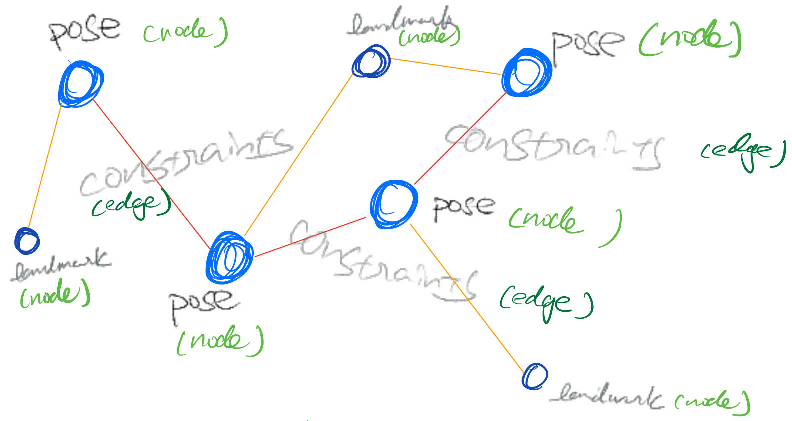


{ edge : dynamic/sensor model
{ node : objective problem

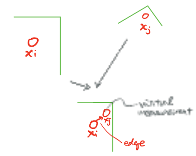
→ solve the over determined problem!

• Edge Creation

- odometry

$O \xrightarrow{\text{edge}} O$
$x_i \qquad x_{i+1}$

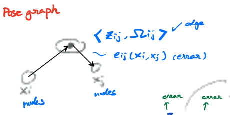
- measurement



edges are then represented as Homogenous coordinates

- odometry
  $x_i^{-1} x_{i+1}$
- observation
  $x_i^{-1} x_j$
  (node i sees node j)

Pose graph



goal: $x^* = \underset{x}{\operatorname{argmin}} \sum_{ij} e_{ij}^T \Omega_{ij} e_{ij}$

error, error, information matrix

remark: why do we write it this way

assumption: constraints are independent of each other

error function:

$e_{ij}(x_i, x_j) = t2v(z_{ij}^{-1}(x_i^{-1} x_j))$

measurement; x_j reference w.r.t. x_i

Gauss-Newton:

- linearization:

$e_{ij}(x+\Delta x) \simeq e_{ij}(x) + J_{ij}\Delta x$

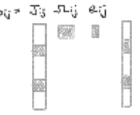
Remark: $J_{ij} = \frac{\partial e_{ij}(x)}{\partial x}$

$J_{ij} = \left[0 \cdots 0 \,\middle|\, \frac{\partial e}{\partial x_i} \,\middle|\, 0 \cdots 0 \,\middle|\, \frac{\partial e}{\partial x_j} \,\middle|\, 0 \cdots \right]$

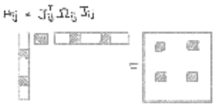
- $b^T = \sum_{ij} b_{ij}^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$ (sparse)

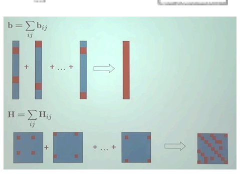
$H = \sum_{ij} H_{ij}^T = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

$b_{ij} = J_{ij} \; \Omega_{ij} \; e_{ij}$



$H_{ij} = J_{ij}^T \Omega_{ij} J_{ij}$





e.g.

$z_{12} = 1m \quad z_{23} = 1m$

$X_1 \to X_2 \to X_3$

$\Omega_{12} = 1 \quad \Omega_{23} = \frac{1}{2}$

3 nodes
2 constraints

- $X_0 = \{x_1, x_2, x_3\} = \{0, 0, 0\}$
  initial guess
- $e_{ij} = z_{ij} - (x_j - x_i)$

$e_{12} = (1 - (0-0)) = 1$
$e_{23} = (1 - (0-0)) = 1$

- $J_{ij} = \left(\frac{\partial e_{ij}}{\partial x_1} \quad \frac{\partial e_{ij}}{\partial x_2} \quad \frac{\partial e_{ij}}{\partial x_3} \right)$

$J_{12} = \left(\frac{\partial e_{12}}{\partial x_1} \quad \frac{\partial e_{12}}{\partial x_2} \quad \frac{\partial e_{12}}{\partial x_3} \right)$

$= (1 \quad -1 \quad 0)$

$J_{23} = (0 \quad 1 \quad -1)$

- $b^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$

$= 1 \cdot 1 \cdot (1, -1, 0) \quad (b_1)$
$+ 1 \cdot \frac{1}{2} \cdot (0, 1, -1) \quad (b_2)$

$= (1 \quad -\frac{1}{2} \quad -\frac{1}{2})$

- $H = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

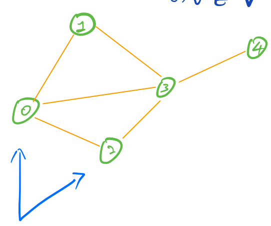
- $\Delta x = -H \backslash b$ → error when $\det(H) = 0$

- change the relative constraints to global one

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

basic graph theory

$G = (V, E)$ E connect u, v; $u, v \in V$

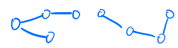


neighbor

$V = \{0, 1, 2, 3, 4\}$

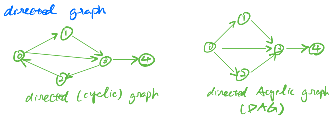
$E = \{(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)\}$

1. neighbor(0) = {0, 1, 2}
2. degree(0) = 3
   degree(2) = 2
3. path $4 \to 3 \to 2 \to 0$
4. cycle $0 \to 1 \to 3 \to 2 \to 0$
5. connectivity :- graph is connected if ∃ path between (u,v)
   $(u,v) \in V$
   - graph is connected when all vertices are connected
   - connected component $V' \subseteq V$

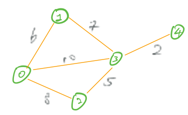


types of graph

1. undirected graph (shown above)
2. directed graph



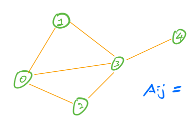
3. weighted graph



4. trees
   1. connected and acyclic
   2. removing edge disconnects graph
   3. adding edge creates a cycle

graph representation

- Adjacency Matrix



$A_{ij} = \begin{cases} 1 & \text{for edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$

$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 1 & 0 \end{array}$

- edge set
  {(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)}

- adjacency list
  0 → [1 2 3]
  1 → [0 3]
  2 → [0 3]
  3 → [0 1 2 4]
  4 → [3]