Exposure corrections for darkroom printing

1 The formula

$$\boxed{\text{corrected time} = 2^{\pm\text{fraction of diaph}} \times \text{exposure time}}$$

A linear scale of EV (Exposure Value *.ie.* diaphragms) for exposure translates to a logarithmically scaled exposure times.

1.1 Example use 1

For instance, I exposed my print at 40 seconds. I want to close the print by making the corners darker. I choose to darken the corners by one diaphragm. I will thus have to expose them at 80 seconds ($2^{+1} \times 40$ s). This means I have to add 40 seconds to the corners in addition to the 40 seconds global exposure.

1.2 Example use 2

I have printed a photo with an exposure time of 12 seconds. The print is a bit over-exposed (*ie* it is too dark): I would like to reduce the exposure by one-fourth of a diaphragm. Then from the table I know I will need to expose for 10.1 seconds ($= 2^{-1/4} \times 12$ s, *ie* about 10 seconds).

2 Corrected times for different diaphragm corrections

-1/2	-1/4	-1/8	0	+1/8	+1/4	+1/2
2.8	**3.4**	**3.7**	**4**	**4.4**	**4.8**	**5.7**
3.5	**4.2**	**4.6**	**5**	**5.5**	**5.9**	**7.1**
4.2	**5**	**5.5**	**6**	**6.5**	**7.1**	**8.5**
4.9	**5.9**	**6.4**	**7**	**7.6**	**8.3**	**9.9**
5.7	**6.7**	**7.3**	**8**	**8.7**	**9.5**	**11.3**
6.4	**7.6**	**8.3**	**9**	**9.8**	**10.7**	**12.7**
7.1	**8.4**	**9.2**	**10**	**10.9**	**11.9**	**14.1**
7.8	**9.2**	**10.1**	**11**	**12**	**13.1**	**15.6**
8.5	**10.1**	**11**	**12**	**13.1**	**14.3**	**17**
9.2	**10.9**	**11.9**	**13**	**14.2**	**15.5**	**18.4**
9.9	**11.8**	**12.8**	**14**	**15.3**	**16.6**	**19.8**
10.6	**12.6**	**13.8**	**15**	**16.4**	**17.8**	**21.2**
11.3	**13.5**	**14.7**	**16**	**17.4**	**19**	**22.6**
12	**14.3**	**15.6**	**17**	**18.5**	**20.2**	**24**
12.7	**15.1**	**16.5**	**18**	**19.6**	**21.4**	**25.5**
13.4	**16**	**17.4**	**19**	**20.7**	**22.6**	**26.9**
14.1	**16.8**	**18.3**	**20**	**21.8**	**23.8**	**28.3**
14.8	**17.7**	**19.3**	**21**	**22.9**	**25**	**29.7**
15.6	**18.5**	**20.2**	**22**	**24**	**26.2**	**31.1**
16.3	**19.3**	**21.1**	**23**	**25.1**	**27.4**	**32.5**
17	**20.2**	**22**	**24**	**26.2**	**28.5**	**33.9**
17.7	**21**	**22.9**	**25**	**27.3**	**29.7**	**35.4**
18.4	**21.9**	**23.8**	**26**	**28.4**	**30.9**	**36.8**
19.1	**22.7**	**24.8**	**27**	**29.4**	**32.1**	**38.2**
19.8	**23.5**	**25.7**	**28**	**30.5**	**33.3**	**39.6**
20.5	**24.4**	**26.6**	**29**	**31.6**	**34.5**	**41**
21.2	**25.2**	**27.5**	**30**	**32.7**	**35.7**	**42.4**
21.9	**26.1**	**28.4**	**31**	**33.8**	**36.9**	**43.8**
22.6	**26.9**	**29.3**	**32**	**34.9**	**38.1**	**45.3**
23.3	**27.7**	**30.3**	**33**	**36**	**39.2**	**46.7**
24	**28.6**	**31.2**	**34**	**37.1**	**40.4**	**48.1**
24.7	**29.4**	**32.1**	**35**	**38.2**	**41.6**	**49.5**
25.5	**30.3**	**33**	**36**	**39.3**	**42.8**	**50.9**
26.2	**31.1**	**33.9**	**37**	**40.3**	**44**	**52.3**
26.9	**32**	**34.8**	**38**	**41.4**	**45.2**	**53.7**
27.6	**32.8**	**35.8**	**39**	**42.5**	**46.4**	**55.2**
28.3	**33.6**	**36.7**	**40**	**43.6**	**47.6**	**56.6**

