More Applications

of

the Pumping Lemma

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The Pumping Lemma:

- Given a infinite regular language L
- there exists an integer m
- for any string $w \in L$ with length $|w| \geq m$
- we can write $w = x\, y\, z$
- with $|x\, y| \leq m$ and $|y| \geq 1$
- such that: $x\, y^i\, z \in L \qquad i = 0, 1, 2, \ldots$

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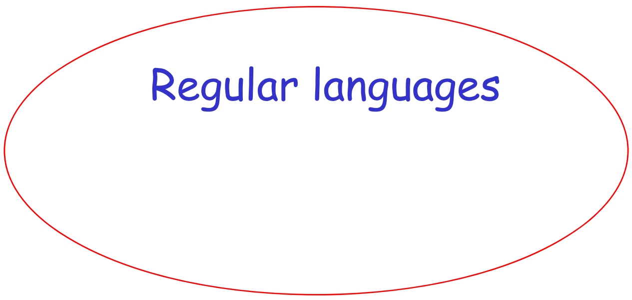
Non-regular languages $L = \{vv^R : v \in \Sigma^*\}$



Theorem: The language

$$L = \{vv^R : v \in \Sigma^*\} \qquad \Sigma = \{a, b\}$$

is not regular

Proof: Use the Pumping Lemma

$$L = \{vv^R : v \in \Sigma^*\}$$

Assume for contradiction
that L is a regular language

Since L is infinite
we can apply the Pumping Lemma

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$$L = \{vv^R : v \in \Sigma^*\}$$

Let m be the integer in the Pumping Lemma

Pick a string w such that: $w \in L$ and

length $|w| \geq m$

We pick $w = a^m b^m b^m a^m$

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Write $a^m b^m b^m a^m = x\ y\ z$

From the Pumping Lemma

it must be that length $|x\ y| \leq m, \quad |y| \geq 1$

$$xyz = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{...a}_{}}^{m}\underbrace{\overbrace{b...b}^{m}\overbrace{b...b}^{m}\overbrace{a...a}^{m}}_{z}$$

Thus: $y = a^k, \quad k \geq 1$

$x\ y\ z = a^m b^m b^m a^m \qquad y = a^k, \quad k \geq 1$

From the Pumping Lemma: $x\ y^i\ z \ \in\ L$

$i = 0, 1, 2, ...$

Thus: $x\ y^2\ z \ \in\ L$

$$x\,y\,z = a^m b^m b^m a^m \qquad y = a^k,\quad k \geq 1$$

From the Pumping Lemma: $x\,y^2\,z \in L$

$$xy^2z = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...a}_{y}...a}^{m\ +\ k}\underbrace{\overbrace{b...b}^{m}\overbrace{b...b}^{m}\overbrace{a...a}^{m}}_{z} \in L$$

Thus: $a^{m+k} b^m b^m a^m \in L$

$$a^{m+k} b^m b^m a^m \in L \qquad k \geq 1$$

BUT: $L = \{vv^R : v \in \Sigma^*\}$

$\Downarrow$

$$a^{m+k} b^m b^m a^m \notin L$$

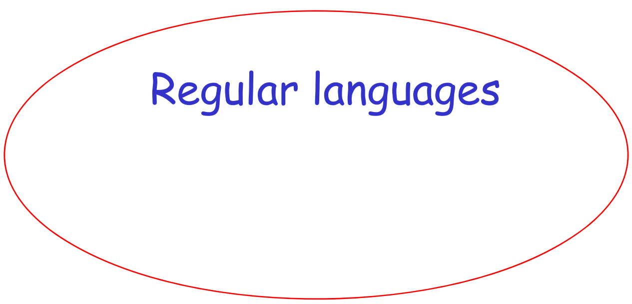
CONTRADICTION!!!

Therefore: Our assumption that L is a regular language is not true

Conclusion: L is not a regular language

Non-regular languages

$$L = \{a^n b^l c^{n+l} : \ n, l \geq 0\}$$



Theorem: The language

$$L = \{a^n b^l c^{n+l} : \; n,l \geq 0\}$$

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$$L = \{a^n b^l c^{n+l} : \ n, l \geq 0\}$$

Let m be the integer in the Pumping Lemma

Pick a string w such that: $w \in L$ and

length $|w| \geq m$

We pick $w = a^m b^m c^{2m}$

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Write $a^m b^m c^{2m} = x\ y\ z$

From the Pumping Lemma

it must be that length $|x\ y| \leq m, \ |y| \geq 1$

$$xyz = \underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...ab...bc...cc...c}_{z}$$

$$\overbrace{a...aa...aa...a}^{m}\overbrace{b...b}^{m}\overbrace{c...cc...c}^{2m}$$

