Generalized Extended State Observer Based Control for Systems With Mismatched Uncertainties

Shihua Li, *Senior Member, IEEE*, Jun Yang, *Member, IEEE*, Wen-Hua Chen, *Senior Member, IEEE*, and Xisong Chen

***Abstract*—The standard extended state observer based control (ESOBC) method is only applicable for a class of single-input–single-output essential-integral-chain systems with matched uncertainties. It is noticed that systems with nonintegral-chain form and mismatched uncertainties are more general and widely exist in practical engineering systems, where the standard ESOBC method is no longer available. To this end, it is imperative to explore new ESOBC approach for these systems to extend its applicability. By appropriately choosing a disturbance compensation gain, a generalized ESOBC (GESOBC) method is proposed for nonintegral-chain systems subject to mismatched uncertainties without any coordinate transformations. The proposed method is able to extend to multi-input–multi-output systems with almost no modification. Both numerical and application design examples demonstrate the feasibility and efficacy of the proposed method.**

***Index Terms*—Disturbance compensation gain, disturbance rejection, generalized extended state observer based control (GESOBC), mismatched uncertainties, multi-input–multi-output (MIMO) system, nonintegral-chain system.**

I. Introduction

VARIOUS uncertainties, including unmodeled dynamics, parameter perturbations, and external disturbances, always bring adverse effects on modern industrial control systems. With the growing interest in high-precision control, the utilization of the disturbance rejection technique is generally required in the controller design. It is well known that feedforward compensation control, which requires the measurement of the disturbance, is one of the most effective disturbance rejection methods. One fact that should be pointed out is that many uncertainties in control systems are unmeasurable; thus, the disturbance estimation technique is particularly crucial for disturbance attenuation.

During the past decades, several elegant approaches had been proposed to estimate disturbances, including the unknown input observer (UIO) [1], the disturbance observer (DOB) [2]–[6], the perturbation observer [7], [8], the equivalent input disturbance (EID) based estimation [9], [10], and the extended state observer (ESO) [11]–[15]. Sliding mode observers originally were developed as a robust observer technique to provide the robust state estimate for linear and nonlinear systems under disturbances [16]–[18]. It was extended to estimate unknown inputs or faults by reconstructing the disturbance using the so-called "equivalent output error injection" in [19]. The sliding-mode DOB was also proposed for disturbance estimation via states and control input information in [20] and [21]. Note that all of these methods are designed based on the model of the plant. A natural doubt may be what does a designer have to know about the plant in order to build the estimator [12]. Among the previously listed approaches, ESO requires the least amount of plant information [22]; in fact, only the system order should be known. Due to such a promising feature, ESO-based control (ESOBC) schemes, also known as active disturbance rejection control, have become more and more popular in recent years. Successful applications of ESOBC in various industrial systems, including robotic systems [23], [24], motion control systems [25]–[31], manipulator systems [32], [33], power converters [34], [35], gyroscopes [36], and flight control systems [37]–[39], have been reported within the past decade.

Although the ESOBC has obtained successful achievements in many practical control systems, it is also noticed that ESOBC mainly has its roots in motion control systems. The potential reason is that the standard ESOBC is only available for integral chain systems which widely exist in motion control systems. Such integral chain form is not necessarily satisfied for general systems, and it is required to transfer the system to meet the standard formulation by coordinate transformations such that the standard ESOBC method can be used. However, as pointed out by Han [14], it is generally not easy to reformulate the problem to satisfy the standard formulation, which becomes one of the most crucial factors restricting the applicability of the ESOBC.

Another factor that severely constrains the application of standard ESOBC method is that the uncertainties in many practical systems may not satisfy the so-called matching condition [40], which implies that the uncertainties act via the same channel as the control input. For example, in flight control systems, the lumped disturbance torques caused by unmodeled dynamics, external winds, parameter perturbations, etc., always affect the states directly rather than through the input channels [41]. Another example in the MAGnetic LEViation (MAGLEV) suspension system is that the track input disturbance acts on a

Manuscript received January 19, 2011; revised May 12, 2011 and August 3, 2011; accepted October 2, 2011. Date of publication December 28, 2011; date of current version July 2, 2012. This work was supported in part by the Program for New Century Excellent Talents in University (NCET- 10-0328), the Natural Science Foundation of Jiangsu Province under Grant BK2009270, the Graduate Innovation Program of Jiangsu Province under Grant CX10B_077Z, and by the Scientific Research Foundation of Graduate School of Southeast University.

S. Li, J. Yang, and X. Chen are with the School of Automation, Southeast University, Nanjing 210096, China (e-mail: lsh@seu.edu.cn; junyang8402@gmail.com; chenxisong@263.net).

W.-H. Chen is with the Department of Aeronautical and Automotive Engineering, Loughborough University, LE11 3TU Leicestershire, U.K. (e-mail: w.chen@lboro.ac.uk).

Digital Object Identifier 10.1109/TIE.2011.2182011

TABLE I
APPLICABILITY OF THE PROPOSED GESOBC AND THE ESOBC

Method	Disturbance	Variable	System
GESOBC	mismatched/matched	MIMO	non-integral-chain
ESOBC	matched	SISO	essential-integral-chain

different channel from the control input [47]. The problem also appears in a permanent magnet synchronous motor system, in which the uncertainties consisting of the parameter variation and the load torque enter the system via different channels from the control inputs [42].

Note that disturbance-based feedforward control for systems with mismatched uncertainties is a longstanding unresolved problem [48], [49]. A generalized ESOBC (GESOBC) method is proposed in this paper to solve the disturbance attenuation problem of a class of nonintegral-chain system with mismatched uncertainties. It is shown that, by properly choosing a disturbance compensation gain, the mismatched uncertainties can be attenuated from the system output. A systematic method is developed for the disturbance compensation gain design. Parameter selection of the proposed method is discussed in detail. In addition, feasible conditions for extending the proposed GESOBC to multi-input–multi-output (MIMO) systems without any coordinate transformations are also investigated. The proposed GESOBC method largely extends the applicability of the ESOBC since it exhibits many superiorities over the standard ESOBC method, which are listed in Table I, where the essential-integral-chain systems refer to the systems that can be converted into integral chain systems by transformation.

The remainder of this paper is organized as follows. In Section II, preliminary results regarding the formulation of the standard ESOBC are presented. Section III investigates the newly proposed GESOBC for general systems with mismatched uncertainties. In Section IV, some related problems about the proposed method are further discussed. Application example and simulations are studied to demonstrate the efficiency of the proposed method in Section V. Finally, the concluding remarks are summarized in Section VI.

II. PRELIMINARY-STANDARD ESOBC

An uncertain system with the order of n under the standard consideration is usually an integral chain system, described by [11]

$$\begin{cases} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ & \vdots \\ \dot{x}_n &= f\left(x_1, \ldots, x_n, \omega(t), t\right) + bu \\ y &= x_1 \end{cases} \tag{1}$$

where $x_1, \ldots, x_n$ are the states, u is the control input, y is the output, $\omega(t)$ is the external disturbance, b is a system parameter, and $f(x_1, \ldots, x_n, \omega(t), t)$ represents the uncertain function, also known as lumped disturbance.

In the framework of ESOBC, an augmented variable [13]

$$x_{n+1} = f\left(x_1, \ldots, x_n, \omega(t), t\right) \tag{2}$$

external disturbance

$x_{n+1} = f(x_1, \ldots, x_n, w(t), t)$ lump disturbance

is introduced to linearize system (1). Combining (1) with (2), the extended state equation is given by

$$\begin{cases} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ & \vdots \\ \dot{x}_n &= x_{n+1} + bu. \\ \dot{x}_{n+1} &= h(t). \\ y &= x_1 \end{cases} \tag{3}$$

with $h(t) = \dot{f}(x_1, \ldots, x_n, \omega(t), t)$.

In order to estimate the states, a linear ESO is designed as [11]

$z_1 \rightarrow x_1$
$z_2 \rightarrow x_2$

$$\begin{cases} \dot{z}_1 &= z_2 - \beta_1(z_1 - y), \\ \dot{z}_2 &= z_3 - \beta_2(z_1 - y), \\ & \vdots \\ \dot{z}_n &= z_{n+1} - \beta_n(z_1 - y) + bu, \\ \dot{z}_{n+1} &= -\beta_{n+1}(z_1 - y) \end{cases} \tag{4}$$

where $z_1, z_2, \ldots, z_n$ and z_{n+1} are estimates of states $x_1, x_2, \ldots, x_n$ and x_{n+1}, respectively, and $\beta_1, \beta_2, \ldots, \beta_{n+1}$ are the observer gains to be designed.

