Adenza

Calypso Inflation Derivatives Analytics

Version 17.0

February 2022 – First Edition

The purpose of this Calypso Analytics guide is to provide an understanding of the models and methods that are implemented in Calypso for inflation derivatives, it will also provide an overview of how to set up and configure the analytics. The primary focus is on analytics for product configuration there will be reference made to separate Calypso documentation.

Revision Date	Comments
February 2022	First edition for version 17.0.

Contents

Section 1. Market Data Generation

1.1 Interpolation Methods

Interpolation implementations usually are defined to be applied to any quantities, such as inflation rates or discount factors, without regard to their type. Thus they are purely mathematical, and the following sections specify the algorithms.

1.1.1 InterpolatorLinear

For value x that lies between x1 and x2 the interpolated value y for given ordinate values y1 and y2 is given by:

$$y(x) = A\,y_1 + B\,y_2 \qquad \text{with} \qquad A = \frac{x_2 - x}{x_2 - x_1} \qquad \text{and} \qquad B = \frac{x - x_1}{x_2 - x_1}$$

1.1.2 InterpolatorLogLinear

For value x that lies between x1 and x2 the interpolated value y for given ordinate values y1 and y2 is given by:

$$y(x) = e^{\ln(y_1) + \frac{(\ln(y_2) - \ln(y_1))(x - x_1)}{x_2 - x_1}}$$

1.1.3 InterpolatorSpline

We implemented the cubic spline algorithm as described in Numerical Recipes 2nd edition (NR), pp.115; the only difference to the NR code is that we allow having an array of ordinates to be processed at once rather than one by one. So in one computation, we can process for example an array of curves rather than a single curve and upon interpolation we will be returned an array of (interpolated) rates rather than a single rate.

1.1.4 Interpolator3DLinear

We implemented the multi-dimensional bilinear interpolation algorithm as described in *Numerical Recipes 2nd edition (NR),* pp.123. Our implementation is for 3 dimensions only.

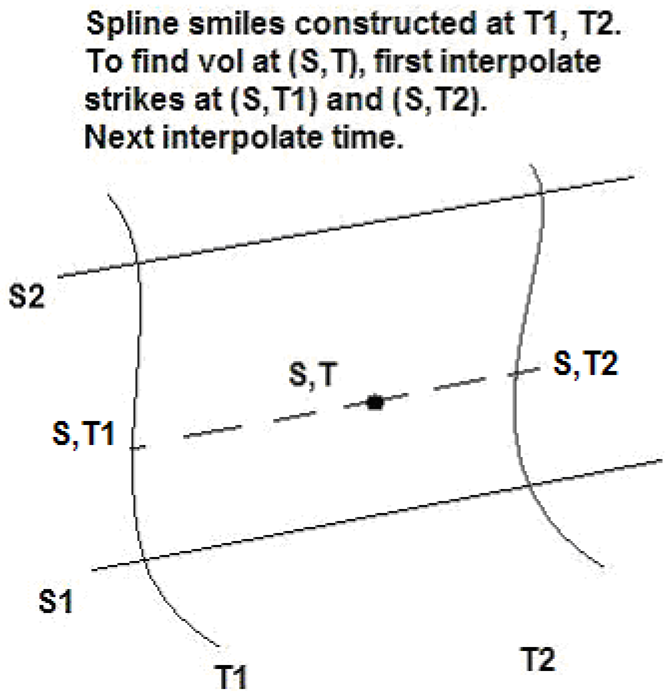
1.1.5 Interpolator3DSpline1D

Same as Interpolator3DLinear except that the points on the third axis are interpolated using a cubic spline, while the others interpolated linearly. Constant extrapolation is used beyond the known points in both dimensions. The most frequent application is to spline interpolation of strikes for volatility surfaces.

For example, for FX options, the spline is used for the delta (strike) axis while linear interpolation for the expiry time axis.The procedure is illustrated below. The "strike" axis for FX options is the delta axis. Define

V(S,T) : Volatility at delta strike S and expiry time T

For each expiry time in the surface, a spline is constructed along the delta axis.



To find the volatility at any delta and expiry (S,T), the following procedure is used:

1. First existing points (S1, T1) and (S2, T2) are found on the surface that obey:

   $$S1 <= S < S2, T1 <= T < T2.$$

2. Splines in the strike dimension exist at the times T1 and T2. Each spline is interpolated upon individually to find the volatility at the strike S for each of those times. The result is two volatilities, V(S, T1) and V(S, T2).
3. Linear interpolation in the time direction is performed on V(S,T1) and V(S,T2) to obtain V(S,T).

1.2 Inflation Curves

Inflation curves are a required input for pricing inflation swaps and inflation caps/floors, this section provides an overview of the construction techniques that are implemented in Calypso.

