Adenza

Calypso FX and Money Market Analytics

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Welcome to the Calypso FX and Money Market Analytics guide which aims to provide an understanding of the analytics that underlie the Calypso pricing capability.

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Section 1. FX Conventions

1.1 Currency Pair Definitions

Primary and Quoting Currencies

A Currency Pair is specified by a *primary currency* and a *quoting currency*. The terminology indicates that FX rates for the pair are given in terms of amount of quoting currency per one unit of primary currency. Then for any amount of primary currency, one converts to quoting currency by multiplication:

$$\text{Quoting Currency Amount} = (\text{FX Rate}) * (\text{Primary Currency Amount})$$

Example: For a EUR/USD pair with EUR defined as the primary currency and USD as the quoting currency, a quote of 1.20 indicates 1.0 EUR can be exchanged for 1.20 USD.

However, it is possible to reverse the definition; in the Calypso Currency Pair window one can set the "Divide" choice, reversing the roles of primary and quoting currency for FX rate quotes. Then primary is converted to quoting currency by dividing by the FX rate, rather than multiplying. However, the same effect can be achieved by defining a Currency Pair with the currencies switched. The "Divide" choice is therefore best avoided except in rare circumstances.

Spot Days

A Currency Pair also requires a specification of the number of days between the quotation date (or trade date) and the Spot Date on which the exchange is to take place at that quote. Typically this is two business days.

The Spot Days can be specified in two ways. It can be established directly for each Currency Pair. Or it can be derived from the default Spot Days assigned to each individual currency, in which case the Spot Days for the pair is defined to be the maximum of the Spot Days of the component currencies.

The date on which the exchange takes place is termed the *Value Date* in the FX market. The Value Date is typically the Spot Date, but can also be one business day after the quotation date (for O/N quotes).

Note: Calypso uses the term "Val date" to denote the day of the market data used in pricing a trade. For FX trades this corresponds to the date the quote is made (price is agreed on) and is not the same as the FX market's "Value Date", which instead corresponds to what Calypso terms the trade's *maturity date* or *settle date*.

Holidays

Each currency has a set of holidays associated with it. The holidays of a Currency Pair is the union of the holidays of the two currencies.

1.2 Tenor Arithmetic

The over-the-counter FX markets provide quotes according to tenor -- 1W, 3M, 1Y, etc. The computation of dates from these tenors differs between the two markets. The following market conventions are used by Calypso in producing dates for FX trades and FX curve generation.

1.2.1 FX Spot Date

Given a quotation date (trade date) the corresponding Spot Date is computed from the Spot Days of the Currency Pair.

Please refer to Calypso Getting Started documentation for details on FX Spot Date calculation.

1.2.2 FX Forward Tenors

Given a tenor and a trade date for an over-the-counter FX forward trade, the following procedure is used to derive the forward settlement date.

1. Find the Spot Date.

2. Obtain the holiday calendar for the Currency Pair. If neither currency of the pair is USD, then extend the holiday calendar by adding the NYC holidays; or, if the pair has a "third calendar" defined, add that holiday calendar rather than NYC. Use the combined holiday calendar in the following steps.

3. Find a trial forward date:

- If the tenor is less than or equal to 6 days, add that number of business days to the Spot Date using the holiday calendar of Step 2.
- If the number of tenor days is greater than 6, add that number of calendar days to the Spot Date. (1W tenor is 7 calendar days, 1M counts as 30 calendar days, etc.).

4. If the forward date of Step 3 is not a business day:

- If the number of tenor days is less than 28, find the next business day.
- If the number of tenor days is 28 or greater, find the next business day using the Modified Following method. If the tenor is given in whole months and the Spot Date was the last business day of a month, then subsequently roll the forward date to the last business day of its month.

5. The result is the forward date.

Section 2. FX Curves

2.1 Generation from Yield Curves

An FX Curve can be generated from a pair of yield curves using the arbitrage-free relationship between forward FX rates and the discount rates of the two currencies.

In the FX Curve application, the user specifies this generation method by un-checking the "Generate from instruments" box and by selecting yield curves for the "Base Curve" and the "Quote Curve." Pictorially:

(Base (Primary) Currency Yield Curve) * (Quoting Currency Yield Curve) → FX Curve

The generation equations are as follows. Define:

- $D_Q(T_{SPOT}, T)$: Discount factor of quoting currency yield curve from spot date to forward date
- $D_B(T_{SPOT}, T)$: Discount factor of primary currency yield curve from spot date to forward date
- R_{SPOT}: Spot FX rate (number of units of quoting currency per single unit of primary currency)
- $R_{fwd}(T)$: Forward FX rate at time T

All quantities also have BID and ASK superscripts to identify the quote side. The equations are:

$$R_{fwd}(T)^{BID} = R_{SPOT}{}^{BID} \frac{D_B(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}}, \quad T > T_{SPOT}$$

$$R_{fwd}(T)^{ASK} = R_{SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}, \quad T > T_{SPOT}$$

For times prior to the spot date,

$$R_{fwd}(T)^{ASK} = R_{SPOT}{}^{ASK} \frac{D_Q(T,T_{SPOT})^{BID}}{D_B(T,T_{SPOT})^{ASK}}, \quad T < T_{SPOT}$$

$$R_{fwd}(T)^{BID} = R_{SPOT}{}^{BID} \frac{D_Q(T,T_{SPOT})^{ASK}}{D_B(T,T_{SPOT})^{BID}}, \quad T < T_{SPOT}$$

Note: Using Bid and Ask spreads as inputs can result in unrealistic spreads on the resulting interest rates. For this reason, many market practitioners prefer to use Mid rates and only add in a spread when making a trade.

For more details on the no-arbitrage equations, please see the section "FX Forward Rate Projection" in FX Valuation chapter.

2.2 FX Points Generator

The FXPoints Generator creates an FX curve based on FX swap points. It generates curve dates, uses Forward FX points (spreads) as quote inputs, and adjusts points for O/N, T/N market conventions.

The generator is very simple, as the swap points are the forward FX points for dates after the spot date. The only calculations are:

- calculation of the settlement dates for the given underlying tenors
- application of the market conventions for O/N and T/N swap points.
- application of the bid/ask market convention for swap points after the spot date.

The rule for swap points after the spot date is:

- If the bid price is greater than the ask price, the quotes are using an implied minus sign. Supply the sign before placing the values on the curve as forward FX points.

The rules for swap points prior to spot are as follows:

- The sign of the points is reversed.
- Bid and Ask are reversed.
- The all-in rate for exchange today is found by summing the O/N and the T/N quotes, if the spot date is two business days from today. For markets in which the spot date is only one business day (e.g., USD/CAD), the O/N quote alone gives today's points.

Reference: FX quote conventions are detailed in *Mastering Financial Calculations*, R. Steiner (Prentice Hall, 1998).

Example. The following are the Quotes for the FX Swap underlyings and the FX Spot Rate. Because the FX Points generator is used, the quotes are expected to be expressed using the market convention of FX Swap points. (Only mid quotes are used here, so the bid/ask sign conventions will not apply.)

FX.USD.JPY.ON	-0.91
FX.USD.JPY	109.1
FX.USD.JPY.TN	-0.32
FX.USD.JPY.1D	-2.28
FX.USD.JPY.1M	-11.205
FX.USD.JPY.2M	-21.85
FX.USD.JPY.3M	-31.95
FX.USD.JPY.6M	-67.56
FX.USD.JPY.9M	-111.34
FX.USD.JPY.1Y	-164.14

From these FX Swap points, the following FX Curve is created on December 5, 2003, whose spot date is December 9, 2003:

Date	Offset	Mid
12/5/2003	0	1.23
12/8/2003	3	0.32
12/9/2003	4	0
12/10/2003	5	-2.28
1/9/2004	35	-11.205

2/9/2004	66	-21.85
3/9/2004	95	-31.95
6/9/2004	187	-67.56
9/9/2004	279	-111.34
12/9/2004	370	-164.14

Here the resulting points are Forward Points, that is, the simple difference from the FX Spot Rate. To find the all-in forward rate, one uses:

$$\text{All} - \text{in Rate} = (\text{Spot Rate}) + (\text{Points on FX Curve}) \,/\, (\text{Points Factor}).$$

So for example, the rate for a USD/JPY exchange that settles on 3/9/2004 is

109.1 + (-67.56) / 100 = 108.4244.

