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Dynamic Information Regimes in Financial Markets

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Abstract

We develop a model of investor information choices and asset prices in which the availability of information about fundamentals is time-varying and responds to investor demand for information. A competitive research sector produces more information when more investors are willing to pay for that research. This feedback, from investor willingness to pay for information to more information production, generates two regimes in equilibrium, one having high prices and low volatility, the other the opposite. The low-price, high-volatility regime is associated with greater information asymmetry between informed and uninformed investors. Information dynamics move the market between regimes, creating large price drops even with no change in fundamentals. In our calibration, the model suggests an important role for information dynamics in financial crises.

1 Introduction

Most research linking investor information acquisition and asset prices assumes a constant information environment. But why should the level of potentially available information remain constant in a market that is perpetually in flux? Changes in technology and regulation can generate persistent shocks to what an investor can learn about company fundamentals; and changes in what can be learned should influence investors as they decide whether to acquire costly information. Pushing this idea a step further, we investigate what happens when the information environment itself changes in response to investor demand for information. In other words, we posit that the news media, financial intermediaries, company executives, regulators, and prominent investors are not simply passive streams of information: the level of information they provide depends on investor demand. We then find that asset prices can change dramatically in response to changes in the supply and demand for information. With greater information production, prices can become more

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volatile and drop sharply. Whereas casual economic intuition suggests that price crashes generate demand for news, we show that the reverse can also happen.

To capture these ideas, we develop a dynamic model of information and asset prices in which the level of available information changes in response to exogenous shocks and endogenous investor demand. In our model, an information shock changes the precision of information about fundamentals. An information shock is neither good news nor bad news – it is simply a change in the *amount* of knowable information. Information shocks can occur in the absence of any shocks to fundamentals. Surprisingly we find that even such pure information shocks can have very large price impacts.

Examples of pure information shocks that are independent of fundamentals include regulatory changes (e.g., the Sarbanes-Oxley Act or Regulation Fair Disclosure), changes in accounting standards, or voluntary disclosure decisions by firms or governments. Such exogenous shocks trigger an endogenous response in the number of investors who choose to become informed. As more investors become informed, more information about fundamentals becomes available. This happens because a competitive information production sector, with a zero marginal cost of transmitting information once it has been discovered, will produce more information when more investors are willing to pay for it. This mechanism magnifies the asymmetry between informed and uninformed investors, it tends to increase price volatility, and it can amplify small information shocks into large price drops. In the model, such price drops result from an endogenous transition of the economy from a low- to a high-information regime.

Information shocks – that is changes in the amount of information being produced in the economy – may coincide with shocks to fundamentals. Bad news about fundamentals can spur demand for more information, which is met with increased information production. This in turn can lead to large price drops and increased volatility. Although in our model such price drops can occur even without a change in fundamentals, they are amplified when an increase in information precision and a decline in fundamentals occur together.

Mamaysky (2022) argues that a portion of the volatility and price drops observed during the early phase of the COVID-19 crisis is attributable to this dynamic. In the language of our model, the large fundamental shock of the coronavirus pandemic also triggered a positive information shock in the form of increased information production, which in turn caused the economy to temporarily transition to a high-information, low-price regime. In our model, information dynamics on their own can produce crisis-like effects, with low prices and high risk premia. Fundamentals shocks that are accompanied by information shocks lead to considerably more severe market disruptions than do fundamentals shocks in isolation.

Our model combines exogenous shocks to the quality of available information, an endogenous response by investors who may choose to become informed at a cost, and feedback from investor information choices to information producers, and ultimately to the amount of information that is produced. Most of our analysis uses a reduced-form representation of the feedback mechanism, but we provide a microfoundation for the mechanism through a competitive information production sector that supplies investor demand for information. Importantly, the presence of an information production sector allows the cost of producing information to be shared among many investors. This generates a key dynamic in our model: more informed investors endogenously lead to more information production. Information about fundamentals falls in three categories: publicly known, privately knowable at a cost, and completely unknowable.[1] In the interest of clarity, we only treat the case in which the fraction of knowable information varies, while the portion of knowable information that is publicly known is fixed.

In more detail, we develop an overlapping generations (OLG) model with a single risky asset, which pays a dividend each period, and a riskless asset. In each period, a new generation of investors observes the information environment, i.e. the current precision of the signal about the end-of-period dividend, decides whether to become informed at a cost, sets optimal demands, and trades to clear an exogenous net supply of shares. Market clearing determines the price. At the end of the period, these investors receive their dividend and sell their shares at the new price. The notion of "generations" should not be taken literally in our setting; the OLG framework simply provides a tractable dynamic setting to model changes in information, and it ensures that investors care about future prices as well as the next dividend.

Crucially, in making their information choices at the start of the period, investors take into account the distribution of exogenous shocks to information precision and the feedback from information choices in the current period to future precision. The future precision will affect the end-of-period asset price and thus investors' capital gains. Incorporating such time variation in information precision into the model is a technical challenge, and we develop a new solution methodology to address it.

[1]Consider the outcome of a drug trial. Publicly known information includes what is written about this drug trial in the popular press, like the New York Times or the Washington Post. Privately knowable but costly information includes what can be learned about this trial from a painstaking analysis of hundreds of technical articles about this and related drugs. Information that is unknowable includes the outcome of the trial relative to expectations.

Using this framework, we show that information shocks can lead to large and persistent drops in prices and increases in volatility. This is the main contribution of our paper. We show that information shocks alone can produce prolonged periods of depressed prices and elevated volatility; we know of no other model in the literature that exhibits this behavior. Why does greater information precision have these adverse effects? Most of the paper is devoted to explaining this pattern, but a key part of the answer is time-varying information asymmetry: greater information precision for informed investors puts the uninformed at a greater disadvantage. Within our framework, feedback from the demand for information to the amount of information available leads to persistence in the information state, which amplifies the effect of information asymmetry. When we shut off the feedback, the effects of information shocks become much more transient.

To examine the magnitude of price effects arising from information shocks, we calibrate our model to stock market data. The equilibrium dynamics of the calibrated model fluctuate between two regimes, one with low volatility and high prices, and one with high volatility and low prices. The high volatility, low price regime is associated with high information production. The model can spend long intervals in each regime. A transition from one to the other can be sudden and result in a price move of over 10%, with no change in fundamentals. The two regimes emerge from investor information choices; we do not impose them in setting up the model. These information effects are present even though our investors are fully rational: they understand that the economy can transition from one regime to the other. In an extension of our model, we allow for information shocks that are correlated with shocks to dividends. We find the effects of greater information precision are amplified when they are accompanied by negative dividened shocks: the information shocks can reinforce the fundamental shocks, leading to larger and longer lasting price moves.

1.1 Contribution to the Literature

The interplay between information and asset prices is often studied through single-period models of the type in Grossman and Stiglitz (1980), Hellwig (1980), Admati (1985), and a large subsequent literature. But there are four important features available in a dynamic model that are inaccessible in a single-period model, and these merit discussion. First, persistent exogenous information shocks are necessary, though it turns out not sufficient, to generate endogenous low- and high-information regimes. A second feature, the feedback generated by our microfoundation is also needed; more investors have to induce more information production. Third, in a single-period model, exogenous shocks are often approximated by changes in model parameters, but such changes are necessarily outside the model and, in particular, not contemplated by the agents in the model. In contrast, our agents' beliefs take into account that the economy can transition between different information regimes; such transitions are therefore a feature of the model itself.

Finally, as in Avdis (2016) and Dutta and Nezlobin (2017), a dynamic model captures two distinct aspects of an increase in available information: greater information reduces uncertainty about the next dividend but can *increase* volatility in future prices and thus in capital gains. The first of these effects is clear — the information we model is information about dividends. To appreciate the second effect, note that in the absence of dividend information, price volatility is driven entirely by supply volatility; but when some investors have dividend information, this information is partly reflected in the price, so a persistent increase in signal precision leads to persistent price volatility. In a single-period model, the price merely determines the cost of a claim to an end-of-period dividend. With overlapping generations, investors earn the change in price over the period as well as a dividend, so the variance in this return affects their investment decisions at the beginning of the period. The two information effects, on dividends and on end-of-period prices, are potentially offsetting and lead to more complex tradeoffs than can be captured in a single-period setting.[2] We will see that this dual role of information in dynamic models can lead to starkly different conclusions than those of static models.

To the best of our knowledge, our model is the first to capture a stochastic information environment, endogenous investor information choices, and feedback from these choices to available information. Spiegel (1998) develops an overlapping generations model in which all investors have the same information. Watanabe (2008) extends Spiegel's (1998) model by introducing asymmetric information. Biais, Bossaerts, and Spatt (2010) also model asymmetric information in an OLG setting. In their model, as in Watanabe's (2008), the fraction of informed investors and the precision of their signals are fixed and exogenous. Wang (1993) develops a continuous-time model of trading among differentially informed investors with a fixed fraction of informed investors and a fixed information environment; Wang (1994) is a discrete-time version of the model that investigates trading volume. The model of Veldkamp (2006) includes a dynamic information market, but its investors are indifferent to end-of-period prices, leading to starkly different implications than our model. The OLG model of Farboodi and Veldkamp (2020) incorporates a changing information environment, but the change is limited to a deterministic increase in investor information processing capacity over time. Signal precision also changes deterministically over time in Brennan and Cao (1997).

[2]This dual role of information is emphasized in the multiperiod models of Avdis (2016) and Dutta and Nezlobin (2017), but those models do not include feedback effects from information choices to the information environment.

Information revelation is at the center of the crisis explanations of Dang, Gorton, and Holmström (2020) and Gorton and Ordoñez (2014). In their accounts, a crisis results when lenders choose to acquire information about borrowers' collateral; with less information available, borrowers with poor collateral have access to credit, and the increased supply of credit sustains higher growth. We work in an entirely different framework, but one contrast is particularly noteworthy. In Gorton and Ordoñez (2014), the information revealed is bad news; following an aggregate shock, some unobservable amount of collateral becomes bad, thus inducing more information acquisition. In our setting, it suffices for the precision of information to change — a shock may bring more news or less news without being specifically good or bad news. An increase in precision leads to a price drop when it magnifies the information asymmetry between informed and uninformed investors, leading the uninformed to reduce their demand for the risky asset. (As we noted previously, negative correlation between information precision and fundamentals amplifies the price drop.)

We present our model in Section 2. Section 3 microfounds our information dynamics and feedback mechanism. Section 4 presents the model solution and our main theoretical results. Section 5 studies changes in the level of knowable information and shows that feedback can lead to two information regimes, using parameters calibrated to market data. To isolate the effect of information dynamics, and to emphasize the potential effect of information alone, we focus the analysis in the main body of the paper on the case of information shocks which are uncorrelated with dividends.[3] Section 6 explores the mechanisms leading to large price changes across regimes and considers information asymmetry, information dynamics, and strategic complementarity in information acquisition. The appendices provide proofs of our theoretical results. An Electronic Companion provides additional proofs and results, as well as details of our calibration and numerical calculations.

2 Model

2.1 Dividends and Timing

A single infinitely-lived security pays a dividend in each period. The dividend paid at the end of period t is given by

$$D_{t+1} = \bar{D} + \rho(D_t - \bar{D}) + M_{t+1} = \underbrace{(1-\rho)\bar{D}}_{\mu_D} + \rho D_t + M_{t+1}. \tag{1}$$

[3]Section EC.5 extends the model to allow for correlated information and dividend shocks.

The innovation M_{t+1} is normally distributed with mean zero and variance σ_M^2. It decomposes as

$$M_{t+1} = \underbrace{m_t + \theta_t}_{\tilde{m}_t} + \epsilon_{t+1}, \tag{2}$$

with the following interpretation: m_t is known to informed investors; θ_t is public information; $\tilde{m}_t$ is the knowable portion of the innovation; and ϵ_{t+1} is unknowable at the beginning of period t. These are mean zero, normally distributed random variables, independent across time,[4] with variances given by

$$\mathsf{var}(\tilde{m}_t) = f_t \mathsf{var}(M) \qquad \text{and} \qquad \mathsf{var}(\epsilon_{t+1}) = (1 - f_t)\mathsf{var}(M), \tag{3}$$

and

$$\mathsf{var}(m_t) = \phi \mathsf{var}(\tilde{m}_t) \qquad \text{and} \qquad \mathsf{var}(\theta_t) = (1 - \phi)\mathsf{var}(\tilde{m}_t). \tag{4}$$

Thus,

$$\begin{aligned} f_t &= \text{fraction of dividend innovation that is knowable;} \\ 1 - \phi &= \text{fraction of knowable part of dividend innovation that is public.} \end{aligned}$$

The parameter ϕ will control the degree of asymmetry between informed and uninformed investors. Higher ϕ corresponds to higher asymmetry.

The economy contains overlapping generations of agents. The new generation is in the market for two periods, t and $t + 1$. Before making investment decisions in period t, all agents observe f_t, θ_t, D_t, (and ϕ), and the time-t informed agents observe m_t.[5] A fraction $\lambda_t \in [0, 1]$ of agents are informed at time t. Becoming informed entails paying a fixed cost c_I. The time-t uninformed agents, representing $1 - \lambda_t$ of the population, in addition to observing θ_t and D_t, also observe the market clearing price P_t. Since the market-clearing price contains information about m_t through the demands of the informed traders, the uninformed also make rational inferences from the price about the innovation m_t. The price is not fully revealing about m_t because of the presence of unobservable supply shocks. In this respect, for a given f_t and ϕ, our information environment is the same as in Grossman and Stiglitz (1980). After observing all available (public or private) information, investors set their demands as functions of the price, which determines the price

[4] More precisely, the innovations M_t are i.i.d. Conditional on f_t, the components m_t, θ_t, ϵ_{t+1} are independent of each other and of all $(m_s, \theta_s, \epsilon_{s+1})$ and f_s for $s \leq t - 1$.

[5] We have solved the model with time-varying ϕ_t but, for clarity, we assume it is constant in this paper.

Time $= t$

First, agents observe f_t and decide whether to become informed at cost c_I

Then simultaneously,

- Agents observe public information P_t, D_t, θ_t
- Informed observe private information m_t
- Agents submit their demands q_t^I,q_t^U
- Price P_t clears the market:

$$\lambda_t q_t^I + (1-\lambda_t) q_t^U = \bar{X} + X_t$$

Time $= t+1$

Agents sell back their shares at price P_{t+1}

Agents earn excess return: $P_{t+1} + D_{t+1} - RP_t$

Figure 1: Sequence of events in each period.

through market clearing. At time $t+1$, investors receive the dividend, sell their shares at the time $t+1$ price, and the process repeats. Figure 1 summarizes the timing of the model; the agent demands $q_t^{I/U}$ and asset supply $\bar{X} + X_t$ are discussed in Section 2.3.

