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ORIGINAL PAPER

A Mathematical Programming Model for Scheduling of Nurses' Labor Shifts

Ebru Yilmaz

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Abstract In this study, a mathematical programming model is proposed for scheduling problem of nurses' labor shifts. The developed mathematical programming model's aim is to minimize nurses' total idle waiting time during a week planning horizon. In this model, investigated constraints are as follows: (1) Maximum total working time a week for each nurse must not be exceeded. (2) After a nurse works a shift, the nurse can be assigned to another shift after two shifts at least. This constraints-set ensures resting of the nurse after the nurse works a shift. (3) Total number of nurses worked for each shift must be controlled with maximum and minimum bounds given for number of nurses for each shift. In this manner, total number of nurses worked for each shift is between maximum and minimum limit-values given for each shift. This constraint ensures flexibility to the user to determine number of nurses for each shift. (4) The decision variable that shows nurse-shift assignment pairs is 0 or 1. In this study, maximum total working time a week for a nurse, total number of nurses in a health service, maximum and minimum numbers of nurses worked a shift are user-specified parameters. In this way, this model can be adapted for the studies with different values of these parameters. In this study, the developed model is illustrated using a numerical example and then LINGO8.0 software is used to ensure the global optimum solution of the developed model. Results and also sensitivity analysis carried out for this example are presented in the study.

E. Yilmaz (✉)
Department of Industrial Engineering,
Faculty of Engineering and Architecture, Cukurova University,
01330 Adana, Turkey
e-mail: eyilmaz@cu.edu.tr
e-mail: ebru_yilmaz_tr@yahoo.com

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Introduction

As it's well known, nurses are health staffs that are needed during 24 h a day and 7 days a week to treat patients in hospitals. This situation requires solution of scheduling problem of nurses' labor shifts to provide dynamic and efficient quality of health service.

In the past, a considerable number of relevant studies on nurse scheduling problem have been found. Soliman [18] mentioned importance of efficient use of nursing resources through patient dependency systems. Smith and Wiggins [17] divided nurse scheduling modeling into three categories: cyclical scheduling approach, heuristic scheduling approach and mathematical programming approach. Cyclical scheduling approach sets up shift and vacation arrangement by the nurse chiefs based on the needs of nurse unit, the regulations of hospital and the number of nursing staffs. Cyclical scheduling approach normally utilizes cyclical scheduling pattern on a fixed time range. For heuristic scheduling approach, the nurse chiefs often construct a decision tree with consideration of nursing staff workforce, nurse service pattern, hospital scheduling policy and other factors and then utilize the scheduling result on a cyclic basis. Mathematical programming approach is a special mathematical model developed to respond to the scheduling problems for different cases. Normally, it is constructed with objective functions and constraint equations and then utilizes appropriate algorithms to solving for the optimal solutions [20].

There are a lot of methods for nurse scheduling problem in the literature. For instance, integer programming, mixed integer programming, goal programming and linear programming were used for solving the nurse scheduling problem: [13, 21] proposed integer programming models. Bell et al [4] developed a mixed integer programming model with a decision support system. Arthur and Ravindran [2], [3, 15, 16] used goal programming models. Jaumard et al. [10] presented a generalized 0–1 linear programming model for the nurse scheduling problem. They used column generation and branch-and-bound to solve their model. Other techniques have also been found for the nurse scheduling problems: There are the papers that used meta-heuristic solution approaches for this problem in the literature. For example, [8] developed a tabu search approach for the scheduling problem of nurses. Brusco and Jacops [5], [7, 19] proposed simulated annealing algorithms. Aickelin and Dowsland [1], [11, 14, 20] developed genetic algorithms. In the literature knowledge-based approaches were also suggested by [6, 9]. And network programming was suggested by [12].

In this study, a mathematical programming model is developed for scheduling problem of nurses' labor shifts. In this developed model, the planning horizon is taken into consideration as a week. In this model, 3 shifts a day are taken into consideration. So, total number of shifts a week is 21. The mathematical programming model's objective developed for this problem is to minimize nurses' total idle waiting time during a week planning horizon. LINGO8.0 packet programming that is an optimization software and has been widely used for many years in the literature is used to ensure this model's global optimum solution. The remainder of this paper is organized as follows: First, the developed mathematical programming model for scheduling of nurses' labor shifts is explained in Section "Method". A numerical example and then sensitivity analysis are presented in Sections "Illustrative example" and "Sensitivity analysis", respectively. Finally, discussions and conclusions of this paper and future researches are presented in Section "Discussions and conclusions".

