Recommender Systems 2

Internet Analytics (COM-308)

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Overview

- Focus on recommendation of text (prose, tags, …)
- Vector space model
  - Each dimension ~ one term (word)
- TF-IDF metric:
  - Frequency in document makes a word important
  - Frequency in many docs makes a word less important
- Probabilistic model for text classification
  - Naïve Bayes: every word is i.i.d. given class
- Smoothing:
  - Dealing with rare words not seen in training

Basic idea

- Recommend to user u items similar to the ones he/she liked before
  - Collaborative filtering: similar = liked by people who like the same stuff as u
  - Content-based: similar = with similar content features as previously liked items
- What features:
  - Context-dependent
  - Images&music: signal properties (hard); meta-information; tags;…
    - Pandora: music genome project, ~ 400 features
  - Text: easiest & most widespread
    - Prose, tags,…

Vector space model

- Compact description of a document
  - Ignores order – “bag of words”
- One dimension per term/word
  - Typically very sparse
- Count vector:
  - f_i = # of occurrences of word i in document
- Note: not reversible, ignores order of words
  - The meaning of a sentence would be lost on a human reader!
  - (a a be human lost meaning of on reader sentence the would!)

Profile from words

- How to create a useful profile of a document?
  - Frequent words are characteristic of "topic"
  - Document A: ("Probability":50, "Markov":20, "Poisson":15,…)
  - Document B: ("Wimbledon":30, "Federer":8, "Nadal":5,…)
- TF: Term Frequency
  - Function of one document j (not the whole corpus)
  - Def: f_{ij} = # of occurrences (frequency) of word i in doc j
  - Def: $TF_{ij} = \dfrac{f_{ij}}{\max_k f_{kj}}$
  - Importance of word i in document j

TF-IDF: A measure of word importance

- Problem:
  - Most frequent terms would be (in English): the, be, to, of, and, a, in, that, have, I, it, for, not, on, with, he, as, you, do, at,…
  - No information, because common to all docs
  - We want words that are frequent **only** in target docs
- IDF: Inverse Document Frequency
  - Function of whole corpus
  - Def: $n_i = \#$ documents j where word i occurs (at least once)
  - Def: $IDF_i = -\log_2 \frac{n_i}{N}$
  - If I know word i, number of bits of information I learn about which document is the target within corpus

TF-IDF vector space model

- Document profile D within a corpus:
  - $TFIDF_{ij} = TF_{ij} \times IDF_i$
  - Take top terms as document profile
  - High score: word frequent in this document, but not in most others
- Vectors are high-dim but sparse
- Refinements: text preprocessing
  - Remove stop words: the, be, to, of, and, a,…
  - Stemming & lemming: transforming
    - "the boy's cars are different colors" -> "the boy car be differ color" [Manning et al.]
  - Vector cutoff to most important terms
  - Allow multi-word terms ("United States")

