Adenza

Calypso Equity Derivatives Analytics

Version 17.0

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The purpose of the Equity Derivatives Analytics guide is to describe the analytics that underlying Equity Derivatives pricing.

Revision Date	Comments
February 2022	First edition for version 17.0.

Contents

Section 1. Market Data Generation

1.1 Dividend Curves

Dividend curves are used for projecting dividends.

Discrete Dividends Generator - Generates dividend yields from discrete dividends and base curves (interest rates).

Implied Dividend Generator - Generate dividend yields from underlying instruments.

Equity Indices Generator - Generates dividend yields from equity indices as underlying instruments, and base curves (interest rates).

Options

Options on equity indexes or single equities. Option prices C_j and P_j for call and put options expiring at times t_j, j = 1,…,N_O.

Bootstrapping with Options

Options can be used on their own or added to a curve that has already been created with near-term futures. The futures curve can be extended with longer-term equity options ("LEAPS").

To find the dividends implied by option prices, one uses the put/call parity at a given strike.

$$C - P = Se^{-qt} - Ke^{-rt}$$

(The right-hand side is the present value of the future payoff S(t) – K of a combined put-call portfolio.) So from the prices of puts P_j and calls C_j at a strike and an interest rate, the "zero" or average dividend rate can be backed out:

$$q_j' = -\frac{C_j - P_j + Ke^{-rt_j}}{S_0 t_j}$$

From these the forward dividend rate can be found as described previously.

1.2 Borrow Curves

Equity Derivatives pricing requires the incorporation of a funding rate into the calculation of a “Forward” value of an underlying. The funding rate will be based on an institution’s cost of “borrowing” the security.

The borrow curve adds either a spread to the spread curve, or calculates a borrow rate. In the case of a spread, the spread curve can either be the prevailing discount curve, or in some cases other curves. For example, in the U.S. the borrow curve would be based off of FEDFUNDS.

The logic is that the pricer first looks for a borrow curve based on the underlying product, then based on the currency (general borrow curve), and then if the borrow curve is not found, use the discount curve (meaning that the spread is 0).

In the trade worksheet, you can overwrite the borrow spread used in pricing by entering a spread in the BORROW_SPREAD transient pricing parameter.

Currently, Equity Derivative pricing algorithms will calculate a forward rate like:

$F = ^{e(r-q)T}$ Where r is from the forecast curve and q is from the dividend curve.

Equity Derivative pricing will change to calculate ALL forward prices as follows:

$F = ^{e(r^{*}-q)T}$ Where r* = borrow rate, and is obtained as follows:

1. r* = r – s, when the borrow curve is a “spread” curve.
2. r* = b, where b is the all in borrow rate, when the borrow curve stores all in rates.

Section 2. Equity Derivatives Pricing

Product	Calypso Product	Description	Pricer
ADR, Equity	ADR, Equity		PricerEquity
Asian Equity Option	EquityStucturedOption	Single exercise date (European). Discrete number of observation dates. Arithmetic average. Average asset price and average strike.	PricerBlack1FMonteCarlo
Barrier Equity Option	EquityStructuredOption	Continuous monitoring of the barrier. Upon exercise, the holder gets a vanilla European option. Single barrier and double barrier.	PricerBlack1FAnalyticBarrier, PricerBlack1FMonteCarlo
Basket Equity Option	EquityStructuredOption	European Vanilla/Bestof/Worstof/Rainbow option.	PricerBlackNFMonteCarlo
CFD	CFDConvertibleArbitrage		PricerCFDConvertibleArbitrage
CFD	CFDDirectional		PricerCFDDirectional
CFD	CFDPairTrading		PricerCFDPairTrading
CFD	CFDRiskArbitrage		PricerCFDRiskArbitrage
Chooser Equity Option	EquityStructuredOption	Single exercise date (European) for both the underlying options and the option on the choice.	PricerBlack1FSemiAnalyticChooser
Cliquet Option	EquityStructuredOption	Single exercise date Cliquet Option on Equity/EquityIndex.	PricerSVJMonteCarloExotic Requires an SVJ volatility surface that uses the Heston volatility model. Please refer to the Calypso Analytics Library document for details.
Compo Equity Option	EquityStucturedOption	Asset currency is different from payoff currency. Forward exchange is floating.	PricerBlack1FAnalyticCompo
Compound Equity Option	EquityStructuredOption	Single exercise date (European) for both the underlying option and the option on option.	PricerBlack1FSemiAnalyticCompound
Digital Barrier Equity Option	EquityStructuredOption	Continuous monitoring of the barrier. Upon exercise, the holder gets a vanilla digital option. Single barrier and double barrier.	PricerBlack1FAnalyticBarrier

Product	Calypso Product	Description	Pricer
Dividend Swap	DividendSwap	A Dividend Swap is an OTC agreement between two counterparties to exchange Realized Dividends versus a Fixed(Strike) Dividend on one or more Forward Dates.	PricerDividendSwap
ELS	EquityLinkedSwap		PricerEquityLinkedSwap, PricerEquityLinkedSwapAccrual
Equity Index Futures	Equity Index Futures		PricerFutureEquity
Lookback Equity Options	EquityStructuredOption	Lookback option.	PricerBlack1FMonteCarlo
Quanto Equity Option	EquityStucturedOption	Asset currency is different from payoff currency. Forward exchange is fixed upon trade creation.	PricerBlack1FAnalyticQuanto
Stock Futures	Equity Futures		PricerFutureEquity
Vanilla (OTC, ETO, Equity Index, Equity, Warrant, Certificate) Option	EquityStructuredOption, ETOEquity, ETOEquityIndex, OTCEquityOption, Warrant Certificate	European, Black-Scholes-Merton with continuous dividend yield.	PricerBlack1FAnalyticVanilla
Vanilla (OTC, ETO, Equity Index, Equity, Warrant, Certificate) Option	EquityStructuredOption, ETOEquity, ETOEquityIndex, OTCEquityOption, Warrant Certificate	European, Black-Scholes-Merton with Escrowed dividend model.	PricerBlack1FAnalyticDiscreteVanilla
Vanilla (OTC, ETO, Equity Index, Equity, Warrant, Certificate) Option	EquityStructuredOption, ETOEquity, ETOEquityIndex, OTCEquityOption, Warrant Certificate	American, Bermudan	PricerBlack1FFiniteDifference

Product	Calypso Product	Description	Pricer
Vanilla (OTC, ETO, Equity Index, Equity) Option	EquityStructuredOption, ETOEquity, ETOEquityIndex, OTCEquityOption	American	PricerBlack1FBinomialVanilla
Vanilla Basket Equity Option	EquityStructuredOption	European Vanilla Basket Option.	PricerBlackNFJuAnalyticVanilla
Vanilla Digital Option	EquityStructuredOption	Cash or nothing / asset or nothing. Single exercise date decides if holder get predefined payoff (cash or asset) or not (European style).	PricerBlack1FAnalyticDigital, PricerBlack1FMonteCarlo
Variance Swap	VarianceSwap	Pricing is based on implied volatility levels found in relevant listed option prices.	PricerCarrLeeVolatilityDerivative

2.1 Equity Structured Option Subtypes

The pricer is based on the subtype of the option.

Vanilla

Gives the buyer the right, but not the obligation, to buy or sell an equity or equity index at a fixed price on or before a specified date.

Asian

Asian or average rate options derive the final spot as the arithmetic or geometric average of a series of pre-specified dates.

- Geometric average options where the average is ((x1…xn)1/n), have a closed form solution, but are far less common in practice than arithmetic averages.
- Arithmetic average options where the average is ∑xn, cannot be valued using a closed form solution. There are approximations (Turnbull and Wakeman 1991), that are fairly accurate, or Monte Carlo simulations can be applied.

Asian option pricing algorithms use the term structure of dividends and volatilities to price the forward resets. You have the option to use a single interest rate, dividend rate, or volatility to price. The Asian option window includes a section to generate the Asian dates, and a section to view the generated dates.

Barrier

Barrier (or Knock) options are standard options whose value depends on whether a certain barrier is reached.

Options can be knocked "in" or "out".

- "In" Barrier options are paid for today but first come into existence if the underlying price hits the barrier before expiration.
- "Out" Barrier options begin as standard options except that the option is knocked out, or becomes worthless, if the barrier is hit.

It is possible to include a previously specified cash rebate, which is paid out if an "In" option is never knocked in, or an "out" option is knocked out.

There are standard closed form pricing formulas for knock options whose knock window extends over the life of the knock. If the knock window extends over part of the life of the option, it must be calculated using a lattice or Monte Carlo.

Digital

The payout is pre-determined at the beginning of the contract, and is paid according to whether the spot level is achieved (or not achieved).

Lookback

An option whose payoff is dependent on the maximum or the minimum of the asset price achieved during a certain period.

Performance

Performance options are of 3 types namely Rainbow, Best Of, and Worst Of.

- Rainbow Option – an option whose payoff is dependent on two or more underlying variables.
- Best Of – a Best Of option pays out on the best performing of a number of assets over a period of time.
- Worst Of – a Worst Of option pays out on the worst performing of a number of assets over a period of time.

Basket

An option may be captured on a basket of equities.

2.2 PricerEquityLinkedSwap

Principle

The MTM valuation methodology estimates the “risk neutral” forward value of all flows and then calculates the present value of each. The sum of all received flows less the paid flows equals the fair value. The driving assumption behind this methodology is that forward calculation represents the no arbitrage point. The contract can be replicated, or hedged, with no cost and/or economic risk.

The valuation methodology is based on market practice and the terms of the deal. If the termination cost is expected to take the inherent value of all flows, current and future, into account the MTM methodology should be used.

Assumptions

We assume that dividends are paid out and not reinvested. We also assume that dividends are paid out upon receipt.

General Formula

A vanilla swap is priced by calculating the present value of all future cash flows. This is described by the following formula:

$$\mathrm{NPV_{leg}} = \sum_{i=1}^{N} convertFX(\mathrm{Cf_i df}_i)$$

Expanding the convertFX function (which converts cash flows in foreign currencies into the trade currency):

$$\mathrm{NPV_{equity\ leg}} = \sum_{i=1}^{N} [(\mathrm{Cfperformance_i df}_i \mathrm{FX}_i)] + \sum_{i=1}^{N} [(\mathrm{Cfdividend_i df}_i \mathrm{FX}_i)]$$

Trade currency of an equity swap is the settlement currency of the performance (price_change) flows. The settlement currency of dividend flows can be different from the performance flows currency. In this case the dividend flows are first discounted using the dividend discount curve, then converted into the trade currency and only then added to the NPV of the equity leg.

Equity and funding legs are valued separately, and the resulting $\mathrm{NPV_{equity\ leg}}$ and $\mathrm{NPV_{funding\ leg}}$ are added to obtain the total NPV.

$$\mathrm{NPV_{swap}} = \mathrm{NPV_{equity\ leg}} + \mathrm{NPV_{funding\ leg}}$$

The trade NPV can be converted to the base currency of the pricing environment. The pricer measure DETAILED_DATA needs to be selected in order to get the NPV calculated in both trade currency and base currency.

The FX rate used for conversion of the NPV into base currency is the current spot rate (spot rate at the Val Date).

