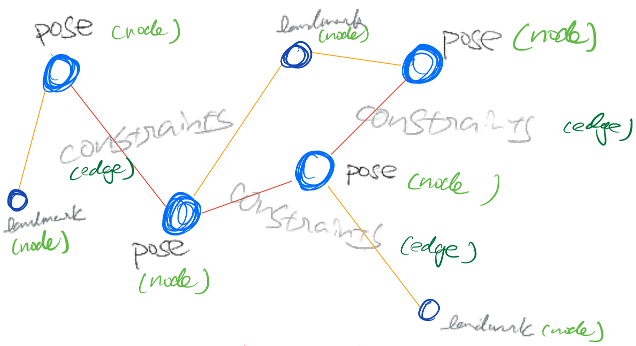
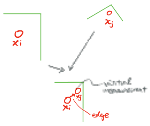


{ edge : dynamic / sensor model
{ node : objective problem
→ solve the over determined problem!

- **Edge Creation**
  - odometry

  $x_i \xrightarrow{edge} x_{i+1}$

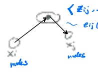
  - measurement



edges are then represented as Homogenous coordinates

- odometry
  $x_i^{-1} x_{i+1}$
- observation
  $x_i^{-1} x_j$
  (node i sees node j)

Pose graph



$\langle z_{ij}, \Omega_{ij} \rangle$ ← edge

~ $e_{ij}(x_i, x_j)$ (error)

goal: $x^* = \underset{x}{\operatorname{argmin}} \sum_{ij} \underbrace{e_{ij}^T}_{error} \underbrace{\Omega_{ij}}_{information\ matrix} \underbrace{e_{ij}}_{error}$

remark: why do we write in this way
A: covariances? Gaussian?
Everything: we're looking at the region of the Gaussian distribution
assumption: constraints are independent of each other

error function:
$e_{ij}(x_i, x_j) = t2v(\underbrace{z_{ij}^{-1}}_{measurement} \underbrace{(x_i^{-1} x_j)}_{x_j\ reference\ w.r.t.\ x_i})$

Gauss-Newton:
- linearization:

$e_{ij}(x + \Delta x) \simeq e_{ij}(x) + J_{ij} \Delta x$

Remark: $J_{ij} = \frac{\partial e_{ij}(x)}{\partial x}$

$e_{ij}(x)$ depends only on x_i & x_j

$e_{ij}(x) = e_{ij}(x_{ij})$

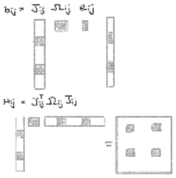
$J_{ij} = \left[0 \cdots 0 \,\middle|\, \frac{\partial e}{\partial x_i} \,\middle|\, 0 \cdots 0 \,\middle|\, \frac{\partial e}{\partial x_j} \,\middle|\, 0 \cdots \right]$

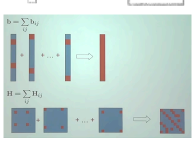
- $b^T = \sum_{ij} b_{ij}^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$ (sparse)

$H = \sum_{ij} H_{ij}^T = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

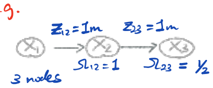
$b_{ij} = J_{ij} \; \Omega_{ij} \; e_{ij}$

$H_{ij} = J_{ij}^T \Omega_{ij} J_{ij}$





e.g.



3 nodes
2 constraints

- $\chi_0 = \{x_1, x_2, x_3\} = \{0, 0, 0\}$

initial guess

- $e_{ij} = z_{ij} - (x_j - x_i)$

$e_{12} = (1 - (0 - 0)) = 1$

$e_{23} = (1 - (0 - 0)) = 1$

- $J_{ij} = \left(\frac{\partial e_{ij}}{\partial x_1} \;\; \frac{\partial e_{ij}}{\partial x_2} \;\; \frac{\partial e_{ij}}{\partial x_3} \right)$

$J_{12} = \left(\frac{\partial e_{12}}{\partial x_1} \;\; \frac{\partial e_{12}}{\partial x_2} \;\; \frac{\partial e_{12}}{\partial x_3} \right)$

$= (1 \;\; -1 \;\; 0)$

$J_{23} = (0 \;\; 1 \;\; -1)$

- $b^T = \sum_{ij} e_{ij}^T \Omega_{ij} J_{ij}$

$= 1 \cdot 1 \cdot (1, -1, 0)$ (b_1)

$+ 1 \cdot 1/2 \cdot (0, 1, -1)$ (b_2)

$= (1 \;\; -1/2 \;\; -1/2)$

- $H = \sum_{ij} J_{ij}^T \Omega_{ij} J_{ij}$

$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1\frac{1}{2} & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$

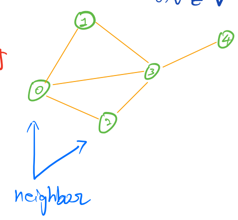
- $\Delta x = -H \backslash b$ → error when $\det(H) = 0$

- change the relative constraints to global one

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

basic graph theory

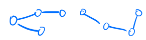
$G = (V, E)$ E connect u, v $u, v \in V$



$V = \{0, 1, 2, 3, 4\}$

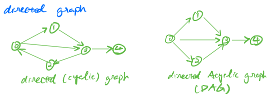
$E = \{(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)\}$

1. neighbor(0) = {0, 1, 2}
2. degree (0) = 3
   degree (2) = 2
3. path 4 → 3 → 2 → 0
4. cycle 0 → 1 → 3 → 2 → 0
5. connectivity :- graph is connected if ∃ path between (u,v) (u,v) ∈ V
   - graph is connected when all vertices are connected
   - connected component $V_i \subseteq V$

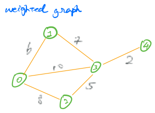


types of graph

1. undirected graph (shown above)
2. directed graph



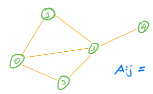
3. weighted graph



4. trees
   1. connected and acyclic
   2. removing edge disconnects graph
   3. adding edge creates a cycle

graph representation

- Adjacency Matrix



$A_{ij} = \begin{cases} 1 & \text{for edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$

$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$

- edge set

{(0,1), (0,2), (0,3), (1,3), (2,3), (3,4)}

- adjacency list

0 → [1 2 3]
1 → [0 3]
2 → [0 3]
3 → [0 1 2 4]
4 → [3]