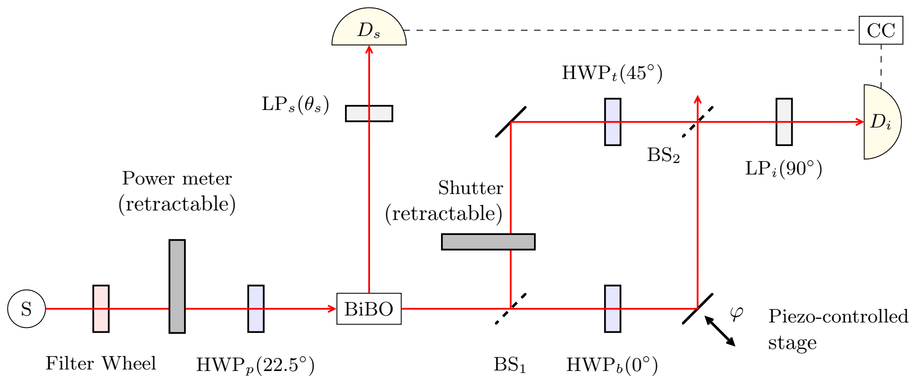

Figure 1: Entangled pair quantum eraser experiment, where the signal photon's polarization carries "which-way" information about the idler's path through the MZI. Setting $\theta_s = 45°$ erases the "which-way" information from the signal, restoring self-interference of the idler in the MZI. The idler does not self-interfere for $\theta_s \in \{0°, 90°\}$, as the which-way info remains on the signal.

Introduction

The existing Lab 6 apparatus seems capable of an even more compelling quantum eraser experiment that uses entangled pairs. Rather than marking the idler with its own "which-way" information, it is possible to mark the *signal* with the idler's path through the MZI. Then by manipulating the signal, we can "erase" the which-way information and restore self-interference of the idler in the MZI.

This experiment can be run simply by inputting new HWP and LP angles into the normal PC control screen. With no changes to any apparatus on the optical table.

We ensure that only horizontally polarized idlers can reach the idler detector D_i via the bottom MZI arm, and only vertical can reach it via the top arm. To reach D_i via the bottom arm, idlers only encounter LP_i which is set to 90° (H) – only H photons entering the bottom arm can pass. To reach D_i via the top arm, idlers pass through MZI HWP_t set to 45° before LP_i – only V photons entering the top arm can pass, as HWP_t will convert them to H. Photons entering the top arm as H will become V polarized and fail to pass LP_i.

Using $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$ entangled pairs means that the signal's polarization marks which path the idler can take through the MZI. This which-way info on the signal prevents the idler from self-interfering in the MZI. Adjusting the signal LP_s to 45° erases the which-way information by projecting the signal's polarization onto D. This restores self-interference of the idler in the MZI.

Operator for Lab 6

In the prelab, we created the MZI operator $\hat{Z}'(\vartheta)$ where ϑ is the MZI HWP$_t$ angle. Let's compose a polarizer operator for the idler polarizer LP_i onto $\hat{Z}'$ to create an operator for the full Lab 6 experiment.

Linear polarizer operator Here's an operator for a LP at angle θ, operating on photons in the H=90°, V=0° basis as we have been:

$$\hat{P}(\theta) = \begin{bmatrix} \sin^2(\theta) & -\sin(\theta)\cos(\theta) \\ -\sin(\theta)\cos(\theta) & \cos^2(\theta) \end{bmatrix}$$

Since the idler LP$_i$ is placed at the horizontal MZI output, we need an operator that applies $\hat{P}$ to the polarization component of the state in spatial state $|\psi_b\rangle$, while leaving the top path $|\psi_t\rangle$ unchanged.

$$\hat{P}'(\theta) = (|\psi_b\rangle\langle\psi_b| \otimes \hat{P}(\theta)) + (|\psi_t\rangle\langle\psi_t| \otimes \hat{I})$$

Lab 6 operator for a single photon And now we can compose $\hat{P}'$ and $\hat{Z}'$ to create an operator that encompasses all of the Lab 6 apparatus:

$$\hat{E}'(\theta, \vartheta) = \hat{P}'(\theta)\hat{Z}'(\vartheta)$$

We will use it with the MZI HWP$_t$ set to $\vartheta = 45°$ and the idler LP$_i$ set to $\theta = 90°$, so

$$\hat{E}'_{45,90} = \hat{E}'(\theta = 90°,\ \vartheta = 45°)$$

Evaluating with single photons As desired, sending single horizontal or vertical polarized photons into this MZI+LP$_i$ apparatus results in constant idler detection probabilities. Such photons can only travel one of the arms, so there is no self-interference.

$$|\langle\psi_b, H|\hat{E}'_{45,90}|\psi_b, H\rangle|^2 = \frac{1}{4}$$
$$|\langle\psi_b, H|\hat{E}'_{45,90}|\psi_b, V\rangle|^2 = \frac{1}{4}$$

Sending in diagonally polarized idlers produces a detection probability that varies with phase delay. Since diagonal polarization is a superposition of $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$, contributions from both arms can self-interfere.

