

When Did Growth Begin?

New Estimates of Productivity Growth in England from 1250 to 1870

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March 2, 2023

Abstract

We provide new estimates of the evolution of productivity in England from 1250 to 1870. Real wages over this period were heavily influenced by plague-induced swings in the population. We develop and implement a new methodology for estimating productivity that accounts for these Malthusian dynamics. In the early part of our sample, we find that productivity growth was zero. Productivity growth began in 1600—almost a century before the Glorious Revolution. We estimate productivity growth of 3% per decade between 1600 and 1760, which increased to 6% per decade between 1770 and 1860. Our estimates attribute much of the increase in output growth during the Industrial Revolution to a falling land share of production, rather than to faster productivity growth. Our evidence helps distinguish between theories of why growth began. In particular, our findings support the idea that broad-based economic change preceded the bourgeois institutional reforms of 17th century England and may have contributed to causing them. We estimate relatively weak Malthusian population forces on real wages. This implies that our model can generate sustained deviations from the “iron law of wages” prior the Industrial Revolution.

JEL Classification: N13, O40, J10

*We thank Venance Riblier and Caleb Wroblewski for excellent research assistance. We thank Bernard Beaudreau, Gregory Clark, Ernesto Dal Bó, Brad DeLong, Edward Nelson, James Robinson, Jean-Laurent Rosenthal, Jan De Vries, Jonathan Vogel, David Weinstein, Jacob Weisdorf and seminar participants at various institutions for valuable comments and discussions. We thank the Alfred P. Sloan Foundation, the Smith Richardson Foundation, and the Program for Economic Research at Columbia University for financial support.

1 Introduction

When did economic growth begin? A traditional view holds that economic growth began with the Industrial Revolution around 1800. Recent work has challenged this view pushing the date of the onset of growth back. Crafts (1983, 1985) and Harley (1982) revised downward previous estimates of growth in Britain during the Industrial Revolution. These new estimates indicate that British output per capita was larger by the mid-18th century than was previously thought implying that substantial growth must have occurred at an earlier date (see also Crafts and Harley, 1992). Acemoglu, Johnson, and Robinson (2005) argue that a First Great Divergence occurred starting around 1500 with Western Europe growing apart from other areas of the world following the discovery of the Americas and the sea route to India. They support this view with data on urbanization rates. Broadberry et al. (2015) argue that growth began even earlier than this. They present new estimates of GDP per person for Britain back to 1270. These data show slow but steady growth in GDP per person from the beginning of their sample. Finally, Kremer (1993) uses world population estimates to argue for positive but glacially slow growth for hundreds of thousands of years.

An important facet of the debate about when growth began is when *productivity* growth began. We contribute to this debate by constructing a new series for productivity growth in England back to 1250. Figure 1 plots our new productivity series (solid black line). Our main finding is that productivity growth in England began in 1600. Between 1250 and 1600, we estimate that productivity growth was zero.¹ We estimate productivity growth of about 3% per decade from 1600 to 1760. Productivity growth then increased to 6% per decade between 1770 and 1860. We attribute much of the increase in output growth during the Industrial Revolution to a fall in the land share of production rather than to an increase in productivity growth.

Our results help distinguish between different theories of *why* growth began. They suggest that researchers should focus on developments proximate to the 16th and 17th centuries. An important debate regarding the onset of growth centers on the role of institutional change. Our results help sharpen this debate. We find that productivity growth began almost a century before the Glorious Revolution and well before the English Civil War. While the institutional changes associated with these events may have been important for subsequent growth, researchers must look to earlier events for the seeds of modern growth. Plausible candidates include the Reformation, the decline of feudalism, the rise of the yeoman, movable type printing and the associated increase in literacy,

¹The positive but glacially slow productivity growth rate implied by Kremer's (1993) population data for the period 1200 to 1500 lies within our credible set.

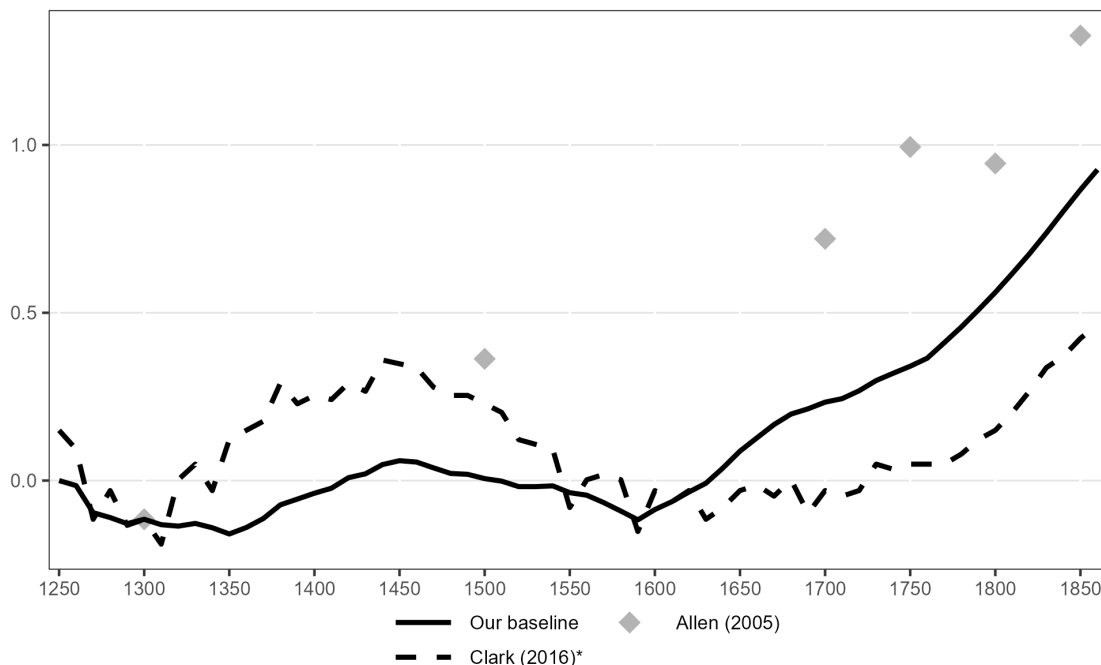


Figure 1: Estimates of Productivity in England

Note: Each series is the natural logarithm of productivity. The series denoted by “Clark (2016)*” is the series from Clark (2016) extended to 1860. We received this series from Clark in private correspondence. Clark’s series estimates TFP for the entire economy based on a dual approach. Allen’s (2005) estimates are for TFP in the agricultural sector using a primal approach. Our baseline productivity series is normalized to zero in 1250. The other two series are normalized to match our baseline series in 1300.

and expansion of international trade. We discuss these in more detail below.

The most comprehensive existing productivity series for England was constructed by Clark (2010, 2016). Clark estimated changes in TFP for the entire English economy from 1209 onward using the “dual approach”—i.e., as a weighted average of changes in real factor prices (e.g., Hsieh, 2002). Figure 1 plots Clark’s series over our sample period (broken black line).² A striking feature of this series is that it implies that productivity in England was no higher in the mid-19th century than in the 15th century. This result does not line up well with other existing (less comprehensive) measures of productivity in England or with less formal assessments of the English economy. For example, Allen (2005) estimates that TFP in agriculture was 162% higher in 1850 than in 1500 (grey diamonds in Figure 1).³ Clark himself commented that if the fluctuations in his series are not measurement error “they imply quite inexplicable fluctuations in the performance of the preindustrial

²Clark (2016) published an update of Clark’s better-known 2010 series for the shorter time period 1250-1600. The series we plot in Figure 1 is Clark’s 2016 series extended to 1860. We received this series from Clark by private correspondence. The 2016 series differs from the 2010 series prior to 1600 due to a new land rent series and because Clark corrected an important error in the 2010 series. We discuss this in more detail in Appendix E.

³Allen (2005) employs the familiar “primal approach” of Solow (1957), i.e., subtracts a weighted average of growth in factor inputs from output growth, but is only able to do this for agriculture and for a few years.

economy.”

Our conclusions about productivity in England are quite different from those of Clark (2010, 2016). According to our estimates, productivity in England was roughly 130% higher in 1850 than in 1500 rather than being essentially unchanged. We also estimate smaller fluctuations in productivity prior to 1600. In particular, our productivity series falls much less between 1450 and 1600. These substantial differences arise from differences in the data and methodology we use. We take the labor demand curve as our starting point and estimate changes in productivity as shifts in the labor demand curve. This means that the key data series that inform our estimates are real wages and measures of labor supply (population and days worked per year). Real wages and population are arguably among the best measured series of all economic time series over our long sample period. In contrast, an important input into Clark’s productivity series is a series for land rents that is very noisy prior to the 18th century. We use data on land rents only after 1760. We also improve on Clark’s treatment of returns to capital by using more data and allowing for measurement error.

Our approach is best understood by considering Figure 2. This figure presents a scatter plot of real wages in England (y-axis) against labor supply in England (x-axis). Variation in labor supply in England is mostly driven by variation in the population, but also affected by variation in days worked per worker. From 1250 to 1300, the population of England (and labor supply) increased and real wages decreased. The period from 1300 to 1450 was a period of frequent plagues—the most famous being the Black Death of 1348. Over this period, the population of England fell by a factor of two resulting in a sharp drop in labor supply. Over this same period, real wages rose substantially. Then, from 1450 to 1600, the population (and labor supply) recovered and real wages fell. In 1630, the English economy was back to almost exactly the same point it was at in 1300.

One way to explain these dynamics between 1300 and 1630 is as movements along a stable labor demand curve with no change in productivity. Had productivity grown between 1300 and 1630, the economy could not have returned to essentially the same point in 1630 as it was in 1300 since the labor demand curve would have shifted up and to the right over the intervening period. Then in the 17th century, something important seems to change. The points start moving off the prior labor demand curve. Specifically, they start moving up and to the right relative to the earlier curve. This suggests that productivity started growing in the 17th century in England.

The basic idea behind our approach is to estimate a labor demand curve for England and then

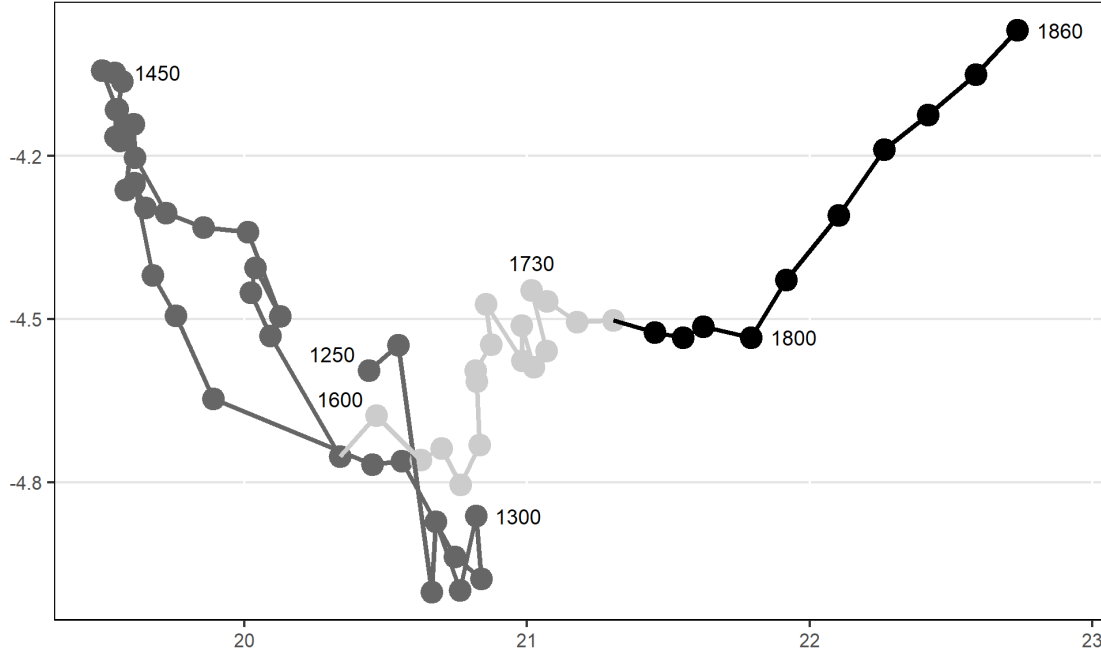


Figure 2: Real Wages and Labor Supply

Note: The figure presents a scatter plot of the logarithm of real wages in England against the logarithm of labor supply in England over the period 1250-1860. The data on real wages are from Clark (2010). Estimates of labor supply are based on our calculations. Labor supply varies mainly due to variation in the population, but also due to changes in days worked per person.

back out productivity growth as shifts in this labor demand curve. To get a better sense for how this approach works, consider the following simple labor demand curve for a pre-modern society

$$W_t = (1 - \alpha)A_t \left(\frac{Z}{L_t} \right)^\alpha,$$

where W_t denotes real wages, A_t denotes productivity, Z denotes land (which is fixed), and L_t denotes labor. The model we consider later in the paper is more general. But the basic challenge we face can be grasped using this simple model. If we take logarithms, this equation becomes

$$w_t = \phi - \alpha l_t + a_t,$$

where lower case letters denote logarithms of upper case letters. Given this equation, a simple-minded empirical approach would be to regress wages on labor and equate the residual from that regression with productivity. In our context, however, this simple-minded approach is not likely to work well because of Malthusian population forces. In a Malthusian world, increases in productivity induce increases in the population and therefore the labor force (see the points after

1630 in Figure 2). This means that in a Malthusian world l_t and a_t are correlated and an OLS estimate of α is likely to yield nonsense.

To overcome this challenge, we take a structural approach. We write down a Malthusian model of the economy which includes both a labor demand curve and a model for the evolution of the population over time as a function of real incomes. We then estimate this model. In other words, we model the endogeneity of population dynamics. This allows us to produce an estimate of the labor demand curve and an estimate of productivity growth that accounts for the endogeneity implied by Malthusian population dynamics. Intuitively, the model implies a series of exogenous population shocks purged of Malthusian population dynamics and these are used to estimate the labor demand curve.

Since our analysis extends into the early industrial era, we must confront the fact that the importance of land as a factor of production fell rapidly with the spread of steam power, which meant that the production of energy was no longer land intensive (Wrigley, 2010). To capture this crucial development, we allow the factor shares in our model to change after the onset of the Industrial Revolution. We use data on land rents to pin down how fast the land share changed after 1760. The modest increase in land rents that we observe in the face of explosive growth in labor and capital after 1760 leads us to estimate a rapidly falling land share of production.

The fact that we allow for this structural transformation implies that the standard way of measuring productivity (a multiplicative A_t in front of a function $F(L_t, K_t, \dots)$) is no longer valid. Following Caves, Christensen, and Diewert (1982) we derive a Malmquist productivity index (Malmquist, 1953) for our setting. The Malmquist index reduces to A_t in the familiar setting of constant factor shares, but remains valid even when the structure of the production function is changing.

Allowing the land share to fall after the onset of the Industrial Revolution has important implications for our estimates of productivity. If we don't allow for this change, we estimate a much larger break in productivity in 1760. Productivity growth is, of course, just a measure of our ignorance. Modelling the shift of production away from land intensive technology allows us to explain a larger part of growth during the Industrial Revolution, leaving less for the residual.

Our estimates also shed light on the lack of real wage growth during the latter part of the 18th century, sometimes referred to as "Engel's Pause" (Engels, 1845, Allen, 2009b). Our Malthusian model implies that during this period real wages were held back by very rapid increases in the population, which in a Malthusian world put downward pressure on the marginal product of

labor. This explanation contrasts with the common idea that the absence of real wage growth during this period resulted from the lion's share of the fruits of technical change going to capital as opposed to labor. This idea has received attention in the modern context in relation to the development of automation and artificial intelligence (Acemoglu and Restrepo, 2019).

In addition to estimates of productivity, our methodology yields estimates of the speed of Malthusian population dynamics in pre-modern England. Our estimates imply that these population dynamics were very slow: a doubling of real incomes led to only a 6 percentage point per decade increase in population growth. Together with our other estimates, this implies that the half-life of a plague-induced drop in the population was roughly 170 years prior to the onset of the Industrial Revolution. As land shares fell after 1760, the Malthusian population dynamics became even slower and weaker. By 1860, we estimate the half-life of population shock to have risen to 420 years. Earlier estimates of the speed of Malthusian population dynamics in England also indicate that they were slow. For example, Lee and Anderson (2002) find a half-life of 107 years, while Crafts and Mills (2009) find a half-life of 431 years. Chaney and Hornbeck (2016) document very slow population dynamics in Valencia after the expulsion of the Moriscos in 1609.

The weakness of the Malthusian population dynamics we estimate imply that our model is consistent with sustained deviations from "the iron law of wages" (i.e., that wages in a Malthusian economy are stuck at subsistence). Modest productivity growth over a few centuries can temporarily overwhelm the Malthusian population dynamics in our model and result in sustained periods of real wages several times higher than at other times. Once productivity growth falters, real wages will slowly fall back to a lower level. But this will take several centuries. Our model can therefore make sense of episodes that historian sometime refer to as "golden ages" or "efflorescences" (Goldstone, 2002).

We are not the first to plot a figure like Figure 2 and argue that it has implications about the evolution of productivity in England. We formalize this intuitive idea and assess what exactly it implies about productivity. Clark (2005, 2007a) discuss informally how shifts in the labor demand curve of a Malthusian model can be informative about the timing of the onset of economic growth. The existing papers most closely related to ours from a methodological point of view are Lee and Anderson (2002) and Crafts and Mills (2009). These papers structurally estimate a Malthusian model of the English economy, as we do. However, their analysis does not incorporate capital or days worked per workers and it does not allow the land share of production to change after the onset of the Industrial Revolution. These differences imply that their productivity series is very

different than our. In addition, their sample period is considerably shorter than ours (theirs starts in 1540 while ours start in 1250). This last point means that they cannot address the question of when growth began.

Our paper is also related to the literature in macroeconomics on the transition from pre-industrial stagnation to modern growth—often referred to as the transition “from Malthus to Solow.” Important papers in this literature include Galor and Weil (2000), Jones (2001), and Hansen and Prescott (2002). Relative to these papers, our work is more empirical. We contribute detailed estimates of the evolution of productivity, while these papers propose theories of how productivity growth rose. Our work is also related to recent work by Hansen, Ohanian, and Ozturk (2020).

Our paper proceeds as follows. Section 2 presents our Malthusian model of the economy. Section 3 discusses the data we use and our estimation strategy. Section 4 presents our results on productivity. Section 5 presents our results on the strength of the Malthusian population force. Section 6 presents our estimates of the population. Section 7 concludes.

2 A Malthusian Model of the Economy

We now present a simple model of the pre-industrial and early industrial English economy. While our focus is on the pre-industrial period and, in particular, the possibility that productivity growth may have begun prior to the Industrial Revolution, our data extends well into the early industrial period. It is therefore important for our model to capture both the character of the pre-industrial economy and the early industrial economy. The role of land in production is particularly important in this regard. Prior to the Industrial Revolution, land was a hugely important factor of production. However, the advent of steam power led to a sharp fall in the role of land in production as fossil fuels substituted for human and animal power in the production of energy (and the role of food production in the economy shrank).

To capture this change, we distinguish between the pre-industrial period and the early industrial period and allow the importance of land in production to change after the onset of the Industrial Revolution. We date the shift from the pre-industrial period to the early industrial period to between 1760 and 1770. This date is dictated by two considerations. First, this is a traditional date for the beginning of the Industrial Revolution. Second, estimating a changing production function requires systematic data on rents and the capital stock. Estimates of the capital stock in England are, as far as we are aware, only available after 1760.

2.1 A Model of the Pre-industrial Economy

We model time as discrete and denote it by a subscript t . Since we use decadal data later in the paper, each time period in the model is meant to represent a decade. Output is produced with land, capital, and labor according to the following production function:

$$Y_t = F_t(Z, K_t, L_t) = A_t Z^\alpha K_t^\beta L_t^{1-\alpha-\beta},$$

where Y_t denotes output, Z denotes land (which is fixed), K_t denotes capital, and L_t denotes labor (in units of worker days). We assume that producers hire workers in a competitive labor market taking wages as given. Producer optimization then gives rise to the following labor demand curve:

$$W_t = (1 - \alpha - \beta) A_t Z^\alpha K_t^\beta L_t^{-\alpha-\beta},$$

where W_t denotes the real daily wage. Taking logarithms of this equation yields

$$w_t = \tilde{\phi} + a_t + \beta k_t - (\alpha + \beta) l_t, \quad (1)$$

where lower case letters denote logarithms of upper case letters and $\tilde{\phi} = \log(1 - \alpha - \beta) + \alpha \log Z$.

We assume that producers accumulate capital to the point where the marginal product of capital is equal to its user cost. This gives rise to the following capital demand equation:

$$r_t + \delta = \beta A_t Z^\alpha K_t^{\beta-1} L_t^{1-\alpha-\beta}, \quad (2)$$

where r_t is the rental rate for capital and δ is the rate of depreciation of capital. Since we do not have data on capital for the pre-industrial period, we use the capital demand equation – equation (2) – to eliminate K_t from the labor demand equation – equation (1). Taking logs of the resulting equation yields

$$w_t = \tilde{\phi}' + \frac{1}{1-\beta} a_t - \frac{\alpha}{1-\beta} l_t - \frac{\beta}{1-\beta} \log(r_t + \delta), \quad (3)$$

where

$$\tilde{\phi}' = \frac{\beta}{1-\beta} \log \beta + \log(1 - \alpha - \beta) + \frac{\alpha}{1-\beta} \log Z.$$

As in all models, productivity is a catch-all variable capturing the influence of all variables that are not explicitly modeled in the production function. This includes changes in technology and in

institutions, but may also capture effects of international trade and colonial exploitation.

We assume that the labor force in the economy is proportional to the population and that each worker works D_t days per year. This implies that

$$L_t = \Lambda D_t N_t,$$

where N_t denotes the population and Λ is a constant. Taking logs of this equation and using the resulting equation to eliminate l_t in equation (3) yields

$$w_t = \phi + \frac{1}{1-\beta} a_t - \frac{\alpha}{1-\beta} (d_t + n_t) - \frac{\beta}{1-\beta} \log(r_t + \delta), \quad (4)$$

where $\phi = \tilde{\phi}' - \alpha/(1-\beta)\lambda$.

A central aspect of our model is the law of motion for the population. Following Malthus (1798), we assume that population growth is increasing in real income:

$$\frac{N_t}{N_{t-1}} = \Omega (W_{t-1} D_{t-1})^\gamma \Xi_t,$$

where Ω is a constant, γ is the elasticity of population growth with respect to real income, and Ξ_t denotes other (exogenous) factors affecting population growth. Taking logarithms of this equation yields

$$n_t - n_{t-1} = \omega + \gamma(w_{t-1} + d_{t-1}) + \xi_t. \quad (5)$$

Malthus argued that both the birth rate and the death rate varied with real income. He described “preventive checks” on population growth that lowered birth rates. These included contraception, delayed marriage, and regulation of sexual activity during marriage. Malthus also described “positive checks” on population growth that raised death rates. These include disease, war, severe labor, and extreme poverty. In our model, the parameter γ captured the elasticity of both birth rates and death rates with respect to income. This parameter therefore captures any tendency of either preventive or positive checks to lower population growth when income falls.

We assume that the logarithm of productivity is made up of a permanent and transitory component:

$$a_t = \tilde{a}_t + \epsilon_{2t}, \quad (6)$$

where

$$\tilde{a}_t = \mu + \tilde{a}_{t-1} + \epsilon_{1t}, \quad (7)$$

$\epsilon_{1t} \sim \mathcal{N}(0, \sigma_{\epsilon_1}^2)$, and $\epsilon_{2t} \sim \mathcal{N}(0, \sigma_{\epsilon_2}^2)$. Both ϵ_{1t} and ϵ_{2t} are independently distributed over time. Here, \tilde{a}_t is the permanent component of productivity, which follows a random walk with drift, while ϵ_{2t} is the transitory component of productivity. The average growth rate of productivity is given by the parameter μ . This is a key parameter in our model. As we describe in more detail below, we allow for structural breaks in μ , i.e., changes in the average growth rate of productivity.

We allow for two types of exogenous population shocks:

$$\xi_t = \xi_{1t} + \xi_{2t}. \quad (8)$$

First, we allow for “plague” shocks:

$$\exp(\xi_{1t}) \sim \begin{cases} \beta(\beta_1, \beta_2), & \text{with probability } \pi \\ 1, & \text{with probability } 1 - \pi \end{cases} \quad (9)$$

These plague shocks occur infrequently (with probability π) but when they occur they kill a (potentially sizable) fraction of the population. The fraction of the population that survives follows a beta distribution $\beta(\beta_1, \beta_2)$. The historical record indicates that plagues ravaged Europe frequently in the 14th and 15th centuries and continued to strike until the 17th century (Gottfried, 1983, Shrewsbury, 1970). Europe had no significant plague outbreak from the late 8th to the mid-14th century. This changed with the Black Death, which probably originated on the steppes of Mongolia, reached China in the 1330s, travelled west through trade routes, and landed in England in the summer of 1348. For three centuries after the Black Death, the plague would reappear every few decades, and wipe out a significant share of the population each time.⁴ In England, the last major outbreak was the Great Plague of London in 1665-66. There is no shortage of potential explanations for the relatively sudden disappearance of plague—collective immunity, better nutrition, changes in the dominant rat species, improved quarantine methods, among others (Appleby, 1980). In addition to the “plague,” smallpox, measles, typhus, and dysentery were frequent

⁴Plague is caused by a bacillus, *Yersinia pestis*, which naturally lives in the digestive tract of rodent fleas (although some epidemiologists contend that human fleas were the main carriers (Appleby, 1980, Dean et al., 2018)). Rat populations thus constitute a reservoir for plague. Periodically, the bacilli would multiply in the fleas’ stomachs, causing them to regurgitate in the rat’s bloodstream. This would kill the rats. Once the rats died, the fleas would turn to other hosts, including humans. This mechanism explains the recurring character of the disease.

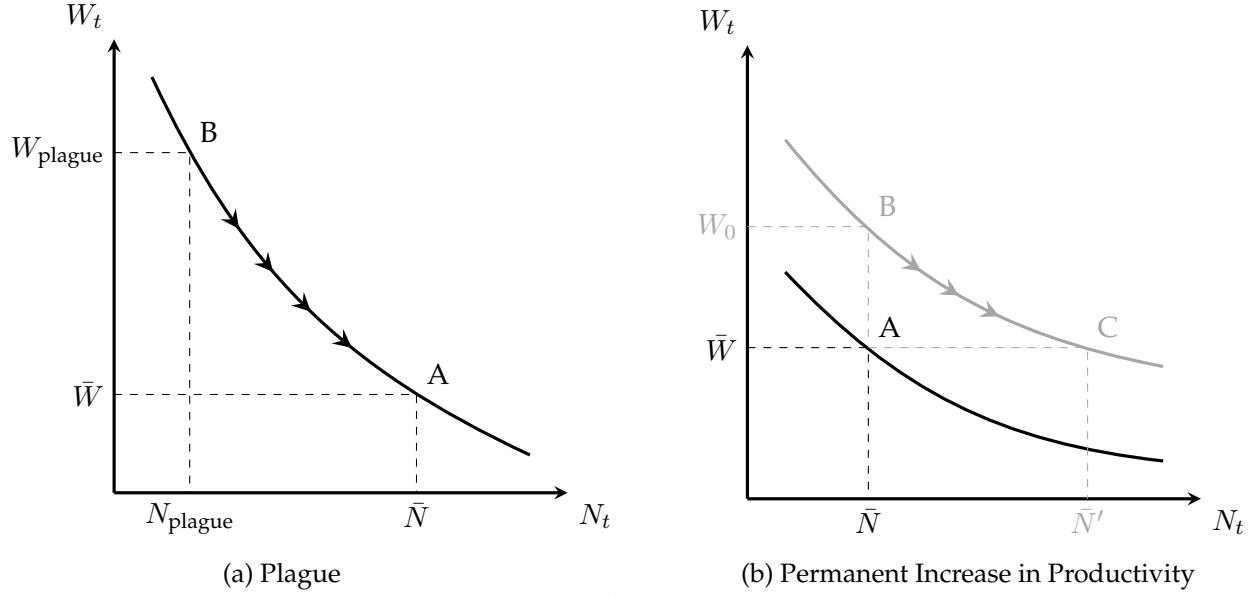


Figure 3: Malthusian Dynamics

mass killers. While they were certainly catastrophic to those afflicted, these outbreaks are ideal from an identification standpoint: they are frequent, sizeable and plausibly unrelated to the state of the economy.

