Some General Thoughts About Broken Symmetry

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This is my most complete summary of the theory of broken symmetry in condense matter systems.

Some general thoughts about broken symmetry

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I. Introduction

It is appropriate to commemorate the discovery of piezoelectricity, one of the most useful of broken symmetry phenomena, with a general discussion of broken symmetry as it manifests itself in condensed matter systems. Arguments from broken symmetry, however, go back even further in French science : Louis Pasteur's deduction that fermentation was a spontaneous life process on the basis of the optical activity of fermentation products is to me one of the most miraculously early and deep insights in the history of science. It is striking that so much of that history has taken place within a few hundred meters of where we now stand.

The more theoretical physicists penetrate the ultimate secrets of the microscopic nature of the universe, the more the grand design seems to be ultimate symmetry and ultimate simplicity. But all of the interesting parts of the universe, at least to us, are, like the earth itself as well as our own bodies, markedly complex and markedly *unsymmetric*. In the most elementary sense, then, we are surrounded by « broken symmetry », the result undoubtedly of some sequence of catastrophes. What I want to do here is to discuss the general rules which govern this process of the development of complexity and the breaking of symmetry in particular kinds of cases.

In particular, I want to point out that there is a complete, rather satisfactory theoretical structure describing one particular type of broken symmetry, namely that which occurs in equilibrium condensed matter systems, such as the crystals which exhibit piezoelectricity. It is this kind of broken symmetry object which allows us to build structures, communicate, make measurements, calculate, locate ourselves — in essence, carry on all the everyday business of life.

It is clear that an equally important kind of broken symmetry occurs in many dissipative systems when they are driven far from equilibrium. These systems exhibit nonlinear instabilities at which new structures appear, which have often been called [1] « dissipative structures » and discussed on a parallel basis with the equilibrium structures of broken symmetry systems. Surely there are obvious parallels between these two types of systems — for instance, they both come under the general umbrella of « catastrophe theory » [2], and the second kind often — but not always — changes the symmetry of the system in which it occurs, if only in some trivial fashion.

What I want to do here is to pin down the properties of condensed matter systems exhibiting broken symmetry in some detail. After all, the existence of broken sym-

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metry in itself may not be of any use or significance : for instance, fully developed turbulence can be thought of as a broken symmetry state, yet seems to be totally chaotic and without definable structure. When one speaks of « dissipative structures » and makes the analogy to equilibrium phases, there is an implied analogy to their structural properties. Thus it is relevant to exhibit these and their underlying sources.

The second part of the talk will consist almost entirely of questions rather than answers. To my mind there exists neither a theoretical nor an experimental basis for deciding whether (or not) dissipative systems have structural properties analogous to equilibrium ones, and I am essentially presenting a program of questions one may ask in this field.

II. Equilibrium broken symmetry and the concept of rigidity

Landau was the first person to emphasize the important role which symmetry plays in the phase transitions of equilibrium condensed matter systems [3]. It is this, rather than Ehrenfest's concept of 1st *vs.* *n*th order, which gives us our most fundamental classification scheme of phases and the most basic theorems of solid state physics.

In modern terms, one envisages a basic symmetry group of the underlying particles and of space, G_0, containing not only such elements as rotations (isotropy of space) and translations (homogeneity of space) but time-reversal invariance, in some cases spin-rotation invariance, etc. Landau observed the very important fact that condensed states often exhibit lower symmetry than G_0 ; for instance, where a molecular liquid is homogeneous and isotropic and exhibits the group G_0, a nematic liquid crystal is anisotropic, a smectic one inhomogeneous. The new group of, for instance, the nematic, G_1, has only rotation symmetry about the director D.

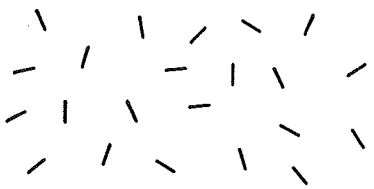

FIG. 1*a*. — Nematic liquid crystal in the disordered state. The line segments represent the rodlike molecules of the nematic. Averaging molecular orientations over macroscopic distances yields zero.