-1	-3/4	-2/3	-1/2	-1/3	-1/4	-1/8	0	1/8	1/4	1/3	1/2	2/3	3/4	1
1	1.2	1.3	1.4	1.6	1.7	1.8	2	2.2	2.4	2.5	2.8	3.2	3.4	4
1.5	1.8	1.9	2.1	2.4	2.5	2.8	3	3.3	3.6	3.8	4.2	4.8	5	6
2	2.4	2.5	2.8	3.2	3.4	3.7	4	4.4	4.8	5	5.7	6.3	6.7	8
2.5	3	3.1	3.5	4	4.2	4.6	5	5.5	5.9	6.3	7.1	7.9	8.4	10
3	3.6	3.8	4.2	4.8	5	5.5	6	6.5	7.1	7.6	8.5	9.5	10.1	12
3.5	4.2	4.4	4.9	5.6	5.9	6.4	7	7.6	8.3	8.8	9.9	11.1	11.8	14
4	4.8	5	5.7	6.3	6.7	7.3	8	8.7	9.5	10.1	11.3	12.7	13.5	16
4.5	5.4	5.7	6.4	7.1	7.6	8.3	9	9.8	10.7	11.3	12.7	14.3	15.1	18
5	5.9	6.3	7.1	7.9	8.4	9.2	10	10.9	11.9	12.6	14.1	15.9	16.8	20
5.5	6.5	6.9	7.8	8.7	9.2	10.1	11	12	13.1	13.9	15.6	17.5	18.5	22
6	7.1	7.6	8.5	9.5	10.1	11	12	13.1	14.3	15.1	17	19	20.2	24
6.5	7.7	8.2	9.2	10.3	10.9	11.9	13	14.2	15.5	16.4	18.4	20.6	21.9	26
7	8.3	8.8	9.9	11.1	11.8	12.8	14	15.3	16.6	17.6	19.8	22.2	23.5	28
7.5	8.9	9.4	10.6	11.9	12.6	13.8	15	16.4	17.8	18.9	21.2	23.8	25.2	30
8	9.5	10.1	11.3	12.7	13.5	14.7	16	17.4	19	20.2	22.6	25.4	26.9	32
8.5	10.1	10.7	12	13.5	14.3	15.6	17	18.5	20.2	21.4	24	27	28.6	34
9	10.7	11.3	12.7	14.3	15.1	16.5	18	19.6	21.4	22.7	25.5	28.6	30.3	36
9.5	11.3	12	13.4	15.1	16	17.4	19	20.7	22.6	23.9	26.9	30.2	32	38
10	11.9	12.6	14.1	15.9	16.8	18.3	20	21.8	23.8	25.2	28.3	31.7	33.6	40
10.5	12.5	13.2	14.8	16.7	17.7	19.3	21	22.9	25	26.5	29.7	33.3	35.3	42
11	13.1	13.9	15.6	17.5	18.5	20.2	22	24	26.2	27.7	31.1	34.9	37	44
11.5	13.7	14.5	16.3	18.3	19.3	21.1	23	25.1	27.4	29	32.5	36.5	38.7	46
12	14.3	15.1	17	19	20.2	22	24	26.2	28.5	30.2	33.9	38.1	40.4	48
12.5	14.9	15.7	17.7	19.8	21	22.9	25	27.3	29.7	31.5	35.4	39.7	42	50
13	15.5	16.4	18.4	20.6	21.9	23.8	26	28.4	30.9	32.8	36.8	41.3	43.7	52
13.5	16.1	17	19.1	21.4	22.7	24.8	27	29.4	32.1	34	38.2	42.9	45.4	54
14	16.6	17.6	19.8	22.2	23.5	25.7	28	30.5	33.3	35.3	39.6	44.4	47.1	56
14.5	17.2	18.3	20.5	23	24.4	26.6	29	31.6	34.5	36.5	41	46	48.8	58
15	17.8	18.9	21.2	23.8	25.2	27.5	30	32.7	35.7	37.8	42.4	47.6	50.5	60
15.5	18.4	19.5	21.9	24.6	26.1	28.4	31	33.8	36.9	39.1	43.8	49.2	52.1	62
16	19	20.2	22.6	25.4	26.9	29.3	32	34.9	38.1	40.3	45.3	50.8	53.8	64
16.5	19.6	20.8	23.3	26.2	27.7	30.3	33	36	39.2	41.6	46.7	52.4	55.5	66
17	20.2	21.4	24	27	28.6	31.2	34	37.1	40.4	42.8	48.1	54	57.2	68
17.5	20.8	22	24.7	27.8	29.4	32.1	35	38.2	41.6	44.1	49.5	55.6	58.9	70
18	21.4	22.7	25.5	28.6	30.3	33	36	39.3	42.8	45.4	50.9	57.1	60.5	72
18.5	22	23.3	26.2	29.4	31.1	33.9	37	40.3	44	46.6	52.3	58.7	62.2	74
19	22.6	23.9	26.9	30.2	32	34.8	38	41.4	45.2	47.9	53.7	60.3	63.9	76
19.5	23.2	24.6	27.6	31	32.8	35.8	39	42.5	46.4	49.1	55.2	61.9	65.6	78
20	23.8	25.2	28.3	31.7	33.6	36.7	40	43.6	47.6	50.4	56.6	63.5	67.3	80
20.5	24.4	25.8	29	32.5	34.5	37.6	41	44.7	48.8	51.7	58	65.1	69	82
21	25	26.5	29.7	33.3	35.3	38.5	42	45.8	49.9	52.9	59.4	66.7	70.6	84
21.5	25.6	27.1	30.4	34.1	36.2	39.4	43	46.9	51.1	54.2	60.8	68.3	72.3	86
22	26.2	27.7	31.1	34.9	37	40.3	44	48	52.3	55.4	62.2	69.8	74	88
22.5	26.8	28.3	31.8	35.7	37.8	41.3	45	49.1	53.5	56.7	63.6	71.4	75.7	90
23	27.4	29	32.5	36.5	38.7	42.2	46	50.2	54.7	58	65.1	73	77.4	92
23.5	27.9	29.6	33.2	37.3	39.5	43.1	47	51.3	55.9	59.2	66.5	74.6	79	94
24	28.5	30.2	33.9	38.1	40.4	44	48	52.3	57.1	60.5	67.9	76.2	80.7	96
24.5	29.1	30.9	34.6	38.9	41.2	44.9	49	53.4	58.3	61.7	69.3	77.8	82.4	98
25	29.7	31.5	35.4	39.7	42	45.9	50	54.5	59.5	63	70.7	79.4	84.1	100
25.5	30.3	32.1	36.1	40.5	42.9	46.8	51	55.6	60.6	64.3	72.1	81	85.8	102
26	30.9	32.8	36.8	41.3	43.7	47.7	52	56.7	61.8	65.5	73.5	82.5	87.5	104
26.5	31.5	33.4	37.5	42.1	44.6	48.6	53	57.8	63	66.8	75	84.1	89.1	106
27	32.1	34	38.2	42.9	45.4	49.5	54	58.9	64.2	68	76.4	85.7	90.8	108
27.5	32.7	34.6	38.9	43.7	46.2	50.4	55	60	65.4	69.3	77.8	87.3	92.5	110
28	33.3	35.3	39.6	44.4	47.1	51.4	56	61.1	66.6	70.6	79.2	88.9	94.2	112
28.5	33.9	35.9	40.3	45.2	47.9	52.3	57	62.2	67.8	71.8	80.6	90.5	95.9	114
29	34.5	36.5	41	46	48.8	53.2	58	63.2	69	73.1	82	92.1	97.5	116
29.5	35.1	37.2	41.7	46.8	49.6	54.1	59	64.3	70.2	74.3	83.4	93.7	99.2	118
30	35.7	37.8	42.4	47.6	50.5	55	60	65.4	71.4	75.6	84.9	95.2	100.9	120

