Stochastic Modeling for Real-Time Kinematic GPS/GLONASS Positioning

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ABSTRACT: *It is well known that for satellite-based high-precision kinematic positioning, the correct integer ambiguities must be identified on-the-fly (OTF). It has also been noted that reliable integer ambiguity resolution is highly dependent on applying correct stochastic models for differenced GPS and GLONASS measurements. Stochastic modeling for real-time kinematic (RTK) positioning, however, is a difficult task to accomplish. In this study, a practical method is proposed for directly estimating the variance and covariance components for the differenced GPS and GLONASS measurements. The applicability of the proposed method for RTK positioning has been tested with both GPS dual-frequency and combined GPS/GLONASS single-frequency datasets. Test results show that using the estimated measurement covariance matrices significantly improves the success rates of ambiguity resolution and the accuracy of positioning results.*

INTRODUCTION

Reliable ambiguity resolution is critical for real-time kinematic (RTK) GPS/GLONASS positioning because centimeter-level positioning accuracy cannot be achieved until the carrier-phase ambiguities have been fixed to their correct integer values. Over the past decade, many different ambiguity resolution techniques have been developed. A review and evaluation of existing ambiguity resolution on-the-fly (OTF) techniques can be found in [1]. The starting point for most ambiguity resolution techniques is the so-called float solution of real-valued ambiguity estimates and their associated statistics, which are used to construct a search window assumed to contain the correct integer ambiguities. The process of searching all possible integer ambiguity combinations within the search window is then performed using a search criterion based on the minimization of the quadratic form of the least-squares residuals. Therefore, reliable ambiguity recovery is highly dependent on the realistic float solution.

The real-valued ambiguity, together with other state parameters, which may include the position, velocity, and acceleration of a moving platform and other parameters of interest, can be estimated using the Kalman filtering technique [2–5] or sequential least squares [1, 6]. With these statistical parameter estimation approaches, the state parameters not only can be estimated recursively in real time, but also have statistically defined optimal properties. It is well known, however, that optimum parameter estimation requires the adopted stochastic models to be suitably correct.

In commonly used procedures for constructing the covariance matrices of the differenced GPS and GLONASS measurements, all the one-way code- or carrier-phase measurements are assumed to be independent and to have the same accuracy. In fact, these assumptions do not fit reality [7, 8]. First, the code- or carrier-phase observations are spatially correlated (the single- and double-difference methods are based explicitly on this fact), which has been shown in [9, 10]. Second, measurements obtained from different satellites cannot have the same variance because of varying noise levels. This is particularly true when the measurements come from different systems in combined GPS and GLONASS positioning. Any deficiency in the stochastic models for the differenced GPS and GLONASS measurements in data processing will inevitably result in unreliable statistics for ambiguity resolution and biased positioning results [11, 8]. In practice, however, stochastic modeling for code- and carrier-phase measurements is not trivial, in particular for real-time applications.

The purpose of this study is to develop a real-time stochastic modeling method for use in RTK GPS/GLONASS positioning. To this end, the commonly used Kalman filtering technique and the

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existing methods for measurement covariance matrix estimation are briefly reviewed. Based on the measurement filtering residuals, a method for estimating the measurement covariance matrix is proposed. Applications of the proposed method are demonstrated with examples from real GPS and combined GPS/GLONASS datasets.

PARAMETER ESTIMATION

As with static positioning, functional and stochastic models are required to estimate the unknown state parameters in kinematic positioning. The functional models include the measurement model:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\epsilon}_k \tag{1}$$

and the dynamic (state) model:

$$\mathbf{x}_k = \boldsymbol{\Phi}_{k,k-1} \mathbf{x}_{k-1} + \boldsymbol{\tau}_k \tag{2}$$

The corresponding stochastic models are further assumed as:

$$E(\boldsymbol{\epsilon}_k \boldsymbol{\epsilon}_i^T) = \begin{cases} \mathbf{R}_k & i = k \\ \mathbf{0} & i \neq k \end{cases}$$

$$E(\boldsymbol{\tau}_k \boldsymbol{\tau}_i^T) = \begin{cases} \mathbf{Q}_k & i = k \\ \mathbf{0} & i \neq k \end{cases}$$

$$\text{and} \quad E(\boldsymbol{\epsilon}_k \boldsymbol{\tau}_i^T) = \mathbf{0} \tag{3}$$

where $\mathbf{z}_k$ is an $n \times 1$ vector of measurements, and n is the number of measurements; $\mathbf{H}_k$ is an $n \times t$ design matrix; $\mathbf{x}_k$ is a $t \times 1$ state parameter vector, and t is the number of state parameters; $\boldsymbol{\epsilon}_k$ is an $n \times 1$ measurement noise vector; $\boldsymbol{\Phi}_{k,k-1}$ is a $t \times t$ state transition matrix; $\boldsymbol{\tau}_k$ is a $t \times 1$ random error vector; $\mathbf{R}_k$ is an $n \times n$ measurement noise covariance matrix; $\mathbf{Q}_k$ is a $t \times t$ process noise covariance matrix; and i, k are the time indices.

