△ Kinematics

△ Generalized coordinates

$q = \begin{bmatrix} q_b \\ q_j \end{bmatrix}$ → base, → joints; $q_b = \begin{bmatrix} q_{bP} \\ q_{bR} \end{bmatrix} \in SE(3)$

△ Generalized velocities / accelerations

$u = \begin{bmatrix} {}_Iv_B \\ {}_Bw_{IB} \\ \dot\varphi_1 \\ \vdots \\ \dot\varphi_{n_j} \end{bmatrix}$, $\dot u = \begin{bmatrix} {}_I a_B \\ {}_B\dot w_{IB} \\ \ddot\varphi_1 \\ \vdots \\ \ddot\varphi_{n_j} \end{bmatrix} \in R^{6+n_j}$

$u = E_{fb}\dot q$, $E_{fb} = \begin{bmatrix} I_{3\times3} & & \\ & E_{R} & \\ & & I_{n_j\times n_j} \end{bmatrix}$

△ P / v of a pt on the robot

- P

${}_I r_{IQ}(q) = {}_I r_{IB}(q) + C_{IB}(q)\,{}_B r_{BQ}(q)$

- V

${}_I v_Q = {}_I v_B + \dot C_{IB}\,{}_B r_{BQ} + C_{IB}\,{}_B\dot r_{BQ}$

$= {}_I v_B + C_{IB}[{}_B w_{IB}]_\times {}_B r_{BQ} + C_{IB}\,{}_B\dot r_{BQ}$

$= {}_I v_B - C_{IB}[{}_B r_{BQ}]_\times {}_B w_{IB} + C_{IB}\,{}_B\dot r_{BQ}$

$= {}_I v_B - C_{IB}[{}_B r_{BQ}]_\times {}_B w_{IB} + C_{IB}\,\frac{\partial {}_B r_{BQ}}{\partial q_j}\dot q_j$

$\dot u = J_{q_j}\dot q_j$

$= \left[I_{3\times3} \;\; -C_{IB}[{}_B r_{BQ}]_\times \;\; C_{IB}\frac{\partial {}_B r_{BQ}}{\partial q_j}(q_j) \right] u$

$= {}_I J_Q(q)\,u$

△ Contact constraints

- C_i: contact point, cannot move

$\begin{cases} {}_I r_{IC_i} = \text{const} \\ {}_I\dot r_{IC_i} = {}_I J_{C_i} u = 0 \\ {}_I\ddot r_{IC_i} = {}_I J_{C_i}\dot u + {}_I\dot J_{C_i} u = 0 \end{cases}$

$J_c = \begin{bmatrix} J_{C_1} \\ \vdots \\ J_{C_{n_c}} \end{bmatrix} \in R^{3n_c \times n_n}$ — dim of $\dot x$, no. of contact pt.

- Nullspace

$0 = \dot r_c = J_c \dot q \Rightarrow \dot q = J_c^+ 0 + N_c \dot q_0 = N_c \dot q_0$

$0 = \ddot r_c = J_c \ddot q + \dot J_c \dot q \Rightarrow \ddot q = J_c^+(-\dot J_c \dot q) + N_c \ddot q_0$

- first priority

task 0: $\dot w_c = J_c \ddot q + \dot J_c \dot q = 0$

task 1: $\dot w_e = J_e \ddot q + \dot J_e \dot q = \dot w_e^*$

- system model due to contact constraints

△ Contact Jacobian

- $J_c = \frac{\partial r}{\partial q}$ → constraints, → base & joints

$= [J_{c,b} \;\; J_{c,j}]$ — relation between base motion & joint motion & constraints

$= \left[\frac{\partial r_c}{\partial q_b} \;\; \frac{\partial r_c}{\partial q_j}\right]$

- $\text{rank}(J_{c,b}) \Rightarrow$ base is fully controllable

$\text{rank}(J_c) - \text{rank}(J_{c,b})$

total no. of independent constraints — no. of constraints contributes to motion of the base

no. of kinematic constraints for joint actuations

e.g.

$q_b = \begin{pmatrix} x \\ y \\ \varphi \end{pmatrix}$, $q_j = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}$

$J_c = R^{4\times7}$ — x,y constraints of 2 contact point

$\text{rank}(J_c) = 4$

$\text{rank}(J_{c,b}) = 3$

$4-3 = 1$ ("1" kinematic constraints of the actuators)

△ generalized coordinates DON'T correspond to the degree of freedom

△ Dynamics

$M\ddot q + b + g = \tau + J^T F_e$ — contact Jacobian, external force, generalized force

$M(q)\dot u + b(q,u) + g(q) = S^T\tau + J_c^T F_c$ — selection matrix, Jacobian

again

$q = \begin{bmatrix} q_b \\ q_j \end{bmatrix}$ un-actuated base, actuated joints

$u_j = S u$

$= S\begin{bmatrix} u_b \\ u_j \end{bmatrix}$

that's why "selection matrix" $= [0, I]\begin{bmatrix} u_b \\ u_j \end{bmatrix}$

$\Rightarrow M\ddot q_j + b + g = \tau + J^T F_e$

$\Rightarrow M\ddot q + b + g = S^T\tau + J_c^T F_c$

$M\dot u + b + g = S^T\tau + J_c^T F_c$ — only joints has torques!

