

# **INTRO to DATA SCIENCE**

## **LECTURE 10: DECISION TREE CLASSIFIERS**

## **LAST TIME:**

- PROBABILITY**
- NAIVE BAYES**
- WORD COUNT MATRICES**

**QUESTIONS?**

**I. DECISION TREES**

**II. BUILDING DECISION TREES**

**III. OPTIMIZATION FUNCTIONS**

**IV. PREVENTING OVERFITTING**

**EXERCISE:**

**V. IMPLEMENTING DECISION TREES WITH SCIKIT-LEARN**

# **I. DECISION TREES**

*Q: What is a decision tree?*

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**non-parametric:** *no parameters, no distribution assumptions*

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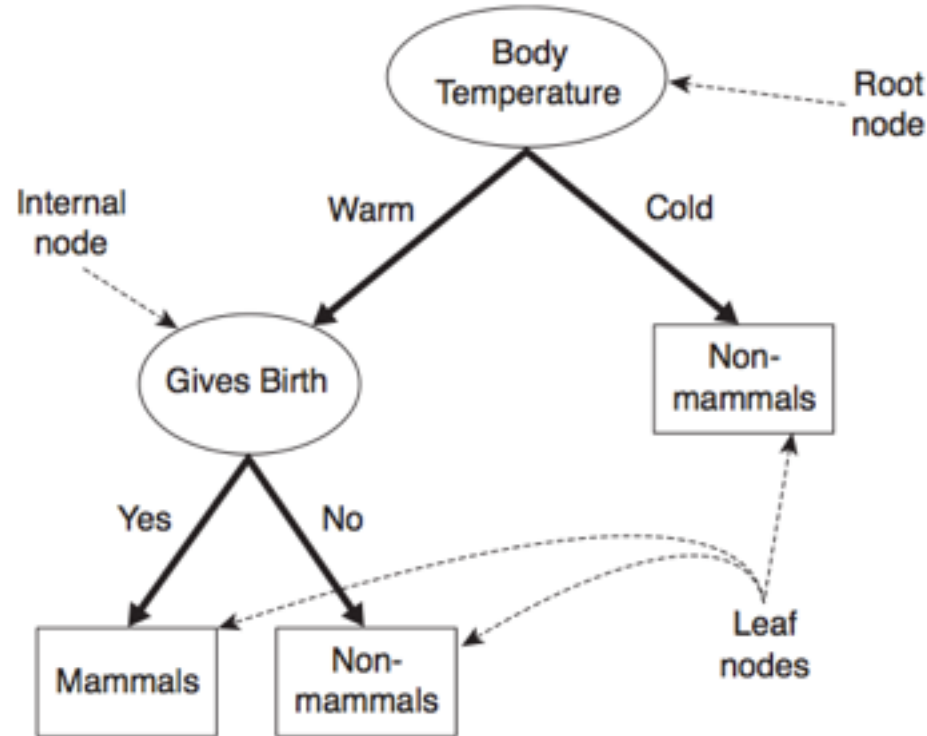
*A: A non-parametric hierarchical classification technique.*

**non-parametric:** *no parameters, no distribution assumptions*

**hierarchical:** *consists of a sequence of questions which yield a class label when applied to any record*



*Q: How is a decision tree represented?*



**Figure 4.4.** A decision tree for the mammal classification problem.

*Q: How is a decision tree represented?*

*A: Using a configuration of **nodes** and **edges**.*

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*A: Using a configuration of **nodes** and **edges**.*

*More concretely, as a multiway tree, which is a type of (directed acyclic) **graph**.*

*In a decision tree, the nodes represent questions (**test conditions**) and the edges are the answers to these questions.*

*The top node of the tree is called the **root node**. This node has 0 incoming edges, and 2+ outgoing edges.*