In kinematic GPS/GLONASS positioning, to shorten the time period of ambiguity recovery, all the available code- and carrier-phase measurements should be used. In the case of processing GPS data only, the double-differenced (DD) code- and carrier-phase measurements are commonly formed to simplify the measurement equations. In the case of processing GLONASS data only or combined GPS/GLONASS data, however, because of the multiple frequencies of the GLONASS signals, the standard DD procedure cannot cancel relative receiver clock error. In this situation, the ambiguity parameters cannot be separated from the receiver clock parameters [12]. To overcome this problem, an optimal option is to use the DD GPS and single-differenced (SD) GLONASS code measurements together with the GPS-GPS and GLONASS-GLONASS DD carrier-phase measurements [12]. Because the receiver clock errors are unstable, it is common to estimate them every epoch [13, 14]. In these situations, the state parameter vector contains the time-dependent parameters (i.e., three components of position and receiver clock bias) and time-independent carrier-phase ambiguity parameters if they have not been resolved.

Based on the mathematical models presented in equations (1)–(3), Kalman filtering techniques can be used to estimate the unknown state parameters. The basic Kalman filtering computation procedure can be described by the standard prediction, filtering, and smoothing equations [15, 16]. For real-time kinematic positioning applications, only the prediction and filtering equations are needed. The predicted state values $\bar{\mathbf{x}}_k$ and their covariance matrix are presented as:

$$\bar{\mathbf{x}}_k = \boldsymbol{\Phi}_{k,k-1} \hat{\mathbf{x}}_{k-1} \tag{4}$$

$$\mathbf{Q}_{\bar{\mathbf{x}}_k} = \boldsymbol{\Phi}_{k,k-1} \mathbf{Q}_{\hat{\mathbf{x}}_{k-1}} \boldsymbol{\Phi}_{k,k-1}^T + \mathbf{Q}_k \tag{5}$$

where $\hat{\mathbf{x}}_{k-1}$ is an optimal estimator of the state parameters at the previous epoch $(k-1)$. Real-time estimates of the state parameters are computed using the following filtering equations:

$$\hat{\mathbf{x}}_k = \bar{\mathbf{x}}_k + \mathbf{G}_k \mathbf{d}_k \tag{6}$$

$$\mathbf{Q}_{\hat{\mathbf{x}}_k} = \mathbf{Q}_{\bar{\mathbf{x}}_k} - \mathbf{G}_k \mathbf{Q}_{\mathbf{d}_k} \mathbf{G}_k^T \tag{7}$$

with

$$\mathbf{G}_k = \mathbf{Q}_{\bar{\mathbf{x}}_k} \mathbf{H}_k^T \mathbf{Q}_{\mathbf{d}_k}^{-1} \tag{8}$$

$$\mathbf{d}_k = \mathbf{z}_k - \mathbf{H}_k \bar{\mathbf{x}}_k \tag{9}$$

$$\mathbf{Q}_{\mathbf{d}_k} = \mathbf{R}_k + \mathbf{H}_k \mathbf{Q}_{\bar{\mathbf{x}}_k} \mathbf{H}_k^T \tag{10}$$

being the *gain matrix*, the *innovation vector*, and its covariance matrix, respectively.

