36) 1) Defining the vector h: Let h be a vector in $L^2(\mathbb{Z})$ with components given by $h_0 = \frac{1}{2}$, $h_1 = \frac{-1}{2}$, $h_k = 0$ otherwise.

2) Convolution in Time domain: Consider the convolution $y = h * x$ for a general x in $L^2(\mathbb{Z})$.

3) Transformation to Frequency domain: In the frequency domain the convolution corresponds to the multiplication $Y(f) = H(f)\,X(f)$, where $H(f) = \sum_{k=-\infty}^{\infty} h_k e^{-2\pi i k f} = \frac{1}{2}(1 - e^{-2\pi i f})$

4) Magnitude of Frequency Response: The Magnitude of $H(f)$ can be expressed as:
$|H(f)| = \left|\frac{1}{2}(1 - e^{-2\pi i f})\right|$, simplifying this gives us:

$$|H(f)| = \frac{1}{2}|1 - e^{-2\pi i f}|$$
$$= \frac{1}{2}\sqrt{1 - 2\cos(2\pi f) + 1}$$
$$= \frac{1}{2}\sqrt{2 - 2\cos(2\pi f)}$$

5) Interpretation of $|H(f)|$: We can see that $|H(f)|$ is zero at $f = 0$, representing that the DC component is blocked, and increases as the frequency f approaches the Nyquist rate $(f = \frac{1}{2})$, indicating that higher frequencies are passed through. Thus, the vector h acts as a high-pass filter under convolution.

37) $$Y(z) = \sum_{k=-\infty}^{\infty} y_k z^{-k}$$
$$= \sum_{k=-\infty}^{\infty} x_{k+r} z^{-k}$$

$k + r = m$ ~~$\to 0$~~, when $k = -\infty$, $m = -\infty + r$ and when $k = \infty$, $m = \infty + r$, the sum becomes

$$Y(z) = \sum_{m=-\infty}^{\infty} x_m z^{-(m-r)}$$

$$= \sum_{m=-\infty}^{\infty} x_m z^{-m} z^{r}$$

Since z^r doesn't depend on the summation index it can be taken out

$$Y(z) = z^r \sum_{m=-\infty}^{\infty} x_m z^{-m}$$

since this is just $Y(z) = z^r \cdot X(z)$ we can conclude the proof