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Continuum models for freeways with two lanes and numerical tests

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Abstract **This paper deals with the continuum modeling of traffic dynamics for freeways with two lanes where the faster vehicles are allowed to travel on both lanes while the slower vehicles are allowed to travel on one lane only. The speed gradient-based momentum equation is used to develop the traffic models for each lane. Using the proposed models, some nonequilibrium phenomena such as small disturbance instability and stop-and-go waves, together with results from numerical tests are investigated. The conditions for keeping the models' linear stability are presented.**

Keywords: traffic flow models, two lanes, momentum equation.

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Lighthill et al.[1] and Richards[2], independently proposed the first-order continuum traffic models, called LWR theory in transportation science. According to this theory, traffic flows on a long homogeneous freeway are governed by a conservation equation as follows:

$$\frac{\partial \rho}{\partial t}+\frac{\partial q}{\partial x}=s(x,t), \qquad (1)$$

where ρ is the traffic density, q is the flow rate, $s(x,t)$ is the generation rate, while x and t represent space and time, respectively,. If the freeway has no on-off ramp, then $s(x,t)=0$, otherwise $s(x,t)\neq 0$. Let v be the space mean speed, a flow-density relationship exists, i.e.

$$q=\rho v. \qquad (2)$$

The LWR theory further assumes that there exists an equilibrium speed-density relationship as follows:

$$v=v_e(\rho). \qquad (3)$$

Combining the above three equations, we can obtain the analytical solutions of various simple traffic flow problems by the method of characteristics, and reveal the shock waves and traffic jams from these solutions[3]. However, the LWR model does not faithfully describe the nonequilibrium traffic flow dynamics since the speed is assumed to be always determined by the equilibrium speed-density (eq. (3)) so that no speed fluctuation around the equilibrium state is allowed. For this reason many scholars have developed high-order continuum traffic flow models that incorporate a dynamics equation or momentum equation representing the car-following behavior. This momentum equation takes the acceleration and inertia of driving into account. Hence the high-order models overcome some deficiencies in the simple continuum models and improve the ability of reproducing complex traffic behavior. Payne[4] employed the car-following theory to derive the following momentum equation:

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=\frac{v_e(\rho)-v}{\tau}-\frac{\nu}{\rho\tau}\frac{\partial \rho}{\partial x}, \qquad (4)$$

where $\nu=-0.5\partial v_e(\rho)/\partial\rho$ is the anticipation coefficient and τ is the relaxation time. Payne's model can be used to analyze such phenomena as broken equilibrium, stop-and-go and phase transition. After Payne's work, a variety of momentum equations have been developed, e.g. Kühne[5], Ross[6], Papaeorgiou et al.[7], Michaloupolos et al.[8]. Recently, Zhang[9] presented a momentum equation as follows:

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=\frac{v_e(\rho)-v}{\tau}-\rho(v_e'(\rho))^2\frac{\partial \rho}{\partial x}. \qquad (5)$$

Zhang's model proposed that traffic phase transitions in the presence of a strong disturbance undergo three stages, namely anticipation-dominant stage, balanced anticipation-relaxation stage and relaxation-dominant stage. Eq. (5) is subject to the second stage. However, the existing high-order models have a fundamental flaw that one of characteristic speeds resultant from the momentum and conservation equations is larger than the macroscopic flow velocity. This implies that vehicles behind influence vehicles ahead[10].

Based on the full velocity difference (FVD) car-following model, Jiang et al.[10,11] proposed a new dynamics equation in which the density gradient term is replaced by a speed gradient term. They showed that the model's characteristic speeds are not larger than the macroscopic flow velocity; hence vehicles respond only to frontal stimulus. The momentum equation is given below:

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=\frac{v_e(\rho)-v}{\tau}+c_0\frac{\partial v}{\partial x}, \qquad (6)$$

where $c_0=\Delta/t_\Delta$ represents the propagation speed of disturbance, Δ is the distance between the following car and the leading car, and t_Δ is the time needed for the backward propagated disturbance to travel a distance of Δ.

In the above models, the following car is not allowed to overtake the leading car. Hence these models are only able to formulate the traffic flow on freeways with single lane. Daganzo[12] developed a continuum theory of traffic dynamics for freeways with two vehicles types and a set of lanes. The theory defines the conservation equation as

$$\frac{\partial P}{\partial t}+\frac{\partial Q}{\partial x}=0, \qquad (7)$$

where $P=(\rho_1,\rho_2), Q=(q_1,q_2)$, ρ_1 and ρ_2, q_1 and q_2, v_1 and v_2 represent the traffic densities, the flow rates and the mean speeds on lane 1 and lane 2, respectively. Note that $Q = Q(P)$, and (7) can be rewritten as

$$\frac{\partial P}{\partial t}+Q'(P)\frac{\partial P}{\partial x}=0, \tag{8}$$

where $Q'(P)$ is the Jacobi matrix of $Q(P)$, i.e.