It should be noted that in GPS/GLONASS kinematic positioning, the precise carrier-phase measurements with code pseudoranges can generate accurate positioning results. In the case of a good satellite constellation with dual-frequency GPS or single-frequency GPS/GLONASS receivers, integer carrier-phase ambiguities can be identified with single-epoch data, and the positioning accuracy then reaches the centimeter level. To avoid the effect of the errors in the process noise covariance matrix on the state estimation, the filter can be set up to operate only on the measurement noise. The basic concept regarding this filter operating mode has previously been discussed [17, 18]. It has been theoretically proven that the filter operating only on the measurement noise has good convergence performance [19]. In this case, the process noise covariance matrix must be properly constructed, i.e., assigning a very large variance (say, 10^6) to the time-dependent state parameters and zero variance to the time-independent carrier-phase ambiguity parameters. With these specific filter settings, the filtering results will not be affected by the dynamics of the moving platform and thus will be identical with sequential least-squares adjustment results. Similar filtering settings can be found in other applications [20, 21]. While the impact of the process noise covariance matrix is eliminated, the measurement covariance matrix should be estimated realistically in real time to obtain reliable parameter estimates.

EXISTING METHODS FOR COVARIANCE MATRIX ESTIMATION

In static positioning, modern statistical methods, such as Minimum Norm Quadratic Unbiased Estimation (MINQUE), have been applied successfully to estimate the covariance matrix for DD GPS measurements [8]. Although the MINQUE method has some very well-defined properties, it requires an iterative procedure, and the number of required iterations is application dependent. Thus, it is unsuitable for real-time kinematic positioning.

Based on the assumption that the accuracy of one-way measurements is dependent on signal-to-noise ratio [7, 22] or satellite elevation angle [20, 23–26], some approximate formulae have been proposed for calculating the standard deviations of one-way GPS measurements. The covariance matrix of the differenced measurements is then formed using the error propagation law, in which, however, the one-way measurements are still treated as statistically independent. On the other hand, constant coefficients in these formulae, estimated empirically using certain datasets under specific observing conditions, may not always fit other datasets very well. Therefore, a suitable method should be one using current data for real-time stochastic modeling.

In the field of automatic control, procedures for the on-line estimation of measurement (and process) noise matrices are termed *adaptive filtering techniques* [27, 28]. Adaptive filtering techniques are divided into four categories: *Bayesian*, *maximum likelihood* (ML), *correlation*, and *covariance-matching* methods. While the correlation method is applicable mainly to constant coefficient systems (i.e., the elements in the design matrix are time-invariant), the Bayesian and ML methods are computationally intensive and cannot realistically be used for real-time data processing [27, 28]. Compared with these methods, the covariance-matching method is computationally more efficient.

The basic concept of the covariance-matching method is to make the elements of the actual innovation covariance matrix consistent with their theoretical values. The actual covariance matrix of the innovation sequence $\mathbf{d}_k$ is calculated approximately by its sample covariance. By matching this estimated sample covariance matrix with its theoretical form as presented by equation (10), the measurement noise covariance matrix is estimated as [28]

innovation covariance

$$\hat{\mathbf{R}}_k = \hat{\mathbf{Q}}_{\mathbf{d}_k} - \mathbf{H}_k \mathbf{Q}_{\bar{\mathbf{x}}_k} \mathbf{H}_k^T = \frac{1}{m} \sum_{i=1}^{m} \mathbf{d}_{k-i} \mathbf{d}_{k-i}^T - \mathbf{H}_k \mathbf{Q}_{\bar{\mathbf{x}}_k} \mathbf{H}_k^T \tag{11}$$

where m is chosen empirically to provide some statistical smoothing. However, as equation (11) is the difference between two positive definite matrices, it cannot be guaranteed that the resulting matrix, $\hat{\mathbf{R}}_k$, is positive definite.

COVARIANCE MATRIX ESTIMATION METHOD BASED ON FILTERING RESIDUALS

To obtain a realistic estimator of the measurement noise covariance matrix, a more precise estimator of the measurement noise level than the innovation sequence should be used. To this end, Kalman filtering estimation formulae are derived in the following using the least-squares method.

By integrating the measurements $\mathbf{z}_k$ and the predicted values of state parameters $\bar{\mathbf{x}}_k$, the optimal estimators of the state parameters $\mathbf{x}_k$ can be obtained using the least-squares technique [2, 15, 16]. The Gauss-Markov models are

$$E(\mathbf{l}_k) = \mathbf{A}_k \mathbf{x}_k \text{ or } \mathbf{l}_k = \mathbf{A}_k \mathbf{x}_k + \mathbf{v}_k \tag{12}$$

$$\mathbf{C}_{\mathbf{l}_k} = \mathbf{P}^{-1} = \begin{bmatrix} \mathbf{R}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{\bar{\mathbf{x}}_k} \end{bmatrix} \tag{13}$$

where

$$\mathbf{l}_k = \begin{bmatrix} \mathbf{z}_k \\ \bar{\mathbf{x}}_k \end{bmatrix}, \mathbf{v}_k = \begin{bmatrix} \mathbf{v}_{\mathbf{z}_k} \\ \mathbf{v}_{\bar{\mathbf{x}}_k} \end{bmatrix}, \mathbf{A}_k = \begin{bmatrix} \mathbf{H}_k \\ \mathbf{E} \end{bmatrix}.$$

