Informed Trading with a Short-Sale Prohibition

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Abstract

Using a rational expectations equilibrium framework, I evaluate the effects of a short-sale prohibition in an economy with asymmetrically-informed investors who are identical except for their information sets. Relative to an economy in which short selling is permitted, the financial market is less informationally efficient under a short-sale ban even when the ban is not binding. This alters the risk-sharing environment and leads to an increase in information acquisition. Additionally, a short-sale prohibition increases market depth. Imposing a cost on short selling instead of a strict prohibition yields similar results. Novel empirical implications are identified.

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1 Introduction

Short-sale constraints exist in many forms. They include search costs to locate securities, fees paid to securities lenders, and margin accounts maintained with broker-dealers. Numerous institutional and legal restrictions on short selling also exist. For example, Almazan et al. (2004) report that a majority of mutual funds restrict themselves from taking short positions, and financial regulations designed to curb short selling in the United States date back to 1934, including an outright ban on short sales in 2008 during the Great Recession.[1] Because short sellers are often well informed and account for a large fraction of trading volume, any constraints that impede short sales can extensively impact markets.[2] Yet, despite their prevalence, many economic implications of short-sale constraints are not well understood. Much of the extant theoretical and empirical literature is either silent or conflicted regarding the impact of short-sale constraints on various economic attributes.[3]

To evaluate the economic effects of short-sale constraints, I construct a rational expectations equilibrium model a la Grossman and Stiglitz (1980) in which short sales are prohibited. Informed traders, uninformed traders, and liquidity traders all trade a single stock and a single bond. All three types of traders are prohibited from shorting the stock but not the bond. Informed traders, who are identical to uninformed traders in every respect except for their information sets, possess an information advantage over uninformed and liquidity traders regarding the future payoff of the stock, and they can trade on the basis of this information advantage. A portion of the informed traders' private information is revealed to uninformed traders through trading.

Relative to an economy in which short selling is permitted, I find that barring short sales de-

[1] A report by the Division of Economic and Risk Analysis (2014) of the U.S. Securities and Exchange Commission (SEC) describes a history of financial regulations related to short selling in the United States. Saffi and Sigurdsson (2011), Beber and Pagano (2013), and Boehmer, Jones, and Zhang (2013) discuss the 2008 short-sale ban. Section 16(c) of the Securities Exchange Act of 1934 currently prohibits officers, directors, and 10% beneficial owners from shorting stock in their firm.

[2] For empirical evidence, see, e.g., Dechow et al. (2001), Bris, Goetzmann, and Zhu (2007), Chang, Cheng, and Yu (2007), Boehmer, Jones, and Zhang (2008), Diether, Lee, and Werner (2009a), Diether, Lee, and Werner (2009b), Saffi and Sigurdsson (2011), Engelberg, Reed, and Ringgenberg (2012), Beber and Pagano (2013), Boehmer, Jones, and Zhang (2013), and Boehmer and Wu (2013).

[3] Reed (2013) provides a review of the existing literature on short selling.

creases the amount of private information conveyed to uninformed traders through trade because a short-sale prohibition may censor informed traders' demand and, thereby, lower the precision of the price signal. Consequently, the market is less informationally efficient under a short-sale ban. Importantly, market efficiency declines even when a ban is nonbinding in equilibrium simply because uninformed traders cannot disentangle whether informed traders are bound by the constraint. This result contrasts with other studies that find that short-sale constraints reduce market efficiency only when such constraints bind (Bai, Chang, and Wang, 2006; Cao, Zhang, and Zhou, 2007).

The drop in market efficiency increases uninformed traders' perceived risk associated with the stock payoff, resulting in a lower average equilibrium stock price and a shift in allocations from uninformed to informed traders, which corresponds to constrained optimal risk sharing.[4] As a consequence, a short-sale ban seldom binds in equilibrium, and, interestingly, a ban does not always bind in cases where informed traders prefer to hold a short position in the absence of a ban. This result, which contrasts with other models (Blocher, Reed, and Van Wesep, 2013; Reed, Saffi, and Van Wesep, 2016; Daniel, Klos, and Rottke, 2017; Weitzner, 2017), has important empirical implications, suggesting that short-sale constraints could potentially influence equilibrium attributes even when such constraints are not observed to bind.

Because the market is less efficient under a short-sale ban, *ceteris paribus*, more traders choose to become informed. Thus, a short-sale ban increases information acquisition even though a ban may prevent traders from fully exploiting their information. This result contrasts with other analyses that find that short-sale constraints decrease information acquisition (Nezafat, Schroder, and Wang, 2017; Liu and Wang, 2018). The increase in the number of informed traders mitigates the detrimental effect on market efficiency caused by a short-sale ban because more information is revealed through trade when more traders are informed.

In an extension of the main analysis, I relax the strict prohibition against short selling and permit

[4] The unconditional average price is lower; however, as discussed below, a short-sale ban tends to result in a higher price in states where the ban is binding for informed traders.

traders to short the stock while incurring a cost. The qualitative effects of imposing a cost on short sales are generally the same as those arising from an outright ban, but the quantitative magnitudes of the effects are smaller. Hence, the results stemming from the imposition a short-sale ban discussed above appear to be qualitatively robust to the imposition of less stringent short-sale restrictions.

The result that a short-sale ban may affect equilibrium attributes even when the ban is non-binding stems from an assumption that uninformed traders cannot discern whether informed traders are bound by the constraint. The model's economic implications should hold as long as at least some uninformed traders cannot discern whether informed traders are bound, but if all uninformed traders could unambiguously infer whether informed traders were bound, then a ban would not affect equilibrium outcomes unless it were binding.

In reality, some passive investors who are involved in securities lending (e.g., Vanguard, DFA) may be able to detect whether other traders are short-sale constrained based on the equilibrium dynamics between lending fees and borrowing demand, but the extent to which these types of investors would incorporate such knowledge into their portfolios is unclear because their portfolios are largely determined by exogenously specified index weights and capital flows from investors who are unlikely to be aware of the real-time dynamics between lending fees and borrowing demand. If passive investment companies were able to incorporate their knowledge of whether short-sale constraints are binding into their portfolios in practice, however, then the model could be interpreted as applying to stocks that are not big constituents of major indexes. While other articles examine the influence of the securities lending market on short selling (Duffie, Gârleanu, and Pedersen, 2002; Blocher, Reed, and Van Wesep, 2013), my model is agnostic as to the source of supply of shares available for shorting. Because uninformed traders in the model do not observe whether informed traders hold a short position, the supply of shares available for shorting could be interpreted as originating from an (unmodeled) investment company or broker who has delegated authority to lend shares, and uninformed traders could be interpreted as retail investors or less-informed institutional investors, e.g., pension funds or insurance companies.

Additionally, there is nothing inherently special about a short-sale prohibition. Indeed, many of the results stem from the creation of an ownership restriction, as a major economic effect of a short-sale ban is to censor informed traders' demand. Thus, many of the results would be qualitatively unaffected by alternative restrictions on ownership, such as, for example, a hypothetical "long-position constraint" or an ownership tax, because such restrictions would censor informed traders' demand like a short-sale ban does. Specifically, market efficiency would likely still decline, which would result in a lower average stock price and a shift in allocations from uninformed to informed traders, similar to the outcome with a short-sale prohibition.

To the best of my knowledge, this article is the first to model a short-sale restriction using a rational expectations equilibrium framework in which the only difference between traders is the quality of their information. A few models analyze the effects of a short-sale constraint when investors have heterogeneous beliefs. For example, Miller (1977) argues that a short-sale prohibition should increase prices when investors' opinions about asset payoffs diverge because equilibrium prices should reflect the views of optimists but not those of pessimists, who are prevented from shorting. Allen, Morris, and Postlewaite (1993) and Chen, Hong, and Stein (2002) echo this view. However, Jarrow (1980) and Gallmeyer and Hollifield (2008) show that stock prices can either rise or fall when investors have heterogeneous beliefs. Empirically, some studies find that short-sale constraints lead to overvaluation (Figlewski, 1981; Jones and Lamont, 2002; Boehme, Danielsen, and Sorescu, 2006; Chang, Cheng, and Yu, 2007), whereas other studies do not (Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013). I find that a short-sale prohibition tends to decrease the unconditional average stock price due to the rise in the uninformed traders' perceived risk. However, such a prohibition tends to result in a higher price conditional on the ban binding informed traders' demand, which is consistent with many theoretical and empirical analyses (Duffie, 1996; Jones and Lamont, 2002; Asquith, Pathak, and Ritter, 2005; Boehme, Danielsen, and Sorescu, 2006; Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013; Blocher, Reed, and Van Wesep, 2013; Daniel, Klos, and Rottke, 2017).

Diamond and Verrecchia (1987) demonstrate that a short-sale prohibition decreases both market efficiency and liquidity because it results in larger spreads. Liu and Wang (2018) also show that a short-sale constraint leads to larger bid-ask spreads. Empirically, several studies report that short-sale restrictions result in larger spreads (Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013) as well as decrease market efficiency (Senchack and Starks, 1993; Bris, Goetzmann, and Zhu, 2007; Saffi and Sigurdsson, 2011; Beber and Pagano, 2013; Boehmer and Wu, 2013). As discussed above, I find that banning short sales decreases the informational efficiency of the market. I also find that a short-sale prohibition increases market depth, which contrasts with other studies that find that short-sale constraints reduce liquidity (Diamond and Verrecchia, 1987; Liu and Wang, 2018). Because informed traders may be bound by the short-sale constraint and hold zero shares of stock in several states of the world when short selling is prohibited, there is a greater likelihood that a marginal change in aggregate demand is due to a change in liquidity trader demand (rather than informed trader demand) under a short-sale ban, so uninformed traders' beliefs about the stock payoff are less influenced by changes in demand when short sales are prohibited. Consequently, liquidity trades have a smaller impact on the stock price, and the market has greater depth.

My model is perhaps most similar to Bai, Chang, and Wang (2006) and Nezafat, Schroder, and Wang (2017), though there are several important differences. In particular, a short-sale ban affects equilibrium outcomes in Bai, Chang, and Wang (2006) only when it binds because the source of camouflage, namely, an endowment shock for informed traders, allows uninformed traders to infer precisely whether informed traders are short-sale constrained. In contrast, the short-sale ban in my model affects equilibrium outcomes even when it is nonbinding because uninformed traders cannot discern whether informed traders are bound in the presence of liquidity traders. This is an economically significant difference that generates novel empirical predictions, as discussed above.[5]

[5]Furthermore, many of the underlying assumptions differ in these other papers. For example, Bai, Chang, and Wang (2006) assume that asset payoffs are uniformly distributed, which results in a learning environment that is not as rich as the one considered in this article. In contrast, asset payoffs in my model are normally distributed, which is

Nezafat, Schroder, and Wang (2017) model an economy in which traders observe conditionally independent signals with identical precision. There is always a fraction of traders for whom a short-sale constraint is nonbinding in their model, so market efficiency is unaffected by a short-sale ban because the price reflects such traders' information. Consequently, the portfolio restrictions created by a short-sale ban reduce the value of information, resulting in less information acquisition. In contrast, a short-sale ban reduces market efficiency in my model with identical signals, increasing the value of becoming informed and, thus, resulting in greater information acquisition. Bai, Chang, and Wang (2006), along with Yuan (2006) and Cao, Zhang, and Zhou (2007), who also explore the effects of short-sale constraints on asset prices using rational expectation equilibrium frameworks, do not examine the effects on information acquisition.

2 Model

The model is an extension of Grossman and Stiglitz (1980) that incorporates a short-sale constraint. Three types of agents exist: informed traders, uninformed traders, and liquidity traders. All three types may be prohibited from taking a short position in a risky asset, depending on the regulatory regime. I consider two different regimes. Under the first regime, short sales are permitted. This setting serves as a benchmark for evaluating the effects of a short-sale ban. Under the second regime, short sales are prohibited. The only difference between the two regulatory regimes is the permissibility of short sales. I evaluate the effects of a strict prohibition against short selling for simplicity. While many short-sale constraints are less restrictive in reality (e.g., search costs and lending fees), outright bans on short selling do exist (e.g., the short-sale prohibition in 2008 during the Great Recession, the current prohibition for statutory insiders, and restrictions on financial institutions such as mutual funds), and shorting certain assets may not be practical (e.g., structured notes). In

consistent with the literature. Yuan (2006) assumes that only a fraction of informed traders are subject to a short-sale constraint but that no uninformed traders are, while Cao, Zhang, and Zhou (2007) assume that short-sale constraints do not apply to uninformed traders. In my model, however, all traders are subject to a short-sale prohibition. Indeed, the only difference between informed and uninformed traders in my model is their information sets.

Section 4.2, I consider an extension of the model where short selling is permitted but costly.

There are two assets in the economy: a risk-free bond and a risky stock. The bond has an exogenous interest rate that, for simplicity, is set to zero. The stock generates a random payoff $\tilde{v} \sim \mathcal{N}(0, \sigma_v^2)$. The stock price is endogenous and denoted by p. The supply of the bond is elastic, and the supply of the stock is normalized to one share. Traders are always permitted to short the bond, but, depending on the regulatory regime, they may be prohibited from shorting the stock.

There are continuums of informed and uninformed traders who have identical preferences characterized by constant absolute risk aversion (CARA) with common risk-aversion coefficient γ. Both types of traders receive identical endowments of the stock and bond, which are denoted by w_s and w_b, respectively. The mass of informed traders is exogenous and denoted by $\lambda \in (0, 1)$; accordingly, the mass of uninformed traders is $1 - \lambda$. I later relax this assumption and allow the mass of informed traders to be determined endogenously in Section 4.1.

Time is indexed by $t \in \{1, 2\}$. Trading occurs at $t = 1$, and consumption takes place at $t = 2$. Let s_i (s_u) and b_i (b_u) denote the respective quantities of the stock and bond held by informed (uninformed) traders from $t = 1$ to $t = 2$. The amount of stock held by each type of trader must be non-negative when short sales are prohibited but can take any value when short sales are permitted.

Before trading at $t = 1$, informed traders privately observe a noisy signal of the stock payoff,

$$x = \tilde{v} + \tilde{\varepsilon}, \tag{1}$$

where $\tilde{\varepsilon} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Informed traders update their beliefs about the stock payoff in a Bayesian fashion. Conditional on the signal x, the Bayesian posterior distribution of the stock payoff is

$$\tilde{v}|x \sim \mathcal{N}\left(\frac{\sigma_v^2 x}{\sigma_v^2 + \sigma_\varepsilon^2}, \frac{\sigma_v^2 \sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right). \tag{2}$$

Uninformed traders do not observe x, but they infer a conditional distribution of $\tilde{x}$ in equilibrium.

In addition to informed and uninformed traders, there also exist liquidity traders who demand a random amount of stock at $t = 1$ denoted by $\tilde{z}$. I assume that $\tilde{v}$, $\tilde{\varepsilon}$, and $\tilde{z}$ are mutually independent. The primary role of liquidity traders in the model is to provide camouflage so that uninformed traders cannot perfectly infer the informed traders' private information. Importantly, however, the presence of liquidity traders prevents uninformed traders from inferring whether informed traders are bound by the short-sale ban in equilibrium. As a consequence, a short-sale ban may affect equilibrium outcomes even when the ban is not binding.