2.2 Information Environment

The innovation of our paper is to allow the information environment, as represented by f_t, to evolve over time in response to exogenous shocks and in response to information decisions made by past generations of investors. We show below that a straightforward microfoundation leads to simple dynamics of information precision: f_t follows an AR(1) process combined with a feedback effect from today's fraction informed λ_t to tomorrow's precision, or

$$f_{t+1} = a_f + \kappa_f (f_t - a_f) + b_f \lambda_t + \epsilon_{f,t+1}, \tag{5}$$

for constants a_f, κ_f and b_f, as well as a noise term $\epsilon_{f,t+1}$. We assume (for now) that the information shocks $\epsilon_{f,t+1}$ and fundamental shocks M_{t+1} are independent. This allows us to cleanly separate the effect of changing information precision from the effect of changing fundamentals; in Section EC.5 we introduce correlation in the shocks. To be consistent with the interpretation of f_t as a measure of signal precision in (3), we need to restrict f_t to values between 0 and 1. We therefore apply a mapping $\Pi_{\mathcal{D}}$ to the right side of this equation, where $\Pi_{\mathcal{D}}$ projects the real line to a finite set

$\mathcal{D} \subseteq [0, 1]$.[6] We thus arrive at our model of the information environment:

$$f_{t+1} = \Pi_{\mathcal{D}} \left(a_f + b_f \lambda_t + \kappa_f (f_t - a_f) + \epsilon_{f,t+1} \right). \tag{6}$$

This specification provides a simple model that captures persistent, stochastic time variation in the information environment and, most importantly, feedback from the fraction informed λ_t to the available information. Section 3 provides a microfoundation for (5) and (6). Section EC.7 in the Electronic Companion develops an alternative to (6) in which f_t is restricted to the unit interval through a logistic mapping. Our results are robust to this alternative model, but the state variable is easier to interpret in (6), so we focus on this case.

Illustration of Dynamics

To illustrate the dynamics of the information state f_t and the implications for the equilibrium asset price P_t, we first consider an example. As detailed in subsequent sections, our dynamic equilibrium includes market clearing and utility maximizing decisions by agents in setting their demands and deciding whether to become informed, taking into account feedback from the fraction informed to the information available. To show where we are headed, we use an example to demonstrate the main innovation of our model. Figure 2 plots an equilibrium path in our economy, subject to a particular set of information and dividend shocks. For this example, we assume information and dividend shocks are uncorrelated, and supply shocks (discussed in the next section) are turned off. Each of the three series in the figure is subject to the same set of shocks, but represents the equilibrium path across different sets of parameter values.

The top panel of Figure 2 shows the evolution of f_t in (6) in blue with baseline parameters $a_f = 0.175$, $b_f = 0.384$, and $\kappa_f = 0.91$ from the calibration in Appendix A. The other model parameters are detailed in Table 1. The red, dash-dotted line turns off the feedback by setting $b_f = 0$. Early in the plotted history, the information state experiences several consecutive positive shocks. Without feedback, it quickly mean-reverts toward $a_f = 0.175$. With feedback, the increase in the fraction informed λ_{t+1} due to a positive information shock $\epsilon_{f,t+1}$ would feed into a higher value of f_{t+2}. (We discuss the behavior of λ_t as a function of f_t in Section 5.1.) This makes f_t remain elevated much longer. The bottom panel shows the consequences for the equilibrium price

[6]For x outside the unit interval, $\Pi_{\mathcal{D}}(x) = \min(1, \max(0, x))$ projects x to $[0, 1]$. Our theoretical and numerical results are simplified by taking $\mathcal{D}$ to be finite, so we take it to be of the form $\{0, 1/(n-1), 2/(n-1), \ldots, 1\}$ and let $\Pi_{\mathcal{D}}(x)$ round $x \in [0, 1]$ up to the closest point in $\mathcal{D}$.

P_t. During the protracted period of elevated f_t, the model with feedback produces substantially lower prices than the model without feedback. This sharp contrast shows the important role of information dynamics, as the dividends are identical in the two cases. In the calibration we interpret each period as a month, so the lower figure shows roughly a 5-year period of depressed prices resulting entirely from information dynamics. Prices do not revert back to the high-price regime until the information state experiences several consecutive negative shocks.

For comparison, the figures show a dashed black line corresponding to no feedback but greater persistence, with $\kappa_f = 0.98$. As expected, greater persistence slows the mean-reversion in f_t. But the price in this case is nearly identical to the case $b_f = 0$, $\kappa_f = 0.91$. Thus, the effect of feedback is qualitatively different from ordinary persistence. Indeed, we will see that in the model with feedback f_t is drawn toward a level of 0.88 as well as to the point $a_f = 0.175$. Feedback endogenously introduces two information regimes, one high and one low. The two regimes in the feedback model are highly persistent, as several f_t shocks are needed in rapid succession to induce transitions from one regime to the other. A transition from the low information (i.e., low f_t) regime to the high one is accompanied by a large drop in price and, we will see later, an increase in volatility.

The vast majority of the asymmetric information literature (see the discussion in Section 1.1) assumes a constant f_t. In this case, the three equilibrium paths in Figure 2 would be identical, since the only difference between the paths is in the behavior of f_t. Even if one extends the standard models in the literature to the case of a stochastic and highly persistent f_t, but with no feedback, the difference in equilibrium price paths would be only of second order (as is the difference between the dash-dotted red and dashed black lines in the bottom panel of the figure). The introduction of feedback, from the number of informed to the information state, leads to a first order difference in the behavior of prices (and volatilities as we discuss later). This large difference in prices occurs even in the absence of any differences in fundamentals, as the dividend and supply shocks across the three equilibrium paths are identical.

Much of this paper is devoted to explaining the large price drop observed in Figure 2. In short, greater information precision increases the information asymmetry between informed and uninformed investors, and the price falls due to a higher risk premium. The feedback in our model makes the high information state persistent thus amplifying the price drop. The mechanism leading to the price drop is further explored in Section 6.

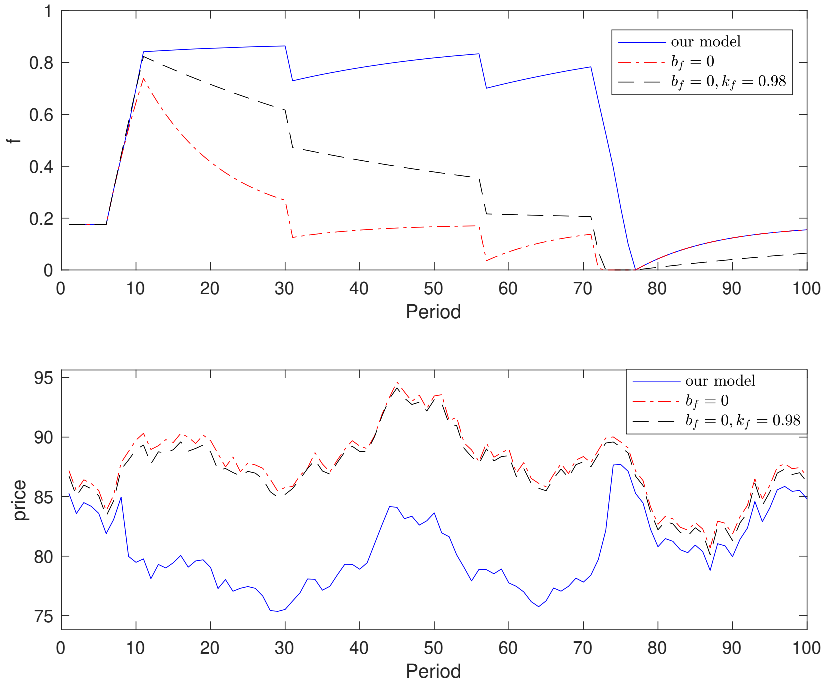

Figure 2: Top: Information state f_t. Bottom: Price P_t. Feedback in f_t creates a prolonged period of depressed prices.

2.3 Investor Optimization Problem

We return to the model formulation. At the beginning of period t, a unit mass of new (young) investors enter the market, each endowed with wealth W_t, known at time t. Investors who enter at time t consume their wealth at $t+1$ and leave the market. For an investor who buys q shares of the risky asset at price P_t at the beginning of period t and sells the shares at the beginning of period $t+1$ at price P_{t+1}, the terminal wealth is given by

$$W_{t+1} = R(W_t - qP_t) + q(D_{t+1} + P_{t+1}) = RW_t + q(D_{t+1} + P_{t+1} - RP_t), \tag{7}$$

where $R > 1$ is the gross return on a riskless asset. It will be convenient to define the per period *net profit* from owning a single share of the stock as

$$\pi_{t+1} \equiv D_{t+1} + P_{t+1} - RP_t, \tag{8}$$

in which case the budget constraint becomes $W_{t+1} = RW_t + q\pi_{t+1}$. The agents set their demands for shares of the risky asset at time t by solving a mean-variance problem

$$J_t^\iota \equiv \max_q \mathsf{E}\left[\mathsf{E}[W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}] - \frac{\gamma}{2}\mathsf{var}(W_{t+1}|\mathcal{I}_t^\iota, f_{t+1})\Big|\mathcal{I}_t^\iota\right], \quad \iota \in \{I, U\}, \tag{9}$$

where $\mathcal{I}_t^U = \{f_t, \lambda_t, D_t, \theta_t, P_t, W_t\}$ is the uninformed agents' information set at time t, $\mathcal{I}_t^I = \mathcal{I}_t^U \cup \{m_t\}$ is the informed agents' information set, and $\gamma > 0$ is a risk aversion parameter. Similar objectives are used in Peress (2010), Van Nieuwerburgh and Veldkamp (2009), and Mondria (2010). Maximizing (9) is equivalent to maximizing

$$\mathsf{E}\left[\psi\left\{\mathsf{E}\left[-\exp(-\gamma W_{t+1})|\mathcal{I}_t^\iota, f_{t+1}\right]\right\}|\mathcal{I}_t^\iota\right], \tag{10}$$

with $\psi(u) = -\log(-u)/\gamma$, if W_{t+1} is conditionally normal, as it will be in our equilibrium. Relative to the standard CARA utility, i.e., $\psi(u) = u$, the present formulation greatly simplifies the model solution, as we explain in Section 4.1. We could allow investors to condition on past values of variables in their information sets in (9), but past information will be irrelevant, given our independence assumptions.

In addition to investor demands for shares of the risky asset, we need to specify the supply. As in the OLG model of Allen, Morris, and Shin (2006), we assume that X_t, the stochastic part of the supply of the risky asset, is independent and identically distributed from one period to the next. As explained in Allen et al. (2006), i.i.d. supply can be interpreted as the result of trading by price-insensitive noise traders who reverse their trades at the end of each period. New investors each period thus only clear a new exogenous supply shock.[7] We assume each X_t is normally distributed with mean zero and variance σ_X^2. We also assume a positive net supply $\bar{X}$ of the risky asset that is constant over time.

2.4 Market Equilibrium and Information Choice

Recall that the state space $\mathcal{D}$ of f_t is finite, so any function of f_t can be represented as a $|\mathcal{D}|$-dimensional vector. Given a function $\lambda : \mathcal{D} \mapsto [0, 1]$, yielding the fraction informed $\lambda(f_t)$, a *market equilibrium* is defined by a price process P_t and demands q_t^I and q_t^U, depending on the price and

[7]Our model extends easily to allow persistent supply shocks, at the expense of adding an additional state variable, which complicates our numerical examples. See Avdis (2016) for a model in which supply persistence influences investors' decisions to become informed.

other time-t information $\mathcal{I}_t^I$ and $\mathcal{I}_t^U$, that clear the market,

$$\lambda_t q_t^I + (1-\lambda_t) q_t^U = \bar{X} + X_t, \tag{11}$$

and for which q_t^ι solve (9), $\iota \in \{I, U\}$, for all t.

Market clearing and investor optimality define a market equilibrium, given a function λ that determines the fraction of investors who are informed. Next, we define what it means for this fraction to be determined endogenously. As in our discussion of Figure 1, we suppose that investors at the beginning of the period can choose to become informed at a cost c_I, incurred at the beginning of the period but after observing the current information state f_t. Investors' decisions to become informed or remain uninformed thus define a mapping from the information state to the fraction informed, which is precisely λ. We will use the following definition:

Definition 2.1 (Endogenous fraction informed)**.** *Given the f_t dynamics in (6), we call λ the endogenous fraction informed if it satisfies the following conditions for each $f \in \mathcal{D}$:*

(i) $\lambda(f) = 0$ *and* $\mathsf{E}[J_t^I - Rc_I | f_t = f] < \mathsf{E}[J_t^U | f_t = f]$*; or*

(ii) $0 \leq \lambda(f) \leq 1$ *and* $\mathsf{E}[J_t^I - Rc_I | f_t = f] = \mathsf{E}[J_t^U | f_t = f]$*; or*

(iii) $\lambda(f) = 1$ *and* $\mathsf{E}[J_t^I - Rc_I | f_t = f] > \mathsf{E}[J_t^U | f_t = f]$*.*

Note that the expectations in Definition 2.1 are taken prior to the agents receipt of their signals. In case (ii), the fraction $\lambda(f)$ is the point at which the marginal investor is indifferent between becoming informed and remaining uninformed. Cases (i) and (iii) cover the possibility that one information choice dominates the other and is therefore selected by all investors.

3 Microfoundation for Information Dynamics

We now provide a microfoundation for the information dynamics in Section 2.2. The microfoundation accomplishes two objectives. First, it justifies our premise that the presence of more informed investors generates more information production. Second, it provides motivation for the form of the dynamics in (5) and (6).

3.1 Supply Response to Increased Demand for Information

To establish that an increase in the number of informed investors generates more information production, we begin with a static model of a market for information. We think of this market as operating within a single period t of the model in Section 2 and determining the dividend innovation M_{t+1}, but we abstract from the specific features of that setting. In the next section, we extend the analysis to multiple periods to generate the information dynamics in (5) and (6). The information market contains $\Lambda > 0$ agents who have committed to spend the cost c_I of becoming informed; these correspond to the informed investors in Section 2.4. These investors must now decide how to optimally spend c_I on the acquisition of information. They may choose to produce information at an exogenously given cost of p_S per unit, or they may buy information at a market price of p_M per unit. In the market for information, agents are price takers and they cannot sell information.

There is also a competitive information production sector consisting of information intermediaries, who, like agents, can directly acquire information at the exogenous cost of p_S per unit.[8] We assume that each intermediary acquires information that is distinct from that acquired by other intermediaries. Unlike agents, intermediaries can sell the information they acquire. In fact, they can sell the same information to multiple agents without incurring additional costs. This is the distinguishing feature of the market for information, as also modeled in Veldkamp (2006). Information intermediaries include media companies and financial research firms who sell the same reports to multiple subscribers under a license that prevents subscribers from reselling the information.

Agents do not have the infrastructure to sell information they find on their own. But this information may be more valuable to an agent because it is not shared. We normalize the per-unit value of information sold by intermediaries to be 1; information produced directly by an agent has a value of $1 + v$ per unit, with $v \geq 0$.[9]

We now describe an equilibrium in this market for information. Agents allocate their budget between intermediaries and privately produced information by maximizing the total value of the information they acquire, solving

$$\max_{0 \leq c_M \leq c_I} \left\{ \frac{c_M}{p_M} + \frac{c_I - c_M}{p_S}(1 + v) \right\}, \tag{12}$$

[8] In the multiperiod setting of Section 3.2 we will think of intermediaries as acquiring information *sources*, with each source producing a unit of information in each period. In a single-period setting, the distinction between a source and the unit of information it provides is unimportant.

[9] The value of information should be endogenous to its use in making investment decisions, but we do not model trading by agents in this section. Our focus is on the supply response to increased demand for information, and we simplify other aspects of the model to highlight this feature.

where c_M is the amount paid to intermediaries.