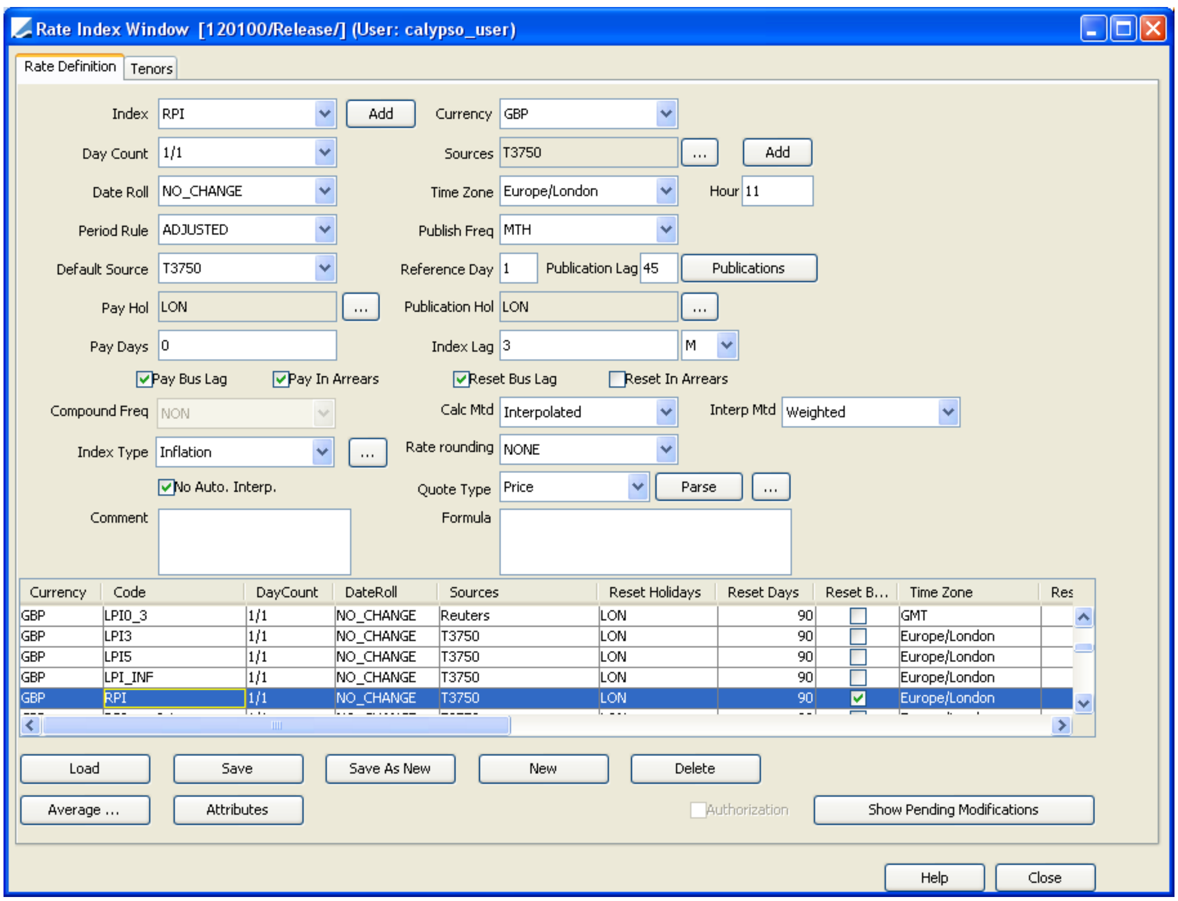
1.2.1 InflationIndexKerkhof Generator

The InflationIndexKerkhof generator is used to generate an inflation curve from underlying instruments using Kerkhof's approach[1]. See the Calypso white paper: Implementation of Kerkhof inflation curve construction for further information on the model description.

The below documentation shows the configuration required for the InflationIndexKerkhof generator.

Rate index definition

Define the inflation index: **Configuration > Interest Rates > Rate index Definitions**. See Calypso documentation for further information on the various configuration detail of the inflation rate index.



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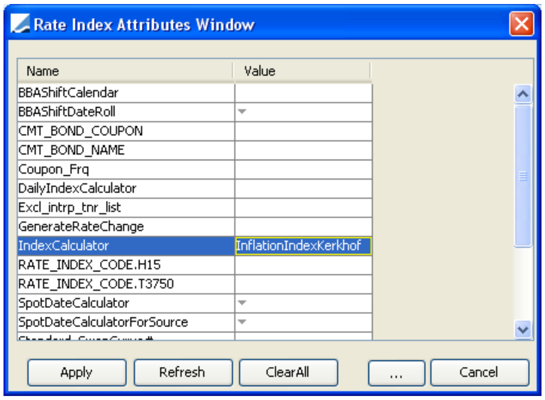
[1] Inflation Derivatives Explained, *J. Kerkhof,* Lehman Brothers, July 2005