$$Q'(P)=\begin{bmatrix} \frac{\partial q_1}{\partial \rho_1} & \frac{\partial q_1}{\partial \rho_2} \\ \frac{\partial q_2}{\partial \rho_1} & \frac{\partial q_2}{\partial \rho_2} \end{bmatrix}. \tag{9}$$

Daganzo first studied the possible relationships between velocities and densities according to the density ratio of two lanes, and then formulated the traffic flow models for each of totally four ρ_1– ρ_2 regions. To graphically carry out analyses, Daganzo adopted simple continuum model only; hence his approach cannot be used to describe the nonequilibrium traffic flow dynamics. Wu[13] proposed a multi-lane traffic model, considering the specific traffic conditions in Chinese cities (e.g. untidy flows and lower speeds on roads). But Wu's model tends to equalize all lanes' densities through averaging them. This conflicts with the real traffic since the density on lane for faster vehicles is generally less than that on lane for slower vehicles.

In this paper, we introduce the momentum equation proposed by Jiang et al.[10,11] into the Daganzo's modeling framework for multi-lane traffic flow, and develop the corresponding high-order continuum model. We study the conditions required for keeping the model's linear stability and analyze traffic waves. Numerical results are presented to demonstrate the model's effectiveness in reproducing multi-lane traffic.

1 The high-order continuum model for two-lane traffic

Suppose that there is a freeway with two lanes: lane 1 is for faster vehicles only and lane 2 can be used by both faster and slower vehicles. The faster vehicles can shift to lane 2 if lane 1 becomes more congested, but the vehicles on lane 2 cannot travel on lane 1. The flow rates, densities and velocities on these two lanes are denoted by q, ρ, v, q_2, ρ_2, v_2, respectively. For simplicity, we further assume that there is no on-off ramp on the freeway.

Let $P=(\rho_1,\rho_2),\ Q=(q_1,q_2),\ V=(v_1,v_2)$, where $q_i=\rho_i v_i,\ i=1,2$. The following operation between P and V is defined for mathematical expression only:

$$Q=PV=(\rho_1,\rho_2)(v_1,v_2)=(\rho_1 v_1,\rho_2 v_2). \tag{10}$$

In equilibrium state, there are relations governing the speeds, flow rates and densities as follows:

$$V=V_e(P), Q=PV_e(P). \tag{11}$$

Furthermore, let $v_{1e}=v_{1e}(\rho_1)$ and $v_{2e}=v_{2e}(\rho_1,\rho_2)$ be decreasing functions with respect to ρ_1 and ρ_2, respectively, and $q_{1e}=q_{1e}(\rho_1)$ and $q_{2e}=q_{2e}(\rho_1,\rho_2)$ be strictly concave functions. Therefore, the speed and flow rate on lane 1 are associated with its own density only, while those on lane 2 depend upon the densities on both lanes. In addition, the two equations $q_1=q_1(\rho_1)$ and $q_2=q_2(\rho_1,\rho_2)$ are independent of each other so that the characteristic speeds generated by these two lanes' traffic flow models are not zero.

Following Daganzo[12], the conservation equation is given below:

$$\frac{\partial P}{\partial t}+\frac{\partial Q}{\partial x}=S(x,t), \tag{12}$$

where $S(x,t)=[s_1(x,t),s_2(x,t)]^T$, $s_1(x,t)$ and $s_2(x,t)$ are the flow generation rates on lane 1 and lane 2, respectively. As there is no on-off ramp on the freeway, we have

$$s_1(x,t)+s_2(x,t)=0. \tag{13}$$

The high-order continuum traffic model proposed by Jiang et al.[10, 11] is applied to each lane, so that

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}=\frac{v_{ie}-v_i}{\tau_i}+c_{i0}\frac{\partial v_i}{\partial x}, \qquad i=1,2, \tag{14}$$

where τ_1 and τ_2 are the relaxation times of two lanes, c_{10} and c_{20} are the propagation speeds of disturbances on two lanes, respectively. Usually, $c_{10} \geq c_{20}$ holds.

Eqs. (12) and (14) can be expressed in a vector manner as follows:

$$\frac{\partial U}{\partial t}+[A]\frac{\partial U}{\partial x}=E, \tag{15}$$

where

$$U=\begin{bmatrix}\rho_1\\ v_1\\ \rho_2\\ v_2\end{bmatrix},\quad [A]=\begin{bmatrix} v_1 & \rho_1 & 0 & 0\\ 0 & v_1-c_{10} & 0 & 0\\ 0 & 0 & v_2 & 2\rho_2\\ 0 & 0 & 0 & v_2-c_{20}\end{bmatrix},$$

$$E=\begin{bmatrix} s_1\\ (v_{1e}-v_1)/\tau_1\\ s_2\\ (v_{2e}-v_2)/\tau_2\end{bmatrix}.$$