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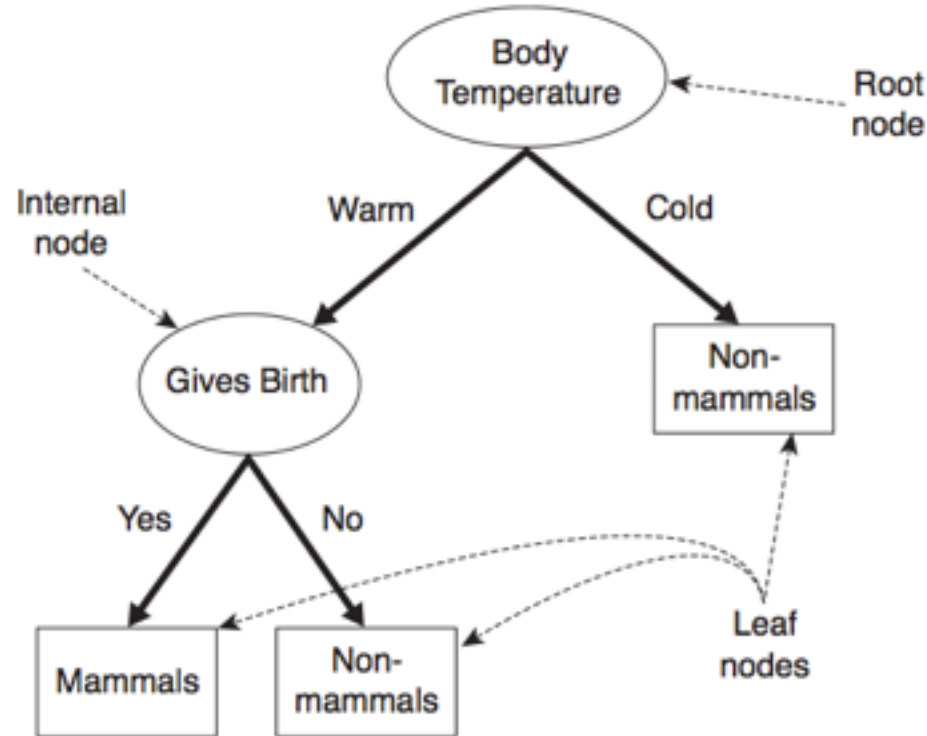
### NOTE

The nodes in our tree are connected by directed edges.

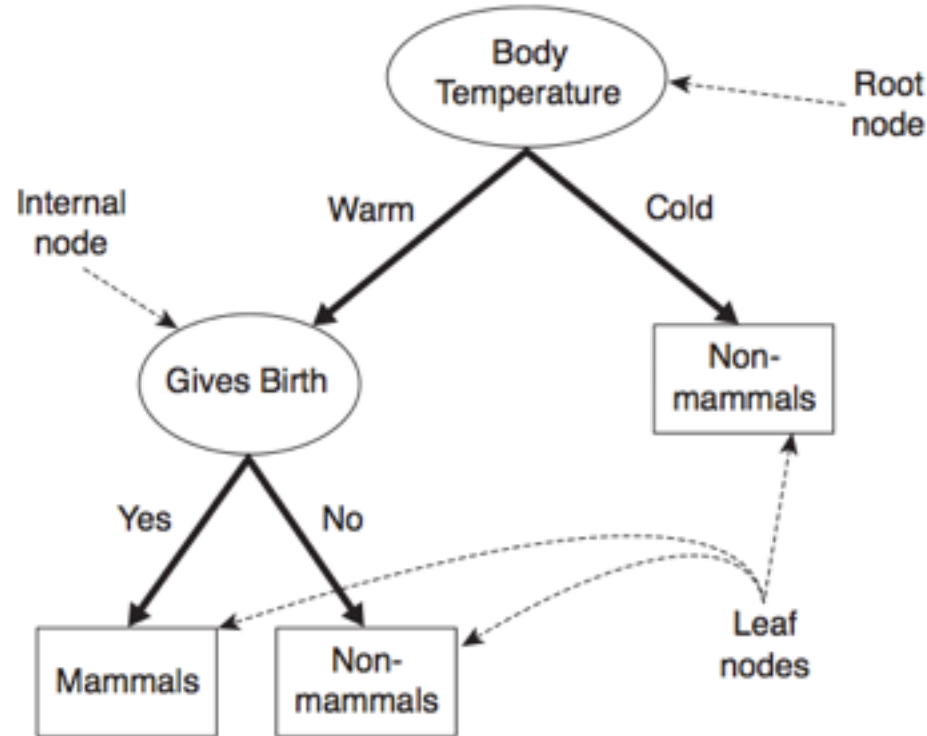
These directed edges lead from parent nodes to child nodes.

