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Matching Mechanisms for Refugee Resettlement†

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Current refugee resettlement processes account for neither the preferences of refugees nor the priorities of hosting communities. We introduce a new framework for matching with multidimensional knapsack constraints that captures the (possibly multidimensional) sizes of refugee families and the capacities of communities. We propose four refugee resettlement mechanisms and two solution concepts that can be used in refugee resettlement matching under various institutional and informational constraints. Our theoretical results and simulations using refugee resettlement data suggest that preference-based matching mechanisms can improve match efficiency, respect priorities of communities, and incentivize refugees to report where they would prefer to settle. (*JEL* C78, D82, J15, J18)

Each year, around 100,000 refugees are resettled to hosting countries, such as the United States, that offer them a path to citizenship. However, few countries have developed systematic resettlement procedures, despite ample evidence that the communities (or *localities*) where refugees are initially resettled matter a great deal for refugees' education, job prospects, and earnings (Åslund and Rooth 2007; Åslund and Fredriksson 2009; Åslund, Östh, and Zenou 2010, 2011; Damm 2014; Bansak et al. 2018; Martén, Hainmueller, and Hangartner 2019).

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Incorporating refugees' preferences over localities matters because refugees have private information about their own skills, abilities, and needs, which can affect the refugee-locality match quality—and which cannot always be directly observed by government authorities. Meanwhile, respecting priorities of localities over refugees can improve integration-relevant outcomes of refugees, ensure the best use of local resources, and build community support for localities to continue participating in resettlement. In this paper, we consider how refugees' preferences and localities' priorities can be incorporated into refugee resettlement processes. We introduce and analyze several approaches that balance between objectives of refugee welfare, incentives, and respect for localities' priorities.

Our analysis draws in part upon classic matching models from contexts such as the matching of students to schools (Abdulkadiroğlu and Sönmez 2003). In school choice, each student takes up exactly one school place. In refugee resettlement, by contrast, families must be kept together, but families have different *sizes*: larger families (e.g., a couple with four children) take up more places than smaller ones (e.g., a single individual). Because localities in practice have inflexible total quotas on the number of refugees they are able to admit, family sizes render most standard matching mechanisms inadequate for refugee resettlement. Additionally, localities often have further requirements that constrain the allocation, such as limits on the respective numbers of single-parent families, refugees speaking a given language, or school-age children they can accommodate. To incorporate these institutional requirements, we model (possibly multidimensional) *knapsack constraints* (Dantzig 1957) that limit the central authority's ability to allocate refugees to localities.

While our work is motivated by refugee resettlement, our model generalizes a number of existing models, such as those used in school choice, college admissions, and resident-hospital matching (see online Appendix G for details). Therefore, our framework has applications beyond refugee resettlement—specifically, to any setting where matching occurs subject to knapsack constraints, such as day care assignment, allocation of teachers to schools, and college admissions with diversity constraints.

Our Theoretical Contribution.—We consider a general model of matching with multidimensional knapsack constraints, in which each family has a (possibly multidimensional) size and each locality has a (possibly multidimensional) capacity. A group of families can only be matched to a given locality if, for every dimension, the total size of the families does not exceed the locality's capacity. All our results and solution concepts are novel even when there is only one dimension. (Indeed, with the exception of Theorems 2 and 4, our results in the main text are not even affected by the number of dimensions.)

We start with the case in which the resettlement agency focuses only on the preferences of refugee families. We show that the Knapsack Top Trading Cycles (KTTC) algorithm—a slight modification of the classical Top Trading Cycles (TTC) algorithm of Shapley and Scarf (1974)—allows us to incorporate knapsack constraints and obtain a Pareto-efficient mechanism in which refugee families have no incentive to misreport their preferences (Proposition 1). In practice, however, resettlement agencies already have existing allocation processes, so we consider how to incorporate preference information into a setting with a baseline allocation, i.e., an *endowment*.

A matching is then *individually rational* if every family is matched to a locality it weakly prefers to its endowment. In this case, because families have different sizes, a Pareto-efficient and individually rational matching cannot be achieved by only using the trading cycles that arise in the KTTC algorithm: it might be necessary to swap sets of families in order to guarantee feasible Pareto improvements. We therefore relax Pareto efficiency by considering *Pareto-improving chains* in which swaps occur at the level of families. We define a matching to be *chain-efficient* if it cannot be improved by carrying out any Pareto-improving chain. We show that there does not exist a chain-efficient and strategy-proof mechanism (Theorem 1). When there is more than one dimension, there does not even exist a strategy-proof mechanism that Pareto improves upon all endowments that are not chain-efficient (Theorem 2). In order to Pareto improve upon an endowment whenever possible, introduce an algorithm, called Knapsack Top Trading Cycles with Endowment (KTTCE), that generalizes the KTTC algorithm. The KTTCE mechanism is strategy-proof and can potentially Pareto improve upon the endowment by carrying out Pareto-improving chains (Theorem 3). If there is only one dimension and larger families have higher priority, then the KTTCE algorithm is additionally guaranteed to improve upon every endowment that is not chain-efficient (Theorem 4).

When priorities of localities also need to be taken into account, new trade-offs arise. In particular, (pairwise) stable matchings may not exist, and determining whether a stable matching exists (or finding a stable matching when one exists) is a computationally intractable problem (McDermid and Manlove 2010; Biró and McDermid 2014). In our model, stable outcomes only exist under fairly strong conditions (e.g., if families are prioritized by size; see Proposition C.4 in online Appendix C). To address the nonexistence and computational shortcomings of stable matchings, we introduce an alternative solution concept called *interference-freeness*, which is related to envy-freeness (Sotomayor 1996; Balinski and Sönmez 1999; Wu and Roth 2018; Kamada and Kojima forthcoming). Envy-freeness eliminates justified envy; that is, it rules out matchings in which a family prefers to its own match a locality that hosts a lower-priority family. Interference-freeness allows a family to be envied as long as none of the capacity it uses can be claimed by higher-priority families. We show that a family-optimal interference-free matching exists and can be found via a modification of the classical Deferred Acceptance (DA) algorithm (Gale and Shapley 1962), which we call the Knapsack Deferred Acceptance (KDA) algorithm (Theorem 5). However, unlike in contexts such as school choice, our modification of DA is manipulable because localities' choice functions (induced by the priorities and constraints) do not satisfy the cardinal monotonicity condition (Alkan 2002; Hatfield and Milgrom 2005). In fact, there is no family-optimal interference-free and strategy-proof mechanism (Proposition 3), implying a trade-off between incentives and efficiency when the designer requires interference-freeness. We therefore develop a strategy-proof and interference-free mechanism, called Threshold Knapsack Deferred Acceptance (TKDA) (Theorem 6).

We use refugee resettlement data from HIAS, a resettlement agency operating in the United States, to illustrate our mechanisms. In our simulations, we assume that localities rank refugees according to their likelihood of employment estimated by Ahani et al. (2021). As refugee preferences are currently not collected, we simulate different preference distributions. We find that (particularly when refugees' preferences are uncorrelated) (i) the KTTCE mechanism makes many families better-off compared to the endowment and (ii) the KDA algorithm is substantially more efficient than the TKDA algorithm.

Impact on Resettlement Practices.—In May 2018, HIAS started to systematically match refugees according to their likelihood of gaining employment while taking various integration constraints into account (Ahani et al. 2021). HIAS's matching software, Annie™ MOORE, is based on a model of matching with multidimensional knapsack constraints that we introduced in our original working paper (Delacrétaz, Kominers, and Teytelboym 2016) but accounts for neither the refugees' preferences over localities nor the priorities of the localities themselves.

Agencies such as HIAS see value in incorporating refugees' preferences into the matching process because refugees are likely to hold private information about their preferences that can affect the quality of matches. As Mark Hetfield, the President and CEO of HIAS, has explained,

> Many Somali refugees initially settled around the country subsequently migrated to Lewiston, Maine. Lewiston has a weak economy but an established Somali community. Consequently, efforts to resettle these refugees elsewhere in the US were less effective than they could have been. Their preferences should have been taken into account from the start. (Roth 2015)

Our KTTCE mechanism constitutes a natural first step in this direction, as it allows us to incorporate preferences into current practices immediately. For example, if HIAS were to collect preferences, it could use the KTTCE mechanism in conjunction with the Annie™ MOORE system it currently uses: Annie™ MOORE would calculate an endowment, and the KTTCE mechanism would identify and carry out mutually beneficial trades.

In the long run, an agency may wish to involve localities more closely in the matching process. Our mechanisms based on interference-freeness thus provide solutions that account for the priorities of localities alongside the preferences of refugees. Priority orders could come from a combination of employment probabilities (as calculated by Annie™ MOORE), administrative policies, and local residents' preferences; accounting for these explicitly may help create goodwill from localities—who may in turn increase their capacity—or at least take advantage of the localities' information about what constitutes a good match.

Relationship to Prior Work.—Matching market design for refugee resettlement was first proposed by Moraga and Rapoport (2014) as a part of a system of international refugee quota trading (Schuck 1997). In the international context of matching refugees to countries, however, the refugee matching market is "thick"—in the sense that any country can be expected to host any family up to its capacity—and can be reasonably modeled as a standard school choice problem (Abdulkadiroğlu and Sönmez 2003; Jones and Teytelboym 2017; Sayedahmed 2022). Jones and Teytelboym (2018) informally introduced the idea of refugee resettlement matching in the national context and pointed out the knapsack constraints and the "thinness" of matching markets that arise on the local level.

