

△ SGD

$$w_t \leftarrow w_t - \alpha \nabla f(w_t)$$

△ Adagrad

$$g_t = \nabla f(w_t)$$

$$G_t = \sum_{\tau=1}^t g_\tau g_\tau^T$$

$$\begin{bmatrix} w_{t+1}^1 \\ w_{t+1}^2 \\ \vdots \\ w_{t+1}^n \end{bmatrix} = \begin{bmatrix} w_t^1 \\ w_t^2 \\ \vdots \\ w_t^n \end{bmatrix} - \alpha \begin{bmatrix} \frac{1}{\sqrt{G_t^{11} + \epsilon}} & 0 & \dots & 0 \\ 0 & \frac{1}{\sqrt{G_t^{22} + \epsilon}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sqrt{G_t^{nn} + \epsilon}} \end{bmatrix} \begin{bmatrix} g_t^{(1)} \\ g_t^{(2)} \\ \vdots \\ g_t^{(n)} \end{bmatrix}$$

△ Adam

$$g_t \leftarrow \nabla f(w_t)$$

$$m_t \leftarrow \beta_1 (m_{t-1}) + (1-\beta_1) g_t$$

$$v_t \leftarrow \beta_2 (v_{t-1}) + (1-\beta_2) g_t g_t^T$$

$$\hat{m}_t \leftarrow \frac{m_t}{1-\beta_1}$$

$$\hat{v}_t \leftarrow \frac{v_t}{1-\beta_2}$$

$$w_{t+1} \leftarrow w_t - \alpha \frac{\hat{m}_t}{\sqrt{\hat{v}_t + \epsilon}} g_t$$

Information gain:

$$H(S) = - \sum_{i=1}^{\text{class}} p(S_i) \log(p(S_i))$$

$$H(S) = \left(\frac{15 \cdot 1}{151} H(S_1) + \frac{136}{151} H(S_2) \right)$$

$$\begin{cases} P[|E_n(h) - E(h)| > \epsilon] \leq 2 |A| e^{-\frac{1}{2} \epsilon^2 N} \\ P[|E_n(h) - E(h)| > \epsilon] \leq 4 m_N(2N) e^{-\frac{1}{2} \epsilon^2 N} \end{cases}$$

dichotomies:
split of set
where two subset
are exclusive,
whose union is original set!

Adagrad

$$g_t = \nabla f(w_t)$$

$$G_t = \sum_{\tau=1}^t g_\tau g_\tau^T$$

$$\begin{bmatrix} w_{t+1}^1 \\ w_{t+1}^2 \\ \vdots \\ w_{t+1}^n \end{bmatrix} = \begin{bmatrix} w_t^1 \\ w_t^2 \\ \vdots \\ w_t^n \end{bmatrix} - \alpha \begin{bmatrix} \frac{1}{\sqrt{G_t^{11} + \epsilon}} & 0 & \dots & 0 \\ 0 & \frac{1}{\sqrt{G_t^{22} + \epsilon}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sqrt{G_t^{nn} + \epsilon}} \end{bmatrix} \begin{bmatrix} g_t^{(1)} \\ g_t^{(2)} \\ \vdots \\ g_t^{(n)} \end{bmatrix}$$

Adam

$$\textcircled{1} g_t = \nabla f(w_t)$$

$$\textcircled{2} m_t = (1-\beta_1) m_{t-1} + \beta_1 g_t$$

$$v_t = (1-\beta_2) v_{t-1} + \beta_2 g_t g_t^T$$

$$\textcircled{3} \hat{m}_t = \frac{m_t}{1-\beta_1}$$

$$\hat{v}_t = \frac{v_t}{1-\beta_2}$$

$$\textcircled{4} w_{t+1} = w_t - \alpha \frac{\hat{m}_t}{\sqrt{\hat{v}_t + \epsilon}} g_t$$

	k					
$\mathcal{B}(N, k)$	1	2	3	4	5	6
N	1	1	2	2	2	2
	2	1	3	4	4	4
	3	1	4	7	8	8
	4	1	5	11	15	16
	5	1	6	16	26	31
	6	1	7	22	42	57

$$m_k(N) \leq \mathcal{B}(N, k) \leq \sum_{i=1}^k \binom{N}{i} \leq N^k$$