$S = [0 \; I]$ — equivalent in different form

$M\dot u + b + g + J_c^T F_c = S^T\tau$ — reaction force contact force, lift

△ Soft contact

physical accuracy v. numerical stability

△ Hard contact

- closed contact

$r^N = 0$

$\dot r^N = 0$

- linear complementary constraint

$\ddot r^N \geq 0$

$F^N \geq 0$

$\ddot r^N F^N = 0$

△ Hard contact of an MBS

- $M\ddot q + b + g + J_c^T F_c = S^T\tau$ — (1)
- $\dot r_c = J_c \dot q = 0$
- $\ddot r_c = J_c\ddot q + \dot J_c\dot q = 0$ — (2)

$\Rightarrow$ from (1)

$\ddot q = M^{-1}(S^T\tau - b - g - J_c^T F_c)$

w/ (2)

$\ddot r_c = J_c[M^{-1}(S^T\tau - b - g - J_c^T F_c)] + \dot J_c\dot q = 0$ — (3)

- get F_c - contact force from (3)

$F_c = (J_c M^{-1} J_c^T)^{-1}(J_c M^{-1}(S^T\tau - (b+g)) + \dot J_c \dot q)$ — given my constraints, actuations, get F_c

∴ w/ (1)

$M\ddot q + b + g + J_c^T F_c = S^T\tau$

$\Rightarrow M\ddot q + b + g + J_c^T(\ldots) = S^T\tau$

$\Rightarrow M\ddot q + b + g + J_c^T(\ldots) = S^T\tau$

define

$N_c = I - M^{-1}J_c^T(J_c M^{-1} J_c^T)^{-1}J_c$

$J_c N_c = J_c - \cancel{J_c M^{-1} J_c^T (J_c M^{-1} J_c^T)^{-1} J_c}$

$= 0$

$\Rightarrow N_c^T M\ddot q + N_c^T(b+g) + N_c^T J_c^T(\ldots) = N_c^T S^T\tau$, with $N_c^T J_c^T = 0$

$\Rightarrow N_c^T M\ddot q + N_c^T(b+g) = N_c^T S^T\tau$

support consistent dynamics — independent of reaction force!

△ impulse transfer @ contact

$\int_{\{t\}} (M\dot u + b + g + J_c^T F_c - S^T\tau)\,dt$

$= M(u^+ - u^-) + J_c^T \hat F_c = 0$ — (1), $\hat F_c$: $F_c \cdot dt$

also

$\dot r_c^+ = J_c u^+ = 0$ — (2)

$\Rightarrow$ w/ (1)

$u^+ - u^- = -M^{-1}J_c^T \hat F_c$ — (3)

$\Rightarrow J_c u^+ - J_c u^- = -J_c M^{-1} J_c^T \hat F_c$

$\Rightarrow J_c u^- = J_c M^{-1} J_c^T \hat F_c$

$\Rightarrow \dot r_c^- = \Lambda_c^{-1} \hat F_c$

$\Rightarrow \hat F_c = \Lambda_c \dot r_c^- = \Lambda_c J_c u^-$ — end-effector inertia

∴ $\Delta u = u^+ - u^-$ (from (3))

$= -M^{-1}J_c^T \hat F_c$

$= -M^{-1}J_c^T \Lambda_c J_c u^-$

$= -M^{-1}J_c^T (J_c M^{-1} J_c^T)^{-1} J_c u^-$ — change in generalized vel.

∴ $u^+ = u^- - M^{-1}J_c^T(J_c M^{-1}J_c^T)^{-1}J_c u^-$

$= [I - M^{-1}J_c^T(J_c M^{-1}J_c^T)^{-1}J_c]\,u^-$

$= N_c u^-$ — post-impact vel

N_c also maps $u^- \to u^+$

- $E_{loss} = \Delta E_{kin}$

$= -\frac{1}{2}\Delta u^T M \Delta u$

$= -\frac{1}{2}\Delta \dot r_c^T \Lambda_c \Delta \dot r_c$

$= -\frac{1}{2}\dot r_c^{-T}\Lambda_c \dot r_c^-$

△ Dynamics control

again

- $M(q)\dot u + b(q,u) + g(q) + J_c^T F_c = S^T\tau$
- $\dot r_c = J_c u$

$\ddot r_c = J_c \dot u + \dot J_c u = 0$

- $F_c = (J_c M^{-1} J_c^T)^{-1}(J_c M^{-1}(S^T\tau - b - g) + \dot J_c u)$
- $N_c = I - M^{-1}J_c^T(J_c M^{-1}J_c^T)^{-1}J_c$
- $N_c^T(M\ddot q + b + g) = N_c^T S^T\tau$

