Computational Statistical Physics

Exercise sheet 01

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Exercise 1.   2-d Ising model

Goal: We start by simulating the 2-d Ising model using the Metropolis-Hastings-based single-spin flip Monte Carlo method.

Write a program for a Monte Carlo simulation to solve the two-dimensional Ising model on the square lattice with periodic boundary conditions. Implement the *single-spin flip* Metropolis-Hastings algorithm for sampling. As you will have to reuse this code for upcoming exercise sheets, it might be worth to make sure that it is well-structured!

Hint: If you get stuck you can use the python skeleton provided.

Task 1: Measure and plot the energy E, the magnetization M, the *magnetic susceptibility* χ and the *heat capacity* C_V at different temperatures T.

Task 2: Determine the critical temperature T_c.

Hint: You should obtain $T_c \approx 2.27$.

Task 3: Study how your results depend on the system size.

Hint: Start with small systems to reduce the computation time.

Task 4 (OPTIONAL): Save computation time by avoiding unnecessary re-evaluations of the exponential function. To achieve this, use an array to store the possible spin-flip acceptance probabilities.

Task 5 (OPTIONAL): Plot the dependence of M on the simulation time at a temperature $T < T_c$.

Hint: For small systems you should be able to observe sign-flips in M.