Macroscopic and Microscopic Assessment of Traffic Rules

Summary goes here

INTERPRETATION OF THE PROBLEM

MODEL

To simplify the model, we consider a stretch of straight two-lane freeway with no on/off ramp. To assess the performance of the given traffic rule, we would like to analyze its impact on traffic flow, road safety, and ability to recovery from traffic jams.

traffic flow

quantity	variable	unit
traffic density on lane 1	ρ_1	cars/m
traffic density on lane 2	ρ_2	cars/m
equilibrium velocity on lane 1	v_{1e}	m/s
equilibrium velocity on lane 2	v_{2e}	m/s

TABLE I. variables used in the macroscopic model

Traffic flow rate is best captured in a macroscopic model with variables shown in Table I. The total amount of traffic flow will then be determined by

$$Q(\rho_1, \rho_2) = \rho_1 \cdot v_1(\rho_1, \rho_2) + \rho_2 \cdot v_2(\rho_1, \rho_2) \tag{1}$$

We further assume that at equilibrium, there is some velocity-density relation. Given this velocity-density relation, the flow at any given junction of the freeway at a given time is completely determined by the local traffic densities ρ_1 and ρ_2.

To derive such a relation, we consider all cars to have uniform length $l(m)$, travel with uniform velocity $v_e(\rho)$ and maintain uniform bumper-to-bumper distance $d(m)$ from their neighbors. At this equilibrium, each car will take up a total space of $d + l$ on one lane of the freeway. Since the highway won't be completely conjested all the time, there will normally be extra free space in between cars in addition to the safe following distance. We can think of this free space as being filled by an invisible vihcle density ρ_0, therefore

$$\rho + \rho_o = \frac{1}{d + l} \tag{2}$$

Incorporating the two-second rule enforced by the New York Sate Department of Motor Vehicles [?], $d = v_e(\rho)t$, where $t = 2s$. Equation (2) can be solved to obtain and expression for $v_e(\rho)$

$$v_e(\rho) = (\frac{1}{\rho + \rho_o} - l)/t \tag{3}$$

We define the capacity (or jamming density) of the road by the density at which traffic stops flowing

$$0 = (\frac{1}{\rho_j + \rho_o} - l)/t \tag{4}$$

$$\Rightarrow \rho_j = \frac{1}{l} - \rho_o \tag{5}$$

This speed-density relationship also imposes a natural speed limit on the road, specifically the speed of traffic at zero density

$$v_{nl} = v_e(0) = (\frac{1}{\rho_o} - l)/t \tag{6}$$

Consider the natural speed limit of a freeway lane to be $50m/s$, about $110mph$ then $\rho_o = \frac{1}{105}$ and $\rho_j = 0.19$

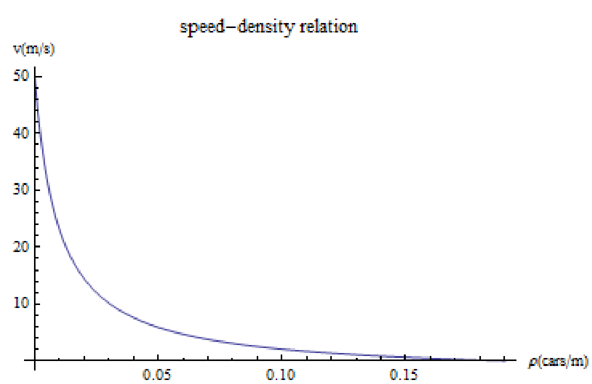

FIG. 1. speed-density relation with a natural speed limit of $50m/s$

With the speed-density relation determined, we can derive the flow of a two-lane freeway using the prescription described in equation 1

safety

We used a similar microscopic model with two lanes to examine the safety issue of the rules imposed. There are a few different criteria that could be used to determine the safety.