The same applies to dates before spot, that is, there is no FX Swap quote convention in the output. So for an exchange that settles today, December 5, 2003, the rate is

109.1 + 1.23/ 100 = 109.1123.

2.3 FX Forward Generator

The FXForward Generator creates an FX curve based on FX Forwards all-in rates. It generates curve dates and spreads as the difference between FX Forward quotes and Spot quotes.

Since FX Forward all-in rates are not the standard quotation method in the marketplace, this generator is less useful than the FXPoints.

Section 3. Interest Rate Curves using FX

FX rates can be used as input to the generation of interest rate curves. The inputs and the result of two types of curve creation are shown in the following table.

Generation Name	Currency 1 curves	Currency 2 curves	Currency 1 vs. Currency 2	Other Allowed Quotes	Resulting Interest Rate Curve
Curve Zero FX Derived	Interest Curve	None	FX Curve	Single-currency swaps	Interest curve in the form of a Zero Curve
Curve Basis	Interest curve (to project both discount and forward rates)	Interest curve (for discounting only)	FX Forward spreads; Cross-Currency Basis Swaps	Any curve underlying	Interest curve in the format of a Basis Curve (for forward rate generation)

3.1 Basis Curve Generation

The basis curve is the most flexible way of producing an interest curve from cross-currency information. Basis curve generation is described fully in the Calypso document, "Interest Rate Derivatives Analytics."

3.2 Curve Zero FX Derived Generation

Curve Zero FX Derived generates an interest rate curve of the quoting currency from the interest rate curve of the primary currency and an FX curve. Pictorially:

(Primary Currency Yield Curve) * (Primary/Quoting FX Curve) → Quoting Currency Yield Curve

The yield curve is derived using the arbitrage-free relationship between forward FX rates and the discount rates of the two currencies. Using the same quantities as described above in the section on "Generation from Zero Curves", the no-arbitrage relations are:

$$D_Q(T_{SPOT},T)^{BID} = \frac{R_{SPOT}{}^{BID}}{R_{fwd}(T)^{BID}} D_B(T_{SPOT},T)^{ASK}, \quad T > T_{SPOT}$$

$$D_Q(T_{SPOT},T)^{ASK} = \frac{R_{SPOT}{}^{ASK}}{R_{fwd}(T)^{ASK}} D_B(T_{SPOT},T)^{BID}, \quad T > T_{SPOT}$$

For the case where $T < T_{SPOT}$, the discount factors are inverted, resulting in:

$$D_Q(T,T_{SPOT})^{BID} = \frac{R_{fwd}(T)^{ASK}}{R_{SPOT}^{ASK}} D_B(T,T_{SPOT})^{ASK}, \quad T < T_{SPOT}$$

$$D_Q(T,T_{SPOT})^{ASK} = \frac{R_{fwd}(T)^{BID}}{R_{SPOT}^{BID}} D_B(T,T_{SPOT})^{BID}, \quad T < T_{SPOT}$$

Be aware that using Bid and Ask spreads as inputs can result in unrealistic spreads on the resulting interest rates. For this reason, many market practitioners prefer to use Mid rates and only add in a spread when making a trade.

For more details on the no-arbitrage equations, please see the section “FX Forward Rate Projection” in the chapter on FX Valuation.

Example. Suppose the following is the zero curve for USD Libor 6M generated on December 5, 2003, with zero rates expressed as Act/360 Annual rates:

Date	Offset	Zero	DF
12/8/2003	3	1.03275	0.999914
12/9/2003	4	1.03275	0.999886
1/9/2004	35	1.15986	0.998879
2/9/2004	66	1.16701	0.997875
3/9/2004	95	1.17879	0.996912
6/9/2004	187	1.27864	0.993422
9/9/2004	279	1.43655	0.989007
12/9/2004	370	1.6233	0.983586
12/9/2005	735	2.38331	0.953049
12/11/2006	1,102	3.01428	0.913102
12/10/2007	1,466	3.48916	0.869651

These discount factors are given as of the curve generation date (Dec 5 2003), but in the FX equations one needs the discount factor based on the FX Spot date, Dec 9 2003. Denoting the curve generation date by T_0, one has:

$$D_B(T_{SPOT},T) = \frac{D_B(T_0,T)}{D_B(T_0,T_{SPOT})}, \quad T > T_{SPOT}$$

The generation also requires a USD/JPY FX curve; for this, use the example curve described above under “FX Points Generator.” The quoting curve calculation is shown in the following table:

Date	FX Fwd Points	FX Forward Rate	Primary Ccy DF	Primary Ccy DF as of Spot	Quoting Ccy DF as of Spot	Quoting Ccy DF

12/5/2003	1.23	109.1123	1	0.999886	0.999998578	1
12/8/2003	0.4	109.104	0.99991438	0.999971	1.00000813	0.99999
12/9/2003	0	109.1	0.99988585	1	1	0.999999
12/10/2003	-2.28	109.0772	*0.99985674*	0.999971	1.000179907	1.000178
1/9/2004	-11.205	108.98795	0.99887948	0.998994	1.000020576	1.000019
2/9/2004	-21.85	108.8815	0.99787513	0.997989	0.999991784	0.99999
3/9/2004	-31.95	108.7805	0.99691227	0.997026	0.999954453	0.999953
6/9/2004	-67.56	108.4244	0.99342201	0.993535	0.9997262106	0.999725
9/9/2004	-111.34	107.9866	0.98900682	0.98912	0.999318085	0.999317
12/9/2004	-164.14	107.4586	0.98358625	0.983699	0.998724259	0.998723

For example, the calculation for 6/9/2004 (June 9 2004) is:

$$D_B(T_{SPOT},T) = \frac{0.99342201}{0.99988585} = 0.993535422.$$

$$R_{SPOT} = 109.1, \quad R_{fwd}(T) = 108.4244$$

$$D_Q(T_{SPOT},T) = \frac{109.1}{108.4244} 0.993535422 = 0.9997262106.$$

$$D_Q(T_0,T) = 0.9997262106 * 0.999999 = 0.999724789.$$

3.2.1 Adding Single-Currency Swaps

Single-currency swaps in the currency of the result curve can be added as underlyings. Using bootstrapping, these swaps will extend the result curve beyond the points generated using the FX Curve.

The bootstrap generator that is used will be the same type as used in the interest curve of the other currency. For example, taking the USD/JPY case, if one has a USD interest rate curve that was generated with the Bootstrap Forwards Generator, then for the FX Derived curve the JPY swaps will extend the resulting JPY curve also using the Bootstrap Forwards algorithm. If the USD had no generator assigned to it, then the default used for the JPY swaps is the standard Bootstrap Generator.

In keeping with the convention in the usual Bootstrap generator, the single-currency swaps take precedence over money market instruments. For example, suppose the FX Curve was created out to 1Y using 1Y FX Forwards, so

that the cash part of the derived curve goes out to 1Y. If one wants to also use swaps starting with a 9M swap, then points of the cash curve after the 9M date are removed, so that all the swap information will be used.

Section 4. FX Valuation

Product	Calypso Product	Pricer
FX Spot	FX, FXTakeup	PricerFX
	FXCash	PricerFXCash
FX Forward	FXForward	PricerFXForward PricerFXForwardHomeBased
	FXNDF	PricerFXNDF
	FXOptionForward	PricerFXOptionForward
	FXOptionSwap	PricerFXOptionSwap
FX Swap	FXSwap	PricerFXSwap

4.1 FX Forward Rate Projection

A foreign exchange rate that applies to a currency exchange taking place on date T is said to be the "Value T" rate. Interest-rate parity relates the FX Spot rate ("Value Spot") with the FX rate for exchange on another date, which can be before or after the spot date.

4.1.1 The Basic No-Arbitrage Argument

Define the following:

- R_{spot} – The spot FX rate defined for settling FX deals on spot date T_{spot}.
- $R_{fwd}(T)$ – The forward FX rate on date T.
- D_Q (T1, T2) – The discount factor for discounting quoting currency from any date T2 to earlier date T1.
- D_B (T1, T2) – The discount factor for discounting base (primary) currency from any date T2 to earlier date T1.

