Practice II. Quantization of audio-visual signals.

1 Objectives

- Characterize the effect of quantification of the audio and speech signals.
- Understand the statistics of the quantification noise.
- Understand the effect of quantification on audio and speech signals.
- Understand the effect of quantification of gray and color images.

Bring Headphones to the laboratory.

2 Previous Study

2.1 Quantization of signals

1. Consider a 3-bits uniform mid-tread quantizer adjusted to the interval [-1,1], as the one explained in class. Quantize the sequence x[n] = [0.7, -0.1, 0.4, 0.6, -1.2].
2. For the example above, compute the quantization error sequence $e[n]=x[n]-x_q[n]$.
3. Compute numerically the quantization SNR as the energy of x[n] divided by the energy of e[n]. (The energy of a sequence x[n] is defined as $E_x = \sum_{\forall n} |x[n]|^2$)
4. Review the proof of the expression $SNR_{dB} = 6B + 4.77 - 20\log_{10}\left(\frac{x_{max}}{\sigma_x}\right)$ for the uniform quantizer in the slides of chapter 2. This formula is derived under some assumptions that the quantization example proposed in question 1 does not satisfy. Which are these assumptions?
5. Figure 2.1 shows the histogram of the sequence y[n]. How long is the sequence y[n]? Which are the maximum and minimum amplitudes of y[n]? Which is approximately the probability that the amplitude takes values in the interval [-0.2,0.2].

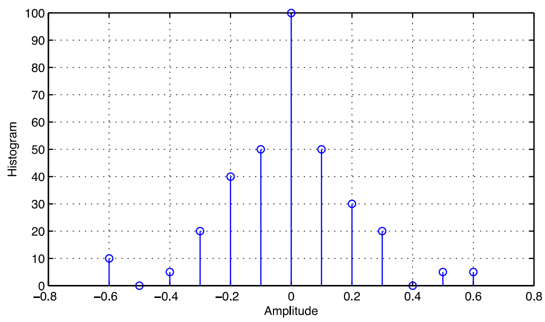


Figure 2.1: Histogram of y[n] in question 5

2.2 Study of color image signals

As it was explained in class, the human eye is not uniformly sensitive to the different wavelengths in the visible spectrum. The following figure provides the human eye luminous efficiency as a function of wavelength in two different ambient conditions: good illuminated scene (photopic vision) and badly illuminated scene (scotopic vision).

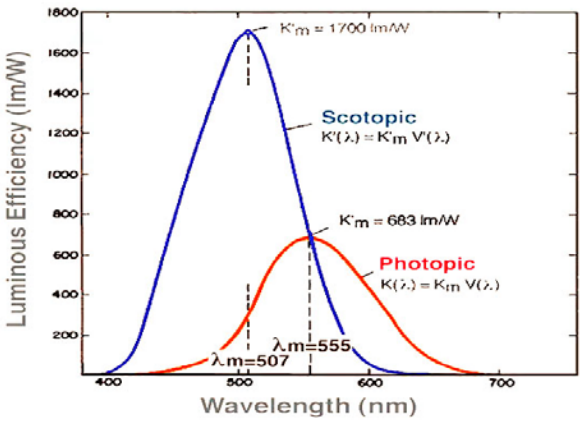


Figure 2.2: Photopic and scotopic luminous efficiency, which provides the average sensitivity of the human eye as a function of the light wavelength (source: http://webvision.med.utah.edu)

6. In view of figure 2.2, discuss which of these three lights is perceived as the brightest by the human eye in case of photopic vision: red light, green light or blue light.

In the laboratory, you will study the effect of quantizing a color image and how the quantization affects differently the color components (red, green, blue). The following figure shows horizontal color bars created by mixing different RGB color components with a gradient luminance.