- inverse dynamics

$\tau^* = (N_c^T S^T)^+ N_c^T(M\ddot q + b + g)$

$\tau^* = (N_c^T S^T)^+ N_c^T(M\ddot q + b + g) + \underline{N(N_c^T S^T)\tau_0^*}$ — multiple solution ⇕ internal force direction

△ inducing QP

$\min_x \| A_i x - b_i \|_2$

w/ $x = \begin{bmatrix} \dot u \\ F_c \\ \tau \end{bmatrix}$

(1) $M(q)\dot u + b(q,u) + g(q) + J_c^T F_c = S^T\tau$

$\Rightarrow A_1 = [\hat M \;\; \hat J_c^T \;\; -S^T]$

$b_1 = -\hat b - \hat g$

(2) $J\dot u + \dot J u = \dot w^*$

$\Rightarrow A_2 = [\hat J_i \;\; 0 \;\; 0]$ — motion tasks

$b_2 = \dot w^* - \hat{\dot J}_i u$

(3) $F_i = F_i^*$

$\Rightarrow A_3 = [0 \;\; I \;\; 0]$ — force tasks

$b = F_i^*$

(4) $\min \|\tau\|_2$ — torque min

$\Rightarrow A = [0 \;\; 0 \;\; I]$

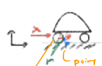
$b = 0$

⟶ then solve QP!

△ case study

(1)

- contact constraints?



point; $q = \begin{bmatrix} x \\ \varphi \end{bmatrix}$ (unactuated base + actuated wheel)

△ point on the wheel

${}_I r_{IP} = \begin{bmatrix} x + r\sin\varphi \\ r + r\cos\varphi \\ 0 \end{bmatrix}$

△ Jacobian

$\frac{\partial r}{\partial q} = \begin{bmatrix} \frac{\partial r_1}{\partial x} & \frac{\partial r_1}{\partial \varphi} \\ \frac{\partial r_2}{\partial x} & \frac{\partial r_2}{\partial \varphi} \\ \frac{\partial r_3}{\partial x} & \frac{\partial r_3}{\partial \varphi} \end{bmatrix} = \begin{bmatrix} 1 & r\cos\varphi \\ 0 & -r\sin\varphi \\ 0 & 0 \end{bmatrix}$

△ contact constraints

vel = 0 ⟵ when P point now to the ground, establishing contact

${}_I\dot r_{IP}\big|_{\varphi=\pi} = J\big|_{\varphi=\pi}\dot q$ — $\dot r = 0$!

$= \begin{bmatrix} 1 & -r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot x \\ \dot\varphi \end{bmatrix} = 0$

$\Rightarrow \dot x - r\dot\varphi = 0$ (rolling constraint)

$\Rightarrow$ possible base motion

$\dot q = J_c^+ 0 + N_c\dot q_0$

$= N_c \dot q_0$ → $N(J_c)$ ∴ $NJ_c = 0$

$= \begin{bmatrix} r \\ 1 \end{bmatrix}\dot q_0$

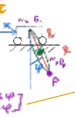
(2)

Floating base system with 12 actuated joint and 6 base coordinates (18DoF)

$J_c = [J_{c,b}\; J_{c,j}]$		1 contact			
total constraints (rank(J_c))	0	3	6	9	12
base constraints (rank$(J_{c,b})$)	0	3	5	6	6
internal constraints (rk(J_c) − rk$(J_{c,b})$)	0	0	1	3	6
uncontrollable DoF (6 − rk$(J_{c,b})$)	6	3	1	0	0

(3) $N_c^T M\ddot q + N_c^T(b+g) = N_c^T S^T\tau$

△ EoM



△ Contact Jacobian

$J_c = [J_{c,b} \;\; J_{c,j}] = \begin{bmatrix} 1 & 2l\cos\varphi \\ 0 & 2l\sin\varphi \end{bmatrix}$

$q_b = \begin{bmatrix} q_b \\ q_j \end{bmatrix} = \begin{bmatrix} x \\ \varphi \end{bmatrix}$

${}_I r_P = \begin{bmatrix} x + 2l\sin\theta \\ -2l\cos\theta \end{bmatrix}$

$\frac{\partial r}{\partial q} = \begin{bmatrix} \frac{\partial r_x}{\partial x} & \frac{\partial r_x}{\partial \varphi} \\ \frac{\partial r_y}{\partial x} & \frac{\partial r_y}{\partial \varphi} \end{bmatrix}$

