

Pushdown Automata PDAs

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Pushdown Automaton -- PDA

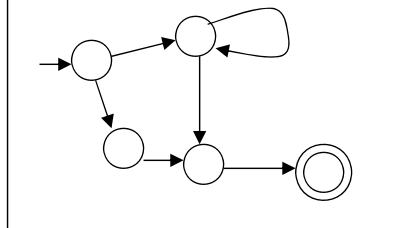
Input String



Stack

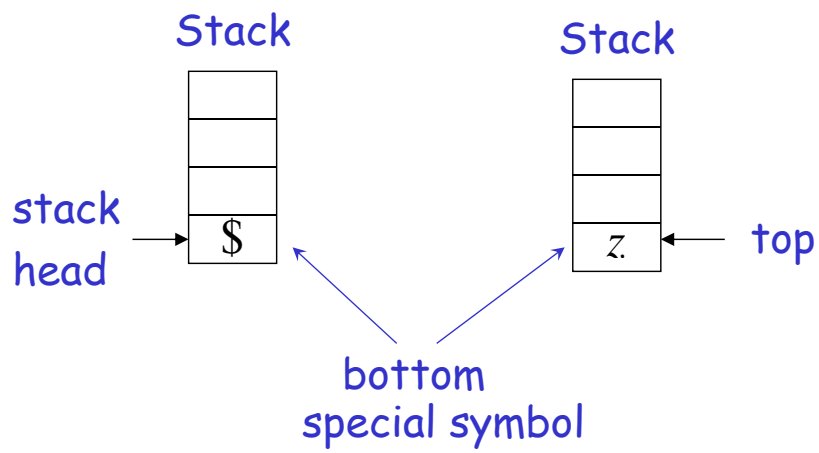


States



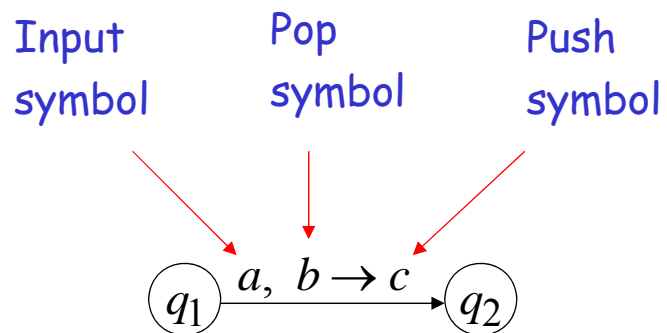
2

Initial Stack Symbol

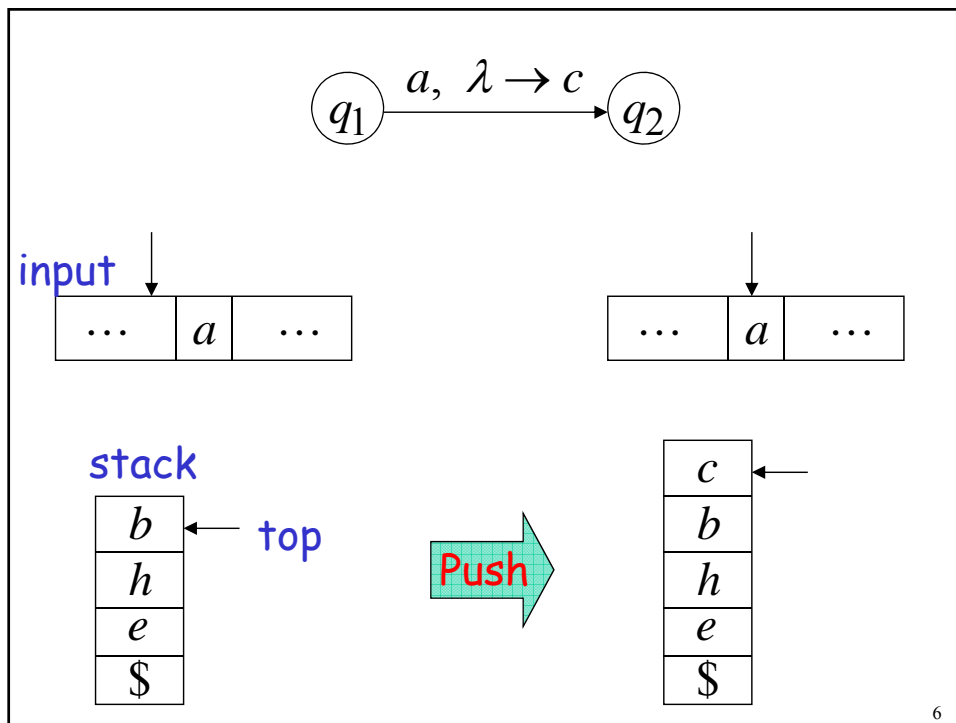
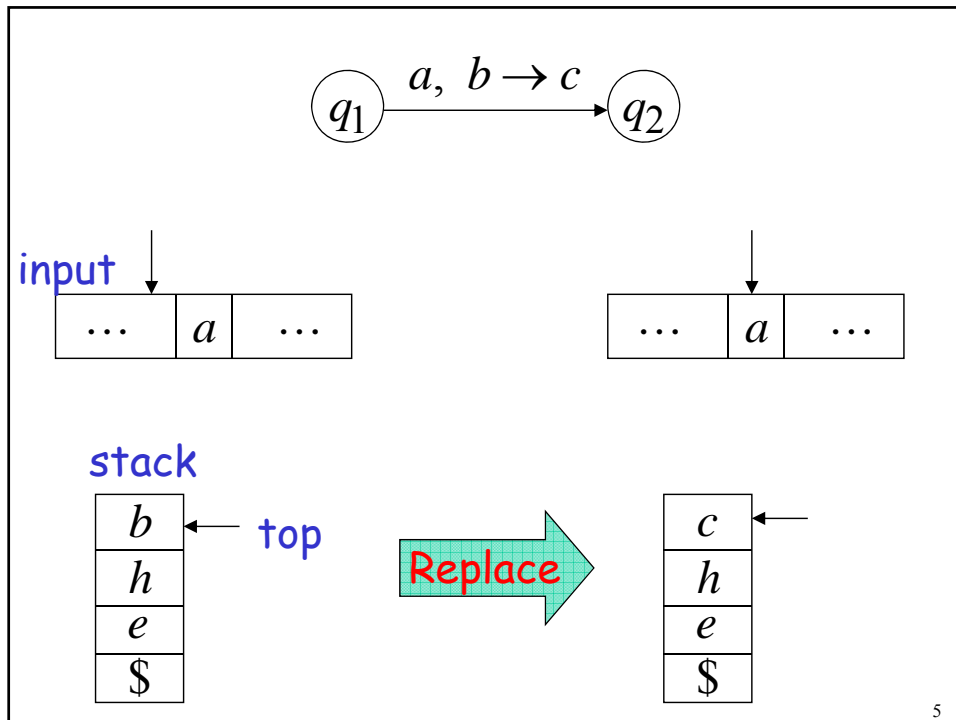


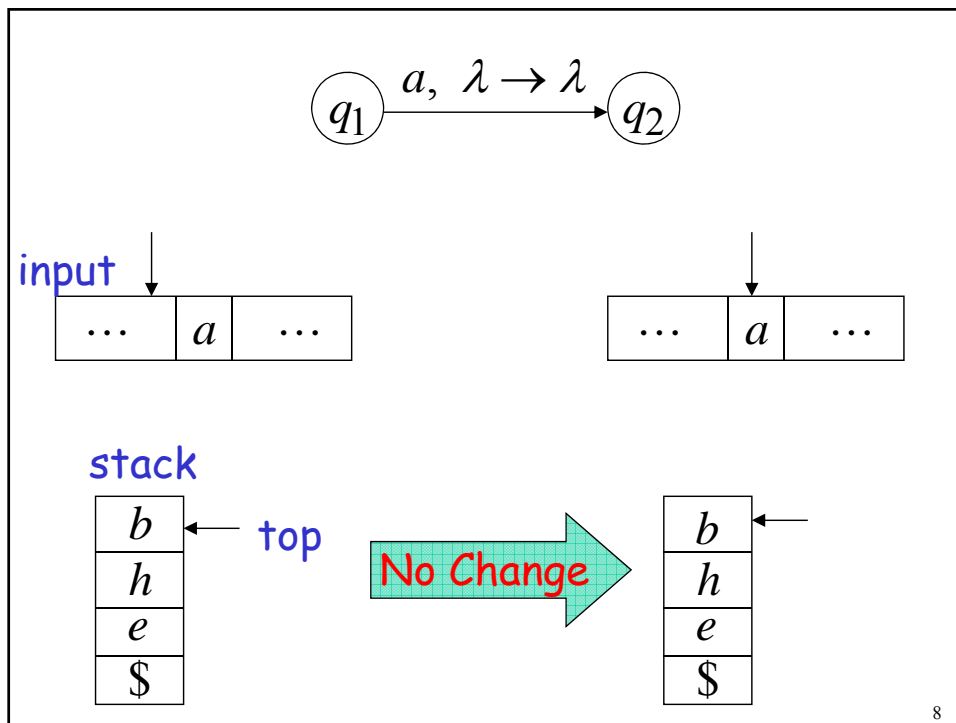
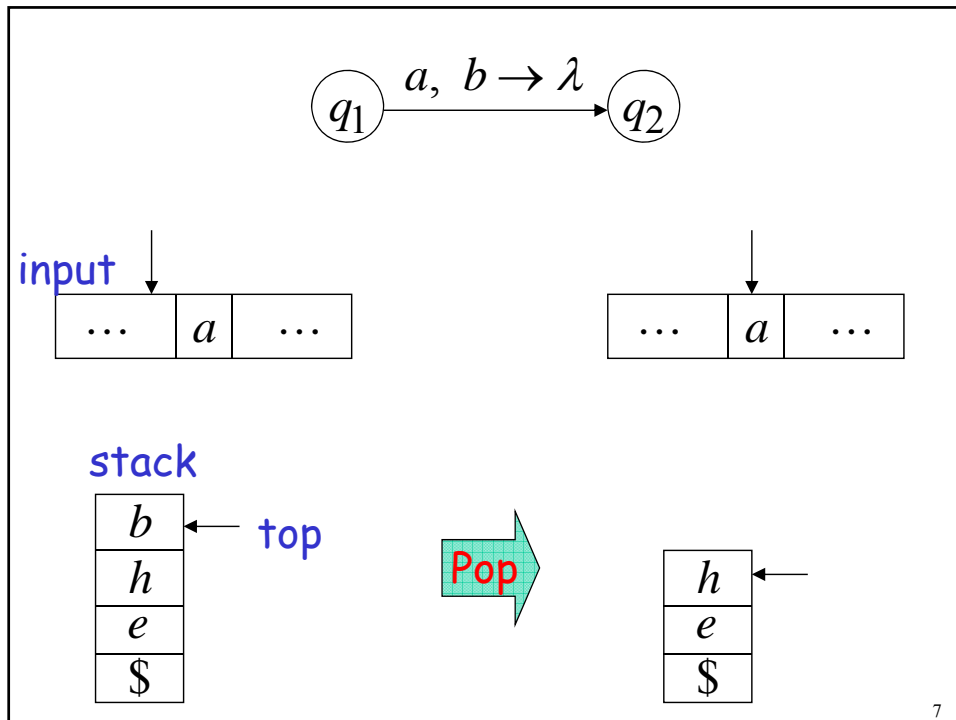
3

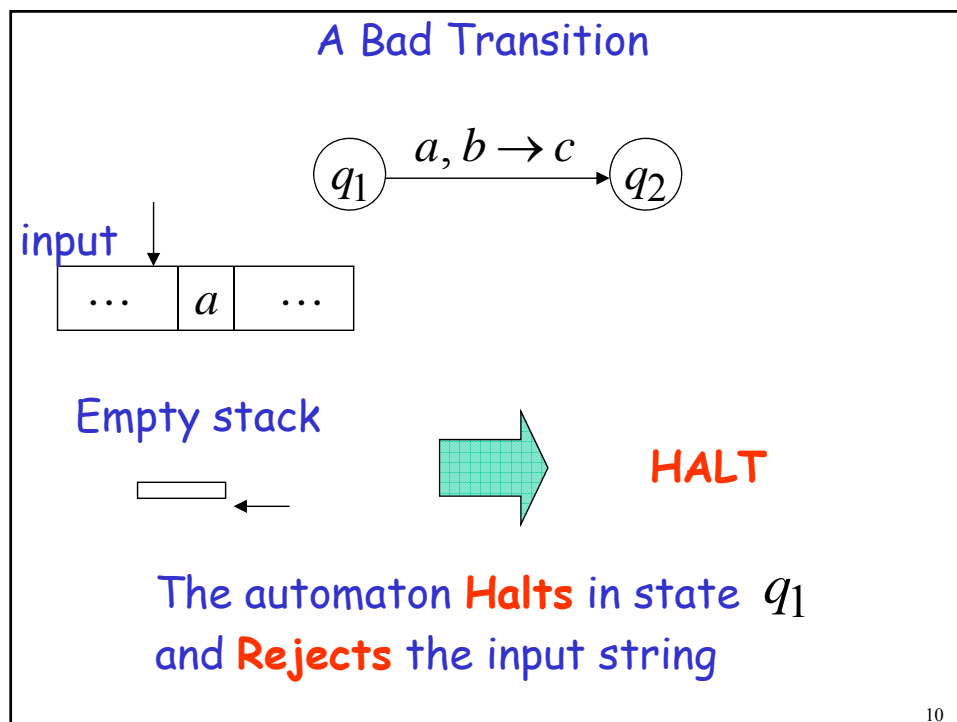
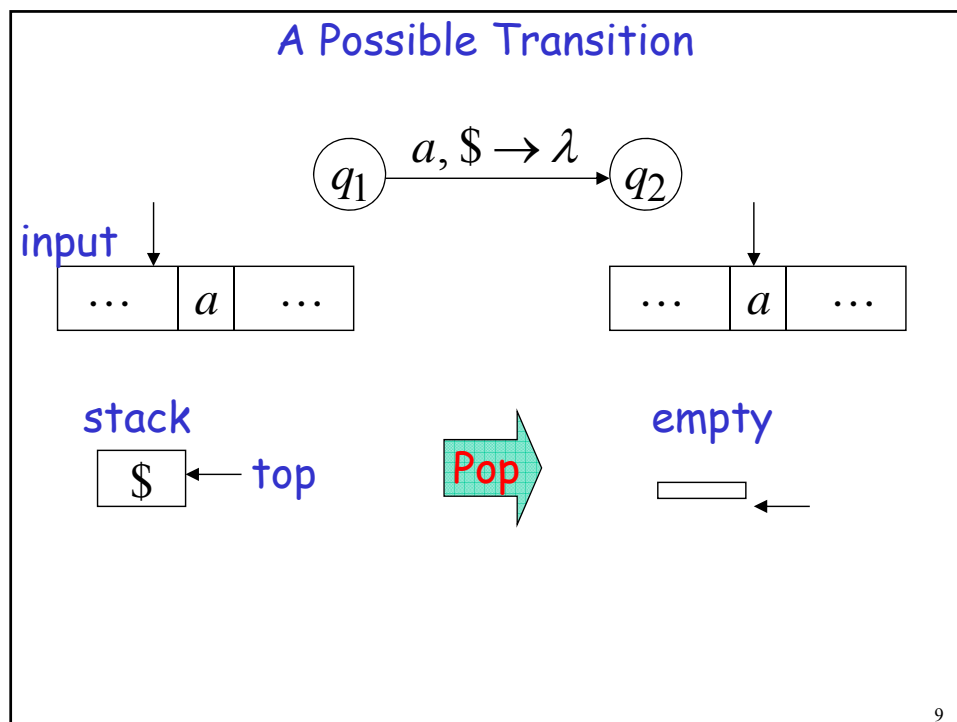
The States