3 A note on exposure times and time-deltas

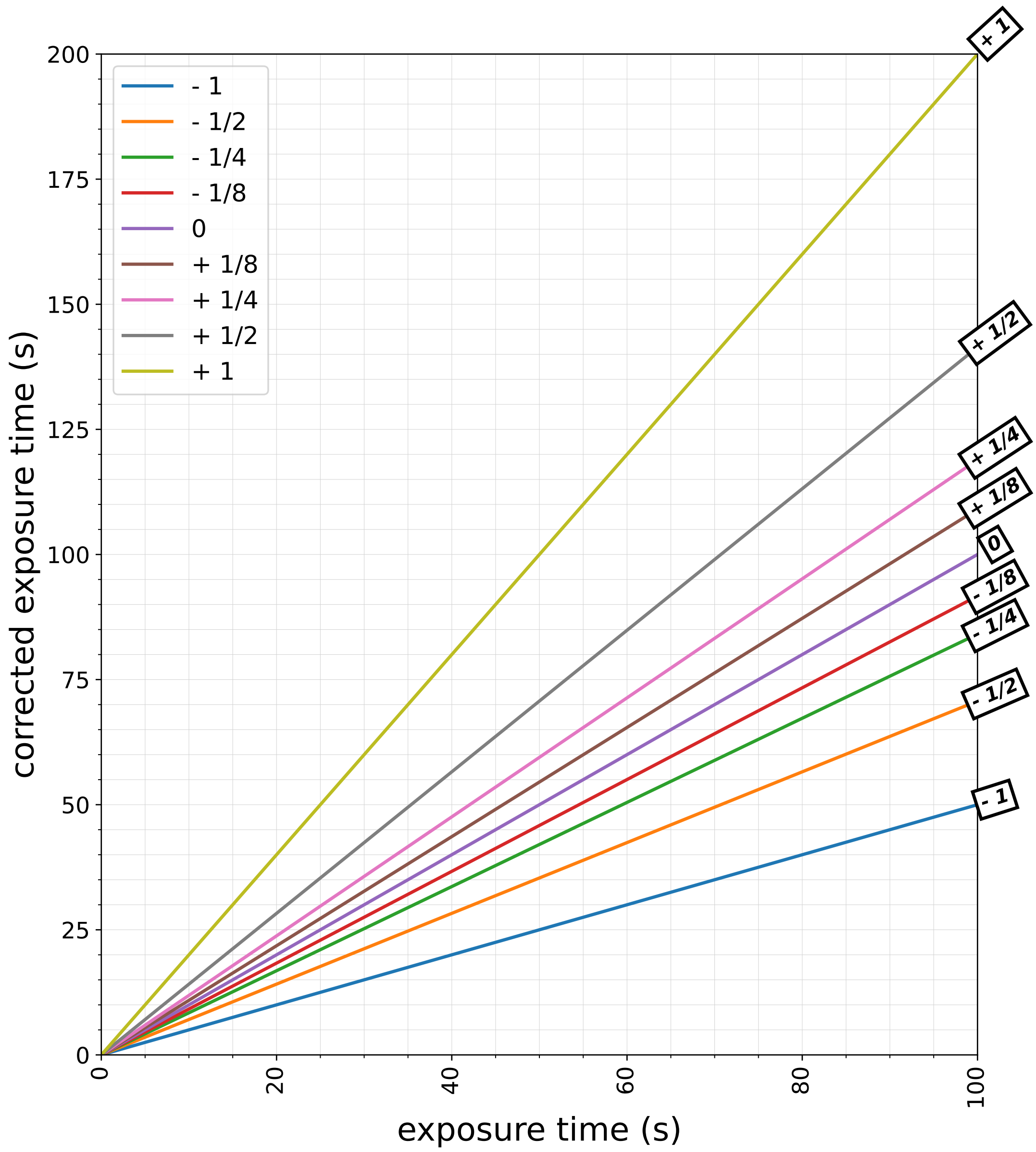
Case study example I print at 10 seconds, I want to add 1/2 of an EV: I will have to print at about 14.1 seconds. Meaning: Based on a 10 seconds exposure, I need to add about 4 seconds exposure to reach one-half of a diaphragm exposure increase. But now if I print at 40 seconds, and I *still* want to add 1/2 of an EV: I will have to add about 16 seconds exposure to reach same one-half of a diaphragm exposure increase.

Important Remark One can note that actually, the multiplicative factor is same between exposure times and time-deltas. In the example: a factor 4. Factor 4 from 10 seconds to 40 seconds. And same factor 4 for time-deltas from additional 4 seconds to additional 16 seconds.