Comparing the model (15) with Daganzo's simple continuum model, we can find that the former contains two momentum equations. This gives the new model ability of capturing the dynamic features of traffic flow in nonequilibrium state. By analyzing the model's characteristic values, we can examine the properties of the characteristic speeds. The character values of the matrix $[A]$ are the solutions of the following equation:

$$\det([A]-\lambda[I])=0, \tag{16}$$

where [I] is a 4×4 identity matrix. Eq. (16) can be rewritten as

$$\begin{vmatrix} v_1-\lambda & \rho_1 & 0 & 0 \\ 0 & v_1-c_{10}-\lambda & 0 & 0 \\ 0 & 0 & v_2-\lambda & 2\rho_2 \\ 0 & 0 & 0 & v_2-c_{20}-\lambda \end{vmatrix}=0. \tag{17}$$

The four character values are: $\lambda_1=v_1$, $\lambda_2=v_1-c_{10}$, $\lambda_3=v_2$, $\lambda_4=v_2-c_{20}$. Hence the characteristic speeds generated from our model are

$$\begin{aligned} &\left(\frac{dx}{dt}\right)_1=v_1,\ \left(\frac{dx}{dt}\right)_2=v_1-c_{10}, \\ &\left(\frac{dx}{dt}\right)_3=v_2,\ \left(\frac{dx}{dt}\right)_4=v_2-c_{20}. \end{aligned} \tag{18}$$

From $c_{i0}\geq 0\ (i=1,2)$, it follows that all characteristic speeds are not larger than their macroscopic flow speeds $v_i\ (i=1,2)$. In other words, the back disturbances cannot propagate forward. In addition, the relation $c_{i0}<v_i$ can be ensured from positive characteristic values, which implies that the propagation speeds of disturbances are not larger than the macroscopic flow speeds.

2 Decomposition of the model

We now consider four possible ρ_1–ρ_2 regions and develop their high-order continuum models according to each region's traffic condition. At present, these four regions are qualitatively identified and our further study is to accurately determine the boundaries of these regions.

(ⅰ) Both ρ_1 and ρ_2 are very small. Both lanes have free flows with vehicle speeds v_{10} and v_{20} respectively (zero-flow speeds, constant and $v_{20}<v_{10}$). So we have $\partial v_i/\partial x=0$ and $v_{ie}=v_{i0}$, i = 1,2. Substituting these into eq. (15) yields

$$\frac{\partial\rho_i}{\partial t}+v_{i0}\frac{\partial\rho_i}{\partial x}=s_i(x,t),\quad i=1,2. \tag{19}$$

Since there is no on-off ramp, so $s_1(x,t)=s_2(x,t)=0$. We can easily derive the analytical solutions about densities from eq. (19).

(ⅱ) ρ_1 is very small while ρ_2 is large. In this region, the traffic on lane 1 is in free-flow sate, having constant speed v_{10} and $\partial v_1/\partial x=0$ and $v_{1e}=v_{10}$; while the traffic on lane 2 is congested, with the speed v_2 (not constant but depends upon ρ_2 only, and $v_2<v_{20}<v_{10}$). Substituting these into eq. (15) yields

$$\frac{\partial\rho_1}{\partial t}+v_{10}\frac{\partial\rho_1}{\partial x}=s_1(x,t), \tag{20}$$

$$\frac{\partial\rho_2}{\partial t}+v_2\frac{\partial\rho_2}{\partial x}+2\rho_2\frac{\partial v_2}{\partial x}=s_2(x,t), \tag{21}$$

$$\frac{\partial v_2}{\partial t}+(v_2-c_{20})\frac{\partial v_2}{\partial x}=\frac{v_{2e}-v_2}{\tau_2}. \tag{22}$$

Note that in this region, the vehicles on lane 1 are not willing to change their lanes, so $s_1(x,t)=0$. Then, $s_2(x,t)=0$ holds too, because there is no on-off ramp on the freeway. With these results, we can derive the analytical solution about lane 1's density from (20), while eqs. (21) and (22) must be numerically solved by finite differences.

(ⅲ) ρ_1 is large and ρ_2 is small. In this region, lane 1 is congested with vehicle speed $v_1(\rho_1)$, and the speed on lane 2 becomes $v_2(\rho_1,\rho_2)$. Rewrite eq. (15) in detail:

$$\frac{\partial\rho_1}{\partial t}+v_1\frac{\partial\rho_1}{\partial x}+\rho_1\frac{\partial v_1}{\partial x}=s_1(x,t), \tag{23}$$

$$\frac{\partial v_1}{\partial t}+(v_1-c_{10})\frac{\partial v_1}{\partial x}=\frac{v_{1e}-v_1}{\tau_1}, \tag{24}$$

$$\frac{\partial\rho_2}{\partial t}+v_2\frac{\partial\rho_2}{\partial x}+2\rho_2\frac{\partial v_2}{\partial x}=s_2(x,t), \tag{25}$$

$$\frac{\partial v_2}{\partial t}+(v_2-c_{20})\frac{\partial v_2}{\partial x}=\frac{v_{2e}-v_2}{\tau_2}. \tag{26}$$

Clearly, if $v_1(x,t)<v_2(x,t)$, the vehicles on lane 1 will move to lane 2, so $s_1(x,t)\neq 0$ and $s_2(x,t)\neq 0$ in this situation, but $s_1(x,t)+s_2(x,t)=0$ has to be always satisfied. It is well known that eqs. (23)—(26) are hard to be analytically solved. We have to use the finite differences or other numerical methods as presented in the next section of this paper.