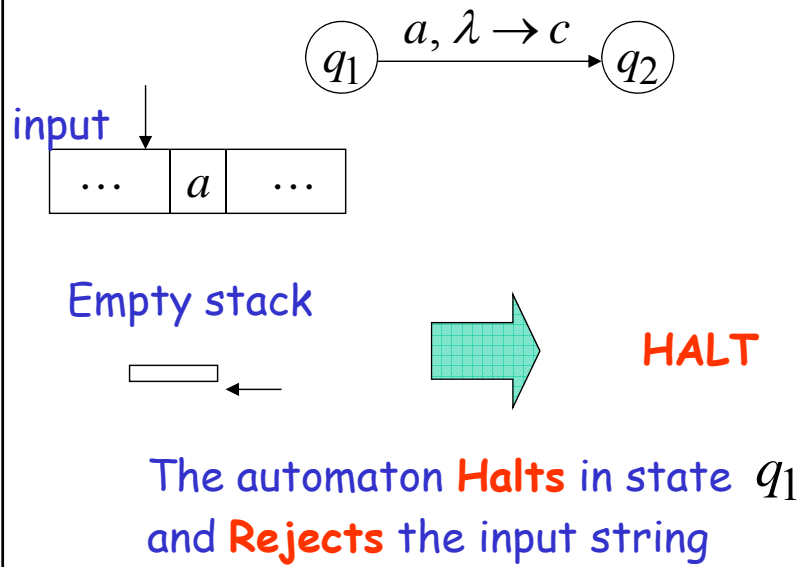
4





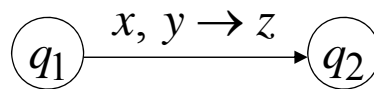


A Bad Transition

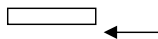


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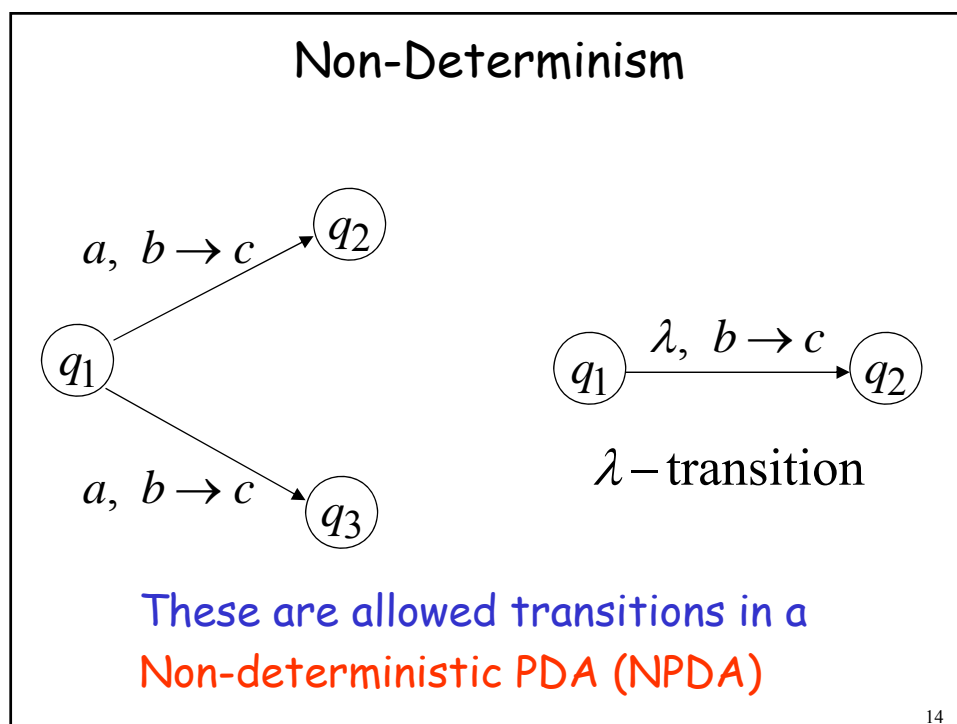
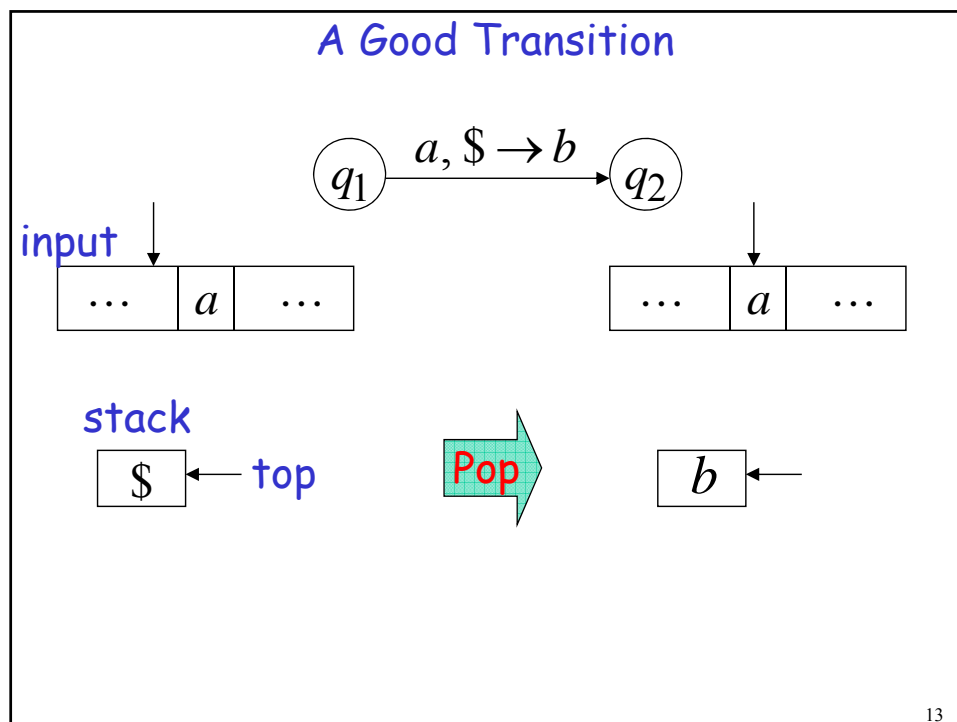
No transition is allowed to be followed
When the stack is empty



Empty stack

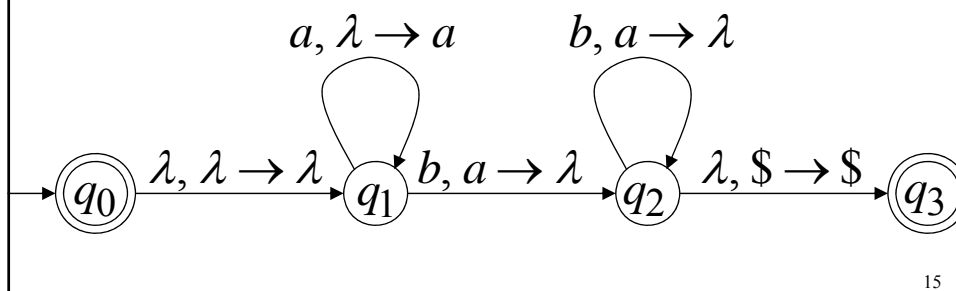


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NPDA: Non-Deterministic PDA

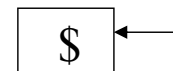
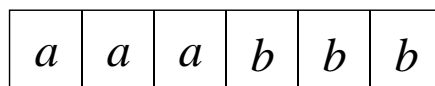
Example:



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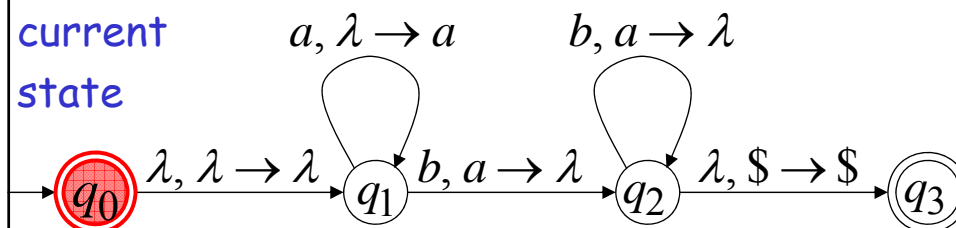
Execution Example: Time 0

Input

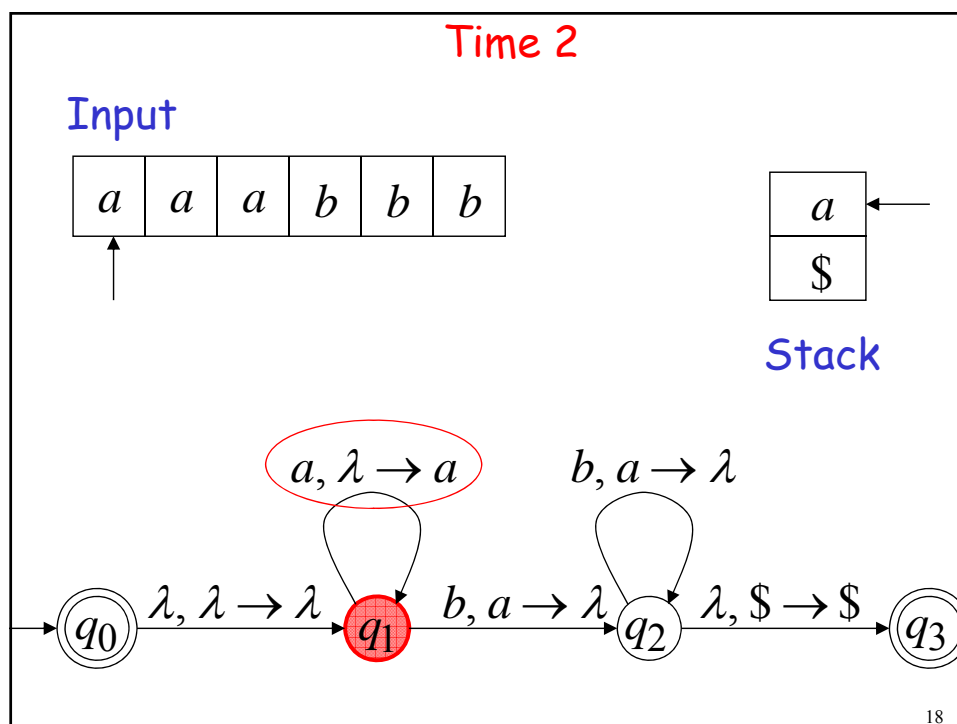
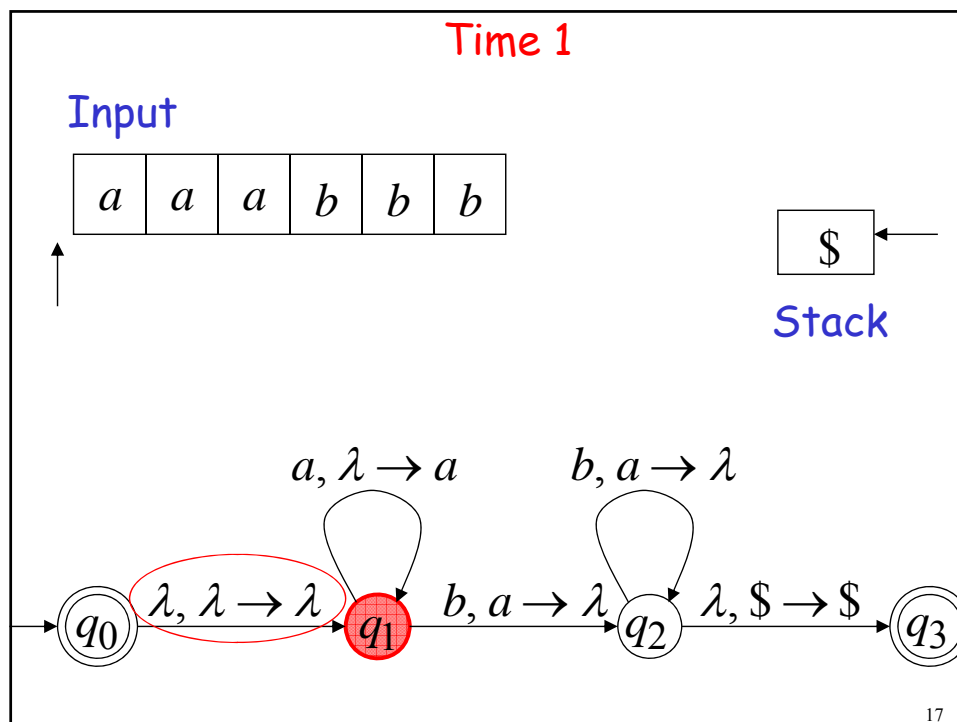