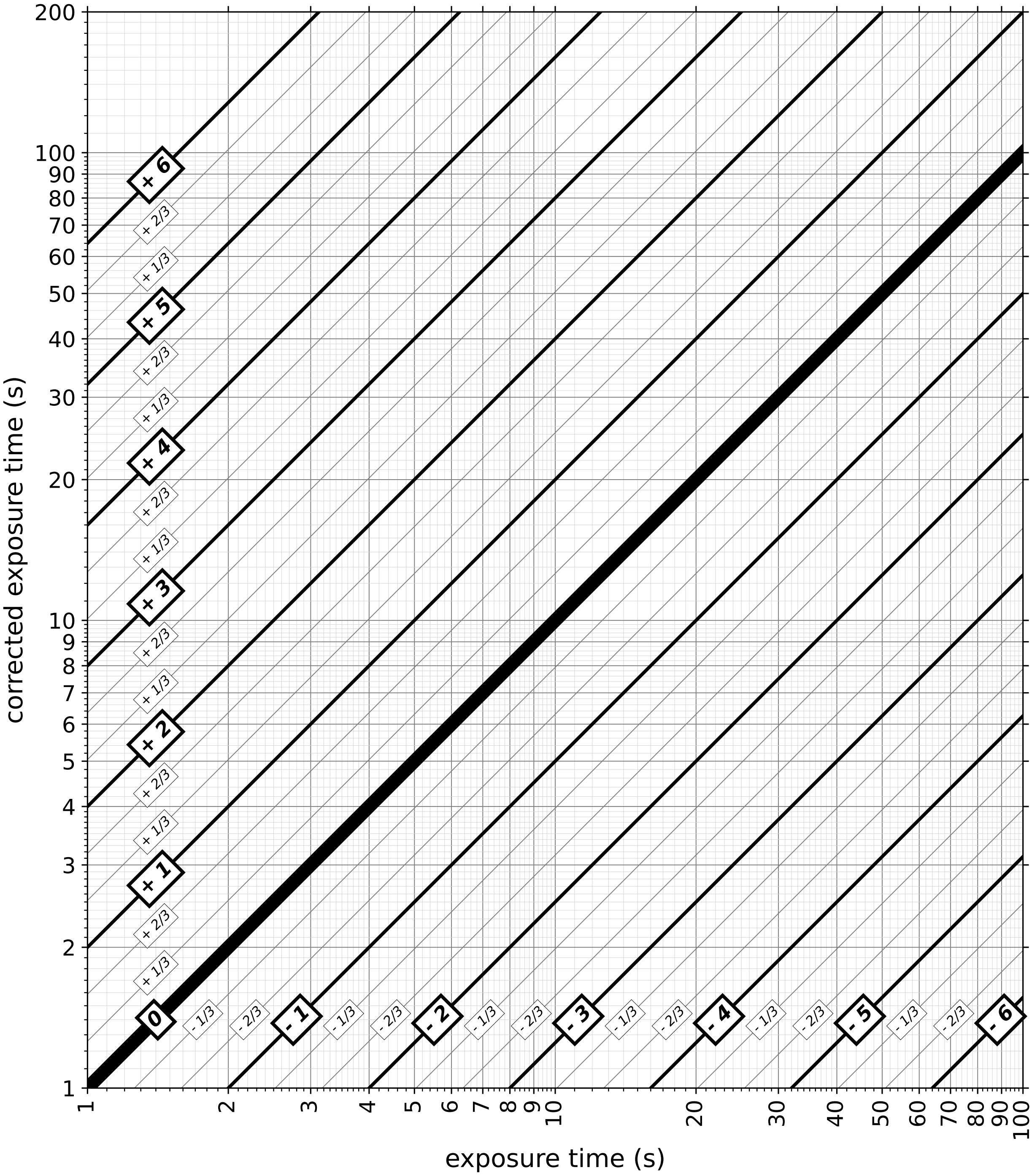
Practical consequences Let's take example of exposing at 30 seconds. Beginners often think that slightly changing the exposition time by few seconds – let's say 31 seconds or 29 seconds – will have an impact on the print. This is wrong thought and such weak time-delta *compared to such large base exposure time* (30s) will have no visible impact at all. From chosen base exposure time: to obtain a visible impact on the print, one has to go to much more important time-deltas.

Summary The larger the exposure time is, the larger the time-delta needed to add or remove same amount of light (expressed in EV) will have to be.

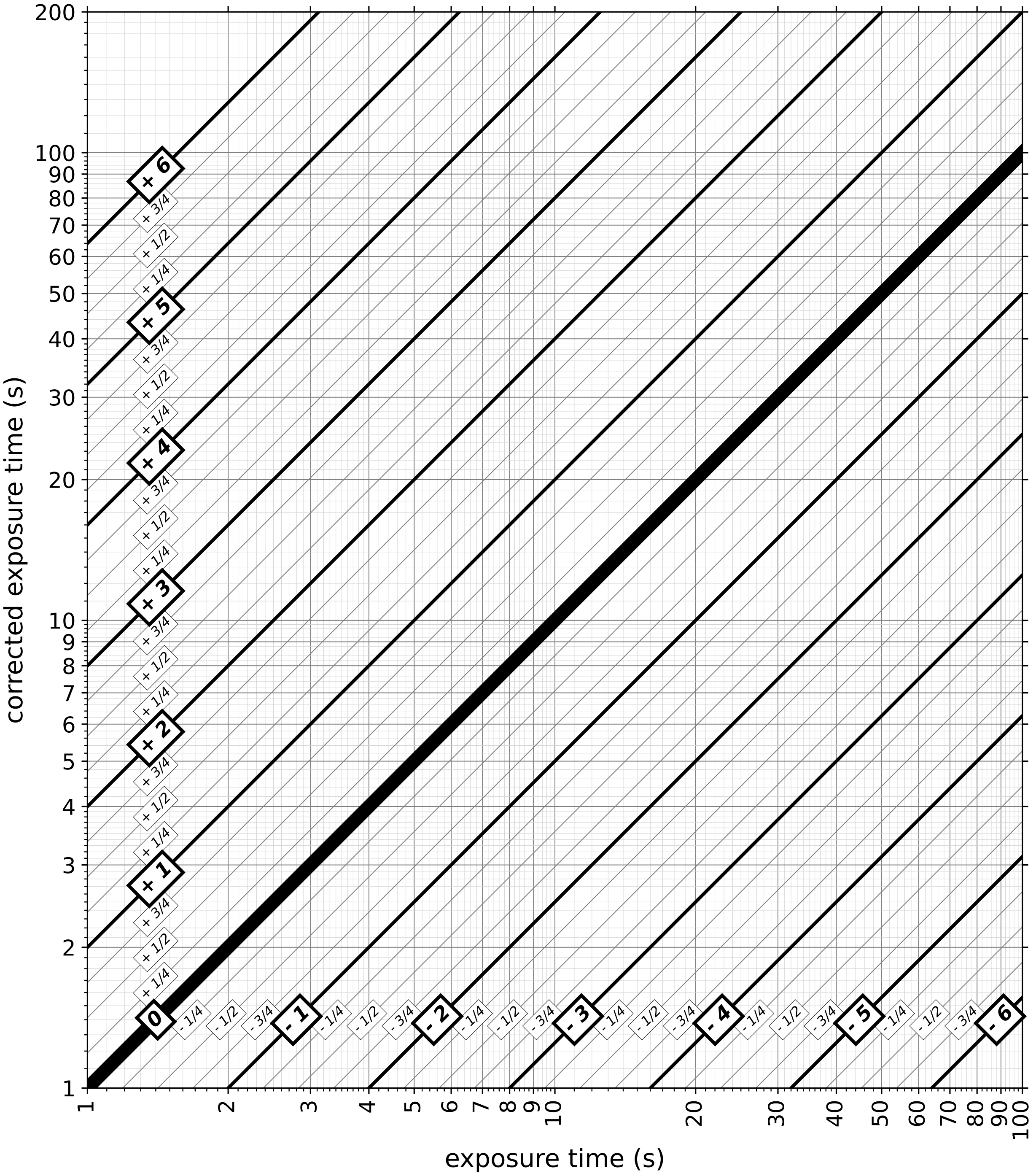
4 Abacus for some EV correction values, linear scale plot



