Adenza

Calypso Yield curves Generation

Version 17.0

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This document describes the generation of yield curves in the Calypso system.

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Contents

Section 1. Curve Types

To define a curve in Calypso one chooses the type of curve, the underlying instruments, the underlying curves if an, the generator and its parameters, and the interpolation. This section provides a description of curve types, and subsequent sections cover the other aspects. The generation algorithms are described afterwards. For details on how to work with the windows please refer to the application user guides.

1.1 CurveZero and CurveBasis

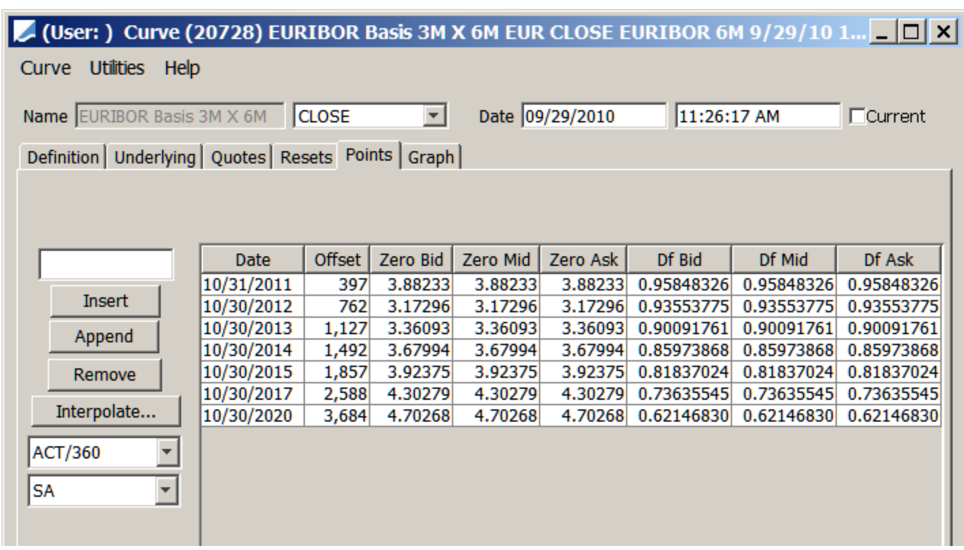
There are two ways to represent curves in Calypso: as a collection of Discount Factors or as a collection of Spreads. The two curve types behave differently for risk purposes.

A **Discount Factor Curve** is the Calypso type **CurveZero**, and stores its points as discount factors. Once this type of curve is generated its points are no longer directly linked to the curves used to create it. If the underlying Base and Discount curves change, the discount factors will not change unless the user deliberately regenerates or rolls the curve.

A **Spread Curve** is the Calypso type **CurveBasis**, and stores its points only as spreads. The discount factors of this curve are not defined except in relation to the Base curve. The spreads on the Spread Curve are added to the zero rates on the Base Curve in order to produce an all-in zero rate, which is converted to a discount factor. When the Base curve changes, the discount factors on a Spread Curve will change automatically, without any deliberate regeneration or rolling by the user.

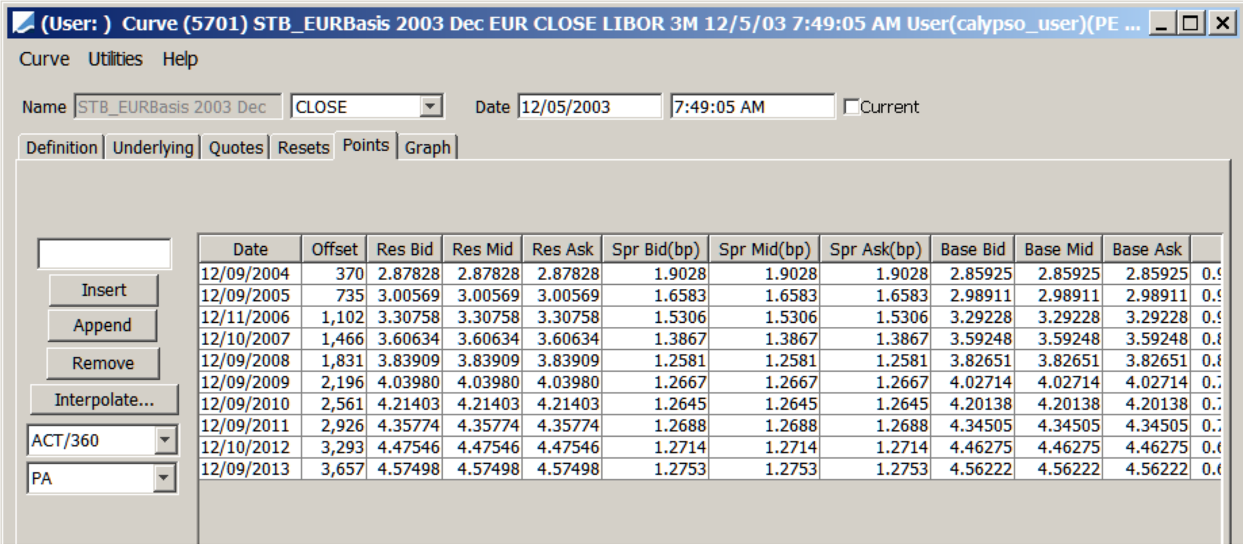
The Spread Curve (CurveBasis) is usually only used when some of the underlying instruments are Basis Swaps. However, Basis Swaps can be used as underlyings in either type of curve. It depends only on the risk relationship one wants to maintain with the underlying curves.

A Discount Factor Curve (CurveZero) is generated by the **CurveZero window**. A Spread Curve (CurveBasis) is generated by the **CurveBasis window**. The distinction between them occurs on the output Points tab. A Discount Factor Curve has discount factor points, as follows:



Date	Offset	Zero Bid	Zero Mid	Zero Ask	Df Bid	Df Mid	Df Ask
10/31/2011	397	3.88233	3.88233	3.88233	0.95848326	0.95848326	0.95848326
10/30/2012	762	3.17296	3.17296	3.17296	0.93553775	0.93553775	0.93553775
10/30/2013	1,127	3.36093	3.36093	3.36093	0.90091761	0.90091761	0.90091761
10/30/2014	1,492	3.67994	3.67994	3.67994	0.85973868	0.85973868	0.85973868
10/30/2015	1,857	3.92375	3.92375	3.92375	0.81837024	0.81837024	0.81837024
10/30/2017	2,588	4.30279	4.30279	4.30279	0.73635545	0.73635545	0.73635545
10/30/2020	3,684	4.70268	4.70268	4.70268	0.62146830	0.62146830	0.62146830

The **CurveBasis** window shows points with the base rates, Spread Curve points, and result zero rates, as seen below:



Date	Offset	Res Bid	Res Mid	Res Ask	Spr Bid(bp)	Spr Mid(bp)	Spr Ask(bp)	Base Bid	Base Mid	Base Ask
12/09/2004	370	2.87828	2.87828	2.87828	1.9028	1.9028	1.9028	2.85925	2.85925	2.85925
12/09/2005	735	3.00569	3.00569	3.00569	1.6583	1.6583	1.6583	2.98911	2.98911	2.98911
12/11/2006	1,102	3.30758	3.30758	3.30758	1.5306	1.5306	1.5306	3.29228	3.29228	3.29228
12/10/2007	1,466	3.60634	3.60634	3.60634	1.3867	1.3867	1.3867	3.59248	3.59248	3.59248
12/09/2008	1,831	3.83909	3.83909	3.83909	1.2581	1.2581	1.2581	3.82651	3.82651	3.82651
12/09/2009	2,196	4.03980	4.03980	4.03980	1.2667	1.2667	1.2667	4.02714	4.02714	4.02714
12/09/2010	2,561	4.21403	4.21403	4.21403	1.2645	1.2645	1.2645	4.20138	4.20138	4.20138
12/09/2011	2,926	4.35774	4.35774	4.35774	1.2688	1.2688	1.2688	4.34505	4.34505	4.34505
12/10/2012	3,293	4.47546	4.47546	4.47546	1.2714	1.2714	1.2714	4.46275	4.46275	4.46275
12/09/2013	3,657	4.57498	4.57498	4.57498	1.2753	1.2753	1.2753	4.56222	4.56222	4.56222

The choice of curve type affects how risk is reported. The Base curve can be varied independently from the Discount Factor Curve, but this is not true for a Spread Curve.

1.2 CurveBasis Interpolation

As introduced above, a CurveBasis defines its points by referring to two components:

- An underlying curve, called the **base curve**
- A set of spreads to the zero rates of the base curve

The CurveBasis can be used as a discount or forward curve for trade valuation, in the same manner as a CurveZero.

A CurveBasis is generated by specifying a base curve and a set of underlying market quotes. The set of zero spreads to the base curve zero rates is computed so that the market quotes are reproduced when the resulting curve is used to value the underlyings.

A **cross-currency** CurveBasis has an additional dependence on curves in another currency. The same CurveBasis window and generator can be used to generate either single-currency or cross-currency basis curves.

1.2.1 CurveBasis Representation of zero rates

A CurveBasis computes its zero rates from the zero spreads it holds and its underlying base curve. The following describes how this combination is done. The generation of the spreads from the underlying market quotes is assumed to already have taken place.

Suppose the zero rates of the underlying base curve are Z_i for dates T_i , and the set of zero spreads s_j on dates t_j. The basis curve defines curve points on *each* of the dates T_i and t_j. The zero rate on any date not falling on these points is found from interpolation of the zero rates on these points, using the interpolation method (e.g., linear) specified for the basis curve.

The curve points of the CurveBasis are defined as follows.

For spread dates:

1. For each spread date t_j, use the base curve to interpolate its zero rate for that date, using the interpolation method defined on that curve. Call this interpolated rate $Z(t_j)$.

2. The combined zero rate of the basis curve at t_j is then defined by adding this interpolated base zero rate to the zero spread:

$$z(t_j) = Z(t_j) + s_j \,.$$

For base curve dates:

1. For each curve point on the base curve at date T_i not equal to some t_j, interpolate $s(T_i)$ from the spreads s_j using linear interpolation on the s_j. (In previous versions of Calypso other spread interpolators than linear were allowed; however this sometimes led to unrealistic curve shapes.)

2. The combined zero rate of the basis curve at T_i is then defined by adding this interpolated spread to the base zero rate:

$$z(T_i) = Z_i + s(T_i)\,, \quad \min(t_j) \; < T_i \; < \; \max(t_j),$$

The spread is assumed constant on dates before the first spread date and after the last spread date.

The base zero rates Z_i can be expressed in terms of a day count and compounding frequency different from those of the spreads, in which case a conversion to those of the spread is performed before the addition is made.

From the basis curve $z(t)$ formed in this way on all points T_i and t_j, discount factors and zero rates can be found in the same manner as any other zero curve.

Note: *In some situations, Xccy basis curve is a special case. If the Xccy basis curve includes some FX forward curve underlyings to generate curve basis, when doing interpolation, the union curve points don't include the base curve points that are before the last FX forward end date, but include the base curve points that are after the last FX forward end date.*

1.2.2 Difference in Generation between CurveZero and CurveBasis

The iterative algorithms and pricers for generating a CurveBasis are the same as those of CurveZero, so one would expect the results to be the same. However their interpolators are different. During the solving and during pricing the interpolation of a CurveBasis will depend on the Base curve.

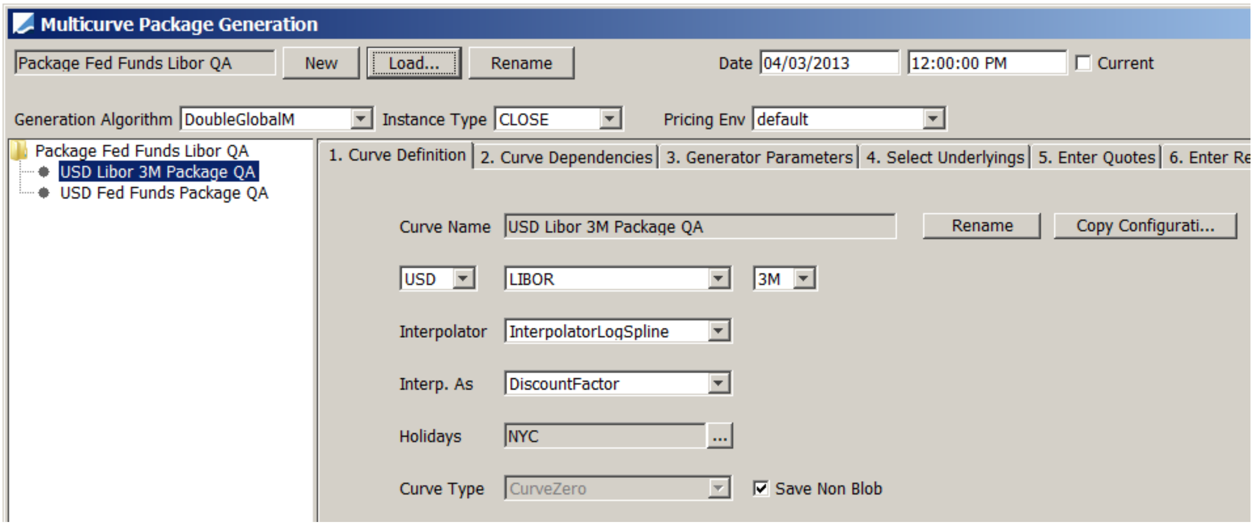
This is particularly true when no short-term instruments were specified for the basis curve that mature prior to the first Basis Swap, as then during solving the short end needs extrapolation backward from the first trial basis swap point. For a CurveBasis this will depend on the extrapolation of the base curve, while for the CurveZero it just depends on the extrapolation of its first discount factor. This can produce somewhat different results between the curve when generated in the zero curve window (Discount Factor curve) versus in the CurveBasis window (Spread curve).

1.3 Simultaneous Curve Generation

Curves can also be defined in a **Multicurve Package**. Curves in the package are of CurveZero (Discount Factor) type and use each other as underlying curves. That is, the instruments in the package require all the curves in the package in order to be priced. All of the curves in the package are generated simultaneously in order to make this possible.

Curves in the package can also use curves that are not in the package as underlyings, and those external curves will not be generated with the package. Details and examples may be found in the section “Simultaneous Curve Generation.”

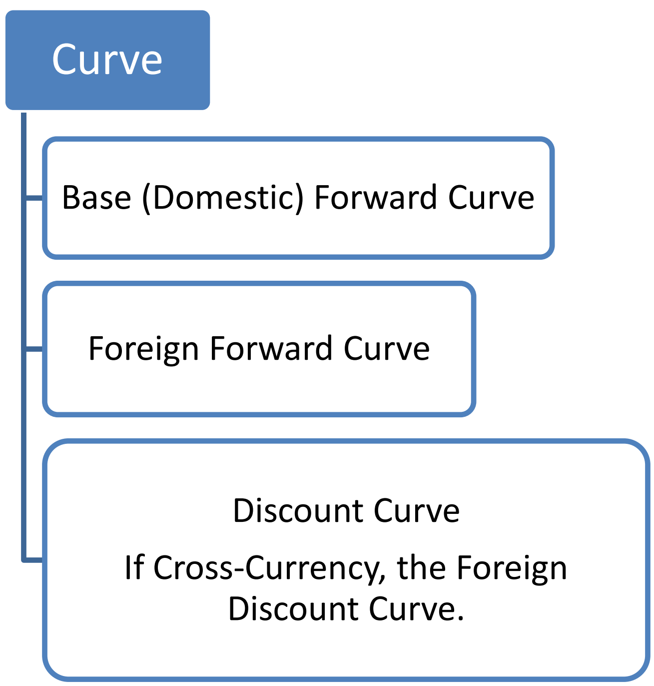
In order to create a Multicurve Package one makes use of the **Multicurve window**, as shown below.



1.4 Curve Dependencies

A curve is generated from a set of underlying instruments. In order to price those instruments several curves will be needed; most typically, for example a Swap, the pricing will need at least one curve for forward rate projection and one curve for discounting. In a single curve window (not Multicurve) only one curve can be generated, so any other curves needed for pricing must be specified by the user. These curves must already have been generated by the system.

Schematically possible dependencies are:



The following cases are typically encountered.

- **Single currency curve without other curve specifications**

  If no Discount curve is specified the generated curve will be used as both the forward and discount curve for pricing its instruments.

- **Single currency curve with Discount curve specification**

  If a Discount curve has been specified then the generated curve will be used as the forward curve for projecting rates of the instruments.

- **Single currency curve with Base curve specification**

  If a Base curve has been specified that curve will be used to project forward rates for Basis Swaps for the rate index that is not the generated curve index. The generated curve will project forward rates for the rate index given in its specification, and will also be used for discounting.

- **Single currency curve with Base and Discount curve specifications**

  The generated curve will project forward rates for the rate index given in its specification. The Discount curve will be used for discounting.

- **Cross-currency discount curve generation with Base and Foreign curve specifications**

  The generated curve will be used for discounting in the currency being generated. The Base curve will be used for forward rates in that currency. The Foreign curve will be used as the forward curve for the foreign currency instrument cashflows; if an additional Discount curve is specified that will be used for discounting the foreign currency flows, otherwise the Foreign curve will be used for both forward projection and discounting.

- **Cross-currency foreign curve generation with Base and Foreign curve specifications**

  This is a lesson common case; usually cross-currency curves generate the discount curve. The choice of generator determines the curve usage. With these specifications, the generated curve will be used for forward rates in the currency being generated. The Base curve will be used for discounting in that currency. The Foreign curves are as above.

- **Simultaneous curve generation**

  In Simultaneous curve (Multicurve package) generation any of the curves in the package can act as Base or Discount curve for the other curves, and be generated with them. Curves that are not in the package but which were already generated and saved can also be chosen as Base or Discount curves, and these will not be generated with the package.

1.5 Chain of Curves

Prior to 2007 it was common to use only one interest rate curve for discounting and for projecting forward rates in all tenors. Since then it was recognized that different credit risks are involved in each of these calculations. Discounting should be performed at the funding rate, which for a collateralized trade is typically the OIS rate. A 6M forward rate is not the same as two successive investments at the 3M rate because a credit default could occur in the first three-month period, making the second one unnecessary.

For this reason, where there was once only a single curve there was now a chain of related curves. If there is a convenient set of market instruments then this chain of curves can be built up one by one, with each curve dependent upon the preceding curves. The following is a typical example of the steps involved.

1. **Build a discount curve using instruments which can be priced with a forward curve the same as the discount curve.**

   An example is a curve built from EONIA OIS swaps that compound daily at the OIS rate, so that its forward rate is the same as the discount rate.

