### Motivation

- +rue-state
- nominal-state
- pron-state

A true-state = nominal-state & creal state

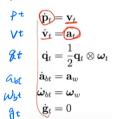
nominal-state -> takes no account in noise/

### Definitions

| Magnitude             | True              | Nominal        | Error                 | Composition  | Measured       | Noise          |                     |
|-----------------------|-------------------|----------------|-----------------------|--|----------------|----------------|---------------------|
| Full state (1)        | $\mathbf{x}_t$    | x              | $\delta x$            | $\mathbf{x}_t = \mathbf{x} \oplus \delta \mathbf{x}$   |                |                |                     |
| Position              | $\mathbf{p}_t$    | p              | $\delta \mathbf{p}$   | $\mathbf{p}_t = \mathbf{p} + \delta \mathbf{p}$        |                |                |                     |
| Velocity              | $\mathbf{v}_t$    | v              | $\delta \mathbf{v}$   | $\mathbf{v}_t = \mathbf{v} + \delta \mathbf{v}$        |                |                | R:                  |
| Quaternion (2,3)      | $\mathbf{q}_t$    | q              | $\delta \mathbf{q}$   | $\mathbf{q}_t = \mathbf{q} \otimes \delta \mathbf{q}$  |                |                | rotation matrix     |
| Rotation matrix (2,3) | $\mathbf{R}_t$    | R              | $\delta \mathbf{R}$   | $\mathbf{R}_t = \mathbf{R}  \delta \mathbf{R}$         |                |                | from body to        |
| Angles vector (4)     |                   | δθ             | 50                    | $\delta \mathbf{q} = e^{\delta \theta/2}$              |                |                | inertial frame      |
|                       |                   |                | 00                    | $\delta \mathbf{R} = e^{[\delta \theta]_{\times}}$     |                |                |                     |
| Accelerometer bias    | $\mathbf{a}_{bt}$ | $\mathbf{a}_b$ | $\delta \mathbf{a}_b$ | $\mathbf{a}_{bt} = \mathbf{a}_b + \delta \mathbf{a}_b$ |                | $\mathbf{a}_w$ |                     |
| Gyrometer bias        | $\omega_{bt}$     | $\omega_b$     | $\delta\omega_b$      | $\omega_{bt} = \omega_b + \delta \omega_b$             |                | $\omega_w$     |                     |
| Gravity vector        | $\mathbf{g}_t$    | g              | $\delta \mathbf{g}$   | $\mathbf{g}_t = \mathbf{g} + \delta \mathbf{g}$        |                |                |                     |
| Acceleration          | $\mathbf{a}_t$    |                |                       |  | $\mathbf{a}_m$ | $\mathbf{a}_n$ |                     |
| Angular rate          | $\omega_t$        |                |                       |  | $\omega_m$     | $\omega_n$     | AUTHORICO CONTROL 4 |

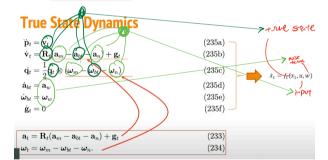
# Dynamic

## · measurement





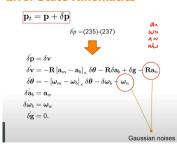
Note:  $a_m,\,\omega_m$  are measuremed in the body-fixed frame  $a_t,\,\omega_t$  are exoressed in the inertial frame



### **Nominal State Kinematics**



## **Error State Kinematics**



# **Discrete-time Nominal States**

Taking the integration of (237) yields the discrete-time form as

$$\mathbf{p} \leftarrow \mathbf{p} + \mathbf{v} \Delta t + \frac{1}{2} (\mathbf{R}(\mathbf{a}_m - \mathbf{a}_b) + \mathbf{g}) \Delta t^2$$
 (260a)

$$\mathbf{v} \leftarrow \mathbf{v} + (\mathbf{R}(\mathbf{a}_m - \mathbf{a}_b) + \mathbf{g}) \,\Delta t \tag{260b}$$

$$\mathbf{q} \leftarrow \mathbf{q} \otimes \overline{\mathbf{q}\{(\omega_m - \omega_b) \, \Delta t\}} \tag{260c}$$

$$\mathbf{a}_b \leftarrow \mathbf{a}_b$$
 (260d)

$$\omega_b \leftarrow \omega_b$$
 (260e)

$$\mathbf{g} \leftarrow \mathbf{g}$$
, (260f)

