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RESEARCH PAPER

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Continuum modeling for two-lane traffic flow*

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Abstract In this paper, we study the continuum modeling of traffic dynamics for two-lane freeways. A new dynamics model is proposed, which contains the speed gradient-based momentum equations derived from a car-following theory suited to two-lane traffic flow. The conditions for securing the linear stability of the new model are presented. Numerical tests are carried out and some nonequilibrium phenomena are observed, such as small disturbance instability, stop-and-go waves, local clusters and phase transition.

Keywords Traffic dynamics · Two-lane traffic flow · Car-following theory · Momentum equation

1 Introduction

Lighthill and Whitham [1] and Richards [2], independently proposed the first-order continuum traffic models, called LWR theory in transportation science. According to this theory, traffic flows on a long homogeneous freeway are governed by the following conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = s(x, t), \tag{1}$$

where ρ is the traffic density, q is the flow rate, $s(x, t)$ is the generation rate, while x and t represent space and time, respectively. If the freeway has no on-off ramp, then $s(x, t) = 0$, otherwise $s(x, t) \neq 0$. Let v be the space mean speed, a flow-density relationship exists, i.e.,

$$q = \rho v. \tag{2}$$

The LWR theory further assumes that there exists an equilibrium speed-density relationship as follows:

$$v = v_e(\rho). \tag{3}$$

Combining the above three equations, we can obtain analytical solutions for various simple traffic flow problems by the method of characteristics, and reveal shock waves and traffic jams from these solutions [3].

However, the LWR model does not faithfully describe the non-equilibrium traffic flow dynamics since the speed is assumed to be always determined by the equilibrium speed-density Eq. (3) so that no speed fluctuation around the equilibrium state is permitted. For this reason many scholars have developed high-order continuum traffic flow models that incorporate a dynamics equation or momentum equation to represent general car-following behavior. This momentum equation takes the acceleration and inertia of driving into account. Hence, the high-order models make up for some deficiencies in the simple continuum models and improve the ability in reproducing complex traffic behavior. Payne [4] employed the car-following theory to derive the following momentum equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e(\rho) - v}{\tau} - \frac{\nu}{\rho\tau}\frac{\partial \rho}{\partial x}, \tag{4}$$

where $\nu = -0.5\partial v_e(\rho)/\partial\rho$ is the anticipation coefficient and τ is the relaxation time. Payne's model can be used to analyze such phenomena as broken equilibrium, stop-and-go and phase transition. After Payne's work, a variety of momentum equations have been developed, e.g., Kühne [5], Ross [6], Papaeorgiou et al. [7], Michaloupolos et al. [8]. Recently, Zhang [9] presented a momentum equation as follows:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e(\rho) - v}{\tau} - \rho(v_e'(\rho))^2\frac{\partial \rho}{\partial x}. \tag{5}$$

It is proposed in Zhang's model that traffic phase transitions in the presence of a strong disturbance undergo three stages,

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namely anticipation-dominant stage, balanced anticipation-relaxation stage and relaxation-dominant stage, and Eq. (5) is applied to the second stage. However, the existing high-order models have a fundamental flaw that one of the characteristic speeds resulted from the momentum and conservation equations is lager than the macroscopic flow velocity. This implies that vehicles would affect the vehicles in front of them [10].

Based on the full velocity difference (FVD) car-following model, Jiang et al. [10,11] proposed a new dynamic equation in which the density gradient term is replaced by a speed gradient term. Recently, Jiang and Wu [12] further analyzed the structural properties of the solutions to the speed gradient traffic flow model. They showed that the model's characteristic speeds are not lager than the macroscopic flow velocity, hence vehicles respond only to their frontal stimulus. The momentum equation is given below:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e(\rho) - v}{\tau} + c_0\frac{\partial v}{\partial x}, \tag{6}$$

where $c_0 = \Delta/t_\Delta$ represents the propagation speed of disturbance, Δ is the distance between the following car and the leading car, and t_Δ is the time needed for the backward propagated disturbance to travel a distance of Δ. Xue and Dai [13] improved Eq. (6) by treating c_0 as a function of the density.