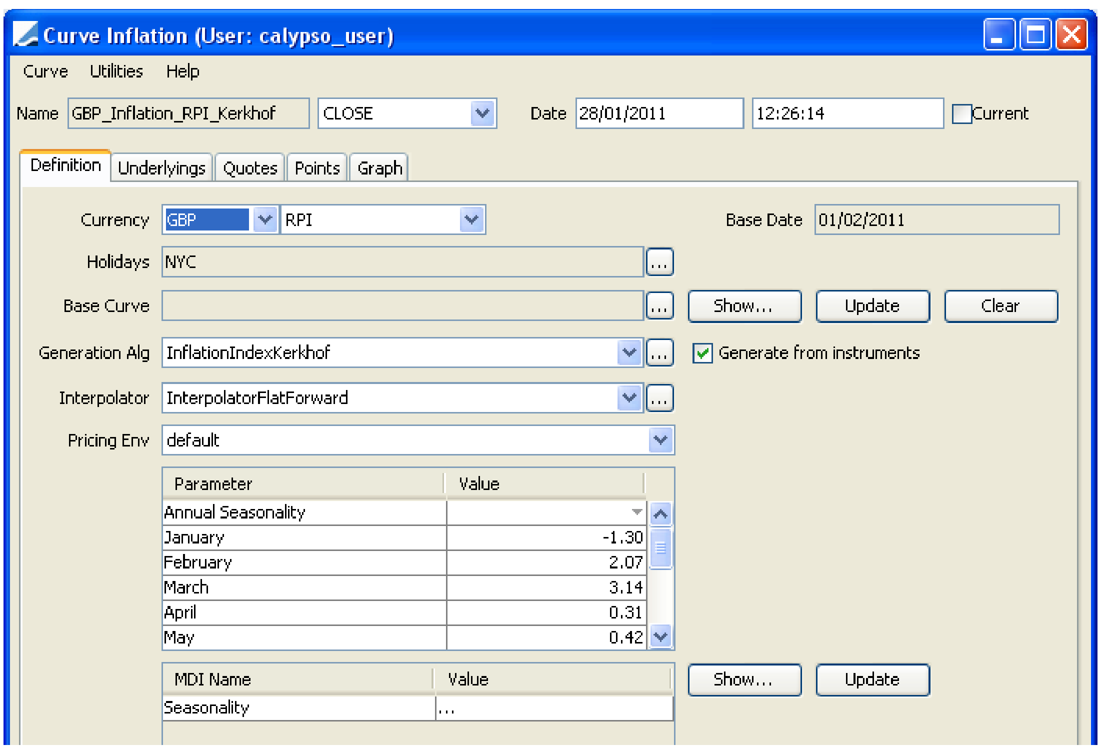
-Note the InflationIndexKerkhof generator supports both interpolated (weighted) and non interpolated (Index Level) inflation indicies.

In the attributes section of the rate index definition of each inflation index to be constructed via the InflationIndexKerkhof generator, it is important the index calculator is set to InflationIndexKerkhof



Inflation Curve definition

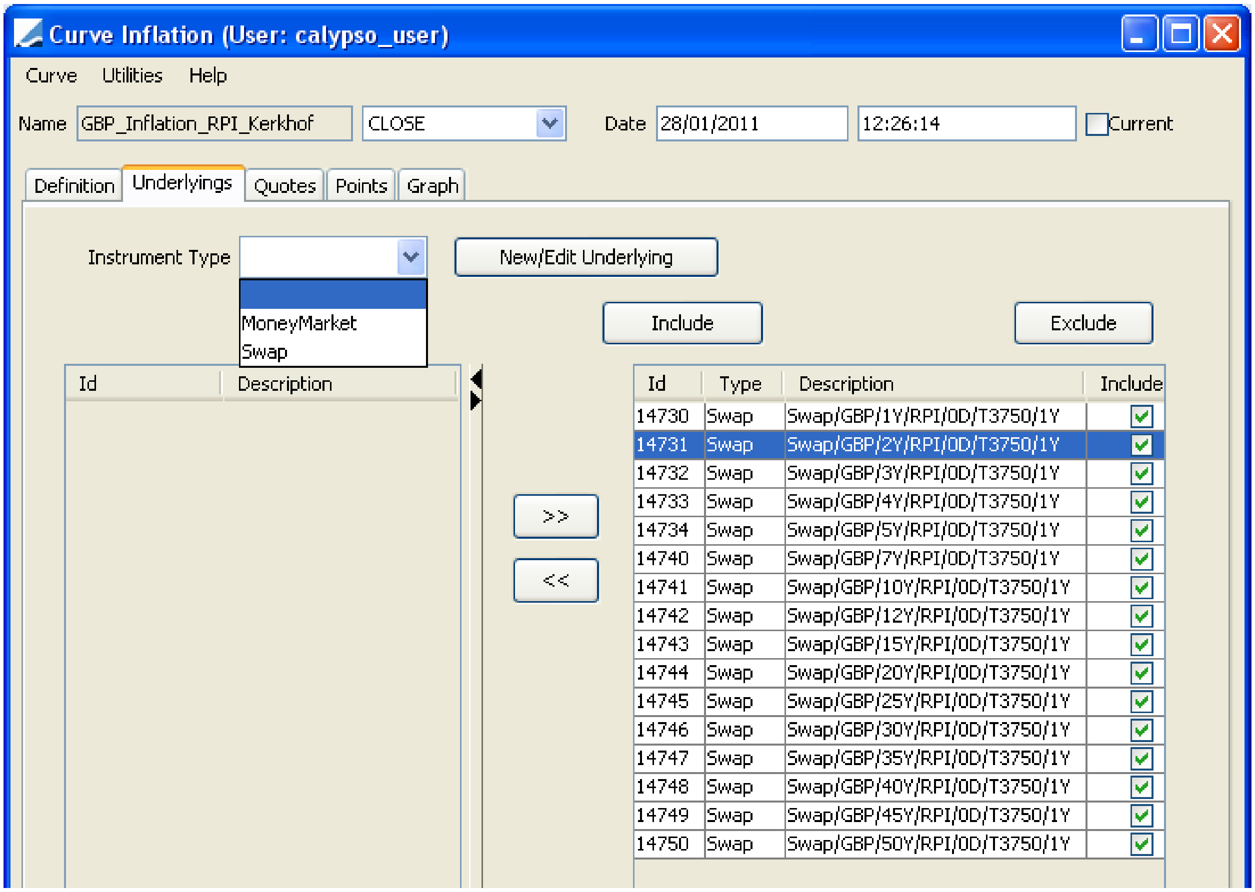
On inflation curve (**Market data > Interest Rate Curves > Inflation Curve**) select the generator InflationIndexKerhof. This is a derived generator, i.e. generate from instruments should be set to true:



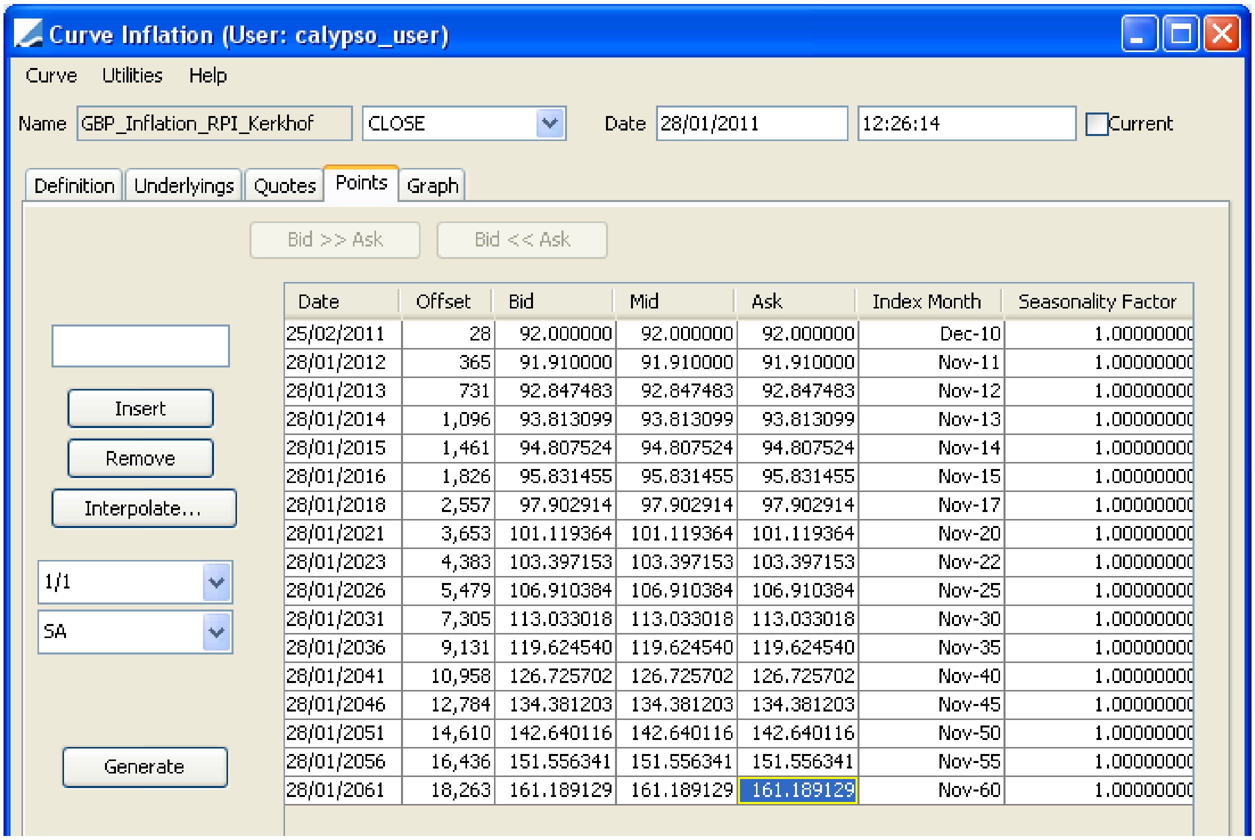
The underlying instruments that are available to construct the curve are zero coupon inflation swaps and MM (levels) for the selected inflation index. The user has the ability to select MM instruments up to the tenor of the first ZC inflation swap selected. This allows the user to control the specific level of the inflation curve at the front end of the curve by entering assumed inflation levels at defined tenors. (The typical use would be to use ZC inflation swaps only).

The user can select the instrument type from the drop down menu, a list of instruments available from the curve underlyings definition will be shown on the left hand panel. The curve underlyings used to construct the curve can then be selected by moving the instrument to the right hand panel using the arrows buttons available.

Note the precise details of the curve underlying can be modified via 'New/Edit underlying'. Ensure the correct definition of curve underlyings before constructing the inflation curve.



After the required curve underlyings are selected and quotes entered, click generate on the point tab. The generated curve points are displayed and shown as reference levels and dates. Note the published inflation levels for the index should also be added to the quote set. If the required inflation levels are not saved to the database a warning message will be displayed.



Seasonality adjustments to the inflation curve can be handled by two methods; both are selected on the definitions tab on inflation curve

1. Monthly adjustment (Enter annualized adjustments)
2. Seasonality curve market data item (MDI)

The seasonality curve defined via in the separate MDI takes precedence.

For method 1, monthly adjustments, set the 'Annual Seasonality' Boolean flag to true and define adjustment on the curve window. Adjustment will be applied by month for every year that is constructed.