Subtracting (3) from (4), the error system is written as

$$\begin{cases} \dot{e}_1 &= e_2 - \beta_1 e_1, \\ \dot{e}_2 &= e_3 - \beta_2 e_1, \\ & \vdots \\ \dot{e}_n &= e_{n+1} - \beta_n e_1, \\ \dot{e}_{n+1} &= -\beta_{n+1} e_1 - h(t) \end{cases} \tag{5}$$

where $e_i = z_i - x_i$ $(i = 1, 2, \ldots, n+1)$ represents the estimation error. By properly choosing the observer gains $\beta_1, \beta_2, \ldots, \beta_{n+1}$, the bounded stability of (5) is guaranteed under the assumption that $h(t)$ is bounded [33].

The standard ESOBC control law is usually designed as [13], [14]

$$u = \boldsymbol{K}_x \boldsymbol{x} - \frac{z_{n+1}}{b} \tag{6}$$

where $\boldsymbol{K}_x$ is the feedback control gain.

III. GESOBC

A. Problem Statement

The standard ESOBC method is possibly not available for the following simple second-order system:

$$\begin{cases} \dot{x}_1 = x_1 - 2x_2 + f\left(x_1, x_2, \omega(t), t\right), \\ \dot{x}_2 = x_1 + x_2 + u. \end{cases} \tag{7}$$

System (7) does not satisfy the standard formulation as (1) in the following two aspects. On the one hand, (7) does not satisfy the integral chain form. On the other hand, the uncertainties $f(x_1, x_2, \omega(t), t)$ enter the system with a different channel from the control input u, i.e., the so-called matching condition is not satisfied. For the aforementioned case, the standard ESOBC law (6) is no longer available. Thus, it is imperative to develop GESOBC for general systems which do not satisfy the standard formulation of system (1).

For the sake of simplicity, the following single-input–single-output (SISO) system with mismatched uncertainties is considered:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}_u u + \boldsymbol{b}_d f\left(\boldsymbol{x}, \omega(t), t\right), \\ \boldsymbol{y}_m = \boldsymbol{C}_m \boldsymbol{x}, \\ y_o = \boldsymbol{c}_o \boldsymbol{x} \end{cases} \tag{8}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n$, $u \in \boldsymbol{R}$, $\omega \in \boldsymbol{R}$, $\boldsymbol{y}_m \in \boldsymbol{R}^r$, and $y_o \in \boldsymbol{R}$ are the state vector, input, external disturbance, measurable outputs, and controlled output, respectively. $f(\boldsymbol{x}, \omega(t), t)$ is the uncertain function in terms of $\boldsymbol{x}$ and ω. $\boldsymbol{A}$ with dimension $n \times n$, $\boldsymbol{b}_u$ with dimension $n \times 1$, $\boldsymbol{b}_d$ with dimension $n \times 1$, $\boldsymbol{C}_m$ with dimension $r \times n$, and $\boldsymbol{c}_o$ with dimension $1 \times n$ are system matrices, respectively.

Remark 1: In (8), uncertainty function $f(\boldsymbol{x}, \omega(t), t)$ represents the lumped disturbance, which is a generalized concept, possibly including external disturbances, unmodeled dynamics, parameter variations, and complex nonlinear dynamics which may be difficult for the feedback part to handle. □

Remark 2: Equation (8) represents a more general class of systems as compared with that of system (1) since system (8) is not confined to integral chain form and may subject to mismatched uncertainties [44]. The matching case is a special case of (8), by simply taking $\boldsymbol{b}_u = \lambda \boldsymbol{b}_d$, $\lambda \in \boldsymbol{R}$. □

B. Composite Control Design

Similar to the standard case in Section II, adding an extended variable

$$x_{n+1} = d = f\left(\boldsymbol{x}, \omega(t), t\right) \tag{9}$$

to linearize system (8), the extended system equation is obtained

$$\begin{cases} \dot{\bar{\boldsymbol{x}}} = \bar{\boldsymbol{A}}\bar{\boldsymbol{x}} + \bar{\boldsymbol{b}}_u u + \boldsymbol{E} h(t) \\ \boldsymbol{y}_m = \bar{\boldsymbol{C}}_m \bar{\boldsymbol{x}} \end{cases} \tag{10}$$

where variables

$$\bar{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x} \\ x_{n+1} \end{bmatrix}$$

$$h(t) = \frac{df\left(\boldsymbol{x}, \omega(t), t\right)}{dt}$$

and matrices

$$\bar{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_{n\times n} & (\boldsymbol{b}_d)_{n\times 1} \\ \boldsymbol{0}_{1\times n} & 0_{1\times 1} \end{bmatrix}_{(n+1)\times(n+1)}$$

$$\bar{\boldsymbol{b}}_u = \begin{bmatrix} (\boldsymbol{b}_u)_{n\times 1} \\ 0_{1\times 1} \end{bmatrix}_{(n+1)\times 1}$$

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{0}_{n\times 1} \\ 1_{1\times 1} \end{bmatrix}_{(n+1)\times 1}$$

$$\bar{\boldsymbol{C}}_m = [\,\boldsymbol{C}_m, \quad 0_{r\times 1}\,]_{r\times(n+1)}.$$

Assumption 1: $(\boldsymbol{A}, \boldsymbol{b}_u)$ is controllable, and $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ is observable. □

Remark 3: A necessary condition of $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ observable is that $(\boldsymbol{A}, \boldsymbol{C}_m)$ is observable. The details can be found in Appendix A. □

For system (10), the ESO is designed as follows:

$$\begin{cases} \dot{\hat{\bar{\boldsymbol{x}}}} = \bar{\boldsymbol{A}}\hat{\bar{\boldsymbol{x}}} + \bar{\boldsymbol{b}}_u u + \boldsymbol{L}(\boldsymbol{y}_m - \hat{\boldsymbol{y}}_m) \\ \hat{\boldsymbol{y}}_m = \bar{\boldsymbol{C}}_m \hat{\bar{\boldsymbol{x}}} \end{cases} \tag{11}$$

where $\hat{\bar{\boldsymbol{x}}} = [\hat{\boldsymbol{x}}^T, \hat{x}_{n+1}]^T$, $\hat{\boldsymbol{x}}$, and $\hat{x}_{n+1}$ are the estimates of the state variable $\bar{\boldsymbol{x}}$, $\boldsymbol{x}$, and x_{n+1} in (10), respectively. Matrix $\boldsymbol{L}$ with dimension $(n+1) \times r$ is the observer gain to be designed.

In the presence of mismatched uncertainties, the standard ESOBC law $u = \boldsymbol{K}_x \boldsymbol{x} - \hat{d}$ (where $\hat{d} = \hat{x}_{n+1}$ and $\boldsymbol{K}_x$ is the feedback control gain) cannot effectively compensate the uncertainties in system (8).

Remark 4: It should be pointed out that the mismatched uncertainties cannot be attenuated completely from the state equation no matter what controller is designed [50]. In this case, one of the most achievable goals is to remove the uncertainties from the output channel in steady state. □

Remark 5: It should be highlighted that, although there are similarities between the proposed GESOBC (or the DOB based on control) and the other observer techniques, including sliding mode based UIOs, they have different focuses, so different design philosophies as the motivations are different. The main objective in GESOBC in this paper is to minimize the influence of the disturbance and uncertainty on the output provided that the disturbance has been estimated; whether or not the unknown disturbances are precisely estimated or whether they are observable from the output is not the major concern. In UIO approaches, including sliding mode observer technique, it mainly concerns if whether an unknown disturbance can be accurately reconstructed, which, in general, may impose a more restricted condition such as observer matching condition [18]. □

The composite control law in this paper is designed as

$$u = \boldsymbol{K}_x \boldsymbol{x} + K_d \hat{d} \tag{12}$$

or

$$u = \boldsymbol{K}_x \hat{\boldsymbol{x}} + K_d \hat{d} \tag{13}$$

where $\boldsymbol{K}_x$ is the feedback control gain and K_d is the disturbance compensation gain, designed as

$$K_d = -\left[\boldsymbol{c}_o(\boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x)^{-1} \boldsymbol{b}_u\right]^{-1} \boldsymbol{c}_o(\boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x)^{-1} \boldsymbol{b}_d. \tag{14}$$

Remark 6: Note that the disturbance compensation gain K_d in (14) is a general case and suitable for both matching and mismatching cases. For the matching case, i.e., $\boldsymbol{b}_u = \lambda \boldsymbol{b}_d$, $\lambda \in \boldsymbol{R}$, it can be obtained from (14) that the disturbance compensation gain reduces to $K_d = -1/\lambda$, which is the same as the standard ESOBC law (6) in most previous literatures. □

The configuration of the proposed GESOBC is shown in Fig. 1.

It will be shown next that the mismatched uncertainties can be eliminated from the output channel in steady state by the proposed control law.