Intermediaries operate in a competitive market. At a market price p_M for a unit of intermediary information, the jth intermediary maximizes its profit by producing N_j units of information, which solves

$$\max_{0 \le N_j \le \bar{N}_j} N_j(p_M \Lambda - p_S),$$

with $\bar{N}_j$ the maximum amount intermediary j can produce. In writing the revenue from each unit as $p_M \Lambda$, we are assuming the intermediary sells its information to all Λ agents; we construct an equilibrium in which this condition holds for every intermediary. The optimal production quantity depends on the revenue and cost. If $p_M \Lambda < p_S$, then $N_j = 0$ is optimal; if $p_M \Lambda > p_S$, then $N_j = \bar{N}_j$ is optimal; and if $p_M \Lambda = p_S$, then every $N_j \in [0, \bar{N}_j]$ is optimal. Since these conditions hold for every intermediary, we can characterize the aggregate production level $N = \sum_j N_j$ from the intermediary sector as the solution to

$$\max_{0 \le N \le \bar{N}} N(p_M \Lambda - p_S), \tag{13}$$

with $\bar{N} = \sum_j \bar{N}_j$, the maximum amount the sector can produce.

The market clears if the total amount agents spend in the market Λc_M equals the total amount intermediaries earn in the market, which is $N \Lambda p_M$, because the same information is sold to all agents. The market clearing condition is thus

$$N p_M = c_M. \tag{14}$$

An equilibrium is a solution c_M, N to the agents' and intermediaries' optimization problems (12) and (13) and a price per unit of information p_M that clear the market by satisfying (14).

We derive an equilibrium under the conditions

$$\Lambda > 1 + v \quad \text{and} \quad \bar{N} \ge c_I \Lambda / p_S. \tag{15}$$

The first condition requires that the number of informed agents Λ is larger than the per-unit value of information produced by an individual agent. This motivates agents to buy information from intermediaries as the cost is shared by sufficiently many investors. We will shortly extend the analysis to cover values of Λ smaller than $1 + v$. The second condition requires the total available information $\bar{N}$ to be greater than the maximum possible amount produced by individual agents, i.e., $c_I\Lambda/p_S$. It ensures that the potential supply of information is sufficiently large to clear the market.

Under condition (15), there exists an equilibrium with

$$c_M = c_I, \quad p_M = p_S/\Lambda, \quad N = c_I\Lambda/p_S. \tag{16}$$

That is, agents spend their full budget buying information from the intermediaries. The market price of information from the intermediaries is $p_M = p_S/\Lambda$, and the total amount of information produced is $N = c_I\Lambda/p_S$. We verify that there exists an equilibrium satisfying (16) as follows. With $p_M = p_S/\Lambda$, we have $1/p_M > (1+v)/p_S$ by the first condition in (15), i.e., the market price of information is sufficiently low. Then, the agents' objective in (12) is maximized at $c_M = c_I$. On the other hand, competition in the intermediary sector suggests that the marginal revenue ($p_M\Lambda$) from producing a unit of information should equal its marginal cost (p_S). This determines the market price of information as $p_M = p_S/\Lambda$, i.e., the production cost p_S is shared by all informed agents. With $p_M\Lambda = p_S$, any $N \in [0, \bar{N}]$ is optimal in (13), and the amount of information produced N is then determined by the market clearing condition (14). Setting $c_M = c_I$ and $p_M = p_S/\Lambda$ in (14) yields $N = c_I\Lambda/p_S$. The second condition in (15) ensures this production level is feasible, i.e, $N \leq \bar{N}$.

In the equilibrium described by (16), the amount of information produced, N, increases with the number of informed agents Λ. This is the key idea underlying our analysis: an increase in the number of informed agents Λ yields an increase in the quantity of information produced N. This happens because intermediaries can sell the same information to multiple agents, so the production cost of information is shared. Note also that in this equilibrium, all informed investors have the same information because they all allocate their full budget to the intermediaries, and each intermediary sells to all Λ agents.

In anticipation of the dynamic setting in Section 3.2, we consider a slight extension of the formulation above. Instead of acquiring units of information, intermediaries acquire information *sources*. A source could be a contact inside a company or it could be a deeper understanding of certain financial statements. The cost incurred by intermediaries in acquiring information is the cost of acquiring a source. Each source produces a new unit of information in each period.[10] In the single-period setting considered thus far, the distinction between acquiring information and

[10]Acquiring a source is like gaining the ability to see a lightbulb; the information provided by the lightbulb — whether the light is on or off — could change each period.

acquiring a source was unimportant. But now we assume that some fraction of sources previously acquired by an intermediary provide new information to the intermediary (at no additional cost) prior to the intermediary's information production in the current period. This corresponds to the persistent part of the information precision f_{t+1} in (5) and (6), as we discuss in the next section.

We refer to information provided to the intermediary sector by the previously acquired sources as bonus information and denote its quantity by N_b. Let the per-unit value of the bonus information to each agent that acquires it be $v_b(\tilde{N}_b)$, where $\tilde{N}_b$ is the total quantity of bonus information acquired by all Λ agents. We assume $v_b(\cdot)$ is a non-increasing and nonnegative function, reflecting the value of bonus information may decrease as it is known by more agents.

Intermediaries provide this bonus information to investors at no cost, but in proportion to how much the investor spends on acquiring new information from intermediaries: an investor who spends c_M receives $N_b c_M / c_I$ of the bonus information. This set-up captures the observation that Wall Street firms commonly provide preferential treatment to their large customers: the larger the client, the more preferential the treatment.[11] The investor's problem (12) thus becomes

$$\max_{0 \leq c_M \leq c_I} \left\{ \frac{c_M}{p_M} + \frac{c_I - c_M}{p_S}(1+v) + \frac{c_M}{c_I} v_b(\tilde{N}_b) N_b \right\}. \tag{17}$$

We extend our previous equilibrium to the optimization problem in (17) under the condition

$$\Lambda > (1+v) - \frac{p_S N_b}{c_I} v_b(N_b \Lambda). \tag{18}$$

As the per-unit value $v_b(\cdot)$ is non-increasing and the total bonus information $\tilde{N}_b$ acquired by agents is bounded above by $N_b\Lambda$ (i.e., providing all bonus information to all agents), $v_b(N_b\Lambda)$ in (18) is a lower bound on the per-unit value of the bonus information $v_b(\tilde{N}_b)$. Given $v_b(\cdot) \geq 0$, (18) is a relaxation of the condition $\Lambda > 1 + v$ in (15). When the value of bonus information is sufficiently large, (18) can hold even with $\Lambda \leq 1 + v$.

Under condition (18), the solutions in (16) continue to characterize an equilibrium for (17): all informed agents will allocate the entire cost $c_M = c_I$ to the intermediaries and obtain the full bonus information N_b. To see this, suppose all other agents spend the full budget c_I. Then, the marginal agent will also choose $c_M = c_I$ in (17) because the derivative of the objective function with respect

[11] See "An Inside Look at Wall Street's Secret Client List," Bloomberg News, March 24, 2016.

to c_M is

$$\frac{1}{p_M} - \frac{1+v}{p_S} + \frac{v_b(\tilde{N}_b)}{c_I} N_b = \frac{\Lambda}{p_S} - \frac{1+v}{p_S} + \frac{v_b(\tilde{N}_b)}{c_I} N_b \geq \frac{1}{p_S}\left[\Lambda - (1+v) + \frac{p_S N_b}{c_I} v_b(N_b \Lambda)\right] > 0,$$

where the equality uses (16), the first inequality uses $v_b(\tilde{N}_b) \geq v_b(N_b\Lambda)$, and the second inequality uses (18).

3.2 Information Dynamics

We now extend the microfoundation of Section 3.1 to multiple periods. Let N_t denote the total information supplied by intermediaries in period t. This total results from three components: new information sources acquired in period t, according to (16); information sources that may remain available from the previous period; and information sources that may become available through exogenous factors. We discuss each of these components.

We denote the information acquired by intermediaries through new sources in period t by $N_t^{(+)}$. Based on (16), $N_t^{(+)}$ takes the form $c_I \Lambda_{Bt}/p_S$, where Λ_{Bt} is the estimated number of informed agents the intermediaries use to set their production. If the intermediaries are myopic, $\Lambda_{Bt} = \Lambda_{t-1}$, meaning that the intermediaries are producing to meet the previous period's demand; if the intermediaries anticipate the change in their demand, we have $\Lambda_{Bt} = \mathsf{E}_{t-1}[\Lambda_t]$. This determines the production level of the intermediaries, which is the first component of the information supply.

The second component, which corresponds to the bonus information from the prior section, is based on persistent changes to the availability of sources. Informed investors may push a company or government to disclose certain information, and once the company or government agrees, it is likely to continue to disclose this information — the source remains available. At some point the policy may change, making the source unavailable. For example, new technology may make certain characteristics of an oil well observable, even if they were unobservable in the past. Similarly, a company may build a canopy over its distribution facility rendering satellite imagery no longer informative.

To capture these ideas, we posit that a fraction p_{au} of previously available information sources become unavailable each period, and a fraction p_{ua} of previously unavailable sources become available. As mentioned, information from sources that persist from the previous period is provided to informed agents at no additional cost. The quantity of this bonus information is given by

$$N_t^{(b)} = p_{ua}(\bar{N} - N_{t-1}) + (1 - p_{au})N_{t-1} = p_{ua}\bar{N} + (1 - p_{au} - p_{ua})N_{t-1}.$$

We assume $p_{ua} + p_{au} < 1$. Then, the quantity of this bonus information increases in the available information in previous period N_{t-1}, leading to persistence in the information environment.

The final component of N_t is an exogenous shock that makes $\epsilon_{N,t}$ information sources available to informed agents. As in our discussion of the second component, these shocks could result from changes in policy or technology; each $\epsilon_{N,t}$ is a one time shock and can be negative. Note that the total information supply N_t must stay between zero and $\bar{N}$, and therefore, at the boundaries, the shock $\epsilon_{N,t}$ is constrained to ensure this happens.[12]

Combining the three components, the evolution of N_t is given by

$$N_t = p_{ua}\bar{N} + (1 - p_{au} - p_{ua})N_{t-1} + \frac{c_I \Lambda_{Bt}}{p_S} + \epsilon_{N,t}.$$

Now set $\kappa_N = 1 - p_{ua} - p_{au}$ and $a_N = p_{ua}\bar{N}/(p_{ua} + p_{au})$. Set $b_N = c_I/p_S$, and consider the myopic case $\Lambda_{Bt} = \Lambda_{t-1}$. Making these substitutions, the evolution of N_t becomes

$$N_t = a_N + \kappa_N(N_{t-1} - a_N) + b_N\Lambda_{t-1} + \epsilon_{N,t}. \tag{19}$$

The above equation has the form in (5), except that it is expressed in terms of the quantity of information available rather than the fraction.

In this section, we have treated information abstractly, as a commodity with the special feature that it can be sold to multiple buyers. To connect this discussion with the model of Section 2, we can think of the information acquired by informed investors as reducing the variance of the dividend innovation M. Each source provides some information about the next dividend. Dividing (19) by $\bar{N}$ and scaling b_N by the size of the population (i.e., the total number of informed and uninformed agents), we get the dynamics of information precision in (5), with the knowable fraction $\mathsf{var}(\tilde{m}_t)/\mathsf{var}(M)$ (i.e., f_t) coinciding with $N_t/\bar{N}$. We further assume that a fraction of $1 - \phi$ of the information N_t is publicly knowable, and obtain the decomposition of $\tilde{m}_t$ into m_t and θ_t in (2).

For tractability, we focus on the dynamics in (5) in which f_t responds to the lagged fraction informed λ_{t-1} (myopic intermediaries). In Section EC.6 of the Electronic Companion, we numer-

[12]If N_t reaches $\bar{N}$, no exogenous change in policy or technology could push N_t above $\bar{N}$; all potentially available sources of information are already available.

ically solve the model with λ_{t-1} replaced by $\mathsf{E}_{t-1}[\lambda_t]$ (fully rational intermediaries) and get very similar results.

4 Model Solution

In this section, we present our model solution. We first take an arbitrary fixed fraction informed $\lambda(f)$, for each information state $f \in \mathcal{D}$, and find a market equilibrium consistent with that function $\lambda(\cdot)$. The role of $\lambda(\cdot)$ is to fully specify the f_t process in (5), though we do not yet require that $\lambda(\cdot)$ corresponds to the optimizing behavior by agents in the model. Then, from a market equilibrium, we give conditions for the fraction informed $\lambda(\cdot)$ to be optimal, in the sense of Definition 2.1. When the conditions for a market equilibrium and an endogenous $\lambda(\cdot)$ are satisfied simultaneously, we say we have an *information equilibrium.*

4.1 Market Equilibrium

Our first step is to show that, for any choice of λ, the model admits a market equilibrium in which the price process takes the form

$$P_t = a(f_t) + b(f_t)m_t + g\theta_t - c(f_t)X_t + dD_t, \tag{20}$$

where g and d are constants, recalling that m_t is known to informed investors, θ_t is public information, and X_t is the stochastic part of the supply of the asset. We sometimes write the coefficient functions as a_t, b_t, c_t, but they depend on t only through f_t.

To characterize investor demands, we need to find the utility of terminal wealth. If prices are given by (20), we can write terminal wealth W_{t+1} in (7) as

$$\begin{aligned} W_{t+1} &= RW_t + q(1+d)D_{t+1} + q(P_{t+1} - dD_{t+1} - RP_t) && (21)\\ &= RW_t + q\Big[(1+d)D_{t+1} + a_{t+1} + b_{t+1}m_{t+1} + g\theta_{t+1} - c_{t+1}X_{t+1} - RP_t\Big]. && (22)\end{aligned}$$

Note that m_{t+1}, θ_{t+1} and X_{t+1} are independent of D_{t+1}, and of any time t information. Recall that, as in (9), $\mathcal{I}_t^\iota$, $\iota \in \{I, U\}$, are the information sets of the informed and uninformed investors. With a view to solving (9), we evaluate the conditional mean of terminal wealth for the two types of investors as

$$\mathsf{E}[W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}] = q\big[(1+d)(\mu_D + \rho D_t + \theta_t + \mathsf{E}[m_t|\mathcal{I}_t^\iota]) + a(f_{t+1}) - RP_t\big] + RW_t, \quad \iota \in \{I, U\}, \tag{23}$$

where $\mu_D = (1-\rho)\bar{D}$. For the conditional variance, we use (21) to write

$$\mathsf{var}(W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}) = q^2(1+d)^2\mathsf{var}[D_{t+1}|\mathcal{I}_t^\iota, f_{t+1}] + q^2\mathsf{var}[P_{t+1} - dD_{t+1}|\mathcal{I}_t^\iota, f_{t+1}]. \tag{24}$$

The challenge in this expression is that the $\mathsf{var}[P_{t+1} - dD_{t+1}|\mathcal{I}_t^\iota, f_{t+1}]$ term depends on the coefficients of the price function in (20) and these are not yet determined. As an intermediate step to overcome this difficulty, we introduce a *conjectured variance* function $V_B(f)$, which we use to rewrite (24) as

$$\mathsf{var}(W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}) = q^2(1+d)^2\left[\mathsf{var}(m_t|\mathcal{I}_t^\iota) + (1-f_t)\sigma_M^2\right] + q^2 V_B(f_{t+1}). \tag{25}$$

Note that (23) and (25) enter an agent's utility function in (9) linearly, which leads to great simplification relative to the standard CARA case of $\psi(u) = u$ in (10).[13]

We posit that investors set their demands using (25), and we then check if the conjectured variance function V_B is correct. Given a conjectured V_B, the following result ensures that we can find a linear price process P_t of the form (20) such that the corresponding optimal demands q_t^I and q_t^U based on (25) clear the market by satisfying (11). The proof of this lemma and expressions for the price coefficients in (20) are provided in Appendix B.