$$|\langle\psi_b, H|\hat{E}'_{45,90}|\psi_b, D\rangle|^2 = \frac{\cos(\delta)}{4} + \frac{1}{4}$$

Extension to signal–idler pairs

Each photon spans a four–dimensional Hilbert space (2 paths $\otimes$ 2 polarisations) so a pair occupies $\mathcal{H}_\mathrm{s} \otimes \mathcal{H}_\mathrm{i}$ (dimension 16). First, let's apply the Lab 6 operator to the idler photon:

$$\hat{\mathcal{E}} = \mathbb{I}_4 \otimes \hat{E}'_{45,90}$$

Apply the signal LP_s at angle θ to the signal photon:

$$\hat{\mathcal{P}}(\theta) = \hat{P}'(\theta) \otimes \mathbb{I}_4$$

Compose them to form the full entangled pair quantum eraser operator:

$$\hat{\mathcal{O}}(\theta) = \hat{\mathcal{E}}\,\hat{\mathcal{P}}(\theta)$$

Input state We prepare the Bell state with both photons in the b-path, noting that the path state has no physical meaning for signal photons:

$$|\Phi^+\rangle = \frac{|\psi_b H\rangle_s |\psi_b H\rangle_i + |\psi_b V\rangle_s |\psi_b V\rangle_i}{\sqrt{2}}.$$

Propagation After the optical elements

$$|\Psi_\mathrm{out}(\theta)\rangle = \hat{\mathcal{O}}(\theta)\,|\Phi^+\rangle.$$

Coincident detection Coincidences are detected when:

(i) the *signal* photon is detected, with no restriction on its path or polarization

(ii) the *idler* photon is found in the b-path showing that it exited the MZI and passed through LP_i to the idler detector D_i, with no polarization restriction.

This is described by the projector

$$\hat{\Pi} = \mathbb{I}_4 \;\otimes\; \big(|\psi_b\rangle\langle\psi_b| \otimes \mathbb{I}_2\big),$$

so that the coincidence probability is

$$P(\theta, \delta) = \langle \Psi_\mathrm{out}(\theta)|\hat{\Pi}|\Psi_\mathrm{out}(\theta)\rangle.$$

Probabilities With the signal LP_s set to $\theta = 0°$ or $90°$ the probability of coincidence detections is constant. The signal carries which-way information about which path the idler took through the MZI, so the idler can not self-interfere.

$$P(0°, \delta) = P(90°, \delta) = \frac{1}{8}$$

With the signal LP_s set to $\theta = 45°$ the probability of coincidence detections varies with the phase delay δ. Projecting the signal onto the diagonal polarization erases the which-way information and restores the idler's self-interference.

$$P(45°, \delta) = \frac{1 - \cos\delta}{8}$$

Experimental Tasks

Task 1: Setup

- Set the pump HWP to 22.5° to generate $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$ pairs.
- Set the MZI HWP to 45° so the upper arm swaps H/V polarizations.
- Set the MZI/idler LP to 90° so only H can pass to the idler detector.
- Open the MZI arm shutter.
- Set acquisition time to 10 sec. Long because we're losing $\approx 7/8$ of photons.
- Set the pump laser power to max.
- Lower the power meter arm.
- Record the power meter reading.

Component	**Setting**	Notes
Idler LP	–	Input is disabled
Signal LP	45°	Varies 45, 0, 90 to turn eraser on/off
Pump HWP	22.5°	Generate $|\Phi^+\rangle$ pairs
MZI LP	90°	Only H photons can reach detector
MZI HWP	45°	Upper arm swaps H/V polarization

Task 2: Scan with eraser on at 45°

- Set the signal LP to 45°, to project the signal polarization onto D and erase which-way info.
- Zero the stage position.
- Lower the power meter arm, acquire counts, raise power meter arm.
- Step through stage positions, acquiring counts at each step until 3 fringes/oscillations in coincidence counts are collected.

Task 3: Scan with eraser off at 0°

- Set the signal LP to 0°, so V signals can pass (idler must use top arm).
- Zero the stage position.
- Lower the power meter arm, acquire counts, raise power meter arm.
- Step through stage positions, acquiring counts at each step. Scan the same range as Task 3.

Task 4: Scan with eraser off at $90°$

- Set the signal LP to $90°$, so H signals can pass (idler must use lower arm).
- Zero the stage position.
- Lower the power meter arm, acquire counts, raise power meter arm.
- Step through stage positions, acquiring counts at each step. Scan the same range as Task 3.

Task 5: Block the signal photon

- Place a card in front of the signal photon, before the signal LP.
- Zero the stage position.
- Lower the power meter arm, acquire counts, raise power meter arm.
- Step through stage positions, acquiring counts at each step. Scan the same range as Task 3.
- Remove the card.

Task 5: Remove the signal LP

- Remove the signal LP.
- Zero the stage position.
- Lower the power meter arm, acquire counts, raise power meter arm.
- Step through stage positions, acquiring counts at each step. Scan the same range as Task 3.