pose_callback : T in

ESKF validation

△

△ pose_callback:

T in $\{I\}$

$\Rightarrow T \Rightarrow \xi_I = T^I_B \, \xi_B$

△ y_ref , x_pose

△ twist_callback:

$\begin{cases} \dot{t} \\ \dot{R} \end{cases}$ in $\{I\}$

$\Rightarrow \begin{cases} \dot{t}^I_B \\ \dot{R}^I_B \end{cases}$

① thrust_pub & control_pub

?

② yaw_ref:

→ set in path?

→ user random give & help modifying.

△ $x, y, z, \phi, \theta, \psi$

$u \; v \; w \quad p \; q \; r$

why your value so weird?

△ PID & MPC

ctrl allocation (+)

different (−)

△ why don't use default control allocation

(thrust_manager.

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau + w$$

$$\rightarrow M_{rb}\dot{v} + M_a\dot{v} + M_{rb}g + C(v)v + D(v)v + g(\eta) = \tau + \xi$$

all in $\mathcal{B}$

$$\rightarrow \underbrace{M_{rb}(\dot{v} + g)}_{\text{IMU measured}} = \tau + \underbrace{\xi}_{\text{external disturbance}} - \underbrace{M_a\dot{v}}_{\text{added mass}} - \underbrace{C(v)v}_{\text{科氏力 force}} - \underbrace{D(v)v}_{\text{damping force}} - \underbrace{g(\eta)}_{\text{gravity + buoyancy}}$$

→ 用這個去 propagate

→ w/ IMU, we can use this

ESKF

△ equation of motion

for any IMU:

$$\begin{cases} \dot{p} = v \\ \dot{v} = R(\tilde{a} - b_a - \eta_a) + g \\ \dot{R} = R(\tilde{\omega} - b_g - \eta_g)^\wedge \\ \dot{b}_g = \eta_{b_g} \\ \dot{b}_a = \eta_{b_a} \\ \dot{g} = 0 \end{cases}$$

△ inducing error state

$$\left.\begin{aligned} p &= \bar{p} + \delta p \\ v &= \bar{v} + \delta v \\ R &= \bar{R} \boxplus \delta R \\ b_g &= \bar{b}_g + \delta b_g \text{ (gyro)} \\ b_a &= \bar{b}_a + \delta b_a \text{ (accel)} \\ g &= \bar{g} + \delta g \end{aligned}\right\} \quad \begin{aligned} \delta\dot{p} &= \delta v \\ \delta\dot{v} &= -R(\tilde{a} - b_a)^\wedge \delta\theta - R\delta b_a - \eta_a + \delta g \\ \delta\dot{\theta} &= -(\tilde{\omega} - b_g)^\wedge \delta\theta - \delta b_g - \eta_g \\ \delta\dot{b}_g &= \eta_g \\ \delta\dot{b}_a &= \eta_a \\ \delta\dot{g} &= 0 \end{aligned}$$

true state, nominal state, error state

$$x = [p, R, v, b_g, b_a, g, \xi]^T$$

△ error state motion of equation

nominal state

$$\begin{cases} p(t+\Delta t) = p(t) + v\Delta t + \frac{1}{2}(R(\tilde{a} - b_a))\Delta t^2 + \frac{1}{2} g \Delta t^2 \\ v(t+\Delta t) = v(t) + R(\tilde{a} - b_a)\Delta t + g\Delta t \\ R(t+\Delta t) = R(t)\,\mathrm{Exp}((\tilde{\omega} - b_g)\Delta t) \\ b_g(t+\Delta t) = b_g(t) \\ b_a(t+\Delta t) = b_a(t) \\ g(t+\Delta t) = g(t) \end{cases}$$

$$\xi(t+\Delta t) = \xi(t)$$

error state

$$\begin{cases} \delta p(t+\Delta t) = \delta p + \delta v \Delta t \\ \delta v(t+\Delta t) = \delta v + (-R(\tilde{a} - b_a)^\wedge \delta\theta - R\delta b_a + \delta g)\Delta t - \eta_v \\ \delta\theta(t+\Delta t) = \mathrm{Exp}(-(\tilde{\omega} - b_g)\Delta t)\delta\theta - \delta b_g \Delta t - \eta_\theta \\ \delta b_g(t+\Delta t) = \delta b_g + \eta_g \\ \delta b_a(t+\Delta t) = \delta b_a + \eta_a \\ \delta g(t+\Delta t) = \delta g. \end{cases}$$

$$\delta\xi(t+\Delta t) = \delta\xi$$

- $\delta x_{k+1} = f(\delta x_k) + w$, $w \sim \mathcal{N}(0, Q)$

$$z = \begin{bmatrix} p \\ R \\ \tau \end{bmatrix} = \begin{bmatrix} p \\ R \\ \underbrace{M_{rb}(\dot{v} + g)}_{\text{no states dep.}} - \xi + \underbrace{M_a\dot{v}}_{\text{no states dep.}} + \underbrace{C(v)v}_{\text{no states dep.}} + D(v)v + g(\eta) \end{bmatrix} + \eta$$

w.r.t. ξ ✓ w.r.t. v ✓ w.r.t.