Andersson and Ehlers (2020) examined a market for allocating private housing to refugees in which landlords have preferences over the sizes of refugee families and over the native languages refugees speak. Our work is the first to offer a formal theory of preference-based matching for refugee resettlement.

Our paper draws upon and contributes to the applied literature on the design and implementation of matching with complex constraints. In our setting, similar to models with "couples" or minimum quotas, stable matchings do not typically exist. Kamada and Kojima (forthcoming) considered many-to-one matching markets under general constraints. While their model is more general, their results are independent of ours, and they focus on the structure of constraints that allow for the existence of feasible, individually rational, and envy-free matchings.

Finally, Nguyen, Nguyen, and Teytelboym (2021) considered a version of our model in which localities have cardinal preferences that arise from an integer optimization problem. However, Nguyen, Nguyen, and Teytelboym (2021) focused on group-stable and near-feasible matchings and do not consider strategic issues that are key for preference-based refugee resettlement.

In some matching market design settings, such as school choice in New Orleans or housing allocation, stability is considered secondary to efficiency. In such cases, the Top Trading Cycles algorithm (Balinski and Sönmez 1999; Abdulkadiroğlu and Sönmez 2003) or its modifications (Pápai 2000; Dur and Ünver 2019) can be used instead of stable mechanisms. Pápai (2003, 2007) analyzed the difficulties of exchange with endowments and multiple goods. In our setting, families are only endowed with at most one locality; however, efficient and strategy-proof mechanisms are similarly hard to find. Abdulkadiroğlu, Pathak, and Roth (2009) showed that no strategy-proof mechanism can improve upon the outcome of the deferred acceptance mechanism. In contrast, our endowment is exogenous, and our negative results rely solely on knapsack constraints.

Organization of the Paper.—In Section I, we describe the institutional context of refugee resettlement in the United States. We state our formal model in Section II. In Section III, we explain how two variations on the Top Trading Cycles algorithm can fully incorporate the preferences of refugees. In Section IV, we propose solutions for the case in which refugee preferences need to be balanced with the priorities of the localities. In Section V, we present simulations based on refugee resettlement data from HIAS. We conclude in Section VI. All proofs, as well as further results and supplemental examples, are in the online Appendix.

I. Institutional Context

Refugees can apply for the US resettlement program directly or be referred by the United Nations High Commissioner for Refugees (UNHCR), often while living in a refugee camp. The refugee resettlement program is managed by the US Refugee Admissions Program (USRAP), which, alongside the UNHCR and the International Organization for Migration, identifies refugees, conducts security and medical checks on all family members, and arranges travel; this process typically takes 18 to 24 months. Then the case is handed over to one of the nine US resettlement agencies, which are responsible for matching the refugee family to a locality. Refugees can declare family members who live in the United States, in which case they are almost certain to join them. But beyond that, resettlement agencies do not collect information about refugees' preferences over initial placements, and instead must make informed guesses about where refugees would fare well.

Resettlement agencies establish their own links to localities that are willing to host refugees. Each fiscal year, agencies review the hosting capacities of their localities. Localities express a variety of hosting constraints to their agencies, the key constraint being the total number of refugees they are able or willing to host. The Department of State approves the quotas for each locality, which can be as high as 200 or as low as 20 refugees per year. Localities cannot exceed the annual quotas. There can also be additional constraints, for example, on the number of refugees from certain countries or regions, the number of children or elderly family members, or the number of refugees with a medical condition. Other (binary) constraints might include whether the community can support single-parent families or disabled refugees. Agencies assign refugees to localities roughly every fortnight. In order to balance the resettlement load, the fortnightly quota for each locality is set proportionally to that locality's annual quota and is treated as a hard constraint (Ahani et al. 2021). Localities do not currently fix priorities over specific types of refugees beyond the constraints they express to agencies. The federal government provides very limited support, with a fixed grant of \$1,125 per refugee to cover the first 90 days of resettlement.

II. Model

There is a finite set of refugee *families* $f \in F$ and a finite set of *localities* $\ell \in L$. We assume that there exists a *null locality* $\emptyset \in L$ that represents being unmatched. There is a finite set D of *dimensions*. The *size* of a family f is the vector $\left(\nu_d^f\right)_{d \in D} \in \mathbb{Z}_{\geq 0}^{|D|} \backslash \{\mathbf{0}\}$.

The vector $\left(\kappa_d^\ell\right)_{d \in D} \in \left(\mathbb{Z}_{\geq 0} \cup \{+\infty\}\right)^{|D|}$ denotes the *capacity* of locality ℓ. We let $\kappa_d^\emptyset = +\infty$ for all dimensions d; i.e., the null locality has infinite capacity. Locality ℓ can *accommodate* a set of families $G \subseteq F$ if $\sum_{g \in G} \nu_d^g \leq \kappa_d^\ell$ for all $d \in D$. Locality ℓ can *accommodate* family f *alongside* $G \subseteq F \backslash \{f\}$ if ℓ can accommodate $G \cup \{f\}$. We assume that every family can be accommodated on its own by at least one locality other than the null.

A (feasible many-to-one) *matching* is a correspondence $\mu : F \cup L \rightrightarrows F \cup L$, such that

(i) each family $f \in F$ is matched to exactly one locality, i.e., $\mu(f) \in L$;

(ii) each locality $\ell \in L$ is matched to a set of families, i.e., $\mu(\ell) \subseteq F$;

(iii) a family $f \in F$ is matched to a locality $\ell \in L$ if and only if the locality is matched to the family, i.e., $\mu(f) = \ell$ if and only if $f \in \mu(\ell)$; and

(iv) each locality $\ell \in L$ can accommodate the families matched to it, i.e., $\sum_{g \in \mu(\ell)} \nu_d^g \leq \kappa_d^\ell$ for every $d \in D$.

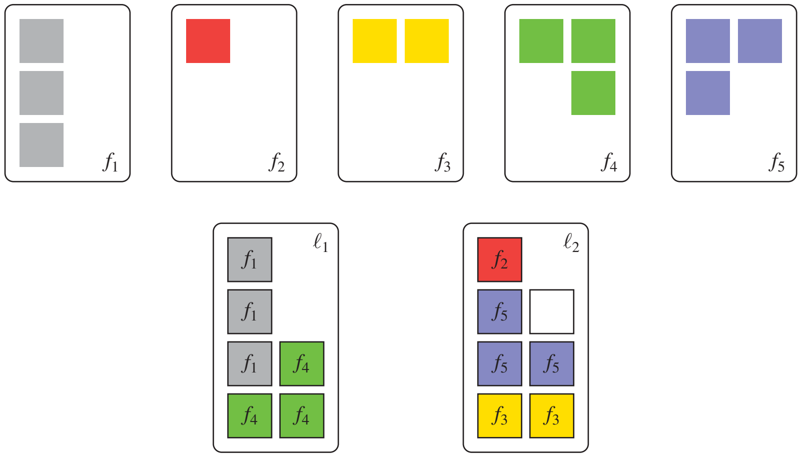

FIGURE 1. MATCHING IN A MARKET WITH TWO-DIMENSIONAL CONSTRAINTS

Notes: There are five families $f_1, \ldots, f_5$, two localities ℓ_1, ℓ_2, and two dimensions, represented by the left and right columns. The sizes of the families are $(3,0)$, $(1,0)$, $(1,1)$, $(1,2)$, and $(2,1)$. The capacities of the localities are $(4,2)$ and $(4,3)$. In the matching pictured, families f_1 and f_4 are matched to locality ℓ_1, and families f_2, f_3, and f_5 are matched to locality ℓ_2.

The first three conditions are standard. Condition (iv) ensures that no locality's capacity is exceeded on any dimension—these are multidimensional knapsack constraints for the matching, and we illustrate them in Figure 1. Our model generalizes a number of existing matching models with complex constraints (see online Appendix G).

Families have *preferences* over localities. We denote by $\succ_f$ the strict ordinal preference list of family f over L and let $\succ = (\succ_f)_{f \in F}$ be the preference profile of families. We write $\ell \succ_f \ell'$ to mean that f strictly prefers ℓ to ℓ' and $\ell \succeq_f \ell'$ to denote that either $\ell \succ_f \ell'$ or $\ell = \ell'$. We assume that each family's least preferred option is being unmatched.

Localities have exogenously fixed *priorities* over families. We let $\rhd_\ell$ be the strict ordinal priority list of locality ℓ over families and let $\rhd = (\rhd_\ell)_{\ell \in L}$ be the ordinal priority profile of the localities. We write $f \rhd_\ell f'$ to mean that f has a higher priority than f' at ℓ. We assume that localities prioritize families they can accommodate on their own: if ℓ can accommodate $\{f\}$ but not $\{f'\}$, then $f \rhd_\ell f'$.

A matching μ is *wasteful* if there exists a family $f \in F$ and a locality $\ell \in L$ such that $\ell \succ_f \mu(f)$ and ℓ can accommodate f alongside $\mu(\ell)$. We say that matching μ *Pareto dominates* matching μ', denoted $\mu \succ \mu'$, if $\mu(f) \succeq_f \mu'(f)$ for all $f \in F$ and $\mu(f) \succ_f \mu'(f)$ for some $f \in F$. We write $\mu \succeq \mu'$ if μ *weakly Pareto dominates* μ', that is, if either $\mu \succ \mu'$ or $\mu = \mu'$. A matching μ is *Pareto-efficient* if there does not exist another matching μ' that Pareto dominates μ.

