given a pin-hole model

$P \rightarrow$ $P \in \mathbb{R}^3$

$P' = Rp + t$

$P_c = \begin{bmatrix} P'_x / P'_z \\ P'_y / P'_z \\ 1 \end{bmatrix}$

$\uparrow R, t$

$\xrightarrow{P_c}$

$u'_c = u_c(1 + k_1 r_c^2 + k_2 r_c^4)$

$v'_c = v_c(1 + k_1 r_c^2 + k_2 r_c^4)$

$\uparrow k_1, k_2$

$\xrightarrow{u'_c, v'_c}$

$u_s = f_x u'_c + c_x$

$v_s = f_y v'_c + c_y$

$\uparrow f_x, f_y, c_x, c_y$

$\rightarrow u_s, v_s$

$$z = h(x, y)$$

meas. pose landmark

∴ error:

$$e = z - h(T, p)$$

∴ @ lm p_j w/ pose T_i

overall error:

$$\frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \| e_{ij} \|^2$$

$$= \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \| z_{ij} - h(T_i, p_j) \|^2$$

→ we are adjusting the pose T & lm p at the same time

→ Bundle Adjustment

∴ in the final problem:

$$x = [T_1, \ldots, T_m, p_1, \ldots, p_n]^T$$

∴ when $x \leftarrow x + \Delta x$

$$\frac{1}{2} \| f(x + \Delta x) \|^2$$

$$\approx \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \| e_{ij} + F_{ij} \Delta \xi_i + E_{ij} \Delta p_j \|^2$$

Partial Derivative w.r.t. T_i Partial Derivative w.r.t. p_j

we then stack all variables altogether:

$$x_c = [\xi_1, \xi_2, \cdots, \xi_m]^T \in \mathbb{R}^{6m}$$

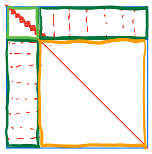
$$x_p = [p_1, p_2, \cdots, p_n]^T \in \mathbb{R}^{3n}$$

$\rightarrow \frac{1}{2} \| f(x + \Delta x) \|^2 = \frac{1}{2} \| e + F \Delta x_c + E \Delta x_p \|^2$

$\rightarrow H \Delta x = g$

- $H = J^T J$ or $J^T J + \lambda I$
- $J = [F \ E]$
- $J^T J = \begin{bmatrix} F^T F & F^T E \\ E^T F & E^T E \end{bmatrix}$

Schur Trick

H = 

$$H = \begin{bmatrix} B & E \\ E^T & C \end{bmatrix}$$

$$\Rightarrow H \Delta x = g \iff \begin{bmatrix} B & E \\ E^T & C \end{bmatrix} \begin{bmatrix} \Delta x_c \\ \Delta x_p \end{bmatrix} = \begin{bmatrix} v \\ w \end{bmatrix}$$