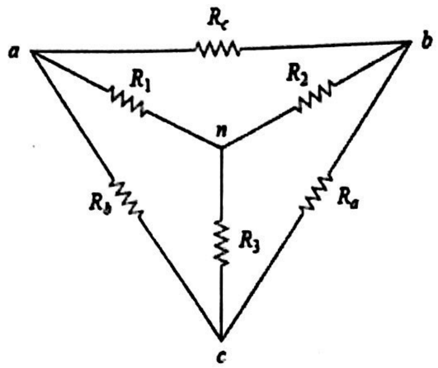
Wye-Delta Transformations



Delta to Wye Conversion

- Each resistor in the Y network is the product of the resistors in the two adjacent branches, divided by the sum of the three resistors.

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}} \qquad \boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}} \qquad \boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

Wye to Δ Conversion Rule:

- Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

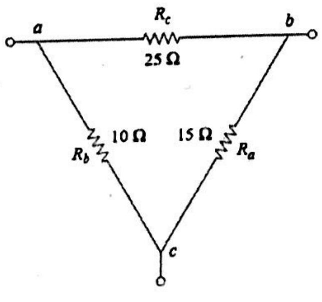
$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} \qquad \boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} \qquad \boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}$$

If $R_1 = R_2 = R_3 = R_Y$, $R_a = R_b = R_c = R_\Delta$, then the Y and Δ networks are said to be balanced.

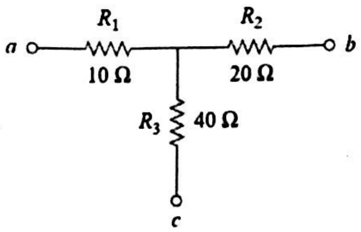
$$\boxed{R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y}$$

Examples:

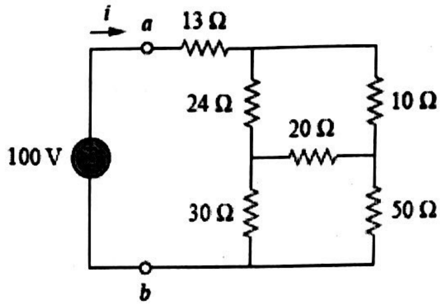
1. Convert the network below to an equivalent Y network.



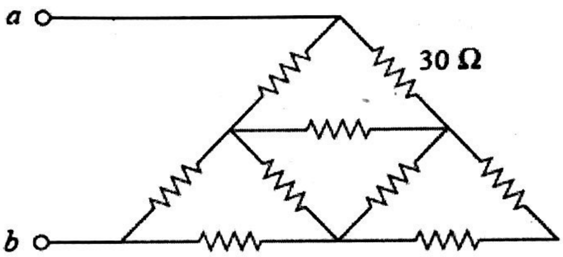
2. Transform the wye network below to a delta network.



3. For the bridge network below, find R_{ab} and *i*.

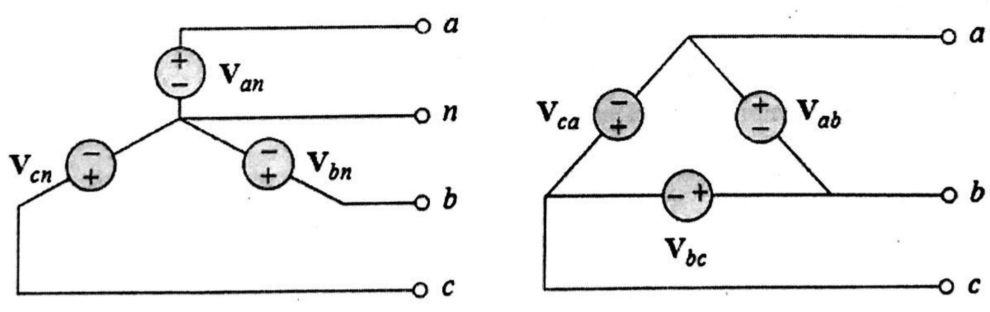


4. Obtain the equivalent resistance R_{ab} of the circuit below. All resistors have a value of 30Ω.



Balanced Three-Phase Connection

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). (Three phase current sources are very scarce.) A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either wye-connected as shown in Fig. (a) or delta-connected as in Fig. (b).



