Tracking the states w/ UUV

1. Data Sync.
2. Bias initialization.
3. eskf model

$\longrightarrow$ IMU_cb

$\longrightarrow$ GPS_cb

$\longrightarrow$ ESKF

$z = h(x)$

$x = \begin{matrix} p \\ v \\ R \end{matrix}$

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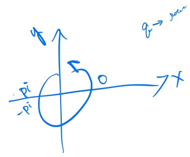
preview = 80

ESKF model

$$x_t = \begin{bmatrix} p_t \\ v_t \\ R_t \\ b_{at} \\ b_{gt} \\ g_t \end{bmatrix} \qquad \underbrace{\tilde{\omega} \quad \tilde{a}}_{\text{IMU}}$$

$$\dot{p}_t = v_t$$
$$\dot{v}_t = R_t(\tilde{a} - b_{at} - \eta_a) + g$$
$$\dot{R}_t = R_t(\tilde{\omega} - b_{gt} - \eta_g)^\wedge$$
$$\dot{b}_{gt} = \eta_{bg}$$
$$\dot{b}_{at} = \eta_{ba}$$
$$\dot{g} = 0$$

subscript w/ t := true state



$$\left.\begin{aligned} p_t &= p + \delta p \\ v_t &= v + \delta v \\ R_t &= R\,\delta R \\ b_{gt} &= b_g + \delta b_g \\ b_{at} &= b_a + \delta b_a \\ g_t &= g + \delta g \end{aligned}\right\}$$

nominal

$$\dot{p}_t = v_t$$
$$\Rightarrow \dot{p} + \delta\dot{p} = v + \delta v$$
$$\therefore \delta\dot{p} = \delta v \quad -①$$
$$\dot{b}_{gt} = \eta_{bg}$$
$$\Rightarrow \cancel{\dot{b}_g} + \delta\dot{b}_g = \eta_{bg} \quad -②$$
$$\dot{b}_{at} = \eta_{ba}$$
$$\Rightarrow \cancel{\dot{b}_a} + \delta\dot{b}_a = \eta_{ba} \quad -③$$
$$\dot{g}_t = 0$$
$$\Rightarrow \cancel{\dot{g}} + \delta\dot{g} = 0 \quad -$$

$$\star \begin{cases} \dot{R}_t = R_t(\tilde{\omega} - b_{gt} - \eta_g)^\wedge & -ⓐ \\ R_t = R \circ \mathrm{Exp}(\delta R) & -ⓑ \end{cases}$$

from ⓑ

$$\dot{R}_t = \dot{R}\,\mathrm{Exp}(\delta\theta) + R\,\dot{\mathrm{Exp}}(\delta\theta)$$
$$= R(\tilde{\omega} - b_{gt})^\wedge \mathrm{Exp}(\delta\theta) + R\,\mathrm{Exp}(\delta\theta)\,\delta\dot{\theta}^\wedge$$

from ⓐ & ⓑ

$$R_t(\tilde{\omega} - b_{gt} - \eta_g)^\wedge$$
$$= R\,\mathrm{Exp}(\delta\theta)(\tilde{\omega} - b_{gt} - \eta_g)^\wedge$$

$$\Rightarrow \begin{cases} R \\ \end{cases}$$

	xy z	Q	R	rotor const.	remark
1330	20N	380 380	1 1	0.02	
1350	20N	380 380	10 10	0.03	
1400	20N	380 480	10 10	0.03	
1410	20N	380 480	15 15	0.03	
1430	20N	300 300	1 1	0.03	
1440	20N	300 480	1 1	0.03	
1450	40N	300 480	1 1	0.03	
1545	40N	300 480	1 1	0.03	(fixed v_B)
1550	40N	300 300	1 1	0.03	
1600	10N	300 330	10 10	0.03	
1600	40	300 480	1 1	0.06	
1700	40	300 480	1 1	0.06	(cancel C(v) in MPC)
1710	40	300/480	1/1	0.03	(cancel C(v) in MPC

EKF Stability proof

△ $E[w_k w_k^T] = Q_k$

$E[v_k v_k^T] = R_k$

△ $x_{k+1} = f(x_k) + w_k$

$z_k = h(x_k) + v_k$

△ @ predict

$\check{x}_k = f(\hat{x}_{k-1})$

$\check{P}_k = F_k \hat{P}_{k-1} F_k^T + Q_k \rightsquigarrow F_k = \left[\frac{\partial f}{\partial x}\right]_{x=\hat{x}_{k-1}}$

@ update

$y_k = z_k - h(\check{x}_k)$

$K_k = \check{P}_k H_k^T (H_k \check{P}_k H_k^T + R_k)^{-1} \rightsquigarrow H_k = \left[\frac{\partial h}{\partial x}\right]_{x=\check{x}_k}$

$\hat{x}_k = \check{x}_k + K_k y_k$

$\hat{P}_k = (I - K_k H_k) \check{P}_k$

證明前假設

△ F_k is non-singular $\forall k \geq 0$

△ R_k, Q_k are bounded from below

where

$$\bar{r} I \preceq R_k \qquad \forall k \geq 0$$
$$\bar{q} I \preceq Q_k$$

$\bar{r}\ \bar{q} > 0$

△ $\check{e}_k = x_k - \check{x}_k$

$\hat{e}_k = x_k - \hat{x}_k$