No.

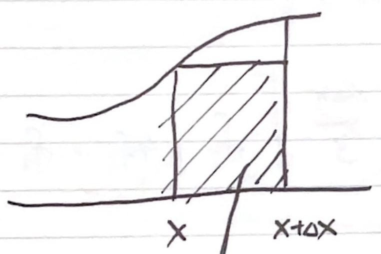
Date

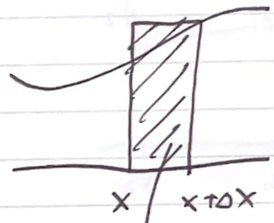
ME564 L16

numerical integration

not accurate,

undershoot or overshoot





$A = f(x) \cdot \Delta x$

left rect

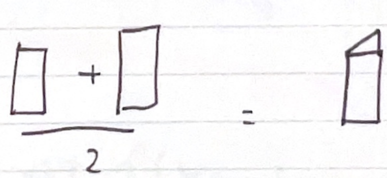
$A = f(x+\Delta x) \cdot \Delta x$

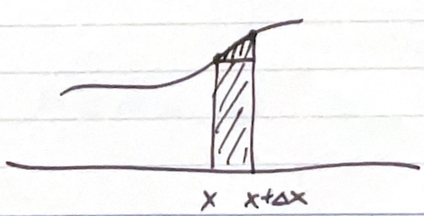
right rect

take average

solve undershooting overshooting

$\frac{1}{2}\left[\left(f(x) \cdot \Delta x\right) + \left(f(x+\Delta x) \cdot \Delta x\right)\right]$ trapezoid rule





$\frac{\Delta x}{2}\left[f(x) + f(x+\Delta x)\right]$

left & right Rectangle

$$\int_a^b f(x)\,dx \approx \sum_{k=0}^{N-1} f(x_k)\Delta x \quad \text{(left-sided)}$$

Local Error: $O(\Delta x^2)$
Global Error: $O(\Delta x)$

taylor expansion

$$\int_{x_0}^{x_0+\Delta x} f(x)\,dx = \int_{x}^{x+\Delta x}\left[f(x_0)+\Delta x\frac{df}{dx}(x_0)+\cdots\right]dx$$

$$= \Delta x f(x_0) + \Delta x^2\frac{df}{dx}(x_0) + \underbrace{\frac{\Delta x^3}{2!}\frac{d^2}{dx^2}f(x_0)+\cdots}_{\text{error } O(\Delta x^2)}$$

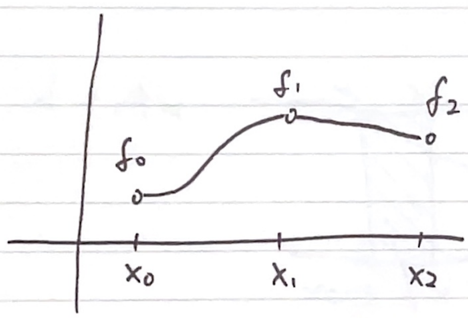
but we take $\frac{b-a}{\Delta x}$ steps!

total error $= O(\Delta x)$

Trapezoidal

Local : $O(\Delta x^3)$
Global : $O(\Delta x^2)$

\>> trapz



$$\int_{x_0}^{x_2} f(x)\,dx = \frac{\Delta x}{3}\left[f_0 + 4f_1 + f_2\right] + \frac{\Delta x^5}{90} f^{(4)}(c)$$

\>> quad

Simpson's Rule Integration

areaSimpson = quad (@(xdummy) spline (x, f, xdummy), a, b)

vector field : ?
$\rightarrow$

$\dot{x} = f(x)$ (Non-linear)

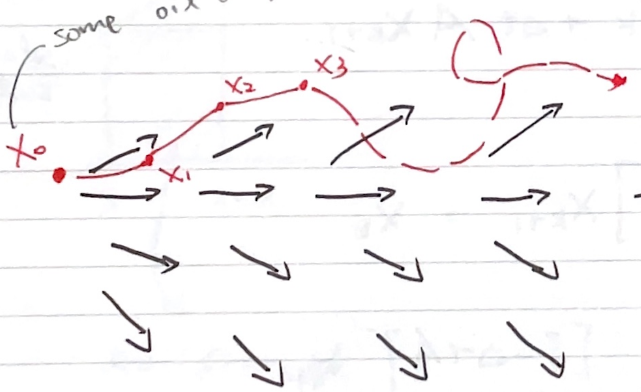
f is a vector field (a field of x)

$\dot{x} = Ax \Rightarrow e^{At} x_0$ (linear)

We are interested in $\rightarrow$ Non-linear numerical solution
$\rightarrow$ numerically obtaining a trajectory

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \ldots \rightarrow x_n$

some oil drop is some gulf



a vector field depends on (t)

$$\dot{x} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\dot{x}_k \approx \frac{x_{k+1} - x_k}{\Delta t} = f(x_k)$$

$$\boxed{x_{k+1} = x_k + \Delta t \cdot f(x_k)}$$

[Forward Euler
not very stable]

if $\dot{x} = Ax \Rightarrow$ $\boxed{\begin{aligned} x_{k+1} &= x_k + \Delta t\, A x \\ &= [I + \Delta t A]\, x_k \end{aligned}}$

linear

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(cont'd)

$$\dot{x}_{k+1} = \frac{x_{k+1} - x_k}{\Delta t}$$ Backward Differene Scheme @ $t = k+1$

$$= f(x_{k+1})$$

$\Downarrow$

$$x_{k+1} = x_k + \Delta t f(x_{k+1})$$

x_{k+1} is implicit defined

implicit Euler

- better stability
- slower to solve

$$\dot{x} = Ax \implies x_{k+1} = x_k + \Delta t A x_{k+1}$$

$$[I - \Delta t A] x_{k+1} = x_k$$

$$x_{k+1} = [I - \Delta t A]^{-1} x_k$$