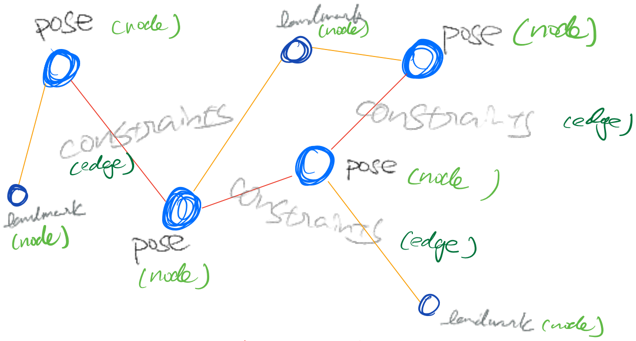
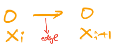


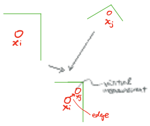
{ edge : dynamic/sensor model
{ node : objective problem
→ solve the over determined problem!

• Edge Creation

• Odometry



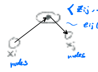
• measurement



edges are then represented as Homogenous coordinates

- odometry
  $X_i^{-1} X_{i+1}$
- observation
  $X_i^{-1} X_j$
  (node i sees node j)

Pose graph



$\langle Z_{ij}, \Omega_{ij} \rangle$ ✓ edge
~ $e_{ij}(x_i, x_j)$ (error)

goal: $x^* = \underset{x}{\operatorname{argmin}} \sum_{ij} \underbrace{e_{ij}^T}_{\text{error}} \underbrace{\Omega_{ij}}_{\text{information matrix}} \underbrace{e_{ij}}_{\text{error}}$

Inverse of covariances

remark: why do we write it this way

assumption: constraints are independent of each other

error function:
$e_{ij}(x_i, x_j) = t2v(\underbrace{Z_{ij}^{-1}}_{\text{measurement}} \underbrace{(x_i^{-1} x_j)}_{x_j \text{ reference w.r.t. } x_i})$

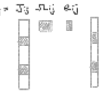
Gauss-Newton:

- linearization:
  $e_{ij}(x + \Delta x) \simeq e_{ij}(x) + J_{ij}\Delta x$
  $J_{ij} = \frac{\partial e_{ij}(x)}{\partial x}$

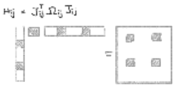
Remark:
$e_{ij}(x)$ depends only on x_i & x_j
$e_{ij}(x) = e_{ij}(x_i, x_j)$
$J_{ij} = \left[0 \cdots 0 \middle| \frac{\partial e}{\partial x_i} \middle| 0 \cdots 0 \middle| \frac{\partial e}{\partial x_j} \middle| 0 \cdots \right]$

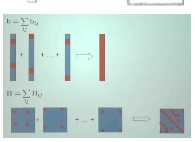
- $b^T = \sum_{ij} b_{ij}^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$ (sparse)
  $H = \sum_{ij} H_{ij}^T = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

$b_{ij} = J_{ij} \; \Omega_{ij} \; e_{ij}$



$H_{ij} = J_{ij}^T \Omega_{ij} J_{ij}$





e.g.

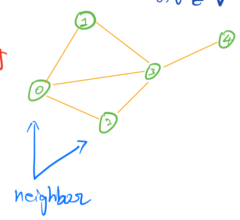
3 nodes
2 constraints

- $\mathcal{X}_0 = \{x_1, x_2, x_3\} = \{0, 0, 0\}$ ← initial guess
- $e_{ij} = Z_{ij} - (x_j - x_i)$
  $e_{12} = (1 - (0 - 0)) = 1$
  $e_{23} = (1 - (0 - 0)) = 1$
- $J_{ij} = \left(\frac{\partial e_{ij}}{\partial x_1} \quad \frac{\partial e_{ij}}{\partial x_2} \quad \frac{\partial e_{ij}}{\partial x_3} \right)$
  $J_{12} = \left(\frac{\partial e_{12}}{\partial x_1} \quad \frac{\partial e_{12}}{\partial x_2} \quad \frac{\partial e_{12}}{\partial x_3} \right)$
  $= (1 \quad -1 \quad 0)$
  $J_{23} = (0 \quad 1 \quad -1)$
- $b^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$
  $= 1 \cdot 1 \cdot (1, -1, 0)$ (b_1)
  $+ 1 \cdot \frac{1}{2} \cdot (0, 1, -1)$ (b_2)
  $= (1 \; -\frac{1}{2} \; -\frac{1}{2})$
- $H = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$
  $= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$
  $= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$
- $\Delta x = -H \backslash b$ → error when $\det(H) = 0$
- change the relative constraints to global one

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

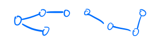
basic graph theory

$G = (V, E)$　E connect u,v　$u, v \in V$



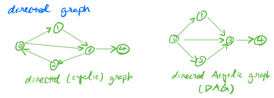
$V = \{0, 1, 2, 3, 4\}$
$E = \{(0,1),$
$(0,2),$
$(0,3),$
$(1,3),$
$(2,3),$
$(3,4)\}$

1. neighbor(0) = {0, 1, 2}
2. degree(0) = 3
   degree(2) = 2
3. path　4 → 3 → 2 → 0
4. cycle　0 → 1 → 3 → 2 → 0
5. connectivity :- graph is connected if ∃ path between (u,v) $(u,v) \in V$
   - graph is connected when all vertices are connected
   - connected component　$V_i \subseteq V$

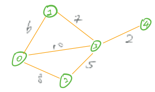


types of graph

1. undirected graph (shown above)
2. directed graph



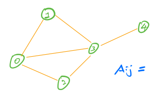
3. weighted graph



4. trees
   1. connected and acyclic
   2. removing edge diconnects graph
   3. adding edge creates a cycle

graph representation

- Adjacency Matrix



$A_{ij} = \begin{cases} 1 & \text{for edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$

$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$

- edge set
  {(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)

- adjacency list
  0 → [1 2 3]
  1 → [0 3]
  2 → [0 3]
  3 → [0 1 2 4]
  4 → [3]