I assume that liquidity-trader demand is drawn from a normal distribution truncated at zero and one, $\tilde{z} \sim \mathcal{TN}(0, \sigma_z^2, 0, 1)$. In Section 4.2, I relax this distributional assumption and demonstrate that the qualitative results are robust to an environment in which liquidity-trader demand is unbounded. The lower truncation point, 0, prevents liquidity traders from shorting the stock, thereby subjecting them to the same regulation as informed and uninformed traders. The upper truncation point, 1, ensures that liquidity-trader demand cannot exceed the outstanding stock supply. Without an upper bound on liquidity-trader demand, aggregate demand could exceed supply when short sales are prohibited, in which case the market would not clear. For consistency, I assume that liquidity-trader demand is doubly truncated even when short sales are permitted. This ensures that any differences between the equilibrium with a short-sale ban and the equilibrium without a ban are not driven by the distribution of liquidity-trader demand.[6] The assumption that liquidity-trader demand is drawn from a truncated normal distribution, as opposed to some other type of distribution, aligns the model as closely as possible with the extant rational expectations literature, which generally assumes that liquidity trades are normally distributed. The assumption can also be motivated by the notion that

[6] Truncating liquidity-trader demand at 0 and 1 implicitly assumes that z reflects the aggregate amount demanded for liquidity purposes. Alternative truncation points could be used if liquidity-trader demand instead was construed as the deviation from non-liquidity-related demand. Under such an alternative construction, aggregate liquidity-trader demand could be negative without violating the short-sale prohibition, but it would still need to be bounded from below because the amount of stock that could be sold without taking a short position would be limited to the traders' endowments. Additionally, liquidity-trader demand would still need to be bounded from above because liquidity providers could not take a short position to satisfy the demand.

liquidity traders are unlikely to short a stock for liquidity reasons in practice because shorting tends to be an expensive way of satisfying liquidity needs, as argued by Diamond and Verrecchia (1987). In contrast, uninformed traders may be willing to short the stock even though they do not possess any private information if they believe they may be able to exploit liquidity traders.

2.1 Equilibrium without a Short-Sale Prohibition

In this section, I describe the equilibrium derivation in an economy without a short-sale prohibition. This setting serves as a benchmark for evaluating the effects of a short-sale ban. To obtain the equilibrium, I first characterize the demand functions for informed and uninformed traders. I then impose a market-clearing condition and demonstrate that the price and allocations that characterize the equilibrium are unique. Although the price and allocations are unique, closed-form analytic expressions are unobtainable, so I explicitly solve for the equilibrium price and allocations numerically.[7] Yuan (2005) and Gallmeyer and Hollifield (2008), for instance, also use numerical methods to study situations where traders face portfolio constraints.

Informed traders face a standard problem. They must allocate their wealth between the stock and bond to maximize their expected utility from consumption c_i subject to a budget constraint:

$$\max_{s_i \in \mathbb{R}} \quad \mathbb{E}\big[-\exp[-\gamma \tilde{c}_i] \,|\, x\big] \tag{3}$$

$$\text{s.t.} \quad \tilde{c}_i = b_i + s_i \tilde{v} \tag{4}$$

$$b_i = w_b + (w_s - s_i)p. \tag{5}$$

[7] Numerical solution methods are necessary because, as shown below, uninformed traders infer information about the stock payoff from informed traders' censored demand. Thus, a short-sale ban creates a situation where uninformed traders update their beliefs based on a signal drawn from a truncated distribution, and closed-form solutions are not attainable when the cumulative distribution function of the signal is not characterized by a closed-form expression (e.g., a normal distribution). Alternatively, a closed-form expression for the stock price is obtainable under different distributional assumptions. For example, Bai, Chang, and Wang (2006) derive an expression for the price of a risky asset when its payoff is uniformly distributed. I nevertheless assume in this article that asset payoffs are normally distributed because (i) normally-distributed payoffs are consistent with the vast majority of the extant literature and (ii) normally-distributed payoffs provide a much richer learning environment for uninformed traders.

An informed trader's demand function is derived by substituting (4) and (5) into (3), integrating over $\tilde{v}$ according to (2), and solving the first-order condition, which gives

$$s_i = \frac{\sigma_v^2 x - (\sigma_v^2 + \sigma_\varepsilon^2)p}{\gamma \sigma_v^2 \sigma_\varepsilon^2}. \tag{6}$$

Uninformed traders do not directly observe x, but they infer a conditional distribution of $\tilde{x}$ given the price. For the market to clear, aggregate demand must equal supply. This implies

$$\lambda s_i + (1-\lambda)s_u + z = 1. \tag{7}$$

Because uninformed traders know their own demand as well as the proportion of informed traders, uninformed traders indirectly observe

$$k \equiv \frac{\lambda[\sigma_v^2 \tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)p]}{\gamma \sigma_v^2 \sigma_\varepsilon^2} + \tilde{z}, \tag{8}$$

which follows from substituting (6) into (7). Note that observing k is equivalent to observing $f \equiv \frac{\lambda}{\gamma \sigma_\varepsilon^2}\tilde{x} + \tilde{z}$ because p is known in equilibrium.[8] Hence, conditional on observing p and k (or, equivalently, f), which is a noisy signal of $\tilde{x}$, uninformed traders infer the following posterior distribution:

$$\tilde{x}|k,p \sim \mathcal{TN}\left(\frac{\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2}\left(k + \frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}p\right)}{\Sigma^2}, \frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}, \gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}(k-1), \gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}k\right), \tag{9}$$

[8]The uninformed traders' ability to infer information about the stock payoff based on their knowledge of their demand and the traders' masses stems from the standard assumption that all traders of a given type are identical. While this assumption enhances tractability, introducing heterogeneity among traders would result in a similar outcome, provided that the heterogeneity could be aggregated into a single signal. Moreover, observing the price is equivalent to observing f because the market-clearing condition is invertible: substituting (6) into (7) yields

$$f - \frac{\lambda(\sigma_v^2 + \sigma_\varepsilon^2)p}{\gamma\sigma_v^2\sigma_\varepsilon^2} + (1-\lambda)s_u = 1,$$

the left hand side of which is strictly decreasing in p because s_u is decreasing in p (see Theorem 1 below).

where

$$\Theta^2 \equiv \frac{\sigma_v^2 + \sigma_\varepsilon^2}{\gamma^2 \sigma_v^4} \tag{10}$$

$$\Sigma^2 \equiv \Theta^2 \left(1 + \frac{\lambda^2(\sigma_v^2 + \sigma_\varepsilon^2)}{\gamma^2 \sigma_\varepsilon^4 \sigma_z^2}\right). \tag{11}$$

The posterior is truncated normally distributed because $\tilde{z}$ is truncated normally distributed.[9]

The updating of beliefs without conditioning on whether informed traders are long or short the stock implicitly assumes that uninformed traders cannot observe whether informed traders are short. This assumption is consistent with the rational expectations equilibrium literature, which generally assumes that less-informed traders observe a normally distributed noisy signal of more-informed traders' demand but not whether those traders are long or short (e.g., Grossman and Stiglitz, 1980). In practice, though, investors may be able to partially infer short interest through lending disclosures and option market activity. However, the disclosure of securities lending activity is a noisy indicator of short selling because many investment companies are required to make only quarterly disclosures of their securities lending activity, and borrowers sometimes borrow securities for reasons other than short selling (e.g., for hedging purposes, to obtain voting rights, or to be the dividend owner of record; see Duffie, Gârleanu, and Pedersen, 2002). Furthermore, the option market for particular stocks may not be sufficiently liquid to enable investors to accurately infer short-sale demand.

The problem for uninformed traders is to maximize their expected utility from consumption c_u

[9]Note that (8) can be rearranged and written as $\tilde{x} = \gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}(k - \tilde{z})$. If $\tilde{z}$ were a non-truncated normal random variable, then standard Bayesian updating would lead to the following posterior distribution:

$$\tilde{x}|k, p \sim \mathcal{N}\left(\frac{\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2}\left(k + \frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}p\right)}{\Sigma^2}, \frac{(\sigma_v^2 + \sigma_\varepsilon^2)\Theta^2}{\Sigma^2}\right).$$

Because truncation of a random variable does not affect the density over the non-truncated interval (aside from scaling by a normalizing constant), (9) is obtained simply by truncating this expression. The lower and upper truncation points in (9) are determined by the respective upper and lower bounds on $\tilde{z}$. A formal derivation of (9) is provided in the Appendix.

by allocating their wealth between the stock and bond subject to a budget constraint:

$$\max_{s_u \in \mathbb{R}} \quad \mathbb{E}\big[- \exp[-\gamma \tilde{c}_u] \,|\, k \big] \tag{12}$$

$$\text{s.t.} \quad \tilde{c}_u = b_u + s_u \tilde{v} \tag{13}$$

$$b_u = w_b + (w_s - s_u)p. \tag{14}$$

Substituting (13) and (14) into (12) and integrating over $\tilde{v}$ and $\tilde{x}$ according to (2) and (9), respectively, allows an uninformed trader's expected utility to be written as[10]

$$-\frac{\Phi\big(\frac{\beta - s_u}{\Sigma}\big) - \Phi\big(\frac{\alpha - s_u}{\Sigma}\big)}{\Phi\big(\frac{\beta}{\Sigma}\big) - \Phi\big(\frac{\alpha}{\Sigma}\big)} \exp\left[-\gamma\left(w_b + w_s p + \frac{\Theta^2}{\Sigma^2}\left[\left(\frac{\lambda \sigma_v^2}{\gamma \sigma_\varepsilon^2 \sigma_z^2} k - p \right) s_u - \tfrac{1}{2}\gamma\sigma_v^2 \left(1 + \frac{\lambda^2}{\gamma^2 \sigma_\varepsilon^2 \sigma_z^2} \right) s_u^2 \right] \right) \right], \tag{15}$$

where

$$\alpha \equiv \Theta^2 \left(-\frac{\gamma^2 \sigma_v^2 \sigma_\varepsilon^2}{\lambda(\sigma_v^2 + \sigma_\varepsilon^2)} k - \gamma p \right) \tag{16}$$

$$\beta \equiv \Theta^2 \left(\frac{\lambda \sigma_v^2}{\sigma_\varepsilon^2 \sigma_z^2} + \frac{\gamma^2 \sigma_v^2 \sigma_\varepsilon^2}{\lambda(\sigma_v^2 + \sigma_\varepsilon^2)} (1 - k) - \gamma p \right) \tag{17}$$

and $\phi(\xi) \equiv \frac{1}{\sqrt{2\pi}} \exp\big[-\frac{\xi^2}{2} \big]$ and $\Phi(\xi) \equiv \frac{1}{2}\big(1 + \mathrm{erf}\big[\frac{\xi}{\sqrt{2}}\big]\big)$ denote the normal probability density and cumulative distribution functions, respectively. The first-order condition yields an expression for the uninformed traders' stock demand, which is uniquely determined in equilibrium.

Theorem 1. *Given a price p and signal k, an uninformed trader's stock demand when short selling*

[10]Specifically, an uninformed trader's expected utility function is represented by

$$-\frac{\displaystyle\int_{\gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}(k-1)}^{\gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}k} \int_{-\infty}^{\infty} \exp\big[-\gamma\tilde{c}_u\big] \exp\left[-\frac{\big(\tilde{v} - \frac{\sigma_v^2 \tilde{x}}{\sigma_v^2+\sigma_\varepsilon^2}\big)^2}{2\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}} \right] \exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)} \right] \exp\left[-\frac{\Big(k - \frac{\lambda[\sigma_v^2\tilde{x} - (\sigma_v^2+\sigma_\varepsilon^2)p]}{\gamma\sigma_v^2\sigma_\varepsilon^2}\Big)^2}{2\sigma_z^2} \right] \mathrm{d}\tilde{v}\,\mathrm{d}\tilde{x}}{\displaystyle\int_{\gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}(k-1)}^{\gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}k} \int_{-\infty}^{\infty} \exp\left[-\frac{\big(\tilde{v} - \frac{\sigma_v^2 \tilde{x}}{\sigma_v^2+\sigma_\varepsilon^2}\big)^2}{2\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}} \right] \exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)} \right] \exp\left[-\frac{\Big(k - \frac{\lambda[\sigma_v^2\tilde{x} - (\sigma_v^2+\sigma_\varepsilon^2)p]}{\gamma\sigma_v^2\sigma_\varepsilon^2}\Big)^2}{2\sigma_z^2} \right] \mathrm{d}\tilde{v}\,\mathrm{d}\tilde{x}},$$

where the last exponential in both the numerator and denominator follows from the convolution of $\tilde{z}$ and k using the relation in (8). Integrating yields (15).

is permitted is characterized by

$$s_u = \frac{1}{\gamma^2\sigma_v^2\Theta^2\left(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\right)}\left[\Theta^2\left(\frac{\lambda\sigma_v^2}{\sigma_\varepsilon^2\sigma_z^2}k-\gamma p\right)-\frac{\left(\phi\left(\frac{\alpha-s_u}{\Sigma}\right)-\phi\left(\frac{\beta-s_u}{\Sigma}\right)\right)\Sigma}{\Phi\left(\frac{\beta-s_u}{\Sigma}\right)-\Phi\left(\frac{\alpha-s_u}{\Sigma}\right)}\right]. \tag{18}$$

Furthermore, s_u is unique and strictly decreasing in p.

In equilibrium, supply must equal demand, i.e., (7) must hold. Because a closed-form expression for s_u does not exist, however, (7) cannot be inverted to obtain a closed-form expression for p.[11] Therefore, I solve for the price and allocations numerically in Section 2.3 below. Because both s_i and s_u are strictly decreasing in p, the equilibrium price and allocations are unique.

Theorem 2. *When short selling is permitted, the stock price p and allocations s_i and s_u that characterize the equilibrium are unique.*

2.2 Equilibrium with a Short-Sale Prohibition

Here, I describe the equilibrium derivation in an economy with a short-sale prohibition. The only difference between this setting and the one analyzed in the previous section is that informed and uninformed traders are now prohibited from shorting the stock. To distinguish this setting from the other, I add a circumflex ($\hat{}$) to some of the variables.

[11] Unlike in a standard Grossman and Stiglitz (1980) setup, where closed-form solutions for equilibrium prices and allocations exist, the truncation of liquidity-trader demand in this setting inhibits the acquisition of closed-form solutions. Because liquidity traders are unable to demand a negative quantity of stock when short sales are prohibited, I assume that liquidity traders do not demand a negative quantity of stock when short sales are permitted to ensure that any differences between the equilibrium in the two settings are not driven by changes in the distribution of liquidity-trader demand. Closed-form expressions for the equilibrium price and allocations are obtainable under the alternative specification analyzed in Section 4.2 where I relax the bounds on liquidity-trader demand.

Informed traders maximize their expected utility subject to a prohibition against short sales:

$$\max_{\hat{s}_i \in \mathbb{R}^+} \quad \mathbb{E}\big[-\exp[-\gamma \tilde{\hat{c}}_i] \,|\, x\big] \tag{19}$$

$$\text{s.t.} \quad \tilde{\hat{c}}_i = \hat{b}_i + \hat{s}_i \tilde{v} \tag{20}$$

$$\hat{b}_i = w_b + (w_s - \hat{s}_i)\hat{p}. \tag{21}$$

Similar to the case where short selling is permitted, an informed trader's demand function is derived by substituting (20) and (21) into (19), integrating over $\tilde{v}$ according to (2), and solving the first-order condition, which yields

$$\hat{s}_i = \max\left\{\frac{\sigma_v^2 x - (\sigma_v^2 + \sigma_\varepsilon^2)\hat{p}}{\gamma \sigma_v^2 \sigma_\varepsilon^2}\,,\, 0\right\}. \tag{22}$$

This demand function is similar to an informed trader's demand in the absence of a short-sale ban, except that here the demand is bounded from below by zero.