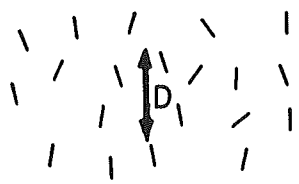

FIG. 1*b*. — For a suitable choice of thermodynamic parameters, the nematic enters the ordered state, with the appearance of a macroscopic order parameter (the director **D**). The system is no longer isotropic, but has chosen a special direction : rotational symmetry has been broken.

Thus phase transitions often involve a change of symmetry. We classify phase transitions into 3 basic classes :

1) Same symmetry in the two phases : as, e.g., liquid-gas, the Mott metal-insulator transition [4]. In this case the transition may be first-order, and the line of first-order transitions can, and often does, end in a critical point where the free energies of the two phases coincide and the transition simply disappears. Too often the formal similarity of this case to certain aspects of case 2) is allowed to obscure the physical difference in principle.

2) G_1 is a subgroup of G_0 : the symmetry of G_0 is « broken ». In this case there can never be a disappearance of the transition line, which may be first or second order depending on details. *Symmetry cannot change continuously* : what I have called the First Theorem of condensed matter physics.

Since the two phases differ in symmetry, they may have the same free energy and the same physical parameters given by the derivatives of the free energy such as $T = \partial F/\partial S$, $P = -\partial F/\partial V$, and yet be distinct : a higher-order transition is permitted, even along a line of critical points, which line may continue into a line of first-order transitions.

3) The uninteresting case is unrelated symmetries G_0 and G_1 ; in this case only a first-order transition is possible, since only by impossible coincidence could all parameters coincide.

In order to do this Landau added one concept which is vitally important to all of these questions : that of the « order parameter » η. Unfortunately, the concept of the order parameter still remains somewhat mysterious, although in most simple cases its choice is obvious. This concept is only of interest in the « broken symmetry » case 2). In case 1), any of the relevant thermodynamic variables will serve to distinguish the degree of difference of the two phases, as for instance in the Mott transition the number of free carriers, etc.

In the broken symmetry case where a loss of symmetry is present, Landau introduces an « order parameter » η indicating the degree of broken symmetry which, as he points out, is usually also the degree to which the system has « ordered » — for instance, in the liquid crystal, the degree to which the molecules are no longer arbitrarily oriented but have now oriented themselves along a specific direction.

In this example, one might use $\langle \cos^2 \theta \rangle - 1/3$ as the order parameter, where θ is the angle of a molecule with the director D. This « order parameter » is very important in many ways, not the least of which is that it is a new thermodynamic variable, and often either is or contains a dynamic one as well.

As far as I know there is no complete characterization of the « order parameter » but it must certainly have the following important properties (and I have the impression that the term is, unfortunately, often used without understanding these restrictions) :

1) It must be a variable which is operated on by the group generators of $G_0/G_1 = H$, the group representing the amount of lost symmetry. This is in fact often called the « group of the order parameter » and defines « order parameter space » [5, 6].

2) Since these group generators — at least for all continuous groups G_0 — are

necessarily *dynamical* as well as *thermodynamic* variables, since they are operators in the Hilbert space of the underlying quantum mechanics, the order parameter must *contain*, at least, a dynamical variable or variables conjugate to these.

3) Finally, the order parameter must be a useful quantitative measure of the degree to which G_0 has been broken. It has then a *thermodynamic* as well as *dynamic* significance. In such cases as order-disorder transitions (described by the Ising model) where the broken symmetry is discrete, this may be its only role : the order parameter can be merely the difference in mean population of the two or more relevant states on a given site.

In the more interesting continuous situation, there are two cases. First, there is the case of spins, where the algebra is totally defined by the spin components, which are themselves the generators of spin rotations. In this case the group generators, S_x, S_y, S_z, are selfcontained, and their mean value $\langle \mathbf{S} \rangle$ can be taken as the order parameter. Since the average of any of these group generators is, by symmetry, a constant of the motion, the ferromagnetic case represents a very rare example — a case in which the order parameter is also a constant of the motion, which has important consequences for the nature of the relevant Goldstone bosons. It is too bad that in illustrating the concept of broken symmetry, the ferromagnetic example is often used : it is extremely atypical.