(a) both constraints active

- $N(J_c) = 0$, no controllable subspace
- contact constraint cannot control the floating base; only change internal force

(b) vertical motion locked

$J_c = [J_{c,b} \;\; J_{c,j}] = [\,0 \;\; 2l\sin\varphi\,]$

$N(J_c) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $N_c^T S^T = [1, 0]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$

- torque has no influence on motion
- only change constraint/reaction/internal force

(c) $J_c = [\,1 \;\; 2l\cos\varphi\,]$

$N_c = \begin{bmatrix} 2l\cos\varphi \\ -1 \end{bmatrix}$

$N_c^T S^T = [2l\cos\varphi \;\; -1]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$

- can change the motion
- change the internal force, will change the motion

△ Control of legged robots

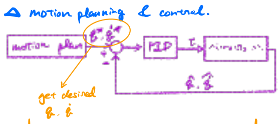
△ Static / dynamic stability

- static
  - ≥ 3 legs on ground
  - joints "freeze", will not fall
  - safe, slow, inefficient
- dynamic
  - < 3 legs on ground
  - will fall if no moving
  - hard, fast, efficient

△ control concepts

- kinematic control
  - high-gain joint position by traj. tracking
- impedance control w/ joint space inverse dynamics
  - low-gain joint control w/ model compensation
- support-consistent inverse dynamic control
  - projection of dynamics & desired acceleration into Nullspace of contact constraints.
- task-space inverse dynamics control
  - directly regulating in "taskspace" as sequential QP

△ motion planning & control.



get desired $q, \dot q$

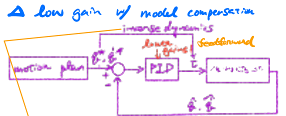
pure kinematic, w/ $q, \dot q$

then based on $q, \dot q$, do tracking

BUT: need accurate modelling of environs, to hv good motion planning

otherwise, robot fail!

△ low gain w/ model compensation

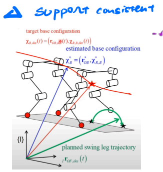


- recall $N_c^T M\ddot q + N_c^T(b+g) = N_c^T S^T\tau$

$\tau = (N_c^T S^T)^+ N_c^T(M\ddot q + b + g)$

support consistent dynamics, can better adapt to unknown terrain.

△ support consistent inverse dynamics



- find the desired acceleration
  - track the swing leg
  - move the base
  - ensure contact constraint

$\Rightarrow$ fully defines the motion of the system

- 12 actuated + 6 actuated = 18 DoF
- 3 swing leg + 6 base motion + 9 contact constraints = 18

$\Rightarrow \ddot q_{des} = \begin{bmatrix} J_F \\ J_B \\ J_c \end{bmatrix}^{-1}\left(\begin{bmatrix} \ddot r_{OF\,des} \\ \dot w_{B\,des} \\ 0 \end{bmatrix} - \begin{bmatrix} \dot J_F \\ \dot J_B \\ \dot J_c \end{bmatrix}\dot q\right)$

more task, then pseudo-inverse, can also hv priority

$\Rightarrow$ after having $\ddot q_{des}$

$\tau = (\hat N_c^T S_c^T) N_c^T(\hat M\ddot q_{des} + \hat b + \hat g)$

△ QP

$\min_x \| A_i x - b_i \|_2$

$x = \begin{bmatrix} \dot u \\ F_c \\ \tau \end{bmatrix}$

s.t.

- $M(q)\dot u + b(q,u) + g(q) + J_c^T F_c = S^T\tau$

$A = [\hat M \;\; \hat J_c^T \;\; -S^T]$ $b = -\hat b - \hat g$

- $J\dot u + \dot J u = \dot w^*$ (motion task)

$A = [\hat J_i \;\; 0 \;\; 0]$ $b = \dot w^* - \hat{\dot J}_i u$

- $F_i = F_i^*$ (force tasks)

$A = [0 \;\; I \;\; 0]$ $b = F_i^*$

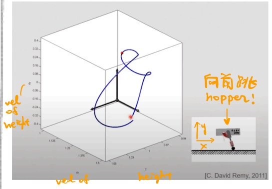
- $\min \|\tau\|_2$ (torque min)

$A = [0 \;\; 0 \;\; I]$ $b = 0$

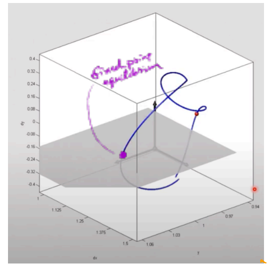
w/ priority

△ Stability of Legged Robot

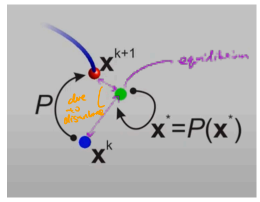
- we want dynamic gait!
- limit cycle analysis: look at things on a step2step basis



vel of height; vel of x; height; 原地跳 hopper!



