Ad Auctions

Internet Analytics (COM-308)

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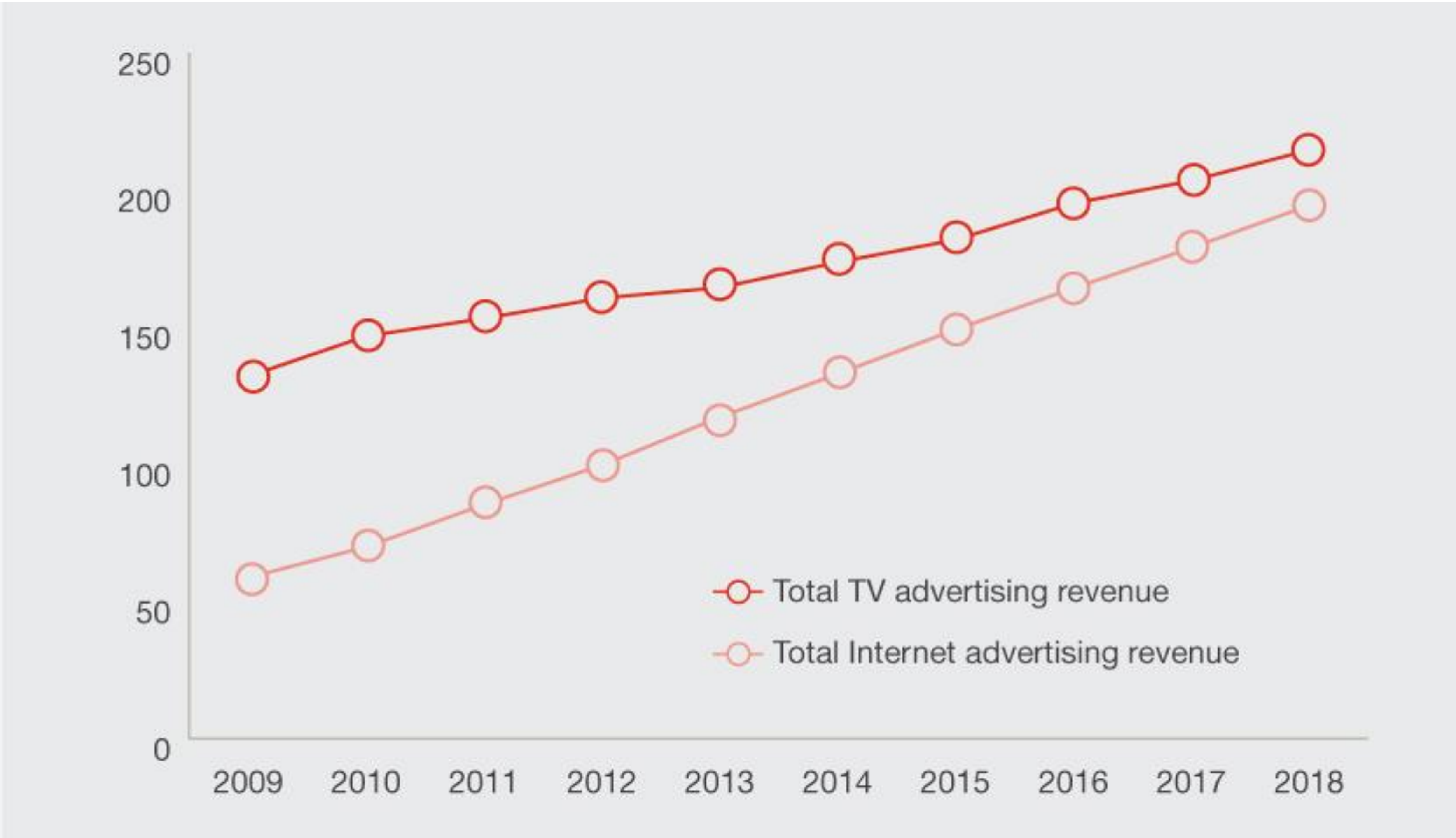
Overview

- Advertisement is **the** Internet business model
  - Plus a bit of subscription, freemium, etc.
- Most ad impressions are auctioned
  - Google AdSense: banner ads on third-party websites
  - Google AdWords: search ads
- Classical auctions: single item
  - First vs second price
- Ad auctions: multiple items = several “positions” on web page
  - Generalized Second Price auction
  - VCG auction