Determination of Future Cashflows $\mathrm{Cf_i}$

Funding Leg INTEREST Flows

For simple floating rate funding cash flows, the value of future flows is calculated using the forecast IR curve.

$$Interest_i = initialnotional * fwd_i yf_i \text{ (swaps without notional adjustment)}$$

$$or$$

$$Interest_i = initialnotional \prod_{j}^{j<i} pricechange_j \, fwd_i yf_i$$

Where:

- $initialnotional$ is the initial notional of the funding leg.
- $pricechange_i$ is the price change factor at each equity price fixing date. For swaps without funding notional adjustment, the price change factor is not taken into account. For swaps with funding notional adjustment, the initial notional plus the sum of all price changes up to the interest calculation date are taken into account.
- yf_i is the time in year fraction (according to the day count) between the start and end of the interest calculation period.

– fwd_i is the forward rate interpolated from the forecast curve (of the funding leg rate index)

Equity Leg PRICE_CHANGE Flows

$$PriceChange_i = quantity * (end\ price * end\ FX\ rate - start\ price * start\ FX\ rate)$$

Where:

- *quantity* is the initial quantity of the equity leg.
- *end price* is the forward equity price as of the price fixing date of the price change flow.
- *end FX rate* is the forward FX rate as of the price fixing date of the price change flow. The FX rate is needed in the case of the settlement currency different from the underlying equity currency.
- *start price* is the forward equity price as of the price fixing date of the previous price change flow.
- *start FX rate* is the forward FX rate as of the price fixing date of the previous price change flow.

The forward equity price is determined based on the assumption that the future net dividend yield will predict the forward equity price.

The calculation takes the current equity price and extrapolates the net yield at the performance fixing date from the dividend and borrow curves (*). These elements are then used to calculate the forward equity price.

Three options exist depending on the type of dividend curve.

- If the dividend curve is of type 'Yield', we calculate the projected price as follows:

  $$ForwardPrice_i = SpotPrice * e^{t\langle r_i - div_i \rangle}$$

  Where:

  - SpotPrice is the price at valuation date taken from the quote set (depends on the pricer parameter QuoteUsage)
  - t is the time in year fraction (according to the day count) between the valuation date and the date of the performance payment (d), where 1.0 corresponds to exactly one year.
  - (*) r_i is the interest rate at date d, interpolated from the borrow curve if there is a borrow curve, or interpolated from the discount curve when no borrow curve is defined
  - div_i is the dividend yield at date d, interpolated from the dividend curve

- If the dividend curve is of type 'Discrete', we calculate the projected price as follows:

  $$ForwardPrice_i = [SpotPrice - PV_{div}]e^{t \times r_i}$$

  Or in other words:

  $$ForwardPrice_i = \left[SpotPrice - \sum_j div_j df_j\right] e^{t \times r_i}$$

  Where:

- SpotPrice is the price at valuation date taken from the quote set (depends on the pricer parameter QuoteUsage)
- t is the time in year fraction (according to the day count) between the valuation date and the date of the performance payment (d), where 1.0 corresponds to exactly one year.
- (*) r_i is the interest rate at future cash flow date d, interpolated from the borrow curve if there is a borrow curve, or interpolated from the discount curve when no borrow curve is defined
- div_j is the future dividend amount specified for date d' where d' < d
- (*) df_j is the discount factor for date d', interpolated from the borrow curve if there is a borrow curve, or interpolated from the discount curve when no borrow curve is defined
- PV_{div} is the present value of all future dividend payments with an ex-dividend date up to and including the date of the performance payment, discounted against the borrow curve if it is present, and otherwise discounted against the discount curve.

DIVIDEND Flows

The dividend flow will depend on:

- dividends related information stored on the underlying equity product level
- dividend settlement currency
- FX rate used to convert dividend flows into dividend settle currency
- retrocession rate (the percentage of dividend performance to be received or paid) stored on the trade level
- dividend payout time (at maturity, funding schedule, perf schedule, upon receipt,…) stored on the trade level
- only the dividends with a declared date > valuation date will be taken into account in the NPV calculation

Determination of Discount Factors $\mathrm{df_i}$

Discount factors are interpolated from the points stored in the zero yield curve that is configured for the product type, product subtype and currency. Please note that you are able to define different zero yield curves for different product types, but that you can also define a single zero yield curve per currency.

Calypso offers four types of interpolation:

- Linear interpolation:
- Log Linear interpolation
- Spline interpolation
- Daily compounding forward rate interpolation

Relevant Pricing Parameters

INCLUDE_FEES

- True or False. True to include fees in pricing, or False otherwise. Default is False.

- This applies to the calculation of NPV and CASH (but not FEES_NPV).

NPV_INCLUDE_CASH

- True or False. True to include cash received or paid on the valuation date, or False otherwise.

INSTANCE_TYPE

- Determines which instance of market data should be used for pricing.
- Default is LAST

QuoteUsage

- Allows specifying the quote side to be used: BID, ASK, MID, LAST, OPEN, CLOSE, HIGH or LOW.

USE_MARKS

- True or False. Default is False. If True, the marks recorded by the EOD_MARKING scheduled task are used in pricing and reporting. If False, the usual pricing mechanism is triggered.

Required Market Data

Zero yield curve (discount curve) - Required for trade currency and dividend settle currency (if different from trade currency).

Forecast curve - Required for the funding leg if this is a floating rate swap, always required for currency of the performance leg.

Dividend curve - Required for the performance leg.

Borrow curve - Optional, used for performance leg.

Equity quotes - Underlying spot price at valuation date, used for forward equity price calculation.

Interest rate quotes.

FX rates - When settlement currency is not equal to underlying equity currency: FX Rate between equity currency and settlement currency at trade valuation date.

2.3 PricerEquityLinkedSwapAccrual

Accrual pricing approach defines the APS value by recognizing only the unrealized performance and financing based on today's value of the underlying asset. Future flows are not considered while pricing.

NPV = Unpaid performance + Unpaid income - Financing Costs

If the deal can be terminated at any time without taking future flows into account, then the Accrual pricing methodology should be used.

2.4 PricerEquity

This pricer retrieves the required (equity) quote from the quote set and uses that to compute the pricer measures.

2.5 PricerCFDRiskArbitrage

These pricers are a custom implementation for equities.

2.6 PricerFutureEquity

Prices equity contracts and equity index contracts.

2.7 PricerOTCEquity

Prices OTC equities.

2.8 PricerBlack1FAnalyticVanilla

Prices OTC options on equities:

- European Vanilla:standard Black-Scholes.
- American Vanilla: single asset binomial tree.

European Vanilla Option

$$O = ISe^{(b-r)T}N(Id_1) - IKe^{-rT}N(Id_2)$$

where

$$d_1 = \frac{\ln(S/K) + (b + \sigma/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Note

- $S = \frac{1}{m}\sum_{i=1}^{m} F_i$ where m is number of discrete observations, F_i is the forward price of the underlying asset at time t_i
- $\sigma^2 = \frac{1}{T}\ln(\frac{M}{S})$ where $M = \frac{1}{m^2}\sum_{i=1}^{m}(F_i^{\,2} e^{\sigma_i^2 T_i} + 2\sum_{i<j} F_i F_j e^{\sigma_i^2 T_i})$

Forward Starting Vanilla Option

For Forward Starting vanilla options we use the following:

- Payoff = $(I(S(T) - \alpha S(t_v))^+$
  - I = 1 gives the formula for Call and I = -1 gives formula for Put
  - T is the option maturity time
  - t_v is the future time when option become valid, $0 \le t_v < T$
  - α is a positive constant
- Formula

  For $t_v > t$ situation:

  $$O(t) = IS(t)e^{-rT}[e^{bT}N(Id_1) - \alpha e^{b(t_v - t)}N(Id_2)]$$

where $d_1 = \dfrac{(t - t_v)b + \ln(1/\alpha) + (b + \sigma/2)T}{\sigma\sqrt{T}}$

$$d_2 = d_1 - \sigma\sqrt{T}$$

For $t \geq t_v$ situation, just treat $S(t_v)$ as a constant strike in normal B-S formula.

2.9 PricerBlack1FAnalyticDiscreteVanilla

New Pricing Parameters:

-DIVIDEND_MODEL: defaults to "Escrowed" (can also be "Continuous"). In Escrowed model the dividends are treated as discrete (for it to make sense, the dividend curve need to be of type discrete, otherwise the discrete part will be 0).

- NUMERIC_RHO_SHIFT: the shift applied in the numerical rho calculation. Default value is 0.01 for 1% parallel shift of rates.

- NUMERIC_RHO2_SHIFT: the shift applied to the discrete dividends in the numerical rho2 calculation. Default value is 0.10 for 10%.

- VALUE_INTRADAYS: true/false to value intraday at expiry using the time to expiry in seconds. For all other days than expiry the time is computed from day to day without taking time of day into account. If false, the price at expiry is obtained using TTE=0. If true, TTE > 0 and a THETA can be computed between current valuation time and 1s before expiration.

The DIVIDEND_YIELD pricing parameter is of course useless when DIVIDEND_MODEL is set to "Escrowed".

Pricer Measures:

This pricer supports the traditional Calypso pricer measures (PRICE,CASH, NPV, etc.). Additionally it supports:

-MODEL: presents various data of the forward Black and Scholes model (including Forward value, dividend value in forward, capitalization factor).

It also currently supports only the following Greeks:

- DELTA
- GAMMA,
- THETA (numeric): For TTE/THETA pricer measure, the volatility surface daycount is used.
- VEGA
- RHO (numeric)
- RHO2 (numeric).

Supported Products:

This pricer can be used on products Equity Structured Option (the new equity option since v11.0), OTC Equity Option, ETO Equity and ETO Equity Index.

The analytic model for this pricer is the Forward Model for Vanilla with Discrete Dividends.

2.10 PricerBlack1FAnalyticQuanto

For Quanto European vanilla option we use:

$$O = E[ISe^{(b-r)T} N(Id_1) - IKe^{-rT} N(Id_2)]$$

where $d_1 = \dfrac{\ln(S/K) + (b + \sigma/2)T}{\sigma\sqrt{T}}$

$d_2 = d_1 - \sigma\sqrt{T}$

Note

- $b = r_f - q - \rho\sigma_S\sigma_E$ q is the continuous dividend yield, r_f is the foreign discount rate, ρ is correlation between asset and domestic exchange rate, σ_S is the volatility of underlying asset, σ_E is the volatility of domestic exchange rate.
- I = 1 gives the formula for Call and I = -1 gives formula for Put
- E is the prefixed domestic exchange rate

2.11 PricerBlack1FAnalyticBarrier

European standard/digital single/double barrier options are priced using the formulas given under Barrier Options.

2.12 PricerBlack1FAnalyticDigital

European style digital vanilla options are priced using the formulas given under Barrier Options.

2.13 PricerBlack1FBinomialVanilla

American style vanilla options or priced using the binomial tree method.

2.14 PricerBlack1FFiniteDifference

European, Bermudan and American options with Vanilla payoffs, and Digitals (observed at expiry, continuously, and at discrete points) can be priced with this pricer. It is based on the Finite Difference methods, calibrated with constant volatility and time dependent rates.

Pricing Parameters:

PricerBlack1FFiniteDifference has 2 additional Pricing Parameters over PricerBlack1FAnalyticVanilla:

- ACCURACY_LEVEL : defaults to 6. 0 is fast/not very accurate, 9 is slow/very accurate. It corresponds to specific refinements of the finite difference grid: a low accuracy level means a relatively small number of space & time grid points where the PDE is solved, a high accuracy means a high number of space & time grid points. The exact number of space and time points depends on the payoff.
- Rannacher: Default scheme to use. It is based on Crank-Nicolson, with a few Euler implicit steps at each discontinuity (for example at maturity for a European, at each exercise date for a Bermudan, etc.). This produces smooth Greeks and will most of the time have a convergence of nearly second order.