Like in the case without a short-sale prohibition, demand must equal supply for the market to clear, which implies

$$\lambda \hat{s}_i + (1-\lambda)\hat{s}_u + z = 1. \tag{23}$$

The market-clearing condition enables uninformed traders to indirectly observe a noisy signal of $\tilde{x}$. Substituting (22) into (23) gives

$$\hat{k} \equiv \max\left\{\frac{\sigma_v^2 \tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)\hat{p}}{\gamma \sigma_v^2 \sigma_\varepsilon^2}\,,\, 0\right\}\lambda + \tilde{z}. \tag{24}$$

The information-inference problem faced by uninformed traders is more complicated when short sales are prohibited because they do not know whether the short-sale constraint is binding for informed traders. However, they can infer the probability that informed traders are short-sale constrained and update their beliefs about the stock payoff accordingly.

Note that there are two possible signals embedded in (24). First, if informed traders are bound by the short-sale constraint, then $\max\left\{\frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\hat{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}, 0\right\} = 0$ and uninformed traders observe

$$\hat{k} = \tilde{z}. \tag{25}$$

Because $\hat{k}$ is uncorrelated with $\tilde{x}$ if informed traders are bound, uninformed traders would infer only that $\sigma_v^2\tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)\hat{p} \leq 0$, which implies $\tilde{x} \leq \gamma^2\sigma_v^4\Theta^2\hat{p}$, if they could discern that informed traders were constrained.[12] Hence, the uninformed traders' posterior distribution of $\tilde{x}$ would simply be a truncation of the prior,

$$\tilde{x}|\hat{k}, \hat{p} \sim \mathcal{TN}\big(0, \sigma_v^2 + \sigma_\varepsilon^2, -\infty, \gamma^2\sigma_v^2\Theta^2\hat{p}\big), \tag{26}$$

if they could infer that informed traders were bound by the short-sale constraint.

Second, if informed traders are not bound by the constraint, then uninformed traders observe

$$\hat{k} = \frac{\lambda[\sigma_v^2\tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)\hat{p}]}{\gamma\sigma_v^2\sigma_\varepsilon^2} + \tilde{z} \tag{27}$$

because $\max\left\{\frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\hat{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}, 0\right\} = \frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\hat{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}$. Like in the absence of a ban, this is equivalent to observing $f \equiv \frac{\lambda}{\gamma\sigma_\varepsilon^2}\tilde{x} + \tilde{z}$ because $\hat{p}$ is known in equilibrium. Additionally, uninformed traders would infer $\sigma_v^2\tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)\hat{p} > 0$, which implies $\tilde{x} > \gamma^2\sigma_v^4\Theta^2\hat{p}$, if they could discern that informed traders were not bound by the ban. Thus, the uninformed traders' posterior distribution of $\tilde{x}$ would be

$$\tilde{x}|\hat{k}, \hat{p} \sim \mathcal{TN}\left(\frac{\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2}\big(\hat{k} + \frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}\hat{p}\big)}{\Sigma^2}, \frac{(\sigma_v^2 + \sigma_\varepsilon^2)\Theta^2}{\Sigma^2}, \gamma^2\sigma_v^2\Theta^2\hat{p}, \gamma^2\sigma_v^2\Theta^2\hat{p} + \frac{\gamma\sigma_\varepsilon^2}{\lambda}\hat{k}\right). \tag{28}$$

if they could deduce that informed traders were not bound. Because uninformed traders cannot discern whether informed traders are bound by the short-sale restriction, however, they are uncertain

[12] Although $\hat{k}$ is uncorrelated with $\tilde{x}$ when informed traders are bound, the price nevertheless depends on both x and z when informed traders are bound because whether they are bound endogenously depends on x and z. In other words, whether informed traders are bound is correlated with $\tilde{x}$, but $\hat{k}$ is uncorrelated with $\tilde{x}$ conditional on the short-sale constraint binding informed traders' demand.

about whether they are observing (25) or (27) in equilibrium and, therefore, must take into account this uncertainty when formulating their beliefs and choosing their stock allocation.

Uninformed traders maximize their expected utility subject to a short-sale restriction:

$$\max_{\hat{s}_u \mathbb{R}^+} \quad \mathbb{E}\big[-\exp[-\gamma\tilde{\hat{c}}_u] \,|\, \hat{k}\big] \tag{29}$$

$$\text{s.t.} \quad \tilde{\hat{c}}_u = \hat{b}_u + \hat{s}_u \tilde{v} \tag{30}$$

$$\hat{b}_u = w_b + (w_s - \hat{s}_u)\hat{p}. \tag{31}$$

Substituting (30) and (31) into (29) and integrating over both $\tilde{v}$ and $\tilde{x}$ (according to either (26) or (28)) provides an expression for an uninformed trader's expected utility conditional on whether informed traders are bound by the short-sale ban. If uninformed traders could infer that informed traders were bound by the ban, then an uninformed trader's conditional expected utility would be[13]

$$-\frac{\Phi\big(\frac{\hat{\delta}+\hat{s}_u}{\Theta}\big)}{\Phi\big(\frac{\hat{\delta}}{\Theta}\big)} \exp\big[-\gamma\big(w_b + (w_s - \hat{s}_u)\hat{p} - \tfrac{1}{2}\gamma\sigma_v^2\hat{s}_u^2\big)\big], \tag{32}$$

[13] An uninformed trader's expected utility is represented by

$$-\frac{\displaystyle\int_{-\infty}^{\gamma^2\sigma_v^2\Theta^2\hat{p}} \int_{-\infty}^{\infty} \exp\big[-\gamma\tilde{\hat{c}}_u\big] \exp\left[-\frac{\big(\tilde{v}-\frac{\sigma_v^2\tilde{x}}{\sigma_v^2+\sigma_\varepsilon^2}\big)^2}{2\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}}\right] \exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)}\right] \exp\left[-\frac{\hat{k}^2}{2\sigma_z^2}\right] \mathrm{d}\tilde{v}\,\mathrm{d}\tilde{x}}{\displaystyle\int_{-\infty}^{\gamma^2\sigma_v^2\Theta^2\hat{p}} \int_{-\infty}^{\infty} \exp\left[-\frac{\big(\tilde{v}-\frac{\sigma_v^2\tilde{x}}{\sigma_v^2+\sigma_\varepsilon^2}\big)^2}{2\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}}\right] \exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)}\right] \exp\left[-\frac{\hat{k}^2}{2\sigma_z^2}\right] \mathrm{d}\tilde{v}\,\mathrm{d}\tilde{x}}$$

if informed traders are constrained, but an uninformed trader's expected utility is represented by

$$-\frac{\displaystyle\int_{\gamma^2\sigma_v^2\Theta^2\hat{p}}^{\gamma^2\sigma_v^2\Theta^2\hat{p}+\frac{\gamma\sigma_\varepsilon^2}{\lambda}\hat{k}} \int_{-\infty}^{\infty} \exp\big[-\gamma\tilde{\hat{c}}_u\big] \exp\left[-\frac{\big(\tilde{v}-\frac{\sigma_v^2\tilde{x}}{\sigma_v^2+\sigma_\varepsilon^2}\big)^2}{2\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}}\right] \exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)}\right] \exp\left[-\frac{\Big(\hat{k}-\frac{\lambda[\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\hat{p}]}{\gamma\sigma_v^2\sigma_\varepsilon^2}\Big)^2}{2\sigma_z^2}\right] \mathrm{d}\tilde{v}\,\mathrm{d}\tilde{x}}{\displaystyle\int_{\gamma^2\sigma_v^2\Theta^2\hat{p}}^{\gamma^2\sigma_v^2\Theta^2\hat{p}+\frac{\gamma\sigma_\varepsilon^2}{\lambda}\hat{k}} \int_{-\infty}^{\infty} \exp\left[-\frac{\big(\tilde{v}-\frac{\sigma_v^2\tilde{x}}{\sigma_v^2+\sigma_\varepsilon^2}\big)^2}{2\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}}\right] \exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)}\right] \exp\left[-\frac{\Big(\hat{k}-\frac{\lambda[\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\hat{p}]}{\gamma\sigma_v^2\sigma_\varepsilon^2}\Big)^2}{2\sigma_z^2}\right] \mathrm{d}\tilde{v}\,\mathrm{d}\tilde{x}}$$

if informed traders are not constrained. The last exponent in both the numerator and denominator of the former expression follows from the convolution of $\tilde{z}$ and $\hat{k}$ according to the relation in (25), whereas the last exponent in the latter expression follows from the relation in (27). Likewise, the limits of integration for the two expression follow from (26) and (28). Integrating these expressions yields (32) and (34), respectively.

where

$$\hat{\delta} \equiv \gamma \Theta^2 \hat{p}. \tag{33}$$

Conversely, if uninformed traders could deduce that informed traders were not bound by the short-sale ban, then an uninformed trader's conditional expected utility would be

$$-\frac{\Phi\left(\frac{\hat{\beta}-\hat{s}_u}{\Sigma}\right) - \Phi\left(\frac{\hat{\alpha}-\hat{s}_u}{\Sigma}\right)}{\Phi\left(\frac{\hat{\beta}}{\Sigma}\right) - \Phi\left(\frac{\hat{\alpha}}{\Sigma}\right)} \exp\left[-\gamma\left(w_b + w_s\hat{p} + \frac{1}{\Sigma^2}\left[\frac{\hat{\beta}}{\gamma}\hat{s}_u - \tfrac{1}{2}\gamma\sigma_v^2\Theta^2\left(1 + \frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\right)\hat{s}_u^2\right]\right)\right], \tag{34}$$

where

$$\hat{\alpha} \equiv \Theta^2\left(-\frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2}{\lambda(\sigma_v^2+\sigma_\varepsilon^2)}\hat{k} - \gamma\hat{p}\right) \tag{35}$$

$$\hat{\beta} \equiv \Theta^2\left(\frac{\lambda\sigma_v^2}{\sigma_\varepsilon^2\sigma_z^2}\hat{k} - \gamma\hat{p}\right). \tag{36}$$

The law of total expectation implies that the uninformed traders' expected utility conditional on $\hat{k}$ and $\hat{p}$ but not conditional on whether informed traders are bound by the short-sale constraint is

$$\begin{aligned} &-\frac{\Phi\left(\frac{\hat{\delta}+\hat{s}_u}{\Theta}\right)\exp\left[-\gamma\left(w_b + (w_s - \hat{s}_u)\hat{p} - \frac{1}{2}\gamma\sigma_v^2\hat{s}_u^2\right)\right]}{\Phi\left(\frac{\hat{\delta}}{\Theta}\right) + \Phi\left(\frac{\hat{\beta}}{\Sigma}\right) - \Phi\left(\frac{\hat{\alpha}}{\Sigma}\right)} \\ &\quad - \frac{\left(\Phi\left(\frac{\hat{\beta}-\hat{s}_u}{\Sigma}\right) - \Phi\left(\frac{\hat{\alpha}-\hat{s}_u}{\Sigma}\right)\right)\exp\left[-\gamma\left(w_b + w_s\hat{p} + \frac{1}{\Sigma^2}\left[\frac{\hat{\beta}}{\gamma}\hat{s}_u - \frac{1}{2}\gamma\sigma_v^2\Theta^2\left(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\right)\hat{s}_u^2\right]\right)\right]}{\Phi\left(\frac{\hat{\delta}}{\Theta}\right) + \Phi\left(\frac{\hat{\beta}}{\Sigma}\right) - \Phi\left(\frac{\hat{\alpha}}{\Sigma}\right)}, \end{aligned} \tag{37}$$

which is obtained by weighting the two conditional utility expressions by the probability that informed traders are bound by the short-sale ban, i.e., weighting (32) by $\frac{\Phi\left(\frac{\hat{\delta}}{\Theta}\right)}{\Phi\left(\frac{\hat{\delta}}{\Theta}\right)+\Phi\left(\frac{\hat{\beta}}{\Sigma}\right)-\Phi\left(\frac{\hat{\alpha}}{\Sigma}\right)}$ and (34) by $\frac{\Phi\left(\frac{\hat{\beta}}{\Sigma}\right)-\Phi\left(\frac{\hat{\alpha}}{\Sigma}\right)}{\Phi\left(\frac{\hat{\delta}}{\Theta}\right)+\Phi\left(\frac{\hat{\beta}}{\Sigma}\right)-\Phi\left(\frac{\hat{\alpha}}{\Sigma}\right)}$. Thus, (37) accounts for the uninformed traders' uncertainty about whether informed traders are bound by the restriction. Differentiating (37) with respect to $\hat{s}_u$ and solving the corresponding first-order condition provides an expression for an uninformed trader's stock demand.

Theorem 3. *Given a price $\hat{p}$ and signal $\hat{k}$, an uninformed trader's demand when short selling is* *prohibited is characterized by the maximum of* 0 *and*

$$\hat{s}_u = \frac{-\Phi\left(\frac{\hat{\delta}+\hat{s}_u}{\Theta}\right)\left(\frac{\phi\left(\frac{\hat{\delta}+\hat{s}_u}{\Theta}\right)\Theta}{\Phi\left(\frac{\hat{\delta}+\hat{s}_u}{\Theta}\right)}+\hat{\delta}\right)-\left(\Phi\left(\frac{\hat{\beta}-\hat{s}_u}{\Sigma}\right)-\Phi\left(\frac{\hat{\alpha}-\hat{s}_u}{\Sigma}\right)\right)\frac{\Theta^2}{\Sigma^2}\left(\frac{\left(\phi\left(\frac{\hat{\alpha}-\hat{s}_u}{\Sigma}\right)-\phi\left(\frac{\hat{\beta}-\hat{s}_u}{\Sigma}\right)\right)\Sigma}{\Phi\left(\frac{\hat{\beta}-\hat{s}_u}{\Sigma}\right)-\Phi\left(\frac{\hat{\alpha}-\hat{s}_u}{\Sigma}\right)}-\hat{\beta}\right)\hat{\Psi}}{\Phi\left(\frac{\hat{\delta}+\hat{s}_u}{\Theta}\right)\left(1+\frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)+\left(\Phi\left(\frac{\hat{\beta}-\hat{s}_u}{\Sigma}\right)-\Phi\left(\frac{\hat{\alpha}-\hat{s}_u}{\Sigma}\right)\right)\frac{\gamma^2\sigma_v^2\Theta^4}{\Sigma^2}\left(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\right)\hat{\Psi}}, \quad (38)$$

where

$$\hat{\Psi} \equiv \exp\left[\frac{\lambda^2\sigma_v^4\Theta^2}{\sigma_\varepsilon^4\sigma_z^2\Sigma^2}\left(\hat{\alpha}-\tfrac{1}{2}\hat{s}_u\right)\hat{s}_u\right]. \quad (39)$$

Like the setting without a prohibition against short sales, a closed-form expression for aggregate demand does not exist when short sales are prohibited. Thus, a closed-form expression for the price does not exist. The equilibrium price is obtained using numerical methods, as discussed below.

2.3 Numerical Solution Method

In the previous subsections, I characterize expressions for the traders' demand functions both with and without a short-sale prohibition. While the demand functions for informed traders are in closed form, the demand functions for uninformed traders are not. A closed-form expression for the uninformed traders' demand—and, hence, the equilibrium stock price—does not exist because the basic nature of a short-sale prohibition gives rise to demand functions that depend on truncated probability distributions. Therefore, I numerically compute the equilibrium price and allocations.

