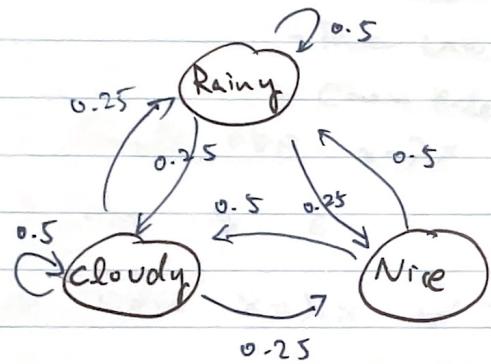


ME564 L1

Weather in Seattle



$$X_{\text{today}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} R \\ N \\ C \end{matrix}$$

$$X_{t+n} = A X_{\text{today}}$$

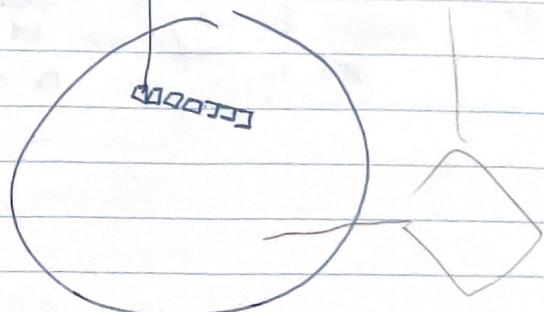
$$A = \begin{bmatrix} 0.5 & 0.5 & 0.25 \\ 0.25 & 0 & 0.25 \\ 0.25 & 0.5 & 0.5 \end{bmatrix} \begin{matrix} \text{Rainy} \\ \text{Nice} \\ \text{Cloudy} \end{matrix}$$

Markov model.

one pixel
one $X_N = A^{N-1} X_1$

$$\begin{matrix} X_1 \\ || \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{matrix} \quad \begin{matrix} X_2 \\ || \\ AX_1 \end{matrix} \quad \begin{matrix} X_3 \\ || \\ A^2 X_1 \end{matrix} \quad \cdots \quad \begin{matrix} X_N \\ || \\ A^{N-1} X_1 \end{matrix}$$

$$\begin{bmatrix} 0.5 \\ 0.25 \\ 0.25 \end{bmatrix}$$



1. Review Calculus

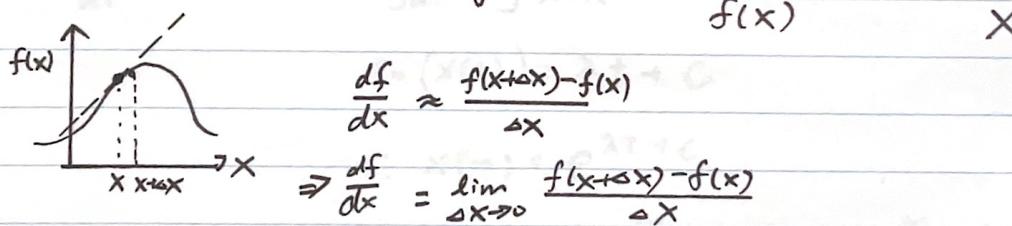
- Derivative
- Power Law
- Chain Rule

2. Simple ODE $\dot{x} = \lambda x$

3. What is e

4. Solve $\dot{x} = \lambda x$ w/ Taylor Series

The Derivative : rate of change of a function w.r.t.



Power Law

$$f(x) = x^n \quad \frac{df}{dx} = nx^{n-1}$$

$$\frac{df}{dx} \approx \frac{1}{\Delta x} [(x + \Delta x)^n - x^n]$$

Pascal Triangle
Binomial Expansion

$$= \frac{1}{\Delta x} \left[x^n + n x^{n-1} \Delta x + \frac{n(n-1)}{2} x^{n-2} (\Delta x)^2 + \dots + (-\Delta x)^n \right]$$

$$= \frac{1}{\Delta x} \left[n x^{n-1} \Delta x + \frac{n(n-1)}{2} x^{n-2} (\Delta x)^2 + \mathcal{O}((\Delta x)^3) \right]$$

on the order of Δx cube & T

$$= n x^{n-1} + \mathcal{O}(\Delta x)$$

on the order of Δx & higher multiplied by at least 2 Δx

$\hookrightarrow = 0$ when $\Delta x \rightarrow 0$

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \quad f' := \frac{df}{dx}$$

e.g. $\sin(x^3)$

ME564 L2 (contd)

X is the size of population of bunnies...

$$\frac{dx}{dt} = \dot{x} = \lambda x$$



change in population, which is proportional to the size of population

ask:

What is the size of pop x as a function of time?

method 1: $\frac{dx}{dt} = \lambda x \Rightarrow \int \frac{dx}{dt} = \int \lambda dt$

$$\ln(x(t)) = \lambda t + C$$

$$\therefore x(t) = e^{\lambda t + C}$$

$$= K e^{\lambda t} \Rightarrow \text{get } K \text{ via I.C}$$

Initial condition

$$x(t=0) = e^0 K = K$$

★ $x(t) = x(0) e^{\lambda t}$

What is e ?