In outline, the basic relationship involves the following steps:

1. Borrow N of currency B on the spot date TSPOT, with agreement to pay at time T the amount

$$\frac{N}{D_B(T_{SPOT}, T)}$$

2. Enter into an FX forward to convert this amount into currency Q on date T, guaranteeing the amount

$$N \frac{R_{FWD}(T)}{D_B(T_{SPOT}, T)}$$

3. Convert the borrowed amount N on the Spot date to currency Q at the spot rate to give RspotN.

4. Invest in currency Q rates until time T to obtain

$$N \frac{R_{SPOT}}{D_Q(T_{SPOT}, T)}$$

5. If there is no arbitrage these amounts must be equal:

$$\frac{R_{fwd}(T)}{D_B(T_{SPOT}, T)} = \frac{R_{SPOT}}{D_Q(T_{SPOT}, T)}$$

4.1.2 FX Forward Mid Rates

If all the steps in the interest-rate parity argument are performed at the Mid rates then the logic is straightforward to apply these relationships to project FX forward rates using two interest rate curves and the FX spot rate.

Many market participants prefer to calculate at the Mid rates and then add on a spread. This is a more stable calculation than one in which the spreads of different curves and FX rates are all used, which can cause a build up of spreads producing an overall spread that is too wide compared to the market.

The MID forward rate for foreign exchange taking place on a date T ("for Value T") is projected from two interest rate curves using these formulas:

- If T >= T_{spot} (forward projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_B(T_{SPOT}, T)^{MID}}{D_Q(T_{SPOT}, T)^{MID}}, \quad T > T_{SPOT}$$

- If T < T_{spot} ("backward" projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_Q(T, T_{SPOT})^{MID}}{D_B(T, T_{SPOT})^{MID}}, \quad T < T_{SPOT}$$

4.1.3 Bid/Ask Rates in the No-Arbitrage Argument

To take the bid/ask spreads into account one needs to look at the steps in the no-arbitrage argument more carefully. There are in fact several different ways of combining trades to produce bid/ask relationships. What follows is the method Calypso has been using through Version 14. In this derivation the spot and the forward FX rates are taken to be on the same side, i.e., either both bid or both ask. In this form the equations are readily related to the usual method of quoting FX swap points in terms of the same side for spot and forward.

1. Borrow N of currency B on the spot date T_{SPOT}. Since one will be borrowing one will pay the Ask interest rate. Thus one agrees to pay at time T the amount

$$\frac{N}{D_B(T_{SPOT},T)^{ASK}}$$

2. Enter into an FX forward to convert this amount into currency Q on date T. Because one is selling base currency this is done at the Bid FX rate. So the maturity payment in Q currency is

$$N\frac{R_{FWD}(T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

3. Convert the borrowed amount N on the Spot date to currency Q. Since one is selling base currency this is done at the Bid FX rate, giving

$$NR_{SPOT}{}^{BID}$$

4. Invest in currency Q rates until time T. Because one is loaning one will receive the Bid interest rate. At time T one will then receive

$$N\frac{R_{SPOT}{}^{BID}}{D_Q(T_{SPOT},T)^{BID}}$$

5. If there is no arbitrage the amount received cannot be greater than the amount paid:

$$\frac{R_{FWD}(T)^{BID}}{D_B(T_{SPOT},T)^{ASK}} \geq \frac{R_{SPOT}{}^{BID}}{D_Q(T_{SPOT},T)^{BID}}$$

In order to use the other sides of the rates one would start by borrowing in the quoting currency Q and use the rates for buying base currency, which are the Ask FX rates. The equation can be found from the one above by reversing the bid and ask and the inequality sign.

4.1.4 FX Forward Bid/Ask Rates

The preceding argument gives limits on the FX bid/ask forward rates. In order to produce a definite number, we use the equals sign in the following. This also has the advantage of guaranteeing the correct bid/ask relationship. (Relationships that do not make these assumptions are previously discussed).

Under these assumptions, bid/ask forward FX rates can be projected from interest rates as follows:

- If T >= T_{spot} (forward projection),

$$R_{fwd}(T)^{BID} = R_{SPOT}{}^{BID}\frac{D_B(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}}, \quad T > T_{SPOT}$$

$$R_{fwd}(T)^{ASK} = R_{SPOT}{}^{ASK}\frac{D_B(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}, \quad T > T_{SPOT}$$

- If T < T_{spot} (“backward” projection),

$$R_{fwd}(T)^{BID} = R_{SPOT}{}^{BID}\frac{D_Q(T,T_{SPOT})^{ASK}}{D_B(T,T_{SPOT})^{BID}}, \quad T < T_{SPOT}$$

$$R_{fwd}(T)^{ASK} = R_{SPOT}{}^{ASK} \frac{D_Q(T,T_{SPOT})^{BID}}{D_B(T,T_{SPOT})^{ASK}}, \quad T < T_{SPOT}$$

Consistency Checks

For these formulas it can be confirmed that the bid/ask definitions are consistent in that they preserve the requirement that bid rates are less than ask rates. That is, if the spot bid rate is less than the spot ask rate, and the bid interest rates are less than the ask interest rates in both currencies – so that the bid discount factors are *greater* than the ask discount factors -- then these formulas guarantee that the FX forward bid will be less than the FX forward ask.

Another check is the behavior for T very near T_{SPOT}. As T approaches T_{SPOT} the discount factors approach 1, and so the forward bid rates approach the spot bid rates, and the forward ask rates approach the spot ask rates.

Inverting the equations

In principle these equations can be rearranged to provide formulas for discount factors given the forward rates. However, doing so can create interest rates where the bid rate is greater than the ask rate. The equations cannot be inverted and at the same time guarantee the relation of bid and ask rates. This is demonstrated in the following sections.

4.1.5 Deriving Interest Rates in One Currency from Interest Rates in Another Currency

Mid Interest Rates

One can rearrange the Mid equations to solve for one of the discount factors rather than the FX forward rate. To derive the quoting currency discount factor relative to spot, one has for forward projection

$$D_Q(T_{SPOT},T)^{MID} = \frac{R_{SPOT}{}^{MID}}{R_{fwd}(T)^{MID}} D_B(T_{SPOT},T)^{MID}, \quad T > T_{SPOT}$$

and for backward projection

$$D_Q(T,T_{SPOT})^{MID} = \frac{R_{fwd}(T)^{MID}}{R_{SPOT}{}^{MID}} D_B(T,T_{SPOT})^{MID}, \quad T < T_{SPOT}\,.$$

Today-based Mid Interest Rates

These give discount factors relative to the Spot date. To find the discount factors relative to Today, T_0, with $T_0 < T_{Spot}$, one can find $D_Q(T_0,T_{Spot})$ using the second equation only if the if the FX rate for Value Today is known, $R_{fwd}(T_0)^{MID}$. If this is not available, one would instead need to extrapolate from the first calculable discount factors.

As an example of an extrapolation method, suppose the first given forward rate is as of T_1 which is after the Spot date. Then one can calculate $D_Q(T_{Spot.}\ T_1)$ from the first equation. Convert this to an interest rate between Spot and T_1 and then assume this rate is constant back to T_0 to derive $D_Q(T_0,T_{Spot})$.

With this factor from Today to Spot known one can transform the preceding equations to discount factors for T with respect to today:

$$D_Q(T_0,T)^{MID} = D_Q(T_0,T_{SPOT})^{MID} D_Q(T_{SPOT},T)^{MID}, \quad T > T_{SPOT}$$

$$D_Q(T_0,T)^{MID} = \frac{D_Q(T_0,T_{SPOT})^{MID}}{D_Q(T,T_{SPOT})^{MID}}, \quad T < T_{SPOT}$$

4.1.6 Bid/Ask Interest Rates

The bid/ask equations for the forward FX rates can be rearranged to give the interest rate equations. However, unlike the FX case, *the interest rate equations do not guarantee the correct bid/ask relationships*.