(ⅳ) Both ρ_1 and ρ_2 are large. In this region, since both lanes are very congested, vehicles on lane 1 will not change to lane 2, i.e. all vehicles travel on their own lanes separately. Then, eq. (15) can be expressed as follows:

$$\frac{\partial\rho_1}{\partial t}+v_1\frac{\partial\rho_1}{\partial x}+\rho_1\frac{\partial v_1}{\partial x}=0, \tag{27}$$

$$\frac{\partial v_1}{\partial t}+(v_1-c_{10})\frac{\partial v_1}{\partial x}=\frac{v_{1e}-v_1}{\tau_1}, \tag{28}$$

$$\frac{\partial\rho_2}{\partial t}+v_2\frac{\partial\rho_2}{\partial x}+2\rho_2\frac{\partial v_2}{\partial x}=0, \tag{29}$$

$$\frac{\partial v_2}{\partial t}+(v_2-c_{20})\frac{\partial v_2}{\partial x}=\frac{v_{2e}-v_2}{\tau_2}. \tag{30}$$

The above equations can be solved by numerical methods.

3 Stability analysis

We are interested in region (iii) because in this region, vehicles on lane 1 may shift to lane 2. Hence, our stability analysis is subject to this region.

Suppose that ρ_{i*} and v_{1*} are the steady-state solutions of eqs. (23)—(26), and let $\rho_i = \rho_{i*} + \xi_i$ and $v_i = v_{i*} + \eta_i$ be the perturbed solutions, where $\xi_i = \xi_i(x,t)$ and $\eta_i = \eta_i(x,t)$ represent small smooth perturbations to the steady-state solutions. Substituting $\rho_i = \rho_{i*} + \xi_i$ and $v_i = v_{i*} + \eta_i$ into eqs. (23)—(26), then taking Taylor series expansions at the point (ρ_{i*}, v_{i*}) and neglecting the higher order terms of ξ_i and η_i, we obtain the following equations:

$$\frac{\partial \xi_1}{\partial t} + v_{1*}\frac{\partial \xi_1}{\partial x} + \rho_{1*}\frac{\partial \eta_1}{\partial x} = 0, \tag{31}$$

$$\frac{\partial \eta_1}{\partial t} + (v_{1*} - c_{10})\frac{\partial \eta_1}{\partial x} = \frac{v'_{1e}(\rho_{1*})\xi_1 - \eta_1}{\tau_1}, \tag{32}$$

$$\frac{\partial \xi_2}{\partial t} + v_{2*}\frac{\partial \xi_2}{\partial x} + 2\rho_{2*}\frac{\partial \eta_2}{\partial x} = 0, \tag{33}$$

$$\begin{aligned}&\frac{\partial \eta_2}{\partial t} + (v_{2*} - c_{20})\frac{\partial \eta_2}{\partial x}\\ &= \frac{(\partial v_{2e}(\rho_{1*}, \rho_{2*})/\partial \rho_1)\xi_1 + v'_{2e}(\rho_{1*}, \rho_{2*})\xi_2 - \eta_2}{\tau_2},\end{aligned} \tag{34}$$

where $v'_{ie} = \dfrac{\partial v_{ie}}{\partial \rho_i}$, $i = 1, 2$. Assume $\partial v_{2e}/\partial \rho_1 << v'_{1e}$. Then the term $(\partial v_{2e}(\rho_{1*}, \rho_{2*})/\partial \rho_1)\xi_1$ in eq. (34) can be neglected. One can eliminate ξ_1 and ξ_2 from eqs. (31)— (34) and obtain

$$(\partial_t + c_1\partial_x)\eta_1 = -\tau_1[(\partial_t + c_{11}\partial_x)(\partial_t + c_{12}\partial_x)]\eta_1, \tag{35}$$

$$(\partial_t + c_2\partial_x)\eta_2 = -\tau_2[(\partial_t + c_{21}\partial_x)(\partial_t + c_{22}\partial_x)]\eta_2, \tag{36}$$

where $c_1 = v_{1*} + \rho_{1*}v'_{1e}(\rho_{1*})$, $c_2 = v_{2*} + 2\rho_{2*}v'_{2e}(\rho_{1*}, \rho_{2*})$, $c_{11} = v_{1*} - c_{10}$, $c_{12} = v_{1*}$, $c_{21} = v_{2*} - c_{20}$, $c_{22} = v_{2*}$. Using the method by Jiang et al.[10], we can prove that model (23)—(26) is linearly stable if and only if $c_{11}, c_{12}, c_{21}, c_{22},$ c_1 and c_2 satisfy the following conditions:

$$c_{11} \le c_1 \le c_{12}, \tag{37}$$

$$c_{21} \le c_2 \le c_{22}. \tag{38}$$

Otherwise, traffic instability will occur, which leads to some complex traffic phenomena like stop-and-go traffic.