Online advertisement market

- Business models for the Internet:
  - (a) Paying for it: subscription, per-use, taxes, …
  - (b) Providing data and consuming advertisement
- "If you're not paying for it, you're the product"
  - Willingness to sacrifice privacy and "paying through attention"
  - Not just about the money, but ease of use
- Ad market:
  - Globally ~ 700bn$/year
  - Online: ~ 150bn$/year (2015)
    - Web: maturing; mobile: big challenge

Online vs TV advertisement market

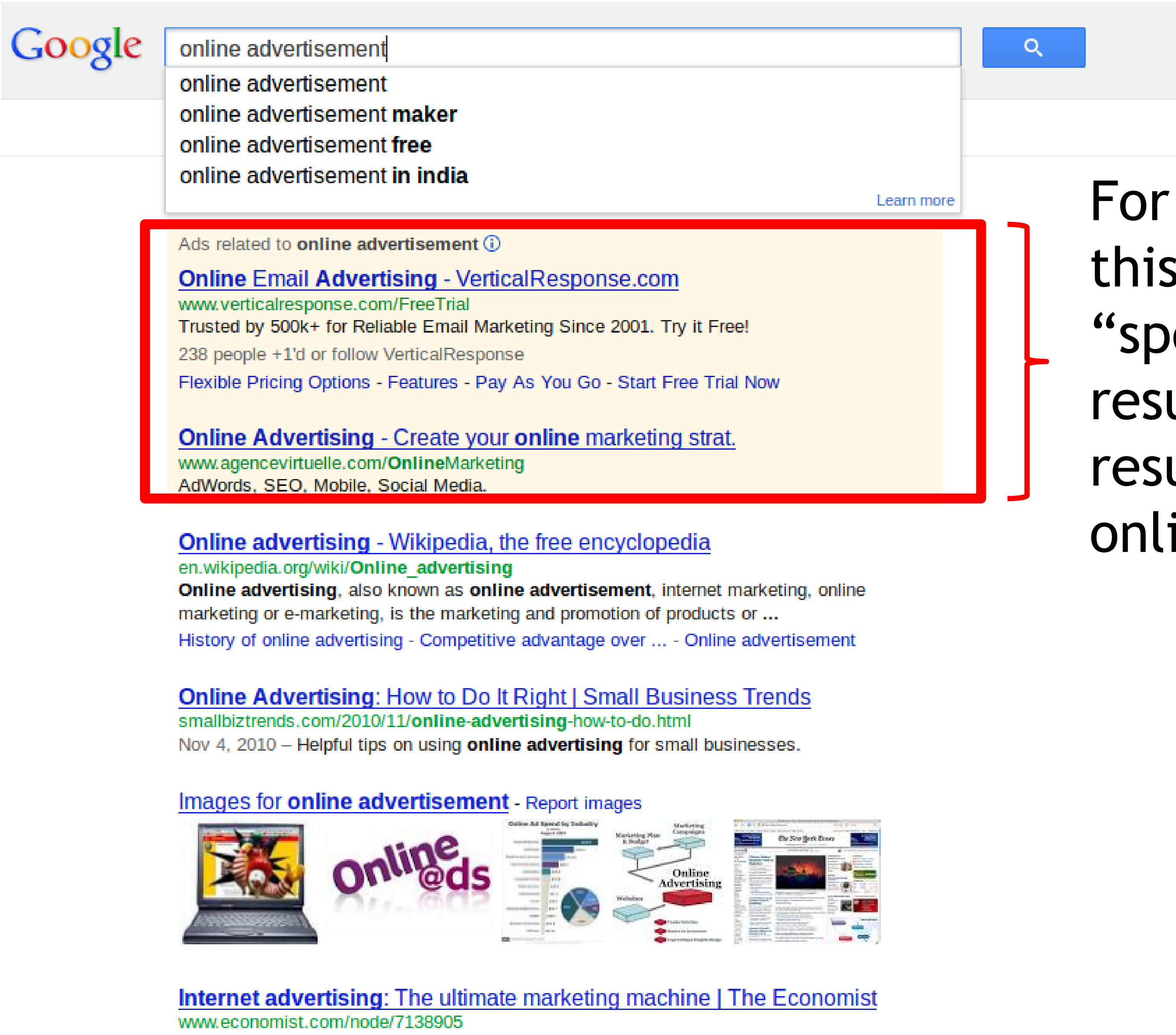


[source: PwC 2014]

