Universität Rostock Traditio et Innovatio

Institute of Communications Engineering

Prof. Dr.-Ing. Sascha Spors
Signal Theory and Digital Signal Processing
Institute of Communications Engineering (INT)
Faculty of Computer Science and Electrical Engineering (IEF)

Lab exercise II in Digital Signal Processing, winter semester 2018/19 (course #24505)

Lecturer: Vera Erbes, email: vera.erbes@uni-rostock.de, room: W: 8.211
Authors: Frank Schultz, Vera Erbes

All 4 lab exercises are due 10th Feb. 2019.

Spectral analysis and system identification

Level conventions: dBu and dBFS

It is useful to introduce the following accurately differentiated voltage level conventions which do not cause confusion as more commonly used conventions do. These conventions are formulated for the unit volt, but can be similarly defined for other physical field-type quantities.

$x(t)$ is a signal that is continuous regarding time and value. For the time period T, the root mean square value is

$$x_{\mathrm{RMS}} = \sqrt{\frac{1}{T}\int\limits_{t_0}^{t_0+T} x^2(t)\mathrm{d}t} \tag{1}$$

and the peak value is

$$x_{\mathrm{PEAK}} = \max(|x(t)|)_T. \tag{2}$$

The root mean square value x_{RMS} has the unit $\mathrm{V_{RMS}}$ and is expressed as a level as

$$20 \cdot \log_{10}\left(\frac{x_{\mathrm{RMS}}}{0.775\,\mathrm{V_{RMS}}}\right) = \dots\ \mathrm{dBu_{RMS}}. \tag{3}$$

The peak value x_{PEAK} has the unit $\mathrm{V_{PEAK}}$ and is expressed as a level as

$$20 \cdot \log_{10}\left(\frac{x_{\mathrm{PEAK}}}{0.775\,\mathrm{V_{PEAK}}}\right) = \dots\ \mathrm{dBu_{PEAK}}. \tag{4}$$

For a sine signal

$$s(t) = \sqrt{2} \cdot 0.775\,V \cdot \sin(\omega t) \tag{5}$$

with the angular frequency ω in $\frac{\mathrm{rad}}{\mathrm{s}}$ and the time t in s, the root mean square value for integration during complete periods $T = k \cdot \frac{2\pi}{\omega} = k \cdot \frac{1}{f}$, $k \in \mathbb{N}_+$, is $s_{\mathrm{RMS}} = 0.775\,\mathrm{V_{RMS}}$ and the peak value (the amplitude of the sine) is $s_{\mathrm{PEAK}} = \sqrt{2} \cdot 0.775\,\mathrm{V}_{\mathrm{PEAK}} \approx 1.0960\,\mathrm{V}_{\mathrm{PEAK}}$. Expressed as levels according to the conventions above this is $0\,\mathrm{dBu}_{\mathrm{RMS}}$ and $3.01\,\mathrm{dBu}_{\mathrm{PEAK}}$. This means that the so-called crest factor of the signal can be calculated directly from these two quantities in dB:

$$\mathrm{Crest}_{\mathrm{dB}} = \mathrm{dBu}_{\mathrm{PEAK}} - \mathrm{dBu}_{\mathrm{RMS}} = 3.01\,\mathrm{dB}. \tag{6}$$

The unit $\mathrm{dBu}_{\mathrm{RMS}}$ that has been introduced here corresponds to the convention for dBu for voltage levels in electrical engineering. Whenever only simple sine signals are used, the separate declaration of root mean square and peak value is waived. But audio signals often exhibit more complex signal structures where it is necessary to state two of the three signal quantities root mean square value, peak value and crest factor.

