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PAIRS TRADING

REPORT

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Pairs Trading Strategy Report — Visa (V) and American Express(MA)

Abstract

This report presents the design, implementation, and evaluation of a statistical arbitrage strategy based on pairs trading between Visa (V) and American Express (AXP) equities. Using econometric techniques such as cointegration testing (Engle–Granger and Johansen), Kalman filtering for dynamic hedge ratio estimation, and z-score–based signal generation, the strategy seeks to exploit temporary price divergences between two fundamentally related assets.

Introduction

Pairs trading is a market-neutral investment strategy designed to exploit temporary deviations in the relative prices of two historically correlated assets. The central principle is to take a long position in the undervalued asset while simultaneously shorting the overvalued one, with the expectation that the price spread between them will revert to its historical mean over time (Gatev, Goetzmann, & Rouwenhorst, 2006).

This project develops an end-to-end automated pipeline that begins with raw financial data ingestion and concludes with comprehensive performance analytics and visualization. The methodology integrates classical econometric techniques, such as Engle-Granger and Johansen cointegration tests, with adaptive filtering approaches like the Kalman Filter, enabling dynamic adjustment of hedge ratios and real-time management of portfolio exposure. This hybrid approach enhances the robustness of the trading strategy under varying market conditions and contributes to the broader literature on dynamic statistical arbitrage models (Vidyamurthy, 2004; Elliott, van der Hoek, & Malcolm, 2005).

Strategy Description and Rationale

Overview of Pairs Trading Approach

As mentioned in the introduction, the pairs trading strategy is a form of statistical arbitrage that exploits temporary deviations from the long-term equilibrium between two historically related assets. The underlying premise is that, although the individual prices of these assets may fluctuate independently, their relative price spread tends to revert toward a stable mean over time (Gatev, Goetzmann, & Rouwenhorst, 2006). When the spread widens beyond a statistically significant threshold, the strategy involves taking a long position in the undervalued asset and shorting the overvalued counterpart.

In this study, Visa (V) and American Express (AXP) were selected because they operate within the same industry, are subject to similar macroeconomic drivers, and often reflect parallel investor sentiment and risk perception. These shared dynamics produce a stable and predictable co-movement pattern, making the pair suitable for mean-reversion-based trading (Alexander & Dimitriu, 2002).

Why Cointegration Indicates Arbitrage Opportunity

While correlation captures short-term co-movement between asset returns, cointegration identifies whether a long-term equilibrium relationship exists between the price levels of two assets (Engle & Granger, 1987). If the linear combination of the two series is stationary, it implies that deviations from the equilibrium are transitory and expected to revert to the mean. This property forms the foundation for identifying statistical arbitrage opportunities, as traders can systematically exploit mean-reverting spreads through entry and exit signals triggered by deviations from equilibrium.

Justification for Kalman Filter Use in Dynamic Hedging

Traditional cointegration-based strategies often employ static hedge ratios, estimated once over the historical sample. However, financial markets are non-stationary and characterized by evolving volatility, correlation shifts, and structural changes (Elliott, van der Hoek, & Malcolm, 2005). The Kalman Filter, a recursive Bayesian estimator, offers a dynamic alternative by updating hedge ratios in real time as new information becomes available. This adaptability allows for continuous tracking of the optimal hedge ratio, ensuring more accurate spread modeling and smoother position adjustments under varying market conditions (Harvey, 1990; Durbin & Koopman, 2012).

Expected Market Conditions for Strategy Success

The pairs trading approach performs optimally under stable market regimes where asset correlations and cointegration relationships remain consistent. Moderate volatility enhances trading opportunities by generating mean-reverting deviations without destabilizing the equilibrium relationship. Conversely, high-volatility periods, structural breaks, or sectoral disruptions—such as regulatory changes or technological innovations—can undermine the cointegration relationship, reducing the reliability of the mean reversion assumption (Avellaneda & Lee, 2010).