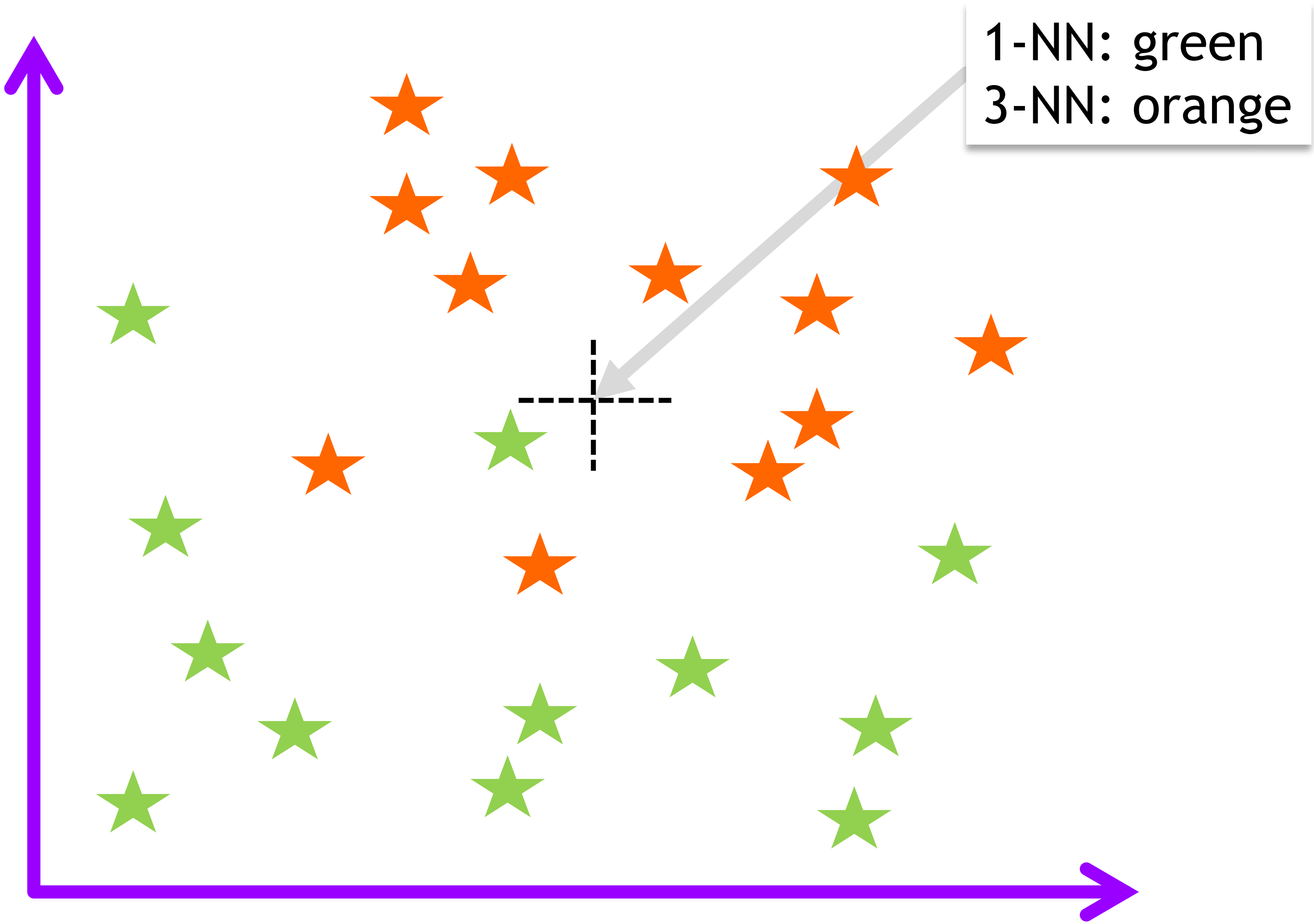
Queries and recommendations

- User profile (query) Q:
  - Explicit: e.g., formulating a query ("north korea")
  - Implicit: ratings (e.g., "likes")
- Explicit:
  - These models are from information retrieval:
    - Searching by query: return most similar docs to query
    - Query terms → TF-IDF vector Q
- Assumption:
  - Likelihood that user Q likes document D ~ $sim(Q,D)$
  - Options for $sim(Q,D)$:
    - $sim(Q,D) = \cos(Q,D)$
    - $sim(Q,D) = \frac{\sum_i (q_i - \bar{q})(d_i - \bar{d})}{\sqrt{\sum (q_i - \bar{q})^2 \sum (d_i - \bar{d})^2}}$