5 Abacus: One third of an EV correction exposure times



6 Abacus: One fourth of an EV correction exposure times



7 Required time-deltas for a test strip linearly distributed over EVs

Quite often making a test strip is made from successive shifted and cumulative exposures with a given base shift time. Doing so, the exposed test strip will be made of various exposure times which will be uniformely distributed.

But one can note the following issue: following root formula provided in section 1, if the exposure times will get uniformely distributed over the test strip, *the amount of light each strip receives will not.*

7.1 Illustration of the problem brought by constant time-deltas:

I first expose the whole paper at 10 seconds, then I build the test strips by adding subsequent 5 seconds exposures (named time delta). This results in a test strip made of 6 strips, each separated by a constant amount of time. 10s, 15s, 20s, 25s, 30s, 35s, 40s.

Such distribution of exposure times results in the following EV distribution

- Strip from 10s to 15s: a bit more than +1/2 EV added.
- Strip from 15s to 20s: now a bit less than +1/2 EV added.
- Strip from 25s to 30s: only about +1/4 EV added.
- Strip from 35s to 40s: somewhere between +1/8 and +1/4 EV added.

Thus – for a constant exposure time delta of 5 seconds – the larger the exposing time of a strip, the smaller the amount of exposure light will be deposited on this strip.

8 The formula

The conceptually correct way to proceed is the following. Test strips have to be distributed in exposure times in such a way *the amount of light each strip receives is constant*. Establishing a math relation is a bit more triky because test strips receive light in a step–by–step cumulative maner. Thus, to build exposure time for a given strip, one has to know the exposure time the previous strip has received, and only the fraction of light necessary to reach the next EV step has to be added for the current exposure. The recursion formula is the following:

$$
\begin{aligned}
\text{Let's note base exposure time} &: t_0 \\
\text{let's note EV step} &: \Delta\Phi \\
\text{let's note index of strip} &: n, n > 0 \\
\text{Then exposure time for strip } n \text{ is} &: \quad \Delta t_n = t_0 \times \left(2^{n\,\Delta\Phi} - 2^{(n-1)\,\Delta\Phi}\right) \\
&\qquad\quad = t_0 \times 2^{n\,\Delta\Phi}\left(1 - 2^{-\Delta\Phi}\right)
\end{aligned}
$$

8.1 Example: necessary times-deltas for a uniform EV distribution

Let's consider 10 seconds base exposure, and a uniform step over EV equal to +1/2 diaph. Then, using the formula we obtain the amount of light to add *for each strip* table below. And the order of maskings for exposing the strip now becomes crucial (see section 10):

Strip index	exposure time	cumulated exposure times	comments
	10 seconds	**10 s**	**this is base exposure of the whole sheet**
1	**add 4.14 seconds**	**14.14 s**	**we fall back on the time correction to add +1/2 EV from 10s exposure.**
2	**add 5.86 seconds**	**20 s**	**the strip represents +1 EV added, logically the exposure of this strip is twice the original time.**
3	**add 8.28 seconds**	**28.28 s**	**the strip represents +1.5 EV added.**
4	**add 11.7 seconds**	**40 s**	**the strip represents +2 EV added, logically it is quadrupled 10 s basetime.**
4	**add 16.57 seconds**	**56.57 s**	**the strip represents +2.5 EV added.**
5	**add 23.4 seconds**	**80 s**	**+3 EV added, logically it is 8 times (2^3) the 10s basetime.**

A test strip built in such a way is *uniformely distributed over exposures*. To obtain such uniform distribution, a non-uniform distribution of exposure times is required. For such non-uniform distribution of exposure times, the order for masking the strips becomes crucial as it leads to two different ways to cumulate times. One is correct while the other is not. This is discussed in following section 10.

