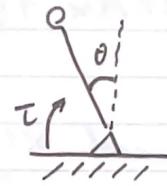


ME564 LB

General Systems  $\dot{x} = Ax + Bu$ 

$$\dot{\theta} = -\sin(\theta) + \tau$$



linear feedback control system

$$\dot{\theta} = \omega$$

$$\dot{\omega} = -\sin \theta + \tau$$

linearize @  $\theta = \pi$ 

$$\frac{d}{dt} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tau$$

$$\dot{x} = Ax + Bu$$

Feedback:

$$\tau = -2\theta - 2\omega$$

$\tau$  linearized @  $\theta = \pi$

$$\frac{d}{dt} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tau$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} \quad \lambda = -1, -1 \rightarrow \text{stable}$$

General Systems  $\dot{x} = Ax + Bu$  solution  $x(t) = ?$

Case I:

$$u(t) = 0, \text{ and } x(0) = x_0$$

$$x(t) = e^{At} x_0 \quad (\text{homogeneous, initial condition response})$$

Case 2:

$$x(0) = 0, \quad u(t) = \delta(t), \quad B = x_0$$

↳ Dirac delta function

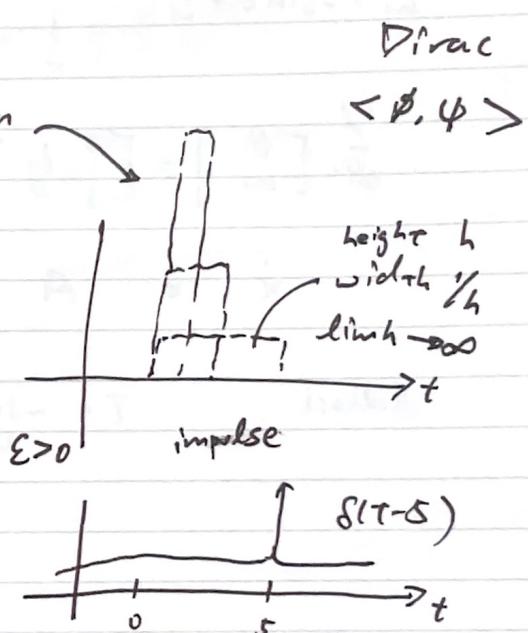
$$x(0) = 0, \quad u(t) = \delta(t-5)$$

↑ impulse function  
@  $t=5$

$$B = x_0$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

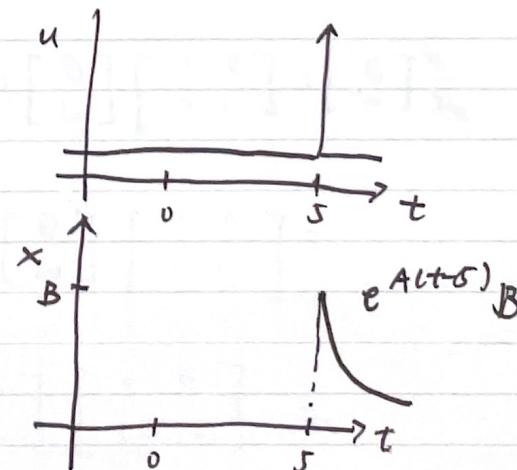
$$\int_{-t}^{\epsilon} \delta(t) dt = 1 \quad \text{for any } \epsilon > 0$$