4 Numerical tests

In this section, using the finite difference method, we numerically solve the model in region (iii) to demonstrate the model's ability in capturing complex traffic phenomena. The difference equations corresponding to eqs. (23)—(26) are as follows, for lane 1:

$$\begin{aligned}\rho_{1i}^{j+1} &= \rho_{1i}^j + \frac{\Delta t}{\Delta x}v_{1i}^j(\rho_{1(i-1)}^j - \rho_{1i}^j)\\ &\quad + \rho_{1i}^j\frac{\Delta t}{\Delta x}(v_{1i}^j - v_{1(i+1)}^j) + \Delta t s_{1i}^j.\end{aligned} \tag{39}$$

If the traffic is heavy, i.e. $v_{1i}^j < c_{10}$, then

$$\begin{aligned}v_{1i}^{j+1} &= v_{1i}^j + \frac{\Delta t}{\Delta x}(c_{10} - v_{1i}^j)(v_{1(i+1)}^j - v_{1i}^j)\\ &\quad + \frac{\Delta t}{\tau_1}(v_{1e} - v_{1i}^j),\end{aligned} \tag{40}$$

otherwise

$$v_{1i}^{j+1} = v_{1i}^j + \frac{\Delta t}{\Delta x}(c_{10} - v_{1i}^j)(v_{1i}^j - v_{1(i-1)}^j) + \frac{\Delta t}{\tau_1}(v_{1e} - v_{1i}^j). \tag{41}$$

For lane 2

$$\begin{aligned}\rho_{2i}^{j+1} &= \rho_{2i}^j + \frac{\Delta t}{\Delta x}v_{2i}^j(\rho_{2(i-1)}^j - \rho_{2i}^j)\\ &\quad + 2\rho_{2i}^j\frac{\Delta t}{\Delta x}(v_{2i}^j - v_{2(i+1)}^j) + \Delta t s_{2i}^j.\end{aligned} \tag{42}$$

If the traffic is heavy, i.e. $v_{2i}^j < c_{20}$, then

$$\begin{aligned}v_{2i}^{j+1} &= v_{2i}^j + \frac{\Delta t}{\Delta x}(c_{20} - v_{2i}^j)(v_{2(i+1)}^j - v_{2i}^j)\\ &\quad + \frac{\Delta t}{\tau_2}(v_{2e} - v_{2i}^j),\end{aligned} \tag{43}$$

otherwise

$$\begin{aligned}v_{2i}^{j+1} &= v_{2i}^j + \frac{\Delta t}{\Delta x}(c_{20} - v_{2i}^j)(v_{2i}^j - v_{2(i-1)}^j)\\ &\quad + \frac{\Delta t}{\tau_2}(v_{2e} - v_{2i}^j).\end{aligned} \tag{44}$$

In the above equations, index *j* represents the time interval and index *i* the road section. In order to investigate the consequences caused by small localized disturbance in an initial homogenous condition, we adopt the following initial variations of the average density ρ_{i0} $(i = 1, 2)$:

$$\begin{aligned}\rho_i(x,0) = \rho_{i0} + \Delta\rho_{i0}\Bigg\{&\cosh^{-2}\left[\frac{160}{L}\left(x - \frac{5L}{16}\right)\right]\\ &-\frac{1}{4}\cosh^{-2}\left[\frac{40}{L}\left(x - \frac{11L}{32}\right)\right]\Bigg\}, \quad i = 1, 2\end{aligned} \tag{45}$$

where L=32.2 (km) is the length of the road section under consideration. The following periodic boundary conditions are adopted:

$$\rho_i(L,t)=\rho_i(0,t), v_i(L,t)=v_i(0,t), \quad i=1,2. \qquad (46)$$

The equilibrium speed-density relationships are as follows:

$$v_{1e}=v_{10}(1-\rho_1/\rho_{1j}), \qquad (47)$$

$$v_{2e}=v_{20}(1-\rho_2/\rho_{2j})(1-(\rho_1+\rho_2)/(\rho_{1j}+\rho_{2j})), \qquad (48)$$

where ρ_{1j} and ρ_{2j} are the jam densities of lane 1 and lane 2, respectively. Assume the initial flows to be in local equilibrium everywhere, i.e. $v_1(x,0)=v_{1e}(\rho_1(x,0))$ and $v_2(x,0)=v_{2e}(\rho_1(x,0),\rho_2(x,0))$. Other parameters are as follows: $\Delta\rho_{10}=0.005$ (veh/m), $\Delta\rho_{20}=0.008$ (veh/m), Δx =100(m), Δt =1(s), v_{10} =40(m/s), v_{20}=30(m/s), c_{10} = 15(m/s), c_{20} =11(m/s), τ_1 =15(s), τ_2 =10(s), ρ_{1j} = 0.15 (vel/m), and ρ_{2j} = 0.2(vel/m). If $v_1<v_2$, let s_1 = $-0.01\rho_1 v_1$, $s_2=0.01\rho_1 v_1$. Using these data to compute c_{11} and c_{12} given below eq. (36), we will find that according to condition (37), the traffic on lane 1 becomes unstable when $\rho_{10}>0.056$ if vehicles on it are not allowed to shift to lane 2. Numerical results show that instability occurs only when $\rho_{10}>0.06$ if lane change is allowed. This means the critical value of causing instability is going up.