Loan L interest rate ' r '

Loan amount: X

$$x(0) = L$$

• compound yearly $\rightarrow x(1) = (1+r)L = (1+r)x(0)$

• compound monthly $\rightarrow x(1) = (1+\frac{r}{12})^{12} x(0)$

$\downarrow \lim \Delta t \approx 0$

$$x(1) = \lim_{n \rightarrow \infty} (1 + \frac{r}{n})^n x(0)$$

$$= e^r x(0)$$

Compound continuously

$$\exp(0.5) = 1.0513 \rightarrow \text{extra } 0.0013!$$

more on e

$$\text{Interest rate } r = 0.05$$

$n=1$	yearly	$X(1) = (1+0.05) X(0)$	$= 1.05 X(0)$
$n=12$	monthly	$X(1) = \left(1 + \frac{0.05}{12}\right)^{12} X(0)$	$= 1.05116 X(0)$
$n=100$	"	$X(1) = \left(1 + \frac{0.05}{100}\right)^{100} X(0)$	$= 1.051257 X(0)$
$n=10^6$	"	$X(1) = \left(1 + \frac{0.05}{10^6}\right)^{10^6} X(0)$	$= 1.051271 X(0)$
$n \rightarrow \infty$	continuously	$X(1) = \left(1 + \frac{0.05}{n}\right)^n X(0)$ $= e^{0.05} X(0)$ $= e^{0.05} X(0)$	$= 1.051271 X(0)$

for sth system $\dot{x} = \lambda x$

$$\frac{dx}{dt} = r x(t) \quad \text{continuous}$$

$$\Delta x = r x(t) \Delta t \quad \text{discrete}$$

e.g. Radioactive Decay

$$\dot{x} = -\lambda x(t)$$

rate \curvearrowright proportional to number of $x(t)$.
is λ

$$x(t) = e^{-\lambda t} x(0)$$

Plutonium half-life = 80 million years = 8×10^7

$$\therefore x(t=80 \times 10^7) = \frac{x(0)}{2} = e^{-\lambda 8 \times 10^7} x(0)$$

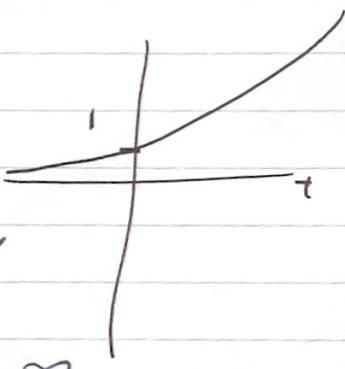
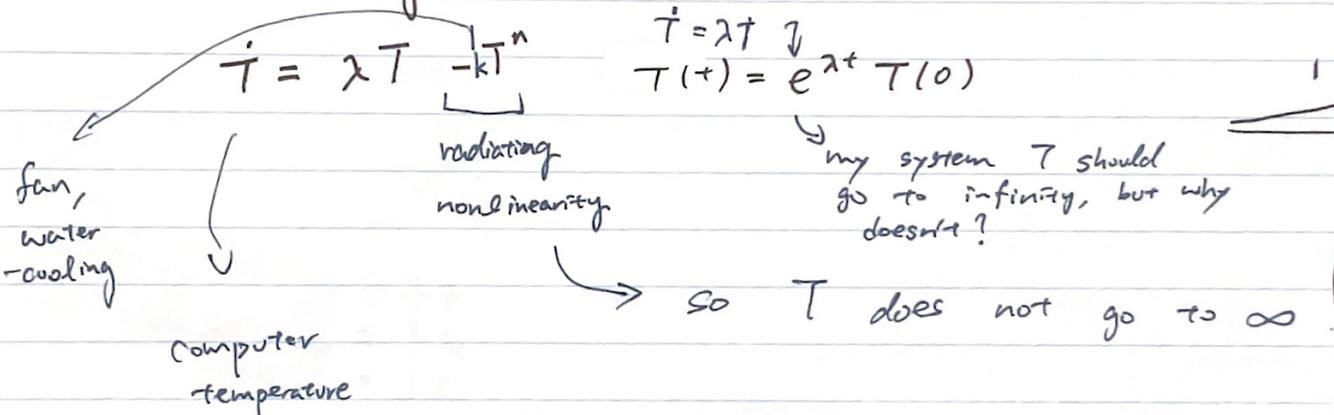
$$\therefore \lambda = \frac{-\ln(1/2)}{8 \times 10^7} \rightarrow \text{decay rate no. } \lambda$$

my plutonium

84
Polonium: $HL \approx 138$ days $\approx .35$ years

$$\lambda = \frac{-\ln(1/2)}{.35}$$

Thermal Runaway lithium-ion for instance



$$\dot{x} = \lambda x \quad . \quad \frac{d}{dx} e^{\lambda x} = \lambda e^{\lambda x}$$

$$x(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + \dots$$

↓
derive $e^{\lambda t}$