In addition to the plague shocks, we allow for a second type of population shocks: $\xi_{2t} \sim \mathcal{N}(0, \sigma_{\xi_2}^2)$. Both of the population shocks are independently distributed over time. Together these population shocks are meant to capture a host of potential influences on population growth, in addition to plagues.

It is useful to consider the dynamics of the model after a plague shock and a productivity shock. Figure 3a depicts the evolution of real wages and the population after a plague. The downward sloping curve in the figure represents the labor demand curve in the economy. Suppose the economy is initially in a steady state at point A, but then a plague strikes that kills a fraction of the population. The economy will then jump from point A to point B. At point B the population is lower reflecting the death caused by the plague and real wages are higher reflecting the higher marginal product of labor of the surviving workers. After the plague, the economy will then gradually move down the labor demand curve until it reaches point A again. This movement occurs because higher real wages lead to positive population growth. As the population grows the economy moves along the labor demand curve and real wages fall. Once the economy is back at point A, real wages are again sufficiently low that population growth is zero.

Figure 3b depicts the evolution of real wages and the population after a productivity shock.

Suppose again that the economy is initially in a steady state at point A . This time, however, suppose a permanent increase in productivity shock occurs. This shifts the labor demand curve out. In the short run, the population is fixed. The economy therefore jumps from point A to point B . Over time after the shock, the economy will then gradually move along the new labor demand curve until it gets to a new steady state at point C . These dynamics are again due to high wages causing positive population growth and a larger and larger population reducing wages until they are back at a point where population growth is zero.

Notice that the dynamics of the economy after these two shocks are quite different. In the case of a plague shock, the population moves sharply on impact but returns to its original level in the long run. In the case of a permanent change in productivity, however, real wages move on impact but not the population, while the population changes over time and ends up at different point than the economy started at. The main empirical challenge we face is distinguishing between labor demand (productivity) shocks and labor supply (plague) shocks. It is these differences in dynamics that help us to distinguish these two empirically.

2.2 A Model of the Early Industrial Economy

The last century of our data covers the early industrial period in England. A crucial development over this period is the rapid fall in the importance of land as a factor of production. The primary driving force in this development was the introduction of the steam engine powered by fossil fuels. This technological advance meant that the production of energy was no longer land intensive (Wrigley, 2010). To capture this change, we allow the exponents in the production function to change after the onset of the Industrial Revolution. A fall in α_t will then capture the fall in the importance of land as a factor of production, and the evolution of β_t will determine to what extent it is capital or labor that increases in importance. The production function we assume for the early industrial period is

$$Y_t = F_t(Z, K_t, L_t) = A_t Z^{\alpha_t} K_t^{\beta_t} L_t^{1-\alpha_t-\beta_t}, \quad (10)$$

which is the same as before except that the exponents α_t and β_t are now time varying. In this case, the labor demand curve – equation (4) – becomes:

$$w_t = \phi_t + \frac{1}{1-\beta_t} a_t - \frac{\alpha_t}{1-\beta_t} (d_t + n_t) - \frac{\beta_t}{1-\beta_t} \log(r_t + \delta), \quad (11)$$

where:

$$\phi_t = \frac{\beta_t}{1 - \beta_t} \log \beta_t + \log(1 - \alpha_t - \beta_t) + \frac{\alpha_t}{1 - \beta_t} \log Z - (\alpha_t + \beta_t)\lambda.$$

The fact that α_t and β_t can change implies that more data is needed to identify the model: an increase in wages for a given level of labor supply could be due to an increase in a_t or a fall in α_t . To address this issue, we make use of the demand curves for land and capital. We assume that potential renters and owners of land will trade until the rental price of land equals its marginal product:

$$S_t = \alpha_t A_t Z^{\alpha_t - 1} K^{\beta_t} L_t^{1 - \alpha_t - \beta_t} \quad (12)$$

where S_t denotes the rental price of land. Capital demand is the same as before – equation (2) – except that α_t and β_t are time varying. Taking logarithms and manipulating the land, capital and labor demand curves yields:

$$s_t = w_t + n_t + d_t - \log Z + \log \alpha_t - \log(1 - \alpha_t - \beta_t) + \lambda \quad (13)$$

$$\log(r_t + \delta) = w_t + n_t + d_t - k_t + \log \beta_t - \log(1 - \alpha_t - \beta_t) + \lambda \quad (14)$$

These two extra equations pin down α_t and β_t . Simple manipulation of the demand curves for labor, land, and capital – which we spell out in detail in Appendix A – establishes three intuitive results. First, an increase in s_t (land rents) holding other variables constant implies an increase in α_t (the land share) and a decrease in both β_t (the capital share), and $1 - \alpha_t - \beta_t$ (the labor share). Second, an increase in r_t (the return on capital) holding other variables constant implies an increase in β_t (the capital share) and a decrease in both α_t (the land share), and $1 - \alpha_t - \beta_t$ (the labor share). Third, an increase in w_t (wages) holding other variables constant implies an increase in $1 - \alpha_t - \beta_t$ (the labor share) and a decrease in both α_t (the land share) and β_t (the capital share). At a mechanical level, we are adding two equations per time period – equations (13) and (14) – and two observables per time period – s_t and k_t .

2.3 Measuring Productivity with Structural Transformation

Allowing the exponents in the production function to change raises another important complication. In this case, A_t is no longer a natural measure of productivity.⁵ Productivity is meant to

⁵One simple way to see this is to consider a change in the units that we use to express labor. Suppose $\tilde{L}_t \equiv \psi L_t$, then: $Y_t = A_t Z^{\alpha_t} K_t^{\beta_t} L_t^{1 - \alpha_t - \beta_t} = (A_t / (\psi^{1 - \alpha_t - \beta_t})) Z^{\alpha_t} K_t^{\beta_t} \tilde{L}_t^{1 - \alpha_t - \beta_t}$. With the new units for labor, it is $A_t / \psi^{1 - \alpha_t - \beta_t}$ rather than A_t that multiplies the factors of production. If α_t and β_t change over time, $A_t / \psi^{1 - \alpha_t - \beta_t}$ will behave differently from A_t . Clearly, a more general concept of productivity is needed. See Appendix B for a discussion of how

capture the rate at which inputs can be converted into outputs. In settings with multiple inputs (or outputs), how to operationalize this concept is ambiguous. In some cases, such as $Y_t = A_t F(X_t)$ where X_t denotes a vector of inputs, all reasonable measures of productivity agree (in this case A_t). But in the more general case of $Y_t = F_t(X_t)$, this is not the case. Caves, Christensen, and Diewert (1982) introduce the notion of a Malmquist productivity index for a quite general case of production technologies, based on ideas in Malmquist (1953). This index uses the notion of the distance between the input-output vector chosen at one point in time and the technological frontier at another point in time. For example, the distance of the input-output vector that is chosen in period $t + 1$ from the time t technological frontier is $D_t(X_{t+1}, Y_{t+1}) = F_{t+1}(X_{t+1})/F_t(X_{t+1})$ – i.e., actual output at time $t + 1$ divided by counterfactual output using the input vector of time $t + 1$ but the time t production function. Analogously, the distance of the input-output vector that is chosen at time t from the time $t + 1$ technological frontier is $D_{t+1}(X_t, Y_t) = F_t(X_t)/F_{t+1}(X_t)$.

Caves, Christensen, and Diewert (1982) recommend using a geometric average of $D_t(X_{t+1}, Y_{t+1})$ and $D_{t+1}(X_t, Y_t)^{-1}$ as a Malmquist index of productivity:

$$\frac{M_t}{M_{t-1}} = \sqrt{\frac{F_t(Z, K_t, L_t)F_t(Z, K_{t-1}, L_{t-1})}{F_{t-1}(Z, K_t, L_t)F_{t-1}(Z, K_{t-1}, L_{t-1})}}, \quad M_0 = 1. \quad (15)$$

We adopt this recommendation. With constant exponents α and β , the growth rates of M_t and A_t are the same (i.e., $M_t/M_{t-1} = A_t/A_{t-1}$). More generally, however, the growth rate of M_t will differ from the growth rate of A_t in important ways. See Appendix B for a more detailed discussion of the Malmquist index.

With the production function (10), the logarithm of the Malmquist index is

$$\hat{m}_t = \hat{a}_t + \hat{\alpha}_t \log Z + \hat{\beta}_t \bar{k}_t - (\hat{\alpha}_t + \hat{\beta}_t)(\bar{d}_t + \bar{n}_t + \lambda), \quad (16)$$

where hats denote deviations from the previous period, $\hat{x}_t = x_t - x_{t-1}$, and bars denote the average of period $t - 1$ and period t , $\bar{x}_t = (x_{t-1} + x_t)/2$. Once again, if α_t and β_t are constant, this expression just collapses to: $\hat{m}_t = \hat{a}_t$.

As in the pre-industrial era, we assume that the logarithm of productivity (m_t) is subject to permanent and transitory shocks in the early-industrial era:

$$m_t = \tilde{m}_t + \epsilon_{2t}, \quad (17)$$

the Malmquist index (introduced below) avoids this issue.

where

$$\tilde{m}_t = \mu + \tilde{m}_{t-1} + \epsilon_{1t}, \quad (18)$$

$\epsilon_{1t} \sim \mathcal{N}(0, \sigma_{\epsilon_1}^2)$, and $\epsilon_{2t} \sim \mathcal{N}(0, \sigma_{\epsilon_2}^2)$. As before, ϵ_{1t} and ϵ_{2t} are independently distributed over time. Here, \tilde{m}_t is the permanent component of productivity, which follows a random walk with drift, while ϵ_{2t} is the transitory component of productivity.

3 Data and Estimation

We reproduce the equations and distributional assumptions of our full model in Appendix C for convenience. We estimate the model using data on wages w_t , the population n_t , days worked d_t , the rate of return on land and rent charges as proxies for $\log(r_t + \delta)$, land rents s_t (after 1760), and the capital stock k_t (after 1760). We do this using Bayesian methods. In particular, we use a Hamiltonian Monte Carlo sampling procedure (Gelman et al., 2013, Betancourt, 2018).⁶ This methodology allows us to easily handle missing data and measurement error in our historical data. The unobservables that we make inference about are the permanent component of productivity \tilde{a}_t (\tilde{m}_t after 1760), the shocks to this component ϵ_{1t} , the transitory shocks to productivity ϵ_{2t} , land rents s_t (before 1760), the capital stock k_t (before 1760), the plague shocks ξ_{1t} , the symmetric population shocks ξ_{2t} , α_t and β_t (after 1760), and the parameters ϕ , α , β , ω , γ , μ , π , β_1 , β_2 , $\sigma_{\epsilon_1}^2$, $\sigma_{\epsilon_2}^2$, and $\sigma_{\xi_2}^2$. Below, we first describe the data we use in more detail as well as assumptions about measurement error in these data. We then discuss the priors we assume for the parameters and structural breaks we allow for.

3.1 Data

All the data we use are decadal averages. In our figures, a data point listed as 1640 refers to the decadal average from 1640 to 1649. We sometimes refer to a variable at a point in time (say 1640) when we mean the decadal average for that decade. In other words, we use 1640 and “the 1640s” interchangeably.

Figure 4 plots the data series we use for real wages in England. This is the series for unskilled building workers from Clark (2010). The main features of this series are a large and sustained rise between 1300 and 1450, a large and sustained fall between 1450 and 1600, some recovery over the 17th century, stagnation during the 18th century, and finally a sharp increase after 1800. Figure A.1

⁶We implement this procedure using Stan (Stan Development Team, 2017).

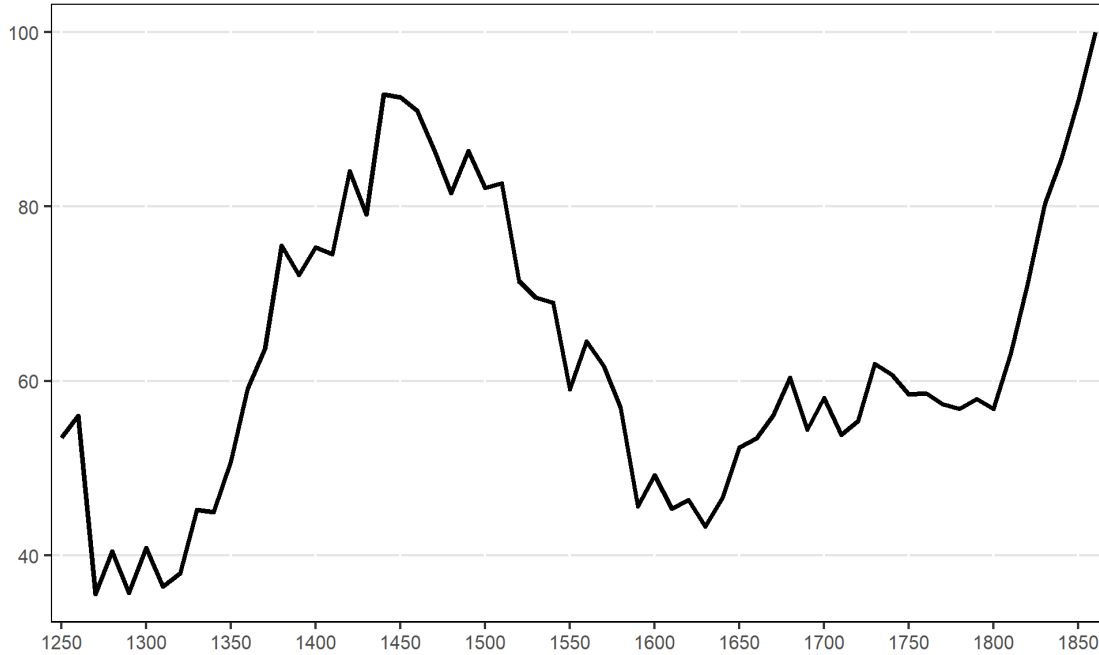


Figure 4: Real Wages in England, 1250-1860

Note: The figure presents estimates of the real wages of unskilled building workers in England from Clark (2010).

compares this series with several other series for real wages in England. This comparison shows that the real wage series we use is quite similar to Clark’s real wage series for farmers. It also largely shares the same dynamics as Clark’s series for craftsman. We have redone our analysis with the farmers and craftsmen series and discuss this analysis in section 4.⁷

Figure 5 presents the population data that we use. For the period from 1540 onward, we use population estimates from Wrigley et al. (1997), which in turn build on the seminal work of Wrigley and Schofield (1981). Sources for population data prior to 1540 are less extensive. Clark (2007b) uses unbalanced panel data on the population of villages and manors from manorial records and penny tithing payments to construct estimates of the population prior to 1540. We build on Clark’s work to construct an estimate of the population before 1540. We cannot directly use Clark’s pre-1540 population series since Clark’s method for constructing his series involves making assumptions about the evolution of productivity.⁸ Since we aim to use the population series to make inference about the evolution of productivity in England, we cannot use a population

⁷Much controversy has centered on the behavior of real wages in England between 1770 and 1850. This debate revolves around the extent to which laborers shared in the benefits of early industrialization (see, e.g. Feinstein, 1998, Clark, 2005, Allen, 2007, 2009b). In Figure A.1, we also plot Allen’s (2007) wage series (which starts in 1770). The figure shows that the differences discussed in the prior literature are modest from our perspective and therefore do not materially affect our analysis.

⁸Appendix D discusses Clark’s method in more detail.

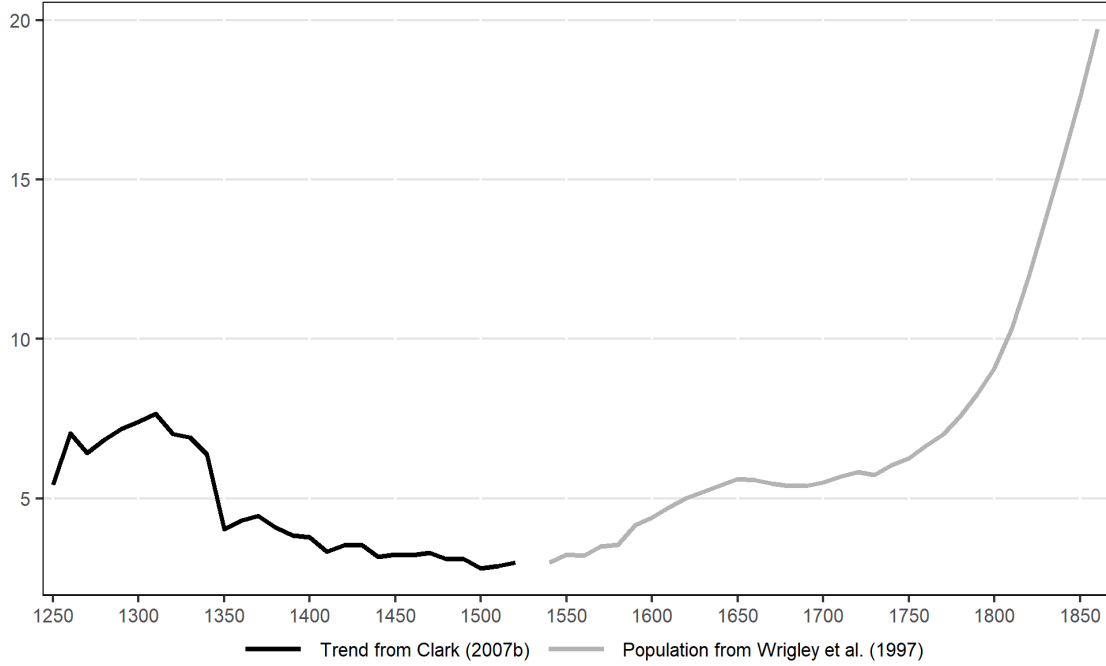


Figure 5: Population Data for England, 1250-1860

Note: The figure presents population estimates for England for the period 1540-1860 from Wrigley et al. (1997) (grey line) and the population trend estimates by Clark (2007b) for the period 1250-1520 (black line). In this figure, the black line is normalized for visual convenience such that its last point is equal to the first point of the grey line.

series that already embeds assumptions about productivity growth. However, as an intermediate input into constructing his pre-1540 population series, Clark estimates a regression of his village and manor level population data on time and village/manor fixed effects. Clark refers to the time effects from this regression as a population trend. We plot this population trend in Figure 5 (normalized for visual convenience). We base our estimates of the population of England prior to 1540 on this population trend series. In section 5, we discuss how this series compares to (lower frequency) population data reported in Broadberry et al. (2015).

The population data plotted in Figure 5 are missing information about the population in 1530 and are also missing a normalization for the population prior to 1540. We assume that the true population is unobserved and estimate it using the equation $n_t = \psi + \tilde{n}_t + \iota_t^n$, where n_t denotes the true unobserved population, \tilde{n}_t denotes our observed population series (Clark's population trend series prior to 1530 and the population series from Wrigley et al. (1997) after 1530), $\iota_t^n \sim t_{\nu_n}(0, \sigma_n^2)$ denotes measurement error, and ψ denotes a normalization constant. We normalize ψ to zero after 1530 and estimate its value for the pre-1530 Clark series.

The second variable that determines the labor supply is days worked d_t . We treat this variable as exogenous and present results for two assumptions about its evolution. Our baseline

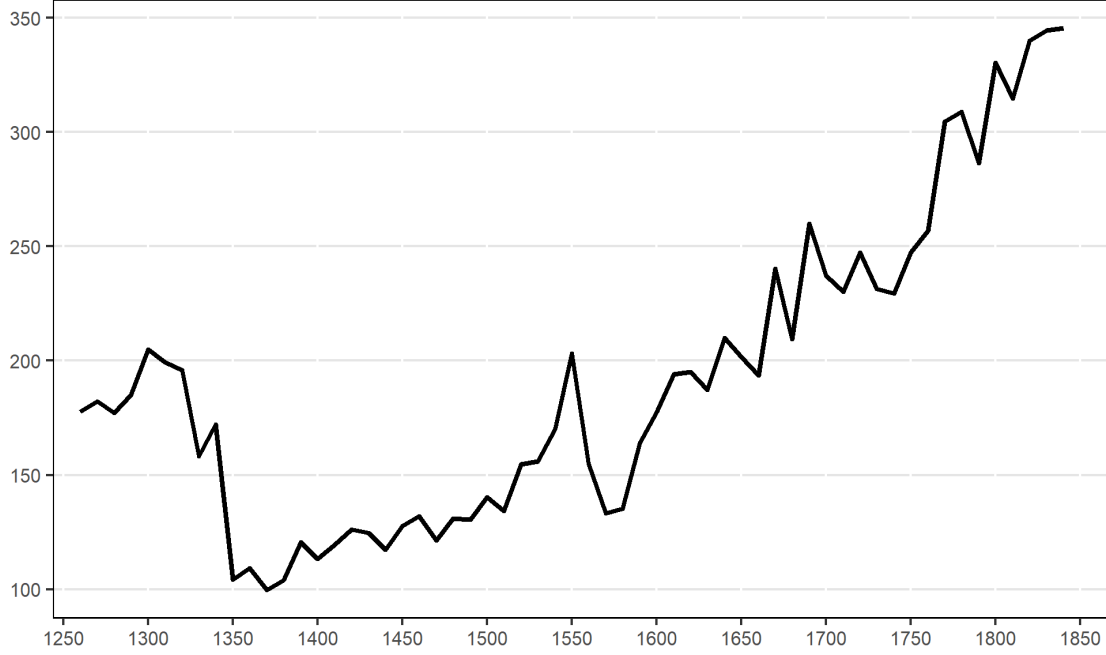


Figure 6: Days Worked per Worker in England, 1260-1840

Note: The figure presents an estimate of the evolution of days worked per worker in England from Humphries and Weisdorf (2019).

estimation is based on the series for days worked from Humphries and Weisdorf (2019). Figure 6 plots this series which indicates that days worked dropped sharply after the Black Death and then started a long upward march. Humphries and Weisdorf's series, thus, indicates that England experienced a large Industrious Revolution (de Vries, 1994, 2008). Since the extent of the Industrious Revolution is quite controversial, we also present results assuming that days worked remain constant throughout our sample period.⁹ In our baseline analysis, we assume that Humphries and Weisdorf's series is measured with error $d_t = \tilde{d}_t + \iota_t^d$, where d_t denotes the true number of days worked per worker, which is unobserved, \tilde{d}_t denotes Humphries and Weisdorf's estimates of days worked, and $\iota_t^d \sim t_{\nu_d}(0, \tilde{\sigma}_d^2)$ denotes the measurement error. Humphries and Weisdorf do not provide estimates for 1250, 1850, and 1860. We extrapolate from the series we have assuming that $d_t = d_{t-1} + \eta_t$ where $\eta_t \sim \mathcal{N}(0, \sigma_d^2)$.

Figure 7 plots data on rates of return on agricultural land and "rent charges" compiled by Clark (2002, 2010).¹⁰ The rate of return on agricultural land is measured as R/P , where R is the

⁹Comparisons of direct estimates by Blanchard (1978) for 1400-1600 and Voth (2000, 2001) for 1760-1830 also support the idea of an Industrious Revolution. Earlier indirect estimates by Clark and Van Der Werf (1998), however, suggest modest changes in days worked over our sample. Humphries and Weisdorf (2019) argue that their new series on the income of workers on annual contracts represents an important improvement relative to the series used by Clark and Van Der Werf (1998).

¹⁰Clark's series on "rent charges" should not be confused with his series on land rents. These are different series.

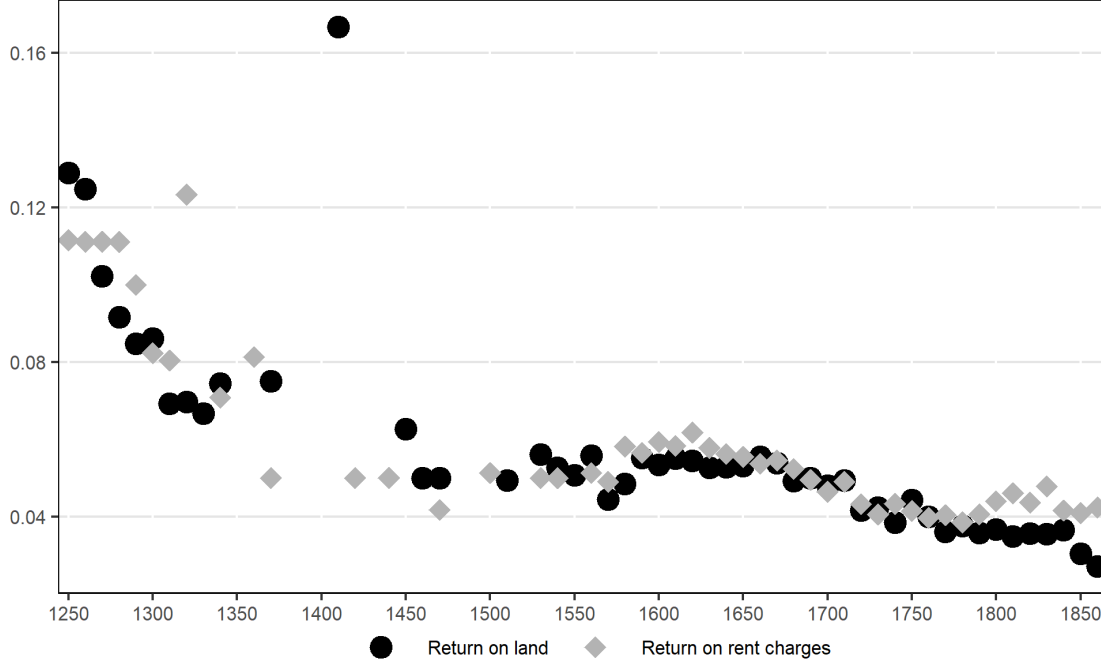


Figure 7: Rates of Return on Land and Rent Charges

Note: The figure plots the data we use on rates of return on land and rent charges. These data are from Clark (2002, 2010).

rent and P is the price of a piece of land. As Clark (2010) explains, “rent charges” were perpetual nominal obligations secured by land or buildings. Again, these are measured as R/P , where R is the annual payment and P is the price of the obligation. See Clark (2010) for more detail. We view each of these series as a noisy measure of the rate of return on capital in England over our sample period. In other words, we assume that $r_t = \tilde{r}_{it} + \nu_{it}^r$, where r_t denotes the true rate of return on capital at time t , \tilde{r}_{it} denotes noisy measure i (either land or rent charges), and $\nu_{it}^r \sim t_{\nu_{ir}}(0, \tilde{\sigma}_{ir}^2)$ denotes the measurement error. In periods when neither measure is available, we assume that the interest rate follows a random walk with truncated normal innovations: $r_t \sim \mathcal{N}_{(0,2)}(r_{t-1}, 0.01^2)$.