In the above models, the following car is not allowed to overtake the leading car. Hence, these models are applicable only to the traffic flow on freeways with single lane. Daganzo [14] developed a continuum theory of traffic dynamics for freeways with two lanes allocated for two sorts of vehicles (lane 1 for faster vehicles and lane 2 for slower vehicles). The theory defines the conservation equation as

$$\frac{\partial P}{\partial t} + \frac{\partial Q}{\partial x} = 0, \tag{7}$$

where $P = (\rho_1, \rho_2)$, $Q = (q_1(\rho_1, \rho_2), q_2(\rho_1, \rho_2))$, ρ_i and q_i represent the traffic density and the flow rate on lane i $(= 1, 2)$, respectively. Daganzo first studied the possible relationships between the velocities and the densities based on the density ratio of the two lanes, then formulated the traffic flow models for each of totally four $\rho_1 - \rho_2$ regions. For carrying out analyses graphically, Daganzo adopted simple continuum model only; hence his approach cannot be used to describe the non-equilibrium traffic flow dynamics. Wu [15] proposed a multi-lane traffic model, considering the specific traffic conditions in Chinese cities (e.g., untidy flows and lower speeds on roads). But Wu's model tends to equalize all lanes' densities through averaging them. This conflicts with the real traffic since the density on the lane allocated for faster vehicles is generally less than that on the lane allocated for slower vehicles. Tang and Huang [16] lately developed a continuum model for freeways with two lanes which applies the momentum equation proposed by Jiang et al. [10, 11] into the Daganzo's modeling framework for multi-lane traffic flow [14]. However, this model allows only the faster vehicles to change lanes.

In this paper, we further extend the work of Tang and Huang [16], allowing that vehicles on both lanes to change their lanes according to the traffic condition on the spot and at that time. In Section 2, we formulate the new model, starting from a car-following theory modified to suit the two-lane traffic flow. In Section 3, we analyze the linear stability of the new model and give the conditions for securing the stability. Numerical results are presented in Section 4, where some nonequilibrium phenomena are investigated, such as small disturbance instability, stop-and-go waves, local clusters and phase transition. Section 5 concludes the paper.

2 Model

The classical car-following model by Gazis et al. [17] can be used to model the motion of vehicles following each other on a single lane without overtaking. In this model, the distance between successive vehicles can be arbitrarily close, which is unrealistic. Bando et al. [18] proposed a car-following model called the optimal velocity model (OVM) that considers the safe velocity determined by the distance from the leading vehicle. Helbing and Tilch [19] improved the OVM and developed a generalized force model. Jiang et al. [20] recently proposed a full velocity difference model for considering the effects of both the distance and the relative speed of two successive vehicles. Theoretically, Jiang et al.'s model is more realistic and accurate than previous ones. The list of contributions on car-following theory is now a long one and still becoming larger. However, these models are only for the single lane traffic. In the case of two-lane traffic, the interactions between vehicles on two lanes must be taken into account. As shown in Fig. 1, the motion of vehicles on lane 1 is affected not only by the distance and the relative speed of two successive vehicles on lane 1 but also by the location of vehicles on lane 2.

Considering the above reasons, we adopt the modified full velocity difference model to describe the motions of vehicles on both lanes, i.e.,

$$\frac{\mathrm{d}u_{n_i+1}(t)}{\mathrm{d}t} = k_i[u_i(\Delta x_i, \Delta y_i) - u_{n_i+1}(t)] + \lambda_i \Delta u_i,\ i = 1, 2, \tag{8}$$

where $\Delta x_i = x_{n_i} - x_{n_i+1}$, $\Delta u_i = u_{n_i} - u_{n_i+1}$, with x_{n_i} (x_{n_i+1}) being the position of the leading (following) cars on lane i, u_{n_i} (u_{n_i+1}) being the speed of the leading (following) cars on lane i, Δy_1 being the distance between the car $n_1 + 1$ and the car $n_2 + 1$, Δy_2 being the distance between the car n_2+1 and the car n_1, k_i being the reaction coefficient, and λ_i being the sensitivity coefficient [20].