- TR-BDF2: an A-stable scheme, like Euler implicit, but also of second order convergence, like Crank-Nicolson. It has the advantage of keeping the greeks always smooth, while preserving accuracy, especially for american options.
- Crank-Nicolson : Can be used for comparison purposes.
- Euler Implicit : Can be used for comparison purposes.
- Lawson-Morris : Not supported, can be used as experimental.

Pricer Measures:

- DELTA : one can double click and see the graph of the delta at valuation date as a function of the spot price.
- GAMMA: one can double click and see the graph of the gamma at valuation date as a function of the spot price.
- PRICE: one can double click and see the graph of the price at valuation date as a function of the spot price, as well as the payoff at maturity.
- VEGA : one can double click and see the graph of the delta at valuation date as a function of the spot price. It is computed numerically with a small volatility perturbation.
- THETA : one can double click and see the graph of the theta at valuation date as a function of the spot price. It is computed numerically with a small time to expiry perturbation.

For more information about the above scheme and their analytics see the referenced paper.

2.15 PricerDividendSwap

NPV of a Dividend Swap is the sum of the NPV of all Non-Paid Flows. The NPV in Payment Currency is equal to:

$$NPV_{PmtCcy} = \sum_{i=Flows} NPVFlow_i$$

Where,

NPVFlowis the NPV, in Payment Currency of the ith Non-Paid Flows.

$$NPVFlow_i = df_i * (RealizedDividends_i + UnrealizedDividends_i) * FX_i$$

Where,

- df = the discount factor from the **Payment** date of the ith flow
- RealizedDividends = the realized dividends from the Start of the ith flow, up to and including the Valuation Date.
- UnrealizedDividends =the expected dividends, from the Dividend Curve, for the period directly **after**the Valuation Date to the end of the Flow.
- FX_i = the FX Rate between the Asset Currency and the Payment currency. This will equal 1 if the they are the same, OR will equal the “FixedFX” rate if this is chosen on the deal.

2.15.1 Unrealized Dividend Calculation Methodology

The calculation of unrealized dividends will be based on the dividend projection style on the Dividend Curve. Currently, projected dividends can be discrete or continuous yield.

Discrete Curves

Unrealized Dividends for a period will be equal to the sum of all projections whose "ex-date" is greater than the Valuation date and falls into Start and End date of the period.

Yield Based Dividend Projections

The yield is presumed to be an annualized return, in percent, that is applied to the Spot level of the underlying. For example, a curve as follows:

Valuation Date: Feb-15-2009

SPX500 Level: 850

Offset	Yield(%)	Dividend Point(Calculated)
1M	2.1%	850 * 2.1% * #days / 365
2M	2.2%	850 * 2.2% * #days / 365
3M	2.3%	850 * 2.3% * #days / 365
18M	2.4%	…
2Y	2.5%	…

Since these are "Spot" dividend yields, then we will have to calculate "Forward" dividend yields for Forward starting Dividend Periods. The following two examples will demonstrate the calculation methodology.

Example #1

Start Period: Jan-01-2009

End Period: Jan-01-2010

Valuation Date: Feb-15-2009

Implied Dividend Point calculation (steps)

- Note unrealized portion of flow: Feb-16-2009 ->Jan-01-2010
- Ask dividend curve for the yield for this period. The curve will interpolate between 3M and 18M. Lets presume it returns 2.33%
- Calculate the implied Dividend Points for the period:
- #days * 2.33% * Spot Level of SPX500 / 365.
- Where #days is the number of Calendar days between Feb-16-2009 and Jan-01-2010.

Example #2

Forward starting Period

Start Period: Jan-01-2010

End Period: Jan-01-2011

Valuation Date: Feb-15-2009

Implied Dividend Point calculation (steps)

- Note unrealized portion of flow: Jan-01-2010 ->Jan-01-2011
- Ask dividend curve for the yield for this period. The curve should calculate the forward using the 1Y, 18M and 2Y points. Lets presume it returns 2.4%
- Calculate the implied Dividend Points for the period:
- #days * 2.4% * Spot Level of SPX500 / 365.
- Where #days is the number of Calendar days between Jan-01-2010 and Jan-01-2011.

2.16 PricerEquityLinkedSwap

Refer to the Credit Derivatives Analytics guide for Total Return Swap.

2.17 PricerEquityLinkedSwapAccrual

Accrual pricing approach defines the ELS value by recognizing only the unrealized performance and financing based on today's value of the underlying asset. Future flows are not considered while pricing.

NPV = Unpaid performance + Unpaid incomes - Financing Costs

If the deal can be terminated at any time without taking future flows into account Accrual pricing methodology should be used.

Use the pricing parameter FIXING_DATE_ACCRUAL.

True or False. Determines when a cash flow is no longer included in the NPV of the swap. True so that cash flow realization is based on the fixing date. False so that the cash flow realization is based on the payment date.

Default is false.

2.18 PricerBlackNFJuAnalyticVanilla

The BlackNFJuAnalyticVanillapricer, supports the valuation of European basket options within the Equity Structured Option product. The model is the n-factor Black-Sholes model, whilst the valuation is based on an accurate analytic approximation of Ju, M. (2002), “Pricing asian and basket options via Taylor expansion”, Journal of Computational Finance, 5(3):79-103.

2.19 PricerBlackNFMonteCarlo

The BlackNFMonteCarlopricer, supports the valuation of European basket options within the Equity Structured Option product. The model is the n-factor Black-Sholes model, whilst the valuation is based on a monte-carlo routine.

2.20 PricerBlack1FSemiAnalyticChooser

The Black1FSemiAnalyticChooser pricersupports the valuation of simple and complex chooser option within the Equity Structured Option product. The model is the 1-factor Black-Sholes model with internal adjusters. The valuation is performed with numerical integration. The internal adjusters assure the limiting cases in pricing the underlying options are handled well.

2.21 PricerBlack1FSemiAnalyticCompound

The Black1FSemiAnalyticCompound pricer supports the valuation of compound options within the Equity Structured Option product. The model is the 1-factor Black-Sholes model with internal adjusters. The valuation is performed with numerical integration. The internal adjusters assure the limiting cases in pricing the underlying options are handled well.

2.22 PricerBlack1FMonteCarlo

The Black1FMonteCarlo pricer supports the valuation of European path dependent options within the Equity Structured Option product. The model is the 1-factor Black-Sholes model, whilst the valuation is based on a monte-carlo routine.

Section 3. Analytics

This chapter described the models used for pricing.

3.1 Black-Scholes Model

The standard Black-Scholes model is used for European style (single exercise) single asset options on equities, indices and futures.

The general form of the Black-Scholes model is used to value European style vanilla options. European options do not give the holder to exercise before maturity and therefore have an analytic solution. For calls and puts, we derive the following equations (see below):

$$C = e^{-r_p t_p}(FN(d_1) - KN(d_2))$$

$$P = e^{-r_p t_p}(KN(-d_2) - FN(-d_1))\ ,$$

where $F = \frac{Se^{-qt_e}}{e^{-r_g t_e}} = Se^{(r_g - q)t_e}$, $d_1 = \frac{\ln\left[\frac{F}{K}\right]}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$ and $d_2 = d_1 - \sigma\sqrt{t_e}$.

This particular variation on the Black-Scholes model accounts for the timing of cashflows encountered in real-world transactions. The model does this by using different time periods for each type of cashflow. The variables are described as follows:

- S is the spot price of the underlying security.
- K is the strike price
- r_p is the continuously compounded risk-free rate with base act/365
- q is the continuously compounded dividend yield with base act/365
- r_g is the continuously compounded growth rate with base act/365

  Typically, periodically compounded rates which can be changed to continuous rates using the following equation:

  $$e^r = \left(1 + \frac{r}{n}\right)^{\frac{1}{n}}$$

  with n being the periodicity.
- σ is the volatility of returns of the underlying security
- t_e is the time period from the valuation date to the option's expiration date, i.e. the time for which the option is traded
- t_p is the time to payment (i.e. from the valuation date to the settlement date (usually two days after expiration)
- N(x) is the cumulative standard normal distribution function

Also, it is important to note that the value from the Option Pricing model may need adjustments to obtain the desired value (e.g. FX, Libor, and Swaptions).

All pricer measures (npv and Greeks) as defined below can be expressed also as of spot date by dividing the pricer measures as of value date by the discount factor between value date and spot date.

Implementation of the European Option Pricing Model

The general equation can be utilized to price various types of instruments. The parameter values are shown in the table below for each instrument. The correct pricing equation can be found by setting the parameters as shown.

Underlying Security	r_p equals	q equals	r_g equals
Stocks w/o dividends	risk-free rate	0	risk-free rate
Stocks w/ dividends	risk-free rate	dividend	risk-free rate
FX	quoting currency's interest rate	base currency's interest rate	quoting currency's interest rate
Futures	risk-free rate	0	0
Libor	risk-free rate	0	0
Swap	risk-free rate	0	0

Black Scholes Model 3-1.

3.1.1 Call Options

NPV

$$C = e^{-r_p t_p}(FN(d_1) - KN(d_2))$$

where

$$F = Se^{(r_g - q)t_e}$$

$$d_1 = \frac{\ln\left[\frac{F}{K}\right]}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t_e} \qquad (1)$$

$$d_2^2 = d_1^2 - 2\sigma d_1\sqrt{t_e} + \sigma^2 t_e$$

$$d_2^2 = d_1^2 - 2\sigma\frac{\ln\left[\frac{F}{K}\right] + \sigma^2 t_e}{\sigma\sqrt{t_e}}$$

$$d_2^2 = d_1^2 - 2\ln\left(\frac{F}{K}\right) \qquad (2)$$

$$\begin{aligned} n(d_2) &= \frac{e^{\frac{-d_2^2}{2}}}{\sqrt{2\pi}} \\ &= \frac{e^{\frac{-d_1^2}{2} + \ln\left(\frac{F}{K}\right)}}{\sqrt{2\pi}} \\ &= \frac{Fe^{\frac{-d_1^2}{2}}}{K\sqrt{2\pi}} \\ &= n(d_1)\frac{F}{K} \qquad (3) \end{aligned}$$

Delta

$$\Delta_c = \frac{\partial C}{\partial S} = e^{-r_p t_p}\left(e^{(r_g-q)t_e}(Sn(d_1)\frac{\partial d_1}{\partial S} + N(d_1)) - Kn(d_2)\frac{\partial d_2}{\partial S}\right)$$

Substituti ng equation (3) gives :

$$\Delta_c = e^{-r_p t_p + (r_g-q)t_e} N(d_1) + e^{-r_p t_p + (r_g-q)t_e} Sn(d_1)\frac{\partial d_1}{\partial S} - e^{-r_p t_p} Kn(d_1)\frac{\partial d_1}{\partial S}\frac{F}{K}$$

$$\Delta_c = e^{-r_p t_p + (r_g-q)t_e} N(d_1) + e^{-r_p t_p + (r_g-q)t_e} Sn(d_1)\frac{\partial d_1}{\partial S} - Se^{-r_p t_p + (r_g-q)t_e} n(d_1)\frac{\partial d_1}{\partial S}$$

$$\frac{\partial d_1}{\partial S} = \frac{\partial}{\partial S}\frac{(\ln Se^{-qt_e} - \ln Ke^{-rt_e})}{\sigma\sqrt{t_e}} = \frac{1}{S\sigma\sqrt{t_e}}$$

$$\Delta_c = e^{-r_p t_p + (r_g-q)t_e} N(d_1)$$

Delta Premium

$$\Delta_{c,premium} = e^{-r_p t_p + (r_g-q)t_e} N(d_1)*100 - e^{-r_p t_p}(FN(d_1) - KN(d_2))$$

