hurm i
ypput

Exam questions (Examples assignments & Tutorial problems)

The unit weight of a Fluid is what is of Importance in Fluid statics analysis

## Fluid-Mechanics

- Fluid is any subst. that begins to Flow at any application of Force.

## Course-outline

1 The Fluid Fundamentals;

a Definitions, units & dimensions

2 Elements of Fluid statics

a Fluid density, pressure, surface tension, Viscosity, Compressibility; manometry Fluid thrust on Immersed plane surfaces, Floatation stability

3 Fluid Dynamics;

a Conservation laws, Continuity eqn, Euler's eqn, Bernoulli's eqn, Fluid power, momentum eqn with applications, Impact jet

4 Introduction to Incompressible viscous Fluid;

a Reynold's number & Simple Flow measurement using Current meter

5 Hydraulic machinery;

a Types of machines, Impulse & rxn turbine, Performance Curve of a Centrifugal pump, turbines, axial pump, specific speed of pumps, multi stage pumps, Characteristic performance Curve of pumps.

## Objectives of CVE221

i) To Introduce the Concepts, theories & applications of Fluid mechanics & also establish their relevance in engineering.

ii) To develop the principles that underlie the Subject.

iii) To ~~apply the~~ demonstrate the applications of these principles in the design of hydraulic machines.

1 Dimensions, units & definitions

Types of units

a) Ancient Greek system
b) Customary system
c) Imperial system
d) Metric system (SI) -- This would be our focus

They're 6 primary units used to describe

Primary/Fundamental quantities

| | Quantities | Unit | Dimension |
|---|---|---|---|
| 1 | Length | Metre, M | L |
| 2 | Mass | Kilogram, Kg | m |
| 3 | Time | Seconds, S | T |
| 4 | Temp | Kelvin, K | $\theta$ |
| 5 | Current | Ampere, A | I |
| 6 | Luminosity | Candela, cd | Cd |

Derived Quantities

| | Quantities | Unit | Dimension |
|---|---|---|---|
| 1 | Force | Newton, N, $Kgm/s^2$ | $MLT^{-2}$ |
| 2 | Energy | Joules $Kgm^2/s^2$ | $ML^2T^{-2}$ |
| 3 | Pressure | Pascal, Pa $Kg/s^2m$ | $ML^{-1}T^{-2}$ |
| 4 | mass Density, $\rho$ | $kg/m^3$ | $ML^{-3}$ |

Specific gravity, S = $\dfrac{\text{Unit weight of substance}}{\text{Unit weight of pure water}}$

| | | | |
|---|---|---|---|
| 5 | Weight density ($\omega$) | $Kg/m^2s^2$ | $ML^{-2}T^{-2}$ |
| 6) | Specific volume, $v$ | $m^3/Kg$ | $M^{-1}L^3$ |
| 7) | Specific gravity, S | unitless | dimensionless |
| 8) | Surface Tension, $T$ | $N/m$, $Kg/s^2$ | $MT^{-2}$ |

## Fluid Statics

A) **Density** is a measure of the quantity of matter in a unit vol of a substance. It can be expressed in 3 different ways namely;

1 Mass Density - This is the mass per unit ~~weight~~ volume at standard temp & pressure. Mathematically,

mass density, $\rho = M/V$ --(1), where M = Mass of Fluid, V = Vol of Fluid

Its unit is Kg/m³. Also, typical values include;

Water($\rho$) = 1000 $kg/m^3$, Air ($\rho$) = 1.23 $kg/m^3$

Mercury ($\rho$) = 13546 $kg/m^3$

2) Weight Density or Unit Weight - it is the weight per unit volume of a Fluid at standard temp & pressure - mathematically, Unit weight denoted by $\omega = W/V$ -- (II)

where W = Total weight, V = Vol of Fluid

$W = mg$ OR $\omega = \dfrac{mg}{V} = \rho \cdot g$ --- (III)

Its unit is ~~[illegible]~~ $N/m^3$. Also, typical values include;

Specific gravity, $S = \dfrac{\text{Unit weight of substance}}{\text{Unit weight of pure water}}$

| | | |
|---|---|---|
| 5 Weight density ($\omega$) | $Kg/m^2s^2$ | $ML^{-2}T^{-2}$ |
| 6) Specific volume, $v$ | $m^3/Kg$ | $M^{-1}L^3$ |
| 7) Specific gravity, S | unitless | dimensionless |
| 8) Surface tension, $\tau$ | $N/m$, $Kg/s^2$ | $MT^{-2}$ |

## Fluid statics

A) **Density** is a measure of the quantity of matter in a unit vol of a substance. It can be expressed in 3 different ways namely;

1 **Mass Density** - This is the mass per unit ~~weight~~ volume at standard temp & pressure. Mathematically,

mass density, $\rho = M/V$ --(1), where $M$ = Mass of Fluid, $V$ = Vol of Fluid

Its unit is $Kg/m^3$. Also, typical values include;

Water($\rho$) = $1000 Kg/m^3$, Air ($\rho$) = $1.23 Kg/m^3$

Mercury ($\rho$) = $13546 Kg/m^3$

2) **Weight Density or Unit Weight** - it is the weight per unit volume of a fluid at standard temp & pressure - mathematically, Unit weight denoted by $\omega = W/V$ - (II)

where W = Total weight, V = Vol of Fluid

W=mg

OR $\omega = \dfrac{mg}{V} = \rho \cdot g$ --- (III)

Its unit is ~~[illegible]~~ $N/m^3$. Also, typical values include;

For $H_2O = 9810\ N/m^3$, Air $= 1.27\ N/m^3$
For mercury, $Hg = 1322300\ N/m^3$

8) Specific gravity, S — is the ratio btw the unit weight of Flud ~~and~~ to the unit weight of a standard Fluid. It is dimensionless & has no unit. Mathematically,

$$S = W_F / W_{S.F} \quad \text{--- (IV)}$$

For liquids, the standard Fluid is $H_2O$ at $4°C$

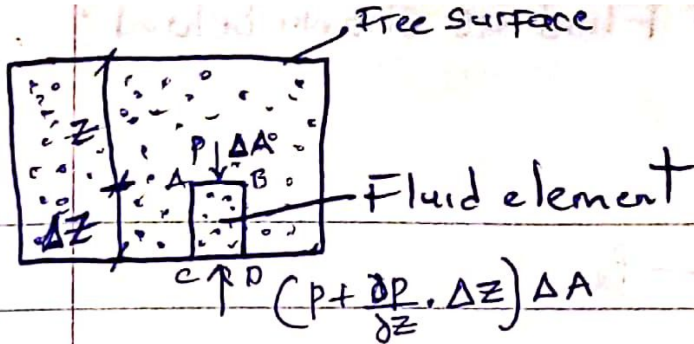
B) Pressure — It is the Force per unit area. Mathematically, pressure, $p = F/A$ --- (V)

The unit of pressure is $N/m^2$, where $1 N/m^2 = 1 Pa = 10^5$ bar

Hydrostatic laws — It states that the rate of increase in pressure in the vertical downward direction is equal to the unit weight of the Fluid. Mathematically, $W = \frac{\partial P}{\partial Z}$ --- (VI), where

Proof

Consider a Fluid element as shown below:




Let $P$ = Intensity of pressure
" $\Delta A$ = x-sectional area of the fluid element.
" $Z$ = distance btw the Free surface & the Fluid element
" $\Delta Z$ = height of the Fluid element

① $F_{AB} = P.\Delta A$
② $F_{CD} = \left(P + \frac{\partial P}{\partial Z}.\Delta Z\right).\Delta A$
③ $W = \omega \times V = \omega \times \Delta A \times \Delta Z$

For equilibrium Condition; $F_{AB} - F_{CD} + W = 0$
Substituting we obtain;

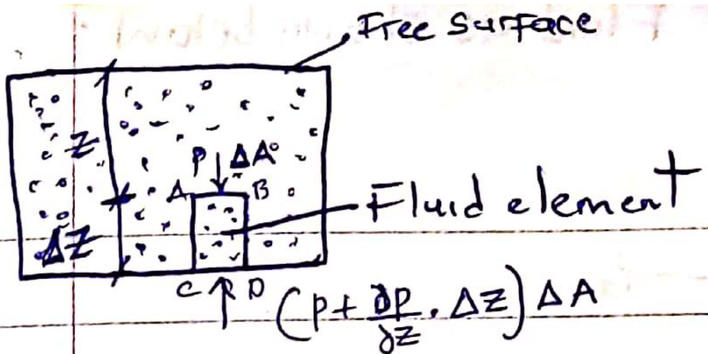
$$P.\Delta A - \left(P + \partial P/\partial Z \cdot \Delta Z\right).\Delta A + \omega \times \Delta A \times \Delta Z = 0$$

$$\cancel{P.\Delta A} - \cancel{P.\Delta A} - \frac{\partial P}{\partial Z}.\Delta A.\Delta Z + W\Delta A.\Delta Z = 0$$

$$-\partial P/\partial Z + \omega = 0 \equiv \omega = \partial P/\partial Z \quad \text{proved ①}$$

Also $\int dp = w\int dZ \equiv P = \omega Z + C$ --- (vii)

Pascal's law - It states that the pressure at any pt in a fluid at rest is the same in all direction.
Mathematically $P_x = P_y = P_z$




Let $p$ = Intensity of pressure
" $\Delta A$ = x-sectional area of the Fluid element.
" $Z$ = distance btw the Free surface & the Fluid element
" $\Delta Z$ = height of the Fluid element

① $F_{AB} = P.\Delta A$
② $F_{CD} = \left(P + \frac{\partial P}{\partial Z}.\Delta Z\right).\Delta A$
③ $W = \omega \times V = \omega \times \Delta A \times \Delta Z$

For equilibrium Condition; $F_{AB} - F_{CD} + W = 0$
Substituting we obtain;

$$P.\Delta A - \left(P + \partial P/\partial Z . \Delta Z\right).\Delta A + \omega \times \Delta A \times \Delta Z = 0$$

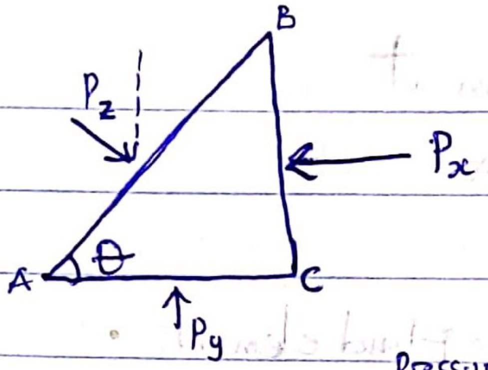
$$\cancel{P.\Delta A} - \cancel{P.\Delta A} - \frac{\partial P}{\partial Z}.\Delta A.\Delta Z + W \Delta A . \Delta Z = 0$$

$$-\partial P/\partial Z + \omega = 0 \equiv \omega = \partial P/\partial Z \quad \text{Proved ✓}$$