Using the method by Liu et al. [21], we can transfer the microscopic variables to the macroscopic ones, as follows:

$$u_{n_i+1}(t) \to v_i(x, t), \qquad u_{n_i}(t) \to v_i(x + \Delta_i, t),$$
$$u_i(\Delta x_i, \Delta y_i) \to v_{ie}(\rho_1, \rho_2), \quad k_i \to 1/\tau_i, \quad \lambda_i \to 1/\sigma_i,$$

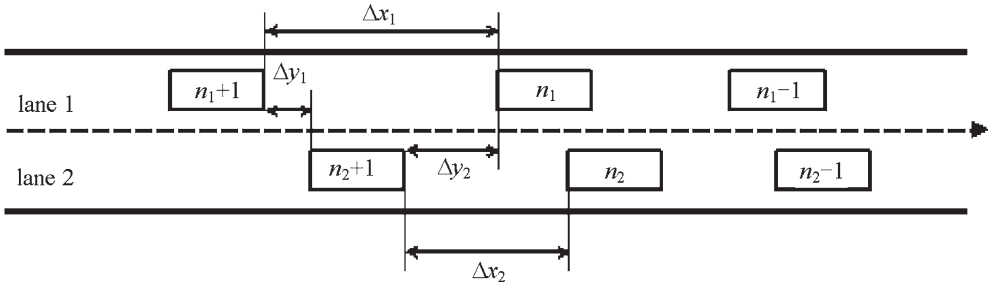

Fig. 1 Sketch of a two-lane traffic

where $i = 1, 2$, τ_i is the relaxation time for vehicles traveling on lane i, σ_i is the time needed for the backward propagated disturbance to travel distances Δ_i on lane i. Applying the above continuous variables to Eq. (8), we have

$$\frac{\mathrm{d}v_i(x,t)}{\mathrm{d}t} = \frac{v_{ie}(\rho_1,\rho_2) - v_i(x,t)}{\tau_i} + \frac{v_i(x+\Delta_i,t) - v_i(x,t)}{\sigma_i}, \quad i = 1, 2. \tag{9}$$

Expanding the right-hand side of Eq. (9), and neglecting high-order terms, we obtain

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = \frac{v_{ie} - v_i}{\tau_i} + \frac{\Delta_i}{\sigma_i}\frac{\partial v_i}{\partial x}, \quad i = 1, 2. \tag{10}$$

The dynamics model for two-lane traffic consists of the following equations:

$$\frac{\partial \rho_i}{\partial t} + v_i\frac{\partial \rho_i}{\partial x} + \rho_i\frac{\partial v_i}{\partial x} = s_i(x,t), \quad i = 1, 2, \tag{11}$$

$$\frac{\partial v_i}{\partial t} + (v_i - c_{i0})\frac{\partial v_i}{\partial x} = \frac{v_{ie} - v_i}{\tau_i}, \quad i = 1, 2, \tag{12}$$

$$s_1(x,t) + s_2(x,t) = 0, \tag{13}$$

where $c_{i0} = \Delta_i/\sigma_i$ represents the propagation speed of the disturbance on lane i. Equation (13) states that there is no on-off ramp in the freeway. But, it does not mean $s_1(x,t) = 0$ and $s_2(x,t) = 0$ must hold for all times and spaces, because lane changes are allowed in this study. Note that the model given by Eqs. (11) and (12) for a specific lane is formally the same as the continuum model proposed by Jiang et al. [10, 11]. However, attention should be paid to such a difference that all velocities in Eqs.(11) and (12) are functions of ρ_1 and ρ_2.