Method

In this study, a mathematical programming model is developed for scheduling problem of nurses' labor shifts In this developed model, the planning horizon is taken into consideration as a week. In this model, 3 shifts a day are taken into consideration. Working time for each shift is 8 h and shifts' working hours are as follows: from 8.00 A.M. to 4.00 P.M., from 4.00 P.M. to 0.00 A.M., and from 0.00 A. M. to 8.00 A.M.. Total number of shifts a week is 21 because of taking into consideration of a week as the planning horizon in this model. In the next section, the developed mathematical programming model is explained in detail. In this problem formulation given in the next section, first, notation and then objective function and constrains are explained in detail.

Developed mathematical programming model

The following notation is used in the model:

Indices:

i nurse index $i=1,.., TN$
j shift index $j=1,..,21$

Decision variable:

$$x_{ij} = \begin{cases} 1, \text{if nurse } i \text{ works shift } j \text{ of a week planning horizon} \\ 0, \text{otherwise} \end{cases}$$

The decision variable shows assigned nurse-shift pairs for all of both nurses and shifts. That's if nurse i is assigned to shift j of a week planning horizon, the decision variable is 1, otherwise 0.

Input parameters:

WH maximum total working time a week for a nurse
TN total number of nurses in a hospital / health service
$N_{j(min)}$ minimum number of nurses worked shift j
$N_{j(max)}$ maximum number of nurses worked shift j

Model:

Objective function:

$$Min\left[(WH^*TN) - \left(8\left(\sum_i \sum_j x_{ij}\right)\right)\right] \tag{1}$$

Subject to:

$$8\left(\sum_j x_{ij}\right) \leq WH \qquad \forall i \tag{2}$$

$$\left\{\begin{array}{l} \sum_{j=1}^{3} x_{ij} \leq 1 \\ \sum_{j=2}^{4} x_{ij} \leq 1 \\ . \\ . \\ . \\ \sum_{j=19}^{21} x_{ij} \leq 1 \end{array}\right\} \qquad \forall i \tag{3}$$

Table 1 The minimum number of nurses worked shift j for a week planning horizon (value of $N_{j(min)}$)

	Shifts of day-1 j=1,2,3	Shifts of day-2 j=4,5,6	Shifts of day-3 j=7,8,9	Shifts of day-4 j=10,11,12	Shifts of day-5 j=13,14,15	Shifts of day-6 j=16,17,18	Shifts of day-7 j=19,20,21
Minimum number of nurses for each shift	3	3	4	4	3	1	1
	2	3	2	2	2	1	1
	1	1	1	1	1	1	1

$$N_{j(\min)} \leq \sum_i x_{ij} \leq N_{j(\max)} \qquad \forall j \qquad (4)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \qquad (5)$$

Equation 1 shows the objective function of the mathematical programming model developed for scheduling problem of nurses' labor shifts. The developed mathematical programming model's objective is to minimize nurses' total idle waiting time during a week planning horizon. The first term in (1) is the maximum total working time a week for all nurses in a hospital / health service. The second term in (1) shows that a shift time (8 h) is multiplied by total number of shifts that all nurses works in a week. In this model, the each shift's time is 8 h. If the shift's time is defined as a variable in input parameters, this model can be adapted for the studies with different shift-time. In this model, it is wanted that the value of objective function is 0 or the closest to 0.

Equation 2 ensures that maximum total working time a week for each nurse must not be exceeded.

Equation 3 consists of a set of some constraints. This set explains that after a nurse works a shift, the nurse can be assigned to another shift after two shifts at least. This set ensures resting of the nurse after the nurse works a shift. For example; if i=1 (nurse's number equals 1), some constraints from this set can be showed as follows:

$$\left.\begin{array}{l} x_{11} + x_{12} + x_{13} \leq 1 \\ x_{12} + x_{13} + x_{14} \leq 1 \\ x_{13} + x_{14} + x_{15} \leq 1 \\ x_{14} + x_{15} + x_{16} \leq 1 \\ . \\ . \\ . \\ x_{119} + x_{120} + x_{121} \leq 1 \end{array}\right. \quad \text{for } i = 1$$

Equation 4 ensures that total number of nurses worked for each shift must be controlled with maximum and minimum bounds given for number of nurses for each shift. In this manner, total number of nurses worked for each shift is between maximum and minimum limit-values given for each shift. This constraint ensures flexibility to the user to determine number of nurses for each shift.

Equation 5 requires the decision variable x_{ij} to be 0 or 1. So, it is said easily that the developed mathematical programming model is a 0–1 integer linear programming model.

In this study, maximum total working time a week for a nurse, total number of nurses in a hospital / health service, maximum and minimum numbers of nurses worked a shift are user-specified parameters. In this way, this model can be adapted for the studies with different values of these parameters.