We use data on land rents and the capital stock after 1760 to infer how α_t and β_t change over this period. These data are plotted in Figure 8. Land rents series we use is an index from Clark (2002, 2010). The capital stock series we use is the net capital stock of Feinstein (1988), expressed in millions of pounds in 1851-1860 prices. This net capital stock series reflects both industrial and agricultural investment. We assume that both of these variables are observed with measurement error $s_t = \tilde{s}_t + \nu_t^s$ and $k_t = \tilde{k}_t + \nu_t^k$, where s_t and k_t denote the true land rent and capital stock, respectively, \tilde{s}_t and \tilde{k}_t denote our noisy measures of land rents and the capital stock, respectively,

A “rent charge” is a rate of return on an asset (i.e., measured in percent), while land rents are a nominal series (i.e., pounds sterling per year).

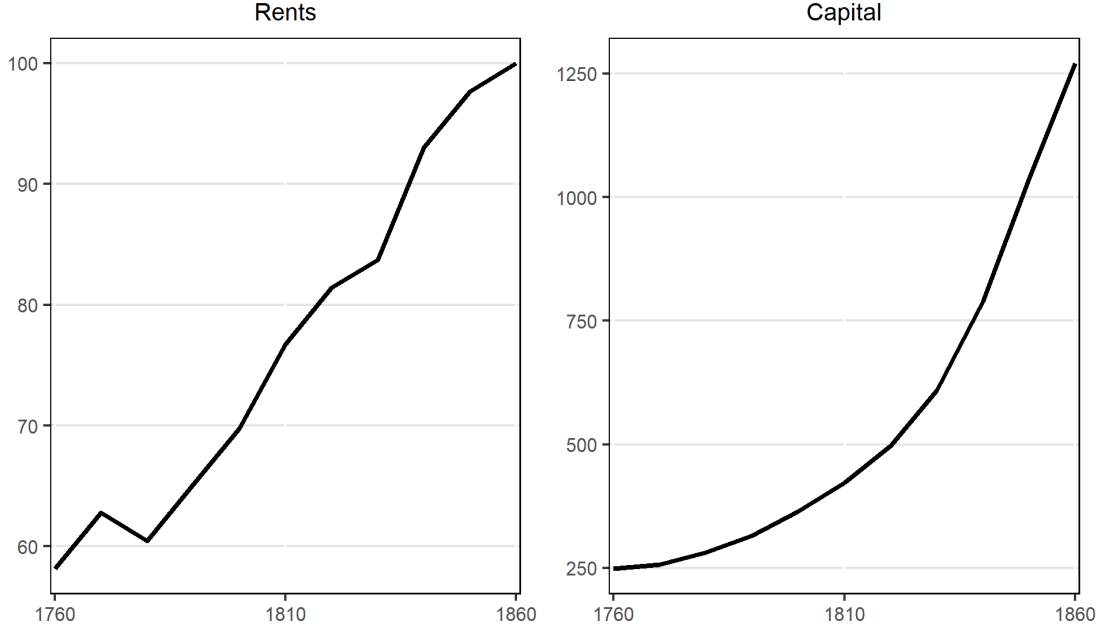


Figure 8: Land Rents and Capital after 1760

Note: The figure plots the data we use on land rents from (Clark, 2002, 2010) and the net capital stock from (Feinstein, 1988, table VIII).

and $\iota_t^s \sim t_{\nu_s}(0, \tilde{\sigma}_s^2)$ and $\iota_t^k \sim t_{\nu_k}(0, \tilde{\sigma}_k^2)$ denote the measurement error in these two variables.

Finally, we need to normalize some of the variables in our model. Labor demand – equation (11) – features $\log Z$ and λ , which we normalize to 0. The observed wage, land rents, and our capital series are indices. They are thus not normalized in a theoretically consistent fashion. For wages, for instance, we would want to observe the marginal product of an additional unit of labor expressed in units of aggregate output. What we observe is an index which is normalized to 100 in 1860. Our Bayesian framework allows us to handle this issue in a straightforward fashion. For wages we assume that $w_t = \varphi^w + \tilde{w}_t$, where \tilde{w}_t is the observed wage, w_t is the true wage, and $\varphi^w \sim \mathcal{N}(0, 100^2)$ is a normalization constant. For land rents and the capital stock, we add the following normalization constants to their measurement equations: $\varphi^s \sim \mathcal{N}(0, 100^2)$ and $\varphi^k \sim \mathcal{N}(0, 100^2)$, respectively.

3.2 Priors

Table 1 lists the priors we assume for the model parameters. In all cases, we choose highly dispersed priors. Most of the priors are self-explanatory. But some comments are in order. The prior for ψ is set such that the peak population before the Black Death is between 4.5 and 6 million with 95% probability. This range encompasses the estimates of Clark (2007b) and Broadberry et al.

Table 1: Priors for Model Parameters

Parameter	Prior	Parameter	Prior
α	$\mathcal{U}(0, 2)$	γ	$\mathcal{U}(-2, 2)$
φ^x	$\mathcal{N}(0, 100^2)$	ψ	$\mathcal{N}(10.86, 0.07^2)$
ω	$\mathcal{N}(0, 1)$	μ	$\mathcal{N}(0, 1)$
μ_{ξ_1}	$\mathcal{U}(0.5, 0.9)$	ν_{ξ_1}	$\mathcal{P}_I(0.1, 1.5)$
π	$\mathcal{U}(0, 0.5)$	δ	$\mathcal{N}_{(0,0.2)}(0.1, 0.05^2)$
$\sigma_{\epsilon_1}^2$	$\text{IF}(3, 0.001)$	$\sigma_{\epsilon_2}^2$	$\text{IF}(3, 0.005)$
$\sigma_{\xi_2}^2$	$\text{IF}(3, 0.005)$	σ_n^2	$\text{IF}(3, 0.005)$
σ_d^2	$\text{IF}(3, 0.005)$	σ_{ri}^2	$\text{IF}(3, 0.005)$
$\tilde{\sigma}_d^2$	$\text{IF}(3, 0.005)$	$\tilde{\sigma}_s^2$	$\text{IF}(3, 0.005)$
$\tilde{\sigma}_k^2$	$\text{IF}(3, 0.005)$	ν_n^{-1}	$\mathcal{U}(0, 1)$
ν_d^{-1}	$\mathcal{U}(0, 1)$	ν_s^{-1}	$\mathcal{U}(0, 1)$
ν_k^{-1}	$\mathcal{U}(0, 1)$	ν_{ir}^{-1}	$\mathcal{U}(0, 1)$

(2015). Rather than specifying priors for β_1 and β_2 , we specify priors for the mean of ξ_1 which we denote $\mu_{\xi_1} = \beta_1/(\beta_1 + \beta_2)$ and the pseudo sample size of ξ_1 which we denote $\nu_{\xi_1} = \beta_1 + \beta_2$. The priors we choose for these parameters follow the recommendations of Gelman et al. (2013, p. 110) for a flat prior for a beta distribution. Figure A.2 plots the prior densities for the standard deviations of ϵ_1 , ϵ_2 , and ξ_2 . In section 4.5, we discuss how varying our priors affects our main results.

To discipline the behavior of α_t and β_t after 1760, we assume that the simplex $(\alpha_t, \beta_t, 1 - \alpha_t - \beta_t)$ follows a Dirichlet distribution with concentration vector $c_s \times (\alpha_{t-1}, \beta_{t-1}, 1 - \alpha_{t-1} - \beta_{t-1})$, where $c_s = 3$. For any value of c_s , this implies that the mean of α_t is α_{t-1} and the mean of β_t is β_{t-1} . The choice $c_s = 3$ implies that, with $\alpha_{t-1} = \beta_{t-1} = 1/3$, the distribution is uniform over simplexes. A smaller value of c_s would concentrate the prior distribution towards the corners of the simplex — draws where one of the coefficients is close to 1 and the others close to 0. A larger value, on the other hand, concentrates the prior towards the mean of the distribution — most draws would be close to $(\alpha_{t-1}, \beta_{t-1}, 1 - \alpha_{t-1} - \beta_{t-1})$. Thus, our prior choice is a way to center the distribution around the previous value of the coefficients, while allowing them to change if the likelihood dictates it.

3.3 Structural Breaks

The post-1540 population data we use is higher quality than our earlier population data. For this reason, we allow for a structural break in the scale and degrees-of-freedom parameters of the measurement error in our population data— σ_n^2 and ν_n , respectively—in 1540. These structural breaks allows us to capture the change in data quality at this point. We also allow for a structural break in the probability of a plague π in 1680. The timing of this break is chosen to immediately follow the Great London Plague of 1665.¹¹ This break is meant to capture the fact that plagues are less frequent in the latter part of our sample. The exact timing of this break does not affect our main results in a material way.

To be able to capture potential changes in average growth of productivity, we allow for two structural breaks in the productivity parameters μ , $\sigma_{\epsilon_1}^2$, and $\sigma_{\epsilon_2}^2$. We set the second of these breaks to occur in 1760 (i.e., first decade with new parameters is 1770s). This break date is meant to capture the onset of the Industrial Revolution. We discuss robustness regarding this assumption in section 4. In addition to this break, we allow for an additional break earlier in the sample. By allowing for this earlier break, we allow for the possibility that there may have been a break in average growth before the Industrial Revolution. We consider a range of possible dates for the first break in μ , $\sigma_{\epsilon_1}^2$, and $\sigma_{\epsilon_2}^2$ between 1550 and 1770. The case of 1770 corresponds to there only being one break (i.e., no break before 1770).¹²

To pin down the timing of the first break, we estimate a mixture model. Since μ , $\sigma_{\epsilon_1}^2$, and $\sigma_{\epsilon_2}^2$ break twice, they take on three values: one for each regime. Let's denote these as $\mu(i)$ with $i \in \{1, 2, 3\}$ (with analogous notation for $\sigma_{\epsilon_1}^2$, and $\sigma_{\epsilon_2}^2$). From the beginning of our sample until 1540, $\mu = \mu(1)$. From 1550 until 1760, $\mu = (1 - I)\mu(1) + I\mu(2)$, where I is an indicator variable that switches from zero to one at the time of the first break. Finally, from 1770 until 1860, $\mu = \mu(3)$. The indicator variable I has a multinomial distribution with probability for switching from zero to one at each date between 1550 and 1760. We also allow for a case where the first break does not happen, i.e., $\mu = \mu(1)$ before 1770 and $\mu(3)$ afterwards. We estimate the probabilities of the multinomial distribution for I . The prior for these probabilities is a Dirichlet distribution with concentration vector $c_b \times (1, \dots, 1)$. We choose a small value for c_b . This ensures that each draw

¹¹Notice that the change in the population between the 1660s and the 1670s is affected by the Great London Plague. So, ξ_{1t} for $t = 1670$ will be affected by the Great London Plague. This is why we assume that ξ_{1t} for $t \geq 1680$ is governed by a different π than earlier values of ξ_{1t} .

¹²The fact that we allow for permanent breaks in productivity growth implies that productivity growth has a unit root component. This allows our model to match the fact that the population is integrated of order two in our sample, which has been emphasized in prior work on this topic (Bailey and Chambers, 1993, Crafts and Mills, 2009).

from the distribution is close to a corner of the distribution, i.e. chooses a specific break date. In particular, we set $c_b = 0.001$.¹³ The output from our estimation of these probabilities is a posterior probability distribution over break dates.

4 When Did Productivity Growth Begin in England?

Our primary object of interest is the evolution of productivity in England over our sample period. We therefore start the discussion of our empirical results by describing our results about productivity. As we discuss above, we allow for two structural breaks in average productivity growth μ over our sample. For the first of these break dates, we estimate the probability that the break occurred at different dates between 1550 and 1770. Figure 9 plots the probability distribution we estimate. The highest estimated probability is for a break in 1600. We estimate a sharp rise in the probability of a break in the late 16th century and a somewhat more gradual fall from 1600 to 1650. The probability of a break occurring before 1640 – i.e., before the onset of the English Civil War – is estimated to be 56%. The probability of a break occurring before 1680 – i.e., before the Glorious Revolution – is 68%.¹⁴

Table 2 presents our estimates of the average growth rate of productivity μ as well as the standard deviation of the productivity shocks σ_{ϵ_1} and σ_{ϵ_2} in our baseline case with two breaks. Our estimates of the other parameters are presented in Table 3, and will be discussed in more detail in section 5. The estimates we present in Table 2 and all subsequent results in the paper are conditional on the first break in the productivity process occurring in 1600, unless otherwise stated. We discuss the sensitivity of our results to this choice later in the section.

The first notable result in Table 2 is that average productivity growth prior to 1600 was zero. Kremer (1993) used data on the growth rate of the world population to argue that growth has been non-zero and increasing for many millennia. The world population estimates he used indicate that world population growth from 1200 to 1500 was 0.6% per decade. In our Malthusian model

¹³For a simple exposition of the Dirichlet distribution and the role of the concentration parameter, see Stan Function Reference, section 23: <https://mc-stan.org/docs/functions-reference/dirichlet-distribution.html>.

¹⁴Our baseline analysis assumes that there is a break at the onset of the Industrial Revolution. Conditional on this, the probability that there is no break before 1770 is estimated to be only 19%. We have also considered the case with a single break. In this case, the probability distribution of the timing of that single break is twin peaked (see Figure A.3). There is a peak in 1810 (Industrial Revolution) and another peak in 1600. Interestingly, the peak in 1600 is much larger than the peak in 1810 and the sum of the probability around the peak in 1600 (54% between 1570 and 1670) is also considerably larger than the sum of the probability around the peak in 1810 (39% between 1760 and 1860). Clearly, our model favors growth beginning in the early 17th century over the traditional notion that it began with the Industrial Revolution.

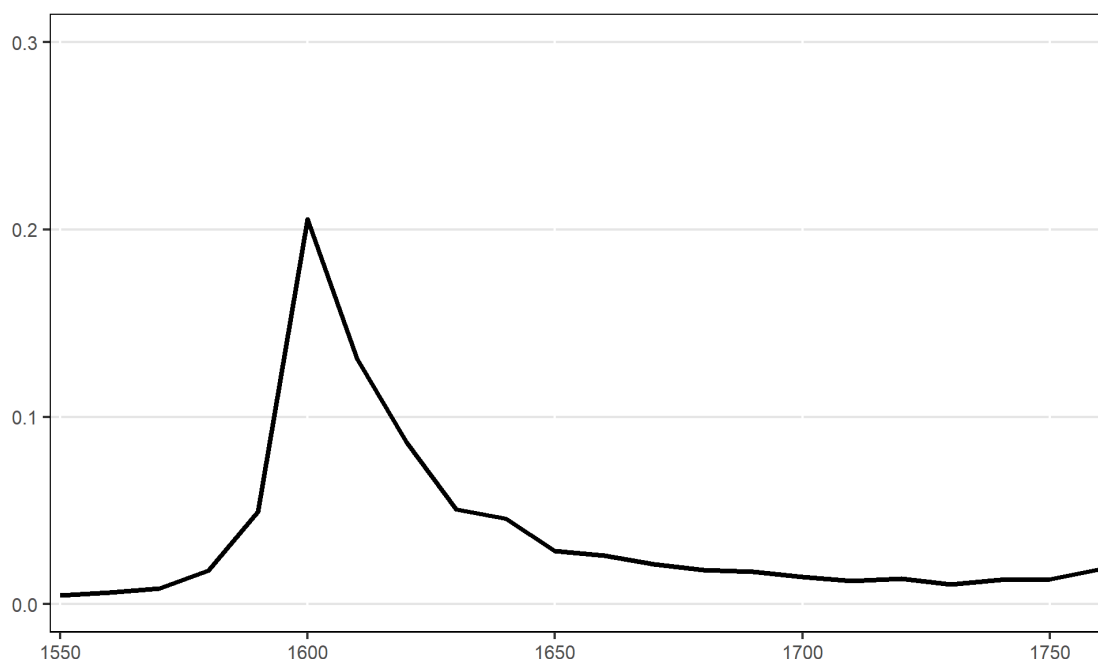


Figure 9: Probability of Different Productivity Growth Break Dates

Note: The figure plots our estimate of the probability that a structural break occurred in the parameters μ , σ_1 , and σ_2 in different decades between 1550 and 1760. The residual probability that no break occurred before 1770 is estimated to be 20%.

(as well as Kremer's model) steady state productivity growth is α times steady state population growth. Our baseline estimate of α is 0.38 (Figure 12). This suggests that world productivity growth was 0.23% per decade over the period 1200 to 1500, i.e., positive but glacial. This slow growth rate is well within the credible set of our pre-1600 estimate of μ .

Our results indicate that sustained productivity growth began in 1600 (or around that time). We estimate μ to have been 3% per decade over the period 1600 to 1760. Productivity growth then increased to 6% per decade from 1770 to 1860. Output growth increased even more, which our estimates attribute to a falling land share after the onset of the Industrial Revolution (more on this below). The period between 1600 and 1760, thus, appears to have been a period of transition in England between an era of total stagnation and an era of modern economic growth.

Figure 10 presents our baseline estimates of the time series evolution of the permanent component of productivity. These estimates indicate that the level of productivity in England was very similar in 1600 to what it had been in the late 13th century. In the intervening period, productivity fluctuated a slight bit. It reached its lowest point right before the Black Death in 1340, then increased and peaked a century later before receding slightly. After 1600, productivity began a sustained increase.

Table 2: Productivity Parameters

	Mean	St Dev	2.5%	97.5%
$\mu_{a,t<1600}$	-0.00	0.01	-0.02	0.01
$\mu_{a,1600\leq t<1770}$	0.03	0.01	0.01	0.04
$\mu_{a,t\geq 1770}$	0.06	0.02	0.03	0.09
$\sigma_{\epsilon_1,t<1600}$	0.04	0.01	0.02	0.07
$\sigma_{\epsilon_1,1600\leq t<1770}$	0.02	0.01	0.01	0.04
$\sigma_{\epsilon_1,t\geq 1770}$	0.02	0.01	0.01	0.04
$\sigma_{\epsilon_2,t<1600}$	0.06	0.01	0.04	0.08
$\sigma_{\epsilon_2,1600\leq t<1770}$	0.04	0.01	0.02	0.05
$\sigma_{\epsilon_2,t\geq 1770}$	0.04	0.01	0.02	0.06

Note: The table presents the mean, standard deviation, 2.5% quantile, and 97.5% quantile of the posterior distribution we estimate for average productivity growth μ in the three regimes and also for the standard deviation of the permanent and transitory productivity shocks ϵ_{1t} and ϵ_{2t} . These statistics are conditional on the first break in the productivity parameters occurring in 1600.

4.1 Productivity and Real Wages

Figure 11 compares our estimate of productivity with the data we use on real wages. This figure illustrates well the importance of accounting for Malthusian population forces when estimating productivity in the pre-industrial era. Through the lens of our Malthusian model, the large changes in real wages prior to 1600 are explained almost entirely by changes in labor supply—the economy was moving up and down a stable labor demand curve as suggested by Figure 2. As a result, changes in productivity were very substantially muted relative to changes in real wages over this period. In sharp contrast, after 1600 productivity increased more rapidly than real wages. Over this period, the population in England grew rapidly as did days worked per worker. Our Malthusian model implies that this expansion of labor supply held back real wages relative to the change in productivity.

The period between 1730 and 1800 is particularly interesting in this context. Over this period, real wages in England fell slightly despite substantial productivity growth. One potential explanation of this is “Engels’ Pause,” i.e., the idea that the lion’s share of the gains from early industrialization went to capitalists as opposed to laborers (Engels, 1845, Allen, 2009b). However, our Malthusian model provides an alternative explanation for this divergence between wages and productivity: Over this period, the population of England grew rapidly. In our Malthusian model, the growth in the population put downward pressure on the marginal product of labor and thus

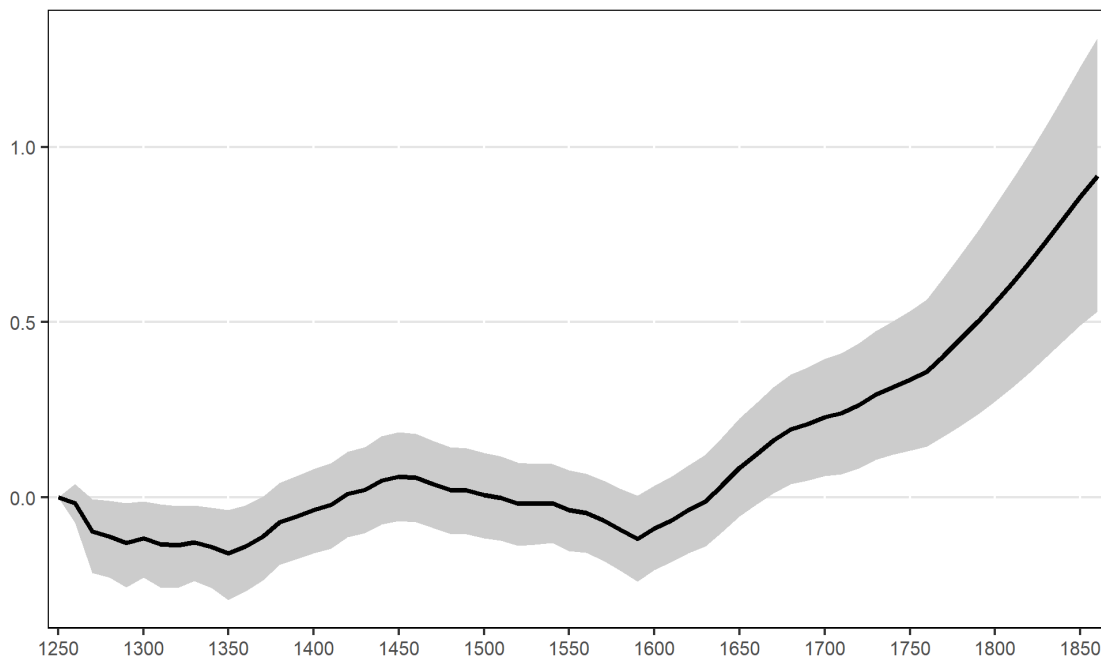


Figure 10: Permanent Component of Productivity

Note: The figure plots our estimates of the evolution of the permanent component of productivity \tilde{a}_t over our sample period. The black line is the mean of the posterior for each period and the gray shaded area is the 90% central posterior interval.

reduced the growth in wages relative to productivity.¹⁵

4.2 Productivity and the Land Share

A feature of our results that may seem curious is that the increase in productivity growth after 1760 – i.e. at the time of the Industrial Revolution – is modest. (It goes from 3% per decade to 6% per decade.) Rather than estimating a substantial break in productivity growth, we find that the Industrial Revolution led to a huge change in the relative importance of different factors of production. Figure 12 plots our estimates of the evolution of α_t and β_t over our sample period – which correspond to the land and capital share under the assumption that the production function is Cobb-Douglas. We estimate a value for α prior to the Industrial Revolution of 0.38 (0.09), and a value for β of 0.17 (0.06).¹⁶ After 1760, we estimate a sharp fall in α_t . By 1860, α_t had fallen by

¹⁵The literature on Engels' Pause has typically focused on the first half of the 19th century as opposed to the second half of the 18th century. The real wage series we use – constructed by Clark (2010) – indicates that real wages actually started growing robustly around 1810. This contrasts with the real wage series of Feinstein (1998) and Allen (2007), which date the onset of rapid real wage growth a few decades later.

¹⁶Our estimate of the land share is substantially higher than Clark's. He estimates a land share of approximately 20% prior to the onset of the Industrial Revolution (see Figure E.6). As we note above, α_t is equal to the land share under the assumption that the production function is Cobb-Douglas. For the period prior to 1760, we need not make this assumption. Equation (1) holds as a log-linear approximation of labor demand for a generic production function. For

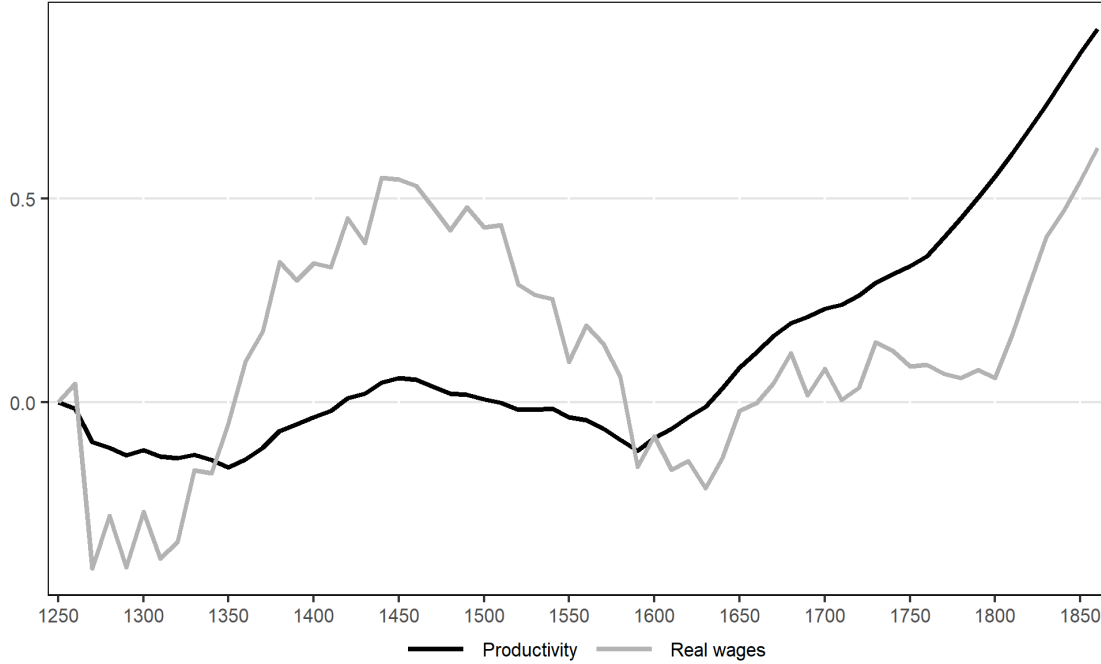


Figure 11: Comparison with Real Wages

Note: The figure plots our estimates of the evolution of the permanent component of productivity \tilde{a}_t along with the real wage series we use.

more than half to a value of 0.15.

This sharp fall in α_t reflects the nature of technical change during the Industrial Revolution. The advent of the steam engine powered by fossil fuels meant that the production of energy was no longer land intensive. This led to a large fall in the land share of output. According to our estimates, the fall in α_t led mostly to an increase in the labor share of production. Between 1760 and 1860, the capital share increased by less than a percentage point, while the labor share increased from 45% to 67%.

To illustrate the importance of the fall in α_t for our estimates of productivity, Figure A.4 plots estimates of productivity under the assumption that α and β are constant throughout the sample period. In this case, we estimate a much sharper increase in productivity growth after 1760 – 10% per decade rather than 6%. Intuitively, with a large land share, the marginal product of labor is sharply downward sloping in labor supply. After 1760, England experienced explosive growth in its population (Figure 5). With a large land share, this imparts strong downward pressure on real wages. Since real wages actually rose over this period, large increases in productivity growth are

instance, if the production function displays a constant elasticity of substitution between land and labor, α depends on both the land share and the elasticity of substitution between labor and land. With an elasticity of substitution between land and labor less than one, $\alpha = 0.38$ implies a land share below 38%. We present this more general derivation in appendix F. Allowing for a production function that is more general than the Cobb-Douglas production function is more complicated after 1760 when the structure of the production function is changing.