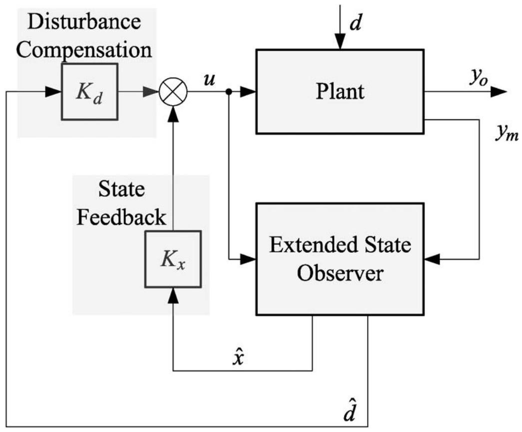


Fig. 1. Configuration of the proposed GESOBC method.

C. Stability and Disturbance Rejection Analysis

Assumption 2: The lumped disturbances satisfy the following conditions: 1) $d(t) = f(x, \omega(t), t) = \bar{f}(\omega(t), t)$; 2) they are bounded; 3) they have constant values in steady state, i.e., $\lim_{t\to\infty} \dot{d}(t) = \lim_{t\to\infty} h(t) = 0$ and $\lim_{t\to\infty} d(t) = D_c$. □

The state and disturbance estimation errors are defined as

$$\boldsymbol{e}_x = \hat{\boldsymbol{x}} - \boldsymbol{x} \tag{15}$$

$$e_d = \hat{d} - d \tag{16}$$

where $\hat{d} = \hat{x}_{n+1}$ represents the estimate of system uncertainties.

Combining (10), (11), and (15), with (16), the estimation error equation is given by

$$\dot{\boldsymbol{e}} = \boldsymbol{A}_e \boldsymbol{e} - \boldsymbol{E} h(t) \tag{17}$$

where

$$\boldsymbol{e} = \begin{bmatrix} \boldsymbol{e}_x \\ e_d \end{bmatrix}, \quad \boldsymbol{A}_e = \bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}_m. \tag{18}$$

The bounded stability of the ESO can be obtained from the following conclusion.

Lemma 1 [45]: Assuming that the observer gain vector $\boldsymbol{L}$ in (11) is chosen such that $\boldsymbol{A}_e$ is a Hurwitz matrix, then the observer error $\boldsymbol{e}$ for the ESO is bounded for any bounded $h(t)$. □

Lemma 2: The following single-input linear system:

$$\dot{x} = Ax + Bu \tag{19}$$

is asymptotically stable if A is a Hurwitz matrix and u is bounded and satisfies $\lim_{t\to\infty} u(t) = 0$. The proof can be found in Appendix B. □

Lemma 3: For system (19), if matrix A is Hurwitz and $\lim_{t\to\infty} u(t) = U_c \neq 0$, the state converges to a constant vector $-A^{-1}BU_c$, i.e., $\lim_{t\to\infty} x(t) = -A^{-1}BU_c$. The result can be easily followed from *Lemma 2* by coordinate transformations. □

1) In the Case of Known States: If the states are available, the composite control law is designed as (21). The stability and disturbance rejection performance is analyzed by the following theorems.

Theorem 1: Suppose that *Assumption 1* is satisfied. The bounded stability of system (8) under the proposed GESOBC law (12) for any bounded $h(t)$ and $d(t)$ is guaranteed if the observer gain $\boldsymbol{L}$ in (11) and the feedback control gain $\boldsymbol{K}_x$ in (12) are selected such that $\boldsymbol{A}_e$ in (18) and $\boldsymbol{A}_f = \boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x$ are Hurwitz matrices, respectively.

Proof: Combining system (8) and composite control law (12) with error system (17), the closed-loop system in the presence of known states is given as

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{e}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_f & \boldsymbol{b}_u \bar{\boldsymbol{K}} \\ \boldsymbol{0} & \boldsymbol{A}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{e} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{b}_d + \boldsymbol{b}_u K_d \\ -\boldsymbol{E} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} h \\ d \end{bmatrix} \tag{20}$$

where $\bar{\boldsymbol{K}} = [\boldsymbol{0}_{1\times n}, K_d]$.

Since both $\boldsymbol{A}_f$ and $\boldsymbol{A}_e$ are Hurwitz matrices, it is obtained that

$$\begin{bmatrix} \boldsymbol{A}_f & \boldsymbol{b}_u \bar{\boldsymbol{K}} \\ \boldsymbol{0} & \boldsymbol{A}_e \end{bmatrix}$$

is also Hurwitz matrix.

It can be concluded from *Lemma 1* that the closed-loop system (26) is bounded-input–bounded-output stable for any bounded $h(t)$ and $d(t)$ if $\boldsymbol{K}$ and $\boldsymbol{L}$ are properly selected. □

Theorem 2: Supposing that *Assumptions 1* and 2 are satisfied, the observer gain $\boldsymbol{L}$ and the feedback control gain $\boldsymbol{K}_x$ are also chosen such that matrices $\boldsymbol{A}_e$ in (18) and $\boldsymbol{A}_f$ are Hurwitz and $\boldsymbol{c}_o \boldsymbol{A}_f^{-1} \boldsymbol{b}_u$ is invertible. For system (8) under control law (12), the lumped disturbances can be attenuated from the output channel in steady state under the proposed GESOBC law (12).

Proof: Substituting control law (12) into system (8) and considering (16), the state is expressed as

$$\boldsymbol{x} = (\boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x)^{-1} \left[\dot{\boldsymbol{x}} - \boldsymbol{b}_u K_d e_d - (\boldsymbol{b}_u K_d + \boldsymbol{b}_d) d \right]. \tag{21}$$

Combining (8) and (14) with (21) gives

$$y_o = \boldsymbol{c}_o (\boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x)^{-1} \dot{\boldsymbol{x}} + \boldsymbol{c}_o (\boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x)^{-1} \boldsymbol{b}_d e_d. \tag{22}$$

It can be observed from (22) that the lumped disturbances are removed from the output channels. Under the given conditions, the following results are obtained from *Lemmas 2 and 3* and *Theorem 1*:

$$\lim_{t\to\infty} \dot{\boldsymbol{x}}(t) = 0, \quad \lim_{t\to\infty} \boldsymbol{e}(t) = \boldsymbol{0}. \tag{23}$$

Combining (22) with (23) yields

$$\lim_{t\to\infty} y_o(t) = 0. \tag{24}$$

□

2) In the Case of Unknown States: If the state variables are unmeasurable, the estimate of both the lumped disturbance and states can be used for control design. In this case, the composite control law is designed as (13). By denoting $\boldsymbol{K} = [\boldsymbol{K}_x, \ K_d]$, a compact expression of the control law (13) can be obtained

$$u = \boldsymbol{K}\hat{\bar{\boldsymbol{x}}}. \tag{25}$$

Theorem 3: Suppose that *Assumption 1* is satisfied. The bounded stability of system (8) under the proposed GESOBC law (13) for any bounded $h(t)$ and $d(t)$ is guaranteed if the observer gain $\boldsymbol{L}$ in (11) and the feedback control gain $\boldsymbol{K}_x$ in (13) are selected such that $\boldsymbol{A}_e$ and $\boldsymbol{A}_f$ are Hurwitz matrices, respectively.

Proof: Combining system (8) and composite control law (13) with error system (17), the closed-loop system is written as

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{e}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_f & \boldsymbol{b}_u\boldsymbol{K} \\ \boldsymbol{0} & \boldsymbol{A}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{e} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{b}_d + \boldsymbol{b}_u K_d \\ -\boldsymbol{E} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} h \\ d \end{bmatrix}. \quad (26)$$

Since both $\boldsymbol{A}_f$ and $\boldsymbol{A}_e$ are Hurwitz matrices, it is easy to prove that matrix

$$\begin{bmatrix} \boldsymbol{A}_f & \boldsymbol{b}_u\boldsymbol{K} \\ \boldsymbol{0} & \boldsymbol{A}_e \end{bmatrix}$$

is also a Hurwitz matrix. The proof is completed by using the result in *Lemma 1*. □

Theorem 4: Supposing that *Assumptions 1* and 2 are satisfied, the observer gain $\boldsymbol{L}$ and the feedback control gain $\boldsymbol{K}_x$ are also chosen such that matrices $\boldsymbol{A}_e$ in (18) and $\boldsymbol{A}_f$ are Hurwitz and $\boldsymbol{c}_o\boldsymbol{A}_f^{-1}\boldsymbol{b}_u$ is invertible. For system (8) under control law (13), the lumped disturbances can be attenuated from the output channel in steady state with the proposed GESOBC law (13).

