NPDAs Accept Context-Free Languages

Theorem:

$$\left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{(Grammars)}\end{array}\right\} = \left\{\begin{array}{c}\text{Languages}\\ \text{Accepted by}\\ \text{NPDAs}\end{array}\right\}$$

Proof - Step 1:

$$\left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{(Grammars)}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{Languages}\\ \text{Accepted by}\\ \text{NPDAs}\end{array}\right\}$$

Convert any context-free grammar G to a NPDA M with: $L(G) = L(M)$

Proof - Step 2:

$$\left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{(Grammars)}\end{array}\right\} \supseteq \left\{\begin{array}{c}\text{Languages}\\ \text{Accepted by}\\ \text{NPDAs}\end{array}\right\}$$

Convert any NPDA M to a context-free grammar G with: $L(G) = L(M)$

Deterministic PDA

DPDA

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Deterministic PDA: DPDA

Allowed transitions:

$$q_1 \xrightarrow{a,\, b \to w} q_2$$

$$q_1 \xrightarrow{\lambda,\, b \to w} q_2$$

(deterministic choices)

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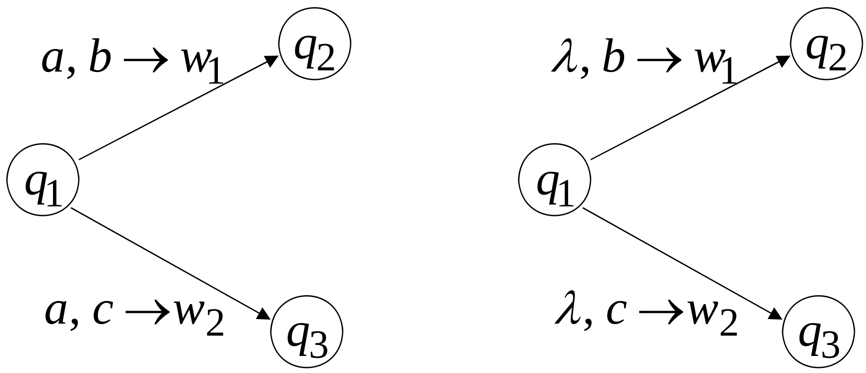
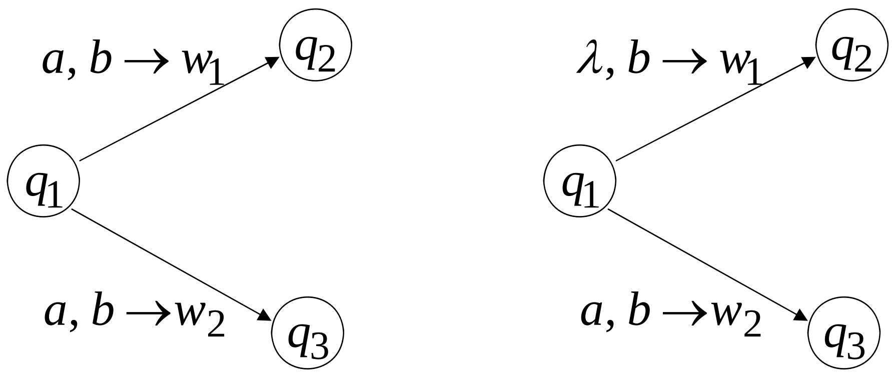
Allowed transitions:



(deterministic choices)

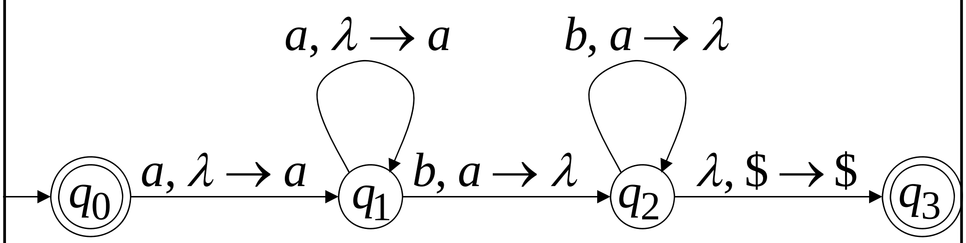
Not allowed:



(non deterministic choices)

