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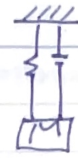
ME564 L12

$\dot{x} = Ax + f(t)$

$\dot{x} = f(x) + g(t)$



Differential Eqns w/ forcing

(*) $\ddot{x} + 3\dot{x} + 2x = 0$ homogeneous

(**) $\ddot{x} + 3\dot{x} + 2x = \underbrace{e^{-3t}}_{\text{forcing term}}$ inhomogeneous

Part 1: solve (*) to find homogeneous solution

$x = e^{\lambda t}$

$\dot{x} = \lambda e^{\lambda t}$

$\ddot{x} = \lambda^2 e^{\lambda t}$

$[\lambda^2 + 3\lambda + 2] e^{\lambda t} = 0$

$(\lambda+2)(\lambda+1) = 0$

$\lambda = -1, -2$

$$\boxed{x(t) = k_1 e^{-t} + k_2 e^{-2t}}$$

Part 2: Solve ** to find particular solution

$x_p(t) = k e^{-3t}$ (guessing will not work in general)

as $e^{\lambda t}$, $\cos t$, $\sin t$ are solutions of differential equations, we will be able to let $x_p(t) = k e^{-3t}$

$x_p(t) = k e^{-3t}$

$\dot{x}_p(t) = -3k e^{-3t}$

$\ddot{x}_p(t) = 9k e^{-3t}$

$(9k - 9k + 2k) e^{-3t} = e^{-3t}$

$2k e^{-3t} = e^{-3t}$

$k = \frac{1}{2}$

$$\boxed{x_p = \frac{1}{2} e^{-3k}}$$

method of undetermined coefficients

Part 3:

$$\boxed{x(t) = \underbrace{k_1 e^{-t} + k_2 e^{-2t}}_{\text{initial condition}} + \underbrace{\frac{1}{2} e^{-3t}}_{\text{forcing.}}}$$

ODE is linear (superposition

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method of variation parameters.

$$\ddot{x}+3\dot{x}+2x=0 \rightarrow \ddot{x}+3\dot{x}+2x=e^{-3t}$$

$$x(t)=k_1e^{-t}+k_2e^{-2t}$$

$$\Downarrow$$

$$x(t)=u_1(t)e^{-t}+u_2(t)e^{-2t} \qquad \text{(assumed)}$$

$$\dot{x}(t)=-u_1(t)e^{-t}-2u_2(t)e^{-2t}+\underbrace{\dot{u}_1(t)e^{-t}+\dot{u}_2(t)e^{-2t}}_{\hookrightarrow \text{ let them } =0 \text{ constraint 1.}}$$

$$\ddot{x}(t)=-\dot{u}_1e^{-t}+u_1e^{-t}-2\dot{u}_2e^{-2t}+4u_2e^{-2t}$$

$$\ddot{x}+3\dot{x}+2x$$

$$\Rightarrow \underbrace{[u_1-3u_1+2u_1]}_{=0}e^{-t}+\underbrace{[4u_2-6u_2+2u_2]}_{=0}e^{-2t}-\dot{u}_1e^{-t}-2\dot{u}_2e^{-2t}=e^{-3t}$$

Constraint 2: $-\dot{u}_1e^{-t}-2\dot{u}_2e^{-2t}=e^{-3t}$

$$\dot{u}_1e^{-t}+\dot{u}_2e^{-2t}=0$$

$$\dot{u}_1e^{-t}+2\dot{u}_2e^{-2t}=-e^{-3t}$$

$$\dot{u}_1(t)=e^{-2t} \qquad u_1(t)=-\frac{1}{2}e^{-2t}+C_1$$

$$\dot{u}_2(t)=-e^{-t} \qquad u_2(t)=e^{-t}+C_2$$

$$x(t)=\left(-\frac{1}{2}e^{-2t}+k_1\right)e^{-t}+\left(e^{-t}+k_2\right)e^{-2t}$$

$$=\frac{1}{2}e^{-3t}+k_1e^{-t}+k_2e^{-2t}$$

Linear systems :

$\dot{x} = Ax + \underline{\underline{Bu}}$

external forcing

if x_1 is a solution : $\dot{x}_1 = Ax_1$

& if x_2 is a solution: $\dot{x}_2 = Ax_2$

then

$x = k_1 x_1 + k_2 x_2$ is also a solution

$$\frac{d}{dt}x = k_1\dot{x}_1 + k_2\dot{x}_2$$

$$Ax = A[k_1x_1 + k_2x_2] = \underset{\dot{x}_1}{\underset{\shortparallel}{\underline{k_1Ax_1}}} + \underset{\dot{x}_2}{\underset{\shortparallel}{k_2Ax_2}}$$

$$= k\dot{x}_1 + k_2\dot{x}_2$$

$\therefore$ $k_1x_1 + k_2x_2$ is a soln.