AAE 6102 Satellite Communication & Navigation

Ch 1 Basics

GPS Architecture

- Space segment — broadcast ranging codes & navigation data (50 bits/second)
- Control segment
  - track/monitor GPS satellites
  - estimate clock & orbit
  - small maneuvers to maintain orbit
  - small clock corrections
  - relocations to compensate SV failure
- User segment — receivers

distributed globally → ensure coverage
- position surveyed exactly

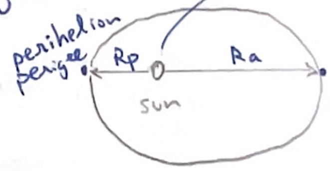
Satellite position representation

Kepler's three law

- The law of orbit

$R_a = a(1+e)$

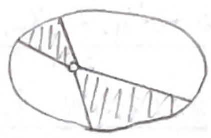
$R_p = a(1-e)$



e = eccentricity of the ellipse

- The law of areas

$$\frac{A_0}{t_1 - t_0} = \frac{A_1}{t_3 - t_1}$$



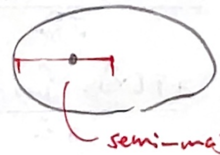
area velocity = const.

- The law of period

$$\frac{R_0^3}{T_0^2} = \frac{R_1^3}{T_1^2}$$

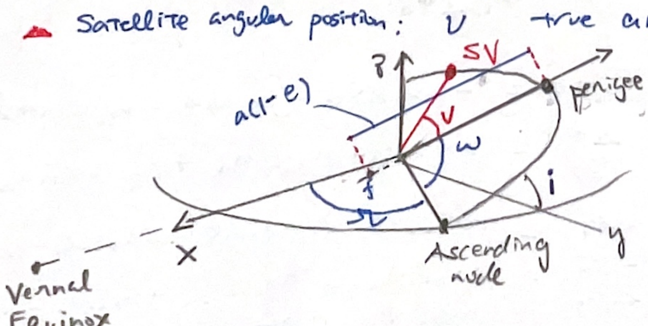
semi-major axis

orbital period



Keplerian Elements

- orbit shape : a semimajor axis; e eccentricity
- orbit orientation relative to earth : i inclination; Ω right ascension of ascending node; ω angle of perigee
- Satellite angular position : ν true anomaly



△ Coordinate Transformation

Keplerian parameters

↓

Peri-focal coordinates

↓

Earth centered inertial

↓

Earth centered earth-fixed (World Geodetic Survey WGS-84)

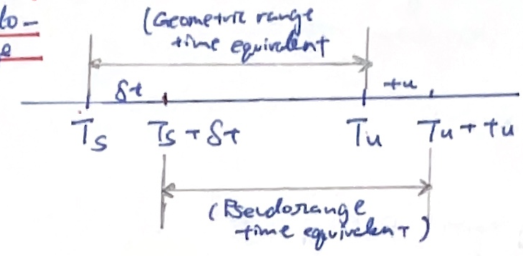
△ factors that affect Satellite orbit/clock

- perturbation forces
  - non sphericity of earth's gravitational potential
  - 3rd body effects
  - solar radiation pressure,
- relativity effects
  - less gravity → time faster
  - faster speed → time slower

Ch 2 Positioning

TOA & GNSS measurement

pseudo-range



$$\rho = c[(T_u + t_u) - (T_s + \delta t)] + \sum \delta$$

$$= \underbrace{c(T_u - T_s)}_{\text{Geometric Range}} + \underbrace{c(t_u - \delta t)}_{\text{Clock offset from system time}} + \underbrace{\sum \delta}_{\text{delay \& noise}}$$