Delta Forward

$$\Delta_{c,forward} = \frac{\partial C}{\partial F} = e^{-r_p t_p}\left((Fn(d_1)\frac{\partial d_1}{\partial F} + N(d_1)) - Kn(d_2)\frac{\partial d_2}{\partial F}\right)$$

Substituti ng equation (3) gives :

$$\Delta_c = e^{-r_p t_p} N(d_1) + e^{-r_p t_p} Fn(d_1)\frac{\partial d_1}{\partial F} - e^{-r_p t_p} Kn(d_1)\frac{\partial d_1}{\partial F}\frac{F}{K}$$

$$\Delta_c = e^{-r_p t_p} N(d_1) + e^{-r_p t_p} Fn(d_1)\frac{\partial d_1}{\partial F} - Fe^{-r_p t_p} n(d_1)\frac{\partial d_1}{\partial F}$$

$$\Delta_c = e^{-r_p t_p} N(d_1)$$

Gamma

$$\Gamma_c = \frac{\partial^2 C}{\partial S^2} = \frac{\partial}{\partial S}\Delta_c$$

$$\Gamma_c = e^{-r_p t_p + (r_g-q)t_e} n(d_1)\frac{\partial d_1}{\partial S}$$

$$\frac{\partial d_1}{\partial S} = \frac{1}{S\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\Gamma_c = \frac{1}{S\sigma\sqrt{t_e}}\left(e^{-r_p t_p + (r_g-q)t_e} n(d_1)\right)$$

Vega

$$\nu_c = \frac{\partial C}{\partial \sigma} = e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial \sigma} - e^{-r_p t_p} Kn(d_2)\frac{\partial d_2}{\partial \sigma}$$

$$\nu_c = e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-r_p t_p} n(d_1)\frac{\partial d_2}{\partial \sigma}\frac{S}{K} e^{r_g t_e - q t_e}$$

$$\nu_c = Se^{-r_p t_p + (r_g - q)t_e} n(d_1)(\frac{\partial d_1}{\partial \sigma} - \frac{\partial d_2}{\partial \sigma})$$

where

$$\frac{\partial d_1}{\partial \sigma} = -\frac{\ln\left[\frac{F}{K}\right]}{\sigma^2 \sqrt{t_e}} + \frac{\sqrt{t_e}}{2}$$

and

$$\frac{\partial d_2}{\partial \sigma} = -\frac{\ln\left[\frac{F}{K}\right]}{\sigma^2 \sqrt{t_e}} - \frac{\sqrt{t_e}}{2}$$

$$\nu_c = e^{-r_p t_p + (r_g - q)t_e} S\sqrt{t_e} n(d_1)$$

rho$_{r_g}$

$$\rho_{c,g} = \frac{\partial C}{\partial r_g} = e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial r_g} + St_e e^{-r_p t_p + (r_g - q)t_e} N(d_1) - Ke^{-r_p t_p} n(d_2)\frac{\partial d_2}{\partial r_g}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-r_g t_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial d_1}{\partial r_g} = \frac{\partial d_2}{\partial r_g} = \frac{t_e}{\sigma\sqrt{t_e}}$$

Substituti ng equation (3) gives :

$$\rho_{c,g} = \frac{t_e e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)}{\sigma\sqrt{t_e}} + St_e e^{-r_p t_p + (r_g - q)t_e} N(d_1) - Ke^{-r_p t_p} n(d_2)\frac{\partial d_2}{\partial r}$$

$$\rho_{c,g} = e^{-r_p t_p + (r_g - q)t_e} S(\frac{t_e n(d_1)}{\sigma\sqrt{t_1}} + t_e N(d_1)) - Ke^{-r_p t_p} n(d_2)\frac{\partial d_2}{\partial r}$$

$$\rho_{c,g} = e^{-r_p t_p + (r_g - q)t_e} S(\frac{t_e n(d_1)}{\sigma\sqrt{t_1}} + t_e N(d_1)) - e^{-r_p t_p} Kn(d_1)\frac{\partial d_1}{\partial r}\frac{S}{K}e^{(r_g - q)t_e})$$

$$\rho_{c,g} = e^{-r_p t_p + (r_g - q)t_e} S(\frac{t_e n(d_1)}{\sigma\sqrt{t_e}} + t_e N(d_1)) - Se^{-r_p t_p + (r_g - q)t_e} n(d_1)\frac{\partial d_1}{\partial r}$$

$$\rho_{c,g} = e^{-r_p t_p + (r_g - q)t_e} St_e N(d_1)$$

rho$_{r_p}$

$$\rho_{c,p} = \frac{\partial C}{\partial r_p} = e^{-r_p t_p} Kt_p N(d_2) - t_p e^{-r_p t_p + (r_g - q)t_e} SN(d_1)$$

rho$_q$

$$\rho_{c,q} = \frac{\partial C}{\partial q} = e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial q} - e^{-r_p t_p + (r_g - q)t_e} St_e N(d_1) - Ke^{-r_p t_p} n(d_2)\frac{\partial d_2}{\partial q}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-rt_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial d_1}{\partial q} = \frac{\partial d_2}{\partial q} = \frac{-t_e}{\sigma\sqrt{t_e}}$$

Substituti ng equation (3) gives :

$$\rho_{c,q} = e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial q} - Ke^{-r_p t_p} n(d_1)\frac{S}{K}e^{(r_g - q)t_e}\frac{\partial d_1}{\partial q} - St_e e^{-r_p t_p + (r_g - q)t_e} N(d_1)$$

$$\rho_{c,q} = -St_e e^{-r_p t_p + (r_g - q)t_e} N(d_1)$$

Theta

Defining $t_1 = T_1 - \tau$, $t_2 = T_2 - \tau$, and $t_3 = T_3 - \tau$:

$$\Theta_c = -\frac{\partial C}{\partial \tau} = e^{-r_p t_p + (r_g - q)t_e}(q + r_p - r_g)SN(d_1) - e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial \tau} + Ke^{-r_p t_p} n(d_2)\frac{\partial d_2}{\partial \tau} - e^{-r_p t_p} Kr_p N(d_2)$$

Substituti ng equation (3) gives :

$$\Theta_c = e^{-r_p t_p + (r_g - q)t_e}(q + r_p - r_g)SN(d_1) - e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\frac{\partial d_1}{\partial \tau} + Ke^{-r_p t_p + (r_g - q)t_e} n(d_1)\frac{S}{K}\frac{\partial d_2}{\partial \tau} - e^{-r_p t_p} Kr_p N(d_2)$$

$$\Theta_c = e^{-r_p t_p + (r_g - q)t_e}(q + r_p - r_g)SN(d_1) + e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\left(\frac{\partial d_2}{\partial \tau} - \frac{\partial d_1}{\partial \tau}\right) - e^{-r_p t_p} Kr_p N(d_2)$$

Given :

$$\frac{\partial d_2}{\partial \tau} - \frac{\partial d_1}{\partial \tau} = \frac{\partial}{\partial \tau}\sigma\sqrt{t_e} = -\frac{\sigma}{2\sqrt{t_e}}$$

$$\Theta_c = e^{-r_p t_p + (r_g - q)t_e}(q + r_p - r_g)SN(d_1) - \frac{e^{-r_p t_p + (r_g - q)t_e} S\sigma n(d_1)}{2\sqrt{t_e}} - e^{-r_p t_p} Kr_p N(d_2)$$

3.1.2 Put Options

NPV

$$P = Ke^{-r_p t_p} N(-d_2) - Se^{-r_p t_p + (r_g - q)t_e} N(-d_1)$$

$$d_1 = \frac{\ln\left[\frac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t_e} \qquad (1)$$

$$d_2^2 = d_1^2 - 2\sigma d_1 \sqrt{t_e} + \sigma^2 t_e$$

$$d_2^2 = d_1^2 - 2\sigma \frac{\ln\left[\frac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right] + \sigma^2 t_e}{\sigma\sqrt{t_e}}$$

$$d_2^2 = d_1^2 - 2\ln\left(\frac{S}{K}e^{r_g t_e - qt_e}\right) \qquad (2)$$

$$\begin{aligned} n(d_2) &= \frac{e^{\frac{-d_2^2}{2}}}{\sqrt{2\pi}} \\ &= \frac{e^{\frac{-d_1^2}{2} + \ln\left(\frac{S}{K}e^{r_g t_e - qt_e}\right)}}{\sqrt{2\pi}} \\ &= \frac{Se^{\frac{-d_1^2}{2}}}{K\sqrt{2\pi}} e^{r_g t_e - qt_e} \\ &= n(d_1)\frac{S}{K}e^{r_g t_e - qt_e} \qquad (3) \end{aligned}$$

Delta

$$\Delta_p = \frac{\partial P}{\partial S} = e^{-r_p t_p}\left(-e^{(r_g-q)t_e}(Sn(-d_1)\frac{\partial -d_1}{\partial S} + N(-d_1)) + Kn(-d_2)\frac{\partial -d_2}{\partial S}\right)$$

Substituti ng equation (3) in the equation above gives :

$$\Delta_p = -e^{-r_p t_p+(r_g-q)t_e}N(-d_1) - e^{-r_p t_p+(r_g-q)t_e}Sn(-d_1)\frac{\partial -d_1}{\partial S} + e^{-r_p t_p}Kn(-d_2)\frac{\partial -d_2}{\partial S}\frac{F}{K}$$

$$\Delta_p = -e^{-r_p t_p+(r_g-q)t_e}N(-d_1) - e^{-r_p t_p+(r_g-q)t_e}Sn(-d_1)\frac{\partial -d_1}{\partial S} - Se^{-r_p t_p+(r_g-q)t_e}n(-d_1)\frac{\partial -d_1}{\partial S}$$

$$\frac{\partial -d_1}{\partial S} = \frac{\partial -d_2}{\partial S} = \frac{\partial}{\partial S}\frac{(\ln Se^{-qt_e} - \ln Ke^{-rt_e})}{\sigma\sqrt{t_e}} = -\frac{1}{S\sigma\sqrt{t_e}}$$

$$\Delta_p = -e^{-r_p t_p+(r_g-q)t_e}N(-d_1)$$

Delta Premium

$$\Delta_{p,premium} = -e^{-r_p t_p+(r_g-q)t_e}N(-d_1)*100 + Ke^{-r_p t_p}N(-d_2) - Se^{-r_p t_p+(r_g-q)t_e}N(-d_1)$$

Delta Forward

$$\Delta_{p,forward} = \frac{\partial P}{\partial F} = e^{-r_p t_p}\left((-Fn(-d_1)\frac{\partial -d_1}{\partial F} - N(-d_1)) + Kn(-d_2)\frac{\partial -d_2}{\partial F}\right)$$

Substituti ng equation (3) gives :

$$\Delta_{p,forward} = -e^{-r_p t_p}N(-d_1) - e^{-r_p t_p}Fn(-d_1)\frac{\partial -d_1}{\partial F} + e^{-r_p t_p}Kn(-d_1)\frac{\partial -d_1}{\partial F}\frac{F}{K}$$

$$\Delta_{p,forward} = -e^{-r_p t_p}N(-d_1) - e^{-r_p t_p}Fn(-d_1)\frac{\partial -d_1}{\partial F} + Fe^{-r_p t_p}n(-d_1)\frac{\partial -d_1}{\partial F}$$

$$\Delta_{p,forward} = -e^{-r_p t_p}N(-d_1)$$

Gamma

$$\Gamma_p = \frac{\partial^2 P}{\partial S^2} = \frac{\partial}{\partial S} - e^{-r_p t_p+(r_g-q)t_e}N(-d_1)$$

$$\frac{\partial -d_1}{\partial S} = \frac{\partial}{\partial S}\frac{-(\ln Se^{-qt_e} - \ln Ke^{-rt_e})}{\sigma\sqrt{t_e}} = -\frac{1}{S\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\Gamma_p = \frac{e^{-r_p t_p+(r_g-q)t_e}n(d_1)}{S\sigma\sqrt{t_e}}$$

Vega

$$\nu_p = \frac{\partial P}{\partial \sigma} = -e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)\frac{\partial - d_1}{\partial \sigma} + e^{-r_p t_p} Kn(-d_2)\frac{\partial - d_2}{\partial \sigma}$$

$$\nu_p = -e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)\frac{\partial - d_1}{\partial \sigma} + Ke^{-r_p t_p} n(-d_1)\frac{\partial - d_1}{\partial \sigma}\frac{S}{K}e^{(r_g - q)t_e}$$

$$\nu_p = -e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)\left(\frac{\partial - d_1}{\partial \sigma} - \frac{\partial - d_2}{\partial \sigma}\right)$$

The difference in the partial derivative s with respect to σ can be derived from equation (1) and is found to be - $\sqrt{t_1}$.