Online advertisement market

- Banner ads:
  - Pay-per-impression (user sees ad)
    - CPM (cost-per-mille)
  - Pay-per-click (PPC) (user takes action)
    - CTR (clickthrough rate = clicks per impression)
    - CPC (cost per click)
- Search ads / sponsored search
  - Google AdWords: bidding for keywords plus other constraints (geographic, max cost,…)
  - Google shows your ads in response to searches, charges for traffic

Ad auctions



For each search, this table of “sponsored search results” is the result of an online auction

Search ads: how to allocate?

- Resource allocation problem:
  - One seller: google, amazon, bing, etc.
  - Many bidders: advertisers buying visibility
  - Several items: multiple locations on results panel
- Auction:
  - Google runs an auction for every search to sell the ad space!
  - Several billion searches per day
  - Very heavy tailed CPC distribution:
    - “mesothelioma” had CPC up to 79 US$ a few years ago – asbestos lawyers

Auctions

- Open vs sealed envelope
  - Public vs private bids
- Open: ascending (English) vs descending (Dutch)
  - Ascending: increase the price until single bidder left
  - Descending: decrease the price until a bidder calls out
- First-price vs second-price
  - First: winning bidder pays highest bid
  - Second: winning bidder pays second-highest bid
- Equivalences:
  - Descending equivalent to sealed first-price
    - Winning bidder calls out when willing to pay the price
  - Ascending equivalent to sealed second-price
    - Winning bidder stops bidding when second-highest drops out

Single-item auctions

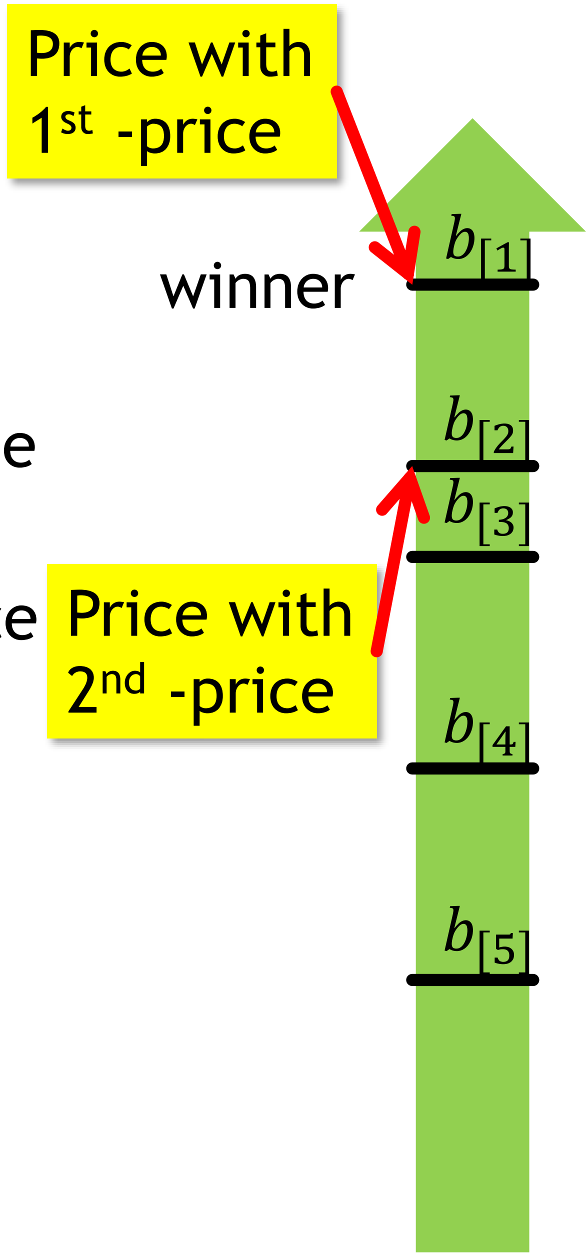
- One item for sale, many potential buyers
  - N bidders
  - Bid of bidder i: b_i
  - Internal valuation of bidder i: v_i
    - Advertisement: revenue generated by ad
    - Art: monetary measure for viewing pleasure/pride/envy of friends/…
  - Price paid by bidder i: p_i
    - 0 if lost, price determined by auction mechanism if won
- Payoff (or utility):
  - $U_i = \begin{cases} 0 & \text{if lost} \\ v_i - p_i & \text{if won} \end{cases}$