9 Table of exposures for a test stript made using $\Delta\Phi = 1/2$ EV step

Listing 1: The python code used to compute the table

```python
import numpy as np
# INITIAL SETTINGS
N_STRIPS = 5  # number of strips
dEV = 0.5     # the step in exposures, here 1/2 EV
BASE_EXPOSURE = 10 #seconds
#============
STRIP_IDX = np.arange(1, N_STRIPS, 1)
EXPOSURE_TIMES = BASE_EXPOSURE * 2**(STRIP_IDX * dEV) * (1 - 2**(-dEV) )
CUMULATED_EXPOSURE_TIMES =  BASE_EXPOSURE +  np.cumsum(EXPOSURE_TIMES)
```

Using python code above, one can build the following exposures table:

Strip index	**Cum. $\Delta\Phi$**	**exposure time 5 s**	**exposure time 10 s**	**exposure time 15 s**	**exposure time 20 s**
1	**1/2**	**+ 2 s**	**+ 4.14 s**	**+ 6.21 s**	**+ 8.3 s**
2	**1**	**+ 2.9 s**	**+ 5.86 s**	**+ 8.79 s**	**<u>+ 11.71 s</u>**
3	**1.5**	**+ 4.14 s**	**+ 8.28 s**	**+ 12.4 s**	**+ 16.57 s**
4	**2**	**+ 5.86 s**	**<u>+ 11.71 s</u>**	**+ 17.57 s**	**+ 23.4 s**
5	**2.5**	**+ 8.28 s**	**+ 16.57 s**	**+ 24.85 s**	**+ 33.1 s**
6	**3**	**<u>+ 11.71 s</u>**	**+ 23.4 s**	**+ 34.15 s**	**+ 46.7 s**

9.1 Comments

Note the similarity of exposure time deltas between cases for base time 5 seconds, 10 seconds, and 20 seconds. Indeed, basetime is being doubled between each of these three cases.

For varying base exposure times obtained through doubling: 5s, 10s, 20s. If a constant additional +11.71 seconds (underlined in the table) exposure is provided,

- it would correspond to adding +3EV for the case of a 5s base exposure,
- while it would correspond to adding only +2EV for the case of 10s base exposure,
- while at last it would correspond to adding only +1EV for the case of a longer 20 seconds exposure.

This interpretation exercise is same as the ones presented in sections 3 and 7.1.

10 Masking order for a test strip

Mainly there are two possible ways for creating a test strip: through progressively masking out the sheet of paper, or through progressively revealing the sheet of paper. Being conscious of the conceptual differences between these two orderings turns out crucial when manipulating non-uniform exposure time-deltas. For the case described here, times-deltas have been provided as a law which progressively *increases* time-deltas. In consequence one has to build the test strip in the following order:

- Make the global exposure,
- And progressively *mask out* strips while adding the progressively increasing time-deltas.

If the list of time-deltas is considered in the reversed order (from the largest time-delta to the shortest), then the correct masking procedure would be the following:

- Make the global exposure, *then fully mask the sheet*,
- And progressively *reveal* the strips while adding the progressively *decreasing* time-deltas.

Any other combination to associate time-deltas and progressive [masking / revealing] the strips would result in a wrong test-strip with exposures cumulated in a wrong reversed order.

11 Tables, Test-strip uniformely distributed on EV scale

11.1 Strip with $\Delta\Phi = 1/3$ EV step

Leftmost column is the base time exposure in seconds to be applied on the whole sheet of paper, then next columns represent the exposure times deltas (in seconds) to add at each strip.

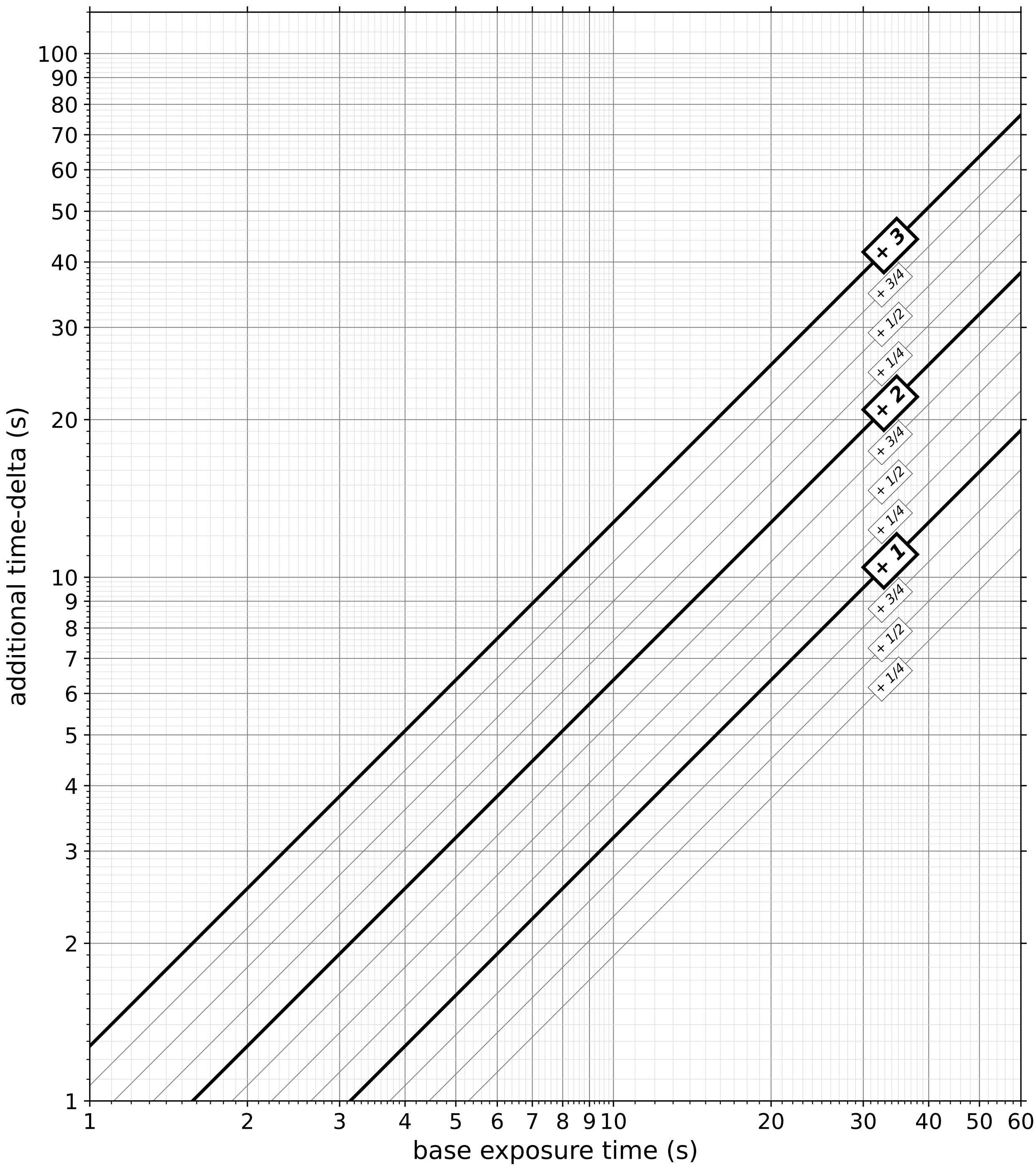
basetime (s)	+ 1/3	+ 2/3	+ 1	+ 1/3	+ 2/3	+ 2	+ 1/3	+ 2/3	+ 3
5	1.300	1.637	2.063	2.599	3.275	4.126	5.198	6.550	8.252
10	2.599	3.275	4.126	5.198	6.550	8.252	10.397	13.099	16.504
15	3.899	4.912	6.189	7.798	9.824	12.378	15.595	19.649	24.756
20	5.198	6.550	8.252	10.397	13.099	16.504	20.794	26.198	33.008
25	6.498	8.187	10.315	12.996	16.374	20.630	25.992	32.748	41.260
30	7.798	9.824	12.378	15.595	19.649	24.756	31.191	39.298	49.512
35	9.097	11.462	14.441	18.194	22.924	28.882	36.389	45.847	57.764
40	10.397	13.099	16.504	20.794	26.198	33.008	41.587	52.397	66.016
45	11.696	14.737	18.567	23.393	29.473	37.134	46.786	58.946	74.268
50	12.996	16.374	20.630	25.992	32.748	41.260	51.984	65.496	82.520
55	14.296	18.011	22.693	28.591	36.023	45.386	57.183	72.046	90.772
60	15.595	19.649	24.756	31.191	39.298	49.512	62.381	78.595	99.024