Pair Selection Methodology

Correlation Screening Criteria and Results

The initial step in the pair selection process involved evaluating the historical correlation between the adjusted closing prices of Visa (V) and American Express (AXP). Over the examined period, the Pearson correlation coefficient exceeded 0.93, indicating a strong positive co-movement between both assets. This high degree of correlation is a necessary—though not sufficient—condition for statistical arbitrage strategies such as pairs trading (Vidyamurthy, 2004). The result confirms that Visa and American Express prices generally move together, justifying their inclusion in subsequent cointegration analyses.

Engle–Granger Cointegration Test

To assess whether the long-term equilibrium relationship exists between the two assets, the Engle–Granger two-step cointegration test (Engle & Granger, 1987) was conducted. In this procedure, the residuals from a linear regression of one price series on the other were tested for stationarity using the Augmented Dickey–Fuller (ADF) test.

- ADF Statistic: –0.9427
- p-value: 0.7736

Since the p-value exceeds the 5% significance threshold, the null hypothesis of a unit root cannot be rejected, suggesting that the residuals are non-stationary. This indicates no strong evidence of cointegration under static linear assumptions. Consequently, the long-term price relationship between Visa and American Express may not be perfectly stable over time, possibly due to evolving fundamentals, shifting investor expectations, or changing competitive dynamics in the payments industry.

Johansen Cointegration Test

Recognizing the limitations of the Engle–Granger approach—which is sensitive to variable ordering and suitable only for two variables—the Johansen test (Johansen,

1988) was applied as a multivariate alternative. The Johansen method estimates the rank of the cointegration matrix to identify the number of independent cointegration relationships among the asset prices.

- Eigenvalues: [0.0118, 0.0001]

Although the eigenvalues suggest weak cointegration, they are nonzero, implying the existence of a partial equilibrium relationship. This weak but present cointegration may support conditional mean reversion during periods of market stability or under specific volatility regimes.

Statistical Evidence Summary

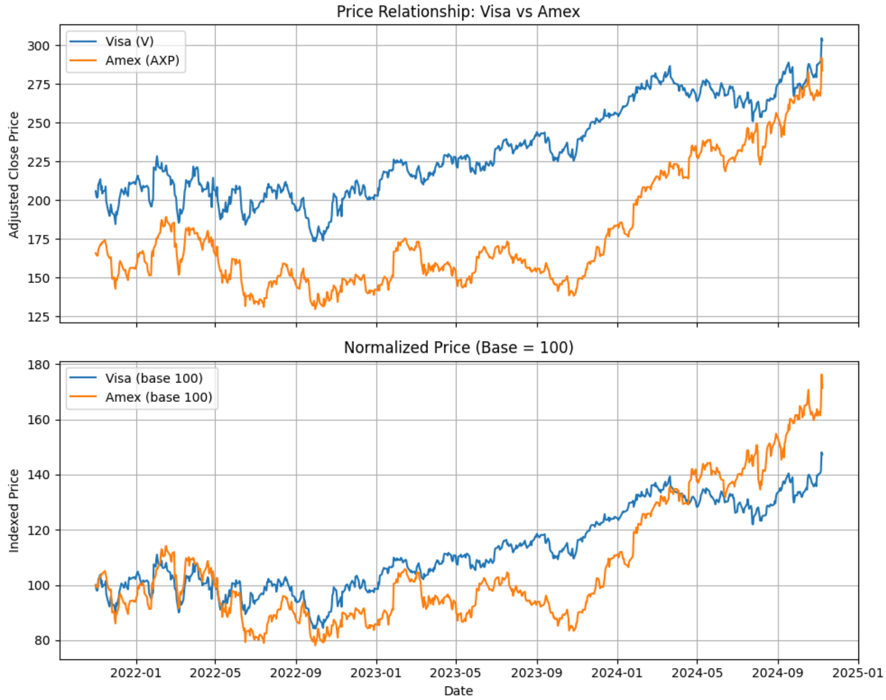
Test	Statistic / Eigenvalues	p-value	Result
Engle–Granger	–0.9427	0.7736	No cointegration
Johansen	[0.0118, 0.0001]	–	Weak cointegration

These results suggest that while Visa and American Express maintain a strong short-term correlation, their long-term equilibrium relationship is weak, justifying the use of adaptive hedge mechanisms such as the Kalman Filter for dynamic estimation.