More typical in its complication is the antiferromagnetic case. Here an additional but discrete symmetry is broken, that of the lattice translation, and the order parameter is not the mean magnetization but the sublattice magnetization, or more generally a Fourier component of the magnetization. While this is not a constant of the motion, it still can be related to generators of the continuous part of the symmetry group.

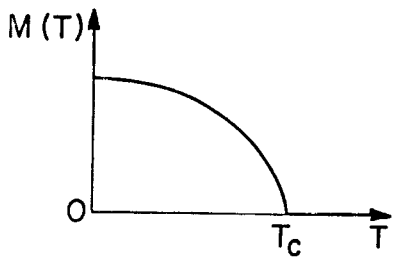

FIG. 2. — Variation of magnetization M with temperature T in a simple ferromagnet. This is a typical second-order phase transition, in which the order parameter grows continuously from zero as T is lowered below a critical temperature T_c.

The cases of density waves — solid lattice or smectic liquid crystal — and superfluidity are more typical. Let us first discuss the simple one, superfluidity [7]. Here the broken symmetry is the gauge symmetry of interactions which locally conserve particle number, and the group generator is then the number operator, which may be written $i\frac{\partial}{\delta\varphi}$, φ the phase or gauge variable. But the phase itself is not a suitable order parameter because it is periodic and its origin is meaningless. Hence it is natural to use $\langle e^{i\varphi(r)} \rangle = \psi(r)$ as an order parameter.

This exhibits the most straightforward type of order parameter. The *phase* of ψ is the relevant dynamical variable, independently of the magnitude which is purely statistical in nature. This is the general rule : the order parameter contains a phase-angle like quantity which is both a *dynamical variable* and *reflects the original symmetry* — in the sense that the free energy cannot depend on the *value* of φ, only on relative values in different parts of the sample. φ may move about freely in the space defined by the group H (in this case the group of gauge symmetry $U(1)$). In the isotropic ferro- and antiferromagnetic cases, « angle » is the spin direction, a dynamical variable free to rotate on the sphere. In real (as opposed to model) anisotropic ferromagnets, the spin retains its dynamical character in such phenomena as domain wall motion and spin waves. The nematic liquid crystal behaves in much the same way.

But the density waves bring in added complications. The phase angle variable in this case is position in space, and it is reasonable to use the Fourier components of the density as order parameters :

$$\rho_G = \langle e^{i\mathbf{G}.\mathbf{r}} \rangle .$$

Clearly, this contains the displacement **u** as a phase parameter :

$$\rho_G(u) = \langle e^{i\mathbf{G}.(\mathbf{r}+\mathbf{u})} \rangle$$

is a density wave displaced by a distance **u**. Thus the strain **u** is the dynamical variable and $| \rho_G |$ the statistical one for this case.

In the crystal lattice case, where the number of independent G's is equal to the dimensionality of space, **u** is the only independent angle variable in a certain sense. This is despite the fact that **G**, of course, also can rotate freely in space. However, as Halperin and Nelson have emphasized [8] (and as was already evident in the old Shockley-Read construction of a grain boundary as a dislocation array) one must introduce a large, finite density of defects in the strain, essentially destroying *positional* order, before *orientational* readjustment of parts of the crystal independently is permitted. Thus in the true lattice case, only the strain is a relevant dynamical object, although the crystal overall may rotate as well.

This is not the case in the cholesteric and smectic, which are perhaps the most difficult to characterize in principle of all ordered states. Here strain **u** *and* **G** are both free angle variables, but there is what Volovik and Mineev [9] have called an « integral constraint » restricting the angular variation of G : namely,

$$\oint G.\mathrm{d}s = 0 .$$

That is, the layers may bend but the distance between layers must be constant. The nature of singularities, at least, in these two cases is not understood as yet.