In addition, if we apply matrixes to analyze character speeds of the system (11)–(12), we find that it can be divided into two independent subsystems whose character speeds are respectively $\lambda_{11} = v_1$, $\lambda_{12} = v_1 - c_{10}$ and $\lambda_{21} = v_2$, $\lambda_{22} = v_2 - c_{20}$. From the above-mentioned, it is found that the character speeds of each subsystem are not larger than their macroscopic flow speeds, which shows that on each lane the rear disturbances cannot propagate forward, but it does not guarantee that in a system consisting of two lanes the rear disturbance will not affect the driving behavior of the leading vehicle because lane changing is allowed in these two-lane traffic system.

3 Linear stability

It is difficult to obtain an analytical result about the stability of two-lane traffic flow when lane changing is allowed. We here assume $s_i(x,t) = 0$ $(i = 1, 2)$ for simplicity. For lane $i (= 1, 2)$, let ρ_{i*} and v_{i*} be the steady-state solutions of the model (11)–(13), $\rho_i = \rho_{i*} + \xi_i$ and $v_i = v_{i*} + \eta_i$ be the perturbed solutions, where $\xi_i = \xi_i(x,t)$ and $\eta_i = \eta_i(x,t)$ represent small smooth perturbations to the steady-state solutions. Substituting $\rho_i = \rho_{i*} + \xi_i$ and $v_i = v_{i*} + \eta_i$ into Eqs. (11) and (12), taking the Taylor series expansions at the point (ρ_{i*}, v_{i*}) and neglecting the higher order terms of ξ_i and η_i, we obtain the following equations:

$$\frac{\partial \xi_i}{\partial t} + v_{i*}\frac{\partial \xi_i}{\partial x} + \rho_{i*}\frac{\partial \eta_i}{\partial x} = 0, \quad i = 1, 2, \tag{14}$$

$$\frac{\partial \eta_1}{\partial t} + (v_{1*} - c_{10})\frac{\partial \eta_1}{\partial x} = \frac{a\xi_1 + b\xi_2 - \eta_1}{\tau_1}, \tag{15}$$

$$\frac{\partial \eta_2}{\partial t} + (v_{2*} - c_{20})\frac{\partial \eta_2}{\partial x} = \frac{c\xi_1 + d\xi_2 - \eta_2}{\tau_2}, \tag{16}$$

where $a = \partial v_{1e}(\rho_{1*},\rho_{2*})/\partial\rho_1$, $b = \partial v_{1e}(\rho_{1*},\rho_{2*})/\partial\rho_2$, $c = \partial v_{2e}(\rho_{1*},\rho_{2*})/\partial\rho_1$, $d = \partial v_{2e}(\rho_{1*},\rho_{2*})/\partial\rho_2$. Eliminating ξ_1 and ξ_2 from Eqs. (14)–(16), we obtain

$$(\partial_t + c_1\partial_x)\eta_1 = -\tau_1[(\partial_t + c_{11}\partial_x)(\partial_t + c_{12}\partial_x)]\eta_1, \tag{17}$$

$$(\partial_t + c_2\partial_x)\eta_2 = -\tau_2[(\partial_t + c_{21}\partial_x)(\partial_t + c_{22}\partial_x)]\eta_2, \tag{18}$$

where $c_1 = v_{1*} + \rho_{1*}(ad - bc)/d$, $c_2 = v_{2*} + \rho_{2*}(ad - bc)/a$, $c_{11} = v_{1*} - c_{10}$, $c_{12} = v_{1*}$, $c_{21} = v_{2*} - c_{20}$, $c_{22} = v_{2*}$. Using the method by Jiang et al. [10,11], we can prove that model (11)-(13) is linearly stable if and only if c_1, c_2, c_{11}, c_{12}, c_{21} and c_{22} satisfy the following conditions:

$$c_{11} \le c_1 \le c_{12} \quad \text{and } c_{21} \le c_2 \le c_{22}. \tag{19}$$

Otherwise, traffic instability will occur, which leads to some complex traffic phenomena like stop-and-go traffic. Equation (19) shows that one lane's stability is affected by its lateral lane's traffic density although no lane changing appears.