Also $\int dp = w\int dZ \equiv P = \omega Z + C$ --- (VII)

Pascal's law - It states that the pressure at any pt in a Fluid at rest is the same in all direction.
Mathematically, $P_x = P_y = P_z$

Consider a small wedge of Fluid as shown below;




$P_x$ is the horizontal ~~Force~~ pressure acting on the Fluid wedge
$P_y$ " " Vertical pressure ~~Force~~ " " " " "
$P_z$ " " Inclined ~~Force~~ Pressure " " " " "
$\theta$ " " Inclination angle or angle of Inclination

$F_{BC} = P_x \cdot BC$

$F_{AC} = P_y \cdot AC$

$F_{AB} = P_z \cdot AB$

$\Sigma F_x = 0$

$F_{BC} = F_{AB} \sin\theta$

$P_x \cdot BC = P_z \cdot AB \sin\theta$

$BC = AB \sin\theta$

$P_x \cdot AB\sin\theta = P_z AB \sin\theta$

$P_x = P_z$ — — (1)

$\Sigma F_y = 0$

$F_{AC} - W = F_{AB} \cos\theta$

Because we are dealing with a very element, the W is neglected. Therefores

$P_y \cdot AC = P_z \cdot AB\cos\theta$

But $AC = AB\cos\theta$

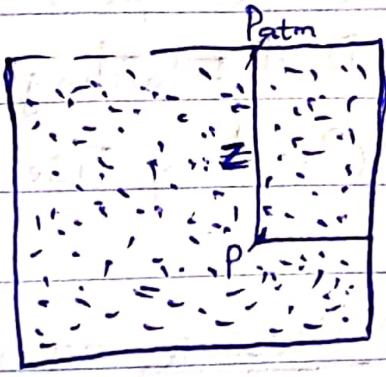
$P_y \cdot AB\cos\theta = P_z \cdot AB\cos\theta$

$\therefore P_y = P_z$ — — — (2)

Combining eqn (1) & (2) we obtain;

$P_x = P_y = P_z$

C) Absolute & Gauge Pressure




$P = \rho g Z + \text{Constant}$ (From the Integration)

at $P = P_{atm}$

$Z = 0$

$P_{atm} = \rho g(0) + \text{Constant}$

$\text{Constant} = P_{atm}$

$\therefore\ P = \rho g Z + P_{atm}$

Absolute pressure, $P_{Abs} = P_{gauge} + P_{atm}$

Typical values of atmospheric pressure;

$101.3\ kN/m^2 = 101.3\ KPa$

$10.3m$ of Water; $760\ mm\ Hg$

D) Surface Tension

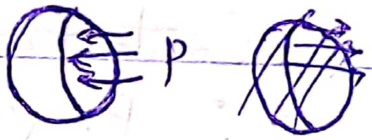
It is the tensile force acting per unit length in a liquid surface, in contact with a gas, ~~or~~ acting on the interface of 2 immiscible liquids so that the surface is acting as a membrane.

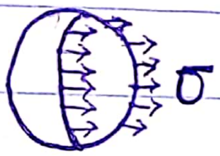
## Practical manifestations of Surface tension

1) Rain droplets
2) Dust Floating on a liquid Surface
3) Capillary rise

1) Pressure on a liquid droplet

Consider a liquid droplet of diameter, d under a pressure P and Surface tension, $\sigma$, where;




d = diameter droplet

P = pressure on droplet

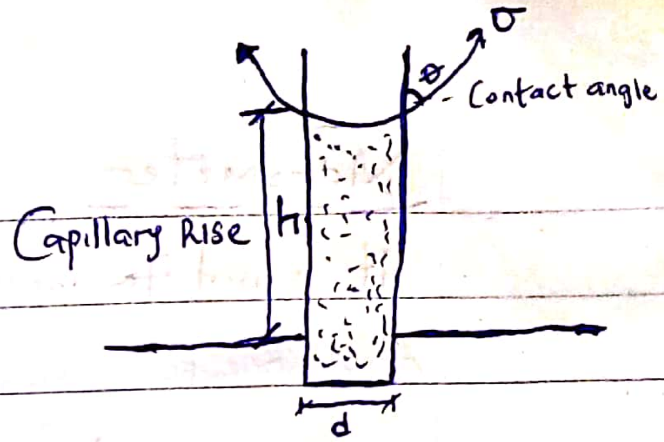
$\sigma$ = Surface Tension

$$F_P = P \times \frac{\pi d^2}{4}, \quad F_s = \sigma \cdot \pi d$$

$$F_P = F_s \quad \therefore \quad P \times \frac{\pi d^2}{4} = \sigma \pi d$$

$$\therefore P = 4\sigma/d$$

2) Capillarity - This is ~~the~~ a phenomenon by which a liquid rises in a tube above or below the general water level. It is due to the combined effect of cohesion & adhesion.




From the diagram, the Vertical Cpt of The Force, $F_s$ is given by; $F_s = \sigma \cdot \pi d \cos\theta$

$W = w \times \pi d^2/4 \times h$

$F_s = W \Rightarrow \sigma \cdot \pi d \cos\theta = \frac{w \cdot \pi d^2}{4} \cdot h$

$$h = \frac{4\sigma\cos\theta}{wd}$$

For glass & water, $\theta$ is approximately equal to zero (i.e $\theta \simeq 0$), while for mercury, $\theta \simeq 140°C$

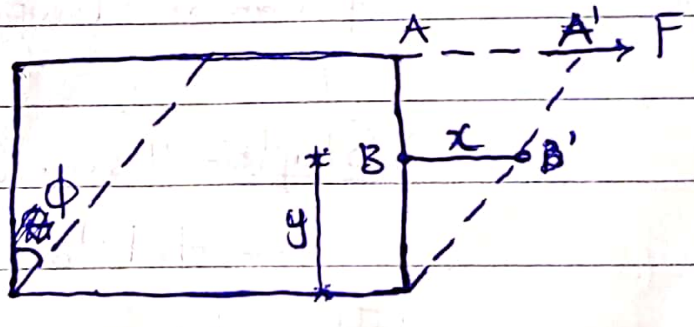
## Viscosity

Viscosity is the ppty of a Fluid that determines its resistance to Flow. It is a measure of the internal Friction that Causes resistance to Flow. Also, it's due to Cohesion & exchange of molecular momentum bt.w the layers of a Fluid.

$\phi$ = Shear strain

$\phi = \frac{x}{y}$. Rate, $\frac{x}{ty} = \frac{u}{y}$

$= \frac{du}{dy}$

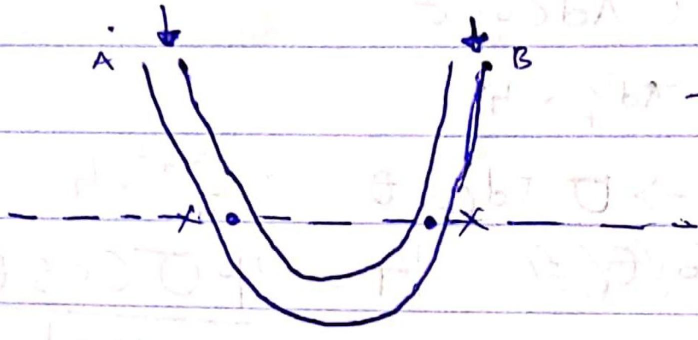



$\tau = F/A$, $\tau \propto \frac{du}{dy}$ or $\tau = \mu \, du/dy$

Scanned with CamScanner

# Manometer

It is used for measuring the pressure of a fluid ~~two types~~ ~~static, kinetic~~



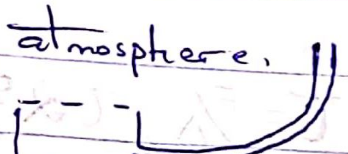

The pressure at both pts (dots) are same

- At the same level of the fluid on both part of the tube, the pressure is same, while at different heights on both sides, results in different pressure.

Pressure of a fluid can be measured by the ffg devices;
1) Manometer (2) mechanical guages

1) Manometer - They're devices used for measuring the pressure at a pt in a fluid, by balancing the column of fluid by the same of ~~other~~ another column of fluid in a pipe or channel. They can be classified into simple & Differential manometers.

a) Simple - It's one which consists of a glass tube whose one end is connected to a pt where pressure is to be measured, & the other end remains open to the atmosphere.



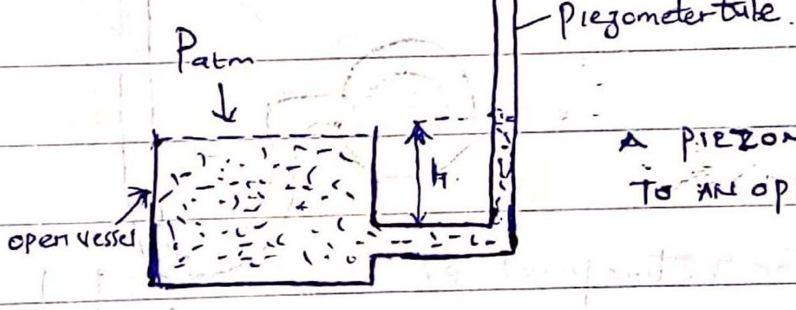
Column

Common types of Simple manometer are;

a) Piezometer (b) U-tube manometer (c) Single Column manometer

a) **Piezometer** - This is a device used to measure liquid pressure in a system by measuring the height to which a Column of the liquid rises against gravity. It Consists of a glass tube inserted into the wall of a vessel or of a pipe Containing liquid, whose pressure is to be measured. The tube extends vertically upward to such a height that liquid Can freely rise without overflowing. The pressure at any pt in the liquid is indicated by the height of the liquid in the tube above that pt, which Can be read on a scale attached to it

ost glass tubes e not scaled, b rather scales re inscribed on hem.




A PIEZOMETER TUBE FITTED TO AN OPEN VESSEL

The Patm affects the ~~type~~ degree of Fluid moving upwards in the tube. The Patm Can be determined by height, h

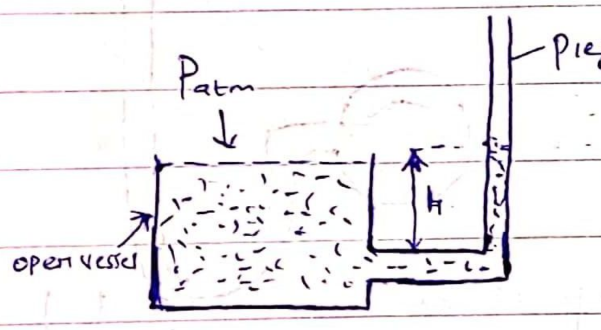
- The Patm = h, i.e increase in pressure = Increase in height.