In equations (12) and (13), $\mathbf{v}_{\mathbf{z}_k}$ is an $n \times 1$ residual vector of measurements $\mathbf{z}_k$; $\mathbf{v}_{\bar{\mathbf{x}}_k}$ is a $t \times 1$ residual vector of the pseudomeasurements $\bar{\mathbf{x}}_k$; $\mathbf{E}$ is a $t \times t$ identity matrix; $\mathbf{C}_{\mathbf{l}_k}$ is an $(n + t) \times (n + t)$ covariance matrix of the measurement vector $\mathbf{l}_k$; and $\mathbf{P}$ is an $(n + 1) \times (n + t)$ weight matrix. With the least-squares principle, the optimal estimators of the state parameters and their covariance matrix are formulated as equations (6) and (7). The advantage of this derivation is that it generates the most desirable least-squares residuals, i.e., the measurement filtering residuals:

$$\mathbf{v}_{\mathbf{z}_k} = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k \tag{14}$$

which obviously is the optimal estimator of the measurement noise level because the estimated values $\hat{\mathbf{x}}_k$ (not the predicted values $\bar{\mathbf{x}}_k$) of the state parameters are used in the computations. To obtain the covariance matrix of the measurement filtering residuals, equation (14) is further derived as

identity.

$$\mathbf{v}_{\mathbf{z}_k} = \mathbf{z}_k - \mathbf{H}_k(\bar{\mathbf{x}}_k + \mathbf{G}_k \mathbf{d}_k) = (\mathbf{E} - \mathbf{H}_k \mathbf{G}_k)\mathbf{d}_k \tag{15}$$

Kalman gain

$z_k - H_k x_k^f$ $\quad$ $Ed_k\, Ed_k^T$ – H_k[illegible]

By applying the error propagation law to the above equation, one obtains

$$\mathbf{Q}_{\mathbf{v}_{\mathbf{z}_k}} = \mathbf{R}_k - \mathbf{H}_k \mathbf{Q}_{\hat{\mathbf{x}}_k} \mathbf{H}_k^T \tag{16}$$

It is noted that equations (10) and (16) are similar. Based on the same philosophy as that used in the covariance-matching method, if the covariance matrix $\mathbf{Q}_{\mathbf{v}_{\mathbf{z}_k}}$ is computed using the measurement filtering residuals from the previous m epochs, the covariance matrix $\mathbf{R}_k$ can be estimated as

$$\begin{aligned}\hat{\mathbf{R}}_k &= \hat{\mathbf{Q}}_{\mathbf{v}_{\mathbf{z}_k}} + \mathbf{H}_k \mathbf{Q}_{\hat{\mathbf{x}}_k} \mathbf{H}_k^T \\ &= \frac{1}{m} \sum_{i=0}^{m-1} \mathbf{v}_{\mathbf{z}_{k-i}} \mathbf{v}_{\mathbf{z}_{k-i}}^T + \mathbf{H}_k \mathbf{Q}_{\hat{\mathbf{x}}_k} \mathbf{H}_k^T \end{aligned} \tag{17}$$

which can be used in the computation of epoch k + 1. In equation (17), m is called *the width of moving windows*. Unlike equation (11), which is constructed with the predicted quantity, equation (17) is based on the estimated parameters. Because equation (17) is the sum of the two positive definite matrices, covariance matrix $\hat{\mathbf{R}}_k$ estimated with equation (17) is always positive definite. A slight disadvantage of equation (17) compared with equation (11) is that it requires some extra computations for both $\mathbf{v}_{\mathbf{z}_k}$ and $\mathbf{H}_k \mathbf{Q}_{\hat{\mathbf{x}}_k} \mathbf{H}_k^T$, which are not generated by the standard Kalman filtering process. Fortunately, the number of these additional computations is small and leads to no significant time delay in data processing. It is noted that equation (17) may also be derived using a maximum likelihood criterion (e.g., [29]).

TEST RESULTS AND ANALYSES

Both GPS dual-frequency and integrated GPS/GLONASS single-frequency datasets were used to test the performance of the above-proposed stochastic modeling method.