A rearrangement of the equations for FX forward rate projection gives

- If $T >= T_{spot}$ (forward projection),

$$D_Q(T_{SPOT},T)^{BID} = \frac{R_{SPOT}^{BID}}{R_{fwd}(T)^{BID}} D_B(T_{SPOT},T)^{ASK}, \quad T > T_{SPOT}$$

$$D_Q(T_{SPOT},T)^{ASK} = \frac{R_{SPOT}^{ASK}}{R_{fwd}(T)^{ASK}} D_B(T_{SPOT},T)^{BID}, \quad T > T_{SPOT}$$

- If $T < T_{spot}$ (“backward” projection),

$$D_Q(T,T_{SPOT})^{ASK} = \frac{R_{fwd}(T)^{BID}}{R_{SPOT}^{BID}} D_B(T,T_{SPOT})^{BID}, \quad T < T_{SPOT}$$

$$D_Q(T,T_{SPOT})^{BID} = \frac{R_{fwd}(T)^{ASK}}{R_{SPOT}^{ASK}} D_B(T,T_{SPOT})^{ASK}, \quad T < T_{SPOT}$$

In the next sections it is shown why this rearrangement can violate bid/ask consistency, and an alternative formula is considered.

Conflict Between Bid/Ask Relationships of FX and Interest Rates

The preceding equations for the interest rates do not guarantee that the bid interest rate is less than the ask rate. This is one reason why it is often more useful to simply use the Mid equations and then add in spreads to the result.

A quick way to visual the conflict is by noting that taking the ratio of the parity equations gives (for T after Spot)

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} = A\frac{D_Q(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}$$

where

$$A = \frac{R_{SPOT}^{ASK}}{R_{SPOT}^{BID}} \frac{D_B(T_{SPOT},T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

In a normal market for spot FX and base currency interest rates

$$A > 1$$

Now if quoting rates were in a normal market,

$$\frac{D_Q(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}} > 1 \ ,$$

which immediately shows the forward FX rates have normal bid/asks,

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > 1.$$

On the other hand, if we were given that the forward FX rates were normal, all we would know about the quoting interest rates would be

$$\frac{D_Q(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}} = \frac{1}{A}\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > \frac{1}{A}$$

As A > 1 the left hand side can be less than 1, so the bid/ask relationship can easily be violated for the quoting interest rates.

In order to have $D_Q(T_{SPOT},T)^{BID} > D_Q(T_{SPOT},T)^{ASK}$, one must have

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > A \quad .$$

So it is not sufficient that the forward ask be greater than the bid; it must be greater by a large enough amount:

$$\left(\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}}\right) / \left(\frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}}\right) > \frac{D_B(T_{SPOT},T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

If the base discount factors correctly have the bid factor greater than the ask factor the right hand side is not less than one. So for the quoting discount factors to have the correct bid/ask relationship it must at least be true that

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > \frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}}$$

In other words, the FX bid/ask spreads must widen as one goes out in time in order for the derived quoting discount factors to potentially have the correct bid/ask relationship. They must also widen by an amount that is governed by the primary currency discount factors.

However, this is *not* a sufficient condition for interest rate bid/ask consistency. The prior equation is more restrictive. Expressed in words, the ratio of the spot bid/ask ratio to the forward bid/ask ratio must be greater than the discount factor bid/ask ratio. If this condition does not hold, the resulting discount factor will have an inverted bid/ask relationship.

The relationship also holds for deriving primary currency discount factors, as is readily seen by repeating the argument.

This inverted relationship does happen when deriving interest rates from real market quotes. This implies either interest rates are driving FX rates rather than vice versa, or else that the market is using Mid rate relationships and adding in spreads without regard to keeping consistency among these equations.

Alternate Formula for Interest Rates gives Inconsistent FX

One can instead use a different no-arbitrage derivation that guarantees derived interest rates will always have the correct bid/ask relationship. But as a consequence, one will find that it also is not consistently invertible. That is, inverting it to find the FX forwards can produce bid FX rates that are greater than ask rates.

Here is the alternate no-arbitrage derivation. Suppose one has an amount N of quoting currency on the Spot date. This can be invested two different ways:

Investment Method 1. Lend at the Bid interest rate to arrive at end amount

$$N / D_Q(T_{SPOT}, T)^{BID}$$

Investment Method 2. Buy base currency at the Ask rate, lend at the Bid base currency interest rate, and sell back base currency at the forward Bid rate, to arrive at

$$N \frac{R_{fwd}(T)^{BID}}{R_{SPOT}{}^{ASK} D_B(T_{SPOT}, T)^{BID}} .$$

Absence of arbitrage is guaranteed if the amount received is not greater than the amount paid. But suppose one argued these two methods of investment should be equal. Setting the two end results to be the same one would have

- Alternate Derivation for T >= T_{spot} (forward projection),

$$\overline{D}_Q(T_{SPOT}, T)^{BID} = \frac{R_{SPOT}{}^{ASK}}{R_{fwd}(T)^{BID}} D_B(T_{SPOT}, T)^{BID}, \quad T > T_{SPOT}$$

$$\overline{D}_Q(T_{SPOT}, T)^{ASK} = \frac{R_{SPOT}{}^{BID}}{R_{fwd}(T)^{ASK}} D_B(T_{SPOT}, T)^{ASK}, \quad T > T_{SPOT}$$

A bar has been placed over the derived discount factor in order to indicate this is a different derivation than the previous one. Note the argument was repeated for the Ask side. The difference between these equations and the previous ones are the switching of the bid and ask sides on R_{SPOT} and D_B. If this switch produces a difference in calculations then the discount factors obtained with this set of equations is not the same as with the previous set.

Dividing the bid equation by the ask equation one finds that the correct bid/ask relation is guaranteed for the discount factor $\overline{D}_Q$ if the other quotes are normal.

$$\frac{\overline{D}_Q(T_{SPOT}, T)^{BID}}{\overline{D}_Q(T_{SPOT}, T)^{ASK}} = A \frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}}$$

with again

$$A = \frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}} \frac{D_B(T_{SPOT},T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

One sees that the A is on the opposite side of the equation than before, showing a different dependence on the bid/ask ratios. So now the previous argument can be followed through to show that in a normal market this guarantees the correct bid/ask rates for $\overline{D}_Q$ but not for the forward rates.

4.1.7 Bid/Ask Limits and Widest Spreads

The Bid/Ask equations make certain assumptions in order to arrive at equalities that can be used in forward rate projection. One can loosen these assumptions to find inequalities that give limits on the bid/ask spreads. The set of inequalities that gives the largest spread will provide the firm limits to arbitrage.

Neither of the formulas derived previously involved full round-trip arbitrage. To perform this, one starts with zero currency, borrows in one currency and repays in the same currency. One compares doing this with and without a currency conversion. If there is a difference beyond the bid/ask spread one has a definite arbitrage opportunity.

Borrow base currency N_B on the spot date at the ask rate. At maturity pay out

$$\frac{N_B}{D_B(T,T_{SPOT})^{ASK}}$$

At spot convert N_B to currency Q at the bid rate (selling base currency), lend at the bid rate, and enter into a forward to convert back to currency B at the ask rate (buying base currency), to receive at maturity

$$\frac{R_{SPOT}{}^{BID} N_B}{R_{fwd}(T)^{ASK} D_Q(T,T_{SPOT})^{BID}}$$

In order to prevent arbitrage, the net cashflows cannot be greater than zero:

$$\frac{R_{SPOT}{}^{BID} N_B}{R_{fwd}(T)^{ASK} D_Q(T,T_{SPOT})^{BID}} - \frac{N_B}{D_B(T,T_{SPOT})^{ASK}} \leq 0$$

This gives a lower limit on the forward FX ask rate:

$$R_{fwd}(T)^{ASK} \geq \frac{R_{SPOT}{}^{BID} D_B(T,T_{SPOT})^{ASK}}{D_Q(T,T_{SPOT})^{BID}}$$

Repeating the process but starting by borrowing in currency Q produces an upper limit on the forward FX bid rate:

$$R_{fwd}(T)^{BID} \leq \frac{R_{SPOT}{}^{ASK} D_B(T,T_{SPOT})^{BID}}{D_Q(T,T_{SPOT})^{ASK}}$$

One sees that this does not guarantee that $R_{fwd}(T)^{BID} \leq R_{fwd}(T)^{ASK}$ as the right hand side of the ask equation is less than the right hand side of the bid equation.

