

1. Taylor series & $\dot{x} = \lambda x$
2. What is Taylor series

3. 2nd order systems

$$\ddot{x} + x = 0$$

- harmonic oscillator, spring-mass
- try Taylor series

$$\dot{x} = ax \Rightarrow x(t) = e^{at} x(0)$$

use Taylor Series: \rightarrow to derive

higher order terms

$$x(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + \dots + \text{h.o.t.}$$

$$\mathcal{O}(t^4) \quad \text{w/ } x(0) = x_0$$

$$\dot{x}(t) = 0 + C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 + \dots$$

$$\therefore C_0 = x_0$$

$$a x = a C_0 + a C_1 t + a C_2 t^2 + a C_3 t^3 + \dots$$

$$C_0 = x_0$$

$$t^0 \Rightarrow C_1 = a C_0 \quad C_1 = a x_0$$

$$t^1 \Rightarrow 2C_2 = a C_1 \quad C_2 = \frac{a^2}{2} x_0$$

$$t^2 \Rightarrow 3C_3 = a C_2 \quad C_3 = \frac{a^3}{3!} x_0$$

$$t^3 \Rightarrow 4C_4 = a C_3 \quad C_4 = \frac{a^4}{4!} x_0$$

$$\vdots$$

$$C_N = \frac{a^N}{N!} x_0$$

$$x(t) = x_0 + x_0 a t + \frac{a^2}{2} t^2 x_0 + \frac{a^3}{3!} t^3 x_0 + \dots$$

$$= \left(1 + at + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \dots \right) x_0$$

$$e^{at}$$

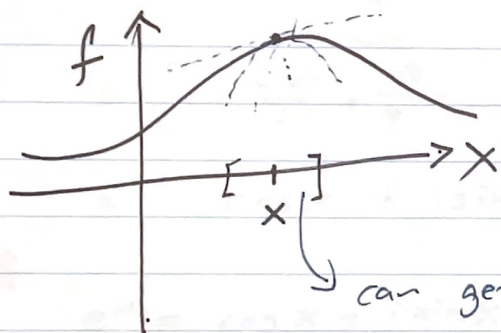
$$\dot{x} = Ax \quad x(t) = e^{At} x_0$$

Taylor Series

smooth function could all be expanded as T.S.

: A function $f(x+\Delta x)$ can be Taylor expanded at

$$f(x+\Delta x) = f(x) + \frac{df}{dx}(x) \cdot \Delta x + \frac{1}{2!} \frac{d^2f}{dx^2}(x) \cdot \Delta x^2 + \frac{1}{3!} \frac{d^3f}{dx^3}(x) \cdot \Delta x^3 + \dots + \frac{1}{n!} \frac{d^n f}{dx^n}(x) \Delta x^n + \dots$$



can get good approximation on the neighborhood.

Equivalent

other representation

 $f(x)$ expanded about a base pt. "a"

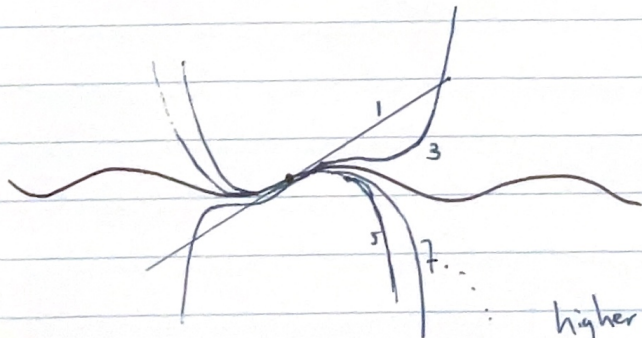
$$f(x) = f(a) + \frac{df}{dx}(a) \underbrace{(x-a)}_{\Delta x} + \frac{d^2f}{dx^2}(a) \frac{(x-a)^2}{2!} + \dots$$

e.g. $f(x) = \sin(x)$ about $a=0$ (maclaren Series)

$$= \sin(0) + \cos(0) \cdot x + \frac{-\sin(0)}{2!} x^2 - \frac{\cos(0)}{3!} x^3 + \frac{\sin(0)}{4!} x^4 + \frac{\cos(0)}{5!} x^5 + \dots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$f(x) = \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$



higher order, higher precision in approximation

generalizing complex number w/ Taylor Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \dots$$

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \dots$$

$$= 1 + ix + \frac{-x^2}{2!} + \frac{-ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} + \dots$$

$$= \left(1 + \frac{-x^2}{2!} + \frac{x^4}{4!} + \dots \right) + i \left(ix - \frac{ix^3}{3!} + \frac{ix^5}{5!} + \dots \right)$$

$$= \cos x + i \sin x$$

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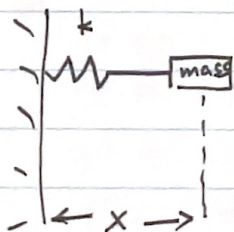
$$\therefore e^{ix} = \cos x + i \sin x$$

Euler's Formula

Second Order ODEs:

$$\ddot{x} + x = 0$$

$$\ddot{x} + \dot{x} + x = 0$$



$$F = ma$$

$$F = -kx$$

$$\Delta -kx = m\ddot{x}$$

$$\ddot{x} = \frac{-k}{m}x$$

method 1:

guess a solution

method 2:

also guess

$$x(t) = e^{\lambda t} \rightarrow \text{plug this into equation}$$



generalize