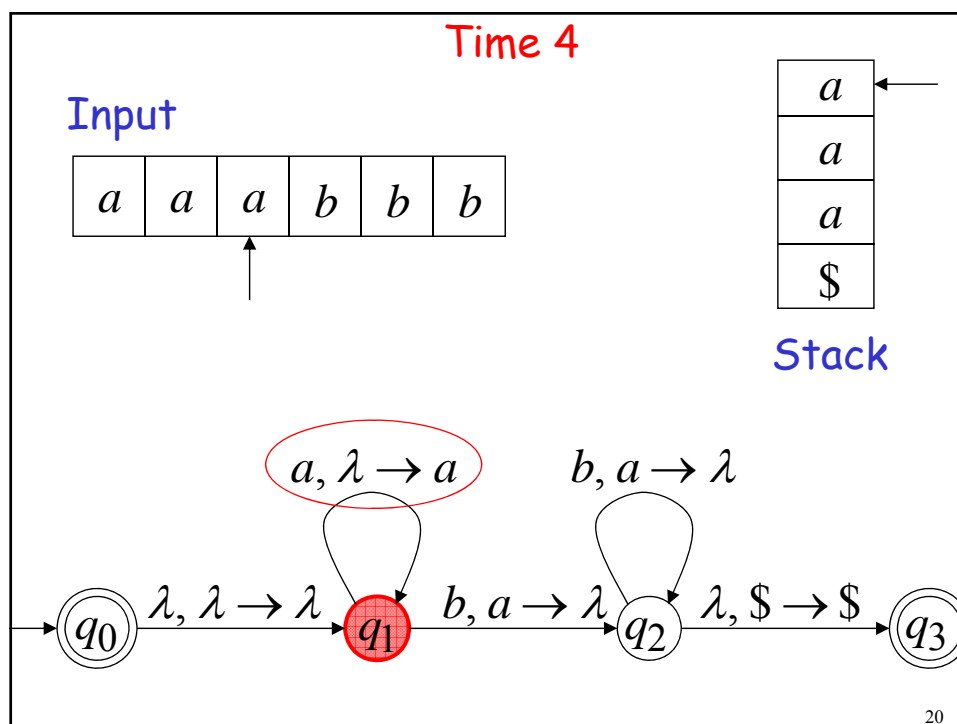
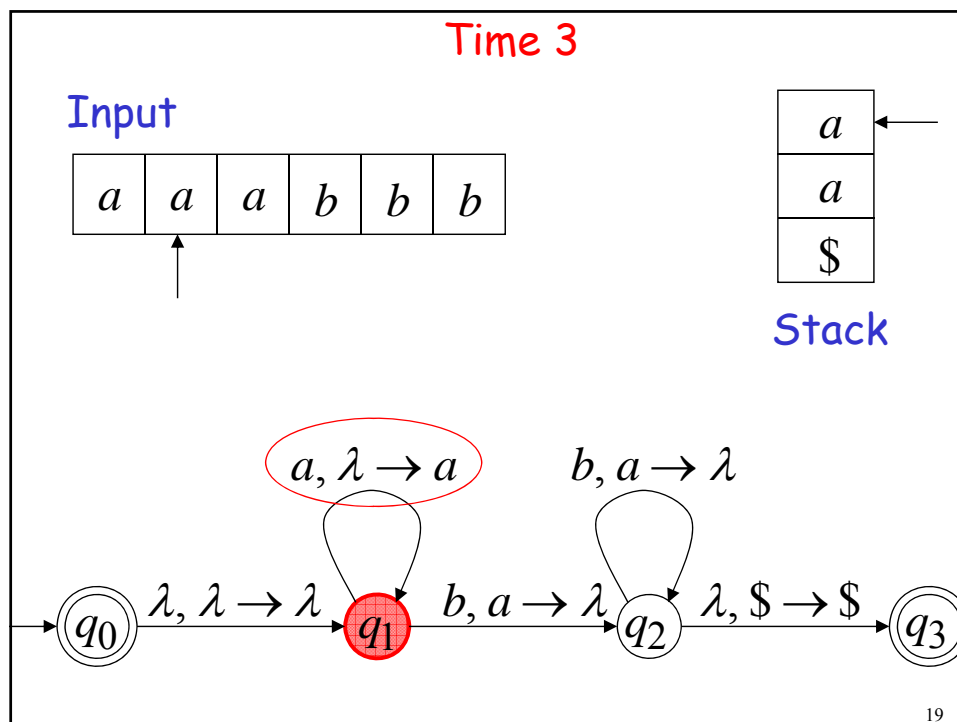
Stack

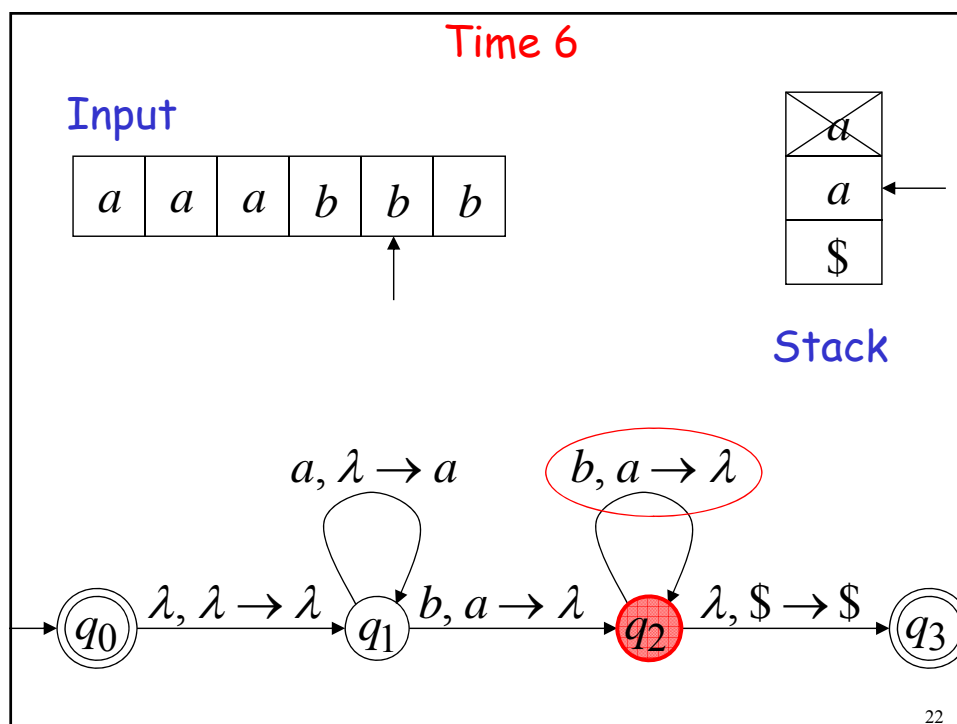
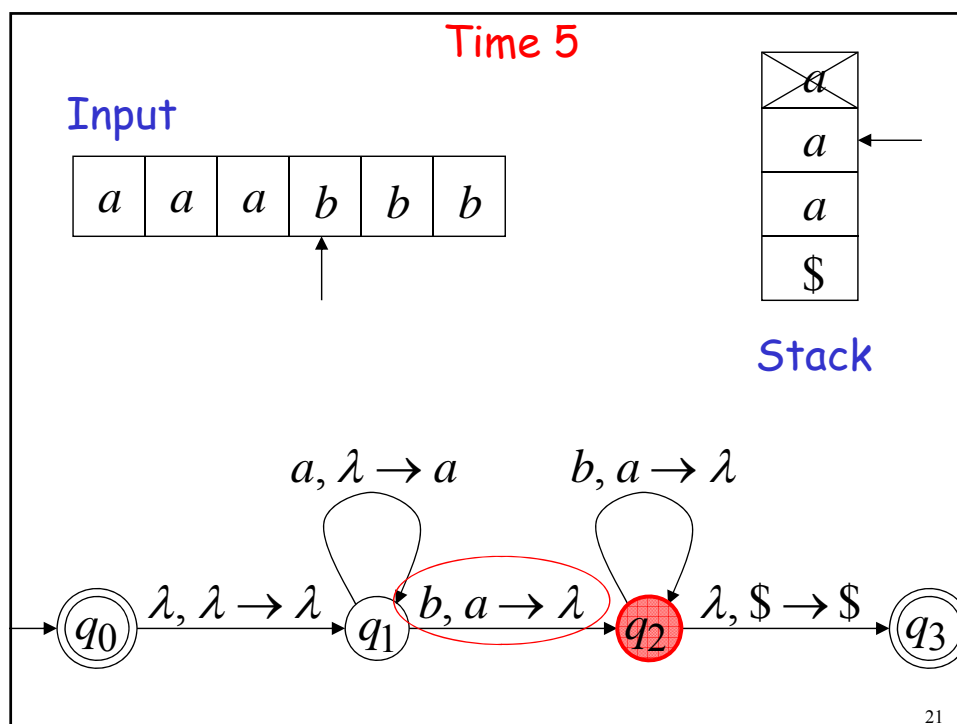
current
state

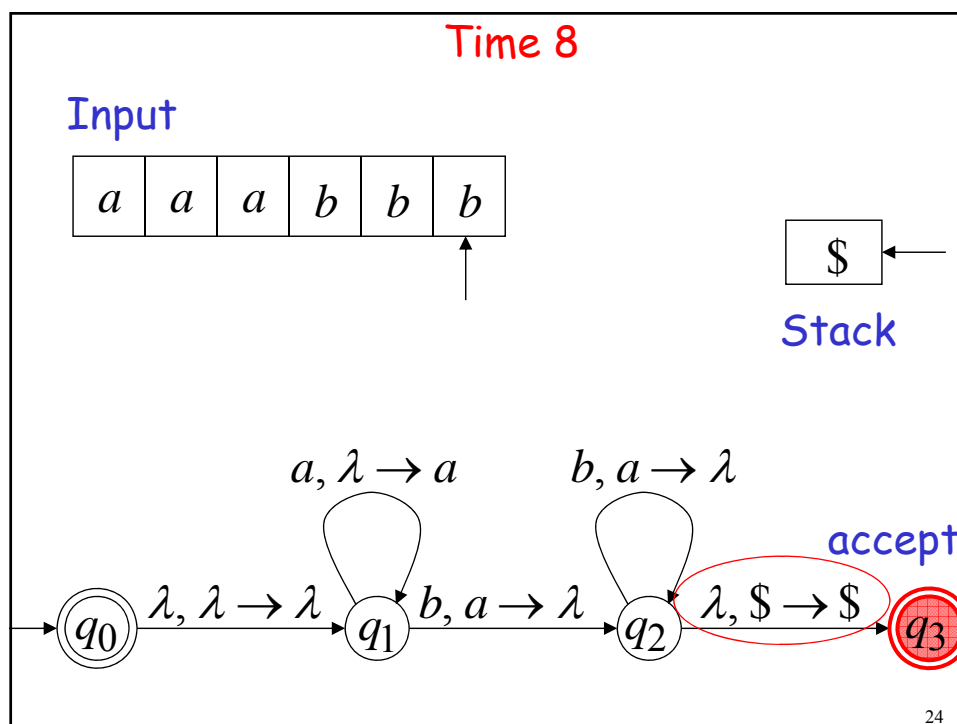
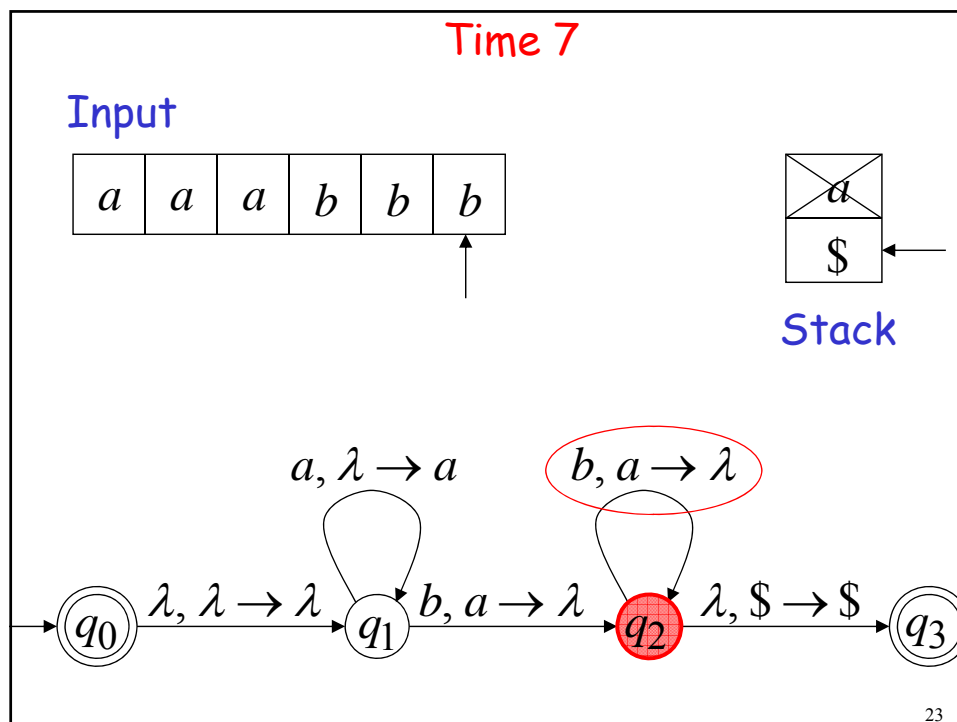


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A string is accepted if there is
a computation such that:

All the input is consumed

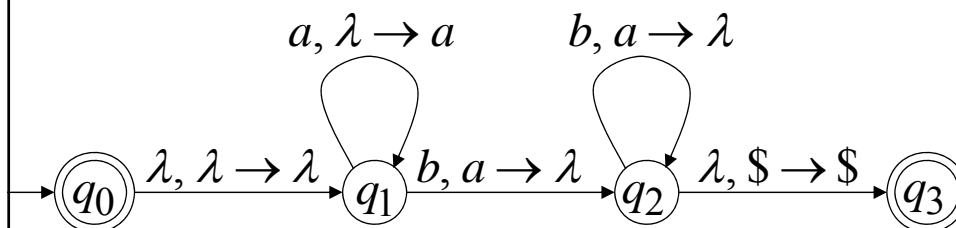
AND

The last state is a final state

At the end of the computation,
we do not care about the stack contents

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The input string *aaabbb*
is accepted by the NPDA:

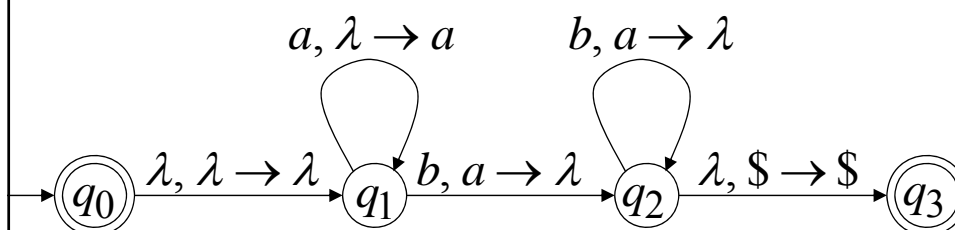


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In general,

$$L = \{a^n b^n : n \geq 0\}$$

is the language accepted by the NPDA:

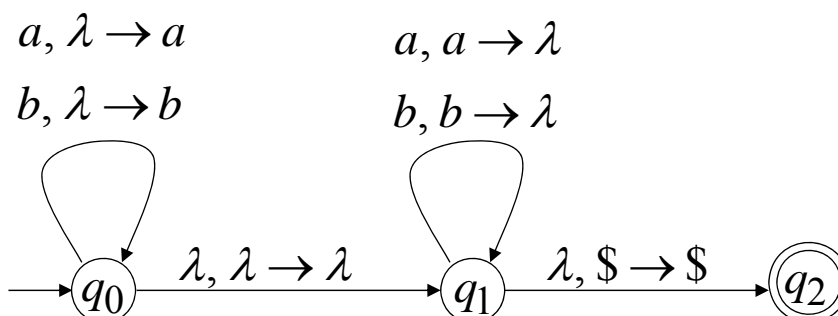


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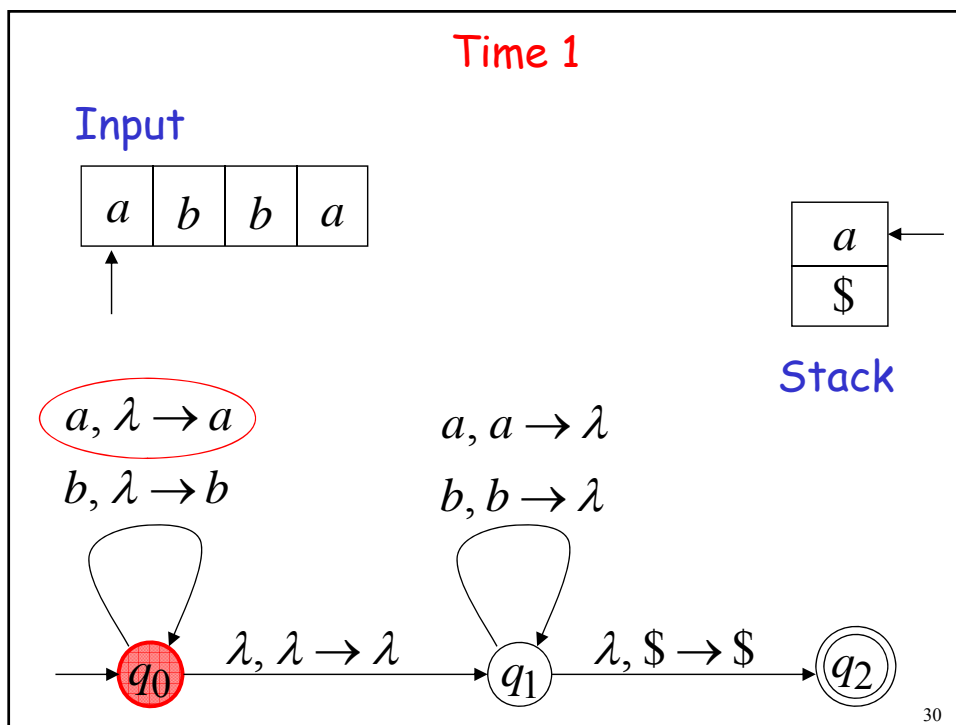
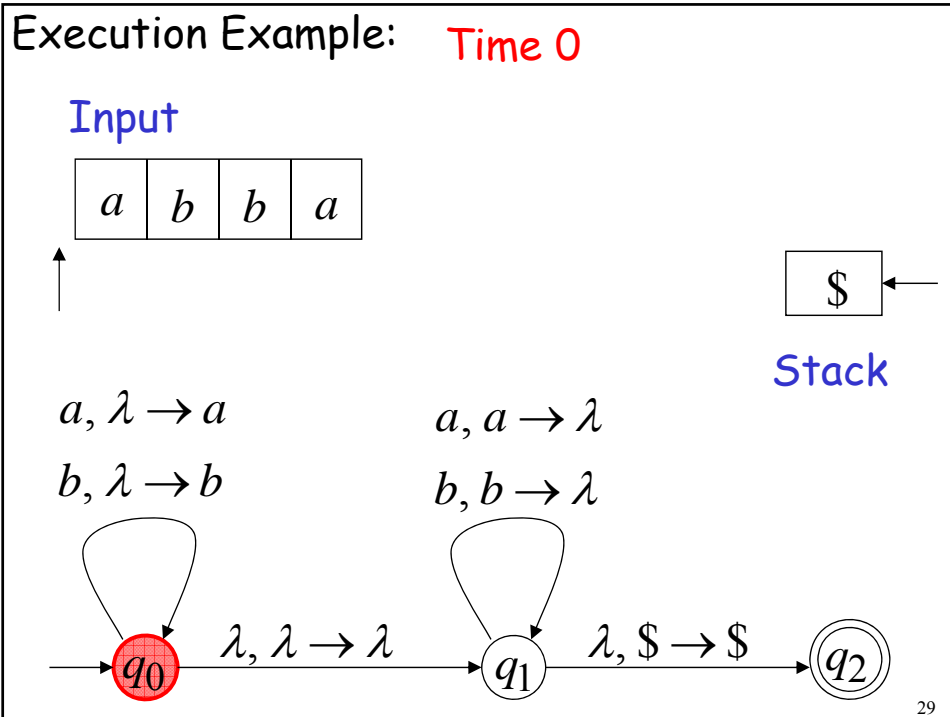
Another NPDA example

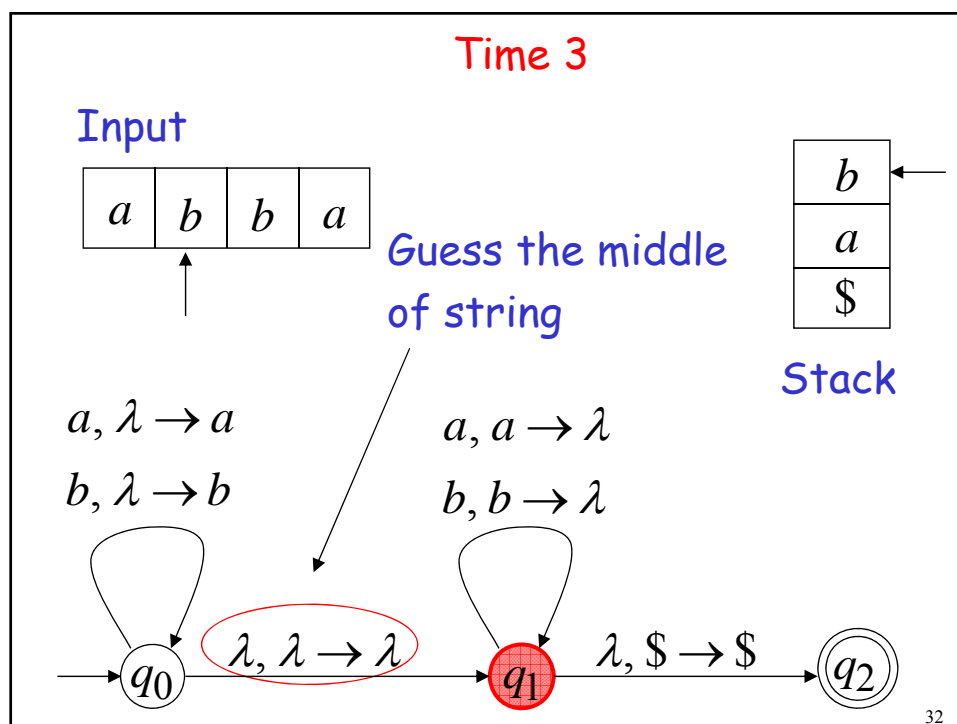
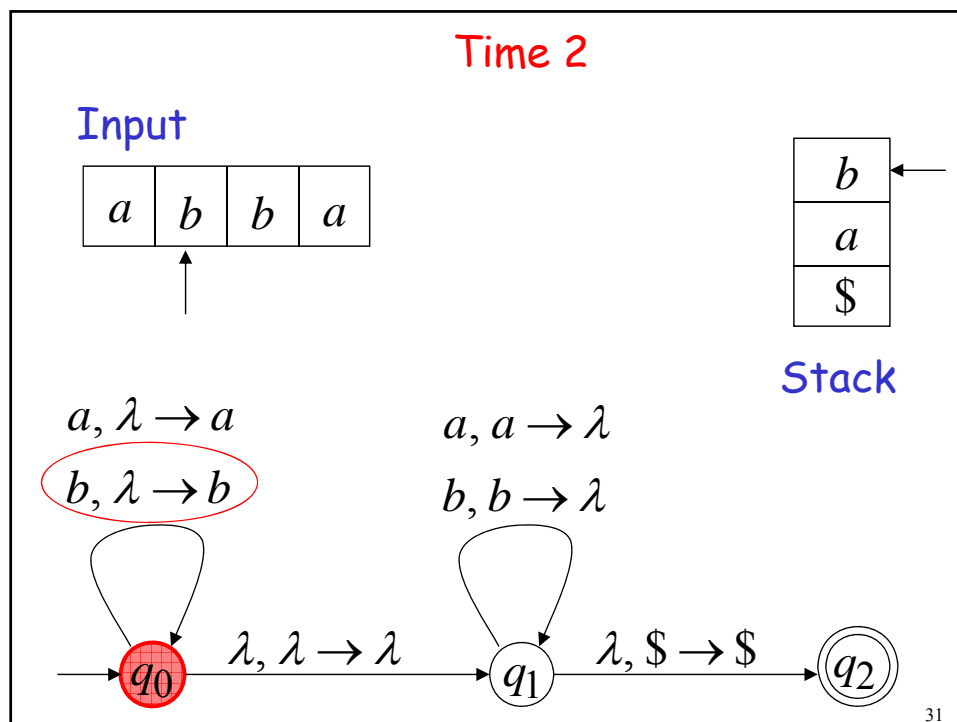
NPDA M

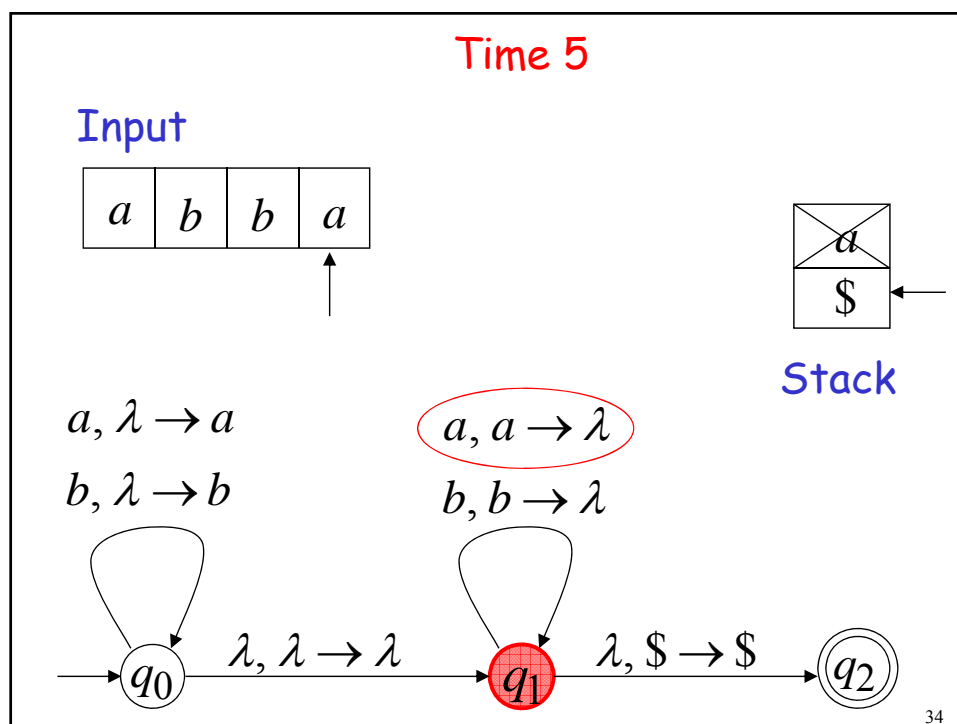
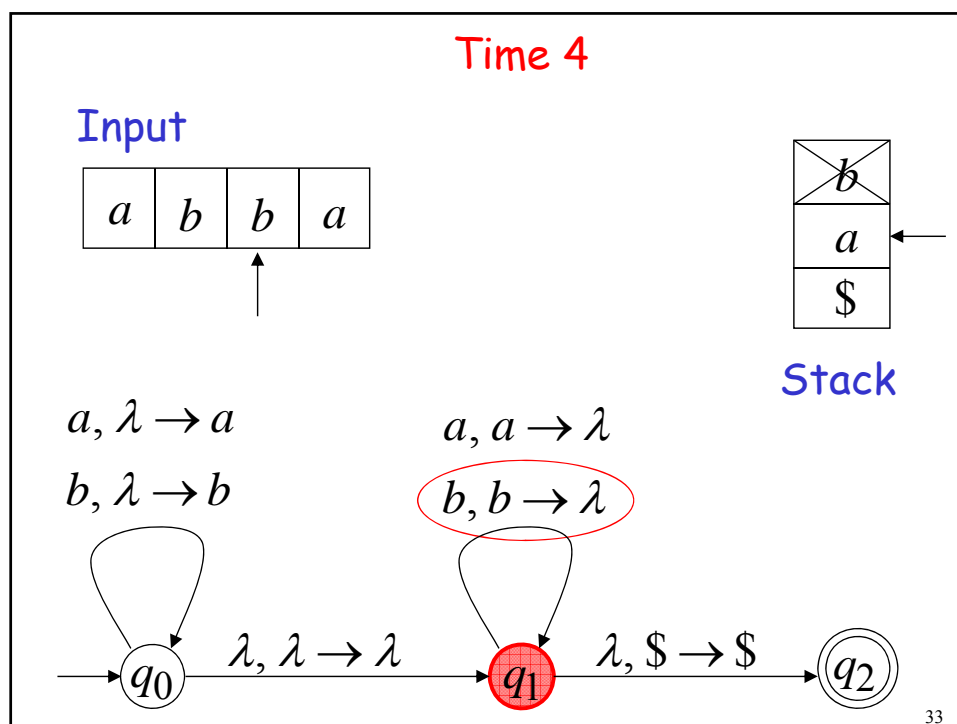
$$L(M) = \{ww^R\}$$

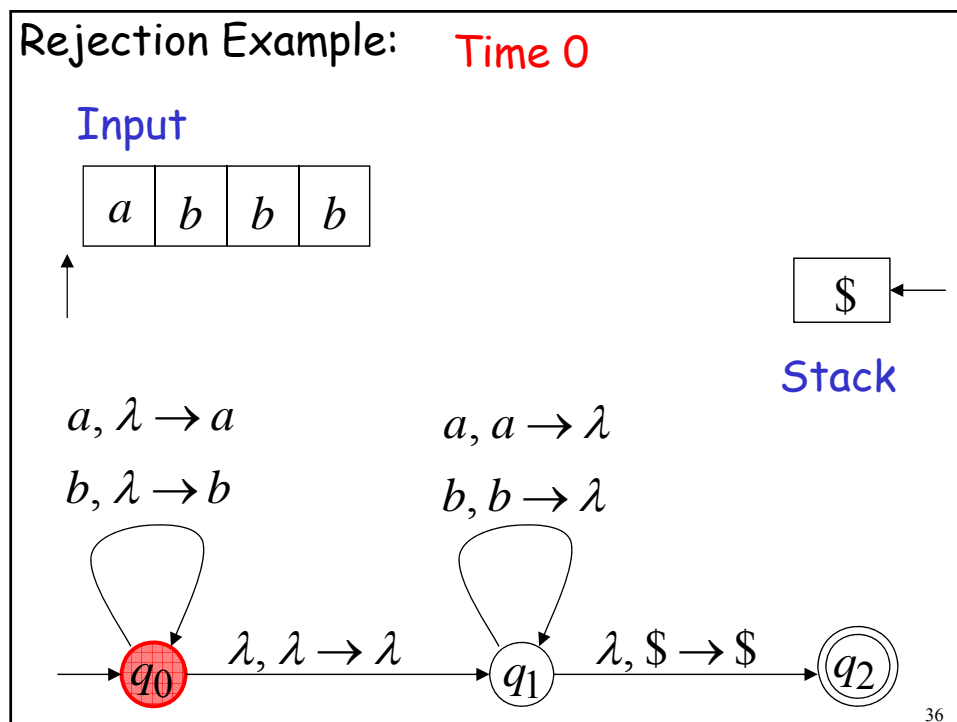
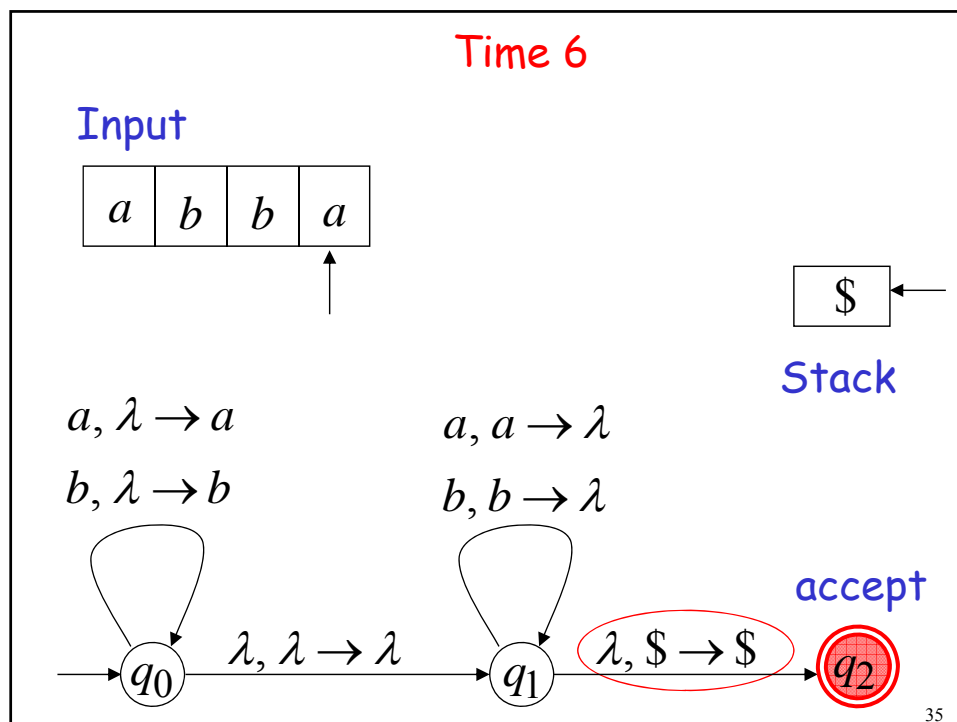