4 Numerical tests

In this section, using the finite difference method, we numerically solve the model to demonstrate the model's ability of capturing complex traffic phenomena. Let index j represent the time interval and index k the road section, then the difference equations corresponding to Eqs. (11)–(12) are as follows:

$$\rho_{ik}^{j+1} = \rho_{ik}^{j} + \frac{\Delta t}{\Delta x} v_{ik}^{j}\left(\rho_{i(k-1)}^{j} - \rho_{ik}^{j}\right) + \rho_{ik}^{j}\frac{\Delta t}{\Delta x}\left(v_{ik}^{j} - v_{i(k+1)}^{j}\right) + \Delta t s_{ik}^{j}, \quad i = 1, 2. \tag{20}$$

If the traffic is heavy, i.e. $v_{ik}^{j} < c_{i0}$, then

$$v_{ik}^{j+1} = v_{ik}^{j} + \frac{\Delta t}{\Delta x}\left(c_{i0} - v_{ik}^{j}\right)\left(v_{i(k+1)}^{j} - v_{ik}^{j}\right) + \frac{\Delta t}{\tau_i}\left(v_{ie} - v_{ik}^{j}\right), \quad i = 1, 2, \tag{21}$$

otherwise

$$v_{ik}^{j+1} = v_{ik}^{j} + \frac{\Delta t}{\Delta x}\left(c_{i0} - v_{ik}^{j}\right)\left(v_{ik}^{j} - v_{i(k-1)}^{j}\right) + \frac{\Delta t}{\tau_i}\left(v_{ie} - v_{ik}^{j}\right), \quad i = 1, 2. \tag{22}$$

In order to investigate the consequences caused by small localized disturbance in an initial homogenous condition, we adopt the following initial variations of the average density ρ_{i0} ($i = 1, 2$) which had been used by Herrmann and Kerner [22]:

$$\rho_i(x, 0) = \rho_{i0} + \Delta\rho_{i0}\left\{\cosh^{-2}\left[\frac{160}{L}\left(x - \frac{5L}{16}\right)\right] - \frac{1}{4}\cosh^{-2}\left[\frac{40}{L}\left(x - \frac{11L}{32}\right)\right]\right\}, \quad i = 1, 2, \tag{23}$$

where $L = 32.2$ km is the length of the road section under consideration. The following periodic boundary conditions are adopted:

$$\rho_i(L, t) = \rho_i(0, t), \quad v_i(L, t) = v_i(0, t), \quad i = 1, 2. \tag{24}$$

The equilibrium speed-density relationships are as follows:

$$v_{1e} = v_{10}\left\{\left(1 + \exp\left[\frac{(\rho_1 + 0.1\rho_2)/(\rho_{1j} + 0.1\rho_{2j}) - 0.25}{0.06}\right]\right)^{-1} - 3.72 \times 10^{-6}\right\}, \tag{25}$$

$$v_{2e} = v_{20}\left\{\left(1 + \exp\left[\frac{(0.1\rho_1 + \rho_2)/(0.1\rho_{1j} + \rho_{2j}) - 0.25}{0.06}\right]\right)^{-1} - 3.72 \times 10^{-6}\right\}, \tag{26}$$

where ρ_{ij} is the jam densities of lane i. Assume the initial flows to be in local equilibrium everywhere, i.e., $v_1(x, 0) = v_{1e}(\rho_1(x, 0), \rho_2(x, 0))$ and $v_2(x, 0) = v_{2e}(\rho_1(x, 0), \rho_2(x, 0))$. Other parameters are as follows: $\Delta\rho_{10} = 0.008$ veh/m, $\Delta\rho_{20} = 0.01$ veh/m, $\Delta x = 100$ m, $\Delta t = 1$ s, $v_{10} = 40$ m/s, $v_{20} = 30$ m/s, $c_{10} = 15$ m/s, $c_{20} = 11$ m/s, $\tau_1 = 15$ s, $\tau_2 = 10$ s, $\rho_{1j} = 0.15$ veh/m, and $\rho_{2j} = 0.2$ veh/m. Note that the jam density of lane 1 is less than that of lane 2.