Therefore, ν can be written as follows :

$$\nu_p = e^{-r_p t_p + (r_g - q)t_e} Sn(d_1)\sqrt{t_e}$$

rho$_{r_p}$

$$\rho_{p,p} = \frac{\partial P}{\partial q} = -t_p e^{-r_p t_p} KN(-d_2) + t_p e^{-r_p t_p + (r_g - q)t_e} SN(-d_1)$$

rho$_{r_g}$

$$\rho_{p,g} = \frac{\partial P}{\partial r_g} = -e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)\frac{\partial - d_1}{\partial r_g} - St_e e^{-r_p t_p + (r_g - q)t_e} N(-d_1) + Ke^{-r_p t_p} n(-d_2)\frac{\partial - d_2}{\partial r_g}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-r_g t_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial - d_1}{\partial r} = \frac{\partial - d_2}{\partial r} = -\frac{t_e}{\sigma\sqrt{t_e}}$$

$$\rho_{p,g} = \frac{t_e e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)}{\sigma\sqrt{t_e}} - St_e e^{-r_p t_p + (r_g - q)t_e} N(-d_1) + Ke^{-r_p t_p} n(-d_2)\frac{\partial - d_2}{\partial r_g}$$

$$\rho_{p,g} = e^{-r_p t_p + (r_g - q)t_e} S(\frac{t_e n(-d_1)}{\sigma\sqrt{t_e}} - t_e N(d_1)) + Ke^{-r_p t_p} n(-d_2)\frac{\partial - d_2}{\partial r_g}$$

Substituti ng equation (3) gives :

$$\rho_{p,g} = e^{-r_p t_p + (r_g - q)t_e} S(\frac{t_e n(-d_1)}{\sigma\sqrt{t_1}} - t_e N(-d_1)) + e^{-r_p t_p} Kn(-d_1)\frac{\partial - d_1}{\partial r_g}\frac{S}{K}e^{(r_g - q)t_e})$$

$$\rho_{p,g} = e^{-r_p t_p + (r_g - q)t_e} S(\frac{t_e n(-d_1)}{\sigma\sqrt{t_e}} - t_e N(-d_1)) + Se^{-r_p t_p + (r_g - q)t_e} n(-d_1)\frac{\partial d_1}{\partial r_g}$$

$$\rho_{p,g} = -e^{-r_p t_p + (r_g - q)t_e} St_e N(-d_1)$$

rho$_q$

The sensitivity of the option price to dividends

$$\rho_{2,p} = \frac{\partial P}{\partial q} = -e^{-r_p t_p + (r_g - q)t_e}(Sn(-d_1)\frac{\partial - d_1}{\partial q} - St_e N(-d_1)) + Ke^{-r_p t_p} n(-d_2)\frac{\partial - d_2}{\partial q}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-rt_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial d_1}{\partial q} = \frac{\partial d_2}{\partial q} = \frac{-t_e}{\sigma\sqrt{t_e}}$$

Substituti ng equation (3) gives :

$$\rho_{2,p} = -e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)\frac{\partial - d_1}{\partial q} + Ke^{-r_p t_p} n(-d_1)\frac{S}{K}e^{(r_g - q)t_e}\frac{\partial - d_1}{\partial q} + St_e e^{-r_p t_p + (r_g - q)t_e} N(-d_1)$$

$$\rho_{2,c} = St_e e^{-r_p t_p + (r_g - q)t_e} N(-d_1)$$

Theta

$$\Theta_p = -\frac{\partial P}{\partial \tau} = e^{-r_p t_p + (r_g - q)t_e}((-q - r_p + r_g)SN(-d_1) + Sn(-d_1)\frac{\partial - d_1}{\partial \tau}) - e^{-r_p t_p} Kn(-d_2)\frac{\partial - d_2}{\partial \tau} + e^{-r_p t_p} Kr_p N(-d_2)$$

Substituti ng equation (3) gives :

$$\Theta_p = e^{-r_p t_p + (r_g - q)t_e}((-q - r_p + r_g)SN(-d_1) + Sn(-d_1)\frac{\partial - d_1}{\partial \tau}) - e^{-r_p t_p} Kn(-d_1)\frac{S}{K} e^{(r_g - q)t_e}\frac{\partial - d_2}{\partial \tau} + e^{-r_p t_p} Kr_p N(-d_2)$$

$$\Theta_p = e^{-r_p t_p + (r_g - q)t_e}(-q - r_p + r_g)SN(-d_1) + e^{-r_p t_p + (r_g - q)t_e} Sn(-d_1)\left(\frac{\partial - d_1}{\partial \tau} - \frac{\partial - d_2}{\partial \tau}\right) + e^{-r_p t_p} Kr_p N(-d_2)$$

Given :

$$\frac{\partial - d_1}{\partial \tau} - \frac{\partial - d_2}{\partial \tau} = \frac{\partial}{\partial \tau} - \sigma\sqrt{t_e} = -\frac{\sigma}{2\sqrt{t_e}}$$

$$\Theta_p = e^{-r_p t_p + (r_g - q)t_e}((-q - r_p + r_g)SN(-d_1) - \frac{e^{-r_p t_p + (r_g - q)t_e} S\sigma n(-d_1)}{2\sqrt{t_e}}) + e^{-r_p t_p} Kr_p N(-d_2)$$

3.2 Forward Model for Vanillas with Discrete Dividends

Notations

*S(t)*Value of the Stock at time *t*.
F(*t*,*T*) Value of the Forward between *t* and *T*.
B(*t*,*T*) Discount factor between *t* and *T*.
RF(*t*,*T*) Repo factor between *t* and *T*. If we suppose that the instantaneous repo rate is constant, equal tor_p, RF(t,T) = $e^{-rp(T-t)}$
C(*t*,*T*) Capitalisation factor between *t* and *T*.
d_{ti}Fixed Dividend paid at the datet_{di}
d(*t*,*T*) Cumulated exponential dividend rate from *t* to *T*.

European Option Pricing

We consider a European Option (Call or Put) with the following characteristics:
K Option Strike.
t_0Pricing Date.
t_pPremium payment date (if it is in the past, then we set 0 t t p).
T Final observation date.
T_dDelivery date.
P_t Option price at time *t*
ε = 1for a call and ε = −1 for a put

$$P_{t_0} = \varepsilon \frac{B(t_0, T_d)}{B(t_0, t_p)}[F(t_0, T)N(\varepsilon d_1) - KN(\varepsilon d_2)$$

with

$$d_1 = \frac{ln\ (F(t_0, T)/K)}{\sigma\sqrt{T - t_0}} + \frac{1}{2}\sigma\sqrt{T - t_0}$$

$$d_2 = \frac{ln\,(F(t_0,T)/K)}{\sigma\sqrt{T-t_0}} - \frac{1}{2}\sigma\sqrt{T-t_0}$$

Computation of the Forward

Dividend yield:

$$F(t,T) = S_t C(t,T) e^{-d(t,T)(T-t)}$$

Fixed Discrete Dividends:

$$F(t,T) = S_t C(t,T) - \sum_{t_{d_i}<T} d_{t_i}\, C(t_{d_i},T)$$

To simplify notations, we have not separated the ex-date from the payment date in the previous formula but it should be clear that the inclusion of the dividend depends on the ex-date and that the capitalization depends on the payment date. Moreover we stress that the repo should be included in the capitalization of the dividend (this point is rather technical and is due to the fact that a borrowing contract is written on a notional, not on a number of shares).

Sensitivities

Delta / Gamma:

Delta is computed as $\frac{\partial P_{t_0}}{\partial F}\frac{\partial F}{\partial S}$.The first term is given by the classical Black Scholes formula, the second depends on the representation of the forward. This approach shall be extended for the computation of the Gamma.

Theta:

Theta is not be computed by Black Scholes analytical formula! In fact, the definition of Theta strongly depends on the approach chosen for the rate (is the Theta computed by keeping the market rate unchanged or by keeping the forward rate unchanged). So, once the time evolution of the curves has been setup, Theta can only be computed by finite difference using the correct day count conventions for each object.

Vega:

Black and Scholes analytical formula can be used.

Rho:

Rho is just indicative since the rate risk is analysed through a risk report that shifts the market rates one by one.

At the deal level, it is computed numerically, by shifting up the whole discount rates by one percent (using the conventions set at the rate curve level).

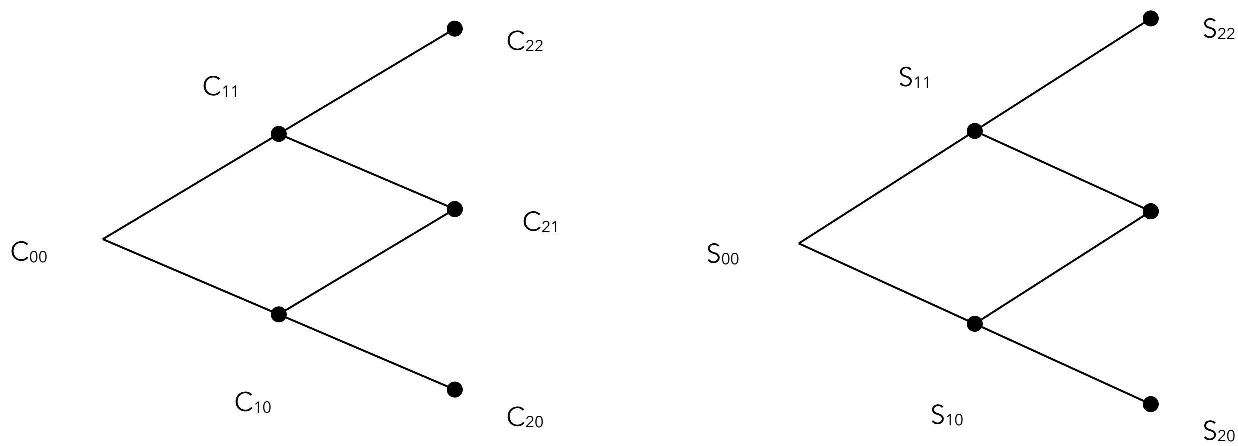
Rho 2:

It is computed numerically by shifting up the dividends not yet announced by ten percent of their expected value.