For method 2, Seasonality curve market data item, select the seasonality curve on the UI then generate the inflation curve as required.

The seasonality adjustments required for the Kerkhof methodology implemented in Calypso are an additive adjustments i.e. the monthly annualized adjustments for a 1 year period add up to 0. See the Calypso user documentation on how to define a seasonality curve.

Section 2. Inflation Trades

The pricing of the following inflation trade types are supported in Calypso.

Types of Trades	Calypso Product	Pricer
Inflation Zero Coupon (ZC) CapFloor	CapFloor	PricerCapFloorInflationBlack
Inflation Swap (vanilla)	Swap	PricerSwap PricerSwapHagan (Pricing model is identical to PricerSwap)

2.1 Inflation Swap

2.1.1 Standard

A standard inflation swap could be one of the following types:

- Fixed-Floating
- Floating-Fixed
- Floating-Floating

The supported characteristics are the following: no amortization structure, the floating leg tenor matches the index payment frequency i.e. zero coupon (0D tenor) or period on period (e.g. year on year YoY).

We will focus on the fixed-floating type of standard swap, since we have both types of swap legs to valuate.

Let's introduce the notations first:

All dates are calculated using the appropriate day count usage (the corresponding curve's day count).

Each cash-flow i starts at date T_i, ends at date T_{i+1}, has a reset date $\tilde{T}_i$ and a payment date $\overline{T}_i$

- N_i is the notional
- yf_i is the year fraction between T_i and T_{i+1}
- df_t is the discount factor between valuation date and t.
- $I(x)$ inflation value at time x.
- K is the fixed rate.

For the fixed leg, the NPV is the following $NPV_{fix} = \sum_{i=0}^{i=N-1} N_i \cdot yf_i \cdot df_{\overline{T}_i} \cdot K$

For the floating leg, the NPV is the following $NPV_{flt} = \sum_{i=0}^{i=N-1} N_i \cdot \frac{I(T_{i+1})}{I(T_i)} \cdot df_{\bar{T}_i}$

2.1.1.1 Convexity Adjustment

Currently for inflation swaps and cap/floors there is no convexity adjustment.

2.2 Inflation CapFloor

An inflation cap/floor is a derivative, similar to an interest rate cap/floor swap, in which the buyer/seller receives/pays payments at the end of each period in which the inflation rate exceeds the agreed strike price.

The option itself can be one of the following types:

- Cap – the option to buy the underlying at a specific value at maturity or exercise date.
- Floor – the option to sell the underlying at a specific value at maturity or exercise date.

Once exercised, the user enters into the corresponding underlying swap, which is in general an inflation swap.

NOTE that this pricer is part of the Calib module. Refer to Calypso Analytics Module documentation for installation information.

2.1.2 PricerCapFloorInflationBlack

In order to price an Inflation CapFloor, PricerCapFloorInflationBlack, uses a modified version of the Black-Scholes model.

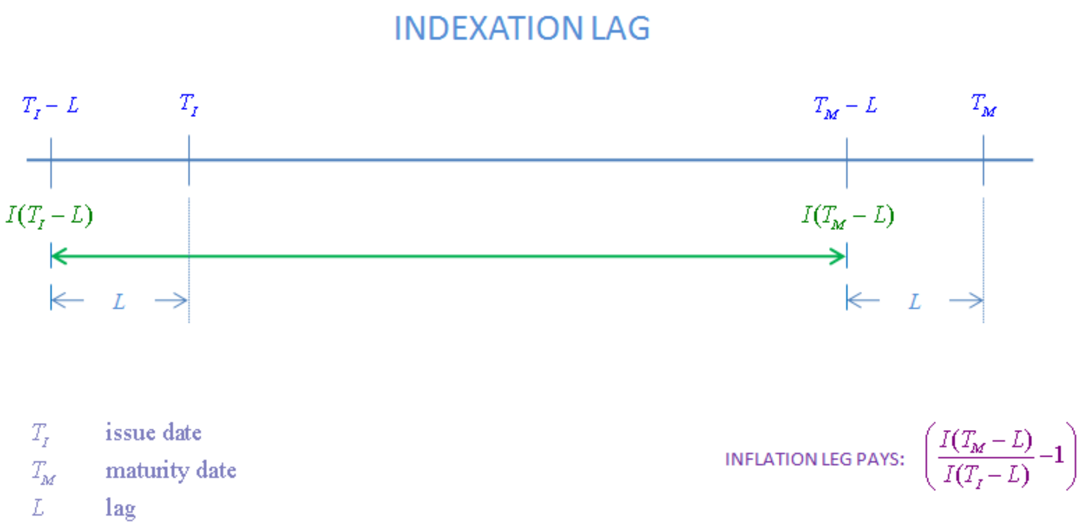
It is important to note that the following pricer is supported only for Zero Coupon CapFloor.

Let's introduce the notation:

- K is the strike of the option
- S is the spot.
- T_S is the start date of the option.
- T_e is the expiration time of the option.
- L is the indexation lag.
- $I(T_S)$ is the inflation value at the start of the option
- $I(T_e)$ is the inflation value at the maturity of the option

- $I(t)$ is the inflation value at the valuation date
- $D(x)$ represents the cumulative normal distribution function at *x*.
- σ is the volatility of the forward swap rate.
- df is the discount factor at T_e.
- N is the notional
- r is the risk free interest rate (for inflation derivatives, it is always zero)

Based on when the valuation date is, before or after start date, different analytics solutions are used. An example of the indexation lag effect respect to the product life time is given as follows:



Due to the indexation lag there are therefore two different analytics approaches: first one, when valuation date occurs before start date, second one, when valuation date occurs after start date.