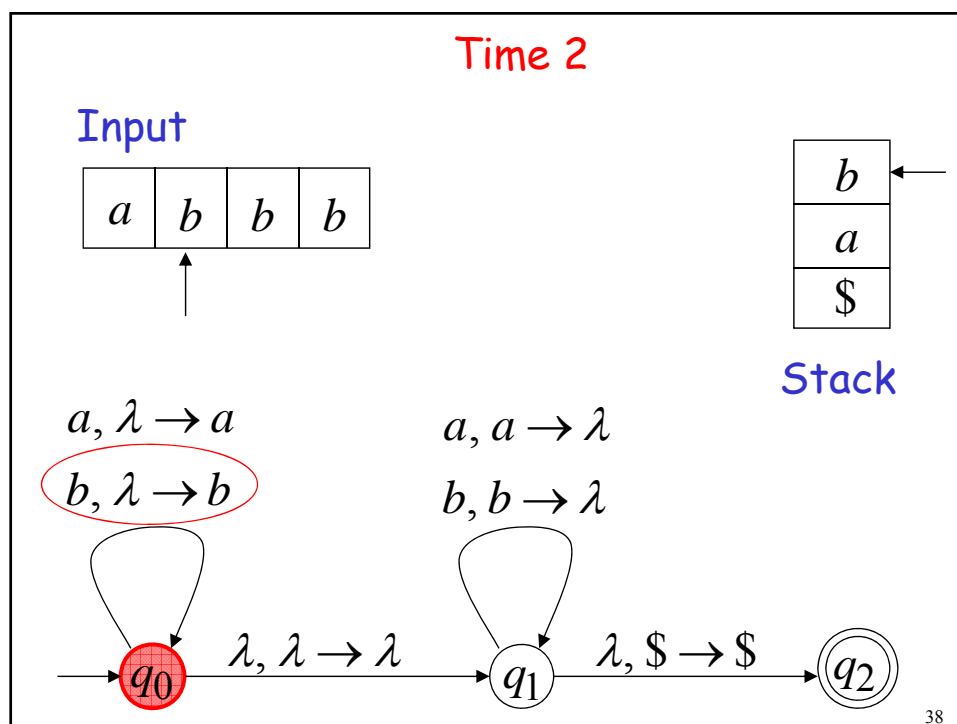
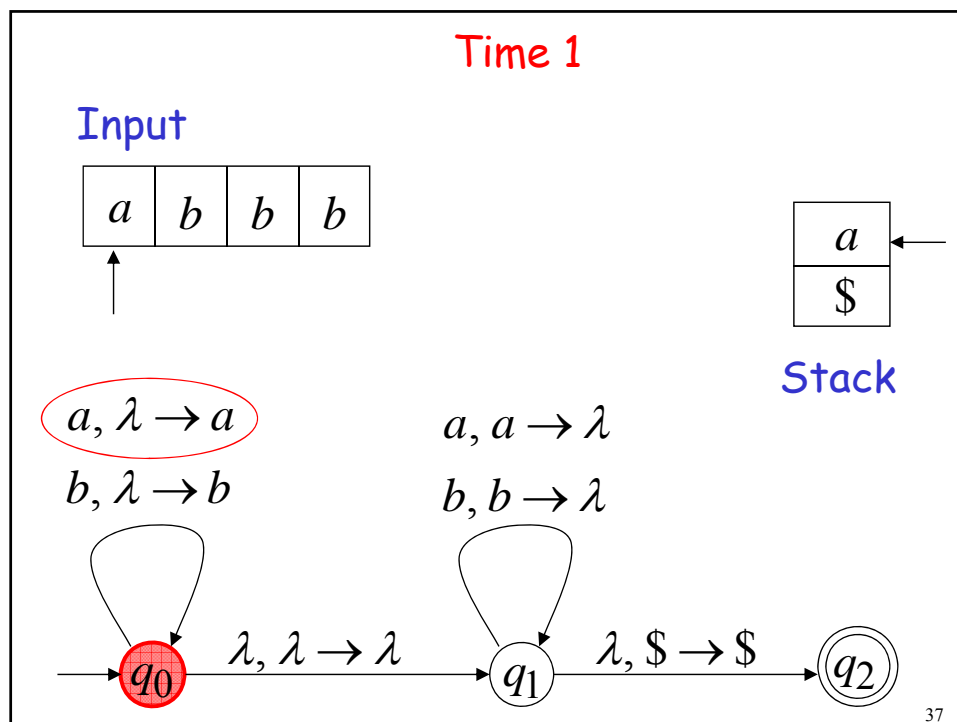
28

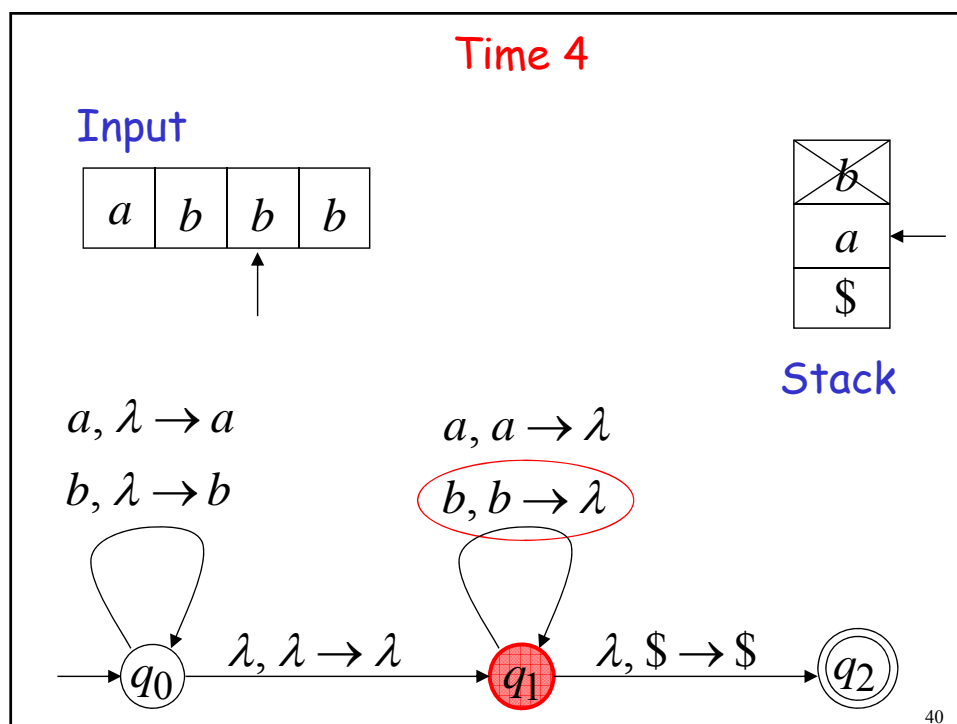
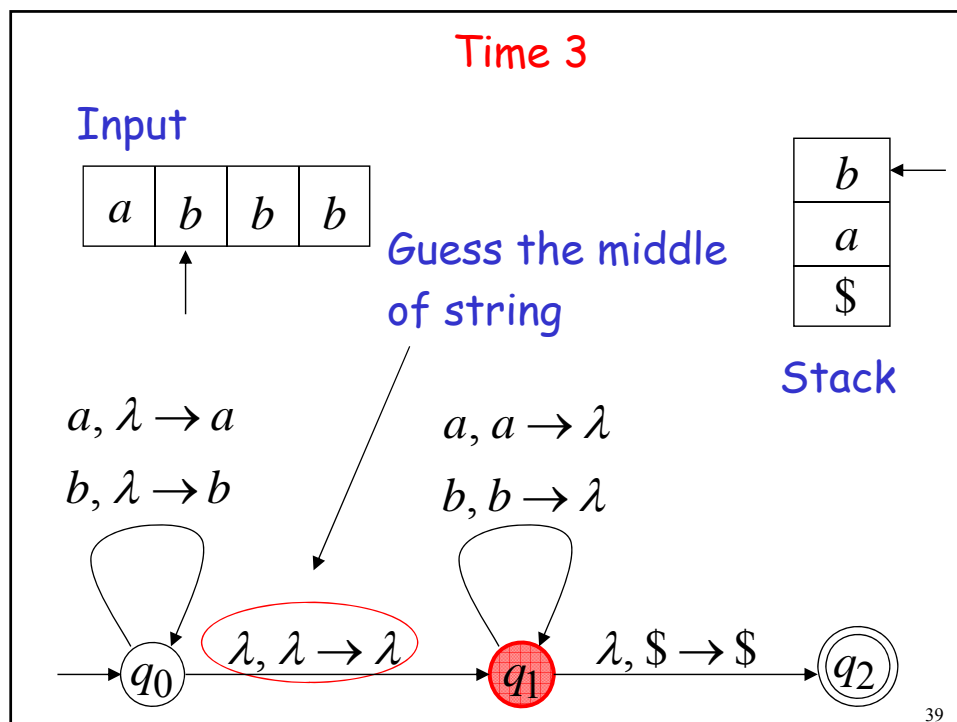


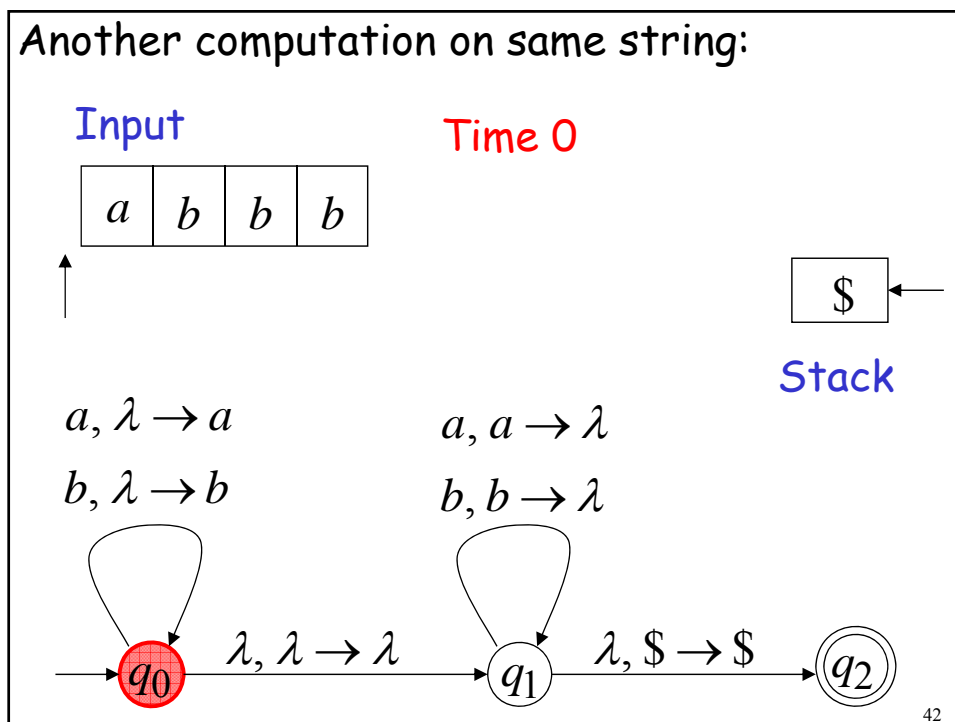
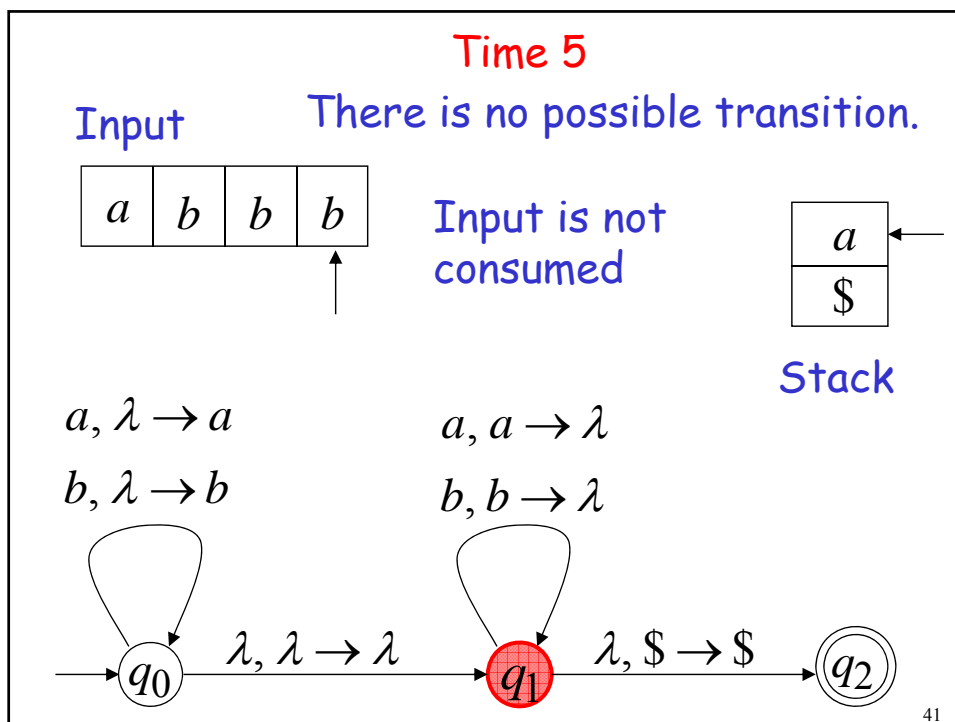