Table 4.1. The vertebrate data set.

| Name          | Body Temperature | Skin Cover | Gives Birth | Aquatic Creature | Aerial Creature | Has Legs | Hibernates | Class Label |
|---------------|------------------|------------|-------------|------------------|-----------------|----------|------------|-------------|
| human         | warm-blooded     | hair       | yes         | no               | no              | yes      | no         | mammal      |
| python        | cold-blooded     | scales     | no          | no               | no              | no       | yes        | reptile     |
| salmon        | cold-blooded     | scales     | no          | yes              | no              | no       | no         | fish        |
| whale         | warm-blooded     | hair       | yes         | yes              | no              | no       | no         | mammal      |
| frog          | cold-blooded     | none       | no          | semi             | no              | yes      | yes        | amphibian   |
| komodo dragon | cold-blooded     | scales     | no          | no               | no              | yes      | no         | reptile     |
| bat           | warm-blooded     | hair       | yes         | no               | yes             | yes      | yes        | mammal      |
| pigeon        | warm-blooded     | feathers   | no          | no               | yes             | yes      | no         | bird        |
| cat           | warm-blooded     | fur        | yes         | no               | no              | yes      | no         | mammal      |
| leopard       | cold-blooded     | scales     | yes         | yes              | no              | no       | no         | fish        |
| shark         |                  |            |             |                  |                 |          |            |             |
| turtle        | cold-blooded     | scales     | no          | semi             | no              | yes      | no         | reptile     |
| penguin       | warm-blooded     | feathers   | no          | semi             | no              | yes      | no         | bird        |
| porcupine     | warm-blooded     | quills     | yes         | no               | no              | yes      | yes        | mammal      |
| eel           | cold-blooded     | scales     | no          | yes              | no              | no       | no         | fish        |
| salamander    | cold-blooded     | none       | no          | semi             | no              | yes      | yes        | amphibian   |



**Figure 4.4.** A decision tree for the mammal classification problem.



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### NOTE

Internal nodes represent test conditions which partition the records at that node.

# REVIEW:

*1. HOW DOES A DECISION TREE CLASSIFY DATA?*

*2. WHAT IS THE DIFFERENCE BETWEEN A ROOT, INTERNAL, AND TREE NODE?*

# **II. BUILDING DECISION TREES**

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*A: Use a **heuristic** algorithm.*

*The basic method used to build (or “grow”) a decision tree is Hunt’s algorithm.*

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**greedy** – *algorithm makes locally optimal decision at each step*

**recursive** – *splits task into subtasks, solves each the same way*

**local optimum** – *solution for a given neighborhood of points*

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*The partitioning decision is made at each node according to a metric called **purity**.*

*A partition is 100% pure when all of its records belong to a single class.*

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**NOTE**

This is the *base case* for the recursive algorithm.

*Consider a binary classification problem with classes  $X$ ,  $Y$ . Given a set of records  $D_t$  at node  $t$ , Hunt's algorithm proceeds as follows:*

*2) If  $D_t$  contains records from both classes, then a test condition is created to partition the records further. In this case,  $t$  is an internal node whose outgoing edges correspond to the possible outcomes of this test condition.*

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*These outgoing edges terminate in **child nodes**. A record  $d$  in  $D_t$  is assigned to one of these child nodes based on the outcome of the test condition applied to  $d$ .*

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*3) These steps are then recursively applied to each child node.*

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### NOTE

Decision trees are easy to interpret, but the algorithms to create them are a bit complicated.



