Modern Freeway Traffic Flow Models

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Background

- We want to model the propagation of cars on a freeway (one lane).
- Historically, the approach is to model vehicle density as a fluid (typically extensions of classical gas models).
  - *Advantage*: Evolution is described by well studied PDEs; traffic has some gas-like properties (conservation of mass, diffusivity, etc.).
  - *Disadvantage*: Fail to capture a lot of physically relevant phenomena.

Macroscopic & Microscopic models

	Macroscopic	**Microscopic**
	models overall behavior (average density, etc.)	each car has its own model
Ex:	Fluid models	Cellular automata
Pro:	Draws on existing tools	Accurate
Con:	limited modeling power	computational cost, hard to analyze

Conservation laws for traffic flow

- Quantity of interest: **car density** u
- Early models were usually extensions of classical gas/fluid models.
- Traffic must satisfy the conservation of mass.
- Most models belong to the family

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = 0$$

where f is the **flux**.

Rate of change = Rate in - Rate out

Lighthill & Whitham

- In 1955, Lighthill and Whitham proposed the first macroscopic traffic model.
- A deterministic conservation law:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uV) = 0$$

- u is **car density**, V is **velocity**

$$V(u) = V_m(1-u)$$

- V_m is the maximum possible speed.
- *This PDE models the large scale behavior of traffic.*

Problems with classical models

- *Cars are not gas particles.*
- *Something is missing from the Lighthill and Whitham model.*
  - It is a good model for very high and very low density.
  - The interesting behavior (causing phase transitions in traffic behavior) occurs in between.
- **Idea:** Go back to the basics (microscopic level). Incorporate more realistic phenomena into a new microscopic model. Build a new macroscopic model from this.

Look-ahead models

- In 2006, Sopasakis and Katsoulakis extended the Lighthill and Whitham model by allowing for vehicles to interact with each other.
- **Look-ahead rule:** *vehicles accelerate less when cars are ahead of them.*

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- Their approach:
  - Start with a discrete (cellular automaton) microscopic model of a single lane freeway.
  - Define the way vehicles behave/interact (how their density changes with time).
  - "Take the limit" to obtain a macroscopic (average) model of traffic density.

Multi-class models

Another recent improvement on classical models (Wong & Wong, 2002)

- Model the fact that *not all people drive the same*.
- Cars are split into several classes. A vehicle's class determine its characteristics in the model (*e.g.*, speed)
- Our model incorporates look-ahead dynamics for multiple classes of cars.
- We build our PDEs from a cellular automaton model.

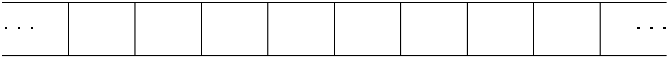
Strategy

We will derive a macroscopic model from a microscopic model.

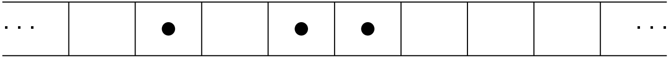
- The microscopic model is a cellular automaton model that has both multi-class features and look-ahead dynamics.
- First derive an evolution equation for the density of vehicles in the semi-discrete (cellular automaton) model.
- After some approximations, obtain a PDE as a scaling limit of the cellular automaton model.

Cellular Automaton traffic models

- Represent a one-lane highway as a lattice $\mathcal{L}$.

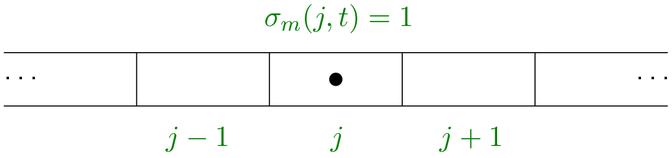


- Label the cells $j, j+1, j+2, \ldots$
- Represent vehicles on freeway as particles inhabiting cells



- Assume we have M different *classes* of vehicles.
  Each class m has a different maximum velocity v_m.
- The configuration of vehicles of class m on the lattice is represented by the variable σ_m:

$$\sigma_m(j,t) = \begin{cases} 1, & \text{if a vehicle of class } m \text{ occupies site } j \in \mathcal{L}, \\ 0, & \text{otherwise.} \end{cases}$$



We are also interested in the whether or not a given site is occupied by *any* vehicle (regardless of class).

$$\sigma(j,t) = \sum_{m=1}^{M} \sigma_m(j,t)$$

$\sigma(j,t) = 1$ if *any* vehicle occupies cell j at time t.