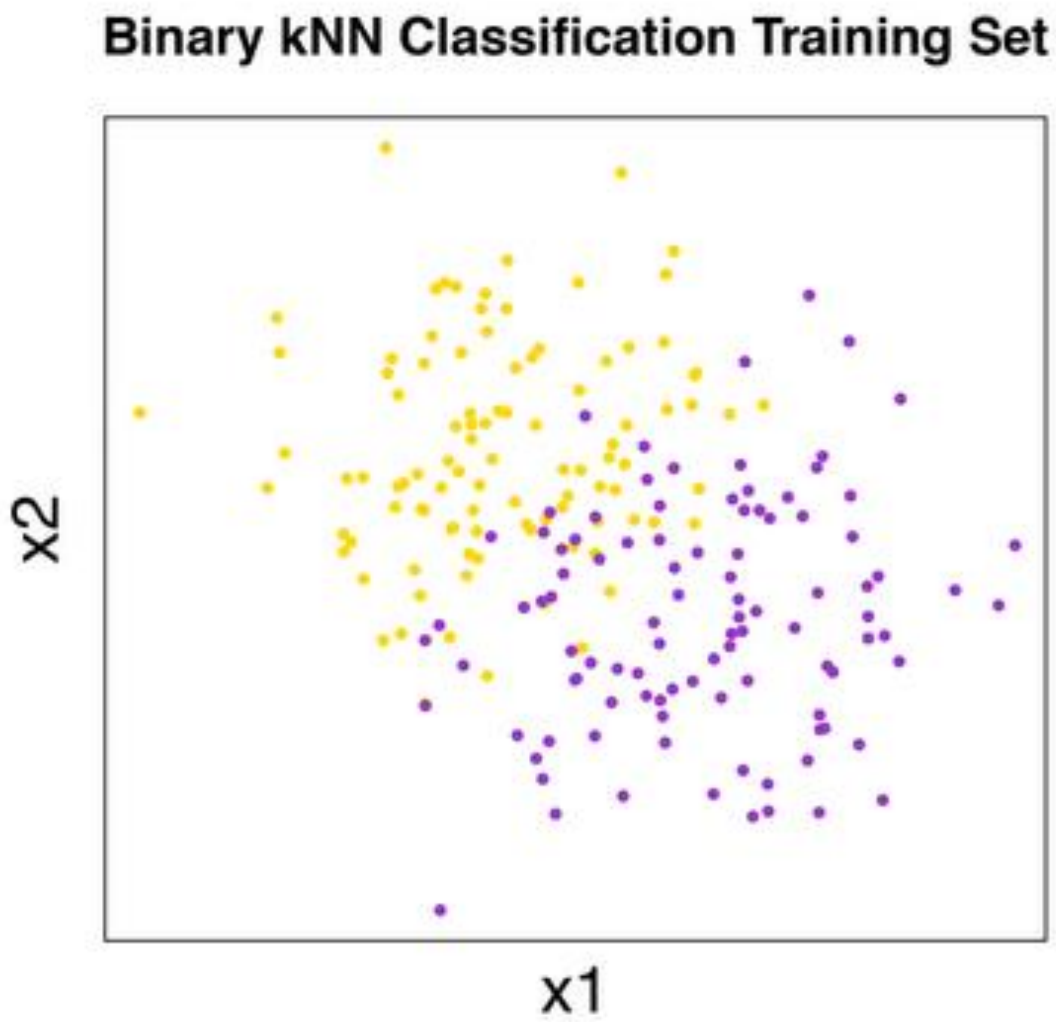
From queries to ratings

- Implicit: user rates documents rather than queries:
  - Treat highly rated/liked docs as “positive queries”, low rated/not liked as “negative queries”
- How to rate a new document D?
  - Classification problem: many methods
  - Generic non-parametric method: kNN (k nearest neighbors)
  - Select k rated docs in Q closest to D according to $sim(Q, D)$; majority in this set is predictor