3.3 Binomial Single Asset Tree

Multi-exercise single asset vanilla options: for FX options, equity options, and future options.

The standard Cox, Ross and Rubinstein version of the Binomial tree model is used to value American-style options. The Binomial Tree model is a discrete version of the continuous Black-Scholes model and discretizes time into equal intervals. At the time t=0, the path of the security can either go up exp(u) or down exp(d) with a probabilities of p_{up} and p_{down}, respectively. A tree of asset prices can then be constructed and the option payoff at each node can be calculated and discounted back to time t=0 to arrive at the option value. The equations for constructing the tree are shown below.



$$u = \exp(\sigma\sqrt{\Delta t_e})$$

$$d = \frac{1}{u}$$

$$p_{up} = \frac{\exp((r_g - q)\Delta t_e) - d}{u - d}$$

$$p_{down} = 1 - p_{up}$$

The American-style option price must be calculated at each node. This is done using the following equations:

At maturity (t = T), the option price is computed as follows :

$$C = \max[S - K, 0]$$

$$P = \max[K - S, 0]$$

Prior to maturity, the American option price is computed as follows :

$$C_{11} = \max[S_{11} - K,\ e^{-r_p t_p}(p_{up} * C_{22} + p_{down} C_{21})]$$

$$P_{11} = \max[K - S_{11},\ e^{-r_p t_p}(p_{up} * P_{22} + p_{down} P_{21})]$$

The variables are described as follows:

- S is the spot price of the underlying security.
- K is the strike price
- r_g, r_p are equal to the continuous discount rates (see table on page 5 for more information)
- q is the continuous dividend yield of the security

  Note that both r and q are continuous rates. Typically, the market quotes periodically compounded rates which can be changed to continuous rates using the following equation:

  $r_{continuous} = [(\ln(r_{periodic}*t+1)] / t$
- σ is the volatility of returns of the underlying security
- t_e is the time period from the valuation date to the option's expiration date
- t_p is the time period from the spot date (typically 2 days after the transaction date) and the settlement date (typically 2 days after expiration)

The price of the option is affected by changes in the various inputs. The change in the option price can be determined by using the formulas shown below.

Delta

Delta is the rate of change of the option value with respect to the underlying's spot price.

$$\Delta_c = \frac{\partial C}{\partial S} = \frac{C_{11} - C_{10}}{S_{00}(u-d)}$$

$$\Delta_p = \frac{\partial P}{\partial S} = \frac{P_{11} - P_{10}}{S_{00}(u-d)}$$

Delta Premium

$$\Delta_{c,premium} = \frac{C_{11} - C_{10}}{S_{00}(u-d)} - C$$

$$\Delta_{p,premium} = \frac{P_{11} - P_{10}}{S_{00}(u-d)} + P$$

Delta Forward

$$\Delta_c = \frac{C_{11} - C_{10}}{S_{00} * e^{t_e(r_g - q)} * (u - d)}$$

$$\Delta_p = \frac{\partial P}{\partial S} = \frac{P_{11} - P_{10}}{S_{00} * e^{t_e(r_g - q)} * (u - d)}$$

Gamma

Gamma represents the rate of change of delta with respect to the spot price. It can be computed as the second derivative of the option premium with respect to spot, or the first derivative of the option's delta. Delta can also be computed with respect to a 1% change in the underlying.

$$\Gamma_c = \frac{\partial^2 C}{\partial S^2} = \frac{\dfrac{C_{22} - C_{21}}{S_{00}(u^2 - 1)} - \dfrac{C_{21} - C_{20}}{S_{00}(1 - d^2)}}{.5(S_{00}u^2 - S_{00}d^2)}$$

$$\Gamma_p = \Gamma_c$$

Theta

Theta measures the sensitivity of the option price with respect to time.

$$\Theta_c = -\frac{\partial C}{\partial \tau} = -\frac{C_{21} - C_{00}}{2\Delta\tau}$$

$$\Theta_p = -\frac{\partial P}{\partial \tau} = -\frac{P_{21} - P_{00}}{2\Delta\tau}$$

Vega

Vega measures the rate of change of the option value with respect to changes in the volatility of the underlying.

$$\nu_c = \frac{\partial C}{\partial \sigma} = \frac{C(\sigma + \Delta\sigma) - C(\sigma - \Delta\sigma)}{2\Delta\sigma}$$

$$\nu_p = \frac{\partial P}{\partial \sigma} = \nu_c$$

rho

rho measures the rate of change of the option value with respect to the risk-free rate.

$$\rho_c = \frac{\partial C}{\partial r} = \frac{C(r + \Delta r) - C(r - \Delta r)}{2\Delta r}$$

$$\rho_p = \frac{\partial P}{\partial r} = \frac{P(r + \Delta r) - P(r - \Delta r)}{2\Delta r}$$

rho₂

rho₂ is the sensitivity of the option price to dividend yield.

$$\rho_{2,c} = \frac{\partial C}{\partial q} = \frac{C(q+\Delta q) - C(q-\Delta q)}{2\Delta q}$$

$$\rho_{2,p} = \frac{\partial P}{\partial q} = \frac{P(q+\Delta q) - P(q-\Delta q)}{2\Delta q}$$

3.4 Turnbull-Wakeman

Analytic approximation for discrete arithmetic Asian options; Reference in one of the books;

3.5 Binary/Digital Options

3.5.1 Cash or Nothing Call Option

NPV

$$C = e^{-r_p t_p} K N(d_2)$$

$$d_2 = \frac{\ln\left[\frac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma\sqrt{t_e}} - \frac{\sigma\sqrt{t_e}}{2}$$

Delta

$$\Delta_c = \frac{\partial C}{\partial S} = Ke^{-r_p t_p} n(d_2) \frac{\partial d_2}{\partial S}$$

$$\frac{\partial d_2}{\partial S} = \frac{\partial}{\partial S} \frac{\ln S - \ln K}{\sigma\sqrt{t_e}} = \frac{1}{S\sigma\sqrt{t_e}}$$

$$\Delta_c = \frac{Ke^{-r_p t_p} n(d_2)}{S\sigma\sqrt{t_e}}$$

Delta Forward

$$\Delta_{c,\,forward} = \frac{\partial C}{\partial F} = Ke^{-r_p t_p} n(d_2)\frac{\partial d_2}{\partial F}$$

$$\frac{\partial d_2}{\partial F} = \frac{\partial}{\partial F}\frac{\ln F - \ln K}{\sigma\sqrt{t_e}} = \frac{1}{F\sigma\sqrt{t_e}}$$

$$\Delta_{c,\,forward} = \frac{Ke^{-r_p t_p - (r_g - q)t_e} n(d_2)}{S\sigma\sqrt{t_e}}$$

Gamma

$$\Gamma_c = \frac{\partial^2 C}{\partial S^2} = \frac{\partial}{\partial S}\frac{Ke^{-r_p t_p} n(d_2)}{S\sigma\sqrt{t_e}}$$

$$\frac{\partial n(d_2)}{\partial S} = \frac{\partial}{\partial S}\frac{-n(d_2) * d_2}{S\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\Gamma_c = \frac{e^{-r_p t_p} Kn(d_2)}{\sigma\sqrt{t_e}}\left(\frac{-n(d_2) * d_2}{S^2\sigma\sqrt{t_e}} - \frac{n(d_2)}{S^2}\right)$$

$$\Gamma_c = -\frac{e^{-r_p t_p} Kn(d_2)}{S^2\sigma\sqrt{t_e}}\left(\frac{d_2}{\sigma\sqrt{t_e}} + 1\right)$$

Vega

$$\nu_c = \frac{\partial C}{\partial \sigma} = e^{-r_p t_p} Kn(d_2)\frac{\partial d_2}{\partial \sigma}$$

$$\frac{\partial d_2}{\partial \sigma} = -\frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} - \frac{\sqrt{t_e}}{2}$$

$$\nu_c = -Ke^{-r_p t_p} n(d_2)\left(\frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} + \frac{\sqrt{t_e}}{2}\right)$$

rho$_{r_p}$

$$\rho_{p,c} = \frac{\partial C}{\partial r_p} = -Kt_p e^{-r_p t_p} N(d_2)$$

rho$_{r_g}$

$$\rho_{g,c} = \frac{\partial C}{\partial r_g} = Ke^{-r_p t_p} n(d_2) \frac{\partial d_2}{\partial r_g}$$

$$\frac{\partial d_2}{\partial r_g} = \frac{t_e}{\sigma\sqrt{t_e}}$$

$$\rho_{g,c} = Ke^{-r_p t_p} n(d_2) \frac{\sqrt{t_e}}{\sigma}$$

rho$_q$

The sensitivity of the option price to dividends.

$$\rho_{2,c} = \frac{\partial C}{\partial q} = Ke^{-r_p t_p} n(d_2) \frac{\partial d_2}{\partial q}$$

$$\frac{\partial d_2}{\partial q} = \frac{-t_e}{\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\rho_{2,c} = -\frac{\sqrt{t_e} Ke^{-r_p t_p} n(d_2)}{\sigma}$$

Theta

Defining $\mathrm{t}_1 = \tau - \mathrm{T}_1, \mathrm{t}_2 = \tau - \mathrm{T}_2$, and $\mathrm{t}_3 = \tau - \mathrm{T}_3$:

$$\Theta_c = -\frac{\partial C}{\partial \tau} = Ke^{-r_p t_p}\left(- n(d_2) \frac{\partial d_2}{\partial \tau} + r_p N(d_2) \right)$$

$$\frac{\partial d_2}{\partial \tau} = \frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{1.5}} - \frac{\sigma}{4\sqrt{t_e}}$$

Combining the above equations gives :

$$\Theta_c = -e^{-r_p t_p} K \left\{ n(d_2) \left(\frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{1.5}} - \frac{\sigma}{4\sqrt{t_e}} \right) - r_p N(d_2) \right\}$$

3.5.2 Cash or Nothing Put Options

NPV

$$P = Ke^{-r_p t_p} N(-d_2)$$

$$d_2 = \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma\sqrt{t_e}} - \frac{\sigma\sqrt{t_e}}{2}$$

$$n(d_2) = \frac{e^{\frac{-d_2^2}{2}}}{\sqrt{2\pi}}$$

Delta

$$\Delta_p = \frac{\partial P}{\partial S} = Ke^{-r_p t_p} n(-d_2) \frac{\partial - d_2}{\partial S}$$

$$\frac{\partial d_2}{\partial S} = \frac{\partial}{\partial S} \frac{\ln S - \ln K}{\sigma\sqrt{t_e}} = \frac{1}{S\sigma\sqrt{t_e}}$$

$$\Delta_p = -\frac{Ke^{-r_p t_p} n(-d_2)}{S\sigma\sqrt{t_e}}$$

Delta Forward

$$\Delta_{p,\,forward} = \frac{\partial P}{\partial F} = Ke^{-r_p t_p} n(-d_2) \frac{\partial - d_2}{\partial F}$$

$$\frac{\partial d_2}{\partial F} = \frac{\partial}{\partial F} \frac{\ln F - \ln K}{\sigma\sqrt{t_e}} = \frac{1}{F\sigma\sqrt{t_e}}$$

$$\Delta_{p,\,forward} = -\frac{Ke^{-r_p t_p - (r_g - q)t_e} n(-d_2)}{S\sigma\sqrt{t_e}}$$