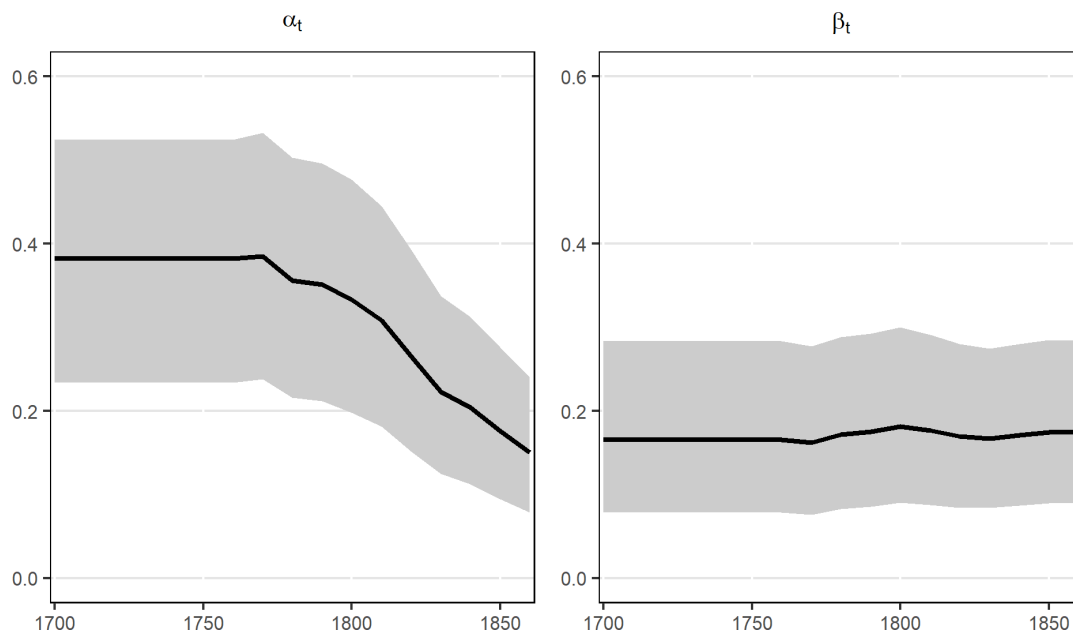


Figure 12: Parameters of the Production Function

Note: The figure plots our estimates of the evolution of the parameters α_t and β_t . They are constant until 1770 by assumption. The black line is the mean of the posterior for each period and the gray shaded area is the 90% central posterior interval.

needed to fit the data. In our baseline analysis, however, the land share is falling after 1760 which implies that the downward pressure on real wages from population growth is smaller and less productivity growth is therefore needed to explain the increase in real wages.

4.3 Capital, Lands Rents, and Comparison with Clark (2010, 2016)

We use data on the capital stock and land rents from 1760 onward. We view these variables as being unobserved prior to that time. Figure 13 plots what our model implies about their evolution prior to 1760. We infer that both capital and land rents fell sharply at the time of the Black Death and then began to rise gradually. The growth rate of both variables was modest prior to 1600, but increased after that time. Our model-inferred series for land rents differs quite a bit from Clark's 2010 and also his updated 2016 series for land rents (see Figure E.4). In particular, Clark's series do not show a sharp drop in land rents after the Black Death. They also show a smaller rise in land rents between 1500 and 1760. But these series are rather noisy prior to the 18th century.

Our estimates of productivity differ substantially from those of Clark (2010, 2016). Appendix E presents a detailed decomposition of the factors leading to the differences. Several factors are important. One is Clark's use of land rent series that are quite noisy prior to the 18th century as we

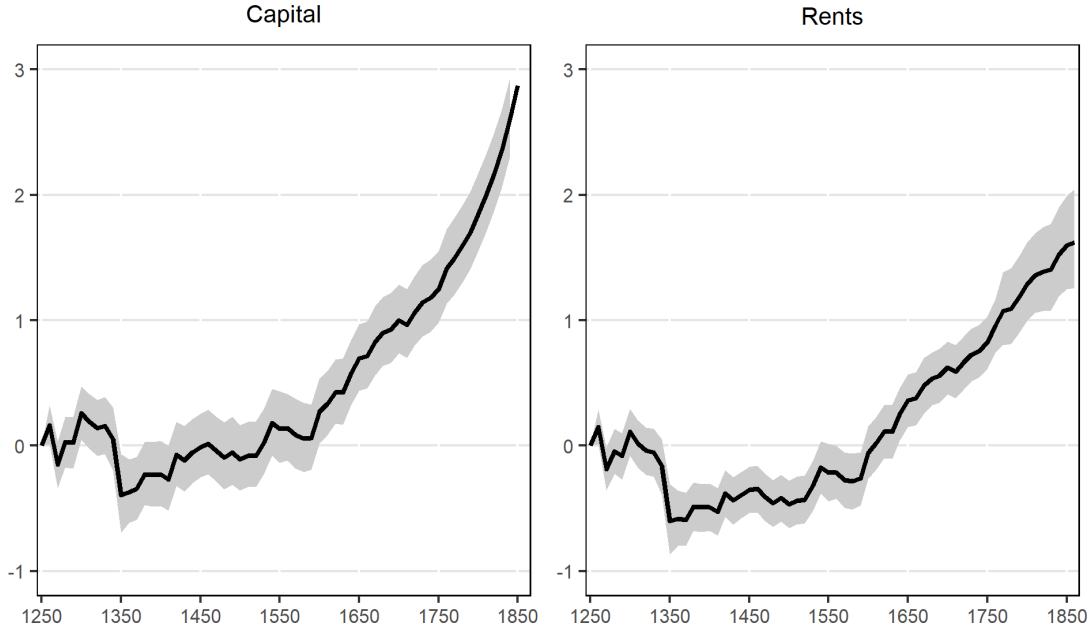


Figure 13: Capital and Rents

Note: The figure plots our estimates of the evolution of the logarithm of capital, k_t , and rents, s_t . They are normalized to 0 in 1250. The black line is the mean of the posterior for each period and the gray shaded area is the 90% central posterior interval.

discuss above. We also improve on Clark’s treatment of the return to capital by using two series (return on agricultural land and “rent charges”) and allowing for measurement error. Another important contributor to the difference between our series and Clark’s (better known) 2010 series is an error in that series that leads to a spurious 25 log point drop between 1540 and 1550.

4.4 From When to Why

By dating the onset of productivity growth, our results help discriminate between competing explanations for *why* growth began. We estimate that sustained productivity growth began in England substantially before the Glorious Revolution of 1688. According to our estimates, productivity in England rose by 33% from 1600 to 1680. North and Weingast (1989) argue that the political regime that emerged in England after the Glorious Revolution—characterized by a power sharing arrangement between Parliament, the Crown, and the common law courts—resulted in secure property rights and rule of law and thereby laid the foundation for economic growth. While the institutional changes associated with the Glorious Revolution may well have been important for growth, our results indicate that the seeds of growth in England were sown earlier.

Our results support explanations of the onset of growth that focus attention on developments

that occurred in the period surrounding 1600. The Reformation is an obvious candidate. In particular, Henry VIII's confiscation of monastic lands was a big shock to land ownership patterns and the land market in England (Heldring, Robinson, and Vollmer, 2021); and England became a favored destination for skilled immigrants fleeing religious persecution on the continent. Also, London experienced an explosion of its population around this time—from 55,000 in 1520 to 475,000 in 1670 (Wrigley, 2010)—likely due to a rapid increase in international trade. English woolen exports expanded rapidly over this period (new draperies) as did intercontinental trade, colonization, and privateering. The British East India Company was founded in 1600 and the Virginia Company founded its first permanent settlement in North America in 1607.

Our finding that the onset of growth preceded both the Glorious Revolution and the English Civil War (1642-1651) lends support to the Marxist view that economic change propelled history forward and drove political and ideological change. Marx (1867) stressed the transition from feudalism to capitalism. He argued that after the disappearance of serfdom in the 14th century, English peasants were expelled from their land through the enclosure movement. That spoliation inaugurated a new mode of production: one where workers did not own the means of production, and could only subsist on wage labor. This proletariat was ripe for exploitation by a new class of capitalist farmers and industrialists. In that process, political revolutions were a decisive step in securing the rise of the bourgeoisie. To triumph, capitalism needed to break the remaining shackles of feudalism. As the *Communist Manifesto* puts it, "they had to be burst asunder; they were burst asunder" (Marx and Engels, 1848, pp. 40-41). Hill (1940, 1961) offers more recent treatments of the political revolutions in England in the 17th century that stress class conflict and their economic origins.

Acemoglu, Johnson, and Robinson (2005) synthesize the Marxist and institutionalist views. They argue that Atlantic trade enriched a merchant class that then demanded secure property rights and secured these rights through the Civil War and Glorious Revolution. This last narrative lines up well with our result that steady growth—perhaps driven by the Atlantic trade—began about half a century before the Civil War. However, we do not detect a radical increase of growth in the immediate aftermath of either the Civil War or Glorious Revolution: 3.1% (1600-1640), 4.1% (1640-1680) and 3.2% (1680-1810).

Allen (1992) argues that a long and gradual process of institutional change in England over the 600-year period from the Norman Conquest to the Glorious Revolution resulted in a situation in the 16th century where the yeoman class had acquired a substantial proprietary interest in

the land, and thus an incentive to innovate. The timing of Allen's 'rise of the yeoman' lines up reasonably well with our estimate of the onset of growth. According to Allen, property rights, rule of law, and personal freedom gradually expanded, and the social order was gradually transformed from a feudal to a capitalist order. From the 12th century, royal courts helped freeholders gain full ownership over their land. After the Black Death, serfdom collapsed as landlord competed for scarce labor. Early enclosures (15th and early 16th centuries) involved brutal evictions and depopulation of manors. The Crown reacted to this by increasing protection of tenant farmers.

The spread of movable-type printing across Europe after 1450 led to a large increase in literacy in England in the 16th and 17th centuries (Cressy, 1980, Houston, 1982), and a huge drop in the price of books (Clark and Levin, 2011). This likely had wide ranging effects on culture. Mokyr (2009, 2016) and McCloskey (2006, 2010, 2016) have argued that the crucial change that caused growth to begin was the emergence of a culture of progress based on the idea that mankind can improve its condition through science and rational thought. Others have stressed a Protestant ethic (Weber, 1905) and Puritanism (Tawney, 1926). The timing of these changes lines up reasonably well with our estimates although it is not straightforward to pinpoint precisely what these theories imply about the timing of the onset of growth.

Bogart and Richardson (2011) stress the importance of the post-Glorious Revolution regime's push to reorganize and rationalize property rights through enclosures, statutory authority acts, and estate acts. While our results contradict the notion that growth began with the Glorious Revolution, the fact that England underwent massive institutional change in the 17th and 18th century may have played an important role in sustaining growth during this period.

Allen (2009a) argues that the Industrial Revolution occurred in Britain around 1800 because innovation was uniquely profitable then and there. His theory relies on growth in the 17th century leading to high real wages in England in the 18th century as well as the development of a large coal industry. High wages and cheap coal made it profitable to invent labor saving technologies in textiles such as the spinning jenny, water frame, and mule, as well as coal burning technologies such as the steam engine and coke smelting furnace. While our theory does not point to the Industrial Revolution as the genesis of economic growth, Allen's theory helps explain how growth was sustained and the particular direction it took that led to the huge fall in the land share we estimate.

4.5 Robustness

Break Dates: Figure A.5 compares the evolution of the permanent component of productivity for four different assumptions about when the breaks to the productivity component occur. Recall that Figure 10 presents the evolution of the permanent component of productivity conditional on the first break occurring in 1600. Figure A.5 compares this baseline set of results with the case where the first break occurs sometime between 1550 and 1760. It also presents results for the case where the first break occurs in 1600 but the second break occurs sometime between 1770 and 1860. Finally, it presents results for a case with a single break that occurs sometime between 1550 and 1860. All four sets of results are very similar.

Days Worked: Our baseline analysis uses Humpries and Weisdorf's (2019) estimates of the evolution of days worked per worker. These estimates imply a substantial Industrious Revolution. Since this conclusion is controversial, Figure A.6 presents an alternative set of estimates for productivity where we instead assume that days worked per worker were constant throughout our sample period. This yields a pattern for productivity that is quite a bit more variable prior to 1600 and in general tracks the real wage much more closely than our baseline results.

Real Wage Data: Our baseline analysis uses real wage data for unskilled builders from Clark (2010). Figure A.7 presents five alternative estimated productivity series where we instead use other wage series. First, we present estimates of productivity using the following day wage series: 1) Clark's (2010) real wages series for farm laborers, 2) Clark's (2010) real wages series for building craftsmen, 3) Allen's (2007) real wage series for the period 1770 onward (with our baseline wage series before that time). We also present estimates of productivity based on the assumption that the builders, farmers, and craftsmen series from Clark (2010) are all noisy signals of the underlying true wage. Finally, we use Humphries and Weisdorf's (2019) annual wage series along with the assumption that days worked are constant. All five of these alternative productivity series are quite similar to our baseline productivity series, although there is some divergence early in the sample period.

Population Data: Our baseline analysis uses population data prior to 1540 from Clark (2010). Figure A.8 presents estimates of productivity using population data from Broadberry et al. (2015) for the period prior to 1540. Broadberry et al.'s (2015) estimates of the population are infrequent and irregular in their frequency. There are quite a few decades for which Broadberry et al. (2015) have no estimate, e.g., they present no estimate between 1450 and 1522. In this robustness analysis, we view the population as an unobserved variable in decades for which we do not have an

estimate from Broadberry et al. (2015). Our results on the evolution of productivity for this case are very similar to our baseline case.

Priors: Figure A.9 presents estimates of productivity using different prior distributions than we use in our baseline analysis. First, we present results for a case where we change the prior on σ_{ϵ_1} —the variance of permanent productivity shocks—to be $\text{IG}(3, 0.005)$, i.e., the same as the prior on the other productivity and population shocks. Second, we present results for a case where we change the prior on ψ —the level of the population prior to 1540—to be $\mathcal{N}(10.86, 10^2)$, i.e., much wider than in our baseline analysis. In both cases, the resulting productivity series are very similar to our baseline results. Other priors are quite dispersed. We find it unlikely that our results are sensitive to making any of these priors even more dispersed.

5 Liberating the Economy from the Iron Law of Wages

In section 4, we estimate a gradual increase in productivity growth μ and a sharp fall in the land share of production α after the onset of industrialization. Here we discuss how both of these developments – as well as our small estimate of the elasticity of population growth to real income γ (see discussion below) – contributed to liberating the economy from the Malthusian “iron law of wages,” i.e., the notion that wages tend to a very low (subsistence) level. We also discuss how our estimates reconcile the Malthusian model with episodes historians have identified prior to the Industrial Revolution when some part of the world has experienced substantial economic growth over a sustained period of time, sometimes several hundred years. Goldstone (2002) refers to these episodes as efflorescences. They include ancient Greece, ancient Rome, Song China, the Islamic golden age, and the golden age of Holland, to name but a few. These episodes have often been used as evidence against the Malthusian model (e.g., Persson, 2008).

It is important to recognize that steady positive productivity growth in a Malthusian model like ours results in a persistent force pushing wages higher. As wages rise, a counteracting force comes into play pushing wages lower (population growth). The strength of the force pushing wages lower is increasing in the level of the real wage. This implies that as wages rise the downward force gets stronger and stronger and eventually chokes off further increases in wages. In other words, there is a steady state real wage for each level of average productivity growth in a Malthusian model. This steady state is not at subsistence. Rather, the steady state real wage is increasing in average productivity growth.

In appendix G, we show that the steady state wage in our Malthusian model is given by

$$\bar{w} = \frac{\mu}{\alpha\gamma} + \text{constant}, \quad (19)$$

As we discussed above, faster productivity growth μ results in a stronger force pushing wages up and therefore a higher steady state wage. The strength of the counteracting force – which we call the Malthusian population force – is governed by two parameters in our model: the land share α and the elasticity of population growth with respect to per capita income γ . Intuitively, α determines the slope of the long-run labor demand curve, i.e., how rapidly real wages fall as population rises, while γ determines how rapidly the population increases when real wages are high.

With zero productivity growth, the steady state real wage is potentially very low. Its level depends on factors outside of the scope of our analysis such as hygiene, the marriage rate, contraceptive technology, and the level of violence in society. With positive productivity growth, the steady state real wage can be much higher. Whether it is depends on the size of α and γ . We discussed our estimates of α in section 4. We next turn to our estimates of γ .

5.1 The Elasticity of Population Growth with Respect to Income

Table 3 presents our estimate of γ (as well as all other parameters not presented in section 4). Recall that γ is the elasticity of population growth with respect to real wages. Our posterior mean estimate for γ is 0.09. This means that, for a 100 log point increase in real wages, population growth per decade increases by 9 log points. While this estimate of γ is relatively small, it is highly statistically significant. In this sense, our estimates strongly support the presence of a Malthusian population force prior to the Industrial Revolution.

One way to interpret the size of γ is to calculate the half-life of population dynamics after an exogenous shock to the level of the population, e.g., due to a plague. Assuming for simplicity that days worked and the return on capital are constant, that all shocks are equal to their average value, and that there's no productivity growth ($\mu = 0$), we show in appendix G that the dynamics of the population after an initial disturbance are given by the following AR(1) process:

$$n_{t+1} = \left(1 - \frac{\gamma\alpha}{1 - \beta}\right) n_t + \text{constant}. \quad (20)$$

The speed of population recovery after a plague-induced decrease is, thus, governed by $1 -$

Table 3: Parameter Estimates

	Mean	St Dev	2.5%	97.5%
<i>Main Parameters</i>				
γ	0.09	0.02	0.04	0.13
ω	-0.03	0.02	-0.07	0.01
<i>Population Parameters</i>				
$\pi_{t < 1680}$	0.15	0.11	0.01	0.43
$\pi_{t \geq 1680}$	0.11	0.10	0.00	0.39
μ_{ξ_1}	0.80	0.10	0.55	0.90
ν_{ξ_1}	2.87	7.18	1.02	11.23
σ_{ξ_2}	0.06	0.01	0.04	0.07
<i>Population Measurement Error Parameters</i>				
$\sigma_{n,t < 1540}$	0.04	0.01	0.02	0.06
$\sigma_{n,t \geq 1540}$	0.03	0.00	0.02	0.04
$\nu_{n,t < 1540}$	17.23	344.62	1.17	60.28
$\nu_{n,t \geq 1540}$	74.30	3280.53	2.17	273.49
<i>Days Worked Parameters</i>				
σ_h	0.08	0.01	0.06	0.10
$\tilde{\sigma}_h$	0.04	0.01	0.03	0.06
ν_h	8.52	1002.41	1.23	15.25

Note: The table presents the mean, standard deviation, 2.5% quantile, and 97.5% quantile of the posterior distribution we estimate for the slope of labor demand α , the elasticity of population growth to income γ , the subsistence wage parameter ω , the probability of a plague shock π , the mean of the plague shock μ_{ξ_1} , the pseudo sample size of the plague shocks ν_{ξ_1} , the standard deviation of the normal population shock σ_{ξ_2} , the scale and degrees of freedom parameters of the population measurement error shocks, σ_n and ν_n , respectively, the standard deviation of inferred changes in days worked σ_h , and the scale and degrees of freedom parameters of the days worked measurement error shocks, $\tilde{\sigma}_h$ and ν_h , respectively.

$\gamma\alpha/(1 - \beta)$ in this case. In particular, the half-life of the population dynamics, i.e., the time it takes the population to recover half of the way back to steady state after a plague-induced drop, is $\log 0.5 / \log(1 - \alpha\gamma/(1 - \beta))$. (The half-life of real wage dynamics is the same.)

Plugging our estimates of the parameters γ , α , and β into the formulas above, we find that the half-life of population and real wage dynamics prior to the Industrial Revolution was roughly 170 years. Since α falls sharply after 1760 and β changed little, the Malthusian population force becomes even weaker after 1760. By 1860, the half-life of population and real wage dynamics have risen to 420 years. These long half-lives imply that the strength of the Malthusian population force was rather weak in England over our sample period – a result that is sometimes referred to as “weak homeostasis.” Prior work has also found weak homeostasis. For example, Lee and Anderson (2002) find a half-life of 107 years, while Crafts and Mills (2009) find a half-life of 431

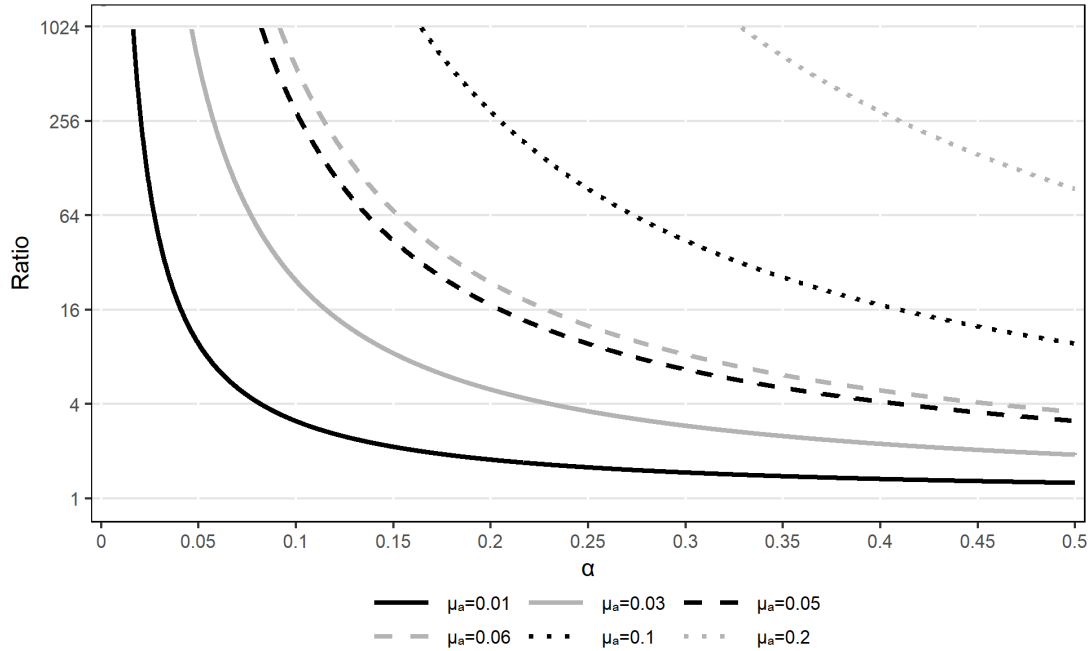


Figure 14: Steady State Wage with Productivity Growth

Note: The figure plots steady state real wage for different average levels of productivity growth (μ) and land shares (α) relative to the steady state real wage with zero productivity growth. The land share varies on the x-axis and each line plots real wages for a given level of average productivity growth. These impulse responses are calculated assuming that all other model parameters are at their posterior mean values.

years.

Another way to gauge the quantitative magnitude of our estimate of γ is to calculate how much the changes in real per capita income in England over our sample prior to the 17th century affected population growth. Between 1270 and 1440, real per capita income in England rose by 70%. Our estimate of γ implies that this increase in per capita income stimulated population growth by a mere 5 percentage points per decade. A doubling of real per capita income would have stimulated population growth by only slightly more, 6 percentage points per decade.

5.2 Prosperity and the Land Share

We can now use equation (19) and our parameter estimates to assess how changes in the economy after the onset of growth in 1600 affected the long-run steady state real wage the economy was tending towards. Figure 14 plots the steady state wage in our Malthusian economy for different values of productivity growth μ and the land share α relative to the steady state wage with zero productivity growth. Each line in the figure gives the steady state wage for a particular value of productivity growth as the value of α varies.

The figure illustrates clearly how important the fall in the land share is for liberating the econ-

omy from the iron law of wages. Consider first how steady state real wages respond to productivity growth when the land share is equal to our pre-industrial estimate of $\alpha = 0.38$. In this case, productivity growth of 3% per decade raises the real wage by a factor of 2 in the long run, while 6% productivity growth raises the real wage by a factor of 6. These results show that our Malthusian model is consistent with substantial, multi-hundred year efflorescences of the kind discussed by Goldstone (2002) if we allow for modest productivity growth. This is true even with the high land share of production we estimate for the pre-industrial era. The small value we estimate for γ and the associated weak homeostasis are key to this result.

As the land share falls, the steady state real wage for any given level of average productivity growth rises sharply. With our estimate of the land share for 1860 ($\alpha = 0.15$), productivity growth of 6% per decade can raise the real wage by a whopping factor of 94 in the long run. In other words, this level of productivity growth would have eventually led to a 94-fold increase in real wages even if the Demographic Transition had not occurred and the Malthusian population force had continued at its 1860 strength. Clearly, a falling land share is a powerful force for liberating the economy from the iron law of wages when productivity growth is positive.

Another way to visualize these effects is to plot the steady state wage relative to the actual wage over time. We do this in Figure 15. Before 1600, the ratio of the steady state wage to the actual wage is relatively stable around 1. Once productivity growth begins, however, the steady state wage jumps higher and the actual wage only gradually catches up. After 1760, the steady state wage begins to rise rapidly as the land share of production falls. By 1860, the steady state wage is more than ten times higher than the actual wage.

5.3 Post-1750 Population Explosion

The modest strength of the Malthusian population force in our model begs the question whether our model can explain, with these parameter values, the large increase in the population of England that occurred after 1750 (see Figure 5). In 1740, the population of England was 6 million. By 1860, it had risen to almost 20 million. The population therefore grew at a compound rate of 10.4% per decade over this 120-year period.