kNN classifier



kNN classifier: learning k



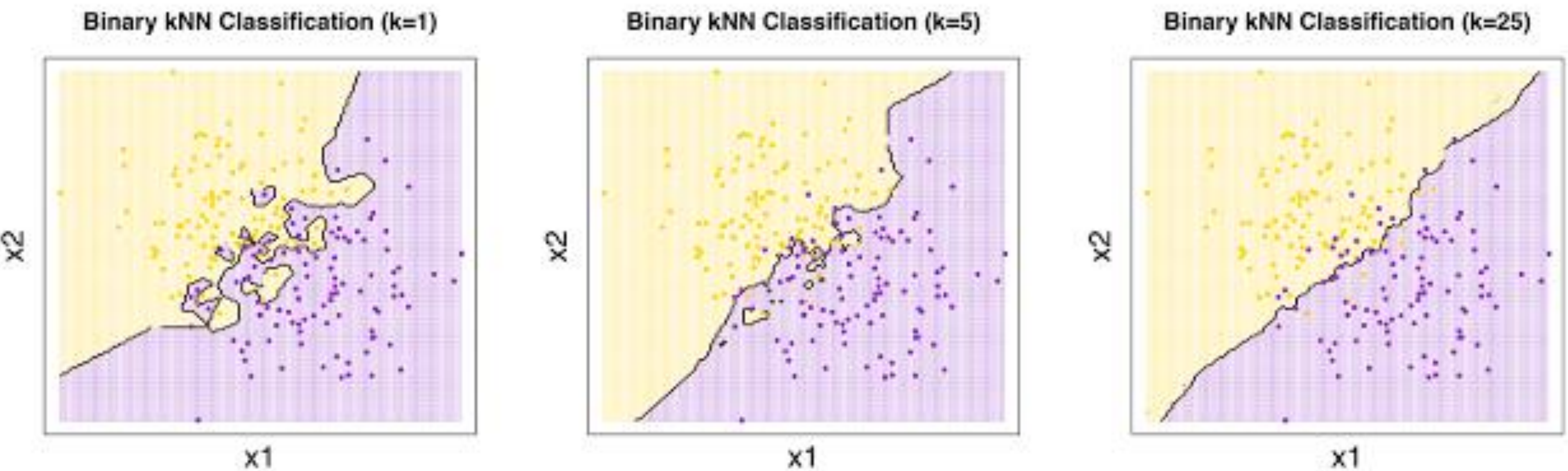
[Burton DeWilde: Data Science Rules (datasciencerules.blogspot.com), Oct 2012]

kNN: impact of k

overfitted

best model

overgeneralizes



[Burton DeWilde: Data Science Rules (datasciencerules.blogspot.com), Oct 2012]

Critique of vector-space approach

- Assumptions implicit in approach
  - “small angle between TF-IDF vectors means document close to query”: intuitively ok, hard to quantify
  - Quantities do not have “physical meaning”, purely heuristic
- We would like a clean model: assumptions, performance measure we can optimize & compare
  - Probabilistic model: rigorous treatment of uncertainty

Probabilistic models

- Significant uncertainty in predictions
  - Quantization effects: like/dislike -> how much?
  - Context: e.g.: dislike right now (mood), or dislike categorically?
  - Errors, confusions, etc.
- Uncertainty → model explicitly as probability
  - Make assumptions explicit
  - Easier to interpret significance
  - Result comes with measure of uncertainty (confidence interval, etc.)

ML: supervised vs unsupervised

- Supervised learning:
  - Given: (input, output) → find input-output map

- Unsupervised learning:
  - Given: (input) → find structure

Supervised ML: classification vs regression

- Classification:
  - Y is a class label
  - Categorical (not necessarily numeric)
  - Example: {spam, not spam}; {blue, green}; political party affiliation inferred from questions
- Regression:
  - Y is (typically) in $\mathbb{R}$
  - Example: temperature tomorrow; ranking for a movie (1…5 stars)

Bayesian inference

- Statistical inference: frequentist (non-Bayesian)
  - Observation Z
  - Model: $p_\theta(z)$: distribution of Z, depending on hidden parameter θ
  - Goal: infer θ from observation(s) of Z
  - Maximum Likelihood estimator: $\hat{\theta} = \max_\theta p_\theta(Z)$
    - Estimated parameter best explains observed data

Bayesian inference

- Statistical inference: Bayesian
  - We know something about θ: prior knowledge about the problem
  - θ is a random variable with a known distribution: prior
  - Model: $p(Z|\theta)$: distribution of Z, conditional on hidden random variable θ
  - Bayes' rule:
$$P(\theta|Z) = \frac{P(\theta, Z)}{P(Z)} = \frac{P(Z|\theta)P(\theta)}{\sum_{\theta'} P(Z|\theta')P(\theta')}$$
  - Maximum A Posteriori (MAP) estimator:
$$\hat{\theta} = \max_{\theta} P(\theta|Z)$$
  - But the full posterior distribution $P(\theta|Z)$ carries additional information!
    - How certain/uncertain are we about θ given data Z