To be certain of bid/ask consistency one needs the bid rate to be less than the right hand side of the ask rate equation. If one makes this an equality then one arrives at the bid/ask equations from the previous sections, which is why those formulas guarantee FX bid/ask consistency.

Another reason to prefer the method of the previous sections is that it provides the widest bid/ask spread for the FX rate. One can check that by rearranging the assignments of bid/ask sides there is no combination that produces a smaller bid or a larger ask.

By rearranging the inequalities one has limits on the interest rates,

$$D_Q(T, T_{SPOT})^{BID} \geq \frac{R_{SPOT}{}^{BID} D_B(T, T_{SPOT})^{ASK}}{R_{fwd}(T)^{ASK}}$$

$$D_Q(T, T_{SPOT})^{ASK} \leq \frac{R_{SPOT}{}^{ASK} D_B(T, T_{SPOT})^{BID}}{R_{fwd}(T)^{BID}}$$

This does not guarantee the bid/ask relationship for discount factors, $D_Q(T, T_{SPOT})^{BID} > D_Q(T, T_{SPOT})^{ASK}$. To be certain of the relationship the ask discount factor must be less than or equal to the right hand side of the bid equation. Turning that into an equality produces the equations of the previous sections, which is why those equations guarantee discount factor bid/ask consistency. This also gives the widest bid/ask spread for the discount factors.

Because of the inequalities one cannot guarantee both discount factor and FX forward rate consistency with a single set of equations.

4.1.8 Triangulated FX Bid/Ask Relationships

Consider the case of a triangulated FX rate, that is, where one converts from base (primary) currency B to quoting currency Q by first converting through the "split" currency X.

The relations between interest and FX rates can be found by writing the equations for each pair and creating the combination exchange.

If only mid rates are used then the dependence on the discount factors of currency X drop out in this process, and one is left with the original relation between the B and Q rates.

If using bid and ask rates the discount factors of currency X do contribute to the end formulas, because in going through currency X one pays the bid ask spread on the currencies, and this cannot be ignored. Define

$R_{B/Q,SPOT}$: the spot rate for converting one unit of B into Q currency

$R_{B/X,SPOT}$: the spot rate for converting one unit of B into X currency

$R_{X/Q,SPOT}$: the spot rate for converting one unit of X into Q currency

with analogous definitions for the forward rates R_{fwd}.

Enter into a forward FX agreement to convert from currency B to X and a second agreement to convert the proceeds from X to Q. In the first trade one is selling B and so is done at the ask of rate B/X; in the second trade one is selling X so it is done at the ask of X/Q. The net of the combined trades gives the ask of a triangulated B/Q rate:

$$R_{B/Q,fwd}(T)^{ASK} = R_{B/X,fwd}(T)^{ASK} R_{X/Q,fwd}(T)^{ASK}$$

The reverse logic holds for the bid rates:

$$R_{B/Q,fwd}(T)^{BID} = R_{B/X,fwd}(T)^{BID} R_{X/Q,fwd}(T)^{BID}$$

Now the equations of Section 1.4 for the two conversions are (for $T > T_{SPOT}$):

$$R_{B/X,fwd}(T)^{BID} = R_{B/X,SPOT}{}^{BID} \frac{D_B(T_{SPOT},T)^{ASK}}{D_X(T_{SPOT},T)^{BID}},$$

$$R_{B/X,fwd}(T)^{ASK} = R_{B/X,SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}},$$

$$R_{X/Q,fwd}(T)^{BID} = R_{X/Q,SPOT}{}^{BID} \frac{D_X(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}},$$

$$R_{X/Q,fwd}(T)^{ASK} = R_{X/Q,SPOT}{}^{ASK} \frac{D_X(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}.$$

Combining the asks to give the triangulated ask rate produces

$$R_{B/Q,fwd}(T)^{ASK} = R_{B/X,SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}} R_{X/Q,SPOT}{}^{ASK} \frac{D_X(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}$$

which can be rewritten by combining the spot rates into a triangulated rate,

$$R_{B/Q,fwd}(T)^{ASK} = R_{B/Q,SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}} \frac{D_X(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}}$$

Similarly for the bid side,

$$R_{B/Q,fwd}(T)^{BID} = R_{B/Q,SPOT}{}^{BID} \frac{D_B(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}} \frac{D_X(T_{SPOT},T)^{ASK}}{D_X(T_{SPOT},T)^{BID}}$$

One sees these are almost the same as the original equations except for multiplication or division by the additional factor

$$\frac{D_X(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}}$$

which takes into account the spread from the buy and sell of the X currency.

If using only mid rates this factor is just 1 and the discount factors of the X currency do not enter into the relations. But when using bid ask rates, if one did not include this factor then the B to Q bid/ask relationships that were guaranteed in the previous equations can fail.

4.2 FX Spot and Forwards

Two pricers are available for pricing FX forward trades, **PricerFXForward** and **PricerFXForwardHomeBased**. For FX spot trades one can use **PricerFX** or **PricerFXForwardHomeBased**. There is presently little difference between PricerFX and PricerFXForward.

4.2.1 PricerFX and PricerFXForward

Net Present Value

The NPV is the sum of the discounted future cashflows combined into a common currency. There are two possibilities depending on the order in which discounting and currency conversion is done:

A. Discount the cashflows using the zero curves of the two currencies and then convert the results to one currency using the FX rate on the value date.

B. Convert the cashflows to a common currency using forward FX rates on the cashflow payment dates and afterwards perform the discounting using the curve of the resulting currency.

In **PricerFXForward/PricerFX** the FX_POINTS parameter distinguishes these two methods. If FX_POINTS is True, then Method B is used, as the forward points are required for doing the future conversion. In PricerFXForwardHomeBased Method B is used, but if forward FX rates are not available in the market data set then Method A is used.

PricerFXForwardHomeBased discounts all cashflows using the discount curve of the Pricing Environment base currency, referred to in this context as the "home" currency. The home currency can be different from either currencies of the FX trade, which are referred to as the trade "base currency" and the trade "quote currency."

The following table summarizes the NPV methods.

Pricer	NPV method	Market Data Needed
PricerFXForward / PricerFX	IF FX_POINTS is False, discount with each ccy curve before conversion (Method A).	- Base ccy zero curve - Quote ccy zero curve - Spot rate
PricerFXForward / PricerFX	IF FX_POINTS is True, convert to trade quote ccy using forward points before discounting with quote ccy curve (Method B).	- Quote ccy zero curve - FX curve Base/Quote - Spot rate
PricerFXForwardHomeBased	Convert cashflows to the home currency prior to discounting using the home discount curve. If direct forward rates are not available use USD as a middle currency. If insufficient FX data is available, revert to Method A.	- Home ccy (pricing environment base ccy) zero curve - FX curves for converting trade base and quote currencies to Home ccy - FX curve for USD to Home ccy conversion, as needed

Pricer	NPV method	Market Data Needed
		- Spot rates between Home and trade base and quote ccy's.

Pricing Parameters

There are some pricing parameters of particular interest to the FX pricers.

FX_POINTS

Values: True/False

Implemented by: PricerFXForward, PricerFX.

Not implemented by PricerFXForwardHomeBased, because this pricer by default uses forward FX points.

This parameter is explained in the preceding section on NPV.

- If True: FX forward points from an FX curve are used to convert future cashflows to a common currency prior to discounting.
- If False: Future FX cashflows are discounting using separate zero curves for each currency, and the result converted to a single currency.

ADJUST_FX_RATE

Values: True/False

Implemented by: All currency conversion routines in the system, including trade windows and reports. Transient parameter override is allowed by PricerFXForward and PricerFX.

Transient parameter override is not allowed by PricerFXForwardHomeBased, which by design uses ADJUST_FX_RATE equal to True to compute an NPV.

When currency conversion is done for the present values of cashflows, there are two choices for the FX rate. The first choice is to employ the spot FX rate quoted on the valuation date (the date for which the present value was computed). The second choice is to use "Today's Rate", which is the FX rate that could be applied on the valuation date itself if the currency exchange were to be completed on that day.