Fixing a set of families and their sizes, a set of localities and their capacities, and a profile of priorities, we define a (direct) *mechanism* as a function φ that takes as input a preference profile and outputs a matching.[1] A mechanism φ is *strategy-proof*

[1] If a mechanism always selects a matching with a certain property (e.g., nonwastefulness), we refer to the mechanism as having that property (e.g., a *nonwasteful mechanism*).

if for each family $f \in F$, there does not exist a report of a preference list $\succ'_f$ such that

$$\varphi(\succ'_f, \succ_{-f})(f) \succ_f \varphi(\succ)(f),$$

where $\varphi(\succ)(f)$ is the locality to which f is matched under the mechanism φ and the preference profile $\succ$. Strategy-proofness requires that families cannot make themselves better-off by misreporting their preferences, irrespective of the reports of other families.

Throughout the paper, we describe all our mechanisms via algorithms: a mechanism takes a preference profile as an input and uses instructions from its corresponding algorithm to produce a matching. We say that a family is *permanently matched* (respectively, *permanently rejected*) to a locality at some step of the algorithm for mechanism φ if by that step it has been established that the pair will (respectively, will not) be matched in the output of φ.

Knapsack Constraints in Practice.—From a practical perspective, it is important to distinguish constraints from preferences. Refugees have preferences over localities, which we allow to be arbitrary. Constraints are independent of preferences and imposed by the localities, in agreement with resettlement agencies and the US Department of State.

In practice, there is a constraint on the total number of refugees a locality can host. If we call this dimension d and consider a family f and a locality ℓ, then ν_d^f is the number of family members in f, and κ_d^ℓ is the total number of refugees ℓ can host.

Resettlement agencies face additional constraints that are introduced by the localities. In the case of HIAS, all of these constraints are currently binary (that is, for each $d' \neq d$, $\kappa_{d'}^\ell = 0$ or $\kappa_{d'}^\ell = \infty$). For example, some localities are equipped to host single-parent families or refugees with particular medical conditions, while others are not. As these additional constraints are binary, we do not require additional dimensions to capture them, so the special case of our model in which $|D| = 1$ is sufficient for introducing preferences into current practice.[2]

However, there are reasons to think that our general model may be useful to apply preference-based matching more widely. First, it is conceivable that additional institutional constraints may be added in the future. In fact, until 2016, there was a constraint on the number of refugees from some regions, and modeling such a situation would require adding a dimension for each of the relevant regions. Second, some of the constraints that HIAS imposes are not actually binary but are treated as such in practice because managing multidimensional constraints manually is difficult. Our model would allow HIAS to handle multidimensional constraints without having to simplify them; for example, a locality with one part-time social worker could perhaps take up to two families needing social care but no more. Third, our model

[2]Instead of adding extra dimensions to account for binary constraints, we can straightforwardly extend the model so that some family-locality pairs can simply be considered to be infeasible. Families would only have preferences over feasible localities, and localities would only rank feasible families. The results in the paper would go through with this minor modification.

allows localities to express logistical constraints. For example, a constraint on the total number of children could reflect the capacity of schools, and a constraint on the number of seniors could reflect the capacity of care homes.

As we mentioned in Section I, while quotas on total number of refugees are set annually, refugees arrive over the course of the year, so that matching takes place roughly every fortnight. The current practice for HIAS is to consider each static matching problem independently: each fortnight, there is a set of families to be matched to a set of localities, and the total number of refugees that a locality can host is set proportionally to its remaining annual quota and treated as a hard constraint (Ahani et al. 2021).[3] Adjusting capacity across matching periods over the course of the year or changing the frequency of the matchings (e.g., to monthly or quarterly) could in principle improve both the employment-based optimization (Ahani et al. 2021, Table 10) and preference-based matching. We focus on the static matching problem and leave dynamic capacity management for future work.

III. Targeting Efficiency

In this section, we propose matching mechanisms that incorporate refugee preferences and give refugees an incentive to report their preferences over localities truthfully.

A. *The Knapsack Top Trading Cycles Mechanism*

Our first mechanism, the *Knapsack Top Trading Cycles* mechanism described in Algorithm 1, is an extension of the Top Trading Cycles mechanism (Shapley and Scarf 1974) to an environment with (possibly multidimensional) knapsack constraints. The only meaningful difference is that, because families have different sizes, a locality may appear "full" to some families and not to others during the course of the algorithm. This modification does not affect the key properties of TTC.

PROPOSITION 1: *The KTTC mechanism is strategy-proof and Pareto-efficient.*

B. *Improving Efficiency from an Endowment*

Resettlement agencies already have existing processes for assigning refugees to localities (Bansak et al. 2018). For example, HIAS matches refugees according to observable characteristics to maximize objectives such as the likelihood of employment (Ahani et al. 2021). One way to incorporate the current objectives of the agencies would be to use them in determining priorities of localities in the KTTC algorithm—our simulations in Section V illustrate this approach. However, agencies might be concerned that some of the refugees could be made worse off

[3] Since the decisions about the timing of arrivals are made by the US government, there is little or no scope for refugees to manipulate the timing (as in Caspari 2019). Moreover, since the annual quotas are fixed at the beginning of the fiscal year and then managed by the resettlement agencies, there is no short-run scope for quota manipulation by localities (as there is, for example, in the setting of Sönmez 1997).

ALGORITHM 1: KNAPSACK TOP TRADING CYCLES (KTTC)

Initialize the *current matching* μ^1 such that $\mu^1(f) = \emptyset$ for all $f \in F$. No families are *permanently matched*.

Round $i \geq 1$

Each locality ℓ *permanently rejects* all families that ℓ cannot accommodate alongside $\mu^i(\ell)$.

Each family f that is not permanently matched points at its most preferred locality among those that have not permanently rejected f. Every family that points at the null is permanently matched to it. If all families are permanently matched, end and output μ^i. Otherwise, continue.

Each nonnull locality ℓ points at its highest-priority family that has not been permanently matched.

At least one cycle appears, and each family and each locality is involved in at most one cycle. Update the current matching to μ^{i+1} by permanently matching each family in a cycle to the locality at which it is pointing.

If all families are permanently matched, end and output μ^{i+1}. Otherwise, continue to Round $i + 1$.

in such an implementation of KTTC than in the current systems based on observables. To make preference-based matching as easy as possible to integrate with existing systems, we extend our TTC-based approach to allow resettlement agencies to use their initial allocations—which we refer to as an *endowment*—as a baseline and allow refugees to express preferences in order to improve upon that baseline.

Consider an exogenous endowment matching μ^E. We say that a matching μ is *individually rational* if $\mu \succeq \mu^E$, that is, if μ weakly Pareto dominates the endowment. We also refer to locality $\mu^E(f)$ as *family f's endowment* and to families $\mu^E(\ell)$ as *locality ℓ's endowment*. In the school choice setting, i.e., when $|D| = 1$ and $\nu_d^f = 1$ for all f (denoting $D = \{d\}$), the TTC mechanism finds an individually rational and Pareto-efficient matching by carrying out one cycle at a time.[4] In contrast, in a setting with knapsack constraints, carrying out one cycle at a time may not achieve Pareto efficiency because some Pareto improvements may require families to swap in groups. For example, two "small" families in one locality might be able to swap simultaneously with a "large" family in another locality, but none of the pairwise swaps between a "small" family and the "large" family would be feasible. We therefore limit the set of Pareto improvements that can be executed.

DEFINITION 1: *Given a matching μ, a* Pareto-improving chain *is a sequence*

$$(f_1, \ell_1, f_2, \ell_2, \ldots, f_n, \ell_n)$$

[4] The KTTC mechanism collapses to the TTC mechanism in the school choice setting.

of distinct families and localities such that

- $\ell_1 \succ_{f_1} \mu(f_1)$;
- *for all* $i = 2, \ldots, n$,
  - ℓ_{i-1} *can accommodate* f_{i-1} *alongside* $\mu(\ell_{i-1}) \setminus \{f_i\}$,
  - $\ell_i \succ_{f_i} \ell_{i-1} = \mu(f_i)$; *and*
- ℓ_n *can accommodate* f_n *alongside* $\mu(\ell_n) \setminus \{f_1\}$.

In any Pareto-improving chain, family f_1 moves to locality ℓ_1, which f_1 prefers to its current locality. Locality ℓ_1, in turn, must be able to accommodate f_1 alongside all families in $\mu(\ell_1)$ except for f_2, which leaves locality ℓ_1 for a more preferred locality ℓ_2. The Pareto-improving chain continues with f_3 moving from ℓ_2 to ℓ_3, and so on. The Pareto-improving chain terminates in one of two ways: either (i) the Pareto-improving chain is *open* and no family leaves the last locality, i.e., $\ell_n \neq \mu(f_1)$; or (ii) the Pareto-improving chain is *closed* and f_1 leaves the last locality, i.e., $\ell_n = \mu(f_1)$.

DEFINITION 2: *A matching is* chain-efficient *if it has no Pareto-improving chain.*

Chain efficiency constitutes a relaxation of Pareto efficiency because it only requires the elimination of Pareto-improving chains, which form a subset of all possible Pareto improvements. In school choice, Pareto efficiency is equivalent to chain efficiency, and the Pareto-efficient TTC mechanism is strategy-proof. In our setting with knapsack constraints, however, there might be a matching that Pareto dominates a chain-efficient matching if there are groups of families that could participate in a Pareto-improving swap that is not a Pareto-improving chain. Moreover, even chain-efficient mechanisms are not strategy-proof.