Each car/particle jumps forward to unoccupied neighboring cells, leaving an empty cell behind.

The rate at which a particle jumps depends on

- particle's class (multi-class model)
- the *density of particles ahead* (look-ahead rule)

Look-ahead interaction potential (linear)

$$J(r) = \begin{cases} \frac{2}{\gamma}\left(1 - \frac{r}{\gamma}\right), & 0 \leq r \leq \gamma, \\ 0, & \text{otherwise.} \end{cases}$$

Closer cars have a greater impact than distant ones.

$\gamma =$ **Look-ahead distance** (a constant parameter)

Transition rates

Between times t and $t + dt$, vehicles of class m try to advance from cell j to cell $j + 1$ at rate c_m:

$$c_m(j, \sigma_m, \sigma) = v_m \sigma_m(j)(1 - \sigma(j+1)) \exp\left[-W(j, \sigma(j,t))\right]$$

- v_m: maximum velocity for class m
- If a vehicle of *any* class occupies cell $j + 1$, you can't advance.
- Slowdown factor: $\exp\left[-W(j, \sigma(j,t))\right]$ depends on $\sigma = \sum_m \sigma_m$, not σ_m:

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- Slowdown factor: $\exp\left[-W(j, \sigma(j, t))\right]$ depends on $\sigma = \sum_m \sigma_m$, not σ_m:

$$W(j, \sigma) = \sum_{k \geq j+2} J(k - j - 2)\sigma(k, t)$$

(*convolution of interaction potential with cars ahead*)

Dynamics of the discrete model

- Each σ_m evolves as a *continuous time stochastic process* on space $\Sigma = \{0,1\}^{\mathcal{L}}$ (set of all possible vehicle configurations)
- Want to use the theory of Markov processes to write an evolution equation for σ_m
- Look-ahead rule $\implies$ σ_m is *not* Markovian!
- The couple $\eta := (\sigma_m, \sigma)$ *is* Markovian; it's transition rate is

$$c(j,\eta) = \begin{cases} c_m(j,\eta), & \text{if } \exists\, m : \sigma(j) = \sigma_m(j) = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Dynamics of the Markov process

- How do we describe the evolution of σ_m?

  Given an initial configuration of (σ_m, σ), how do we expect the vehicles to be distributed at some later time?

- *Transition kernel* P_t of the process: for $f \in L^\infty(\{0,1\}^{\mathcal{L}} \times \{0,1\}^{\mathcal{L}})$

$$P_t f(\eta) = \sum_{\eta'} f(\eta') P\{\eta_t = \eta' \mid \eta_0 = \eta(0)\}$$

  $P_t f$ gives the expected value of $f(\eta)$ at future time t.

- η Markovian $\implies$ Chapman-Kolmogorov equation:

$$P_{t+s} f(\eta) = P_t P_s f(\eta)$$

  (so-called "memoryless" property)

Dynamics of the Markov process

From another point of view:

- Regard P_t as an operator acting on a function space.
- $P_{t+s} = P_t P_s \implies \{P_t\}_{t\geq 0}$ is a *semigroup*
- This perspective is used to write a differential equation describing the evolution of (functions of) our process.
  The evolution equation is written in terms of the *infinitesimal generator* associated with the semigroup $\{P_t\}$.

Dynamics of the Markov process

- By definition, the generator of the Markov process $\eta = (\sigma_m, \sigma)$ is the limit
$$Gf = \lim_{t \to 0^+} \frac{P_t f - f}{t}$$
for all functions for which this limit exists.
- G describes the infinitesimal evolution of the process in time. It gives the "derivative" of our expected future value.