$$x(t) = x(0) + \int_0^t [Ax(\tau) + Bu(\tau)] d\tau$$

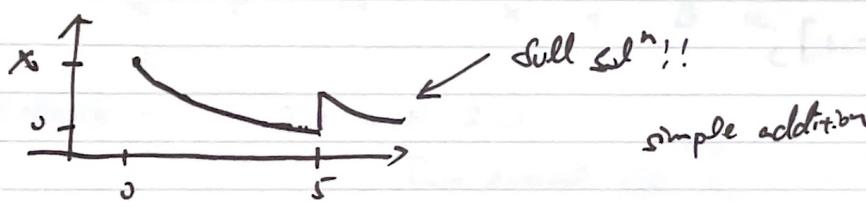
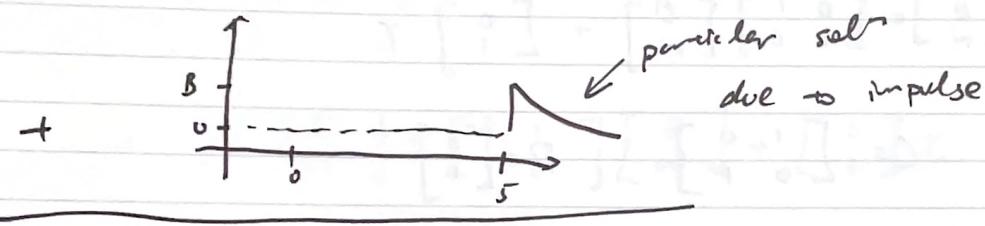
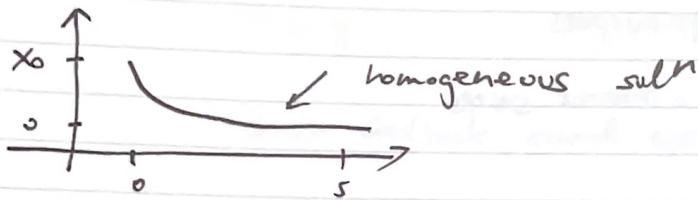
$$x(5^+) = B$$

$$\Rightarrow x(t) = e^{A(t-5)} B$$



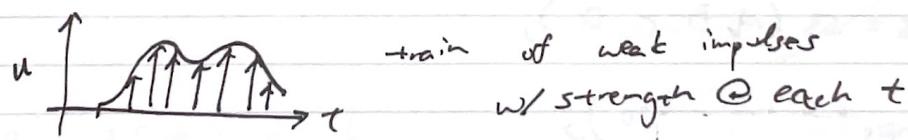
Case 3:

$$u(t) = \delta(t-5), \quad B, \quad x(0) = x_0$$

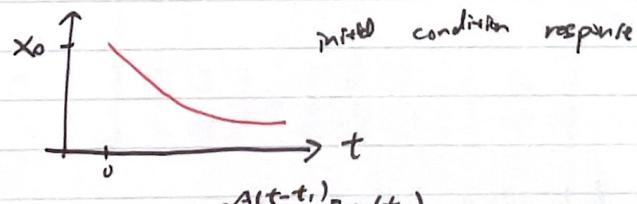


$$\text{General systems } \dot{x} = Ax + Bu \quad \text{soln } x(t) = ?$$

Convolution



$$Bu(\tau) \delta(t-\tau)$$



$$+ \uparrow Bu(t_1) \delta(t-t_1) \quad e^{A(t-t_1)} Bu(t_1)$$

$$x(t) = e^{At} x_0 + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau$$

$$+ \uparrow Bu(t_2) \delta(t-t_2) \quad e^{A(t-t_2)} Bu(t_2)$$

↓ convolution integral

$$e^{At} * Bu(t)$$

$$\text{e.g. } \dot{x} = -2x + u$$

$$x_0 = 10$$

$$u = 5 \sin(t)$$

$$\therefore x(t) = 10e^{-2t} + \int_0^t e^{-2(t-\tau)} 5 \sin(\tau) d\tau$$

In Matlab

$$\dot{x} = Ax + Bu \quad u = \text{inputs}$$

$$y = Cx + Du \quad y = \text{outputs}$$

 $x$  = internal states

$$\ddot{\theta} = -\sin \theta$$

$$\frac{d}{dt} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tau$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

```

>> A = [0 1; -1 -0.1];
>> B = [0; 1];
>> C = eye(2);
>> D = [0; 0];
>> sys = ss(A, B, C, D);
>> impulse(sys, 100);
>> t = 0:0.01:50;
>> u = 0*t;
>> u(1001:2000) = (1:1000)/10000;
>> u(2001:3000) = (1000-(1:1000))/10000;
>> plot(t, u)
>> lsim(sys, u, t)

```