In this model, a week planning horizon is taken into consideration. The total number of shifts and maximum total working time for a nurse can be determined according to different planning horizons. In this way, this model can be adapted for the studies with different planning horizon.

Table 2 The maximum number of nurses worked shift j for a week planning horizon (value of $N_{j(max)}$)

	Shifts of day-1 j=1,2,3	Shifts of day-2 j=4,5,6	Shifts of day-3 j=7,8,9	Shifts of day-4 j=10,11,12	Shifts of day-5 j=13,14,15	Shifts of day-6 j=16,17,18	Shifts of day-7 j=19,20,21
Maximum number of nurses for each shift	5	4	5	5	4	2	1
	3	4	3	3	4	1	1
	2	2	2	2	2	1	1

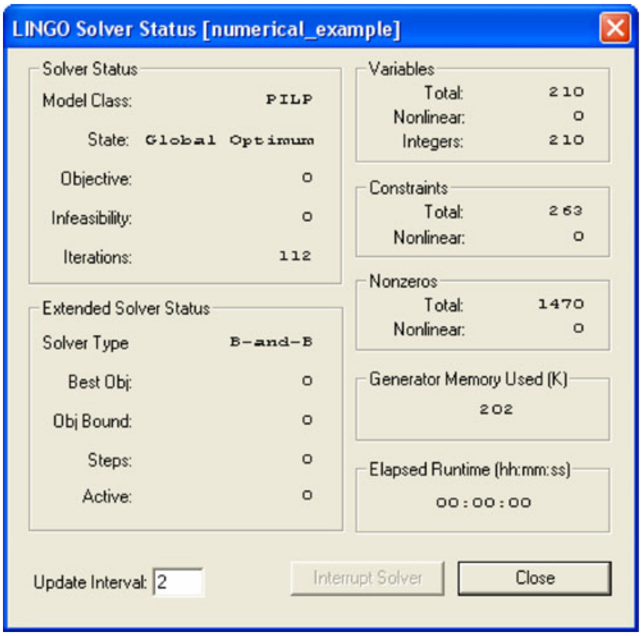

Fig. 1 An interface of the global optimum solution obtained using LINGO8.0 for this problem

In the next section, application of the developed mathematical programming model is ensured using a numerical example.

Illustrative example

We assume that total number of nurses (*TN*) is 10 for system with shift in a hospital. Moreover, maximum total working time a week for a nurse (*WH*) is 40 h in this hospital. Tables 1 and 2 show minimum and maximum numbers of nurses worked a shift for a week planning horizon, respectively.

LINGO8.0 packet programming that is an optimization software and has been widely used for many years in the literature is used to ensure this model's global optimum solution. The numerical example is solved by a branch-and-bound method (B&B) under LINGO8.0 software on a Personal Computer including Intel® Core™2 Duo CPU E6550@ 2.33 GHz, 2.34 GHz processors and 1.00 GB RAM.

An interface of the global optimum solution obtained using LINGO8.0 is showed in Fig. 1. As shown in Fig. 1, we obtain the value of the objective function as 0. 0 is the best value that can be founded for all numerical examples. Moreover, this global optimum solution is obtained in a short time.

Table 3 shows the decision variables' values according to this global optimum solution. In other words, Table 3 shows nurses and shifts that these nurses can be assigned. This situation is shown using 1 in Table 3. For example, for nurse-1 (that's if $i=1$), the shifts that the nurse can be assigned are 2, 6, 10, 13 and 16, respectively.

In the next section, the sensitivity analysis carried out for this problem is showed.

Sensitivity analysis

Sensitivity analysis is used for determining of the effects of changes in model's parameters. We carried out the sensitivity analysis using changing of maximum total working time a week for a nurse (*WH*) to determine the changeability of global optimum solution. In Table 4, the values of the objective function obtained according to the different values of *WH* are shown.

Figure 2 is obtained using Table 4. This figure shows the change of the objective function value obtained according to the different values of *WH*. As shown in Fig. 2, the

Table 3 The shifts that the nurses can be assigned according to global optimum solution

Nurses (values of *i*)	Shifts (values of *j*)																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0
2	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0
3	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0
4	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0
5	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	1
7	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0
8	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0
9	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0
10	0	1	0	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0

Table 4 The values of the objective function obtained according to the different values of *WH*

Test No.	Value of *WH*	Value of objective function
1	40	0
2	41	10
3	42	20
4	43	30
5	44	40
6	45	50
7	46	60
8	47	70
9	48	32
10	49	42
11	50	52
12	51	62
13	52	72
14	53	82
15	54	92
16	55	102
17	56	112
18	57	122
19	58	132
20	59	142
21	60	152

objective function value is on the increase until *WH* equals 48 h. But, the objective function value decreases when *WH* equals 48 h. Then, it is seen from this figure that the objective function value is on the increase again because of increasing of *WH*.