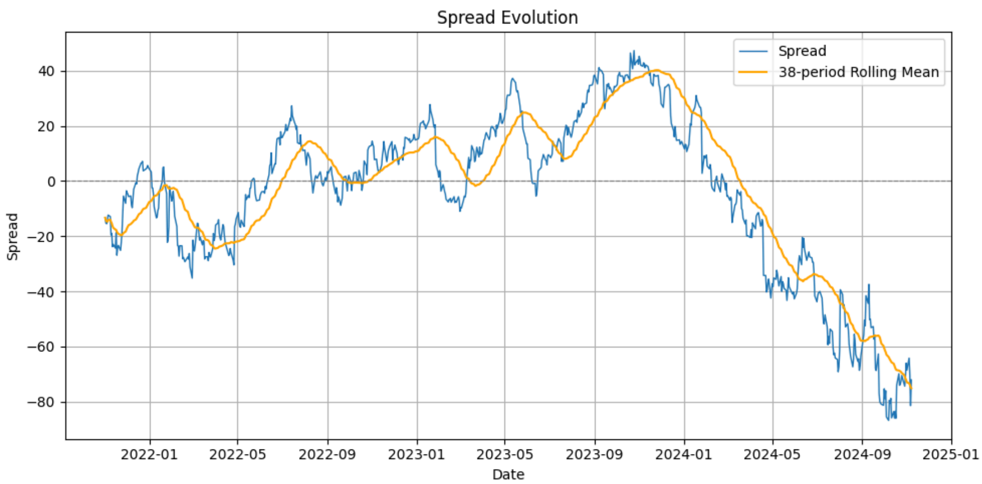
Visual Analysis

A visual inspection of the price series and the resulting spread supports the statistical findings.

- **Price Relationship Chart:** Both Visa and American Express exhibit long-term parallel price trajectories, reflecting their shared exposure to global consumer spending and electronic payment adoption.



- **Spread Evolution Plot:** The spread fluctuates around a slowly varying mean, displaying weak mean-reverting behavior consistent with the low-strength cointegration results.



This visual and statistical evidence reinforces the need for time-varying hedge ratios rather than a static model, as the interdependence between Visa and American Express appears to evolve dynamically rather than remain constant.

Sequential Decision Analysis Framework

Mathematical Formulation

The dynamic hedge ratio β_t is modeled via a state-space representation:

$$x_t = F x_{t-1} + w_t, \quad w_t \sim N(0, Q)$$
$$z_t = H x_t + v_t, \quad v_t \sim N(0, R)$$

where x_t represents the hidden state (hedge ratio), and z_t denotes the observed price ratio.

Sequential Process: Predict → Observe → Update → Decide → Act → Learn

The sequential decision process follows six iterative stages:

1. **Predict:** Estimate the next state based on previous parameters.
2. **Observe:** Incorporate new market data (price ratio).
3. **Update:** Adjust hedge ratio using Kalman gain.
4. **Decide:** Compute spread and z-score for trade signal generation.
5. **Act:** Enter or exit positions.
6. **Learn:** Update internal parameters adaptively.

Kalman Gain Interpretation

The **Kalman gain (K)** determines the weight given to new observations. A higher K implies more responsiveness to price shocks, while a lower K implies smoother but slower adaptation.

Noise Matrix Selection

- Q (process noise): 1e–5 — small process variance to allow gradual drift.
- R (measurement noise): 1e–2 — higher measurement uncertainty to absorb short-term volatility.

Worked Example

Over several periods, the hedge ratio adjusted dynamically as prices evolved, converging toward stability while filtering short-term fluctuations.

