1 Motivation

- +rue-state
- nominal-state
- error-state

△ true-state = nominal-state ⊕ crear state

△ nominal-state → takes no account in noise

midel imperfection

Definitions

Magnitude	True	Nominal	Error	Composition	Measured	Noise		
Full state (1)	\mathbf{x}_t	x	$\delta \mathbf{x}$	$\mathbf{x}_t = \mathbf{x} \oplus \delta \mathbf{x}$				
Position	\mathbf{p}_t	p	$\delta \mathbf{p}$	$\mathbf{p}_t = \mathbf{p} + \delta \mathbf{p}$				
Velocity	\mathbf{v}_t	v	$\delta \mathbf{v}$	$\mathbf{v}_t = \mathbf{v} + \delta \mathbf{v}$			R:	
Quaternion (2,3)	\mathbf{q}_t	q	$\delta \mathbf{q}$	$\mathbf{q}_t = \mathbf{q} \otimes \delta \mathbf{q}$			rotation matrix	
Rotation matrix (2,3)	\mathbf{R}_t	R	$\delta \mathbf{R}$	$\mathbf{R}_t = \mathbf{R} \delta \mathbf{R}$			from body to	
Angles vector (4)			$\delta \boldsymbol{\theta}$	$\delta \mathbf{q} = e^{\delta \theta/2}$ $\delta \mathbf{R} = e^{[\delta \theta]_{\times}}$			inertial frame	
Accelerometer bias	\mathbf{a}_{bt}	\mathbf{a}_b	$\delta \mathbf{a}_b$	$\mathbf{a}_{bt} = \mathbf{a}_b + \delta \mathbf{a}_b$		\mathbf{a}_w		
Gyrometer bias	$oldsymbol{\omega}_{bt}$	ω_b	$\delta\omega_b$	$oldsymbol{\omega}_{bt} = oldsymbol{\omega}_b + \delta oldsymbol{\omega}_b$		ω_w		
Gravity vector	\mathbf{g}_t	g	$\delta \mathbf{g}$	$\mathbf{g}_t = \mathbf{g} + \delta \mathbf{g}$				
Acceleration	\mathbf{a}_t				\mathbf{a}_m	\mathbf{a}_n		
Angular rate	ω_t				ω_m	ω_n	NETWORKED CONTROL ROBOTICS LAD	4

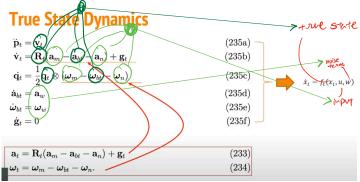
Dynamic

· measurement

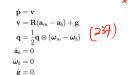




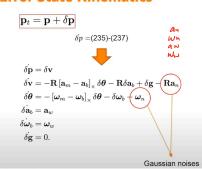
Note: $a_m,\,\omega_m$ are measuremed in the body-fixed frame $a_t,\,\omega_t$ are exoressed in the inertial frame



Nominal State Kinematics



Error State Kinematics



Discrete-time Nominal States

Taking the integration of (237) yields the discrete-time form as

$$\mathbf{p} \leftarrow \mathbf{p} + \mathbf{v} \Delta t + \frac{1}{2} (\mathbf{R}(\mathbf{a}_m - \mathbf{a}_b) + \mathbf{g}) \Delta t^2$$
 (260a)

$$\mathbf{v} \leftarrow \mathbf{v} + (\mathbf{R}(\mathbf{a}_m - \mathbf{a}_b) + \mathbf{g}) \,\Delta t \tag{260b}$$

$$\mathbf{q} \leftarrow \mathbf{q} \otimes \mathbf{q} \{ (\boldsymbol{\omega}_m - \boldsymbol{\omega}_b) \, \Delta t \}$$
 (260c)

$$\mathbf{a}_b \leftarrow \mathbf{a}_b \tag{260d}$$

$$\omega_b \leftarrow \omega_b$$
 (260e)

$$\mathbf{g} \leftarrow \mathbf{g}$$
, (260f)

Discrete-time Error States

Taking the integration of (238) yields the discrete-time form as

$$\delta \mathbf{p} \leftarrow \delta \mathbf{p} + \delta \mathbf{v} \Delta t \qquad (261a)$$

$$\delta \mathbf{v} \leftarrow \delta \mathbf{v} + (-\mathbf{R} [\mathbf{a}_m - \mathbf{a}_b]_{\times} \delta \boldsymbol{\theta} - \mathbf{R} \delta \mathbf{a}_b + \delta \mathbf{g}) \Delta t + \mathbf{v_i} \qquad (261b)$$

$$\delta \boldsymbol{\theta} \leftarrow \mathbf{R}^{\top} \{ (\boldsymbol{\omega}_m - \boldsymbol{\omega}_b) \Delta t \} \delta \boldsymbol{\theta} - \delta \boldsymbol{\omega}_b \Delta t + \mathbf{v_i} \qquad (261c)$$

$$\delta \mathbf{a}_b \leftarrow \delta \mathbf{a}_b + \mathbf{a_i} \qquad (261d)$$

