Report for Exercise 01

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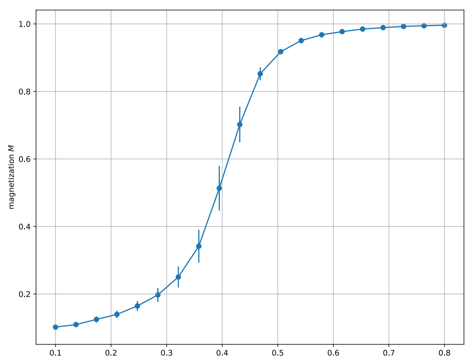
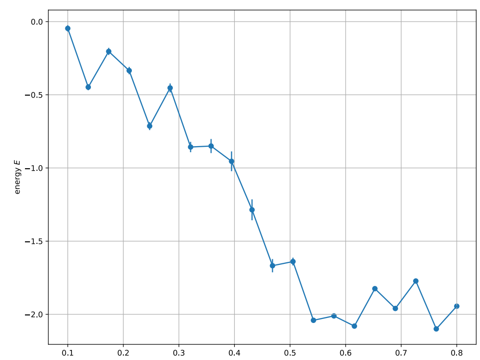
1 Task 1

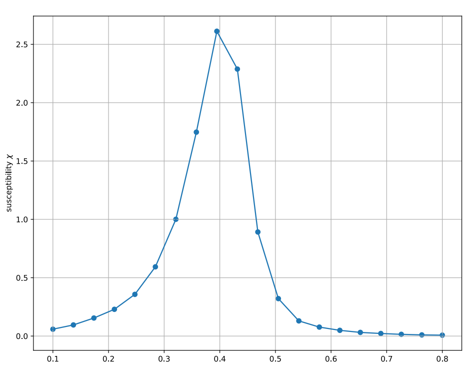
Figure 1 shows the plots of the magnetization M (fig. 2a), the energy E (fig. 2b), the magnetic susceptibility χ (fig. 1c) and the heat capacity C_c (fig. 1d) at different temperatures T, which we obtained by using the $M(RT)^2$ algorithm to simulate a two dimensional Ising model of length $L = 10$.



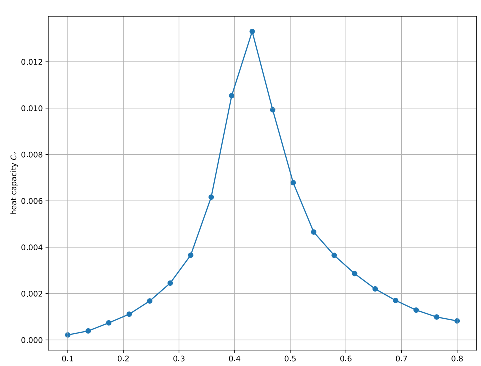
(a) plot of the magnetization M



(b) plot of the energy E



(c) plot of the susceptibility χ



(d) plot of the heat capacity C_v

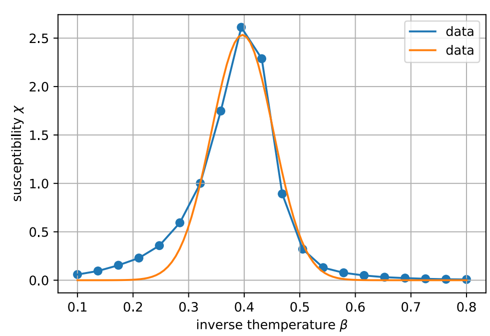
Figure 1: Plots for different measures obtained form a 2-d ising model of size 10.

The plots make sense in the way that they all show how the quantities change due to the expectet phase transition and especially the order parameter which is the magnetization (fig. 2a) varies continuously near the critical temperature which proofs the point that the 2-dimensional ising model performs a second order phase transition. From all plots one can see that they vary in the inverse temperature interval $[0.3, 0.5]$ which indeed includes the analytical critical temperature at which the phase transition occurs which is $\beta_c \approx 0.44$. The energy plot (fig. 2b) shows big deviations compared to the other plots. I was not able to identify the reason for that or if that is an error.