Figure 16 compares the evolution of the population in England from 1750 to 1860 with the predicted evolution of the population from our model. We construct the predicted evolution by taking the evolution of real wages and days worked in England as given and simulating the evolution of the population using equation (5) starting from its actual value in 1740 and assuming

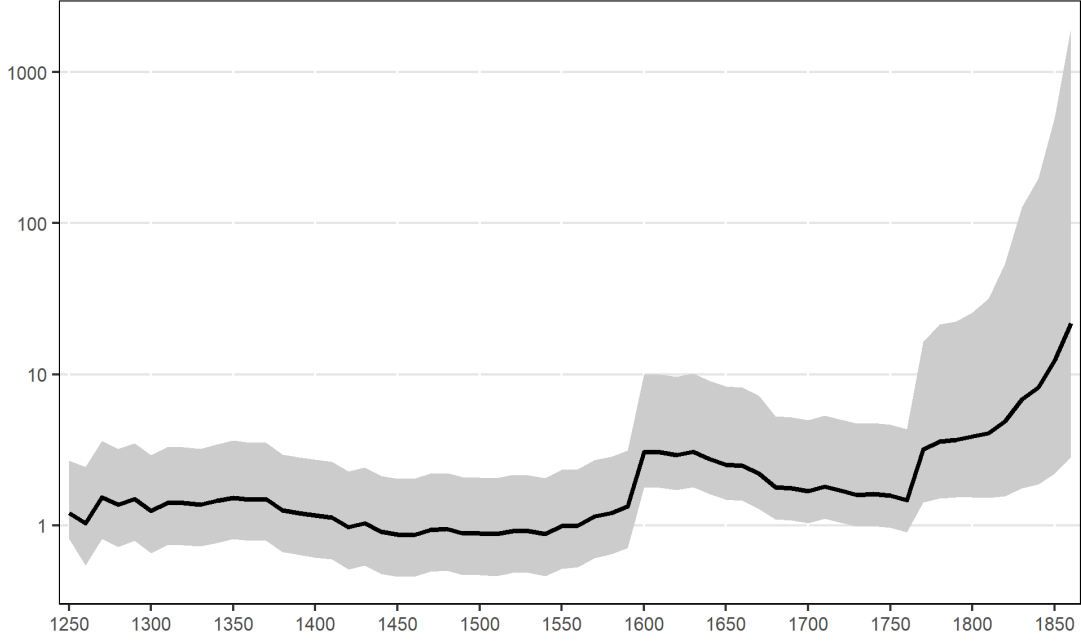


Figure 15: Steady State Wage Relative to Actual Wage

Note: The figure plots the ratio of the steady state wage – given by equation (47) – and the actual wage over time. Note that the steady state wage is a function of days worked. In the figure, we use the days worked at each date as the d^* in equation (47). The black line is the median of the posterior for each period and the gray shaded area is the 90% central posterior interval.

no population shocks. This analysis shows that in fact our model can explain the vast majority of the rapid increase in the population between 1740 and 1860. This may seem surprising given the weak Malthusian population force and the somewhat modest increase in real wages over this period. However, per capita income in England over this period rose much more than real wages did because days worked increased quite substantially (see Figure 6).

6 Plagues and the Population

Figure 17 plots our estimate of the evolution of the population of England from 1250 to 1550 along with prior estimates from Clark (2007a), Clark (2010), and Broadberry et al. (2015). Our estimates are very similar to Clark's. This implies that our estimation procedure largely validates the assumptions Clark makes regarding the evolution of productivity in constructing his population estimates. The estimates of Broadberry et al. (2015) are substantially lower early in the sample period, but then gradually converge.

The evolution of the population in England over our sample period is heavily affected by plagues. Our model captures plagues (and other influences on the population other than changes

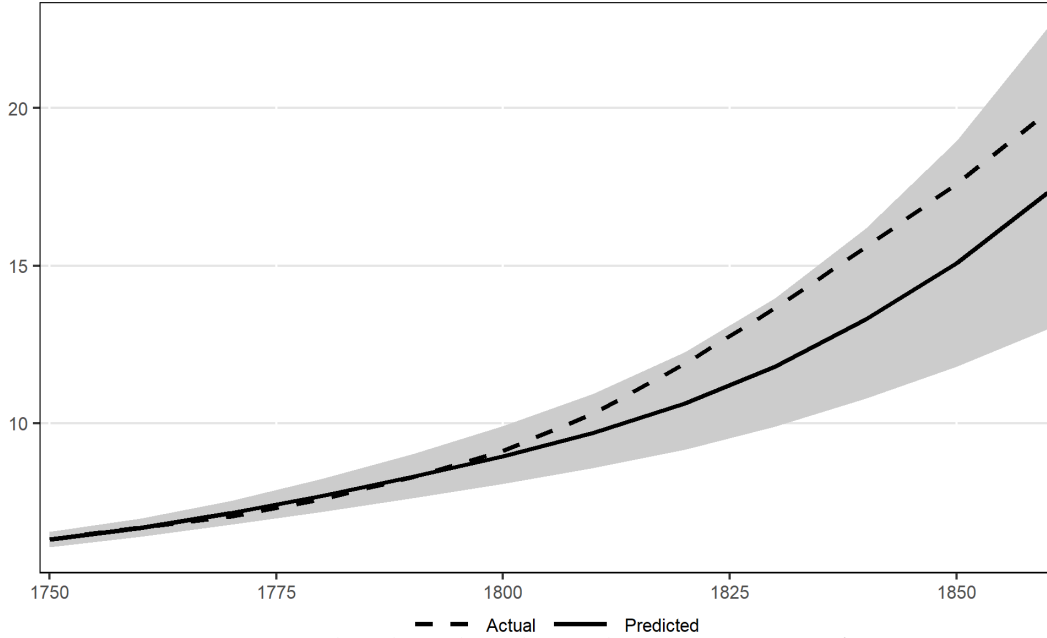


Figure 16: Actual and Predicted Population Dynamics after 1750

Note: The dashed line is the evolution of the population in England. The solid line is the predicted evolution of the population in England from our Malthusian model. In calculating this line, we take the evolution of real wages and days worked in England as given and simulate the evolution of the population using equation (5) starting from its actual value in 1740. The gray shaded area is the 90% central predictive interval given our estimates of α and γ .

in real income) through the shocks ξ_{1t} and ξ_{2t} . Figure 18 plots the evolution of the sum of these population shocks over our sample period. The largest population shock by far is the Black Death of 1348. We estimate that the population shocks associated with the Black Death lead the population of England to shrink by 32%. But Figure 18 also makes clear that England faced steady population headwinds—i.e., persistent negative population shocks—from the early 14th century until about 1500.

7 Conclusion

In this paper, we use a Malthusian model to estimate the evolution of productivity in England from 1250 to 1870. Our principal finding is that productivity growth began in 1600. Before 1600, productivity growth was zero. We estimate a growth rate of productivity of 3% per decade between 1600 and 1760 and an increase to 6% per decade between 1770 and 1870. These results indicate that sustained growth in productivity began well before the Glorious Revolution and Industrial Revolution. They point in particular to the early 17th century as a crucial turning point for productivity growth in England, a result that helps distinguish between competing lines of

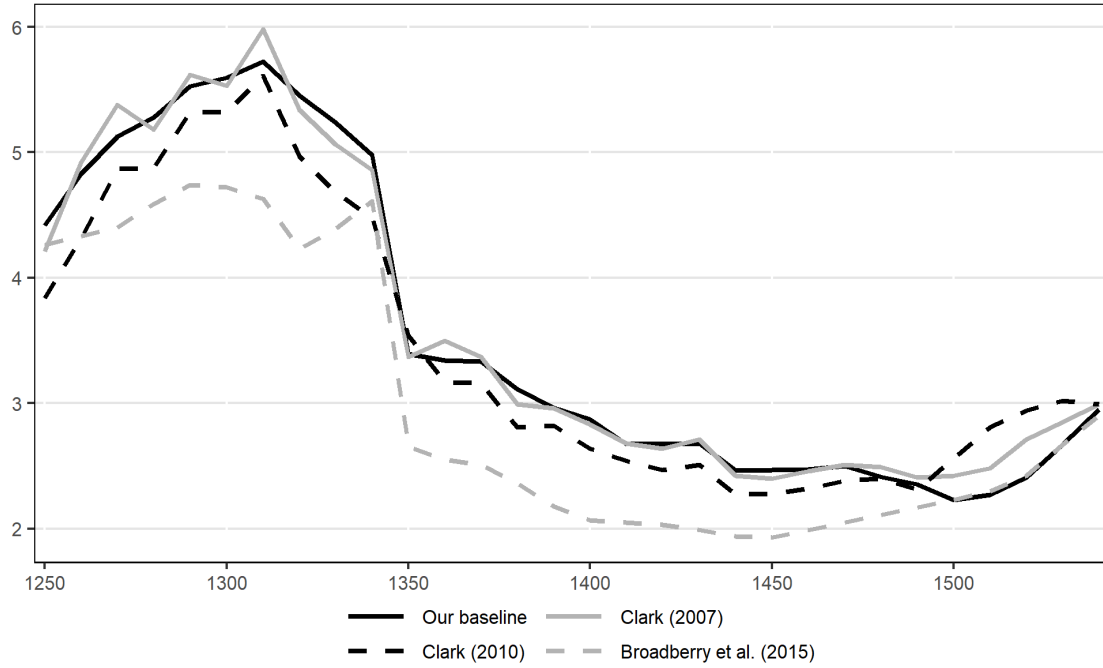


Figure 17: Comparison of Population Estimates for England

Note: The figure plots our estimates of the evolution of the population of England along with estimates from Clark (2007a), Clark (2010), and Broadberry et al. (2015)

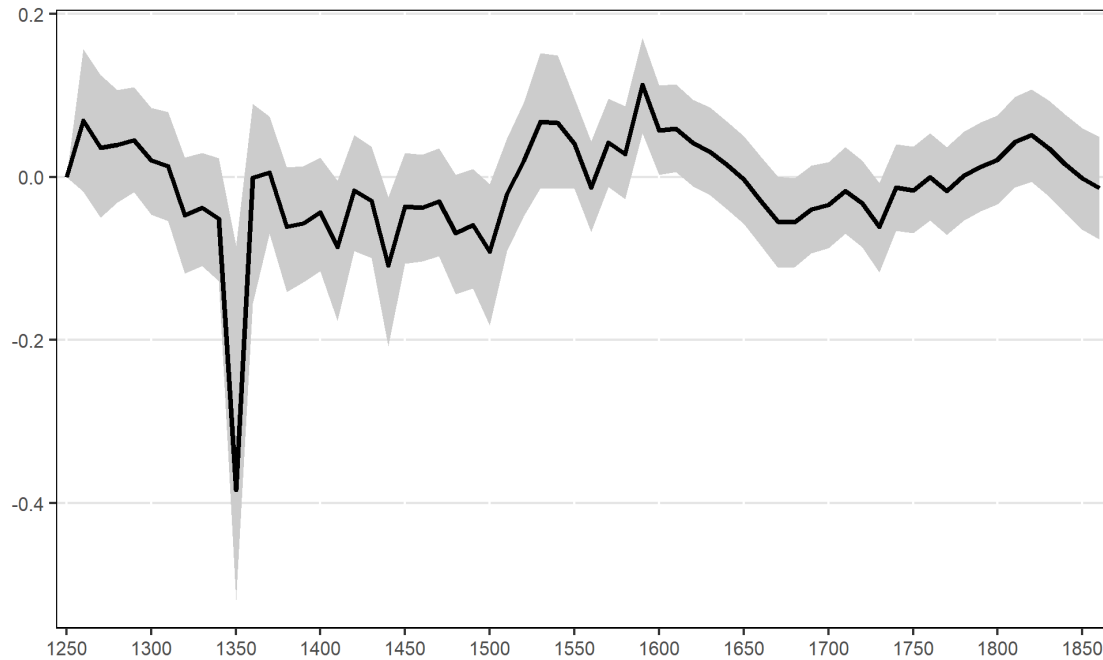


Figure 18: Population Shocks

Note: The figure plots our estimates of the population shocks hitting the English economy over our sample period, i.e., $\xi_{1t} + \xi_{2t}$. The black line is the mean of the posterior for each period and the gray shaded area is the 90% central posterior interval.

thought regarding the ultimate causes of the emergence of growth.

We attribute the high output growth of the Industrial Revolution only partly to productivity growth. A second important factor was a rapidly falling land share of production associated with the transition to steam power fueled by coal. We also use our model to estimate the strength of the Malthusian population force in pre-Industrial England. We estimate that this force was rather weak. The half-life of the response of real wages after a plague induced decrease in the population was about 170 years.

A Identification of α_t and β_t

Consider the demand curves for labor, land, and capital in the early-industrial era:

$$W_t = (1 - \alpha_t - \beta_t)A_t Z^{\alpha_t} K_t^{\beta_t} L_t^{-\alpha_t - \beta_t}, \quad (21)$$

$$S_t = \alpha_t A_t Z^{\alpha_t - 1} K_t^{\beta_t} L_t^{1 - \alpha_t - \beta_t}, \quad (22)$$

$$r_t + \delta = \beta_t A_t Z^{\alpha_t} K_t^{\beta_t - 1} L_t^{1 - \alpha_t - \beta_t}. \quad (23)$$

We begin by dividing land demand and capital demand by labor demand:

$$\frac{S_t}{W_t} = \frac{\alpha_t}{1 - \alpha_t - \beta_t} \frac{L_t}{Z}, \quad (24)$$

$$\frac{r_t + \delta}{W_t} = \frac{\beta_t}{1 - \alpha_t - \beta_t} \frac{L_t}{K_t}. \quad (25)$$

Manipulating equation (24) yields

$$\alpha_t = X_t - X_t \beta_t, \quad (26)$$

where

$$X_t = \frac{S_t/W_t}{(L_t/Z) + (S_t/W_t)}.$$

Manipulation equation (25) yields

$$\alpha_t = Y_t - Y_t \beta_t, \quad (27)$$

where

$$Y_t = \frac{(r_t + \delta)/W_t}{(L_t/K_t) + ((r_t + \delta)/W_t)}.$$

Solving equations (26) and (27) for α_t and β_t yields

$$\alpha_t = X_t \frac{1 - Y_t}{1 - X_t Y_t}, \quad (28)$$

$$\beta_t = Y_t \frac{1 - X_t}{1 - X_t Y_t}, \quad (29)$$

and we then also have that

$$1 - \alpha_t - \beta_t = \frac{(1 - X_t)(1 - Y_t)}{1 - X_t Y_t}. \quad (30)$$

Consider a case where S_t (land rents) goes up while all other variables remain constant. This increases X_t but leaves Y_t unchanged. As a consequence, α_t increases and both β_t and $1 - \alpha_t - \beta_t$ decrease.¹⁷

Next, consider a case where r_t (rental rate of capital) goes up while all other variables remain constant. This increases Y_t but leaves X_t unchanged. As a consequence, β_t increases and both α_t and $1 - \alpha_t - \beta_t$ decrease.

Finally, consider a case where W_t (wage) goes up while all other variables remain constant. This decreases both X_t and Y_t . As a consequence, both α_t and β_t decrease and $1 - \alpha_t - \beta_t$ increases.¹⁸

B The Malmquist Productivity Index

The concept of productivity is meant to measure the ratio of output to inputs (Diewert and Nakamura, 2007). In situations with more than one inputs (or outputs), the exact way in which this basic concept is operationalized is ambiguous. In some special cases, all reasonable measures of productivity will agree. This is, for example, the case if production is assumed to take the following form $Y_t = A_t F(X_t)$, where Y_t denotes output and X_t denotes a vector of inputs. In this case, A_t is the natural measure of productivity. In the more general case of $Y_t = F_t(X_t)$ the definition of productivity is less clear cut.

Caves, Christensen, and Diewert (1982) introduce the notion of a Malmquist productivity index for a quite general case of production technologies, based on ideas in Malmquist (1953). The discussion below builds on the exposition of these concepts in Färe et al. (1994). Consider a production technology S_t that transforms inputs $X_t \in \mathbb{R}_+^N$ into output $Y_t \in \mathbb{R}_+$: $S_t = \{(X_t, Y_t) : X_t \text{ can produce } Y_t\}$. Written in terms of a production function $Y_t = F_t(X_t)$, we have $S_t = \{(X_t, Y_t) : Y_t \leq F_t(X_t)\}$. In other words, S_t defines the set of all feasible input-output vectors.

Caves, Christensen, and Diewert (1982) define the Malmquist productivity index in terms of the distance function $D_t(X_s, Y_s) = \inf\{\theta : (X_s, Y_s/\theta) \in S_t\}$. The distance $D_t(X_s, Y_s)$ is then the minimum multiplicative proportion by which Y_s needs to be scaled down for the input-output vector (X_s, Y_s) to be feasible with time t technology. For example, if period s is a later period than

¹⁷The derivative of $(1 - X_t)/(1 - X_t Y_t)$ with respect to X_t is $-(1 - Y_t)/(1 - X_t Y_t)^2$, which is negative.

¹⁸The total derivative of $1 - \alpha_t - \beta_t$ with respect to W_t is: $-\left((1 - Y_t)^2 \times \partial X_t / \partial W_t + (1 - X_t)^2 \times \partial Y_t / \partial W_t\right) / (1 - X_t Y_t)^2$. Since X_t and Y_t are both decreasing in W_t , this derivative is positive. For $1 - \alpha_t - \beta_t$ to increase, α_t or β_t must decrease. Manipulating equations (24) and (25), we have: $\alpha_t / \beta_t = S_t / (r_t + \delta) \times Z / K_t$. Since the ratio of α_t over β_t is constant and at least one of them decreases, both must decrease.

period t and technology is “more advanced” at this later period, (X_s, Y_s) may be feasible using technology S_s , but $Y_s/D_t(X_s, Y_s)$ with $D_t(X_s, Y_s) > 1$ may be the largest output that is feasible given input use X_s and the inferior technology S_t .

Given the definition of S_t , the distance is the smallest θ such that $Y_s/\theta \leq F_t(X_s)$, which means $D_t(X_s, Y_s) = Y_s/F_t(X_s)$. Under our maintained assumptions in this paper, $D_t(X_t, Y_t) = 1$, i.e., the output actually produced at time t with inputs X_t is exactly feasible. (More generally, one can imagine production at time t being inside the technical frontier at time t . In this case, $D_t(X_t, Y_t) < 1$.)

Next consider $D_t(X_{t+1}, Y_{t+1})$, i.e., the distance of the input-output vector at time $t + 1$ from the technical frontier at time t . Applying the definition of the distance function we have that $Y_{t+1}/D_t(X_{t+1}, Y_{t+1}) = F_t(X_{t+1})$, which implies

$$D_t(X_{t+1}, Y_{t+1}) = \frac{Y_{t+1}}{F_t(X_{t+1})} = \frac{F_{t+1}(X_{t+1})}{F_t(X_{t+1})}.$$

This is very intuitive: The distance of the time $t + 1$ technology from the time t technology evaluated at the time $t + 1$ input-output vector is simply the output at time $t + 1$, i.e., $F_{t+1}(X_{t+1})$, divided by what output would be if the input vector at time $t + 1$ were used with the time t technology, i.e., $F_t(X_{t+1})$.

A Malmquist index for productivity growth between periods t and $t + 1$ that uses the production technology of time t as a reference technology is then defined as

$$M_{t,t+1}^t \equiv \frac{D_t(X_{t+1}, Y_{t+1})}{D_t(X_t, Y_t)} = \frac{F_{t+1}(X_{t+1})/F_t(X_{t+1})}{1} = \frac{F_{t+1}(X_{t+1})}{F_t(X_{t+1})}.$$

We can also consider $D_{t+1}(X_t, Y_t)$, i.e., the distance of the input-output vector at time t from the technical frontier at time $t + 1$. Applying the definition of the distance function, we have that $Y_t/D_{t+1}(X_t, Y_t) = F_{t+1}(X_t)$, which implies

$$D_{t+1}(X_t, Y_t) = \frac{Y_t}{F_{t+1}(X_t)} = \frac{F_t(X_t)}{F_{t+1}(X_t)}.$$

A Malmquist index for productivity growth between periods t and $t + 1$ that uses the production technology of time $t + 1$ as a reference technology is then defined as

$$M_{t,t+1}^{t+1} \equiv \frac{D_{t+1}(X_{t+1}, Y_{t+1})}{D_{t+1}(X_t, Y_t)} = \frac{1}{F_t(X_t)/F_{t+1}(X_t)} = \frac{F_{t+1}(X_t)}{F_t(X_t)}.$$

Caves, Christensen, and Diewert (1982) recommend defining the Malmquist index as the geometric average of $M_{t,t+1}^t$ and $M_{t,t+1}^{t+1}$. In this case the Malmquist index becomes

$$M_{t,t+1} \equiv \left(\frac{D_t(X_{t+1}, Y_{t+1})}{D_t(X_t, Y_t)} \frac{D_{t+1}(X_{t+1}, Y_{t+1})}{D_{t+1}(X_t, Y_t)} \right)^{1/2} = \left(\frac{F_{t+1}(X_{t+1})}{F_t(X_{t+1})} \frac{F_{t+1}(X_t)}{F_t(X_t)} \right)^{1/2}.$$

This definition avoids favoring the technology in one of the two periods over the other.

B.1 Normalization and the Malmquist Index

As we discuss in footnote 5 in the body of the paper, one symptom of A_t not being a good measure of productivity in the case where the functional form of the production function changes over time is that the growth rate of A_t will be sensitive to the choice of normalization of the inputs to production. This is not the case for the Malmquist index.

To illustrate this, consider again the change in the unit in which labor is expressed that we discussed in footnote 5: $\tilde{L}_t \equiv \psi L_t$. In this case we have that

$$F_t(Z, K_t, L_t) \equiv A_t Z^{\alpha_t} K_t^{\beta_t} L_t^{1-\alpha_t-\beta_t} = \tilde{A}_t Z^{\alpha_t} K_t^{\beta_t} \tilde{L}_t^{1-\alpha_t-\beta_t} \equiv \tilde{F}_t(Z, K_t, \tilde{L}_t), \quad (31)$$

where

$$\tilde{A}_t \equiv \frac{A_t}{\psi^{1-\alpha_t-\beta_t}}$$

Clearly, if α_t or β_t vary over time, the growth rates of A_t and \tilde{A}_t will not be the same.

The Malmquist index, however, suffers no such issue. Since, by equation (31), $F_t(Z, K_t, L_t) = \tilde{F}_t(Z, K_t, \tilde{L}_t)$, this equation immediately implies that the Malmquist index remains the same. In fact, any rewriting of the production function that leaves the mapping from input to output unchanged, i.e. that does not change the production possibility frontier, implies the same Malmquist index because the formula for the Malmquist index only depends on output for some quantities of inputs.

We can illustrate this point by deriving an expression for the Malmquist index in terms of the observables in our model – equation (16) – for both F_t and \tilde{F}_t and denoting the associated indices

as m_t and \tilde{m}_t :

$$\begin{aligned}
\hat{m}_t &= \hat{a}_t + \hat{\alpha}_t \log Z + \hat{\beta}_t \bar{k}_t - (\hat{\alpha}_t + \hat{\beta}_t) \bar{l}_t \\
&= \hat{a}_t + (\hat{\alpha}_t + \hat{\beta}_t) \log \psi + \hat{\alpha}_t \log Z + \hat{\beta}_t \bar{k}_t - (\hat{\alpha}_t + \hat{\beta}_t) (\bar{l}_t + \log \psi) \\
&= \hat{\hat{a}}_t + \hat{\alpha}_t \log Z + \hat{\beta}_t \bar{k}_t - (\hat{\alpha}_t + \hat{\beta}_t) \bar{\bar{l}}_t \\
&= \hat{\hat{m}}_t.
\end{aligned}$$

Recall that hats denote deviations from the previous period, $\hat{x}_t = x_t - x_{t-1}$, and bars denote the average of period $t - 1$ and period t , $\bar{x}_t = (x_{t-1} + x_t)/2$. To go from the first to the second line, we added and subtracted the normalization that transforms \bar{l}_t into $\bar{\bar{l}}_t$: $(\hat{\alpha}_t + \hat{\beta}_t) \log \psi$. In the third line, this time-varying normalization is absorbed by the A residual, $\hat{\hat{a}}_t$, and $\bar{l}_t + \log \psi$ is converted to $\bar{\bar{l}}_t$. From this we see that while the A residual is normalization-dependent the Malmquist index is not.

C Model Equations

We reproduce the equations and distributional assumptions of our full model here for convenience:

$$\begin{aligned}
w_t &= \phi_t + \frac{1}{1-\beta_t} a_t - \frac{\alpha_t}{1-\beta_t} (d_t + n_t) - \frac{\beta_t}{1-\beta_t} \log(r_t + \delta) \\
\phi_t &= \log \beta_t + \log(1 - \alpha_t - \beta_t) + \frac{\alpha_t}{1-\beta_t} z - (\alpha_t + \beta_t) \lambda \\
s_t &= w_t + n_t + d_t - z + \log \alpha_t - \log(1 - \alpha_t - \beta_t) \\
k_t &= w_t + n_t + d_t - \log(r_t + \delta) + \log \beta_t - \log(1 - \alpha_t - \beta_t) \\
n_t &= n_{t-1} + \omega + \gamma(w_{t-1} + d_{t-1}) + \xi_{1t} + \xi_{2t} \\
\hat{m}_t &= \hat{a}_t + \hat{\alpha}_t z + \hat{\beta}_t \bar{k}_t - (\hat{\alpha}_t + \hat{\beta}_t) (\bar{d}_t + \bar{n}_t) \\
m_t &= \tilde{m}_t + \epsilon_{2t} \\
\tilde{m}_t &= \mu + \tilde{m}_{t-1} + \epsilon_{1t}
\end{aligned}$$

$$\exp(\xi_{1t}) \sim \begin{cases} \beta(\beta_1, \beta_2), & \text{with probability } \pi \\ 1, & \text{with probability } 1 - \pi \end{cases}$$

$$\epsilon_{1t} \sim \mathcal{N}(0, \sigma_{\epsilon_1}^2), \quad \epsilon_{2t} \sim \mathcal{N}(0, \sigma_{\epsilon_2}^2), \quad \xi_{2t} \sim \mathcal{N}(0, \sigma_{\xi_2}^2)$$

Before 1760, α_t and β_t are assumed to be constant. This implies that the sixth equation collapses to $\hat{a}_t = \hat{m}_t$ before 1760. As a result, rents s_t and the capital stock k_t only appear in the equations that define them (the third and fourth equations). This is also the period for which we do not have data on s_t and k_t . For this period, we therefore use the third and fourth equations to estimate s_t and k_t .

Below we reproduce the assumptions we make about measurement error and normalizations in our data:

$$\begin{aligned} w_t &= \varphi^w + \tilde{w}_t \\ n_t &= \psi + \tilde{n}_t + \iota_t^n \\ d_t &= \tilde{d}_t + \iota_t^d \\ r_t &= \tilde{r}_{it} + \iota_{it}^r \\ s_t &= \varphi^s + \tilde{s}_t + \iota_t^s, \\ k_t &= \varphi^k + \tilde{k}_t + \iota_t^k, \end{aligned}$$

Here, the variables with tilde's are the measured variables, while the variables without tilde's are the true variables, $\varphi^w \sim \mathcal{N}(0, 100^2)$, $\varphi^s \sim \mathcal{N}(0, 100^2)$, and $\varphi^k \sim \mathcal{N}(0, 100^2)$ are normalization constants, and $\iota_t^n \sim t_{\nu_n}(0, \sigma_n^2)$, $\iota_t^d \sim t_{\nu_d}(0, \tilde{\sigma}_d^2)$, $\iota_{it}^r \sim t_{\nu_{ir}}(0, \tilde{\sigma}_{ir}^2)$, $\iota_t^s \sim t_{\nu_s}(0, \tilde{\sigma}_s^2)$, and $\iota_t^k \sim t_{\nu_k}(0, \tilde{\sigma}_k^2)$ capture measurement error. A few additional details regarding missing observations are given in the main text.

D Clark's Population Series

As we discuss in the main text, Clark (2007b) uses unbalanced panel data on the population of villages and manors from manorial records and penny tithing payments to construct estimates of the population prior to 1540. Clark starts by running a regression of this data on time fixed effects and manor/village fixed effects. He refers to the time fixed effects from this regression as a population trend series.

Clark's population trend series does not provide information on the overall level of the population prior to 1540, only changes in the population (i.e., a normalization is needed). In addition, Clark's microdata is sufficiently unreliable for the 1530s that Clark does not make use of his estimated population trend for that decade. Clark uses the following procedure to surmount these problems. First, he regresses his population trend on real wages from 1250 to 1520, and separately regresses the Wrigley et al. (1997) population series on wages from 1540 to 1610. He observes

that the R^2 in both regressions are high and that they yield similar slope coefficients. He concludes from this that (i) the English economy moved along stable labor demand curves during both subsamples and (ii) these two labor demand curves had similar slopes.

