Report for Exercise 02

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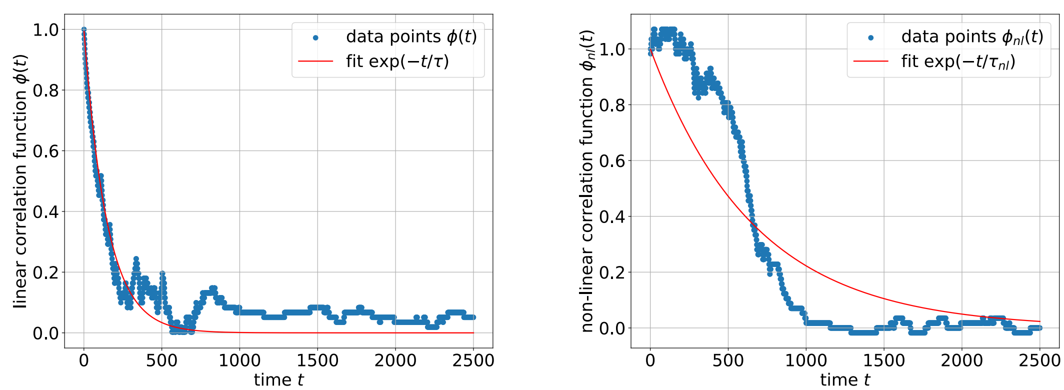
1 Task 1

Figure 1 shows an example plots of the linear and non liner correlation function obtained from metropolis single flip updates on a 3d Ising model grid of size $L = 5$ at inverse temperature $\beta = 0.4$ where t is the number of single flip updates using the metropolis algorithm starting from a random grid. For both functions I plotted an exponential function $\exp(-\frac{t}{\tau})$. For the linear plot (fig. 1a)) I obtained $\tau = 147.6$ and for the non-linear plot (fig. 1b) $\tau_{nl} = 665.5$.



(a) plot of the liner correlation function $\phi(t)$ (b) plot of the non-liner correlation function$\phi_{nl}(t)$

Figure 1: Plots and fits for the non liner and linear correlation functions for a 3d grid of size $L = 5$ at inverse temperature $\beta = 0.4$ where t is the number of single flip updates using the metropolis algorithm starting from a random grid.

To obtain more precise results for different temperatures I averaged over ten measures of the correlation lengths for each temperature. The results can be seen in table 1.

inverse temperature β	0.4	0.5	0.6	0.7
linear correlation time τ	143.5 ± 27.0	139.1 ± 30.7	175.5 ± 86.9	170.2 ± 61.9
non-linear correlation time τ_{nl}	616.9 ± 292.0	590.9 ± 447.9	522.0 ± 397.0	672.3 ± 386.5

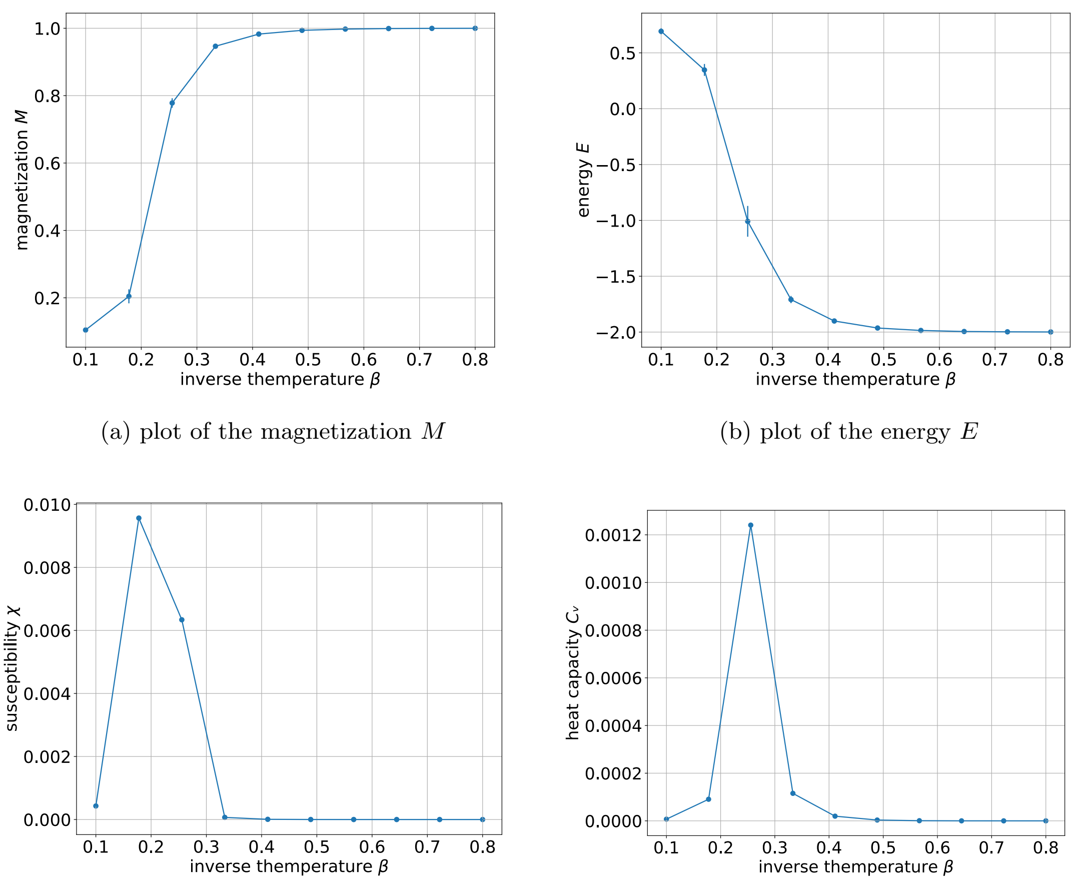
Table 1: averaged correlation times over ten measures for different inverse temperatures β.

The results in table 1 for the linear correlation time are looking reasonable. The variance of the non-linear correlation time on the other hand is quite large where I don't know if this is due to an error somewhere or if it makes sense.

We can use the correlation times to calculate the number of steps needed to reach equilibrium $n_e^{th} = 3\tau \approx 430.5$ and the number of samples we need to discard, i.e. the number of subsweeps $n_0 = 3\tau_{nl} =\approx 1850.7$.

2 Task 2

Figure 2 shows the plots of the magnetization M (fig. 2a), the energy E (fig. 2b), the magnetic susceptibility χ (fig. 2c) and the heat capacity C_c (fig. 2d) at different temperatures T, which we obtained by using the $M(RT)^2$ algorithm to simulate a three dimensional Ising model of length $L = 5$.



(a) plot of the magnetization M

(b) plot of the energy E

(c) plot of the susceptibility χ

(d) plot of the heat capacity C_v

Figure 2: Plots for different measures obtained form a 3-d ising model of size 5.

3 Task 3

We can obtain the critical temperature from the plots in figure 2. We can see that the phase transition happens for $\beta \in [0.2, 0.3]$. I assume that $\beta = 0.23$ is the critical inverse temperature which corresponds to a critical temperature $T_c \approx 4.34$ which lies quite close to the real critical temperature of $T_{analytical} \approx 4.51$. To improve the result obtained from the plots one could increase the system size which would result in sharper peaks and steps.