Figures 1—6 show the temporal evolution of traffic with localized perturbations $\Delta\rho_{10}=0.005$ (veh/m) and $\Delta\rho_{20}=0.008$ (veh/m) and different average densities ρ_{i0} $(i=1,2)$. The scales of space x, time t and density ρ are 100 m, 1 s and veh/m, respectively. The findings from these figures are summarized below:

(ⅰ) The density changes occur on lane 2 more frequently and more easily than that on lane 1;

(ⅱ) In Fig. 1 with parameters $\rho_{10}\le 0.03$ and $\rho_{20}\le 0.035$, the traffic flow densities on both lanes are very low, and hence the perturbation is dissipated without any amplification;

(ⅲ) In Figs. 2—3 with parameters $0.03<\rho_{10}\le 0.055$ and $0.035<\rho_{20}\le 0.06$, the density on lane 1 is very low, but on lane 2 small perturbations are amplified, leading to traffic instability;

(ⅳ) In Figs. 4—5 with parameters $0.055<\rho_{10}\le 0.06$ and $0.06<\rho_{20}\le 0.08$, the density on lane 2 is still low, while on lane 2 some local clusters appear (the situation of multiple clusters corresponds to a stop-and-go traffic);

(ⅴ) In Fig. 6 with parameters $\rho_{10}>0.06$ and $\rho_{20}>0.08$, a single but small cluster caused by small perturbations on lane 1 is observable and the stop-and-go traffic on lane 2 becomes very serious.

(ⅵ) The space-time evolution of the cluster on lane 1, although small, takes a turn for the shape when $\rho_{10}\ge 0.07$, and the case is the same with lane 2 when

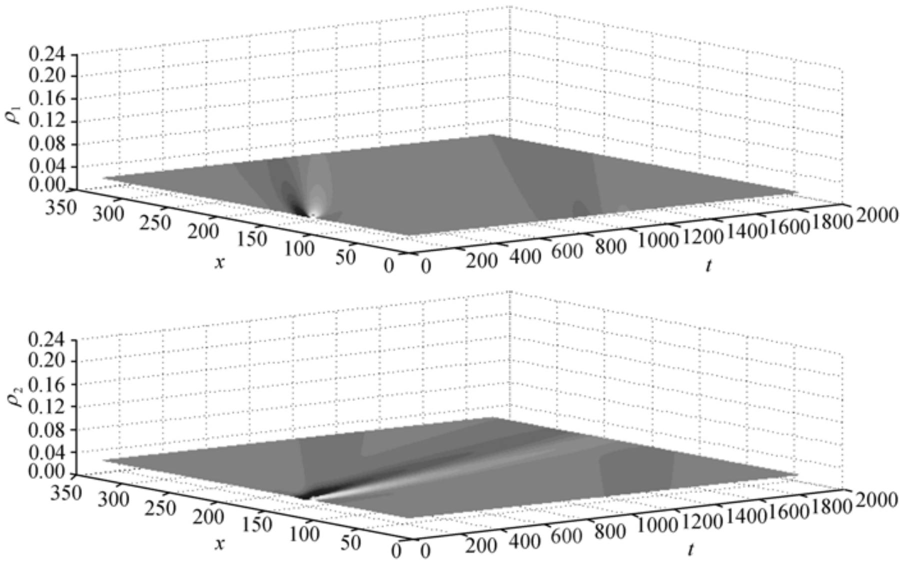

Fig. 1. $\rho_{10}=0.03$, $\rho_{20}=0.035$.