Single-item auction

- Valuation, price, payoff
  - Valuation v_i: depends only on bidder (personal preference, business case, etc.)
  - Price $p_i = p_i(b_1, b_2, \dots, b_N)$: depends on everybody's bid through the auction mechanism
- Strategy:
  - Each bidder selects bid b_i that maximizes $U_i(b_1, b_2, \dots, b_N)$
  - b_i too low: risk not winning the auction
  - b_i too high: risk paying too much

Why second-price auction?

- Intuition:
  - First price seems natural and reasonable:
    - Bid what you are willing to pay = value
  - Second-price seems manipulable:
    - Bid very high to win, pay only 2nd price
- Theory:
  - First price: bidding value means zero payoff → must bid less than value
  - Second-price: bidding too high is a bad strategy, if others follow the same strategy → will pay above value, negative payoff



Truthful bidding in second-price

- Theorem: in a second-price auction, truthful bidding ($b = v$) is a dominating strategy
  - Dominating: regardless of what strategy other players use, best strategy for myself
- Proof: assume I bid b' instead of $b = v$
  - Case $b' < v$ ("under-bidding"):
    - Affects outcome only if 2nd-highest bid $b_{[2]}$ is $b' < b_{[2]} < v$ → auction lost, $U = 0$ instead of $U = v - p = v - b_{[2]} \geq 0$
  - Case $b' > v$ ("over-bidding"):
    - Affects outcome only if highest bid $b_{[1]}$ is $v < b_{[1]} < b'$ → auction won, $U = v - p = v - b_{[1]} < 0$ instead of $U = 0$

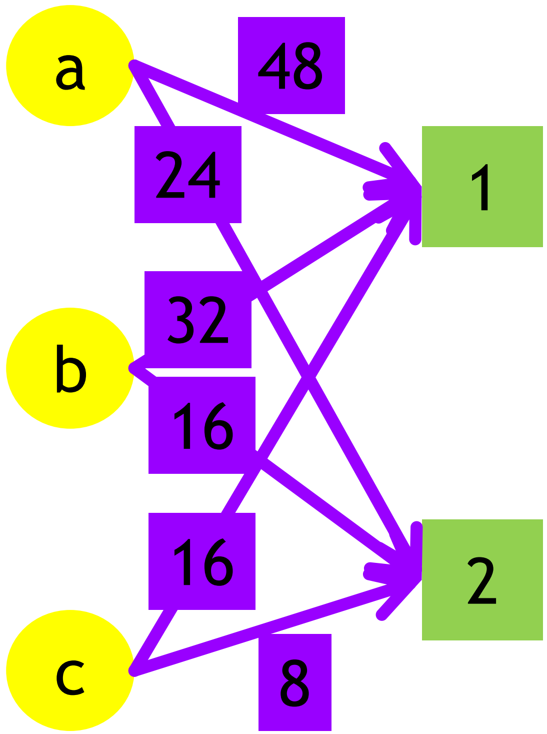
Second price below b'

Multiple-item auctions

- K spaces for sale (decreasing value)
- Bid vector: $b_i = (b_{i1}, b_{i2}, \dots, b_{iK})$
- Allocation:
  - Maximum matching: M maximizes $\sum_{(i,j)\in M} b_{ij}$
- Generalized Second Price (GSP):
  - Winner of kth item pays winning bid for $(k+1)$st item
  - Simple, used in Google AdWords
- Vickrey-Clarke-Groves (VCG):
  - Bidder i pays its "damage" (externality in economics-speak) on everybody else
  - More on this later…