An equation for σ_m

Some terminology

- **Def:** "Rate kernel" $\alpha(\eta, \eta') =$ *rate at which the process transitions from η to η'.*
  ("The time derivative of $P_t\eta$")
- **Def:** "$\sigma_m^{j,j+1}$:" the configuration resulting from a vehicle of class m moving from j to $j+1$ (and no other jumps taking place),

$$\sigma_m^{j,j+1} = \begin{cases} \sigma_m(k,t), & j \neq k, k-1 \\ \sigma_m(k+1,t), & j = k, \\ \sigma_m(k-1,t), & j = k-1. \end{cases}$$

An equation for σ_m

- We can apply G to *any* bounded function f of the process $\eta = (\sigma_m, \sigma)$. Pick $f(\eta) = \sigma_m$.

$$
\begin{aligned}
G(\sigma_m(j,t)) &= \lim_{t \to 0^+} \frac{P_t \sigma_m - \sigma_m}{t} = \sum_{\eta'} \alpha(\eta, \eta')[\sigma'_m - \sigma_m] \\
&= \sum_{j \in \mathcal{L}} c_m(j, \sigma_m, \sigma)[\sigma_m^{j,j+1} - \sigma_m] \\
&= -c_m(j, \sigma_m, \sigma) + c_m(k-1, \sigma_m, \sigma)
\end{aligned}
$$

An equation for σ_m

$$G(\sigma_m(j,t)) = c_m(j-1,\sigma_m,\sigma) - c_m(j,\sigma_m,\sigma)\sigma_m(j)$$

Rate in - Rate out

- Can we analyze, qualitatively, the macroscopic behavior of the traffic without dealing with the details of the discrete model?
- *Want*: the expected density of vehicles, expressed as a function of space and time, namely $E\sigma_m(j,t)$.
- *Approach*:
  - Write a differential equation for $E\sigma_m(j,t)$.
  - Let the cell size shrink to zero.
  - This should give us a PDE describing the unknown function we are after.

Evolution equations

- By the definition of the generator,

$$\frac{d}{dt}E\sigma_m(j,t) = EG\sigma_m(j,t).$$

- E denotes expectation with respect to the transition probability measure P.
- By our discrete conservation law (rate in minus rate out),

$$\begin{aligned}\frac{d}{dt}E\sigma(j,t) &= -E[c_m(j,t,\sigma)] + E[c_m(j-1,t,\sigma)] \\ &= -E[v_m\sigma_m(j,t)(1-\sigma(j+1,t))\exp\left[-W(j,\sigma)\right]] \\ &\quad + E[v_m\sigma_m(j-1,t)(1-\sigma(j,t))\exp\left[-W(j-1,\sigma)\right]].\end{aligned}$$

Moving to a continuous spatial variable

- Now allow lattice sites $j \in \mathcal{L}$ to have fixed length h (instead of 1).
- Define $u_m(x,t)$, $x \in \mathbb{R}$, as piecewise linear (in space) interpolations of the expectations $E\sigma_m(j,t)$ for each class $1 \leq m \leq M$.
- $u_m(x,t)$ is continuous in x and linear on each cell (*i.e.*, for $x \in (jh,(j+1)h)$).
- The look-ahead interaction potential affects u_m through the *integral*

$$W(x,u) = \int_{x+2h}^{\infty} J(y-x-2h)u(y,t)\,dy$$

where

$$u(x,t) := \sum_{m=1}^{M} u_m(x,t).$$

Approximations

Our "rate in-rate out" equation, in terms of $u_m(x,t), u(x,t)$ is

$$\frac{\partial}{\partial t} u_m(j,t) = -E[v_m u_m(j,t)(1-u(j+h,t))\exp\left[-W(j,u)\right]] \\ + E[v_m u_m(j-h,t)(1-u(j,t))\exp\left[-W(j-h,u)\right]].$$

Two approximations make this equation analytically tractable:

(i) the weak propagation of chaos, and

(ii) a Taylor series expansion.

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Propagation of chaos

We assume the so-called *weak propagation of chaos* for our process.

- "Chaos" is statistical independence.
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- "Chaos" is statistical independence.
- In a system that weakly propagates chaos, *randomness in the initial configuration is passed on to future configurations.*
- If particle positions are approximately independent initially, then they are well approximated as independent in the future. (This is reasonable; can be justified heuristically.)