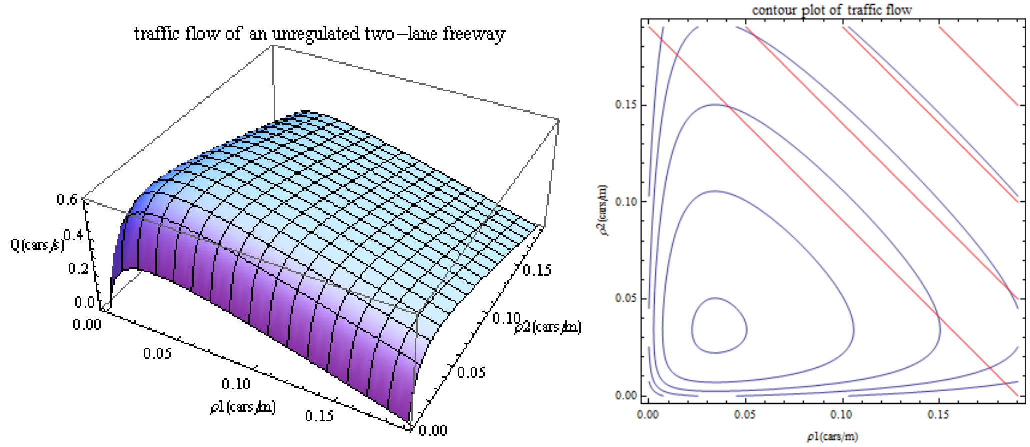

FIG. 2. traffic flow as a function of lane 1 and lane 2 traffic density on a two-lane freeway

The contour plot provides a good prediction for how the traffic rule might affect the amount of traffic flow. In light traffic, $\rho_1 + \rho_2$ is small. As seen in the bottom left corner of the contour plot of traffic flow, any bias in one lane over the other will reduce traffic flow. In this regime, as traffic increase, vehicles should utilize both lanes in the same way to maximize the total amount of traffic flowing through the freeway. However, as traffic satuate the road capacity, bias in the densities of the the roads does not affect traffic flow in a significant manner. This is demonstrated in the contour plot through an overlap of the contours lines of flow(blue) and of total traffic(red) on the road.

In over model, we consider mainly the two following, total amount of slow down distance by all affected cars and the number of cars that need to slow down.

Assumption of safety analysis

Before analyzing the model, there are a few assumptions that we have made to simplify the model. Firstly, we assume each vehicle is a point, so the length is negligible. Secondly, to analyze the road performance of the cars with different traveling speed, we assume that the cars have two types of speed, v_1 and v_2 with $v_2 > v_1$. In a traveling equilibrium of a lane, we assume that the cars are separated by a distance of d that is related to the speed of the car on that lane v, following time t and the traffic density on that lane ρ. In particular, since cars are traveling in equilibrium condition, the speed is constant (except those cars that want to overtake others). The following time t refers to the safe time difference that

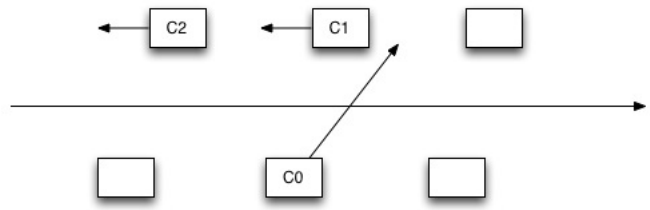

FIG. 3. car overtaking illustration of unruled two lane traffic

drivers deem for two adjacent cars to pass through the same spot of the road, which is usually around 1 to 2 seconds. Therefore, drivers will keep at least $d_m = vt$ distance from the car in front. However, usually drivers will not stick to the distance $d_m = vt$, but rather keep a distance that is a little bit further for extra safety. Hence, in equilibrium, the actual difference between two cars is $d_s = vt + d_0$, where d_0 is the extra safety distance, and it is a function of lane traffic density ρ, thus $d_0 = f(\rho)$. According to our previous analysis, d_0 has a inverse relationship with ρ, i.e. when ρ is low, c is large.

Analysis of safety model on both ruled and unruled traffic

We then consider the performance difference between the two situations (one with rule applied and one without). First consider the case where drivers are free to drive on both left and right lane, which is the case where no rule is enforced. In this case, when a car, C_0, overtakes another one, it needs to shift to the other lane and push back cars on that lane. In particular, in order to keep at least a safe distance $d_m = vt$ from the previous car, C_0 has to slow down and until the difference with the from car is d_m. This will suppress the distance with the car behind C_1, which will cause the car to slow down. Therefore, the action of overtaking will cause a series of slowing down of cars until all have reached at least a minimum safety distance d_m.