Clark next makes the assumption that there was no productivity growth between 1520 and 1540—the labor demand curve did not shift during this time. This allows him to extrapolate the relationship that he finds in the post-1540 data to the earlier sample, and infer both the population in 1530 and the missing normalization from the level of real wages. Clark also uses the fitted values for the population from his labor demand curve as an alternative estimate of the population and averages this with the trend series to get what he calls the “best” estimate of population before 1540.

E A Comparison with Clark (2010, 2016)

Our approach to estimating productivity in England from the 13th to 19th centuries yields quite different results than the most comprehensive existing estimates by Clark (2010, 2016). Here, we consider from where the differences arise. We break this discussion into three parts. First, we discuss Clark’s dual approach and differences between his 2010 series and his 2016 series. Second, we discuss how Clark’s dual approach relates to our Malmquist approach. Third, we discuss differences that arise from the fact that our approach has different implications for the evolution of factor prices and factor shares than Clark’s approach.

A summary of our conclusions is as follows. First, Clark (2010) made an error in calculating the growth rate of his index from 1540 to 1550 which contributes to the difference between this series and our series. Clark (2016) corrects this error. Second, Clark’s dual approach is approximately equal to our Malmquist approach. Differences between these approaches do not contribute much to the difference between our results and Clark’s. Third, differences in the factor prices and factor shares implied by our approach, relative to those used by Clark, explain the remaining differences in the evolution of productivity. In particular, the rental price of land we infer differs markedly from Clark’s land rent series.

E.1 Clark’s Dual Approach and Differences Between Clark (2010) and Clark (2016)

Clark (2010, 2016) employs a “dual approach” to estimating productivity. Specifically, his estimate

of the growth rate of productivity is

$$\frac{E_t}{E_{t-1}} = \left(\frac{S_t}{S_{t-1}} \right)^{s_{Z,t-1}} \left(\frac{r_t + \lambda}{r_{t-1} + \lambda} \right)^{s_{K,t-1}} \left(\frac{W_t}{W_{t-1}} \right)^{s_{L,t-1}} \frac{1 - \tau_{t-1}}{1 - \tau_t}. \quad (32)$$

where we use E_t (for efficiency) to denote the dual estimate of productivity, λ is a risk premium, τ_t is the share of national income paid in indirect taxes, and $s_{Z,t-1}$, $s_{K,t-1}$, and $s_{L,t-1}$ are time-varying estimates of the land share, capital share, and labor share, respectively.¹⁹

Clark's 2016 productivity series is an updated version of his better known 2010 productivity series for the sample period 1250-1600. Clark has shared with us the file he used to construct his 2016 series by private correspondence. This file extends his 2016 series from 1600 to 1860 and contains the component series Clark uses to construct this series. Our discussion here is based on these series. For the period after 1600, the new productivity series coincides with Clark's 2010 series.

Figure E.1 plots Clark's 2010 productivity series (solid gray line) and Clark's 2016 productivity series extended to 1860 using the file Clark shared with us (broken black line). We refer to the extended 2016 series as "Clark (2016)*". These series differ for two reasons. First, Clark's 2010 series contains an error in the growth rate from 1540 to 1550. This error creates a 25 log point spurious drop in the 2010 series. Clark's 2016 series corrects this error. Second, Clark's 2016 series incorporates a new land rent series for the period 1250-1600. Both of these changes make Clark's 2016 series more similar to our productivity estimate (solid black line in Figure E.1) than his 2010 series.

The Malmquist index we use for our baseline estimates uses average factor shares rather than lagged factor shares. Using average factor shares is also recommended by Barro and Sala-i-Martin (2004, p. 435). We can modify Clark's dual approach – equation (32) – to use average factor shares as follows:

$$\frac{E_t}{E_{t-1}} = \left(\frac{S_t}{S_{t-1}} \right)^{\bar{s}_{Z,t}} \left(\frac{r_t + \lambda}{r_{t-1} + \lambda} \right)^{\bar{s}_{K,t}} \left(\frac{W_t}{W_{t-1}} \right)^{\bar{s}_{L,t}} \frac{1 - \tau_{t-1}}{1 - \tau_t}. \quad (33)$$

As in the main text, the bar on top of each s signifies an average between $t - 1$ and t : $\bar{s}_{Z,t} = (s_{Z,t-1} + s_{Z,t})/2$ and similarly for $\bar{s}_{K,t}$ and $\bar{s}_{L,t}$.

The fourth line plotted in Figure E.1 is productivity growth estimated using equation (33) and

¹⁹The discussion in Clark (2010, 2016) suggests that Clark estimates the level of productivity rather than its growth rate. However, data Clark has shared with us (discussed below) makes clear that he, in fact, estimates growth rates of productivity. This distinction is important as the level formula Clark discusses in his 2010 and 2016 papers does not provide a valid measure of productivity when factor shares are allowed to vary over time. See footnote 5 in the main text for more detail on this point.

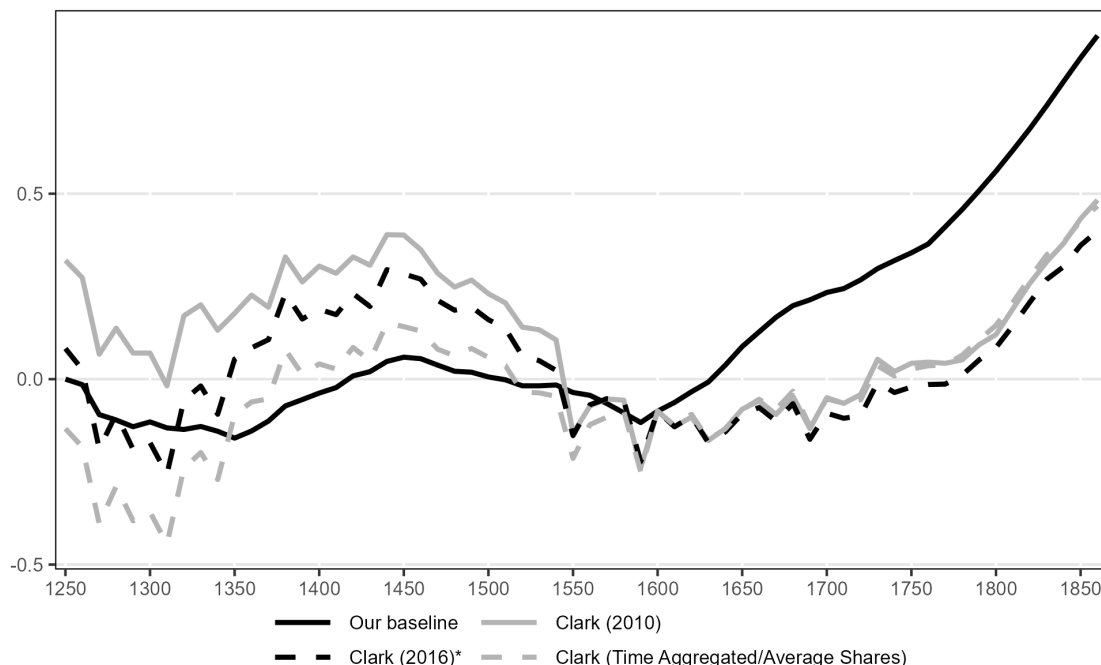


Figure E.1: A Comparison between Clark (2010) and Clark (2016)

Note: The Figure plots four productivity series. The solid black line is our baseline Malmquist index. The solid gray line is Clark’s (2010) original productivity series. The broken black line – labeled “Clark (2016)*” – is Clark’s (2016) productivity series extended to 1860. We obtained this series from Clark in private correspondence. The broken gray line is an estimate of productivity using equation (32) with decadal data, i.e., this series moves to average shares and time aggregates relative to the Clark (2016)* series. The latter three series are normalized to be equal to the Malmquist index in 1600.

Clark’s data series for factor prices and factor shares (broken gray line). This line also differs from the two Clark series because of time aggregation. Clark estimates productivity using equation (32) at an annual frequency and then averages over decades. To be consistent with our approach in the rest of the paper, we average the data over each decade and then use equation (32) to estimate productivity at a decadal frequency. We see that moving from lagged to average factor shares and decadal time aggregation results in estimates of productivity that are lower early in the sample. This difference is mostly due to the switch to average shares – time aggregation only makes a small difference. These changes result in a productivity series that is closer to ours between 1350 and 1600.

E.2 The Dual Approach versus the Malmquist Approach

We next show that our Malmquist index and the dual approach are equal up to a first-order approximation. To see this, we go back to equation (16), which we reproduce here for convenience:

$$\hat{m}_t = \hat{a}_t + \hat{\alpha}_t \log Z + \hat{\beta}_t \bar{k}_t - (\hat{\alpha}_t + \hat{\beta}_t) \hat{l}_t.$$

In this equation, bars denote arithmetic averages across period $t - 1$ and t and hats denote differences between the two periods. Rearranging this equation yields²⁰

$$\hat{m}_t = \hat{y}_t - \bar{\beta}_t \hat{k}_t - (1 - \bar{\alpha}_t - \bar{\beta}_t) \hat{l}_t. \quad (34)$$

The right-hand side is the primal measure of the growth rate of productivity, i.e., the Solow residual (Solow, 1957). Here, weights are given by the arithmetic average of the factor shares across the two periods. To go from the primal measure to the dual measure, we can follow Hsieh (2002) and start from the fact that the value of output must equal payments to factors: $Y_t = S_t Z + (r_t + \delta)K_t + W_t L_t$. Taking a log-linear approximation of this expression at times $t - 1$ and t around a situation where factor shares are the averages of the two periods yields the following expression:

$$\hat{y}_t = \bar{\alpha}_t \hat{s}_t + \bar{\beta}_t \left(\log \left(\frac{r_t + \delta}{r_{t-1} + \delta} \right) + \bar{k}_t \right) + (1 - \bar{\alpha}_t - \bar{\beta}_t) (\hat{w}_t + \hat{l}_t),$$

²⁰The derivation is

$$\begin{aligned} \hat{m}_t &= a_t - a_{t-1} + \frac{1}{2} (\alpha_t \log Z + \beta_t k_t + (1 - \alpha_t - \beta_t) l_t - (\alpha_{t-1} \log Z + \beta_{t-1} k_{t-1} + (1 - \alpha_{t-1} - \beta_{t-1}) l_{t-1})) \\ &\quad - \frac{1}{2} (\alpha_{t-1} \log Z + \beta_{t-1} k_t + (1 - \alpha_{t-1} - \beta_{t-1}) l_t - (\alpha_t \log Z + \beta_t k_{t-1} + (1 - \alpha_t - \beta_t) l_{t-1})) \\ &= a_t + \alpha_t \log Z + \beta_t k_t + (1 - \alpha_t - \beta_t) l_t - (a_{t-1} + \alpha_{t-1} \log Z + \beta_{t-1} k_{t-1} + (1 - \alpha_{t-1} - \beta_{t-1}) l_{t-1}) \\ &\quad - \frac{1}{2} (\alpha_t \log Z + \beta_t k_t + (1 - \alpha_t - \beta_t) l_t - (\alpha_{t-1} \log Z + \beta_{t-1} k_{t-1} + (1 - \alpha_{t-1} - \beta_{t-1}) l_{t-1})) \\ &\quad - \frac{1}{2} (\alpha_{t-1} \log Z + \beta_{t-1} k_t + (1 - \alpha_{t-1} - \beta_{t-1}) l_t - (\alpha_t \log Z + \beta_t k_{t-1} + (1 - \alpha_t - \beta_t) l_{t-1})) \\ &= \hat{y}_t - \bar{\beta}_t \hat{k}_t - (1 - \bar{\alpha}_t - \bar{\beta}_t) \hat{l}_t. \end{aligned}$$

For the first equality, we just use the definition of the bar and hat symbols. For the second equality, we add and subtract the expression contained in the line that follows the second equal sign. The third equality is again a straightforward use of the bar and hat symbols.

where we have dropped higher order terms. Combining this equation and equation (34), we obtain

$$\hat{m}_t = \bar{\alpha}_t \hat{s}_t + \bar{\beta}_t \log \left(\frac{r_t + \delta}{r_{t-1} + \delta} \right) + (1 - \bar{\alpha}_t - \bar{\beta}_t) \hat{w}_t. \quad (35)$$

This equation shows that the log-change in the Malmquist index is equal to the dual measure of productivity growth up to a first order approximation.

The productivity measures in equation (35) differ in some details from the ones plotted in Figure E.1. First, the left-hand-side of equation (35) is \hat{m}_t , the change in m_t . The productivity measure plotted as our baseline estimate in Figure E.1 (solid black line) is \tilde{m}_t rather than m_t . Recall that \tilde{m}_t is the permanent component of productivity (see equations (17)–(18)). Our baseline estimate in Figure E.1, thus, filters out some high frequency variation in productivity, which makes it smoother than estimates based on the dual approach.

Clark's dual approach also differs in a few details from the right-hand side of equation (35). Clark's dual approach does not incorporate capital depreciation (δ), but it includes a risk premium (λ) and taxes (τ_t) that are not incorporated into equation (35). The similarity (and difference in details) between the right-hand side of (35) and Clark's dual approach can be more easily seen by taking logarithms of equation (33):

$$\hat{e}_t = \bar{s}_{Z,t} \hat{s}_t + \bar{s}_{K,t} \log \left(\frac{r_t + \lambda}{r_{t-1} + \lambda} \right) + \bar{s}_{L,t} \hat{w}_t - \log \left(\frac{1 - \tau_t}{1 - \tau_{t-1}} \right) \quad (36)$$

Comparing this equation to equation (35), notice that in our model, α_t , β_t , and $1 - \alpha_t - \beta_t$ are the land, capital, and labor shares, while in equation (36) these are denoted by $s_{Z,t}$, $s_{K,t}$ and $s_{L,t}$, respectively. The two formulas are, thus, the same apart from δ being replaced by λ , and the presence of τ_t in equation (36).

These details turn out not to make much of a difference. To see this, Figure E.2 plots a dual measure of productivity using the formula in equation (33) but with our factor price and factor share series (broken black line). In other words, this measure of productivity, differs from ours only in terms of method, not data. We see that the resulting productivity series tracks our baseline productivity series very closely. The only difference is that our baseline measure is smoother at high frequency reflecting the fact that it filters out high-frequency movements in productivity.

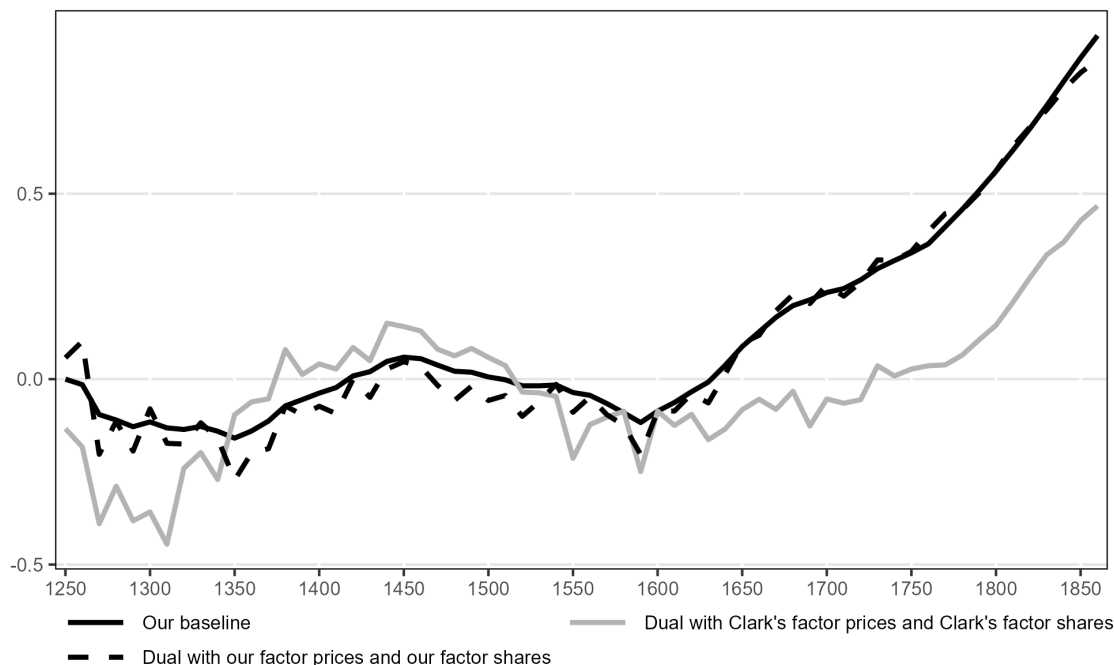


Figure E.2: Our Productivity Measure Compared with the Dual Approach

Note: The Figure plots three productivity series. The solid black line is our baseline Malmquist index. The solid gray line is the index constructed with Clark's factor prices and factor shares using equation (33). This is the same line as the one we label "Clark (Time Aggregation/Average Shares)" in Figure E.1. The dashed black line is the index constructed with our factor prices and factor shares using equation (33). The latter two series are normalized to be equal to the Malmquist index in 1600.

E.3 Factor Prices and Factor Shares

We now turn to the role played by differences in the factor price and factor share series used by Clark relative to those implied by our analysis. Since we have shown above that the dual approach and the Malmquist approach are virtually equivalent, we will carry out the rest of the analysis using the dual approach for concreteness. In particular, we will calculate productivity using equation (33) with different combinations of Clark's and our factor price and factor share series. In the case of Clark's series, we will use Clark's 2016 series extended to 1860. We have already plotted two such cases in Figure E.2. The solid gray line uses Clark's factor price and factor share series, while the broken black line uses our factor price and factor share series. Next, we consider intermediate cases.

A complication that arises if we seek a decomposition of the remaining difference between our productivity index and Clark's – the solid gray line and the broken black line in Figure E.2 – into the share explained by factor prices and the share explained by factor shares is that the productivity indexes we are considering are non-linear. This implies that the difference in question is not simply the sum of the effect of changing the factor prices, on the one hand, and the effect of

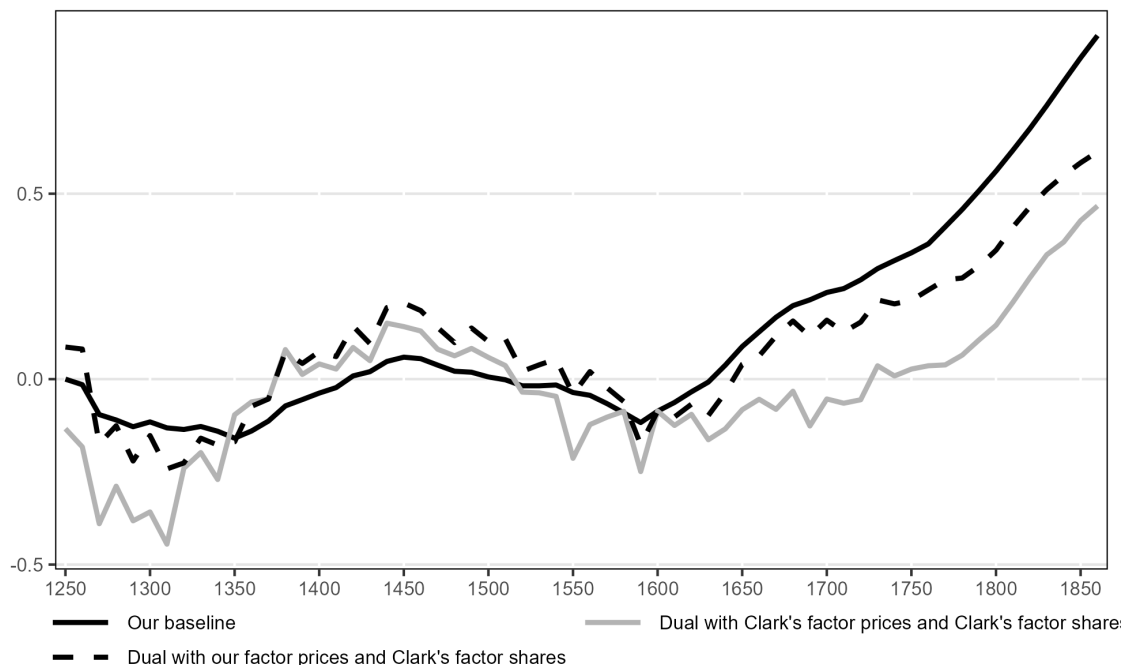


Figure E.3: Contribution of Factor Prices to Differences in Productivity Estimates

Note: The Figure plots three productivity series. The solid black line is our baseline Malmquist index. The solid gray line is the index constructed with Clark's factor prices and factor shares using equation (33). The dashed gray line is the index constructed with Clark's factor prices and Clark factor shares using equation (33). The latter two series are normalized to be equal to the Malmquist index in 1600.

changing the factor shares, on the other hand. Rather, there is also an interaction term, which is non-trivial.

With this in mind, we begin by considering how changing the factor price series alone affects the productivity series. Figure E.3 plots a dual estimate of productivity using Clark's factor shares but our factor price series (broken black line). The difference between the solid gray line and the broken black line in Figure E.3 is thus due to moving from Clark's factor price series to our factor price series. Focusing on the period after 1600, we see that this change explains a sizable portion of the difference between our results and the series using Clark's factor prices and factor shares. Quantitatively, it explains 32% of the difference in productivity growth from 1600 to 1860.

Figure E.4 plots Clark's factor price series (solid gray lines) and our factor price series (solid black lines). In the case of land rents, we also plot the series used in Clark (2010) (broken gray line). Our real wage series looks very similar to Clark's. The differences in our productivity estimates are therefore not arising from real wages. The raw real interest rate date we use is also similar to that used by Clark. However, we allow for measurement error in real interest rates and make use of two return series (rates of return on land and rent charges). This implies that our real interest rate series is substantially smoother in the early part of our sample and around 1600. In particular,

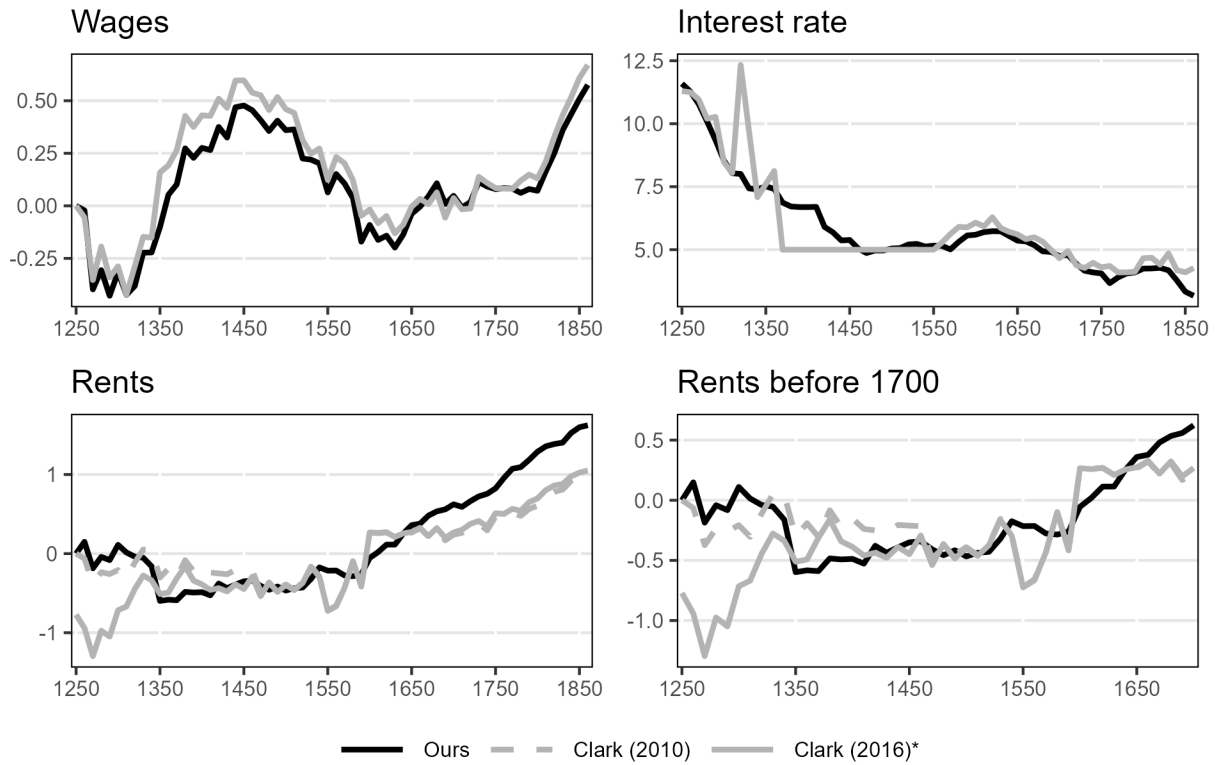


Figure E.4: A Comparison of Price Series

Note: The top two panels plots the wage and interest rate series used in our analysis and used by Clark (2016). The bottom two panels plot the land rent series used by Clark (2010, 2016) and the land rents that are implied by our analysis.

Clark's interest rate series is constant between 1370 and 1540, reflecting Clark's choice of how to interpolate over a period of relatively sparse data, while our series falls more gradually over the early part of this period.

For land rents, we use the same data as Clark after 1760 but choose to infer land rents from the model prior to 1760. Our inferred series differs quite a bit from Clark's series. Clark's data is quite noisy over this early period. One difference is that our series shows a larger increase between 1500 and 1760 than Clark's data. Another, particularly notable, difference is that we infer a substantial drop in land rents after the Black Death in 1350, while Clark's series does not show such a drop. It is odd that Clark's series does not show a drop in land rents after the Black Death since the labor-land ratio fell by a very large amount. But measuring land rents in the 14th century is difficult due to the complexity of the relationship between landlords and tenants in a feudal era. It is also notable that Clark's 2016 series for land rents differs quite substantially from his earlier 2010 land rent series for the period prior to 1500. From 1250 to 1500, the 2016 series increases by 47%, while the 2010 series falls by 68%.