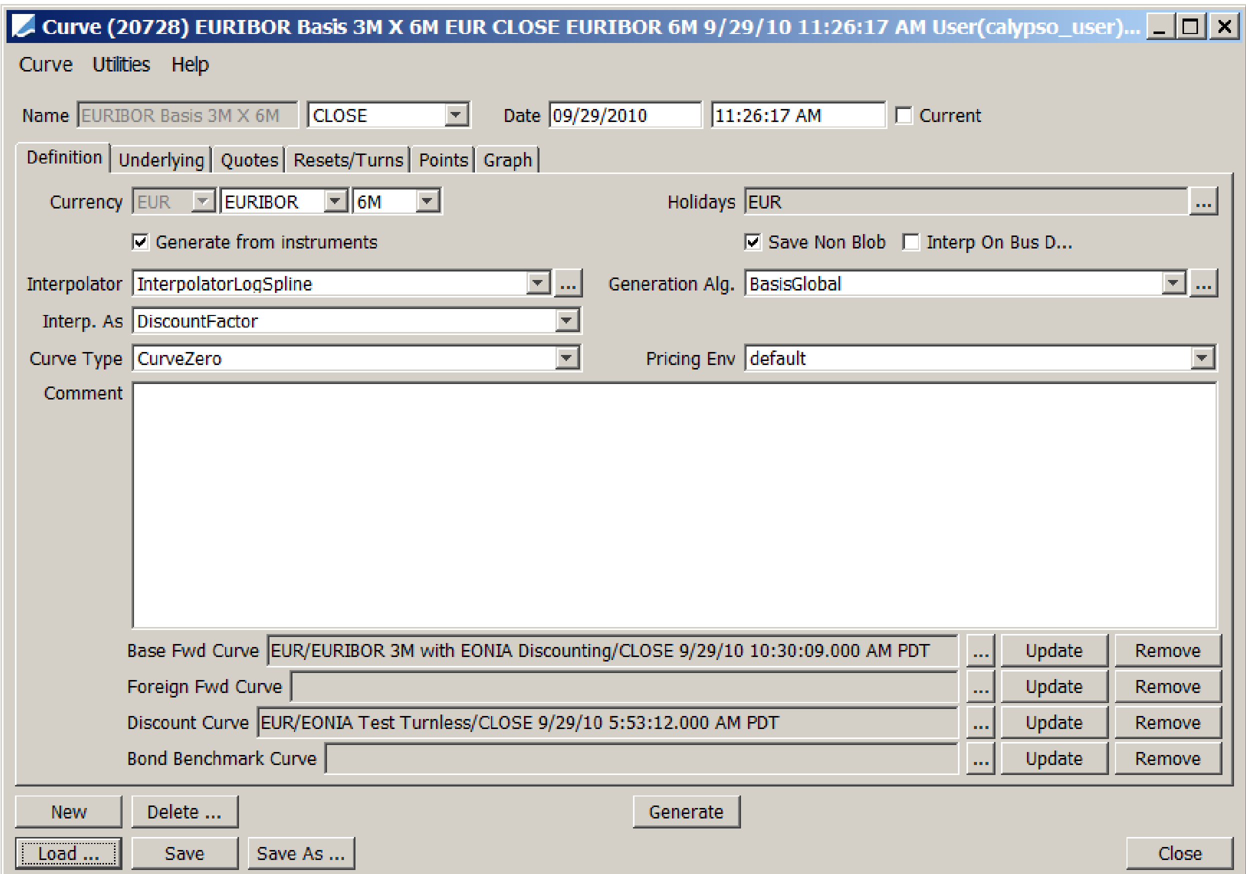
2. **Build a forward curve using vanilla swaps and the discount curve previously created.**

For example, vanilla 3M EURIBOR swaps quoted in collateralized form will be discounted with the EONIA curve, so one can solve for the 3M EURIBOR forward curve that recovers the swap quotes.

3. **Build a forward curve of another tenor using basis swaps and the previously created curves.**

   Pricing the market 3MX6M EURIBOR basis swaps requires the EONIA discount card and the 3M and 6M EURIBOR curves. As the first two curves have already been created, the 6M curve can be generated that prices the basis swaps to the market.

The following is a screenshot of the curve window for creating the 6M curve of the example just described. The generated curve will be used as the forward curve for 6M EURIBOR, while the 3M EURIBOR Base curve will be used for the opposite leg of the Basis Swap instruments, and the EONIA curve will be used for discounting cashflows on both legs.



If the market instruments are not as cooperative then the curves cannot be built up in sequence. This occurs when each instrument in the market requires two or more curves in order to be priced. Then there is no alternative but to build the set of curves simultaneously. For details, please refer to the section "Simultaneous Curve Generation."

1.6 How to Validate Curve Results

1.6.1 Round-Trip Pricing

A curve is correctly generated if it passes the test of round-trip pricing. This is done in three steps:

- Make a trade on one of the underlying instruments of the curve. The trade's value date is the same as the curve date. Set the trade's notional to 1,000,000.
- Apply a Pricing Environment to the trade that uses the curve and also any curves that were involved in the generation. The curves should have the same usage as during generation, for example, projecting forward rates for a particular index. Ensure the curves have all the same dates as used during generation. The Time Zone of the pricing environment must be the same as the Time Zone of the Pricing Environment used during generation.
- Make sure any rate fixings are available to the trade in the same fashion as used in curve generation. Alternately, select the Reset from Curve pricing parameter (if available) to that any fixings are ignored.
- Price the trade to find the Breakeven Rate, if that is an available pricer measure. Or, put the quote on the trade and price it to find the NPV.
- Compare the result to the quote for that instrument used in curve generation. Round-trip pricing is considered successful if either of the following holds: (a) the Breakeven rate is within 0.1 basis points of the input rate, or (b) the NPV is 0.01 or less when the input quote is placed on the trade.

The practical measure for generation success is the typical bid/ask spread of that instrument's market. If the calibration error is less than the bid/ask spread it is considered to have no economic significance.

1.6.2 Validation Trade

An automatic method to make a validation trade is to launch it from the curve window itself. On the Underlyings tab, select the desired underlying and press CTRL + Double-click. The trade will pop up.

However, note that the pricing environment on that trade is the one specified on the curve. It does not necessarily have the generated curve in it. You may need to select an environment you want to use for testing that has the generated curve configured for pricing that instrument. All other curves used in generation must also be in that environment and have the same date as used during generation.

The generated curve must be saved before it will be picked up by the pricing environment.

The trade will look for any needed reset fixings, so be aware of the Quote Set it is using. Verify that the reset in the Quote Set is the same as the reset used in curve generation. The reset on a swap trade can be seen in the Cashflow tab. Compare this to any reset on the curve.

There are parameters that affect the source of the reset. During curve generation, a swap underlying with Manual First Reset will use a reset given in the curve Quotes table. In the Swap trade window the use of a reset, when the fixing date is the value date, depends on the Pricing Environment parameter **RESET_FROM_CURVE**. If RESET_FROM_CURVE is True, then today's reset will be projected from the curve rather than using a given value; if this is False, the reset will be obtained from the Quote Set. In that case, one must make sure the Quote Set value of the reset is the same as that supplied on the curve. Refer to the discussion *Rate Resets: Manual First Reset Parameter* in the "Curve Underlyings" section of this document for more information.

Other trade and curve parameters need to be aligned in order for round-trip pricing to be confirmed. These include the trade pricing parameters ADJUST_FX and FORECAST_FX_RATE and the curve generator approximation parameter Daily Average Swap Fast Approx. For further information, see the section below, *Troubleshooting Round-Trip Pricing Problems*.

1.6.3 Automatic Calibration Check

If using a Global generator the fastest way to validate a curve is to use the automatic calibration check.

The Log category "**GenGlobal**" provides the calibration check. Configure the log to use this category and then generate the curve. During generation the process will automatically create trades and test them against the newly-generated curve.

Manual pricing of the trade, if properly done, will produce results identical to those of GenGlobal.

For more information on validating the Global generators see the section "Validation and Accuracy" in the chapter "Global Generator."

1.6.4 Daily Points

To view the points for every day of the curve, use the Log category "**DailyPoints**." Apply it to the Log configuration before generation, then generate. The points will appear in the log when the generation is complete, from which they can be copied and pasted into Excel.

1.6.5 Special Considerations

FRA interpolation

In **Forward Rate Agreements**, the curve generation technique does not always match the pricers. A typical FRA quote is considered to represent the forward rate between the start and end dates of the FRA. Curves are generated under this assumption. But actual settlement (and therefore pricing) of a FRA is not drawn from a curve, and does not necessarily match a quoted fixing; instead, interpolation of two fixings is required. For example, due to various differences between date conventions, a '3M' FRA may have a period length of 93 days, but the 3M Libor that resets on the FRA reset date may have only 91 days, so to obtain the 93-day rate the settlement process interpolates linearly with the 6-month reset. This interpolation procedure of a 3M and 6M rate is not currently being used in curve generation with FRAs, in favor of the faster method of directly applying the quote to the FRA period (93 days in this case). This produces some discrepancies with the pricer whenever FRA periods don't align exactly with Libor periods. (Calypso is considering allowing the user to choose the more complete method in curve generation; however, that brings up the issue that 3M and 6M rates should be projected from different curves.)

Futures Bid/Ask Quotes

In **Futures Bid/Ask quotes**, the prices need to be entered into the window so as to align with the yield Bid/Ask quotes of other instruments. This means a Futures Ask price must be placed in the Bid column (representing the lower rates), and the Futures Bid price must be placed in the Ask column.

Bid/Ask Basis Swap Quotes

Basis curves generated from **Bid/Ask basis swap quotes** find their Mid curves by averaging the Bid and Ask curves, *not* by generation directly from the Mid quotes. So the Mid curves will not reprice to the Mid quotes. By contrast, zero curves generated without basis swap quotes do generate Mid curves directly from Mid quotes. This is to manage the propagation of spreads. (See further comments on Bid/Ask curves, below.)

1.6.6 Troubleshooting Round-Trip Pricing Problems

If one is finding round-trip pricing appears to be failing, here are some things to verify:

- The Trade that was created is identical to the one created by the generator during its calibration, including all characteristics, trade date, value date, and all other dates

- The Pricing Environment used in pricing the Trade is the same as the curve's pricing environment, including the same parameters, market quotes and associated curves (discount curve, base curve, etc.)
- The Pricing Environment is configured to use the generated curve and all the curves that were used during that curve's generation (this may require updating the environment).
- Check the dates on the curves used in pricing, as some may have been rolled before being used in the trade window.
- Check the fixing rates on the cashflow tab in the trade window to make sure they are the same as the fixing rates used in the curve. You made need to prevent the use of a fixing quote by selecting the RESET_FROM_CURVE pricing parameter in the trade window, if that is available.
- **ADJUST_FX** must be set to True on cross-currency trades. This is the setting that is assumed during curve generation, as all NPVs are calculated on the curve date (requiring adjusting the spot FX rate to the curve date). Also, in the Pricing Env for the trade there cannot be a curve assigned to the **ADJUST_FX usage** unless it is the curve that is being validated, else there will be a conflict between how the FX spot rate was adjusted during curve generation and how it is being adjusted in the validation trade.
- If the curve has daily average underlyings and has the generator parameter **Daily Average Swap Fast Approx** set to True, then usually round trip pricing will not be exact. The curve was generated with an approximation, but the pricing is done with an exact daily calculation (unless the averaging on the trade definition is removed).
- If a cross-currency swap has mark-to-market principal resets, the trade pricing parameter FORECAST_FX_RATE needs to be set to True. The forecasting provides the calculation of the principal resets; otherwise, no principal payments will be computed. To avoid possible mistakes, it is recommended all Pricing Environments have this value set to True.
- Check if there are different Time Zones for the user and the Pricing Environment. This can cause the dates to be off by one day. Carefully compare the dates of the swap with those on the curve to see if there is a mismatch.
- For Basis Swaps note there are two breakeven rates, one for each leg. When testing one must check the quote against the correct leg. The simplest way to do this is to use the Pricer Measure DETAILED_DATA, which shows the breakdown of results for each leg. To verify, check which leg the quote is defined on by looking at the Curve Underlying definition for the basis swap, and compare with the corresponding leg in the trade.
- If using a Global generator, run the "GenGlobal" log and view the calibration check results. If that check shows a much smaller error than one sees when testing manually it is almost certain some mechanical error is being made in the manual test. Review all the procedures listed above to identify the problem.

Section 2. Curve Underlyings

2.1 Common Instruments

The instruments used to build the curve are termed Curve Underlyings and include the types

- Money Market Deposits
- FRAs
- Deposit Futures
- Interest Rate Swaps: fixed/floating Single Currency or Cross-Currency
- Basis Swaps: floating/floating Single Currency or Cross-Currency
- Basis Swaps quoted as the difference between two fixed/floating swaps
- Bonds
- Swaps quoted as Benchmark Bond plus Spread
- FX Forwards
- FX Futures
- Futures or swaps whose quotes are spreads over reference futures or swaps

Specific Curve Underlyings by currency, rate index, and other parameters are created by the user. With each Curve Underlying there is a quote name associated to a standard market quote.

2.2 Trades

For curve generation a trade is made from each Curve Underlying. The trade date and value date are set to be the curve date, and the spot date is calculated from the trade date.

The basic curve generation logic is to find a curve that prices these trades back until the input quote is recovered.

The trade can be viewed using the Underlyings tab of the trade window using CTRL+DBL-CLICK on the name of the underlying. This brings up the trade window and is useful in verifying the curve generation process.

Note that the Pricing Environment used by this trade is the same as that used on the curve, which does not include the curve itself unless that curve has already been generated and saved and configured for use in the Pricer Configuration of the environment. (See the Round Trip Pricing validation section.)

2.3 Single-Period Instruments

A single-period instrument is a Curve Underlying that has one continuous interest period defined by a start date and an end date. The instrument's market quote defines the interest rate applicable between those dates. These include Money Market Deposits, Deposit Futures, and Forward Rate Agreements.

The following describes the most common single-period underlyings. Variations of these can also be used. For example, for FRAs and Money Markets fixed start and end dates can be entered rather than relative tenors.

2.3.1 Money Market Deposits

Money Market Curve Underlyings have an interest start date on the Spot Date, usually two business days after the curve date; the end date is found by adding the tenor of the underlying to the Spot Date and adjusting for holidays and weekends, according to the convention defined in the underlying. To cover the period between the curve date and the Spot Date, one usually begins a curve with an O/N (overnight) rate and possibly also a T/N (tomorrow next) rate; otherwise, the rate over the gap will need to be extrapolated.

The discount factor is related to a standard money market rate *R* by

$$D = \frac{1}{1 + Rt}$$

where the time period extends from the Spot Date to the deposit maturity date.

If the rate is a discount rate, the relation is

$$D = 1 - Rt$$

The special case of BU252 money markets, currently only implemented for BRL, is

$$D = \frac{1}{(1 + R)^{B/252}}$$

where B is the number of business days between Spot Date and the maturity date.

2.3.2 Deposit Futures

Deposit Futures Curve Underlyings are derivatives of an underlying term deposit that starts on the Spot Date after the futures settlement date and continues for the tenor of the deposit. The term of the underlying deposit determines the forward period and this is used in curve generation. Some textbooks simplify the yield curve calculation by assuming the futures rate applies to the period between futures settlement dates but as this produces inaccuracies Calypso does not employ this approximation.

The unadjusted forward rate is given by

$$R = 1 - \frac{Futures\ Quote}{100}$$

The forward rate can be corrected for convexity. The rate is then used to create a discount factor as described in the previous section on Money Market Deposits.

The convexity adjustment is described in the next section.

2.3.3 Deposit Futures Convexity Adjustments

Convexity adjustments to futures prices can be entered manually or can be generated by Calypso using a simple model. This is governed by two parameters, Use Future Convexity and Use Manual Future Convexity

A. Manually Adjusted

For manually entered convexities, set

- Use Future Convexity = True
- Use Manual Future Convexity = True

The convexity adjustment in the Quotes tab is displayed in basis points. The forward rate used in curve generation is

$$Forward\ rate = 1 - \frac{Futures\ Quote}{100} - \frac{Convexity\ Entry}{10000}$$

Example

In curve window suppose one has

- Futures quote = 98.52
- Convexity entry = 1.06

The unadjusted forward rate is then

$$1\text{-}.9852{=}0.0148$$

This decimal represents a forward rate of 1.48%.

The convexity-adjusted forward rate is

$$1 - .9852 - 0.000106 = 0.014694$$

This is a decimal. As a percent the adjusted forward rate is 1.4694%.The curve is generated so as to reproduce this adjusted forward rate.

Note: One can use the log category "CurveUnderlyingFuture" while generating the curve to see the unadjusted rate and the adjusted value.

B. Volatility Adjusted

To allow Calypso to compute the convexity adjustments, set

- Use Future Convexity = True
- Use Manual Future Convexity = False or blank

This calculation requires a volatility surface. This needs to have already been created and assigned to the appropriate deposit futures in the PricerConfig "Product Specific" tab.

The basic algorithm for calculating convexity adjusted forward rate is as follows:

- Find the Future Rate, R:

$$R = 1 - \frac{Futures\ Quote}{100}$$

- Convert the Future Rate (R) into a continuously compounded Future rate (R_c):

$$R_c = m\ log(1 + \frac{R}{m})$$

- Obtain the volatility *v* from the cap volatility surface defined in the Pricer Configuration for the Pricing Environment used by the curve. This volatility if for the caplet maturing at the underlying deposit start date of the futures contract with strike equal to the futures rate R_c.
- The convexity adjustment is calculated using the Ho-Lee model, which is the Hull-White model with zero mean-reversion. If the cap surface holds lognormal volatility (Black-Scholes), then the volatility is adjusted by the futures rate to approximate a normal volatility. The adjustment for lognormal *v* is

  $$c_{adj} = 0.5t_1t_2R_c^2v^2$$

  If the cap surface holds normal volatility (basis point volatility, "BP vol"), then the adjustment is

  $$c_{adj} = 0.5t_1t_2v^2$$

- Apply the convexity value to adjust the continuously compounded future Rate (R_c) to a continuously compounded forward rate (F_c).

$$F_c = R_c - c_{adj}$$

- Convert the continuously compounded forward rate *Fc* to a simple forward rate *F* with the compounding frequency equal to the Tenor of the contract:

$$F = m(\exp(\frac{F_c}{m}) - 1)$$

Where:

t_1 : Underlying future start date

t_2 : Underlying future end date (i.e. t1 + tenor of future contract)

R : Future rate (Simple compounding)

R_c : Continuously compounded Future Rate

F : Forward rate (Simple compounding)

F_c : Continuously compounded Forward Rate

v : Volatility at Underlying future start date

m : Compounding Frequency, assumed given by the Tenor of the contract

There are variations on this algorithm that can use a correlation.

Convexity Log Category

Detailed information about the convexity adjustment can be found by using the Log category "CurveUnderlyingFuture" during curve generation. There are two lines associated with a futures contract containing separately these values:

- "Unadjusted rate" – the input quote (as rate: 100 -futures price)
- "Applied rate" – the input quote minus Convexity.

The difference between these will give the convexity that was used.

Display of Convexity Values

Whether entered manually or computed by the system, the convexity values applied to the futures rates are displayed on the Points tab in the Convexity columns. This display needs some explanation.

In the CurveZero window, the convexity is applied to any dates on the curve that fall within the futures period to which the convexity is applied. So one might see multiple rows with the same convexity – this just means that one futures contract covers all the dates in that row, and the convexity was applied to that one contract.

The CurveBasis window does not display convexity on the Points tab. The Log category described at the end of the previous section should be used instead.

In the Multicurve window, the convexity display has been improved over the single curve window so that multiple rows do not appear for one convexity. The convexity points are mapped to the corresponding future end dates.

2.3.4 Forward Rate Agreements

Forward Rate Agreement Curve Underlyings assign their quotes to be the forward rates between the start and end dates of the FRA period. Determination of these start and end dates follows the convention common in the major markets, as follows:

FRA Start Date = Spot Date plus Start Tenor, rolled by Modified Following

FRA End Date = Spot Date plus End Tenor, rolled by Modified Following

For example, a 9X12 FRA will have its start date that is 9 months after the spot date and an end date 12 months after the spot date. Contrast this with a futures type of calculation in which after calculation of the start date the end date is found by adding the 3M rate index tenor to the start date. Because of calendar accidents these procedures can result in slightly different end dates. Calypso uses the former convention. Since the FRA rate is applied exactly to the period between the start and end dates the rate will not always correspond with the Libor rate for the same start date, because of the potentially different end date. In these cases the parties of the trade often agree to interpolate between two rate indexes, for example, the 1M and the 3M Libor.

For more information about validating the FRA results, see the section "Validation and Accuracy" in the Global Generator chapter.

2.4 Multi-Period Instruments

In generating with multi-period instruments such as swaps and bonds, Calypso adds only a single point to the curve at the last date on which a discount factor needs to be known to price the instrument. This last date is usually the maturity date, but it may be the end date of the last forward rate period, as a discount factor is needed on that date in order to project the forward rate. Discount factors on all points prior to the last added point are interpolated as needed using the curve's interpolation method.

The generation procedure for a swap is as follows:

1. A swap trade is created with the quoted rate as its fixed rate.
2. A guess is made for the discount factor as of the last date of the instrument. This creates a candidate curve.
3. The swap is priced with the candidate curve to find Net Present Value implied by that curve.
4. From the calculated Net Present Value, an improved guess of the discount factor is made.
5. Steps 2 – 4 are repeated until the solver reaches a termination condition. For more details please see the section below on the BootStrap generator.

2.4.1 Example swap types

The Curve Underlyings for Swaps and Basis Swaps can handle a number of variations. Some interesting types are listed below.

