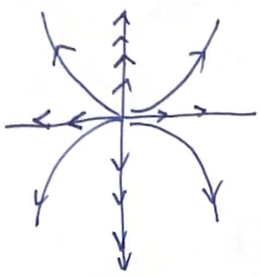
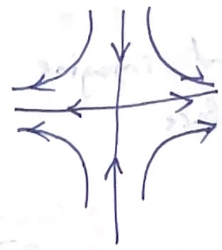
ME564 L10

Overview of 2×2 $\dot{x} = Ax$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\lambda = \pm i$



unstable source



saddle

mixed of stable & unstable.

neutrally stable

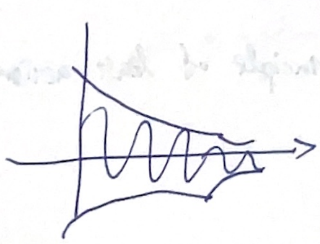
$A = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$ $\lambda = -1 \pm i$

Real $(\lambda) < 0$
↳ stable

stable spiral

$e^{\lambda t} = e^{(-1 \pm i)t} = e^{-t} e^{\pm it}$

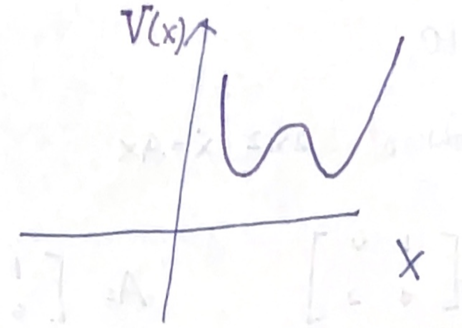
decay sin/cos



$A = \begin{bmatrix} -0.1 & 0 \\ 1 & -0.11 \end{bmatrix}$ eigen values that are really close together.

e-vec close to parallel.

Particle in a potential well
& linearizing nonlinear ODEs



Consider a potential $V(x)$ and imagine sliding a 'bead' on this surface

F=ma $\Rightarrow$ Force is $F = -\frac{\partial V}{\partial x}$ $\quad m\ddot{x} = -\frac{\partial U}{\partial x}$ $\quad m=1$

$$\boxed{\ddot{x} = -\frac{\partial U}{\partial x}}$$

Let's write Lagrange

Kinetic energy $T = \frac{1}{2}\dot{x}^2$

pot. $U(x)$

Lagrangian: $L(x,\dot{x}) = T(\dot{x}) - U(x)$

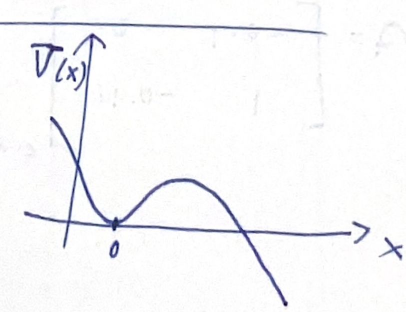
Euler Lagrangian Equation: $\frac{d}{dt}\frac{\partial L}{\partial x} - \frac{\partial L}{\partial x} = 0$

principle of least action

$$\ddot{x} = \frac{\partial L}{\partial x} = -\frac{\partial U}{\partial x}$$

Ex: $\ddot{x} = -x + x^2$ $\quad$ (i.e. $V(x) = \frac{x^2}{2} - \frac{x^3}{3}$)

integrating



$\downarrow$

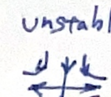
$\left.\begin{aligned}\dot{x} &= v\\ \dot{v} &= -x + x^2\end{aligned}\right\}$ fixed points $\longrightarrow$ $x = 0, 1$, $v = 0$ $\quad \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}$

$\xi_1 \begin{bmatrix}1\\1\end{bmatrix}$ $\xi_2 \begin{bmatrix}1\\-1\end{bmatrix}$