Example: Max-Likelihood vs Bayesian

- Medical test
  - You take a medical test whose accuracy is 90% - that is, prob. test gives right result = 0.9
  - Frequentist:
    - $P(pos|sick) = 0.9; P(pos|healthy) = 0.1$
    - ML: $Z = pos$ → $\hat{\theta} = sick$
    - Test comes back positive → you conclude you are sick

Example: ML vs Bayesian

- Medical test:
  - Bayesian:
    - Medical test; prior = one in a million: $P(sick) = 10^{-6}$
    - If test comes back positive:
    - $P(sick|pos) = \frac{P(pos|sick)P(sick)}{P(pos|sick)P(sick)+P(pos|healthy)P(healthy)}$
    - $P(sick|pos) \cong 0.9 \times 10^{-5}$
    - You conclude you are very likely healthy!
  - Watch out: doctors apparently don’t know this!

Naïve Bayes classifier

- Need a probabilistic model for a document
- Simplest model:
  - Naïve = independent terms (features)
  - Each word is generated according to i.i.d. distribution

$$P(Z_1, \dots, Z_n | \theta) = \prod_i P(Z_i | \theta)$$

- Hidden variable:
  - Relevant (good, G) or not relevant (bad, B)
- Observable variable:
  - Message = set of words $(z_1, z_2, \dots, z_n)$
- Classify message into (G, B)
- Model $p(Z|\{G, B\}), p(\{G, B\})$:
  - Learn from data

Example: naïve Bayes classifier

- Training set:

Get nice watch	Cheap replica watch
New York rocks!	New cheap loan
Watch for rocks	Get lottery million
	Million dollar watch

- Prior: $P(\theta = G) = \frac{3}{7}; P(\theta = B) = \frac{4}{7}$
- Conditional word distributions $P(Z|\theta)$:

Z	get	nice	watch	new	york	rocks	for	cheap	replica	loan	lottery	million	dollar	perfect
$9 \times P(Z\|G)$	1	1	2	1	1	2	1	0	0	0	0	0	0	0
$12 \times P(Z\|B)$	1	0	2	1	0	0	0	2	1	1	1	2	1	0

Example: naïve Bayes classifier

- Classifying sentences $M = (Z_1, Z_2, Z_3, \dots)$:
  - «get new watch»:

$$P(G|M) =$$

$$= \frac{P(Z_1|G)P(Z_2|G)P(Z_3|G)P(G)}{P(Z_1|G)P(Z_2|G)P(Z_3|G)P(G) + P(Z_1|B)P(Z_2|B)P(Z_3|B)P(B)} =$$

$$= \frac{9^{-3} \cdot 1 \cdot 1 \cdot 2 \cdot 3/7}{9^{-3} \cdot 1 \cdot 1 \cdot 2 \cdot \frac{3}{7} + 12^{-3} \cdot 1 \cdot 1 \cdot 2 \cdot 4/7} = 0.64$$

Z	get	nice	watch	new	york	rocks	for	cheap	replica	loan	lottery	million	dollar	perfect
$9 \times P(Z\|G)$	1	1	2	1	1	2	1	0	0	0	0	0	0	0
$12 \times P(Z\|B)$	1	0	2	1	0	0	0	2	1	1	1	2	1	0

Example: naïve Bayes classifier

- Classifying sentences $M = (Z_1, Z_2, Z_3, \ldots)$:
  - «cheap replica rocks»:

$$P(G|M) =$$

$$= \frac{\boxed{=0}\; P(Z_3|G)P(G)}{\boxed{=0}\; P(Z_3|G)P(G) + P(Z_1|B)P(Z_2|B)\; \boxed{=0}\; P(B)}$$

  - Undefined!