The algorithm is as follows. First, I draw values of v, ε, and z from their respective distributions. Next, I conjecture a price and use it to compute informed traders' demand with either (6) or (22). I then compute the signal observed by uninformed traders with either (8) or (24), which I subsequently use to compute the uninformed traders' demand with either (18) or (38). Last, I update the price (using a numerical solver) until excess demand equals zero. I repeat this algorithm for 25,000 pseudo-random draws of v, ε, and z and values of λ ranging from 0.05 to 0.95.

Table I lists the parameter values for the numerical analysis.[14] I report results for three different

[14]Parameter values are selected to illustrate the qualitative effects of a short-sale ban; the model is not calibrated.

Table I: Parameter Values.

Variable	Symbol	#1	#2	#3
Variance of $\tilde{v}$	σ_v^2	1	1	1
Variance of $\tilde{\varepsilon}$	σ_ε^2	0.10	0.25	0.10
Variance of $\tilde{z}$	σ_z^2	0.10	0.10	0.05
Risk aversion coefficient	γ	1	1	1
Stock endowment	w_s	1	1	1
Bond endowment	w_b	0	0	0

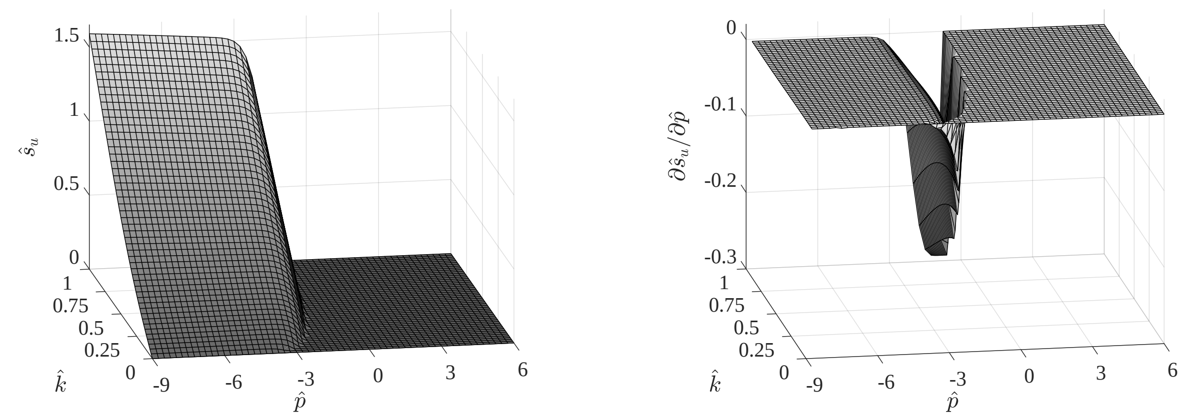
parameterizations to demonstrate how the various parameters influence the equilibrium outcomes. I also verify the robustness of the results over a wider range of parameters; to maintain a streamlined exposition, however, I do not report the robustness results, which are qualitatively consistent with those presented below.[15] Informed and uninformed traders are identical in every respect except their information sets. In all of the parameterizations, I set the traders' risk aversion coefficient, γ, equal to 1. Informed and uninformed traders each receive a stock endowment of 1 and a bond endowment of 0. Because the mass of informed traders is λ and the mass of uninformed traders is $1 - \lambda$, the aggregate endowment of stock and bonds in the economy is 1 and 0, respectively. I normalize the variance of the stock payoff, σ_v^2, at 1 but consider values for the variance of the noise component of the informed traders' private signal, σ_ε^2, ranging from 0.10 to 0.25.[16] I also consider variances of liquidity-trader demand ranging from 0.05 to 0.10.

2.4 Existence and Uniqueness

As a preliminary matter, I numerically demonstrate that a unique equilibrium exists with a short-sale prohibition (recall, the equilibrium without a short-sale ban is unique, as stated in Theorem 2).

[15] I verify that the qualitative results are robust to various combinations of parameter values for σ_ε^2 and σ_z^2, ranging from 0.05 to 0.50 and from 0.01 to 0.20, respectively.

[16] These values of σ_ε^2 provide informed traders with an information advantage that is large enough to generate interesting results. If there is not a sufficient degree of information asymmetry, then a short-sale ban has little effect on equilibrium outcomes because traders almost never short the stock. In the limit as $\sigma_\varepsilon^2 \to \infty$, traders are symmetrically (un)informed and perfect risk sharing occurs.



(a) $\hat{s}_u$

(b) $\partial\hat{s}_u/\partial\hat{p}$

Figure 1: Demand Function. (a) An uninformed trader's demand when short sales are prohibited and the corresponding partial derivative with respect to the stock price are plotted in (a) and (b), respectively. The parameter values correspond to parameterization #1 and $\lambda = 0.5$.

First, note that the informed traders' demand function, which is given by (22), is continuous and monotonically non-increasing in the stock price (i.e., $\partial\hat{s}_i/\partial\hat{p} \leq 0$). Provided that the short-sale ban is not binding, the informed traders' demand is monotonically decreasing in the stock price.

Because the analytic expression for the uninformed traders' demand function, which is given by (38), is not directly interpretable, I plot the demand function (Figure 1(a)) and its partial derivative with respect to $\hat{p}$ (Figure 1(b)) over a range of values for the stock price and the uninformed traders' signal $\hat{k}$.[17] Figures 1(a) and 1(b) indicate that an uninformed trader's demand function is continuous and monotonically non-increasing in the stock price and monotonically decreasing when the short-sale constraint is not binding.

Together, the continuous and monotonic nature of the informed and uninformed traders' demand functions indicate that aggregate stock demand is continuous and monotonically non-increasing in

[17]The figures are generated using the baseline parameterization #1 with $\lambda = 0.5$. Other parameterizations and values for $\lambda \in (0, 1)$ produce similar results. The demand is reported for $\hat{k} \in [0, 1]$ because, in equilibrium, $\hat{k} = \lambda\hat{s}_i + z$ must be within the unit interval. Over all of the parameterizations and the 25,000 pseudo-random draws, the minimum and maximum equilibrium prices are approximately -4.3 and 4.0, respectively.

the stock price. Furthermore, aggregate demand appears to be monotonically decreasing in the price, provided that the short-sale constraint is not binding for both types of traders. This suggests that an equilibrium exists when short selling is prohibited. Moreover, the equilibrium seems to be unique when at least one type of trader is not short-sale constrained.[18]

3 Effects of a Short-Sale Prohibition

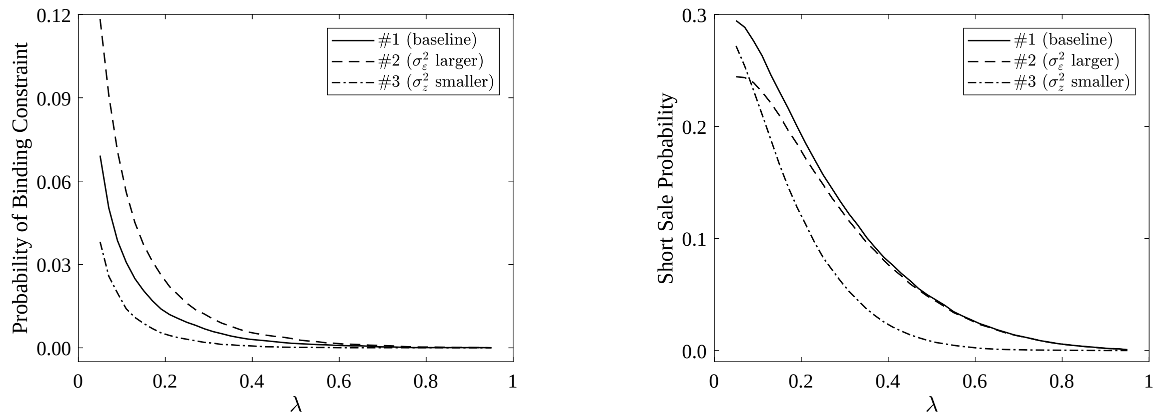
In this section, I discuss the impact of a short-sale prohibition when the mass of informed traders is exogenous. Such a setting is relevant for understanding the effects of barring short sales in cases where traders may not be able to easily or quickly augment their information sets, such as the temporary ban on short selling imposed by the SEC and other financial regulators around the world in 2008. I later evaluate the effects of a short-sale ban when information acquisition is endogenous.

3.1 Short Selling and Binding Constraints

A short-sale ban is seldom binding for informed traders in equilibrium. As shown in Figure 2(a), the probability of a ban constraining informed traders is roughly 0% – 12%, depending on the parameterization and the fraction of traders that are informed, λ. A short-sale ban tends to bind when the stock price is higher and informed traders' private information is more negative, i.e., when the stock is "overvalued." However, because the traders are risk averse, the desire to share risk usually dissuades informed traders from selling all of their shares, even when the stock is somewhat overvalued. Although the constraint binds with only a small probability, prohibiting short sales has a considerable impact on equilibrium outcomes because the *possibility* that informed traders are constrained affects uninformed traders' beliefs, as discussed below in Section 3.2.

Notably, a short-sale ban is not always binding in cases where informed traders prefer to hold

[18] Although multiple equilibria exist when $z = 1$, as any price that results in both types of traders holding no stock can support an equilibrium in this case, the probability that $z = 1$ is zero because z is drawn from $\mathcal{TN} \sim (0, \sigma_z^2, 0, 1)$. Thus, the (continuous) equilibrium appears to be almost surely unique (cf. Pálvölgyi and Venter, 2015).



(a) Binding Constraint

(b) Short Selling

Figure 2: Likelihood of Binding Constraint and Short Selling. The probability that a short-sale prohibition is binding for informed traders is plotted in (a). The probability that informed traders hold a short position in the absence of a short-sale prohibition is plotted in (b).

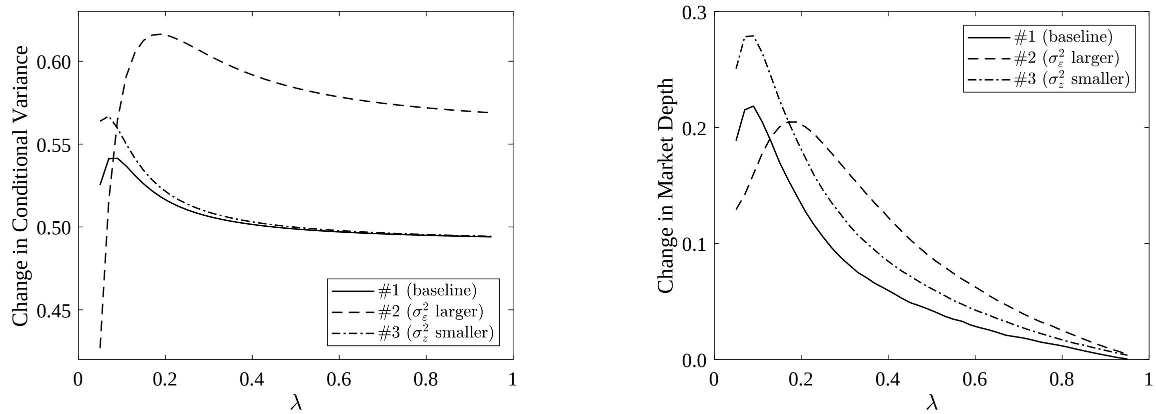
a short position in the absence of a ban, as can be seen by comparing Figure 2(a) with Figure 2(b), which plots the probability that informed traders hold a short position when short selling is permitted. This is because, as discussed below in Sections 3.2 and 3.4, a short-sale ban increases uncertainty for uninformed traders, which results in a lower stock price and greater demand for informed traders on average. The probability that traders in the model sell short is compatible with empirical evidence reported by Dechow et al. (2001) and Asquith, Pathak, and Ritter (2005) (at least when λ is not too small), who find that short interest tends to be small in practice.

The finding that a short-sale ban does not always bind when traders would otherwise hold a short position without a ban has two important empirical implications, although testing them could prove to be challenging. First, from a policy perspective, observed short interest may not provide an accurate estimate of the extent to which a proposed short-sale constraint would bind in practice. Second, short-sale constraints may influence equilibrium attributes even when the constraints are not observed to bind, which contrasts with other studies that suggest that a short-sale constraint affects prices only when it binds (e.g., Blocher, Reed, and Van Wesep, 2013; Reed, Saffi, and Van Wesep, 2016; Daniel, Klos, and Rottke, 2017; Weitzner, 2017). Both of these qualitative empirical implications should hold provided that at least some uninformed traders cannot discern whether informed traders are bound, but the quantitative impact will depend on the degree to which a short-sale ban alters uninformed traders' beliefs.

The likelihood of informed traders being bound by a short-sale constraint depends on the degree of information asymmetry, the volatility of liquidity trades, and the mass of informed traders. A short-sale ban is more likely to bind when informed traders' private information is less precise (σ_ε^2 is larger in parameterization #2 than in parameterization #1) or when liquidity trades have a larger variance (σ_z^2 is smaller in parameterization #3 than in parameterization #1). Because uninformed traders face less adverse selection and liquidity trades provide more camouflage in these cases, informed traders are able to trade more shares of stock and are, thus, more likely to sell all of their shares. Furthermore, because the price tends to more accurately reflect the fundamental value of the stock when more traders are informed, the likelihood of there being an overvaluation of the stock high enough to persuade informed traders to sell all of their shares declines as λ increases.

3.2 Market Efficiency

The informational efficiency of the market reflects the degree to which informed traders' private information is conveyed to uninformed traders through trade. Notably, a short-sale ban may alter market efficiency even when the ban is nonbinding because the possibility that informed traders' demand is censored affects how uninformed traders update their beliefs. This result contrasts with much of the extant literature, which typically finds that a short-sale constraint either does not affect market efficiency (e.g., Nezafat, Schroder, and Wang, 2017) or affects market efficiency only when the constraint binds (e.g., Bai, Chang, and Wang, 2006; Cao, Zhang, and Zhou, 2007). The model's qualitative implications should be robust as long as at least some uninformed traders cannot discern whether informed traders are bound by the short-sale ban, but a short-sale prohibition would not



(a) Market Efficiency

(b) Market Depth

Figure 3: Market Efficiency and Market Depth. The difference between the average conditional variance of $\tilde{x}$ and the difference between average market depth with and without a short-sale prohibition are plotted in (a) and (b), respectively.

affect uninformed traders' beliefs or other equilibrium attributes unless it were binding if *all* uninformed traders could discern whether informed traders were bound.

Figure 3(a), which plots the change in the uninformed traders' conditional variance of $\tilde{x}$ arising from a short-sale ban, shows that uninformed traders face greater uncertainty when short selling is prohibited. Thus, measuring market efficiency as the precision (or inverse of the variance) of uninformed traders' beliefs about informed traders' private information, as in Spiegel and Subrahmanyam (1992) and Lenkey (2014), a short-sale ban tends to result in a less efficient market (i.e., $\mathbb{V}[\tilde{x}|\hat{k}]^{-1} < \mathbb{V}[\tilde{x}|k]^{-1}$). The finding that uninformed traders observe a less precise signal of the informed traders' private information and, as a result, face greater uncertainty regarding the stock payoff under a short-sale ban is consistent with other studies (e.g., Bai, Chang, and Wang, 2006; Cao, Zhang, and Zhou, 2007), but, as stated above, the distinction arises because I find that a short-sale ban may affect market efficiency even when the ban is not binding.