Thus: $y = a^k, \quad k \geq 1$

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$x\ y\ z = a^m b^m c^{2m}$ $\qquad y = a^k, \quad k \geq 1$

From the Pumping Lemma: $x\ y^i\ z \in L$

$i = 0, 1, 2, \ldots$

Thus: $x\ y^0\ z = xz \in L$

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$x\ y\ z = a^m b^m c^{2m}$ $\qquad y = a^k, \quad k \geq 1$

From the Pumping Lemma: $xz \in L$

$$xz = \underbrace{a\ldots a}_{x}\underbrace{\overbrace{a\ldots a}^{m-k}\!\!\!\!\!\!\!\!\overbrace{b\ldots b}^{m}\overbrace{c\ldots cc\ldots c}^{2m}}_{z} \in L$$

Thus: $a^{m-k} b^m c^{2m} \in L$

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$$a^{m-k}b^m c^{2m} \in L \qquad k \geq 1$$

BUT: $L = \{a^n b^l c^{n+l} : n,l \geq 0\}$

$$\Downarrow$$

$$a^{m-k}b^m c^{2m} \notin L$$

CONTRADICTION!!!

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Therefore: Our assumption that L is a regular language is not true

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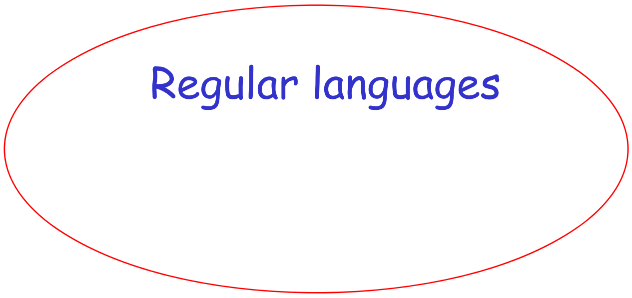
Non-regular languages $L = \{a^{n!}:\ n \geq 0\}$



Theorem: The language $L = \{a^{n!}:\ n \geq 0\}$

is not regular

$$n! = 1 \cdot 2 \cdots (n-1) \cdot n$$

Proof: Use the Pumping Lemma

$$L = \{a^{n!}:\ n \geq 0\}$$

Assume for contradiction
that L is a regular language

Since L is infinite
we can apply the Pumping Lemma

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$$L = \{a^{n!}:\ n \geq 0\}$$

Let m be the integer in the Pumping Lemma

Pick a string w such that: $w \in L$

length $|w| \geq m$

We pick $w = a^{m!}$

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Write $a^{m!} = x\ y\ z$

From the Pumping Lemma
it must be that length $|x\ y| \le m, \quad |y| \ge 1$

$$xyz = a^{m!} = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}a...a}^{m}\overbrace{a...aa...a}^{m!-m}$$

(where z is the part after y: $a...aa...aa...a$)

Thus: $y = a^k, \quad 1 \le k \le m$

$x\ y\ z = a^{m!}$ $\qquad y = a^k, \quad 1 \le k \le m$

From the Pumping Lemma: $x\ y^i\ z \ \in\ L$

$i = 0, 1, 2, ...$

Thus: $x\ y^2\ z \ \in\ L$

$x\,y\,z = a^{m!}$ $\qquad$ $y = a^k, \quad 1 \le k \le m$

From the Pumping Lemma: $x\,y^2\,z \in L$

$$xy^2z = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...a}_{y}a...a}^{m+k}\overbrace{a...a}^{m!-m} \in L$$

Thus: $a^{m!+k} \in L$

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$a^{m!+k} \in L$ $\qquad$ $1 \le k \le m$

Since: $L = \{a^{n!} : n \ge 0\}$

$\Downarrow$

There must exist p such that:

$$m!+k = p!$$

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However: $m!+k \le m!+m$ for $m > 1$

$$\le m!+m!$$

$$< m!m + m!$$

$$= m!(m+1)$$

$$= (m+1)!$$

$$\Downarrow$$

$$m!+k < (m+1)!$$

$$\Downarrow$$

$m!+k \neq p!$ for any p

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$a^{m!+k} \in L$ $\quad 1 \le k \le m$

BUT: $L = \{a^{n!}: n \ge 0\}$

$$\Downarrow$$

$$a^{m!+k} \notin L$$

CONTRADICTION!!!

