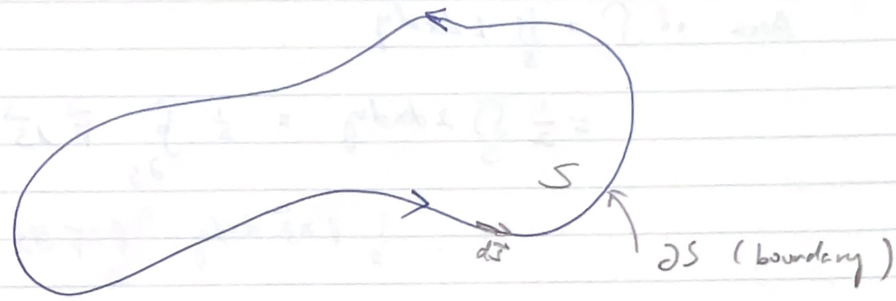
ME564
L25

Stoke's Theorem

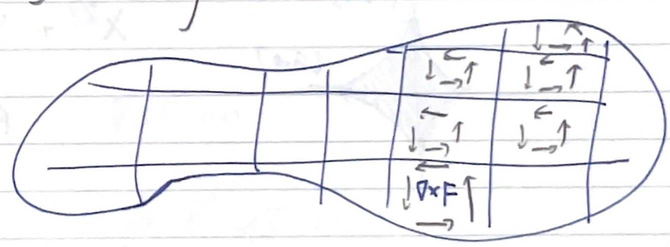


$$\oint_{\partial S} \vec{F} \cdot d\vec{s} = \iint_{S} \nabla \times \vec{F} \cdot ds$$

→ curl from interior boxes cancelled out

→ left with tangent surfaces vectors

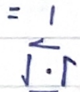
Stokes theorem



in 2D

$\nabla \cdot \vec{F} = 1$

$\nabla \times \vec{F} = 1$



Green's Theorem

$$\vec{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

vector function

F_1: scalar function

F_2: scalar function.

$$\int_{\partial S} \vec{F} \cdot ds$$

$$= \int_{\partial S} F_1 dx + F_2 dy = \iint_{S} \nabla \times \vec{F} \cdot ds = \iint \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\, dy$$

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1st Use : compute area

let $\vec{F} = \begin{bmatrix} -y \\ x \end{bmatrix}$ $\therefore$ $\nabla \times \vec{F} = 2$

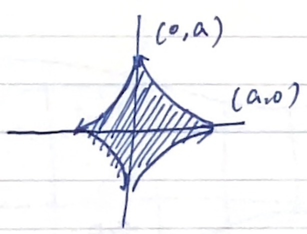
Area of $S = \iint_S 1\, dx\, dy$

$= \frac{1}{2} \iint_S 2\, dx\, dy = \frac{1}{2} \oint_{\partial S} \vec{F}\, d\vec{s} = \frac{1}{2} \oint_{\partial S} -y dx + x dy$

$\iint_S \nabla \times \vec{F}\, dx dy = \oint_{\partial S} \vec{F}\, ds$

$\therefore$ Area of $S = \frac{1}{2} \oint_{\partial S} x dy - y dx$

e.g. Hypocycloid. areas



$x^{2/3} + y^{2/3} = a^{2/3}$

parametrize :

$x = a \cos^3(\theta)$

$y = a \sin^3(\theta)$

$\theta = 0 \to 2\pi$

$dx = -3a\cos^2(\theta)\sin(\theta)d\theta$

$dy = 3a\sin^2(\theta)\cos(\theta)d\theta)$

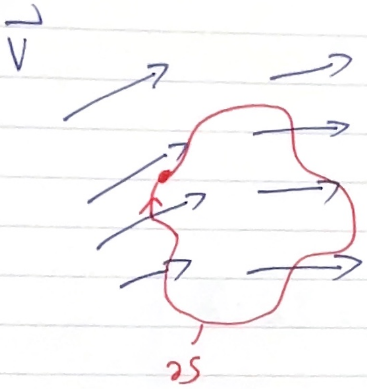
$A = \frac{1}{2} \int_{\partial A} x\, dy - y\, dx$

$= \frac{1}{2} \int_0^{2\pi} \left[3a^2 \sin^2 \cos^4 + 3a^2 \sin^4 \cos^2 \right] d\theta$

$= \frac{3}{8} \pi a^2$

2nd use : Physics w/ rotation. e.g. - Kelvin's circulation theorem
- lift on helicopter

3rd use , Physics w/o rotation



no curl

$$\oint_{\partial S} \vec{F} \cdot ds = \iint_{\partial} \nabla \times \vec{F} \, dx\,dy = 0$$

if $\nabla \times \vec{F} = 0$

then $\oint_{\partial S} \vec{F} \cdot d\vec{s} = 0$

for all closed curves "∂S"

then $\vec{F}$ is called a (conservative vector field!!!)

e.g. gravitational force field

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$$\nabla \times (\nabla \varphi) = 0$$

$\vec{F}$

$$\vec{F} = \begin{bmatrix} \partial\varphi/\partial x \\ \partial\varphi/\partial y \\ \partial\varphi/\partial z \end{bmatrix} = \frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}$$

dx dy dz

if $\vec{F} = \nabla\varphi$, then $\vec{F}$ is conservative

(if vector field is a gradient of some scalar function)

$$\oint_C \vec{F} \cdot d\vec{s} = 0$$