Tracking the states w/ UUV

1. Data Sync.
2. Bias initialization.
3. eskf model

IMU cb →

GPS cb →

ESKF →

$z = h(x)$

$x = \begin{matrix} P \\ V \\ R \end{matrix}$

preview = 80

ESKF model

$x_t = \begin{bmatrix} p_t \\ v_t \\ R_t \\ b_{at} \\ b_{gt} \\ g_t \end{bmatrix}$ $\quad \tilde{\omega} \ \tilde{a}$ IMU

$\dot{p}_t = v_t$

$\dot{v}_t = R_t(\tilde{a} - b_{at} - \eta_a) + g$

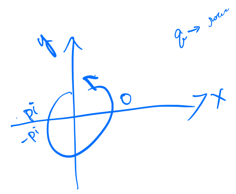
$\dot{R}_t = R_t(\tilde{\omega} - b_{gt} - \eta_g)^\wedge$

$\dot{b}_{gt} = \eta_{bg}$

$\dot{b}_{at} = \eta_{ba}$

$\dot{g} = 0$

Subscript w/ t := true state



$p_t = p + \delta p$

$v_t = v + \delta v$

$R_t = R\,\delta R$

$b_{gt} = b_g + \delta b_g$

$b_{at} = b_a + \delta b_a$

$g_t = g + \delta g$

nominal

$\dot{p}_t = v_t$

$\Rightarrow \dot{p} + \delta\dot{p} = v + \delta v$

$\therefore \delta\dot{p} = \delta v \quad — ①$

$\dot{b}_{gt} = \eta_{bg}$

$\Rightarrow \cancel{\dot{b}_g} + \delta\dot{b}_g = \eta_{bg} \quad — ②$

$\dot{b}_{at} = \eta_{ba}$

$\Rightarrow \cancel{\dot{b}_a} + \delta\dot{b}_a = \eta_{ba} \quad — ③$

$\dot{g}_t = 0$

$\Rightarrow \cancel{\dot{g}} + \delta\dot{g} = 0 \quad —$

*
$$\begin{cases} \dot{R}_t = R_t(\tilde{\omega} - b_{gt} - \eta_g)^\wedge & — ⓐ \\ R_t = R \circ \mathrm{Exp}(\delta R) & — ⓑ \end{cases}$$

from ⓑ

$$\dot{R}_t = \dot{R}\,\mathrm{Exp}(\delta\theta) + R\,\dot{\mathrm{Exp}}(\delta\theta)$$
$$= R(\tilde{\omega} - b_{gt})^\wedge \mathrm{Exp}(\delta\theta) + R\,\mathrm{Exp}(\delta\theta)\,\delta\dot{\theta}^\wedge$$

from ⓐ & ⓑ

$$R_t(\tilde{\omega} - b_{gt} - \eta_g)^\wedge$$
$$= R\,\mathrm{Exp}(\delta\theta)(\tilde{\omega} - b_{gt} - \eta_g)^\wedge$$

$\Rightarrow \begin{cases} R \\ \end{cases}$

	xy z	Q	R	rotor const.	remark
1330	20N	380 380	1 1	0.02	
1350	20N	380 380	10 10	0.03	
1400	20N	380 480	10 10	0.03	
1410	20N	380 480	15 15	0.03	
1430	20N	300 300	1 1	0.03	
1440	20N	300 480	1 1	0.03	
1450	40N	300 480	1 1	0.03	
1545	40N	300 480	1 1	0.03	(fixed V_B)
1550	40N	300 300	1 1	0.03	
1600	10N	300 300	10 10	0.03	
1600	40	300 480	1 1	0.06	
1700	40	300 480	1 1	0.06	(cancel C(v) in MPC)
1710	40	300/480	1/1	0.03	(cancel C(v) in MPC)

EKF Stability proof

- $E[w_k w_k^T] = Q_k$

  $E[v_k v_k^T] = R_k$

- $x_{k+1} = f(x_k) + w_k$

  $z_k = h(x_k) + v_k$

- @ predict

  $\check{x}_k = f(\hat{x}_{k-1})$

  $\check{P}_k = F_k \hat{P}_{k-1} F_k^T + Q_k$

  $F_k = \left[\frac{\partial f}{\partial x}\right]_{x=\hat{x}_{k-1}}$

  @ update

  $y_k = z_k - h(\check{x}_k)$

  $K_k = \check{P}_k H_k^T (H_k \check{P}_k H_k^T + R_k)^{-1}$

  $H_k = \left[\frac{\partial h}{\partial x}\right]_{x=\check{x}_k}$

  $\hat{x}_k = \check{x}_k + K_k y_k$

  $\hat{P}_k = (I - K_k H_k) \check{P}_k$

證明前假設

- F_k is non-singular $\forall k \geq 0$

- R_k, Q_k are bounded from below

  where

  $$\bar{r} I \preceq R_k, \quad \bar{q} I \preceq Q_k \quad \forall k \geq 0$$

  $\bar{r}, \bar{q} > 0$

- $\check{e}_k = x_k - \check{x}_k$

  $\hat{e}_k = x_k - \hat{x}_k$