– Daily Averaging Basis Swaps - One side can be a daily averaging floating rate leg and the other side a vanilla floating rate leg.
- Basis Two Swap – The representation of a basis swap as two back-to-back fixed-versus-floating swaps, with the spread quotes as the difference between the fixed rates.
- Mark-to-Market XCCY Swaps – Allows specification of a periodic reset of notional on one or the other leg of a cross-currency swap, according to the FX rate on the reset date, with an accompanying exchange of principle.
- Fixed-versus-Floating XCCY Swaps – An extension of vanilla fixed-versus-floating swaps to allow different currencies on the legs, used in emerging markets.
- SIFMA (BMA) Swaps – Ability to specify weekly averaging swaps as curve underlyings without needing to define a special rate index or calculator.

2.4.2 Rate Resets: Manual First Reset Parameter

The index rate fixings needed by the swaps in the curve are shown on the Resets tab of the curve window. Their values will be populated from the Quote Set, if available. The user can also manually enter these values.

The first rate index fixing needed swaps may not yet have been published at the time of curve generation. The handling of known or unknown first fixings is determined by the checkbox **"Manual First Reset"** on the Swap and BasisSwap Curve Underlying definitions.

Manual First Reset set to False (box is unchecked)

A known fixing will not be used during generation. The needed rate is projected from the curve.

This applies whether the fixing date is on the curve generation date or before it. So a past reset, which is sometimes needed (see below) will be ignored, even if it is already present in the database.

Manual First Reset set to True (box is checked)

If a fixing rate is given in the curve Resets table, and the fixing date is the same as the curve generation date, that fixing rate will be used; otherwise, the rate is projected from the curve.

Note in particular: If there is no reset fixing on the curve, a reset will no longer be automatically sought for in the database. It is assumed that the user does not want to use a reset if there is none on the curve itself. In P&L and sensitivity reports the new behavior prevents inconsistencies when rolling and shifting curves. This behavior is new in Version 15.

<u>Case of past resets:</u> If the fixing date is before the curve generation date, the quote in the curve's Resets table will *not* be applied. The reset in the window is undated and so always assumed to be on the curve date. In this case, the quote will be obtained from the dataserver, and if it cannot be found, a market data exception will be thrown. A missing past reset will *not* be projected from the curve when Manual First Reset is True.

If the past reset quote is found from the dataserver, the quote will *not* be displayed in the curve window Resets table. Currently the Resets table only displays quotes for the curve generation date. In future versions this quote will be made visible, but will not be editable.

Rate Projection

In all cases, when needed, the rate is projected from the curve being built, if it is the needed index curve. The latest guess for the curve at each iteration of the solver is used for projection.

Example of Past Reset

A fixing date that is prior to the curve generation curve date can occur if the curve is generated on a date that is not a business day for the fixing institution. For example, a USD Libor curve could be generated in New York on Easter Monday, which is a UK holiday but not a US holiday. The spot date for Libor swaps will be two London business days later, which is the following Wednesday. However, the reset day for Wednesday will be two London business days previous, which falls on the preceding Friday. Thus the fixing on Friday falls before the curve generation date on Monday. The curve will have to obtain the Friday fixing from the system quote database, or else throw an exception, if Manual First Reset is True. But the fixing will not be used if Manual First Reset is false – the past rate will be projected instead, which can be accomplished because the forward period for the fixing still falls after the curve generation date.

2.5 Spread Underlyings

Spread underlyings are curve underlying instruments whose quote is a spread to another curve underlying. The Spread underlying has a type that defines the instrument it creates for curve generation.

The most frequently used Spread underlying is a Swap whose rate is defined as a spread to a benchmark bond yield, such as a US Treasury bond. These are also referred to as "Treasury plus spread" or "Bond plus spread" swaps. There are two ways to obtain the bond yield, either from a quote or a benchmark bond zero curve. For risk purposes, the spread to a benchmark curve provides clearer sensitivities to the separate spread and bond components.

A Spread underlying can also be any of these types:

- Swap
- Money Market
- FRA
- Future

For example, a Money Market spread underlying represents a money market trade. Its properties, such as currency, maturity, and so on, are defined to be the same as another "reference" Money Market curve underlying which has already been entered into the system. Two quotes enter into the Spread instrument's rate: a quote for the Spread and a quote for the reference instrument. In this example, the total interest rate of the Money Market trade is the rate of the reference Money Market trade plus the quote for the Spread underlying.

It is also possible to use a quote for a different reference instrument than the one used to assign the properties of the Spread trade (currency, maturity, etc.). In this case, the properties of the reference quote instrument are ignored in curve generation, and only the value of the quote itself is used.

The handling of bond plus spread swaps are described in the next sections.

2.5.1 Swap Spread On Bond Quote (Bond Yield Plus Spread)

A Spread underlying of type "Swap Spread On Bond Quote" is a swap whose fixed rate is derived by adding a quoted spread to the yield of a specified bond. The quote for the Spread is in basis points. The attributes of the swap are defined on the Spread object and although the payment frequency, day count, and so on may differ from those of the bond, the market convention is that the quoted spread is not adjusted for those attributes. That is, one simply has:

Swap all-in fixed rate = Bond yield + Spread

without any further transformation of the rate according to day count, etc. By convention, market practitioners allow for these differing attributes when they provide a quote for the spread.

In curve generation the yield is calculated from a given bond price using the usual pricer for that bond (unless the yield is quoted directly, so that no calculation is needed). Then spread is added to the yield and a swap is created with that yield plus spread value as the fixed rate. In the BootStrap the procedure for finding a discount factor at the end of the swap is performed as described under "Multi-Period Instruments."

The bond can be a relative benchmark bond – for example, the 10Y on-the-run US Treasury -- or a specific issue. Two bonds can be associated with the Spread as well. For the case of two bonds, the all-in rate is found as follows:

1. The yield is found for each bond using the bond pricer. Call these y_A and y_B.

2. Identify the bond time intervals to be used for interpolation. Call these t_A and t_B. They are found as follows:
   - If both bonds are relative bonds, so that they are associated with benchmark tenors – for example, 5Y and 7Y – those tenors are used as the time intervals for interpolation. Note these tenors will usually not be exactly the same as the remaining time to maturity of the bonds.
   - If at least one of the bonds is a specific issue, use the *original* maturity of each bond, at the time of issue. Note again this will not be the same as the remaining time to maturity of the bonds.
3. Identify the Spread time interval, which is the length of the tenor of the Spread definition – for example, 8Y, or 7Y6M. Call this t_S.
4. Calculate the interpolated yield for the Spread's tenor using linear interpolation:

$$y = y_A + (y_B - y_A)\frac{t_S - t_A}{t_B - t_A}.$$

5. Calculate the all-in swap rate by adding the spread:

Swap all-in fixed rate = y + Spread

The reset rate parameters to a Spread underlying in the same fashion as for swaps.

2.5.2 Swap Spread On Bond Curve (Bond Implied Yield Plus Spread)

A Spread underlying of type "Swap Spread On Bond Curve" obtains its reference bond yield from a given benchmark bond curve, rather than a bond quote. Otherwise the calculations are the same as for the Swap Spread on Bond Quote. The advantage of this type is that it allows risk sensitivity reports to separate yield from spread risk in a transparent fashion.

The benchmark bond curve must be supplied in the curve definition. On the curve window, the Benchmark Curve is a field on the Definition tab. If the curve is not supplied, the generation cannot proceed.

Because the bond curve is given, there no reference bond quote will be used from the curve's Quotes list. By contrast, the other type, the Swap Spread on Bond Quote, does used a bond quote (one appears automatically in the curve window's Quotes table) and does not use the benchmark curve.

The bond benchmark curve is a zero curve of discount factors, not a bond par curve of par yields. It is expected to have been generated using bond quotes (either price or yield). During curve generation, the definition of the benchmark bond is used to solve for the par yield given the benchmark zero curve. The process is the same as would be done in a bond trade window to solve for the yield. First the dirty price of the bond is found by discounting the cashflows using the benchmark curve. Then the standard bond solver is employed to solve for the yield given this dirty price. In the curve generator, this yield is added to the quote for the Swap Spread on Bond Curve to obtain the swap's fixed rate. Generation then proceeds as for a normal swap.

If the reference bond is one of the bonds from which the benchmark curve was generated, then the yield solved from the benchmark curve should be the same as the input quote used for the benchmark generation. This is just the round-trip pricing guarantee for curve generation. Slight differences might arise through rounding effects. If the benchmark curve is not current, so that it had to be rolled without regeneration before used by the swap curve, then there may also be differences between the solved yield and the input quote.

Bond Yield and Spread Risk Sensitivities

In Calypso sensitivity reports, deltas are found by bumping the quote on each underlying in every curve used in pricing a portfolio. This process provides clear bond hedges when using the underlyings Swap Spread on Bond Curve, because the benchmark bond curve is one of the curves that will appear in the report.

Sensitivity calculations proceed in this way. Each bond quote will be perturbed, the benchmark bond curve regenerated, and the result used in the swap curve. The altered yield of the bond flows through to the swap rates used in curve generation, and the resulting curve then gives a change in the portfolio NPV when pricing. Thus bond deltas are then evident.

As for the swap curve, the Swap Spread quotes are then bumped in turn while the benchmark curve is held constant, resulting in the effect on NPV when just the spread changes but not the bond yield.

The risk report then shows separate entries for each bond and swap quote in the two curves.

Section 3. Generation Algorithms

The algorithms for generating curves in Calypso make use of the trade pricers defined in the system. A direct successive-approximations approach is taken: a curve is found that prices the trades back to their market values when using the pricers. Calypso does not use approximations or simplifications for producing curves.

3.1 List of Generators

The following generators are available. They will be discussed in the following sections.

- BootStrap
- BasisBootStrap
- Global
- BasisGlobal
- BootStrapForwards
- BootStrapISDA
- FX Derived

3.2 General Procedure

The following steps summarize the generation procedure.

1. **Create the Trades.**

   The generator creates a trade from each underlying instrument in the curve. The trade date is set equal to the curve date.

2. **Apply the Quotes.**

   The market quotes are applied to the trade. For example, for a vanilla swap the fixed coupon is set equal to the swap quote.

3. **Obtain the Pricers.**

   For each trade obtain a Pricer. This will usually be a standard Calypso Pricer, such as PricerSwap, although it can be overridden using the pricing environment's pricer configuration.

4. **Make a Trial Curve.**

   Make a trial guess for the curve.

5. **Price the Trades.**

   Price the trade(s) as of the curve date with the trial curve, using the trade's Pricer and the curve's pricing environment.

6. **Improve the Trial Curve.**

   Use the result to improve the trial guess of the curve.

7. **Iterate.**

   Repeat Steps 5 and 6 until the price using the curve is sufficiently close to the market price, or until an iteration limit is reached.

The two generators **BootStrap** and **Global** differ in how they perform the sequence of trial curve construction and pricing.

The **BootStrap** generator treats each trade one at a time, making a trial curve of sufficient length to price the trade and iterating until a satisfactory solution is found, then adding in the trade of next longer maturity and repeating the process.

The **Global** generator first makes a trial curve out to the longest maturity trade then prices all the trades with that trial. It updates the trial guess based on the result and then prices all of the trades again.

Section 4. BootStrap Generator

The Bootstrap Generator generates a zero curve one instrument at a time, in order of time to maturity, calibrating the curve successively to each instrument.

Typical user-selected underlying instruments are Money Market, FRA, Future, Swap, Bond Plus Spread, Bond. The algorithm supports future convexity correction (manual spreads or using correlation-driven adjustments) and turn rate adjustments.

Generator parameters govern the use of underlyings and quotes, and the choice of variations in the generation procedure. The parameters for the BootStrap generator are described in the section on the Global Generator Parameters.

4.1 The BootStrap Procedure

Calypso orders the instruments, first adding single-period rate instruments and then multi-period "priced" instruments; single-period instruments with forward end dates falling on or after the maturity of any multi-period instrument are excluded from the bootstrapping procedure.

1. Identify the end date of each instrument. This is the last date for which it would require discount factors from a curve. For example, for a swap the end date is the later of the last cashflow payment date and the last forward period end date (the last date on which the last rate reset depends).
2. Sort the instruments in order from earliest end date to longest end date.
3. Separate the "single-period instruments" money market, futures, and FRA instruments from the rest. For each of these instruments, find their start and end dates, and calculate the discount factor between these dates that reproduces the market quote. This creates a set of forward discount factors.
4. Combine the forward discount factors found in Step 3 by handling any gaps or overlaps between the instrument periods. The section "Gaps and Overlaps of Single-Period Instruments" (see below) describes the handling of these periods in detail.
5. This creates a curve out to the last date of the simple single-period instruments. Now handle the longer swaps and bonds. These are sorted in order from shortest to longest maturity.
6. Starting with the shortest instrument in the list, extend the curve that has been constructed so far by adding a point at that instrument's end date. *The end date of an instrument is the last date that affects its value, which will either be the instrument maturity date or the last forward period end date, whichever is later*. Set a trial value for the discount factor on that date.
7. Calculate the NPV of the instrument with the trial curve, using the Calypso Pricer for that trade. If a Discount curve has been specified, then the trial curve is used to project the forward rates of the trade only; otherwise the trial curve is used also for discounting.
8. Compare the NPV to the market value. For swaps fair market value is zero. Using the difference between the NPV and the market value, make an improved guess of the trial discount factor.
9. Repeat Steps 7 and 8 until termination criteria for the solving algorithm are found. The goal is to price a swap to less than 0.01 NPV per 1,000,000 notional.
10. Proceed to the next longer instrument on the curve. Extend the curve that has been constructed so far by adding a point at that instrument's end date. Set a trial value for the discount factor on that date.
11. Repeat steps 7 through 10 until the instrument list is completed.

4.2 Solver for BootStrap

The algorithm used for solving for zero NPV is a Brent root-finder. Any similar solver can in principle be used; Calypso's version has proved a robust method that balances accuracy and performance.

Sometimes during generation the user will receive a message that a solution does not exist. This is *not* an error message, it is a statement of fact: the solver has determined there is no mathematical solution for the given inputs. The user should re-examine the setup for errors in quotes or instrument definition. In the majority of cases this is due to bad quotes: these can easily be inconsistent if they do not come from actual market quotes.

4.3 Spline Calibration in BootStrap

It is important to note that when using a spline interpolator the procedure as described will fail to calibrate to all instruments simultaneously. This is because the interpolator is non-local. Even though the shorter swaps may be calibrated, when longer swaps are added their values will alter the spline that is drawn throughout the curve, and so change the discount factors on intermediate cashflow points of the swaps (those not on the swap end date and so not represented by points on the curve). So the longer swaps will destroy the calibration of the shorter swaps.

To eliminate this problem of the BootStrap method one must set the generator parameter **"Generate on all flow points"** to be True. Then after each swap is calibrated a point is placed on the curve at every cashflow point and every forward period star and end date. This fixes the curve at those points so that later swaps cannot alter it.

4.4 Gaps and Overlaps of Single-Period Instruments

Each single-period instrument produces a "forward interval" consisting of two dates; the interval's discount factor between its start and end date is found from the market rate, producing a forward rate for this interval. When all the single-period instruments have been computed in this fashion the result is a set of forward intervals which can be arbitrarily related to each other in terms of gaps and overlaps.

These intervals are sorted by start date and secondarily by end date (if start dates are equal then the end dates are considered; if end dates are also equal, the earlier instrument in the curve specification list is given precedence); then the intervals are processed one by one. This checks the relation of each successive interval with the next earlier one and applies an appropriate method to eliminate any gap or overlap between them. When completed, the result is a set of contiguous, non-overlapping forward intervals, each one having a forward rate across its interval, which are then used to compute a zero curve.

The following is a list of the cases and how they are handled in the BootstrapGenerator. The earlier of the two intervals to be compared will be called Interval A, the later will be Interval B. Sorting guarantees that the start and end dates of Interval A are both less than or equal to the corresponding dates of Interval B. The discount factor between two dates t1 and t2 will be designated F(t1, t2), and the start date of Interval A is denoted A1, the end date A2; thus F(A1,A2) and F(B1, B2) are known inputs to the procedure.

<u>First interval added to the curve</u>. If there is a gap between the Interval A's start date and the curve's valuation (start) date, a new interval to cover the gap is created and the continuous ACT/365 forward rate of the first interval is applied to it. Thus this uses "pull-back constant forward interpolation."

<u>Interval B has start and end dates both the same as Interval A</u>. Interval B is discarded. The next interval after it will be compared to Interval A. (Note then that the resulting curve will probably not reproduce the input price of the instrument correctly that associated to the discarded interval.)

<u>Interval B has the same start date as Interval A.</u>

←-------A--------→←New→

←-------------B--------------→

Remove Interval B and insert an interval between the end date of Interval A and the end date of Interval B (the previous sorting and cases guarantee a positive time interval). The rate over the new interval is determined by standard arbitrage to retain both forward rates, F(A1, A2)*F(A2,B2) = F(B1, B2). This preserves the pricing of both instruments. (The picture above shows both the original A and B intervals and the newly created interval; after this step, the intervals left for curve building are the contiguous, non-overlapping intervals A and New.)

Interval B has the same end date as Interval A.

←-------------A---------------→

←New→←-------B---------→

Remove Interval A. Add a new interval from the start date of Interval A to the start date of Interval B (sorting and previous cases guarantee a positive time interval). The discount factor over the new interval is determined to preserve both forward rates: F(A1,B1)*F(B1,B2) = F(A1, A2).

Interval B is wholly contained within Interval A.

←-----------------A------------------→

←New→←------B------→←New→

Remove Interval A. Create two new intervals from the start of A to the start of B, and from the end of B to the end of A. Set the forward rate over the two new intervals to be equal, and solve for this rate so as to preserve the forward rates over A and B: F(A1,B1)*F(B1,B2)*F(B2,A2) = F(A1,A2).

Interval B overlaps the end of Interval A.

←-------------A---------------→

←--------------B--------------→

←---New----→←----New--→←---New----→

Remove both Intervals A and B. Replace with three new intervals, covering A1 to B1, B1 to A2, and A2 to B2. To do this there is a different procedure from the other methods, in that first Interval A is added to the earlier intervals and a temporary zero curve is generated out to Interval A's end date, A2. Using this temporary curve the discount factors from the curve's start date to the interval start dates A1 and B1 are found, i.e., F(0,A1), F(0, B1). Then the temporary curve is discarded.

The discount factors over the new intervals, F(A1,B1), F(B1,A2) , and F(B1,A2) intervals are determined by arbitrage: F(0,A1)*F(A1,B1)=F(0,B1); F(A1,B1)*F(B1,A2) = F(A1,A2); and F(B1,A2)*F(A2,B2) = F(B1,B2). Notice that this preserves the forward rates on Intervals A and B; the temporary curve only serves to provide a method for introducing a forward rate on the partial interval from A1 to B1.

Interval B starts after the end date of Interval A.

←----------A---------→←New→←-------B---------→

This is a gap. Create a new interval to fill the gap from A2 to B1. The forward rate over the new interval is set equal to the forward rate over Interval A. Thus this is a "pull forward constant forward rate extrapolation" method.

Summary

Note that all of these methods preserve the forward rates over existing intervals, and do not necessarily preserve the zero rates of the curve built so far. Adding instruments to a curve one by one you have the alternative of finding discount factors using the curve's interpolation method rather than preserving forward rates over intervals. Calypso's methods preserve the forward rates, thus the existing curve is changed, not merely extended, when each successive single-period instrument is added.

Example of Gap Handling for a Single Money Market Underlying

As an example of gaps, consider a simple case where the curve is built with only one money market instrument, a 1M Libor with quoted rate of 3.00%. As will be seen, this creates *two* points on the curve, one each at the start and the end date of the interest rate period.

The generation goes through the following steps:

1. One notes the rate applies to a deposit that begins on the Spot Date, which is the Curve Date plus two business days. The deposit ends on the Spot Date plus 1M. The generator calculates these dates and regards the 3.00% as a forward rate for the interval (Spot Date, Spot Date + 1M).

2. There is now a gap between the Curve Date and the Spot Date. This is the situation described above under First interval added to the curve. To find a rate for the interval between these dates extrapolation must be used. In this case, the steps are:

a) Convert the simple-interest ACT/360 Libor rate to a continuous ACT/365 rate. The formula used is:

$$1 + 0.03\left(\frac{T}{360}\right) = \exp\left(R\frac{T}{365}\right)$$

where T is the number of actual days in the 1M deposit. This formula is solved for the continuous rate R.

b) Assign the rate R to the interval between the Curve Date and the Spot Date.