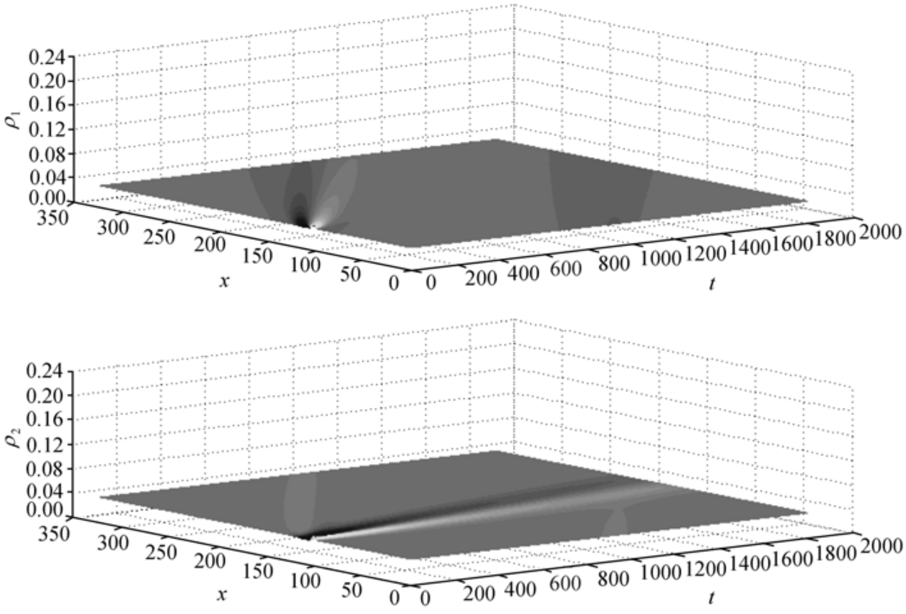

Fig. 2. $\rho_{10} = 0.036$, $\rho_{20} = 0.042$.

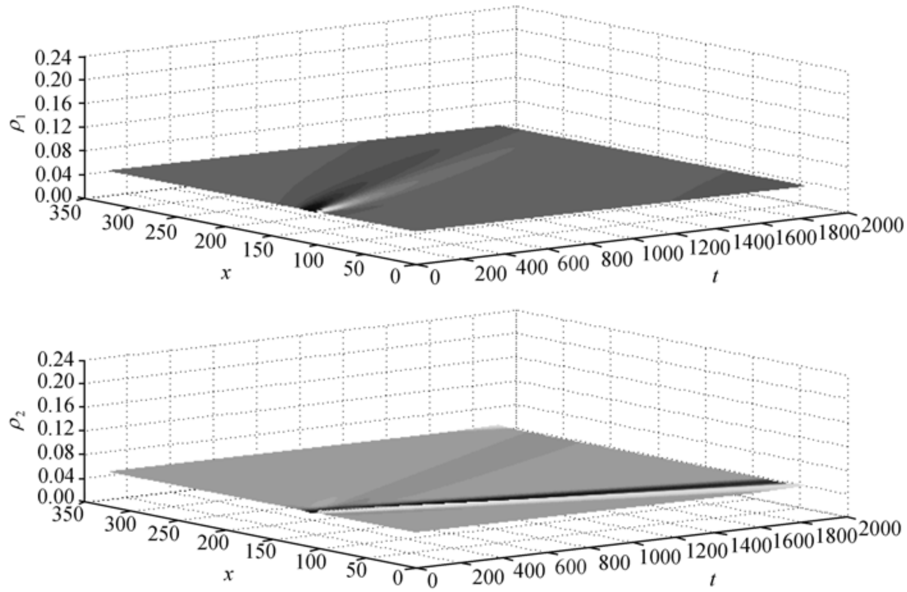

Fig. 3. $\rho_{10} = 0.055$, $\rho_{20} = 0.06$.

$\rho_{20} \geq 0.042$. When the initial densities are low, the perturbations on both lanes are dissipated quickly, leading vehicles to quicken their travels automatically, so the clusters stretch forward in shape. When densities become larger, the leading vehicles' accelerations are restricted and the clusters start to stretch backward in shape. The turning of clusters in shape appears later on lane 1 than on lane 2. This is because vehicles on lane 1 can shift to lane 2 so that lane 2 becomes congested more easily. This phenomenon cannot be observed on single lane freeway.

5 Conclusions

The previously developed high-order continuum models were mainly based on traffic situation with single lane only, but not suitable for analyzing the traffic flows on freeways with multiple lanes. Daganzo[12]TPP proposed the

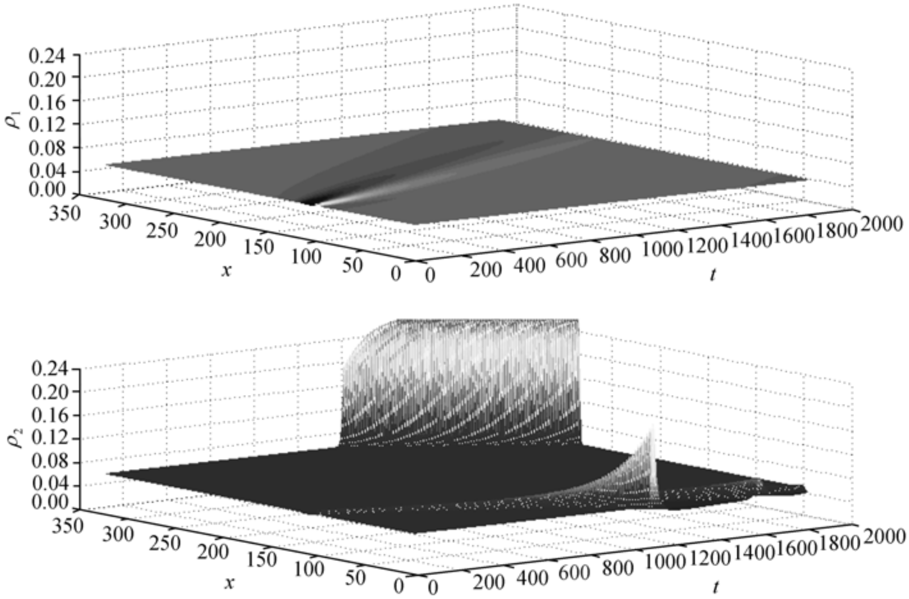

Fig. 4. $\rho_{10} = 0.06$, $\rho_{20} = 0.07$.