Lemma 4.1. *Under the f_t model (6), for any conjectured variance function $V_B(\cdot) > 0$, and any $\lambda(\cdot)$ taking values in $[0,1]$, there exists a linear price process of the form (20), with $d = \rho/(R-\rho)$ and $g = 1/(R-\rho)$, such that the corresponding optimal demands q_t^I and q_t^U clear the market.*

We define a market equilibrium to be a price process P_t and optimal demands q_t^I and q_t^U in which the conjectured variance is "correct." Using the dividend process in (1)–(4) and the price function in (20), the correctness condition becomes

$$V_B(f) = b(f)^2\phi f\sigma_M^2 + g^2(1-\phi)f\sigma_M^2 + c(f)^2\sigma_X^2, \qquad \forall f, \tag{26}$$

[13] With CARA utility and conditionally normal W_{t+1}, the expectation in (10) would be

$$-\mathsf{E}[\exp(-\gamma\mathsf{E}[W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}] + (\gamma^2/2)\mathsf{var}(W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}))|\mathcal{I}_t^\iota].$$

Given the non-standard conditional distributions of $a(f_{t+1})$ and $V_B(f_{t+1})$, the outer expectation and first order conditions with respect to q must then be evaluated numerically, making solution of the model considerably harder.

as can be seen by comparing the last term in (24) and (25). If (26) holds, then investors' conjectures about how the variance of $P_{t+1} - dD_{t+1}$ depends on f_{t+1} are consistent with the price process. We call V_B *self-consistent* if it satisfies (26). We cannot define V_B through (26) because b, g, and c are determined by V_B through (25).

Given a conjectured variance V_B, Lemma 4.1 yields a price process, and given a price process we can calculate a new V_B using (26). A self-consistent V_B is a fixed point of this iterative process. Once we have a self-consistent V_B, Lemma 4.1 delivers a market equilibrium.

We provide sufficient conditions that ensure the existence of a self-consistent V_B and thus of a market equilibrium under mild restrictions on model parameters. Section EC.2 gives our most general parameter restrictions. For simplicity, here we state a special case that is easy to verify and holds in our numerical examples.

Proposition 4.1. *Suppose that $R \in [1, 1.2]$ and the model parameters satisfy*

$$(1+d)\gamma\sigma_M\sigma_X = \frac{R\gamma\sigma_M\sigma_X}{R-\rho} \leq 0.28. \tag{27}$$

Then for any fixed $\lambda(\cdot)$, there exists a market equilibrium with prices of the form in (20) and a self-consistent V_B given by (26).

The point of condition (27) is that we need $(1+d)\gamma\sigma_M\sigma_X$ to be small (see in particular footnote 15), and with the mild bounds on R we can show that 0.28 is small enough. This condition translates to upper bounds on market volatilities σ_M and σ_X, dividend persistence ρ, and risk aversion γ. See Section EC.2 of the Electronic Companion for more general conditions and the proof.[14]

Two special cases of Proposition 4.1 are worth mentioning. If we fix $f_t \equiv 0$ and $\lambda \equiv 0$ we have an OLG model without asymmetric information, similar to the one in Spiegel (1998). As in Spiegel's (1998) model, the price coefficients can be expressed through solutions of quadratic equations.[15] If

[14] In our calibration, we mainly think of a single time period as a month. Over a longer horizon, we would expect larger values of σ_M, σ_X, and R; the effect on ρ and γ and therefore on condition (27) is less clear. The more general conditions in Section EC.2 may hold over longer horizons even if (27) does not.

[15] In this case, the equation for a self-consistent variance conjecture reduces to solving a quadratic equation with two real roots, which is given by:

$$V_B^2 + \left[2(1+d)^2\sigma_M^2 - \frac{R^2}{\gamma^2\sigma_X^2}\right]V_B + (1+d)^4\sigma_M^4 = 0.$$

The two roots describe two market equilibria, one with high price variance and one with low price variance, and we need an upper bound on the left side of (27) to ensure that both roots are positive. However, the high variance equilibrium is unstable under arbitrarily small parameter perturbations; only the low variance equilibrium is robust to such changes. In our numerical experiments, we find that if we start from a low value of $V_B(\cdot)$ we converge to the low variance equilibrium.

we fix f_t and λ at constant strictly positive values, we get a model similar to Watanabe's (2008), which has asymmetric information but a fixed information environment and no feedback.

4.2 Information Equilibrium

Proposition 4.1 shows the existence of a market equilibrium with an exogenous $\lambda(\cdot)$. We now endogenize $\lambda(\cdot)$, in the sense of Definition 2.1. We need to find, for each f, a $\lambda(f)$ that makes investors indifferent between paying the cost c_I of becoming informed or staying uninformed; if no such λ exists, we set λ equal to 0 or 1 according to Definition 2.1. The following proposition, proved in Section EC.3, shows that this procedure does indeed generate an endogenous fraction informed.

Proposition 4.2. *Suppose the shocks $\epsilon_{f,t}$ have a density. Then for any strictly positive variance conjecture there exists an endogenous fraction informed λ in the sense of Definition 2.1.*

The condition that the shocks $\epsilon_{f,t}$ have a density may be surprising, given that f_t is restricted to a finite set. But this condition will ensure that conditional expectations of the form $\mathsf{E}[V_B(f_{t+1})|f_t = f]$ change continuously with λ; see Section EC.3 of the Electronic Companion. Continuity in λ is helpful in verifying Definition 2.1 (ii). In our numerical results, we allow discrete shocks; these could be approximated arbitrarily closely by shocks with a density.

Given fraction informed λ, Proposition 4.1 ensures that we can find a self-consistent variance V_B, and given V_B and the initial λ, Proposition 4.2 ensures that we can find an endogenous λ. In Section EC.1, we formulate a result that supports the simultaneous existence of V_B and λ, which requires expanding the definition of admissible values for λ. In our numerical solution of the model, we iteratively update V_B and λ, as explained below, and we find that this procedure converges quickly to an information equilibrium.

4.3 Numerical Solution of the Model

Figure 3 shows how we solve the model numerically. We start from an arbitrary V_B and λ satisfying the conditions in Lemma 4.1. The proof of Lemma 4.1 is constructive and yields coefficient functions $b(\cdot)$ and $c(\cdot)$, as required for the algorithm. We then solve for endogeneous fraction informed $\lambda^*(f)$, at each f, by equating expected utilities as in Definition 2.1 (ii) or applying the boundary cases (i) or (iii) in the definition. In these calculations of $\lambda^*(f)$, the expected utilities in (9) (or more precisely their difference $J_t^I - J_t^U$) are calculated using the substitution in (25). With the new $\lambda^*(\cdot)$, we use (26) to update V_B. We repeat these steps until the change in V_B is small, which happens

start with a conjectured variance $V_B^0(f)$, $f \in \mathcal{D} = \{0, 1/(n-1), \dots, 1\}$;
repeat
  set $V_B^1 \leftarrow V_B^0$;
  for each $f \in \mathcal{D}$, solve for optimal $\lambda^*(f)$, $b(f)$, and $c(f)$ given V_B^1;
  set V_B^0 equal to the right-hand side of (26) given λ^*;
until $||V_B^1 - V_B^0|| < \varepsilon$;
solve for $a(f)$, $f \in \mathcal{D}$, following Appendix B.2;

Figure 3: Calculation of information equilibrium

quickly in our numerical experiments. As indicated in Figure 3, the $a(f)$ coefficient in the price (20) is not needed for the iterative process and can be calculated using the final V_B and λ.[16]

5 Analysis of the Model

Our model is based on the microfoundation in Section 3 which shows that as more investors become informed, more information becomes available. This type of feedback can arise at the onset of market stress in response to heightened investor attention. In this section, we will show that these dynamics can lead to periods of low and high volatility and high and low prices driven purely by changes in the information state, with no change in fundamentals, as illustrated in Figure 2. In other words, we can generate transitions similar to business cycles or even financial crises through changes in the level of information alone, without necessarily the release of negative information.

5.1 Dynamics of Information Precision

To provide insight into the model, we develop the numerical example of Figure 2. A single period in our model is one month, and our dividend process is calibrated to the S&P 500 index. The model parameter values are given in Table 1, and details of the calibration are in Appendix A. The solid line in Figure 4 shows λ as a function of f. We calculate this curve by starting from a flat variance conjecture V_B and iteratively updating V_B and λ as discussed in Section 4.

At low levels of information precision f, the figure shows a flat section where $\lambda(f) = 0$; with little information available, no investor chooses to bear the cost of becoming informed. Once f increases to just above 0.4, we have a positive fraction of investors informed, and this fraction generally increases with the precision f.[17]

[16]The equation for solving $a(\cdot)$ is derived in the proof of Lemma 4.1 and shown in Appendix B.2.
[17]For some parameter values, at f near 1 we have a small decline in $\lambda(f)$. The possibility of a decline in $\lambda(f)$ as

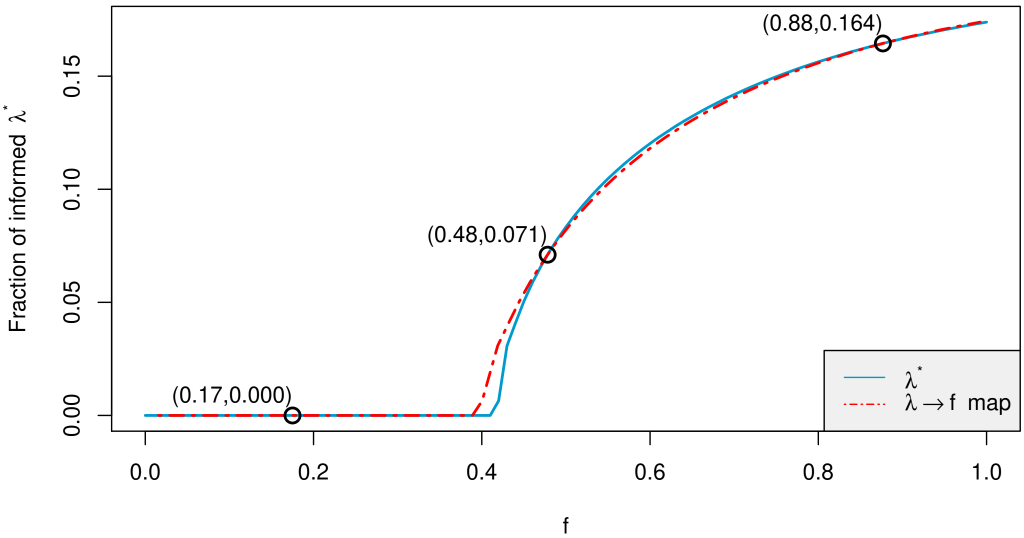

Figure 4: The solid line shows the fraction informed $\lambda(f_t)$ in information state f_t, and the dashed line shows the mapping from λ to f_{t+1} without exogenous shocks. Each circle shows a point where $f_t = f_{t+1}$ when the shocks in (6) are zero, labeled with its (f, λ) value. The figure uses parameter values from Table 1.

To interpret the dashed line in Figure 4, we shut off the exogenous shocks in the evolution of f_t by setting $\epsilon_{f,t+1} \equiv 0$ in (6). The dashed line then shows the mapping from λ to the next value of f. That is, starting from any $f_t = f$ on the horizontal axis, reading up to the solid line then across to the dashed line and back down to the horizontal axis yields f_{t+1}. Points where the two lines cross are fixed-point combinations of $(f, \lambda(f))$ in a model without exogenous shocks. In other words, the three circled points in the figure are cases where $f_{t+1} = f_t$ when $\epsilon_{f,t+1} = 0$.

Consider, for example, the circled point near $f = 0.48$, $\lambda(f) = 0.071$. Starting at that f, the endogenous fraction informed $\lambda(f)$ is precisely the value that keeps the information state at f under the evolution in (6) without exogenous shocks. The model still has feedback from λ to f (and f to λ), but f_t remains fixed. The same argument applies to the intersection near $f = 0.88$. In the lower left, the curves intersect throughout an interval where $\lambda(f) = 0$, and we have a fixed point at $(a_f, 0)$ because the dynamics in (6) drive f_t to a_f when $\lambda_t = \epsilon_{f,t+1} = 0$.

If we keep $\epsilon_{f,t} = 0$ and start the evolution of f_t near 0.88, it will move toward 0.88; and if we start the evolution near 0.175, f_t will move toward 0.175. In contrast, the point $f = 0.48$ is an

f increases reflects the dual roles of information in a multiperiod model. Becoming informed benefits an investor by reducing uncertainty about the end-of-period dividend. However, as more investors become informed, the variance of the end-of-period asset price increases, so the net effect on the variance of an investor's end-of-period wealth is indeterminate.

unstable fixed point: starting to the left of this point will drive f_t to 0.175, and starting to the right will drive f_t to 0.88. When we reintroduce the shocks $\epsilon_{f,t}$, we therefore expect f_t to spend long periods near 0.175 and long periods near 0.88.

These observations highlight the importance of exogenous shocks in our model. Without exogenous shocks in (6), once f_t reaches one of the three fixed points, it stays there forever — there are no transitions between regimes. Moreover, the three fixed points include the unstable fixed point near $f = 0.48$. In this sense, the model without shocks is potentially misleading: in the dynamic setting, we will have persistent regimes near 0.175 and 0.88 but not near 0.48.

This behavior explains the pattern we saw in Figure 2. An initial set of positive shocks increase signal precision f_t. With feedback dynamics, f_t stays near 0.88 for a long time: once the fraction informed $\lambda(f_t)$ is high, the demand for information keeps f_t high. Eventually, exogenous negative shocks decrease f_t sufficiently that it moves toward 0.175. The effect of feedback and exogenous shocks is therefore to endogenously create two regimes (corresponding to the two stable fixed points in Figure 4). This pattern is clearly absent in a static model with $f_t \equiv f$. We have not yet explained why the high f_t regime is associated with low prices and, as we will see, with high volatility. That explanation will come in Section 5.3.

Figure 5 provides additional information on the stochastic dynamics of f_t. The left panel shows the steady-state distribution of f_t (indicated by the blue circles in the left panel of the figure), calculated using a Markov chain representation.[18] The distribution is bimodal, showing that the economy spends the majority of its time in the vicinity of the two stable fixed points from Figure 4, and confirming the presence of two regimes.

If we fix λ at its mean value of 0.0731, which effectively turns off the feedback effect because the fraction informed no longer responds to the information state, the steady-state distribution (shown by red triangles) becomes unimodal — we no longer get two regimes. Similarly, we will get a unimodal distribution centered at $a_f = 0.175$ in an economy with $b_f = 0$.

The right panel shows that the two regimes in the feedback model are highly persistent, in the sense that the cumulative probability of transitioning from one to the other remains low, even after many periods. The probability of transitioning within 240 periods is only about 6–8%.

[18]See Section EC.4 of the Electronic Companion for details.