Proof: Combining (8) with (13) gives

$$\boldsymbol{x} = (\boldsymbol{A} + \boldsymbol{b}_u\boldsymbol{K}_x)^{-1}\left[\dot{\boldsymbol{x}} - \boldsymbol{b}_u\boldsymbol{K}\boldsymbol{e} - (\boldsymbol{b}_u K_d + \boldsymbol{b}_d)d\right]. \quad (27)$$

Based on (8), (27), and (14), the output can be represented as

$$y_o = \boldsymbol{c}_o(\boldsymbol{A}+\boldsymbol{b}_u\boldsymbol{K}_x)^{-1}(\dot{\boldsymbol{x}} - \boldsymbol{b}_u\boldsymbol{K}_x\boldsymbol{e}_x) + \boldsymbol{c}_o(\boldsymbol{A}+\boldsymbol{b}_u\boldsymbol{K}_x)^{-1}\boldsymbol{b}_d e_d. \quad (28)$$

Considering (23), the same result as (24) can be obtained from (28). □

Remark 7: The first condition in *Assumption 2* is used for rigorous stability analysis. In general, the lumped disturbances may contain some state variables. As known for the disturbance estimator based control community, including DOBC [2]–[4], ESOBC [11]–[14], UIO-based control [1], and EID-based control [9], [10], it is difficult to prove the stability for this general case of $d(t) = f(x, \omega(t), t)$. One may doubt the availability of the proposed method in this case. In many practical engineering systems, the dominated dynamics can be stabilized by the feedback control, and the state uncertainties contained in the lumped disturbance $d(t) = f(x, \omega(t), t)$ are relatively weak, which will not affect the system stability. In this case, such uncertainties can be regarded as part of the lumped disturbance and can be handled by the proposed method. This is possibly the major reason for the prevalence of using the disturbance estimator based control to compensate plant uncertainties in real engineering systems. Although we have not given a rigorous result in this case due to technical difficulty, the effectiveness of our method in such a case has been illustrated by numerical simulation examples in this paper. □

IV. Further Discussions

A. Extension to MIMO System

For the purpose of comparison with the standard ESOBC, only a SISO system with uncertainties in single channel is considered in Section III. The proposed GESOBC method is able to extend to MIMO system with almost no modification. Here, the MIMO system may include multiple disturbances in different channels. A general MIMO system is described as

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_u\boldsymbol{u} + \boldsymbol{B}_d\boldsymbol{f}\left(\boldsymbol{x}, \boldsymbol{\omega}(\boldsymbol{t}), t\right), \\ \boldsymbol{y}_m = \boldsymbol{C}_m\boldsymbol{x}, \\ \boldsymbol{y}_o = \boldsymbol{C}_o\boldsymbol{x} \end{cases} \quad (29)$$

where $\boldsymbol{x} \in \boldsymbol{R}^n$, $\boldsymbol{u} \in \boldsymbol{R}^m$, $\boldsymbol{y}_m \in \boldsymbol{R}^r$, $\boldsymbol{y}_o \in \boldsymbol{R}^p$, and $\boldsymbol{f} \in \boldsymbol{R}^q$.

1) Solvability of the Disturbance Compensation Gain: The disturbance compensation gain in (14) is no longer available since $\boldsymbol{C}_o(\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{K}_x)^{-1}\boldsymbol{B}_u$ is possibly noninvertible or even not a square matrix. In this case, it can be verified that an alternative but more general condition

$$\boldsymbol{C}_o(\boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{K}_x)^{-1}\boldsymbol{B}_u\boldsymbol{K}_d = -\boldsymbol{C}_o(\boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{K}_x)^{-1}\boldsymbol{B}_d \quad (30)$$

must be satisfied to guarantee the feasibility of the proposed method.

The disturbance compensation gain $\boldsymbol{K}_d$ can be solved from (30) if the following rank condition holds:

$$\begin{aligned} &rank\left(\boldsymbol{C}_o(\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{K}_x)^{-1}\boldsymbol{B}_u)\right) \\ &= rank\left(\left[\boldsymbol{C}_o(\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{K}_x)^{-1}\boldsymbol{B}_u, -\boldsymbol{C}_o(\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{K}_x)^{-1}\boldsymbol{B}_d\right]\right. \end{aligned} \quad (31)$$

2) Conditions in Assumption 1: Another factor that possibly influences the feasibility of the proposed GESOBC for MIMO systems is that the condition in *Assumption 1* may not be satisfied with the increased number of the lumped disturbances. Consider a system with multiple lumped disturbances, depicted by

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1, x_2, \omega, t), \\ \dot{x}_2 = -2x_1 - x_2 + u + f_2(x_1, x_2, \omega, t), \\ y_m = x_1. \end{cases} \quad (32)$$

It can be easily verified that $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ is not observable for (32), and the proposed GESOBC is unavailable. However, it is also noticed that the problem becomes feasible as more output information is accessible, which is shown by the following example that is similar with (32) but with more measurable outputs:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1, x_2, \omega, t), \\ \dot{x}_2 = -2x_1 - x_2 + u + f_2(x_1, x_2, \omega, t), \\ y_{1m} = x_1, \\ y_{2m} = x_2. \end{cases} \quad (33)$$

It can be demonstrated that $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ is observable for (33) now.

Generally speaking, besides the conditions in *Assumption 1*, condition (31) should be satisfied to guarantee feasibility of the GESOBC for the MIMO system. If $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ is not observable, one may make the problem feasible by seeking more output information.

B. Parameter Design for GESOBC

As for the proposed GESOBC method, there are mainly three parameters, including feedback control gain $\boldsymbol{K}_x$, observer gain $\boldsymbol{L}$, and disturbance compensation gain K_d, to be designed. A fixed way to determine K_d has been given in (14).

The most important designing parameters are $\boldsymbol{K}_x$ and $\boldsymbol{L}$. As discussed in *Theorems 1–4*, the necessary conditions are $\boldsymbol{L}$ and $\boldsymbol{K}_x$ should be designed to guarantee the stability of the closed-loop system. However, these conditions are not sufficient. The reason lies in the idea that the lumped disturbances would be a function of the states, which can only be estimated if the observer dynamics is faster than the closed-loop dynamics. The same argument for the state observer based control method is available.

It can be found from Section III that the poles of ESO and the closed-loop system are eigenvalues of matrix $\boldsymbol{A}_e = \bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}_m$ and $\boldsymbol{A}_f = \boldsymbol{A} + \boldsymbol{b}_u \boldsymbol{K}_x$, respectively. If $(\boldsymbol{A}, \boldsymbol{b}_u)$ is controllable and $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ is observable, the poles of both the closed-loop system and the ESO can be placed arbitrarily.

To make the observer dynamics quicker than that of the closed-loop system, poles of ESO should be placed much more far away from the origin than those of the closed-loop system.

V. Application Example and Simulations

A. Simple Numerical Example

To demonstrate the efficiency of the proposed GESOBC scheme, a second-order uncertain nonlinear system with mismatching condition is considered

$$\begin{cases} \dot{x}_1 = x_2 + e^{x_1} + w, \\ \dot{x}_2 = -2x_1 - x_2 + u, \\ y = x_1. \end{cases} \tag{34}$$

By denoting $\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$, $\boldsymbol{b}_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\boldsymbol{b}_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\boldsymbol{C}_m = \boldsymbol{c}_o = [\mathbf{1}\ 0]$, and $f(\boldsymbol{x}, w(t), t) = e^{x_1} + w$, it can be observed that system (34) has the formulation of (8).

To guarantee the convergence of ESO, the observer gain vector in (11) is chosen as $\boldsymbol{L} = [14 \quad -66 \quad 125]^T$ such that the related ESO poles are $p_{\text{eso}} = [-5 \quad -5 \quad -5]^T$. The feedback control gain in this example is designed as $\boldsymbol{K}_x = [-4 \quad -4]$. The poles of the closed-loop system regardless of the uncertainties are $p_{cl} = [-2 \quad -3]^T$ under such feedback control gain. The disturbance compensation gain can be calculated according to (14), giving as $K_d = -5$. Considering that the states are unmeasurable, the composite control law (13) is employed. The initial states of system (34) are $\boldsymbol{x}_0 = [1 \quad 0]^T$. The external disturbance $\omega = 3$ acts on the system at $t = 6$ s. The control objective is to remove the uncertainties from the output channel. Here, the setpoint of the output is zero during the simulation. The response curves of the real and estimated states and their estimate errors are shown in Figs. 2–4.

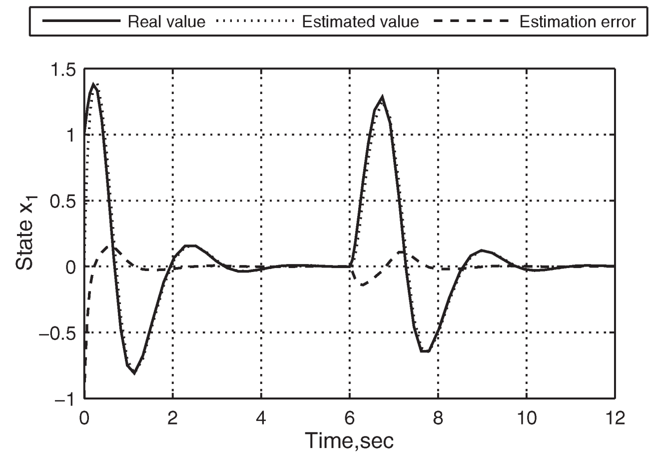


Fig. 2. Response curves of the real and estimated values of state x_1.