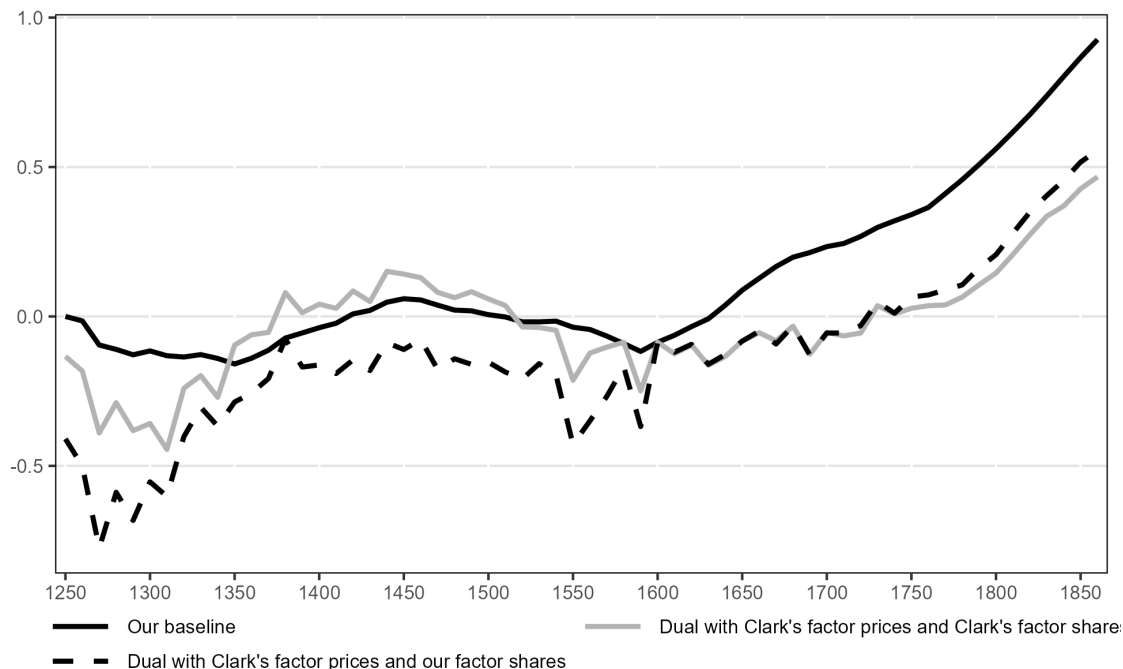


Figure E.5: Contribution of Factor Shares to Differences in Productivity Estimates

Note: The Figure plots three productivity series. The solid black line is our baseline Malmquist index. The solid gray line is the index constructed with Clark's factor prices and factor shares using equation (33). The dashed gray line is the index constructed with Clark's factor prices and our factor shares using equation (33). The latter two series are normalized to be equal to the Malmquist index in 1600.

Turning to factor shares, Figure E.5 plots a dual estimate of productivity using Clark's factor prices but our factor share series (broken black line). The difference between the solid gray line and the broken black line in Figure E.5 is thus due to moving from Clark's factor shares series to our factor share series. For the period after 1600, this change has a minimal effect. Prior to 1600, the differences are larger. Shifting to our factor share results in a sharp rise of the productivity series from 1250 to 1400. This reflects the increase in Clark's 2016 rent series (which both the solid gray and broken black lines are using). This increase receives larger weight in the broken black line because we estimate a larger land share (see Figure E.6). However, all the productivity estimates using the dual approach prior to 1600 are quite volatile at high frequency. Figures E.1, E.2, E.3, and E.5 taken together indicate that the difference between Clark's series and our series before 1600 is a complicated combination of the effects of factor prices, factor shares, their interaction, time aggregation, and average factor share versus lagged factor shares.

Figure E.6 compares our estimates of factor shares (black lines) with Clark's (gray lines). The largest difference is for the land share. We estimate a substantially larger land share than Clark. Recall that in our model the land share is related to the slope of the labor demand curve. Our estimate of the land share is therefore backed out from our estimate of the slope of the labor

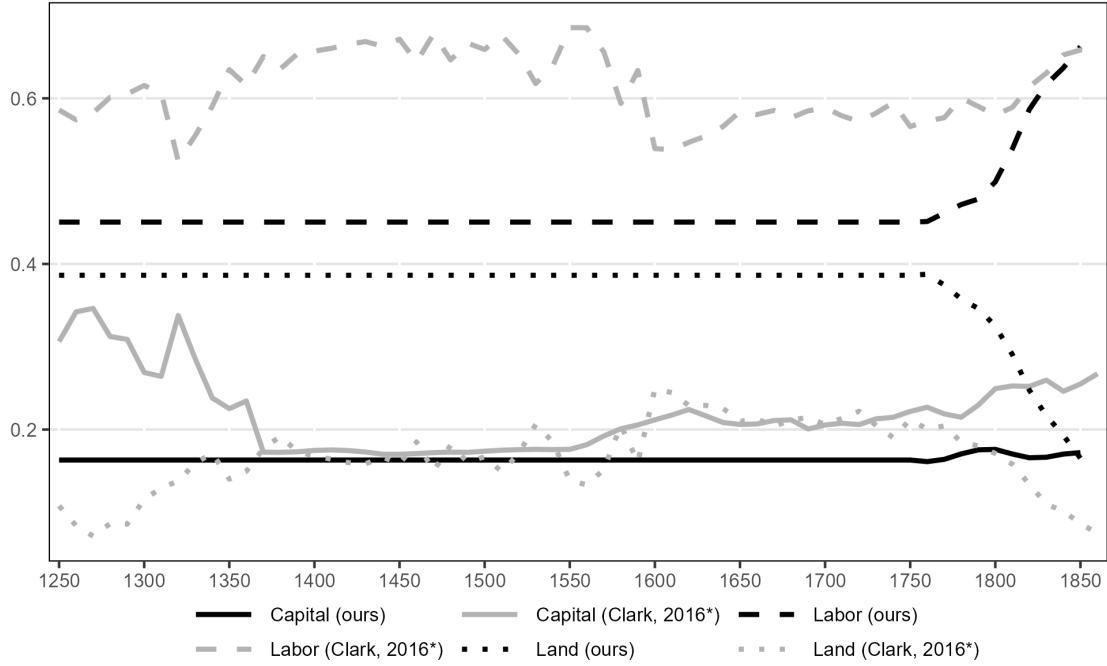


Figure E.6: Factor Shares

Note: The figure presents the factor shares implied by our analysis (black lines) and those used by Clark (2016) extended to 1860 (gray lines). We obtained the latter series from Clark in private correspondence.

demand curve. Clark constructs his factor shares in a very different fashion. His basic approach is to calculate payments to factors by multiplying factor prices with the quantity of those factors. The challenge with this approach is that Clark does not have much data on factor quantities. This forces him to make strong assumptions (educated guesses) about the factor quantities.

For instance, Clark's estimate of payments to labor is: $W_t \times 300 \times \nu N_t$, where W_t is the average daily wage, 300 is the assumed number of days worked, N_t is population, and ν is the fraction of the population that is economically active, which he assumes to be 34%. Clark's assumption that days worked are constant over the entire sample period contrasts sharply with the estimate of Humphries and Weisdorf (2019). Also, it is not clear why he chose 300 days. Earlier work often chose 250. Finally, the notion that the fraction of the population that was economically active was constant over our sample is also a strong assumption. In particular, an important literature has highlighted variation in marriage patterns over our sample and associated variation in the employment of women (De Moor and van Zanden, 2010, Voigtländer and Voth, 2013).

Similarly, to construct payments to capital, Clark makes educated guesses on the stock of housing, improvements to land, livestock, etc. He estimates payments to land by multiplying the rent index with a fixed stock of land (28.24 million acres) before the 1840s and direct estimates from tax returns after this date. With factor payments estimated in this manner, each factor's share can be

obtained by dividing payments accruing to that factor by payments accruing to all factors. Clark's estimates of factor share are, thus, based on a number of strong empirical conjectures. A curious aspect of his estimates is that he estimates a large capital share in the 13th century that then falls by about half in the 14th century (mostly before the Black Death). While we favor our methodology over Clark's, we are cognizant of the fact that these different methodologies sit on different sides of a trade-off: we rely more heavily on the structure of our model, while Clark's estimates hinge more heavily on a myriad of empirical hypotheses.

F More General Production Function for Pre-Industrial Era

Consider the concave production function

$$Y_t = A_t F(Z, L_t, K_t) \quad (37)$$

The first-order conditions are

$$W_t = A_t F_L(Z, L_t, K_t)$$

$$r_t + \delta = A_t F_K(Z, L_t, K_t)$$

where δ is the depreciation rate of capital.

Taking logs in the FOC

$$w_t = a_t + \log(F_L(Z, L_t, K_t)) \approx \tilde{\phi}' + a_t + \frac{L F_{LL}}{F_L} l_t + \frac{K F_{LK}}{F_L} k_t \quad (38)$$

$$\log(r_t + \delta) = a_t + \log(F_K(Z, L_t, K_t)) \approx \tilde{\phi}'' + a_t + \frac{L F_{LK}}{F_K} l_t + \frac{K F_{KK}}{F_K} k_t \quad (39)$$

Solving for k_t in equation (39)

$$k_t = \tilde{\phi}''' + \frac{F_K}{K F_{KK}} (\log(r_t + \delta) - a_t) - \frac{L F_{LK}}{K F_{KK}} l_t \quad (40)$$

Substituting into equation (38)

$$w_t \approx \tilde{\phi} + \left(1 - \frac{F_K F_{LK}}{F_L F_{KK}}\right) a_t + \frac{L}{F_L F_{KK}} (F_{LL} F_{KK} - F_{LK}^2) l_t + \frac{F_K F_{LK}}{F_L F_{KK}} \log(r_t + \delta)$$

Which can be rewritten

$$w_t \approx \tilde{\phi} + \left(1 + \tilde{\beta}\right) a_t - \tilde{\alpha} l_t - \tilde{\beta} \log(r_t + \delta) \quad (41)$$

where

$$\begin{aligned} \tilde{\alpha} &= -\frac{L}{F_L F_{KK}} (F_{LL} F_{KK} - F_{LK}^2) \\ \tilde{\beta} &= -\frac{F_K F_{LK}}{F_L F_{KK}} \end{aligned}$$

Equation (41) shows that a_t is identified up to a first-order approximation. This result does not require a Cobb-Douglas production function, not even constant returns to scale.

F.1 CES Case

Consider the production function

$$Y_t = A_t \left[\alpha'^{\frac{1}{\sigma}} Z^{\frac{\sigma-1}{\sigma}} + (1 - \alpha')^{\frac{1}{\sigma}} (L_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where σ denotes the elasticity of substitution between land and labor in production. Optimal choice of labor by land owners gives rise to the following labor demand curve

$$W_t = (1 - \alpha')^{\frac{1}{\sigma}} A_t \left[\alpha'^{\frac{1}{\sigma}} \left(\frac{Z}{L_t} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha')^{\frac{1}{\sigma}} \right]^{\frac{1}{\sigma-1}}.$$

A log-linear approximation of this equation yields

$$w_t = \phi - \alpha l_t + a_t,$$

where

$$\alpha = \left[\sigma \left(1 + \left(\frac{1 - \alpha'}{\alpha'} \right)^{\frac{1}{\sigma}} \left(\frac{L}{Z} \right)^{\frac{\sigma-1}{\sigma}} \right) \right]^{-1}$$

and L is the level of labor we linearize around. Notice that $\alpha \rightarrow \alpha'$ when $\sigma \rightarrow 1$.

It is furthermore easy to show that with the CES production function given above, the labor share of output is

$$\bar{L}S = 1 - \left[1 + \left(\frac{1 - \alpha'}{\alpha'} \right)^{\frac{1}{\sigma}} \left(\frac{L}{Z} \right)^{\frac{\sigma-1}{\sigma}} \right]^{-1}.$$

Combining these last two equations, we get that

$$\alpha = \frac{1 - \bar{L}S}{\sigma}.$$

G Impulse Response Functions

G.1 Dynamics After Change in Productivity Growth

Our Malthusian model implies that an increase in productivity growth will result in higher steady state wages. To see this, we first abstract for notational simplicity from all the shocks in our model. More precisely, we set the value of all shocks equal to their mean. The mean value of ϵ_{1t} , ϵ_{2t} , and ξ_{2t} is zero. The mean value of ξ_{1t} , however, is $E\xi_{1t} = \pi(\psi\beta_1) - \psi(\beta_1 + \beta_2)$, where $\psi(\cdot)$ is the digamma function. We furthermore, assume that days worked and the interest rate are constant at d^* and r^* .

Given these assumptions, our model simplifies to:

$$w_t = \phi + \frac{1}{1-\beta}\tilde{a}_t - \frac{\alpha}{1-\beta}(n_t + d^*) - \frac{\beta}{1-\beta}\log(r^* + \delta) \quad (42)$$

$$n_t - n_{t-1} = \omega + \gamma(w_{t-1} + d^*) + E\xi_{1t} \quad (43)$$

$$\tilde{a}_t = \mu + \tilde{a}_{t-1}. \quad (44)$$

We can use equation (42) to eliminate w_t in equation (43). This yields:

$$n_t - n_{t-1} = \omega + \gamma\phi + \frac{\gamma}{1-\beta}\tilde{a}_{t-1} - \frac{\alpha\gamma}{1-\beta}n_{t-1} - \frac{\beta\gamma}{1-\beta}\log(r^* + \delta) + \gamma\frac{1-\alpha-\beta}{1-\beta}d^* + E\xi_{1t}.$$

This equation can be rewritten as

$$n_{t+1} = \left(1 - \frac{\gamma\alpha}{1-\beta}\right)n_t + \frac{\gamma}{1-\beta}\tilde{a}_{t-1} + \text{constant}. \quad (45)$$

Next, we subtract α times the second-to-last equation from equation (44) and rearrange. This yields:

$$\tilde{a}_t - \alpha n_t = \mu - \kappa + \frac{1-\alpha\gamma-\beta}{1-\beta}(\tilde{a}_{t-1} - \alpha n_{t-1}), \quad (46)$$

where

$$\kappa = \alpha \left(\omega + \gamma\phi + \gamma\frac{1-\alpha-\beta}{1-\beta}d^* - \frac{\beta\gamma}{1-\beta}\log(r^* + \delta) + E\xi_{1t} \right).$$

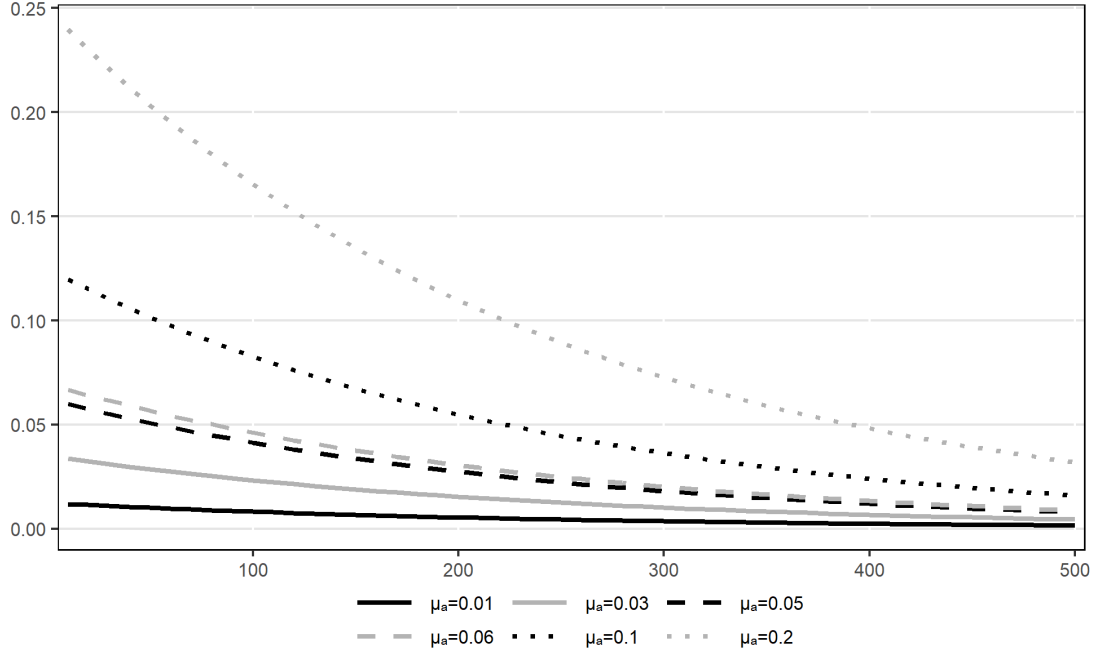


Figure G.1: Real Wage Growth After an Increase in Productivity Growth

Note: Each line plots the growth rate of real wages over time after an increase in productivity growth from $\mu = 0$ to a higher value. These impulse responses are calculated assuming that all model parameters are at their posterior mean values and α is equal to our pre-Industrial estimate of 0.38.

This shows that $\tilde{a}_t - \alpha \times n_t$ follows an $AR(1)$ and therefore settles down to a steady state in the long run as long as $|(1 - \alpha\gamma - \beta)/(1 - \beta)| < 1$. The steady state value of $\tilde{a}_t - \alpha n_t$ is $(\mu - \kappa)(1 - \beta)/(\alpha\gamma)$ and (using equation (42)) the steady state real wage is

$$w^* = \frac{\mu}{\alpha\gamma} - d^* - \frac{\omega}{\gamma} - \frac{E\xi_{1t}}{\gamma} \quad (47)$$

We see from this that the steady state real wage in our Malthusian economy is increasing in the productivity growth rate μ and the extent to which this is the case is influenced by the strength of the Malthusian population force as summarized by $\alpha\gamma$.

Figures G.1 and G.2 present impulse responses to a change in productivity growth that show quantitatively how much changes in productivity growth increase wages over time according to our model when α is set to our pre-Industrial estimate ($\alpha = 0.38$). For each impulse response, we start the economy off in a steady state with zero productivity growth ($\mu = 0$). At time zero in the figures, productivity growth increases. In Figure G.1, we show the evolution of the growth rate of wages (log change) over the subsequent 500 years. In Figure G.2, we show the evolution of the level of wages relative to its earlier steady state level over the subsequent 1000 years. In both figures, we assume that all other shocks are constant at their mean values.

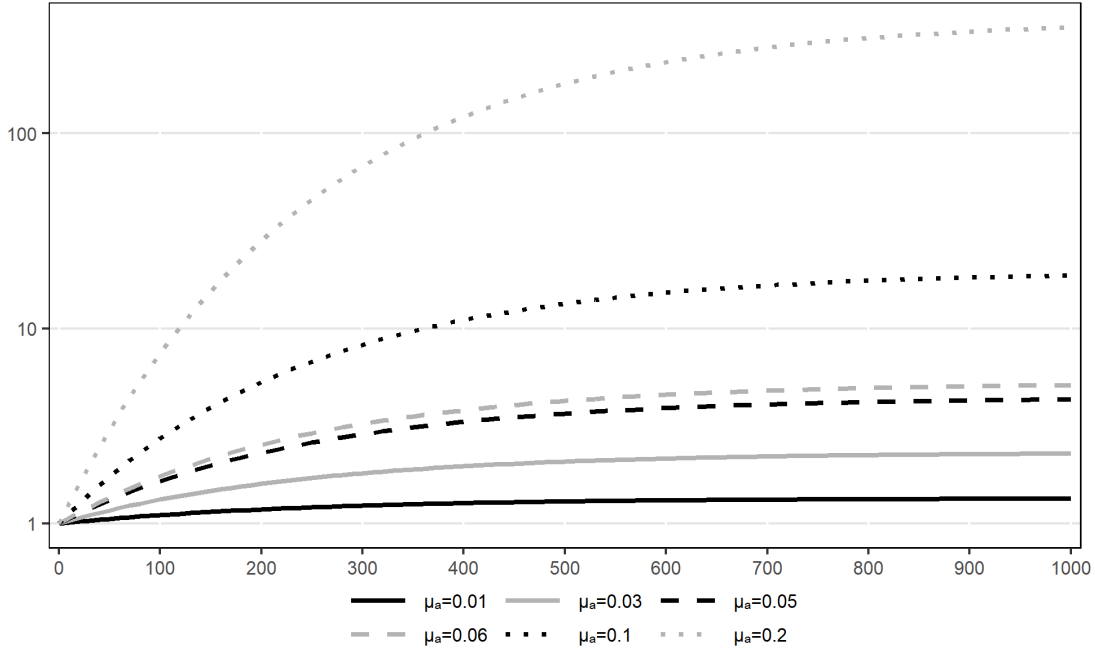


Figure G.2: Evolution of Real Wages After an Increase in Productivity Growth

Note: Each line plots the evolution of real wages over time after an increase in productivity growth from $\mu = 0$ to a higher value. These impulse responses are calculated assuming that all model parameters are at their posterior mean values and α is equal to our pre-Industrial estimate of 0.38.

In Figure G.1, we see that the growth rate of wages is initially equal to the change in productivity. As wages rise and the Malthusian population force kicks in, the growth rate of wages falls. This process takes a very long time due to the weakness of the Malthusian population force. As we discussed above, the half-life of wage growth is roughly 170 years when the land share is at its pre-1760 value. The fact that wage growth continues for hundreds of years after a change in productivity implies that the cumulative increase in wages is substantial. In Figure G.2, we can read off the long-run effect of higher productivity growth on wages. For a “modern” productivity growth rate of $\mu = 0.1$, we find that the long-run effect on the level of wages is an increase of a factor of 20.

G.2 Dynamics after Change in α

We now study the impulse response function of our Malthusian economy to a change in α . The thought experiment is the following: before time 0, the economy is on a balanced growth path with α constant and equal to α^H . At time 0, the value of α falls to α^L . β is constant at all times. Like before, we shut shocks down by setting them equal to their expected value. The permanent

component of the Malmquist index follows its law of motion throughout the experiment:

$$\tilde{m}_t = \mu + \tilde{m}_{t-1}. \quad (48)$$

Since the economy is on the balanced growth path before time 0, the derivations of section G.1 apply and we have for all $t < 0$:

$$w_t = \frac{\mu}{\alpha^H \gamma} - d^* - \frac{\omega}{\gamma} - \frac{E\xi_{1t}}{\gamma}$$

$$\tilde{a}_t - \alpha^H n_t = \frac{(\mu - \kappa^H)(1 - \beta)}{\alpha^H \gamma},$$

where κ^H is the value of κ when $\alpha = \alpha^H$. Similarly, since α is constant for $t \geq 1$, equations (42)–(44) hold and so does equation (46). Therefore, the convergence result for $a_t - \alpha^L n_t$ and w_t , $t > 0$, apply with $\alpha = \alpha^L$.

At time 0, things are more subtle as the change in α implies that equation (44) is replaced by equation (48). Combining equation (43) at time 0 and the formula for w_t with $t < 0$, we know n_0 :

$$n_0 = n_{-1} + \frac{\mu}{\alpha^H}.$$

Invoking equation (16), we can solve for a_0 :

$$\tilde{a}_0 = \tilde{a}_{-1} + \mu + (\alpha^H - \alpha^L) (\log Z - d^* - \bar{n}_0 - \lambda), \quad (49)$$

where we have used the fact that β is constant, $a_t = \tilde{a}_t$, $m_t = \tilde{m}_t$, and $\hat{m}_t = \mu$. Finally, w_0 is given by equation (42) with $\alpha = \alpha^L$.

We show the impulse response functions of W_t and N_t in Figures G.3 and G.4. We set α^H , the value of α before time 0, to 0.38, which is the posterior mean before 1770 and show the results for various values of α^L . The lowest one, 0.15, is the posterior mean for α_t in the last decade of the sample. For simplicity, we set $\mu = 0$ so that population has a well-defined steady state. Both variables are expressed as a multiple of their steady state value with $\alpha = \alpha^H$. Note that, by assumption, the variables are in the latter steady state before time 0.

Real wages jump on impact. Since productivity, defined as the Malmquist index, is held constant throughout, there is no change in output at time 0 and this jump is entirely explained by the

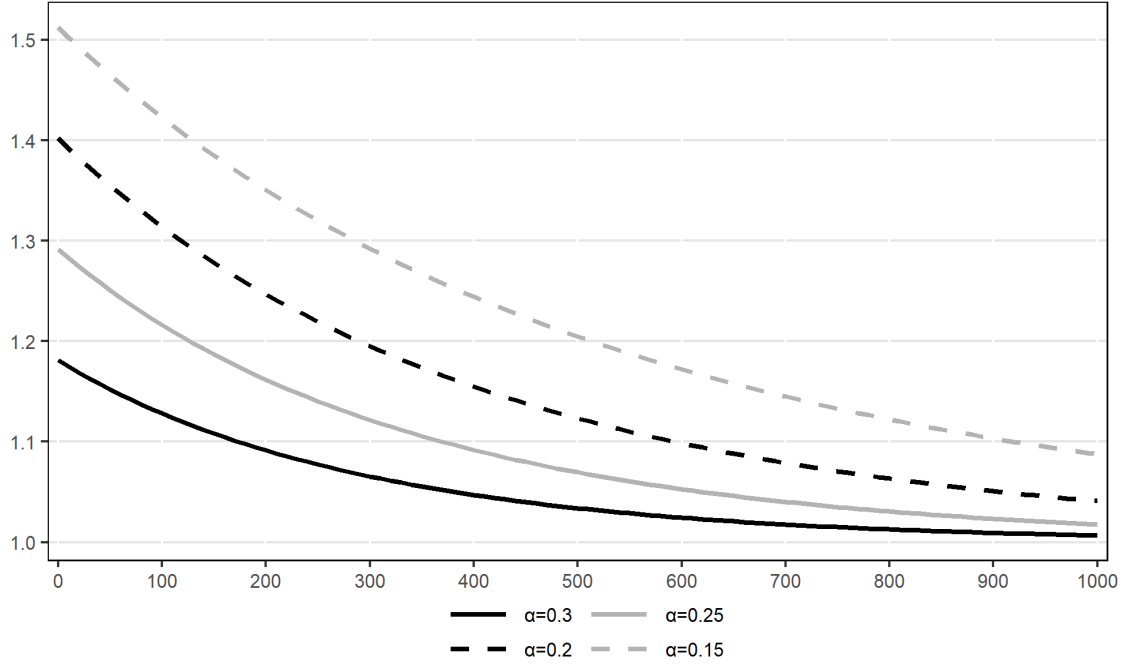


Figure G.3: Response of W_t to a Change in α

Note: The figure plots the response of the real wage (W_t) to a drop in α from its posterior mean before 1770 (0.38) to the value in the legend. W_t is expressed in multiple of its steady state value before the drop.

increase in the labor share.²¹ Indeed, a drop in α from 0.38 to 0.15 implies a 51% increase in the labor share, which is exactly the increase in W_t on impact. From time 1 onward, population increases which pushes the wage down to the old steady state—without growth ($\mu = 0$), the steady state wage doesn't depend on α .

Population is predetermined at time 0, so it does not change on impact. As income rose in period 0, however, it starts increasing in period 1 and slowly converges to a permanently higher level. With a larger labor share, a bigger population can be sustained in steady state.

²¹Formally, the change in output is:

$$\hat{y}_0 = \hat{a}_0 + \hat{\alpha}_0 \log Z + \beta \hat{k}_0 - \hat{\alpha}_0(n_{-1} + d^* + \lambda) = \frac{1}{1 - \beta} \left(\hat{a}_0 + \hat{\alpha}_0 (\log Z - (n_{-1} + d^* + \lambda)) \right) = 0,$$

where we used the fact that $n_0 = n_{-1}$ when $\mu = 0$ in the first equality, the capital demand in the second one, and equation (49) in the third one.