delay & noise = Trop + Ion + K_{rec} + K^{sat} + ε

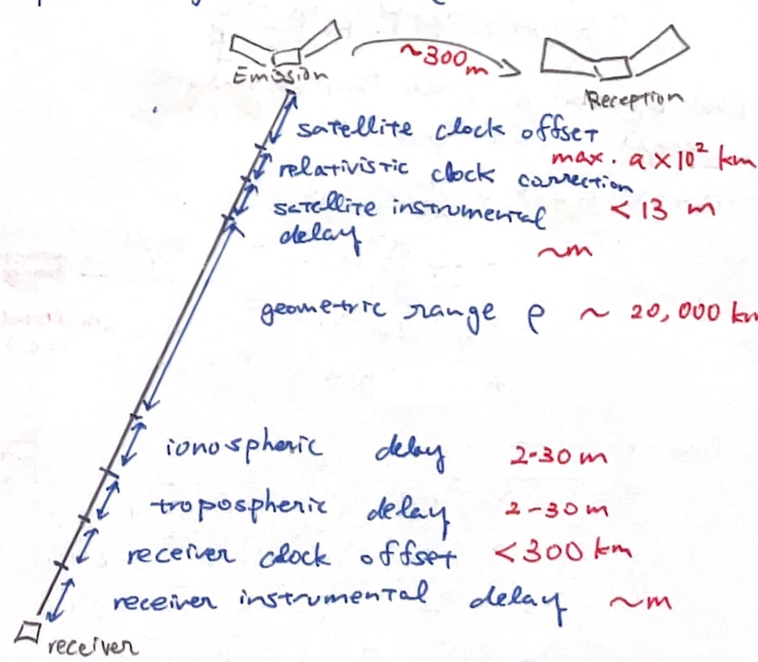
- $T_u + t_u$, $T_s + \delta t$ are obtained from
  - Ranging codes &
  - non-sychronized clock

carrier phase

$$\rho = \lambda \cdot \Delta\varphi = \lambda(\underbrace{\varphi_r(T_2)}_{\text{phase of signal reception}} - \underbrace{\varphi^s(T_1)}_{\text{phase of signal transmission}})$$

- much more precise
- yet w/ ambiguity on integer of existing wavelengths

pseudo-range error source



Least Squares

$x \in R^n$ (tracking state)

$y \in R^m$ (observation state)

$$\begin{cases} y = Hx + n \\ \hat{x} = \underset{x}{\operatorname{argmax}} \; P(y|x) \end{cases}$$

$n \sim N(0, \sigma)$

$$\therefore \hat{x} = \underset{x}{\operatorname{argmax}} \frac{1}{(2\pi\sigma)^{N/2}} e^{-\frac{1}{2\sigma^2}\|y-Hx\|^2}$$

$$= \underset{x}{\operatorname{argmin}} \|y - Hx\|^2$$

$\therefore$ we set $\frac{d}{d\hat{x}}\|y - H\hat{x}\|^2 = 2H^T H\hat{x} - 2H^T y$

$$\therefore \hat{x} = (H^T H)^{-1} H^T y$$

△ weighted version

$$\hat{x} = \underset{x}{\operatorname{argmax}} \frac{1}{(2\pi)^{\frac{N}{2}} |R_n|^{\frac{1}{2}}} e^{-\frac{1}{2}(y-Hx)^T R_n^{-1}(y-Hx)}$$

$$= \underset{x}{\operatorname{argmin}} (y-Hx)^T R_n^{-1}(y-Hx)$$

$$\therefore \hat{x} = (H^T R_n^{-1} H)^{-1} H^T R_n^{-1} y$$

linearization (Single Point Positioning)

△ recall Taylor Series

$$f(x+\Delta x) = f(x) + \frac{df}{dx}(x)\cdot\Delta x + \frac{1}{2!}\frac{d^2 f}{dx^2}(x)\Delta x^2 + \frac{1}{3!}\frac{d^3 f}{dx^3}(x)\Delta x^3 + O(\Delta x^4)$$

recall pseudo-range

$$\rho = \underbrace{c(T_u - T_s)}_{\text{geometry range}} + c(t_u - \delta t) + \Sigma\delta \text{ (neglected here)}$$

$$= r + c(\underbrace{t_u}_{\text{receiver offset}} - \underbrace{\delta t}_{\text{satellite offset}}) \quad \text{known here (broadcasted)}$$

$$= r + ct_u$$

△ linearization to solve

$$\begin{aligned}
\rho_1 &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + ct_u \\
\rho_2 &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + ct_u \\
\rho_3 &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + ct_u \\
\rho_4 &= \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + ct_u
\end{aligned}$$

known known

4 unknowns

$$\triangle \rightarrow \rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u$$

$$= f(x_u, y_u, z_u, t_u)$$

assume a known, approximate position location $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ & $\hat{t}_u$ (time bias)

$$\hat{\rho}_j = \sqrt{(x_j - \hat{x}_u)^2 + (y_j - \hat{y}_u)^2 + (z_j - \hat{z}_u)^2} + c\hat{t}_u$$

$$= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) \quad \text{— } (*)$$

$$\triangle \rightarrow x_u = \hat{x}_u + \Delta x_u \qquad z_u = \hat{z}_u + \Delta z_u$$

$$y_u = \hat{y}_u + \Delta y_u \qquad t_u = \hat{t}_u + \Delta t_u$$

$$\therefore f(x_u, y_u, z_u, t_u) = f(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u, \hat{t}_u + \Delta t_u)$$