Test Description

The first dataset was collected in a marine kinematic GPS positioning test. The test was carried out on July 18, 1997, at the Fremantle port in Perth, Australia, using two Trimble 4000SSE dual-frequency receivers. The rover receiver antenna was mounted on a boat and was moving around the offshore test area (about 7.3 km away from the reference station). The trajectory of the roving antenna for this test is shown in Figure 1. During the 10 min of the test, seven satellites were tracked. The data collection rate was 1 Hz.

Another dataset was collected on a 1.2 km (static) baseline on February 16, 1998, in Perth, Australia, using two Ashtech GG24 GPS/GLONASS receivers. The data span was 26 min with a data interval of 1 s. During the whole session of observation, seven GPS and five GLONASS satellites were tracked.

The true carrier-phase ambiguities were first recovered for each dataset from the whole session. Then, ambiguity resolutions were performed OTF for each segment of 1 epoch. In data processing, the initial standard deviations for the one-way pseudorange and carrier-phase measurements were defined as 1.0 m and 0.05 cycles, respectively. The width of moving windows for estimating the measurement covariance matrix was set to 8 epochs, and the initial estimate was calculated using measurement filtering residuals from the first 8 epochs, with ambiguities being fixed to correct values. In this study, covariance between the different groups of measurements, such as carrier phase and code or L1 and L2 code, is assumed to be zero, which can speed up the filtering process.

Comparison of Various Modeling Methods

It has been found that the existing covariance matching method expressed by equation (11) does not generate a positive definite covariance matrix. For purposes of comparison, a stochastic model based on satellite elevation angles was also tested

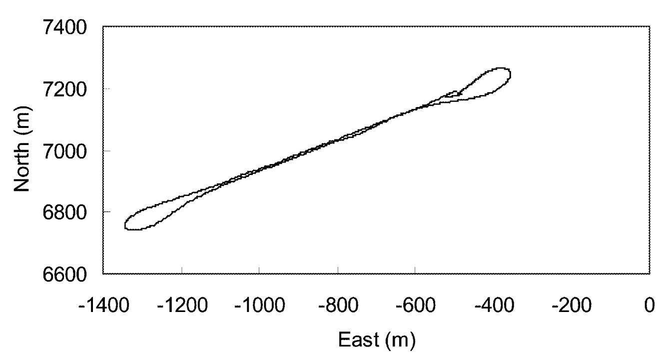

Fig. 1 – Roving Antenna Trajectory for Marine GPS Kinematic Positioning Test

for the GPS dataset (in the literature, an empirical formula is not available for Ashtech GG24 receivers). According to [20, 25], the standard deviation of the one-way code measurements can be approximated by an exponential function of satellite elevation h as:

$$\sigma(\mathrm{h}) = \mathrm{a}_0 + \mathrm{a}_1 \cdot \exp\left(-\frac{\mathrm{h}}{\mathrm{h}_0}\right) \tag{18}$$

where the constant terms a_0, a_1, and h_0, are derived empirically for Trimble 4000SSE receivers as 0.12 m, 1.1 m, 14 deg for L1 code measurements, and 0.14 m, 3.0 m, 11 deg for L2 code measurements, respectively [25]. The standard deviations for the L1 and L2 carrier-phase measurements are computed as 1/100 of the code standard deviations [30]. The covariance matrices for the DD measurements are constructed using the error propagation law. For the DD L2 code measurements of satellite pair space vehicles (SVs) 19-2, standard deviations that were preset, calculated using the satellite elevation and estimated using equation (17), are shown in Figure 2. It is easy to see that there are some differences among these three stochastic models. Actually, the accuracy of the measurements may be influenced by many factors. The stochastic model based on satellite elevation cannot properly reflect true errors in some practical situations [9, 31].

On the other hand, as expected, estimated measurement covariances may be negative (with commonly used stochastic models, these covariances are always positive). For instance, in the case of GPS/GLONASS single-frequency data, the estimated covariance matrix for SD GLONASS code measurements (at epoch 100) is

$$\begin{bmatrix} 1.805 & 0.033 & -0.0263 & 0.983 & -0.405 \\ & 1.368 & 0.736 & 0.435 & -0.233 \\ & & 6.408 & 0.855 & -1.230 \\ & & & 1.515 & -0.631 \\ & & & & 0.479 \end{bmatrix} (\mathrm{m}^2)$$

where the correlation coefficients range from -0.74 to 0.59. In the commonly used or elevation-based stochastic models, however, these correlation coefficients are always equal to zero. Therefore, it is not realistic to assume that all the one-way measurements are independent. It is further noted that, as shown in Figure 3, the measurements do have differing accuracy, and that some GLONASS measurements may be more accurate than GPS measurements. Thus, in combined GPS/GLONASS data processing, simply deweighting the GLONASS measurements will inevitably lead to unreliable results.