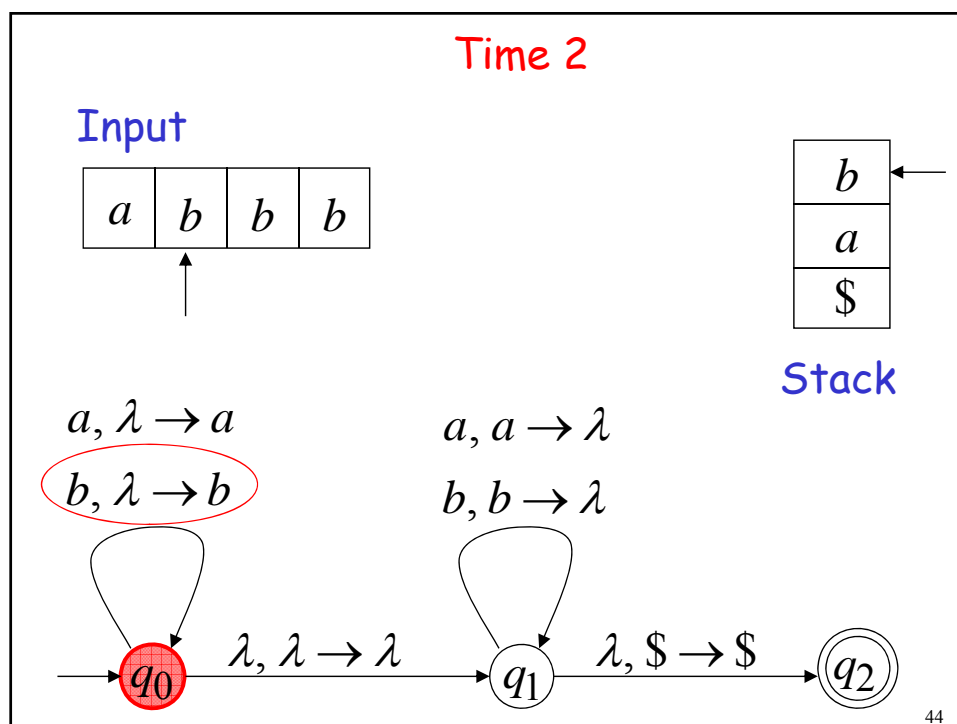
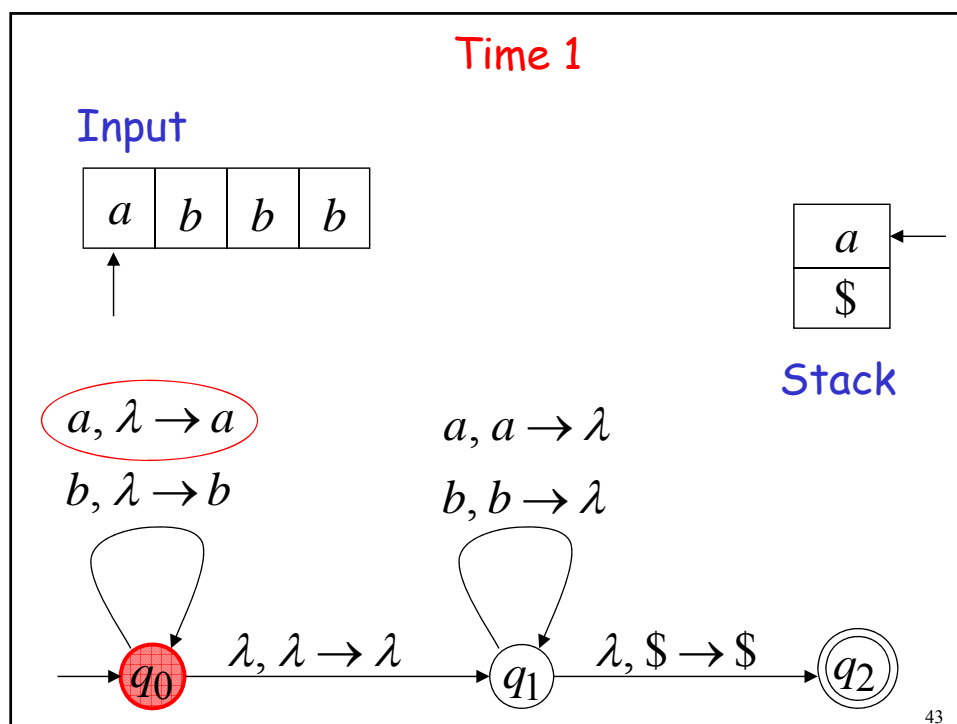


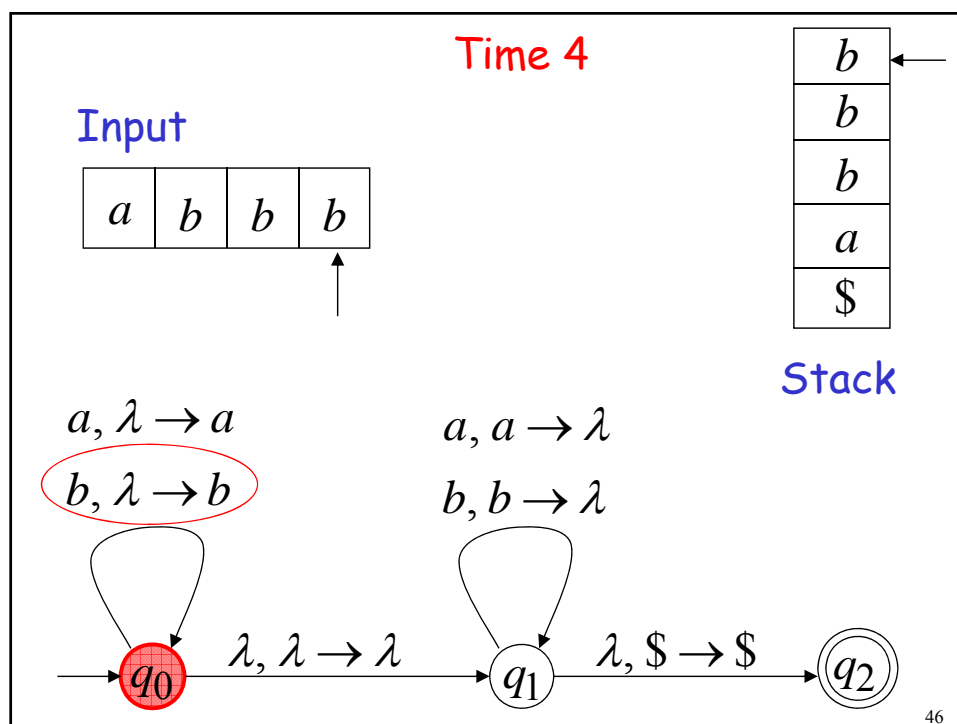
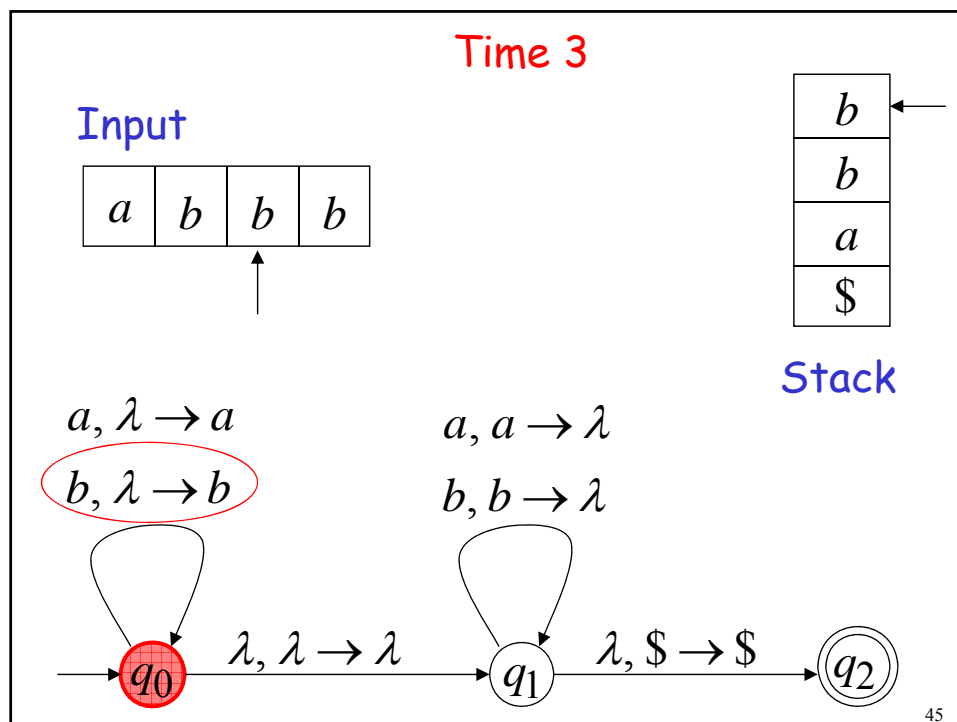


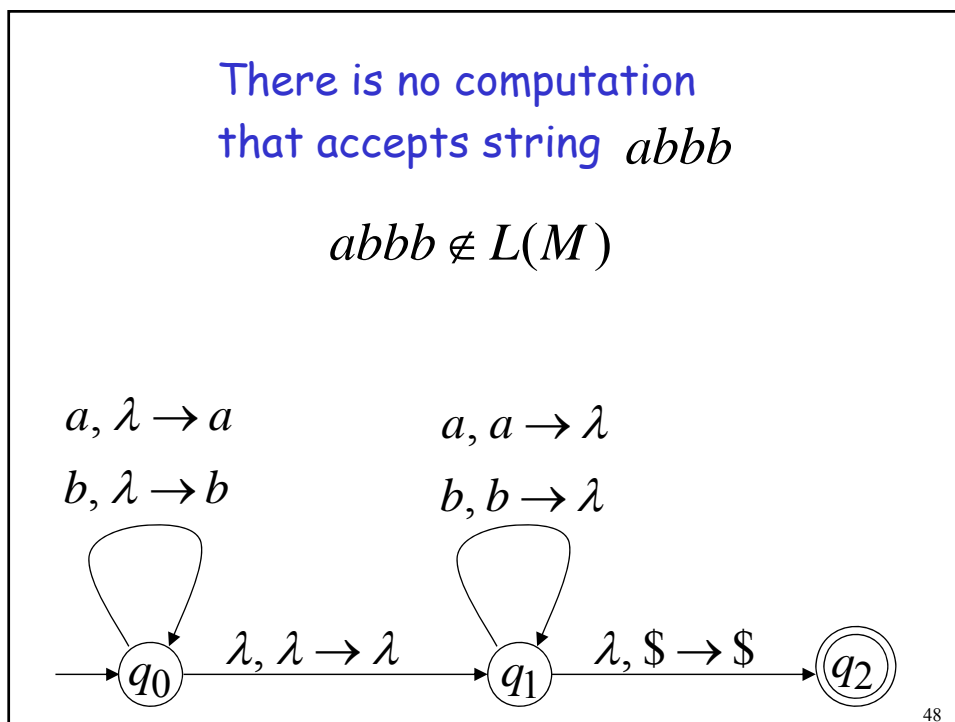
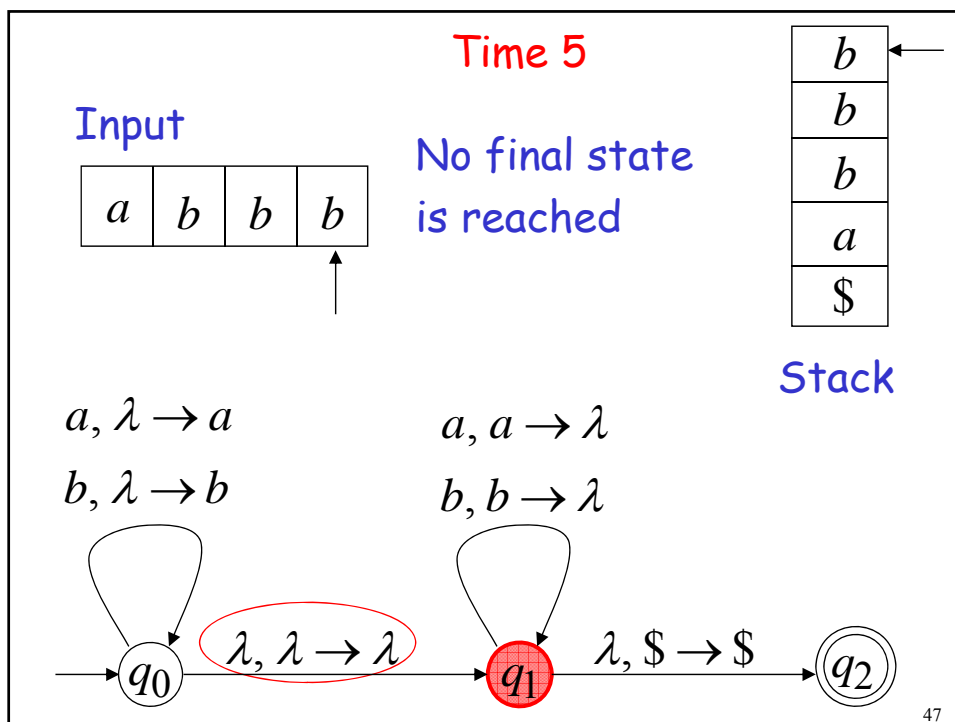












A string is rejected if there is
no computation such that:

All the input is consumed

AND

The last state is a final state

At the end of the computation,
we do not care about the stack contents

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In other words, a string is rejected
if in every computation with this string:

The input cannot be consumed

OR

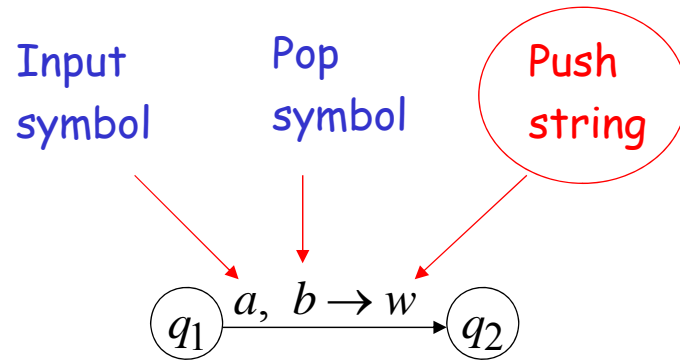
The input is consumed and the last
state is not a final state

