

ME564 L20

Local  $\mathcal{O}(\Delta t^5)$   
global  $\mathcal{O}(\Delta t^4)$   $\rightarrow$  proportional to  $\Delta t$

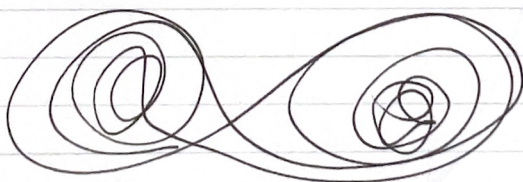
$$\frac{1}{\Delta t} \times \mathcal{O}(\Delta t^5) \rightarrow \mathcal{O}(\Delta t^4)$$

Not true to Chaotic System!

Integrating Chaotic system, just a little local integration error could lead to total Chaos (exponential growth)

$\xi e^{\lambda t} \rightarrow$  Lyapunov exponent

but still saturated at some points like Lorenz tucker



$\xi e^{\lambda t}$

$10^{-16} e^t$

$\uparrow$   
machine precision

assume  $\lambda = 1$

we can get

when  $t = ?$

will we stop to tolerate our error



How to evaluate precision?

→ out is smaller

Symplectic  
variational

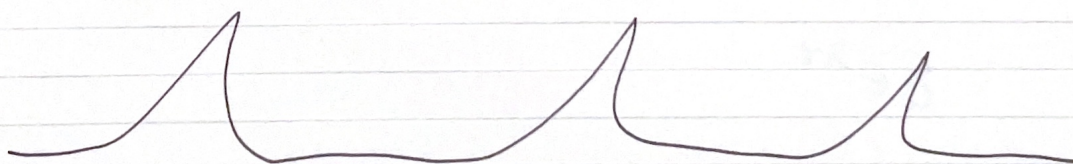
Integrators

Hamiltonian  
Lagrangian

~~$H = T + V \Rightarrow$~~

$$H(q, p) = T + V \Rightarrow \begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} \end{aligned}$$

maintain some conservation properties



$$\frac{d}{dt} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega_1 \\ \omega_2 \end{bmatrix} = f \left( \begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega_1 \\ \omega_2 \end{bmatrix} \right)$$

double pendulum & three-body problem