As important and little-appreciated caution must be added on the dynamics of the density wave situation. The « strain » or « position » variable « u » is that of the density wave itself, which only *usually*, not always, means the position of the whole substance. As Overhauser first pointed out for spin density waves [10], and Leggett for solids [11], the mass being carried as the density wave moves by the density wave fluctuations or « phasons », *may* or *may not* be the whole mass density. This question

is a subtle and complex one related to the question of Mott or Wigner metal-insulator transitions. If there is a true energy gap for excitation of particles in the self-consistent lattice potential — as there is for all real solids, but not for smectics or electron density waves in some cases — the lattice carries all the mass.

From the order parameter and its dynamic nature flow many of the useful and important properties of condensed broken symmetry systems : the Goldstone and Higgs boson excitations, the long-range elastic-like forces (such as Suhl-Nakamura interactions in magnets) but most important of all the property I call *generalized rigidity*. Important examples of generalized rigidity are true rigidity, superconductivity, superfluidity, and hysteresis in magnets.

The order parameter, which is invariably a thermal average of some local quantity, can be defined locally, at least if its variation is sufficiently slow. The magnetization direction of an antiferromagnet, or the phase of a superfluid, can vary from place to place in a given sample. It is only reasonable to suppose that the extra free energy caused by such variation grows only slowly as the rate of variation increases : i.e.,

$$F = F_0(|\eta|, T, - \; -) + F_1(|\varphi|, T) : (\nabla\varphi)^2 + \cdots + \alpha(\nabla|\eta|)^2 + \cdots .$$

We write the gradient terms only schematically, as far as their tensor character goes.

This free energy determines the degree of fluctuation of η and hence φ by conventional Gibbs theory (treating η as a conventional thermodynamic variable whose local average value we can constrain at will for purposes of calculating F). F_0 does not depend on φ at all, by our original symmetry. The $(\nabla\varphi)^2$ term is necessary if the broken symmetry ordered state is to be stable. This equation implies the rigidity property. First, it is clear that with even a very small force applied only locally we can move φ about at will in the whole sample. This is because the dependence of F_0 on the origin of φ vanishes by symmetry, so that φ can move about at will ; while the $\nabla\varphi$ term will enforce uniform φ. Equally, without breaking down ordering, i.e., increasing F_0 at least locally, a force applied on φ at one end of the sample will be transmitted to the other : this is *rigidity*, and I consider it one of the key consequences of broken symmetry in condensed systems, since only with rigidity can structures be formed of these systems, or information or energy be transmitted through them.

III. Generalization of the order parameter and broken symmetry concept, especially « dissipative structures »

In several special cases one has attempted to define an « order parameter » which does not have the full properties, for instance, that of being a dynamical variable — the present version of the order parameter in the spin glass is one example [12]. It is not at all clear, at least theoretically and probably experimentally, that true rigidity exists in this system. My own preference is to leave such aberrant cases to one side and recognize that the analogy is a dangerous one.

It is for this reason that I am disturbed by the common uses of the terms « broken symmetry », « order parameter », and « dissipative structure » in the theory of nonlinear instabilities of driven systems. The attempt is to draw the analogy with equilibrium structures of broken-symmetry systems. It is proposed that if such pro-

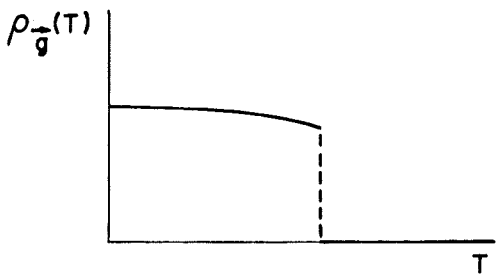

FIG. 3. — In a first-order transition, such as the liquid to solid crystal transition shown here, the order parameter will exhibit a discontinuous jump at the transition with an associated release (or absorption) of latent heat.

perties existed, they would have important consequences in our understanding of the self-organization of living systems [13, 14].

One can argue endlessly about words rather than meanings in this area so I would like to make a very clear distinction between what one might hope to be a *useful* « dissipative structure » as opposed to something which, while it has broken symmetry per se, and contains visible structure and something which might be described as an order parameter, is nonetheless an artefact which does not have the properties which might be useful for purposes of self-organization.