THEOREM 1: *There is no strategy-proof, individually rational, and chain-efficient mechanism.*

The Proof of Theorem 1 is by counterexample; it requires only one dimension and family sizes of at most 2. The intuition is that different Pareto-improving chains can interfere with each other; hence, families can sometimes select the Pareto-improving chain they prefer by manipulating their preferences. As Pareto efficiency implies chain efficiency, Theorem 1 immediately implies that there is no strategy-proof, individually rational, and Pareto-efficient mechanism.

Theorem 1 implies a trade-off between efficiency and strategy-proofness when the designer wants to Pareto improve upon an endowment. This trade-off does not exist in school choice and is a direct consequence of the fact that families have different sizes. But can a strategy-proof mechanism guarantee even a single Pareto improvement upon an endowment? To formalize this idea, we say that a mechanism φ *Pareto improves upon an endowment* μ^E, if $\varphi(\succ) \succ \mu^E$, that is, if the mechanism returns a matching that Pareto dominates the endowment.

THEOREM 2: *If* $|D| > 1$, *there is no strategy-proof mechanism that Pareto improves upon every endowment that is not chain-efficient.*

Theorem 2 considers *any* kind of Pareto improvements, whether or not they are Pareto-improving chains. Therefore, the result directly implies that when $|D| > 1$, it may not be possible to find any Pareto-improving chains—even if they exist—without giving families an incentive to misrepresent their preferences. This means that strategy-proofness may preclude *all* trade in matching markets with multidimensional knapsack constraints.

The intuition for the impossibility result in Theorem 2 is that when $|D| > 1$, families cannot necessarily be compared by sizes: one family can be larger than another family on one dimension but smaller on another dimension. Therefore, different Pareto improvements can be executed with different families, depending on the dimension in which capacity needs to be freed. There can then be situations where, for each Pareto improvement, there is a family that can misreport its preferences in order to block that Pareto improvement and ensure that instead a Pareto improvement is carried out that matches the family with a more preferred locality. We next present a mechanism that can overcome the negative result of Theorem 2 when families can be compared by sizes (which is always the case when $|D| = 1$).

C. *KTTC with Endowment*

We now present an extension of the KTTC mechanism that attempts to Pareto improve upon an endowment. As in the KTTC algorithm, the *KTTC with Endowment* algorithm (Algorithm 2) looks for trading cycles; however, the KTTCE algorithm checks whether the cycles that appear are feasible.[5] A cycle is feasible if each locality ℓ can replace family f' at which ℓ is pointing by family f that is, in turn, pointing at ℓ; i.e., ℓ can accommodate f alongside $\mu^i(\ell)\setminus\{f'\}$. In general, endowments can cause trading cycles to be infeasible.[6] If some trading cycles are feasible, then we match families to the localities they are pointing at in the cycles, just as in the KTTC algorithm. The key step—the Rejection Stage—deals with the case when none of the cycles are feasible. In the Rejection Stage, we pick a family f (at random or according to some exogenous rule) among those at which at least one locality is pointing. Family f is permanently rejected by each locality ℓ where f cannot be accommodated alongside families that are currently matched to ℓ except for the family at which ℓ is pointing. Since f is not necessarily rejected by all localities, the Rejection Stage leaves an opportunity for f to be involved in a feasible cycle in a subsequent round of the algorithm. It is worth noting that any feasible cycle found by the KTTCE algorithm corresponds to at least one Pareto-improving chain.[7] Therefore, the KTTCE algorithm attempts to

[5] This is not particularly challenging from a computational point of view because it simply requires verifying whether each locality in the cycle can replace the family pointing at that locality by the family at which the locality points. Thus, the KTTCE algorithm works in polynomial time.

[6] If $|D| = 1$ and $v_d^f = 1$ for all $f \in F$ and $d \in D$, then all trading cycles are feasible. Moreover, without endowments, all trading cycles are also feasible. The infeasibility typically occurs when a "large" family f points at a locality ℓ, which itself points at a "small" family f'. In the KTTCE algorithm, family f is allowed to point at locality ℓ because ℓ may be able to accommodate f in a subsequent round if some other "large" families leave; however, f might not be able to be matched to ℓ in the current round.

[7] In particular, a feasible cycle can be broken into multiple open Pareto-improving chains if some localities point at families not in their endowment.

ALGORITHM 2: KTTC WITH ENDOWMENT (KTTCE)

Initialize the *current matching* μ^1 such that $\mu^1(f) = \mu^E(f)$ for all $f \in F$. No families are *permanently matched.*

Round $i \geq 1$

Each locality ℓ *permanently rejects* all families that ℓ cannot accommodate alongside families that are permanently matched to ℓ.

Each family f that is not permanently matched points at its most preferred locality among those that have not permanently rejected f. Every family that points at the null is permanently matched to it. If all families are permanently matched, end and output μ^i. Otherwise, continue.

Each nonnull locality ℓ points at its highest-priority family that has not been permanently matched.

At least one cycle appears, and every family and every locality is involved in at most one cycle. Label the families and localities in any such cycle $f_1 \rightarrow \ell_1 \rightarrow f_2 \rightarrow \ell_2, \ldots, f_n \rightarrow \ell_n \rightarrow f_1$.

A cycle is *feasible* if, for all $j = 1, \ldots, n$, ℓ_j can accommodate f_j alongside $\mu^i(\ell_j) \setminus \{f_{j+1}\}$ (letting $f_{n+1} = f_1$).

If one or more cycles are feasible, continue to the *Matching Stage*. Otherwise, continue to the *Rejection Stage*.

Matching Stage: Update the current matching to μ^{i+1} by matching every family in a feasible cycle to the locality at which it is pointing; all these families become permanently matched.

If all families are permanently matched, end and output μ^{i+1}. *O*therwise, continue to Round $i + 1$.

Rejection Stage: Pick one family f (at random or according to some exogenous rule) at which at least one locality is pointing. Permanently reject f from all localities to which f cannot be matched; i.e., ℓ permanently rejects f if ℓ cannot accommodate f alongside $\mu^i(\ell) \setminus \{f'\}$ (where f' is the family at which ℓ is pointing).

If f is permanently rejected by the locality at which f is pointing, let $\mu^{i+1} = \mu^i$ and continue to Round $i + 1$. Otherwise, pick another family that has not been picked yet and repeat the Rejection Stage.

improve upon the endowment by carrying out successive Pareto-improving chains (each of which may be open or closed).

THEOREM 3: *The KTTCE mechanism is strategy-proof and individually rational.*

The KTTCE mechanism preserves strategy-proofness because (i) in the Matching Stage, families point at their most preferred localities (as in KTTC) and (ii) in the Rejection Stage, the permanent rejections from localities do not depend on reported preferences.

Effectively, the KTTCE algorithm adds a Rejection Stage to each round of the KTTC algorithm in order to deal with infeasible cycles created by the endowment. The following proposition formalizes this point by showing that when all families are endowed with the null locality, the outcomes of the KTTC and KTTCE algorithms coincide.

PROPOSITION 2: *If* $\mu^E(f) = \emptyset$ *for all* $f \in F$, *then* $\mu^{KTTC} = \mu^{KTTCE}$.

We illustrate the KTTCE algorithm with an example in online Appendix E.1.

As Theorem 2 shows, whenever $|D| > 1$, no strategy-proof mechanism (e.g., KTTCE) can be guaranteed to find any Pareto improvement upon an endowment that is not chain-efficient. Therefore, without further restrictions, the KTTCE mechanism might simply output the endowment even if Pareto-improving chains exist.

D. *Guaranteeing Pareto Improvements in the KTTCE Mechanism*

We now introduce a condition on family sizes that guarantees that there exist priority profiles for which the KTTCE mechanism improves upon every endowment that is not chain-efficient.

DEFINITION 3: *Sizes are* monotonic *if for any two families* f *and* f' *and any two dimensions* d *and* d', *having* $\nu_d^f > \nu_d^{f'}$ *implies that* $\nu_{d'}^f \geq \nu_{d'}^{f'}$.

Having monotonic sizes rules out that family f is larger than f' in one dimension but smaller than f' in another dimension. When $|D| = 1$, sizes are always monotonic; however, when $|D| > 1$, the monotonicity condition states that if f is larger than f' in one dimension, then f is weakly larger in all other dimensions. We next provide a class of priority profiles that guarantees that the KTTCE mechanism improves upon any endowment that is not chain-efficient.

DEFINITION 4: *Under monotonic sizes, a priority profile is* lexicographic *if, for every* $f, g \in F$ *and every* $\ell \in L \backslash \{\emptyset\}$,

- *families in* ℓ*'s endowment have higher priority, i.e.,* $f \in \mu^E(\ell)$ *and* $g \notin \mu^E(\ell)$ *implies* $f \rhd_\ell g$; *and*
- *within* ℓ*'s endowment, larger families have higher priority, i.e.,* $f, g \in \mu^E(\ell)$ *and* $\nu_d^f > \nu_d^g$ *for some* $d \in D$ *imply* $f \rhd_\ell g$.

Lexicographic priorities imply that each locality prioritizes all families that are in its endowment over those that are not, and that, among the families in its endowment, each locality prioritizes families in decreasing order of sizes. There are no restrictions about how any locality ranks any two families with the same size or any two families not in its endowment.