**Note** - It is the Simplest form of manometer & it measures guage pressure only. It also only measure +ve pressures, when the guage pre-

Common types of Simple manometer are;
a) Piezometer (b) U-tube manometer (c) Single Column manometer

a) Piezometer - This is a device used to measure liquid pressure in a system by measuring the height to which a Column of the liquid rises against gravity. It Consists of a glass tube inserted into the wall of a vessel or of a pipe Containing liquid, whose pressure is to be measured. The tube extends vertically upward to such a height that liquid Can freely rise without overflowing. The pressure at any pt in the liquid is indicated by the height of the liquid in the tube above that pt, which can be read on a scale attached to it

Most glass tubes are not scaled, but rather scales are inscribed on them.




A PIEZOMETER TUBE FITTED TO AN OPEN VESSEL

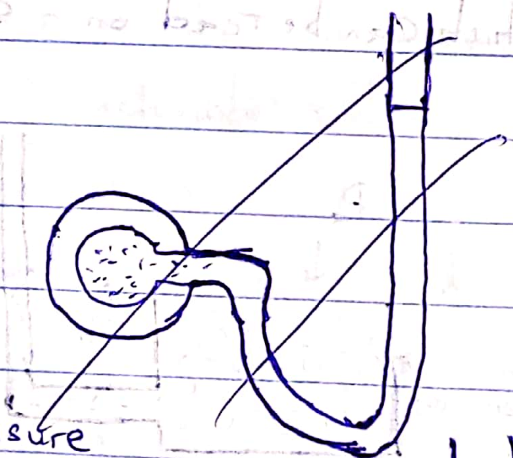
The Patm affects the ~~upp~~ degree of Fluid moving upwards in the tube. The Patm Can be determined by height, H

- The Patm = H, i.e Increase in pressure = Increase in height.

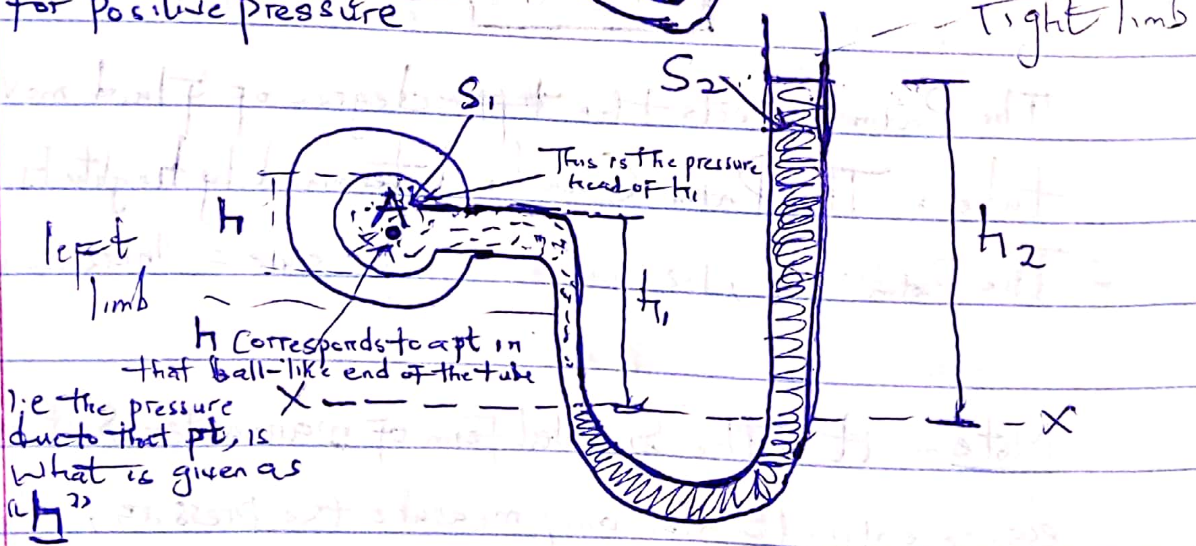
Note - It is the Simplest Form of manometer & it measures guage pressure only. It also only measure +ve pressures, when the guage pre ssure is higher than the ambient pressure.

2) U-tube Manometer - They're used in measuring large pressures in lighter liquids which are to be measured with very long tubes. Also, gas pressure can be measured with this manometer unlike the piezometer, because a gas forms no free atmospheric surface. It consists of a glass tube bent in a U-shape, one end being connected to a pt. where pressure is to be measured & the other end left open to the atmosphere.

Note - It measures both -ve & +ve pressure.



For positive pressure



Let A be the pt at which pressure is to be measured
" X--X is the datum line
" $h_1$ = height of the light liquid
" $h_2$ = height of the heavy liquid
" $S_1$ = Specific gravity of the light liquid
" $S_2$ = " " " " heavy liquid

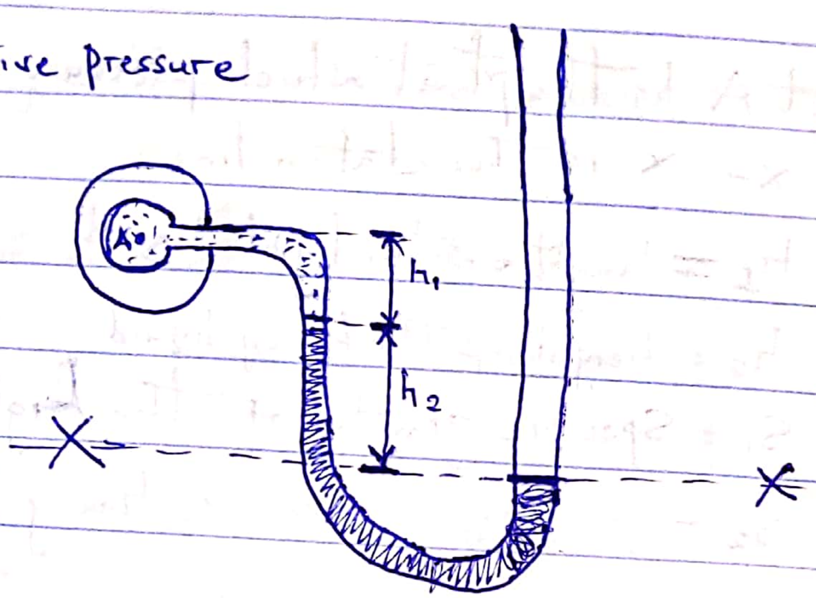
The pressures in the left limb & the right limb above the dat
um line X—X are equal, just as pressures at 2 pts at the s
ame level in a Continuous homogenous liquid are equal.

- From the diagram above, since the pressure on the left & ri
ght limbs are equal, the total pressure on the left limb
Comprising the pressure w.r.t that in the Space, A related to
its specific height, h & the pressure due to $h_1$ = $h_2$,
i.e $h + h_1 = h_2$ ≏ $P_h + P_{h_1} = P_{h_2}$

OR

Pressure head above X-X in left = $h + h_1S_1$
" " " " in right = $h_2S_2$
∴ Since they're equal, $h + h_1S_1 = h_2S_2 \equiv h = h_2S_2 - h_1S_1$ —(i)

For negative pressure




Pressure head above X–X in left $= h + h_1S_1 + h_2S_2$

" " " " " Right $= 0$

$\therefore h + h_1S_1 + h_2S_2 = 0 \equiv h = -h_1S_1 - h_2S_2$ ...(11)

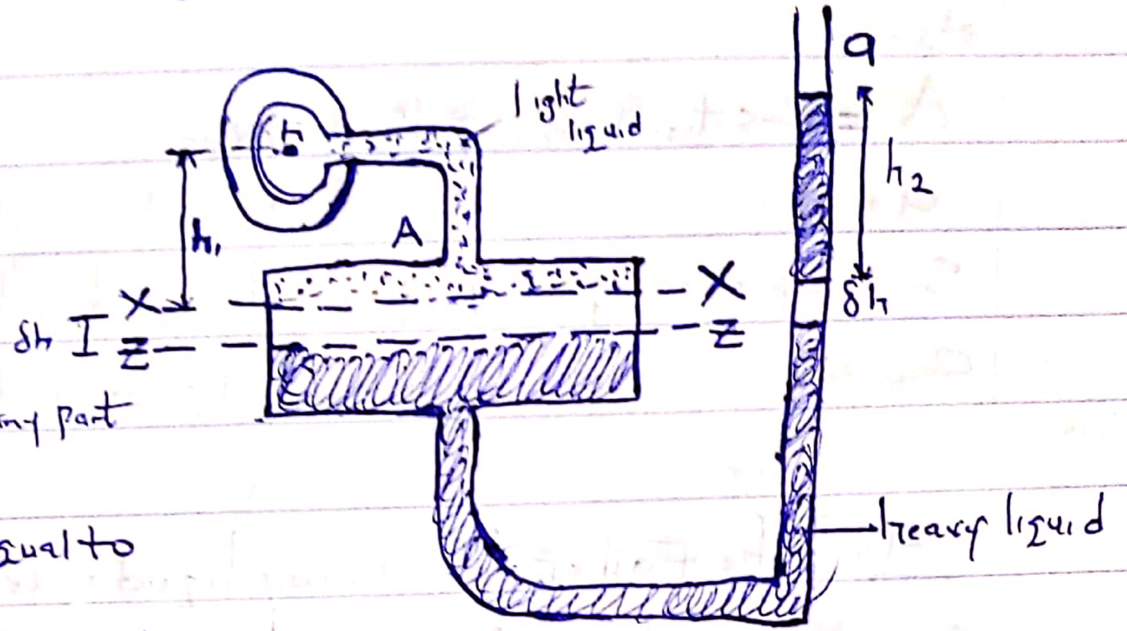
c) Simple Column Manometer – This is a modified type of U-tube manometer in which a shallow reservoir having a large x-sectional area (at abt 100 times) as compared to the area of the tube it's connected to. Why we have a shallow reservoir connected is because;

*) For any variation in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected. And the pressure is the indicated by the height of the liquid in the other limb.

There are 2 types of Simple Column manometers & they're;

1) Vertical Single Column manometer

2) Inclined Single Column manometer




olume of that tiny part
given by
$V = A \times \delta h$
and is also equal to
$a \times h_2$
$\therefore V = A \times \delta h = a \times h_2$.