A short-sale ban's effect on efficiency is more pronounced when informed traders have a smaller information advantage and when liquidity trades are less volatile. In the former case, prohibiting short sales has a greater effect when σ_ε^2 is larger because a short-sale ban binds more frequently, which means that informed traders' demand is censored more often.[19] Additionally, conditional on informed traders being bound by the constraint, uninformed traders face greater uncertainty regarding the informed traders' private information when σ_ε^2 is larger (see (26)). In the latter case, censoring informed traders' demand has a bigger impact when σ_z^2 is smaller because although a ban binds less frequently, the signal $\hat{k}$ is more informative—and, therefore, censoring the signal increases uncertainty to a greater extent—when σ_z^2 is smaller (see (28)).

3.3 Market Depth

Following Kyle (1985) and Leland (1992), I compute market depth as the inverse of the change in price in response to a marginal increase in liquidity-trader demand (i.e., $(\partial p/\partial z)^{-1}$). Under this measure, a market is deeper if a liquidity trade has a smaller impact on the equilibrium price. Because a closed-form expression for the price is unattainable, I compute the derivative numerically.

Figure 3(b), which plots the difference between average market depth when short sales are forbidden and when short sales are permitted, indicates that market depth tends to increase under a short-sale ban. Because there are several states in which informed traders hold zero shares of stock when short-sales are prohibited, there is a greater likelihood that a marginal change in aggregate demand stems from a change in liquidity trader demand as compared to when short sales are allowed. Consequently, uninformed traders' beliefs about the stock payoff are less affected by changes in aggregate demand. Furthermore, uninformed traders trade less aggressively under a short-sale ban because the decrease in market efficiency increases their uncertainty. Together, these two effects

[19] Although the ban binds with low probability when λ is large, the possibility of informed traders being bound increases the uninformed traders' conditional variance of $\tilde{x}$ because there is a non-trivial possibility that the informed traders' private information is extremely negative. Moreover, the magnitude of the effect is non-monotonic in the mass of informed traders λ for the following reason. When λ is small, a short-sale ban may have a smaller effect on efficiency because little information is revealed through trade when there are few informed traders. As λ increases up to a certain point, there is a greater impact from imposing a short-sale ban because there are more informed traders whose trades would reveal a greater amount of information if their demand were uncensored as in the absence of a ban. When λ is sufficiently large, the effects of a short-sale ban are less substantial because the short-sale constraint binds less often.

result in a deeper market when short sales are prohibited.

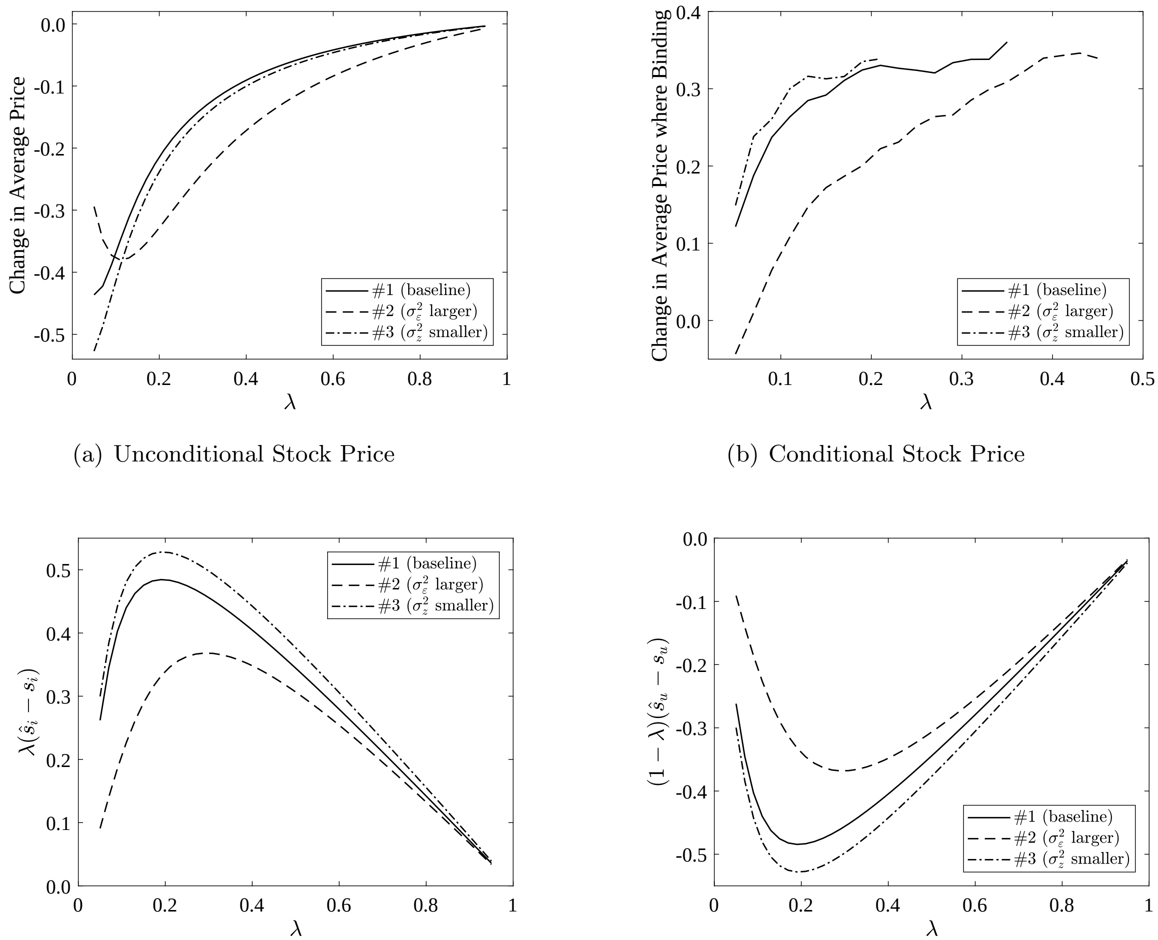
Although market depth is not synonymous with liquidity, the finding that depth increases under a short-sale ban contrasts with much of the extant literature, which generally finds that liquidity (measured as the size of the bid-ask spread) decreases when short selling is restricted (e.g., Diamond and Verrecchia, 1987; Liu and Wang, 2018). It is also important to note that barring short sales could potentially have a detrimental effect on market depth in practice unless market makers, who provide much liquidity by taking temporary short positions, were exempt from such a ban.

3.4 Prices and Allocations

Figure 4(a), which plots the difference between the average stock price with a short-sale prohibition and the average price without a prohibition, shows that the average price is lower when short sales are banned. As discussed in Section 3.2, a short-sale ban increases uninformed traders' uncertainty about the stock payoff. The increased uncertainty results in a lower average price because uninformed traders face greater risk and are, therefore, willing to pay less to hold a given amount of stock. For the market to clear, the price must fall on average. A short-sale ban tends to have a greater effect on the average equilibrium price in cases where uninformed traders experience a greater increase in uncertainty. Bai, Chang, and Wang (2006) and Cao, Zhang, and Zhou (2007) document a similar effect, but in their models the price may decline only if a short-sale constraint binds, whereas I find that the price may decline even when a ban is not binding because uninformed traders are uncertain about whether the ban binds.[20]

Although a short-sale ban leads to a lower price on average, restricting short sales tends to result in a higher price in cases where the constraint is binding, as shown in Figure 4(b), which plots the

[20] More specifically, Bai, Chang, and Wang (2006) show that a short-sale constraint can lead to a lower average price when the mass of informed traders is small but a higher average price when the mass of informed traders is larger. Presumably, this is because a short-sale constraint affects market efficiency in their model only when it binds, so the reduction in informational efficiency dominates any allocational effects when the mass of informed traders is small but not when the mass is larger. In contrast, market efficiency declines in my model even when the short-sale ban is nonbinding, so informational effects tend to outweigh any allocational effects regardless of the traders' masses.



(a) Unconditional Stock Price

(b) Conditional Stock Price

(c) Informed Traders' Allocation

(d) Uninformed Traders' Allocation

Figure 4: Stock Price and Allocations. The difference between the unconditional average stock price with and without a short-sale prohibition is plotted in (a). The difference between the average stock price in states (defined by realizations of $\tilde{x}$ and $\tilde{z}$) where a short-sale prohibition is binding for informed traders and the average stock price in the same states without a short-sale prohibition is plotted in (b). The difference between the average amount of stock held with and without a short-sale prohibition by informed and uninformed traders, respectively, is plotted in (c) and (d).

difference between the average price in states (defined by the realizations of $\tilde{x}$ and $\tilde{z}$) where informed traders are bound by the constraint and the average price in the same states when short selling is permitted.[21] This conditional effect on the price, which is consistent with prior theory and much empirical evidence, arises from the fact that a short-sale ban prevents the price from fully reflecting the informed traders' negative information.[22] Despite the fact that a short-sale ban tends to raise prices when it binds, the unconditional average price is lower because the ban seldom binds in equilibrium and the increase in uncertainty, *ceteris paribus*, drives down the price.

Figures 4(c) and 4(d), respectively, plot the differences between the informed and uninformed traders' average allocations with and without a short-sale prohibition. Informed traders tend to hold more stock when short sales are banned whereas uninformed traders tend to hold less. The shift in allocations is due to the decline in market efficiency. Because barring short sales increases uninformed traders' uncertainty about the stock payoff, they face more risk when short selling is prohibited. This raises the aggregate level of perceived risk in the economy. Although informed traders do not face additional uncertainty, they share in some of the increased risk faced by uninformed traders by holding additional shares of stock, thereby enabling the perceived risk in the economy to be shared more efficiently. Thus, optimal risk sharing under a short-sale prohibition entails an increase in risk exposure for both informed and uninformed traders. A short-sale ban impacts equilibrium allocations to a greater extent when σ_z^2 is smaller because uninformed traders experience a greater increase in uncertainty and informed traders are willing to absorb a substantial amount of that risk by holding more stock. In contrast, banning short sales has a smaller effect on allocations when σ_ε^2 is larger because informed traders are less willing to bear additional risk when their signal x is less precise.

[21] To mitigate small sample effects, I restrict attention to parameterizations where the short-sale ban binds for at least 100 simulated realizations of $\tilde{x}$ and $\tilde{z}$.

[22] See, e.g., Miller (1977), Jarrow (1980), Allen, Morris, and Postlewaite (1993), Chen, Hong, and Stein (2002), Jones and Lamont (2002), Boehme, Danielsen, and Sorescu (2006), Beber and Pagano (2013), Blocher, Reed, and Van Wesep (2013), Boehmer, Jones, and Zhang (2013), Daniel, Klos, and Rottke (2017); but cf. Barlevy and Veronesi (2003), Hong and Stein (2003), Marin and Olivier (2008).

4 Extensions

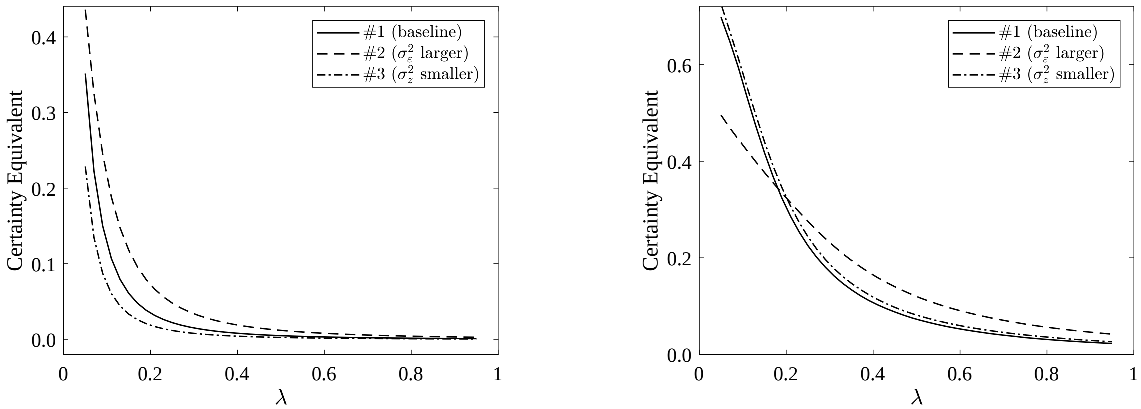
I analyze two extensions of the model. First, I evaluate the effects of a short-sale ban when information acquisition is endogenous in Section 4.1. I then demonstrate the robustness of the results when short selling is permissible but costly in Section 4.2.

4.1 Endogenous Information Acquisition

Here, I evaluate the effects of a short-sale ban when traders can choose whether to become informed. For brevity, I focus on results that differ from those in Section 3. The differences in results are driven by the fact that a short-sale ban induces more traders to become informed.

To determine the mass of traders who choose to become informed, I first compute a certainty equivalent of being informed both with and without a short-sale prohibition when the mass of informed traders λ is exogenous. This involves computing a trader's ex post utility for each simulated realization of $\tilde{v}$, $\tilde{\varepsilon}$, and $\tilde{z}$ by substituting the simulated stock payoff v and the appropriate numerical values for the price and allocations into the trader's objective function. A trader's ex ante expected utility is the average ex post utility across all simulated realizations. The certainty equivalent of being informed is the amount of additional wealth that must be endowed to an uninformed trader such that the trader's ex ante expected utility is the same as an informed trader's. A positive (negative) certainty equivalent indicates that a trader is better (worse) off when informed (uninformed).

As shown in Figures 5(a) and 5(b), which plot the certainty equivalents of being informed when short sales are permitted and prohibited, respectively, informed traders attain a greater expected utility than uninformed traders when information acquisition is exogenous, regardless of whether a short-sale ban exists. Consequently, all traders will choose to become informed unless they incur a sufficiently large information-acquisition cost. I assume that traders incur a fixed cost to become informed, which could be thought of as, for example, a search cost or a monetary cost to acquire information. I take a reduced-form approach rather than modeling an explicit cost function. Accordingly,



(a) Short Sales Permitted　　(b) Short Sales Prohibited

Figure 5: Value of Information. The certainty equivalent of being informed when short sales are permitted and prohibited is plotted in (a) and (b), respectively.

for any given value of $\lambda \in (0, 1)$, I assume that the information market is in equilibrium. This means that an informed trader's expected utility, after accounting for the cost of information acquisition, equals an uninformed trader's expected utility (in other words, the cost of acquiring information for a given λ equals the certainty equivalent of being informed for that value of λ). If their expected utilities were not equal, then some uninformed traders would choose to become informed, or vice versa, until their expected utilities were equal. I also assume that the cost of acquiring information is independent of whether short sales are allowed. Hence, in equilibrium the mass of informed traders when short sales are prohibited, $\hat{\lambda}$, must be such that the certainty equivalent of being informed when short sales are prohibited is the same as when short sales are permitted, i.e.,

$$\log[-\hat{U}_i(\hat{\lambda})] - \log[-\hat{U}_u(\hat{\lambda})] = \log[-U_i(\lambda)] - \log[-U_u(\lambda)], \tag{40}$$

where U_i and U_u, respectively, denote the informed and uninformed traders' expected utilities.