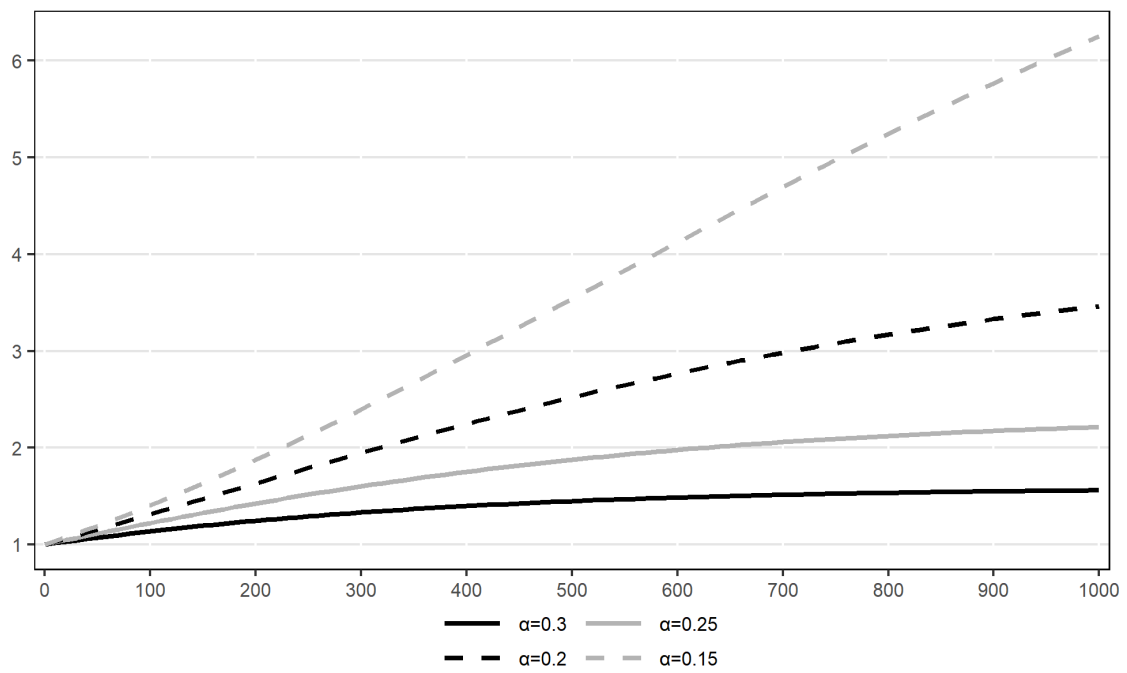


Figure G.4: Response of N_t to a Change in α

Note: The figure plots the response of population (N_t) to a drop in α from its posterior mean before 1770 (0.38) to the value in the legend. N_t is expressed in multiple of its steady state value before the drop.

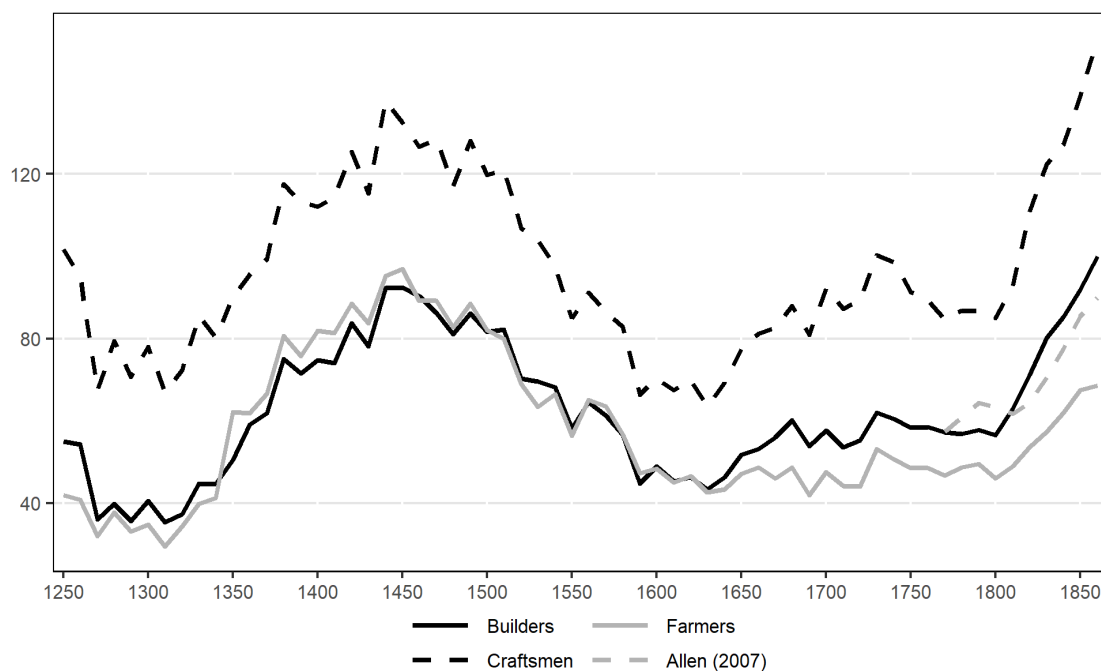


Figure A.1: A Comparison of Real Wage Measures in England, 1250-1860

Note: The figure presents four estimates of the real wages in England. Three are from Clark (2010): builders, farmers, and craftsmen. The remaining series is from Allen (2007). The builders series is the series we use in our main analysis. The builders series is normalized to 100 in 1860. The levels of the farmers and craftsmen series indicate differences in real earnings relative to builders. The Allen (2007) series is normalized to equal the builders series in 1770.

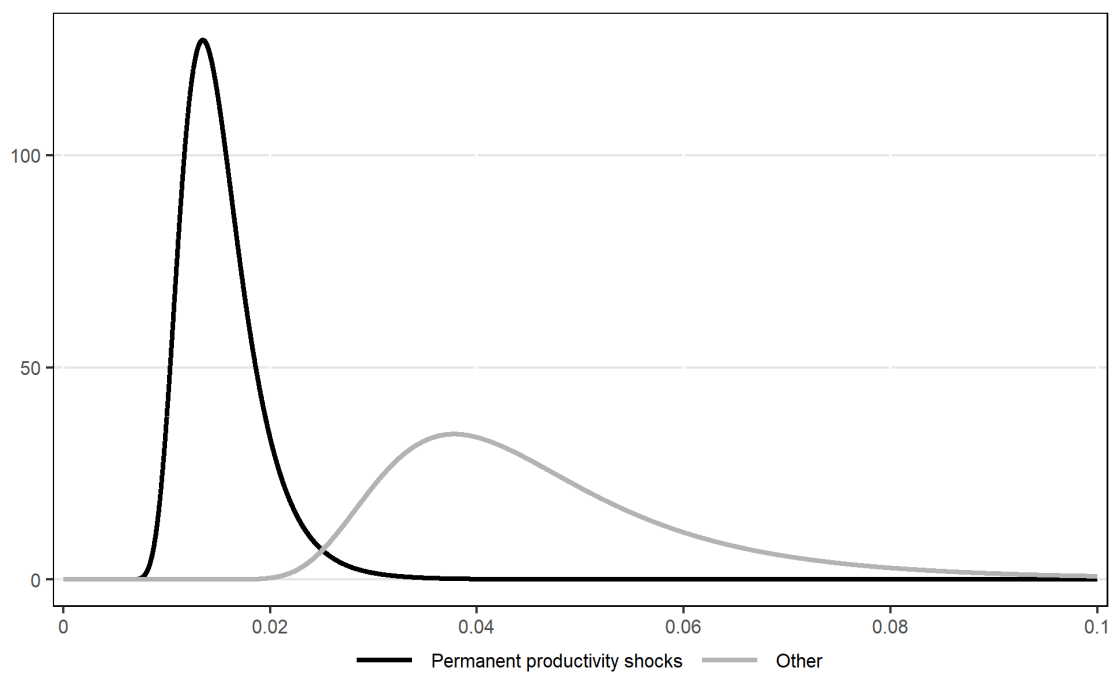


Figure A.2: Prior Densities for Standard Deviations

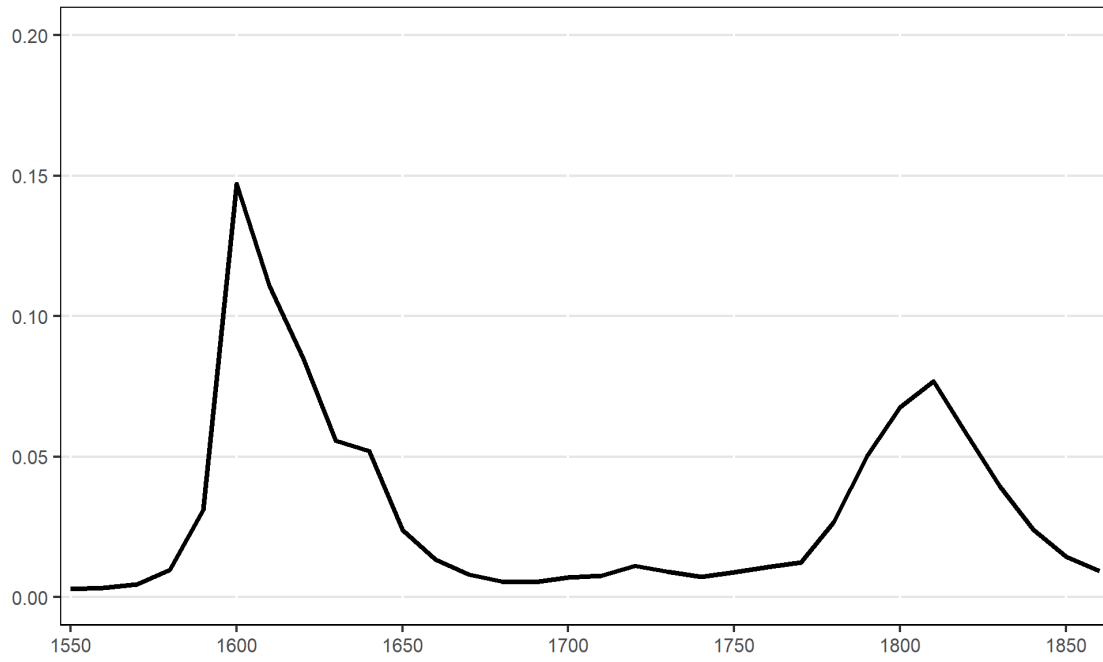


Figure A.3: Probability Distribution over Break Dates with a Single Break

Note: The figure presents the probability distribution over break dates when we allow for a single break between 1550 and 1860.

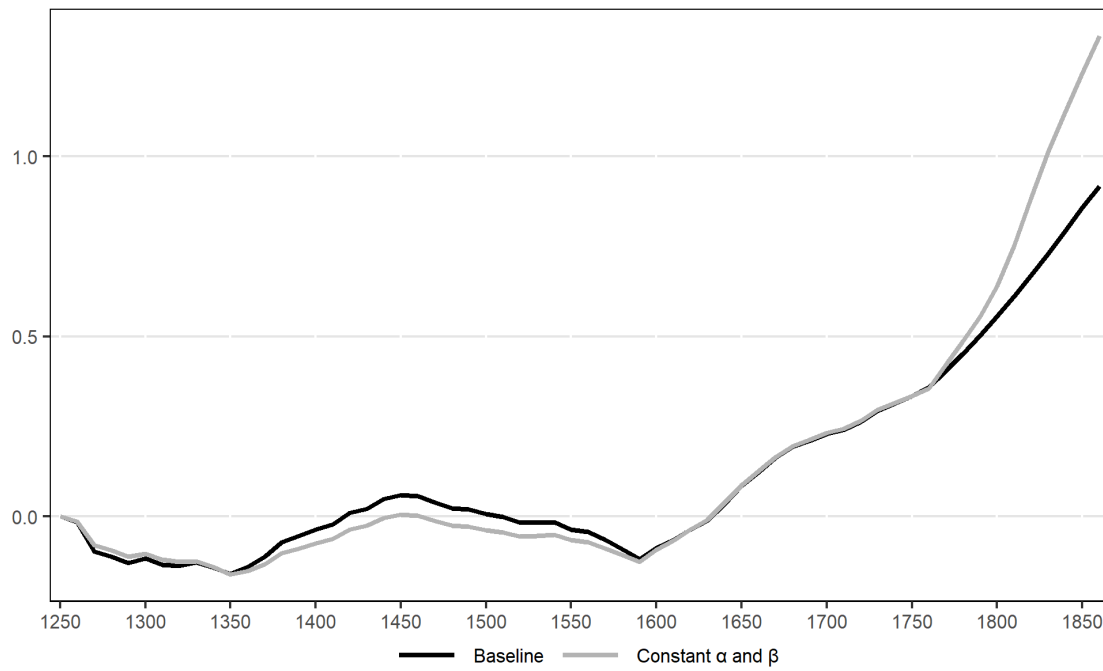


Figure A.4: Productivity with Constant α and β

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity \tilde{m}_t with estimates using an unchanging production function.

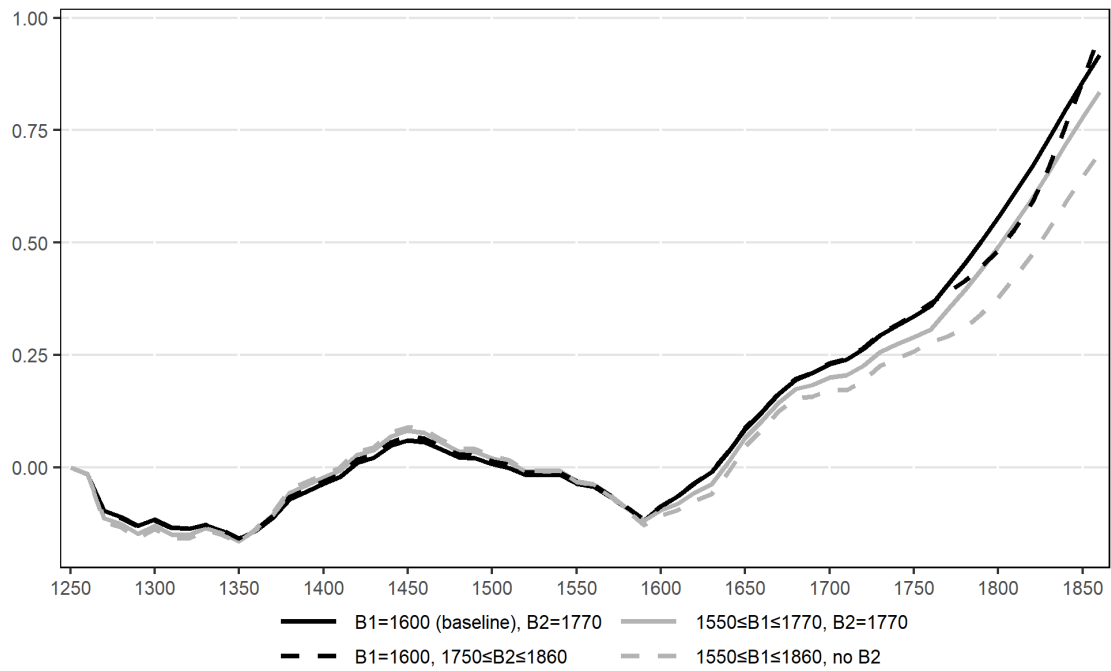


Figure A.5: Productivity Allowing for Different Break Dates

Note: The figure compares estimates of the evolution of the permanent component of productivity \tilde{m}_t when we allow for different break dates. B1 and B2 stand for breaks 1 and 2.

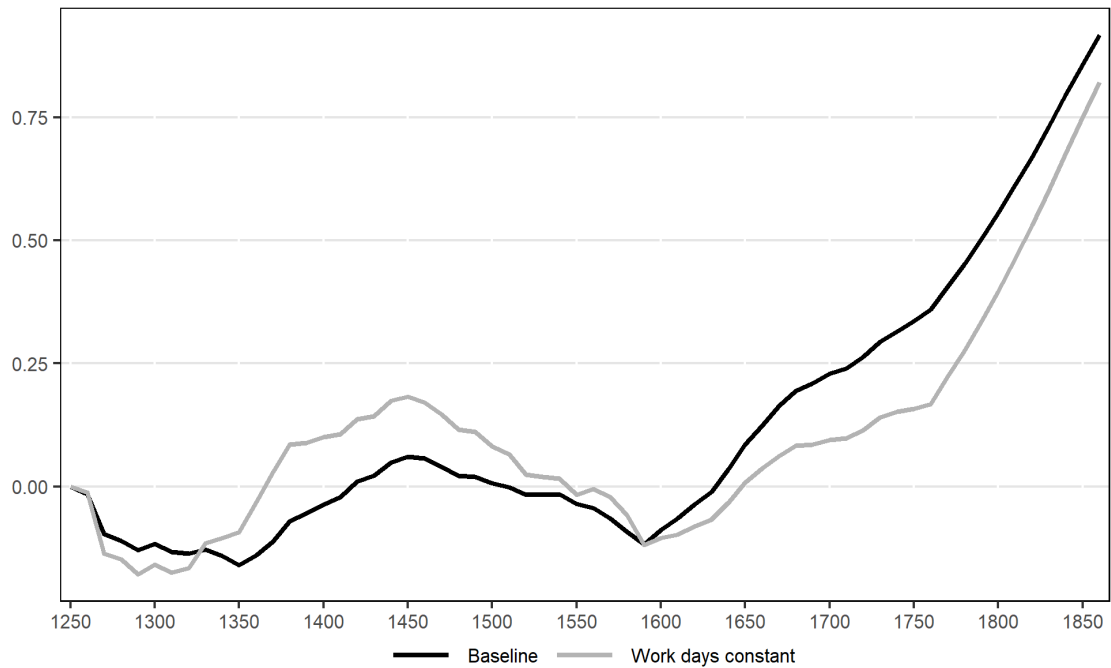


Figure A.6: Productivity with Constant Days Worked

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity \tilde{m}_t with alternative estimates where we assume that days worked per workers were constant over our sample period.

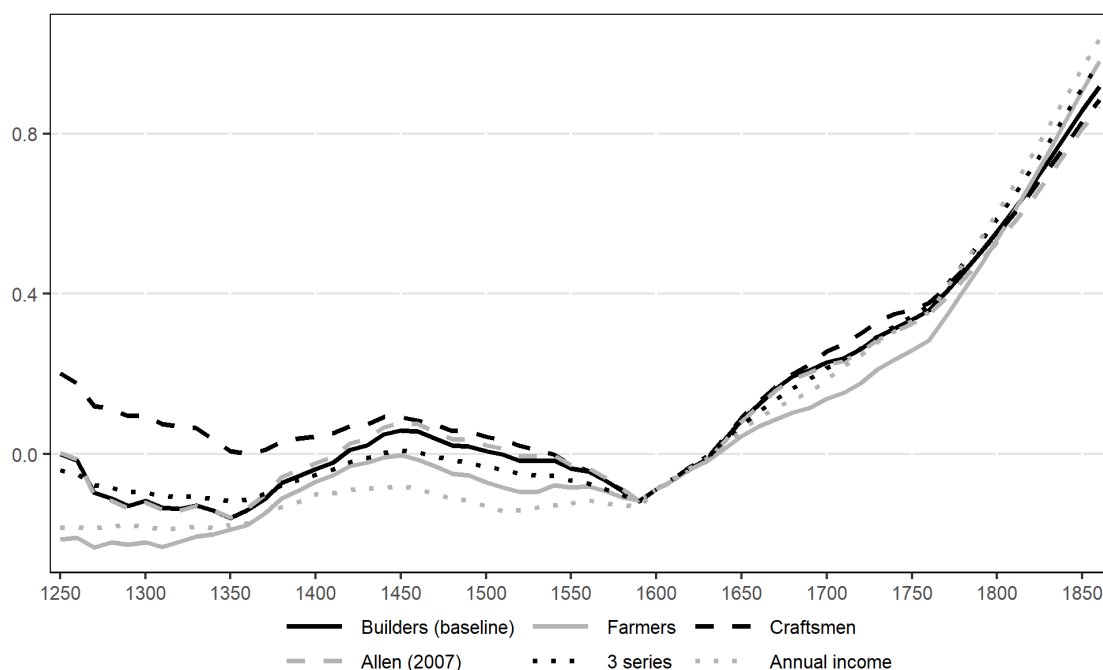


Figure A.7: Productivity using Alternative Wage Series

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity \tilde{m}_t with estimates using different wage series. The “Farmers” series is the farm worker series from Clark (2010), the “Craftsmen” series is the building craftsmen series from Clark (2010), the “Allen (2007)” series uses Allen’s (2007) series from 1770 onward (but our baseline wage series before that). Finally, we present estimates of productivity based on the assumption that the builders, farmers, and craftsmen series are all noisy signals of the true underlying wage. These estimates are labeled “3 series”.

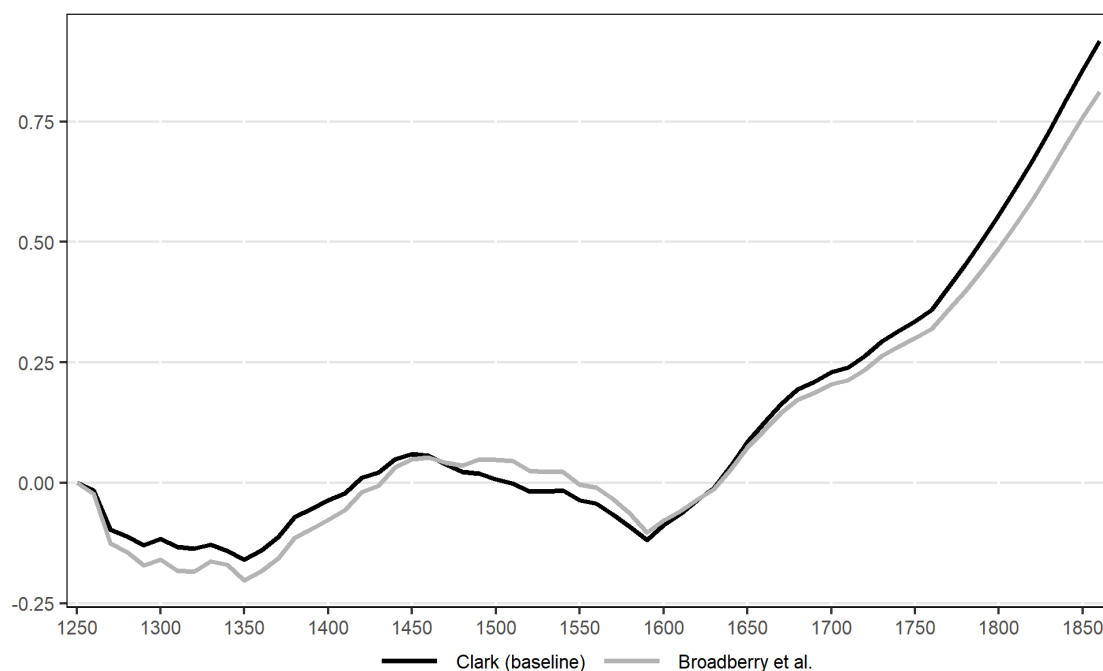


Figure A.8: Productivity using Different Population Data

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity \tilde{m}_t with estimates using data on the population of England prior to 1540 from Broadberry et al. (2015).

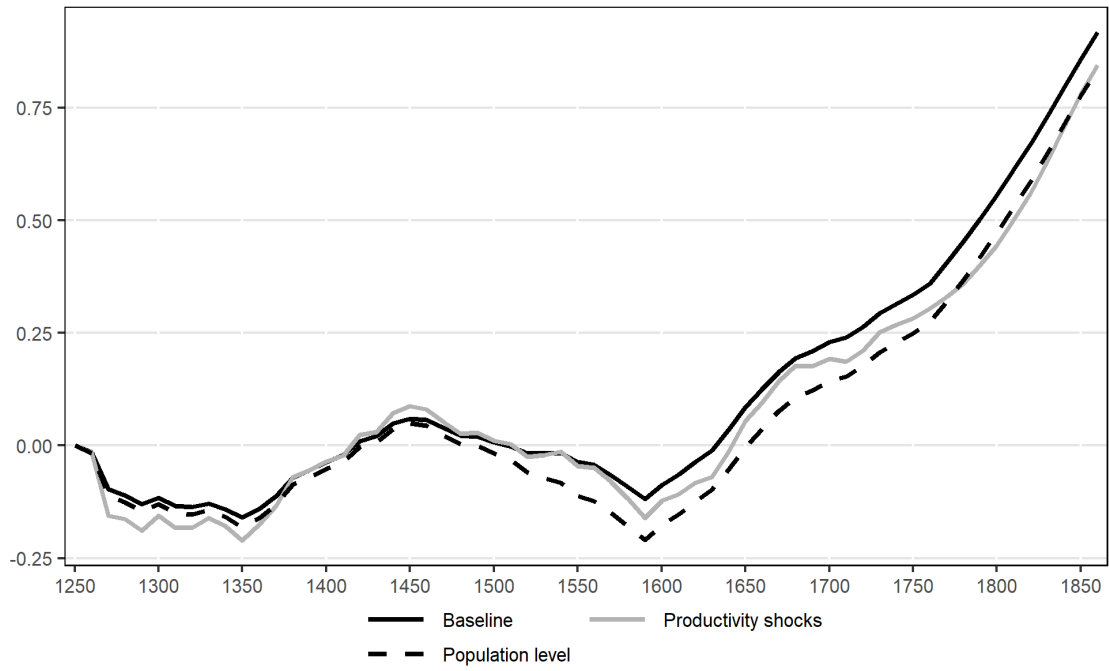


Figure A.9: Productivity using Different Priors

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity \tilde{m}_t with estimates using different prior distributions. The “Productivity shocks” series changes the prior on σ_{ϵ_1} to be $\Pi(3, 0.005)$, i.e., the same as the prior on the other productivity and population shocks. The “Population level” series changes the prior on ψ to be $\mathcal{N}(10.86, 10.0)$.

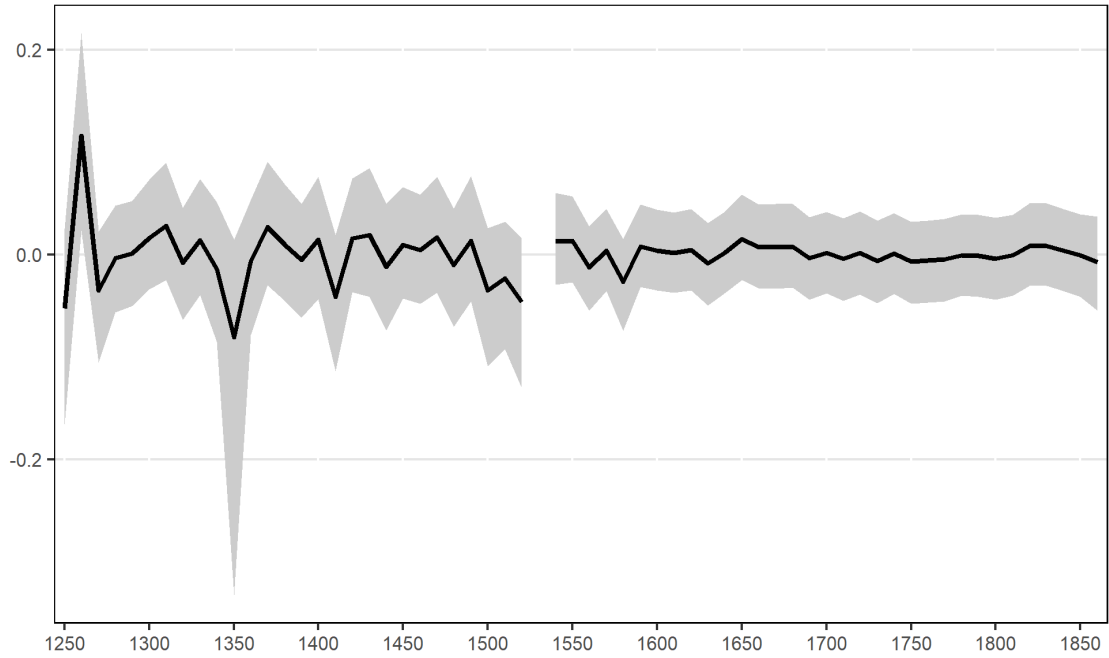


Figure A.10: Measurement Error in Population Data

Note: The figure plots our estimate of the measurement error in our population data ι_t^n .

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