If sizes are monotonic and priorities are lexicographic, at the start of the KTTCE algorithm, each locality ℓ points at the largest family in its endowment. Suppose that ℓ permanently rejects a family f that is not in its endowment. This means that f cannot be accommodated at ℓ even when the *largest* family in ℓ's endowment has been removed. Therefore, the endowment does not admit a Pareto-improving chain in which f moves to ℓ. If the outcome of the KTTCE algorithm is the endowment, then f has been permanently rejected by every preferred locality. This means that the endowment has no Pareto-improving chains involving family f. By extending the preceding argument to every family, we can see that the KTTCE algorithm only returns the endowment when the endowment is chain-efficient.

THEOREM 4: *If sizes are monotonic and priorities are lexicographic, then the KTTCE mechanism Pareto improves upon every endowment that is not chain-efficient.*

Since neither individual rationality nor strategyproofness depends on the localities' priorities, Theorem 4 tells us that as long as family sizes are monotonic, we can adjust the pointing order of localities to follow some lexicographic priorities so that the KTTCE mechanism Pareto improves upon every endowment that is not chain-efficient.[8] Theorem 4 contrasts with the impossibility result of Theorem 2 by highlighting the possibility of strategy-proof Pareto improvement upon endowments that are not chain-efficient in the cases when there is only one dimension (because in that case sizes are monotonic) or when family sizes have sufficient structure.[9]

Moreover, Theorem 4 is relevant for current practice: HIAS currently uses Annie™ MOORE with only one dimension—the only constraint is on the total number of refugees—in its resettlement processes (Ahani et al. 2021). In Section V, we simulate realistic resettlement settings with one and three dimensions and show that the KTTCE mechanism finds Pareto improvements even when Theorem 4 does not apply.

IV. Accounting for Priorities

Beyond accounting for refugees' preferences, there are good reasons for taking the priorities of localities seriously as well. However, the mechanisms introduced in Section III are not guaranteed to satisfy the priorities of localities. In this section, we offer mechanisms that respect the priorities of localities in addition to the preferences of refugee families. We introduce two pieces of notation that will be useful throughout the section. First, given any family f and any locality ℓ, we denote by $\hat{F}_\ell^f = \{g \in F : g \rhd_\ell f\}$ the set of families that have a higher priority than f at ℓ. Second, given any family f and any matching μ, we denote by $\hat{F}_\mu^f = \{g \in F : g \rhd_{\mu(f)} f \text{ and } \mu(f) \succeq_g \mu(g)\}$ the set of families that have a higher priority than f at $\mu(f)$ and weakly prefer $\mu(f)$ to their own match.

A. *Interference-Free Matchings*

In order to capture the importance of priorities in our multidimensional setting, we introduce a novel criterion for respecting priorities—*interference-freeness*.

[8] While the KTTCE mechanism improves upon every endowment that is not chain-efficient, it does not improve upon every endowment that is not Pareto-efficient. In fact, Proposition B.1 in online Appendix B shows that even if $|D| = 1$, there is no strategy-proof mechanism that Pareto improves upon every endowment that is not Pareto-efficient.

[9] We leave open the question of finding a maximally efficient, strategy-proof, and individually rational mechanism.

To formalize the idea, recall that a locality ℓ can accommodate a family f alongside a set of families $G \subseteq F \backslash \{f\}$ if $\nu_d^f + \sum_{g \in G} \nu_d^g \leq \kappa_d^\ell$ for all $d \in D$; we relax this definition as follows.

DEFINITION 5: *Locality $\ell \in L$ can* weakly accommodate *family $f \in F$ alongside $G \subseteq F \backslash \{f\}$ if, for all $d \in D$, either*

$$\nu_d^f = 0 \quad or \quad \nu_d^f + \sum_{g \in G} \nu_d^g \leq \kappa_d^\ell.$$

Definition 5 relaxes the original concept of accommodation by only taking into account the dimensions in which f takes up at least one unit of capacity. If $|D| = 1$, then weak accommodation is equivalent to accommodation; however, if $|D| > 1$, then ℓ may be able to weakly accommodate f alongside G but unable to accommodate f alongside G.[10] Intuitively, weak accommodation means that f does not compete for ℓ's capacity with the families in G: for each dimension d, either f does not require any units of d or there are enough units of d for $G \cup \{f\}$.

DEFINITION 6: *A family f* interferes *with a matching μ if $\mu(f)$ cannot weakly accommodate f alongside $\hat{F}_\mu^f$. A matching μ is* interference-free *if no family interferes with it.*

Interference-free matchings respect priorities in the sense that, for any family f matched to a locality ℓ, higher-priority families that prefer ℓ to their current match do not have any claim to the capacity that f uses: even if all of these families were to move to ℓ, that capacity would still be available to f. By contrast, if f interferes, then some of the capacity it is using could be claimed by a higher-priority family. Interference-free matchings may be wasteful, which unfortunately is unavoidable due to a stark trade-off in matching markets with sizes between respecting priorities and eliminating waste—as Delacrétaz (2019) showed, if waste must be eliminated, then one cannot put any bound on the number of units that a family could claim at a more preferred locality. Moreover, the underuse of capacity may well be tolerable in the refugee resettlement context because it can be used for the next cohort of resettled refugee families.

Two solution concepts that exist in the literature and relate to respecting priorities are *stability* (Roth 1984; Abdulkadiroğlu and Sönmez 2003) and *envy-freeness* (in the sense of eliminating justified envy; see Sotomayor 1996; Balinski and Sönmez 1999; Wu and Roth 2018; Kamada and Kojima forthcoming). While stability is arguably the most common solution concept in matching theory, in the presence of sizes, stable matchings are not guaranteed to exist; moreover, finding a stable matching even when one exists is a computationally intractable problem (McDermid and Manlove 2010). Envy-freeness is a useful concept to ensure that priorities are respected because it precludes *any* priority violation. In the presence of multidimensional constraints, however, envy-freeness is too strong and generates unnecessary waste. Interference-freeness relaxes envy-freeness by tolerating priority violations

[10] Note that a family f can be weakly accommodated alongside G but not accommodated alongside G only if G itself cannot be accommodated.

that do not pose any problem because the lower-priority family does not interfere with higher-priority families. We formally define stability and envy-freeness and detail their relationship with interference-freeness and nonwastefulness in online Appendix C.[11] Our simulations (see online Appendix F.2) show that the efficiency gain associated with using interference-freeness over envy-freeness can be large.

B. *A Family-Optimal Interference-Free Mechanism*

We now prove the existence of the (unique) family-optimal interference-free matching by introducing an algorithm that finds this matching in polynomial time. In each round of our Knapsack Deferred Acceptance algorithm (Algorithm 3), all families propose to their favorite localities that have not yet permanently rejected them. A locality ℓ tentatively accepts a proposing family f if ℓ can weakly accommodate f alongside families with a higher priority from which ℓ has received a proposal (in this round or a previous one); otherwise, ℓ permanently rejects f. By construction, families that have already proposed to ℓ can only be matched to ℓ or a less preferred locality, which implies that if ℓ permanently rejects f, then f cannot be matched to ℓ in any interference-free matching. In each nonterminal round of the KDA algorithm, at least one family is permanently rejected, so the algorithm eventually terminates. The KDA algorithm matches each family to its most preferred locality in any interference-free matching, which yields the following result.[12]

THEOREM 5: *The output of the KDA algorithm is the unique family-optimal interference-free matching.*

We now illustrate the KDA algorithm and discuss its incentive properties.

EXAMPLE 1: *There are four families, four localities, and one dimension. The preferences, priorities, sizes, and capacities are*

$$\succ_{f_1}: \ell_2, \ell_1, \ldots, \quad \succ_{f_2}: \ell_1, \ell_3, \ell_4, \ldots, \quad \succ_{f_3}: \ell_1, \ell_2, \ldots, \quad \succ_{f_4}: \ell_1, \ell_3, \ldots,$$
$$\rhd_{\ell_1}: f_1, f_2, f_3, f_4, \quad \rhd_{\ell_2}: f_3, f_1, \ldots, \quad \rhd_{\ell_3}: f_4, f_2, \ldots, \quad \rhd_{\ell_4}: \ldots,$$
$$\nu = d_1 \begin{array}{c} \begin{matrix} f_1 & f_2 & f_3 & f_4 \end{matrix} \\ \begin{pmatrix} 1 & 2 & 1 & 1 \end{pmatrix} \end{array}, \qquad \kappa = d_1 \begin{array}{c} \begin{matrix} \ell_1 & \ell_2 & \ell_3 & \ell_4 \end{matrix} \\ \begin{pmatrix} 2 & 1 & 2 & 5 \end{pmatrix} \end{array}.$$

The KDA algorithm—the execution of which is displayed in Table 1—calculates the family-optimal interference-free matching:

$$\begin{pmatrix} f_1 & f_2 & f_3 & f_4 \\ \ell_1 & \ell_4 & \ell_2 & \ell_3 \end{pmatrix}.$$

We now show that the KDA mechanism is not strategy-proof. To see this, suppose that family f_2 reports locality ℓ_3 to be its first choice. Then, all families are tentatively

[11] Envy-freeness implies interference-freeness, while stability is logically independent of the other two concepts.

[12] The existence of an interference-free matching, which is directly implied by Theorem 5, does not follow from the existence of the family-optimal envy-free matching (Wu and Roth 2018; Kamada and Kojima forthcoming), as the two concepts are distinct. However, we show in online Appendix C.2 that the family-optimal interference-free matching coincides with the family-optimal envy-free matching when $|D| = 1$.