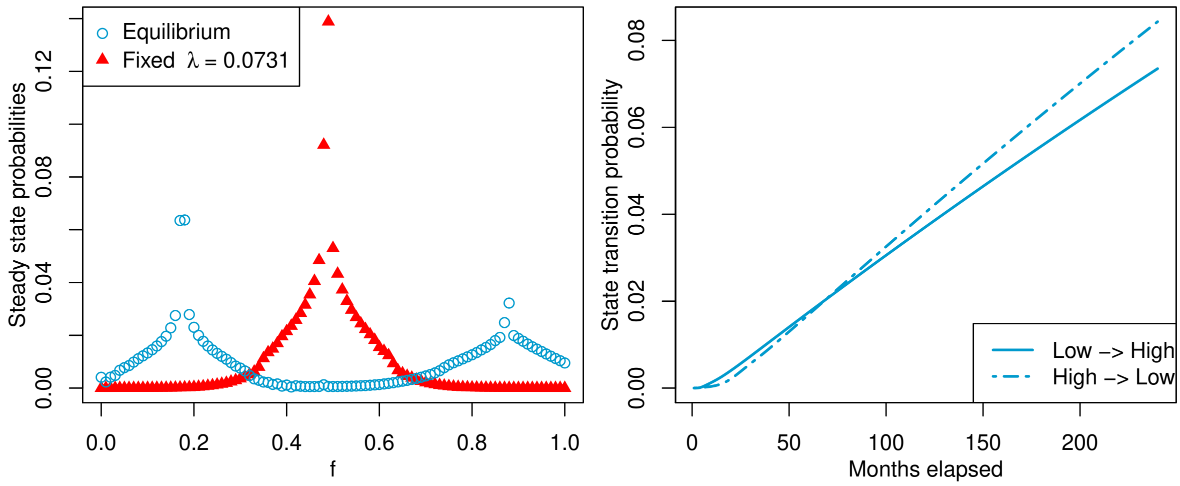

Figure 5: The left panel shows the equilibrium steady state distribution of f_t. The right panel shows the low-to-high regime transition probability $\mathsf{P}[f_{t+i} > 0.5|f_t = .17]$ (solid line) and the high-to-low regime transition probability $\mathsf{P}[f_{t+i} < 0.5|f_t = .88]$ (dashed line) as a function of i (measured in months). The figure uses parameter values from Table 1.

5.2 Price Drops and Volatility Spikes

Figure 6 shows model quantities calculated using the parameters in Table 1. Three of the panels show the price coefficient functions a, b, and c from (20). The upper right panel shows the b/c ratio, which reflects the amount of information that prices contain about fundamentals (the higher the ratio, the more responsive prices are to changes in m_t relative to changes in X_t). For low f, when no investors are informed, no dividend information is reflected in the price, and $b = 0$. As f increases to the point where some investors become informed, b and c increase, which drives up the price variance. The increase in c reflects a higher risk compensation for accommodating supply shocks and is attributable to higher price variance and a growing informational disadvantage of the uninformed relative to the informed.

The net effect of the increase in b and c is that prices become more informative about fundamentals as signal precision increases, i.e., b/c increases with f. This monotonicity of b/c parallels an empirical finding in Brancati and Macchiavelli (2019) that prices become more information-sensitive when information precision increases. As shown in Appendix B.1, we have

$$\frac{b_t}{c_t} = \frac{\lambda_t(1+d)}{\gamma q_D^I}, \tag{28}$$

where q_D^I is the expected conditional variance of the net profit for informed investors. From (28) we can see that the price informativeness ratio b/c increases with the signal precision f primarily because the fraction informed $\lambda(f_t)$ increases with f.[19] Equation (28) also shows why the shape of b/c resembles that of $\lambda(f_t)$ in Figure 4. As pointed out by Goldstein and Yang (2015), b/c also equals the trading intensity of informed investors [20]. The increasing b/c thus suggests that informed investors in aggregate trade more aggresively in response to their signal m_t as f increases.

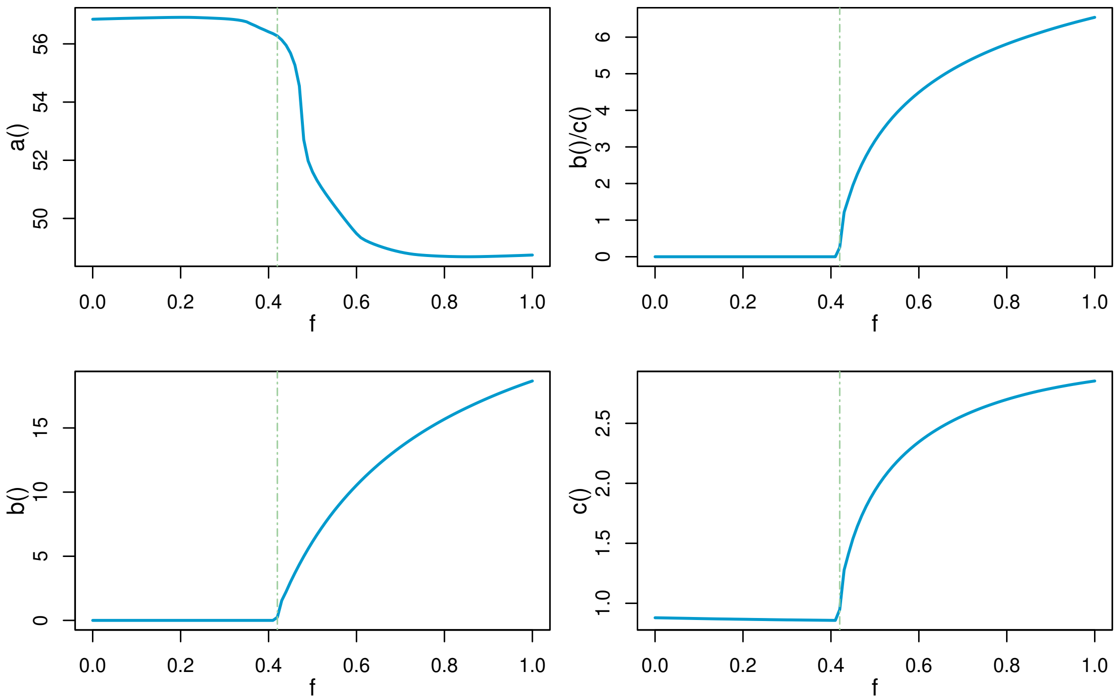

Figure 6: Price coefficients $a(f)$, $b(f)$, and $c(f)$ and trading intensity $b(f)/c(f)$. The vertical, dashed line shows where λ becomes positive. The figure uses parameter values from Table 1.

The upper left panel of Figure 6 shows that $a(f)$ drops sharply as signal precision f increases. The left panel of Figure 7 shows the resulting effect on the expected stock price, which from (20) is

$$P_0 \equiv a(f) + d\bar{D}. \tag{29}$$

The price response is dramatic: a small increase in f leads to a price drop of 10%. We will explain the price drop in Section 5.3.

[19]The behavior of the conditional variance q_D^I, which varies much less with f, is discussed in the next section.

[20]At a fixed λ, the market clearing condition is $\lambda q^I(m,P) + (1-\lambda)q^U(P) = X$ where q^I (q^U) is the demand of the informed (uninformed). Differentiating with respect to X gives an expression for $\partial P/\partial X$. Then differentiating the market clearing condition with respect to m and using $\partial P/\partial X$, shows that $\lambda \times \partial q^I/\partial m = -\partial P/\partial m/\partial P/\partial X$. When P is given by (20), this equals b/c.

As $b(f)$ and $c(f)$ increase with f, the price variance $V_B(f)$ in (26) increases with f. This can be seen in the right panel of Figure 7. Thus, the price drop associated with an increase in information precision f is accompanied by a spike in volatility. In the low-f region where $b(f) = 0$, $V_B(f)$ is below 0.5; but for f near 0.88, $V_B(f)$ exceeds 1.5, so the change in information regime produces more than a three-fold increase in price variance.

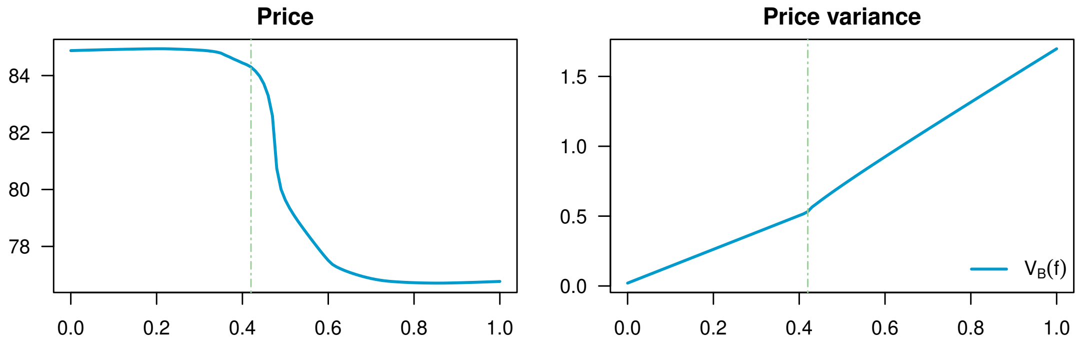

Figure 7: Left: Equilibrium expected price P_0 from (29). Right: Variance V_B from (26). Both are plotted against f. The figure uses parameter values from Table 1.

It is customary to associate large declines in market prices with the arrival of bad news. Following a 10% decline (the price drop in Figure 7) in an individual stock price or the overall market, one would expect media and expert accounts of what bit of bad news — a product failure, a CEO scandal, a change in government policy — triggered the fall. But in our setting it is simply more news — in the form of increased precision f_t — that drives investors, not necessarily good or bad news.[21]

The potential for increased volatility from increased information has policy implications. A regulatory change that leads to persistently higher information precision for informed investors is potentially destabilizing in times of market stress.Interestingly, in their analysis of disclosure of the results of regulatory stress tests for banks, Goldstein and Leitner (2018) conclude that disclosure is valuable only under adverse conditions.[22] Our results do not conflict but rather reflect different considerations, as the objective in Goldstein and Leitner (2018) is optimal risk sharing among banks, and the information disclosed in their setting separates weak and strong banks. In Ahnert and Georg (2018), the possibility of ex post information spillovers between banks can lead to an ex

[21]In a macro context, Cesa-Bianchi and Fernandez-Corugedo (2018) find that an increase in economic uncertainty results in a *decrease* in the risk premium, which is consistent with our results.

[22]The information disclosed about regulatory stress tests is disclosed publicly, but the design of scenarios and the interpretation of the results are technical matters that are arguably accessible only to informed investors who have acquired the necessary expertise.

ante reduction in systemic risk.

We have thus far kept the dividend shocks and information shocks independent of each other. This separation has allowed us to isolate the impact of a more precise signal for informed investors from the impact of specifically positive or negative signals. In practice, events that fuel investor demand for greater information may plausibly be accompanied by adverse effects on fundamentals. Although we do not have a specific theoretical foundation for such a linkage, to explore this possibility we examine (in Section EC.5) the effect of a negative correlation in shocks to f_t and shocks to dividends. As expected, this correlation amplifies the resulting price drop.

5.3 Decomposing Price and Volatility

The price drop in Figure 7 is driven by the drop in the $a()$ curve in Figure 6. In Appendix B.2, we show that $a_t = a(f_t)$ can be decomposed into two components,

$$a_t = \frac{(1+d)\mu_D}{R-1} - \sum_{i=1}^{\infty} \frac{1}{R^i} \mathsf{E}_t[\pi_{t+i}], \tag{30}$$

where π_{t+i} is the net profit from holding one share of the stock from $t+i-1$ to $t+i$, as in (8), and E_t denotes conditional expectation given f_t. From this expression, we can express the $a_t + dD_t$ component of the price P_t as

$$a_t + dD_t = \sum_{i=1}^{\infty} \frac{1}{R^i} \mathsf{E}[D_{t+i}|D_t] - \sum_{i=1}^{\infty} \frac{1}{R^i} \mathsf{E}[\pi_{t+i}|f_t],$$

which is the present value of all future expected dividend payments minus a discount reflecting the expected present value of all future net profits. In the context of the Campbell (1991) and Vuolteenaho (2002) return variance decomposition, the second term in a_t (or $a_t + dD_t$) represents the effect of a time-varying discount rate on stock prices. To understand how a change in signal precision f_t creates a price drop, we need to understand the effect of f_t on the second term in a_t.

The stock's expected net profit over a single period is given by[23]

$$\mathsf{E}_t[\pi_{t+1}] = \underbrace{\gamma}_{\text{risk aversion}} \times \underbrace{\bar{X}}_{\text{asset supply}} \times \underbrace{\left(\lambda \frac{1}{q_D^I} + (1-\lambda)\frac{1}{q_D^U}\right)^{-1}}_{\text{average uncertainty}}, \tag{31}$$

[23]This is shown in equation (52) of Appendix B.2. This expression generalizes the corresponding quantity derived from equation (A10) in Grossman and Stiglitz (1980).

where q_D^I and q_D^U are the expected conditional variances of the net profit for informed and uninformed investors,

$$q_D^{I/U} = \mathsf{E}\left[\mathsf{var}(\pi_{t+1}|\mathcal{I}_t^{I/U}, f_{t+1})\middle|\, \mathcal{I}_t^{I/U}\right].$$

They account for the uncertainty in both next period's dividend and price level. Equation (31) thus reflects the average return uncertainty faced by investors, weighted by the fractions of informed and uninformed in the economy, and scaled by $\gamma\bar{X}$.

The left panel of Figure 8 shows q_D^I (solid line) and q_D^U (dashed). The shape of the two curves reflects the tradeoff engendered by increased information precision. When f is low, an increase in f decreases the expected variance of net profits for informed and uninformed investors because more is known about next period's dividend, the D_{t+1} term in (8). As long as f is low enough so that the fraction informed $\lambda(f) = 0$ (to the left of the vertical, dashed line in the graphs) this is the only effect, and higher information precision lowers uncertainty. However, past the no-informed point, with f large enough that $\lambda(f) > 0$, the uncertainty of next period's net profit starts to increase, due to the increasing variance of P_{t+1} in the expression for π_{t+1} in (8). This effect outweighs the decrease in the variance of next period's dividend, and thus increases the conditional variances of the net profit. For high enough f the increased information about next period's dividend begins to dominate, and the expected conditional variance begins to fall again. This pattern depends crucially on the dynamic structure of our model: in a single-period setting, where investors care about the next dividend but not future prices, more precise information always reduces investment uncertainty.

Through (31), the common shape of q_D^I and q_D^U is inherited by $\mathsf{E}_t[\pi_{t+1}]$, as illustrated in the right panel of Figure 8. Recall from our discussion of Figures 4 and 5 that f_t spends most of its time near $f = 0.175$ or near $f = 0.88$. $\mathsf{E}_t[\pi_{t+1}]$ is greater near $f = 0.88$ than it is near $f = 0.175$, which indicates an increase in the expected profit from holding the stock when moving from the low- to the high-information regime. This increase in expected profit is associated with the decrease in the current stock price seen in the left panel of Figure 7.

However, the change in expected net profit across regimes is quite small, as indicated by the vertical scale in the right panel of Figure 8. How does a small change in expected profit get amplified into a 10% price drop? The answer lies in the combination of the price discount reflected in (30) and the persistence of the two information regimes.