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Therefore: Our assumption that L is a regular language is not true

Conclusion: L is not a regular language

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Lex

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Lex: a lexical analyzer

- A Lex program recognizes strings

- For each kind of string found
  the lex program takes an action

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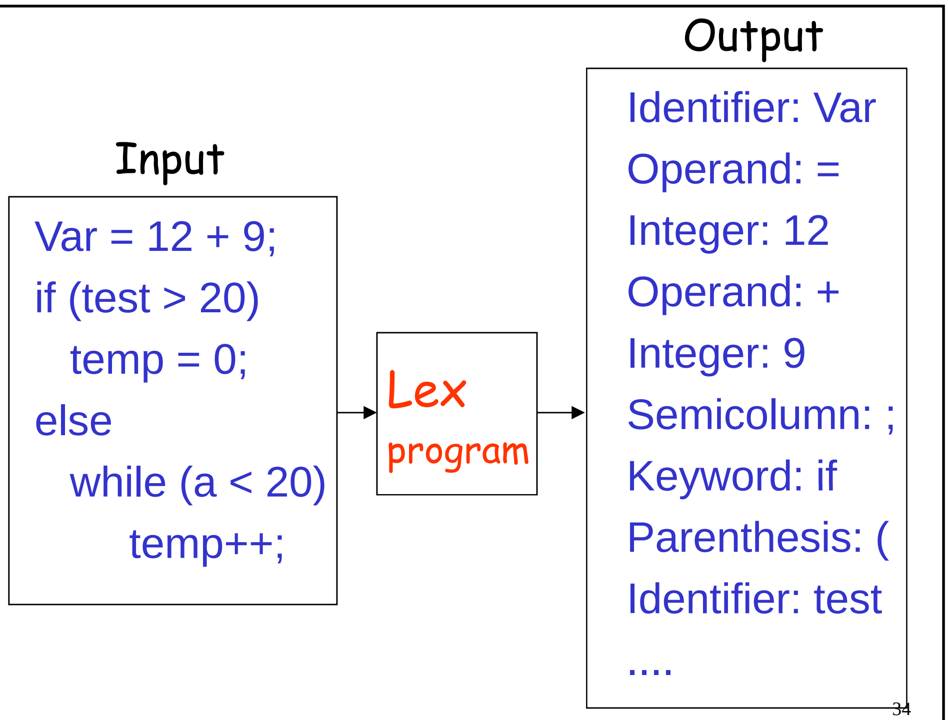

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In Lex strings are described with regular expressions

Lex program

Regular expressions

"+"
"-"   /* operators */
"="

"if"
"then"   /* keywords */

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Lex program

Regular expressions

(0|1|2|3|4|5|6|7|8|9)+   /* integers */

(a|b|..|z|A|B|...|Z)+   /* identifiers */

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integers

(0|1|2|3|4|5|6|7|8|9)+

[0-9]+

identifiers

(a|b|..|z|A|B|...|Z)+

[a-zA-Z]+

Each regular expression
has an associated action (in C code)

Examples:

Regular expression	Action
\n	linenum++;
[0-9]+	printf("integer");
[a-zA-Z]+	printf("identifier");

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Default action: ECHO;

Prints the string identified to the output

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A small lex program

```
%%
[ \t\n]          ;    /*skip spaces*/

[0-9]+           printf("Integer\n");

[a-zA-Z]+        printf("Identifier\n");
```

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Input	Output
1234 test var 566 78 9800	Integer Identifier Identifier Integer Integer Integer

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Another program

```
%{
int linenum = 1;
%}
%%
[ \t]           ;    /*skip spaces*/
\n              linenum++;
[0-9]+          prinf("Integer\n");
[a-zA-Z]+       printf("Identifier\n");
.               printf("Error in line: %d\n",
                       linenum);
```

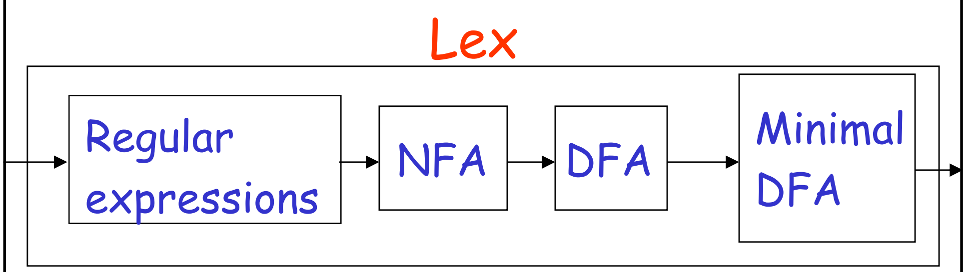

Input	Output
1234 test var 566 78 9800 + temp	Integer Identifier Identifier Integer Integer Integer Error in line: 3 Identifier

Lex matches the longest input string

Example: Regular Expressions "if"
"ifend"

Input:	ifend	if
Matches:	"ifend"	"if"

Internal Structure of Lex



The final states of the DFA are associated with actions