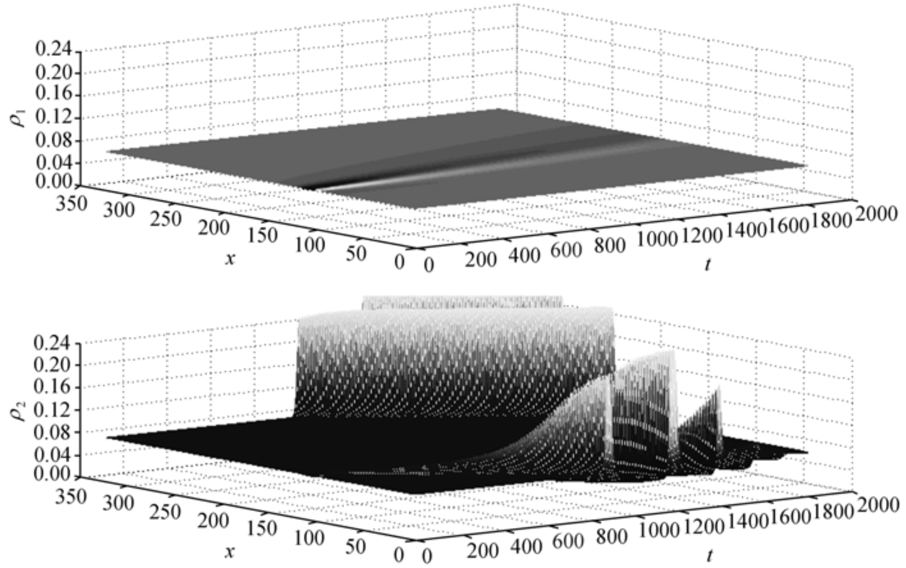

Fig. 5. $\rho_{10} = 0.07$, $\rho_{20} = 0.08$.

traffic flow models for two lanes, but without considering the effects by travel acceleration and driving inertia, hence they failed in faithfully describing non-equilibrium traffic flow dynamics. In this paper, we developed a new modeling framework for traffic flows on freeways with two lanes, by incorporating a dynamic equation with speed gradient. The numerical results show that the new model can correctly describe the movement and amplification of small perturbations, and reproduce complex traffic phenomena like stop-and-go. The new model is still simple in comparison with the traffic reality although it considers some flow interactions between lanes. Our on-going research is to develop a multi-lane model that integrates more factors affecting lane choice, overtaking through

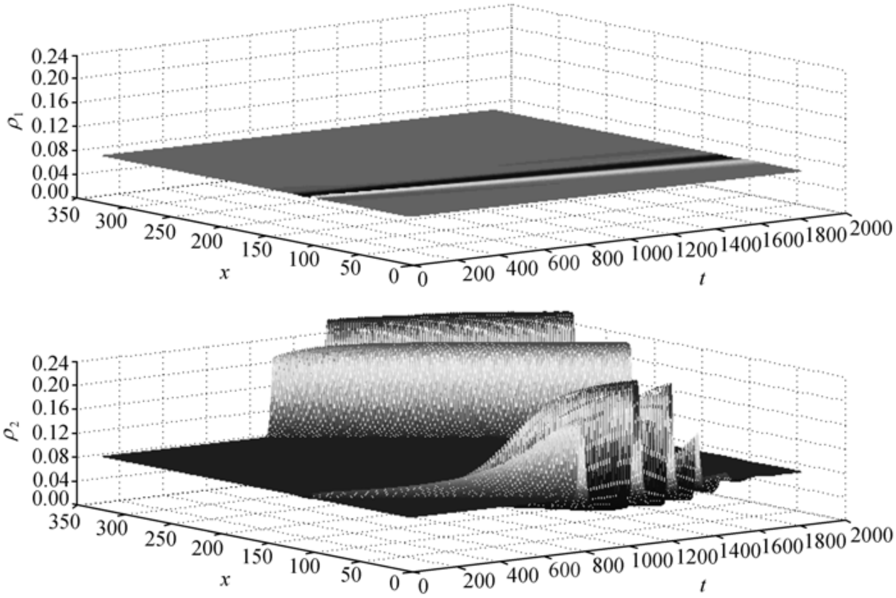

Fig. 6. $\rho_{10} = 0.08$, $\rho_{20} = 0.09$.

changing lane and car-following.

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