fixed point equilibrium



equilibrium; due to disturbance

- Poincaré map $x_{k+1} = P(x_k)$
- fixed point $x^* = P(x^*)$
- linearization $\Delta x_{k+1} = \frac{\partial P}{\partial x}\Delta x_k$

$\text{Eig}\left(\frac{\partial P}{\partial x}\right) < 1$

$\Rightarrow$ stable

△ Passive walker efficiency

$\frac{E_{used}}{\text{work}} = \frac{mgh}{mgd} = \frac{h}{d}$

step 1: move base

try to move the base as good as possible.

But not to violate the basics!

$\min_{\ddot q} \| \dot w_{B,des}(\tau) - J_B\ddot q - \dot J_B\dot q \|$

$M\ddot q + b + g + J_c^T F_c = S^T\tau$ — EoM

$J_c\ddot q + \dot J_c\dot q = 0$ — contact constraints

$F_{c,n,i} > F_{c,min}$ — minimal contact force

$\mu F_{c,n,i} > \|F_{c,t,i}\|_2$ — contact force in friction cone

step 2: move swing leg

try to swing it as good as possible, but not to violate step 1:

$\min_{\ddot q} \| \ddot r_{OF,des}(\tau) - J_F\ddot q - \dot J_F\dot q \|$

$C_1 = \ddot x_{B,des}(\tau) - J_B\ddot q - \dot J_B\dot q$ — step 1 not affected

$M\ddot q + b + g + J_c^T F_c = S^T\tau$ — EoM

$F_{c,n,i} > F_{c,min}$ — minimal contact force

$\mu F_{c,n,i} > \|F_{c,t,i}\|_2$ — contact force in friction cone

step 3: minimize $\|\tau\|_2$ + $\|F_{c,i}\|$

△ ANYmal Case Study

< 10hz
Navigation → Motion Planning 10~100 hz
- motion
- foothold
- reference torque

main research

Tracking ~khz
- follow the plan
- safety constraints
- reactive behavior

relatively mature

Joint Control
- joint torque
- joint position

Plant → proprioception
- joint position
- IMU
- contact sensor

→ state estimation

exteroception 30hz
- camera → SLAM
- lidar

△ Tracking of ANYmal
- inverse dynamics-based
- whole-body control
- get the "u"

– floating-base model

- $q = \begin{bmatrix} {}_I r_{IB} \\ q_{IB} \\ q_j \end{bmatrix}$ generalized coordinates $u = \begin{bmatrix} {}_I v_B \\ {}_B \omega_{IB} \\ \dot{q}_j \end{bmatrix}$ vel

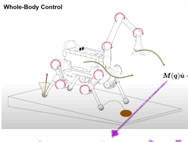
- $M(q)\dot{u} + h(q,u) = S^T\tau + J_S^T(q)\lambda$ (b+g) external force / ground contacts

$\lambda = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_{n_c} \end{bmatrix}$

$J_S = \begin{bmatrix} J_{E_1} \\ \vdots \\ J_{E_{n_c}} \end{bmatrix}$

- ${}_I J_{E_k} = [I \;\; -C_{IB} \cdot {}_B r^{\times}_{BE_k} \;\; 0 \cdots {}_I J_{BE_k} \cdots 0]$

Whole-Body Control



$\tau = ?$

$M(q)\dot{u} + h(q,u) = S^T\tau + J_S^T(q)\lambda$

$\lambda = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_{n_c} \end{bmatrix} \quad J_S = \begin{bmatrix} J_{E_1} \\ \vdots \\ J_{E_{n_c}} \end{bmatrix}$

$\begin{bmatrix} M_b \\ M_j \end{bmatrix}\dot{u} + \begin{bmatrix} h_b \\ h_j \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} + \begin{bmatrix} J_{S_b}^T \\ J_{S_j}^T \end{bmatrix} \frac{F_s}{\lambda}$

get τ base on $\dot{u}$ & λ

⇒ $\xi = \begin{bmatrix} \dot{u} \\ \lambda \end{bmatrix}$ ⇒ break down into tasks

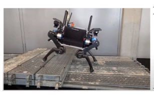
- EoM
- contact constraints
- force/torque limits

$\begin{cases} W(A\xi - b) = 0 \\ W'(D\xi - f) \le 0 \end{cases}$

(solve the tasks parallelly / hierarchically

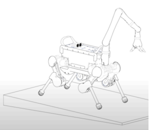
i.e., PROJECT THE CONSTRAINTS INTO THE NULL SPACE OF HIGHER PRIORITY TASKS!)