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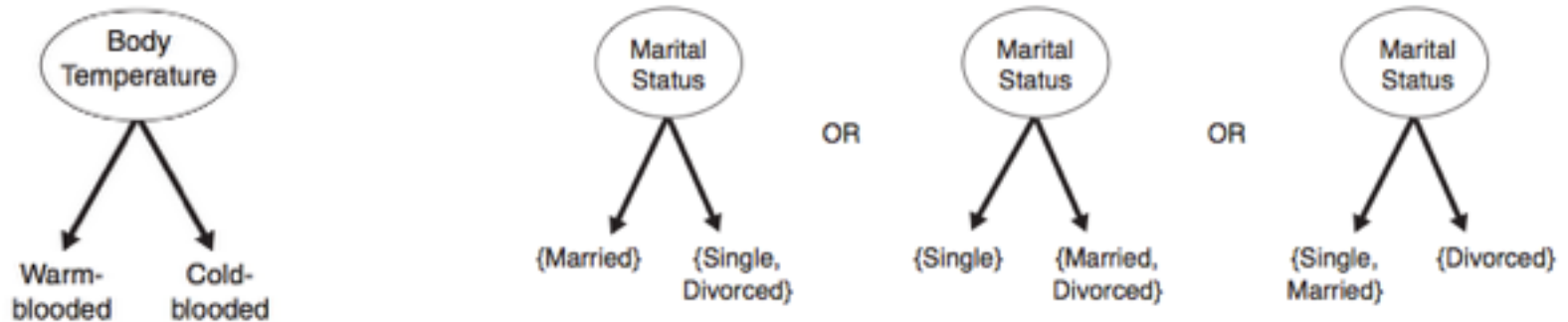


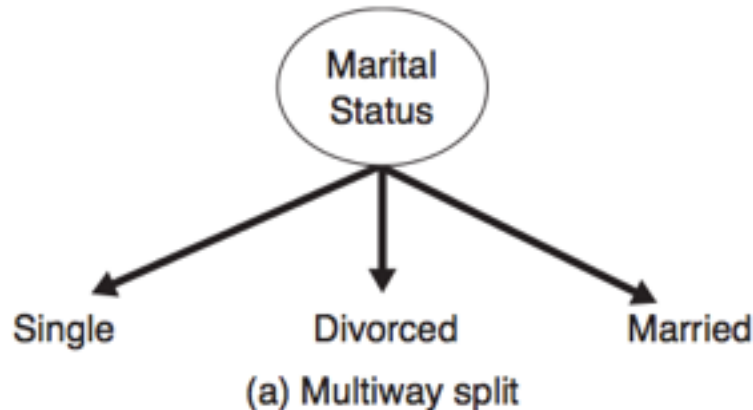
Figure 4.8. Test condition for binary attributes.

(b) Binary split {by grouping attribute values}

*Q: How do we partition the training records?*

*A: There are a few ways to do this.*

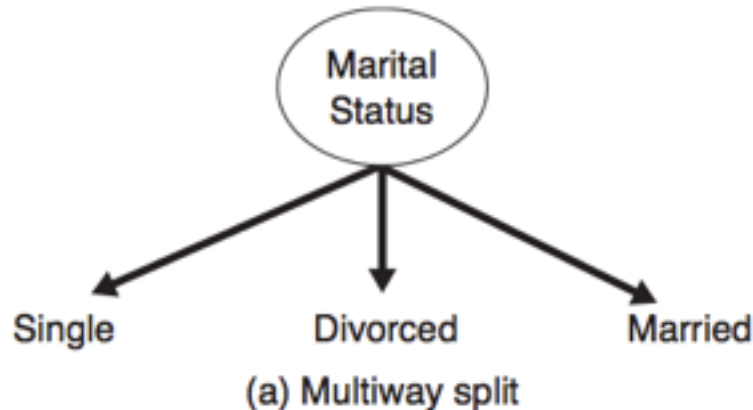
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### NOTE

Multiway splits can produce purer subsets, but may lead to overfitting!