3. Find the discount factor at the Spot Date using the forward rate between the Curve Date and the Spot Date. This is:

$$DF(CurveDate, SpotDate) = \exp\left(-R\frac{S}{365}\right)$$

where S is the number of calendar days to the Spot Date; S = 2 if there are no holidays or weekends. This point is added to the curvev.

4. Find the discount factor on Spot Date + 1M using the discount factor on the Spot Date and the forward rate on the interval (Spot Date, Spot Date + 1M). This happens to be the same rate R in this case, because of the extrapolation that had to be performed. Thus:

$$DF(CurveDate, SpotDate + 1M) = DF(CurveDate, SpotDate)DF(SpotDate, SpotDate + 1M)$$
$$= \exp\left(-R\frac{S}{365}\right)\exp\left(-R\frac{T}{365}\right).$$

This is the second point added to the curve. This gap method has simply applied the 1M rate over the entire period from Curve Date to the maturity date.

Note that the resulting curve has two points, one at Spot Date and one at Spot Date + 1M, and is not the same as would be produced by having only a single point at Spot Date + 1M and none at Spot. The reason is that differing interpolation schemes would result in different discount factors at the Spot Date, which would result in changes to the interest rate between Spot and Spot + 1M; as a consequence the 1M money market would not be guaranteed to price back to the input quote.

4.5 BasisBootStrap Generator

The BasisBootStrap generator makes use of other curves besides a discount curve so that basis swaps (floating versus floating swaps) can be priced.

A single-currency basis swaps requires two curves for projecting forward rates and one discount curve. The curve that is being generated is called the *basis curve* and is used to forecast the leg of the swap whose rate index corresponds to the Rate Index and Tenor settings in the curve definition. The other leg of the swap, the one with a Rate Index and Tenor different from the curve being generated, has its forward rates forecast from the *base curve* specified in the curve definition. The legs are termed the *basis leg* and *base leg,* respectively. If there is no leg of the Curve Underlying that matches the generating curve's Rate Index and Tenor then the side of the underlying termed the 'Basis' leg is assigned to the generating curve. Otherwise the names Base and Basis on the Curve Underlying have no significance, as either leg can be used as either base or basis leg depending on which curve is being generated (see note below on version change).

Thus the curves used in generating with a single-currency basis swap are as follows:

Non-generating leg (base leg, opposite the generated curve leg):

- Discount curve: discount curve if specified, else base curve
- Forecast curve: base curve

Generating leg (basis leg):

- Discount curve: discount curve if specified, else base curve
- Forecast curve: basis curve (generated curve)

The basis curve is solved for in this manner:

1. Find the NPV of the base leg using the base curve. This is the target NPV.

2. Choose a trial discount factor. If the curve type is CurveZero, add this trial discount factor to the curve at the basis leg's end date. If the type is CurveBasis convert the factor to a trial zero rate spread before including it in the basis curve. Use this amended basis curve to forecast the basis leg forward rates. Use this curve and the base and discount curves to price the basis swap.

3. Repeat Step 2, varying the trial spread until the NPV of the basis leg matches the target NPV:

NPV (generating leg) = NPV (non-generating leg)

The spread or discount factor that causes this to be true becomes the point that extends the basis curve.

4.6 BasisBootStrap: Cross-currency Curves

Cross-currency curve generation differs from that of single-currency curves in two ways. First, as the legs are in different currencies, two more curves need to be specified in order to provide the forward rates and discount factors in the currency opposite to the curve being generated. Second, the curve being generated is a discount curve rather than a forward curve as it was in the single-currency case. A forward curve in that currency therefore needs to be specified also.

4.6.1 Generation from cross-currency basis swaps

A single-currency basis swaps requires two curves for projecting forward rates and one discount curve. The curve that is being generated is called the *cross-currency curve* and is used to discount the swap leg whose currency matches the curve definition, termed the *domestic currency*. That leg has its forward rates forecast with the *base domestic curve*. The other leg of the swap is priced with the foreign discount curve and the foreign forward curve. If the foreign discount curve is not specified then the foreign forward curve will be used for foreign discounting. On the Curve Underlying the names Base and Basis have no significance for the cross-currency swaps (see note below on version change).

The curves used in generating with a cross-currency basis swap are as follows:

Domestic leg (currency the same as the generated curve):

- Discount curve: cross-currency curve (generated curve)
- Forecast curve: base domestic curve

Foreign leg:

- Discount curve: foreign discount curve if specified, else foreign forward curve
- Forecast curve: foreign forward curve

The basis curve is solved for in this manner:

1. Find the NPV of the foreign leg using the foreign curves. Convert this to domestic currency at the FX spot rate. This is the target NPV.

2. Forecast the forward rates on the domestic leg using the domestic base curve.

3. Choose a trial discount factor. If the curve type is CurveZero, add this trial discount factor to the trial curve at the domestic swap leg's end date. If the type is CurveBasis convert the factor to a trial zero rate spread before including it in the curve. Use this amended basis curve to discount the domestic leg.

4. Repeat Step 3, varying the trial spread until the NPV of the basis leg matches the target NPV. The spread that causes this to be true becomes the zero spread that extends the basis curve.

4.6.2 Principal Exchange on Cross-Currency Basis Swaps.

It is important to select the correct Principal Exchange type on the cross-currency basis swap Curve Underlying. Otherwise the curve generation is likely to fail.

All standard cross-currency swaps have at least a **Final Exchange** of principal to reduce FX risk. Most major currencies also have periodic **Mark-to-Market Exchanges** of principal which occur with each swap interest payment, which resets the notional of one of the legs to match the prevailing FX rate. The leg whose notional is to be reset needs to be specified on the Curve Underlying definition.

4.6.3 Cross-Currency Curve Generation Parameters

When building a cross-currency curve using either BasisBootStrap or BasisGlobal, there are two parameters that affect the generation.

FX Forward Quote Type

Allowed values:

- Points
- All-In Forward

Specifies whether the FX Forward underlyings are quoted by Forward Points or All-In Forward Rate. They are related by

$$\textit{All-In Forward Rate} = \textit{Spot Rate} + \textit{Forward Points}$$

The values on the Quote tab of the curve window are interpreted accordingly. Forward Points quotation is most common in FX rates spanning the US, UK, Europe, Japan, Canada, and Australia, while in other countries the All-In Forward Rate quotes may be more frequently found.

Interpolate FX Curve

Allowed values: True/False

When set to "True", discount factors will be computed using interest rate parity for each day covered by any FX Forward underlyings which usually is no more than a year. These daily points will be created at the time of curve generation and stored, rather than interpolated on the fly, to maximize speed of curve lookup during pricing. In the curve window, the user will see these points for every day out to the end of the FX Forwards.

All other parameters not specific to cross-currency are described in the section on the Global Generator Parameters.

4.6.4 FX Forwards and Interest Rate Parity

FX Forwards can be used in cross-currency curve generation. The quotes are interpreted as FX Swap points that are added to an FX spot rate to find the all-in forward rate . The generation over mixed instruments proceeds by first dealing with the FX forwards, then any money market and futures underlyings, and then the basis swaps.

Generation with the FX uses interest-rate parity to combine the foreign discount curve with the FX forward quotes to produce a discount curve in the domestic currency. This curve is afterwards extended using the remaining non-FX-Forward underlyings.

Interest-rate parity defines a relationship among the discount factors of the two curves, the FX Spot rate, and the FX Forward rates.

The formula is straightforward if one is not using Bid and Ask rates. Many market participants prefer to calculate at the Mid rates and then add on a spread. This is a more stable calculation than one in which the Bid/Ask spreads of different curves and FX rates are all used, which can cause a combination of spreads that is too wide compared to the market. In addition, the no-arbitrage arguments for Bid and Ask rates can only produce inequalities, leaving an ambiguity in the curve definition. Attempts to resolve the ambiguity can result in crossed markets, that is, where either the interest rates have Bid rates greater than Ask rates.

The MID forward rate for foreign exchange taking place on a date T (“for Value T”) is projected from two interest rate curves using these formulas:

- If T >= T_{spot} (forward projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_B(T_{SPOT}, T)^{MID}}{D_Q(T_{SPOT}, T)^{MID}}, \quad T > T_{SPOT}$$

- If T < T_{spot} (“backward” projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_Q(T, T_{SPOT})^{MID}}{D_B(T, T_{SPOT})^{MID}}, \quad T < T_{SPOT}$$

The quantities are defined as follows:

- R_{spot} – The spot FX rate defined for settling FX deals on spot date T_{spot}.
- $R_{fwd}(T)$ – The forward FX rate on date T.
- D_Q (T1, T2) – The discount factor for discounting quoting currency from any date T2 to earlier date T1.
- D_B (T1, T2) – The discount factor for discounting base (primary) currency from any date T2 to earlier date T1.

For a derivation of the equations and a thorough explanation of the problems with Bid/Ask spreads, please see the Calypso White Paper "Relationships among FX Forward Rates and Interest Rates."

4.6.5 Cross-Currency Forward Curve Generators

When using cross-currency swaps, the BasisBootStrap generator creates a discount curve. It is possible to generate a forward curve instead of a discount curve. This requires a different choice of generator. The choices are:

- **XccyForwardBootStrap**
- **XccyForwardGlobal**

If one of these is selected, then the generation arrangement is as follows.

- **"Base Forward" curve in the domestic currency**, used to ***discount*** cashflows on the domestic leg of the swap
- **"Foreign" curve in the foreign currency,** used to project forward rates on the foreign leg of the swap.
- **Discount curve in the foreign currency**, used to discount cashflows on the foreign leg of the swap

  If the Discount curve is not specified, then the Foreign curve will be used as the Discount curve
- **The forward curve in domestic** currency is being solved, and is used to ***project forward rates*** on the domestic leg of the swap.

4.7 Shortcomings of BootStrap

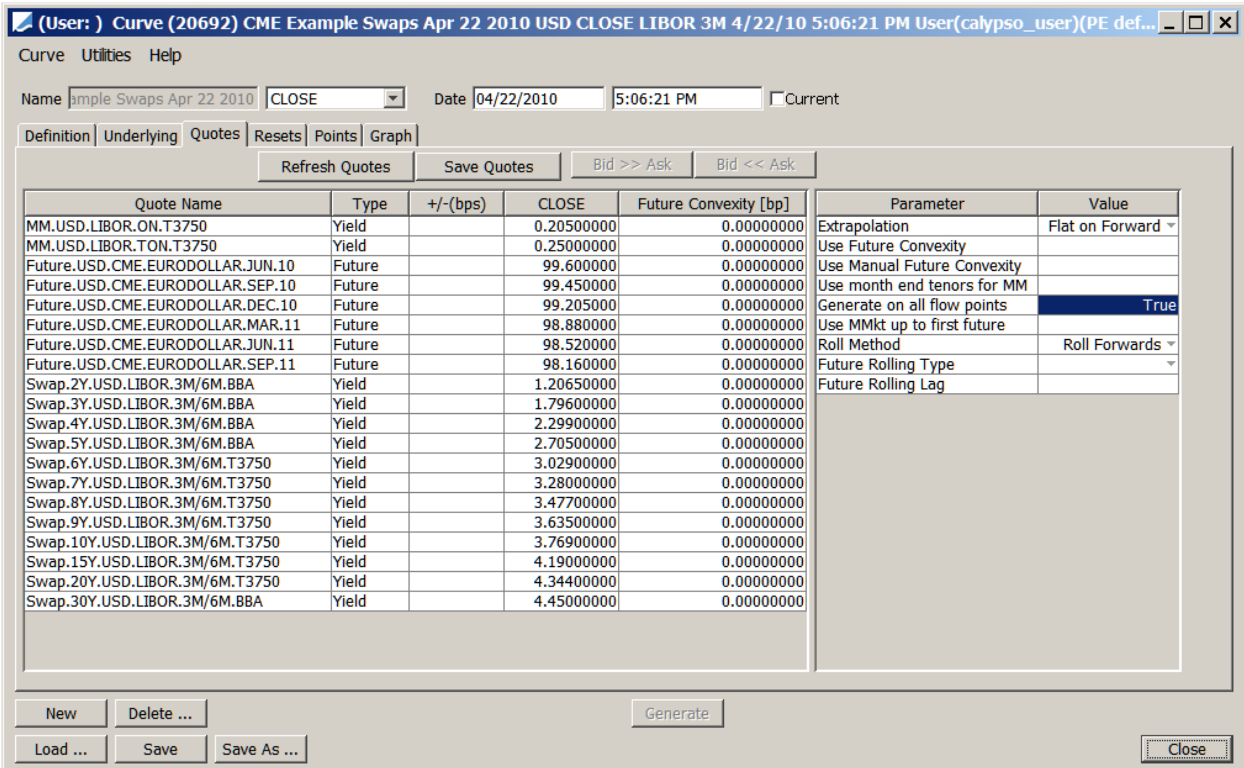
The BootStrap generator guarantees a curve that prices the instruments to the market quotes. In doing so, however, it does not impose any requirements on the smoothness of the forward rates.

The first type of non-smoothness is that combining of money markets, futures, and FRAs using gap and overlap methods during the bootstrap is not designed to create smooth forward rates at the joins. As a result, often spikes or cusps in the forward rates are created instead.

The second type comes from non-local interpolators such as a cubic spline or monotone convex. These are joined together in a non-smooth as the generation proceeds along the curve toward longer instruments. This often creates unreasonable behavior in the forwards across the swaps. One can obtain smoother forwards for the spline by setting the parameter “Generate on all flow points” to False, but as a result the calibration will be poorer for some instruments.

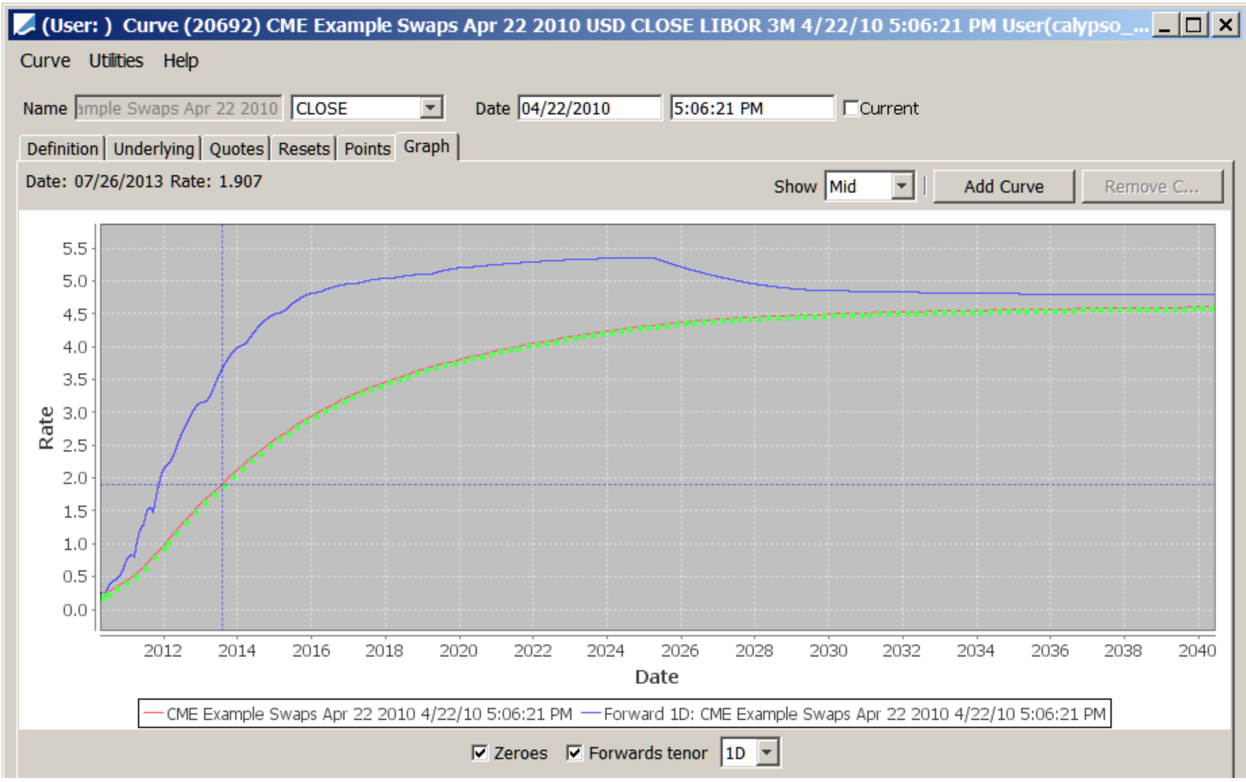
The forward rates can be viewed on the Graph tab of the curve window. In the following screenshots the underlying instruments, quotes, and resulting points are shown for a typical USD curve. This was created with the BootStrap generator with spline interpolation on the log of discount factors. The graph shows the zero rates and the 1-day forward rates.

To overcome these limitations the **Global** generator was created to provide smooth forward rates.

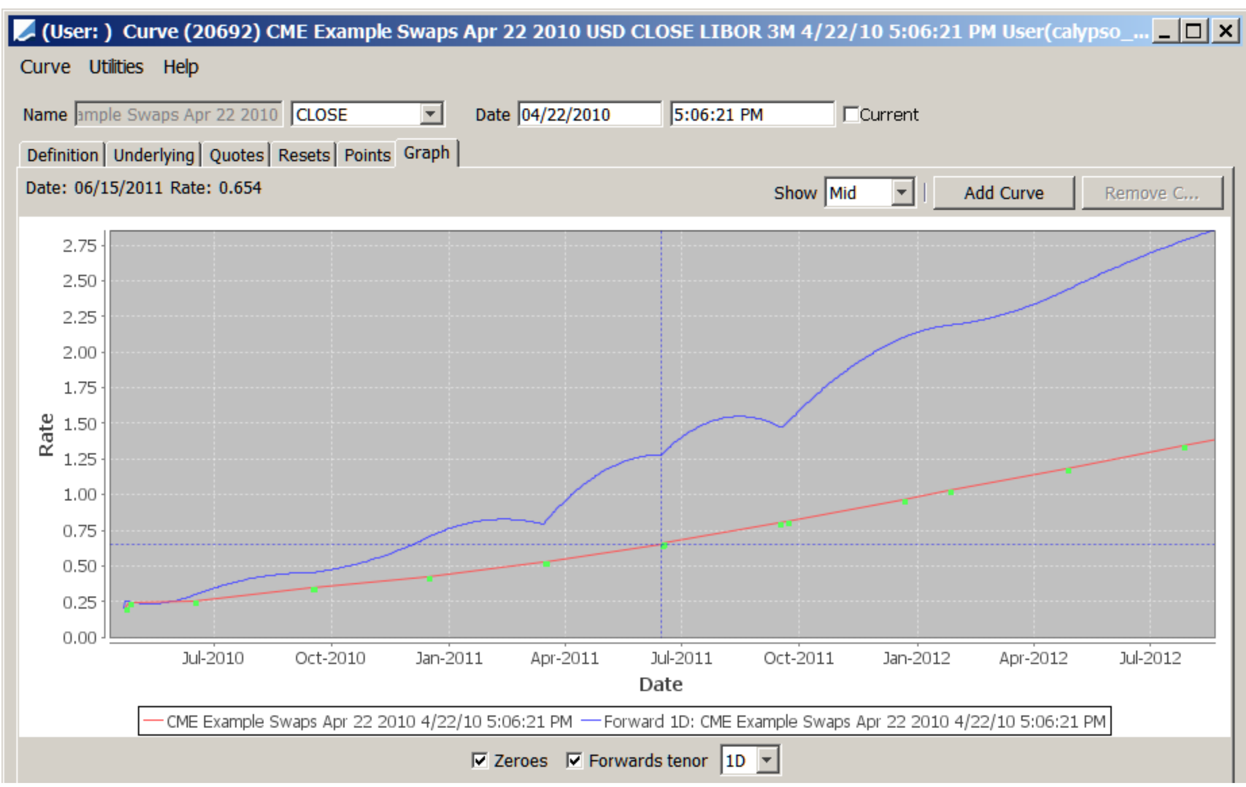


Quote Name	Type	+/-(bps)	CLOSE	Future Convexity [bp]
MM.USD.LIBOR.ON.T3750	Yield		0.20500000	0.00000000
MM.USD.LIBOR.TON.T3750	Yield		0.25000000	0.00000000
Future.USD.CME.EURODOLLAR.JUN.10	Future		99.600000	0.00000000
Future.USD.CME.EURODOLLAR.SEP.10	Future		99.450000	0.00000000
Future.USD.CME.EURODOLLAR.DEC.10	Future		99.205000	0.00000000
Future.USD.CME.EURODOLLAR.MAR.11	Future		98.880000	0.00000000
Future.USD.CME.EURODOLLAR.JUN.11	Future		98.520000	0.00000000
Future.USD.CME.EURODOLLAR.SEP.11	Future		98.160000	0.00000000
Swap.2Y.USD.LIBOR.3M/6M.BBA	Yield		1.20650000	0.00000000
Swap.3Y.USD.LIBOR.3M/6M.BBA	Yield		1.79600000	0.00000000
Swap.4Y.USD.LIBOR.3M/6M.BBA	Yield		2.29900000	0.00000000
Swap.5Y.USD.LIBOR.3M/6M.BBA	Yield		2.70500000	0.00000000
Swap.6Y.USD.LIBOR.3M/6M.T3750	Yield		3.02900000	0.00000000
Swap.7Y.USD.LIBOR.3M/6M.T3750	Yield		3.28000000	0.00000000
Swap.8Y.USD.LIBOR.3M/6M.T3750	Yield		3.47700000	0.00000000
Swap.9Y.USD.LIBOR.3M/6M.T3750	Yield		3.63500000	0.00000000
Swap.10Y.USD.LIBOR.3M/6M.T3750	Yield		3.76900000	0.00000000
Swap.15Y.USD.LIBOR.3M/6M.T3750	Yield		4.19000000	0.00000000
Swap.20Y.USD.LIBOR.3M/6M.T3750	Yield		4.34400000	0.00000000
Swap.30Y.USD.LIBOR.3M/6M.BBA	Yield		4.45000000	0.00000000

Parameter	Value
Extrapolation	Flat on Forward
Use Future Convexity	
Use Manual Future Convexity	
Use month end tenors for MM	
Generate on all flow points	True
Use MMkt up to first future	
Roll Method	Roll Forwards
Future Rolling Type	
Future Rolling Lag	



The following is a magnified view of the futures portion of the curve. The oscillations in the forward rates is due to the handling of gaps and overlaps between underlying futures deposit periods.