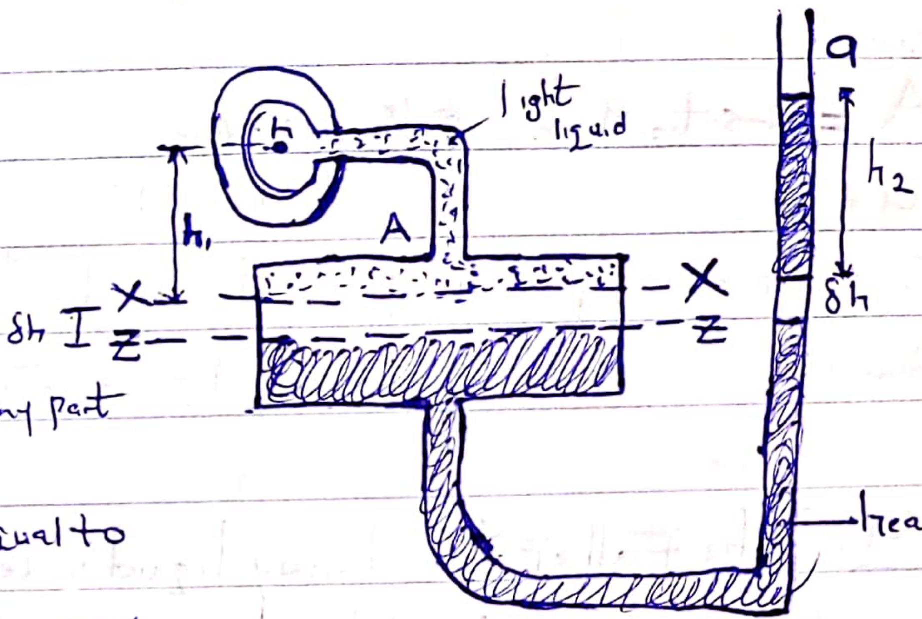
1) <u>Vertical SCM</u>: Let X-X be the datum line in the reservoir when the Single Column manometer is not Connected to the pipe. Now Consider the manometer is Connected to a pipe Containing light liquid under a Very high pressure. The pressure in the pipe will Force the light liquid to push the heavy liquid in the reservoir downwards. As the area of the reservoir is Very large, the Fall of the heavy liquid level will be Very small. This downward movement of the heavy liquid in the reservoir will Cause a Considerable rise of the heavy liquid in the right limb.

Let $h_1$ = Height From Centre of the pipe above X--X

$h_2$ = Rise of heavy liquid in the right limb.

$\delta h$ = Fall of heavy liquid level on the reservoir.

2) Inclined Single Column manometer




Volume of that tiny part is given by
$V = A \times \delta h$
and is also equal to
$a \times h_2$
$\therefore V = A \times \delta h = a \times h_2$

1) Vertical SCM: Let X-X be the datum line in the reservoir when the Single Column manometer is not Connected to the pipe. Now Consider the manometer is Connected to a pipe Containing light liquid under a Very high pressure. The pressure in the pipe will Force the light liquid to push the heavy liquid in the reservoir downwards. As the area of the reservoir is Very large, the Fall of the heavy liquid level will be Very small. This downward movement of the heavy liquid in the reservoir will Cause a Considerable rise of the heavy liquid in the right limb.

Let $h_1$ = Height From Centre of the pipe above X--X
$h_2$ = Rise of heavy liquid in the right limb.
$\delta h$ = Fall of heavy liquid level in the reservoir.

h is the pressure in the pipe expressed in terms of head of water.

A = X-sectional area of the reservoir.

a = ,, ,, ,, ,, tube on the right limb

$S_1$ = Specific gravity of light liquid in the pipe.

$S_2$ = ,, ,, ,, heavy liquid in the pipe.

Note - The fall of the heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

Since $V = A \times \delta h = a \times h_2$

$$\therefore \delta h = \frac{a \times h_2}{A}$$

Considering pressure head above the datum line Z --Z, Pressure head on the left limb = $h + h_1 S_1 + \delta h S_1$

Pressure head on the right limb = $h_2 S_2 + \delta h S_2$

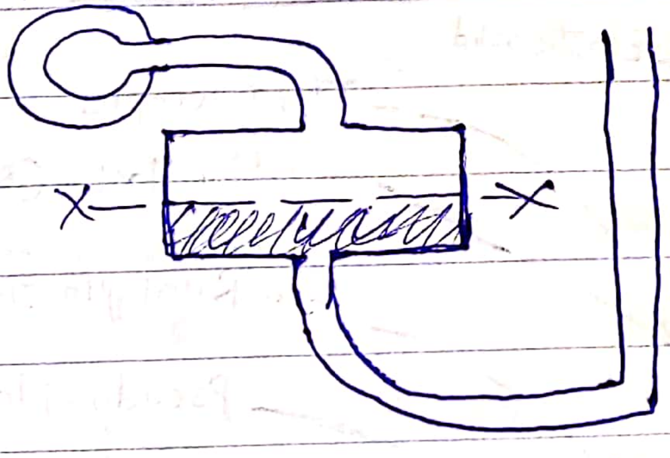
$\therefore$ Since they're equal

$$h + h_1 S_1 + \delta h S_1 = h_2 S_2 + \delta h S_2$$

$$h = h_2 S_2 + \delta h S_2 - h_1 S_1 - \delta h S_1$$

$$h = S_2(h_2 + \delta h) - S_1(h_1 + \delta h)$$

Example: $S_1 = 0.8$ (light liquid), $S_2 = 13.6$ (Mercury), $h_1 = 300mm$, $h_2 = 500mm$




From the diagram above showing a Single Column manometer Connected to a pipe, ~~of Sp~~ Containing liquid of specific gravity, $S_1 = 0.8$, the ratio of area of reservoir to that of the limb (right) is 100 ($A/a = 100$). Find the pressure of the pipe, $S_2 = 13.6$

Ans $= 64.98 KN/m^2$

Soln

$S_1 = 0.8$, $S_2 = 13.6$ $A/a = 100$, $h_1 = 300mm$ (0.3m), $h_2 = 500mm$ (0.5m), $H = ?$

$$h = S_2(\delta h + h_2) - S_1(\delta h + h_1)$$

$$\delta h = \frac{a \times h_2}{A} = \frac{1 \times 500}{100} = 5 \times 10^{-3} m$$

$$h = 13.6(5 + 500) - 0.8(5 + 300)$$

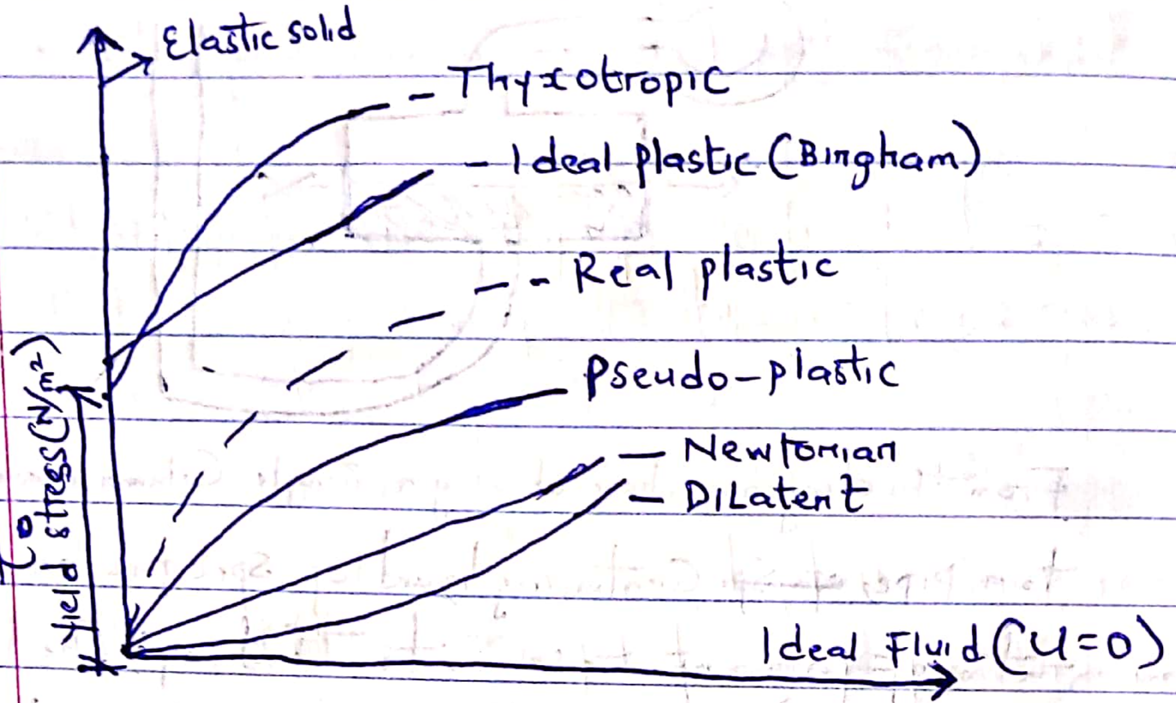
$$= 13.6(505) - 0.8(305) = 6624$$

$$h = 13.6(5\times10^{-3} + 0.5) - 0.8(5\times10^{-3} + 0.3)$$

$$= 6.868 - 0.244 = 6.624$$

$$H = -h_1 S_1 - h_2 S_2 = -0.3 \times 0.8 - 0.5.$$

# Newtonian & Non-Newtonian Fluid




Variation of shear stress with respect to velocity Gradient

Each of these lines can be represented by;
$\tau = A + B\left(\frac{du}{dy}\right)^n$, i.e a general eqn for all the lines

1) **Newtonian Fluid** are those that follow newton's viscosity eqn, $\tau = \mu\left(\frac{du}{dy}\right)$. Examples are water, air & Kerosene

2) **Non-newtonian Fluids** are those that don't follow the newton's viscosity eqn.

3) **Ideal plastic**- Shear stress reaches a certain minimum threshold before flow commences. examples are sewage sludge, drilling ~~mud~~ mud. Also, for ideal plastics;
$A = \tau_0$, $B = \mu$, $n = 1$ ; $\tau = \tau_0 + \mu \frac{du}{dy}$

4) <u>Pseudo-plastic</u> - Here, no minimum shear stress is required & viscosity decreases with the rate of shear stress. e.g Most Colloidal Suspension Such as Clay, milk, Cement, etc.

5) <u>Real plastic</u> - The behaviour lies btw the ideal plastic & the pseudo-plastic.