Z	get	nice	watch	new	york	rocks	for	cheap	replica	loan	lottery	million	dollar	perfect
$9 \times P(Z\|G)$	1	1	2	1	1	2	1	0	0	0	0	0	0	0
$12 \times P(Z\|B)$	1	0	2	1	0	0	0	2	1	1	1	2	1	0

Problem with unseen training terms

- Sparsity problem:
  - If alphabet of words is large w.r.t. training set, there are some words z we never see (e.g., $z =$"mesonoxian")
    - Estimate: $P(\text{mesonoxian}|\{G,B\}) = 0$
  - If target message contains "mesonoxian":

$$P(\{G,B\}) = \frac{P(z|\theta)P(\theta)}{\sum_{\theta'} P(z|\theta')P(\theta')} = \frac{0}{0}$$

- Problem:
  - We estimate a distribution from a very small set of samples – a form of overfitting
  - How to correctly estimate very rare words?
- Approach 1:
  - Ignore unseen words → simple, but crude; throws away information

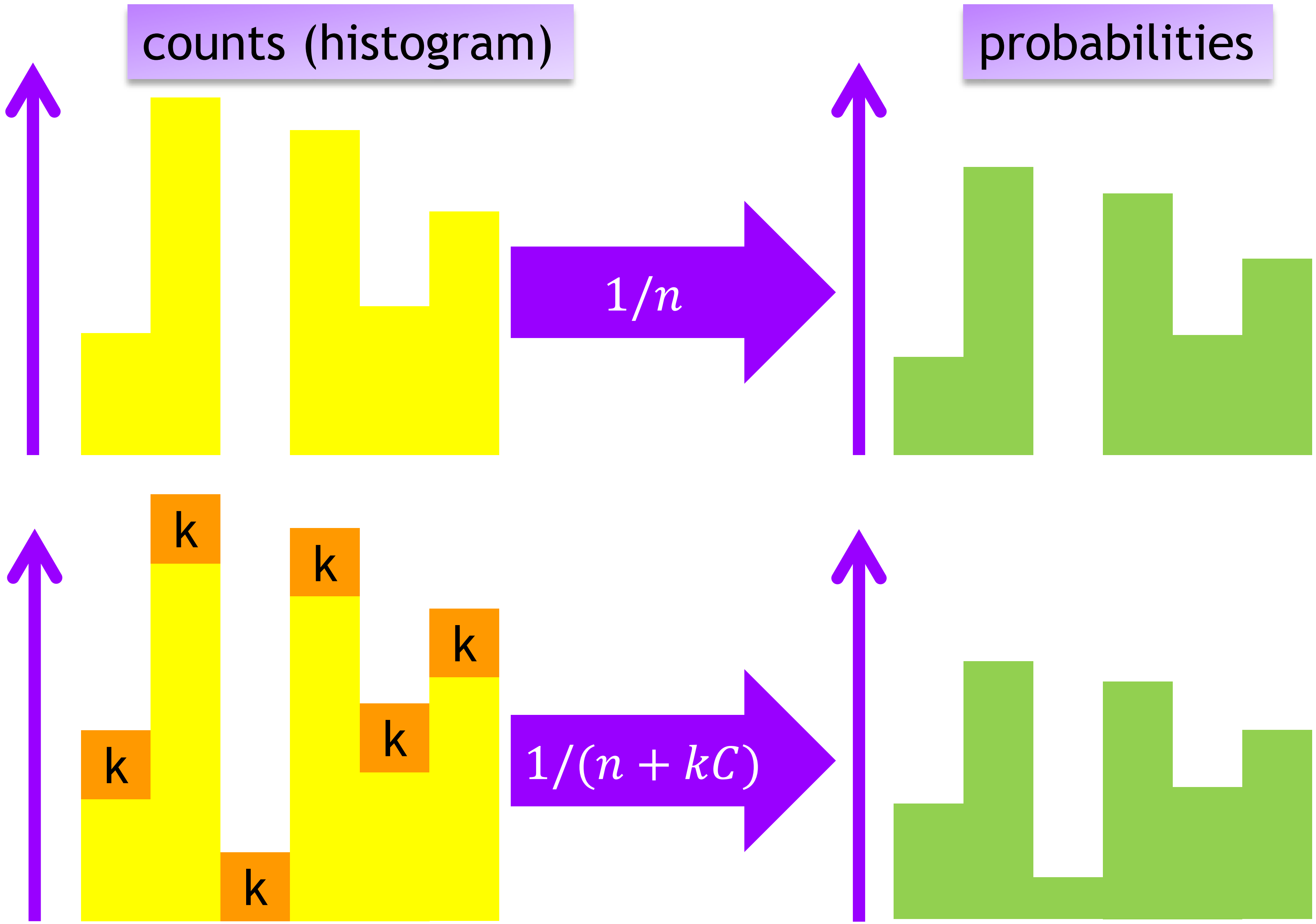
Laplace smoothing

- Idea: assume every word occurs at least once
  - Aka "additive smoothing", "add-one smoothing"
- Bias towards uniform distribution
  - A form of regularization
- Estimate of a distribution over domain $D = \{d_1, \ldots, d_C\}$ from data set $\{x_1, x_2, \ldots, x_n\}$
  - Unsmoothed: $p(X = x) = \frac{|\{x: x = d_i\}|}{n}$ (n=# samples)
  - Smoothed: assume k "fake" observations for each class

$$p(X = d_i) = \frac{|\{x: x = d_i\}| + k}{n + kC}$$

  - Empty dataset $(n = 0)$ → $P(Z|\theta)$ uniform
  - Large dataset $(n \gg 1)$ → smoothed $P(Z|\theta) \cong$ unsmoothed $P(Z|\theta)$

Laplace smoothing



Example: Laplace-smoothed classifier

- Sentence M =«cheap replica rocks»:

$$P(G|M) =$$

$$= \frac{P(Z_1|G)P(Z_2|G)P(Z_3|G)P(G)}{P(Z_1|G)P(Z_2|G)P(Z_3|G)P(G) + P(Z_1|B)P(Z_2|B)P(Z_3|B)P(B)} =$$