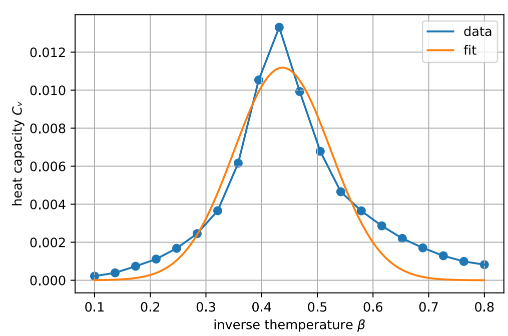
2 Task 2

Now we want to estimate the critical temperature from our plots in fig. 1 which we then can compare to the analytical critical temperature $T_c \approx 2.27$. For the estimation of the critical temperature I fitted a normal distribution both to the plot of the magnetic susceptibility (fig. 1c) and the heat capacity (fig. 1d), which can be seen in figure 2. I average over the means from the fits to get an estimate for the critical inverse temperature $\beta_c \approx 0.417 \pm 0.029$, which relates to a critical temperature of

$$T_c \approx 2.39 \pm 0.18$$



(a) fit of the magnetic susceptibility χ



(b) fit of the heat capacity C_v

Figure 2: Fits of the susceptibility and the heat capacity in order to estimate the critical temperature T_c of the phase transition.

The analytical critical temperature $T_{analyt.} \approx 2.27$ lies within the confidence interval of our approximation and the error of our simulation is $|T_c - T_{analyt.}| = 0.12$ which is a quite good approximation for a lattice of size 10. To improve the simulation we could simply increase the grid size which would also increase the run time.

3 Task 3

For the observation of the dependence of the system on the system size I simulated the ising model for size 5,10,15 and 25. In figure 3, I plotted the magnetization for the different grid sizes.

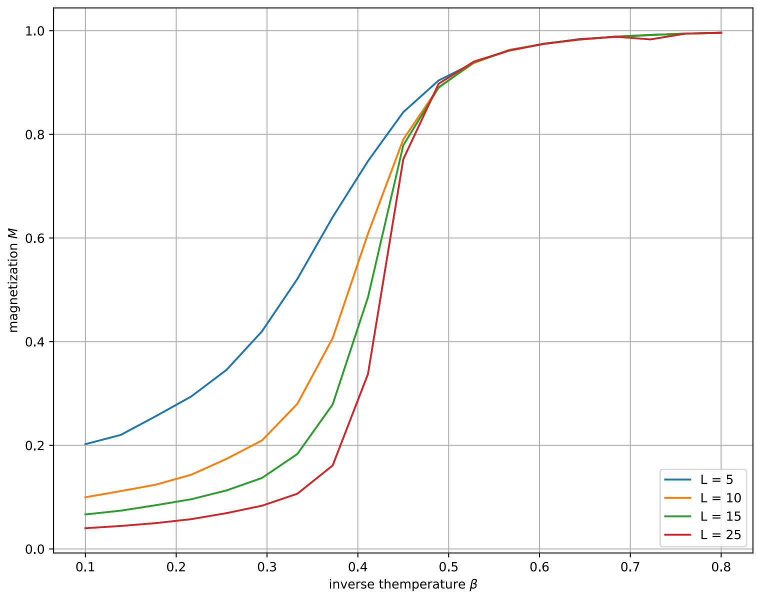


Figure 3: magnetization for different grid sizes

As one expect you can observe in fig. 3 that the magnetization converges towards the step function for an infinite grid which would have its step at the critical inverse temperature $\beta_c \approx 0.44$. Therefore the simulated critical temperature converges toward the analytical critical temperature. This proves the point that if we want more precise measures we need to increase the grid size. The disadvantage of this is that the computation time scales with the increasing grid size.