Propagation of chaos

- *Why do we want to assume this?* It permits us to approximate the joint probability distribution of $\{\sigma_m\}_{m=1}^{\infty}$ as a product measure:

$$E[\sigma_m(j,t)\sigma(j+1,t)] \approx E\sigma_m(j,t)E\sigma(j+1,t)$$

- *Remark*: Weak propagation is *not* about the independence of transition probabilities, which are highly coupled.
  This assumption does not suppress the look-ahead dynamics.

A Taylor series approximation

By Taylor's Theorem (with integral remainder),

$$\exp\left[-EW(x,u)\right] = \exp\left[-W(x,u)\right] + R(x),$$

where

$$|R(x)| \leq |EW(x,u) - W(x,u)|.$$

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As the number of vehicles increases and as the look ahead distance grows, we expect the above approximation to become increasingly accurate (by *formally* using the law of large numbers). Taking the expectation (with respect to P) in the Taylor approximation, we have

$$E[\exp\left[-W(x,u)\right]] \approx E[\exp\left[-EW(x,u)\right]] = \exp\left[-EW(x,u)\right].$$

Semi-discrete evolution equation

- With our approximations, we have

$$\begin{aligned}\frac{\partial}{\partial t}u_m(x,t) = &-c_m u_m(x,t)(1-u(x+h,t))\exp\left(-W(x,u)\right)\\ &+c_m u_m(x-h,t)(1-u_m(x,t))\exp\left(-W(x-h,t)\right).\end{aligned}$$

Semi-discrete evolution equation

- With our approximations, we have

$$\frac{\partial}{\partial t}u_m(x,t) = -c_m u_m(x,t)(1-u(x+h,t))\exp\left(-W(x,u)\right) + c_m u_m(x-h,t)(1-u_m(x,t))\exp\left(-W(x-h,t)\right).$$

- Define the flux F_m for each vehicle class m:

$$F_m(u_m,u,x,t) := v_m u_m(x-h,t)(1-u(x,t))\exp\left(-W(x,u)\right)$$

Then

$$\frac{\partial}{\partial t}u_m(x,t) = F_m(u_m,u,x,t) - F_m(u_m,u,x+h,t).$$

$$\frac{\partial}{\partial t}u_m(x,t) = \textbf{Rate in - Rate out}$$

Rescaling

- Cell size h is now fixed, but we'll later let h become arbitrarily small.
- Accordingly, rescale time $t \to \tau = th$. (For convenience, denote the "new" time τ again by t.) Then, the conservation law becomes

$$\frac{\partial}{\partial t} u_m(x,t) = \frac{1}{h} \left(F_m(u_m, u, x, t) - F_m(u_m, u, x+h, t) \right).$$

- Before we can pass to a PDE limit, we need to establish some regularity for the flux F_m.

Regularity of the flux

Want to show that $F_m(u_m, u, \cdot, t)$ has a weak derivative.

Claim

For fixed $u(\cdot, t) \in W^{1,p}(\mathbb{R})$, $1 \leq p \leq \infty$, *the function* $W(\cdot, u) \in C^1(\mathbb{R})$.

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By direct differentiation,

$$\begin{aligned}\frac{\partial}{\partial x} W(x,t) = \int_{x+2h}^{x+2h+\gamma} \frac{\partial}{\partial x} J(y - x - 2h) u(y,t)\, dy \\ + J(\gamma) u(x + 2h + \gamma, t) \\ - J(0) u(x + 2h, t).\end{aligned}$$

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For fixed $u_m(\cdot,t), u(\cdot,t) \in W^{1,p}(\mathbb{R})$, $1 \le p \le \infty$, *the function* $F_m(u_m, u, \cdot, t) \in W^{1,p}(\mathbb{R})$.

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Fix $1 \leq p < \infty$. Expanding $F_m(u_m, u, x, t)$,

$$\begin{aligned} F_m(u_m, u, x, t) &= c_m u_m(x,t)(1 - u(x,t))\exp\left(-W(x,u)\right) \\ &= c_m u_m \exp\left(-W(x,u)\right) \\ &\quad - c_0 u_m(x,t)u(x,t)\exp\left(-W(x,u)\right). \end{aligned}$$

$W(\cdot, u) \in C^1(\Omega)$, so it suffices to show that $uu_m \in W^{1,p}(\Omega)$. Take a test function $\varphi \in C_0^\infty(\Omega)$. Take $\{v_k\}, \{w_k\} \subset C^\infty(\overline{\Omega})$ so that $v_k \to u_m$, $w_k \to u$ in $W^{1,p}(\Omega)$ as $k \to \infty$.