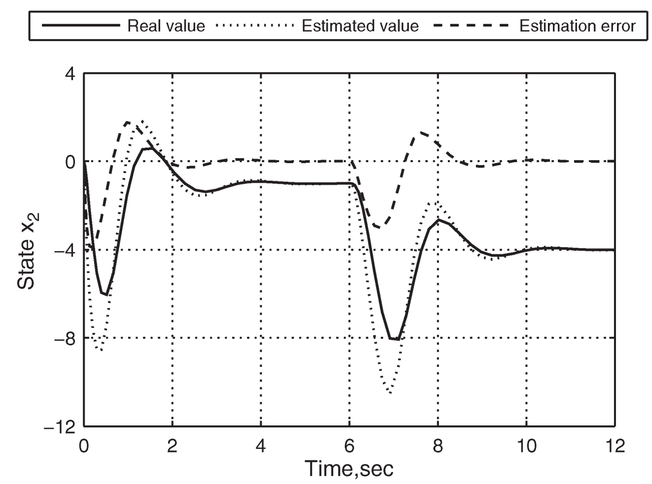


Fig. 3. Response curves of the real and estimated values of state x_2.

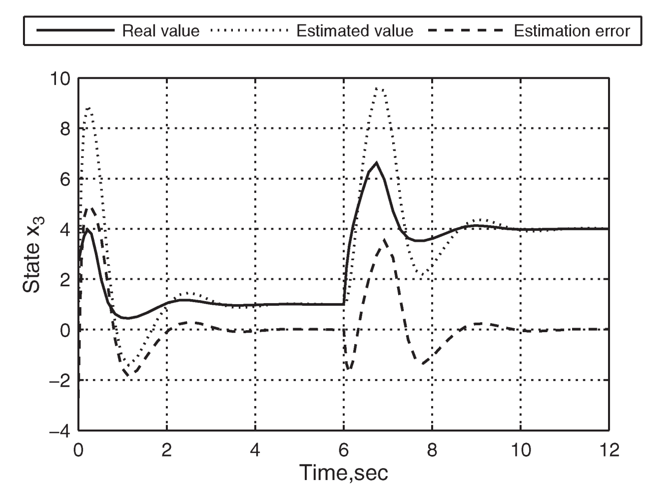


Fig. 4. Response curves of the real and estimated disturbance $x_3 = d$.

The corresponding time history of the control input is shown in Fig. 5.

It can be observed from Fig. 2 that the output converges to the setpoint quickly in the presence of both uncertainties and external disturbances. As shown in Figs. 2–4, the estimation errors of ESO converge to zero for all of the states in such case of uncertainties.

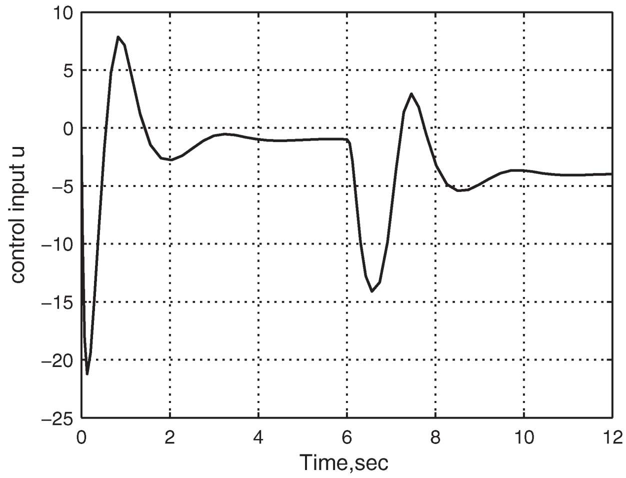


Fig. 5. Time history of the control input u.

TABLE II
PARAMETERS OF THE MAGLEV SUSPENSION SYSTEM

Parameters	Meaning	Value
M_s	Carriage Mass	1000kg
F_o	Nominal force	9810 N
G_o	Nominal air gap	0.015m
R_c	Coil's Resistance	10Ω
B_o	Nominal flux density	1T
L_c	Coil's Inductance	0.1H
I_o	Nominal current	10A
N_c	Number of turns	2000
V_o	Nominal voltage	100V
A_p	Pole face area	0.01m^2

B. Application Example

Disturbance rejection of a MAGLEV system is studied in this part. The dynamic model of the MAGLEV system is given by [47].

1) Nonlinear Model: The complete nonlinear model for the MAGLEV suspension system is given by

$$B = K_b \frac{I}{G} \tag{35}$$

$$F = K_f B^2 \tag{36}$$

$$\frac{dI}{dt} = \frac{V_{\text{coil}} - IR_c + \frac{N_c A_p K_b}{G^2}\left(\frac{dz_t}{dt} - \frac{dZ}{dt}\right)}{\frac{N_c A_p K_b}{G} + L_c} \tag{37}$$

$$\frac{d^2 Z}{d^2 t} = g - \frac{K_f}{M_s}\frac{I^2}{G^2} \tag{38}$$

$$\frac{dG}{dt} = \frac{dz_t}{dt} - \frac{dZ}{dt} \tag{39}$$

where variables I, z_t, Z, (dz_t/dt), (dZ/dt), G, B, and F denote the current, the rail position, the electromagnet position, the rail vertical velocity, the electromagnet vertical velocity, the air gap, the flux density, and the force, respectively. Signal V_{coil} is the voltage of the coil. Other symbols in (35)–(39) are system parameters listed in Table II.

2) Linearized MAGLEV Suspension Model: The linearization of the MAGLEV suspension is based on small perturbations around the operating points. The following definitions are used, in which the lower case letters define a small variation around the operating point and the subscript "o" refers to the operating condition

$$B = B_o + b \tag{40}$$

$$F = F_o + f \tag{41}$$

$$I = I_o + i \tag{42}$$

$$G = G_o + (z_t - z) \tag{43}$$

$$V_{\text{coil}} = V_o + u_{\text{coil}} \tag{44}$$

$$Z = Z_o + z. \tag{45}$$

The linearized state-space equation of the MAGLEV suspension model is expressed as

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_u u + \boldsymbol{B}_d d, \\ \boldsymbol{y}_m = \boldsymbol{C}_m \boldsymbol{x}, \\ y_o = \boldsymbol{c}_o \boldsymbol{x} \end{cases} \tag{46}$$

where the states are the linearized current, vertical electromagnet velocity, and air gap, i.e., $\boldsymbol{x} = [i\ \dot{z}\ (z_t - z)]^T$, the input $u = u_{\text{coil}}$ is the voltage, and the track input $d = \dot{z}_t$ is the rail vertical velocity. Suppose that the measurable outputs are the air gap and the vertical velocity, i.e., $y_{m1} = z_t - z$, and $y_{m2} = \dot{z}$. The controlled variable is the variation of air gap, i.e., $y = z_t - z$. The detailed linearization procedure can be found in [47]; here, the state matrix $\boldsymbol{A}$, the input matrix $\boldsymbol{B}_u$, the disturbance matrix $\boldsymbol{B}_d$, and the output matrices $\boldsymbol{C}_m$ and $\boldsymbol{c}_o$ are given directly

$$\boldsymbol{A} = \begin{bmatrix} \frac{-R_c}{L_c + K_b N_c \frac{A_p}{G_o}} & \frac{-K_b N_c A_p I_o}{G_o^2\left(L_c + K_b N_c \frac{A_p}{G_o}\right)} & 0 \\ -2K_f \frac{I_o}{M_s G_o^2} & 0 & 2K_f \frac{I_o^2}{M_s G_o^3} \\ 0 & -1 & 0 \end{bmatrix} \tag{47}$$

$$\boldsymbol{B}_u = \begin{bmatrix} \frac{1}{L_c + K_b N_c \frac{A_p}{G_o}} \\ 0 \\ 0 \end{bmatrix} \tag{48}$$

$$\boldsymbol{B}_d = \begin{bmatrix} \frac{K_b N_c A_p I_o}{G_o^2\left(L_c + K_b N_c \frac{A_p}{G_o}\right)} \\ 0 \\ 1 \end{bmatrix} \tag{49}$$

$$\boldsymbol{C}_m = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{50}$$

$$\boldsymbol{c}_o = [0\ \ 0\ \ 1]. \tag{51}$$

The major external disturbance in MAGLEV system is the deterministic inputs to the suspension in the vertical direction. Such deterministic inputs are the transitions onto the track gradients. The deterministic input components considered here are referred to [47] and are shown in Fig. 6. They represent a gradient of 5% at a vehicle speed of 15 m/s, while the jerk level is 1 m/s^3.

It can be observed from (46), (48), and (49) that the disturbances enter the system via a different channel from that of the control input. In other words, the disturbances in the MAGLEV system are mismatching ones. The control specifications of the MAGLEV system under consideration of the deterministic track input are given in Table III [47].