Gamma

$$\Gamma_p = \frac{\partial^2 P}{\partial S^2} = \frac{\partial}{\partial S} - \frac{Ke^{-r_p t_p} n(-d_2)}{S\sigma\sqrt{t_e}}$$

$$\frac{\partial d_2}{\partial S} = \frac{\partial}{\partial S} \frac{-n(d_2) * d_2}{S\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\Gamma_p = \frac{e^{-rt_2} Kn(d_2)}{\sigma\sqrt{t_e}} \left(\frac{n(d_2) * d_2}{S^2\sigma\sqrt{t_e}} + \frac{n(d_2)}{S^2} \right)$$

$$\Gamma_p = \frac{e^{-r_p t_p} Kn(d_2)}{S^2\sigma\sqrt{t_e}} \left(\frac{d_2}{\sigma\sqrt{t_e}} + 1 \right)$$

Vega

$$\nu_p = \frac{\partial P}{\partial \sigma} = e^{-r_p t_p} Kn(d_2) \frac{\partial - d_2}{\partial \sigma}$$

$$\frac{\partial d_2}{\partial \sigma} = -\frac{\ln\left[\frac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} - \frac{\sqrt{t_e}}{2}$$

$$\nu_p = Ke^{-r_p t_p} n(d_2) \left(\frac{\ln \frac{Se^{-qt_e}}{Ke^{-r_g t_e}}}{\sigma^2\sqrt{t_e}} + \frac{\sqrt{t_e}}{2} \right)$$

rho$_{r_p}$

$$\rho_{p,p} = \frac{\partial P}{\partial r_p} = -t_p Ke^{-r_p t_p} N(-d_2)$$

rho$_{r_g}$

$$\rho_{g,p} = \frac{\partial P}{\partial r_g} = Ke^{-r_p t_p} n(d_2) \frac{\partial - d_2}{\partial r}$$

$$\frac{\partial - d_2}{\partial r_g} = -\frac{t_e}{\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\rho_{g,p} = -\frac{e^{-r_p t_p} Kn(d_2)\sqrt{t_e}}{\sigma}$$

rho$_q$

The sensitivity of the option price to dividends:

$$\rho_{2,p} = \frac{\partial P}{\partial q} = Ke^{-r_p t_p} n(d_2) \frac{\partial - d_2}{\partial q}$$

$$\frac{\partial d_2}{\partial q} = \frac{-t_e}{\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\rho_{2,p} = \frac{\sqrt{t_e} Ke^{-r_p t_p} n(d_2)}{\sigma}$$

Theta

Defining $\mathrm{t}_1 = \tau - \mathrm{T}_1$, $\mathrm{t}_2 = \tau - \mathrm{T}_2$, and $\mathrm{t}_3 = \tau - \mathrm{T}_3$:

$$\Theta_p = -\frac{\partial P}{\partial \tau} = -e^{-r_p t_p} K\left(n(d_2)\frac{\partial - d_2}{\partial \tau} - r_p N(-d_2)\right)$$

$$\frac{\partial d_2}{\partial \tau} = \frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{\,1.5}} - \frac{\sigma}{4\sqrt{t_e}}$$

Combining the above equations gives :

$$\Theta_p = -e^{-r_p t_p} K\left\{ n(d_2)\left(\frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{\,1.5}} - \frac{\sigma}{4\sqrt{t_e}} \right) - r_p N(-d_2)\right\}$$

3.5.3 Asset or Nothing Binary Call Option

NPV

$$C = Se^{(r_g - q)t_e - r_p t_p} N(d_1)$$

$$d_1 = \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$n(d_1) = \frac{e^{\frac{-d_1^2}{2}}}{\sqrt{2\pi}}$$

Delta

$$\Delta_c = \frac{\partial C}{\partial S} = e^{(r_g - q)t_e - r_p t_p}\left(Sn(d_1)\frac{\partial d_1}{\partial S} + N(d_1) \right)$$

$$\frac{\partial d_1}{\partial S} = \frac{\partial}{\partial S}\frac{\ln S - \ln K}{\sigma\sqrt{t_e}} = \frac{1}{S\sigma\sqrt{t_e}}$$

Combining the equations above gives :

$$\Delta_c = e^{(r_g - q)t_e - r_p t_p}\left(N(d_1) + \frac{n(d_1)}{\sigma\sqrt{t_e}} \right)$$

Delta Forward

$$\Delta_{c,\,forward} = \frac{\partial C}{\partial F} = e^{-r_p t_p}\left(Fn(d_1)\frac{\partial d_1}{\partial S} + N(d_1) \right)$$

$$\frac{\partial d_1}{\partial F} = \frac{\partial}{\partial S}\frac{\ln F - \ln K}{\sigma\sqrt{t_e}} = \frac{1}{F\sigma\sqrt{t_e}}$$

Combining the equations above gives :

$$\Delta_{c,\,forward} = e^{-r_p t_p}\left(N(d_1) + \frac{n(d_1)}{\sigma\sqrt{t_e}} \right)$$

Gamma

$$\Gamma_c = \frac{\partial^2 C}{\partial S^2} = \frac{\partial}{\partial S} e^{(r_g - q)t_e - r_p t_p}\left(N(d_1) + \frac{n(d_1)}{\sigma\sqrt{t_e}} \right)$$

$$\Gamma_c = e^{(r_g - q)t_e - r_p t_p}\left(\frac{1}{\sigma\sqrt{t_e}}\frac{\partial n(d_1)}{\partial S} + n(d_1)\frac{\partial d_1}{\partial S} \right)$$

The partial derivative s are calculated as follows :

$$\frac{\partial d_1}{\partial S} = \frac{\partial}{\partial S}\frac{(\ln Se^{-qt_e} - \ln Ke^{-r_g t_e})}{\sigma\sqrt{t_1}} = \frac{1}{S\sigma\sqrt{t_e}}$$

$$\frac{\partial n(d_1)}{\partial S} = \frac{n(d_1) * -d_1}{S\sigma\sqrt{t_e}}$$

Combining the above equations gives :

$$\Gamma_c = \frac{e^{(r_g - q)t_e - r_p t_p} n(d_1)}{S\sigma\sqrt{t_e}}\left(1 - \frac{d_1}{\sigma\sqrt{t_e}} \right)$$

Vega

$$\nu_c = \frac{\partial C}{\partial \sigma} = e^{(r_g - q)t_e - r_p t_p} Sn(d_1)\frac{\partial d_1}{\partial \sigma}$$

$$\frac{\partial d_1}{\partial \sigma} = -\frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} + \frac{\sqrt{t_e}}{2}$$

Combining the above equations gives :

$$\nu_c = e^{(r_g - q)t_e - r_p t_p} Sn(d_1)\left(-\frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} + \frac{\sqrt{t_e}}{2}\right)$$

rho_{r_p}

$$\rho_{c,p} = \frac{\partial C}{\partial r_p} = -t_p e^{(r_g - q)t_e - r_p t_p} SN(d_1)$$

rho_{r_g}

$$\rho_{c,g} = \frac{\partial C}{\partial r_g} = t_e \times Se^{(r_g - q)t_e} e^{-r_p t_p} N(d_1) + Se^{(r_g - q)t_e} e^{-r_p t_p} \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial r_g}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-r_g t_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial d_1}{\partial r_g} = \frac{\sqrt{t_e}}{\sigma}$$

$$\rho_{c,g} = t_e \times Se^{(r_g - q)t_e} e^{-r_p t_p} N(d_1) + Se^{(r_g - q)t_e} e^{-r_p t_p} n(d_1)\frac{\sqrt{t_e}}{\sigma}$$

$$\rho_{c,g} = Se^{(r_g - q)t_e} e^{-r_p t_p} t_e\left(N(d_1) + \frac{n(d_1)}{\sigma\sqrt{t_e}}\right)$$

rho_q

$$\rho_{2,c} = \frac{\partial C}{\partial q} = -t_e \times Se^{(r_g-q)t_e} e^{-r_p t_p} N(d_1) + Se^{(r_g-q)t_e} e^{-r_p t_p} \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial q}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-r_g t_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial d_1}{\partial q} = -\frac{\sqrt{t_e}}{\sigma}$$

$$\rho_{2,c} = -t_e \times Se^{(r_g-q)t_e} e^{-r_p t_p} N(d_1) - Se^{(r_g-q)t_e} e^{-r_p t_p} n(d_1) \frac{\sqrt{t_e}}{\sigma}$$

$$\rho_{2,c} = -Se^{(r_g-q)t_e} e^{-r_p t_p} t_e \left(N(d_1) + \frac{n(d_1)}{\sigma\sqrt{t_e}} \right)$$

Theta

$$\Theta_c = -\frac{\partial C}{\partial \tau} = e^{(r_g-q)t_e - r_p t_p}(q + r_p - r_g) SN(d_1) - e^{(r_g-q)t_e - r_p t_p} Sn(d_1) \frac{\partial d_1}{\partial \tau}$$

$$\frac{\partial d_1}{\partial \tau} = \frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{1.5}} + \frac{\sigma}{4\sqrt{t_e}}$$

Combining the equations above gives :

$$\Theta_c = e^{(r_g-q)t_e - r_p t_p}(-q + -r_p + r_g) SN(d_1) - e^{(r_g-q)t_e - r_p t_p} Sn(d_1) \left(\frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{1.5}} + \frac{\sigma}{4\sqrt{t_e}} \right)$$

3.5.4 Asset or Nothing Put Options

NPV

$$P = Se^{(r_g - q)t_e - r_p t_p} N(-d_1)$$

$$d_1 = \frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$n(d_2) = \frac{e^{\frac{-d_2^2}{2}}}{\sqrt{2\pi}}$$

Delta

$$\Delta_p = \frac{\partial P}{\partial S} = e^{(r_g - q)t_e - r_p t_p}\left(Sn(-d_1)\frac{\partial - d_1}{\partial S} + N(-d_1)\right)$$

$$\frac{\partial - d_1}{\partial S} = \frac{\partial}{\partial S}\frac{\ln S - \ln K}{\sigma\sqrt{t_e}} = -\frac{1}{S\sigma\sqrt{t_e}}$$

Combining the equations above gives :

$$\Delta_p = e^{(r_g - q)t_e - r_p t_p}\left(N(-d_1) - \frac{n(-d_1)}{\sigma\sqrt{t_e}}\right)$$

Delta Forward

$$\Delta_{p,\,forward} = \frac{\partial P}{\partial F} = e^{-r_p t_p}\left(Fn(-d_1)\frac{\partial - d_1}{\partial S} + N(-d_1)\right)$$

$$\frac{\partial - d_1}{\partial F} = \frac{\partial}{\partial S}\frac{\ln S - \ln K}{\sigma\sqrt{t_e}} = -\frac{1}{F\sigma\sqrt{t_e}}$$

Combining the equations above gives :

$$\Delta_{p,\,forward} = e^{-r_p t_p}\left(N(-d_1) - \frac{n(-d_1)}{\sigma\sqrt{t_e}}\right)$$