The right panel of Figure 5 shows that transitions between $f_t \approx 0.175$ and $f_t \approx 0.88$ are rare. Figure 8 suggests the inequality $\mathsf{E}_t[\pi_{t+1}|f_t = 0.88] > \mathsf{E}_t[\pi_{t+1}|f_t = 0.175]$. As a consequence of

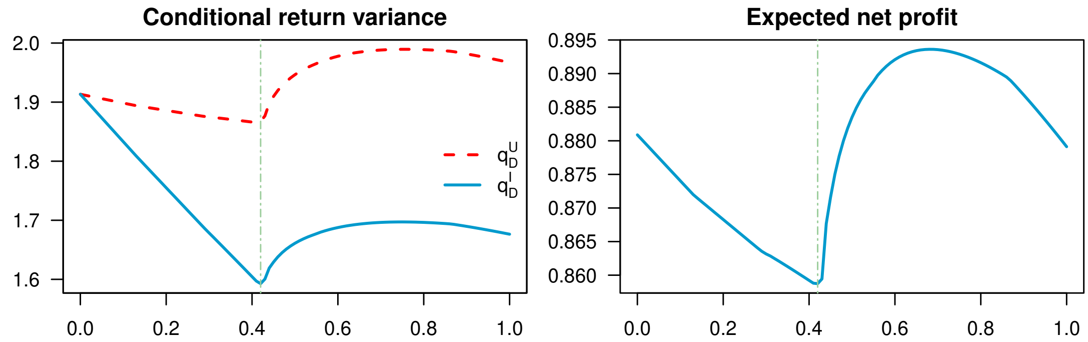

Figure 8: Left: Conditional expected return variance for informed, q_D^I, and uninformed, q_D^U. Right: Expected investment net profit $\mathsf{E}_t[\pi_{t+1}]$ per period. Both are plotted against f. The figure uses parameter values from Table 1.

the persistence in regimes, we expect this inequality to extend to $\mathsf{E}_t[\pi_{t+i}|f_t = 0.88] > \mathsf{E}_t[\pi_{t+i}|f_t = 0.175]$, for large i. The present value of such terms is subtracted from the price P_t through the a_t coefficient in (30). Thus, even a relatively small single-period difference in expected profits around $f = 0.175$ and $f = 0.88$ gets amplified into a large change in the price because the f_t process spends long periods in the low- and high-information regimes.[24]

Are such infrequent regime transitions plausible? Barro (2009) estimates country level crises occur with a 1.7% per year probability. Assuming independence across time, a given country has a 29% (i.e., $1-(1-0.017)^{20}$) probability of experiencing at least one crisis over a 20-year period. As we saw in Figure 5, the probability of a low to high state transition in our model is approximately 7% over a 20-year period. Our calibration therefore suggests that one out of four country-level crises may be accompanied by the information-driven price drop of our model.

6 Exploration of the Mechanism

Section 6.1 analyzes how informational asymmetry and the dynamics of the information process impact price and volatility cycles in our model. Section 6.2 contrasts our model with models of strategic complementarity in information acquisition.

[24]The same argument predicts a sharp decline in the $a()$ curve around the unstable fixed point near $f = 0.48$ in Figure 4. Starting to the right of 0.48, f_t will tend to move toward 0.88, whereas starting to the left of 0.48, f_t will tend to move toward 0.175.

6.1 The Role of Information Asymmetry and Dynamics

In our baseline calibration, large price drops occur when the economy transitions from low- to high-information states. An important parameter in the model is ϕ, the fraction of knowable information that is private. The upper left panel of Figure 9 compares equilibrium average prices P_0 from (29) for different values of ϕ. In our base case (black solid curve), $\phi = 0.35$ and there is a large price drop as f moves from the low- to the high-information regime.

When $\phi = 0$ and all knowable information is public, the economy is characterized by no information asymmetry — the knowable information is equally known to all agents. The price curve corresponding to this no-asymmetry case is the highest one, indicating the smallest price discount relative to the present value of future dividends. The $\phi = 0.6$ case represents a high degree of informational asymmetry, and corresponds to the lowest price curve. This reflects a large risk premium needed to induce the informationally disadvantaged uninformed agents to participate in risk sharing, regardless of the information state f_t. Only for intermediate values of ϕ do we see a lot of variation in the price curve. The price starts out high for low fs and drops for high fs.

The reason that prices in the cases $\phi = 0$ and $\phi = 0.6$ do not change much with f can be seen from Figure 10, which shows the steady-state distribution of f in the different ϕ models. Changing ϕ affects the steady-state distribution because it impacts the endogenous $\lambda(f_t)$. When $\phi = 0$, there are no informed investors since all knowable information is public. With $\lambda = 0$ in (6), any positive $\epsilon_{f,t+1}$ shock quickly decays, pulling f_t back to its low-information fixed point of 0.175. This dynamic is seen in the unimodal distribution (left panel), with the peak centered at $a_f = 0.175$. Similarly, when $\phi = 0.6$, much of the knowable information is private, and $\lambda(f)$ is high. Via the b_f term in the dynamics of f_{t+1} in (6), a relatively high λ produces a steady state distribution that is unimodal at $f = 1$ (right panel). Any negative $\epsilon_{f,t+1}$ quickly dissipates as f is pulled back to one. In these cases, $\mathsf{E}_t[\pi_{t+i}|f_t = f]$ will be close to either $\mathsf{E}_t[\pi_{t+i}|f_t = 0.175]$ or $\mathsf{E}_t[\pi_{t+i}|f_t = 1]$ for any f, and the $a(f)$ term in P_0 – see (30) – is consequently insensitive to f.

For intermediate values of ϕ, the equilibrium λ curve is also in an intermediate range, and the tendencies of f_t towards a_f and towards the high-information fixed point are balanced. The steady state f_t distribution becomes bimodal (middle panel of Figure 10). In this case, a sequence of shocks can occasionally push the economy from one information regime to the other. But both regimes are persistent. As in Section 5.3, this persistence amplifies differences in expected net profit $\mathsf{E}_t[\pi_{t+1}]$ at different values of f to produce large price changes.

As the above discussion has emphasized, the dynamics of f_t in (6) play an important role in

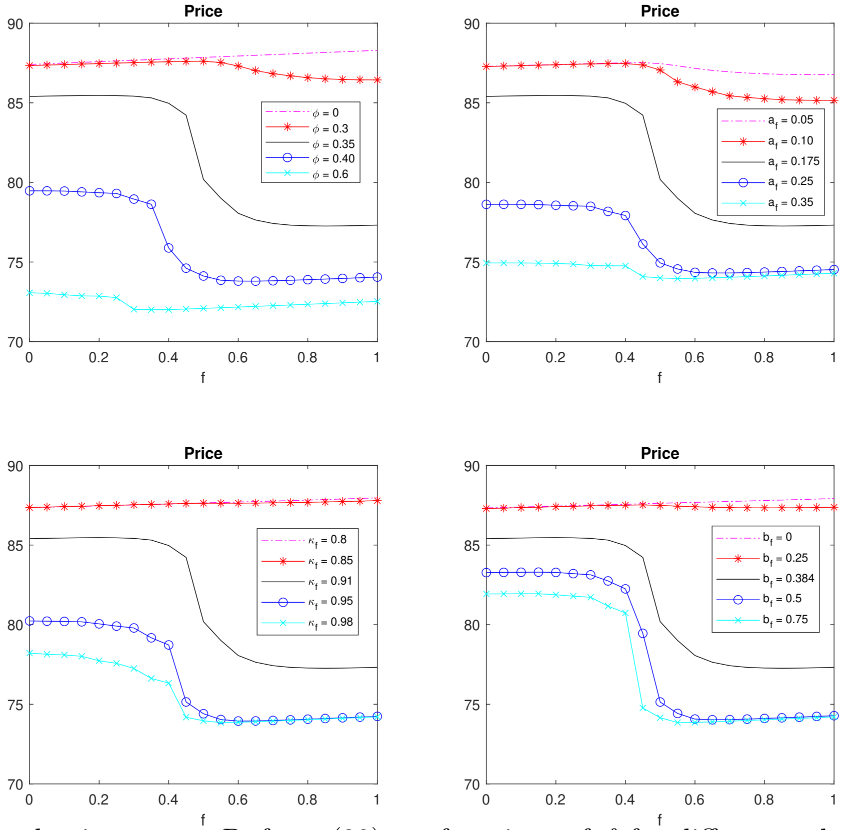

Figure 9: Expected price curves P_0 from (29), as functions of f for different values of ϕ, a_f, κ_f, and b_f. The middle black line in all charts corresponds to our base case parameters in Table 1.

creating a large price drop in the high information state. The key ingredient necessary for market cycles is that both the low- and high-information asymmetry regimes are reachable and persistent (as was the case for intermediate levels of ϕ). Similar logic suggests that intermediate levels of a_f, κ_f, and b_f are necessary for price cycles because they allow for endogenously persistent and reachable low- and high-information states. Recall from (6) that the parameter a_f represents the long-term information precision in the absence of informed investors, and a larger κ_f makes information precision more persistent. According to our microfoundation, a larger b_f can be generated by a lower information production cost p_S. In this sense, a larger b_f can be interpreted as a more active information production or media sector.

The impact of the f_t dynamics on the price level is confirmed in the other three panels of Figure 9. The price stays high when a_f, b_f, or k_f is small. In these cases, the steady state f_t distribution is unimodal at the low-information fixed point, with low information asymmetry between informed and uninformed. On the other hand, when these parameters are large, the resulting

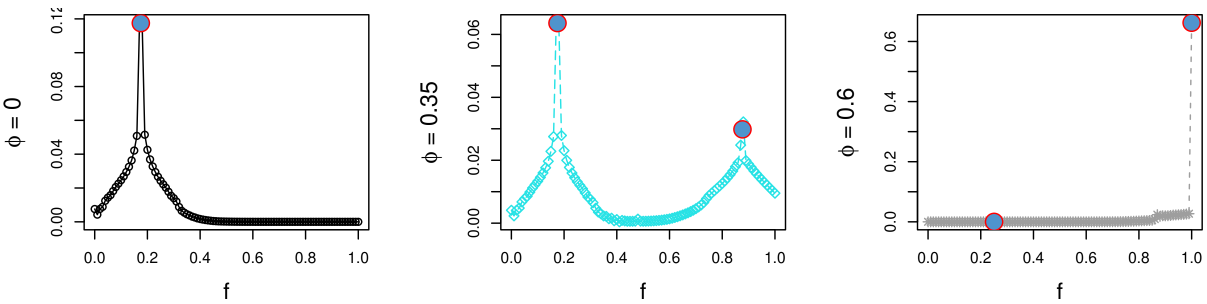

Figure 10: Steady-state distribution of f_t across different levels of ϕ. The figure uses parameter values from Table 1. The solid circles indicate fixed points of the $f \to \lambda^*$ mapping.

steady state f_t distribution is unimodal near $f = 1$, leading to high information asymmetry, low price level, and a smaller price drop between regimes. These effects are consistent with the f_t dynamics in (6), where larger values of a_f, b_f (when $\lambda_t > 0$), or κ_f (when $f_t > a_f$) all increase f_{t+1}, making the high information state more likely.[25]

Figure 11 repeats the analysis shown in Figure 9 but for the b/c (price informativeness) curve, rather than for the average price. The upper left panel shows that, at every f, price informativeness increases with ϕ. Not surprisingly, with a more informative private signal, prices better reflect fundamental information: at $\phi = 0$ the b/c curve equals zero, and for high ϕ the b/c curve grows outwards to the upper left. The other three panels show that the f_t dynamics – crucial for generating price cycles in the model – have relatively small effects on price informativeness. This pattern is consistent with our discussion of $a(f)$ in Section 5.3. The persistence of the two f_t regimes impacts prices primarily through the conditional expectations of all future profits (i.e., the $\mathsf{E}_t[\pi_{t+i}]$ terms in 30). However, the informativeness ratio $b(f)/c(f)$ does not depend on these terms.

Our focus thus far has been on the impact of information precision on average prices. In Section EC.8 of the Electronic Companion, we contrast the impact from an increase in f to what happens when fundamental volatility (σ_M) rises. Both shocks decrease prices, but they have different impacts on price variance and on the fraction informed. This suggests that the two shocks can be distinguished empirically, which is explained at the end of Section EC.8.

[25]In Section EC.7.1 of the Electronic Companion, we show that when we use a logistic mapping for signal precision process, the price drop becomes smaller when the logistic mapping is more “stretched out”, i.e., the signal precision is less sensitive to underlying state variable.

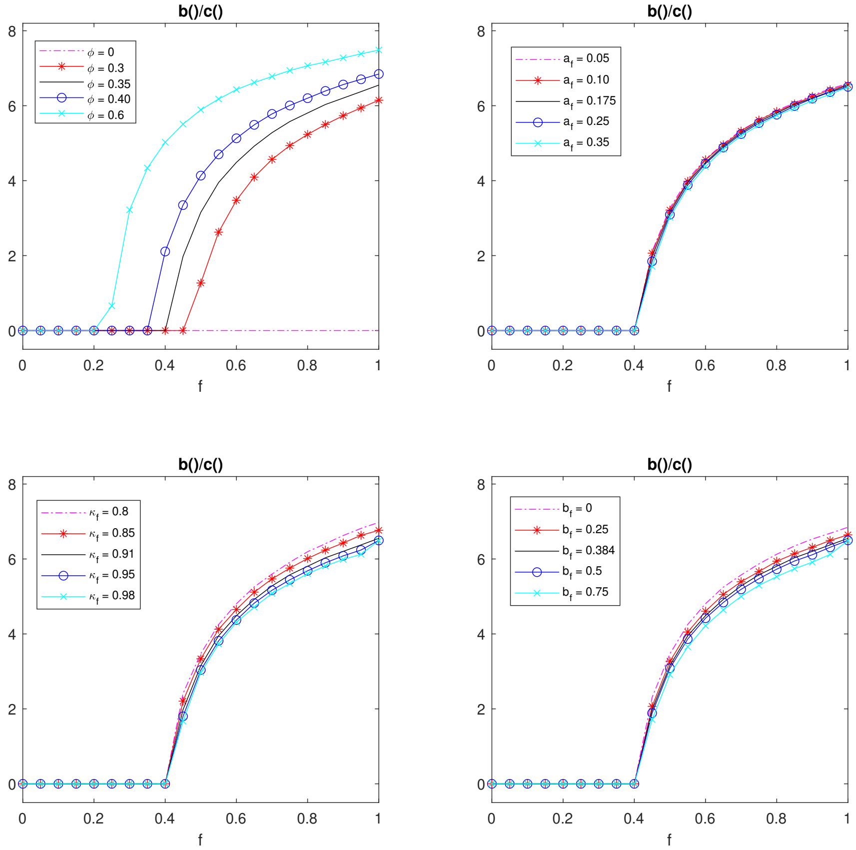

Figure 11: The figure shows $b()/c()$ curves as functions of f for different values of ϕ, a_f, κ_f, and b_f. The middle black line in each chart corresponds to our base case parameters in Table 1.

6.2 The Value of Becoming Informed

A key property of the Grossman and Stiglitz (1980) setting is that the value of becoming informed decreases as the number of informed investors increases. Subsequent work has investigated conditions in which the value of becoming informed increases as more investors become informed. Sources of this type of strategic complementarity identified in the literature include high fixed costs and low marginal costs in information production (Veldkamp 2006); certain deviations from normally distributed uncertainty (Chamley 2007); settings in which investors learn about supply as well as cash flows (Ganguli and Yang 2009 and Avdis 2016); other settings with multiple sources of information (Manzano and Vives 2011 and Goldstein and Yang 2015); and settings in which information acquisition affects cash flows (Dow et al. 2017). With few exceptions, these are static models, but they often result in multiple equilibria, with different asset prices in different equilibria.