*Q: How do we partition the training records?*

*A: There are a few ways to do this.*

*For continuous features, we can use either method:*

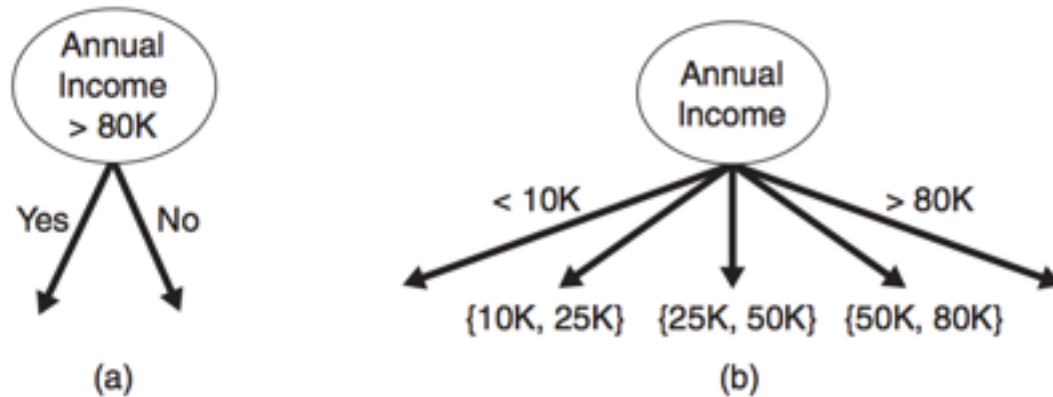
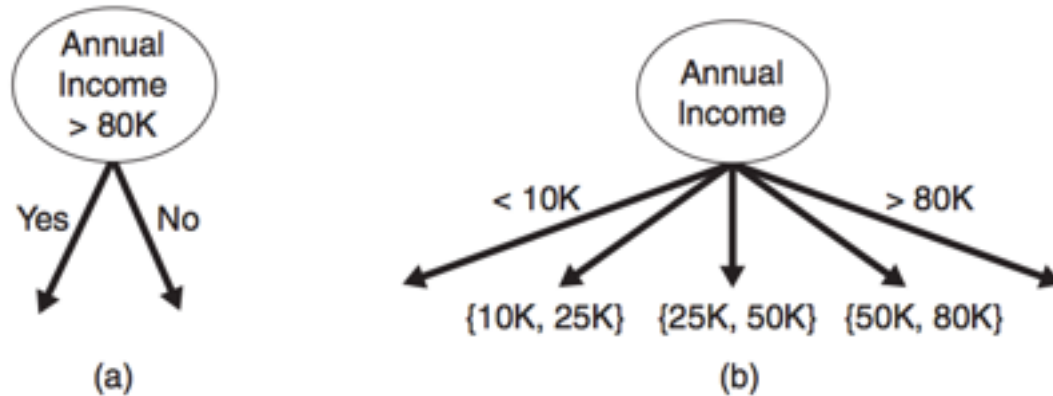


Figure 4.11. Test condition for continuous attributes.

*Q: How do we partition the training records?*

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### NOTE

There are optimizations that can improve the naïve quadratic complexity of determining the optimum split point for continuous attributes.

Figure 4.11. Test condition for continuous attributes.



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*A: Recall that no split is necessary (at a given node) when all records belong to the same class.*

*Therefore we want each step to create the partition with the highest possible purity.*

*We need an objective function to optimize!*

# **III. OPTIMIZATION FUNCTIONS**

*We want our objective function to measure the gain in purity from a particular split.*

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*Therefore we want it to depend on the class distribution over the nodes (before and after the split).*

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*For example, let  $p(i \mid t)$  be the probability of class  $i$  at node  $t$  (eg, the fraction of records labeled  $i$  at node  $t$ ).*



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*For example, let  $p(i \mid t)$  be the probability of class  $i$  at node  $t$ , i.e. the fraction of records labeled  $i$  at node  $t$ .*

### NOTE

We are using the frequentist definition of probability here!