From the above graphical illustrations(WHICH GRAPHICS???), we notice that the car C_0 is pushed back by $d_m - d_0$, C_1 is pushed back by $d_m - 2d_0$ and the i^{th} car is pushed back by $d_m - id_0$. The car C_i will not slow down if $d_m - id_0 \leq 0$, i.e. $d_m \leq id_0$. Therefore, the number of cars slowed down $n \approx d_m/d_0$ when $d_0 << d_m$.

Thus, we would be able to calculate the total amount of braking distance made by all cars after C_0

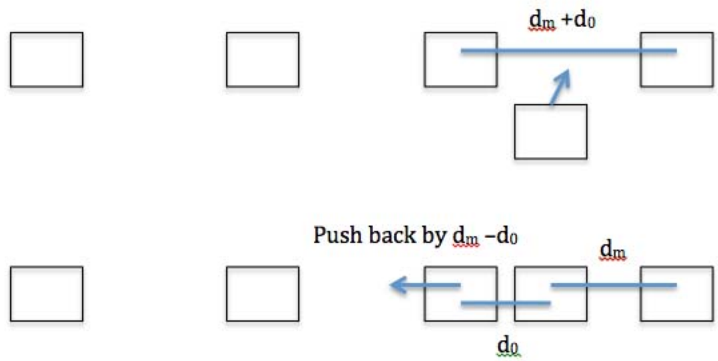

FIG. 4. illustration of minimum distance pushed back

$$d_t = d_m - d_0 + d_m - 2d_0 + \cdots + d_m - nd_0 \tag{7}$$

$$d_t = nd_m - \frac{(d_0 + nd_0)(n)}{2} \tag{8}$$

$$d_t = \frac{d_m^2}{d_0} - \frac{(d_0 + d_m)(d_m)}{2d_0} \tag{9}$$

$$d_t = \frac{d_m^2 - d_0 d_m}{2d_0} \tag{10}$$

In particular, if the $d_t \leq 0$, i.e. no car needs to slow down, we have $d_m \leq d_0$, which means that when the minimum safe distance is less than the extra safe distance, no car will slow down.

Now, let's take a look at the case if the rule is applied to the two lane traffic situation. In this case, the right lane is the regular lane and the left lane is the passing lane, which suppose to have a higher speed. When a car, C_0, is trying to take over the car(s) in front, it needs to shift to the left lane and then shift back to the right lane after overtaking a number of cars. Suppose C_0 only overtake one car C_1, then when C_0 shifts back to the right lane, C_1 probably needs a slow down if $d_m > d_0$. However, we notice that because C_0 has shifted out of the right lane in the first place, it has left a space of $2(d_s)$ behind C_1. Notice that, the slow down distance of C_1 after C_0 has shifted back to the is $d_m - d_0$. Therefore, the distance remain behind C_1 after it finished slowing down is

$$
\begin{aligned}
& 2(d_s) - (d_m - d_0) && (11)\\
&= 2(d_m + d_0) - (d_m - d_0) && (12)\\
&= d_m + 3d_0 && (13)\\
&\geq d_m && (14)
\end{aligned}
$$

Therefore, in this case, C_1 is the only affected car. And the slow down distance is $d_m - d_0$

If we increase the number of car that C_0 overtakes, we know that the first car overtaken by C_0 needs to slow down by $d_m - id_0$, which is less than $d_m + 3d_0$. Thus, the maximum number of cars affected is $max(d_m/d_0, i)$. This shows that the number of cars slowed down is no more than the previous case. In addition, the total amount of slow down distance is still

$$d_t = d_m - d_0 + d_m - 2d_0 + \cdots + d_m - id_0 \tag{15}$$

However, since $i \leq n$, the total slow down distance is less or equal to the previous case.