ALGORITHM 3: KNAPSACK DEFERRED ACCEPTANCE (KDA)

Round $i \geq 1$

Step 1: Each family proposes to its favorite locality that has not permanently rejected it yet.

Step 2: Each locality tentatively accepts a proposing family if the locality can weakly accommodate that family alongside all families with a higher priority that are proposing to or have been permanently rejected by that locality; otherwise, the locality permanently rejects that family.

Step 3: If at least one family has been permanently rejected in Step 2, continue to Round $i+1$. Otherwise, permanently match each family to the locality at which the family is proposing and end.

TABLE 1—THE KDA ALGORITHM APPLIED TO EXAMPLE 1

		Round 1		Round 2		Round 3		Round 4		Round 5	
f_1	$\rightarrow$	ℓ_2	✓	ℓ_2	✗	ℓ_1	✓	ℓ_1	✓	ℓ_1	✓
f_2	$\rightarrow$	ℓ_1	✓	ℓ_1	✓	ℓ_1	✗	ℓ_3	✗	ℓ_4	✓
f_3	$\rightarrow$	ℓ_1	✗	ℓ_2	✓	ℓ_2	✓	ℓ_2	✓	ℓ_2	✓
f_4	$\rightarrow$	ℓ_1	✗	ℓ_3	✓	ℓ_3	✓	ℓ_3	✓	ℓ_3	✓

Note: $f \rightarrow \ell$ means f proposes to ℓ; ✗ is a rejection; ✓ is a tentative acceptance.

accepted in the first round since locality ℓ_1 has two units of capacity and families f_3 and f_4 each require one unit; f_1 and f_2 are the only families that propose to ℓ_2 and to ℓ_3, respectively. The KDA outcome is

$$\begin{pmatrix} f_1 & f_2 & f_3 & f_4 \\ \ell_2 & \ell_3 & \ell_1 & \ell_1 \end{pmatrix}.$$

The following result is an immediate consequence of Example 1 and Theorem 5.

PROPOSITION 3: *There is no strategy-proof and family-optimal interference-free mechanism.*

The driving force behind Proposition 3 is that the *choice function* of localities induced by the KDA algorithm (i.e., the rule that determines, in each round, which families are tentatively accepted and permanently rejected) does not satisfy the cardinal monotonicity condition (Alkan 2002; Fleiner 2003).[13] Cardinal monotonicity—which requires that the number of tentatively accepted families grows monotonically with the number of proposing families—is crucial for designing strategy-proof matching mechanisms (Hatfield and Milgrom 2005; Hatfield, Kominers, and Westkamp 2021).

[13] In other contexts, cardinal monotonicity has been referred to as "size monotonicity" (Alkan and Gale 2003) and the "Law of Aggregate Demand" (Hatfield and Milgrom 2005; Hatfield, Kominers, and Westkamp 2021).

ALGORITHM 4: Threshold Knapsack Deferred Acceptance (TKDA)

Round $i \geq 1$

Step 1: Each family f proposes to its favorite locality ℓ that has not permanently rejected f yet.

Step 2: Each locality ℓ permanently rejects any proposing family f if f*'s* priority rank among all families that are proposing to ℓ is strictly greater than f*'s* threshold at ℓ (calculated by Algorithm 5).

Step 3: If at least one family has been permanently rejected in Step 2, continue to Round $i + 1$. Otherwise, permanently match each family to the locality to which that family is proposing and end.

C. *An Interference-Free and Strategy-Proof Mechanism*

In this section, we introduce the Threshold Knapsack Deferred Acceptance algorithm (Algorithm 4). We show that the TKDA mechanism is interference-free and strategy-proof.

The key difference between the TKDA and KDA algorithms lies in the choice function that decides whether a proposal is tentatively accepted or permanently rejected. In order to ensure that the TKDA mechanism is strategy-proof, the choice function of localities needs to satisfy the cardinal monotonicity condition. That is, for each locality, the choice function ensures that as the set of proposing families expands, the number of families that are tentatively accepted by the locality weakly increases; because of this additional constraint, a locality may permanently reject a family even though the locality can weakly accommodate the family alongside all higher-priority proposing families. Consequently, the outcome of the TKDA algorithm may not be family-optimal within the class of interference-free matchings.[14]

At a high level, the choice function works as follows (see Step 2 of Algorithm 4). In each round, each locality assigns every family (not just those that are proposing) a *threshold*. For each proposing family, the locality then compares the family's threshold with its priority rank among proposing families. The family is tentatively accepted so long as its priority rank does not exceed its threshold; otherwise, the family is permanently rejected.

The key technical step in the TKDA algorithm is the calculation of the threshold for each family-locality pair (f, ℓ) (Algorithm 5). The two extreme cases are simple. If ℓ can weakly accommodate f alongside all higher-priority families, f's threshold for ℓ is ∞, meaning that ℓ will tentatively accept a proposal from f no matter which other families are proposing. If ℓ cannot weakly accommodate f alongside those higher-priority families that are proposing, then f's threshold for ℓ is 0, meaning that

[14] In contrast to the KTTC mechanism, it is straightforward to construct an endowment version of the KDA and TKDA mechanisms by adjusting priorities such that each locality prioritizes families in its endowment over other families. With that adjustment, each family would be guaranteed—under either mechanism—to be matched to either its endowment or a more preferred locality.

ALGORITHM 5: THRESHOLD CALCULATOR

For each locality $\ell \in L$, let Π_ℓ be the set of families that are currently proposing to ℓ.

Step 1: For each family $f \in F$ and each locality $\ell \in L$, calculate the *temporary threshold* of f at ℓ, denoted by $\tilde{\theta}_\ell^f$, as follows:

- If ℓ can weakly accommodate f alongside $\hat{F}_\ell^f$, let $\tilde{\theta}_\ell^f = \infty$.
- If ℓ cannot weakly accommodate f alongside $\Pi_\ell \cap \hat{F}_\ell^f$, let $\tilde{\theta}_\ell^f = 0$.
- Otherwise, find the unique $n \in \mathbb{Z}_{>0}$, such that
  (i) ℓ can weakly accommodate f alongside all sets of families $G \subseteq \hat{F}_\ell^f$ such that $|G| = n - 1$ and $\left(\Pi_\ell \cap \hat{F}_\ell^f\right) \subseteq G$, and
  (ii) ℓ cannot weakly accommodate f alongside a set of families $G' \subseteq \hat{F}_\ell^f$ such that $|G'| = n$ and $\left(\Pi_\ell \cap \hat{F}_\ell^f\right) \subseteq G'$;

  and let $\tilde{\theta}_\ell^f = n$.

Step 2: For each family $f \in F$ and each locality $\ell \in L$, calculate the threshold of f at ℓ, θ_ℓ^f, as follows:

- If $\tilde{\theta}_\ell^f = \infty$, let $\theta_\ell^f = \infty$.
- Otherwise, let $\theta_\ell^f = \min_{g \in \hat{F}_\ell^f \cup \{f\}} \left\{\tilde{\theta}_\ell^g\right\}$.

ℓ will permanently reject any proposal from f; in fact, in that case, ℓ would also reject f in the KDA algorithm.

The intermediate case—in which ℓ can weakly accommodate f alongside all higher-priority families that are proposing to ℓ but not alongside all higher-priority families—is more delicate. In the KDA algorithm, ℓ would tentatively accept f; however, as we next explain, this may violate the cardinal monotonicity condition, and the thresholds allow us to identify when this is the case. Locality ℓ first assigns a temporary threshold $n \in \mathbb{Z}_{>0}$ to family f for which (i) ℓ can weakly accommodate f alongside all such sets containing $n - 1$ families but (ii) ℓ cannot weakly accommodate f alongside at least one such set of n families. The temporary threshold can be interpreted as the largest priority rank for which we can guarantee that the family can be weakly accommodated in all subsequent rounds alongside all higher-priority families that are proposing. That is, if f proposes to ℓ in a subsequent round and is one of the top n proposing families, then ℓ can weakly accommodate f alongside all higher-priority families that are proposing; however, if n or more higher-priority families propose, then depending on which those families are, ℓ may not be able to weakly accommodate f alongside them. Family f's threshold for locality ℓ is then set to the lowest number among the temporary thresholds of f and all higher-priority families. This adjustment is what creates additional rejections compared to the choice function of the KDA algorithm. The rationale is that allowing f to have a larger threshold than a higher-priority family could lead to a violation of the cardinal monotonicity condition. For example, suppose that f and a higher-priority family f' propose, f's threshold is 2, and f''s threshold is 1. Both families might need to be permanently rejected in a later round if a family g with a higher priority than f' proposes, which would violate the cardinal monotonicity condition. What our rule does is lower f's threshold to 1 when only f and f' propose, so that f is permanently

TABLE 2—TKDA ALGORITHM APPLIED TO THE MARKET FROM EXAMPLE 1 WITH TRUTHFUL REPORTING

		Round 1			Round 2			Round 3			Round 4			Round 5		
f_1	$\rightarrow$	ℓ_2	[1]	✓	ℓ_2	[0]	✗	ℓ_1	[∞]	✓	ℓ_1	[∞]	✓	ℓ_1	[∞]	✓
f_2	$\rightarrow$	ℓ_1	[1]	✓	ℓ_1	[1]	✓	ℓ_1	[0]	✗	ℓ_3	[0]	✗	ℓ_4	[∞]	✓
f_3	$\rightarrow$	ℓ_1	[0]	✗	ℓ_2	[∞]	✓	ℓ_2	[∞]	✓	ℓ_2	[∞]	✓	ℓ_2	[∞]	✓
f_4	$\rightarrow$	ℓ_1	[0]	✗	ℓ_3	[∞]	✓	ℓ_3	[∞]	✓	ℓ_3	[∞]	✓	ℓ_3	[∞]	✓

Note: $f \rightarrow \ell$ means f proposes to ℓ; ✗ is a rejection; ✓ is a tentative acceptance; thresholds are in square brackets.

