bmatrix} B & E \\ E^T & C \end{bmatrix}\begin{bmatrix} \Delta x_c \\ \Delta x_p \end{bmatrix} = \begin{bmatrix} v \\ w \end{bmatrix}$$