4.8 BootStrap with Linear Forwards: BootStrapForwards

4.8.1 Linearity Requirement for Forward Rates

In generating zero curves, the standard Calypso bootstrapping algorithm employs market-quoted swaps by solving for the zero rate on the swap maturity date which produces zero net present value for the swap. In doing this calculation, any discount factors and zero rates required for pricing the swap are found by interpolation. The user can choose to interpolate on either zero rates or on discount factors by specifying the environment parameter INT_CURVE_INTERP_RATE_B.

If the user wishes instead to interpolate on forward rates, another bootstrapping algorithm is available. The main feature of this algorithm is **that forward rates used by the swap are required to fall along a line**. This is not quite the same as forcing the curve to be piecewise linear in forward rates, for two reasons:

- Only the actual forward rates needed by the swap fall on a line, not every forward rate within the swap's lifetime.
- Only swaps and basis swaps implement this requirement; cash and futures are solved for as in the standard zero-rate bootstrap.

To employ this bootstrapping method, select **BootStrapForwards** in the generation algorithm field of the CurveZero application.

4.8.2 The BootStrapForwards Generation Procedure

The BootStrapForwards extends a zero curve with a swap using the following procedure:

1. A forward rate for the last floating-rate cashflow of the swap is guessed as a trial rate.
2. For every floating-rate cashflow in the swap, a forward rate is **linearly interpolated** between the last known forward rate from the zero curve and the trial rate. Linear interpolation is used for forwards regardless of the interpolation method defined in the curve application.
3. From the interpolated forward rates, discount factors are found on the start and end of the forward rate periods. In case of gaps, extrapolation across gaps is done using
4. To find the discount factors on all cashflow payment dates, where they differ from forward start and end dates, the interpolation method defined in the curve application is used. The two most commonly used methods are *linear zero rate interpolation* or *compound daily forward interpolation*.
5. Find the NPV of the swap using these discount factors. Adjust the trial forward rate until this NPV is zero.
6. Finally, all of the discount factors used in pricing the swap are stored in the curve. This differs from the zero rate bootstrapping, which only adds one point for each swap. The BootStrapForwards algorithm thus produces curves with many more points on them than the BootStrap algorithm. As a result, it is possible for a user to change interpolation methods after curve generation while still maintaining the swap NPV at zero.

4.8.3 The Algorithm in Detail

In this section, the generation will be described for a curve constructed from money market instruments to 1Y, then two semi-annual swaps of maturity 2Y and 3Y.

<u>Notation:</u>

$i = 0, 1,\ldots, 5$: label for the 6 cashflows of the swaps to 3Y.

FS_i, FE_i : forward period start and end dates for the ith cashflow

fw_i : forward rate of ith cashflow, thus for the interval (FS_i, FE_i)

PD_i : payment date for the ith cashflow

$D(t)$: discount factor for date t

Note that most of the time, but not always, $FE_i = PD_i$.

1. Start with the 1Y money market curve. Add the 2Y swap.

2. Guess fw_3. This is the last needed forward rate of the 2Y swap.

3. The first reset fw_0 of the swap is known from the 1Y curve, but fw_1 may not be known if PD_1 or FE_1 falls after the last date of curve. In this case use *extrapolation* (zero rate or compound daily forward) from the discount factors (not the forwards) of the curve to find the discount factor on PD_1 and/or FE_1 as needed

1Y swap is added as an underlying to determine fw1 so that the 1Y swap is priced par.

4. Draw a line between the known rate fw_1 and the guessed rate fw_3. Read off fw_2 from this line. Thus fw_1, fw_2 and fw_3 lie on a line. This satisfies the piecewise linear forward requirement.

5. Using fw2 and fw3, calculate discount factors on the forward rate start and end dates FS_2, FE_2, FS_3, and FE_3. For example,

$$D(FE_2) = \frac{D(FS_2)}{1 + fw_2 * (FE_2 - FS_2)}.$$

In this equation, if $D(FS_2)$ is unknown, interpolate it from existing known discount factors (zero rate interpolation or compound daily forward).

6. Using the discount factors computed in Step 5 along with the yield curve, interpolate the discount factors on the payment dates PD_2 and PD_3. In the infrequent case where $PD_3 > FE_3$, there will be extrapolation rather than interpolation.

7. Price the 2Y swap, and repeat the guess of fw_3 until the NPV of the swap is zero. The yield curve has now been extended to the later of FE_3 and PD_3.

Now add the 3Y swap:

8. Guess fw_5.

9. Draw a line between the previously solved fw_3 and the guessed fw_5. Read off fw_4 from this line. Thus fw_3, fw_4, and fw_5 lie on a line. This satisfies the piecewise linear forward requirement.

10. Using fw_3, fw_4, and fw_5, calculate discount factors on FS_4, FE_4, FS_5, and FE_5, using the same type of equation as in Step 5.

11. Using the discount factors computed in Step 10 along with the yield curve, interpolate the discount factors on the payment dates PD_4 and PD_5 (zero rate or compound daily forward interpolation). Again, in the rare case where $PD_5 > FE_5$, there will be extrapolation rather than interpolation.

12. Price the 3Y swap, and repeat the guess of fw_5 until the NPV of the swap is zero. The yield curve has now been extended to the later of FE_5 and PD_5.

Thus the two requirements -- linearity of forward rates and NPV of the swaps being zero -- are satisfied.

4.8.4 Compound Daily Forward Interpolation (Linear Log Discount Factor Interpolation)

The Compound Daily Forward interpolation method is the preferred interpolation method for use with the Bootstrap Forwards generator. It finds discount factors on a date between two dates with known discount factors by assuming the forward rate between the known dates is constant. It is identical to the LogLinear interpolation on target Discount Factors, the only difference being that the one does not have to specify the target type for the Compound Daily Forward. For more information, see the section "Interpolation Methods."

4.9 BootStrapISDA

BootStrapISDA is a simple generator used as the conventional curve in converting between premium and quoted coupons for fixed-coupon credit default swaps. It only uses money market deposits and vanilla swaps. For further information please refer to the Calypso credit derivatives guides.

4.10 FX Derived

The FX Derived generator converts a discount curve to a discount curve in another currency by means of an FX Curve containing FX Forward points. The generator can only be used in the dedicated FX Derived Curve Window. The algorithm is the same interest rate parity equation as used in the cross-currency Basis BootStrap generator.

The generator is most useful in situations where the discount and forward curves are the same, for the generator only uses discount curves. As the FX Derived generator does not make use of forward curves it cannot price cross-currency basis swaps correctly. In prior versions of Calypso basis swaps could be used in the generator, but this is no longer possible, as it is expected forward curves are needed.

It is recommended that the BasisBootStrap or BasisGlobal generators be used instead to build cross-currency curves that mix FX Forwards with swap instruments.

Section 5. Global Generator

5.1 Global Generator

The Global generator overcomes the weaknesses in the BootStrap generator in terms of smooth forwards. It also provides a more flexible method that allows special features to be added to the curves, such as turn rates and central bank meeting dates, that are not available with the BootStrap generator.

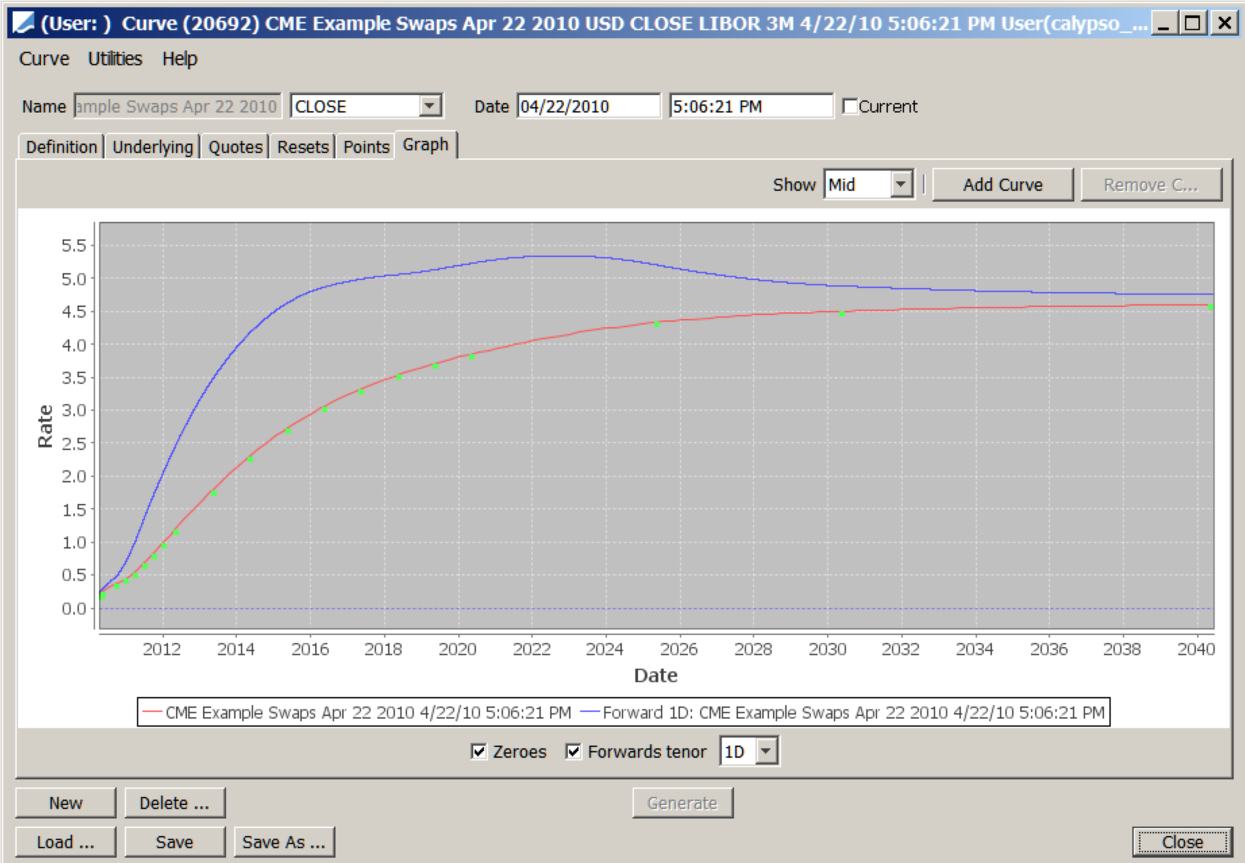
The Global generator begins in the same fashion as the BootStrap generator but then iterates the procedure across the whole curve, repeating the calibration until agreement with market quotes are obtained while simultaneously guaranteeing smoothness. Smoothness is enforced because each trial curve is smooth by construction, according to the chosen interpolator. The interpolator choices that provide the smoothest forward rates are:

- **InterpolatorLogSpline on Discount Factors** (cubic spline on the log of discount factors)
- **InterpolatorMonotoneConvex** (this is always on discount factors regardless of user choice of target)

The Global generator solves the curve with the following steps.

1. Create a trade and pricer for each instrument, as described above.
2. Create a trial guess of the entire curve using the BootStrap generator. Approximations will be used for faster generation of this step, as the guess need not be precise. The curve will extend to the end date of the longest-maturity instrument.
3. Price every trade using the trial curve. This calculated quantity is usually NPV but it can vary depending on the instrument type. As with the BootStrap generator, if a Discount curve has been specified then the trial curve is used only for forward rate projection; otherwise the trial curve is used for discounting also.
4. Compare the calculated price with the target price of each trade. Using a multidimensional optimization algorithm, use these comparisons to produce an improved trial curve.
5. Repeat Steps 3 and 4 until one of the termination criteria of the optimization algorithm is met.

Compare this result to that shown in the section “Shortcomings of BootStrap.” Applying the Global generator to the same curve example pictured in the BootStrap description one sees the forwards are smooth with no unnatural dips or cusps:



5.1.1 Multidimensional Optimization Algorithm

The Global generator does not depend critically on the choice of numerical algorithm used for the optimization search procedure. That algorithm is a generic method that can be swapped out of the generator construction. Currently Calypso employs the Levenberg-Marquardt algorithm.

The algorithm will halt when the curve cannot be improved in a reasonable amount of time. This prevents the generation from going on indefinitely, but it could result in a solution that is not as optimal as possible; this depends on the self-consistency of the input market quotes.

5.1.2 Validation and Accuracy

Automatic Calibration Check

To automatically validate and view the calibration accuracy, please use the Log category "**GenGlobal**". To do so, select Main Entry > Utilities > Maintenance > Log > Configure Log, and type in "GenGlobal". Then generate a curve in a curve window. Afterward, select Main Entry > Utilities > Maintenance > Log > Show Log and look for the GenGlobal entry, and click on the entry that begins with the words "Calibration Check." This will show the difference of the NPV and breakeven rates (or discount factors for futures/fras) from their expected values.

As the Global generator solves for the best curve for both short-dated and long-dated instruments, NPV is not as useful a criteria for calibration success as it was for the BootStrap. Instead the calibration is considered successful if the input market quotes are matched with less than a basis point difference. Typically the difference is far less.

The user can view the calibration quality through the Log category "GenGlobal". This will display the NPV of each trade versus the target NPV and compare the input market quote with the pricer's calculation of the breakeven rate using the curve. For futures the implied discount factor is compared rather than the rate. (Money market deposits, which are guaranteed to have the correct discount factor, are not included in the calibration.) *The Log can slow down the generator significantly, so do not have it on during normal use*.

The following table shows the GenGlobal log for the curve example shown above. Note that the breakeven rates are satisfied to within 0.00001 of a basis point.

3 NPV vs Par:	-2.41E-04	BE DF:	0.997146	Quoted DF:	0.997146	Diff:	-2.41E-10
4 NPV vs Par:	3.55E-05	BE DF:	0.996232	Quoted DF:	0.996232	Diff:	3.55E-11
5 NPV vs Par:	-0.00183	BE DF:	0.99537	Quoted DF:	0.99537	Diff:	-1.83E-09
6 NPV vs Par:	5.56E-06	BE Rate:	0.012065	Quoted Rate:	0.012065	Diff:	2.81E-12
7 NPV vs Par:	-5.12E-05	BE Rate:	0.01796	Quoted Rate:	0.01796	Diff:	-1.74E-11
8 NPV vs Par:	-2.68E-05	BE Rate:	0.02299	Quoted Rate:	0.02299	Diff:	-5.43E-10
9 NPV vs Par:	-3.64E-05	BE Rate:	0.02705	Quoted Rate:	0.02705	Diff:	2.19E-10
10 NPV vs Par:	-4.00E-05	BE Rate:	0.03029	Quoted Rate:	0.03029	Diff:	1.60E-11
11 NPV vs Par:	-4.30E-05	BE Rate:	0.0328	Quoted Rate:	0.0328	Diff:	-2.70E-11
12 NPV vs Par:	-4.52E-05	BE Rate:	0.03477	Quoted Rate:	0.03477	Diff:	1.24E-09
13 NPV vs Par:	-4.89E-05	BE Rate:	0.03635	Quoted Rate:	0.03635	Diff:	-3.91E-11
14 NPV vs Par:	-5.42E-05	BE Rate:	0.03769	Quoted Rate:	0.03769	Diff:	1.46E-09
15 NPV vs Par:	8.91E-05	BE Rate:	0.0419	Quoted Rate:	0.0419	Diff:	1.27E-10
16 NPV vs Par:	2.17E-04	BE Rate:	0.04344	Quoted Rate:	0.04344	Diff:	-1.49E-09
17 NPV vs Par:	-3.34E-04	BE Rate:	0.0445	Quoted Rate:	0.0445	Diff:	-4.11E-11

Daily Points

To view the points for every day of the curve, use the Log category "**DailyPoints**." Apply it to the Log configuration before generation, then generate. The points will appear in the log when the generation is complete, from which they can be copied and pasted into Excel.

Accuracy

The generators solve curves to an accuracy taking into account market practicality and performance demands. As swap bid/ask spreads are typically at one-quarter to one-half a basis point, the generator calibration error goal in the breakeven rate is one-tenth of a basis point. This can vary with the maturity of the underlying instrument. For short-term instruments, the goal is to arrive at an NPV error of less than 0.01 per 1,000,000 notional; in terms of a break even rate, over short maturities such as a week this can result in a somewhat larger basis point error than 0.1 bp, but the difference is immaterial when applied to such short periods. For very long-term instruments the correct break-even rate is more important and the calibration error tends to be very much better than 0.1 bp – but then the NPV error is allowed by the generators to be larger than 0.01 per 1,000,000, because even a very small breakeven rate calibration error applied over a long maturity has an effect -- yet the difference is non-material. This is to emphasize that the presence of non-material calibration errors is an expected part of a numerical system, and should not be taken as indicating any larger mathematical or conceptual problem.

Special Considerations

When using **Central Bank dates**, calibration will not be as accurate as possible if one has set the Jump Stability to a maximum of 100. To ensure calibration accuracy, set the stability value to 0 (which then does not guarantee a well-behaved curve). In addition, if the underlying instruments occur more frequently than central bank dates there is insufficient information to produce an exact calibration, so a best fit is found instead.

5.2 BasisGlobal

The BasisGlobal generator provides the same functionality as the BasisBootStrap generator with the replacement of the global algorithm for the bootstrap. As mentioned in the section on Cross-Currency Basis Swaps, one difference is that the BasisGlobal generator can handle Mark-to-Market cross-currency swaps where the reset notional is on either the domestic or the foreign leg.

5.3 Generator Parameters

Each generator has a set of parameters that provide for refinements of the algorithm. The following is the table of parameters for the Global generator with recommendations and descriptions.

Shaping method parameters are described in the next section. The parameters related to the Central Bank Calendar are described in that section and in the chapter on overnight indexed swaps.