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*The maximum purity partition is given (eg) by the distribution:*

$$p(0 \mid t) = 1 - p(1 \mid t) = 1$$

*Some measures of impurity include:*

$$\text{Entropy}(t) = - \sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t),$$

$$\text{Gini}(t) = 1 - \sum_{i=0}^{c-1} [p(i|t)]^2,$$

$$\text{Classification error}(t) = 1 - \max_i [p(i|t)],$$

*Note that each measure achieves its max at 0.5, min at 0 & 1.*

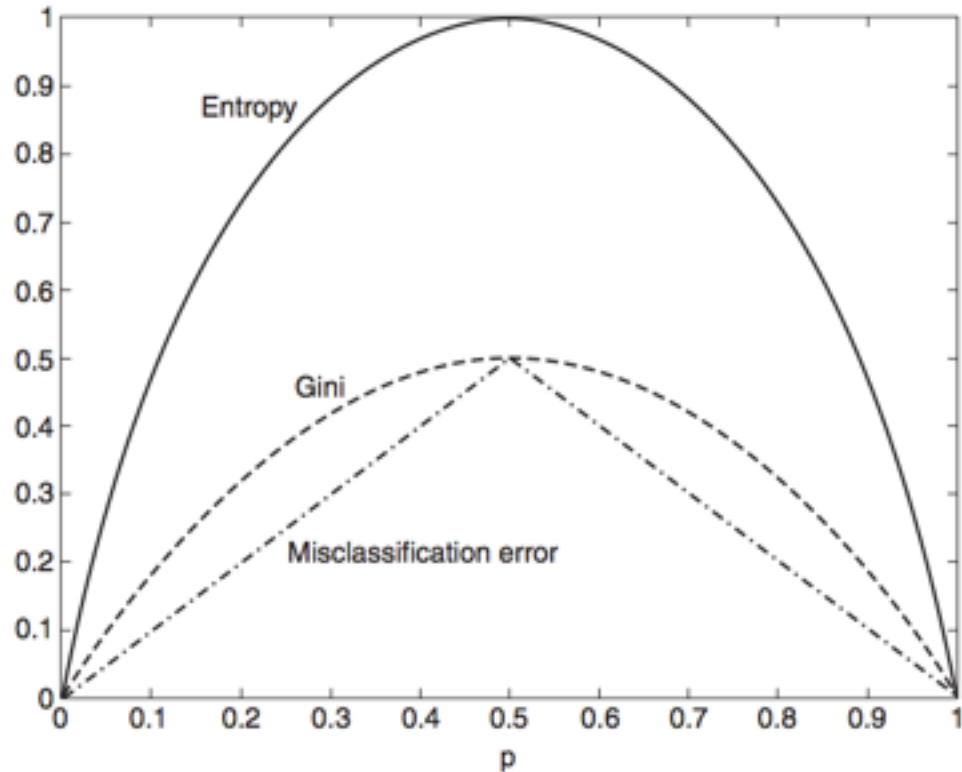


Figure 4.13. Comparison among the impurity measures for binary classification problems.

*Note that each measure achieves its max at 0.5, min at 0 & 1.*

### NOTE

Despite consistency, different measures may create different splits.