It is also noted that in Figures 2 and 3, the estimated standard deviations change quickly. This may be caused by using a short window (8 epochs), which provides a limited amount of information. If the window size is large, the estimated standard deviations change smoothly, but may not produce better ambiguity resolution results (see Table 1).

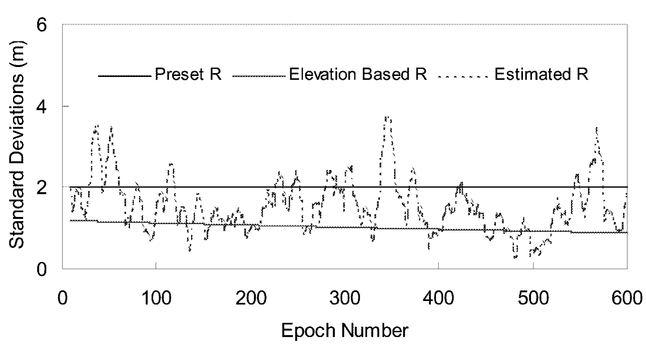

Fig. 2 – Standard Deviations of DD L2 Code Measurements for Satellite Pair SV 15-7 in the GPS Dual-Frequency Data

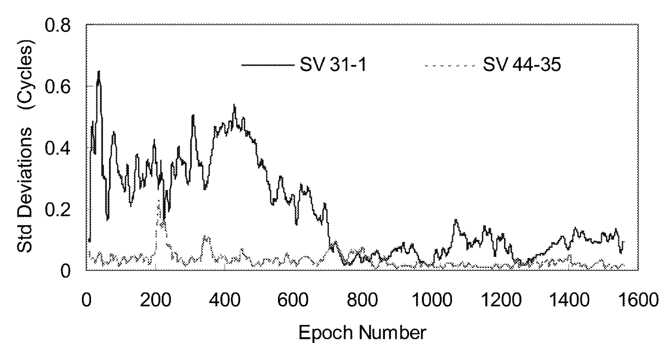

Fig. 3 – Estimated Standard Deviations of DD Carrier Phases for GPS/GLONASS Single-Frequency Data (SVs 35 and 44 are GLONASS satellites)

Table 1 — Impacts of Width of Moving Window on Ambiguity Resolution

Dataset	Width	6	7	8	9	10	30	120
GPS	Averaged F-ratio	3.0	16.2	14.3	11.9	11.1	6.0	4.7
	Averaged Ws	2.3	9.3	8.7	8.0	7.8	5.4	4.4
	Success Rate (%)	59.8	100.0	100.0	100.0	100.0	99.8	97.9
GPS/GLONASS	Averaged F-ratio	3.5	5.2	13.3	12.0	11.1	6.0	4.1
	Averaged Ws	2.4	3.5	6.6	6.3	6.2	4.3	3.3
	Success Rate (%)	62.7	96.9	100.0	100.0	100.0	99.5	94.2

Critical Statistics in Ambiguity Resolution

A realistic estimation of measurement covariance matrices provides reliable statistics for ambiguity resolution. To demonstrate this concept more clearly, both the solutions with the preset and estimated covariance matrix were generated. The *ambiguity dilution of precision* (ADOP), which is defined by [32], is illustrated in Figures 4 and 5, which indicate that with the estimated covariance matrix, the accuracy of the estimated ambiguities was greatly improved. As a consequence, the ambiguity search volume was significantly reduced, and a faster search process could be expected [4, 32]. It is also noted, however, that overall, the elevation-based stochastic model decreased the ADOP values only slightly.

The improved float ambiguity estimates are of great importance for ambiguity validation testing, which is a critical step in ambiguity resolution. The results regarding the commonly used ambiguity validation test statistic *F-ratio* [4] are shown in Figures 6 and 7. In all cases, the ambiguity validation test statistics with the estimated measurement covariance matrices are much larger than those with the preset measurement covariance matrices. If the critical value of the F-ratio is chosen to be 2.0, as is commonly accepted [4], the success rates of ambiguity resolution using the preset (default) covariance matrices are only 49 and 21 percent for the GPS and GPS/GLONASS datasets, respectively. The success rates of ambiguity resolution using the estimated covariance matrices reach 100 percent, however. On the other hand, in the case of the GPS dataset, if the elevation-based measurement covariance matrix is used, the success rate of ambiguity resolution is as low as 68 percent, which is slightly higher as compared with results using the preset measurement covariance matrix.