- examples



Priority	Task
1	Equations of Motion
2	No contact motion
3	Torque limits
4	Friction cone and λ modulation
5	Desired torso x, y position
6	Swing foot motion tracking
7	Limb kinematic limits
8	Main body yaw
9	Main body height
10	Main body roll and pitch
11	Contact force minimization

1	Equations of Motion
1	Force/torque limits
1	No motion at contact points
2	CoM motion tracking
2	Torso angular motion
2	Foot motion tracking
2	End-effector motion tracking
2	End-effector force tracking
2	Torso orientation adaptation
3	Contact force minimisation



- Then after the optimization:

WE GET ξ, $\xi = \begin{bmatrix} \dot{u} \\ \lambda \end{bmatrix}$

GET τ!

⇒ $\tau^d = M_j(q)\dot{u}^* + h_j(q,u) - J_{S_j}(q)\lambda^*$

⇒ impedance

$\tau^{cmd} = \tau^d + k_p\tilde{q} + k_D\dot{\tilde{q}}$

△ Planning of ANYmal
- motion
- foothold
- reference torque

s.t. complex environments

- Traj. Opt / MPC
  - dynamics complexity? the "resolution" of the dynamics.
  - integrated/sequential optimization? w.r.t. timing/foothold locations

we are converging!
- complex/high-resolution dynamics
- integrated versions of optimization.

– foothold optimization

- first optimize the footholds
  i.e., what gait are we going to choose?

$\min_{\xi} \; \frac{1}{2}\xi^T R \xi + c^T\xi$

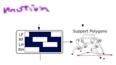
s.t. $D\xi \le f$

$\xi = [p_x, p_y, \ldots p_{x_4}\, p_{y_4} \ldots]$

$\dot{\xi} = f(\xi)$?

- then optimize motion

• w/ gait → polygons of support



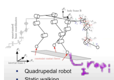
• zero-moment point (ZMP)

△ Motion planning + control
MPC (all in one)

△ Reinforcement Learning
(end 2 end)

△ case study

Kinematics of Floating Base / Mobile Systems



- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

1. How many generalized coordinates?
2. How many base coordinates?
3. How many actuated joint coordinates?
4. How many contact constraints?

1. 12 + 6 = 18
2. 6
3. 12
4. 9 (@ this frame)

5. Write down the contact constraint
6. How many DoFs remain adjustable?
7. Which DoFs remain adjustable?

5. $\begin{cases} {}_I\dot{r}_{OP_i} = {}_I J_{OP_i}\dot{q} = 0 \\ {}_I\ddot{r}_{OP_i} = {}_I J_{OP_i}\ddot{q} + {}_I\dot{J}_{OP_i}\dot{q} = 0 \\ i = 1,2,3 \end{cases}$

△ ${}_I J_{OP_i} = 3[\overset{18}{\quad}]$

$9\begin{bmatrix} \overset{18}{3[\quad]} \\ 3[\quad] \\ 3[\quad] \end{bmatrix}$

⇒ 9 independent constraints

△

6 independent base constraints

3 DoF swing leg

3 internal force direction

8. Given a desired swing velocity ${}_I\dot{\mathbf{r}}^{des}_{OP_{F4}}$ what is the generalized velocity?

$\dot{\mathbf{q}} = f\left(\mathbf{q}, {}_I\dot{\mathbf{r}}^{des}_{OP4}\right)$

9. Is it unique?
10. Is it possible to follow the desired swing trajectory without moving the joints of leg 4? How?

8. $\begin{bmatrix} {}_I\dot{r}_{OP_1} = {}_I J_{OP_1}\dot{q} = 0 \\ {}_I\dot{r}_{OP_2} = {}_I J_{OP_2}\dot{q} = 0 \\ {}_I\dot{r}_{OP_3} = {}_I J_{OP_3}\dot{q} = 0 \\ {}_I\dot{r}_{OP_4} = {}_I J_{OP_4}\dot{q} = {}_I\dot{r}^{*}_{OP_4} \end{bmatrix}$

$12[\overset{18}{\quad}]$

9. NOT UNIQUE

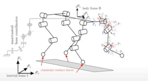
10. yes.

$J_1 = 12[\overset{18}{J}] \quad w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{r}^* \end{bmatrix}$

$J_2 = 3[\overset{18}{0 \;\; J_3}] \quad w_2 [0]$

15 constraints, → not unique

Kinematic Singularity



Task 1:
- contact constraints
- foot motion

Task 2:
- base position
- base orientation

A) Single Task

B) Multiple Tasks

- There exist different formulations for "moving the foot (task1) while keeping the base position and orientation (task2)". Write down the solution for A and B!
- What is the difference?
- What happens in singular configs?