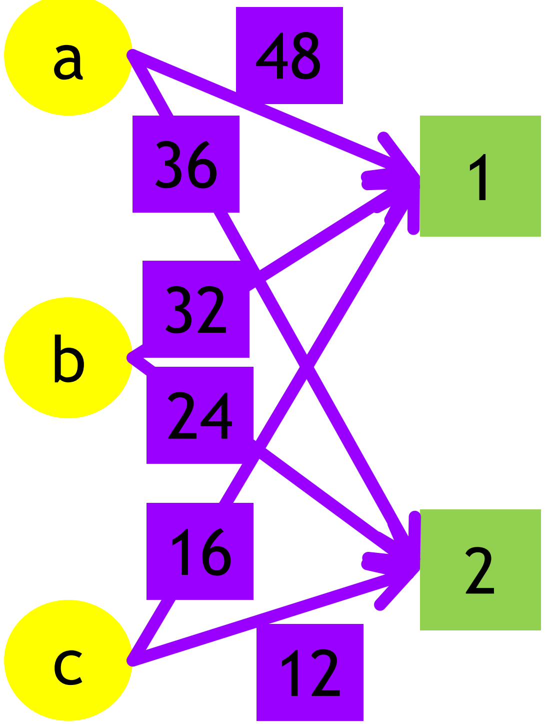
Generalized Second-Price (GSP)

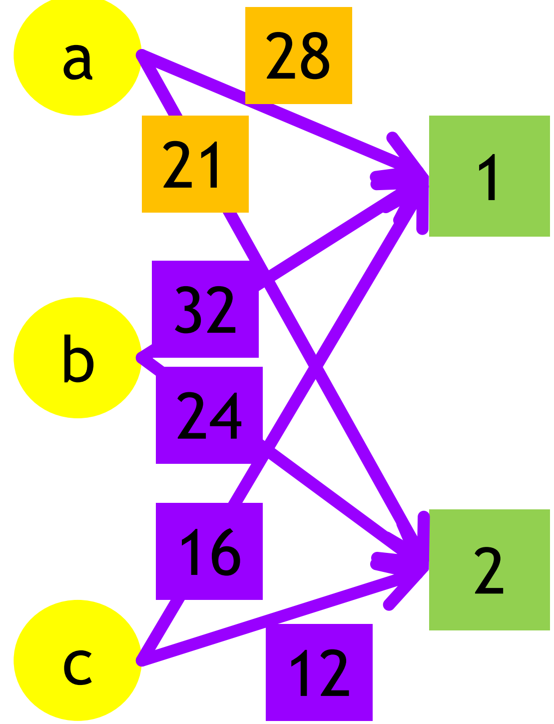
- Ad auctions:
  - Value = CTR x value per click
  - Normally CTR decreases with list, value-per-click assumed independent of position
  - Maximum matching = {(highest bidder, 1st position), (2nd highest bidder, 2nd position), …}
- Example:
  - Assume CTR of (4%,2%) and valuations per click of (12,8,4)
  - Prices (for truthful bids $b = v$):
    - a pays 32
    - b pays 8



Non-truthful bidding in GSP

- GSP is not incentive-compatible – example:





Truthful bidding:

a: $u_a = v_a - p_a =$
$= 48 - 32 = 16$

Tactical bidding:

a: $u_a = v_a - p_a =$
$= 36 - 12 = 24$

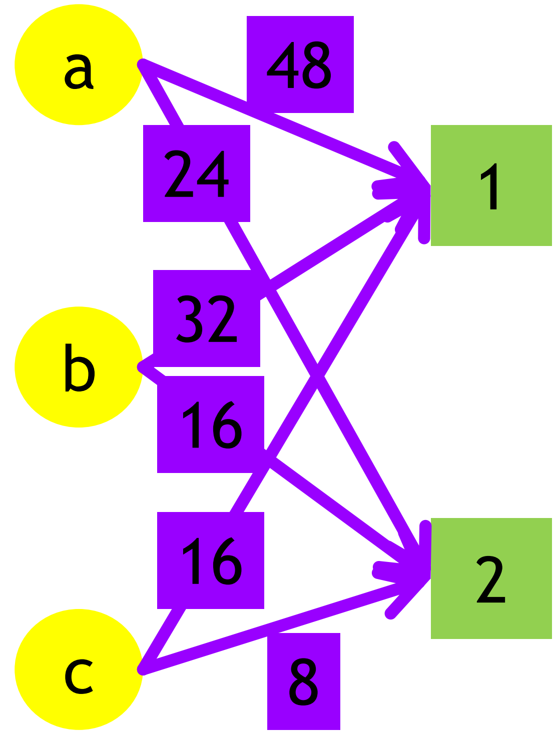
VCG (Vickrey-Clarke-Groves) auction

- Suppose everybody bids truthfully: $b_{ij} = v_{ij}$
- Total valuation (for best matching): $V = \sum_M v_{ij}$
- Def: if bidder i gets item j: $V_{i \leftarrow j} = \sum_{M-(i,j)} v_{ij}$
  - i.e., the value of all bidder-item pairs except for (i, j)
  - Total valuation $V = v_{ij} + V_{i \leftarrow j}$
- Def: valuation without i: V_{-i}
  - Best total valuation if bidder i is completely removed (different matching M')
- Price:
  - Compute matching M
  - $p_i = V_{-i} - V_{i \leftarrow j}$
  - Interpretation: price for i is the decrease in total valuation for everybody else due to i's participation