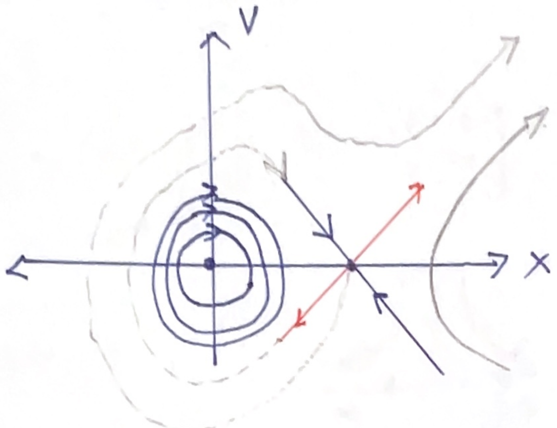
$\frac{Df}{D\begin{bmatrix}x\\v\end{bmatrix}} = \begin{bmatrix}0 & 1\\ -1+2x & 0\end{bmatrix}$; $\quad$ FP1 $\begin{bmatrix}0\\0\end{bmatrix}$ $\frac{Df}{D\begin{bmatrix}x\\v\end{bmatrix}} = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ $\quad$ FP2 $\begin{bmatrix}1\\0\end{bmatrix}$ $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

 center

unstable

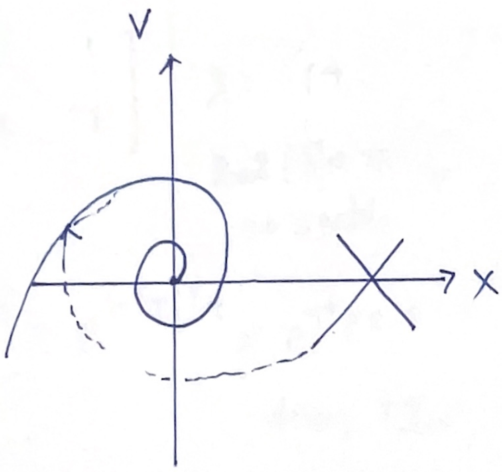


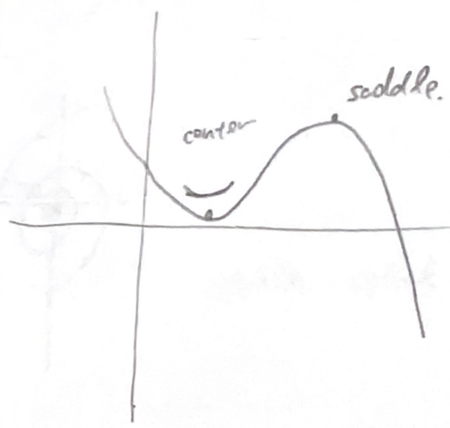
$\lambda = \pm 1$



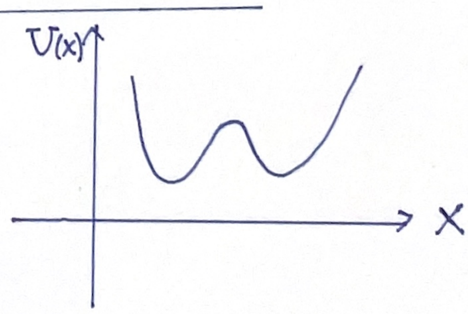
if add damping

$$\ddot{X} = -X + X^2 - \boxed{\dot{X}}$$





$$\ddot{X} = -X^3 + X - \underbrace{\dot{X}}_{\text{optional damping}}$$



$$\dot{x} = y$$
$$\dot{y} = x - x^3 (-y)$$

FPs:

$y = 0$

$x = 0 \pm 1$

$-\frac{1}{4}x^4 + \frac{1}{2}x^2 +$

$$\frac{Df}{DX} = \begin{bmatrix} 0 & 1 \\ 1-3x^2 & 0 \end{bmatrix}$$

FPs:

$$\underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\substack{\text{saddle} \\ \pm 1}} \quad \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix}}_{\text{centers: } \pm i}$$

$$\frac{Df}{DX} = \begin{bmatrix} 0 & 1 \\ 1-3x^2 & -1 \end{bmatrix}$$