OR

The stack head moves below the
bottom of the stack

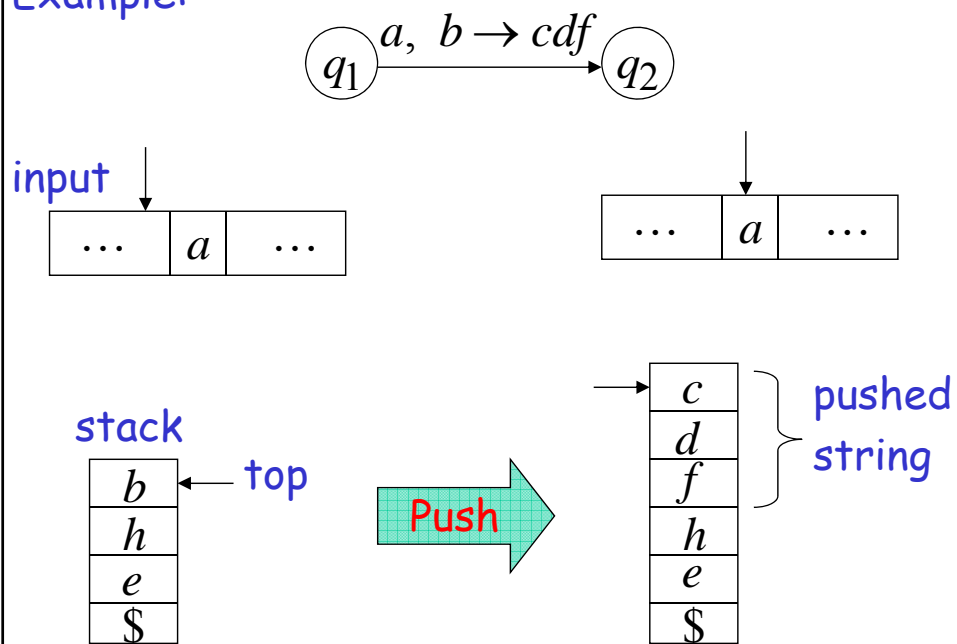
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Pushing Strings



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Example:



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Another NPDA example

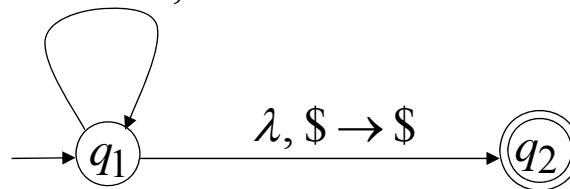
NPDA M

$$L(M) = \{w : n_a = n_b\}$$

$a, \$ \rightarrow 0\$$ $b, \$ \rightarrow 1\$$

$a, 0 \rightarrow 00$ $b, 1 \rightarrow 11$

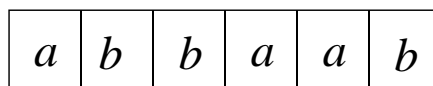
$a, 1 \rightarrow \lambda$ $b, 0 \rightarrow \lambda$



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Execution Example: Time 0

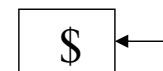
Input



$a, \$ \rightarrow 0\$$ $b, \$ \rightarrow 1\$$

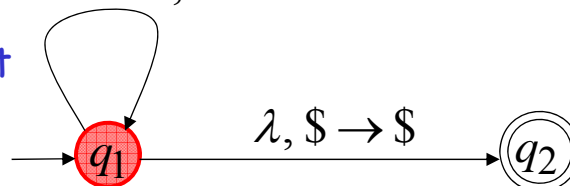
$a, 0 \rightarrow 00$ $b, 1 \rightarrow 11$

$a, 1 \rightarrow \lambda$ $b, 0 \rightarrow \lambda$

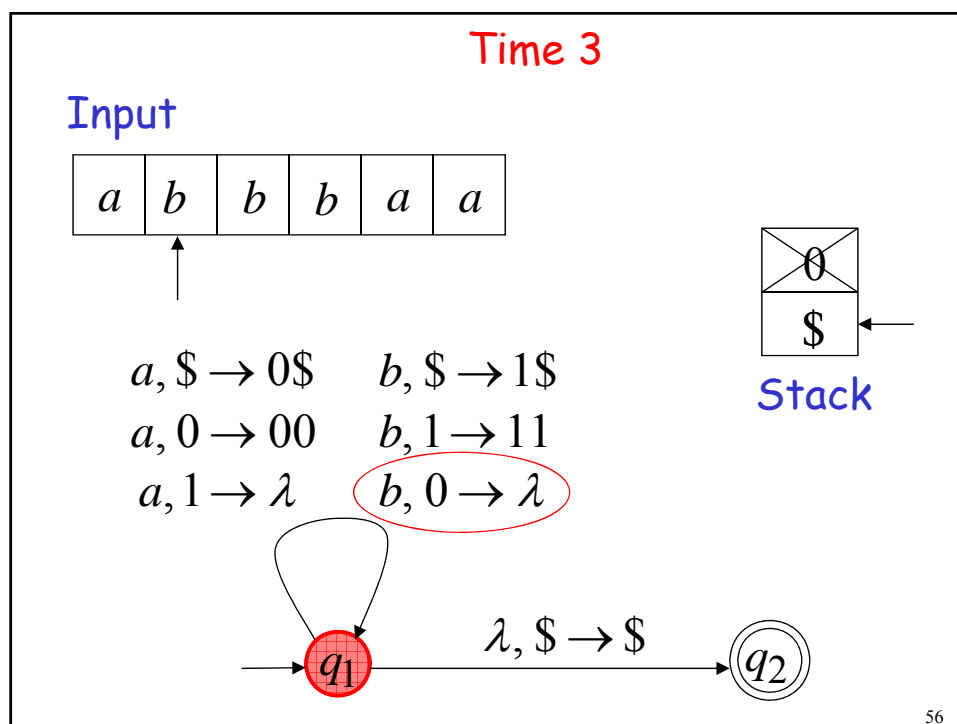
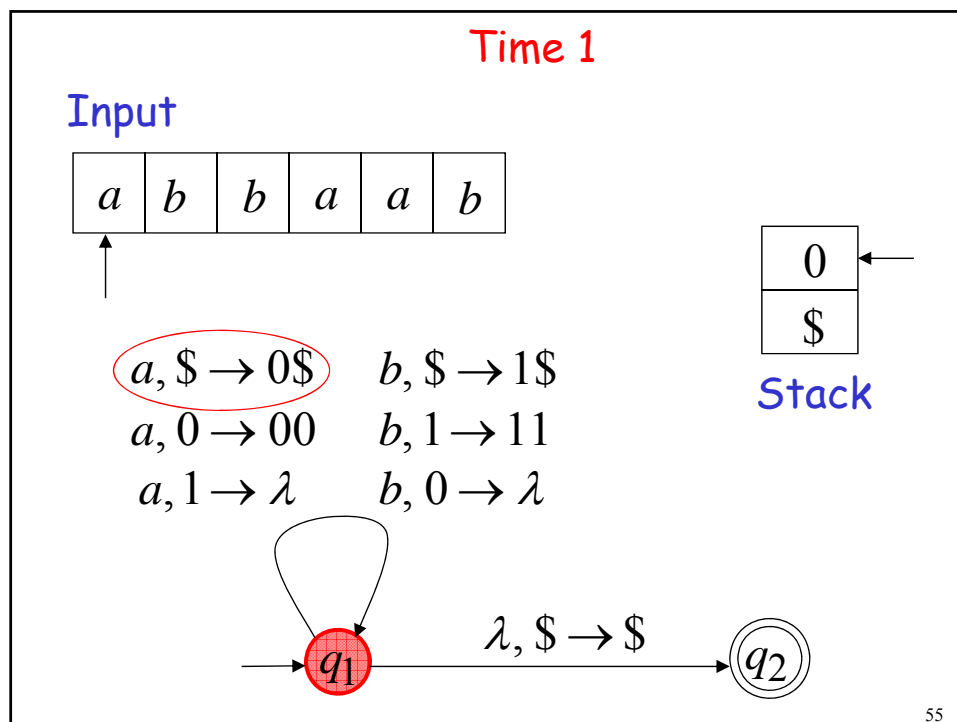


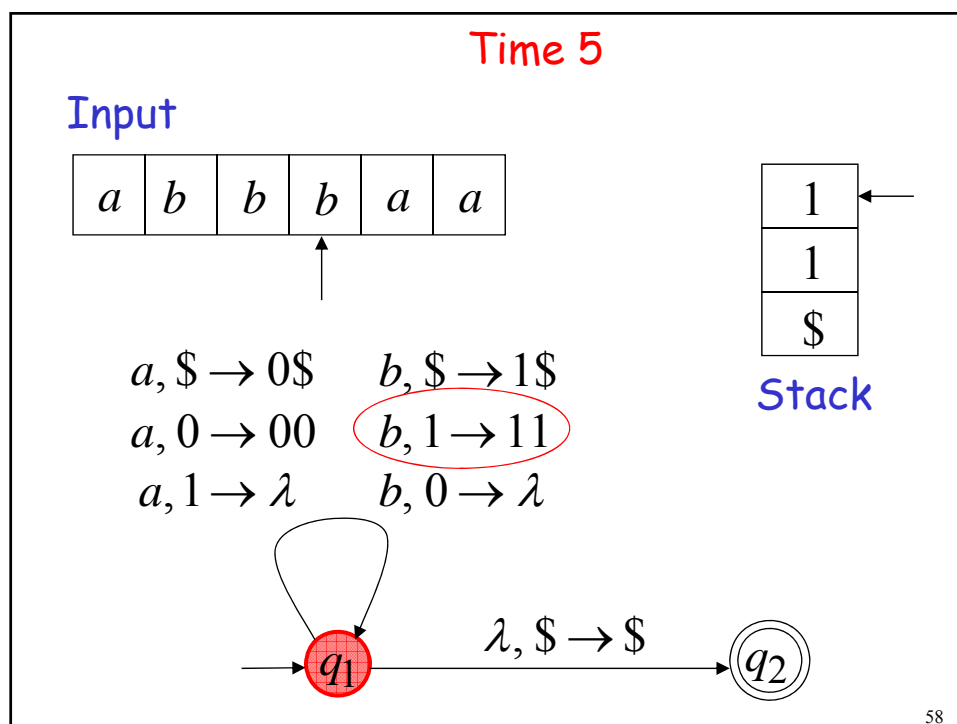
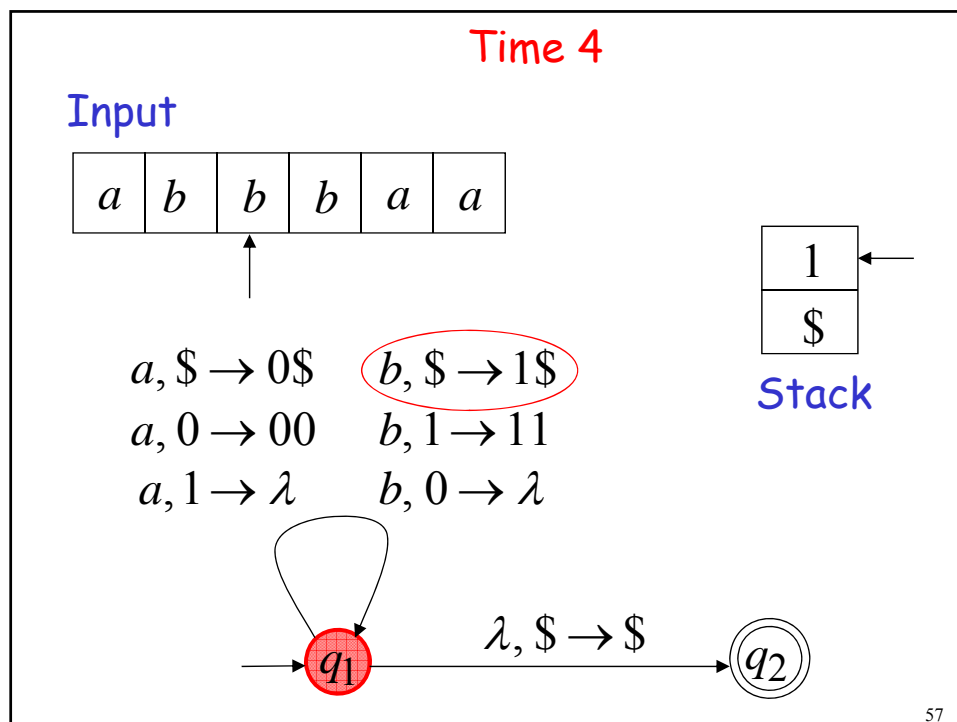
Stack

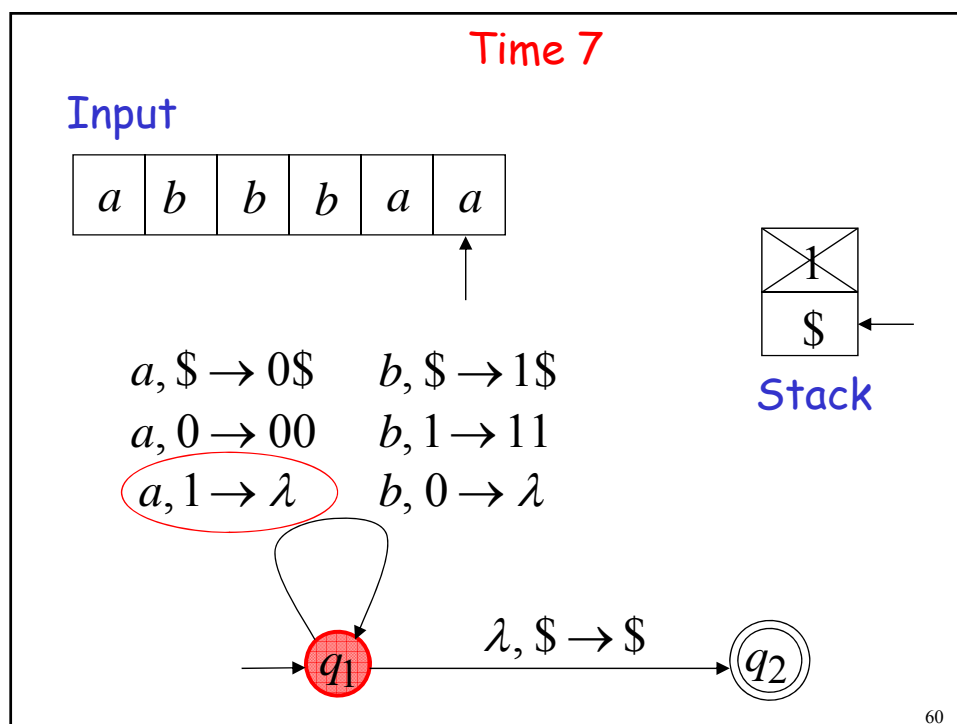
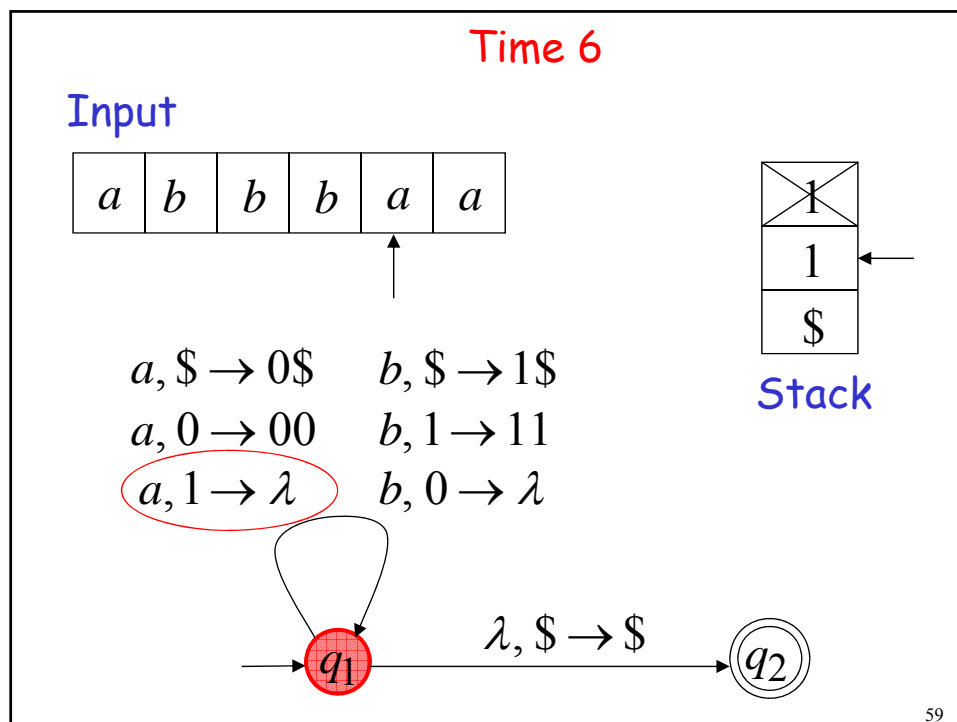
current
state