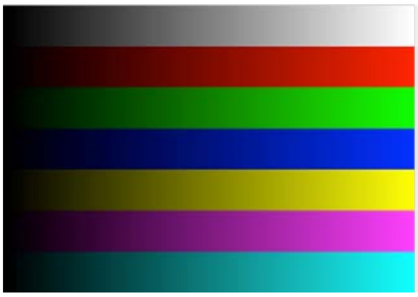


Figure 2.3: Horizontal color gradients (8 bits per color component)

7. Observe the different color bars and state the color components (RGB) that are different than zero in each of the bars by filling the following table (mark with a X the RGB components that are different from 0 in each of the horizontal bars):

Horizontal bar	R	G	B
white			
red			
green			
blue			
yellow			
magenta			
cyan			

Note: You can see the additive mixture of colors in slide 7.15 of theme 7

3 Laboratory

3.1 Uniform quantization of audio signals

The objective of this section is to assess the effect of the quantization on audio signals.

First we will load a few seconds of a musical piece which has been quantized with 16 bits, i.e. 65536 levels. The dynamic range of the signal is in-between -1 and 1. Plot the shape of an interval and, using your headphones, play the entire musical piece.

```
FileName='music.wav';                                 % Define the filename
[y,Fs] = audioread(FileName);                         % Read a stereo wav file
x=(y(:,1)+y(:,2))/2;                           % Add left/right channels to get a mono sound
Interval = 90000:100000;                              % Define a temporal interval
figure(1); plot(Interval,x(Interval)); axis tight;    % Plot the signal in the interval
sound(x,Fs); pause                                    % Listen to audio
```

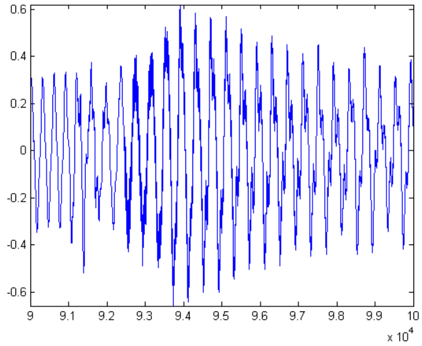


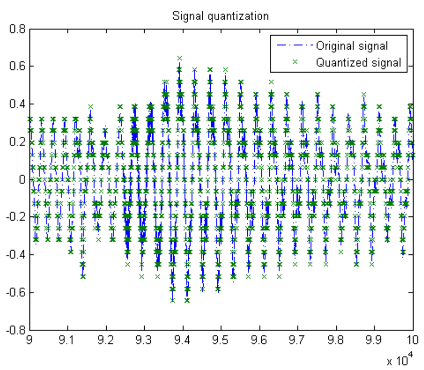
Figure 3.1: Example of an audio signal

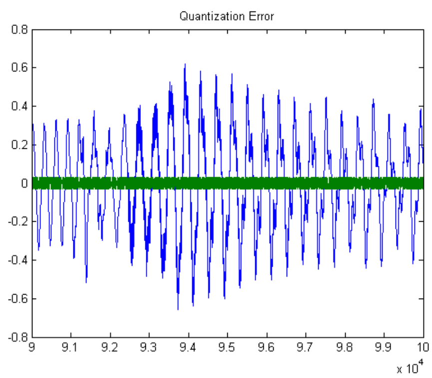
Next we will quantize the audio with different number of bits using the mid-tread uniform quantization scheme. In this scheme, one quantization level is centered around the zero value and has an odd number of quantization bins. The mathematical formula defining the quantization can be written as:

$$Sign(x)\left[Round\left(\frac{|x|}{\Delta}\right)\right]\Delta, \quad \Delta = \frac{2X_{max}}{2^B - 1}$$

Where B is the number of bits and $X_{max}$the highest possible absolute value of the signal (here $X_{max} = 1$). Let us quantize the audio signal in the interval 90.000 – 100.000 and display it as well as the quantization error:

```
Nbits = 5;                                      % Number of bits
Delta = 2/(2^Nbits -1);                         % Compute the quantization step
quantized1=round(x/Delta) * Delta;              % Quantize
figure(2); plot(Interval,x(Interval),'-.',Interval,quantized1(Interval),'x')
legend('Original signal','Quantized signal'); title('Signal quantization')
figure(3); plot(Interval,x(Interval),Interval,x(Interval)-quantized1(Interval));
title('Quantization Error')
```





Figures 3.2 and 3.3: Original and quantized signals (left) and original and quantization error (right)

1. With Nbits=5, how many quantization levels are used to represent the interval [-1, 1]? Measure the dynamic range of the quantization noise in Fig. 3.3. Is it what you were expecting from a theoretical point of view? What is the representation level of samples that are close to 0?