TABLE 3—TKDA ALGORITHM APPLIED TO THE MARKET FROM EXAMPLE 1 WITH A MISREPORT

		Round 1			Round 2			Round 3		
f_1	$\rightarrow$	ℓ_2	[1]	✓	ℓ_2	[1]	✓	ℓ_2	[1]	✓
f_2	$\rightarrow$	ℓ_3	[1]	✓	ℓ_3	[0]	✗	ℓ_4	[∞]	✓
f_3	$\rightarrow$	ℓ_1	[1]	✓	ℓ_1	[1]	✓	ℓ_1	[1]	✓
f_4	$\rightarrow$	ℓ_1	[1]	✗	ℓ_3	[∞]	✓	ℓ_3	[∞]	✓

Note: $f \rightarrow \ell$ means f proposes to ℓ; ✗ is a rejection; ✓ is a tentative acceptance; thresholds are in square brackets.

rejected. This affects efficiency because f is permanently rejected even though f could be tentatively accepted without interfering; however, it ensures that the cardinal monotonicity condition is satisfied. The calculation of thresholds can be done in polynomial time, which ensures that the TKDA algorithm is practical even for large markets.[15]

THEOREM 6: *The TKDA mechanism is strategy-proof and interference-free.*

In Example 1, the TKDA algorithm follows the same deferred acceptance procedure and produces the same matching as the KDA algorithm (see Table 2).

But even though both algorithms produce the same matching, the TKDA algorithm removes the incentive to misreport its preferences that f_2 has in the KDA algorithm. Suppose that family f_2 manipulates its preference report to $\succ_{f_2} : \ell_3, \ell_4, \ldots$. Under the KDA algorithm, such a manipulation allows family f_2 to be matched to locality ℓ_3 instead of ℓ_4. This is no longer possible in the TKDA algorithm, as we illustrate in Table 3.

The TKDA mechanism achieves strategy-proofness by ensuring that its choice function satisfies the cardinal monotonicity condition. This constraint comes with an efficiency cost, as it may require some localities to reject additional families; in fact, a direct consequence of Proposition 3 is that the TKDA algorithm will not always produce the family-optimal interference-free matching. Theorem 6 remains silent on how efficient the TKDA mechanism is. While we leave open the search for a maximally efficient strategy-proof mechanism, we provide a detailed discussion of

[15] To calculate n, take each dimension for which f's size is at least 1 and order the families with a higher priority from largest to smallest size for that dimension. Starting from the largest family, add one family at a time until the families' total size for that dimension (including f's) exceeds the capacity. A number of families is obtained in this way for each dimension, and n is the minimum among these numbers.

the efficiency properties of the TKDA mechanism in online Appendix D. In online Appendix D.1, we compare the choice function of the TKDA algorithm to other choice functions that satisfy the cardinal monotonicity condition. We first adapt the definitions of envy-freeness and interference-freeness to choice functions of localities. We then show that the TKDA choice function is weakly more efficient than any envy-free choice function; moreover, when $|D| = 1$, it maximizes the number of tentatively accepted families within the class of interference-free choice functions. In online Appendix D.2, we derive an explicit lower bound for the efficiency of the TKDA algorithm. Then in online Appendix D.3, we propose a strategy-proof and interference-free mechanism that improves upon the TKDA mechanism in terms of efficiency when $|D| > 1$ by identifying violations of the cardinal monotonicity condition that can be tolerated.

V. Data and Simulations

Our theoretical results leave important empirical questions open about how our mechanisms would perform in practice. To shed some light on these questions, we simulate our mechanisms using data from HIAS. We begin with the analysis of a one-dimensional environment in which each locality only has a constraint on the number of refugees—this is closest to HIAS's current practice. We then consider how our matching mechanisms perform in a three-dimensional setting.[16]

Data.—We use anonymized data on refugees resettled by HIAS from October 2016 to October 2017. There are 839 refugees partitioned into 329 families that were matched across 20 localities. To preserve anonymity, our data on refugees contain only the number of senior, (working-age) adult, and child family members. Following Ahani et al. (2021) and Nguyen, Nguyen, and Teytelboym (2021), we conservatively treat each locality's quota as the total number of refugees that HIAS actually resettled in that locality in 2017.[17] The total number of refugees resettled in a locality varied between 5 and 99. There were 332 children, 498 adults, and 9 seniors in our data. The largest family had 8 members, and there were 157 families with only 1 member. We use expected employment estimates from Ahani et al. (2021) to determine priorities: for each family, we add the employment likelihoods of all working-age adults to determine the expected employment *weight* of each family, and localities rank families from highest to lowest expected employment weight. Since families that have high employment weights in some localities are likely to have high employment weights in other localities, induced priorities are naturally correlated. We make an exception for the KTTCE mechanism, where localities give a higher priority to families in their endowment (but otherwise rank families in decreasing likelihood of employment).

[16] Data and replication package are available via the Inter-university Consortium for Political and Social Research (Delacrétaz, Kominers, and Teytelboym 2023).

[17] The annual quotas were abruptly adjusted following a change in the presidential administration in January 2017.

Table 4—Types of Simulated Preferences

		Priorities	
		Correlated with Preferences	Uncorrelated with Preferences
Preferences	Correlated	Type 4	Type 1
	Uncorrelated	Type 3	Type 2

Simulated Preferences.—As HIAS does not currently collect preferences from refugees, we have to simulate preferences for our analysis. We assume that the utility of family f associated with being assigned to locality ℓ takes the form

$$U_{f\ell} = \delta V_{f\ell} + \beta Y_\ell + \gamma E_{f\ell},$$

where $V_{f\ell}$ is the (normalized to $[0,1]$) employment weight of family f in locality ℓ, Y_ℓ is a locality-specific term, and $E_{f\ell}$ is a pure noise term. Terms Y_ℓ (for each locality) and $E_{f\ell}$ (for each family-locality pair) are drawn independently from $\text{Unif}(0,1)$ in each simulation round. We consider four types of preferences, which are summarized in Table 4.

Type 1: $\beta = 1, \delta = \gamma = 0$. In each simulation round, we draw a value for each locality that determines all of the family preferences. Therefore, preferences are perfectly correlated among families but uncorrelated with the priorities of localities.

Type 2: $\gamma = 1, \beta = \delta = 0$. In each simulation round, we draw a value for each family-locality pair. Therefore, preferences are uncorrelated among families and uncorrelated with priorities.

Type 3: $\delta = \gamma = 1, \beta = 0$. In each simulation round, family preferences are a sum of their employment weight and a noise term. Therefore, preferences are uncorrelated among families but correlated with priorities.

Type 4: $\delta = \beta = 1, \gamma = 0$. In each simulation round, family preferences are a sum of their employment weight and a locality-specific term. Therefore, preferences are correlated among families and correlated with priorities.

For each preference type, in each of the 100 simulation rounds, we computed the outcome of the KDA, TKDA, KTTC, and KTTCE algorithms (where in KTTCE, the endowment is the employment-maximizing allocation).

A. *Performance of the KTTCE Mechanism*

We begin by analyzing the performance of the KTTCE mechanism, as we see that mechanism as representing a first step toward incorporating preferences in refugee resettlement processes. We take the employment-maximizing matching that HIAS

Table 5—Number (Fraction) of Families Made Better-Off by the KTTCE Mechanism (One-Dimensional Constraints)

Preference type	Type 1	Type 2	Type 3	Type 4
Families made better-off	6.1 (1.9%)	46.3 (14.1%)	45.9 (14.0%)	24.2 (7.4%)

Note: Averages over 100 simulation rounds.

currently uses as the endowment; that is, each family is endowed with the locality to which it is matched under the matching that maximizes total refugee employment. Each locality prioritizes the families that are in its endowment over those that are not and, within each tier, ranks families from highest to lowest expected employment. Note that in this setting, Theorem 4 does not apply because priorities are not lexicographic. Therefore, our theoretical results do not guarantee that the KTTCE mechanism would find any Pareto improvements from the endowment.

The results are presented in Table 5. The number of families that the KTTCE mechanism makes better-off is small for Type 1 preferences, with just over six families helped on average. The reason is that Type 1 families cannot trade with each other, as they have identical preferences; therefore, the only improvements that the KTTCE mechanism finds are closed chains, in which a family takes advantage of unused capacity in a more preferred locality. The KTTCE mechanism performs best with Type 2 and Type 3 preferences because family preferences are uncorrelated, which creates the possibility of many mutually beneficial trades.

Overall, the simulation results in Table 5 are encouraging—they suggest that the KTTCE mechanism may find at least some Pareto improvements in practice, even when it is not guaranteed to do so.[18]

B. *Comparison of the KTTC, KDA, and TKDA Mechanisms*

We next turn to our three remaining mechanisms: KTTC, KDA, and TKDA. We compare those mechanisms in terms of interference, average priority rank, and efficiency. The results are presented in Table 6 and Figure 2. For completeness, we also include outcomes for the KTTCE mechanism; however, since it takes the endowment as an input, the KTTCE mechanism's performance cannot be directly compared to that of the other three mechanisms.