Our simulation results show that there exist four basic ranges of ρ_{10} and ρ_{20}, each associated with quite different flow pattern and lane change behavior. When $\rho_{10} \le 0.07$ and $\rho_{20} \le 0.08$ or $\rho_{10} > 0.07$ and $\rho_{20} > 0.08$, the number of vehicles changing from lane 1 to 2 is almost equal to that changing from 2 to 1, so the net lane changing rate is zero. When $\rho_{10} \le 0.07$ and $\rho_{20} > 0.08$, or $\rho_{10} > 0.07$ and $\rho_{20} \le 0.08$, one lane has light traffic and the other heavy traffic, vehicles may change their lanes if they want. In this study, we select the range $\rho_{10} \le 0.07$ and $\rho_{20} > 0.08$ to validate our model (11)-(13). The conditions for changing lane are: $s_2 = -0.8\rho_2 v_2/\Delta x$ and $s_1 = -s_2$ if $\rho_1 \le 0.07$, $\rho_2 > 0.08$, $s_1 = -\rho_1 v_1/\Delta x$, $s_2 = -s_1$ if $\rho_1 > 0.07$, $\rho_2 \le 0.08$.

Figures 2-7 show the temporal evolution of traffic with localized perturbations $\Delta\rho_{10} = 0.008$ veh/m, $\Delta\rho_{20} = 0.01$ veh/m and different ρ_{i0} (i = 1,2). The scales of space x, time t and density ρ_i are 100 m, 1 s and veh/m, respectively. The findings from these figures are summarized below:

(i) In Fig. 2 with $\rho_{10} = 0.057$ and $\rho_{20} = 0.081$, the traffic on each lane is basically stable. We also carried out the simulations without permit of lane change, and found that terrific jams appear on both lanes. Hence, allowing lane change may improve the stability of traffic flows.