By continuity of the inner product in $L^2(\Omega)$ (as u, u_m are elements of $L^2(\Omega)$),

$$
\begin{aligned}
\int_\Omega \varphi' u_m u \, dx &= \lim_{k\to\infty} \int_\Omega \varphi' v_k w_k \, dx \\
&= -\lim_{k\to\infty} \int_\Omega \varphi \left(w_k \frac{\partial}{\partial x} v_k + v_k \frac{\partial}{\partial x} w_k \right) \, dx \\
&= -\int_\Omega \varphi \left(u \frac{\partial}{\partial x} u_m + u_m \frac{\partial}{\partial x} u \right) \, dx.
\end{aligned}
$$

That $\frac{\partial}{\partial x}(u_m u)$ is in $L^p(\Omega)$ is obvious from the fact that u_m, u are bounded and their derivatives are in $L^p(\Omega)$.

One more claim

We need one more tool in order to pass to the PDE limit.

Claim

If $f \in L^p(\Omega)$, $g \in L^{p'}(\Omega)$, $1/p + 1/p' = 1$, *and if* g *is compactly supported on* Ω, *then*

$$\int_\Omega f(x) \left(\frac{g(x+h) - g(x)}{h} \right) dx = - \int_\Omega g(x) \left(\frac{f(x) - f(x-h)}{h} \right) dx,$$

for all sufficiently small h *(*h *is our freeway cell size).*

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Proof.

PDE II, Homework 2, Problem #1. □

PDE Limit

$$\frac{\partial}{\partial t} u_m(x,t) = \frac{1}{h}\left(F_m(u_m, u, x, t) - F_m(u_m, u, x+h, t)\right)$$

- Multiply by an arbitrary but fixed test function $\varphi \in C_0^\infty(\mathbb{R})$.
- Integrate over $\mathbb{R}$ in the spatial variable.

$$\int_{\mathbb{R}} \varphi(x) \frac{\partial}{\partial t} u_m(x,t)\, dx = \int_{\mathbb{R}} \varphi(x) \frac{1}{h}\left(F_m(u_m, u, x, t) - F_m(u_m, u, x+h, t)\right)\, dx$$

- Apply PDE II Homework 2, Problem #1.

PDE Limit

- Apply PDE II Homework 2, Problem #1.

$$\int_{\mathbb{R}} \varphi(x) \frac{\partial}{\partial t} u_m(x,t)\, dx = \int_{\mathbb{R}} \frac{1}{h} \left(\varphi(x) - \varphi(x-h)\right) F_m(u_m, u, x, t)\, dx$$

This holds for all small $h > 0$. Take $h \to 0^+$.

PDE Limit

Taking $h \to 0^+$,

$$\int_{\mathbb{R}} \varphi(x) \frac{\partial}{\partial t} u_m(x,t)\, dx = \int_{\mathbb{R}} \varphi'(x) F_m(u_m, u, x, t)\, dx.$$

By definition of the weak derivative of $F_m(u_m, u, \cdot, t)$,

$$\int_{\mathbb{R}} \varphi(x) \frac{\partial}{\partial t} u_m(x,t)\, dx = -\int_{\mathbb{R}} \varphi(x) \frac{\partial}{\partial x} F_m(u_m, u, x, t)\, dx.$$

Test function φ was arbitrary, so,

PDE Limit

$$\frac{\partial}{\partial t}u_m(x,t) + \frac{\partial}{\partial x}F_m(u_m,u,x,t) = 0,$$

where

$$F_m(u_m,u,x,t) = v_m u_m(x,t)(1-u(x,t))\exp\left[\int_x^\infty -J(y-x)u(y,t)dy\right]$$

- This conservation law describes how the expected density of class m vehicles evolves in space and time on a macroscopic level.