$J_1 = 12[\overset{18}{J}] \quad w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{r}^* \end{bmatrix}$

$J_2 = 6\begin{bmatrix} \overset{18}{{}_I J_{OB}} \\ {}_I J_{OROT} \end{bmatrix} \quad w_2 = [0]$

(A)

$\dot{q} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}^{+} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

↳ if no inverse

⇒ violate all constraints

(B)

$\dot{q} = J_1^{+} w_1 + N_1\dot{q}_0$

$J_2\dot{q} = J_2(J_1^{+}w_1 + N_1\dot{q}_0) = w_2$

$\dot{q}_0 = (J_2 N_1)^{+}[w_2 - J_2 J_1^{+} w_1]$

△ summary for floating-base

- EoM

$M\ddot{q} + b + g + J_c^T F_c = S^T\tau$ —(1)

- contact constraint

$\dot{r}_c = J_c\dot{q} = 0$

$\ddot{r}_c = J_c\ddot{q} + \dot{J}_c\dot{q} = 0$ —(2)

- w/ (1) & (2)

$\ddot{q} = M^{-1}[S^T\tau - (b + g + J_c^T F_c)]$

⇒ $\ddot{r}_c = J_c M^{-1}[S^T\tau - (b+g+J_c^T F_c)] + \dot{J}_c\dot{q} = 0$

- $J_c M^{-1}[S^T\tau - (b+g+J_c^T F_c)] + \dot{J}_c\dot{q} = 0$ — try to get this

⇒ $J_c M^{-1}[S^T\tau - (b+g+J_c^T F_c)] = -\dot{J}_c\dot{q}$

⇒ $[S^T\tau - (b+g+J_c^T F_c)] = -M J_c^{-1}\dot{J}_c\dot{q}$

⇒ $J_c^T F_c + b + g - S^T\tau = M J_c^{-1}\dot{J}_c\dot{q}$

⇒ $J_c^T F_c = M J_c^{-1}\dot{J}_c\dot{q} - b - g + S^T\tau$

⇒ $F_c = J_c^{-T}\{M J_c^{-1}\dot{J}_c\dot{q} + [S^T\tau - (b+g)]\}$

⇒ $F_c = J_c^{-T}MJ_c^{-1}\dot{J}_c\dot{q} + J_c^{-T}MJ_c^{-1}J_cM^{-1}[S^T\tau - (b+g)]$

⇒ $F_c = [J_c^{-T}MJ_c^{-1}][\dot{J}_c\dot{q} + J_cM^{-1}[S^T\tau - (b+g)]]$

$= (J_cM^{-1}J_c^T)^{-1}(J_cM^{-1}[S^T\tau - (b+g)] + \dot{J}_c\dot{q})$

- $F_c = (J_cM^{-1}J_c^T)^{-1}(J_cM^{-1}[S^T\tau - (b+g)] + \dot{J}_c\dot{q})$

- recall EoM $M\ddot{q} + b + g + J_c^T F_c = S^T\tau$

$M\ddot{q} + b + g + J_c^T(J_cM^{-1}J_c^T)^{-1}(J_cM^{-1}[S^T\tau - (b+g)] + \dot{J}_c\dot{q}) = S^T\tau$

⇒ & w/ $N_c = [I - M^{-1}J_c^T[J_cM^{-1}J_c^T]^{-1}J_c)$

$N_c^T = (I - J_c^T[J_cM^{-1}J_c^T]^{-1}J_cM^{-1})$

⇒ & $N_c^T J_c^T(J_cM^{-1}J_c^T)^{-1}$

⇒ $(I - J_c^T[J_cM^{-1}J_c^T]^{-1}J_cM^{-1})J_c^T(J_cM^{-1}J_c^T)^{-1}$

⇒ $J_c^T(J_cM^{-1}J_c^T)^{-1} - J_c^T\cancel{[J_cM^{-1}J_c^T]^{-1}J_cM^{-1}J_c^T}(J_cM^{-1}J_c^T)^{-1}$

⇒ $J_c^T(J_cM^{-1}J_c^T)^{-1} - J_c^T(J_cM^{-1}J_c^T)^{-1} = 0$

- w/ $N_c[J_c^T(J_cM^{-1}J_c^T)^{-1}]$ ⇒ $J_cN_c = 0$

we have

⇒ $N_c^T M\ddot{q} + N_c^T(b+g) = N_c^T S^T\tau$

This is called "support consistent dynamics"

projector: project to nullspace

△ remarks

- how can we do ID control for fixed base sys?