Listen to the quantized sound:

```
sound(quantized1,Fs)
```

2. Describe the distortion you perceive. How would you rate the signal quality in a scale from 1 to 5?

Plot the SNR as a function of the number of bits (Nbits). Show both the actual SNR (SNR_a) and the theoretical SNR (SNR_t):

```
for Nbits = 1:16;                          % Number of bits
    Delta = 2/(2^Nbits -1);                % Compute the quantization step
    quantized1=round(x/Delta) * Delta;     % Quantize
    SNR_a(Nbits)=10*log10((x'*x)/((x-quantized1)'*(x-quantized1)));    % actual SNR
    SNR_t(Nbits)=6*Nbits+4.77-20*log10(1/std2(x));                     % theoretical SNR
end
figure(4),plot(1:16,SNR_a,'b-',1:16,SNR_t,'g-'); grid; axis 'tight';
legend('Actual SNR','Theoretical SNR'); title('SNR as a function of Nbits');
```

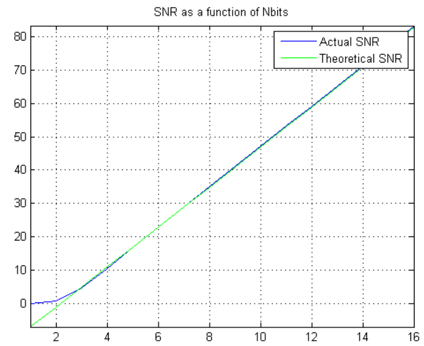


Figure 3.4: SNR as a function of Nbits

3. Describe the difference between both functions. Could you explain why the actual SNR is zero when Nbits=1?

In order to gain some insight on the difference between both SNR plots, consider the histogram of the quantization noise. Change the number of bits in the following code and study the histogram of the quantization error for different values of Nbits (for instance, 2, 4, 8, 14).

```
Nbits =14;                         % Number of bits
Delta = 2/(2^Nbits -1);            % Compute the quantization step
quantized1=round(x/Delta) * Delta; % Quantize
figure(5); hist(x-quantized1,100)  % Display the error histogram
```

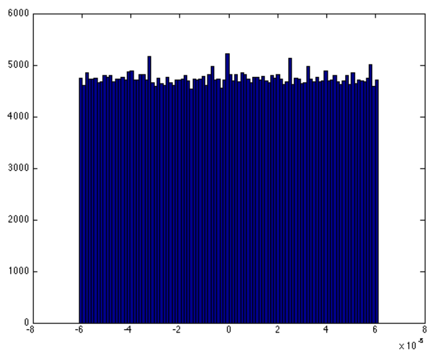


Figure 3.5: Histogram of the quantization error for Nbits=14

4. *For* which set of bit numbers is the assumption of uniform density for the noise quantization approximately valid? How does this result relate with figure 3.4?

Now, listen to the quantized music for different number of quantization bits:

```
Nbits =14;                          % Number of bits
Delta = 2/(2^Nbits -1);             % Compute the quantization step
quantized1=round(x/Delta) * Delta;  % Quantize
sound(quantized1,Fs)
SNR=10*log10((x'*x)/((x-quantized1)'*(x-quantized1)))
```

5. Give the number of quantization bits and the corresponding SNR value at which you start to perceive the distortion. Give the number of bits and the corresponding SNR value at which you would say that the distortion is unacceptable.

3.2 µ-Law quantization of speech signals

One of the problems with uniform quantization is that the maximum amplitude of the quantization error is the same no matter how big or small the samples are. This property is not suitable when we deal with the speech signal, because its amplitude distribution is far of being uniform. µ-law compression/expansion is a way to obtain lower quantization errors for the more probable low amplitude values of the speech signal and, therefore, producing a lower quantization noise power. In order to test the usefulness of the µ-law quantization we will load a speech signal originally quantized at 16bits and a sample rate of 16kHz. Load the signal into MATLAB and listen to it:

```
[x,Fs] = audioread('speech.wav');          % Read the speech wav file
interval = 5750:49930;
sound(x(interval),Fs); pause;
```

First of all we will verify that, generally, the probability distribution of speech signals follows a Laplace distribution:

```
figure(6); hist(x(interval),100);
```