$\therefore$ TS of $f(x_u, y_u, z_u, t_u)$:

$$\begin{aligned}
f(x_u, y_u, z_u, t_u) &= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u}\Delta x_u \\
&+ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u}\Delta y_u \\
&+ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u}\Delta z_u \\
&+ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u}\Delta t_u \\
&+ O(\Delta^2) \quad \text{— } (☆)
\end{aligned}$$

(from above)

$$\left.\begin{aligned}
\frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u} &= -\frac{x_j - \hat{x}_u}{\hat{r}_j} \\
\frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u} &= -\frac{y_j - \hat{y}_u}{\hat{r}_j} \\
\frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u} &= -\frac{z_j - \hat{z}_u}{\hat{r}_j} \\
\frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u} &= c \\
\hat{r}_j &= \sqrt{(x_j - \hat{x}_u)^2 + (y_j - \hat{y}_u)^2 + (z_j - \hat{z}_u)^2}
\end{aligned}\right\} (\#)$$

△ put (#) & (*) into (☆)

we get

$$\rho_j = \hat{\rho}_j - \overbrace{\frac{x_j - \hat{x}_u}{\hat{r}_j}}^{a_{xj}}\Delta x_u - \overbrace{\frac{y_j - \hat{y}_u}{\hat{r}_j}}^{a_{yj}}\Delta y_u - \overbrace{\frac{z_j - \hat{z}_u}{\hat{r}_j}}^{a_{zj}}\Delta z_u + c\Delta t_u$$

$$\Rightarrow \rho_j - \hat{\rho}_j = -a_{xj}\Delta x_u - a_{yj}\Delta y_u - a_{zj}\Delta z_u + c\Delta t_u$$

$$\Rightarrow \Delta\rho_j = a_{xj}\Delta x_u + a_{yj}\Delta y_u + a_{zj}\Delta z_u - c\Delta t_u$$

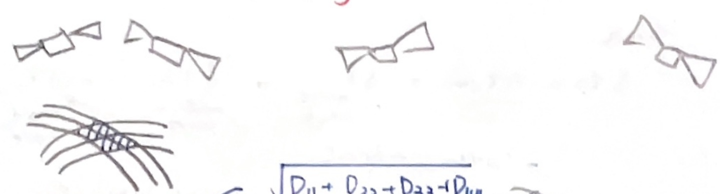
$$\Rightarrow \begin{aligned}
\Delta\rho_1 &= a_{x1}\Delta x_u + a_{y1}\Delta y_u + a_{z1}\Delta z_u - c\Delta t_u \\
\Delta\rho_2 &= a_{x2}\Delta x_u + a_{y2}\Delta y_u + a_{z2}\Delta z_u - c\Delta t_u \\
\Delta\rho_3 &= a_{x3}\Delta x_u + a_{y3}\Delta y_u + a_{z3}\Delta z_u - c\Delta t_u \\
\Delta\rho_4 &= a_{x4}\Delta x_u + a_{y4}\Delta y_u + a_{z4}\Delta z_u - c\Delta t_u
\end{aligned}$$

$$\Rightarrow \Delta\rho = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \end{bmatrix} \quad H = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ a_{x3} & a_{y3} & a_{z3} & 1 \\ a_{x4} & a_{y4} & a_{z4} & 1 \end{bmatrix} \quad \Delta x = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ -c\Delta t_u \end{bmatrix}$$

$\Rightarrow \Delta\rho = H\Delta x \longrightarrow$ could be > 4 over-determined

- get Δx (can use least-square)
- get $(x_u, y_u, z_u, t_u) = (\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) + \Delta x$

Dilution of Precision (DOP)