where

△ △

$$D(v)v = D_L + D_{NL}(v)$$

$$\frac{\partial D(v)v}{\partial \delta v} = \frac{\partial D_L v}{\partial \delta v} + \frac{\partial}{\partial \delta v} D_{NL}(v)v$$

- $D_L v = D_L(\bar{v} + \delta v)$

$\therefore \frac{\partial D_L v}{\partial \delta v} = D_L$

- $D_{NL}(v)v = D_{NL}(\bar{v} + \delta v)(\bar{v} + \delta v)$

$$= \begin{bmatrix} -X_{u|u|} \, |u + \delta u| \, (u + \delta u) \\ -Y_{v|v|} \, |v + \delta v| \, (v + \delta v) \\ -Z_{w|w|} \, |w + \delta w| \, (w + \delta w) \end{bmatrix} \approx \begin{bmatrix} -X_{u|u|} \cdot f_p(u + \delta u) \\ -Y_{v|v|} \cdot f_p(v + \delta v) \\ -Z_{w|w|} \cdot f_p(w + \delta w) \end{bmatrix}.$$

$$\begin{bmatrix} q \\ \dot q \\ \tau_d \end{bmatrix} =$$

$$\begin{matrix} \dot q \\ \ddot q \\ \dot\tau_d \end{matrix} = \begin{bmatrix} \dot q \\ M(q)^{-1}(\tau - \tau_d - H(q,\dot q)) \\ \dot\tau_d \end{bmatrix}$$

$q - (q + \dot q \tau)$

$$\therefore \frac{\partial D_{NL}(v)v}{\partial \delta v} = \begin{bmatrix} -X_{u|u|} \frac{\partial}{\partial \delta u} f_p \\ -Y_{v|v|} \frac{\partial}{\partial \delta v} f_p \\ -Z_{w|w|} \frac{\partial}{\partial \delta w} f_p \end{bmatrix}$$

$21\times21\times21\times9\ (9\times21 \cancel{\times21\times21\times21}\times9 + 9\times9)$

$21\times21\cancel{\times21\times9\ \times9}\times9$

21×9 9×1

$$x = [P\ v\ R\ b_a\ b_g\ g\ \xi]^T$$

$$z = [P\ R\ \tau]^T$$

$$\begin{bmatrix} \frac{\partial P}{\partial P} & \frac{\partial P}{\partial v} & \frac{\partial P}{\partial R} & \frac{\partial P}{\partial b_a} & \frac{\partial P}{\partial b_g} & \frac{\partial P}{\partial g} & \frac{\partial P}{\partial \xi} \\ \frac{\partial R}{\partial P} & \frac{\partial R}{\partial v} & \frac{\partial R}{\partial R} & \frac{\partial R}{\partial b_a} & \frac{\partial R}{\partial b_g} & \frac{\partial R}{\partial g} & \frac{\partial R}{\partial \xi} \\ \frac{\partial \tau}{\partial P} & \frac{\partial \tau}{\partial v} & \frac{\partial \tau}{\partial R} & \frac{\partial \tau}{\partial b_a} & \frac{\partial \tau}{\partial b_g} & \frac{\partial \tau}{\partial g} & \frac{\partial \tau}{\partial \xi} \end{bmatrix} — A.$$

$$\tau_B = \underbrace{M_{rb}\left(\begin{bmatrix}\tilde a \\ \tilde w\end{bmatrix} - \begin{bmatrix} b_a \\ b_g \end{bmatrix}\right)}_{①} \underbrace{- \xi}_{②} + \underbrace{M_a\left(\begin{bmatrix}\tilde a \\ \tilde w\end{bmatrix} - \begin{bmatrix} b_a \\ b_g \end{bmatrix}\right)}_{③} + \underbrace{C(v)v}_{④} + \underbrace{D(v)v}_{⑤} + \underbrace{g(\eta)}_{⑥}$$

$$= mI[\tilde a - b_a] + \begin{bmatrix} 0 & mz_g & 0 \\ -mz_g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}[\tilde w - b_g]$$ ① M_{rb}

$- \xi_B$ ② external ξ

$+ \begin{bmatrix} -X_{\dot u} & & \\ & -Y_{\dot v} & \\ & & -Z_{\dot w} \end{bmatrix} [\tilde a - b_a]$ ③ added mass

$+ [0] V_B + \begin{bmatrix} 0 & Z_{\dot w} w & Y_{\dot v} v \\ Z_{\dot w} w & 0 & -X_{\dot u} u \\ -Y_{\dot v} v & X_{\dot u} u & 0 \end{bmatrix} V_B$ ④ central.