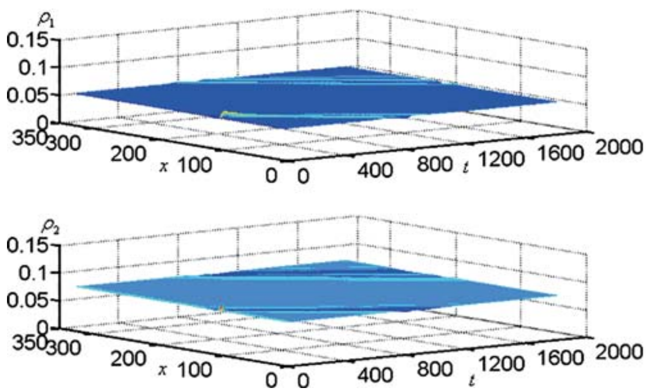

Fig. 2 $\rho_{10} = 0.057$, $\rho_{20} = 0.081$

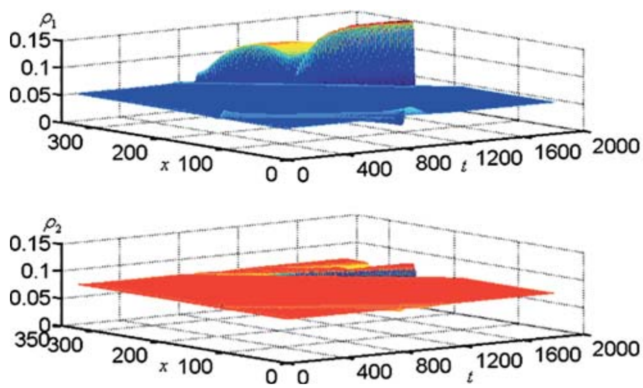

Fig. 3 $\rho_{10} = 0.056$, $\rho_{20} = 0.081$

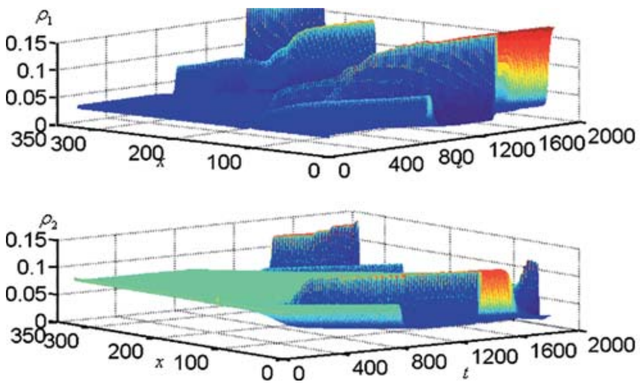

Fig. 4 $\rho_{10} = 0.035$, $\rho_{20} = 0.085$

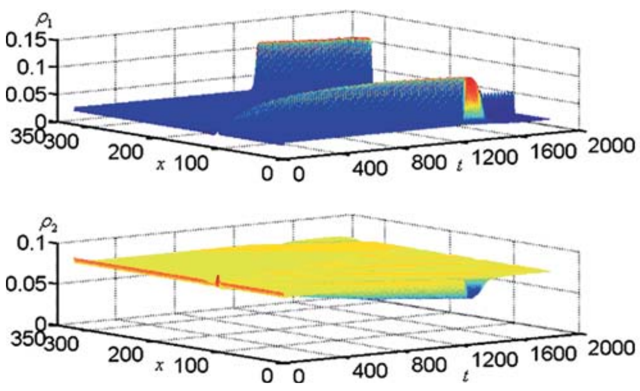

Fig. 5 $\rho_{10} = 0.025$, $\rho_{20} = 0.085$

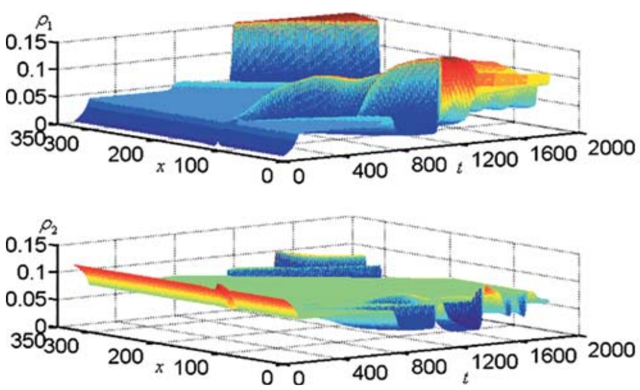

Fig. 6 $\rho_{10} = 0.01$, $\rho_{20} = 0.118$

(ii) In Fig. 3 with $\rho_{10} = 0.056$ and $\rho_{20} = 0.081$, one local cluster appears on lane 1 while the traffic on lane 2 is relatively stable although a slight jam can be observed. This implies that the excessive lane change (vehicles on lane 2 first shift to lane 1, then a few of them go back to lane 2) results in local jams.

(iii) When ρ_{10} declines to 0.035 and ρ_{20} increases to 0.085, more vehicles on lane 2 shift to lane 1, hence the stop-and-go waves can be observed on lane 1, see Fig. 4. Light waves looking like clusters appear on lane 2 due to the limited acceptance capacity of lane 1.

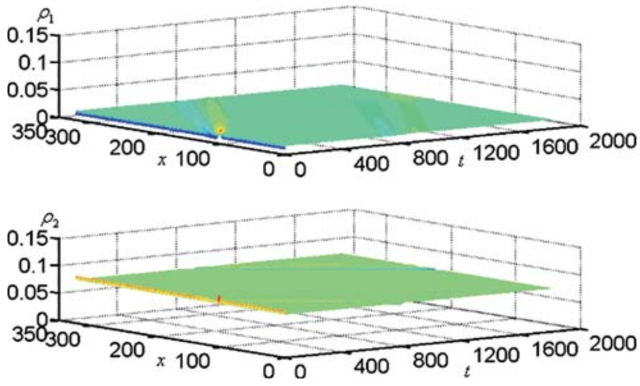

Fig. 7 $\rho_{10} = 0.01$, $\rho_{20} = 0.085$

(iv) When ρ_{10} further declines to 0.025 while ρ_{20} remains unchanged, the things become better than (iii), as seen from Fig. 5. The stop-and-go traffic still exists on lane 1 but it is not so serious as that in Fig. 4. The traffic on lane 2 is basically stable since lane 1 attracts considerable vehicles from lane 2 (the lower ρ_{10}-value let lane 1 have this ability).