$$= \frac{23^{-3} \cdot 1 \cdot 1 \cdot 3 \cdot 4/9}{23^{-3} \cdot 1 \cdot 1 \cdot 3 \cdot 4/9 + 26^{-3} \cdot 3 \cdot 2 \cdot 3 \cdot 5/9} = 0.37$$

- Advantages:
  - We can compute an estimate for any message
  - For small training sets → avoids overfitting

Z	get	nice	watch	new	york	rocks	for	cheap	replica	loan	lottery	million	dollar	perfect
$23 \times P(Z\|G)$	2	2	3	2	2	3	2	1	1	1	1	1	1	1
$26 \times P(Z\|B)$	2	1	3	2	1	1	1	3	2	2	2	3	2	1

RecSys: content vs collaborative

Pros
Independent of other users → no cold start problem for new items (item comes with features)
Independent of other users → can recommend for unique tastes, no "trend to average"
Can provide reasons for recommendation (e.g., matching keywords)

Cons
Multimedia etc.: hard to identify features
Independent of other users → no discovery, no surprises
Cold start problem for new user

- In practice: combination
  - Lack of ratings, few users → rely more on content
  - Lots of users, few tags → collaborative

Summary

- Content: text, tags, user comments, subtitles,…
- Collaborative filtering vs content-based:
  - Blind to content vs blind to other users
- Classical approaches from information retrieval:
  - Vector space models, similarity metrics
- Modern probabilistic approaches from ML:
  - Naïve Bayes, language models (n-grams), word embeddings
- Other application for naïve Bayes: spam filtering
  - $P(B) \cong 0.8 \ldots 0.9$

References

- [A. Rajaraman, J. D. Ullman: Mining of Massive Datasets, Cambridge, 2012 (chapter 9)]
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- [W. B. Croft, D. Metzler, T. Strohman: Search Engines – Information Retrieval in Practice, Addison Wesley, 2010 (chapters 7&10)]
- [Ch. D. Manning, P. Raghavan, H. Schütze: Introduction to Information Retrieval, Cambridge, 2008]