The ADJUST_FX_RATE parameter determines whether to adjust the current quoted spot rate to obtain the rate for settlement on the quote date rather than on the spot date (two business days after the quote date).

– If True: "Today's Rate" is used for conversion of currency. This rate is obtained from the quoted spot rate by discounting using the two discount curves for the relevant currencies. The usual no-arbitrage condition is used to obtain the adjusted rate, as discussed in section “The Basic No-Arbitrage Argument” (Note that an alternative method would be to look up Today's Rate from an FX curve, but this choice is not currently available).

- If False: Use the quoted spot rate to convert currency.

Note that the term “valuation date" used here, the date on which NPV is taken, should not be confused with the FX market terminology where “Value Date” is another term for “Settle Date” or “Maturity Date.”

ZD_PRICING

Values: True/False

Implemented by: PricerFXForward, PricerFX.

The choice is not allowed by PricerFXForwardHomeBased, which by design uses the equivalent of ZD_PRICING equal to True.

This parameter also appears in some other Pricers.

- If True: NPV is computed as of the valuation date of the trade window or report. (This is sometimes referred to as the "zero day", hence the term ZD_PRICING.)
- If False: NPV is computed as of the spot settlement date relative to the valuation date.

USE_FX_MID

Values: True/False

Implemented by: All FX pricers.

- If True: The Mid rate of the FX quote is used for all trades.
- If False: The Bid or Ask rate of the FX quote is used depending on the direction of the trade from the point of view of the user. If the base currency is being purchased, the Ask rate is used; if the base currency is being sold, the Bid rate is used. In making this determination the FX rate from the quote set is first converted, if necessary, to the standard from expressing units of quote currency that are equivalent to one unit of base currency.

DISCOUNT_PRIMARY_CCY

Values: True/False

Implemented by: FX Forward and FX Swap pricers

Controls whether NPV computation of FX Forward and FX Swap is discounted by the primary or secondary currency. This parameter is limited to FX Forward and FX Swap products.

- If True: NPV is discounted by the primary currency. The calculation formula is:

$$NPV = \frac{(R_{fwd} - R_c) * Notional_{primary} * DF_{primary}}{R_{fwd}}$$

- If False: This is the default. NPV is discounted by the secondary currency. The calculation formula is:

$$NPV = \frac{(R_{fwd} - R_c) * Notional_{primary} * DF_{secondary}}{R_{fwd}}$$

  Where,

  R_{fwd} is the FX forward rate;

  R_c is the FX contract rate;

  $Notional_{primary}$ is the notional of the primary currency with sign (based on sell or buy);

  $DF_{secondary}$ is the discounting factor of secondary currency discount curve

  $DF_{primary}$ is the discounting factor of primary currency discount curve

4.2.2 PricerFXForwardHomeBased

This section describes in greater detail the algorithm used by PricerFXForwardHomeBased to compute Net Present Value.

This pricer is designed to serve the following requirements:

- the preference for using the discount curve of the home currency over other yield curves, whether or not the home currency is part of the trade
- the need to find forward rates for currency pairs that are not frequently quoted.

There are potentially four currencies involved in the pricer's computation of NPV:

- Trade currencies, the currencies bought and sold, consisting of the Primary (Base) currency and the Quoting currency
- Home (reporting) currency, used across portfolios for risk management and accounting; this is the Pricing Environment "Base" Currency
- Split currency, either USD or EUR, used to define FX rates for a currency pair that is not directly quoted in the marketplace.

Step 1. Forward cashflow conversion to Home currency.

Each cashflow is converted to the Home currency (whether or not the Home currency is one of the currencies in the trade) at the forward FX rate on the cashflow payment date. The rate is drawn from the FX Curve for the cashflow currency vs. home currency pair:

Rfwd = Rspot + forward points for cashflow payment date

If the FX Curve is missing:

If there is no FX curve for the cashflow currency vs. Home currency, the forward rate is obtained through the FX curves with respect to USD. Thus, to convert from CCY1 to HOME when there is no CCY1/HOME curve, the pricer first converts to USD using the CCY1/USD curve and then to HOME using the USD/HOME curve.

If the CCY1/USD curve is missing, then no conversion is done. The pricer defaults to using the zero curve for CCY1 for discounting.

If the FX Curve is too short:

If the FX Curve does not extend to the date of the cashflow payment, the forward rate is not used. Instead discounting is done with the yield curve for the cashflow's currency.

Note this is equivalent to the "FX_POINTS equals false" method in PricerFXForward. The alternative possibility, extrapolation of the FX Curve using interest rate curves, is not currently implemented.

The system uses the Points Factor corresponding to the currency pair of the actual FX curve that is used in the calculations. Example for the JPY/USD currency pair: if there is a JPY/USD curve, it uses the JPY/USD curve with the JPY/USD Points Factor. But if there is no JPY/USD curve, it looks for a USD/JPY curve and uses it with the USD/JPY Points Factor.

Step 2. Discount Factor Calculation

The Home currency discount curve is used for all discounting. If the conversion cannot be performed then the cashflow's curve is used for discounting. This is equivalent to falling back on the "FX_POINTS equals False" method in PricerFXForward.

Step 3. Conversion to Trade Quote Currency

The pricer returns the NPV in the trade's quote currency. To convert the present value from the Home currency to trade quote currency, the rate on the valuation date, "Today's Rate", is used. This is obtained when possible from the FX curve relating the Home currency and the quote currency, going through USD if necessary.

Equivalence to ZeroCurveFXDerived

It should be noted that finding NPV with PricerFXForwardHomeBased is nearly equivalent to making use of zero curves that have been derived from the Home currency curve using a CurveFX. A curve created in this way is termed a ZeroCurveFXDerived, and there is a curve generator application of this name. If one creates a set of such zero curves, then using PricerFXForward with FX_POINTS set to False, and ADJUST_FX_RATE and ZD_PRICING set to True, will produce NPV that is almost equivalent to using PricerFXForwardHomeBased.

Differences between the methods because of differing methods for adjusting the spot rate to Today's Rate; and handling of the cases where derived zero curves cannot be created is up to the user.

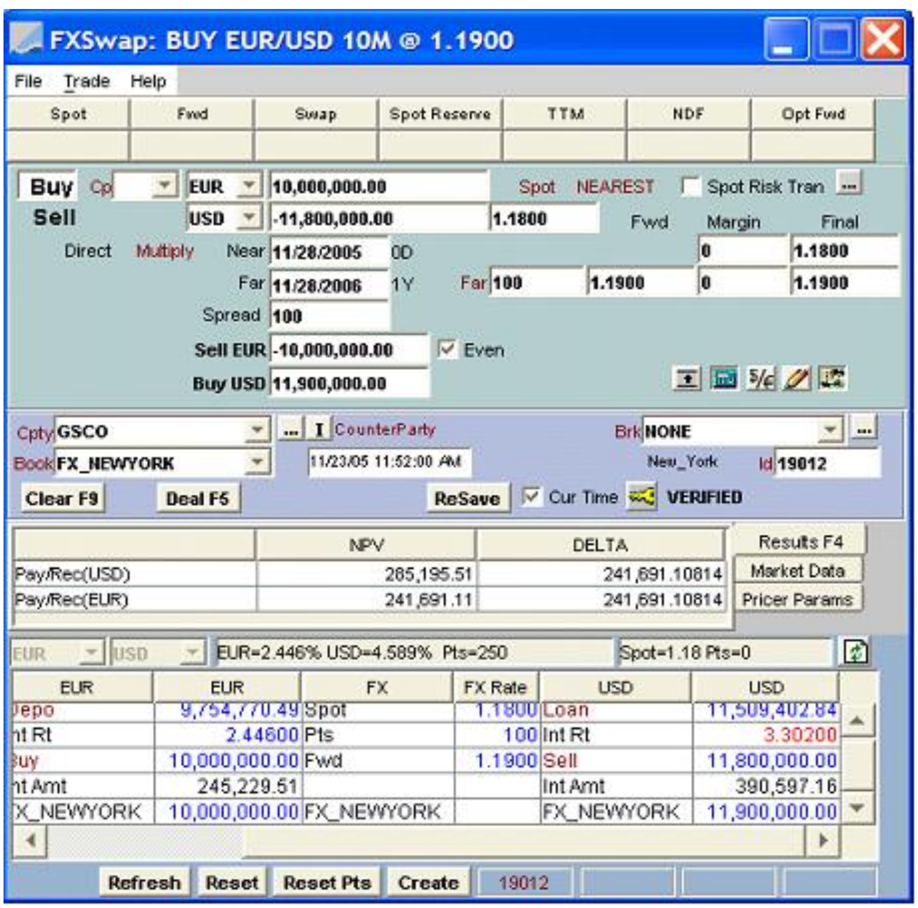
4.3 FX Swaps

FX Swap trades are valued using **PricerFXSwap**, which in fact does nothing but delegate the computation to the spot and forward pricers. Thus:

Whatever pricers are specified in PricerConfig for the FX and FXForward trades will be used for FX Swap trades.