As shown in Figures 8 and 9, the above results were verified by using a newly developed ambiguity validation test statistic, *W-ratio* (Ws) [33]. The advantage of this new statistic is that the confidence

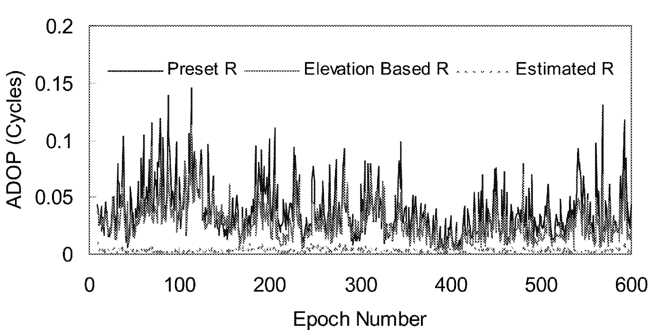

Fig. 4 – Ambiguity Dilution of Precision (ADOP) for GPS Dual-Frequency Data

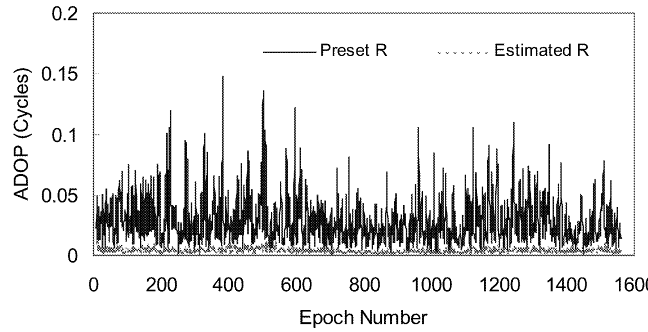

Fig. 5 – Ambiguity Dilution of Precision (ADOP) for GPS/GLONASS Single-Frequency Data