$+ \begin{bmatrix} -X_u & & \\ & -Y_v & \\ & & -Z_w \end{bmatrix} [V_B] + \begin{bmatrix} -X_{u|u|} & \lvert u \rvert\, u \\ -Y_{v|v|} & \lvert v \rvert\, v \\ -Z_{w|w|} & \lvert w \rvert\, w \end{bmatrix}$ ⑤ damping

$+ [g + B]$ ⑥

$\alpha \quad \tau_I = R\tau_B$

$$\tau_I = mI\,R(\tilde a - b_a) + M_{rbR}\,R(\tilde w - b_g)$$
$$- \xi_I$$
$$+ M_a R(\tilde a - b_a)$$
$$+ R[V_B]_\times V_B$$
$$+ R[D_L V_B + D_{NL} f(V_B)]$$
$$+ [g + B]$$

$$H = \frac{\partial h}{\partial x} \frac{\partial x}{\partial \delta x}$$

$$\begin{bmatrix} P \\ v \\ R \\ \xi \end{bmatrix} = \begin{bmatrix} P \\ v \\ R \\ \xi \end{bmatrix} + PH^T(HPH^T + R)^{-1}\begin{bmatrix} P \\ R \\ \tau \end{bmatrix}$$

g

$$\tau_B = mI(\tilde a - b_a)$$
$$+ M_{rbR}(\tilde w - b_g)$$
$$- \xi_B$$
$$+ M_a(\tilde a - b_a)$$
$$\alpha \leftarrow + [V_B]_\times V_B$$
$$\beta \leftarrow + D_L V_B + D_{NL} f(V_B)$$

w.r.t. V_I, R

$$\boxed{\underbrace{z - h(x)}_{\text{error term}}}$$

0.95

$$\delta\xi = \tau - \underbrace{(一坨[illegible])}_{\text{no one depends on } \delta x}$$

①

$$\frac{\partial \tau_I}{\partial V_I} : \frac{\partial}{\partial V_I}(R \cdot V_B \times V_B + R D_L V_B + R D_{NL} f_p(V_B))$$
$$= \frac{\partial}{\partial V_I} V_I \times V_B + \frac{\partial}{\partial V_I} D_L V_I + \frac{\partial}{\partial V_I} R D_{NL} f_p(R^{-1} V_I)$$
$$= \cancel{V_I^\times J(V_B)} - V_B^\times J(V_I) + D_L + R D_{NL}$$
$$= -[V_B]_\times + D_L$$

C_{rb}

$$\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

$$[v]_\times = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

$C_{rb} =$

- in your path,
  u actually give $\dot{psi} = 0$
  this could cause a problem

$$[a]_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \left(-\begin{bmatrix} 0 & -m\cdot w & mv \\ mw & 0 & -mu \\ -mv & mu & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right)$$

$$= \begin{bmatrix} mwq - mvr \\ -mwp + mur \\ mvp - muq \end{bmatrix}$$

$mvs \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

$u = 1.27$
$v = 0.047$
$w = -0.157$

$p = 0.10$
$q = -0.05$
$r = 0.93$

6	u
7	v
8	w
9	p
⋮	q
⋮	r

$$\begin{bmatrix} -m\cdot r\cdot v + m\cdot q\ w \\ m\cdot r\cdot u - m\cdot \overset{1.37}{p}\ w \\ -m\cdot q\cdot u + m\cdot p\ v \end{bmatrix}$$

$$\begin{bmatrix} 0.707 & 0.707 & -0.707 & -0.707 \end{bmatrix} \begin{bmatrix} 385 \\ 409 \\ -625 \\ -170 \\ -279 \\ -280 \end{bmatrix}$$

$\tau = Kt$

$$\boxed{\begin{aligned} \tau_x = {} & m_{rb} \cdot a_x \\ & + m_a \cdot a_x \\ & + C(v_B)v_B \\ & - \xi \\ & + D(v_B)v_B. \end{aligned}}$$

$\xi = -\tau$

$z - h(x)$.

$$\check{\xi} + K\left(\tau - \underbrace{(M_{rb}\dot{v} - \check{\xi} + M_a\dot{v} - D(v)v - g)}_{} \right)$$

$$\hat{\xi} = \underset{0}{\check{\xi}} + K\cdot\left(\tau - M_{rb}\dot{v} + 0 - M_a\dot{v} - D(v)v - g\right)$$

$$\xi = M_{rb}\dot{v} - (\tau - Dv(v) - M_a\dot{v} - g)$$
$$= M_{rb}\dot{v} - \tau + Dv(v) + M_a\dot{v} + g.$$

z-hx

?

7

$$\xi = M_{rb} - (\bar{\tau} - D - M_a - g)$$

code raw

$$\Rightarrow \xi = M_{rb} \underbrace{- \tau}_{z} + D + M_a + g.$$

$$\Rightarrow \boxed{\tau = M_{rb} + D + M_a + g - \xi}$$

$$z - h(x).$$

$$= \tau - (M_{rb} + D + M_a + g \cancel{- \check{\xi}})$$

$$\Rightarrow \tau - M_{rb} - D - M_a - g \cancel{+ \xi}$$

$$\check{x} + K(z - hx)$$

@ init

$x = -0.221723$

$y = 1.6379$

$z = -19e4$

$V = 1.00184$

-0.328515

-0.0067426