One can use PricerFX for spot and PricerFXForward for forwards; alternately one can use PricerFXForwardHomeBased for both spot and forward trades. Having configured these, the valuing of FX swap trades follows as a consequence.

Following is a pricing example.



FX Swap Valuation									
Value Date	1/28/2022								
				Near Leg	**Forward Leg**				
Primary Ccy	EUR		**Date**	11/28/2005	11/28/2006				
Quoting Ccy	USD		**Primary Amt(EUR)**	10,000,000.00	(10,000,000.00)				
FX Rate	1.18		**Quoting Amt(USD)**	(11,800,000.00)	11,900,000.00				
			FX Rate	1.18	1.19				
			Zero (EUR)	2.44%	2.44%		2.44%		
			Zero(USD)	4.59%	4.59%		4.59%		
			Time(yrs)	-16.1781	-15.1781				
			Theo Fwd FX	0.7705	0.7958				
			Theo Points	-4094.66	-3841.56				
			Actual Points	0	100	**Net(Native)**	**Net(EUR)**	**Net(USD)**	
			NPV (EUR)	$ 14,846,890.01	$ (14,488,597.71)	$ 358,292.30	$ 358,292.30	$ 422,784.91	
			NPV (USD)	$ (24,786,760.16)	$ 23,875,933.99	$ (910,826.17)	$ (771,886.59)	$ (910,826.17)	
			NPV (Net)				$ (413,594.29)	$ (488,041.26)	
7.389056099	20.085537		**PV01(EUR)**	24,038.86	(22,007.61)				
0.367879441			**PV01(USD)**	(40,132.68)	36,266.60				
							Forward	Spot	
148.4131591						**Pay/Rec**	11,514,804.49	11,800,000.00	285,195.51
				$ 17,519,330.21	$ (17,096,545.30)	**Pay/Rec(EUR)**	9,758,308.89	10,000,000.00	241,691.11
				$ (7,267,429.95)	$ 6,779,388.69				
					$ 17,096,545.30				

4.4 FX Cash

The FX Cash products specify a series of FX Forward trades. The NPV of the FX Cash product is just the sum of the NPVs of the FX Forward trades. Thus in pricing FX Cash trades the same methodology as pricing FX Forward trades is used.

Two pricers are available, depending on which method of valuing an FX Forward is desired. **PricerFXCash** will use the methodology of PricerFXForward, while **PricerFXCashHomeBased** will use the methodology of PricerFXForwardHomeBased.

4.5 FX NDF

Before the reset price is set and before reset date time, if the settlement is in quoting currency, the net present value of the FX NDF trade is the same as pricing an FX trade. If the settlement is in primary currency, the value is the primary amount * (1 - trade price / forward FX rate), discounted to valuation date using the primary discount curve."

Discounting is always done with the settlement currency discount curve whether that currency is primary or quoting.

After reset date & time and after the reset price is set, if the settlement is in quoting currency, the value in quoting currency is primary amount * (reset price – trade price), which is then discounted to valuation date using the quoting discount curve. If the settlement is in primary currency, the value is the primary amount - (primary amount * trade price / reset price), which is then converted to quoting currency using the forward price, and discounted to valuation date using the quoting discount curve.

4.6 FX Option Forward

The FX Option Forward product gives discretion to one of the trade parties to convert currency at any time within a specified period. The pricers for this perform a simple computation using the user's input for a projected settle date. On that settle date it is assumed that any outstanding balance will be converted. Thus the FX Option Forward is converted to a simple FX Forward trade for pricing purposes.

How this effective FX Forward trade is priced depends on which of the two pricers one has chosen. **PricerFXOptionForward** will use the methodology of PricerFXForward, while **PricerFXOptionForwardHomeBased** will use the methodology of PricerFXForwardHomeBased.

4.7 FX Window Forward

A Window Forward trade is treated like a letter of credit, whereby either the bank or the customer can 'Grant' the option at any time between the start date and the settle date. This process is called the Take Up. You may enter multiple take ups until the outstanding balance is zero.

<u>The product is priced in two steps:</u>

Step 1 – Find the Worst Date

Step 2 – Create an FX Forward with Settle Date on the Worst Date and, like for Flexi Forwards, delegate the pricing to the **PricerFXForward**.

<u>The Worst Date is calculated in the following manner:</u>

Step 1 – Find a list of all candidate dates, which include:

- Window Start Date
- Window End Date
- If Window Start Date < Spot Date < Window End Date, then add the Spot Date
- If FX_POINTS = True, add all the points on the FX Curve such that Window Start Date < Curve Point Date < Window End Date
- If FX_POINTS = False, add the union of all the points of both discount curves (primary and quoting) such that Window Start Date < Curve Point Date < Window End Date

Step 2 – Get the FX points for all candidate dates (either from the FX curve or from the 2 discount curves, depending on the value of FX_POINTS)

Step 3 – The worst date is:

- If buying Primary / selling Quoting: the candidate date with the maximum FX points
- If selling Primary / buying Quoting: the candidate date with the minimum FX points

The log category for review is `FXWorstPointsRetriever`.

4.8 RHO Calculation

For FX Forwards and FX NDFs, RHO is calculated as follows:

Step 1 - Temporarily set pricing parameter FX_POINTS to false, so NPV calculations are done with 2 discount curves, and the FX points curve is ignored

Step 2 - Calculate the NPV of the trade. Let's call it NPV_base. Note that this NPV is calculated without the FX curve.

Step 3 - Depending on the value of Pricing Parameter RHO_SHIFT_UNDERLYINGS, get a shifted curve for the Primary currency like so:

- If RHO_SHIFT_UNDERLYINGS = True, shift the quotes of the curve underlyings of the Primary currency discount curve up by 1 bp and regenerate the curve
- If RHO_SHIFT_UNDERLYINGS = False, shift the zero rates of the Primary currency discount curve up by 1 bps (parallel shift)

Step 4 - Price the trade with the shifted curve, to get a shifted NPV, NPV_shifted

Step 5 - RHO = 100 * (NPV_shifted – NPV_base)

For RHO2, the methodology is exactly the same, except that instead of the Primary currency curve, it is the Secondary (a.k.a. Quoting) currency curve that is shifted.

So, RHO and RHO2 are measures of the NPV sensitivity to 1% changes in the Primary currency and Secondary currency yield curves, respectively. The sensitivity is measured based on a 1 bp shift, but since the sensitivity is rescaled by a factor of 100, it is a first-order change in NPV due to a 100 bps shift.

Section 5. Money Market Instruments

PricerSimpleMM is an empty class. PricerIntraDayMM and PricerCashCommodity extend PricerSimpleMM and only implement some back office specific functions. PricerCallNotice extends PricerCash and implements its own method to calculate accrual, but there is only some implementation difference, not analytically. Hence, PricerSimpleMM is the only pricer class needed to be explored.

PricerSimpleMM

SimpleMM Pricer evaluates either fixed rate or floating rate loan/deposit. Keep in mind it's from bank's prospective. So the first cash flow usually is negative if it's a loan. When evaluating floating SimpleMM, an index rate and a spread should be specified. If the interest payment is due on the start date, the Discount check box needs to be selected.

In PricerSimpleMM's process method, following analytic measures are calculated:

– NPV/Price/Marginal Call
– B/E rate
– Accrual/Accrual_BC/Accrual_First/Indemnity_Accrual
– PV01
– Fees_NPV

Pricing Measures

Following is the description of some measures.

