

ME564 L15

From last time:

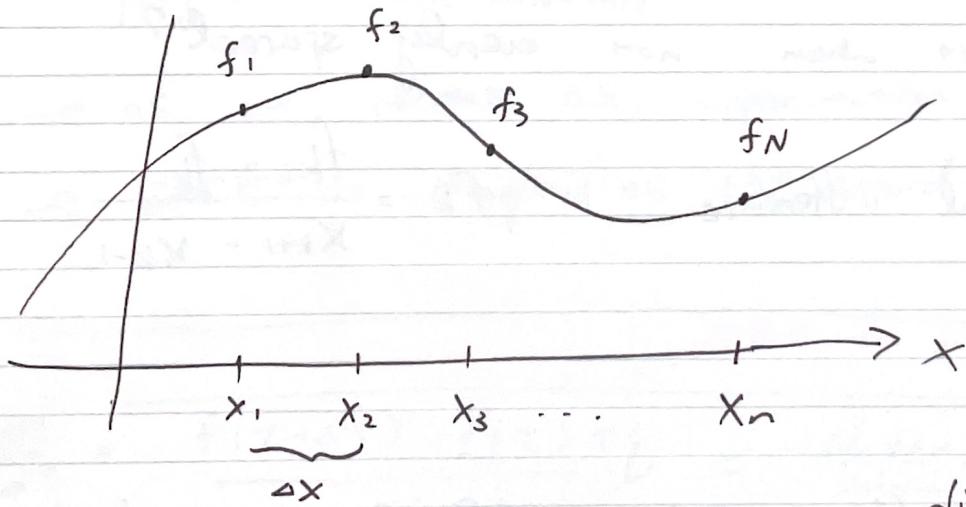
Finite Difference approximation to $f'(x)$

$$\text{- Forward diff } f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x} + O(\Delta x)$$

$$\text{- Back diff } f'(x) \approx \frac{f(x) - f(x-\Delta x)}{\Delta x} + O(\Delta x)$$

$$\text{- central diff } f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O(\Delta x^2)$$

2nd order Diff. : $f(x)$, $f(x+\Delta x)$, $f(x+2\Delta x)$



discrete
evenly spaced data

$$f_k = f(x_k)$$

two vectors of data

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \quad \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix}$$

$$= f(k\Delta x)$$

} calculate
 $\frac{df}{dx}$?

(cont'd)

$$\frac{df}{dx} ?$$

$$f'_k = \frac{f_{k+1} - f_{k-1}}{2\Delta x}$$

= central difference
on the interior pts.

- forward on first pt.
- back on last pt.

How about when not evenly spaced?

central difference:

$$f'_k = \frac{f_{k+1} - f_{k-1}}{x_{k+1} - x_{k-1}}$$

differentiation enlarges error!

numerical smoothing...

Error $\sim \mathcal{O}(\Delta x)$

Compute time $\sim \mathcal{O}(\frac{1}{\Delta x})$

10^{-16} } computer limit precision, ϵ machine precision

Round-off error

double $\Rightarrow 8$ bytes

$\gg 5 + 10^{-17} = 5$ in computer
 $(= 5.000\dots 0)$



plug into finite difference

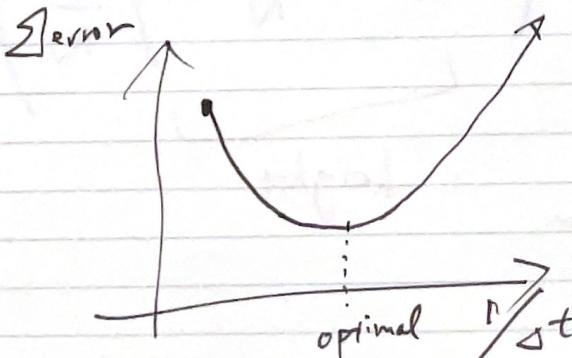
\rightarrow as we decrease Δx , approximation gets better

\rightarrow until it magnifies the round-up error - worse

this one matters

$$\frac{df}{dt} = \frac{f(t+\Delta t) - f(t) + \epsilon}{\Delta t + \epsilon} = \frac{f(t+\Delta t) - f(t)}{\Delta t} + \frac{\epsilon}{\Delta t}$$

$$= f' + \underbrace{\mathcal{O}(\Delta t)}_{\text{taylor series}} + \underbrace{\frac{\epsilon}{\Delta t}}_{\text{machine precision}}$$



FD

$$\Delta t \approx 10^{-5}$$

$(\Delta t \rightarrow \text{smaller})$

Σ Error

$$\frac{\partial \text{Error}(\Delta t)}{\partial \Delta t} = 0$$

Integrator!

$$\frac{dx}{dt} = f(x)$$

$$\frac{x_{k+1} - x_k}{\Delta t} = f(x_k)$$



$$x_{k+1} = x_k + \Delta t \cdot f(x_k)$$

reman?

Lebesgue Integral

$$\int_a^b f(x) dx = \lim_{N \rightarrow \infty} \sum_{k=1}^N \left(f\left(a + \frac{b-a}{N} k\right) \frac{b-a}{N} \right)$$

Right-sided

Rectangle
formula

No this
in computer

height

$$= \lim$$

$$\lim_{\Delta x \rightarrow 0} \sum_{k=1}^N f(x_{1k}) \Delta x$$

no this
in numerical
calculus

or

$$\lim_{\Delta x \rightarrow 0} \sum_{k=1}^N f(x_{1k}) (x_k - x_{k-1})$$