6) <u>Dilatent Fluid</u> - Here, viscosity increases with the rate of shear deformation, e.g quick sand, Custard, etc. For this Fluid; $A = 0$, $B = \mu$; $n > 1$ $\therefore \tau = \mu(du/dy)^n$ $(n > 1)$

7) <u>Thyxotropic</u> - Initial minimum shear stress is required before flow is initiated, and thereafter, viscosity reduced with the length of time shear is applied. e.g ~~Gelled~~ Jelly groundnut oil, Jelly paint, butter, most jelly materials, Yoghurt, etc. Here,

$A = \tau_o$, $B = \mu$, $n < 1$

$\therefore \tau = \tau_o + \mu(du/dy)^n$ $(n < 1)$

8) <u>Rheopectic substances</u> - This is the general name for all the substances, ~~such that whose~~ whose viscosity decreases with the length of time shear stress is applied.

9) <u>Visco-elastic material</u> - behaves newtonian, but becomes plastic if there's a sudden increase in shear stress.

ost lubricants such as engine oil & grease exhibit thus beha
iour.

10) Ideal Fluid - Viscosity of such fluid = zero.

11) Elastic Solid - Even if the ~~st~~ shear stress goes to infini
the fluid doesn't flow.

## Fluid thrust on immersed plane surfaces

~~Fluid~~ Common features of pressure in a static fluid are;

a) Hydrostatic vertical pressure distribution; shows that pressure increases with depth.

b) Pressure at any pt in a static fluid acts equally in al directions.

c) Pressures at the same depth in a continuous fluid are equal.

d) Pressure acting on a boundary acts at right angle to that boundary.

Note: Total pressure & Force (represented as F) are used interchangeably.

Total pressure is the force exerted on a surface when it is in contact with a fluid.

Scanned with CamScanner

The Centre of Pressure is the pt of application of the total pressure.

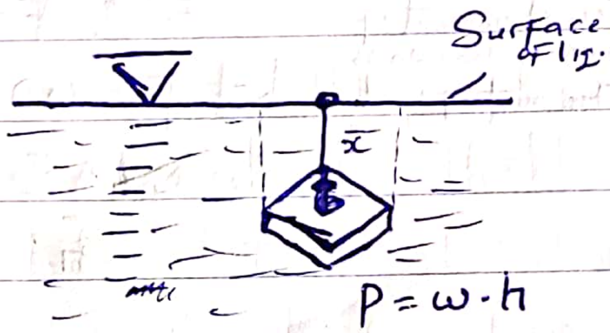
These 2 Cpts listed above can be determined for the ffg cases:

i) Horizontally Immersed Surface

ii) Vertically Immersed Surface

iii) Inclined Immersed Surface

i) <u>Horizontally I.S</u> -




Fluid thrust = Weight of fluid ~~above~~ on the horizontal fluid

Mathematically, $F = W(\omega \times V)$

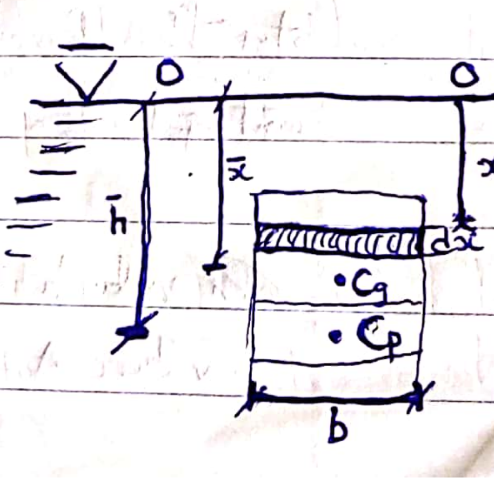
$W = \omega \times V$ $\therefore F = \omega \times V$

where $V = A \times \bar{x}$

where A = area of the horizontal surface

$\therefore F = \omega A \bar{x}$

2) <u>Vertically I.S</u> -




Let's consider a strip of thickness dx, pressure at every pt on this strip, $p = \omega x$

$F = \omega x \cdot A$, where $A = b \times dx$

$F = \omega x \cdot b \cdot dx$

$= w \int b \cdot dx \cdot x$

~~Let~~ Let $\bar{h}$ = distance btw the Centre of pressure (Cp) ~~&~~ the Free water ~~Surface~~ Surface. So that, ~~the mmt of the force~~

Where Cp = Centre of pressure
" Cg = Centre of gravity

$F \times \bar{h} = w \int b \cdot dx \cdot x^2$

$F \times \bar{h} = w \int x^2 \cdot b \cdot dx$

Also, Since $\int x^2 \cdot b\,dx = I_{00}$

Another name for the Second moment of area is known as the mmt of Inertia.

$\therefore F \times \bar{h} = w I_{00}$

but $F = wA\bar{x}$

$\therefore \cancel{w} A\bar{x} \times \bar{h} = \cancel{w} I_{00}$

$\bar{h} = I_{00} / A\bar{x}$

## Parallel axis theorem

This theorem gives the mmt of Inertia abt any axis, given the mmt of Inertia abt a parallel axis passing through the Centroid of the plane Surface & the perpendicular distance btw the 2 ~~distances~~ axes. Mathematically;

mmt of Inertia abt any axis = mmt of Inertia abt the Centroid $+ Ah^2$

which becomes; $I = I_c + Ah^2$ - - - .

Where A = Area of Inertia abt the plane surface, h = Perpendicular distance btw two axes.

From $\bar{h} = I_{o}/A\bar{x}$ or $I_{oo}/A\bar{x}$

$$\therefore \bar{h} = \frac{I_c + A\bar{x}^2}{A\bar{x}} \equiv \frac{I_c}{A\bar{x}} + \bar{x}$$

3) Inclined plane Surface

From $F = WA\bar{x}$

$$\bar{h} = \frac{I\sin^2\theta}{A\bar{x}} + \bar{x}$$

(prove the eqn [Assignment])

i.e Find an expression for the total pressure & position of Centre of pressure for an inclined plane surface.

Example 1

A rectangle plane surface 2m wide & 3m deep is vertically immersed in water. Determine the F & $\bar{h}$ if:

$A = 6m^2$

i) The upper edge of the plane surface coincides with the free water surface.

ii) The upper edge is 2.5m below the free water surfaces.

Soln

(i) $A = 6m^2$,

From $F = WA\bar{x} = \rho g A\bar{x} = 1000 \times 9.81 \times 6 \times 1.5$

$= 88290N \simeq 88.2KN$

Scanned with CamScanner

b) $\bar{h} = \dfrac{I_c + A\bar{x}^2}{A\bar{x}}$

Position of Centre of pressure, $\bar{h}$

$\bar{h} = I_c / A\bar{x} + \bar{x}$

$I_c = \dfrac{bd^3}{12}$

$= \dfrac{2 \times 3^3}{12} = 4.5 m^4$

$\bar{h} = \dfrac{4.5 + 6(1.5)^2}{6(1.5)}$

$= 2m$

ii) $F = WA\bar{x} \simeq \rho g A\bar{x} = 1000 \times 9.81 \times 6 \times (2.5 + 1.5)$

$= 235440 N \simeq 235.44 KN$

b) $\bar{h} = \dfrac{I_c + A\bar{x}^2}{A\bar{x}}$

$I_c = 4.5 m^4$

$\therefore \bar{h} = \dfrac{4.5 + 6(4)^2}{24}$

$= 4.1875 m$

1) Prove $F = WA\bar{x}$

Pressure Intensity, $P = \rho g h$

Pressure Force $dF = P \times dA = \rho g h \times dA$

$dF = \rho g h \times dA$, Total Force thus becomes;

$F = \int \rho g h . dA$, but $\dfrac{h}{y} = \dfrac{\bar{h}}{\bar{y}} = \dfrac{h^*}{y^*} = \sin\theta$

$h = y\sin\theta$

$F = \int \rho g \times y \sin\theta . dA = \rho g \sin\theta \int y dA$

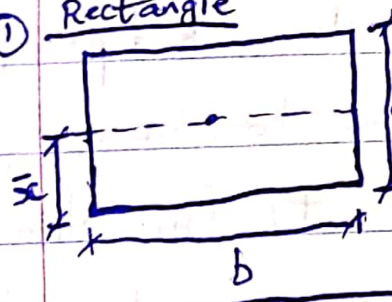
$F = \rho g \sin\theta (\bar{y} . A) = \rho g . \bar{y} \sin\theta . A = \rho g \bar{h} A$

Since $W = \rho g$ and $\bar{h} \simeq \bar{x}$

$\therefore F = WA\bar{x}$

# Properties of Common Shapes

## 1) Rectangle




$A = bd$

$\bar{x} = d/2$

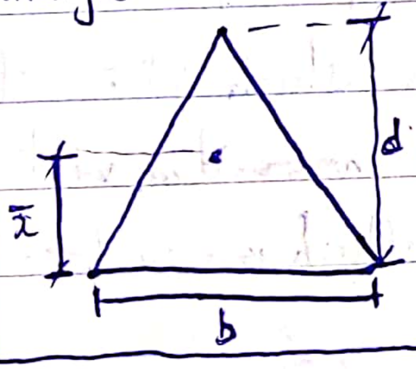
$I_c = bd^3/12$

To obtain $I_b$ (mmt about the base) we obtain;

$I_b = bd^3/3$ $\quad I_b = I_c + A\left(\frac{b}{2}\right)^2$

$= \frac{bd^3}{12} + \frac{3bd^3}{12} = bd^3/3$ (proved)