Gamma

$$\Gamma_p = \frac{\partial^2 P}{\partial S^2} = \frac{\partial}{\partial S} e^{(r_g - q)t_e - r_p t_p} \left(N(-d_1) - \frac{n(-d_1)}{\sigma\sqrt{t_e}} \right)$$

$$\Gamma_p = e^{(r_g - q)t_e - r_p t_p} \left(n(-d_1)\frac{\partial - d_1}{\partial S} - \frac{1}{\sigma\sqrt{t_e}} \frac{\partial n(-d_1)}{\partial S} \right)$$

because

$$\frac{\partial - d_1}{\partial S} = \frac{\partial}{\partial S} \frac{(\ln S - \ln K)}{\sigma\sqrt{t_e}} = -\frac{1}{S\sigma\sqrt{t_e}}$$

and

$$\frac{\partial n(-d_1)}{\partial S} = \frac{n(-d_1) * -d_1}{S\sigma\sqrt{t_e}}$$

The above equations can be combined to give :

$$\Gamma_p = -\frac{e^{(r_g - q)t_e - r_p t_p} n(-d_1)}{S\sigma\sqrt{t_e}} \left(1 - \frac{d_1}{\sigma\sqrt{t_e}} \right)$$

Vega

$$\nu_p = \frac{\partial P}{\partial \sigma} = e^{-r_p t_p} S n(-d_1) \frac{\partial - d_1}{\partial \sigma}$$

$$\frac{\partial d_1}{\partial \sigma} = -\frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} + \frac{\sqrt{t_e}}{2}$$

Combining the above equations gives :

$$\nu_c = -e^{(r_g - q)t_e - r_p t_p} S n(-d_1) \left(-\frac{\ln\left[\dfrac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{\sigma^2\sqrt{t_e}} + \frac{\sqrt{t_e}}{2} \right)$$

rho$_{r_p}$

$$\rho_{p,p} = \frac{\partial P}{\partial r_p} = -t_p e^{(r_g - q)t_e - r_p t_p} S N(-d_1)$$

rho$_{r_g}$

$$\rho_{g,p} = \frac{\partial P}{\partial r_g} = e^{-r_p t_p} S e^{(r_g - q)t_e} n(-d_1) \frac{\partial - d_1}{\partial r_g}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-r_g t_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial - d_1}{\partial r_g} = -\frac{\sqrt{t_e}}{\sigma}$$

Combining equations gives :

$$\rho_{g,p} = -\frac{e^{(r_g - q)t_e - r_p t_p} Sn(d_1) t_e^{1.5}}{\sigma}$$

rho$_q$

$$\rho_{2,p} = \frac{\partial P}{\partial q} = e^{(r_g - q)t_e - r_p t_p} Sn(-d_1) \frac{\partial - d_1}{\partial q}$$

$$d_1 = \frac{\ln Se^{-qt_e} - \ln Ke^{-rt_e}}{\sigma\sqrt{t_e}} + \frac{\sigma\sqrt{t_e}}{2}$$

$$\frac{\partial - d_1}{\partial q} = \frac{\sqrt{t_e}}{\sigma}$$

Combining the above equations gives :

$$\rho_{2,p} = -\frac{e^{(r_g - q)t_e - r_p t_p} Sn(d_1) t_e^{1.5}}{\sigma}$$

Theta

Defining $t_1 = \tau - T_1$, $t_2 = \tau - T_2$, and $t_3 = \tau - T_3$:

$$\Theta_p = -\frac{\partial P}{\partial \tau} = e^{(r_g - q)t_e - r_p t_p}(q + r_p - r_g) SN(-d_1) - e^{(r_g - q)t_e - r_p t_p} Sn(-d_1) \frac{\partial - d_1}{\partial \tau}$$

$$\frac{\partial d_1}{\partial \tau} = \frac{r_g - q}{\sigma\sqrt{t_e}} - \frac{\ln\left[\frac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{1.5}} + \frac{\sigma}{4\sqrt{t_e}}$$

Combining the equations above gives :

$$\Theta_p = e^{(r_g - q)t_e - r_p t_p}(-q - r_p + r_g) SN(-d_1) - e^{(r_g - q)t_e - r_p t_p} Sn(-d_1)\left(-\frac{r_g - q}{\sigma\sqrt{t_e}} + \frac{\ln\left[\frac{Se^{-qt_e}}{Ke^{-r_g t_e}}\right]}{2\sigma t_e^{1.5}} - \frac{\sigma}{4\sqrt{t_e}}\right)$$

3.6 Barrier Options

There are several types of Barrier Options that are implemented in the Calypso system and are discussed in the following section.

3.6.1 Simple Barrier Options

The Simple Barrier Option is a European style option whose payoff depends on the spot, strike and barrier levels. This particular option does not have a continuous barrier found in the Standard Barrier Option. The payout for a simple barrier call is described as follows:

- if S < K the payoff is 0
- if K < S < Barrier the payoff is S-K
- if S > Barrier the payoff is 0

and can be graphed as below:

Where K is the strike, H is the barrier, and S is the spot price.

Calculating the price and sensitivities of the Simple Barrier Option can be done by creating a portfolio of options which replicate the payoff of a Barrier Option. The portfolios for recreating simple barrier options are provided below.

Simple Barrier Call = Plain Vanilla Call Option $_{\text{with strike K}}$ – Asset or Nothing Binary Option $_{\text{with strike = barrier}}$

Simple Barrier Put = Plain Vanilla Put Option $_{\text{with strike K}}$ – Cash or Nothing Binary Option $_{\text{with strike = barrier}}$ – Plain Vanilla Put option $_{\text{with strike = barrier}}$

3.6.2 Standard Barrier Options

Standard Barrier Options have at least one continuous barrier. Each barrier in the Standard Barrier Option determines either when the option becomes live (knock-in) or when the option is ceases to exist (knock-out). A reverse barrier option is an option that has a barrier that is in the money. A knock-in barrier allows activates the option. If an option has a knock-in barrier, the spot price must cross this barrier in order for the option to be activated. If this barrier threshold is not met, then the option expires worthless at its pre-specified expiration date. There are both up and in and down and in options. The up and in option has a barrier that is higher than the spot price at issue and the down and in has a spot price that is higher than the barrier.

If the spot price crosses a knock-out barrier level, then the option ceases to exist and the payoff immediately becomes zero. A down and out barrier option ceases to exist when the spot price moves below the barrier and an up and out barrier option ceases to exist when the spot price moves above the barrier.
The equations for the standard barrier options are shown below.

for H > K :

$$\text{call}_{\text{up and in}} = Se^{-qt_3}N(x_1) - Ke^{-rt_2}N(x_1 - \sigma\sqrt{t_1}) - Se^{-qt_3}\left(\frac{H}{S}\right)^{2\lambda}(N(-y) - N(-y_1))$$
$$+ Ke^{-rt_2}\left(\frac{H}{S}\right)^{2\lambda-2}(N(-y+\sigma\sqrt{t_1}) - N(-y_1+\sigma\sqrt{t_1}))$$

$$\text{call}_{\text{up and out}} = \text{call}_{\text{plain vanilla}} - \text{call}_{\text{up and in}}$$

$$\text{put}_{\text{down and out}} = 0$$
$$\text{put}_{\text{down and in}} = \text{put}_{\text{plain vanilla}}$$

For H ≥ K :

$$\text{call}_{\text{down and out}} = Se^{-qt_3}N(x_1) - Ke^{-rt_2}N(x_1 - \sigma\sqrt{t_1}) - Se^{-qt_3}\left(\frac{H}{S}\right)^{2\lambda}N(y_1)$$
$$+ Ke^{-rt_2}\left(\frac{H}{S}\right)^{2\lambda-2}N(y_1 - \sigma\sqrt{t_1})$$

$$\text{call}_{\text{down and in}} = \text{call}_{\text{plain vanilla}} - \text{call}_{\text{down and out}}$$

$$\text{put}_{\text{up and in}} = -Se^{-qt_3}\left(\frac{H}{S}\right)^{2\lambda}N(-y) + Ke^{-rt_2}\left(\frac{H}{S}\right)^{2\lambda-2}N(-y-\sigma\sqrt{t_1})$$

$$\text{put}_{\text{up and out}} = \text{put}_{\text{plain vanilla}} - \text{put}_{\text{up and in}}$$

for H < K :

$$\text{put}_{\text{down and in}} = -Se^{-qt_3}N(-x_1) + Ke^{-rt_2}N(-x_1 + \sigma\sqrt{t_1}) + Se^{-qt_3}\left(\frac{H}{S}\right)^{2\lambda}(N(y) - N(y_1))$$
$$- Ke^{-rt_2}\left(\frac{H}{S}\right)^{2\lambda-2}(N(y-\sigma\sqrt{t_1}) - N(y_1-\sigma\sqrt{t_1}))$$

$$\text{put}_{\text{down and out}} = \text{put}_{\text{plain vanilla}} - \text{put}_{\text{down and in}}$$

for H ≤ K :

$$\text{call}_{\text{down and in}} = Se^{-qt_3}\left(\frac{H}{S}\right)^{2\lambda} N(y) - Ke^{-rt_2}\left(\frac{H}{S}\right)^{2\lambda-2} N(y-\sigma\sqrt{t_1})$$

$$\text{call}_{\text{down and out}} = \text{call}_{\text{plain vani lla}} - \text{call}_{\text{down and in}}$$

$$\text{call}_{\text{up and out}} = 0$$

$$\text{call}_{\text{up and in}} = \text{call}_{\text{plain vani lla}}$$

$$\text{put}_{\text{up and out}} = -Se^{-qt_3} N(-x_1) + Ke^{-rt_2} N(-x_1+\sigma\sqrt{t_1}) + Se^{-qt_3}\left(\frac{H}{S}\right)^{2\lambda} N(-y_1) - Ke^{-rt_2}\left(\frac{H}{S}\right)^{2\lambda-2} N(-y_1+\sigma\sqrt{t_1})$$

$$\text{put}_{\text{up and in}} = \text{put}_{\text{plain vani lla}} - \text{put}_{\text{up and out}}$$

where :

$$\lambda = \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}$$

$$y = \frac{\ln(\frac{H^2}{SX})}{\sigma\sqrt{t_1}} + \lambda\sigma\sqrt{t_1}$$

$$y_1 = \frac{\ln(\frac{H}{S})}{\sigma\sqrt{t_1}} + \lambda\sigma\sqrt{t_1}$$

$$x_1 = \frac{\ln(\frac{S}{H})}{\sigma\sqrt{t_1}} + \lambda\sigma\sqrt{t_1}$$

3.6.3 Digital Single - Barrier Options

Digital Barrier Options are trigger-type Cash-or-Nothing Options whose payoff occurs if the barrier is touched during the lifetime of the option. The cash payoff can occur either when the barrier is breached or at the maturity of the option. The formulas for these types of options can be found in Haug, pp. 92-95.

3.6.4 Digital Double - Barrier Options

The cash payoff can occur either when the barrier is breached or at the maturity of the option. There are both knock-in and knock-out options. Currently, the Wystup approximation for double-no-touch options and the Pelsser approximation for double-touch options with instant payoff are implemented.

3.6.5 Adjustment for Discrete Barriers

Many options contracts specify the observations as daily closing prices. If this is the case, a correction can be made to the barrier level. Calypso implements the Broadie, Glasserman and Kou (1995) method, also found in Haug, p. 85. The barrier levels are adjusted by exp(0.5826 * volatility * sqrt(observations per year). The upper barrier is multiplied by this factor and the lower divided by the factor. The user can enter the number of observations per year for both Standard and Digital Barrier Options.

3.7 Asian Options

Asian Options are priced by Monte Carlo simulation.

3.8 LookBack Options

Fixed and Floating lookback options are priced by Monte Carlo simulation.