Parameter Name	Recommendation	Description
Extrapolation	Flat on Forward	Extrapolation method: constant forwards or constant zero rates. See the "Additional Information" section.
Use Future Convexity		If True, convexity adjustments will be applied to the futures contracts. If false, no adjustments will be used, even if specified in the Quotes table. This is used in conjunction with the parameter "Use Manual Future Convexity."
Use Manual Future Convexity		If True, the convexities entered on the Quotes tab will be applied to the futures. If False, convexity will be computed using a volatility surface. Use in conjunction with "Use Future Convexity." For the computation, see the section on "Deposit Futures Convexity Adjustments."
Use month end tenors for MM		See the "Additional Information" section.
Generate on all flow points	TRUE	When using Global generator, improves initial guess, but otherwise not used.
Use MMkt up to first future		See the "Additional Information" section.
Roll Method	Roll Forwards	Method for moving the curve to a future date when updating a pricing environment.

Parameter Name	Recommendation	Description
Future Rolling Type		See the "Additional Information" section.
Future Rolling Lag		See the "Additional Information" section.
Interpolate FX Curve		Only appears on BasisGlobal and BasisBootStrap. Described in the section "Cross-Currency Curve Generation Parameters."
FX Forward Quote Type		Only appears on BasisGlobal and BasisBootStrap. Described in the section "Cross-Currency Curve Generation Parameters."
LAST Generates MID Only	True	If curve is LAST type, generate only MID rather than MID/BID/ASK.
Daily Average Swap Fast Approx	True	If True, an approximation will be used for daily averaging swaps that will make curve generation significantly faster for those instruments.
Monotone Convex Require Positive		(For Monotone Convex Interpolation only) If True, then all forward rates are forced to be positive on the dates solved for in the curve. If False, forward rates are allowed to be negative.
Central Bank Calendar		A Calypso Calendar Definition holding central bank meeting dates.
Central Bank Last Tenor	1y	The amount of the calendar to use. Used with "Central Bank Calendar" parameter.
Jump Constraint Method	2	Method for handling case of multiple meeting dates between instrument maturities. Used with "Central Bank Calendar" parameter.
Jump Stability		A stabilization factor to reduce effect of noisy or inconsistent input quotes. Used with "Central Bank Calendar" parameter.

Parameter Name	Recommendation	Description
		100 = Maximum guaranteed stability but can have sizable calibration error. 0 = Minimum guaranteed stability but calibration is exact.

5.3.1 Additional Information on Generator Parameters

This section describes in more detail some of the generator parameters.

Extrapolation

Extrapolate points that fall past the last generated point on the curve – the last maturity date of an underlying instrument.

- **Flat on Zero** – The zero rate of the last generated point is assigned to every following point. The zero rate curve is flat past the end of the curve. This causes the forward rates to drop down to the zero rate as the end of the curve is crossed.
- **Flat on Forward** – The one-day forward rate at the end of the curve is assumed to hold for every following point. The curve of forward rates is flat past the end of the curve and has no drop at the start of the extrapolation section.

Recommended value: **Flat on Forward.** This produces a forward curve without discontinuities.

Use month end tenors for MM

(Rarely used.) Enforces end-of-month dates if the underlying does not do so. If the start date (spot date) of the money market deposit is the last business day of the month, then adding a month-based tenor (1M, 2M, 3M…) to this date will result in the last business day of the corresponding month. This convention is more easily specified using the "END_MONTH" date roll on the curve underlying. Rate indexes also can be specified with this date roll through an "end-to-end" checkbox on the Tenors tab of the Rate Index window, so that forward rates are properly aligned. These are the recommended methods of defining the date roll, so the generator parameter should be left at False.

Recommended value: **False.** The date roll is best given specifically in the curve underlyings.

Use MMkt up to first future

- **True** - The strip of money market deposits will be truncated when they overlap the futures instruments. The last money market deposit to be used will be the earliest one whose end date falls in the first futures' period.
- **False** - All of the money market deposits will be used in curve generation. Their values will be blended with the forward rates of the futures instruments. The BootStrap generator will use the methods for handling overlaps of forward periods, while the Global generator will fit a curve producing the money market and the futures rates.

Recommended value: **True.** This prevents unexpected conflicts and irregularities caused by the overlap of many different instruments.

Roll Method

An important parameter that identifies how a curve is rolled from an earlier date to a later date for use in pricing. It is **not** used in curve generation. When pricing, the Pricing Environment will look up the most recent curve prior to the value date and time. If the curve was generated prior to the value date, the Pricing Environment will "roll" it to the value date according to the method defined on the curve.

- **Roll Forwards**
  - **For CurveZero** - Rolls the curve to the value date by discarding all points before that date and dividing the discount factors by the discount factor on the value date. This preserves the forward rates between dates. The dates on the curve points are not changed, other than discarding the points that had dates prior to the value date.
  - **For CurveBasis** - The spreads on the CurveBasis itself are retained on their dates, but any points whose dates fall prior to the roll-to date are discarded. This alone does not preserve forwards, as the result depends on the rolling of the base curve. The base curve will be rolled using the method DEFINED ON THE BASE CURVE. If the method on the base curve is Roll Forwards, then the resulting discount factors on the CurveBasis will follow the Roll Forwards logic. But if the base curve has a different roll method, the CurveBasis will have to reflect that method. In short, both the base curve and the CurveBasis must have their roll methods set to "Roll Forwards" in order to achieve the rolling method that preserves the forwards of the resultant base + spread curve.
- **Roll Points** – The number of calendar days between the curve date and the value date is found, and this number of days is added to the date of each point in the curve. This translates the discount factors forward in time without change of shape.
- **Regenerate** – The curve date is changed to the value date and time and regenerated. The result will be the same as if the user manually changed the date and generated the curve.

Recommended value: **Roll Forwards.** This is faster than the Regenerate method and keeps the forward rates between dates.

Future Rolling Type

Specifies the method for rolling the futures strip. The first futures in the strip will be rolled to the first futures that is not expired as of the future rolling date. The rolling date is determined by one of the following choices - if no choice is made, Default is used:

- **Default** – Use the curve date as the rolling date.
- **Last Trading Day** – Roll off futures on the last trading day.
- **Lag Days** – The rolling date is obtained by adding to the curve date the number of days specified in the Future Rolling Lag parameter.

Recommended value: No choice or Default. This is up to the user's policy.

Future Rolling Lag

An integer specifying the days of lag in the futures rolling. This parameter is only used if the Future Rolling Type is specified to be Lag Days.

Daily Average Swap Fast Approx

If True, an approximation will be used for daily averaging swaps that will make curve generation significantly faster for those instruments. If False, the full valuation used by the swap pricer will be employed. Curves that include Fed Fund versus Libor basis swaps will benefit from setting this value to True. The result will not exactly reprice to the input quote when pricing in the trade window, due to the approximation, but the error is less than a basis point for 30Y swaps at current interest rate levels (2015). No convexity adjustment is applied to the approximation. Reference: "Valuation of Arithmetic Average of Fed Funds Rates," Katsumi Takada (September 2011), http://ssrn.com/abstract=1981668.

5.4 Global Generator Shaping Parameters

The shaping parameters provide additional control over the shape of the curve, especially on the short end. A Base or Discount Curve can be used to determine the shape.

The parameter "Number of Curve Sections" specifies how many sections are being defined.

If the number is 1 or undefined, there is only one section which is governed by the parameter "Shaping Method" or the "Central Bank Calendar." The Shaping Method has several choices, some of which, like the Central Bank Calendar perform a hard-coded distinction between a short and long end.

If the number of sections select is 2, then a new "short" or "stub" end is added in front of the section defined by the Shaping Method. This new short section is governed by the various "short end" parameters, especially Shaping Method Short End.

Note that in this way it is possible to actually define three different sections: the initial short end, then the longer Shaping Method which itself can divide the curve into an early and a later part, depending on the choices.

In the following, first the summary of the parameter menus is given, then a more detailed description of each choice. Finally, there are examples of useful combinations of parameters and a description of the curves they produce. The Central Bank Calendar method and associated parameters are not included, as they are well described elsewhere – except note that if that calendar is chosen, then only that method is used, and no other shaping parameters will have an effect.

Description of the shaping choices

Shaping Method

The Shaping Method parameter defines different types of curve generation.

Basic

No special shaping is applied. This is the original method that simply solves for the end points of each instrument.

Spot Zero

The Spot Zero method handles an initial gap before the spot date, if any, by extrapolating from the subsequent curve so as to preserve smoothness.

When the curve underlying instruments all have start dates that fall after the curve date – for example, a curve of spot-starting swaps and deposits -- there is a gap in information before the earliest start date (spot date). Some algorithm is needed to fix the discount factor on the earliest start date. The original or "Basic" method perform initial gap extrapolation using a constant zero rate derived from later points. In that case the forward rate behavior near the curve date is uncontrolled and can turn out to be unreasonable. The Spot Zero method instead solves for a discount factor at the spot date (or earliest start date) so that the curve is has zero forward rate slope on that date. This gives smooth forward rate behavior near the curve date.

The Spot Zero method affects spline and monotone convex interpolators but not linear-based interpolators. Linear interpolators will always perform initial spot extrapolation using a constant zero rate, and smoothness is not an issue. In spline-like interpolators, sudden kinks or drops over an initial gap would occur unless the extrapolation method respects the smoothness of the rest of the curve.

The algorithm of the Spot Zero method is as follows.

1. A trial curve is made with a trial discount factor at each curve underlying maturity point and on the earliest start date of the curve underlyings. This earliest start date will be the spot date unless additional underlyings, such as O/N and T/N, are supplied. Call the earliest start date T_1 (expressed in years after the curve date) and its trial discount factor DF_1.

2. The "forward" rate over the initial gap is found. This is just the simple zero rate from the curve date (which is T = 0, discount factor of 1) to the first known date T_1:

$$F_0 = \frac{1}{T_1}\left(\frac{1}{DF_1} - 1\right)$$

3. Now on the trial curve interpolate the discount factor at the date one calendar day after T_1. Call this T_2. So the first step is to draw the interpolation curve (e.g., spline) through the trial curve points. Then use the interpolation curve to find the discount factor at day T_2; call this DF_2. Now calculate the simple one-day (annualized) forward rate from T_1 to T_2:

$$F_1 = \frac{1}{(T_2 - T_1)}\left(\frac{DF_1}{DF_2} - 1\right)$$

4. Find the difference between the zero rate to T1 and the one day forward based at T1:

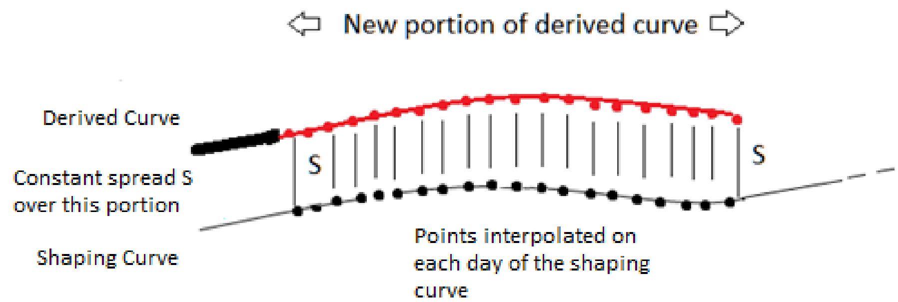
Forward Difference = $F_0 - F_1$.

Note that $Z_0$is also the "forward" rate from the curve date to T_1. So the Forward Difference measures the change in forward rate before and after T_1. The goal is to have this Forward Difference be zero.

5. Repeat Steps 1-4 with another trial curve, including new trial values for DF_1 and DF_2 until the Forward Difference is as small as possible within solver tolerance.

Daily Forwards Spread

For each part of the curve that is extended by an underlying instrument, solve for a constant spread to the one-day forward rates on the discount or base curve. See the following picture. The points 'S' are end points of the underlying instruments of the derived curve.



In more detail, suppose D(t1) is the discount factor on the derived curve at t_1, and a forwards spread of s_{12} was solved for between dates t_1 and t_2. For any time t between t_1 and t_2, the discount factor is

$$D(t) = \frac{D_B(t_1)}{D_B(t_2)} \exp\left(-s_{12}(t - t_1)\right)$$

where $D_B(t_1)$ and $D_B(t_2)$ are the discount factors of the shaping curve at t_1 and t_2. This makes the forward rate of the derived curve between t_1 and t_2 equal to the implied constant rate of the shaping curve plus the spread s_{12}.

The shaping curve is specified by the generator parameter of that name, and has choices of “Base” and “Discount.” This parameter is only available on the BasisGlobal, DoubleGlobal and TripleGlobal generators. A discount curve can be specified with any kind of generator. If there is no discount or base curve specified, there is no shaping curve and this method cannot be used.

Only one curve in the Multicurve package can use the daily forwards spread method. If one attempts to define two curves with the spread method, an error message will be received. Theoretically it should be possible to have two curves that are both spread to a third curve, or two curves that depend upon each other as spreads but over different curve sections. These enhancements are planned for future versions.

Later Priority (Backwards Bootstrap or FRA Stub)

This method is most often used to truncate money market instruments such as a FRA to a short “stub” that ends at the start of the first Futures contract. This is chosen if the user believes the Futures rate is a better projection of the forward rate at that time than is the prior money market rate.

More generally, for the initial Money Market Deposits, FRAs and Futures, this method gives priority to the *later-starting* instruments whenever they overlap with *earlier-starting* instruments. This is equivalent to beginning a bootstrap at the maturity date of the last of these instruments and proceeding backwards, using earlier-starting instruments to extend the curve earlier in time. For this reason this is also known as the ***Backwards Bootstrap*** method.

You can exclude a FRA from the Later Priority shaping method by setting Priority = 1 in the Underling tab. The start date of a FRA is only used as a pillar if Priority = 0.

Later Priority LogLinear DF

Applies the same bootstrapping methodology as "Later Priority" but with LogLinear interpolation on DFs.

Shaping Curve (only in BasisGlobal generator)

The Shaping Curve is used with the “Daily Forwards Spread” method. These choices are only available in the BasisGlobal generator, where there is more than one possible shaping curve. In the Global generator, the shaping curve can only be the Discount curve, so there is no choice necessary; if no Discount curve is supplied, the method cannot be used and an error message is displayed.

Choices:

Discount

Use discount curve to derive shape of short end using daily forward spreads.

Base

Use base curve to derive shape of short end, using basis swap underlyings to find the daily forward spreads to the base curve.

Shaping Horizon

Specifies how much of the short end gets its shape from the “Daily Forwards” shaping method. The rest of the curve – the long end – is solved for as usual without extra shaping.

Values are tenors, e.g., 3M, 6M, 1Y, 2Y. One can type in *any* tenor of this format – it does not need to be one of the tenors in the list.

Number of Curve Sections

This was described above. To summarize:

- 1 means just the “Shaping Method” is used across the whole curve.
- 2 means another short or stub method is added, which is governed by the “Shaping Method Short End”

Shaping Method Short End

The choices have the same meaning as those described above. There is one additional choice, ‘Same as Long End’, which is the default. There is no choice for the ‘Daily Forwards Spread’ method, because the 1-section Shaping Method already separates the curve into long and short ends when using that method. So the choices are:

- Same as Long End
- Basic
- Spot Zero
- Later Priority

Shaping Horizon Short End

Defines where the short end shaping ends and the long end (‘Shaping Method’) begins. Choices are:

- Same as Long End: default, meaning there is no separate short end
- SPOT, 6M, 1Y, 2Y… But the user can type in *any* tenor of this format – it does not need to be one of the tenors in the list. IMPORTANT NOTE: THE TENOR SHOULD NOT BE THE SAME AS ONE OF THE INSTRUMENT MATURITIES IN THE CURVE. The reason is that, due to date arithmetic variations, a 6M instrument might end up falling before or after the '6M' short end point, and change this relative location from day to day. The simplest way to avoid this is to use an odd tenor, such as 170D or 190D rather than 6M.
- [enter date] A fixed date can be entered, using the same format as elsewhere in the user's system, such as in the curve window's Date field.

Short End Interpolator

Choices:

Curve Definition - The short end interpolator will be the same as the interpolator defined on the "Definition" tab of the curve window. This is the default.

LogLinear - The LogLinear on Discount Factor interpolation will be used until the end of the Shaping Horizon Short End, after which the interpolator from the curve window "Definition" tab is used. This is the only choice provided at present. This allows the use of the local interpolation on the short end and a spline interpolator on the long end, to avoid the use of spline at very short dates where risk can be too sensitive to movements of the spline.

Examples of Combinations of Choices

The following are some of the most commonly used sets of choices for shaping parameters.

Selections	Result
Number of Sections: 1 **Shaping Method: Later Priority** **Central Bank Calendar: Any existing calendar**	Backwards bootstrap is performed on MMkt, FRA and Futures, and LogLinear on DF interpolation is used over this section. The rest of the curve is a standard curve generation using the interpolator in the curve definition. All other generator parameters are ignored. Central bank calendar dates are used to define the short end of the curve out to the Central Bank Last Tenor. Any other shaping parameters are ignored.
Number of Sections: 1 **Shaping Method: Daily Forwards Spread**	The Daily Forwards Spread method is used from the beginning of the curve out to the date indicated by the Shaping Horizon. In the Global generator the Discount curve is used as the shaping curve, and must be supplied. After the horizon date, normal curve generation proceeds using the interpolator on the curve window.
Number of Sections: 2 **Shaping Method Short End: Spot Zero**	This is a way to have the Daily Forwards Spread method begin at a future date, rather than at the start of the curve (as occurs in the 1-section method). Ordinary generation is used out to the

Selections	Result
Shaping Horizon Short End: 2Y **Short End Interpolator: Curve Definition** **Shaping Method: Daily Forwards Spread** **Shaping Horizon: 10Y**	2Y horizon as indicated, then the spread method begins using the discount curve (if Global generator), out to the indicated 10Y horizon, then ordinary curve generation takes over. To have the entire long end of the curve use the spread method, select a horizon of 100Y.
Number of Sections: 2 **Shaping Method Short End: Daily Forwards Spread** **Shaping Horizon Short End: 2Y** **Shaping Method: Basic**	This produces the same result as choosing a 1-section Shaping Method as Daily Forwards Spread. That is, the short end is Daily Forwards Spread out to 2 years, then standard ('Basic') curve generation takes over for the rest of the curve. The other parameters are ignored.
Number of Sections: 2 **Shaping Method Short End: Spot Zero** **Shaping Horizon Short End: 2Y** **Short End Interpolator: LogLinear DF** **Shaping Method: Basic**	Standard curve generation is used on the short end with LogLinear on Discount Factor interpolation out to 2Y, then standard generation is used for the rest of the curve with the interpolator defined on the curve Definition tab.
Number of Sections: 2 **Shaping Method Short End: Spot Zero** **Shaping Horizon Short End: 2Y** **Short End Interpolator: LogLinear DF** **Shaping Method: Daily Forwards Spread** **Shaping Horizon: 100Y**	Standard curve generation is used on the short end with LogLinear on Discount Factor interpolation out to 2Y, then the Daily Forwards Spread method is used for the rest of the curve. Note the interpolator defined on the curve is never used, as both the short and long end methods have their own embedded interpolation.
Number of Sections: 2 **Shaping Method Short End: Later Priority** **Shaping Method: Daily Forwards Spread** **Shaping Horizon: 100Y**	The backwards bootstrap with LogLinear DF interpolation is used for the MMkt, FRA and Futures on the short end, then the Daily Forwards spread method is used for the rest of the curve. Note the interpolator defined on the curve is not used.

Section 6. Interpolators

6.1 Interpolator Choices On Rates or Discount Factors

Generation of a curve depends on the choice of interpolator as well as the generation algorithm. The interpolation method is chosen independently of the generator. Interpolation methods are:

- Linear
- Log Linear
- Natural Cubic Spline
- Natural Cubic Spline on Logarithms ("LogSpline")
- Monotone Convex (Hyman-Hagan-West monotonic spline)

The mathematics of interpolation are described in the following sections.

The interpolators can act on:

- Zero Rates (Using the Day Count and Frequency on the curve's Points tab)
- Discount Factors

Not all combinations will result in a valid curve. For example, **negative zero rates are allowed** so a curve with this property will fail when using the logarithm on zero rates.

The application of the interpolators to Zero Rates or to Discount Factors is specified by selecting the 'Target' of the intepolator on the curve window. The target choice called **"Default"** does NOT mean this is the Calypso preferred target; instead it refers to the *user's* preferred target that is defined in the rate interpolation System Environment parameter **INT_CURVE_INTERP_RATE_B** (the B is for Boolean). This parameter can be set to one of these:

Y : yield curves will interpolate on zero rates, using whatever interpolator is chosen

N : yield curves will interpolate on discount factors, using whatever interpolator is chosen.