11.2 Strip with $\Delta\Phi = 1/4$ EV step

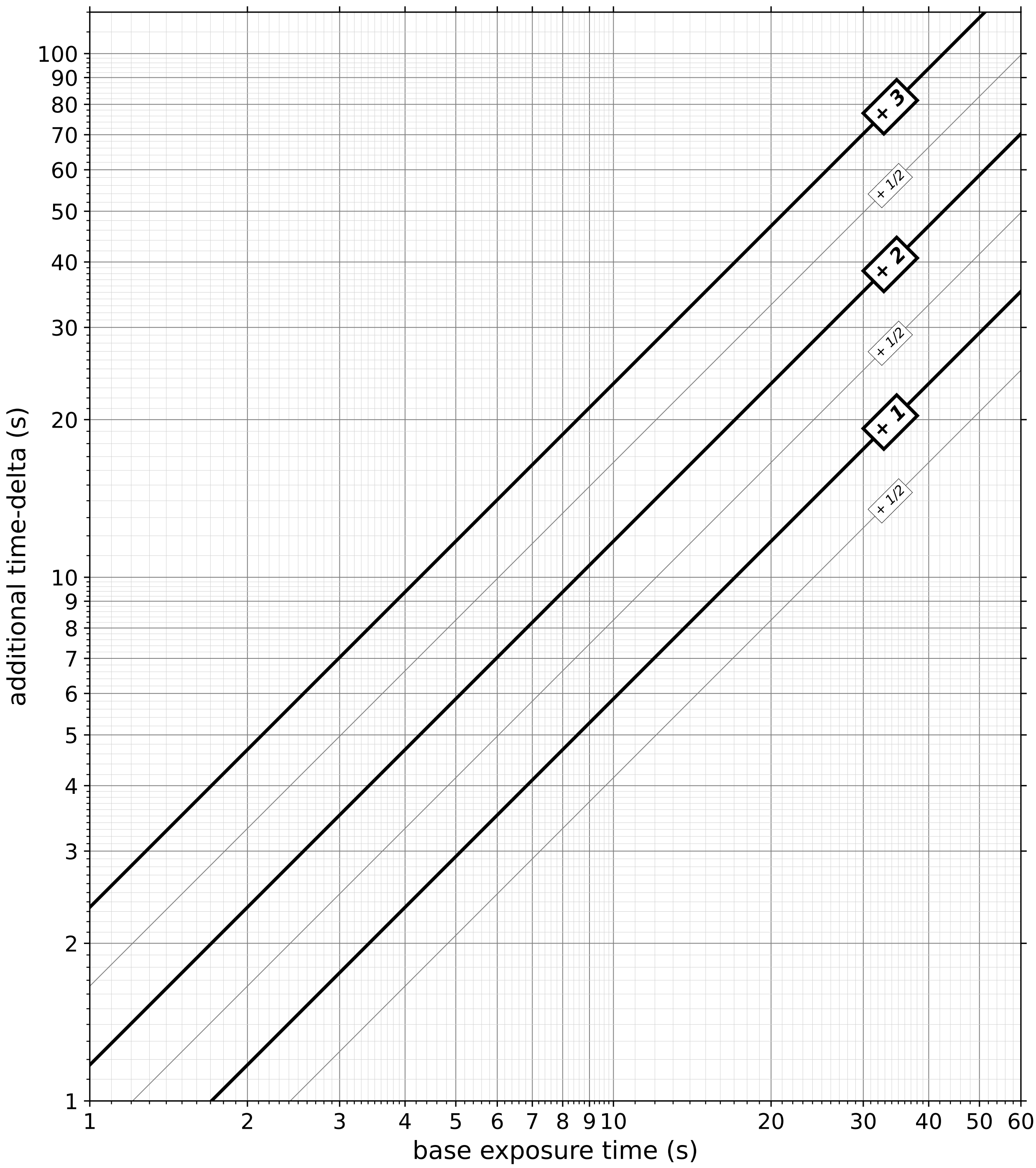
Leftmost column is the base time exposure in seconds to be applied on the whole sheet of paper, then next columns represent the exposure times deltas (in seconds) to add at each strip.