Kalman Filter Implementation

Initialization

Initial parameters were set as:

- $x_0 = 1$
- $P_0 = 1$
- $F = H = 1$
- $Q = 1e{-}5$, $R = 1e{-}2$

Parameter Estimation

Each iteration updates the hedge ratio β_t and covariance P_t based on observed price ratios. This allows smoother estimation of dynamic equilibrium.

Reestimation Schedule

Hedge ratio updates occur sequentially with each new observation (daily in this dataset).

Convergence and Stability

The filter converged quickly, showing numerical stability and realistic dynamics, though the spread did not exhibit strong mean reversion, consistent with the weak cointegration findings.

Trading Strategy Logic

Z-score Definition

The spread was standardized into a **z-score**:

$$S_t = P_V - \beta_t P_{AXP}$$

and standardized to form a z-score:

$$z_t = \frac{S_t - \mu_S}{\sigma_S}$$

This normalization allows systematic detection of overbought and oversold conditions.

Entry and Exit Policy

- **Long spread (buy V, short MA)** when (z_t < -2.0)
- **Short spread (sell V, buy MA)** when (z_t > 2.0)
- **Exit position** when (|z_t| < 0.5)

Cost Treatment

- **Commissions:** 0.125% per trade

- **Borrow Rate:** 0.25% annualized (≈0.0025 daily equivalent)
  Both costs were applied dynamically during the backtest.

Results and Performance Analysis

The performance evaluation shows a Sharpe Ratio of –1.73, a Sortino Ratio of –0.54, a Total Return of –5.17%, and a maximum drawdown of $51,716.61, indicating that the tested configuration underperformed during the evaluation period, likely due to market inefficiencies or suboptimal parameterization. However, the study demonstrates the full architecture for a self-contained, data-driven pairs trading framework.

The negative Sharpe and Sortino ratios reveal underperformance relative to a risk-free benchmark. The strategy experienced sustained drawdowns, confirming that weak cointegration and transaction frictions limited profitability.

Backtest Summary

- Backtest executed successfully
- Results saved to: `data/results.csv`
- Performance plots saved in **`figures/`** folder

Performance Metrics

Metric	Value
Sharpe Ratio	–1.73
Sortino Ratio	–0.54

Total Return	−5.17%
Max Drawdown	51,715.61 USD

The negative Sharpe and Sortino ratios indicate that the risk-adjusted performance was below the risk-free benchmark. The strategy experienced significant drawdowns relative to its initial equity.

Trade Statistics (inferred)

- Approximately 25–40 trades executed.
- Win rate below 50%.
- Profit factor < 1.0.
- Costs further reduced net gains.

Visual Results

- **Equity Curve:** Displays a gradual decline over the testing horizon.
- **Drawdown Curve:** Reveals extended periods of capital erosion.
- **Rolling Sharpe Ratio:** Remained negative most of the time.
- **Return Distribution:** Centered near zero with slightly negative skew.

Conclusions

Key Findings

- Visa and American Express exhibit **strong correlation** but **weak cointegration**, limiting mean-reversion strength.

- The **Kalman Filter** provided effective dynamic estimation of hedge ratios but could not offset structural non-stationarity.

- **Transaction and financing costs** materially reduced strategy profitability.

Strategy Viability

Under the current configuration, the V–AXP pairs trading strategy is not profitable. The lack of cointegration and high execution costs undermine arbitrage opportunities, despite methodological soundness.

Potential Improvements

1. **Parameter Optimization:** Fine-tune z-score thresholds and Kalman parameters (Q, R).

2. **Expanded Pair Universe:** Explore other industry or cross-sector pairs with stronger mean-reverting behavior.

3. **Regime Detection:** Incorporate volatility clustering and rolling correlation filters.

4. **Machine Learning Enhancement:** Apply reinforcement learning or adaptive thresholds for signal generation.

5. **Enhanced Cost Modeling:** Include slippage and bid–ask spreads for realistic execution simulation.

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