$$\delta \boldsymbol{\omega}_b \leftarrow \delta \boldsymbol{\omega}_b + \mathbf{v_i} \qquad (261e)$$

$$\delta \mathbf{g} \leftarrow \delta \mathbf{g} \qquad (261f)$$

og vog .		
$\mathbf{V_i} = \sigma_{ ilde{\mathbf{a}}_n}^2 \Delta t^2 \mathbf{I}$	$[m^2/s^2]$	(262)
$\Theta_{\mathbf{i}} = \sigma_{\tilde{\omega}_n}^2 \Delta t^2 \mathbf{I}$	$[rad^2]$	(263)
$\mathbf{A_i} = \sigma_{\mathbf{a}_w}^2 \Delta t \mathbf{I}$	$[m^2/s^4]$	(264)
$oldsymbol{\Omega_i} = \sigma_{oldsymbol{\omega}_w}^2 \Delta t \mathbf{I}$	$[rad^2/s^2]$	(265)



(283f)

ESKF v.s. EKF

ESKF		EKF
$\begin{split} &\hat{\delta x} \leftarrow F_x(x,u_m) \cdot \hat{\delta x} \\ & P \leftarrow F_x P F_x^\top + F_i Q_i F_i^\top \; , \end{split}$	(268) (269)	$\begin{split} A &= \frac{\partial f}{\partial x} _{\hat{\mathcal{R}}_{k-1}, \mathcal{U}_{k-1}} \text{ and } C = \frac{\partial g}{\partial x} _{\hat{\mathcal{R}}_k} \\ \hat{\mathcal{L}}_k^- &= f(\hat{\mathcal{R}}_{k-1}, \mathcal{U}_{k-1}) \\ P_k^- &= A_k P_{k-1} A_k^T + Q \end{split}$
$\mathbf{K} = \mathbf{P}\mathbf{H}^{\top}(\mathbf{H}\mathbf{P}\mathbf{H}^{\top} + \mathbf{V})^{-1}$ $\hat{\delta}\hat{\mathbf{x}} \leftarrow \mathbf{K}(\mathbf{v} - h(\hat{\mathbf{x}}_t))$	(274) (275)	$\mathcal{K}_{k} = P_{k}^{-}C^{T}(CP_{k}^{-}C^{T} + R)^{-1}$ $\hat{x}_{\nu} = \hat{x}_{\nu}^{-} + \mathcal{K}_{\nu}(\gamma_{\nu} - \hat{\gamma}_{\nu})$
$\mathbf{p} \leftarrow (\mathbf{I} - \mathbf{K}(\mathbf{y} - n(\mathbf{x}_t)))$ $\mathbf{P} \leftarrow (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}$	(276)	$x_k = x_k + \mathcal{R}_k(y_k - y_k)$ $P_k = (I - \mathcal{R}_k C)P_k^-$

Injection of the Observed Error into the Nominal State

6.2 Injection of the observed error into the nominal state

After the ESKF update, the nominal state gets updated with the observed error state using the appropriate compositions (sums or quaternion products, see Table 3),

 $\mathbf{g} \leftarrow \mathbf{g} + \hat{\delta \mathbf{g}}$

 $\mathbf{x} \leftarrow \mathbf{x} \oplus \hat{\mathbf{s}} \mathbf{x} , \tag{282}$ that is, $\begin{aligned} \mathbf{p} \leftarrow \mathbf{p} + \delta \hat{\mathbf{p}} & (283a) \\ \mathbf{v} \leftarrow \mathbf{v} + \delta \hat{\mathbf{v}} & (283b) \\ \mathbf{q} \leftarrow \mathbf{q} \otimes \mathbf{q} \{ \delta \boldsymbol{\theta} \} & (283c) \\ \mathbf{a}_b \leftarrow \mathbf{a}_b + \delta \hat{\mathbf{a}}_b & (283d) \\ \boldsymbol{\omega}_b \leftarrow \boldsymbol{\omega}_b + \delta \hat{\boldsymbol{\omega}}_b & (283e) \end{aligned}$

Error State Reset

Let us call the error reset function g(). It is written as follows,

$$\delta \mathbf{x} \leftarrow g(\delta \mathbf{x}) = \delta \mathbf{x} \ominus \hat{\delta \mathbf{x}}$$
, (284)

where \ominus stands for the composition inverse of \oplus . The ESKF error reset operation is thus,

$$\delta \hat{\mathbf{x}} \leftarrow 0$$
 (285)

$$\mathbf{P} \leftarrow \mathbf{G} \mathbf{P} \mathbf{G}^{\top}$$
. (286)

where G is the Jacobian matrix defined by,

$$\mathbf{G} \triangleq \frac{\partial g}{\partial \delta \mathbf{x}}\Big|_{\delta \hat{\mathbf{x}}}$$
 (287)

$$\mathbf{G} = \begin{bmatrix} \mathbf{I}_{6} & 0 & 0 \\ 0 & \mathbf{I} - \begin{bmatrix} \frac{1}{2} \hat{\delta} \hat{\boldsymbol{\theta}} \end{bmatrix}_{\times} & 0 \\ 0 & 0 & \mathbf{I}_{\theta} \end{bmatrix}. \quad (288)$$