PRICE – "Price" denotes the Market Price of the security.

NPV – The Net Present Value (NPV) of a series of cash flows is the sum of the present values of each of the cash flows, some or all of which may be negative.

$$NPV = \sum_{i=0}^{N-1} C_i \times df_i$$

– N is total number of cash flow
– C_i is the *i*th cash flow
– df_i is the *i*th discount factor corresponding to the *i*th cash flow

NPV uses Trade, SimpleMM, PricerSimpleMMInput, PricingEnv, PricerMeasure classes. An integer curveSide, a Boolean to indicate if include fee and the valuation date.

The curveSide integer in process function has been settled to QuoteSet.MID, which means the discount curve and forecast curve will use the middle value of the ask and bid prices of quotes.

Process delegates to NPV calculation task to function computeNPV, which in turn delegates the task to function pvFixed and function pvFloating to calculate fixed rate SimepleMM and floating rate Simple MM.

In pvFixed, it first checks if the inputted Trade object has the valid CashFlowSet, if not, it let the object to generate and calculate cashflows. (Details of cashflow generation and calculation see swap and bond pricer documents) Then in a loop, pvFixed accumulatesd the present value of each cash flow. The discount factor is retrieved from the discount curve in the PricingEnv object and the cash flow amount is retrieved using a helper function getCashFlowAmount.

In pvFloating, the only difference from pvFixed is that in the loop the cash flow amount is retrieved using a helper function forecastFlow.

ACCRUAL – Accrued Interest is the proportion of interest or coupon earned on an investment from the previous coupon payment date until the value date.

ACCRUAL_FIRST – "Accrual First" denotes the linear accrual including the day of calculation.

CASH – "Cash" denotes the sum of all the cash flows occurring on the Valuation Date.

NOTIONAL – In a Bond Futures contract, the bond bought or sold is a standardized, non-existent, notional bond, as opposed to the actual bonds that are deliverable at maturity. Contracts for differences also require a notional principal amount on which settlement amount can be calculated.

ACCRUAL_BO – Apart from Bonds, Accrual_BO is the same as Accrual. For Bonds, Accrual_BO is the Accrual computed on Valuation Date for same Date (as opposed to the Accraul of the Bond that computes the value adjusted for the settle days).

B/E_Rate – The B/E Rate, or the Break Even rate is the average Interest rate at which a zero profit is recorded. In our system, this would be represented by an NPV = 0.

Break even rate is an interest/coupon rate that makes the NPV of the simple money market zero. In the case of fixed rate simple money market, it's the fixed rate which makes the NPV zero.

$$\sum_{i=0}^{N-1}(P \times BVR \times df_i) + P \times df_{N-1} - P = 0$$

– N is the total number of cash flow
– P is the notional principle
– BVR is the bireak even rate
– df_i is the ith discount factor

In the case of floating rate simple money market, break even rate is the spread.

$$\sum_{i=0}^{N-1}\left[P \times (IR_i + BVR) \times df_i\right] + P \times df_{N-1} - P = 0$$

– IR is the index rate

Break Even Rate uses function solveForBreakEvenRate and class SolverSimpleMM.

SolverSimpleMM's solve function implement an iterative algorithm to solve the equation of BER described in algorithm section.

PV01 – "Present Value of an 01 (PV01)" or "Dollar Value of an 01 (DV01)" or "the value of a basis point" is the change in price due to a 1 basis point change in yield. This is usually expressed as appositive number.

There are also several back office measures such as Funding_MTM, etc.

Pricer Parameters

– MMKT_From_Quote
  - Values – True of False.
  - Usage – When set to true, the product would be priced from quote.
– Instance_Type
  - Values – Close, Open, Last.

  - Usage: Determines the type of quote to use in quote set, for example, when Close is chosen, close quote will be used.
- Repo_rate
  - Values – Double.
  - Usage – If the valuation date is before the settle date , Pricer is going to get a repo rate for forward pricing. The repo rate can come from the pricing parameters. If it is not set there, then pricer will need to find the discount and repo curves in order to produce the repo rate.
- Include_fees
  - Values – True or False.
  - Usage – If set to true, the pricer measures such as NPV and Fees_NPV will reflect the actual fees involved in the trade.
- ZD_Pricing
  - Values – True or False.
  - Usage – If set to true, the price will be discounted to the valuation date from settle date.
- Check_Funding_Rates
  - Values – True or False.
  - Usage – If set to true, the pricer will check if funding rates are set or implemented.
- NPV_Including_Cash
  - Values – True or False.
  - Usage – If set to true, and if the there is cash flow on valuation date, the pricer will add/subtract the cash.
- NPV_Including_Cost
  - Values – True or False.
  - Usage – If set to true, the NPV would reflect settlement cost.

Section 6. Bond and Money Market Futures

Product	Calypso Product	Description	Pricer
Bond Future	FutureBond		PricerFutureBond
Money Market Future	FutureMM		PricerFutureMM PricerFutureMMSpecific
Swap Future	FutureSwap		PricerFutureSwap
Bond Future Option	FutureOptionBond		PricerFutureOptionBond
Money Market Future Option	FutureOptionMM		PricerFutureOptionMM

6.1 PricerFutureBond

NPV — General Formula

Future Price = Forward Bond Price / Cheapest To Deliver Factor. The Forward Bond Price is the price of the selected cheapest to deliver bond at the future delivery date (including accrual adjustment which accounts for those coupons that fall between the valuation date and the future delivery date). The cheapest to deliver factor accounts for the difference between the actual bond used at delivery, and the synthetic 8%, 15 year non-callable benchmark bond.

6.2 PricerFutureMM and PricerFutureMMSpecific

NPV — General Formula

Future Price = 100 – forward yield. The forward yield is derived from the zero curve by calculating the (annually compounded) forward rate between the maturity date of the futures contract and the maturity date of the rate underlying the futures contract. Currently we have not implemented the Eurodollar convexity adjustment which is given by:

$$\Delta Futures\mathrm{Pr}ice = -0.5\sigma^2 t_1 t_2$$

With t_1 being the maturity date of the futures contract

And t_2 being the maturity date of the rate underlying the futures contract

PricerFutureMM should be used for future contracts with relative underlying, and PricerFutureMMSpecific should be used for future contracts with specific underlying. This can be achieved in the pricer configuration using the contract name as a subtype.

6.3 PricerFutureSwap

Same as PricerFutureMM with a swap underlying.

6.4 PricerFutureOptionBond and PricerFutureOptionMM

NPV — General Formula European Style

This pricer implements the Black Scholes formula as follows:

– Formula for call

$$c = e^{-rT}(FN(d_1) - X\,N(d_2))$$

With

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

T : Time Period from valuation date to exercise date,

F : Futures price

X : Strike

– Formula for put:

$$p = e^{-rT}(XN(-d_2) - FN(-d_1))$$

– Greeks: Greeks are defined as follows with p being the npv of the call or put, respectively:

$$Delta = \frac{\partial p}{\partial F}$$

$$Gamma = \frac{\partial^2 p}{\partial F^2}$$

$$Vega = \frac{\partial p}{\partial \sigma}$$

$$Theta = \frac{\partial p}{\partial \tau}$$

$$Rho = \frac{\partial p}{\partial r}$$

NPV — General Formula American Style

This pricer implements currently a binomial (recombining tree). Details of this standard implementation can be found in *Options, Futures and Other Derivative Securities by John Hull 4th Edition (2000)*, pp.213. In the near future, an implied trinomial tree for faster convergence will replace this binomial tree. The numerical results will be the same but the computation speed will still further improve. The Greeks (Delta, Gamma, Vega, Theta, Rho) are computed numerically and scaled the same way so that they exactly agree with the Greeks of the Black-Scholes formula. The results of a European option evaluated on the tree and with Black-Scholes are consistent if the number of steps is 1000 or more.

PricerFutureOptionMMBpVol

The settings are exactly the same as for PricerFutureOptionMM, except that you have to provide a bp volatility surface (simple or derived), or a transient bp volatility.

6.5 Future Option Volatility Surfaces

Future Option Generator – Underlying instruments (money market future option, bond future option) are quoted as prices. Generates an implied volatility surface.