In the proposed GESOBC method, the observer gain is chosen as

$$\boldsymbol{L} = \begin{bmatrix} 79516 & -2370 \\ 62.3 & 125.2 \\ 130 & -1 \\ 4000 & 0 \end{bmatrix}. \tag{52}$$

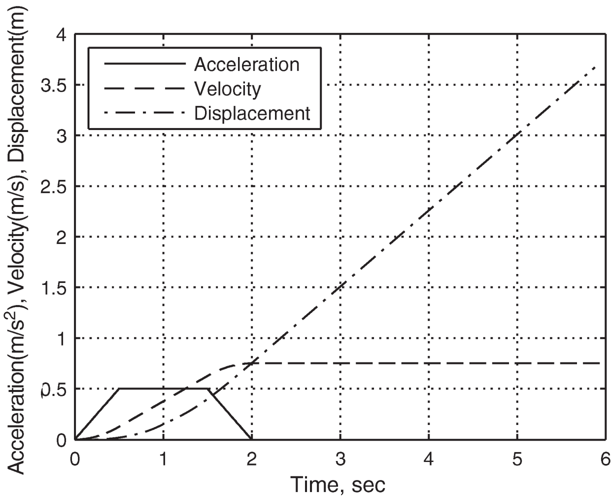


Fig. 6. Track input to the suspension with a vehicle speed of 15 m/s and 5% gradient.

TABLE III
CONSTRAINTS FOR THE MAGLEV SUSPENSION SYSTEM

Constraints	Value
Maximum air gap deviation, $((z_t - z)_p)$	$\leq$0.0075m
Maximum input coil voltage, $((u_{coil})_p)$	$\leq$300V$(3I_oR_c)$
Settling time, (t_s)	$\leq$3s
Air gap steady state error, $((z_t - z)_{e_{ss}})$	=0

The feedback control gain is designed as $\boldsymbol{K}_x = [-60.6$ V/A, 591 V$\cdot$s/m, 40061 V/m$]$. The disturbance compensation gain can be calculated according to (14), giving $K_d = -591.2$ V$\cdot$s/m.

The response curves of both the output and the input of the suspension system under the proposed GESOBC method are shown in Fig. 7. The response curves of the corresponding states are shown in Fig. 8. The disturbance and its estimate are shown in Fig. 9.

It can be observed from Fig. 7(a) that the maximum air gap deviation is less than 0.005 m, the settling time is shorter than 2.4 s, and there is no steady-state error. All of these performances satisfy the design requirements listed in Table III. As shown in Fig. 7(b), the maximum input voltage in such case is about 30 V. The response curves in Fig. 8 show that both the current and the vertical electromagnet velocity vary smoothly and approach to the desired equilibrium points quickly. As shown in Fig. 9, the ESO can estimate the disturbance timely and accurately. The results demonstrate that the proposed GESOBC method has achieved satisfying performance in rejecting such practical disturbance.

VI. CONCLUSION

The standard ESOBC method is only available for a class of SISO essential-integral-chain systems with disturbances/uncertainties satisfying the so-called matching condition. By appropriately developing a disturbance compensation gain, a GESOBC method has been proposed for general systems with mismatched uncertainties and nonintegral-chain form. The proposed method can be extended to MIMO systems with almost no modification. Both numerical and application examples have been designed and simulated to demonstrate the feasibility and efficiency of the proposed method.

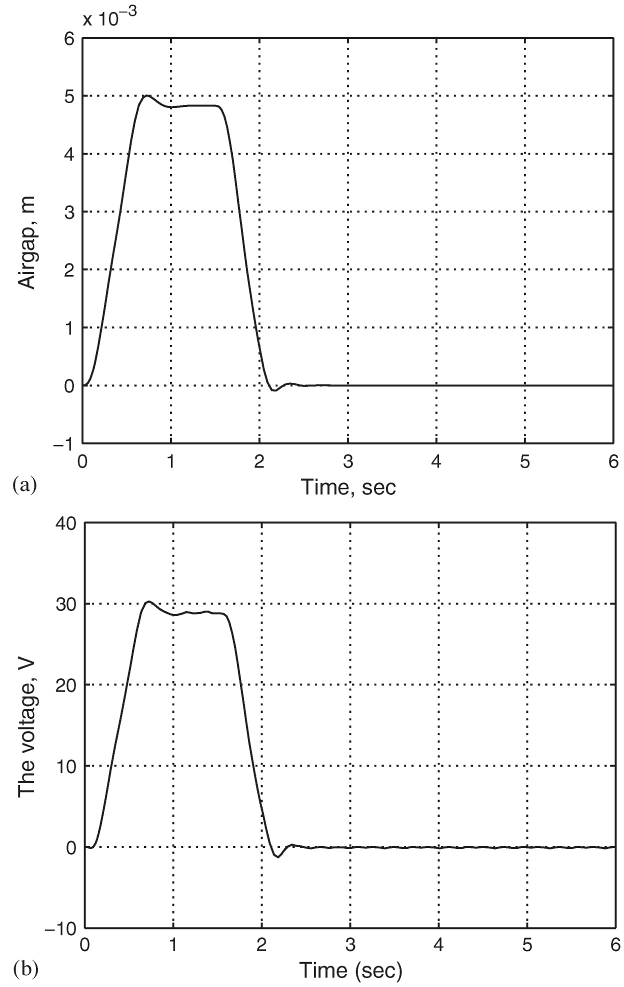


Fig. 7. Response curves of the input and output in the presence of deterministic track input. (a) Air gap $z_t - z$. (b) Voltage of the coil u_{coil}.

APPENDIX

A. Detailed Interpretation of Remark 3

The observability matrices of $(\boldsymbol{A}, \boldsymbol{C}_m)$ and $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}_m)$ are

$$\boldsymbol{P}_o = \begin{bmatrix} \boldsymbol{C}_m \\ \boldsymbol{C}_m\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}_m\boldsymbol{A}^{n-1} \end{bmatrix} \tag{53}$$

$$\bar{\boldsymbol{P}}_o = \left[\begin{array}{c|c} \boldsymbol{C}_m & 0 \\ \boldsymbol{C}_m\boldsymbol{A} & \boldsymbol{C}_m\boldsymbol{b}_d \\ \boldsymbol{C}_m\boldsymbol{A}^2 & \boldsymbol{C}_m\boldsymbol{A}\boldsymbol{b}_d \\ \vdots & \vdots \\ \boldsymbol{C}_m\boldsymbol{A}^{n-1} & \boldsymbol{C}_m\boldsymbol{A}^{n-2}\boldsymbol{b}_d \\ \hline \boldsymbol{C}_m\boldsymbol{A}^{n} & \boldsymbol{C}_m\boldsymbol{A}^{n-1}\boldsymbol{b}_d \end{array}\right]. \tag{54}$$

Assume that $(\boldsymbol{A}, \boldsymbol{C}_m)$ is not observable. Without loss of generality, $\boldsymbol{C}_m\boldsymbol{A}^{n-1}$ in matrix $\boldsymbol{P}_o$ can be expressed as

$$\boldsymbol{C}_m\boldsymbol{A}^{n-1} = \lambda_1\boldsymbol{C}_m + \lambda_2\boldsymbol{C}_m\boldsymbol{A} + \cdots + \lambda_{n-1}\boldsymbol{C}_m\boldsymbol{A}^{n-2} \tag{55}$$

where $\lambda_i (i = 1, \ldots, n-1)$ represents constant coefficients.

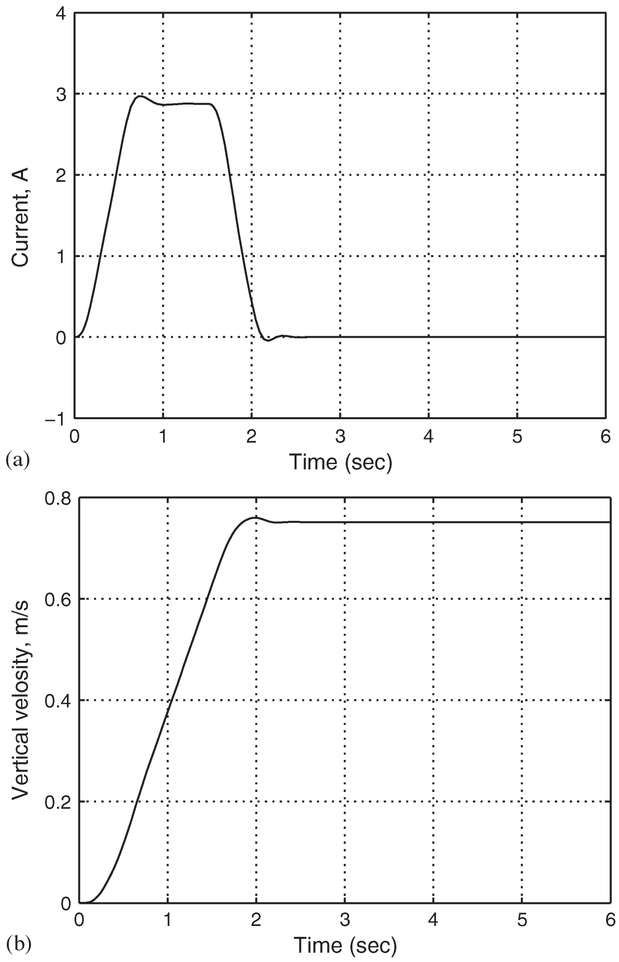


Fig. 8. Response curves of the states in the presence of deterministic track input. (a) Current i. (b) Vertical electromagnet velocity $\dot{z}$.