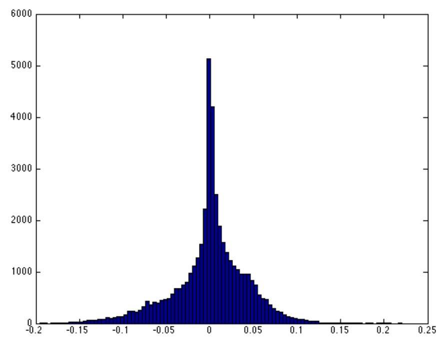


Figure 3.6: Probability distribution of the speech sample

A convenient way of describing µ-law quantization is depicted in Figure 3.7. In this representation, a µ-law compressor precedes a uniform quantizer. The combination (inside the dotted box) is a µ-law quantizer (which performs as a non-uniform quantizer).

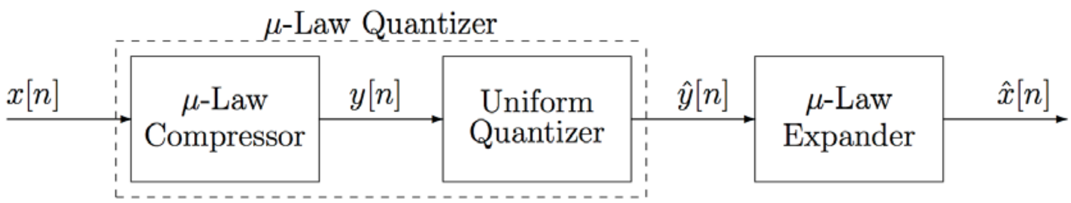
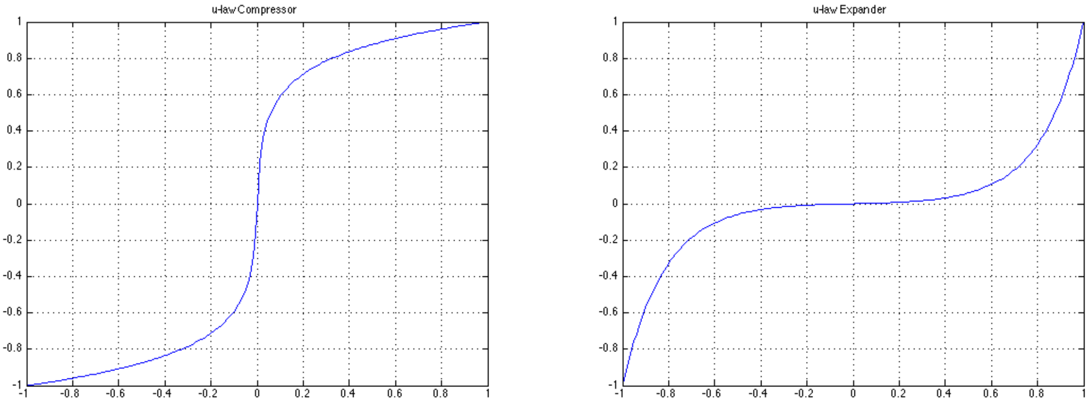


Figure 3.7: µ-law quantization scheme

The μ-law compressor is defined by the equation $y[n] = \mathrm{sgn}(x[n])\frac{\ln(1+\mu|x[n]|)}{\ln(1+\mu)}$ and the μ-law expander with $\hat{x}[n] = \mathrm{sgn}(\hat{y}[n])\frac{1}{\mu}\left((1+\mu)^{|\hat{y}[n]|} - 1\right)$ (both equations apply when the dynamic range of the signal is -1 to 1). We will represent both the compressor and expander functions of the μ-law quantizer with μ=255 (standard value used in telephony):

```
y = -1:0.01:1;
u = 255;
C = sign(y).*log(1+u*abs(y))/log(1+u);                % u-law compressor
figure(8), plot(y,C),grid;title ('u-law Compressor');
E = sign(y).*((1+u).^abs(y)-1)/u;                     % u-law expander
figure(9), plot(y,E),grid;title ('u-law Expander');
```



Figures 3.8 and 3.9: μ-law compressor (left) and expander (right) transformations

6. From Figs. 3.8 and 3.9, what happens to the low amplitude samples of x[n]? And to the high amplitude values? What happens to the quantization error in either case?