(a) (b)

Let us consider the wye-connected voltages in Fig.(a) for now. The voltages Van, Vbn, and Vcn are respectively between lines a, b, and c, and the neutral line n. These voltages are called phase voltages. If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120°, the voltages are said to be balanced. This implies that,

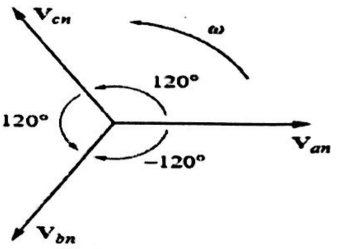
$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

Since the three-phase voltages are 120° out of phase with each other, there are two possible combinations. One possibility is shown and expressed mathematically as

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$
$$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}$$
$$\mathbf{V}_{cn} = V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ}$$

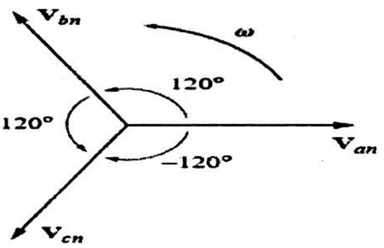


Where: Vp = effective or rms value of the phase voltages

This is known as the abc sequence or positive sequence. In this phase sequence, Van leads Vbn, which in turn leads Vcn.

The other possibilityis called the acb sequence or negative sequence. For this phase sequence, Van leads Vcn, which in turn leads Vbn. It is shown and is given by:

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$
$$\mathbf{V}_{cn} = V_p\underline{/-120^\circ}$$
$$\mathbf{V}_{bn} = V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ}$$



The phase sequence is the time order in which the voltages pass through their respective maximum values. The phase sequence is determined by the order in which the phasors pass through a fixed point in the phase diagram.

A balanced load is one in which the phase impedances are equal in magnitude and in phase.

For a balanced wye-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

where Zy is the load impedance per phase. For a balanced deltaconnected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

where ZΔ is the load impedance per phase in this case.

Recall that a wye-connected load can be transformed into a deltaconnected load, or vice versa.

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

- Y-Y connection (i.e., Y-connected source with a Y-connected load).
- Y-Δ connection.
- Δ-Δconnection.
- Δ-Yconnection

Example Problems:

1) Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}\text{ V}, \qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}\text{ V}, \qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}\text{ V}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

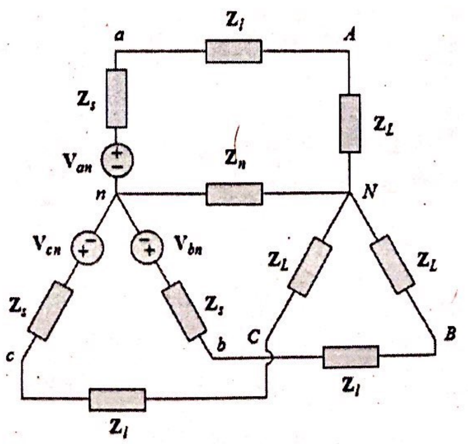
2)

Given that $\mathbf{V}_{bn} = 110\underline{/30^\circ}$ V, find $\mathbf{V}_{an}$ and $\mathbf{V}_{cn}$, assuming a positive (*abc*) sequence.

Answer: $110\underline{/150^\circ}$ V, $110\underline{/-90^\circ}$ V.

Balanced Wye-Wye Connection

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



A balanced Y-Y system, showing the source, line, and load impedances

Consider the balanced four-wire Y-Y system in the figure, where a Y-connected load is connected to a Y-connected source. We assume a balanced load so that load impedances are equal. Although the impedanceZy is the total load impedance per phase, it may also be regarded as the sum of the source impedanceZs, line impedanceZl and load impedance ZL for each phase, since these impedances are in series. As illustrated, Zsdenotes the internal impedance of the phase winding of the generator; Zlis the impedance of the line joining a phase of the source with a phase of the load; ZL is the impedance of each phase of the load; and Zn is the impedance of the neutral line. Thus, in general

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

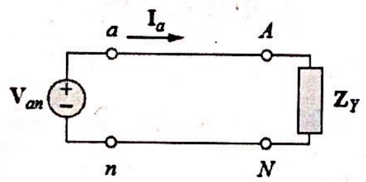
The magnitude of the line voltages VL is√3 times the magnitude of the phase voltagesVp, or

$$V_L = \sqrt{3}V_p$$

Where:

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$



Single-phase equivalent circuit

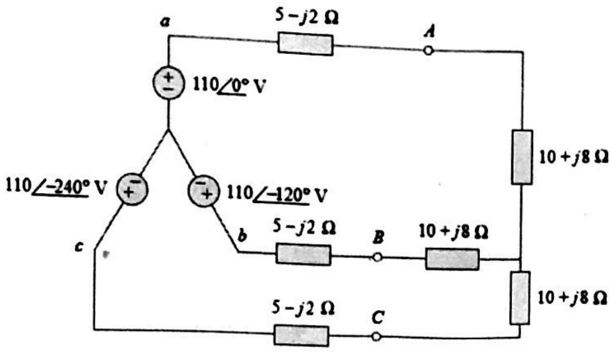
An alternative way of analyzing a balanced Y-Y system is to do so on a "per phase" basis. We look at one phase, say phase a, and analyze the single-phase equivalent circuit. The single-phase analysis yields the line currentIa, as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

FromIa, we use the phase sequence to obtain other line currents. Thus, as long as the system is balanced, we need only analyze one phase. We may do this even if the neutral line is absent, as in the three-wire system.