safety model on light and heavy traffic comparison

Now, let's consider the difference between light and heavy traffic. From our assumption, as the traffic density ρ increase, d_0 decrease. Therefore, the number of cars slowed down $n \approx d_m/d_0$ is increased. This explains that as the traffic become heavier, the chance for a safety risk is increased. In light traffic, we notice that $d_m \approx d_0$, and then in this case, both situation (ruled and unruled) has no need for slowing down. Thus, the difference in safety issue is insignificant.

traffic disturbance

An important measure of the robustness of a traffic rule is its ability to resolve traffic disturbance. Here we adopt a hybrid of car-following and classic kinematic wave theory theory from literature [1]. In this theory, classic LWR kinematcs wave model [2, 3] is combined with a variation of car-following theory [4], giving the following four equations that describe the temperal and spatial variations of the traffic density as well as average velocity of vehicles on the two lanes of the freeway.

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1}{\partial x} v_1 + \rho_1 \frac{\partial v_1}{\partial x} = s_1 \tag{16}$$
$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x} = \frac{v_{1e} - v_1}{\tau_1} + c_{10} \frac{\partial v_1}{\partial x} \tag{17}$$
$$\frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2}{\partial x} v_2 + \rho_2 \frac{\partial v_2}{\partial x} = s_2 \tag{18}$$
$$\frac{\partial v_2}{\partial t} + v_2 \frac{\partial v_2}{\partial x} = \frac{v_{2e} - v_2}{\tau_2} + c_{20} \frac{\partial v_2}{\partial x} \tag{19}$$

where we take $v_{1e} = v_{2e}$ to be the equilibrium velocity-density relations derived from previous sections (equation (**??**)). The source terms for lane 1 and lane 2 must satiesfy $s_1 + s_2 = 0$ by the assumption of no on/off ramp. They will be determined by the traffic rule. Adhering to the assumption that the two roads are identical, we choose $\tau_1 = \tau_2 = 10s$ and $c_{10} = c_{20} = 11m/s$ according to the stability contraint proposed by Tang and Huang[1].

We impose periodic boundary condition and initial condition that represents a traffic disturbance in lane 1 to observe traffic behavior after the disturbance. $\rho_1(x, 0)$ is the functional form of a traffic disturbance suggested in [1]. L is the length of freeway of interest. ρ_{10} is the average vehicle density on lane 1 and $\Delta\rho_{10}$ is the size of the disturbance.

$$\rho_1(0, t) = \rho_1(L, t) \tag{20}$$
$$v_1(0, t) = v_1(L, t) \tag{21}$$
$$\rho_2(0, t) = \rho_2(L, t) \tag{22}$$
$$v_2(0, t) = v_2(L, t) \tag{23}$$
$$\rho_1(x, 0) = \rho_{10} + \Delta\rho_{10} \left(cosh^{-2}(\frac{160}{L}(x - \frac{5L}{16})) - \frac{1}{4} cosh^{-2}(\frac{40}{L}(x - \frac{11L}{32})) \right) \tag{24}$$
$$v_1(x, 0) = v_{1e}(\rho_1(x, 0)) \tag{25}$$
$$\rho_2(x, 0) = \rho_{20} \tag{26}$$
$$v_2(x, 0) = v_{2e}(\rho_2(x, 0)) \tag{27}$$
$$\tag{28}$$

Equations (16-27) are solved using Mathematica 9.0. Four sets of parameters were chosen to represent

1. light traffic, no rule: $\rho_{10} = \rho_{20} =$

2. heavy traffic, no rule:

3. light traffic, keep-right:

4. heavy traffic, keep-right:

RESULT

DISCUSSION

[1] TQ Tang and HJ Huang. Continuum models for freeways with two lanes and numerical tests. *CHINESE SCIENCE BULLETIN*, 49(19):2097–2104, OCT 2004.

[2] G. B. Lighthill, M. H.; Whitham. On kinematic waves II: A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society*, 229:317–345, 1955.

[3] PI RICHARDS. SHOCK-WAVES ON THE HIGHWAY. *OPERATIONS RESEARCH*, 4(1):42–51, 1956.

[4] R Jiang, QS Wu, and ZJ Zhu. A new continuum model for traffic flow and numerical tests. *TRANSPORTATION RESEARCH PART B-METHODOLOGICAL*, 36(5):405–419, JUN 2002.