In our dynamic setting, large price changes occur within the model, rather than through a

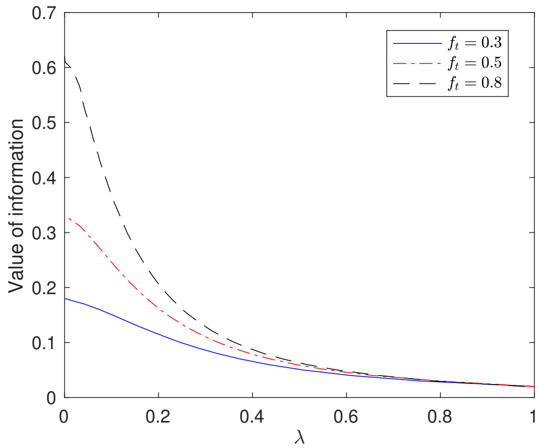

Figure 12: The curves show how the equilibrium value of becoming informed varies with the fraction informed $\lambda(f)$, at different levels of f. As more investors become informed, the value of becoming informed decreases.

change of equilibrium selection in a static model. But we will see that the contrast with earlier work goes beyond this feature. In order to explore strategic complementarity, we vary the cost c_I of becoming informed and recalculate the equilibrium.[26] For each c_I, we get a new $\lambda(\cdot)$ function; we can fix a value of f and see how $\lambda(f)$ varies with c_I. Because $\lambda(f)$ is the equilibrium fraction informed, c_I is precisely the value of becoming informed when $f_t = f$.

Figure 12 shows the results of these calculations at three levels of f. The curves are calculated by varying c_I and recomputing $\lambda(f)$. But we read them in reverse, i.e. with $\lambda(f)$ on the horizontal axis, to see how the equilibrium value of becoming informed varies with the fraction informed $\lambda(f)$. The curves are decreasing, indicating that the value of becoming informed decreases as more investors become informed. In this respect, our model shows the behavior in Grossman and Stiglitz (1980) and differs from the large class of subsequent models that exhibit strategic complementarity in the value of becoming informed.

The underlying source of the feedback effect in our f_t dynamics (6), as developed through the microfoundation in Section 3, is the response of a competitive information production sector to increased demand when the marginal cost of transmitting already discovered information is zero. A similar cost structure of information production is highlighted by Veldkamp (2006) as a source of strategic complementarity, so it is interesting to contrast the implications of the two models. The models have different objectives and differ in many respects; but, most notably, in Veldkamp

[26]In this experiment, we assume the per-unit information cost p_S changes accordingly to offset the impact of c_I on information production. Thus, the dynamics of f_t in (5) and (6) are unchanged.

(2006) higher prices are associated with a larger fraction of informed investors, whereas we saw in Section 5.2 that in our model an increase in λ_t can precipitate a large price drop.

Complementarity in Veldkamp (2006) arises because the price of information in equilibrium is declining in the number of informed investors; with more informed investors, there is the same amount of information, but at a lower per capita cost. In our case, the unit price of information production and the amount paid by each investor for information are fixed; therefore with more informed investors, more information gets produced. This difference leads to information cycles in our model, and these are absent in Veldkamp (2006).

Another key feature driving the difference is the dual role of information in our dynamic model discussed in Section 5.3: more precise information decreases dividend uncertainty but can increase future price variance. The second effect is absent in Veldkamp (2006), where investors earn dividends but do not earn capital gains from reselling their shares, making them indifferent to price variance. With no dependence on the next period's prices, the analysis reduces to a sequence of single-period problems.

To see that end-of-period prices drive the difference in our conclusions we can work backwards as follows. The price drop in our model is driven by the drop in $a(f)$ in (30), which, through (31), is driven by the increase in the conditional variances q_D^U and q_D^I of π_{t+1} shown in Figure 7. If the return π_{t+1} in (8) did not include P_{t+1}, the conditional variances q_D^U and q_D^I would instead decrease with f as the uncertainty in D_{t+1} decreases, eliminating the price drop we see with increasing f.

7 Conclusion

We have developed a model of a financial market in a dynamic information environment and shown that information dynamics can have a profound effect on prices. The model combines exogenous shocks to the level of potentially available information, an endogenous response by investors, and microfounded feedback from investor information choices to the information environment through information production by a competitive research sector. The dynamic structure of the model leads to a dual role for information, in which greater information reduces uncertainty about the next dividend but may increase price variance.

We show that the equilibrium dynamics of our model, calibrated to market data, are characterized by two regimes: a low information regime with high prices and low volatility, and a high information regime with low prices and high volatility. The key dynamic in the model is that the low- and high-information regimes, which arise endogenously, are both reachable and persistent. A transition from the first regime to the second is reminiscent of a financial crisis but with no change in fundamentals — the price drop is driven by the dynamics of information and an increase in information asymmetry. In the case of correlated information and fundamentals shocks, an increase in information production can meaningfully increase the price impact of adverse dividend shocks.

Our model points to an important role for information dynamics in financial crises. At the onset of a crisis, growing investor demand for information met with increasing production of non-public information can drive down prices and increase volatility. The effect can be counteracted by making costly information public to reduce the asymmetry between informed and uninformed investors.

A Model Calibration

In calibrating the model to the aggregate market, we take one period in the model to represent one month. We estimate a monthly dividend process of the form (1) using daily dividend data for the S&P 500 index from 1998–2018, then aggregating this up to the quarterly level (to mitigate seasonality effects), and estimating an ARMA(1,1) process for the quarterly dividend. From this we back out the monthly parameters $\rho = 0.967$ and $\sigma_M = 0.0471$; see Section EC.9 for details.

We adopt the normalization $\bar{D} = 1$ and $\bar{X} = 1$, so dividends and share supplies are measured in units of their monthly averages. We calibrate σ_X^2 to match monthly turnover, meaning the number of shares traded per month divided by the shares outstanding. Recall from Section 2.3 that in each period t, investors buy the new supply X_t originating from liquidity demanders, and investors from the previous period sell back X_{t-1} shares to the previous period's liquidity demanders unwinding their trades. The total trading volume in period t is therefore $|X_t| + |X_{t-1}|$. Using the normality of the supply shocks, the expected volume per period becomes

$$\mathsf{E}[|X_t| + |X_{t-1}|] = 2\sigma_X \sqrt{\frac{2}{\pi}} \approx 1.596\sigma_X. \tag{32}$$

In the Electronic Companion, we find that the average monthly turnover of the Dow Industrials index is 0.0458.[27] To model a period of stress, we assume that turnover, or the turnover expectation of market participants, is 5.69 times higher than normal, so $\sigma_X = 5.69 \times 0.0458 \times 1/2 \times \sqrt{\pi/2} = 0.163$.[28]

[27] Lo and Wang (2000, Table 3) show that from 1987-1996, weekly turnover on a value-weighted index of NYSE and AMEX common shares was 1.25%. Therefore the monthly turnover on this index was $52/12 \times 1.25\% = 5.42\%$.

[28] The monthly trading volume of SPY, which tracks the S&P 500 index and is one of the most liquid exchange-

We use a monthly gross risk-free return of $R = 1.0015$ and set the risk-aversion parameter at $\gamma = 0.46$.[29] This yields an annualized excess return of roughly 15%, which is not unreasonable for periods of stress. We choose a per month cost of being informed of $c_I = 0.2627$, which should be compared with a monthly aggregate average dividend of 1 (since $\bar{X} = \bar{D} = 1$). This high cost of information is comparable to the 2/20 fee structure of many hedge funds, and leads to an equilibrium number of informed of under 18% of the overall population.

For the dynamics of the f_t process in (6), we set $a_f = 0.175$, $\kappa_f = 0.91$ and $b_f = 0.384$. In the context of our information production microfoundation (see Section 3), these choices of a_f and κ_f imply the probability of a unit of information transitioning from available to unavailable is roughly five times larger than the probability of transitioning from unavailable to available. A value of κ_f close to 1 makes the information state persistent by making both of the above probabilities small, and a positive b_f produces positive feedback from the fraction informed to the level of accessible information. When f_t is low, $\lambda(f_t) = 0$, and f_t is pulled toward a level of $a_f = 0.175$.[30]

To get a rough estimate of ϕ, we consider analysts' forecasts of quarterly earnings for the S&P 500 index. In a given quarter, we take the mean quarter-ahead forecast (across analysts) as public information, and we interpret the variance of the forecast error (across analysts) as a measure of private information. Taking the variance (over time) of the mean forecast and the mean (over time) of the forecast error variance gives us a decomposition of the total earnings variance into the public fraction $1 - \phi$ and the private fraction ϕ. Over the 10-year period 2012–2021, this gives us a ϕ value in the range of 26.9–31.5% (see Section EC.9 for details). This is likely an underestimate in the sense that uninformed investors are unlikely to have better information than the consensus forecast, but informed investors may have better information than participating analysts. We use $\phi = 35\%$ as our base case and examine the effect of varying this parameter.

Beyond these qualitative considerations, these specific parameters were chosen to produce plausible model dynamics. Finally, for the shocks $\epsilon_{f,t+1}$, we use a three-point distribution taking values $\{-0.135, 0, 0.135\}$ with probabilities $\{0.03, 0.94, 0.03\}$, so shocks are rare. Model parameters are summarized in Table 1.

traded funds spiked by a factor of five or more during the Global Financial Crisis (GFC) and during the COVID-19 pandemic. In December 2005, before the start of the GFC, the monthly SPY trading valume was 1.074 billion shares. In January 2008, the monthly trading volume – which would be even higher later in the year – was 6.108 billion, for a ratio of 5.687.

[29] As explained in Biais et al. (2010), p.1518, an appropriate magnitude for the risk aversion parameter in a CARA model is unclear. We pin down γ indirectly through the expected return on the risky asset. The choice $\gamma = 0.46$ fits well within the wide range of values used in prior work. For example, Farboodi and Veldkamp (2020) uses 0.05, and Veldkamp (2006) uses 2.5.

[30] Fama and French (2000) show R^2's of year-ahead firm-level earnings forecasts are between 5% and 20%.

Our results are robust to changes in model parameters. For a wide range of values in our non-dividend parameters (since ρ and σ_M are estimated from actual data) in Table 1, there exists a ϕ close to its base value of 0.35 which generates the bimodal f_t distribution and the large price drops that we discuss below. In fact, in many cases the resultant price drops are larger than the one that occurs under our base parameterization.[31]

$\bar{D}$	$\bar{X}$	R	σ_X	ρ	σ_M	γ	c_I	ϕ	$\mathsf{P}[\epsilon_f \neq 0]$	ϵ_f	a_f	b_f	κ_f
1	1	1.0015	0.1629	0.967	0.0471	0.46	0.2627	0.35	0.03×2	0.135	.175	.384	.91

Table 1: Calibrated parameters for model (6)

B Market Equilibrium

B.1 Proof of Lemma 4.1

Investor Demands for the Risky Asset

We prove Lemma 4.1 by solving explicitly for the coefficients of the price in (20). To allow for arbitrary V_B, we write the investor optimization problem (9) as

$$\hat{J}_t^\iota \equiv \max_q \mathsf{E}\left[\mathsf{E}[W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}] - \frac{\gamma}{2}\hat{\mathsf{var}}(W_{t+1}|\mathcal{I}_t^\iota, f_{t+1})\Big|\mathcal{I}_t^\iota\right], \quad \iota \in \{I, U\}, \tag{33}$$

where, using (25),

$$\hat{\mathsf{var}}(W_{t+1}|\mathcal{I}_t, f_{t+1}) = q^2(1+d)^2\left[\mathsf{var}(m_t|\mathcal{I}_t) + (1-f_t)\sigma_M^2\right] + q^2 V_B(f_{t+1}). \tag{34}$$

If the conjectured variance V_B is self-consistent, then (34) yields the conditional variance, but (33) makes explicit investors' objectives with arbitrary V_B.

We can write the terminal wealth in (7) as $W_{t+1} = RW_t + q\pi_{t+1}$. Recalling that W_t is known to time-t investors, we set $\hat{\mathsf{var}}(\pi_{t+1}|\mathcal{I}_t, f_{t+1}) = \hat{\mathsf{var}}(W_{t+1}|\mathcal{I}_t, f_{t+1})/q^2$ to get

$$\hat{\mathsf{var}}(\pi_{t+1}|\mathcal{I}_t, f_{t+1}) = (1+d)^2\left[\mathsf{var}(m_t|\mathcal{I}_t) + (1-f_t)\sigma_M^2\right] + V_B(f_{t+1}). \tag{35}$$

[31]This analysis is available from the authors.

The first-order condition for optimality in (33) becomes

$$q_t^\iota = \frac{1}{\gamma} \frac{\mathsf{E}\left\{\mathsf{E}[\pi_{t+1}|\mathcal{I}_t^\iota, f_{t+1}]|\,\mathcal{I}_t^\iota\right\}}{\mathsf{E}\left\{\hat{\mathsf{var}}(\pi_{t+1}|\mathcal{I}_t^\iota, f_{t+1})|\,\mathcal{I}_t^\iota\right\}} \equiv \frac{1}{\gamma}\frac{q_N^\iota}{q_D^\iota}, \tag{36}$$

where q_N^ι is the conditional expectation of the net profit, and q_D^ι is the expectation of its conditional variance, given a price variance of V_B. Through (35), the conditional variances, reflecting the cash-flow component and the time-t price variance, take the form

$$q_D^I = (1+d)^2(1-f_t)\sigma_M^2 + \mathsf{E}_t V_B(f_{t+1}), \quad q_D^U = q_D^I + (1+d)^2 \mathsf{var}(m_t|P_t, \theta_t). \tag{37}$$

Evaluating the conditional mean in the numerator of (36) as in (23), the demands for time-t informed and uninformed agents become

$$\begin{aligned} q^I &= \frac{q_N^I}{\gamma q_D^I} = \frac{1}{\gamma q_D^I}\Big[(1+d)(\mu_D + \rho D_t + \theta_t + m_t) + \mathsf{E}_t a(f_{t+1}) - RP_t\Big], \\ q^U &= \frac{q_N^U}{\gamma q_D^U} = \frac{1}{\gamma q_D^U}\Big[(1+d)(\mu_D + \rho D_t + \theta_t + \mathsf{E}[m_t|P_t, D_t, \theta_t]) + \mathsf{E}_t a(f_{t+1}) - RP_t\Big]. \end{aligned} \tag{38}$$

For the informed, we have used the fact that $\mathsf{E}[m_t|\mathcal{I}_t^I] = m_t$ and $\mathsf{var}(m_t|\mathcal{I}_t^I) = 0$. For the uninformed, we evaluate (38) using[32] $\mathsf{E}[m_t|P_t, \theta_t] = K_t(b_t m_t - c_t X_t)$ and

$$\mathsf{var}(m_t|P_t, \theta_t) = \phi f_t \sigma_M^2(1 - K_t b_t) = \phi f_t \sigma_M^2 (1 - \mathcal{R}_t^2), \tag{39}$$

with

$$K_t = \frac{\mathsf{cov}(m_t, P_t|\theta_t, D_t)}{\mathsf{var}(P_t|\theta_t, D_t)} = \frac{b_t \phi f_t \sigma_M^2}{b_t^2 \phi f_t \sigma_M^2 + c_t^2 \sigma_X^2} \qquad \text{and} \qquad \mathcal{R}_t^2 \equiv K_t b_t. \tag{40}$$

Market Clearing and Price Coefficients

We now impose market clearing (11), taking λ as given. We substitute investor demands q^ι in (11), use the price function from (20), and collect terms. We do not have to expand q_D^I or q_D^U in the