DPDA example

$$L(M) = \{a^n b^n : n \geq 0\}$$



The language $L(M) = \{a^n b^n : n \geq 0\}$

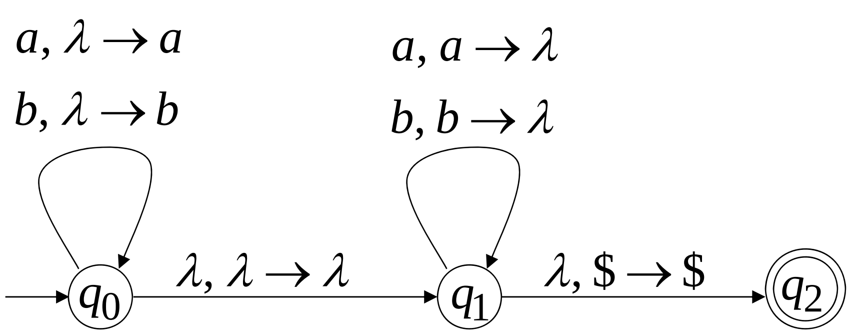
is **deterministic context-free**

Definition:

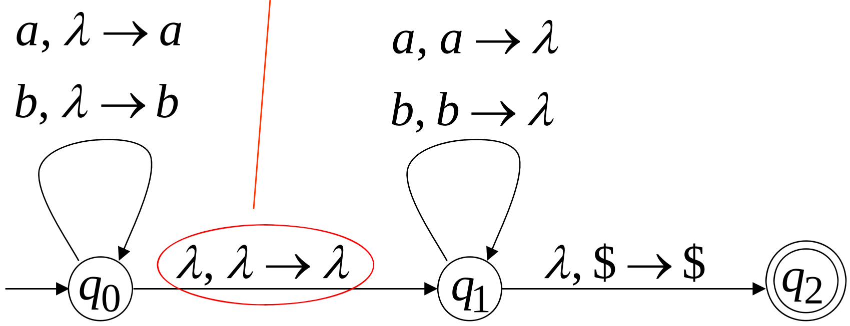
A language L is **deterministic context-free** if there exists some DPDA that accepts it

Example of Non-DPDA (NPDA)

$$L(M) = \{ww^R\}$$



Not allowed in DPDAs



NPDAs

Have More Power than

DPDAs

It holds that:

$$\left\{\begin{array}{c}\text{Deterministic}\\\text{Context-Free}\\\text{Languages}\\\text{(DPDA)}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{Context-Free}\\\text{Languages}\\\text{NPDAs}\end{array}\right\}$$

Since every DPDA is also a NPDA

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We will actually show:

$$\underset{L \notin}{\left\{\begin{array}{c}\text{Deterministic}\\\text{Context-Free}\\\text{Languages}\\\text{(DPDA)}\end{array}\right\}} \subset \underset{L \in}{\left\{\begin{array}{c}\text{Context-Free}\\\text{Languages}\\\text{(NPDA)}\end{array}\right\}}$$

We will show that there exists
a context-free language L which is not
accepted by any DPDA

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The language is:

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\} \qquad n \geq 0$$

We will show:

- L is context-free
- L is **not** deterministic context-free

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$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Language L is context-free

Context-free grammar for L :

$S \rightarrow S_1 \mid S_2$ $\qquad \{a^n b^n\} \cup \{a^n b^{2n}\}$

$S_1 \rightarrow aS_1 b \mid \lambda$ $\qquad \{a^n b^n\}$

$S_2 \rightarrow aS_2 bb \mid \lambda$ $\qquad \{a^n b^{2n}\}$

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Theorem:

The language $L = \{a^n b^n\} \cup \{a^n b^{2n}\}$

is **not** deterministic context-free

(there is **no** DPDA that accepts L)

Proof: Assume for contradiction that

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

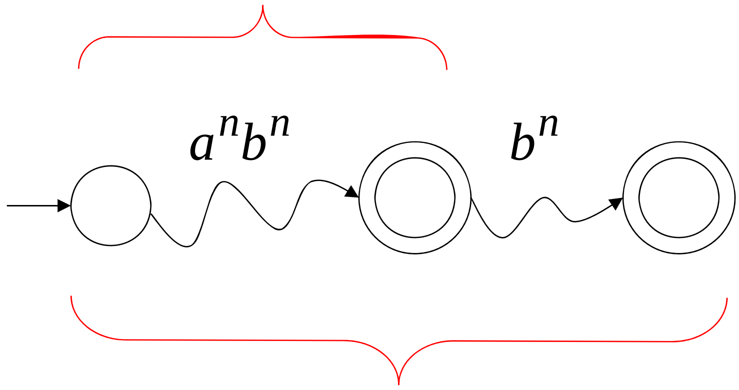
is deterministic context free

Therefore:

there is a DPDA M that accepts L

DPDA M with $L(M)=\{a^n b^n\} \cup \{a^n b^{2n}\}$

accepts $a^n b^n$



accepts $a^n b^{2n}$

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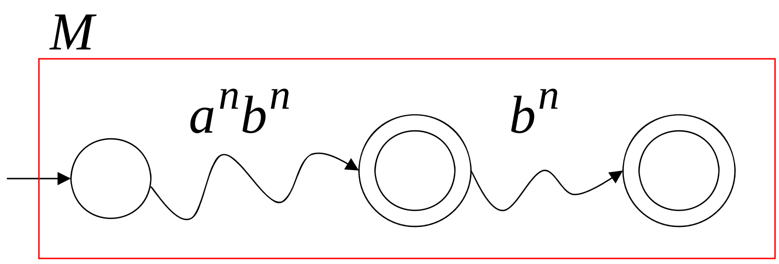
DPDA M with $L(M)=\{a^n b^n\} \cup \{a^n b^{2n}\}$

Such a path exists because of the determinism



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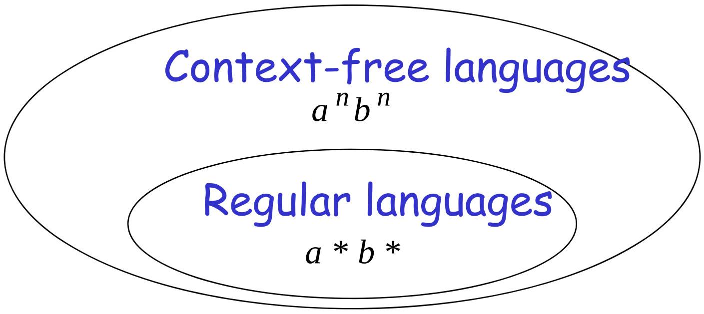
Fact 1: The language $\{a^n b^n c^n\}$ is not context-free



(we will prove this at a later class using pumping lemma for context-free languages)

Fact 2: The language $L \cup \{a^n b^n c^n\}$ is not context-free

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

(we can prove this using pumping lemma for context-free languages)

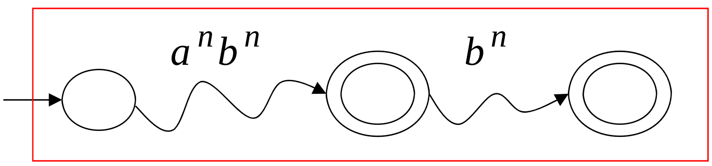
We will construct a NPDA that accepts:

$$L \cup \{a^n b^n c^n\}$$

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

which is a contradiction!

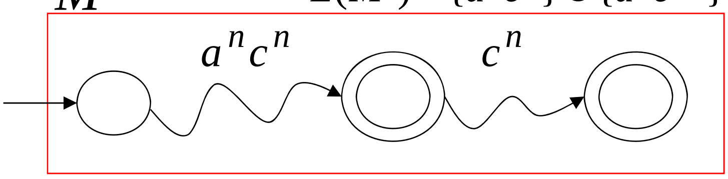
M $\qquad L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$



Modify M

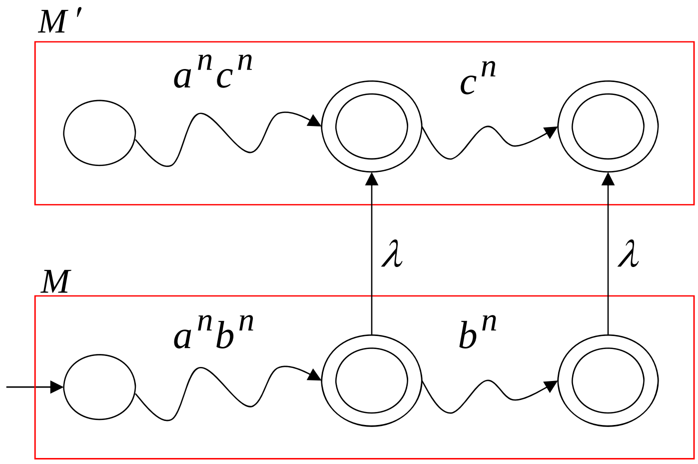
Replace b with c

M' $\qquad L(M') = \{a^n c^n\} \cup \{a^n c^{2n}\}$



The NPDA that accepts $L \cup \{a^n b^n c^n\}$

Connect final states of M'
with final states of M



Since $L \cup \{a^n b^n c^n\}$ is accepted by a NPDA

it is context-free

Contradiction!

(since $L \cup \{a^n b^n c^n\}$ is not context-free)

Therefore:

Not deterministic context free

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

There is **no** DPDA that accepts

End of Proof