In this short paper the properties of the measure for the *optimism* in an interval based entropy comparison are displayed.

1 Definition

Let the base interval be $[a, b]$ and the one that it is compared to $[c, d]$. The measure for the optimisim is then defined to

$$o = \frac{a-c}{d+|a-c|}$$

Furthermore to ensure a proper value we define

$$o = 0 :\Leftrightarrow a - c = 0$$

2 Properties

2.1 Varying c and fixed a and d

Let us assume that we have the borders a and d fixed and c is varying. There are three different cases for special values of c.

The most simple case is for $c = a$,

$$o = \frac{a-c}{d+|a-c|} = \frac{a-a}{d+a-a} = 0 \quad .$$

Another case of interest is when $c \approx d$, i.e. the compared to interval almost collapses to a point. In this case we also need to consider the relation between a and d.

Let us first have a look at the absolute value of o

$$\begin{aligned} |o| &= \left| \frac{a-d}{d+|a-d|} \right| \\ &= \frac{|a-d|}{d+|a-d|} \\ &= \begin{cases} \frac{a-d}{a} & \text{if } a > d, \\ \frac{d-a}{2d-a} & \text{if } a \leq d. \end{cases} \\ &\leq 1 \quad \forall a, d \quad . \end{aligned}$$

With this in mind we are able to derive bounds for o

$$-1 \leq \frac{a-d}{2d-a} \leq o \leq \frac{a-d}{a} \leq 1$$

For this view the addition to the definition is necessary to avoid 0-valued denominators.

Finally the extreme case when $c = 0$ is worth pointing out. As we already covered the point where $c = d$ in the above we assume that $c < d$.

$$o = \frac{a - c}{d + |a - c|} = \frac{a}{d + a}$$

Again we can derive bounds for o depending on the values of a and d.

$$0 \leq o < 1.$$

The lower bound is met for $a = 0$ and the upper holds for $a = 1$ and d very close to 0.

2.2 Varying a and fixed c and d

There are 3 different case which need to be considered, excluding the trivial of $a \to c$:

- $a \to 0$
- $a \to d$
- $a \to 1$

Also the case when the compared to interval collapses to a single point is excluded. However, the results still apply yet the strict inequalities do not longer hold and they change to equalities instead.

For the first case we have

$$0 > o \to \frac{-c}{d + c} > -1 \quad \text{for } a \to 0,$$

while the second one is not more complicated as we get

$$0 < o \to \frac{d - c}{2d - c} < 1 \quad \text{for } a \to d,$$

and finally for $a \to ent^*$, with ent^* being the entropy of the uniform distribution, we obtain

$$\frac{d - c}{2d - c} < o \to \frac{ent^* - c}{d + ent^* - c} < 1 \quad \text{for } a \to 1.$$

2.3 Varying d and fixed a and c

At first we consider the case when $d \to c$, i.e. the interval is going to collapse. It is very similar to the case of $c \to d$ which is reflected in the results:

$$-1 \leq \frac{a - c}{2c - a} \leq o \leq \frac{a - c}{a} \leq 1 \quad ,$$

where o is only negative for $a < c$ and positive for $a > c$.

Another quite interesting aspect is when $d \to a$ as with $a > d$ the intervals do not overlap,

$$0 < o \to \frac{a - c}{2a - c} < 1 \quad \text{for } d \to a.$$

The last case worth considering is when d is nearing ent^*, then the value of o increases as we like to penalize broad intervals