Scenario 1: Valuation Date before Start Date

For the case when Valuation Date is before start date, the NPV is computed as follows:

Effective Strike (K_e):

$$K_e = (1+k)^{(Ts-Te)}$$

Spot (S):

$$S = \frac{I(T_e)}{I(T_S)}$$

Black-Scholes Method (for call):

$$d_1 = \frac{\ln\left(\frac{S}{K_e}\right) + \left(r + \frac{\sigma^2}{2}\right)(\Delta T)}{\sigma\sqrt{\Delta T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K_e}\right) + \left(r - \frac{\sigma^2}{2}\right)(\Delta T)}{\sigma\sqrt{\Delta T}}$$

$$B = D(d_1)S - D(d_2)K$$

Then finally scale and discount the value obtained from the Black-Scholes model:

$$NPV = df * B * N$$

For the put option, the same approach is used, with the Black-Scholes formula changed for the put option case.

Please refer to section 3 of the IRD Analytics Guide for more details regarding Black Scholes model.

Scenario 2: Valuation Date after Start Date

For the case when Valuation Date is after Lag, the NPV is computed as follows:

Effective Strike (K_e):

$$K_e = \left[\frac{I(T_S)}{I(t)}\right](1+k)^{(Ts-Te)}$$

Spot (S):

$$S = \frac{I(T_e)}{I(t)}$$

Black-Scholes Method (for call):

$$d_1 = \frac{\ln\left(\frac{S}{K_e}\right) + \left(r + \frac{\sigma^2}{2}\right)(\Delta T)}{\sigma\sqrt{\Delta T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K_e}\right) + \left(r - \frac{\sigma^2}{2}\right)(\Delta T)}{\sigma\sqrt{\Delta T}}$$

$$B = D(d_1)S - D(d_2)K$$

Then finally scale and discount the value obtained:

$$NPV = \left[\frac{I(T_e)}{I(T_S)}\right] df * B * N$$

For the put option, the same approach is used, with the Black-Scholes formula changed for the put option case.

Please refer to section 3 of the IRD Analytics Guide for more details regarding the Black-Scholes model.

Calypso Configuration Notes

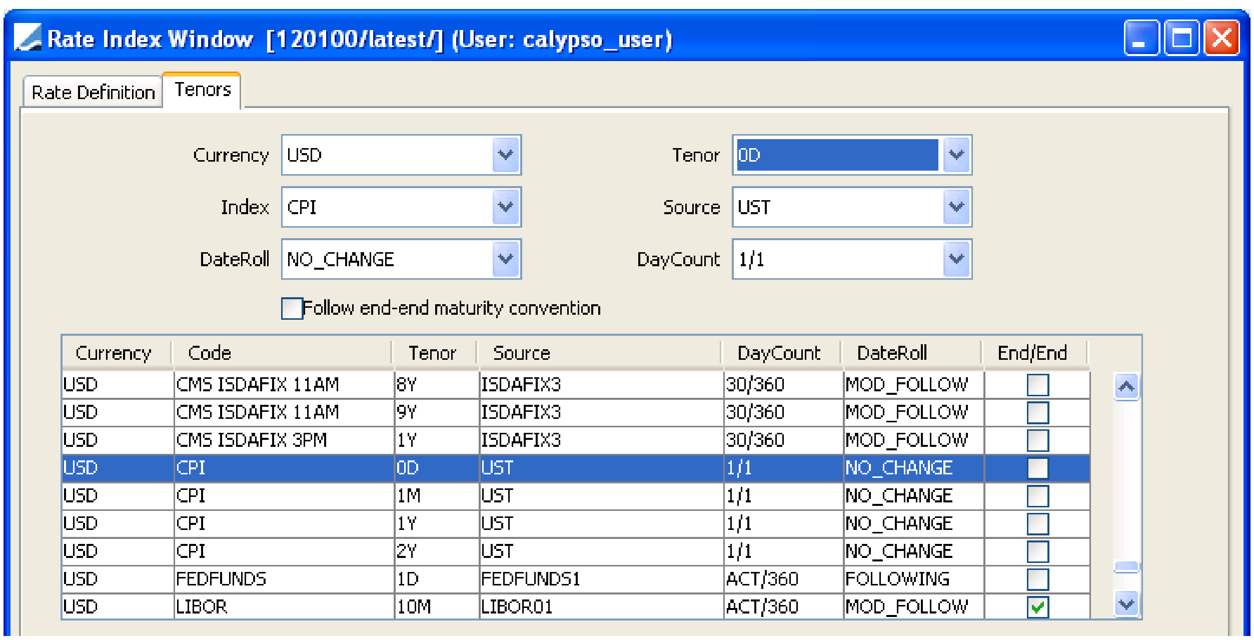
In order to use PricerCapFloorInflationBlack it is necessary to ensure that the following configurations has been setup correctly.