There appears to be a unique equilibrium in the information market for any given λ. Regardless of whether short selling is permitted, the certainty equivalent of being informed is monotonically

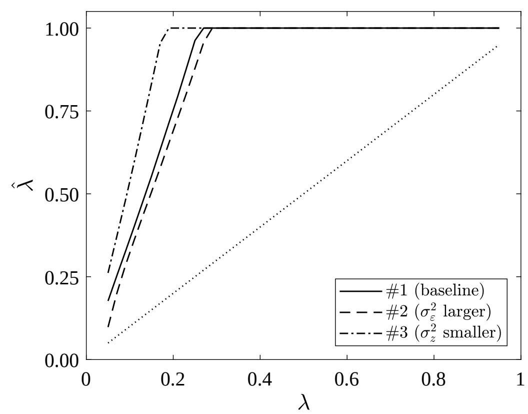
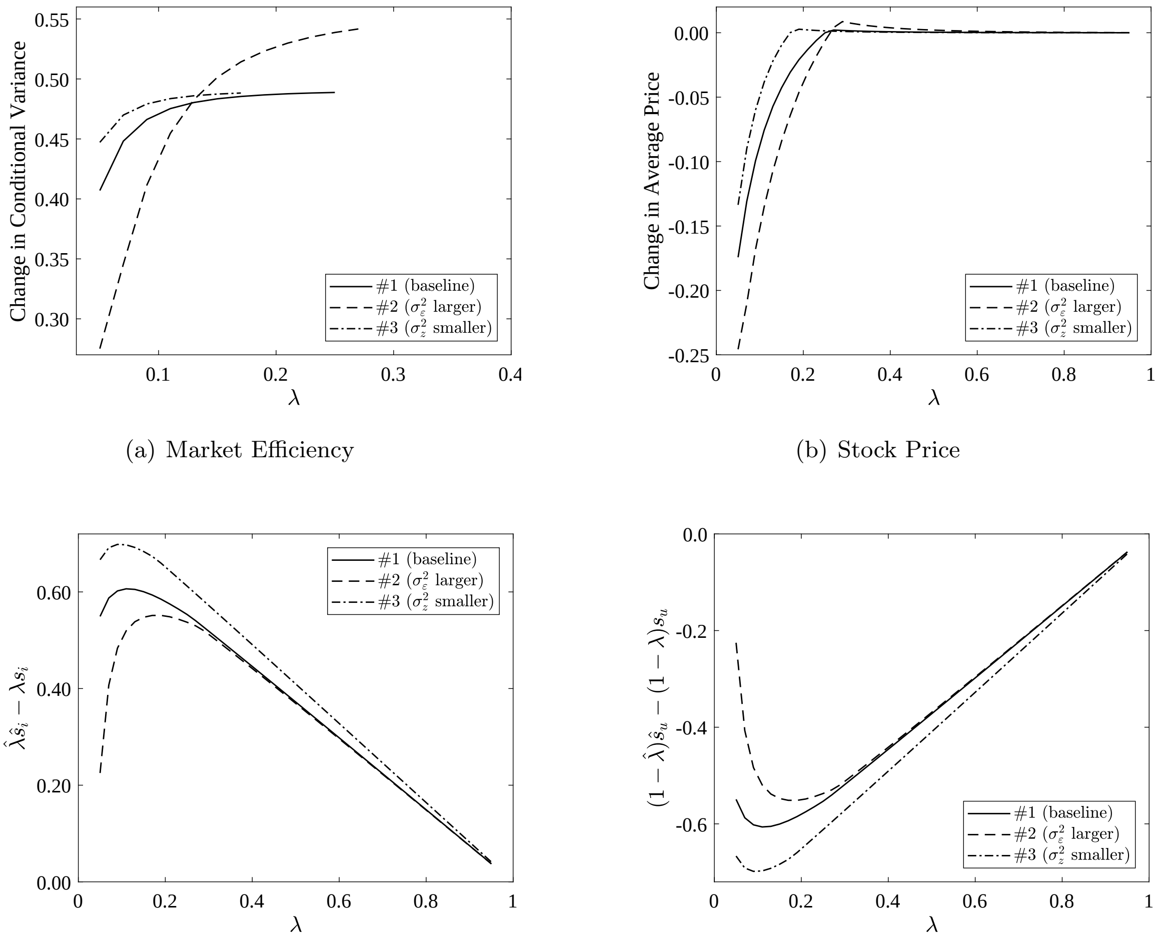


Figure 6: Mass of Informed Traders with Endogenous Information Acquisition. The mass of informed traders when short sales are prohibited is plotted. The dotted line represents the mass of informed traders when short sales are permitted.

decreasing in the fraction of informed traders, as Figure 5 shows.[23] Thus, for any given λ, there appears to be a unique $\hat{\lambda}$ such that either the certainty equivalent of being informed in the presence of a short-sale ban is the same as the certainty equivalent in the absence of a ban or the certainty equivalent is such that all traders choose to become informed.

A short-sale prohibition leads to more traders becoming informed, as illustrated by Figure 6, which plots the mass of traders who choose to become informed when short sales are banned for each given exogenous mass of informed traders when short sales are permitted. Perhaps surprisingly, a short-sale ban induces more information acquisition even though a ban on short selling may prevent traders from fully exploiting their information in cases where the ban is binding. The underlying economic reason is due to a short-sale ban's effect on the information environment. Because market efficiency declines under a short-sale ban (holding λ fixed; see Figure 3(a)), uninformed traders are able to infer less information from the equilibrium price, so information is more valuable when short selling is prohibited. Thus, more traders become informed.

[23] I verify the monotonicity of the two certainty equivalents by numerically computing the partial derivatives (which are negative) of the certainty equivalents with respect to λ, but I do not report the derivatives for conciseness.



(a) Market Efficiency

(b) Stock Price

(c) Informed Traders' Allocation

(d) Uninformed Traders' Allocation

Figure 7: Market Efficiency, Stock Price, and Allocations with Endogenous Information Acquisition. The difference between the average conditional variance of $\tilde{x}$ and the difference between the average stock price with and without a short-sale prohibition when information acquisition is endogenous is plotted in (a) and (b), respectively. The difference between the average amount of stock held with and without a short-sale prohibition by informed and uninformed traders, respectively, when information acquisition is endogenous is plotted in (c) and (d).

The finding that more traders acquire information under a short-sale ban contrasts with other articles that find that short-sale restrictions decrease information acquisition (Nezafat, Schroder, and Wang, 2017; Liu and Wang, 2018). In those other articles, informed traders acquire a less precise signal when subject to short-sale constraints because such constraints limit the traders' ability to exploit information and, therefore, reduce its usefulness. However, those articles do not permit short-sale constraints to influence the number of informed traders. In contrast, by endogenizing the mass of informed traders and allowing uninformed traders to choose to become informed, I show that information acquisition increases under a short-sale ban.

Because there is a larger mass of informed traders when information acquisition is endogenous, more information is revealed through trading. Consequently, relative to the setting where information acquisition is exogenous, a short-sale ban binds less frequently and tends to have a smaller impact on market efficiency when information acquisition is endogenous, which can be seen by comparing Figures 3(a) and 7(a).[24] As a result, uninformed traders' uncertainty regarding $\tilde{x}$ increases to a lesser extent, which means that there is less additional perceived risk to be shared. Furthermore, when λ is sufficiently large, the aggregate uncertainty about $\tilde{x}$ decreases under a short-sale ban because many more traders become informed. Thus, a short-sale prohibition results in a lower average stock price when information acquisition is endogenous and λ is small but a higher average price when λ is larger, as shown in Figure 7(b). The average equilibrium stock allocations are also affected by endogenous information acquisition, which is evident from comparing Figures 4(c) and 4(d) with 7(c) and 7(d), respectively. When information acquisition is endogenous, there is a greater increase (decrease) in the average amount of stock held by informed (uninformed) traders when a short-sale ban is imposed because more traders choose to become informed.

4.2 Costly Short Selling

In practice, many short-sale constraints impose a cost on traders but are less restrictive than an outright ban. In this section, I evaluate how imposing a cost on short selling affects equilibrium outcomes. I add a macron ($^-$) to the appropriate variables to distinguish this setting from those

[24] Figure 7(a) is truncated because no traders are uninformed under a short-sale ban when λ is sufficiently large, as shown in Figure 6.

analyzed above. To streamline the analysis, I briefly describe how the derivations of the traders' demand functions differ from those in the other settings. I then discuss the results.

For tractability, I assume that traders incur a quadratic monetary cost to short the stock, i.e., $\frac{1}{2}\eta\bar{s}_i^2$, where $\eta \in \mathbb{R}^{++}$. This cost function reflects the reality that short-selling costs are higher when there is greater shorting demand. Additionally, I relax the distributional constraints on liquidity-trader demand and assume that such demand is unbounded, i.e., $\tilde{z} \sim \mathcal{N}(0, \sigma_z^2)$.[25]

Incorporating the cost into an informed trader's budget constraint and following a procedure analogous to those described in Sections 2.1 and 2.2 yields an informed trader's demand function:

$$\bar{s}_i = \max\left\{\frac{\sigma_v^2 x - (\sigma_v^2 + \sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}, \frac{\sigma_v^2 x - (\sigma_v^2 + \sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2 + \eta(\sigma_v^2 + \sigma_\varepsilon^2)}\right\}. \tag{41}$$

The first term on the right-hand side (RHS) of (41) is identical to (6) and represents an informed trader's demand if she does not short the stock; the second term reflects the cost of shorting and represents her demand if she takes a short position. The market-clearing condition enables uninformed traders to indirectly observe the following noisy signal of x:

$$\bar{k} \equiv \max\left\{\frac{\sigma_v^2 \tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}, \frac{\sigma_v^2 \tilde{x} - (\sigma_v^2 + \sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2 + \eta(\sigma_v^2 + \sigma_\varepsilon^2)}\right\}\lambda + \tilde{z}. \tag{42}$$

If informed traders are short, then $\max\left\{\frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}, \frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2+\eta(\sigma_v^2+\sigma_\varepsilon^2)}\right\} = \frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2+\eta(\sigma_v^2+\sigma_\varepsilon^2)}$, so uninformed traders observe $\frac{\lambda[\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}]}{\gamma\sigma_v^2\sigma_\varepsilon^2+\eta(\sigma_v^2+\sigma_\varepsilon^2)} + \tilde{z}$ (which is informationally equivalent to observing $\bar{f} \equiv \frac{\lambda\sigma_v^2}{\gamma\sigma_v^2\sigma_\varepsilon^2+\eta(\sigma_v^2+\sigma_\varepsilon^2)}\tilde{x} + \tilde{z}$ because $\bar{p}$ is observable) and would infer $\tilde{x} < \gamma^2\sigma_v^4\Theta^2\bar{p}$ if they could discern that

[25] An equilibrium with costly short selling may not exist for all realizations of $\tilde{x}$ and $\tilde{z}$ if liquidity-trader demand is doubly truncated as in Section 2.2. The reason is that truncated liquidity-trader demand can lead to a large discontinuity in uninformed traders' beliefs (and, hence, demand) where $\bar{k}$ crosses a truncation point. For instance, if $\tilde{z} \sim \mathcal{TN}(0, \sigma_z^2, 0, 1)$, then uninformed traders are unsure whether informed traders are long or short when $\bar{k} \leq 1$ and may take a short position themselves; however, uninformed traders know with certainty that informed traders are not short when $\bar{k} > 1$ and, therefore, will not short the stock. As a result of this discontinuity, a price that clears the market may not exist. This issue does not arise when short selling is prohibited (rather than costly) because the traders cannot short the stock in that case. As discussed in Section 2, $\tilde{z}$ must be bounded when short sales are banned because traders would be unable to take a short position to satisfy large liquidity-trader demand.

informed traders were short. Hence, the uninformed traders' posterior beliefs would be given by

$$\tilde{x}|\bar{k},\bar{p} \sim \mathcal{TN}\left(\frac{\lambda\sigma_v^2(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2\left(\bar{k}+\frac{\lambda(\sigma_v^2+\sigma_\varepsilon^2)}{\Upsilon}\bar{p}\right)}{\sigma_z^2\Upsilon\Omega^2}, \frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Omega^2}, -\infty, \gamma^2\sigma_v^2\Theta^2\bar{p}\right), \tag{43}$$

where

$$\Upsilon \equiv \gamma\sigma_v^2\sigma_\varepsilon^2 + \eta(\sigma_v^2+\sigma_\varepsilon^2) \tag{44}$$

$$\Omega^2 \equiv \Theta^2\left(1+\frac{\lambda^2\sigma_v^4(\sigma_v^2+\sigma_\varepsilon^2)}{\sigma_z^2\Upsilon^2}\right) \tag{45}$$

if they could deduce that informed traders were short. Conversely, if informed traders are not short, then $\max\left\{\frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}, \frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2+\eta(\sigma_v^2+\sigma_\varepsilon^2)}\right\} = \frac{\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}}{\gamma\sigma_v^2\sigma_\varepsilon^2}$, so uninformed traders observe $\frac{\lambda[\sigma_v^2\tilde{x}-(\sigma_v^2+\sigma_\varepsilon^2)\bar{p}]}{\gamma\sigma_v^2\sigma_\varepsilon^2}+\tilde{z}$ (which is informationally equivalent to observing $f \equiv \frac{\lambda}{\gamma\sigma_\varepsilon^2}\tilde{x}+\tilde{z}$) and would infer $\tilde{x} \geq \gamma^2\sigma_v^4\Theta^2\bar{p}$ if they could discern that informed traders were not short. In this case,

$$\tilde{x}|\bar{k},\bar{p} \sim \mathcal{TN}\left(\frac{\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2}\left(\bar{k}+\frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}\bar{p}\right)}{\Sigma^2}, \frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}, \gamma^2\sigma_v^2\Theta^2\bar{p}, \infty\right) \tag{46}$$

would characterize the uninformed traders' beliefs. Because traders incur a cost to short the stock, an uninformed trader's demand, which is obtained by a process similar to those described in Sections 2.1 and 2.2 and detailed in the appendix, depends on whether she takes a short position.