As shown in Table 4 and Fig. 2, if *WH* equals 50 h, the objective function value is 52. So, for all nurses the total working time that is the second term in Eq. 1 is 448 h $(50*10 - 52 = 448\,\mathrm{h})$ for a week planning horizon. Thus, for all nurses the total number of shifts that all nurses work during a week planning horizon is found as $\frac{448}{8} = 56$ because working time of each shift is 8 h. In that case, four nurses work five shifts and six nurses work six shifts during a week planning horizon.

Discussions and conclusions

In this study, a mathematical programming model is developed for scheduling problem of nurses' labor shifts. In developed model, the planning horizon is taken into consideration as a week. In this model, 3 shifts a day are taken into consideration. Working time for each shift is 8 h. If the shift's time is defined as a variable in input parameters, this model can be adapted for the studies with different shift-time. Total number of shifts a week is 21 because of taking into consideration of a week as a planning horizon in this model. The total number of shifts and maximum total working time for a nurse can be determined according to different planning horizons. In this way, this model can be adapted for the studies with different planning horizon too.

The objective of the model developed for this problem is to minimize nurses' total idle waiting time during a week planning horizon. In this model, investigated constraints are as follows:

- Maximum total working time a week for each nurse must not be exceeded.
- After a nurse works a shift, the nurse can be assigned to another shift after two shifts at least. This constraints-set ensures resting of the nurse after the nurse works a shift.
- Total number of nurses worked for each shift must be controlled with maximum and minimum bounds given for number of nurses for each shift. In this manner, total

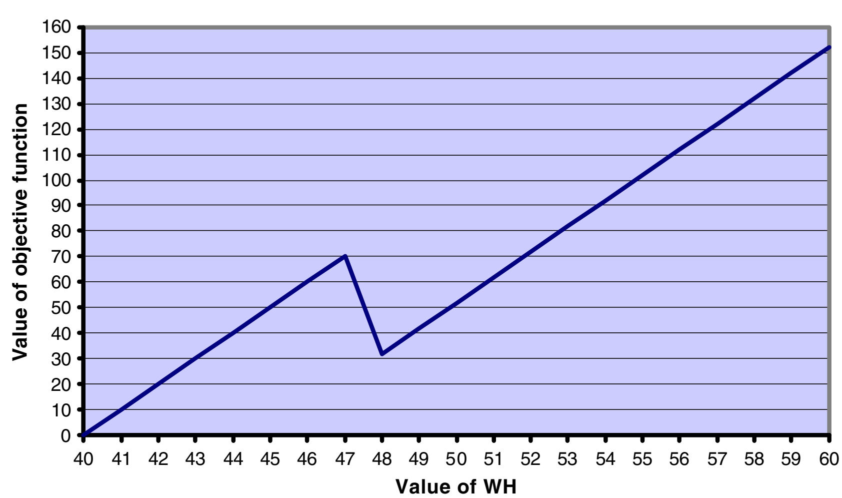

Fig. 2 The change of objective function value according to the different values of *WH*

number of nurses worked for each shift is between maximum and minimum limit-values given for each shift. This constraint ensures flexibility to the user to determine number of nurses for each shift.

- The decision variable x_{ij} is 0 or 1.

In this study, maximum total working time a week for a nurse, total number of nurses in a hospital / health service, maximum and minimum numbers of nurses worked a shift are user-specified parameters. In this way, this model can be adapted for the studies with different values of these parameters.

In this study, the developed model is illustrated using a numerical example and then LINGO8.0 software is used to ensure the global optimum solution of the developed model. We obtained the value of the objective function as 0 solving the example with the total standard-working time (40 h) a week for a nurse. 0 is the best value that can be founded for all numerical examples. Moreover, this global optimum solution is obtained in a short time. Also, in this study, we carried out the sensitivity analysis using changing of maximum total working time a week for a nurse (*WH*) to determine the changeability of this optimum solution.

In the further research, for the scheduling problem of nurses' labor shifts, the other factors can be investigated too: For example, it can be avoided that same nurses work same shifts continuously for the scheduling problem with longer time planning horizon. Moreover, when a nurse can not work a shift unexpectedly, the other (additional) nurse (s) instead of the nurse can be assigned from the same hospital or the different hospitals. So, nurses can be taken into consideration as full-time nurses and part-time nurses in this further research. But, these factors can not be included in the mathematical programming models easily. Therefore, the other approaches such as knowledge-based approaches can be used together with the mathematical programming models so as to form integrated and / or hierarchical models.

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