In each simulation round, we count the number of *interference violations* of a given matching μ; that is, the number of family pairs (f_1, f_2) such that f_1 prefers $\mu(f_2)$ to $\mu(f_1)$, f_1 has a higher priority than f_2 at $\mu(f_2)$, and f_2 interferes with μ.[19] In line with Theorems 5 and 6, which show that the KDA and TKDA mechanisms are interference-free, both mechanisms have zero interference violations. For the KTTC mechanism, the number of interference violations is very small for Type 1

[18] In online Appendix F.1, we also test the effects of changing the order of rejected families in the Rejection Stage of KTTCE algorithm and find some evidence in favor of rejecting families in the order from largest to smallest.

[19] As the ordering of the two families matters, there are $329^2 = 108{,}241$ possible pairs.

Table 6—Outcomes of KTTCE, KTTC, KDA, and TKDA Algorithms (One-Dimensional Constraints)

Preference type	Type 1	Type 2	Type 3	Type 4
Interference violations				
KTTCE	19,397 (17.9%)	16,200 (15.0%)	16,487 (15.2%)	18,506 (17.1%)
KTTC	41 (0.0%)	895 (0.8%)	893 (0.8%)	3,734 (3.4%)
KDA	0	0	0	0
TKDA	0	0	0	0
Average priority rank				
KTTCE	107	106	107	106
KTTC	36	101	100	80
KDA	32	51	51	37
TKDA	21	19	19	20
Number of matched families				
KTTCE	324 (98.6%)	324 (98.4%)	324 (98.4%)	324 (98.6%)
KTTC	307 (93.2%)	312 (95.0%)	312 (95.0%)	309 (93.8%)
KDA	304 (92.3%)	314 (95.6%)	315 (95.7%)	308 (93.5%)
TKDA	275 (83.8%)	265 (80.7%)	265 (80.5%)	271 (82.2%)
Fraction of unfilled capacity				
KTTCE	0.1%	0.1%	0.1%	0.1%
KTTC	7.0%	5.0%	5.5%	6.8%
KDA	10.4%	7.2%	7.7%	9.7%
TKDA	18.6%	20.8%	21.6%	20.1%

Notes: Averages over 100 simulation rounds (and, for average priority rank, over all localities). Numbers (percentages) are rounded to the nearest integer (to 1 d.p.).

preferences; the intuitive reason for this is that the preferences of all families are identical, so interference violations may only occur when a larger family cannot be accommodated in a locality but a smaller family with a lower priority can. The number increases for Type 2 and Type 3 preferences because preference heterogeneity means that more priority trades are possible. Type 4 preferences capture an intermediate "worst-case" scenario for interference violations: as preferences are correlated, there are not many opportunities for trade; however, since priorities and preferences are correlated, many families that do not get their top-choice localities are likely to envy interfering lower-priority families that ended up in these localities.

The second panel in Table 6 shows the average priority rank of matched families across all localities. Localities end up matched to higher-priority families on average in the KDA and TKDA mechanisms compared to the KTTC and KTTCE mechanisms—which is as expected since the KTTC and KTTCE mechanisms do not explicitly take priorities into account. While we only report the averages across localities, the pattern in Table 6 also holds in each locality. Since a similar number of families are matched under the KTTC and KDA mechanisms (third panel in Table 6), it is likely that localities would favor KDA over KTTC. However, the relative ranking of the TKDA and KTTC mechanisms is less clear for localities because many fewer families are matched under TKDA than under KTTC.

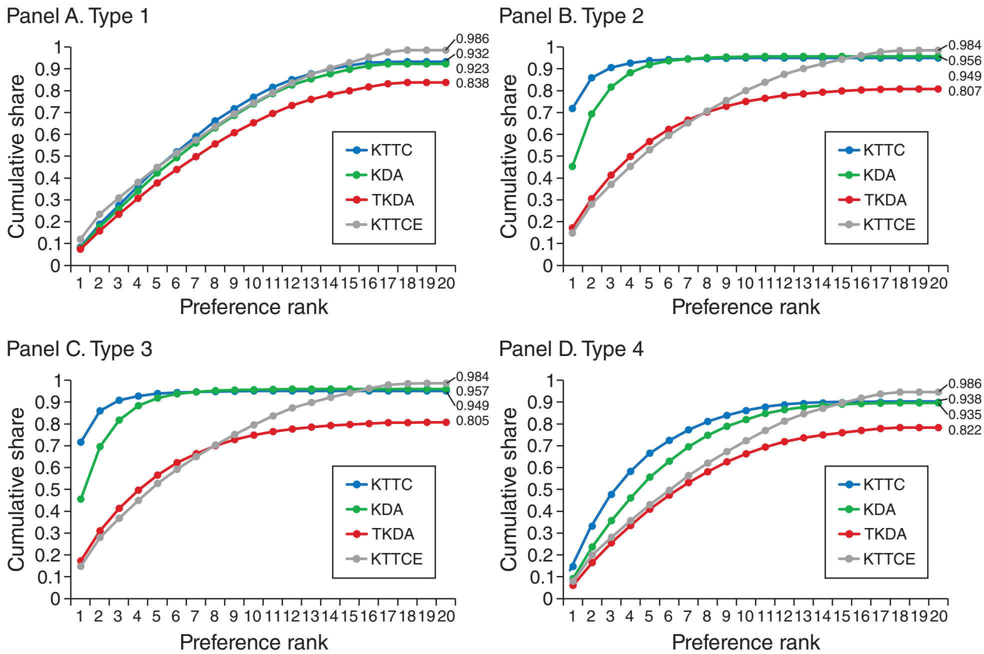

FIGURE 2. PREFERENCE DISTRIBUTIONS FOR OUTCOMES UNDER DIFFERENT MATCHING MECHANISMS (ONE-DIMENSIONAL CONSTRAINTS)

Notes: Labeled numbers: fractions of matched families. Averages over 100 simulation rounds.

Figure 2 illustrates the efficiency of the mechanisms and shows that the KDA mechanism is substantially more efficient than the TKDA mechanism, especially when preferences are uncorrelated (Type 2 and Type 3).[20]

In addition, the bottom two panels in Table 6 show that the KDA mechanism matches more families and leaves less unfilled quota than the TKDA mechanism. Overall, our simulations suggest that the strategy-proofness of TKDA could come at a significant efficiency cost. Our theoretical results are silent about the efficiency ranking between the KTTC and KDA mechanisms—and indeed, our simulations suggest that, depending on the preference type, some families can be better-off under either mechanism. However, as the KTTC mechanism is Pareto-efficient, we should expect it to be more efficient "on average" than the Pareto-inefficient KDA mechanism. Figure 2 confirms this intuition for all four preference types. However, Figure 2 also suggests that the difference between the KTTC and KDA mechanisms in our setting is small relative to the difference between the KDA and TKDA

[20]For preference Types 2–4, the KTTCE mechanism performs substantially worse than the KDA and KTTC mechanisms because it starts with the endowment allocation. The performances of the KTTCE and TKDA mechanisms are comparable in terms of rank distribution, but the former has the advantage of matching more families. With Type 1 preferences, the KTTC, KTTCE, and KDA mechanisms perform similarly. As all families have the same preferences, all three algorithms essentially collapse to a version of the serial dictatorship procedure in which the first-preference locality picks some families, the second-preference locality picks some families among the remaining ones, and so on. While the procedures are not identical—in KTTC, families are picked based on priorities only, in KTTCE, families are picked based on the endowment and then priorities, and in KDA, families are picked based on priorities and interference-freeness—there is very little difference in terms of rank distribution.

mechanisms. Consequently, our simulations suggest that in practice, the efficiency cost imposed by interference-freeness may not be particularly large.

In online Appendix F.2, we repeat our simulations in a setting with three-dimensional constraints and find qualitatively similar results to the one-dimensional setting. Moreover, we consider versions of the KDA and TKDA mechanisms that impose envy-freeness instead of interference-freeness. We find that both mechanisms are considerably more efficient under interference-freeness (Figure F.2 in the online Appendix).

VI. Conclusion

Refugee resettlement presents a real opportunity for marketplace design: policymakers and resettlement agencies are already working with market design experts to improve matching outcomes in ways that—if we do this work well—stand to improve the lives of millions of disenfranchised people worldwide (Andersson 2019; Jones and Teytelboym 2018; Kominers, Teytelboym, and Crawford 2017; Roth 2018). Recent efforts have focused on maximizing short-run employment outcomes; we show how to take this work further by integrating refugees' preferences and localities' priorities into the assignment process. As we have highlighted, the trade-off among maximizing refugee welfare, respecting localities' priorities, and strategy-proofness is exacerbated by the presence of (possibly multidimensional) knapsack constraints in refugee resettlement matching.

Incorporating refugees' preferences into refugee matching will serve not just to improve the quality of assignment outcomes but also to give refugees a larger role in the resettlement process. Moreover, collecting information about refugees' preferences will enable us to better understand what constitutes a high-quality refugee-locality match. Locality priorities are similarly important: if we want localities to be willing to host a large number of refugees, then we must do what we can to respond to their desires and constraints. The hope is that a well-designed resettlement matching system will increase localities' overall willingness to host refugees, boosting resettlement overall.

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