Theorem 4. *Given a price $\bar{p}$ and signal $\bar{k}$, an uninformed trader's demand when short selling is* *costly is characterized by*

$$\bar{s}_u = \begin{cases} \dfrac{-\Phi\big(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\big)\frac{1}{\Omega^2}\left(\frac{\phi\left(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\right)\Omega}{\Phi\left(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\right)}+\bar{\delta}\right)+\Phi\big(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\big)\frac{1}{\Sigma^2}\left(\frac{\phi\left(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\right)\Sigma}{\Phi\left(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\right)}+\bar{\beta}\right)\bar{\Psi}}{\Phi\big(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\big)\frac{\gamma^2\sigma_v^2\Theta^2}{\Omega^2}\big(1+\frac{\lambda^2\sigma_v^4\sigma_\varepsilon^2}{\sigma_z^2\Upsilon^2}\big)+\Phi\big(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\big)\frac{\gamma^2\sigma_v^2\Theta^2}{\Sigma^2}\big(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\big)\bar{\Psi}} & \text{if } \bar{s}_u \geq 0 \quad (47) \\[2ex] \dfrac{-\Phi\big(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\big)\frac{1}{\Omega^2}\left(\frac{\phi\left(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\right)\Omega}{\Phi\left(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\right)}+\bar{\delta}\right)+\Phi\big(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\big)\frac{1}{\Sigma^2}\left(\frac{\phi\left(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\right)\Sigma}{\Phi\left(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\right)}+\bar{\beta}\right)\bar{\Psi}}{\Phi\big(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\big)\frac{\gamma\Theta^2}{\Omega^2}\big(\eta+\gamma\sigma_v^2+\frac{\lambda^2\sigma_v^4}{\sigma_z^2\Upsilon}\big)+\Phi\big(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\big)\big(\gamma\eta+\frac{\gamma^2\sigma_v^2\Theta^2}{\Sigma^2}\big)\big(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\big)\bar{\Psi}} & \text{if } \bar{s}_u < 0, \quad (48) \end{cases}$$

where

$$\bar{\alpha} \equiv \Theta^2\left(-\frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2}{\lambda(\sigma_v^2+\sigma_\varepsilon^2)}\bar{k}-\gamma\bar{p}\right) \tag{49}$$

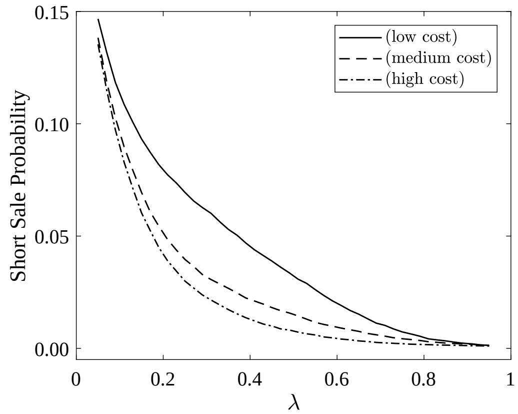
$$\bar{\beta} \equiv \Theta^2\left(\frac{\lambda\sigma_v^2}{\sigma_\varepsilon^2\sigma_z^2}\bar{k}-\gamma\bar{p}\right) \tag{50}$$

$$\bar{\delta} \equiv \Theta^2\left(\gamma\bar{p}-\frac{\gamma\lambda\sigma_v^4}{\sigma_z^2\Upsilon}\bar{k}\right) \tag{51}$$

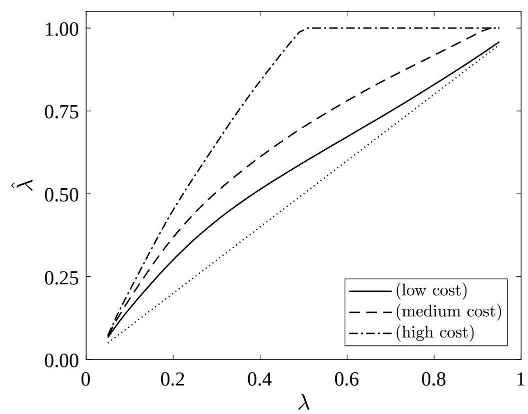
$$\bar{\Psi} \equiv \exp\left[\frac{\eta\lambda^2\sigma_v^4(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\sigma_\varepsilon^4\Sigma^2\big[\lambda^2\sigma_v^4(\sigma_v^2+\sigma_\varepsilon^2)+\sigma_z^2\Upsilon^2\big]}\big[\gamma\sigma_v^2\sigma_\varepsilon^2\big(\bar{\beta}-\tfrac{1}{2}\bar{s}_u\big)+\Upsilon\big(\bar{\alpha}-\tfrac{1}{2}\bar{s}_u\big)\big]\bar{s}_u\right]. \tag{52}$$

The numerical procedure used to evaluate the impact of imposing a cost on short sales is analogous to that described in Section 2.3. To demonstrate robustness, I analyze economies with low ($\eta = 1$), medium ($\eta = 3$), and high ($\eta = 10$) shorting costs. The remaining parameter values correspond to the baseline parameterization #1.

In general, imposing a cost on short sales generates outcomes that are qualitatively the same but not as strong as those resulting from an outright ban. Figure 8(a) shows that informed traders hold a non-positive amount of stock when short sales are costly more frequently than they hold zero shares in the presence of a ban. This result highlights the influence of uninformed traders' uncertainty on the informed traders' propensity to sell short. While the magnitude of an informed trader's short position is directly affected by a short-sale cost, the likelihood of holding a short position is not.



(a) Binding Constraint

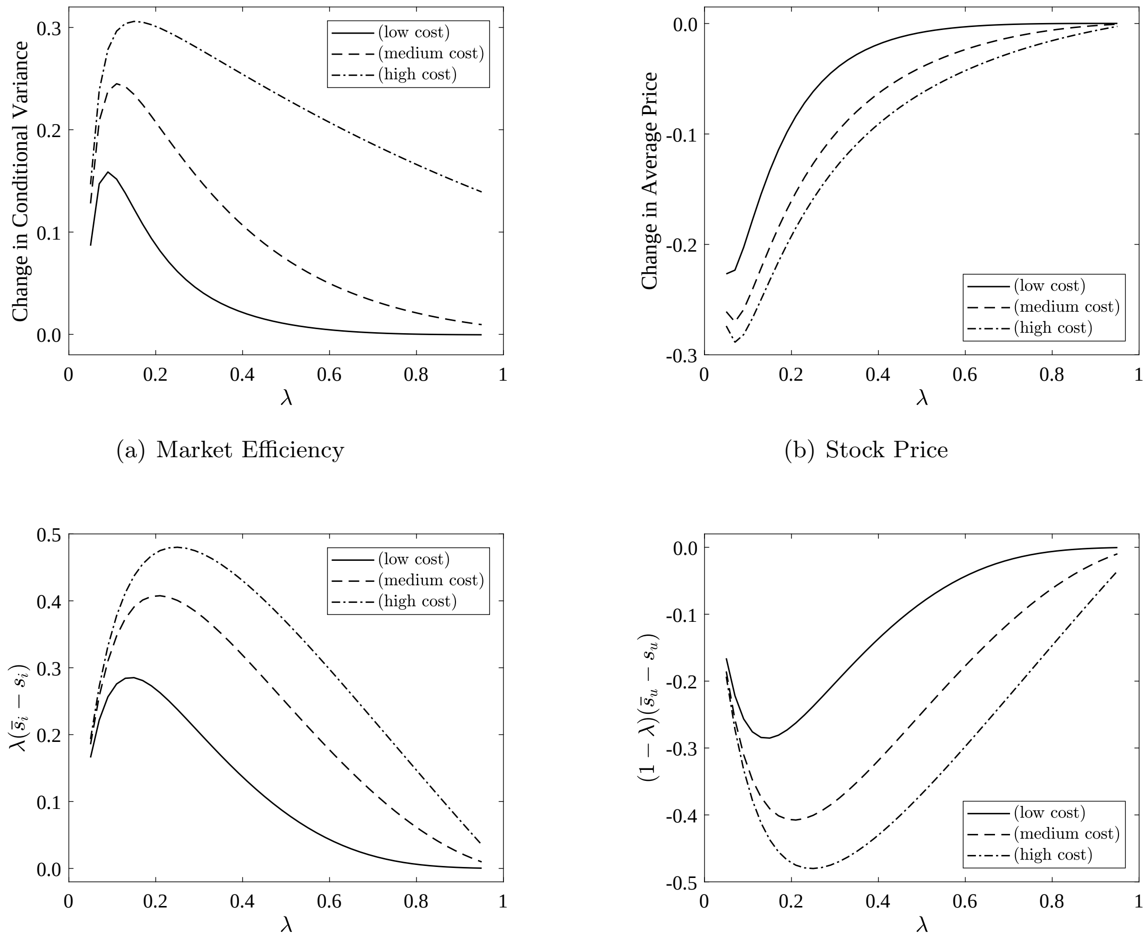


(b) Mass of Informed Traders

Figure 8: Likelihood of Binding Constraint and Mass of Informed Traders with Costly Short Selling. The probability that a short-sale cost constraint is binding for informed traders is plotted in (a). The mass of informed traders with costly short selling is plotted in (b).

Indeed, an informed trader's demand, as given by (41), is a continuous function of x and $\bar{p}$, and the cost of short selling, η, influences the size of an informed trader's short position but not whether she sells short. Rather, the decision to take a short position is indirectly influenced by the short-sale cost's effect on the price $\bar{p}$, which in turn is affected by the uninformed traders' uncertainty.

Comparing the two possible signals in (42) indicates that the trading outcome contains a less precise signal when informed traders sell short, and the precision of the signal conditional on selling short, $\frac{\Omega^2}{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}$, is decreasing in the short-sale cost η. Thus, a short-sale cost lowers market efficiency, as Figure 9(a) shows. This raises the value of information and leads to an increase in the mass of informed traders, as Figure 8(b) shows. However, because a short-sale cost does not completely censor informed traders' demand, the magnitudes of the effects are smaller than when short selling is prohibited. The decline in market efficiency results in a lower stock price, as depicted by Figure 9(b), and a shift in allocations from uninformed to informed traders, as Figures 9(c) and 9(d) illustrate, for the same reasons discussed in Section 3.4.



(a) Market Efficiency

(b) Stock Price

(c) Informed Traders' Allocation

(d) Uninformed Traders' Allocation

Figure 9: Market Efficiency, Stock Price, and Allocations with Costly Short Selling. The difference between the average conditional variance of $\tilde{x}$ and the difference between the average stock price with costly and costless short selling is plotted in (a) and (b), respectively. The difference between the average amount of stock held with costly and costless short selling by informed and uninformed traders, respectively, is plotted in (c) and (d).

5 Conclusion

I evaluate the effects of a short-sale ban using a rational expectations equilibrium framework. Although a short-sale ban seldom binds, constraining short sales affects many equilibrium attributes because it alters uninformed traders' beliefs even when informed traders are not bound by the constraint. Relative to a market in which short selling is permitted, I find that barring short sales reduces market efficiency but increases depth. Additionally, more traders acquire information when short sales are forbidden. Relaxing the strict prohibition and imposing a finite cost on short selling yields similar results.

While this article extends the literature on trading with portfolio constraints under asymmetric information, a few caveats are in order. First, a short-sale ban could affect the distribution of liquidity trades, which in turn could affect equilibrium attributes, if liquidity traders can choose which assets to trade to satisfy their liquidity needs. However, the extent to which the qualitative conclusions would change may be small if a market-wide ban on short selling has a similar effect on all stocks. Second, a short-sale prohibition's effect on information acquisition could generate positive spillover effects if the information affects corporate investment decisions.

References

Allen, Franklin, Stephen Morris, and Andrew Postlewaite. 1993. "Finite Bubbles with Short Sale Constraints and Asymmetric Information." *Journal of Economic Theory* 61 (2):206–229.

Almazan, Andres, Keith C. Brown, Murray Carlson, and David A. Chapman. 2004. "Why Constrain Your Mutual Fund Manager?" *Journal of Financial Economics* 73 (2):289–321.

Asquith, Paul, Parag A. Pathak, and Jay R. Ritter. 2005. "Short Interest, Institutional Ownership, and Stock Returns." *Journal of Financial Economics* 78 (2):243–276.

Bai, Yang, Eric C. Chang, and Jiang Wang. 2006. "Asset Prices Under Short-Sale Constraints." Working Paper, Massachusettes Institute of Technology.

Barlevy, Gadi and Pietro Veronesi. 2003. "Rational Panics and Stock Market Crashes." *Journal of Economic Theory* 110 (2):234–263.

Beber, Alessandro and Marco Pagano. 2013. "Short-Selling Bans Around the World: Evidence from the 2007-09 Crisis." *Journal of Finance* 68 (1):343–381.

Blocher, Jesse, Adam V. Reed, and Edward D. Van Wesep. 2013. "Connecting Two Markets: An Equilibrium Framework for Shorts, Longs, and Stock Loans." *Journal of Financial Economics* 108 (2):302–322.

Boehme, Rodney D., Bartley R. Danielsen, and Sorin M. Sorescu. 2006. "Short-Sale Constraints, Differences of Opinion, and Overvaluation." *Journal of Financial and Quantitative Analysis* 41 (2):455–487.

Boehmer, Ekkehart, Charles M. Jones, and Xiaoyan Zhang. 2008. "Which Shorts are Informed?" *Journal of Finance* 63 (2):491–527.

———. 2013. "Shackling Short Sellers: The 2008 Shorting Ban." *Review of Financial Studies* 26 (6):1363–1400.

Boehmer, Ekkehart and Juan (Julie) Wu. 2013. "Short Selling and the Price Discovery Process." *Review of Financial Studies* 26 (2):287–322.

Bris, Arturo, William N. Goetzmann, and Ning Zhu. 2007. "Efficiency and the Bear: Short Sales and Markets Around the World." *Journal of Finance* 62 (3):1029–1079.

Cao, H. Henry, Harold H. Zhang, and Xin Zhou. 2007. "Short-Sale Constraint, Informational Efficiency and Asset Price Bias." Working Paper, University of Texas at Dallas.

Chang, Eric C., Joseph W. Cheng, and Yinghui Yu. 2007. "Short-Sales Constraints and Price Discovery: Evidence from the Hong Kong Market." *Journal of Finance* 62 (5):2097–2121.

Chen, Joseph, Harrison Hong, and Jeremy C. Stein. 2002. "Breadth of Ownerhip and Stock Returns." *Journal of Financial Economics* 66 (2-3):171–205.

Daniel, Kent, Alexander Klos, and Simon Rottke. 2017. "Overpriced Winners." Working Paper, Columbia University.

Dechow, Patricia M., Amy P. Hutton, Lisa Meulbroek, and Richard G. Sloan. 2001. "Short-Sellers, Fundamental Analysis, and Stock Returns." *Journal of Financial Economics* 61 (1):77–106.

Diamond, Douglas W. and Robert E. Verrecchia. 1987. "Constraints on Short-Selling and Asset Price Adjustment to Private Information." *Journal of Financial Economics* 18 (2):277–311.

Diether, Karl B., Kuan-Hui Lee, and Ingrid M. Werner. 2009a. "It's SHO Time! Short-Sale Price Tests and Market Quality." *Journal of Finance* 64 (1):37–73.

———. 2009b. "Short-Sale Strategies and Return Predictability." *Review of Financial Studies* 22 (2):575–607.

Division of Economic and Risk Analysis. 2014. *Short Sale Position and Transaction Reporting.* U.S. Securities and Exchange Commission.

Duffie, Darrell. 1996. "Special Repo Rates." *Journal of Finance* 51 (2):493–526.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen. 2002. "Securities Lending, Shorting, and Pricing." *Journal of Financial Economics* 66 (2-3):307–339.

Engelberg, Joseph E., Adam V. Reed, and Matthew C. Ringgenberg. 2012. "How are Shorts Informed? Short Sellers, News, and Information Processing." *Journal of Financial Economics* 105 (2):260–278.

Figlewski, Stephen. 1981. "The Informational Effects of Restrictions on Short Sales: Some Empirical Evidence." *Journal of Financial and Quantitative Analysis* 16 (4):463–476.

Gallmeyer, Michael and Burton Hollifield. 2008. "An Examination of Heterogeneous Beliefs with a Short-Sale Constraint in a Dynamic Economy." *Review of Finance* 12 (2):323–364.

Grossman, Sanford J. and Joseph E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70 (3):393–408.

Hayashi, Fumio. 2000. *Econometrics.* Princeton University Press.

Hong, Harrison and Jeremy C. Stein. 2003. “Differences of Opinion, Short-Sales Constraints, and Market Crashes.” *Review of Financial Studies* 16 (2):487–525.

Jarrow, Robert. 1980. “Heterogeneous Expectations, Restrictions on Short Sales, and Equilibrium Asset Prices.” *Journal of Finance* 35 (5):1105–1113.

Jones, Charles M. and Owen A. Lamont. 2002. “Short-Sale Constraints and Stock Returns.” *Journal of Financial Economics* 66 (2-3):207–239.

Kyle, Albert S. 1985. “Continuous Auctions and Insider Trading.” *Econometrica* 53 (6):1315–1335.

Leland, Hayne E. 1992. “Insider Trading: Should It Be Prohibited?” *Journal of Political Economy* 100 (4):859–887.

Lenkey, Stephen L. 2014. “Advance Disclosure of Insider Trading.” *Review of Financial Studies* 27 (8):2504–2537.

