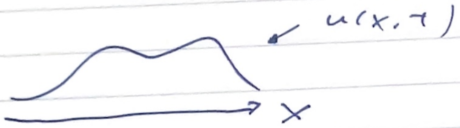
ME564 L26

$$\left.\begin{aligned}\dot{x} &= Ax\\ \dot{x} &= f(x)\\ x&(t)\end{aligned}\right\}\text{ ODEs}$$

$\dot{x} = Ax$
$z = Tx$ change of coords.
$\dot{z} = T^{-1}ATz$
$= Dz$

$$\left.\begin{aligned}\frac{\partial u}{\partial t} &= c\frac{\partial u}{\partial x}\\ u&(x,t)\end{aligned}\right\}\text{ PDEs}$$

Fourier
Laplace. change of coords



Potential Flow

consider a fluid, which is incompressible, & irrotational $\to \vec{V}(x,t)$ let $\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

① steady : $\frac{\partial \vec{V}}{\partial t} = 0 \longrightarrow \therefore \vec{V}(x)$

② incompressible: $\nabla \cdot \vec{V} = 0$ divergence $= 0$ $\quad \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} = 0$

③ irrotational: $\nabla \times \vec{V} = 0$ curl $= 0$ $\quad \frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} = 0$

} simple PDE we can solve!

if $\vec{V} = \nabla \varphi$ — is a scalar function, scalar field

& φ satisfies Laplace's Equation $\nabla^2\varphi = 0$

then $\vec{V}$ satisfies ② & ③

∇^2 is called "The Laplacian"

$$\nabla^2\varphi = \nabla\cdot(\nabla\varphi)$$

$$= \nabla \cdot \begin{bmatrix} \partial\varphi/\partial x \\ \partial\varphi/\partial y \end{bmatrix} = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}$$

$$\boxed{\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0}$$

PDE

(cont'd)

$$\nabla \times (\nabla \varphi) = 0$$

for all φ

$$\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \nabla \varphi = \begin{bmatrix} \frac{\partial \varphi}{\partial x} \\ \frac{\partial \varphi}{\partial y} \end{bmatrix}$$

incomprissible

$$\nabla \cdot \vec{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y}$$

$$= \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}$$

$$= 0$$

laplacian

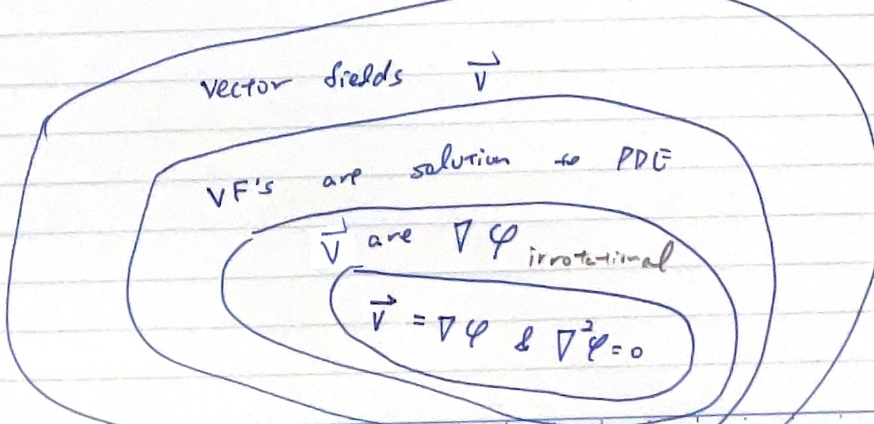
irrotational

$$\nabla \times \vec{V} = \frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y}$$

$$= \frac{\partial^2 \cancel{\varphi}}{\cancel{\partial y}\, \partial x} - \frac{\partial^2 \cancel{\varphi}}{\cancel{\partial x}\, \partial y}$$

$$= 0$$

Airplanes before computers



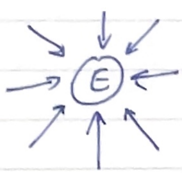
No.

Date

Laplace's Equation

$$\boxed{\nabla^2 \varphi = 0}$$

① Gravitation (away from mass sources)



$\vec{F} = -\nabla P$ where $P = \frac{-mMG}{r}$

verify that $\nabla^2 P = 0$ (away from mass sources)

② Electrostatics (away from point charges)

③ heat conduction (steady - state) T (temperature)

$$\frac{\partial T}{\partial t} = c^2 \nabla^2 T \longrightarrow \nabla^2 T = 0$$

 $\longrightarrow$ $\boxed{T = const}$ $t \to \infty$

$T = k$ this satisfy $\nabla^2 T = 0$

④ potential flow