## 2) Triangle

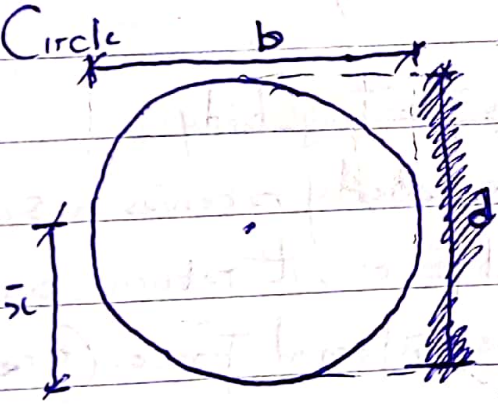



$A = \frac{1}{2}bd$

$\bar{x} = d/3$

$I_c = bd^3/36 \qquad I_b = bd^3/12$

## 3) Circle

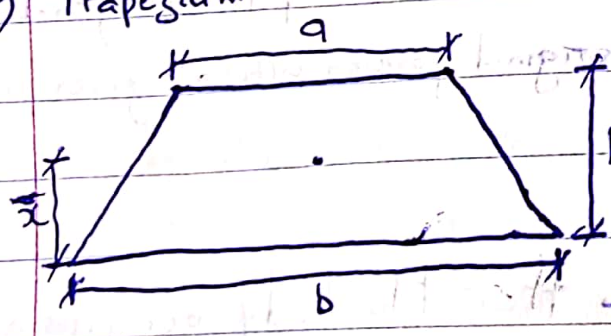



$A = \frac{\pi d^2}{4}$

$\bar{x} = d/2$

$I_c = \frac{\pi d^4}{64}$

## 4) Trapezium




$A = \left(\frac{a+b}{2}\right)h$

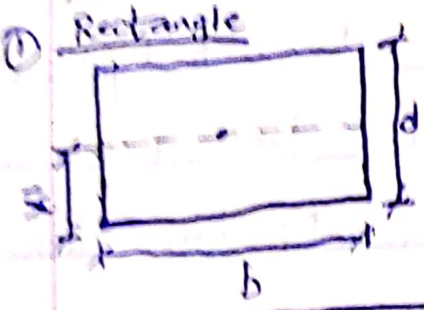
$\bar{x} = \left(\frac{2a+b}{a+b}\right)\left(\frac{h}{3}\right)$

$I_c = \left[\frac{a^2+4ab+b^2}{36(a+b)}\right] \times h^3$

Scanned with CamScanner

# Properties of Common Shapes

## 1) Rectangle




$A = bd$

$\bar{x} = d/2$

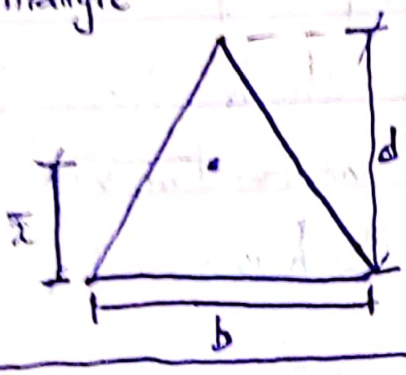
$I_c = bd^3/12$

To obtain $I_b$ (mmt about the b axe) we obtain:

$I_b = bd^3/3$ or $I_b = I_c + A\left(\frac{b}{2}\right)^2$

$= \frac{bd^3}{12} + \frac{3bd^3}{12} = bd^3/3$ (proved)

## 2) Triangle

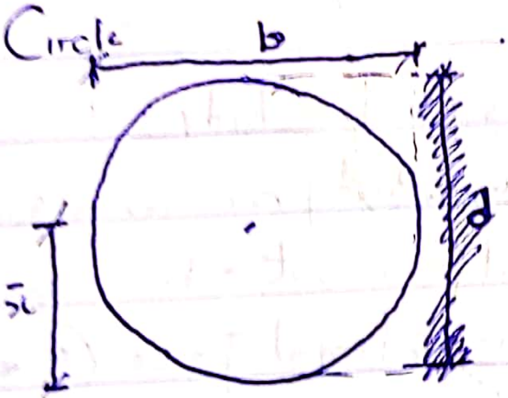



$A = \frac{1}{2}bd$

$\bar{x} = d/3$

$I_c = bd^3/36$ $\quad I_b = bd^3/12$

## 3) Circle

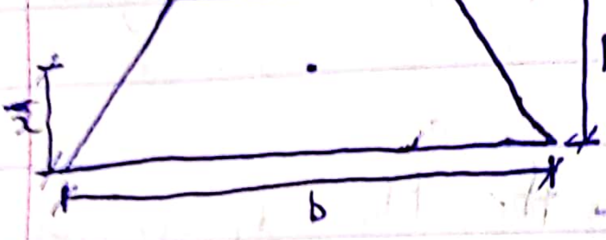



$A = \frac{\pi d^2}{4}$

$\bar{x} = d/2$

$I_c = \frac{\pi d^4}{64}$

## 4) Trapezium




$A = \left(\frac{a+b}{2}\right)h$

$\bar{x} = \left(\frac{2a+b}{a+b}\right)\left(\frac{h}{3}\right)$

$I_c = \left[\frac{a^2+4ab+b^2}{36(a+b)}\right] \times h^3$

# Floatation and stability

Loss of weight is experienced when a body is partially o wholly Immersed in a Fluid. The loss of weight is due to Force acting opposite the action of gravity. This phenomenon is K own as buoyancy, and the force is Known as buoyant F ce. The magnitude of buoyant Force Can be determ ned by archimedes principles which states that when a b ody is partially or wholly immersed in water, it is buoy or lifted up by a Force which is equal to the weight water displaced.

## Types of Stability of a Floating body

1) Stable equilibrium – When a body receives a small angular d splacement due to external Forces, it returns back to its orig inal position due to Some Internal Forces (gravity & buoyancy)

2) Unstable equilibrium – Here, the body heels Farther away & never returns to its original position when given a small ang lar displacement

3) Neutral Equilibrium – Here, the body occupies a new positio & remains at rest when given a small angular displaceme nt.

## Floatation and stability

Loss of weight is experienced when a body is partially or wholly Immersed in a Fluid. The loss of weight is due to Force acting opposite the action of gravity. This phenomenon is known as buoyancy, and the force is Known as buoyant force. The magnitude of buoyant Force Can be determined by archimedes principles which states that when a body is partially or wholly immersed in water, it is buoyed or lifted up by a Force which is equal to the weight of water displaced.

### Types of Stability of a Floating body

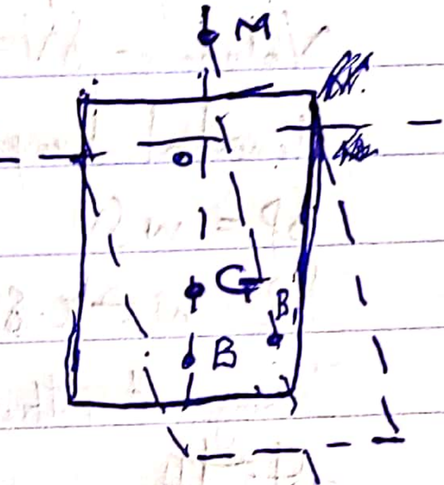
1) Stable equilibrum - When a body receives a small angular displacement due to exteternal forces, it returns back to its original position due to Some Internal Forces (gravity & buoyancy)

2) Unstable equilibrum - Here, the body heels farther away & never returns to its original position when given a small angular displacement

3) Neutral Equilibrum - Here, the body occupies a new position & remains at rest when given a small angular displacement.

## Meta-Center and Meta-Centric height

When a body is given a Small angular displacement, it begins to oscillate. The pt abt which the body oscillates is Known as the meta-Center.




Meta-center is practically defined as the intersection btw a line or an axis passing through the Centre of gravity & the original Centre of buoyancy, ~~when the body is in tilt rather position.~~ and the Centre of buoyancy of the new position when the body is in tilt position.

Centre of buoyany is the pt of application of the buoyant Force.

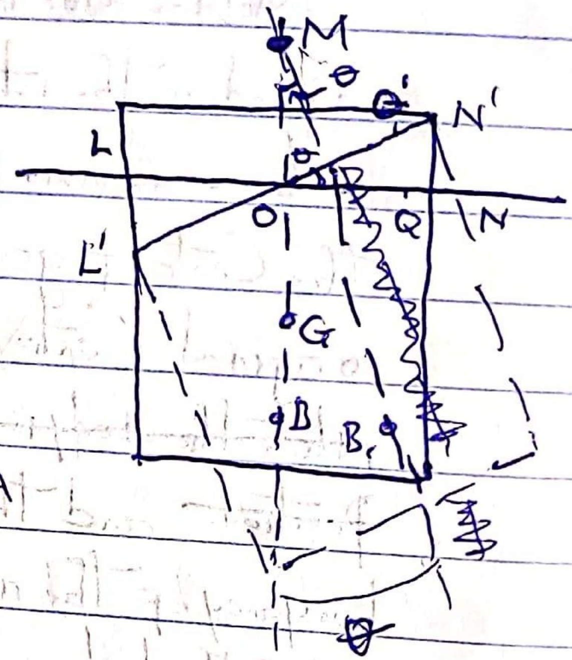
Meta-Centre height is the distance btw the meta-Centre & the Centre of gravity (GM)

For stable equlibrum, the meta-Center M is above the Centre of gravity. While for unstable equlibrum, the meta-Centre is ~~above~~ below. For neutral, the meta-Center Coincides with the Centre of gravity.

# Determination of Meta-Centric height

Let's Consider an elemental prism making an angle $\theta$ with the origin O at a distance $x$ from the origin. ~~Heig~~ ~~of~~ Height of the prism = $x\theta$

Volume = $\delta V = \theta x \cdot \delta A$




Elemental buoyant Forces

$\delta P = w\,\delta V$

$\delta P_B = w\,\theta x \cdot \delta A$

Taking mmt of the buoyant Force abt the origin:

$\int x \cdot \delta P_B = \int w\theta x^2 \delta A = w\theta \int x^2 \delta A$

But $I = x^2 \delta A$

$\therefore x \cdot \delta P_B = w\theta I$

$x \cdot \delta P_B$ = Total change in mmt & is given as:

$x \cdot \delta P_B = P_B \times BB_1$

Note That:

$BB_1 = BM \times \theta$

But $P_B = wV$

$\therefore x \cdot \delta P_B = w \cdot V \times BM\theta = w\theta I = w \cdot V \times BM \cdot \theta$

$\therefore BM = I/V$

Knowing that $GM = BM \pm BG$

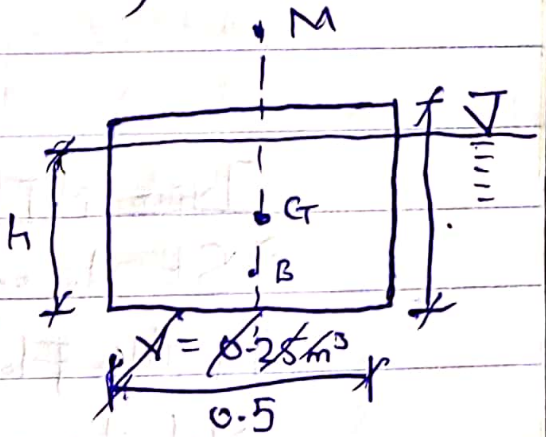
Note - Use -ve when G is above B & use +ve when G is below B

## Example 2

The specific gravity of a floating block of wood in water is 0.75. Find the meta-centric height of the body if the volume is $1 \times 0.5 \times 0.4$.