It is recommended, for clarity, to not use Default as the target, but instead explicitly select Zero Rates or Discount Factors for a curve.

Please note that if using Zero Rate interpolation the Zero Rates are defined by means of the Day Count and Frequency that the user selects on the Points tab of the curve. The user must be careful to make a choice that reflects the desired interpolation rates. For example it is likely one does not want to use compounding, in which case the choice NON should be made for the compounding frequency. Some Day Count choices will give nonsensical answers upon interpolation due to their strange mathematical properties, such as "1/1" or 30/360.

6.2 Smooth and Local Forwards

To obtain smooth forward rates, the recommended interpolators are:

- Cubic Spline on Discount Factors
- Cubic Spline on Log of Discount Factors
- Monotone Convex

The Monotone Convex method published by Hagan and West fits a piecewise polynomial to forward rates. It is described in a later section where the difference between it and the cubic spline method for risk is discussed.

6.3 InterpolatorLinear

For value x that lies between x1 and x2 the interpolated value y for given ordinate values y1 and y2 is given by:

$$y(x) = A\,y_1 + B\,y_2 \qquad \text{with} \qquad A = \frac{x_2 - x}{x_2 - x_1} \qquad \text{and} \qquad B = \frac{x - x_1}{x_2 - x_1}$$

6.4 InterpolatorLogLinear

For value x that lies between x1 and x2 the interpolated value y for given ordinate values y1 and y2 is given by:

$$y(x) = e^{\ln(y_1) + \frac{(\ln(y_2) - \ln(y_1))(x - x_1)}{x_2 - x_1}}$$

6.5 InterpolatorSpline

We implemented the cubic spline algorithm as described in Numerical Recipes 2nd edition (NR), pp.115; the only difference to the NR code is that we allow having an array of ordinates to be processed at once rather than one by one. So in one computation, we can process for example an array of curves rather than a single curve and upon interpolation we will be returned an array of (interpolated) rates rather than a single rate.

6.6 Interpolator3DLinear

We implemented the multi-dimensional bilinear interpolation algorithm as described in *Numerical Recipes 2nd edition (NR),* pp.123. Our implementation is for 3 dimensions only.

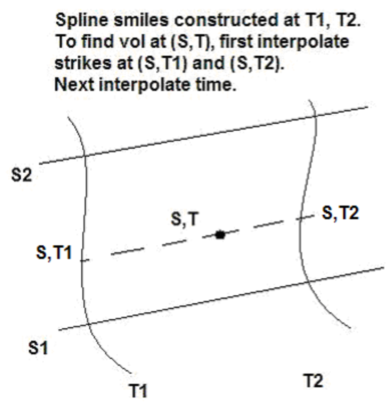
6.7 Interpolator3DSpline1D

Same as Interpolator3DLinear except that the points on the third axis are interpolated using a cubic spline, while the others interpolated linearly. Constant extrapolation is used beyond the known points in both dimensions. The most frequent application is to spline interpolation of strikes for volatility surfaces.

For example, for FX options, the spline is used for the delta (strike) axis while linear interpolation for the expiry time axis.The procedure is illustrated below. The "strike" axis for FX options is the delta axis. Define

V(S,T) : Volatility at delta strike S and expiry time T

For each expiry time in the surface, a spline is constructed along the delta axis.



To find the volatility at any delta and expiry (S,T), the following procedure is used:

1. First existing points (S1, T1) and (S2, T2) are found on the surface that obey:

   S1 <= S < S2, T1 <= T < T2.

2. Splines in the strike dimension exist at the times T1 and T2. Each spline is interpolated upon individually to find the volatility at the strike S for each of those times. The result is two volatilities, V(S, T1) and V(S, T2).
3. Linear interpolation in the time direction is performed on V(S,T1) and V(S,T2) to obtain V(S,T).

6.8 InterpolatorDailyCompFwdRate

The InterpolatorDailyCompFwdRate is equivalent to **LogLinear interpolation on Discount Factors**. The only difference is that InterpolatorDailyCompFwdRate does not use the value of the Target of the interpolator or of the system parameter **INT_CURVE_INTERP_RATE_B.**

InterpolatorDailyCompFwdRate is the preferred interpolator for use with BootstrapForwards curve generation.

The algorithm is Compound Daily Forward interpolation. This method finds discount factors on a date between two dates with known discount factors by assuming the forward rate between the known dates is constant.

Let $D(t_1)$ and $D(t_2)$ be the known discount factors on dates t_1 and t_2, respectively. We want to find the discount factor on a date t that falls between t_1 and t_2.

The assumption is that for each day between t_1 and t_2 there is a constant rate at which interest will compound. That is, there is a daily rate *f* so that:

$$D(t_1)\frac{1}{(1+f)^{(t_2-t_1)}} = D(t_2).$$

The difference $t_2 - t_1$ is expressed in days. To interpolate on t, first find this implied rate *f* from this equation. Then use that rate to find the discount factor on t:

$$D(t) = D(t_1)\frac{1}{(1+f)^{(t-t_1)}}.$$

Or, more directly, one has:

$$D(t) = \left(D(t_1)^{(t_2-t)} D(t_2)^{(t-t_1)} \right)^{1/(t_2-t_1)} .$$

The equivalence with LogLinear on Discount Factors is seen immediately by taking the logarithm of this expression, to give:

$$\ln D(t) = \frac{t_2 - t}{t_2 - t_1} \ln D(t_1) + \frac{t - t_1}{t_2 - t_1} \ln D(t_2) \ .$$

6.9 Monotone Convex (Quadratic Forwards) Interpolation in Calypso

The Monotone Convex method published by Hagan and West fits a piecewise polynomial to forward rates. It is a variation on the method of piecewise quadratic continuous forwards.

On the curve window, when choosing Monotone Convex there is no need to specify the interpolation target Zero Rates or Discount Factors, as the method in fact interpolates on instantaneous continuous forward rates. To do this it will always use discount factors to find forwards on pillar dates from which the interpolation proceeds.

6.9.1 References

The following references explain the Monotone Convex algorithm in detail. The next section provides a summary.

"Interpolation Methods for Curve Construction", Patrick Hagan and Graeme West, Applied Mathematical Finance,Vol. 13, No. 2, 89–129 (June 2006)

"Methods for Constructing a Yield Curve", Patrick Hagan and Graeme West, Wilmott Magazine, June 2008

"Construction of Hedges for Interest Rate Sensitive Instruments", Graeme West

"Interpolation", Calypso Technology White Paper, October 2010

"Curve Construction Effect on Deltas", Calypso Technology Excel Spreadsheet

6.9.2 Outline of Monotone Convex Interpolation

This section summarizes the interpolation method.

As with other interpolation methods, one starts with a curve that is given as a set of discount factors on a set of "pillar" dates. The goal of is to calculate the instantaneous forward rate on any date between the pillar dates. This is done by creating a quadratic function that gives the forwards between each pair of known discount factors.

During curve generation, successive trial sets of discount factors are made, and for each trial set the intervening forward rates, and hence discount factors, are found in order to price the given set of trades. The curve is solved when the trial set of discount factors and their interpolated quadratic forwards price the input trades with sufficient accuracy.

The procedure for producing a quadratic forward interpolation function *f(t)* for any time *t* is as follows.

1. Between each pair of discount factors, find the constant instantaneous forward rate, using $f_i^c = -\frac{1}{(t_{i+1}-t_i)} \ln \left(\frac{D(t_i)}{D(t_{i+1})}\right)$

for the constant forward rate between interval end dates t_i and t_{i+1}. Doing this between every pair of interval dates produces a piecewise-constant forward curve. (Extrapolation is used after the last date.)

2. To introduce quadratic smoothing, in each interval the rate of Step 1 is assigned to the date at the midpoint of the interval. Thus set

$$f\left(\frac{t_i + t_{i+1}}{2}\right) = f_i^c$$

3. The instantaneous forwards $f(t_i)$ at the endpoints of the intervals $\{t_i\}$ are next determined by linear interpolation from the midpoint values of Step 2. So far, one has defined three f(t) at the end points and the midpoints of each interval.

4. On the *ith* interval, the forward rates for other points are chosen to be of the form

$$f_i(\tau) = A_i + B_i\tau + C_i\tau^2$$

where the time parameter along the *ith* interval is

$$\tau = \frac{t - t_i}{t_{i+1} - t_i}, 0 \leq \tau \leq 1$$

The forwards are chosen to satisfy three requirements to match the average rates and the end rates of Steps 1 and 2:

$$f_i(t_i) = f(t_i)$$

$$f_i(t_{i+1}) = f(t_{i+1})$$

$$\frac{1}{(t_{i+1} - t_i)} \int_{t_i}^{t_{i+1}} f_i(t)dt = f_i^c$$

These three equations are readily solved for the three unknowns.

5. If all that is desired are the piecewise-quadratic forwards, then this completes the construction.

However, we now proceed to modify the solution in order to add two more economic requirements.

- Monotonicity. This is the requirement that if three successive constant forwards f_i^c are increasing, then the interpolated instantaneous forwards between them are also increasing. Similarly, if the three are decreasing then all the instantaneous forwards are increasing.
- Convexity. The convexity requirement is that if the constant forwards of Step 1 are all positive, then the smoothed quadratic forwards should also all be positive. (This is an optional requirement which Calypso allows the user to switch off, as is common in the current rates environment.)

Following Hagan and West, the modification involves changing the interpolation method on an interval from pure quadratic to a combination of quadratic and linear. This introduces additional degrees of freedom so that the constraints of Step 4 can still be satisfied, along with the monotonicity and convexity requirements.

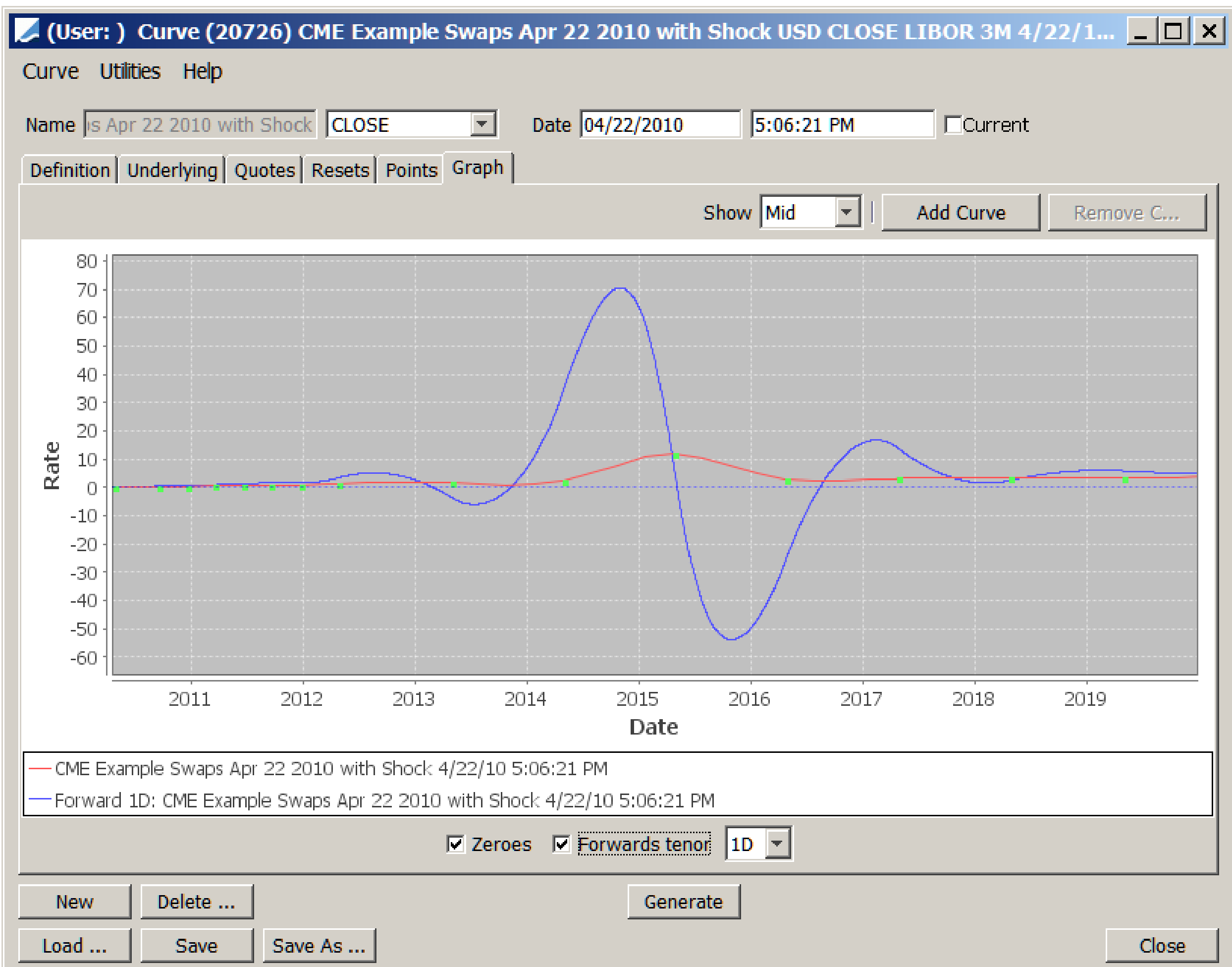
The Hagan-West modification is described in the references given above. This leads to improved behavior under local perturbations.

6.9.3 Behavior Under Local Perturbations

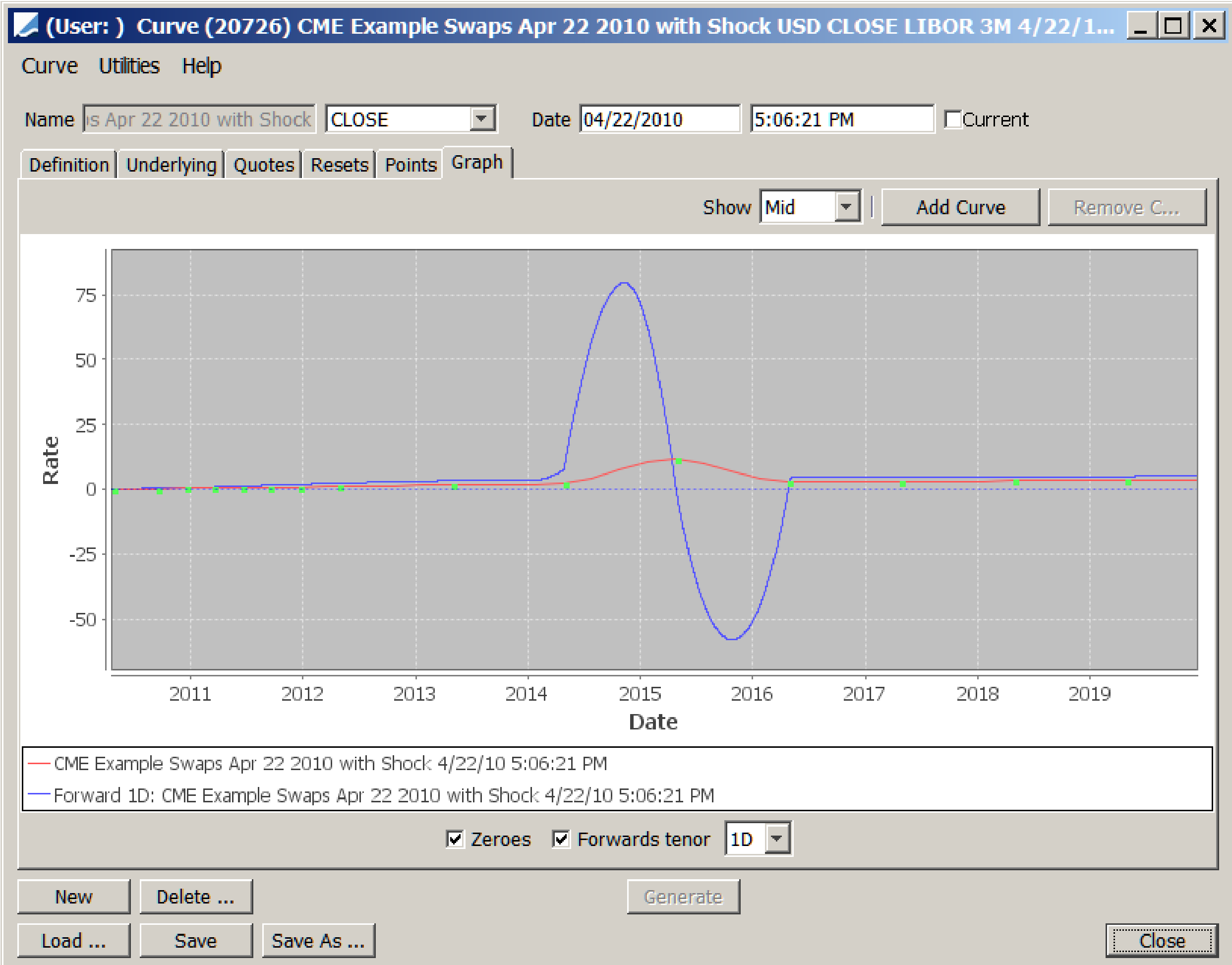
The principle benefit of Monotone Convex over the Log Spline method (cubic spline on the logarithm of discount factors) is the locality of forward rate shifts with respect to changes in market quotes. This results in more realistic risk analysis. When a shift is made in a market quote and the curve regenerated, a cubic spline will not only shift at the maturity date of the market quote but also will have spurious oscillations up and down the curve.

The following graphs illustrate this point. These show the USD curve of the previous examples created with the Global method, but the five-year swap quote has been set to 10.0% -- a very large shift for illustration purposes. In the first graph the curve was generated with a cubic spline on the log discount factors. Note how oscillations in the one-day forward rates extend out three years to either side of the shifted maturity. In the second graph, Monotone Convex interpolation was used, and one can see that the shift in the forward rates ends at the next swap maturities to either side of the 5Y swap.

Cubic spline:



Monotone Convex:



The Monotone Convex interpolation is not a perfect solution. At times produce unexpected gammas when doing bump-and-revalue risk calculations. For more information see the section below, "Obtaining Correct Swap Delta and Gamma with Monotone Convex Interpolation."

6.9.4 Negative Forward Rates

The method can either require all forward rates to be positive, or allow negative rates if that makes a smoother curve.

The choice is provided in the curve generator parameter **"Monotone Convex Require Positive".** If this is specified to be True, then all forward rates are forced to be positive on the dates solved for in the curve. If False, forward rates are allowed to be negative. Negative rates can allow the forward curve to be smoother at the expense of breaking the no-arbitrage property of forward rates.

6.9.5 Obtaining Correct Swap Delta and Gamma with Monotone Convex Interpolation

One aspect of monotone convex interpolation is that sensitivities should not be calculated with the conventional method of shifting an underlying quote up and down and regenerating the curve each time. That conventional method (called "Sequential" shifting in Calypso's scenario designer) causes two problems for monotone convex: first, the sum of the deltas found in this way do not equal the single delta found when a simultaneous shift of the quotes is made to the curve. Second, the gamma for an off-market swap tends to be much larger than for other interpolation methods, often 100 times greater in magnitude. This is due to the regime-changing that is inherent in the monotone convex algorithm (more details on this below). *To obtain correct deltas and gammas, an alternate method must be used.*

Two alternative approaches are recommended: Cumulative Delta and Box Bumping.

In the Cumulative Delta approach one starts from a given curve, bump up the first underlying and find the delta with respect to the original curve, next keep that first bump and now also bump the second underlying, and find the delta of that curve with respect to the original, and so on. One can make a scenario report that computes these deltas - see the "Cumulative" choice for the perturbation method, which is on the same menu as "Sequential" and "Simultaneous". Do this up and down, and at the end subtract the cumulative deltas of successive bumps, and one has a delta per underlying. The resulting individual swap deltas are also known as "Ripple Deltas." In this approach the monotone convex does give sensible answers because it forces the sum of the deltas to be equal to the delta of a full parallel-shift curve (something that fails in the monotone convex case). Also because of the smoother interpolation across the bumped portion the monotone convex could handle the change without shifting to the different regimes within its algorithm.