The two properties which I would consider essential for self-organization are :

a) Autonomy.

b) Rigidity.

Both of these are properties of condensed broken symmetry systems, and we may ask if they are exhibited in any well-understood dissipative systems.

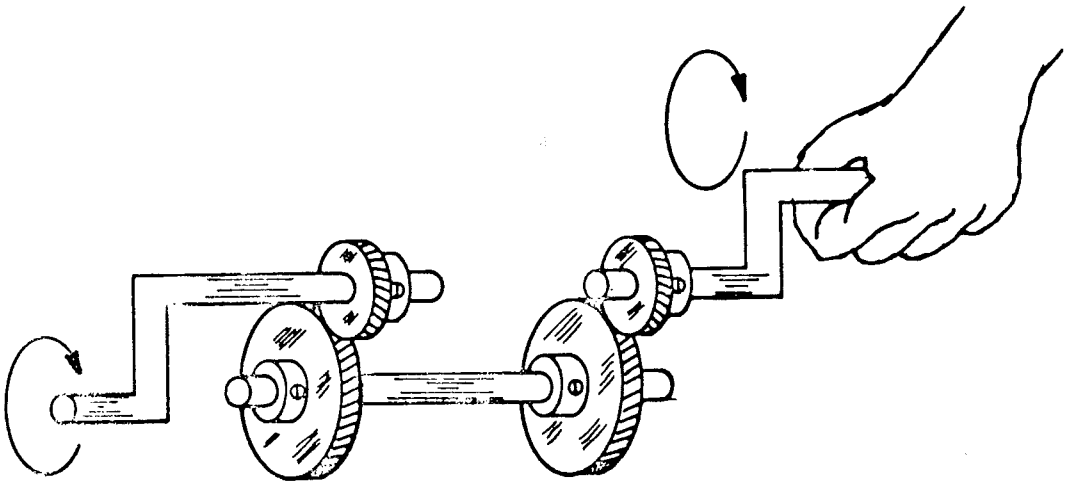

FIG. 4. — Illustration (somewhat schematic) of generalized rigidity. An external force (the crank) couples to the order parameter at one end of the system, represented as a gear. A change in the order parameter at any point in the ordered system is transmitted to all other parts of the system (first gear turns the second gear). The second gear turns the second crank : a force has been transmitted from one end of the system to the other *via* the order parameter.

By autonomy of a structure I mean that its space or time structure should not be predetermined in terms of the scale of the external boundary conditions (as opposed to the microscopic scale of atoms or molecules, for instance). An example of a dissipative system which *does* have autonomy of scale is a dye laser or any laser where the precise mode of oscillation is not predetermined by careful mode selection techniques : the wavelength of light is an autonomous microscopic scale irrelevant to the scale of the apparatus. This is not the case in such classical systems as Bénard or Couette convection cells, which are controlled in size by one of the apparatus dimensions. Autonomy is necessary if one is to speak of *self*-organization as opposed to predetermined organization.

The second property, that of rigidity, it seems to me is also essential. If the structure is to be stable, carry out actions, and, above all, to serve as a *substrate for information*, it is essential that it be rigid : that is, that it have the two properties of (1) having internal degrees of freedom which are not predetermined; (2) having a freedom which may be stably manipulated and which can exert action at a distance.

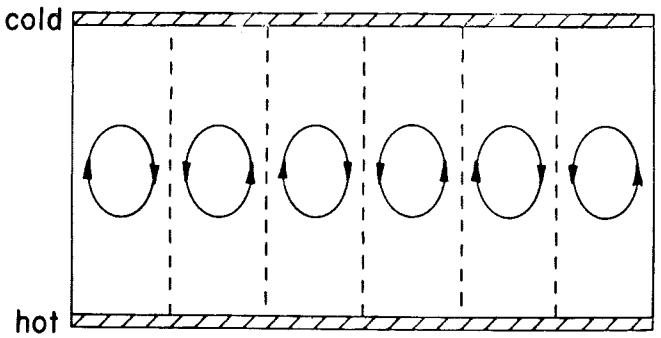

FIG. 5. — The Bénard instability in rectangular geometry. A layer of fluid between two horizontal rectangular plates is heated from below. When a sufficient thermal gradient is reached between top and bottom plates, convection arises in the form of rolls. In this cutaway edge-on view, the arrows represent the fluid velocity.

