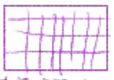
TRAJ-OPT.

So far

△ Dynamic Programming.



put it over a state-space;
not friendly to high dimensional stuff

△ linearization;
solve locally optimal policy;
only when linearization is good.

△ Lyapunov analysis
extension: effective estimates of stable regions

△ SOS. → DP.

Question: mainstream?

TO DEAL w/ High Dim.

find optimal control sol^n w/ single initial condition.

traj expressed in $x(t)$ $u(t)$ w.r.t. x_0

PROBLEM:

$$\min_{u(\cdot)} \ell_f(x(t_f)) + \int_{t_0}^{t_f} \ell(x(t), u(t))\, dt$$

s.t. $\dot{x}(t) = f(x(t), u(t))$

$x(t_0) = x_0$

$t \in [t_0, t_f]$

$x \in \mathcal{X}$

$u \in \mathcal{U}$

NOW: continuous → discrete

WHAT IS THE PARAMETERIZATION

IDEA: parameterization on (t)! single dimension

- Direct Transcription

$$\min_{x[\cdot], u[\cdot]} \ell_f(x[N]) + \sum_{n=0}^{N-1} \ell(x[n], u[n])$$

s.t. $x[n+1] = Ax[n] + Bu[n]$

$x[0] = x_0$

$x \in \mathcal{X}$

$u \in \mathcal{U}$

$x[1] = Ax[0] + Bu[0]$

$x[2] = A(Ax[0] + Bu[0]) + Bu[1]$

- Direct Shooting

$$\min_{u[\cdot]} \ell_f(x[N]) + \sum_{n=0}^{N-1} \ell(x[n], u[n])$$

s.t. $x[n+1] = Ax[n] + Bu[n]$

$x[0] = x_0$

$x \in \mathcal{X}$

$u \in \mathcal{U}$

- Continuous Time

$$x[n+1] = x[n] + \int_{t_n}^{t_n+h} [Ax(t) + Bu[n]]\, dt$$

here assume control inputs are const. @ $t = n_0$ "zero-order hold".

$$= e^{Ah} x[n] + A^{-1}(e^{Ah} - I) Bu[n].$$

or use integrator...!

CONVEX

NON-CONVEX

- Direct Transcription

$$\min_{x[\cdot], u[\cdot]} \ell_f(x[N]) + \sum_{n=0}^{N-1} \ell(x[n], u[n])$$

s.t. $x[n+1] = f(x[n], u[n])$

$x[0] = x_0$

- Direct Shooting

$$\min_{u[\cdot]} \ell_f(x[N]) + \sum_{n=0}^{N-1} \ell(x[n], u[n])$$

s.t. $x[n+1] = f(x[n], u[n])$

$x[0] = x_0$

- Continuous Time

$$x[n+1] = x[n] + \int_{t[n]}^{t[n+1]} f(x(t), u(t))\, dt$$

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variable-step integration

control integration error

e.g. Euler-forward

RK45

sometimes has discontinuous gradients

?

∴ let $t[\cdot]$ also be decision variable.

- Direct Collocation

now your decision variables:

"sample values of $u(t)$ $x(t)$"

⟹ break points of the spline.