For time-discrete signals a similar level convention can be formulated: $x[n]$ is a sequence with $x \in \mathbb{R},\ n \in \mathbb{Z}$. The root mean square value without unit is

$$x_{\mathrm{RMS}} = \sqrt{\frac{1}{N} \sum_{n}^{n+N} x^2[n]}, \tag{7}$$

with $N \in \mathbb{Z}$, and the peak value is

$$x_{\mathrm{PEAK}} = \max(|x[n]|)_N. \tag{8}$$

The root mean square value x_{RMS} expressed as a level is

$$20 \cdot \log_{10}\left(\frac{x_{\mathrm{RMS}}}{1}\right) = \ldots\ \mathrm{dBFS}_{\mathrm{RMS}}, \tag{9}$$

i.e. the level is normalised to 1 as digital audio signals are typically coded in a value range $-1 \le x < 1$ and therefore can exhibit a root mean square value of 1 as a maximum (e.g. for a full-scale square wave).
The peak value x_{PEAK} expressed as a level is

$$20 \cdot \log_{10}\left(\frac{x_{\mathrm{PEAK}}}{1}\right) = \ldots\ \mathrm{dBFS}_{\mathrm{PEAK}}, \tag{10}$$

i.e. it exhibits a normalisation to 1 as well.
For a time-discrete sine signal

$$s[n] = \sin(\Omega n) \tag{11}$$

with the normalised angular frequency Ω in rad and $n \in \mathbb{Z}$ the root mean square value is $s_{\mathrm{RMS}} = \frac{1}{\sqrt{2}}$ for summation over complete periods $N = k \cdot \frac{2\pi}{\Omega}$ and the peak value $s_{\mathrm{PEAK}} = 1$, provided that N is even. This corresponds to $-3.01\,\mathrm{dBFS}_{\mathrm{RMS}}$ and $0\,\mathrm{dBFS}_{\mathrm{PEAK}}$. The common convention for dBFS complies with $\mathrm{dBFS}_{\mathrm{PEAK}}$, but only for sine signals, while there might be confusion for signals with different crest factors. To remedy this, quantities such as "FS Square Wave" and "FS Sine Wave" have been suggested which might only be an improvement for the worse.

Exercise 1: Signals for excitation, spectra and signal parameters (10 points)

For the sampling frequency 48 kHz, three different time-discrete signals of length $N = 96001$ samples shall be generated: white noise, pink noise and a sine signal. The signal characteristics and different presentations of their spectra are to be compared.

a) Generate (e.g. in Matlab) a sine signal `sn` with the frequency $f = 1$ kHz and the above specified sampling rate and length. The amplitude of the sine signal shall be 1. **(0.5 points)**

b) Generate a white noise `wn` with a normally distributed probability density function with the above specified sampling rate and length. **(0.5 points)**

c) Generate a pink noise `pn` with an energy distribution that is proportional to $\frac{1}{f}$ with the above specified sampling rate and length. To this end perform the following steps:

- Calculate the complex spectrum `WN` of wn with an FFT.
- Generate the vector `PNAmpl` for the *amplitude* characteristic $\frac{1}{\sqrt{f}}$. Pay attention to the position of frequencies in the FFT data.
- Apply the pink noise characteristic `PNAmpl` to the spectrum `WN` by multiplication. Set the amplitude for $f = 0$ Hz to zero in the resulting spectrum (i.e. there shall be no direct component).
- Generate the complete spectrum `PN` that exhibits even symmetry for amplitude and odd symmetry for phase (i.e. the right-sided spectrum is the complex conjugate of the left-sided spectrum) so that after an inverse FFT a real pink noise `pn` results. Pay attention to the frequency positions in an FFT spectrum of odd length.

(2 points)

d) Ensure that the generated signals `sn`, `wn` and `pn` are zero-mean. **(0.5 points)**

e) Normalise all signals so that they have a root mean square value of $-3\,\mathrm{dBFS_{RMS}}$. **(0.5 points)**

f) State the peak value in $\mathrm{dBFS_{PEAK}}$ and the crest factor in dB for all signals. **(1 point)**

g) Plot the spectrum as it would be displayed by an FFT analyser: Calculate the complex spectra for all three signals with an FFT and plot the absolute magnitude spectra that are normalised to sine amplitudes with $\frac{2}{N}$ in $\mathrm{dBFS_{PEAK}}$ over logarithmic frequencies $1\,\mathrm{Hz} < f < \frac{fs}{2}$. Pay attention to correct labelling of the axes and use a legend. **(1.5 points)**

h) Plot the spectrum as it would be displayed by a Real-time analyser (RTA) as it is discribed in the following: Simulate how an RTA with octave band filtering would calculate and display the spectra. You can find the centre frequencies f_c as well as upper and lower corner frequencies f_l and f_h for nine octave bands in the following table:

f_l / Hz	44.194	88.388	176.78	353.55	707.11	1414.2	2828.4	5656.9	11314
f_c / Hz	62.5	125	250	500	1000	2000	4000	8000	16000
f_h / Hz	88.388	176.78	353.55	707.11	1414.2	2828.4	5656.9	11314	22627