I am unaware of any work in the literature which demonstrates these properties in any well-understood dissipative system. Most of the conventional hydrodynamic systems which show regular roll patterns are not really autonomous. In these systems one often defines an « order parameter » which is the inhomogeneous component of velocity or flow, and under sufficiently restrictive conditions a kind of Gibbs free energy functional of the order parameter can be derived which gives the equations of motion near the instability by differentiation. But the assumptions which go into this derivation seem to preclude its general use, or the derivation of rigidity properties from it; nor is it clear that this « order parameter » has the real properties of the condensed systems order parameters, such as freedom to move within an order parameter space, locality, etc.

In lasers and in turbulent systems, as well as in some chemical oscillation systems, autonomy seems to be available but not order or rigidity; in general, these systems

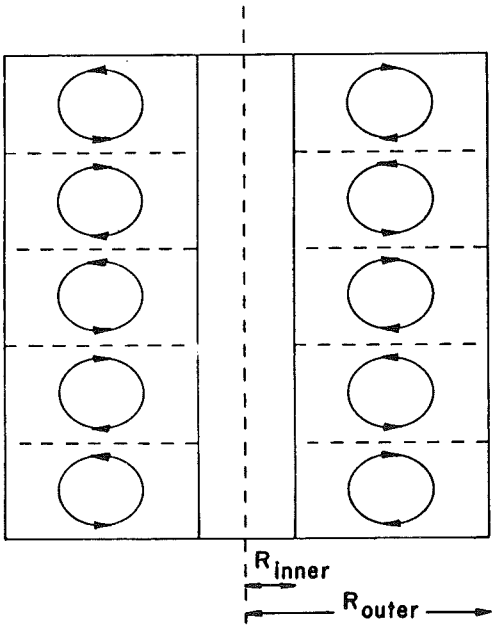

FIG. 6. — Couette flow : A fluid is placed between two cylinders with different rotational velocities about their axes. When the velocity gradient exceeds a critical value, rolls of vortices form. In this view the cylinder is cut along its length.

appear to be very chaotic. Whether this is a general state of affairs needs to be studied more carefully.

If one enquires how living systems seem to go about building autonomous, rigid structures, one finds a fascinating mixture of dissipative and condensation processes at work. One seems to see dissipative stages initiating condensation, such as the formation of membranes, for example, and the condensed systems in turn controlling dissipative stages. Haken has emphasized this structure. It is not clear that the present idea of « dissipative structures » in the theory of nonlinear systems is at all relevant to this process.

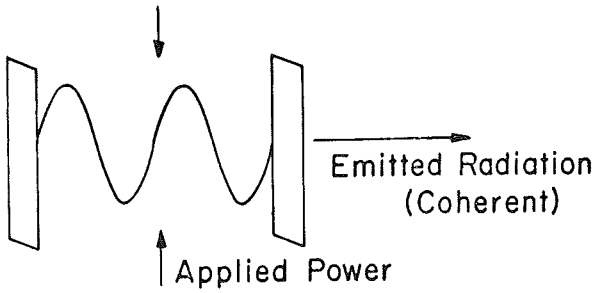

FIG. 7. — In a laser, a standing wave of excitation density is set up between two end plates, or mirrors, resulting in emission of a beam of coherent radiation.

Acknowledgments

The above ideas have evolved over many years and months, and in discussions with many people. Their present form is much influenced by recent discussions with D. Stein, L. Sneddon, A. Iberall and H. Soodak. Some of the ideas about broken symmetry evolved in a lecture course given at Quing Hua University, and I am grateful for their hospitality and the assistance of Charlies Xie.

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