# **Discrete-time Error States**

Taking the integration of (238) yields the discrete-time form as

| $\delta \mathbf{p} \leftarrow \delta \mathbf{p} + \delta \mathbf{v}  \Delta t$   | (261a) |
|--|--------|
| $\delta \mathbf{v} \leftarrow \delta \mathbf{v} + (-\mathbf{R} \left[ \mathbf{a}_m - \mathbf{a}_b \right]_{\times} \delta \boldsymbol{\theta} - \mathbf{R} \delta \mathbf{a}_b + \delta \mathbf{g}) \Delta t + \mathbf{v_i}$ | (261b) |
| $\delta \boldsymbol{\theta} \leftarrow \mathbf{R}^{\top} \{ (\boldsymbol{\omega}_m - \boldsymbol{\omega}_b) \Delta t \} \delta \boldsymbol{\theta} - \delta \boldsymbol{\omega}_b \Delta t + \boldsymbol{\theta_i} $         | (261c) |
| $\delta \mathbf{a}_b \leftarrow \delta \mathbf{a}_b + (\mathbf{a_i})$  | (261d) |
| $\delta oldsymbol{\omega}_b \leftarrow \delta oldsymbol{\omega}_b + oldsymbol{\widetilde{\omega}_i}$   | (261e) |
| $\delta \mathbf{g} \leftarrow \delta \mathbf{g}$ .   | (261f) |

| $og \leftarrow og$ .   |   |                                  | (2011)   |
|--|---|----------------------------------|--|
| $\begin{aligned} \mathbf{V_i} &= \sigma_{\mathbf{a}_n}^2 \Delta t^2 \mathbf{I} \\ \mathbf{\Theta_i} &= \sigma_{\omega_n}^2 \Delta t^2 \mathbf{I} \\ \mathbf{A_i} &= \sigma_{\mathbf{a}_w}^2 \Delta t \mathbf{I} \\ \mathbf{\Omega_i} &= \sigma_{\omega_w}^2 \Delta t \mathbf{I} \end{aligned}$ | $[m^2/s^2] \ [rad^2] \ [m^2/s^4] \ [rad^2/s^2]$ | (262)<br>(263)<br>(264)<br>(265) | Integration of covariance matrices   |
|  |   |                                  | AND ADDRESS OF THE PARTY OF THE |

### **ESKF v.s. EKF**

th

| ESKF   |                | EKF  |
|--|----------------|--|
| $\begin{split} & \delta \mathbf{x} \leftarrow \mathbf{F}_{\mathbf{x}}(\mathbf{x}, \mathbf{u}_m) \cdot \delta \mathbf{x} \\ & \mathbf{P} \leftarrow \mathbf{F}_{\mathbf{x}} \mathbf{P}  \mathbf{F}_{\mathbf{x}}^\top + \mathbf{F}_{\mathbf{i}}  \mathbf{Q}_{\mathbf{i}}  \mathbf{F}_{\mathbf{i}}^\top  , \end{split}$ | (268)<br>(269) | $\begin{split} A &= \frac{\partial f}{\partial x} _{\hat{S}_{k-1}, W_{k-1}} \text{ and } \mathcal{C} = \frac{\partial g}{\partial x} _{\hat{S}_{k}} \\ \hat{S}_{k}^{-} &= f(\hat{S}_{k-1}, W_{k-1}) \\ P_{k}^{-} &= A_{k}P_{k-1}A_{k}^{T} + Q \end{split}$ |
| $\mathbf{K} = \mathbf{P}\mathbf{H}^{\top}(\mathbf{H}\mathbf{P}\mathbf{H}^{\top} + \mathbf{V})^{-1}$<br>$\delta \mathbf{x} \leftarrow \mathbf{K}(\mathbf{y} - h(\hat{\mathbf{x}}_t))$   | (274)<br>(275) | $\mathcal{K}_k = P_k^- C^T (C P_k^- C^T + R)^{-1}$ $\hat{x}_k = \hat{x}_k^- + \mathcal{K}_k (y_k - \hat{y}_k)$   |
| $P \leftarrow (I - KH)P$   | (276)          | $P_k = (I - \mathcal{K}_k C) P_k^-$  |