Now we will test the use of the μ-law compressor to quantize speech signals and compare it with the uniform quantizer from the point of view of the produced quantization noise power. However, using directly the SNR does not capture the advantages of using the μ-law quantizer on speech signals. A better measure is the Segmental Signal-to-Noise Ratio (SNRseg). Instead of working on the whole signal, it calculates the average of the SNR values of short segments (15 to 30 ms) in which the signal dynamic range is practically constant. It is given by:

$$SNRseg = \frac{1}{M}\sum_{m=0}^{M-1} SNR_m$$

where M is the number of segments of the signal and SNR_m is the classical SNR measured in the m^{th} segment of the speech signal. SNRseg is closer to the subjective perception of speech signals, because it gives an average of the SNR for unvoiced (low energy) and voiced (high energy) speech sounds, and provides therefore a better prediction of the μ-law quantizer advantage.

We will plot the SNRseg of the quantized speech signal of low and high power (the power is changed by amplifying the signal) for different number of bits and we will compare the efficiency of the μ-law quantizer vs. that of the uniform quantizer (Notice that the amplifying gain affects the signal dynamic range correspondingly).

```
s = x(interval);
C = sign(s).*log(1+u*abs(s))/log(1+u);         % u-law compressor
for Nbits = 2:15;                              % Number of bits
    Delta = 2/(2^Nbits -1);                    % Compute the quantization step
    qU = round(s/Delta) * Delta;               % Quantize Uniform
    q = round(C/Delta) * Delta;                % Quantize u-law
```

```
    E = sign(q).*((1+u).^abs(q)-1)/u;              % u-law expander
    SNR_UNIFORM(Nbits-1)=snr_seg(s,qU,Fs*20/1000);
    SNR_ULAW(Nbits-1)=snr_seg(s,E,Fs*20/1000);
end
figure(10);hold on; plot(2:15,SNR_UNIFORM,'r'); plot(2:15,SNR_ULAW,'b');
legend('SNRseg (uniform quantizer)','SNRseg (u-law quantizer)');
title('SNRseg for the LOW power signal'); grid on;

s = 4.5*x(interval);
C = sign(s).*log(1+u*abs(s))/log(1+u);          % u-law compressor
for Nbits = 2:15;                               % Number of bits
    Delta = 2/(2^Nbits -1);                     % Compute the quantization step
    qU = round(s/Delta) * Delta;                % Quantize Uniform
    q = round(C/Delta) * Delta;                 % Quantize u-law
    E = sign(q).*((1+u).^abs(q)-1)/u;           % u-law expander
    SNR_UNIFORM(Nbits-1)=snr_seg(s,qU,Fs*20/1000);
    SNR_ULAW(Nbits-1)=snr_seg(s,E,Fs*20/1000);
end
figure(11); hold on; plot(2:15,SNR_UNIFORM,'r'); plot(2:15,SNR_ULAW,'b');
legend('SNRseg (uniform quantizer)','SNRseg (u-law quantizer)');
title('SNRseg for the HIGH power signal'); grid on;
```