$\sqrt{D_{11}+D_{22}+D_{33}+D_{44}}$

- PDOP/GDOP $= \sqrt{D_{11}+D_{22}+D_{33}}$
- HDOP $= \sqrt{D_{11}+D_{22}}$
- VDOP $= \sqrt{D_{33}}$
- TDOP $= \sqrt{D_{44}}/c$

$H\Delta x = \Delta p$

$$H = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{xn} & a_{yn} & a_{zn} & 1 \end{bmatrix}$$

$\Delta x = (H^TH)^{-1}H^T\Delta p$

$\mathrm{cov}(dx) = (H^TH)^{-1}\sigma^2_{UERE}$ user equivalent range error

$$(H^TH)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}$$

→ better geometry, less DOP
→ VDOP > HDOP, no variation vertically
→ precise → concentrated data
accurate → ground-truth-closed data

SV clock correction

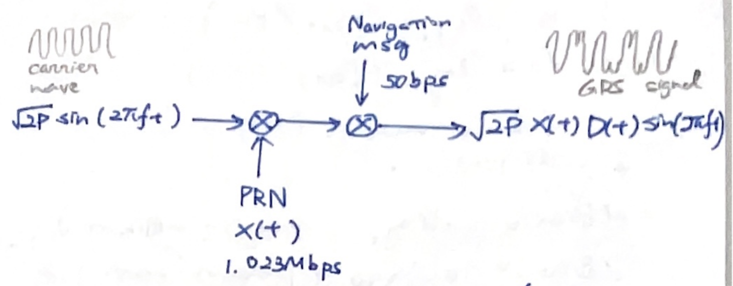
▵ satellite clock offset
- monitored by groud station network
- rebroadcasted to receiver

▵ clock correction model
- thru polynomial
- polynomial coefficients a_0, a_1, a_2
- prediction model (hours to days)
- reference by T_{oc} → operation center

Ch3 GPS/GNSS signal

Signal Structure

- signal = carrier · wave + ephemeris, satellite clock/correction, navigation msg, PRN code for Time of Arrival



- carrier wave @ 1575.42MHz, L1, is modulated by
  - coarse/acquisition (C/A) code civilians
  - precision/secure (P/Y) code military

PRN (Pseudo Random Noise) Code

▵ for measurement of time of arrival (TOA)
▵ generation pattern: Gold Code
▵ 1 second : 50 bit
→ 1000 ms : 50 bit
→ 20 ms : 1 bit
→ 20 ms : 20 repetition of C/A
→ 1 ms : 1 repetition of C/A
→ 1 repetition of C/A : 1023 $\frac{1}{0}$

▵ C/A : 1.023 M cps
P/Y : 10.23 M cps

Signal @ L-band lower frequency gets easier to be affected by ionosphere

Auto-correlation

① Satellite transmits code-phase with timestamp $T^S(n)$
② Receiver receives code-phase with time stamp $T_R(n)$
n here stands for epoch n
③ on timeline of "Receiver", a replica code-phase is generated at $T^S(n)$ on timeline "receiver"
④ conduct auto-correlation by shifting the replica code-phase to match received code-phase on timeline receiver
⑤ the shifted Δt is the inferred propagation time
⑥ pseudorange $= c\cdot\Delta t$
⑦ this pseudorange $= c[(T_u+t_u)-(T_s+\delta t)] + \Sigma\delta$

Carrier Phase : resolve fractional difference / integer ambiguity (N)
could get accuracy of 1-2cm

Power of GPS

- Bel $B = \log_{10}(P_{out}/P_{in})$
- Decibel $dB = \log_{10}(P_{out}/P_{in})$
- set reference value (P_{in})
  1 milliwatt :

$$dBm = 10\log_{10}(\text{power in milliwatt})/1\text{ milliwatt}$$
$$dBW = 10\log_{10}(\text{power in watt})/1\text{ watt}$$

△ Path Loss

- transmit power is 27W
- received power reduced by 10^{16}

-130 dBm ~ -160 dBm < Thermal Noise Power -110 dBm < WIFI BT Phones

△ Antenna Gain : ratio of $\frac{\text{entire sphere area}}{\text{spot area}}$

Ch4 Differential Positioning & Precise Positioning

Errors

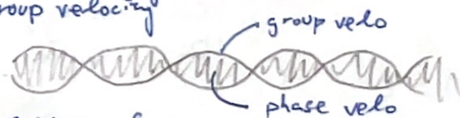
$$\delta t_D = \delta t_{atm} + \delta t_{\substack{noise\\inter.}} + \delta t_{\substack{mp\\multi-path}} + \delta t_{\substack{hw\\hardware}}$$