Soln

$\cancel{BM = I/V}$




$\cancel{I} = \cancel{bd^3/12} = \cancel{\dfrac{0.5 \times 0.4^3}{12}} =$

$\cancel{BM} \cancel{=} \cancel{I}$

## FLUID DYNAMICS: objectives

- Introduce concepts necessary for the analysis of fluid in motion.
- Identify the difference btw steady/non-steady, uniform/non-uniform flows & compressible/incompressible flow
- Demonstrate streamlines & streamtubes.
- Introduce the continuity principle through the conservation of mass & control volume.
- Derive bernoulli equation or energy equation
- To demonstrate the practical uses of bernoulli's eqn & continuit

...ction

y eqn in the analysis of Flow.

- Derive momentum eqn For a Fluid.
- Demonstrate how momentum eqn & the principles of Conservation of momentum are used to predict the Forces induced by a Flowing Fluid.

Types of Flow

1) Steady or Unsteady Flow; A Flow is said to be stead... if the Flow Characteristics, ~~are such that~~ Such as Velocity, acceleration, density, etc, do not Change with time. It Can be expressed mathematically as;

$$\frac{\partial v}{\partial t}=0,\ \frac{\partial a}{\partial t}=0,\ \frac{\partial \rho}{\partial t}=0,$$

Whereas Unsteady Flow is one whose Flow Characteristics Change with time. It can be expressed mathematically as;

$$\partial v/\partial t \neq 0,\ \partial a/\partial t \neq 0,\ \partial \rho/\partial t \neq 0$$

2) Uniform/non-uniform – A Flow is said to be uniform if the Flow characteristics do not Change with distance. Mathematically, $\frac{\partial v}{\partial s}=0,\ \frac{\partial a}{\partial s}=0,\ \frac{\partial \rho}{\partial s}=0$

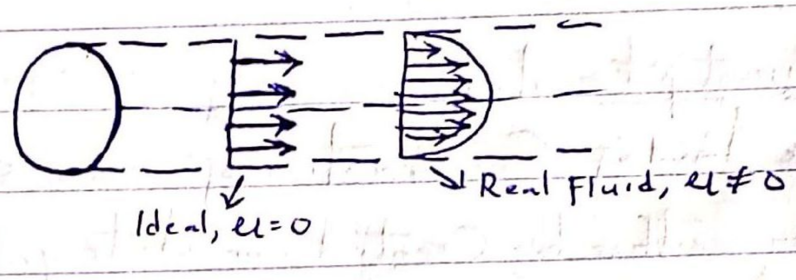
Whereas, in non-uniform Flow, the Flow characteristics change with distance. i.e $\partial v/\partial s \neq 0$, $\partial a/\partial s \neq 0$, $\partial \rho/\partial s \neq 0$

3) <u>Compressible/incompressible</u> - A Fluid is said to be Compressible, if the density Changes under pressure. It Can be expressed mathematically as: $\partial \rho/\partial p \neq 0$

Whereas for the Incompressible, the density doesn't change with pressure. Mathematically, $\partial \rho/\partial p = 0$

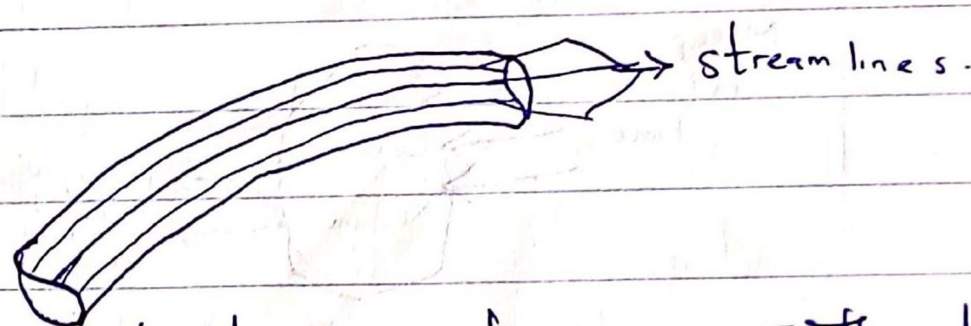
Fluid & Flow are used Interchangeably.

4) <u>Ideal & real Fluid/Flow</u> -




<u>Stream lines & Stream tubes</u>

Streamlines are lines joining pts of equal Velocity (Velocity Contour)

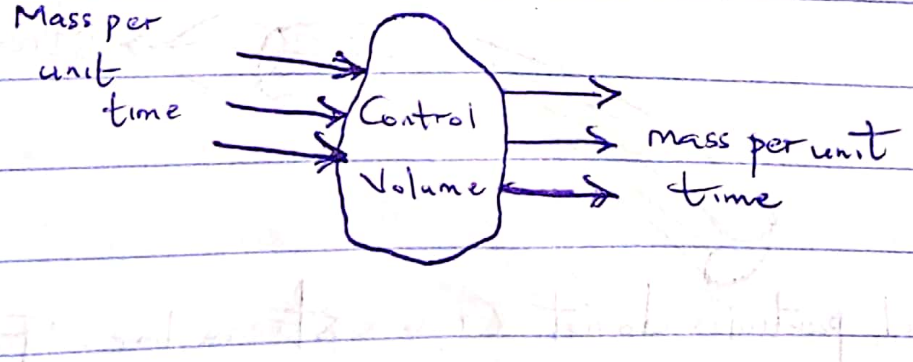



Fluid particles do not Cross a stream line. If they do, then e's has been a change in velocity. Also streamlines don't cross each other. But if they do, the implication is that fluid particles at that pt they meet, they will have different velocities. This means that a fluid particle enter a streamline & stays in the stream line. It is also important to note that in the case of steady flow, streamlines change.

A **streamtube** is an imaginary tubular (tube-like) surface formed by streamlines along the axis of flow. Streamtubes are used to visualize the pattern of flow.

## Continuity Equation

The law of conservation of mass states that matter can neither be created nor destroyed, it only changes from one form to another. This principle can be applied in the analysis of control volume. It can also be applied on the analysis of streamtubes.



## Types of Flow rate

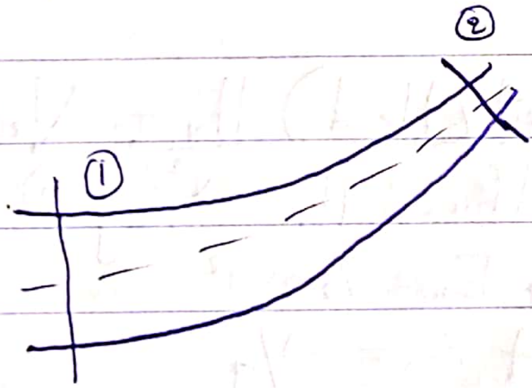
) <u>Mass Flow rate</u> - the mass per unit time. It Can also be defined as the quantity of matter Passing through a section in a unit time. It Can be ~~measured~~ denoted by;

$Q_m = m/t$.

) <u>Volume Flow rate</u> - It is the Volume per unit time. It Can also be Called <u>Flow rate</u>, & is denoted by Q. Mathematically, it has a relationship with the mass Flow rate i.e. $Q = V/t$, but $V = m/\rho$

$\therefore Q = m/\rho \times t = Q_m/\rho$ or $Q_m = Q \cdot \rho$

<u>Continuity Eqn</u>; Anything that enters (during a Flow) will Eventually Come out.




$Q_{m_1} = \rho u_1 A_1$

$\therefore Q = V/t = \frac{A \cdot L}{t} = Au$

$Q_{m_2} = \rho u_2 A_2$

For an Incompressible Fluid; $\rho_1 = \rho_2 = \rho = U_1A_1 = U_2A_2$

<u>Energy Possessed by a Flowing Fluid</u>

Types are;

① Potential – due to height
② Kinetic – due to Velocity
③ Pressure – due to work done by pressure.

① <u>Potential (or potential head)</u> – head is energy per unit weight

Potential head, $H_p = Z (m)$, where $z$ is the datum.

Z is in metres.

Proof; From $E_p = mgh$ or $mgZ$

But, $mg = W$

$E_p = WZ$ $\therefore H_p = E_p/W = Z$ ---- ①

2) <u>~~Kinetic~~ Kinetic (Velocity Head), $H_v$</u> – Velocity head is denot & expressed as thus; $H_v = V^2/2g$ (m)

Proof; From $E_K = \frac{1}{2}mv^2$

But $m = W/g$

$\therefore E_K = WV^2/2g$

$H_K = E_K/W = V^2/2g$ ---- ⑪

3) <u>Pressure (pressure head)</u> – Also Known as Pressure energy is

witness

---

denoted & expressed as; $H_{pr} = P/w$

where $w$ = specific weight.

Proof;

$$FL = PAL = PV$$

But $W = \omega \cdot V$ $\therefore$ $V = W/\omega$

$$FL = P\left(\frac{W}{\omega}\right) = PW/\omega$$

$$E_{pr} = PW/\omega \quad \#$$

$$\therefore \quad E_{pr}/W = P/\omega \ (m) \quad - - (111)$$

## Bernoulli's Equation

It states that for an ideal incompressible fluid, when the flow is steady & continuous, the sum of pressure head, velocity & potential head is constant along a stream line. Mathematically;

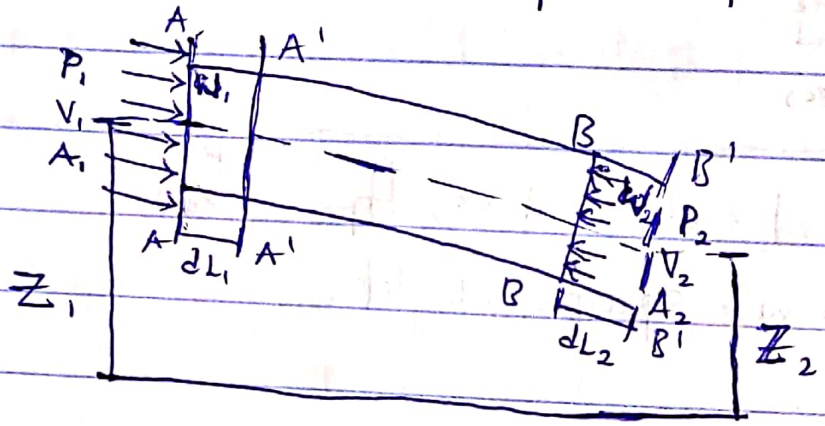
$$P/\omega + V^2/2g + Z = \text{Constant}$$

For real fluid; $\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2 + H_L$

where $H_L$ is loss of head, since something must be lost during

Flow, ~~$H_L$~~ ~~thus~~ becomes what was lost.