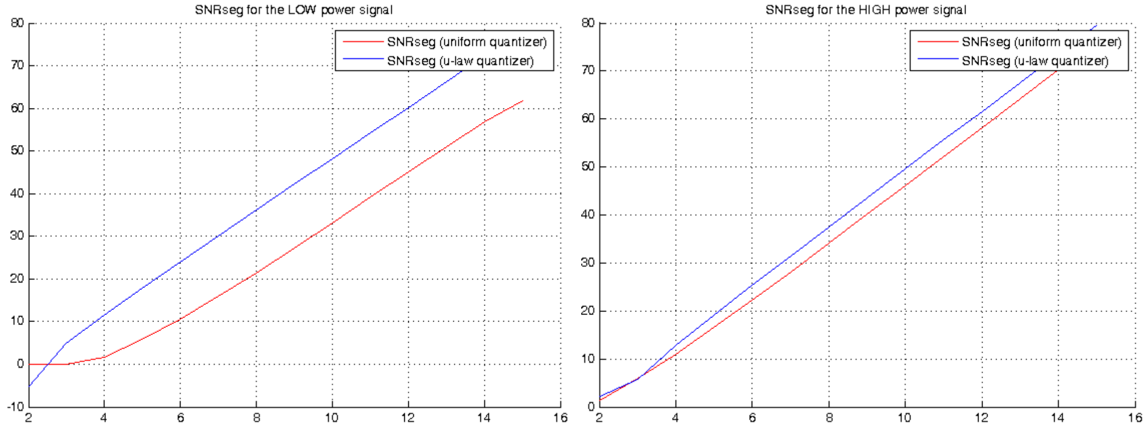


Figure 3.10 and 3.11: SNRseg as a function of Nbits for the uniform and µ-law quantizer

7. From the observation of Figures 3.10 and 3.11, could it be said that for a given Nbits the µ-law quantizer approximately yields the same SNRseg for both the low and high power speech signals? And the uniform quantizer? Why?

In order to analyze the behavior of the noise for the uniform and the µ-law quantizers, plot the quantization noise histogram for both quantizers with Nbits=8 and the two different signal powers:

```
Nbits=8;
Delta = 2/(2^Nbits -1);                         % Compute the quantization step
gain=[1,4.5];                                   % Factors of amplification
r = max(abs(x));                                % Dynamic range of the speech signal

p=0;
figure(12);
for i = 1:size(gain,2);
    s = gain(i)*x(interval);                    % the signal is amplified

% Quantization
    qU = round(s/Delta) * Delta;                % Quantize Uniform
    C = sign(s).*log(1+u*abs(s))/log(1+u);      % u-law compressor
    q = round(C/Delta) * Delta;                 % Quantize u-law
    E = sign(q).*((1+u).^abs(q)-1)/u;           % u-law expander

% Display the error histograms
    p=p+1;
    noise_sigma = std2(s-qU);
    subplot(2,2,p),hist(s-qU,100);
    title(['Uniform, gain=',num2str(gain(i)),', noise sigma=',num2str(noise_sigma)]);
    p=p+1;
    noise_sigma = std2(s-E);
    subplot(2,2,p),hist(s-E,100);
    title(['u-law, gain=',num2str(gain(i)),', noise sigma=',num2str(noise_sigma)]);
end
```

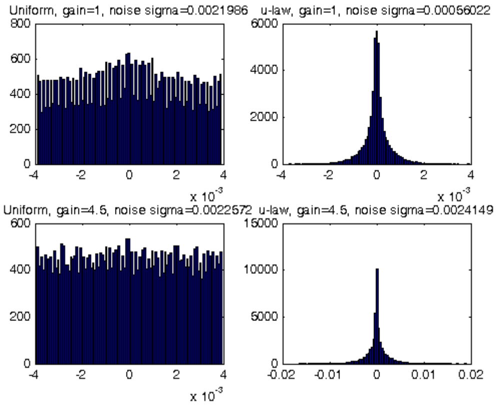


Figure 12: Noise histograms for the uniform and µ-law quantizer with of Nbits

8. Describe in general terms how the histogram of the quantization noise changes when the signal power increases. More specifically, how do the dynamic range and the power of the noise change? Is the behavior different for the uniform and µ-law quantizers? Why?

3.3 Image quantization

In this section we will use the uniform quantization studied in section 3.1 to quantize images.

$$\left[Round \left(\frac{|image(m,n)|}{\Delta} \right) \right] \Delta, \quad \Delta = \frac{Image_{max}}{2^B - 1}$$

We will use the Lena image for the quantization:

```
image_lena = imread('lena.bmp');
figure(13); imshow(image_lena,[]);
```

Figure 3.13: Original Lena image

The original image is stored with 8 bits per pixels. Let us start by a uniform quantization with 4 bits:

```
Nbits = 4;                                                  % Number of bits
Delta = 255/(2^Nbits -1);                                   % Compute the quantization step
image_lena_quantized = uint8(round(double(image_lena)/Delta)*Delta);     % Quantize
figure(13); imshow(image_lena_quantized,[]);
```

In image processing, the signal to noise ratio is generally characterized by the noise power assuming that the signal power is always equal to its maximum value (here 255 for an 8 bit image). This measure is called the Peak Signal to Noise Ratio: $PSNR = 10\,Log10\left(\frac{255^2}{||noise||^2}\right) dB$. Let us compute the PNSR of the quantized image:

```
error = double(image_lena)-double(image_lena_quantized);
10*log10(255^2 / std2(error)^2)
```

9. Experiment with different number of bits (Nbits) from Nbits=8 downwards. Give the number of quantization bits and the corresponding PSNR value at which you start to perceive the distortion (false contours). Give the number of bits and the corresponding PSNR value at which you would say that the distortion is inacceptable.

