- Recall

$f(x, W) = Wx + b$ → (image vector, weight)

- TODO
  - define a loss function
  - algorithm to do optimization

- dataset

$\{(x_i, y_i)\}_{i=1}^N$ x_i is image, y_i is label

- cost function

$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$

- multi-class SVM

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Multiclass SVM loss:

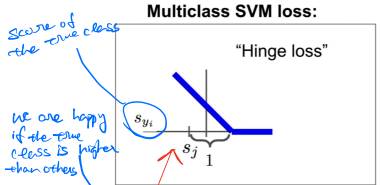
Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$
$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Multiclass SVM loss:



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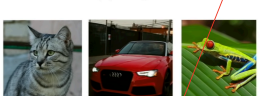

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9		

Multiclass SVM loss:

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= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$
$= \max(0, 1.3 - 4.9 + 1) + \max(0, 2.0 - 4.9 + 1)$
$= 0$

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$
$= \max(0, 2.2 - (-3.1) + 1) + \max(0, 2.5 - (-3.1) + 1)$
$= 12.9$

$\frac{1}{3}(2.9 + 12.9) = 5.27$

Q: What's gonna happen if score for car is changed a bit?
A: loss will not change. will still be 0 for car and total loss still the same.

- min/max for loss?
  A: min = 0, max = ∞
- @ init. W, W is small, all s ≈ 0. what is the loss?
  A: no. classes - 1
- what if the sum includes correct class?
  A: loss + 1
- if we use sum instead of mean?
  A: Quantitatively changed
- what if we use $L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$
  A: algo different

W not good, continue to optimize!

- is optimal W unique?
  A: No, 2W is also optimal

- Regularization

$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(f(x_i, W), y_i) + \lambda R(W)$

Data Loss — Regularization: Model should be simple so it works on data

trade-off hyperparameter

Regularization

λ = regularization strength (hyperparameter)

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1) + \boxed{\lambda R(W)}$$

In common use:

L2 regularization	$R(W) = \sum_k \sum_l W_{k,l}^2$
L1 regularization	$R(W) = \sum_k \sum_l \lvert W_{k,l} \rvert$
Elastic net (L1 + L2)	$R(W) = \sum_k \sum_l \beta W_{k,l}^2 + \lvert W_{k,l} \rvert$
Max norm regularization (might see later)	
Dropout (will see later)	
Fancier: Batch normalization, stochastic depth	

L2 Regularization (Weight Decay)

$x = [1, 1, 1, 1]$

$R(W) = \sum_k \sum_l W_{k,l}^2$

$w_1 = [1, 0, 0, 0]$
$w_2 = [0.25, 0.25, 0.25, 0.25]$

(If you are a Bayesian: L2 regularization also corresponds MAP inference using a Gaussian prior on W)

$$w_1^T x = w_2^T x = 1$$

- softmax classifier (multinomial logistic regression)

Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$ where $s = f(x_i; W)$ softmax

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

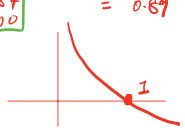
$L_i = -\log P(Y = y_i | X = x_i)$

$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$

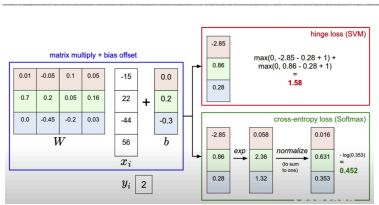
cat	**3.2**
car	5.1
frog	-1.7

exp → 24.5, 164.0, 0.18 → normalized → 0.13, 0.87, 0.00

$L_i = -\log(0.13) = 0.89$



Q: min/max of "loss"
A: 0 ∞
practical, will $L_i = 0$ is hard to get as we need $\frac{e^{s_{y_i}}}{\sum_j e^{s_j}} = 1$ and need $s_{y_i} \to \infty$
∴ will never get to 0



Recap

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

Softmax

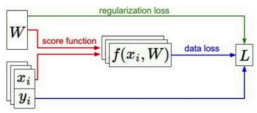
$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

SVM

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Full loss

$$L = \frac{1}{N}\sum_{i=1}^{N} L_i + \underbrace{R(W)}_{\text{regularization}}$$



- How do we do it? Optimization
  - Strategy: Random search
  - Follow the slope: Gradient Descent

```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

step_size: hyperparameter (learning rate); weights_grad: step

Monte-Carlo estimate of some expectation

Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

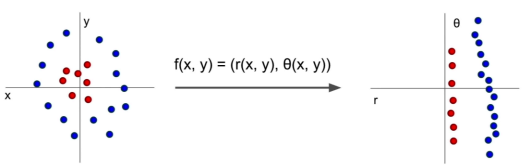
Approximate sum using a **minibatch** of examples
32 / 64 / 128 common

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

- Image Features (instead of raw data)

Image Features: Motivation



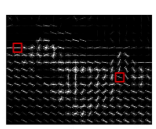
Cannot separate red and blue points with linear classifier

After applying feature transform, points can be separated by linear classifier

- image color histogram
- how about MLMI reader?

Example: Histogram of Oriented Gradients (HoG)

Divide image into 8x8 pixel regions
Within each region quantize edge direction into 9 bins



Example: 320x240 image gets divided into 40x30 bins; in each bin there are 9 numbers so feature vector has 30*40*9 = 10,800 numbers

Example: Bag of Words