(v) Let ρ_{10} be even more small, i.e., $\rho_{10} = 0.01$, and ρ_{20} be larger than 0.085, i.e., $\rho_{20} = 0.118$. We find that the things turn worse, as seen from Fig. 6, where the traffic on each lane is highly unstable. On lane 1, a serious jam spreads to long distance and exists for a long while. On lane 2, some local densities are less than 0.118 because of over lane-changing.

(vi) Let ρ_{10} remains at 0.01 and ρ_{20} changes back to 0.085. Figure 7 shows that the traffic on both lanes becomes stable again. This is because that ρ_{10} and ρ_{20} are relatively small and only a few vehicles on lane 2 shift to lane 1.

Next, we search all combinations of ρ_{10} and ρ_{20} that leads to the above 6 states, each shown by one figure. This is based on a vast amount of simulation data. In Fig. 8, the range $\rho_{10} \leq 0.07$ and $\rho_{20} > 0.08$ is divided into 6 areas, each of them corresponds to one state (i.e., one figure depicted above). Note that the area-state match shown in Fig. 8 is not very accurate because of the figure identification difficulty. It is verified that the phase transition on lane 1 occurs when the $\rho_{10} - \rho_{20}$ combination changes from Fig. 7-area to Fig. 5-area, or from Fig. 2-area to Fig. 3-area. The Fig. 3-area is so narrow that the ρ_{10}-ρ_{20} combination is easily transformed into Fig. 6-area from Fig. 2-area, thus the phase transition happens on both lanes.

The above findings reveal such a fact that the two-lane traffic is much more complicated than the single lane traffic due to the existence of lane change. The lane change may improve the traffic in some cases, however it may worsen the traffic in some other cases.

5 Conclusions

The previously developed high-order continuum models are mainly subject to single lane traffic and not suitable for ana-

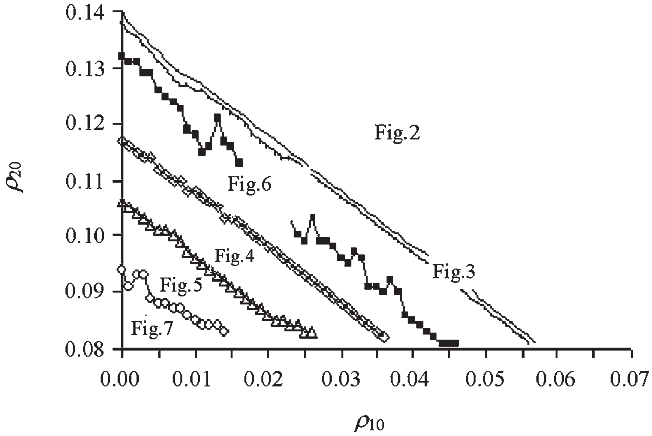

Fig. 8 Six areas, each corresponding to one state or one previous figure

lyzing the traffic flows on freeways with multiple lanes. In this paper, we propose a new dynamic model for two-lane traffic by considering the lane change. This model contains the speed gradient-based momentum equations which are derived from a car-following theory modified to suit two-lane traffic flow. The conditions for securing the linear stability of the new model are presented. Numerical results show that the new model can correctly describe the vehicle movement and the amplification of small perturbations, and reproduce some complex nonequilibrium phenomena such as stop-and-go, ghost jams, local clusters and phase transition. The findings obtained in this study reveal that the two-lane traffic is much more complicated than the single lane traffic due to the existence of lane change. The lane changing may improve the traffic in some cases, however it may worsen the traffic in some other cases.

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