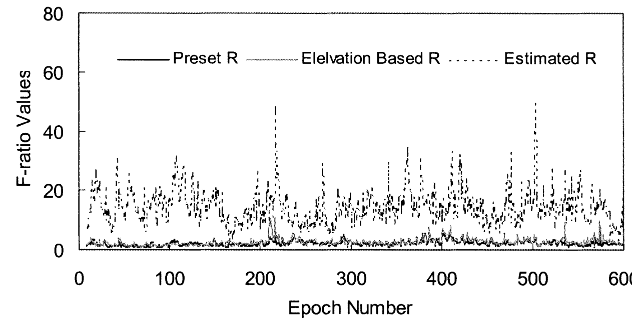

Fig. 6 – F-ratio Values for GPS Dual-Frequency Data

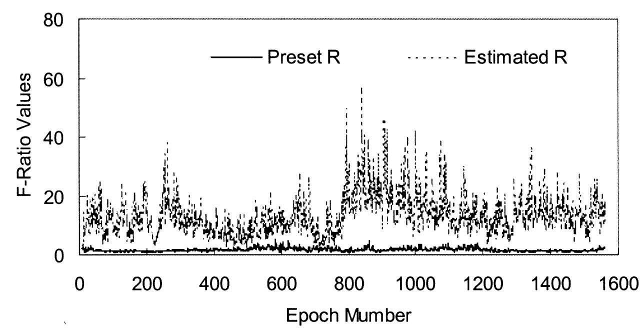

Fig. 7 – F-ratio Values for GPS/GLONASS Single-Frequency Data

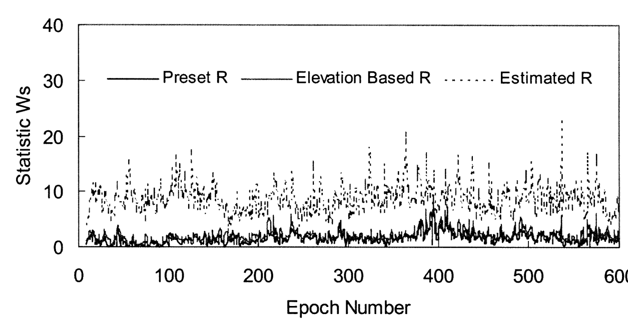

Fig. 8 – Ambiguity Discrimination Test Statistic Ws for GPS Dual-Frequency Data

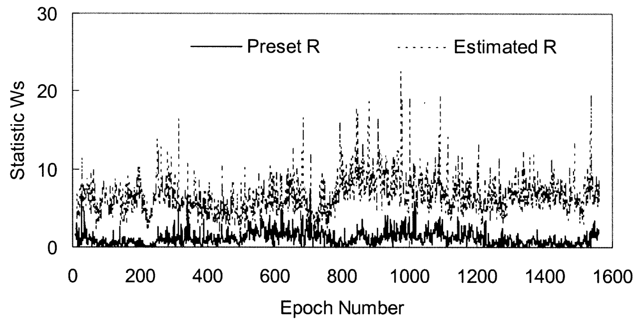

Fig. 9–Ambiguity Discrimination Test Statistic Ws for GPS/GLONASS Single-Frequency Data

level of the ambiguity resolution can be evaluated. For instance, in the case of the GPS dataset, the averaged confidence levels of the ambiguity resolution for the three different stochastic models are 0.87, 0.92, and 1.00, respectively.

Impact on Baseline Estimation

A realistic stochastic model can also improve the accuracy of positioning solutions. In the case of the GPS/GLONASS dataset, for example, at epoch 9, when the filtering process was using the preset measurement covariance matrix, the standard deviations of (x, y, z) coordinates were 0.008, 0.016, and 0.012 m, respectively. When the filtering process was using the estimated measurement covariance matrix, the standard deviations of the coordinates were down to 0.006, 0.013, and 0.009 m, respectively. The differences in the coordinates between the two solutions are around 0.005 m.

Optimal Width of the Moving Windows

It is important to note that, to estimate the measurement covariance matrix, both the preset standard deviations for the one-way measurements and the width of the moving windows must be specified. Some test results indicate that small changes in the preset standard deviations—for example, using 1.0 m instead of 2.0 m for the GLONASS pseudorange measurements—do not significantly influence the success rate of ambiguity resolution. The reason for this is that the influence of the preset standard deviations will be to some extent tuned out in the process of estimating the measurement covariance matrix. The optimal width of the moving window, however, needs to be identified. To this end, some further tests for various widths were conducted. The averaged ambiguity validation test statistics and success rates of ambiguity resolution are shown in Table 1, which reveals that for the tested GPS and GPS/GLONASS datasets, the optimal values for the width of the moving window are, respectively, 7 and 8. Testing on several other datasets revealed that the optimal width varies from 7 to 15, and depends slightly on the redundancy of positioning systems. In a practical implementation, the default width parameter can be set to, for example, 8 or 10; the optimal value for current positioning operation can then be determined adaptively in real-time data processing.

CONCLUSIONS

In GPS/GLONASS kinematic positioning, stochastic models play a critical role in reliable ambiguity resolution and precise positioning. The common practice of assuming that the one-way (code- or carrier-phase) measurements are statistically independent and have the same accuracy is certainly not realistic and thus leads inevitably to unsuitable measurement covariance matrix estimates. Stochastic modeling for code- and carrier-phase measurements, however, is not trivial for real-time applications. Because rigorous methods (such as MINQUE) cannot be used given their extensive computation requirements, more practical methods for RTK GPS/GLONASS positioning need to be developed and tested.

Although the satellite elevation-based stochastic model can easily be used in real-time data processing and provides good performance in some situations, it may not significantly improve the success rate of ambiguity resolution. An efficient method for measurement covariance matrix estimation, based on the postfit filtering residuals, has been proposed. With such a method, the measurement noise covariance matrix can be estimated in real time. By using an estimated covariance matrix, the reliability of ambiguity resolution and the accuracy of kinematic positioning can be significantly improved. Using the preset covariance matrix, the success rate of single-epoch ambiguity resolution is just 49 and 21 percent for the tested GPS dual-frequency and combined GPS/GLONASS single-frequency datasets, respectively. In contrast, the success rate of single-epoch ambiguity resolution using the estimated measurement covariance matrix can reach 100 percent.

Finally, it should be noted that stochastic modeling in a practical real-time application could be a difficult task. There are still some theoretical and implementation issues that need future investigation, such as modeling of the temporal correlations and determination of the optimal width of the moving window in real time.

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