# Injection of the Observed Error into the Nominal State

### 6.2 Injection of the observed error into the nominal state

After the ESKF update, the nominal state gets updated with the observed error state using the appropriate compositions (sums or quaternion products, see Table 3)

|         | $\mathbf{x} \leftarrow \mathbf{x} \oplus \hat{\delta \mathbf{x}}$ ,                          | (282)             |  |
|---------|--|-------------------|--|
| nat is, |  |                   |  |
|         | $\mathbf{p} \leftarrow \mathbf{p} + \hat{\delta \mathbf{p}}$                                 | (283a)            |  |
|         | $\mathbf{v} \leftarrow \mathbf{v} + \hat{\delta \mathbf{v}}$                                 | (283b)            |  |
|         | $\mathbf{q} \leftarrow \mathbf{q} \otimes \mathbf{q} \{ \delta \hat{\boldsymbol{\theta}} \}$ | (283c)            |  |
|         | $\mathbf{a}_b \leftarrow \mathbf{a}_b + \delta \hat{\mathbf{a}}_b$                           | (283d)            |  |
|         | $oldsymbol{\omega}_b \leftarrow oldsymbol{\omega}_b + \delta \hat{oldsymbol{\omega}}_b$      | (283e)            |  |
|         | $\mathbf{g} \leftarrow \mathbf{g} + \hat{\delta \mathbf{g}}$                                 | (283f) worked con |  |

# **Error State Reset**

Let us call the error reset function g(). It is written as follows,

$$\delta \mathbf{x} \leftarrow g(\delta \mathbf{x}) = \delta \mathbf{x} \ominus \hat{\delta \mathbf{x}}$$
, (28)

where  $\ominus$  stands for the composition inverse of  $\oplus$ . The ESKF error reset operation is thus,

$$\delta \mathbf{x} \leftarrow 0$$
 (285)  
 $\mathbf{P} \leftarrow \mathbf{G} \mathbf{P} \mathbf{G}^{\top}$ . (286)

$$\mathbf{P} \leftarrow \mathbf{G} \, \mathbf{P} \, \mathbf{G}^{\top} \; .$$
 where  $\mathbf{G}$  is the Jacobian matrix defined by,

$$\mathbf{G} \triangleq \left. rac{\partial g}{\partial \delta \mathbf{x}} \right|_{\delta \mathbf{\hat{x}}} \ .$$

Similarly to what happened with the update Jacobian above, this Jacobian is the identity on all diagonal blocks except in the orientation error. We give here the full expression and proceed in the following section with the derivation of the orientation error block,  $\partial \delta \theta^+/\partial \delta \theta = \mathbf{I} - \left[\tfrac{1}{2} \hat{\theta} \theta\right]_\times,$ 

$$\mathbf{G} = \begin{bmatrix} \mathbf{I}_{6} & 0 & 0 \\ 0 & \mathbf{I} - \begin{bmatrix} \frac{1}{2} \hat{\delta \boldsymbol{\theta}} \end{bmatrix}_{\times} & 0 \\ 0 & 0 & \mathbf{I}_{a} \end{bmatrix}. \quad (288)$$