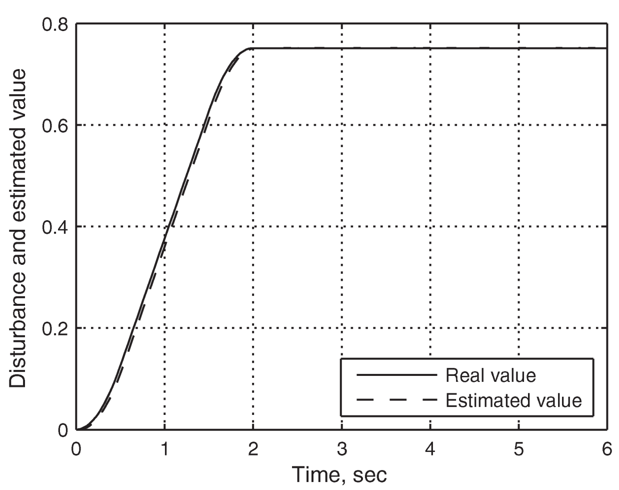


Fig. 9. Curves of the disturbance and its estimated value.

Combining (54) with (55) gives

$$[\boldsymbol{C}_m \boldsymbol{A}^n \boldsymbol{C}_m \boldsymbol{A}^{n-1} \boldsymbol{b}_d] = \lambda_1 [\boldsymbol{C}_m \boldsymbol{A} \boldsymbol{C}_m \boldsymbol{b}_d] + \lambda_2 [\boldsymbol{C}_m \boldsymbol{A}^2 \boldsymbol{C}_m \boldsymbol{A} \boldsymbol{b}_d] + \cdots + \lambda_{n-1} [\boldsymbol{C}_m \boldsymbol{A}^{n-1} \boldsymbol{C}_m \boldsymbol{A}^{n-2} \boldsymbol{b}_d]. \quad (56)$$

From (54) and (56), it can be derived that $rank(\bar{\boldsymbol{P}}_o) < n$. $\square$

B. Proof of Lemma 2

According to the final-value theorem [46], if all poles of $sX(s)$ lie in the left half s plane, $\lim_{t\to\infty} x(t)$ exists. Since $u(t)$ is bounded and satisfies $\lim_{t\to\infty} u(t) = 0$, all poles of $sU(s)$ lie in the left half s plane. In addition, all poles of $(sI - A)^{-1}$ also lie in the left half s plane since A is a Hurwitz matrix. To this end, all poles of $sX(s)$ lie in the left half s plane. Thus, $\lim_{t\to\infty} x(t)$ exists.

Using the final-value theorem yields

$$\begin{aligned}
\lim_{t\to\infty} x(t) &= \lim_{s\to 0} sX(s) \\
&= \lim_{s\to 0} s(sI - A)^{-1} U(s) \\
&= \lim_{s\to 0} (sI - A)^{-1} \cdot \lim_{s\to 0} sU(s) \\
&= \lim_{s\to 0} (sI - A)^{-1} \cdot \lim_{t\to\infty} u(t). \qquad (57)
\end{aligned}$$

Since A is a Hurwitz matrix, $\lim_{s\to 0}(sI - A)^{-1}$ is bounded. Using the condition $\lim_{t\to\infty} u(t) = 0$, it can be obtained from (57) that

$$\lim_{t\to\infty} x(t) = 0. \qquad (58)$$

$\square$

REFERENCES

[1] C. Johnson, "Accommodation of external disturbances in linear regulator and servomechanism problems," *IEEE Trans. Autom. Control*, vol. AC-16, no. 6, pp. 635–644, Dec. 1971.

[2] K. Ohishi, M. Nakao, K. Ohnishi, and K. Miyachi, "Microprocessor-controlled dc motor for load-insensitive position servo system," *IEEE Trans. Ind. Electron.*, vol. IE-34, no. 1, pp. 44–49, Feb. 1987.

[3] T. Umeno and Y. Hori, "Robust speed control of dc servo motors using modern two degrees-of-freedom controller design," *IEEE Trans. Ind. Electron.*, vol. 38, no. 5, pp. 363–368, Oct. 1991.

[4] W.-H. Chen, D. J. Ballance, P. J. Gawthrop, and J. O'Reilly, "A nonlinear disturbance observer for robotic manipulators," *IEEE Trans. Ind. Electron.*, vol. 47, no. 4, pp. 932–938, Aug. 2000.

[5] W.-H. Chen, "Disturbance observer based control for nonlinear systems," *IEEE ASME Trans. Mechatron.*, vol. 9, no. 4, pp. 706–710, Dec. 2004.

[6] L. Guo and W.-H. Chen, "Disturbance attenuation and rejection for systems with nonlinearity via DOBC approach," *Int. J. Robust Nonlinear Control*, vol. 15, no. 3, pp. 109–125, Feb. 2005.

[7] S. Kwon and W. K. Chung, "Robust performance of a multiloop perturbation compensator," *IEEE/ASME Trans. Mechatronics*, vol. 7, no. 2, pp. 190–200, Jun. 2002.

[8] S. Kwon and W. K. Chung, "A discrete-time design and analysis of perturbation observer for motion control applications," *IEEE Trans. Control Syst. Technol.*, vol. 11, no. 3, pp. 399–407, May 2003.

[9] J.-H. She, M. Fang, Y. Ohyama, H. Hashimoto, and M. Wu, "Improving disturbance-rejection performance based on an equivalent-input-disturbance approach," *IEEE Trans. Ind. Electron.*, vol. 55, no. 1, pp. 380–389, Jan. 2008.

[10] J.-H. She, X. Xin, and Y. Pan, "Equivalent-input-disturbance approach—Analysis and application to disturbance rejection in dual-stage feed drive control system," *IEEE/ASME Trans. Mechatronics*, vol. 16, no. 2, pp. 330–340, Apr. 2011.

[11] J. Han, "Extended state observer for a class of uncertain plants," *Control Decis.*, vol. 10, no. 1, pp. 85–88, 1995.

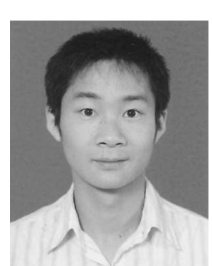
[12] Z. Gao, Y. Huang, and J. Han, "An alternative paradigm for control system design," in *Proc. 40th IEEE Conf. Decis. Control*, 2001, vol. 5, pp. 4578–4585.

[13] D. Sun, "Comments on active disturbance rejection control," *IEEE Trans. Ind. Electron.*, vol. 54, no. 6, pp. 3428–3429, Dec. 2007.