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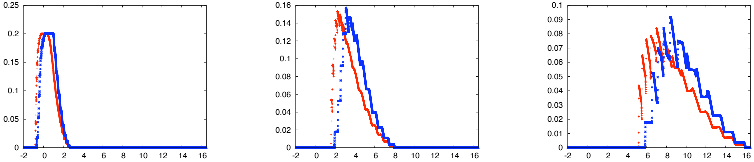
- This conservation law describes how the expected density of class m vehicles evolves in space and time on a macroscopic level.
- The features of the microscopic model are *manifested in the flux* F_m.
- For $m = 1, 2, \ldots, M$, we have a system of conservations laws with coupled fluxes.

Numerical examples

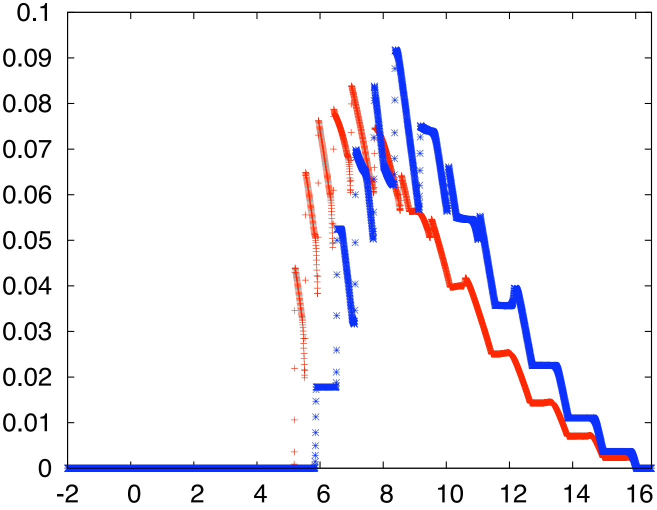
Look-ahead ($\gamma = 3$) vs. No look-ahead

Density profiles of $u(x, t)$



$t = 0.5$ $\quad$ $t = 1.5$ $\quad$ $t = 3$

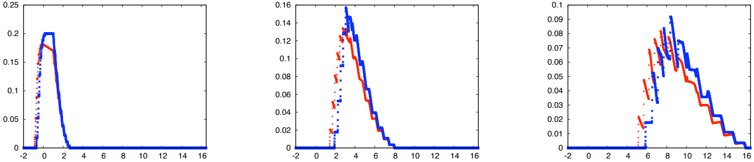
Density profile of $u(x, 3)$, $\gamma = 0.1$



$t = 3$

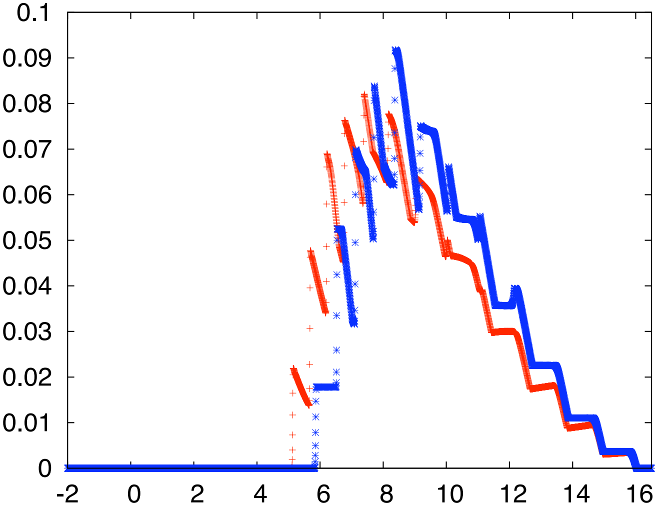
Look-ahead ($\gamma = 3$) vs. No look-ahead

Density profiles of $u(x,t)$



$t = 0.5$ $\quad$ $t = 1.5$ $\quad$ $t = 3$

Density profile of $u(x, 3)$, $\gamma = 3$



$t = 3$

Future work

- Comparison with Monte Carlo simulations
- Comparison with empirical data
- Extension of model to multi-lane freeways

Thank You:

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