[32]Say $E[m|P] = K(bm + cX)$ for $K = \mathsf{cov}(m, P)/\mathsf{var}(P)$ and $\mathsf{var}(m|P) \equiv \mathsf{var}(m - E[m|P])$. Since $m - E[m|P] = (1 - Kb)m - KcX$ then $\mathsf{var}(m - E[m|P]) = (1 - Kb)^2\mathsf{var}(m) + K^2c^2\mathsf{var}(X)$. This equals $(1 - 2Kb + K^2b^2)\mathsf{var}(m) + K^2c^2\mathsf{var}(X) = (1 - 2Kb)\mathsf{var}(m) + K^2(b^2\mathsf{var}(m) + c^2\mathsf{var}(X))$. Note that $K = b\mathsf{var}(m)/(b^2\mathsf{var}(m) + c^2\mathsf{var}(X))$ and therefore $K^2(b^2\mathsf{var}(m) + c^2\mathsf{var}(X)) = b^2\mathsf{var}^2(m)/(b^2\mathsf{var}(m) + c^2\mathsf{var}(X)) = Kb\mathsf{var}(m)$. And therefore $\mathsf{var}(m|P) = (1 - Kb)\mathsf{var}(m)$.

following because these depend on f_t but not on D_t, m_t, θ_t, or X_t. Equation (11) becomes

$$\lambda q_D^U\Big[(1+d)(\mu_D+\rho D_t+\theta_t+m_t)+\mathsf{E}_t a(f_{t+1})-RP_t\Big]+$$
$$(1-\lambda)q_D^I\Big[(1+d)(\mu_D+\rho D_t+\theta_t+\mathsf{E}[m_t|P_t,\theta_t])+\mathsf{E}_t a(f_{t+1})-RP_t\Big]=\gamma q_D^I q_D^U X_t+\gamma q_D^I q_D^U \bar{X}. \quad (41)$$

Collecting the D_t terms and then the θ_t terms yields the constants

$$d=\frac{\rho}{R-\rho} \qquad \text{and} \qquad g=\frac{1}{R-\rho}. \quad (42)$$

Collecting the m_t terms in (41) we get

$$b_t=\frac{1+d}{R}\times\frac{\lambda q_D^U+(1-\lambda)q_D^I K_t b_t}{\lambda q_D^U+(1-\lambda)q_D^I}. \quad (43)$$

This leads to

$$b_t=\frac{\lambda(1+d)q_D^U}{R[\lambda q_D^U+(1-\lambda)q_D^I]-(1+d)(1-\lambda)q_D^I K_t}.$$

Collecting the X_t terms and simplifying (mainly dividing the resulting equation by above), we find

$$b_t/c_t=\frac{\lambda(1+d)}{\gamma q_D^I}, \quad \text{with } c_t=\frac{\gamma q_D^U}{R} \text{ if } \lambda=0. \quad (44)$$

We can now combine these equations to solve for b and c through the following steps, each of which involves only known quantities on the right side:

$$\begin{aligned}
q_D^I(f) &= (1+d)^2(1-f)\sigma_M^2+\mathsf{E}[V_B(f_{t+1})|f_t=f], && (45)\\
r(f) &= \lambda(f)(1+d)/(\gamma q_D^I(f)), && (46)\\
\mathcal{R}^2(f) &= r^2(f)f\phi\sigma_M^2/\left(r^2(f)f\phi\sigma_M^2+\sigma_X^2\right), && (47)\\
q_D^U(f) &= q_D^I(f)+(1+d)^2 f\phi\sigma_M^2(1-\mathcal{R}^2(f)), && (48)\\
b(f) &= \frac{1+d}{R}\frac{\lambda(f)q_D^U(f)+(1-\lambda(f))q_D^I(f)\mathcal{R}^2(f)}{\lambda(f)q_D^U(f)+(1-\lambda(f))q_D^I(f)}, && (49)\\
c(f) &= \begin{cases} b(f)/r(f), & \lambda(f)>0;\\ \gamma q_D^U(f)/R, & \lambda(f)=0.\end{cases} && (50)
\end{aligned}$$

Equation (45) restates the first line of (37); (46) is the ratio in (44); (47) rewrites the expression for $\mathcal{R}^2$ in (40); (48) follows from the second line of (37); (49) and (50) come from (43) and (44).

Note that the function $a(\cdot)$ from (20) plays no role in the above steps for solving b and c. We show how to solve it in the next section.

B.2 Solving for the $a(\cdot)$ curve

We now solve the function $a_t = a(f_t)$. Collecting the constant terms in (41) yields

$$a_t = \frac{1}{R}\left[(1+d)\mu_D - \frac{\gamma q_D^I q_D^U}{\lambda q_D^U + (1-\lambda) q_D^I}\bar{X} + \mathsf{E}_t a(f_{t+1})\right], \tag{51}$$

with $a(\cdot)$ appearing on both sides of the equation. Using the price function in (20) and the net profit in (8), we get

$$\begin{aligned}
\mathsf{E}_t[\pi_{t+1}] = \mathsf{E}_t[D_{t+1} + P_{t+1} - RP_t] &= \mathsf{E}_t[\mu_D + \rho D_t + a_{t+1} + d(\mu_D + \rho D_t) - Ra_t - RdD_t] \\
&= (1+d)\mu_D + \mathsf{E}_t[a_{t+1}] - Ra_t \\
&= \gamma\bar{X}\frac{q_D^I q_D^U}{\lambda q_D^U + (1-\lambda) q_D^I},
\end{aligned} \tag{52}$$

where the third step follows from the definition of d in (42) and the fourth step follows from a_t in (51). Using (52) we can rewrite a_t in (51) as

$$\begin{aligned}
a_t &= \frac{1}{R}\left[(1+d)\mu_D - \mathsf{E}_t[\pi_{t+1}] + \mathsf{E}_t a(f_{t+1})\right] \\
&= \frac{1}{R}\left[(1+d)\mu_D - \mathsf{E}_t[\pi_{t+1}] + \mathsf{E}_t\left\{\frac{1}{R}\left[(1+d)\mu_D - \mathsf{E}_{t+1}[\pi_{t+2}] + \mathsf{E}_{t+1} a(f_{t+2})\right]\right\}\right] \\
&= \cdots = \frac{(1+d)\mu_D}{R-1} - \sum_{i=1}^{\infty}\frac{1}{R^i}\mathsf{E}_t[\pi_{t+i}].
\end{aligned} \tag{53}$$

Because $|\mathcal{D}|$ is finite, $|\mathsf{E}_t[\pi_{t+i}]|$ is bounded and the expression in (53) is well-defined and finite. The quantities in (45)–(50) and (52) are all functions solely of the information state f, so the conditional expectations in (51) and (53) are taken with respect to the evolution of the information state in (6), for given λ. Equation (53) shows a_t is equal to the present value of all future expected dividend payments minus a discount reflecting the expected present value of all future net profits. We show how to calculate the expectation in (53) in Section EC.4 of the Electronic Companion.

C Information Equilibrium

In this section we discuss the procedure for solving for an endogenous λ given a conjectured variance V_B. Given the demands in (38) and V_B, we have, for $\iota \in \{I, U\}$,

$$\begin{aligned} \mathsf{E}\big[\mathsf{E}[W_{t+1}|\mathcal{I}_t^\iota, f_{t+1}]\big|\mathcal{I}_t^\iota\big] &= q^\iota \times q_N^\iota + RW_t = \frac{(q_N^\iota)^2}{\gamma q_D^\iota} + RW_t, \\ \mathsf{E}\big[\mathsf{var}(W_{t+1}|\mathcal{I}_t^\iota, f_{t+1})\big|\mathcal{I}_t^\iota\big] &= (q^\iota)^2 \times q_D^\iota = \frac{(q_N^\iota)^2}{\gamma^2 q_D^\iota}. \end{aligned} \tag{54}$$

We can therefore write the agent's value function in (9), conditional on $\mathcal{I}_t^\iota$ as

$$J_t^\iota = RW_t + \frac{1}{2\gamma}\frac{(q_N^\iota)^2}{q_D^\iota}, \quad \iota \in \{I, U\}. \tag{55}$$

To find an endogenous λ in the sense of Definition 2.1, we need to evaluate the conditional expectation of $J_t^I - J_t^U$ given the information state f_t. We can pull the denominators q_D^ι out of the conditional expectation because we know from (45) and (48) that they are purely functions of the information state. For the numerator terms, using the demands from (38), the price process from (20) and the condition on a_t in (51), it is straightforward to show that

$$q_N^\iota = \mathsf{E}_t[\pi_{t+1}] + (1+d)\mathsf{E}[m_t|\mathcal{I}_t^\iota] - Rb_t m_t + Rc_t X_t,$$

where $\mathsf{E}_t[\pi_{t+1}]$ — which is a function of λ — is given by (52). The θ_t and D_t terms drop out, as do the terms involving a_t.[33] Note that q_N^ι equals the expected net profit $\mathsf{E}_t[\pi_{t+1}]$, which only conditions on f_t, adjusted for the information set of agent $\iota \in \{I, U\}$.

Since $\mathsf{E}[m_t|\mathcal{I}_t^I] = m_t$ we have

$$\mathsf{E}[(q_N^I)^2|f_t] = (\mathsf{E}_t[\pi_{t+1}])^2 + (1+d-Rb_t)^2\phi f_t \sigma_M^2 + R^2 c_t^2 \sigma_X^2. \tag{56}$$

And from $\mathsf{E}[m_t|\mathcal{I}_t^U] = K_t b_t m_t - K_t c_t X_t$ we have that

$$\begin{aligned} \mathsf{E}[(q_N^U)^2|f_t] &= (\mathsf{E}_t[\pi_{t+1}])^2 + [(1+d)K_t b_t - Rb_t]^2\phi f_t\sigma_M^2 + [Rc_t - (1+d)K_t c_t]^2\sigma_X^2 \\ &= (\mathsf{E}_t[\pi_{t+1}])^2 + [(1+d)K_t - R]^2(b_t^2\phi f_t\sigma_M^2 + c_t^2\sigma_X^2). \end{aligned} \tag{57}$$

[33] In particular, we do not need to evaluate $a(\cdot)$ to find the endogenous $\lambda(\cdot)$, which is useful in solving the model numerically.

Combining these expressions with J_t^ι in (55), we get an expression for the difference in conditional expectations

$$\Delta_f = \mathsf{E}[J_t^I - Rc_I | f_t = f] - \mathsf{E}[J_t^U | f_t = f]. \tag{58}$$

When this difference is positive, the marginal investor has an incentive to become informed. For a given f we numerically solve for the $\lambda \in [0, 1]$ which sets $\Delta_f = 0$. If this difference is always strictly positive we set $\lambda = 1$, and if it is always strictly negative we set $\lambda = 0$.

References

Allen, F., Morris, S. and Shin, H.S., 2006, "Beauty contests and iterated expectations in asset markets," *Review of Financial Studies* 19(3), 719–752.

Admati, A., 1985, "A noisy rational expectations equilibrium for multi-asset securities markets," *Econometrica*, 53 (3), 629–657.

Ahnert, T. and Georg, C.P., 2018, "Information contagion and systemic risk." *Journal of Financial Stability*, 35, 159-171.

Avdis, E., 2016, "Information tradeoffs in dynamic financial markets," *Journal of Financial Economics*, 122, 568–584.

Barro, R.J., 2009, "Rare disasters, asset prices, and welfare costs," *American Economic Review* 99(1), 243–264.

Biais, B., Bossaerts, P. and Spatt, C., 2010, "Equilibrium asset pricing and portfolio choice under asymmetric information," *Review of Financial Studies* 23(4), 1503–1543.

Brancati, E., and M. Macchiavelli, 2019, "The information sensitivity of debt in good and bad times," *Journal of Financial Economics* 133, 99–112.

Brennan, M.J. and Cao, H.H., 1997, "International portfolio investment flows," *Journal of Finance* 52(5), 1851–1880.

Campbell, J., 1991, "A variance decomposition for stock returns," *Economic J.* 101, 157–179.

Cesa-Bianchi, A. and E. Fernandez-Corugedo, 2018, "Uncertainty, financial frictions, and nominal rigidities: A quantitative investigation," *J. Money, Credit and Banking*, 50 (4), 603–636.

Chamley, C., 2007, "Complementarities in information acquisition with short-term trades," *Theoretical Economics* 2(4), 441–467.

Dang, T.V., Gorton, G. and Holmström, B., 2020, "The information view of financial crises," *Annual Review of Financial Economics*, 12, 39–65.

Dutta, S., and A. Nezlobin, 2017, "Information disclosure, firm growth, and the cost of capital," *Journal of Financial Economics* 123, 415–431.

Dow, J., I. Goldstein, and A. Guembel, 2017, "Incentives for information production in markets where prices affect real investment," *J. European Econ. Assoc.* 15(4), 877–909.

Fama, E. and K. French, 2000, "Forecasting profitability and earnings," *Journal of Business*, 73 (2), 161–175.

Farboodi, M. and Veldkamp, L., 2020, "Long run growth of financial technology," *American Economic Review*, 110(8), 2485–2523.

Ganguli, J.V. and Yang, L., 2009, "Complementarities, multiplicity, and supply information," *Journal of the European Economic Association* 7(1), 90–115.

Goldstein, I. and Leitner, Y., 2018, "Stress tests and information disclosure," *Journal of Economic Theory*, 177, 34–69.

Goldstein, I. and Yang, L., 2015, "Information diversity and complementarities in trading and information acquisition," *Journal of Finance*, 70, 1723–1765.

Gorton, G. and Ordoñez, G., 2014, "Collateral crises," *Amer. Econ. Rev.* 104(2), 343–378.

Grossman, S. and J. Stiglitz, 1980, "On the impossibility of informationally efficient markets," *American Economic Review*, 70(3), 393–408.

Hellwig, M., 1980, "On the aggregation of information in competitive markets," *Journal of Economic Theory*, 22, 477–498.

Lo, A. and J. Wang, 2000, "Trading volume: Definitions, data analysis, and implications of portfolio theory," *Review of Financial Studies*, 13(2), 257–300.

Mamaysky, H., 2022, "News and markets in the time of COVID-19," working paper.

Manzano, C. and X. Vives, 2011, "Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity," *Journal of Mathematical Economics* 47(3), 346–369.

Mondria, J., 2010, "Portfolio choice, attention allocation, and price comovement," *Journal of Economic Theory*, 145 (5), 1837–1864.

Peress, J., 2010, "The tradeoff between risk sharing and information production in financial markets," *Journal of Economic Theory* 145(1), 124–155.

Spiegel, M., 1998, "Stock price volatility in a multiple security overlapping generations model," *Review of Financial Studies* 11(2), 419–447.

Van Nieuwerburgh, S. and Veldkamp, L., 2009, "Information immobility and the home bias puzzle," *Journal of Finance*, 64(3), 1187–1215.

Veldkamp, L., 2006, "Media frenzies in markets for financial information," *American Economic Review* 96(3), 577–601.

Vuolteenaho, T., 2002. "What drives firm-level stock returns?," *J. Finance* 57(1), 233–264.

Wang, J., 1993, "A model of intertemporal asset prices under asymmetric information," *Review of Economic Studies*, 60, 249–282.

Wang, J., 1994, "A model of competitive stock trading volume," *J. Pol. Econ.* 102, 127–168.

Watanabe, M., 2008, "Price volatility and investor behavior in an overlapping generations model with information asymmetry," *Journal of Finance* 63(1), 229–272.