[14] J. Han, "From PID to active disturbance rejection control," *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 900–906, Mar. 2009.
[15] Y. Xia, P. Shi, G. P. Liu, D. Rees, and J. Han, "Active disturbance rejection control for uncertain multivariable systems with time-delay," *IET Control Theory Appl.*, vol. 1, no. 1, pp. 75–81, Jan. 2007.
[16] A. Benchaib, A. Rachid, E. Audrezet, and M. Tadjine, "Real-time sliding-mode observer and control of an induction motor," *IEEE Trans. Ind. Electron.*, vol. 46, no. 1, pp. 128–138, Feb. 1999.
[17] A. J. Koshkouei and A. S. I. Zinober, "Sliding mode controller-observer design for SISO linear systems," *Int. J. Syst. Sci.*, vol. 29, no. 12, pp. 1363–1373, Dec. 1998.
[18] C. Edwards and S. Spurgeon, *Sliding Mode Control: Theory and Applications*. London, U.K.: Taylor & Francis, 1998.
[19] T. Floquet, C. Edwards, and S. K. Spurgeon, "On sliding mode observers for systems with unknown inputs," in *Proc. Int. Workshop Var. Struct. Syst.*, 2006, pp. 5–7.
[20] C. E. Hall and Y. B. Shtessel, "Sliding mode disturbance observer-based control for a reusable launch vehicle," *J. Guid. Control Dyn.*, vol. 29, no. 6, pp. 1315–1328, Nov./Dec. 2006.
[21] Y.-S. Lu, "Sliding-mode disturbance observer with switching-gain adaptation and its application to optical disk drives," *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3743–3750, Sep. 2009.
[22] Q. Zheng, L. Q. Gao, and Z. Gao, "On stability analysis of active disturbance rejection control for nonlinear time-varying plants with unknown dynamics," in *Proc. 46th IEEE Conf. Decis. Control*, 2007, vol. 12–14, pp. 3501–3506.
[23] J. Su, H. Ma, W. Qiu, and Y. Xi, "Task-independent robotic uncalibrated hand–eye coordination based on the extended state observer," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 34, no. 4, pp. 1917–1922, Aug. 2004.
[24] J. Su, W. Qiu, H. Ma, and P.-Y. Woo, "Calibration-free robotic eye–hand coordination based on an auto disturbance-rejection controller," *IEEE Trans. Robot.*, vol. 20, no. 5, pp. 899–907, Oct. 2004.
[25] Y. X. Su, C. H. Zheng, and B. Y. Duan, "Automatic disturbance rejection controller for precise motion control of permanent-magnet synchronous motors," *IEEE Trans. Ind. Electron.*, vol. 52, no. 3, pp. 814–823, Jun. 2005.
[26] S. H. Li and Z. G. Liu, "Adaptive speed control for permanent-magnet synchronous motor system with variations of load inertia," *IEEE Trans. Ind. Electron.*, vol. 56, no. 8, pp. 3050–3059, Aug. 2009.
[27] H. X. Liu and S. H. Li, "Speed control for PMSM servo system using predictive function control and extended state observer," *IEEE Trans. Ind. Electron.*, vol. 59, no. 2, pp. 1171–1183, Feb. 2012.
[28] S. H. Li, H. X. Liu, and S. H. Ding, "A speed control for a PMSM using finite-time feedback control and disturbance compensation," *Trans. Inst. Meas. Control*, vol. 32, no. 2, pp. 170–187, Apr. 2010.
[29] G. Feng, Y.-F. Liu, and L. Huang, "A new robust algorithm to improve the dynamic performance on the speed control of induction motor drive," *IEEE Trans. Power Electron.*, vol. 19, no. 6, pp. 1614–1627, Nov. 2004.
[30] Y. X. Su, C. H. Zheng, D. Sun, and B. Y. Duan, "A simple nonlinear velocity estimator for high-performance motion control," *IEEE Trans. Ind. Electron.*, vol. 52, no. 4, pp. 1161–1169, Aug. 2005.
[31] D. Wu and K. Chen, "Design and analysis of precision active disturbance rejection control for noncircular turning process," *IEEE Trans. Ind. Electron.*, vol. 56, no. 7, pp. 2746–2753, Jul. 2009.
[32] Y. X. Su, B. Y. Duan, C. H. Zheng, Y. F. Zhang, G. D. Chen, and J. W. Mi, "Disturbance-rejection high-precision motion control of a Stewart platform," *IEEE Trans. Control Syst. Technol.*, vol. 12, no. 3, pp. 364–374, May 2004.
[33] S. E. Talole, J. P. Kolhe, and S. B. Phadke, "Extended-state-observer-based control of flexible-joint system with experimental validation," *IEEE Trans. Ind. Electron.*, vol. 57, no. 4, pp. 1411–1419, Apr. 2010.
[34] Y. Lu, K. W. E. Cheng, and S. L. Ho, "Auto-disturbance-rejection control for phase-shifted resonant converter," *Proc. Inst. Elect. Eng.—Elect. Power Appl.*, vol. 153, no. 5, pp. 711–718, Sep. 2006.
[35] B. Sun and Z. Gao, "A DSP-based active disturbance rejection control design for a 1-kW H-bridge dc–dc power converter," *IEEE Trans. Ind. Electron.*, vol. 52, no. 5, pp. 1271–1277, Oct. 2007.
[36] Q. Zheng, L. Dong, D. H. Lee, and Z. Gao, "Active disturbance rejection control for MEMS gyroscopes," *IEEE Trans. Control Syst. Technol.*, vol. 17, no. 6, pp. 1432–1438, Nov. 2009.
[37] Y. Huang, K. Xu, J. Han, and J. Lam, "Flight control design using extended state observer and non-smooth feedback," in *Proc. 40th IEEE Conf. Decis. Control*, 2001, vol. 1, pp. 223–228.
[38] S. Li, X. Yang, and D. Yang, "Active disturbance rejection control for high pointing accuracy and rotation speed," *Automatica*, vol. 45, no. 8, pp. 1854–1860, Aug. 2009.
[39] Y. Xia, Z. Zhu, M. Fu, and S. Wang, "Attitude tracking of rigid spacecraft with bounded disturbances," *IEEE Trans. Ind. Electron.*, vol. 58, no. 2, pp. 647–659, Feb. 2011.
[40] B. R. Barmish and G. Leitmann, "On ultimate boundedness control of uncertain systems in the absence of matching condition," *IEEE Trans. Autom. Control*, vol. AC-27, no. 1, pp. 153–158, Feb. 1982.
[41] W.-H. Chen, "Nonlinear disturbance observer-enhanced dynamic inversion control of missiles," *J. Guid. Control Dyn.*, vol. 26, no. 1, pp. 161–166, Jan./Feb. 2003.
[42] Y. Abdel-Rady and I. Mohamed, "Design and implementation of a robust current-control scheme for a PMSM vector drive with a simple adaptive disturbance observer," *IEEE Trans. Ind. Electron.*, vol. 54, no. 4, pp. 1981–1988, Aug. 2007.
[43] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. London, U.K.: Prentice-Hall, 1996.
[44] W.-H. Chen and L. Guo, "Analysis of disturbance observer based control for nonlinear systems under disturbances with bounded variation," in *Proc. Control*, Bath, U.K., Sep. 2004, pp. 1–5.
[45] Z. Gao, "Active disturbance rejection control: A paradigm shift in feedback control system design," in *Proc. Amer. Control Conf.*, 2006, pp. 2399–2405.
[46] K. Ogata, *Modern Control Engineering*, 3rd. Englewood Cliffs, NJ: Prentice-Hall, 1997.
[47] K. Michail, "Optimised configuration of sensing elements for control and fault tolerance applied to an electro-magnetic suspension system," Ph.D. dissertation, Univ. Loughborough, Leicestershire, U.K., Oct. 2009.
[48] W.-H. Chen, D. J. Ballance, P. J. Gawthrop, J. J. Gribble, and J. O'Reilly, "Nonlinear PID predictive controller," *Proc. Inst. Elect. Eng.—Control Theory Appl.*, vol. 146, no. 6, pp. 603–611, Nov. 1999.
[49] R. Errouissi, M. Ouhrouche, and W.-H. Chen, "Robust nonlinear generalized predictive control of a permanent magnet synchronous motor with an anti-windup compensator," in *Proc. IEEE Int. Symp. Ind. Electron.*, 2010, pp. 3184–3189.
[50] A. Isidori, *Nonlinear Control Systems: An Introduction*, 3rd ed. New York: Springer-Verlag, 1995.

Shihua Li (M'05–SM'10) was born in Pingxiang, China, in 1975. He received the B.S., M.S., and Ph.D. degrees in automatic control from Southeast University, Nanjing, China, in 1995, 1998, and 2001, respectively.

Since 2001, he has been with the School of Automation, Southeast University, where he is currently a Professor. His main research interests include nonlinear control theory with applications to robots, spacecraft, ac motors, and other mechanical systems.



Jun Yang (M'11) was born in Anlu, China, in 1984. He received the B.S. degree from the Department of Automatic Control, Northeastern University, Shenyang, China, in 2006 and the Ph.D. degree in control theory and control engineering from the School of Automation, Southeast University, Nanjing, China, in 2011.

He is currently a Lecturer with the School of Automation, Southeast University. His research interests include disturbance estimation and compensation, and advanced control theory and its application to flight control systems and motion control systems.

Wen-Hua Chen (M'00–SM'08) received the M.S. and Ph.D. degrees from the Department of Automatic Control, Northeast University, Shenyang, China, in 1989 and 1991, respectively.

From 1991 to 1996, he was a Lecturer and then Associate Professor with the Department of Automatic Control, Nanjing University of Aeronautics and Astronautics, Nanjing, China. He held a research position and then became a Lecturer in control engineering with the Centre for Systems and Control, University of Glasgow, Glasgow, U.K., from 1997 to 2000. He is currently a Senior Lecturer in fight control systems with the Department of Aeronautical and Automotive Engineering, Loughborough University, Leicestershire, U.K. He has published about 120 papers, including 50 journal papers. His research interests include the development of advanced control strategies and their applications in aerospace engineering.

Xisong Chen was born in Quanjiao, China, in 1970. He received the B.S. degree from the Department of Electronic Science and Engineering, Jilin University, Changchun, China, in 1993 and the M.S. and Ph.D. degrees in control theory and control engineering from the School of Automation, Southeast University, Nanjing, China, in 2001 and 2009, respectively.

He is currently a Professor with the School of Automation, Southeast University. His research interests include advanced process control, disturbance rejection theory, and their applications to process control systems.