- atmosphere
- receiver noise + interference
- multipath
- receiver hardware offset

△ Satellite Clock Error
- offset of SV time to GPS time
- curve-fit to model the clock correction (polynomial coefficients in navmsg)
- not location-dependent for users

△ Ephemeris Error
- due to perturbation forces
- curve-fit to model the error
- location-dependent for users (increases with separation) base station receiver

△ Ionosphere Delay
- 電離層
- affects electromagnetic wave propagation
- dispersive medium 15XX MHz
  - $f > f_p \simeq 10^6$ Hz can travel thru
  - phase velocity will exceed group velocity



  - amount of retardation of V_{group} = advance of carrier phase w.r.t. free-space propagation
  - GPS : PRN & navmsg delayed than carrier phase
  - → ionospheric divergence
- removed by dual receiver
- depend on frequency f, elevation angle φ' (@ iono. pierce pt), total electron content TEC
- location-dependent for users (increases w/ separation) base station receiver

△ Troposphere Delay
- 對流層
- nondispersive, but with refraction

dry (hydrostatic)	wet (nonhydrostatic)
- 90% to delay	- lesser to delay
- predict accurately	- difficult to predict
- extends to 40km height	- extends to 10km height
- utilize troposphere model to predict	

basically depends on weather

- location-dependent for users

△ Receiver Noise

- thermal noise jitter
- interference/jamming (malicious signal)

△ Multipath

- 

- depends on environment
  - receiver location
  - satellite elevation angle
  - receiver signal processing
  - antenna gain pattern
  - signal characteristic
- improvements :
  - detect → received signal pattern
    thru → signal/noise density ratio
    to see → reflection/refraction
  - signal processing :
    reduce early/late correlator
  - antenna :
    increase directivity

△ Hardware Bias

- Satellite Bias
  - signals and carrier frequency not totally synced.
- Receiver/Equipment Bias
  - signals are delayed in hardware

1. from above, usually (single point), dominan pseudo-range error :
"Ionospheric Delay"
→ could be removed w/ dual receiver

2. spatially correlated :
- Ephemeris Errors
- Ionospheric Error
- Tropospheric Error
- multipath

Real-Time Kinematic (RTK)

- base receiver + rover receiver (Z2)
- base receiver process GPS signals
- rover receiver process GPS signals
- base station broadcast its measurement
  rover receiver
- rover receiver could then resolves some error like ionospheric errors
- harnesses both pseudo-range (code) & carrier phase
  * thru base correction & RTK algo
- m → cm
  m → cm
  * single-band baseline < 10 km
  multi-band baseline < 60 km
  * base unit could be provided thru "local service provider"

△ typical algo.

1. KF
2. unknowns : X_rover, V_rover, ambg.
3. measurements : double differenced carrier phase
   double differenced pseudo-range
4. Integer ambg. solution : a. EKF update
   b. solve ambg. improve accuracy / converge time

Precise Point Positioning (PPP)

- single receiver
- relies on precise satellite clock / orbit products correction
  ↑ from global stations.
- cm-level.
- PPP performance > RTK

Ch5 Augmentation Systems

differential positioning (relative positioning)

- 2 receivers are close
- track difference between the 2
  - can be tracked more accurately
  - iono + tropo + clock + ephemeris errors are cancelled.
- $\Delta X = (P_1, t_1) - (P_2, t_2)$
  ↳ know ΔX, know reference get rover.
- base station, with high-performance GPS receiver, could calculate error/correction & send correction to rover.
- DGPS = reference station + data link + user receiver
- Local Area Differential GPS
  Wide Area Differential GPS

Ground Based Augmentation Systems (GBAS)
- pseudo-range corrections
- use/don't use
- Aviation
- DGPS

Satellite Based Augmentation Systems (SBAS)
- Aviation
- DGPS
- iono., clock, ephemeris.
- GEO
- monitored / non monitored
- use / don't use

- send corrections
- send integrity msgs.