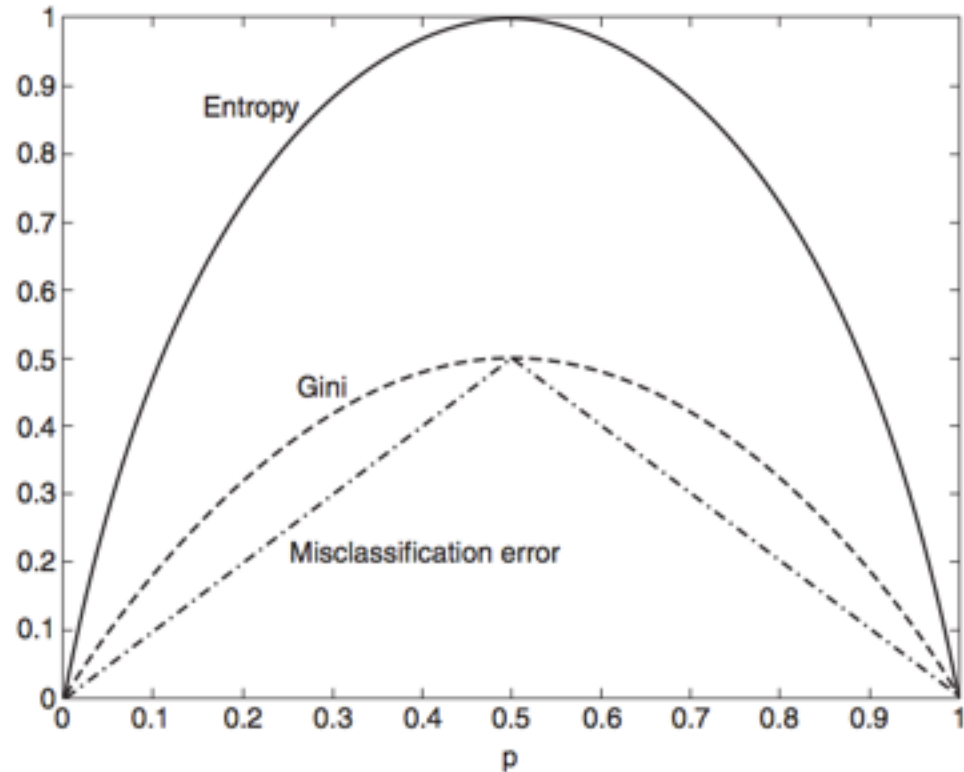


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*Q: Why is this true?*

*Impurity measures put us on the right track, but on their own they are not enough to tell us how our split will do.*

*Q: Why is this true?*

*A: We still need to look at impurity before & after the split.*

*We can make this comparison using the **gain**:*

$$\Delta = I(\text{parent}) - \sum_{\text{children } j} \frac{N_j}{N} I(\text{child } j)$$

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*(Here  $I$  is the impurity measure,  $N_j$  denotes the number of records at child node  $j$ , and  $N$  denotes the number of records at the parent node.)*

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*(Here  $I$  is the impurity measure,  $N_j$  denotes the number of records at child node  $j$ , and  $N$  denotes the number of records at the parent node.)*

*When  $I$  is the entropy, this quantity is called the **information gain**.*

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*One way of dealing with this is to restrict the algorithm to binary splits only (CART).*

*Another way is to use a splitting criterion which explicitly penalizes the number of outcomes (C4.5)*



*We can use a function of the information gain called the **gain ratio** to explicitly penalize high numbers of outcomes:*

$$\text{gain ratio} = \frac{\Delta_{info}}{-\sum p(v_i) \log_2 p(v_i)}$$

*(Where  $p(v_i)$  refers to the probability of label  $i$  at node  $v$ )*

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$$\text{gain ratio} = \frac{\Delta_{info}}{-\sum p(v_i) \log_2 p(v_i)}$$

**NOTE**

This is a form of regularization!

*(Where  $p(v_i)$  refers to the probability of label  $i$  at node  $v$ )*

# **IV. PREVENTING OVERFITTING**

*In addition to determining splits, we also need a stopping criterion to tell us when we're done.*

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*This is correct in principle, but would likely lead to overfitting.*

*One possibility is **pre-pruning**, which involves setting a minimum threshold on the gain, and stopping when no split achieves a gain above this threshold.*

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*This prevents overfitting, but is difficult to calibrate in practice (may preserve bias!)*



*Alternatively we could build the full tree, and then perform **pruning** as a post-processing step.*

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*To prune a tree, we examine the nodes from the bottom-up and simplify pieces of the tree (according to some criteria).*

*Complicated subtrees can be replaced either with a single node, or with a simpler (child) subtree.*

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*The first approach is called **subtree replacement**, and the second is **subtree raising**.*