Liu, Hong and Yajun Wang. 2018. “Asset Pricing Implications of Short-Sale Constraints in Imperfectly Competitive Markets.” *Management Science*, forthcoming.

Marin, Jose M. and Jacques P. Olivier. 2008. “The Dog That Did Not Bark: Insider Trading and Crashes.” *Journal of Finance* 63 (5):2429–2476.

Miller, Edward M. 1977. “Risk, Uncertainty, and Divergence of Opinion.” *Journal of Finance* 32 (4):1151–1168.

Nezafat, Mahdi, Mark Schroder, and Qinghai Wang. 2017. “Short-Sale Constraints, Information Acquisition, and Asset Prices.” *Journal of Economic Theory* 172:279–312.

Pálvölgyi, Dömötör and Gyuri Venter. 2015. “Multiple Equilibria in Noisy Rational Expectations Economies.” Working Paper, University of Cambridge.

Reed, Adam, Pedro A. C. Saffi, and Edward D. Van Wesep. 2016. “Short Sales Constraints and the Diversification Puzzle.” Working Paper, University of North Carolina.

Reed, Adam V. 2013. “Short Selling.” *Annual Review of Financial Economics* 5:245–258.

Saffi, Pedro A. C. and Karl Sigurdsson. 2011. “Price Efficiency and Short Selling.” *Review of Financial Studies* 24 (3):821–852.

Senchack, A. J., Jr. and Laura T. Starks. 1993. “Short-Sale Restrictions and Market Reaction to Short-Interest Announcements.” *Journal of Financial and Quantitative Analysis* 28 (2):177–194.

Spiegel, Matthew and Avanidhar Subrahmanyam. 1992. “Informed Speculation and Hedging in a Noncompetitive Securities Market.” *Review of Financial Studies* 5 (2):307–329.

Weitzner, Gregory. 2017. “The Term Structure of Short Selling Costs.” Working Paper, University of Texas at Austin.

Yuan, Kathy. 2005. “Asymmetric Price Movements and Borrowing Constraints: A Rational Expectations Equilibrium Model of Crisis, Contagion, and Confusion.” *Journal of Finance* 60 (1):379–411.

———. 2006. “The Price Impact of Borrowing and Short-Sale Constraints.” Working Paper.

Appendix

Derivation of Equation (9)

First, note that the unconditional variance of $\tilde{x} = \tilde{v} + \tilde{\varepsilon}$ is $\sigma_v^2 + \sigma_\varepsilon^2$ and the unconditional variance of $\tilde{f} \equiv \frac{\lambda}{\gamma\sigma_\varepsilon^2}\tilde{x} + \tilde{z}$ is $\frac{\lambda^2(\sigma_v^2+\sigma_\varepsilon^2)}{\gamma^2\sigma_\varepsilon^4} + \sigma_z^2$. The unconditional means of $\tilde{x}$ and $\tilde{f}$ are both zero. Then, the posterior density of $\tilde{x}$ conditional on k and p is proportional to

$$\begin{aligned}
&\frac{\frac{1}{\sqrt{2\pi(\sigma_v^2+\sigma_\varepsilon^2)}\sqrt{2\pi\sigma_z^2}}\exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)}\right]\exp\left[-\frac{\tilde{z}^2}{2\sigma_z^2}\right]}{\frac{1}{\sqrt{2\pi\left(\frac{(\sigma_v^2+\sigma_\varepsilon^2)\lambda^2}{\gamma^2\sigma_\varepsilon^4}+\sigma_z^2\right)}}\exp\left[-\frac{\tilde{f}^2}{2\left(\frac{(\sigma_v^2+\sigma_\varepsilon^2)\lambda^2}{\gamma^2\sigma_\varepsilon^4}+\sigma_z^2\right)}\right]}\\
&=\frac{\frac{1}{\sqrt{2\pi(\sigma_v^2+\sigma_\varepsilon^2)}\sqrt{2\pi\sigma_z^2}}\exp\left[-\frac{\tilde{x}^2}{2(\sigma_v^2+\sigma_\varepsilon^2)}\right]\exp\left[-\frac{(\tilde{f}-\frac{\lambda}{\gamma\sigma_\varepsilon^2}\tilde{x})^2}{2\sigma_z^2}\right]}{\frac{1}{\sqrt{2\pi\left(\frac{(\sigma_v^2+\sigma_\varepsilon^2)\lambda^2}{\gamma^2\sigma_\varepsilon^4}+\sigma_z^2\right)}}\exp\left[-\frac{\tilde{f}^2}{2\left(\frac{(\sigma_v^2+\sigma_\varepsilon^2)\lambda^2}{\gamma^2\sigma_\varepsilon^4}+\sigma_z^2\right)}\right]}\\
&=\frac{1}{\sqrt{2\pi\frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}}}\exp\left[-\frac{\left(\tilde{x}-\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2\Sigma^2}\left(k+\frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}p\right)\right)^2}{2\frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}}\right],
\end{aligned} \tag{A.1}$$

where the first equality results from substituting $\tilde{z} = \tilde{f} - \frac{\lambda}{\gamma\sigma_\varepsilon^2}\tilde{x}$ (which follows immediately from the definition of $\tilde{f}$) and the second equality results from algebra. Next, (8) implies $\tilde{x} \geq \gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}(k-1)$ because $\tilde{z}$ is bounded from above by 1. Similarly, (8) implies $\tilde{x} \leq \gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}k$ because $\tilde{z}$ is bounded from below by 0. Integrating (A.1) over the region defined by these bounds provides a normalizing constant,

$$\frac{1}{\sqrt{2\pi\frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}}}\int_{\gamma^2\sigma_v^2\Theta^2p+\frac{\gamma\sigma_\varepsilon^2}{\lambda}(k-1)}^{\gamma^2\sigma_v^2\Theta^2p+\frac{\gamma\sigma_\varepsilon^2}{\lambda}k}\exp\left[-\frac{\left(\tilde{x}-\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2\Sigma^2}\left(k+\frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}p\right)\right)^2}{2\frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}}\right]\mathrm{d}\tilde{x} = \Phi\left(\frac{\beta}{\Sigma}\right) - \Phi\left(\frac{\alpha}{\Sigma}\right),$$

where α and β are defined by (16) and (17), respectively. Because the density of a truncated random variable is given by the density of the corresponding non-truncated random variable normalized by the probability mass of the non-truncated interval (see, e.g., Hayashi, 2000), the posterior density $\tilde{x}|k,p$ is

$$\frac{1}{\left(\Phi\left(\frac{\beta}{\Sigma}\right)-\Phi\left(\frac{\alpha}{\Sigma}\right)\right)\sqrt{2\pi\frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}}}\exp\left[-\frac{\left(\tilde{x}-\frac{\lambda\gamma\sigma_v^4\Theta^4}{\sigma_\varepsilon^2\sigma_z^2\Sigma^2}\left(k+\frac{\lambda\gamma\sigma_v^2\Theta^2}{\sigma_\varepsilon^2}p\right)\right)^2}{2\frac{(\sigma_v^2+\sigma_\varepsilon^2)\Theta^2}{\Sigma^2}}\right],$$

for $\tilde{x} \in \left[\gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}(k-1)\,,\, \gamma^2\sigma_v^2\Theta^2 p + \frac{\gamma\sigma_\varepsilon^2}{\lambda}k\right]$, which is equivalent to (9). □

Proof of Theorem 1

I first show that (18) uniquely characterizes an uninformed trader's demand. Differentiating (15) with respect to s_u and solving the first-order conditions yields (18). Rearranging allows (18) to be rewritten as

$$0 = s_u + \frac{\left(\phi\left(\frac{\alpha-s_u}{\Sigma}\right) - \phi\left(\frac{\beta-s_u}{\Sigma}\right)\right)\Sigma}{\Phi\left(\frac{\beta-s_u}{\Sigma}\right) - \Phi\left(\frac{\alpha-s_u}{\Sigma}\right)} + \frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2\Sigma^2}{\sigma_v^2+\sigma_\varepsilon^2}s_u - \Theta^2\left(\frac{\lambda\sigma_v^2}{\sigma_\varepsilon^2\sigma_z^2}k - \gamma p\right) \tag{A.2}$$

$$= \mathbb{E}[\tilde{h}|\alpha < h < \beta] + \frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2\Sigma^2}{\sigma_v^2+\sigma_\varepsilon^2}s_u - \Theta^2\left(\frac{\lambda\sigma_v^2}{\sigma_\varepsilon^2\sigma_z^2}k - \gamma p\right), \tag{A.3}$$

where $\tilde{h} \sim \mathcal{N}(s_u, \Sigma^2)$ is some arbitrary random variable and (A.3) follows from well-known properties of truncated normal distributions. Differentiating the RHS of (A.3) with respect to s_u gives

$$\begin{aligned}
&1 + \frac{\frac{\alpha-s_u}{\Sigma}\phi\left(\frac{\alpha-s_u}{\Sigma}\right) - \frac{\beta-s_u}{\Sigma}\phi\left(\frac{\beta-s_u}{\Sigma}\right)}{\Phi\left(\frac{\beta-s_u}{\Sigma}\right) - \Phi\left(\frac{\alpha-s_u}{\Sigma}\right)} - \left(\frac{\phi\left(\frac{\alpha-s_u}{\Sigma}\right) - \phi\left(\frac{\beta-s_u}{\Sigma}\right)}{\Phi\left(\frac{\beta-s_u}{\Sigma}\right) - \Phi\left(\frac{\alpha-s_u}{\Sigma}\right)}\right)^2 + \frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2\Sigma^2}{\sigma_v^2+\sigma_\varepsilon^2} \\
&= \frac{1}{\Sigma^2}\mathbb{V}[\tilde{h}|\alpha < h < \beta] + \frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2\Sigma^2}{\sigma_v^2+\sigma_\varepsilon^2} \\
&> 0,
\end{aligned}$$

where the inequality follows from the fact that $\mathbb{V}[\,\cdot\,] \geq 0$. Because the limit of the RHS of (A.3) approaches $-\infty$ as $s_u \to -\infty$, the limit of the RHS approaches ∞ as $s_u \to \infty$, and the RHS is strictly increasing in s_u, there exists a unique value of s_u that satisfies (A.2). Hence, the uninformed traders' stock demand is unique.

Next, I show that s_u is strictly decreasing in p. Define $\alpha' \equiv \alpha + \gamma\Theta^2 p$, $\beta' \equiv \beta + \gamma\Theta^2 p$, and $\tilde{h}' \equiv \tilde{h} + \gamma\Theta^2 p$. Then, (A.3) can be rewritten as

$$0 = \mathbb{E}[\tilde{h}'|\alpha' < h' < \beta'] + \frac{\gamma^2\sigma_v^2\sigma_\varepsilon^2\Sigma^2}{\sigma_v^2+\sigma_\varepsilon^2}s_u - \frac{\lambda\sigma_v^2\Theta^2}{\sigma_\varepsilon^2\sigma_z^2}k. \tag{A.4}$$

Because $\mathbb{E}[\tilde{h}'|\alpha' < h' < \beta']$ is strictly increasing in both p and s_u, a lower value of s_u satisfies (A.4) when p is larger. Hence, s_u is strictly decreasing in p. □

Proof of Theorem 2

The informed traders' demand, λs_i, where s_i is given by (6), is clearly strictly decreasing in p. Theorem

1 implies that the uninformed traders' demand, $(1-\lambda)s_u$ is also strictly decreasing in p. Hence, aggregate demand is strictly decreasing in p. Therefore, there exists a unique p that satisfies the market-clearing condition, defined by (7). $\square$

Proof of Theorem 3

Differentiating (37) with respect to $\hat{s}_u$ and rearranging the first-order condition gives (38). $\square$

Proof of Theorem 4

An uninformed trader's expected utility is derived by substituting an appropriate budget constraint (accounting for the shorting cost, where relevant) into the objective function and integrating over $\tilde{v}$ according to (2) and $\tilde{x}$ according to either (43) or (46). If uninformed traders are not short but informed traders are, then an uninformed trader's expected utility is

$$-\frac{\Phi\left(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\right)}{\Phi\left(\frac{\bar{\delta}}{\Omega}\right)}\exp\left[-\gamma\left(w_b+w_s\bar{p}+\frac{1}{\Omega^2}\left[-\frac{\bar{\delta}}{\gamma}\bar{s}_u-\tfrac{1}{2}\gamma\sigma_v^2\Theta^2\left(1+\frac{\lambda^2\sigma_v^4\sigma_\varepsilon^2}{\sigma_z^2\Upsilon}\right)\bar{s}_u^2\right]\right)\right]. \tag{A.5}$$

If neither uninformed nor informed traders are short, then an uninformed trader's expected utility is

$$-\frac{\Phi\left(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\right)}{\Phi\left(\frac{\bar{\beta}}{\Sigma}\right)}\exp\left[-\gamma\left(w_b+w_s\bar{p}+\frac{1}{\Sigma^2}\left[\frac{\bar{\beta}}{\gamma}\bar{s}_u-\tfrac{1}{2}\gamma\sigma_v^2\Theta^2\left(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\right)\bar{s}_u^2\right]\right)\right]. \tag{A.6}$$

If both uninformed and informed traders are short, then an uninformed trader's expected utility is

$$-\frac{\Phi\left(\frac{\bar{\delta}+\bar{s}_u}{\Omega}\right)}{\Phi\left(\frac{\bar{\delta}}{\Omega}\right)}\exp\left[-\gamma\left(w_b+w_s\bar{p}+\frac{1}{\Omega^2}\left[-\frac{\bar{\delta}}{\gamma}\bar{s}_u-\tfrac{1}{2}\gamma\sigma_v^2\Theta^2\left(1+\frac{\lambda^2\sigma_v^4\sigma_\varepsilon^2}{\sigma_z^2\Upsilon}\right)\bar{s}_u^2\right]-\tfrac{1}{2}\eta\bar{s}_u^2\right)\right]. \tag{A.7}$$

If uninformed traders are short but informed traders are not, then an uninformed trader's expected utility is

$$-\frac{\Phi\left(\frac{\bar{\beta}-\bar{s}_u}{\Sigma}\right)}{\Phi\left(\frac{\bar{\beta}}{\Sigma}\right)}\exp\left[-\gamma\left(w_b+w_s\bar{p}+\frac{1}{\Sigma^2}\left[\frac{\bar{\beta}}{\gamma}\bar{s}_u-\tfrac{1}{2}\gamma\sigma_v^2\Theta^2\left(1+\frac{\lambda^2}{\gamma^2\sigma_\varepsilon^2\sigma_z^2}\right)\bar{s}_u^2\right]-\tfrac{1}{2}\eta\bar{s}_u^2\right)\right]. \tag{A.8}$$

Weighting (A.5) and (A.6) by $\frac{\Phi\left(\frac{\bar{\delta}}{\Omega}\right)}{\Phi\left(\frac{\bar{\delta}}{\Omega}\right)+\Phi\left(\frac{\bar{\beta}}{\Sigma}\right)}$ and $\frac{\Phi\left(\frac{\bar{\beta}}{\Sigma}\right)}{\Phi\left(\frac{\bar{\delta}}{\Omega}\right)+\Phi\left(\frac{\bar{\beta}}{\Sigma}\right)}$, respectively, yields an expression for an uninformed trader's expected utility when she does not short. Differentiating this expression with respect to $\bar{s}_u$ and rearranging the first-order condition gives (47). A similar method using (A.7) and (A.8) yields (48). $\square$