Calculate for all three signals the energy normalised to sine amplitudes in the nine octave bands from the FFT spectra $X[n]$ according to the following equation:

$$20 \cdot \log_{10}\left(\sqrt{\sum_{n=N_1}^{N_2}\left|\frac{2}{N_{\mathrm{FFT}}}X[n]\right|^2}\right) = \ldots\ \mathrm{dBFS_{PEAK}}. \tag{12}$$

The length of the FFT vector is $N_{\mathrm{FFT}} = 96001$ and N_1 and N_2 are the indices for the upper and lower corner frequencies of each octave band. Plot the results in $\mathrm{dBFS}_{\mathrm{PEAK}}$ for all three signals over the logarithmic frequencies $1\,\mathrm{Hz} < f < \frac{fs}{2}$ with markers (not as points connected by a line). Pay attention to correct labelling of the axes and use a legend. **(2.5 points)**

i) Explain why the spectra of the two diagrams from g) and h) have to be interpreted differently. **(1 point)**

Exercise 2: Transfer function with FFT deconvolution (10 points)

The transfer function of the time-discrete two-channel LTI system `sys.p` depicted in fig. 1 is to be examined in Matlab. The system consists of a measurement system and a device under test (DUT) in series. The output signal of the measurement system `xosys` and the overall output signal `xo` are available separately. The system `sys.p` is to be excited by a sine sweep and will respond with two two-channel output signals: `[xosys,xo] = sys(xi,fs)`.

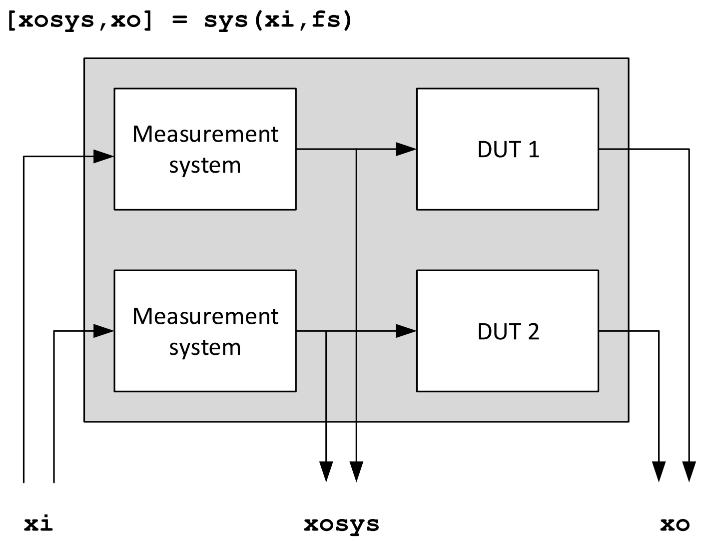

Figure 1: Signal flow of the system

a) Load the sine sweep `Emphasis_FFT18_48K.wav` that is available for download into Matlab, construct a two-channel vector `xi` from it and generate the output signals `[xosys,xo]` by calling the function `sys.p` (have a look at the file `sys_help.m` for usage of the function). Hint: Adopt the sampling frequency of the WAV file. **(2 points)**

b) Calculate the FFT spectra of the signals `xi`, `xosys` and `xo` and the transfer function of the measurement system and of the DUT by linear deconvolution. **(2 points)**

c) Plot the amplitude and phase responses of the transfer functions as well as the group delays and the impulse responses. Pay attention to correct labelling of the axes. **(2 points)**

d) Characterise the amplitude response of the measurement system (types of filter, corner/-centre frequencies, filter orders/Q factor/band width). **(2 points)**

e) Characterise the amplitude response of the DUT (types of filter, corner/centre frequencies, filter orders/Q factor/band width). Discuss the behaviour of the DUT regarding phase response and group delay in frequency and time domain. **(2 points)**