Summary of Extended Kalman filter

given a dynamic & measurement model :

$$x_{k+1} = f(x_k, u_k, w_k)$$
$$y_k = h(x_k, v_k)$$

△ Jacobian for a pinhole model during bundle adjustment

△ pinhole model :

$$\underbrace{s_i}_{\text{depth}} u_i = \underbrace{K}_{\text{Intrinsic}} \underbrace{T}_{SE(3)} \underbrace{P_i}_{\text{3D points}}$$

△ recall objective :

$$u_i - \frac{1}{s_i} K T P_i$$

intermediate variable

△ let $P' \triangleq$ PCD in camera frame

$$P' = (TP)_{1:3} = [X', Y', Z']^T$$

$$\therefore \ su = KTP$$
$$\Rightarrow sv = KP'$$
$$\Rightarrow \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$
$$\therefore \ u = f_x \frac{X'}{Z'} + c_x$$
$$v = f_y \frac{Y'}{Z'} + c_y$$

△ $e = u - \frac{1}{s} KTP$

$$\frac{\partial e}{\partial \delta\xi} = \lim_{\delta\xi \to 0} \frac{e(\delta\xi \oplus \xi) - e(\xi)}{\delta\xi} = \underbrace{\frac{\partial e}{\partial P'}}_{1} \underbrace{\frac{\partial P'}{\partial \delta\xi}}_{2}$$

△ $e = \begin{bmatrix} u - f_x \frac{X'}{Z'} + c_x \\ v - f_y \frac{Y'}{Z'} + c_y \end{bmatrix}$

recall Jacobian: $\frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_n} \end{bmatrix}$

1

$$\therefore \frac{\partial e}{\partial P'} = \begin{bmatrix} \frac{\partial e_1}{\partial X'} & \frac{\partial e_1}{\partial Y'} & \frac{\partial e_1}{\partial Z'} \\ \frac{\partial e_2}{\partial X'} & \frac{\partial e_2}{\partial Y'} & \frac{\partial e_2}{\partial Z'} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{f_x}{Z'} & 0 & \frac{f_x X'}{Z'^2} \\ 0 & -\frac{f_y}{Z'} & \frac{f_y Y'}{Z'^2} \end{bmatrix} \quad — \ 1$$

let $' := [P'; 1]$

2

$$\frac{\partial P'}{\partial \delta\xi} = \frac{\partial TP}{\partial \delta\xi}$$

Lie Groups

$$J = \frac{Df(\mathcal{X})}{D\mathcal{X}} = \lim_{\tau \to 0} \frac{f(\mathcal{X} \oplus \tau) \ominus f(\mathcal{X})}{\tau} \in \mathbb{R}^{n \times m}$$
$$= \lim_{\tau \to 0} \frac{\mathrm{Log}[f(\mathcal{X})^{-1} f(\mathcal{X} \mathrm{Exp}(\tau))]}{\tau}$$

e.g. remark: cannot differentiate w.r.t. R, as R is a matrix. hence, use Lie algebra on the local tangent plane

$$= \lim_{\delta\xi \to 0} \frac{\exp(\delta\xi^\wedge)\exp(\xi^\wedge) P - \exp(\xi^\wedge) P}{\delta\xi}$$

$$\lim_{h \to 0} \frac{e^h e^x - e^x}{h} = \lim_{h \to 0} \frac{(1+h)e^x - e^x}{h} = \lim_{h \to 0} \frac{he^x}{h} = e^x$$

$$= \lim_{\delta\xi \to 0} \frac{(I + \delta\xi^\wedge)\exp(\xi^\wedge) P - \exp(\xi^\wedge) P}{\delta\xi}$$

$$= \lim_{\delta\xi \to 0} \frac{\delta\xi^\wedge \exp(\xi^\wedge) P}{\delta\xi}$$

$\exp(\xi^\wedge) P = Tp = Rp + t$

$$= \lim_{\delta\xi \to 0} \frac{\begin{bmatrix} \delta\omega^\wedge & \delta\rho \\ 0^T & 0 \end{bmatrix} \begin{bmatrix} Rp + t \\ 1 \end{bmatrix}}{\delta\xi}$$

recall Jacobian: $\frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_n} \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_n} \end{bmatrix}$

$$= \lim_{\delta\xi \to 0} \frac{\begin{bmatrix} \delta\omega^\wedge (Rp + t) + \delta\rho \\ 0^T \end{bmatrix}}{\delta\xi}, \quad \delta\xi \to \begin{bmatrix} \delta\rho \\ \delta\omega \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial \delta\rho}(\delta\omega^\wedge(Rp+t) + \delta\rho) & \frac{\partial}{\partial \delta\omega}(\delta\omega^\wedge(Rp+t) + \delta\rho) \\ \frac{\partial}{\partial \delta\rho} 0 & \frac{\partial}{\partial \delta\omega} 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & -(Rp+t)^\wedge \\ 0 & 0 \end{bmatrix}, \quad \omega^\wedge = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & -P'^\wedge \\ 0 & 0 \end{bmatrix}$$

- Here $P' = [x', y', z']$

$$\therefore \frac{\partial P'}{\partial \delta\xi} = [I \ \ -P'^\wedge]$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & Z' & -Y' \\ 0 & 1 & 0 & -Z' & 0 & X' \\ 0 & 0 & 1 & Y' & -X' & 0 \end{bmatrix}$$

- as $\frac{\partial e}{\partial P'} = \begin{bmatrix} -\frac{f_x}{Z'} & 0 & \frac{f_x X'}{Z'^2} \\ 0 & -\frac{f_y}{Z'} & \frac{f_y Y'}{Z'^2} \end{bmatrix}$

& $\frac{\partial P'}{\partial \delta\xi} = \begin{bmatrix} 1 & 0 & 0 & 0 & Z' & -Y' \\ 0 & 1 & 0 & -Z' & 0 & X' \\ 0 & 0 & 1 & Y' & -X' & 0 \end{bmatrix}$

$$\therefore \frac{\partial e}{\partial \delta\xi} = \begin{bmatrix} -\frac{f_x}{Z'} & 0 & \frac{f_x X'}{Z'^2} \\ 0 & -\frac{f_y}{Z'} & \frac{f_y Y'}{Z'^2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & Z' & -Y' \\ 0 & 1 & 0 & -Z' & 0 & X' \\ 0 & 0 & 1 & Y' & -X' & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-f_x}{Z'} & 0 & \frac{f_x X'}{Z'^2} & \frac{f_x X' Y'}{Z'^2} & -f_x - \frac{f_x X'^2}{Z'^2} & \frac{f_x Y'}{Z'} \\ 0 & \frac{-f_y}{Z'} & \frac{f_y Y'}{Z'^2} & f_y + \frac{f_y Y'^2}{Z'^2} & \frac{-f_y X' Y'}{Z'^2} & -\frac{f_y X'}{Z'} \end{bmatrix}$$