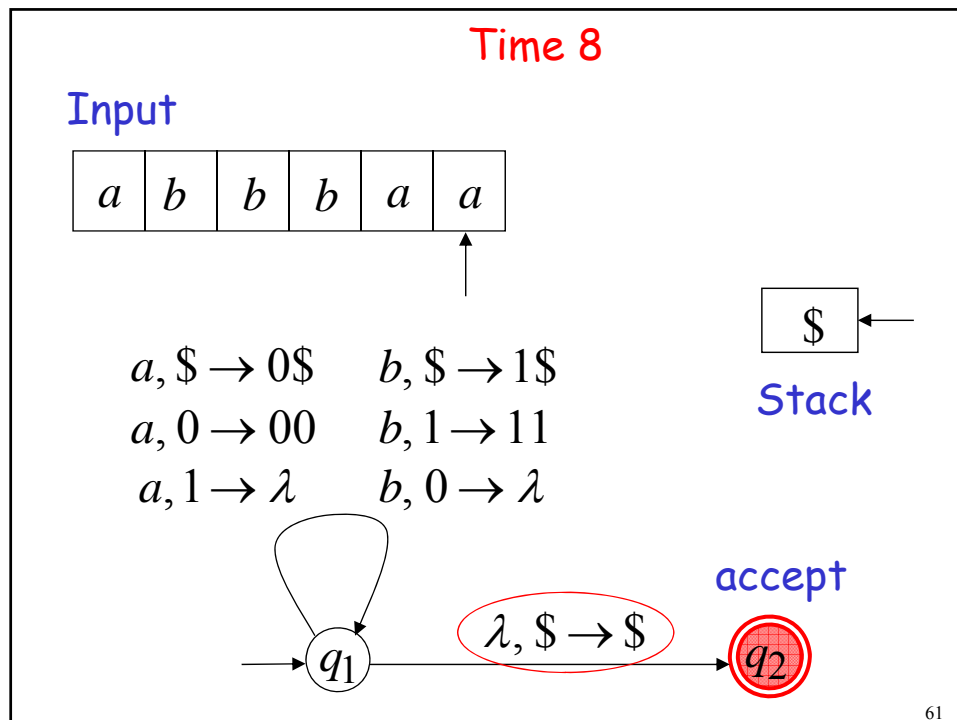


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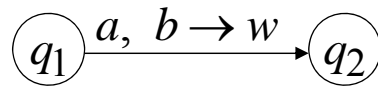






Formalities for NPDAs

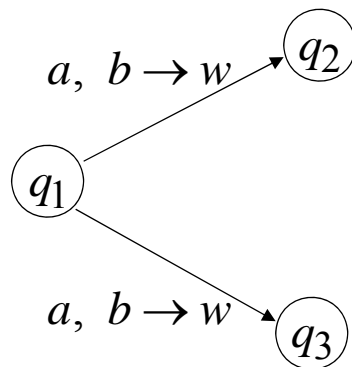
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Transition function:

$$\delta(q_1, a, b) = \{(q_2, w)\}$$

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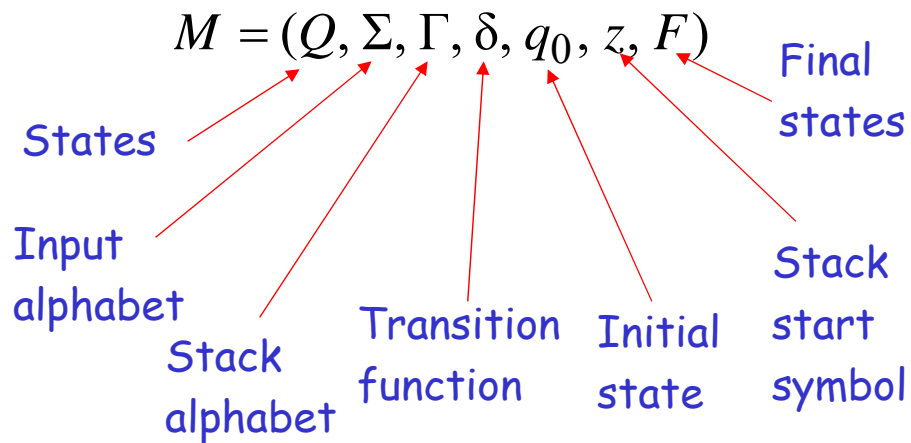
Transition function:

$$\delta(q_1, a, b) = \{(q_2, w), (q_3, w)\}$$

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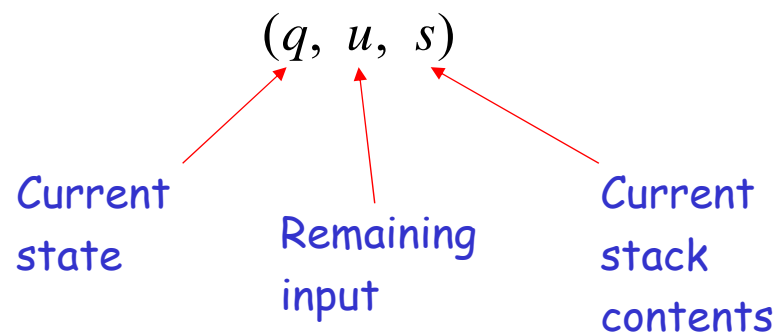
Formal Definition

Non-Deterministic Pushdown Automaton NPDA

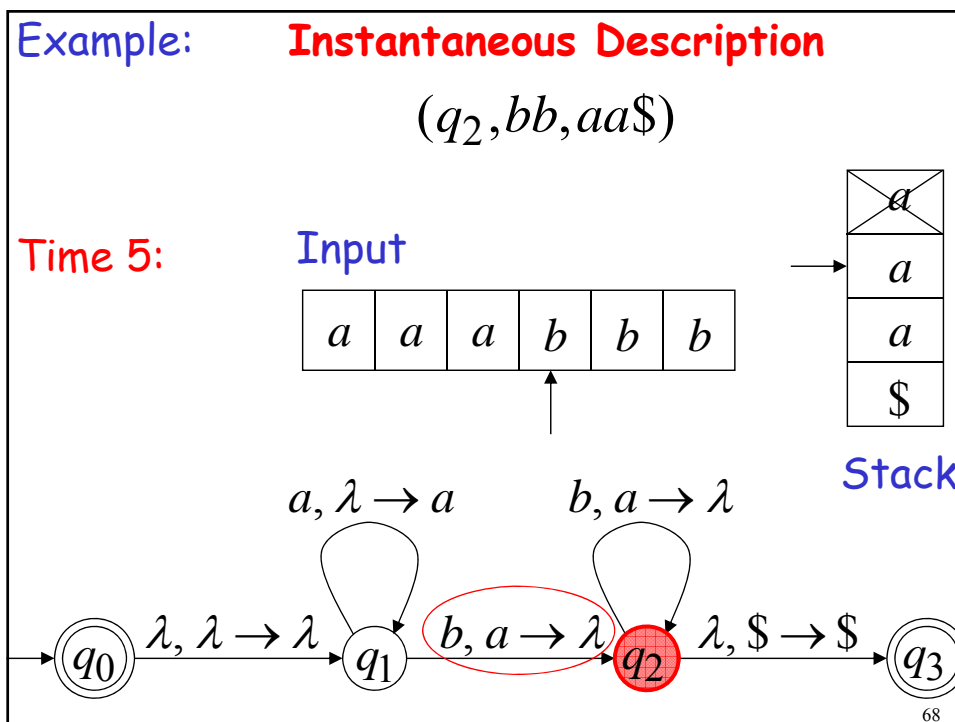
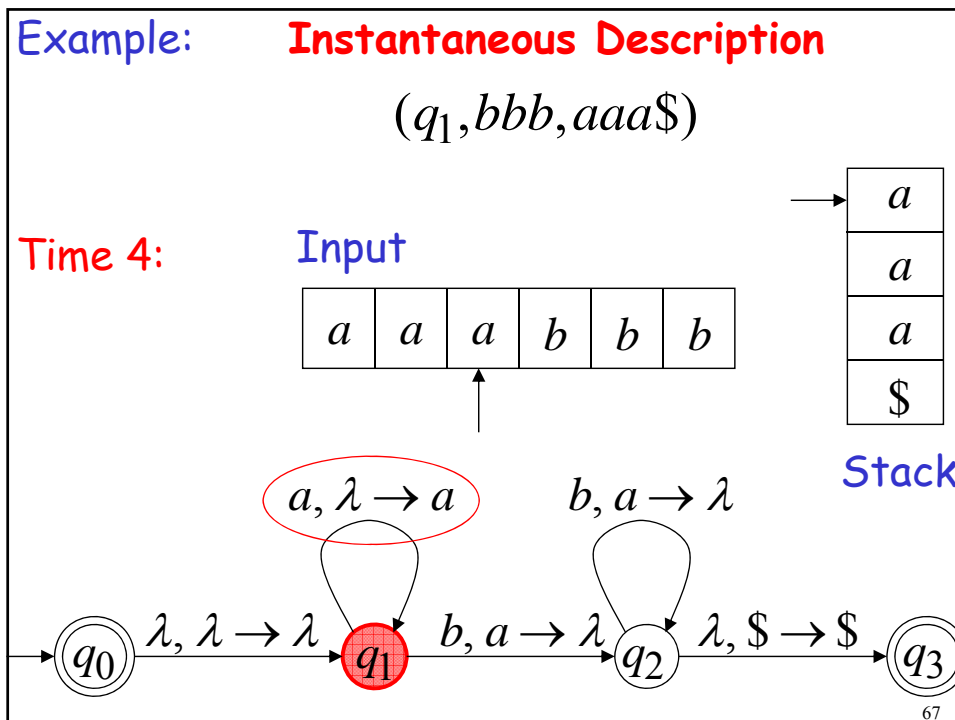


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Instantaneous Description



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We write:

$$(q_1, bbb, aaa\$) \succ (q_2, bb, aa\$)$$

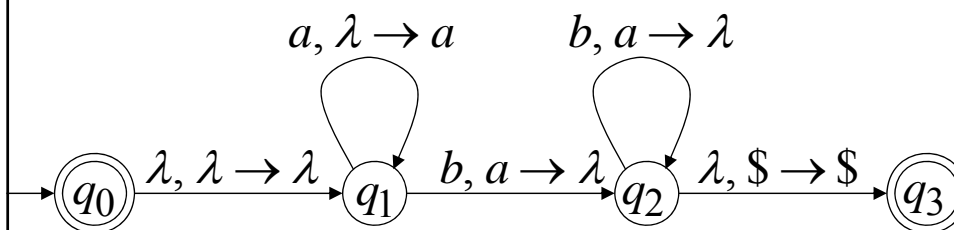
Time 4

Time 5

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A computation:

$$\begin{aligned} &(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ \\ &(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ \\ &(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$) \end{aligned}$$



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$$\begin{aligned}
 &(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ \\
 &(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ \\
 &(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$)
 \end{aligned}$$

For convenience we write:

$$(q_0, aaabbb, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$

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Formal Definition

Language $L(M)$ of NPDA M :

$$L(M) = \{w : (q_0, w, s) \overset{*}{\succ} (q_f, \lambda, s')\}$$

Initial state

Final state

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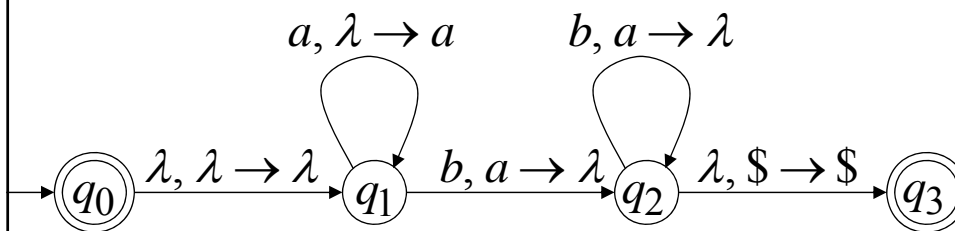
Example:

$$(q_0, aaabbb, \$) \stackrel{*}{\succ} (q_3, \lambda, \$)$$



$$aaabbb \in L(M)$$

NPDA M :



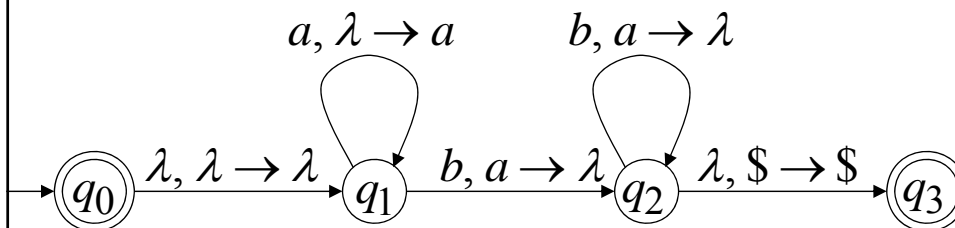
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$$(q_0, a^n b^n, \$) \stackrel{*}{\succ} (q_3, \lambda, \$)$$



$$a^n b^n \in L(M)$$

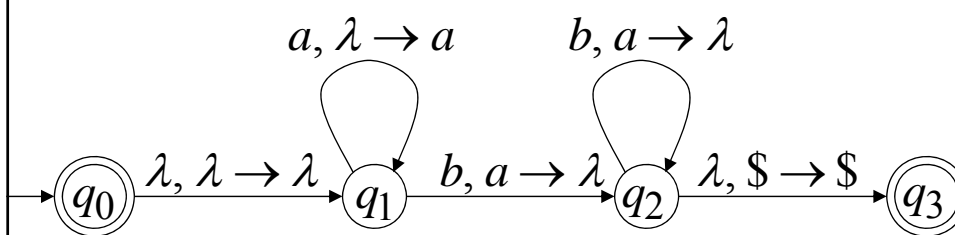
NPDA M :



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Therefore: $L(M) = \{a^n b^n : n \geq 0\}$

NPDA M :



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