⇒ $M\ddot{q} + b + g = \tau$

⇒ $\tau^* = \hat{M}\ddot{q}^* + \hat{b} + \hat{g}$

↳ how the system should move? i.e., desired $\ddot{q}$

- for floating-base systems

$M\ddot{q} + b + g + J_c^T F_c = S^T\tau$

- we need to figure out this first.
- we are constrained in $\ddot{q}$
- but chicken-&-egg.
  → how to get rid of it?

$0 = J_c\ddot{q}^* + \dot{J}_c\dot{q}$

$\ddot{q}^* = J_c^{+}(-\dot{J}_c\dot{q})$

$= -J_c^{+}\dot{J}_c\dot{q} + N_c\ddot{q}_0$

project into another space
- w/ no. of constraints (no. of forces)
  DoF < (generalized coordinate & DoF)
- project to subspace

THAT IS WHY

$N_c^T M\ddot{q} + N_c^T(b+g) = N_c^T S^T\tau$

⇒ $\tau^* = [N_c^T S^T]^{+} N_c^T(M\ddot{q}^* + b + g)$

w/ redundancy

cannot be random, must confined to constraints

usually throw into

⇒ $\tau^* = [N_c^T S^T]^{+}N_c^T(M\ddot{q}^* + b + g) + \text{null}[N_c^T S^T]\tau_0$

$\text{null}[N_c^T S^T]$

$= [I - (N_c^T S^T)^{+} N_c^T S^T]$

△ getting the contact force

- QR decomposition

EoM

$M\ddot{q} + b + g + J_c^T F_c = S^T\tau$

⇒ let $J_c^T = Q[R^T \; 0]^T$ $[Q^T = Q^{-1}]$

⇒ $Q^T[M\ddot{q} + b + g] + [R^T \; 0]^T F_c = Q^T S^T\tau$

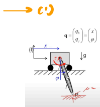
⇒ $\begin{cases} S_cQ^T[M\ddot{q} + b + g] + RF_c = S_cQ^TS^T\tau^* & (1) \\ S_uQ^T[M\ddot{q} + b + g] = S_uQ^TS^T\tau^* & (2) \end{cases}$

order

1. motion planning
   get $\ddot{q}^*$ (confined w/ contact constraints)
2. EoM projected w/ Null-projection
   get τ^* ⟶ (2)
3. w/ τ^*, $\ddot{q}^*$,
   get F_c ⟶ (1)

△ case study

- vertical motion locked @ P



soln

- EoM M b g $+ J_c^T F_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\tau_a$ S^T

- motion constraints

△ $\ddot{y}_p = 0$ $r_{p_y} = -2l\cos\varphi$

⇒ $J_c = \frac{\partial r}{\partial q} = \left[\frac{\partial r}{\partial q_b} \;\; \frac{\partial r}{\partial q_j}\right]$

$= [0 \;\; 2l\sin\varphi]$

△ $0 = J_c\ddot{q} + \dot{J}_c\dot{q}$ $N_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\ddot{q} = -J_c^{+}\dot{J}_c\dot{q} + (I - J_c^{+}J_c)\ddot{q}_0$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

△ $N_c^T M\ddot{q} + N_c^T(b+g) = N_c^T S^T\tau$

⇒ $N_c^T M(-J_c^{+}\dot{J}_c\dot{q} + N_c\ddot{q}_0) + \cancel{N_c^T(b+g)} = \cancel{N_c^T S^T\tau}$

⇒ $(m_c + m_p)\ddot{x}_0 = 0$ ⇒ $\ddot{x}_0 = 0$

free moving,

but your current torque does fuck all!

you can change your torque! $N(N_c^T S^T) = N(0) = 1$

→ but does fuck-all!

horizontal lock @ P

$r_{p_x} = x + 2l\sin\varphi$

△ constraints $\ddot{r}_{p_x} = J_c\ddot{q} + \dot{J}_c\dot{q} = 0$

$J_c = [J_{c,b} \;\; J_{c,j}] = \left[\frac{\partial r}{\partial q_b} \;\; \frac{\partial r}{\partial q_j}\right]$ $N_c = \begin{bmatrix} 2l\cos\varphi \\ -1 \end{bmatrix}$

$= [1 \;\; 2l\cos\varphi]$

△ kinematics

$\ddot{q} = -J_c^{+}\dot{J}_c\dot{q} + \underbrace{(I - J_c^{+}J_c)}_{N_c}\ddot{q}_0$

$N_c^T S^T = [2l\cos\varphi \;\; -1]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$

△ $N_c^T M(-J_c^{+}\dot{J}_c\dot{q} + N_c\ddot{q}_0) + N_c^T(b+g) = -\tau_a$

基於這個 → 得 τ_a

△ no redundancy for τ_a

as $N(N_c^T S^T) = N(-1) = 0$

→ whatever you are doing in τ, affects x & y!