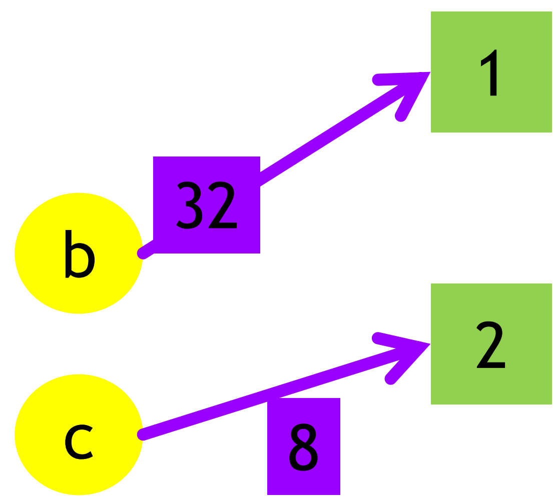
VCG pricing: example

- Allocation identical
- Prices:
  - $p_a = (32 + 8) - (16 + 0) = 24$
    - 16 for reducing b from 32 → 16
    - 8 for reducing c from 8 → 0
  - $p_b = 8$:
    - 8 for reducing c from 8 → 0
- Note: VCG with single item is equivalent to second-price
  - $V_{i \leftarrow 1} = 0$
  - $V_{-i} = b_2$

$V = 64$:



$V_{-a} = 40$:



Truthful bidding in VCG

- Theorem: in a VCG auction, truthful bidding ($b = v$) is a dominating strategy
- Proof:
  - Suppose i bids non-truthfully for some item j
  - $V = v_{ij} + V_{i\leftarrow j} \geq v_{ix} + V_{i\leftarrow x}$ for all $x \neq j$, because V is max valuation (best matching)
  - Subtract V_{-i} from both sides:
    $v_{ij} + V_{i\leftarrow j} - V_{-i} \geq v_{ix} + V_{i\leftarrow x} - V_{-i}$
  - $\underbrace{v_{ij} - p_{ij}}_{\text{payoff when truthful}} \geq \underbrace{v_{ix} - p_{ix}}_{\text{payoff when not truthful}}$

Summary

- Online advertisement: scarce resources + competing interests → auction
- Classification: open/sealed; ascending/descending; 1st/2nd price
- Truthful bidding: bid = value
- Second-price: truthful bidding is dominant strategy
- Multi-item:
  - GSP: very simple rule, but not incentive-compatible (truthful not dominant); used by Google AdWords, very high frequency
  - VCG: incentive-compatible

References

- [M. Chiang: Networked Life, Cambridge 2012 (chapter 2)]

Internet Analytics: conclusion

- Types of data: all about people / user-generated
  - Social and info networks, likes & preferences, text & language
  - Real & diverse datasets for labs
- Learning outcomes:
  - Key **models** to characterize data in social web, social media, and mobile apps
    - E.g.: $G(n,p)$; latent factors; vector space; bidding…
  - Key **methods**: prediction, filtering, ranking, searching, selling
    - E.g.: link prediction; graph sampling; PCA; topic models; auction mechanisms; …
  - **Tools** of the trade: distributed (non-sql) data processing
    - Spark (make sure to put on resume! ;-))

Internet Analytics: conclusion

- Fields:
  - Data Mining: dim reduction; streaming
  - Machine Learning: learning, prediction; regularization; Bayesian networks
  - Network Science: net structure, evolution, processes
  - Graph theory & probability: random graph models; MCMC
- Breadth & straddling fields:
  - Depth was often limited → probe further!
  - No textbook with full coverage → will be built over time
- Where to go from here?
  - Master Specialization in Data Science
  - Master in Data Science
- **Final exam: Wed, 08:15-11:00**

Thanks & good luck!

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