Consider an ideal incompressible Flow as shown below;




The item in the Figure is tapering.

Let $P_1$ = Pressure at A-A

$V_1$ = Velocity at ,,

$A_1$ = X-sectional area at A-A

$Z_1$ = ~~datum~~ height of the X-section from the datum

$P_2 =$
$V_2 =$
$A_1 =$
$Z_1 =$
} Corresponding Values at B-B

~~Note- The~~

$\therefore \; W_1 = W_2 = W$

But $W = \omega\cancel{V} = \omega A L$

$\therefore \; \cancel{V/L = \omega A} \;\; W/\omega = AL$

$W_1 = \omega A_1 L_1 \;,\; W_2 = \omega A_2 L_2$

eqn then ~~...~~ we obtain; $\cancel{w} A_1 L_1 = \cancel{w} A_2 L_2$

Workdone by pressure at A-A, $W_{p_1} = P_1 A_1 \times dL_1$

$W_{p_2} = - P_2 A_2 dL_2$ (workdone by pressure at B-B)

Total workdone by pressure, $W_{TP}$ is thus given by;

$W_{TP} = P_1 A_1 dL_1 - P_2 A_2 dL_2$

Since $V_1 = V_2$

$\therefore P_1 A_1 dL_1 - P_2 A_1 dL_1$

$A_1 dL_1 (P_1 - P_2)$

$\therefore W/\omega (P_1 - P_2)$

Loss of potential ; $W(Z_1 - Z_2)$

Gain in Kinetic Energy; $W\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$

Loss of potential head + Workdone by pressure =
~~Kinetic~~ Gain in Kinetic or Velocity head

i.e $W(Z_1 - Z_2) + \frac{W(P_1 - P_2)}{\omega} = W\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$

dividing through by W & re-arranging;

$$Z_1 + \frac{P_1}{\omega} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\omega} + \frac{V_2^2}{2g}$$

Assignment
~~Example~~ - Derive Euler's eqn from the first principle & use it to prove bernoulli's eqn.

## Momentum eqn

Momentum eqn which is based on the law of Conservation of mass, states that the net Force acting on a body of Fluid = Change in momentum per unit time in the direction of the Force. This eqn is based on newton's Second law of motion, which relates the Sum of Forces acting on a Fluid element to acceleration or rate of change in momentum.

As per Newton's law, $F = Ma$, where;

$F$ = Force or Magnitude of Force on a Fluid element.

$M$ = mass of Fluid element

$a$ = acceleration in the direction of Force.

But $a = dV/dt$

So that $F = m\,dv/dt$ or $F = d(Mv)/dt$

The last eqn above is known as the momentum principle eqn and it Can also be written as $Fdt = d(mv)$

This eqn is Known as the Impulse momentum eqn, which states that the Impulse of a Force, $F$ acting on a Fluid mass, $m$ in a short time Interval $dt$ is equal to the Change in momentum, $d(mv)$

## Applicatns of Momentum Eqn

1) It is used to determine forces due to change in flow direction, magnitude, both. Such problems can be found in

i) pipe bend

ii) Reducers.

iii) Moving Vane

iv) Jet propulsion

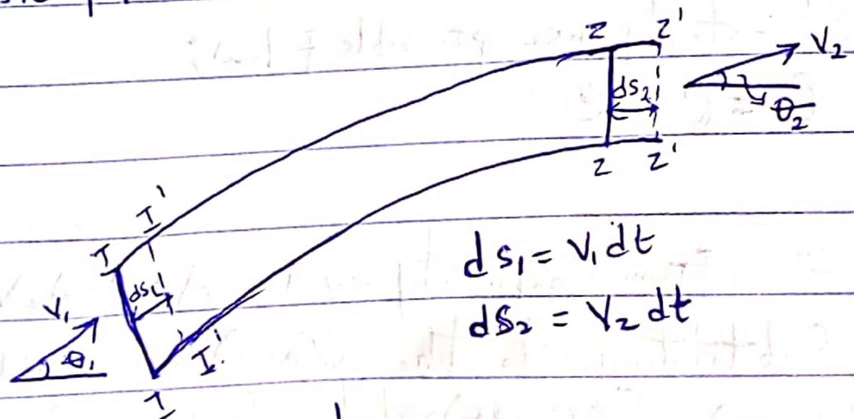
2) It's used to determine change in flow characteristics due to abrupt change in flow section. Such problems can be seen in;

i) Sudden enlargement in pipes

ii) Hydraulic Jump in open ~~chambers~~ Channels.

## Derivation of momentum or Impulse momentum equation

Consider a ~~horizo~~ stream tube carrying steady incompressible flow as shown below




$ds_1 = V_1 dt$

$dS_2 = V_2 dt$

Let $V_1$ = Velocity at entrance.

$\rho_1$ = Fluid density at entrance
$V_2$ = Velocity at exit
$\rho_2$ = Fluid density at exit
Mass at Section I--I and I'--I' =
Mass at Section Z--Z and Z'--Z'
The Implication of the above mass relationship is that ; $\rho_1 A_1 dS_1 = \rho_2 A_2 dS_2$

Momentum at entrance = $(\rho_1 A_1 dS_1) \times V_1$
But $dS_1 = V_1 dt$
$\therefore$ Momentum, $MV_I = (\rho_1 A_1 V_1 dt) \times V_1$

Similarly, $MV_Z = (\rho_2 A_2 V_2 dt) \times V_2$
$\therefore \; d(MV) = M_Z V_Z - M_I V_I$
$= [(\rho_2 A_2 V_2 dt) \times V_2] - [(\rho_1 A_1 V_1 dt) \times V_1]$

For steady incompressible Flow;
$\rho_1 = \rho_2 = \rho$

Also, From Continuity eqn, $A_1 V_1 = A_2 V_2 = Q$
Substituting into the d(MV) eqn, We obtain;
$d(MV) = [(\rho_2 Q\, dt) \times V_2] - [(\rho_1 Q\, dt) \times V_1]$

$\rho Q (V_2 - V_1) dt$

From the Impulse momentum eqn, $F dt = d(mv)$

$F dt = \rho Q (V_2 - V_1) \times dt$

Also, $\rho = w/g$

$F = \frac{wQ}{g}(V_2 - V_1)$ -- This is the Fluid Version of Newton's second law of motion.

Horizontal Cpt of the Velocities;

$V_1 \cos\theta_1$ and $V_2 \cos\theta_2$

Vertical Cpt of the Velocities;

$V_1 \sin\theta_1$ and $V_2 \sin\theta_2$

Horizontal cpt of the Force thus becomes;

$$F_x = \frac{WQ}{g}(V_2 \cos\theta_2 - V_1 \cos\theta_1)$$

Vertical Cpt of the Force;

$$F_y = \frac{WQ}{g}(V_2 \sin\theta_2 - V_1 \sin\theta_1)$$

Force required to hold pipe or Conduit in place thus becomes;

$$F_x = \frac{WQ}{g}(V_1 \cos\theta_1 - V_2 \cos\theta_2)$$

$$F_y = \frac{WQ}{g}(V_1 \sin\theta_1 - V_2 \sin\theta_2)$$

Supplementing the above Forces with hydrostatic Forces; $P_1A_1$ and $P_2A_2$; so that

$$\cancel{P_1A_1}/ax \quad F_x = \frac{WQ}{g}(V_1\cos\theta_1 - V_2\cos\theta_2) + P_1A_1\cos\theta_1 - P_2A_2\cos\theta_2$$

$$F_y = \frac{WQ}{g}(V_1\sin\theta_1 - V_2\sin\theta_2) + P_2A_2\sin\theta_2 - P_1A_1\sin\theta_1$$

The resultant Force on the bend ~~thus~~ thus becomes;

$$F_R = \sqrt{F_x^2 + F_y^2}$$

~~Th~~ it's direction thus becomes;

$$\alpha = \tan^{-1}(F_y/F_x)$$

## Example

In a 45° bend, a horizontal air duct of X-section $1m^2$ reduces to $0.5m^2$. Find the magnitude & the direction of the Force needed to hold the air duct in place. If the Vel and pressure at the $1m^2$ Section are 10m/s and $30kN/m^2$. Assume the specific weight of air to be $0.0116\ kN/m^3$

## Solution

At Section 1; $V_1 = 10m/s$, $P_1 = 30kN/m^2$, $\theta_1 = 0$

$w = 0.0116\ kN/m^3$

At Section 2 ; $A_2 = 0.5m^2$, $\theta_2 = 45^\circ$, $V_2 = ?$, $P_2 = ?$

From Continuity eqn ; $Q = A_1V_1 = A_2V_2$

$1 \times 10 = 0.5 V_2$

$V_2 = 10/0.5 = 20 m/s$

By applying bernoulli's eqn ; $P_1/w + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$

Since the pipe is horizontal, therefore $Z_1 = Z_2$

$P_1/w + \frac{V_1^2}{2g} = P_2/w + V_2^2/2g$

$\frac{30 \times 10^3}{0.0116 \times 10^3} + \frac{10^2}{2 \times 9.81} = \frac{P_2}{0.0116 \times 10^3} + \frac{20^2}{2 \times 9.81}$

$2586.21 + 5.0968 = 0.086 P_2 + 20.387$

$0.086 P_2 = 2570.92$

$P_2 = 29894.4 N/m^2 \simeq 29.9 kN/m^2$

Now, $F_x = \frac{wQ}{g}(V_1 \cos\theta_1 - V_2 \cos\theta_2) + P_1 A_1 \cos\theta_1 - P_2 A_2 \cos\theta_2$

$F_x = \frac{0.0116 \times 10^3 (10)}{9.81}\left[10 \cos 0 - 20 \cos 45\right] + 30(10^3) \times 1 \cos 0 - 29.9(10^3) \cos 45$

$F_y = \frac{wQ}{g}(V_1 \sin\theta_1 - V_2 \sin\theta_2) + P_1 A_1 \sin\theta_1 - P_2 A_2 \sin\theta_2$

$\therefore F_x = 19.46 kN$

$F_y = \frac{0.0116 \times 10^3 (10)}{9.81}\left[10 \sin 0 - 20 \sin 45\right] + 30(10^3) \times 1 \times \sin 0 - 29.9(10^3) \sin 45$

$= -10.7072 kN$

$F_y = -10.7072\,kN$

$F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{19.41^2 + (-10.7072)^2}$

$= \sqrt{491.392}$

$= 22.167\,kN$

$$\alpha = \tan^{-1}\left(\frac{-10.7072}{19.41}\right) = -28.89^\circ$$

Scanned with CamScanner