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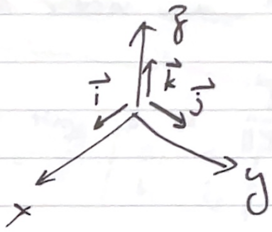
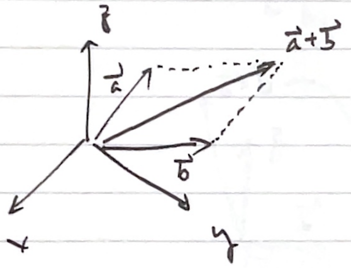
ME564 L21

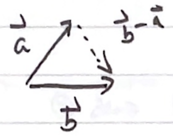
Overview of vectors in 3D

- inner product
- norm
- cross product

Consider a 3-D space







before $\underline{x}$ vector

now $\vec{x}$ vector.

$\vec{i}, \vec{j}, \vec{k}$ unit vectors (unit length)

form an orthonormal basis

(Euclidean spaces)

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

$\vec{i}\ \vec{j}\ \vec{k}$ orthonormal basis

$$\vec{a}+\vec{b} = (a_1+b_1)\vec{i} + (a_2+b_2)\vec{j} + (a_3+b_3)\vec{k}$$

orthonormal (normalized)

orthogonal (non-normalized)

Can write vectors as 3x1 matrices:

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \vec{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Inner Product

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ (scalar)

$\langle \vec{a}, \vec{b} \rangle$ (Dirac "bra" "ket")

$$\vec{a}^T\vec{b} = \begin{bmatrix} a_1 & \cancel{a_2} & a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

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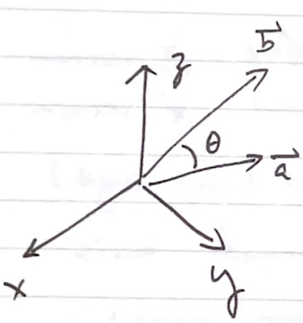
length of $\vec{a} = \sqrt{a_1^2+a_2^2+a_3^2}$ (i, j, k are orthogonal)

norm : $\|\vec{a}\| = \sqrt{\vec{a}\cdot\vec{a}}$ $\quad\quad \vec{a}^T a = [a_1 \; a_2 \; a_3]\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$

$= (\vec{a}\cdot\vec{a})^{1/2}$

can normalized a vector $\vec{a}$

by $\frac{\vec{a}}{\|\vec{a}\|}$ is a unit vector

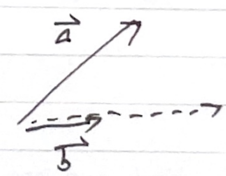


$\vec{a}\cdot\vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta$

if parallel $\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|$

if perpendicular $\vec{a}\cdot\vec{b} = 0$

project $\vec{a}$ into $\vec{b}$ direction



length of is $\vec{a}\cdot\vec{b}$ (assume $\|\vec{b}\| = 1$)

$(\vec{a}\cdot\vec{b})\vec{b}$ $\quad$ $\vec{a}$ proj. onto $\vec{b}$

if $\|b\| \neq 1$ then $\frac{(\vec{a}\cdot\vec{b})\vec{b}}{\|\vec{b}\|^2}$

Cauchy-Schwarz Inequality

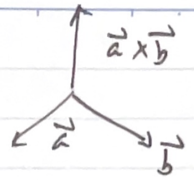
for all $\vec{a}, \vec{b}$

$|\vec{a}\cdot\vec{b}| \leq \|\vec{a}\| \, \|\vec{b}\|$

Triangle Inequality

$\|\vec{a}+\vec{b}\| \leq \|a\| + \|b\|$

cross - product

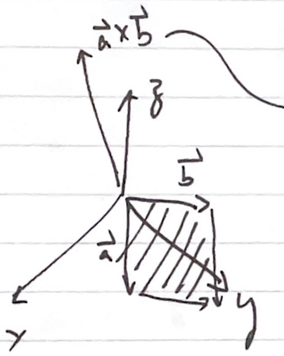


$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$

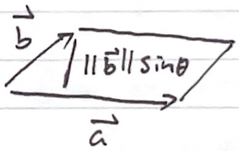
$\vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$

$\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \overbrace{\vec{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \vec{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \vec{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}^{\text{a vector}}$$



norm = area of $\vec{a}$ $\vec{b}$ ▱



$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta$$

$$\begin{bmatrix} \vec{a} \times \vec{b} \\ \\ \\ \end{bmatrix}\begin{array}{l} i \\ j \\ k \end{array} = \underbrace{\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}}_{\vec{a}_\times} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

skew - symmetric

$\nabla \times$ (curl)

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times$$