ICS4U – Solving a Polynomial in One Variable

## What are polynomials?

Polynomials in x are expressions, which, in standard form, look like: . The constants represent the coefficients, which are allowed to be zero or negative when necessary. In order to qualify as polynomials, exponents on x are always non-negative whole numbers. Since , z is called a constant term, and normally has no x. The polynomial could have been written as: . There can be any number of terms up to infinity. For this assignment, we will set the maximum at 8 terms.

You already know some examples: lines (such as ) and quadratics (such as ). Cubics are another example:

We refer to lines as an “order-1 polynomial”; quadratics as an “order-2 polynomial” and cubics as an “order-3 polynomial”. Indeed, we always know the order of a polynomial by looking at the highest exponent on x.

In addition, when we write these polynomials in standard form, we see that the terms are arranged in decreasing order of exponent. This is important. And something else to notice is that the maximum number of terms of any polynomial of order n is n+1. So, an order-4 polynomial will have at most 5 terms, and an order-5 polynomial will have at most 6 terms.

It is possible, if we are strictly adhering to standard form, to know everything about a polynomial by ONLY look at the coefficients. Suppose someone gave you this list of coefficients: . We see that the first term (known as the lead term) has a coefficient of 1, and a constant term of . The constant term is the term without an “x”. We see 5 coefficients for 5 terms, which means the polynomial is order-4. We would then conclude that the polynomial described by these coefficients is .

A list such as which has 7 terms, is order 6. If we wrote out all of the terms for this example, we would get: . We usually leave out the terms with coefficients of zero to obtain:   
.

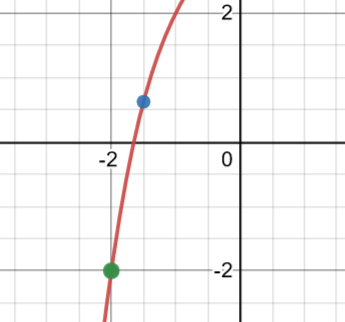
#### Summary:

* A list of numbers represent the coefficients of ordered terms of a polynomial in standard form
* Missing terms are filled in with zeroes
* Negative terms are represented by negative coefficients

## What it means to “solve” a polynomial

Solving a polynomial P(x) means finding the value of x which makes P(x) = 0. This was true when you used the quadratic formula to do the same thing with those polynomials. While order-2 can sometimes be solved, order-3 and order-4 polynomials have messy-looking formulas, and for polynomials of order-5 and above, no formulas exist at all.

For your program, you are not going to apply formulas. Instead you are going to apply an algorithm, which makes use of a theorem known as The Intermediate Value Theorem. The thinking behind this is that if and , and , then there must be some value between and where .

In this polynomial graph, for example, suppose and . We can see that the “blue” point is above the x axis (meaning ), while the “green” point is below the x axis (meaning ). Assuming there are no breaks in the function between and , then there has to be an x in between and which causes the function to cross the x-axis (meaning that for some where , there must be a where ). In other words, a point must exist to provide a solution. We just need to find .

To obtain this graph on Desmos, I entered the polynomial . Using the normal methods students learn in Advanced Functions class regarding the Rational Zeroes Theorem and the Factor Theorem using synthetic division won’t work for finding the x-intercept of this polynomial. Surely, we can see that the function crosses the x-axis *somewhere* between -2 and -1.5.

The programming solution is a kind of guessing and checking that will be done by the computer instead of by a human. So, we need an algorithm. I suggest something simple. This is based on the idea that if and generate points on a function’s graph, then if you average and , then you will generate an that is also on the function, assuming an unbroken curve. This is the intermediate value theorem. The idea being, that, in theory at least, generates a y value closer to 0 than before.

### ALGORITHM:

* You ask the user to input the coefficents of the polynomial one one line, separated by spaces. The maximum number of terms is 8. Avoid polynomials with double roots.
* You ask the user to input initial values for and , where is negative, and is positive. The user should have at least an approximate prior knowledge of the graph (use Desmos).
* **START**
* Plug into the polynomial to obtain .
* If , then . Go back to **START**
* Else if , then . Go back to **START**
* Else if , then a solution is found at and is shown to the user. Program exits.

Sample session

Please input your coefficients: **1 2 3 4**  
Please input your first x (where y<0): **-2**  
Please input your second x (where y>0): **-1.5**  
A solution was found at: **x = -1.65063**

Suggested code for entering a list of coefficients is given below.