basetime (s)	+ 1/4	+ 1/2	+ 3/4	+ 1	+ 1/4	+ 1/2	+ 3/4	+ 2
5	0.946	1.125	1.338	1.591	1.892	2.250	2.676	3.182
10	1.892	2.250	2.676	3.182	3.784	4.500	5.352	6.364
15	2.838	3.375	4.014	4.773	5.676	6.750	8.027	9.546
20	3.784	4.500	5.352	6.364	7.568	9.000	10.703	12.728
25	4.730	5.625	6.689	7.955	9.460	11.250	13.379	15.910
30	5.676	6.750	8.027	9.546	11.352	13.500	16.055	19.092
35	6.622	7.875	9.365	11.137	13.244	15.750	18.731	22.275
40	7.568	9.000	10.703	12.728	15.137	18.001	21.406	25.457
45	8.514	10.125	12.041	14.319	17.029	20.251	24.082	28.639
50	9.460	11.250	13.379	15.910	18.921	22.501	26.758	31.821
55	10.406	12.375	14.717	17.501	20.813	24.751	29.434	35.003
60	11.352	13.500	16.055	19.092	22.705	27.001	32.110	38.185

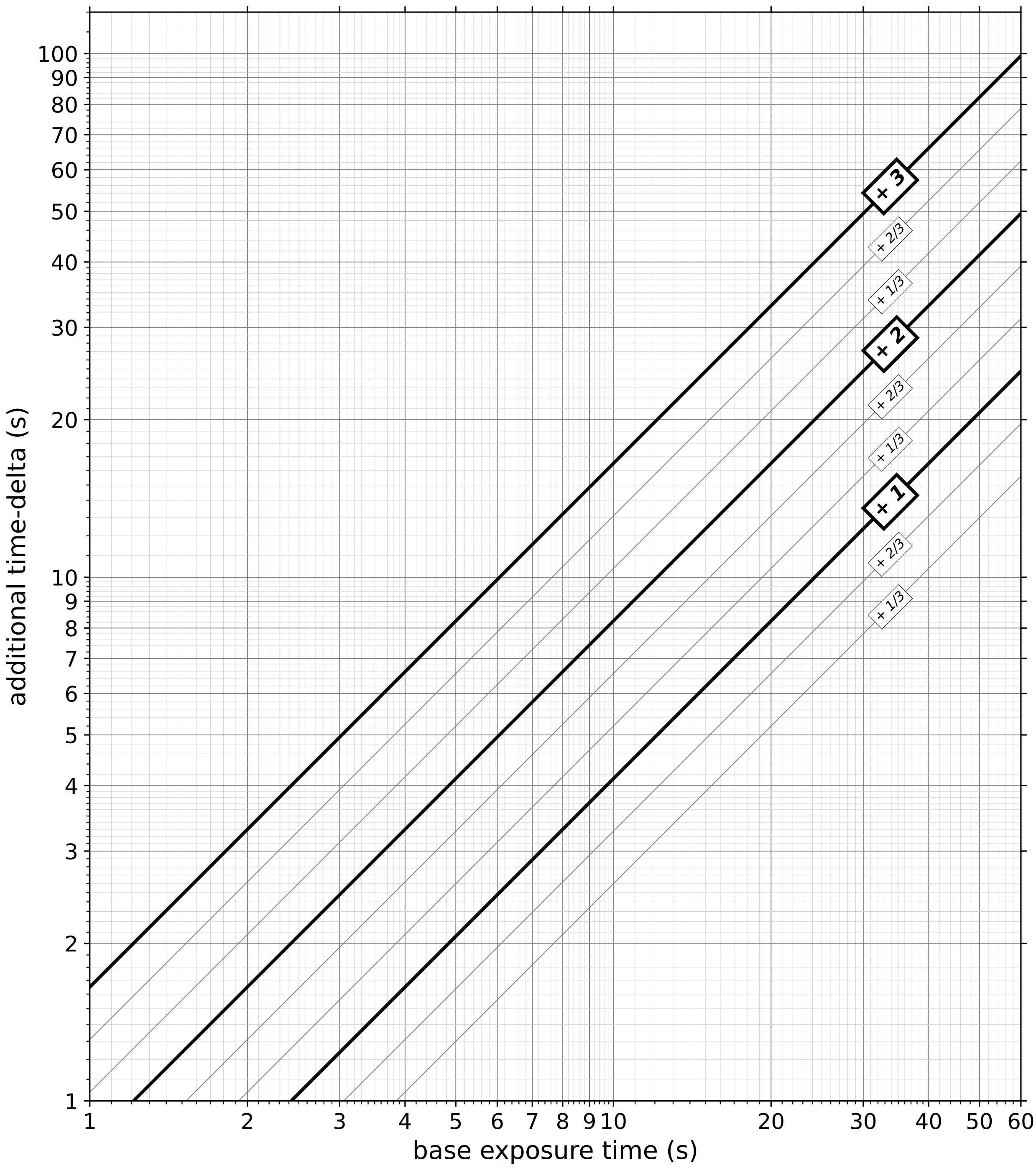
12 Abacus, Test-strip uniformely distributed on EV scale with 1/4 EV step. Time-deltas to add after a base exposure time.



13 Abacus, Test-strip uniformely distributed on EV scale with 1/2 EV step. Time-deltas to add after a base exposure time.



14 Abacus, Test-strip uniformely distributed on EV scale with 1/3 EV step. Time-deltas to add after a base exposure time.



15 Git latest history

1 modified but uncommited files.

Short hash	Date	Description
75e85ec	2024-09-03	lot of updates and changes
1df2301	2024-09-03	lot of updates, changes and renames for greater clarity