Example Problems:

1) Calculate the line currents in the three-wire Y-Y system



Solution:

The three-phase circuit is balanced; we may replace it with its single-phase equivalent circuit. We obtain Iafrom the single-phase analysis, as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ$. Hence,

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

Since the source voltages are in positive sequence, the line currents are also in positive sequence:

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

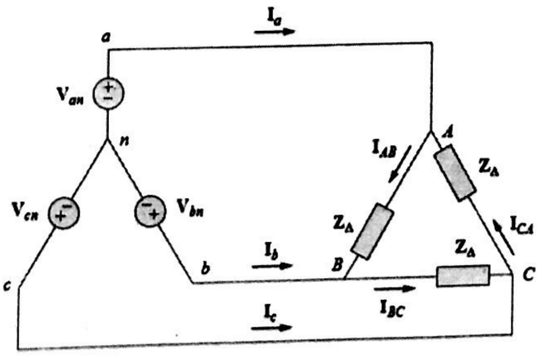
2)

A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3\ \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19\ \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7\ \Omega$ per phase. Assuming a positive sequence for the source voltages and that $\mathbf{V}_{an} = 120\angle 30^\circ$ V, find: (a) the line voltages, (b) the line currents.

Answer: (a) $207.85\angle 60^\circ$ V, $207.85\angle -60^\circ$ V, $207.85\angle -180^\circ$ V, (b) $3.75\angle -8.66^\circ$ A, $3.75\angle -128.66^\circ$ A, $3.75\angle -111.34^\circ$ A.

Balanced Wye-Delta Connection

A balanced Y-Δ system consists of a balanced Y-connected source feeding a balanced Δ-connected load.



Balanced Y- connection

The magnitude of the line current IL is$\sqrt{3}$ times the magnitude of the phase currentIp, or

$$I_L = \sqrt{3} I_p$$

Where:

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

An alternative way of analyzing the Y-Δ circuit is to transform the Δ-connected load to an equivalent Y-connected load. Using the Δ-Y transformation formula,

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

After this transformation, we now have a Y-Y system. The three-phase Y-Δ system can be replaced by the single-phase equivalent circuit. This allows us to calculate only the line currents. The phase currents are obtained and utilizing the fact that each of the phase currents leads the corresponding line current by 30°.

Example Problems:

1)

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

Solution 1:

The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\angle 26.57^\circ\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30^\circ = 100\sqrt{3}\angle 10^\circ + 30^\circ = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\angle 40^\circ \text{ V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle 13.43^\circ - 30^\circ$$
$$= 33.53\angle -16.57^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ \text{ A}$$

Solution 2:

Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\text{ A}$$

as above. Other line currents are obtained using the abc phase sequence.

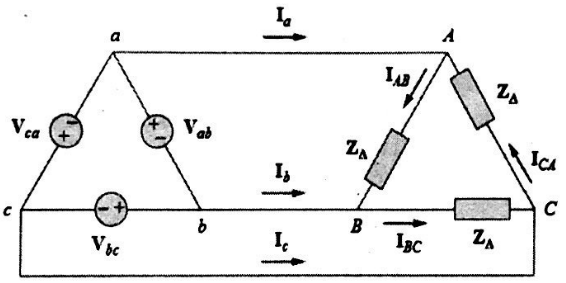
2)

One line voltage of a balanced Y-connected source is $\mathbf{V}_{AB} = 240\underline{/-20^\circ}$ V. If the source is connected to a Δ-connected load of $20\underline{/40^\circ}\ \Omega$, find the phase and line currents. Assume the *abc* sequence.

Answer: $12\underline{/-60^\circ}$ A, $12\underline{/-180^\circ}$ A, $12\underline{/60^\circ}$ A, $20.79\underline{/-90^\circ}$ A, $20.79\underline{/-150^\circ}$ A, $20.79\underline{/30^\circ}$ A.

Balanced Delta-Delta Connection

A balanced Δ-Δ system is one in which both the balanced source and balanced load are Δ-connected.



