Supervised learning for 2D Ising model

In this exercise we want to study the phase transition of the 2D Ising model on a square lattice using supervised learning (Nature Physics, **13**, 431 (2017)). The Ising model is governed by

$$H = -J \sum_{\langle i,j \rangle} s_i s_j \,, \tag{1}$$

and we choose $J = 1$. As you know, in 2D this model shows a phase transition at $T_c \approx 2.27$, separating an ordered from a disordered phase. We want to use a fully connected feed-forward neural network to classify these two phases.

1. Adapt your solution to exercise 1 to generate spin configurations both above and below the transition temperature for a 2D Ising model of a fixed size, e.g. $L = 30$. These configurations will serve to train and validate the neural network.

2. Install TensorFlow. This is most easily achieved using pip. More information can be found **here**. We also want to use Keras, which is provided as part of TensorFlow via `tensorflow.keras`.

3. Implement a fully connected neural network with one input layer, one hidden layer with 100 neurons and an output layer. Use sigmoid neurons, binary cross entropy cost function and train using the Adam method for stochastic optimization. You can find a good tutorial to implement a fully connected neural network **here**.

4. Train the network with some of the generated configurations. Validate the network with the remaining configurations. Study the output layer and the accuracy of the network. Experiment with the number of epochs and the batch size to obtain the best results.