

- $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$ (total derivative)
 $f = f(x, y, z)$

I. Regression (not classification)

- Recall: linear model

$$W^T X = \sum w_i x_i \rightarrow \begin{matrix} \text{homogeneous} \\ \text{additivity} \end{matrix}$$

- $x_n \in \mathbb{R}^d$: feature sample for a sample

$y_n \in \mathbb{R}$: observed output

simple: find a function $h(x) = W^T x$ to approximate y

- measure error $(h(x) - y)^2$ (cost function)

$$\Delta \text{ training error: } E_{\text{train}}(h) = \frac{1}{N} \sum_{n=1}^N (h(x_n) - y_n)^2$$

$$\ggg w = A \setminus x$$

$$\Delta \|W^T x - y\|_2^2$$

$$w^* = (X^T X)^{-1} X^T y \quad (\text{closed-form solution})$$

\hookrightarrow invertible/non-invertible
 (often $d > N$)

\downarrow
 pseudo-inverse

also: complexity sparse/dense

2 - Logistic Regression

→ Binary Classification

• input : $x_1, x_2, \dots, x_n \in \mathbb{R}^d$

output : $y_1, y_2, \dots, y_n \in \{+1, -1\}$

a: training = take the sign of some value function

$$\text{sign}(f(x_i)) \approx y_i$$

b: using logistic hypothesis

$$P(y=1|x) = \theta(w^T x) \text{ where}$$

$$\text{where } \theta(s) = \frac{1}{1+e^{-s}}$$

$$P(y=1|x) + P(y=-1|x) = 1$$

$$1 - \underbrace{\frac{e^{w^T x}}{1+e^{w^T x}}}_{\theta(w^T x)} = \frac{1+e^{w^T x}}{1+e^{w^T x}} - \frac{e^{w^T x}}{1+e^{w^T x}} = \underbrace{\frac{1}{1+e^{w^T x}}}_{\theta(-w^T x)}$$

question: difference between likelihood
probability

check: monotonic increase

linear model/non-linear :

depends on
how you combine
your weights &
features

linearity : homogeneous
additivity

c. hinge loss

- Empirical Risk Minimization

△ $f_W(x)$: decision function to be learned

W is the parameters

△ General empirical risk minimization

$$\underset{W}{\text{minimize}} \quad \frac{1}{N} \sum_{n=1}^N \text{loss}(f_W(x_n), y_n)$$