A balanced delta-delta connection

The line voltages are the same as the phase voltages. Assuming there is no line impedances, the phase voltages of the delta-connected source are equal to the voltages across the impedances; that is

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Hence, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta}$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30°, the magnitude of the line current IL is √3times the magnitude of the phase current,

$$I_L = \sqrt{3} I_p$$

An alternative way of analyzing the Δ-Δ circuit is to convert both the source and the load to their Y equivalents. We already know that $\mathbf{Z}_Y = \mathbf{Z}_\Delta/3$. To convert a Δ-connected source to a Y-connected source, see the next section.

Example Problems:

1)

A balanced Δ-connected load having an impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\angle 0^\circ$ V. Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle -36.87^\circ\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle 0^\circ}{25\angle -36.87} = 13.2\angle 36.87^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 13.2\angle 156.87^\circ \text{ A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = (13.2\angle 36.87^\circ)(\sqrt{3}\angle -30^\circ)$$

$$= 22.86\angle 6.87^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 22.86\angle -113.13^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 22.86\angle 126.87^\circ \text{ A}$$

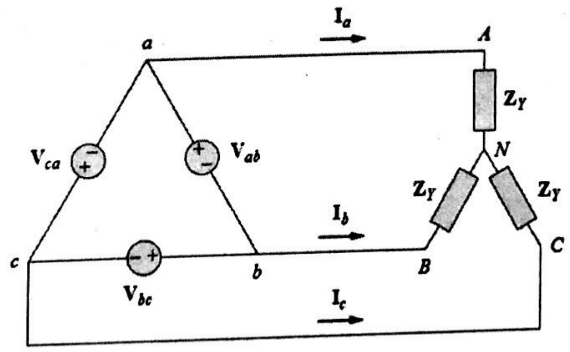
2)

A positive-sequence, balanced Δ-connected source supplies a balanced Δ-connected load. If the impedance per phase of the load is $18 + j12\ \Omega$ and $\mathbf{I}_a = 19.202\angle 35^\circ$ A, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $11.094\angle 65^\circ$ A, $240\angle 98.69^\circ$ V.

Balanced Delta-Wye Connection

A balanced Δ -Y system consists of a balanced Δ-connected source feeding a balanced Y-connected load.



A balanced Δ-Y connection

Consider the circuit in the figure. Again, assuming the abc sequence,

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y}$$

From this, we obtain the other line currents $\mathbf{I}_b$ and $\mathbf{I}_c$ using the positive phase sequence, i.e., $\mathbf{I}_b = \mathbf{I}_a\angle{-120^\circ}$, $\mathbf{I}_c = \mathbf{I}_a\angle{+120^\circ}$. The phase currents are equal to the line currents.

Another way to obtain the line currents is to replace the delta-connected source with its equivalent wye-connected source. We found that the line-to-line voltages of a wye-connected source lead their corresponding phase voltages by 30°. Therefore, we obtain each phase voltage of the equivalent wyeconnected source by dividing the corresponding line voltage of the delta-connected source by √3and shifting its phase by -30°. Thus, the equivalent wye-connected source has the phase voltages,

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle{-30^\circ}$$

$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\angle{-150^\circ}, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\angle{+90^\circ}$$

Alternatively, we may transform the wye-connected load to an equivalent delta-connected load. This results in a delta-delta system, which can be analyzed. Note that,

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \frac{V_p}{\sqrt{3}}\angle{-30^\circ}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle{-120^\circ}, \qquad \mathbf{V}_{CN} = \mathbf{V}_{AN}\angle{+120^\circ}$$

As stated earlier, the delta-connected load is more desirable than the wye-connected load. It is easier to alter the loads in any one phase of the delta-connected loads, as the individual loads are connected directly across the lines. However, the delta-connected source is hardly used in practice, because any slight imbalance in the phase voltages will result in unwanted circulating currents.

Example Problems:

1)

A balanced Y-connected load with a phase impedance of $40 + j25\ \Omega$ is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as reference.

Solution: (Next Page)

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ = 121.2\angle -30^\circ\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.12\angle 32^\circ} = 2.57\angle -62^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 2.57\angle -178^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

which are the same as the phase currents.

2)

In a balanced Δ-Y circuit, $\mathbf{V}_{ab} = 240\angle 15^\circ$ and $\mathbf{Z}_Y = (12 + j15)\ \Omega$. Calculate the line currents.

Answer: $7.21\angle -66.34^\circ$ A, $7.21\angle -173.66^\circ$ A, $7.21\angle 53.66^\circ$ A.

The table below presents a summary of the formulas for phase currents and voltages and line currents and voltages for the four connections. Students are advised not to memorize the formulas but to understand how they are derived. The formulas can always beobtained by directly applying KCL and KVL to the appropriate three-phase circuits.

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

[1] Positive or abc sequence is assumed.