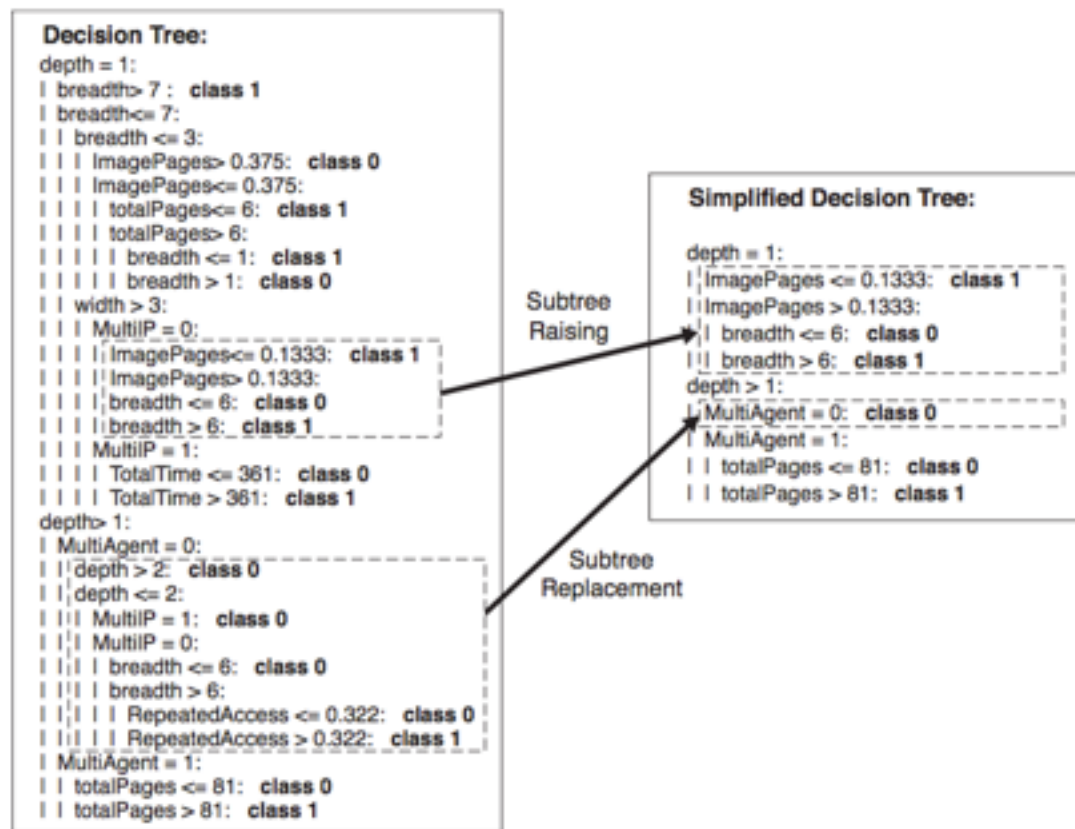


Figure 4.29. Post-pruning of the decision tree for Web robot detection.

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*Pay careful attention to what is going in and out of your model!*

```
>>> X_test_features = X_test[features].values
>>> clf.score(X_train_features, X_train['IsBadBuy'].values)
0.99998042593172565
>>> clf.score(X_test_features, X_test['IsBadBuy'].values)
0.78131993605846084
```

## **INTRO TO DATA SCIENCE**

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# **PROS AND CONS**



*When will you want to use decision trees, and when should you not expect good performance?*

*Easy to understand why a classification was made.*

*Less data preparation necessary.*

*Easy to mix numerical, categorical data.*

*Fast to predict.*

*Susceptible to overfitting.*

*Possibly chaotic sensitivity to training data.*

*Typically, finding a globally optimal tree is intractable: must resort to heuristics to find local optima.*

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# **ENSEMBLE METHODS**

*Some of the dangers of decision trees can be mitigated by combining ensembles of individual trees trained on subsets of the data or feature space. Often, this means sacrificing the ability to interpret the steps leading to a classification.*

*Look out for random forests. Most Kaggle competitions are won with these.*

*Train several tree classifiers on small subsets of the feature space.  
Take the most common prediction among the trees as the final output.*

*Pros: fast to train, harder to overfit, typically very good classification performance*

*Cons: best possible fit is slightly worse, obfuscates the classification path.*

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**DISCUSSION/LAB**