Inflation index tenor (rate index definition)

Window Location: **Configuration > Interest Rates > Rate Index Definitions**

The desired inflation tenor in the rate index definition must be configured. To add a specific tenor to an inflation index, it is necessary to select the inflation index, the required tenor from the tenor box and save.

Please see below as example:



If the trade window was open, before or during this rate index definition process, it is necessary to close and reopen the screen to see the new tenors on the trade window.

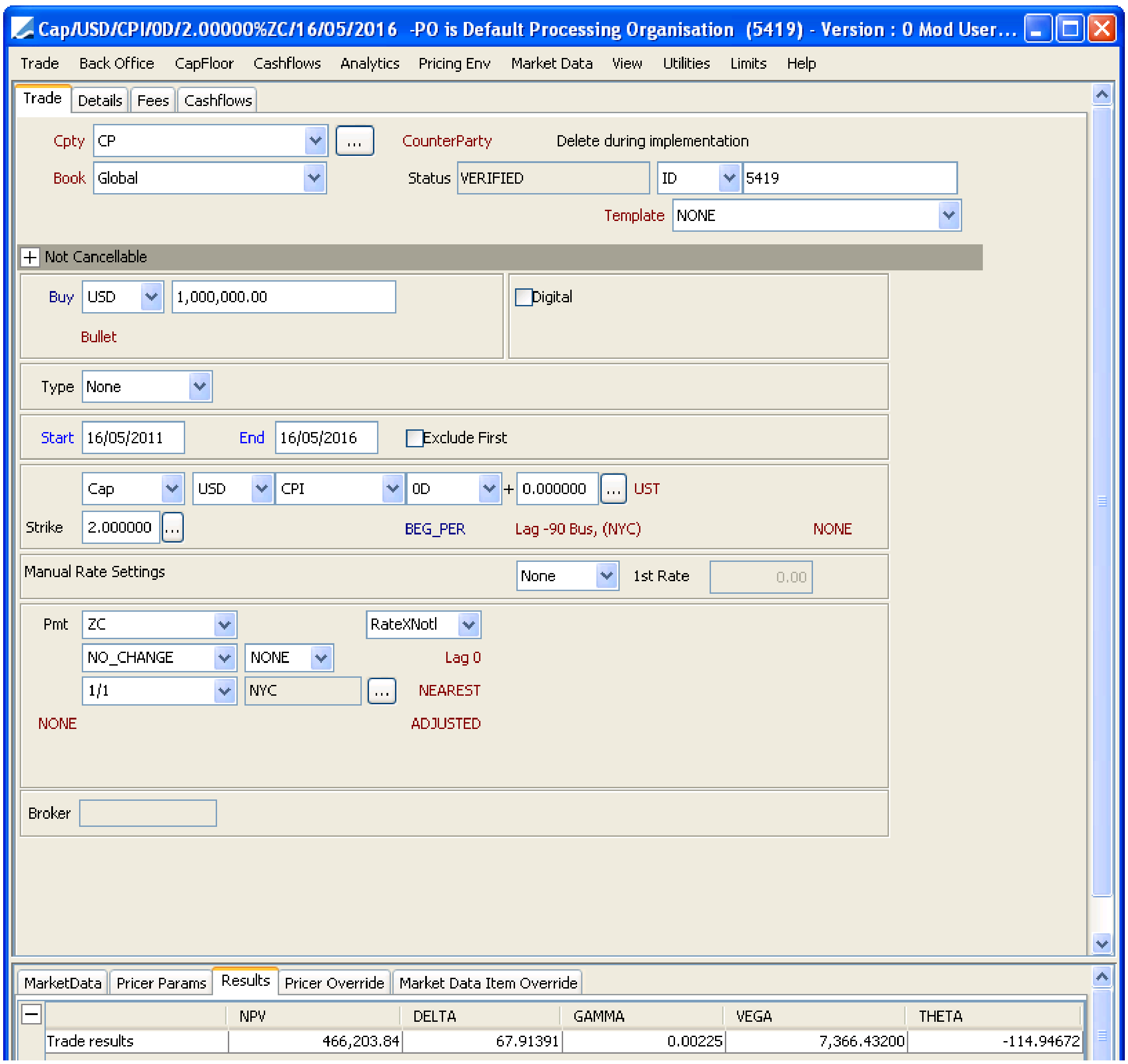
Trade Panel Configuration

As mentioned previously the PricerCapFloorInflationBlack only supports zero coupon trades, therefore it is necessary to ensure that the configuration in the trade panel is for a Zero Coupon case.

The key elements that should be configured correctly are:

- Pmt = ZC (Zero Coupon)
- RateXNotl (selected on the box on the right to Pmt)
- Inflation Index
- Tenor

Please see below trade screen as example of such configuration:



Results

Cashflow analysis

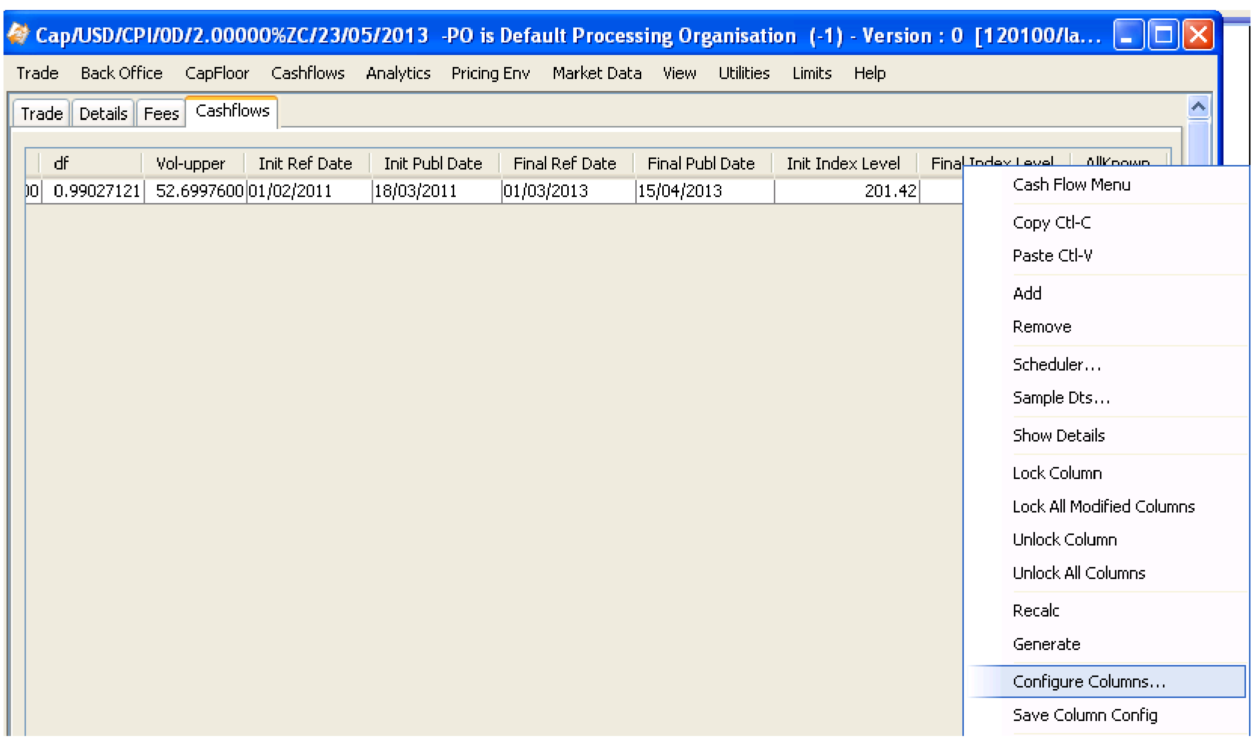
Main results are displayed in the trade screen results tab, however more details regarding the cashflows results can be observed in the cashflows tab.

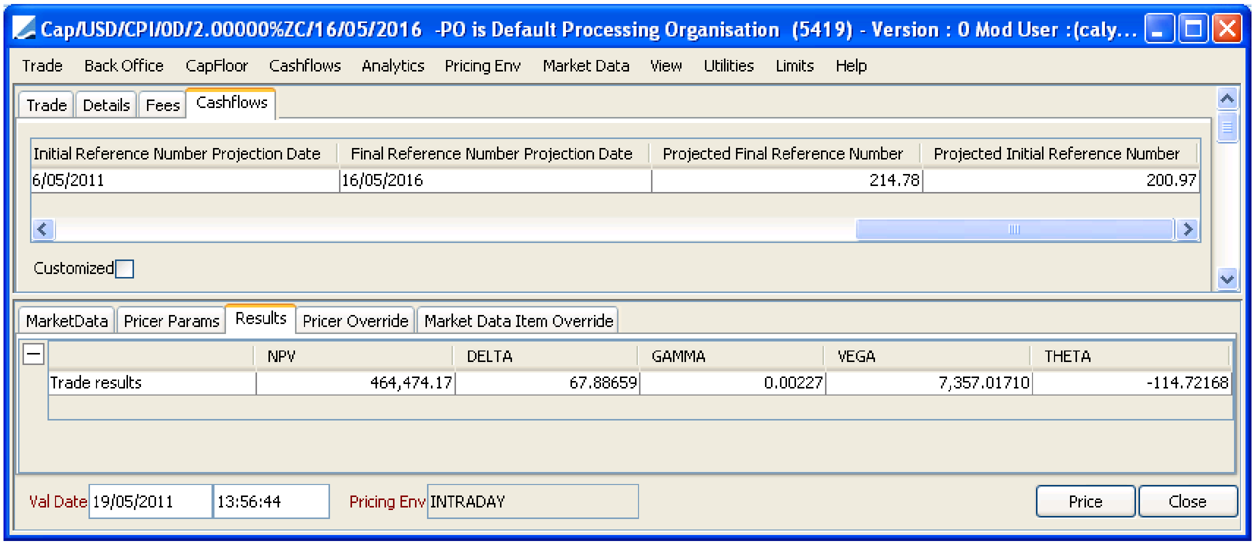
Some of the key outputs that could be observed in this type of trade are:

- Projected Initial Reference Number
- Projected Final Reference Number
- Initial Reference Number Projection Date
- Final Reference Number Projection Date

Such columns do not appear automatically in the cashflow tab, in order to obtain such value it is necessary

1. Select Cashflows Tab
2. Right-click on the cashflow
3. Select "Configure Columns"
4. Insert to the right Columns the variables required





Note: The above columns show information on the reference numbers and dates are actually used in pricing. The lagged inflation index levels (initial and final) are also available to the user.