# APPENDIX

In this section, we are going to prove Therorem 1.
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We first state existing results from [?], which are going to support building the proof. The authors in [?] introduced a typed extension on parametric synthesis techniques for Markov Chains. A typed parametric Markov Chain (tpMC) admits expressions instead of constants as transition probabilities. In [?, Theorem 3] it is shown that the feasibility problem from tpMCs is decidable in PSPACE.

To solve the OWP instance, we compute the corresponding tpMC such that every well-defined instantiation of the tpMC corresponds to an induced CTMC $C[i]$ by applying the instantiation function $i$. Then the OWP instance has a solution iff the feasibility problem of the tpMC has a solution (decidable [?, Theorem 3]). We first show how to compute the embedded pDTMC of a given pCTMC $C$, which is a combination of **??** and **??**.

**Definition 1** (Embedded pDTMC). *Given a pCTMC $\mathcal{C} = (S, \iota, G, V, \mathcal{R}, rew)$ and the corresponding number of workers for each role $num_r$, a variable indicating the desired number of workers $var_r$, the frequency and the rate of each transition $s \xrightarrow{r} s'$ given a role $freq(s, s', r)$ and $rat(s, s', r)$ the embedded pDTMC is the pDTMC $\mathcal{M}_{emb} = (S, \iota, G, V, \mathcal{P}^{emb(C)}, rew^{emb(C)})$, where*

$$\mathcal{P}^{emb(C)}(s, s') = \begin{cases} \frac{R(s,s')}{E(s)} & if E(s) \neq 0 \\ 1 & if E(s) = 0 \ and \ s = s' \\ 0 & otherwise \end{cases}$$

$$rew^{emb(C)} = \frac{1}{E(s)} \cdot rew(s), \ where$$

$$V = var_r \ (\textbf{??})$$

$$\mathcal{R}(s, s') = \sum_{\forall r} var_r \cdot \frac{rat(s, s', r)}{num_r} \ (\textbf{??})$$

$$E(s) = \sum_{s' \in S} R(s, s')$$

Now, given the embedded pDTMC we can construct the corresponding tpMC as follows:

**Definition 2** (corresponding tpMC). *Given the embedded pDTMC $\mathcal{M}_{emb} = (S, \iota, G, V, \mathcal{P}^{emb(C)}, rew^{emb(C)})$ and a budget $B \in \mathbb{N}_{\geq 1}^{|V|}$ the corresponding tpMC $(S, \iota, G, V_M, \mathcal{P}^{emb(C)}, rew^{emb(C)})$ is given by*

$$V_M = \bigcup_{v \in V} V_M(\mathbb{R}^v_{\leq B_v})$$

$$V_M(\mathbb{R}^v_{\leq B_v}) = var_v$$

Hence, the corresponding tpMC is the same as the pDTMC, but it also imposes the constraint that each parameter $v \in V$ is bounded on top by the budget $B_v$. Note that the authors in [?] only consider equality constraints, but the ETR also supports inequalities of polynomials (Proof of Lemma 1 in [?] can also include inequalities).

The feasibility problem of tpMCs states:

**Definition 3** (Feasibility Problem for tpMCs [?]). *Given a tpMC $D$ and a threshold $\tau \in \mathbb{Q}_{\geq 0}$, does there exist a well-defined instantiation $i$ such that $ExpRew(D[i]) \leq \tau$.*

Hence, our problem is encoded as a feasibility problem in tpMCs, which is decidable in PSPACE, hence, our problem is also decidable in PSPACE.