Now we will quantize a color image. We will first create an image with horizontal gradients to observe the effect of the quantization on the different color components. The original image will use 8 bits for each (red, green and blue) color component and it is the same it has been revised in the previous study:

```
gradsize = 2;
barsize = round(256*gradsize/7);
bar = kron(0:255,ones(barsize,gradsize));                % scale bar line
bar0 = zeros(barsize,256*gradsize);                      % zero bar line

gradients(:,:,1) = uint8([bar;bar;bar0;bar0;bar;bar;bar0]);    % RED component
gradients(:,:,2) = uint8([bar;bar0;bar;bar0;bar;bar0;bar]);    % GREEN component
gradients(:,:,3) = uint8([bar;bar0;bar0;bar;bar0;bar;bar]);    % BLUE component
figure(14), imshow(gradients); title('ORIGINAL');
```

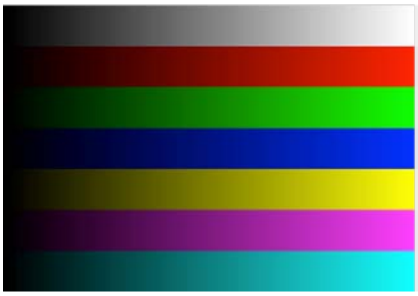


Figure 3.14: Horizontal color gradients (8 bits per color component)

Let us quantize each color component separately and create two different images, one where only the green component is quantized and another one where only the blue component is quantized. We will use 5 bits for the quantization.

```
Nbits = 5;                                           % Number of bits
Delta = 255/(2^Nbits -1);                            % Compute the quantization step
g = uint8(round(double(gradients(:,:,2))/Delta)*Delta);   % Quantize G
b = uint8(round(double(gradients(:,:,3))/Delta)*Delta);   % Quantize B
gradients_quantized_g = gradients; gradients_quantized_g(:,:,2) = g;
figure(15),imshow(gradients_quantized_g),title('GREEN QUANTIZATION');
gradients_quantized_b = gradients; gradients_quantized_b(:,:,3) = b;
figure(16),imshow(gradients_quantized_b),title('BLUE QUANTIZATION');
```

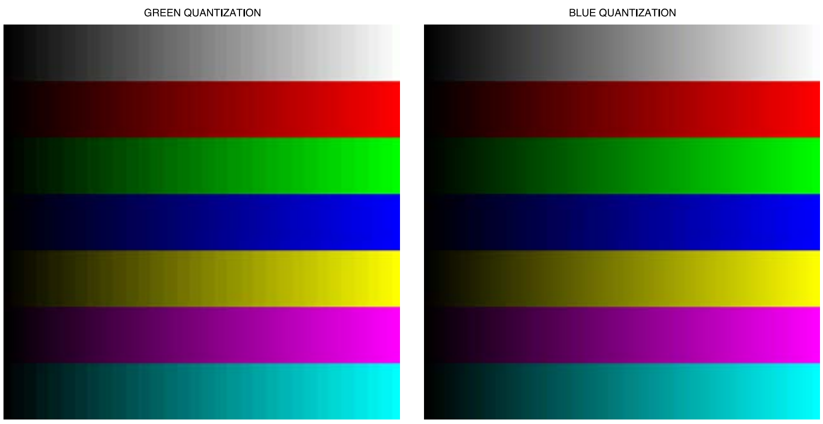


Figure 3.15 and 3.16: Green and blue quantization of the image

10. Which of the gradient bars get affected in each green and blue quantization? Why? Why do seem the bars to be more affected under the green quantization than under the blue quantization? (Hint: take into account your answer to the questions 6 and 7 of the previous study).