The other sensitivity approach is the “Box Bumping” method recommended by the authors of the monotone convex method in communications with Calypso. The method is described in the paper of Hagan and West. One chooses forward buckets and bumps the forward rate of the generated curve in each bucket, then reprices both the off-market swap at hand and the underlying instruments from which the curve was formed. A sensitivity matrix is formed for each underlying instrument versus each bucket, from which the hedge recommendations for the off-market swap can be found through matrix inversion. For details, see the aforementioned paper (or inquire with Calypso).

Mathematical remark: The anomalous gamma is caused by a crossing of certain regime boundaries that exist in the monotone convex algorithm. The algorithm deals with several different cases for adjacent forward rates that involve various sign conditions. In the published algorithm these are the cases involving 'go' and 'g1', which depend on the slope between adjacent forwards. An example of a condition is "g0 < 0 and 0 < g1 and g1 < -0.5 * g0". When a curve quote is bumped up and down for delta/gamma calculations, the forwards change and therefore g0 and g1 change. When this happens the conditions that g0 and g1 satisfy can fall into different cases of the algorithm: there is nothing to prevent this. In the test case that replicates the anomalous gamma I was able to verify explicitly that when the curve is shifted up the g0, g1 relations fell into "zone iii" of the algorithm, while when it is shifted down the relations fall into "zone i". The shift in the relation is very sensitive to the forwards, especially conditions such as "g1 < -0.5 * g0", which can be satisfied or violated by tiny amounts. As a result, in essence an up movement uses a different interpolation algorithm than a down movement, resulting in an added effect on gamma. This is intrinsic to the algorithm.

6.10 InterpolatorLogMonotonicSplineNaturalHyman89

Dougherty et al. (1989) propose another algorithm to preserve monotonicity that has better properties on non monotonic data than Hyman (1983) by relaxing slightly the constraints on the derivatives. This is what we call the Hyman89 monotonic filter.

You can contact Calypso Support to obtain the Calypso white paper “Interpolation” that contains more details about the interpolation methods.

6.11 InterpolatorLogMonotonicCubicBessel

This is a piecewise cubic interpolator on the logarithm of some values that enforces local monotonicity by limiting the first derivative magnitude.

A Bessel Cubic (sometimes called Parabolic Blending) is a local method but is only C1, not C2 like a standard cubic spline. The interpolation provides only an O(dx^3) approximation since the derivatives are O(dx^2). See Carl de Boor "A practical guide to splines" second edition p42.

The monotonicity is applied following Dougherty, Edelman & Hyman "Nonnegativity-, Monotonicity- or Convexity-Preserving Cubic and Quintic Hermite Interpolation" (1989).

The boundaries are "natural": second derivatives = 0.

Sometimes we have function values available, but no derivatives, and we still want a smooth interpolation. In such cases we can still use cubic Hermite interpolation if we can somehow estimate the derivatives. This can be done in many ways, but one common choice is to use the slope of the parabola interpolating the data at three consecutive data-points.

6.12 InterpolatorMonotoneConvexLogTensionSpline

This interpolator enforces Monotonicity and Convexity in Tension Spline interpolation.

Section 7. OIS Curves

7.1 Overnight Indexed Swaps

Overnight lending rates are seen by the market to be nearly without credit default risk, and swaps on compounded or average overnight rates have become popular as a way to trade riskless interest rates. A typical short-term overnight indexed swap (OIS) will pay a single fixed payment at maturity in exchange for one payment at maturity equal to the daily compounded or daily averaged fixing of the overnight rate. Swap over one year will typically have an annual exchange of payments.

The overnight forward and discount rates are seen to be the same, so overnight indexed swaps can be used to create a curve that acts as both a forward and discount curve for swap pricing. This is the classic method of curve creation that was commonly done with Libor rates and bonds. Market practice has been changing to use the OIS rates to create reliable discount curves, and then Libor swaps to create Libor forward curves using OIS discounting.

Overnight rates track the target rate of the central bank, e.g., the European Central Bank or the Federal Reserve. The rates are typically constant until the central bank meets to determine changes in the target rate (every month for the ECB, about every six weeks for the Federal Reserve). When a new rate is announced there is a jump up or down in the overnight rate. The market anticipates this forward rate structure and prices OIS accordingly. Thus a yield curve generation method should take this market expectation into account.

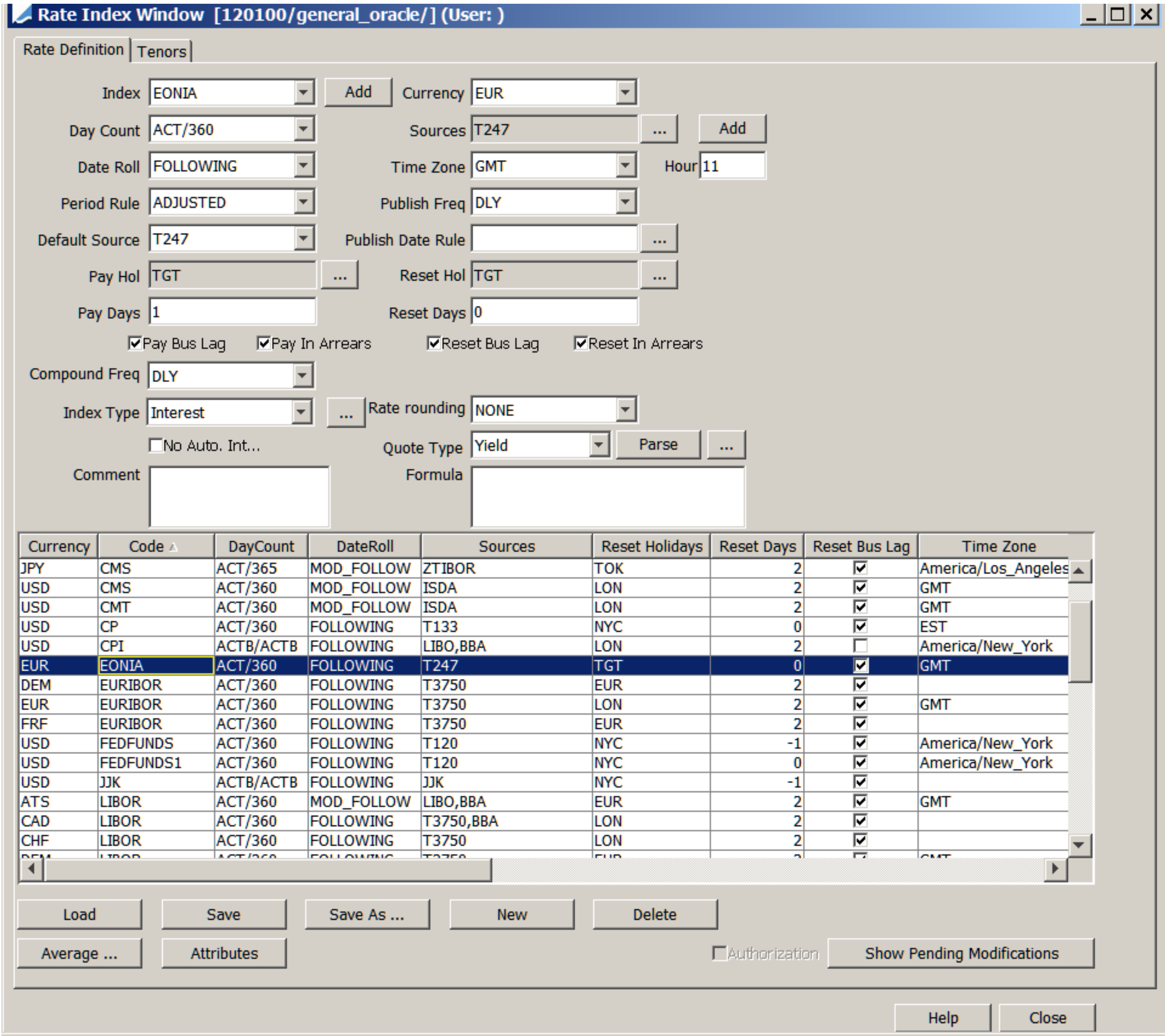
Calypso allows a specification of a central bank meeting calendar and will apply the calendar during curve generation. The set of one-day forward rates it creates will have the expected “stair-step” structure with jumps on the central bank meeting dates. Only the Global generator has this capability.

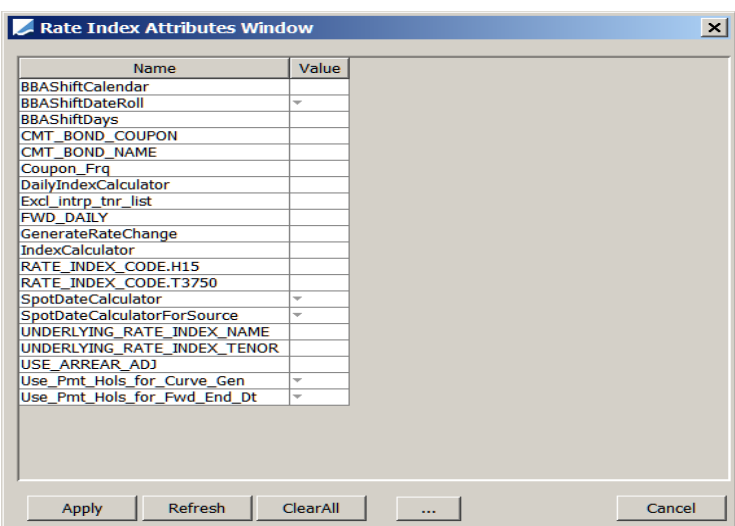
The following section demonstrates this construction.

7.2 OIS Curve Construction

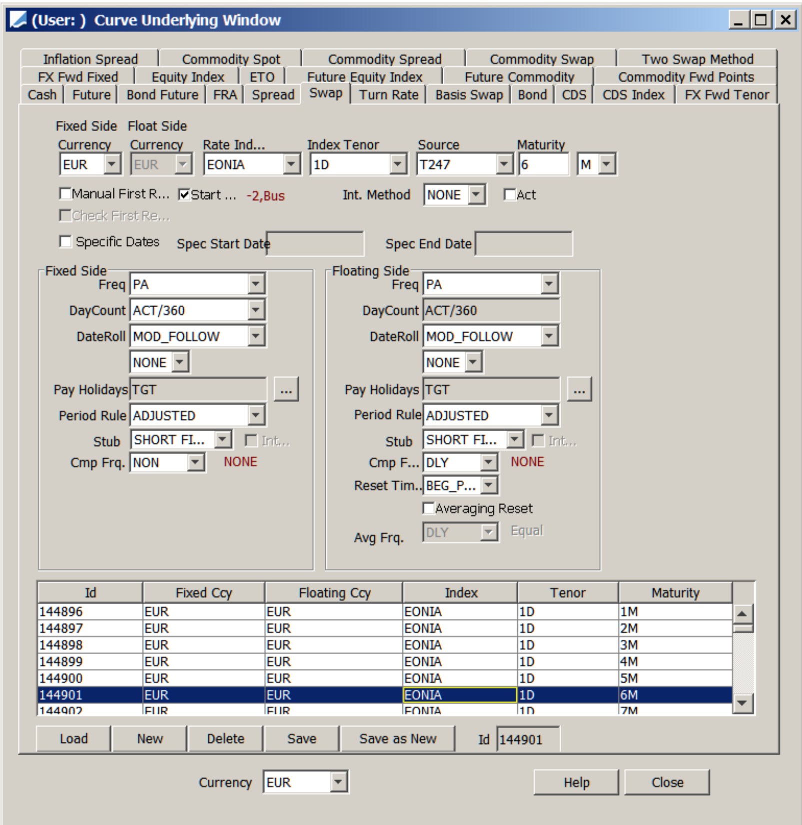
Daily compounded overnight indexed swap curve construction will be described here using EONIA as an example.

The EONIA rate index is defined as shown below. The most important point is that no Index Calculator or Daily Index Calculator should be defined in the “Attributes” panel. (This is a change from prior versions of Calypso. In the current version, the compounding attribute is defined directly on the Curve Underlying and the Trade rather than by means of a rate index calculator.)

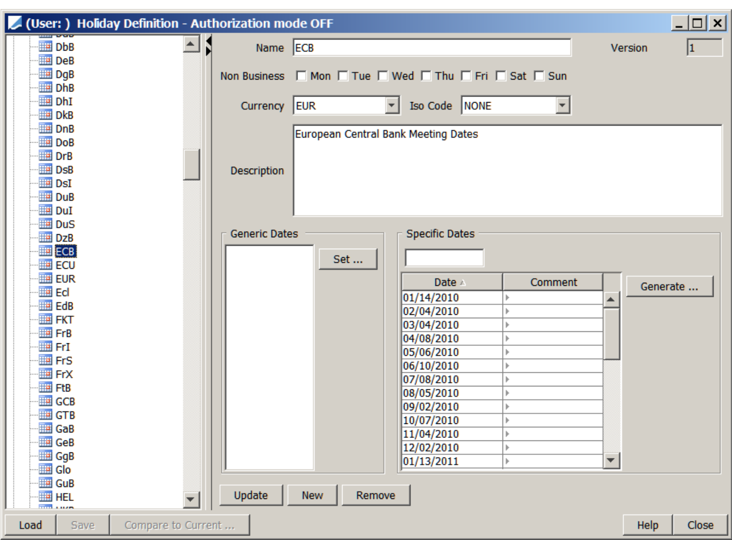

Rate Index Window [120100/general_oracle/] (User:)
Rate Definition
Tenors
Index EONIA
Add
Currency EUR
Day Count ACT/360
Sources T247
Add
Date Roll FOLLOWING
Time Zone GMT
Hour 11
Period Rule ADJUSTED
Publish Freq DLY
Default Source T247
Publish Date Rule
Pay Hol TGT
Reset Hol TGT
Pay Days 1
Reset Days 0
Pay Bus Lag
Pay In Arrears
Reset Bus Lag
Reset In Arrears
Compound Freq DLY
Index Type Interest
Rate rounding NONE
No Auto. Int...
Quote Type Yield
Parse
Comment
Formula
Currency Code DayCount DateRoll Sources Reset Holidays Reset Days Reset Bus Lag Time Zone
JPY CMS ACT/365 MOD_FOLLOW ZTIBOR TOK 2 America/Los_Angeles
USD CMS ACT/360 MOD_FOLLOW ISDA LON 2 GMT
USD CMT ACT/360 MOD_FOLLOW ISDA LON 2 GMT
USD CP ACT/360 FOLLOWING T133 NYC 0 EST
USD CPI ACTB/ACTB FOLLOWING LIBO,BBA LON 2 America/New_York
EUR EONIA ACT/360 FOLLOWING T247 TGT 0 GMT
DEM EURIBOR ACT/360 FOLLOWING T3750 EUR 2
EUR EURIBOR ACT/360 FOLLOWING T3750 LON 2 GMT
FRF EURIBOR ACT/360 FOLLOWING T3750 EUR 2
USD FEDFUNDS ACT/360 FOLLOWING T120 NYC -1 America/New_York
USD FEDFUNDS1 ACT/360 FOLLOWING T120 NYC 0 America/New_York
USD JJK ACTB/ACTB FOLLOWING JJK NYC -1
ATS LIBOR ACT/360 MOD_FOLLOW LIBO,BBA EUR 2 GMT
CAD LIBOR ACT/360 FOLLOWING T3750,BBA LON 2
CHF LIBOR ACT/360 FOLLOWING T3750 LON 2
Load
Save
Save As ...
New
Delete
Average ...
Attributes
Authorization
Show Pending Modifications
Help
Close


Rate Index Attributes Window
Name
Value
BBAShiftCalendar
BBAShiftDateRoll
BBAShiftDays
CMT_BOND_COUPON
CMT_BOND_NAME
Coupon_Frq
DailyIndexCalculator
Excl_intrp_tnr_list
FWD_DAILY
GenerateRateChange
IndexCalculator
RATE_INDEX_CODE.H15
RATE_INDEX_CODE.T3750
SpotDateCalculator
SpotDateCalculatorForSource
UNDERLYING_RATE_INDEX_NAME
UNDERLYING_RATE_INDEX_TENOR
USE_ARREAR_ADJ
Use_Pmt_Hols_for_Curve_Gen
Use_Pmt_Hols_for_Fwd_End_Dt
Apply
Refresh
ClearAll
...
Cancel

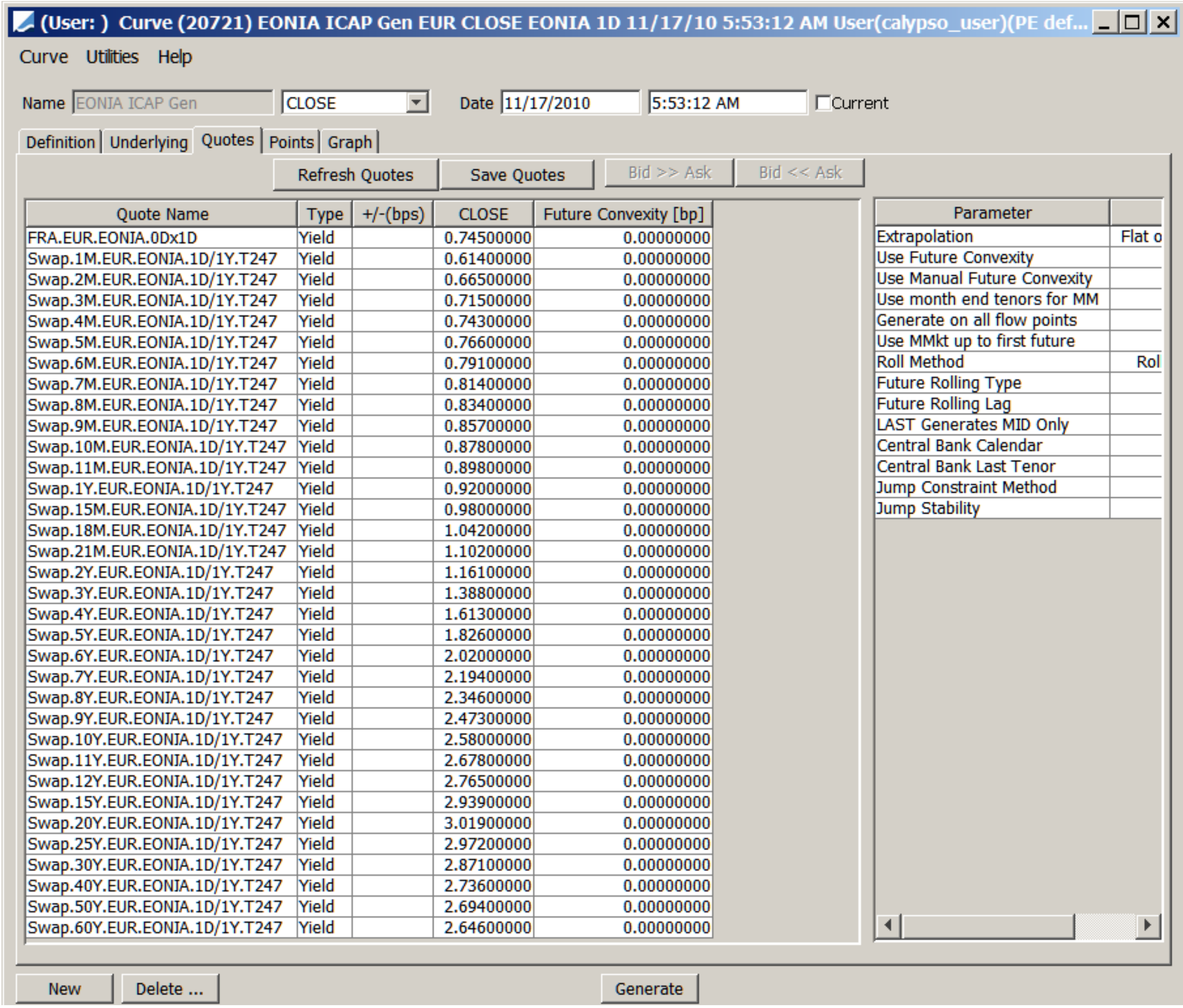
To construct an EONIA curve in Calypso, define swaps as curve underlyings on the EONIA index, as shown in the following screenshot. A curve is constructed from quotes for these swaps and, typically, a forward-rate agreement for the current overnight rate (which may differ from the day's fixing, depending on exogenous events).



The central bank meeting dates are stored in a Calypso calendar:



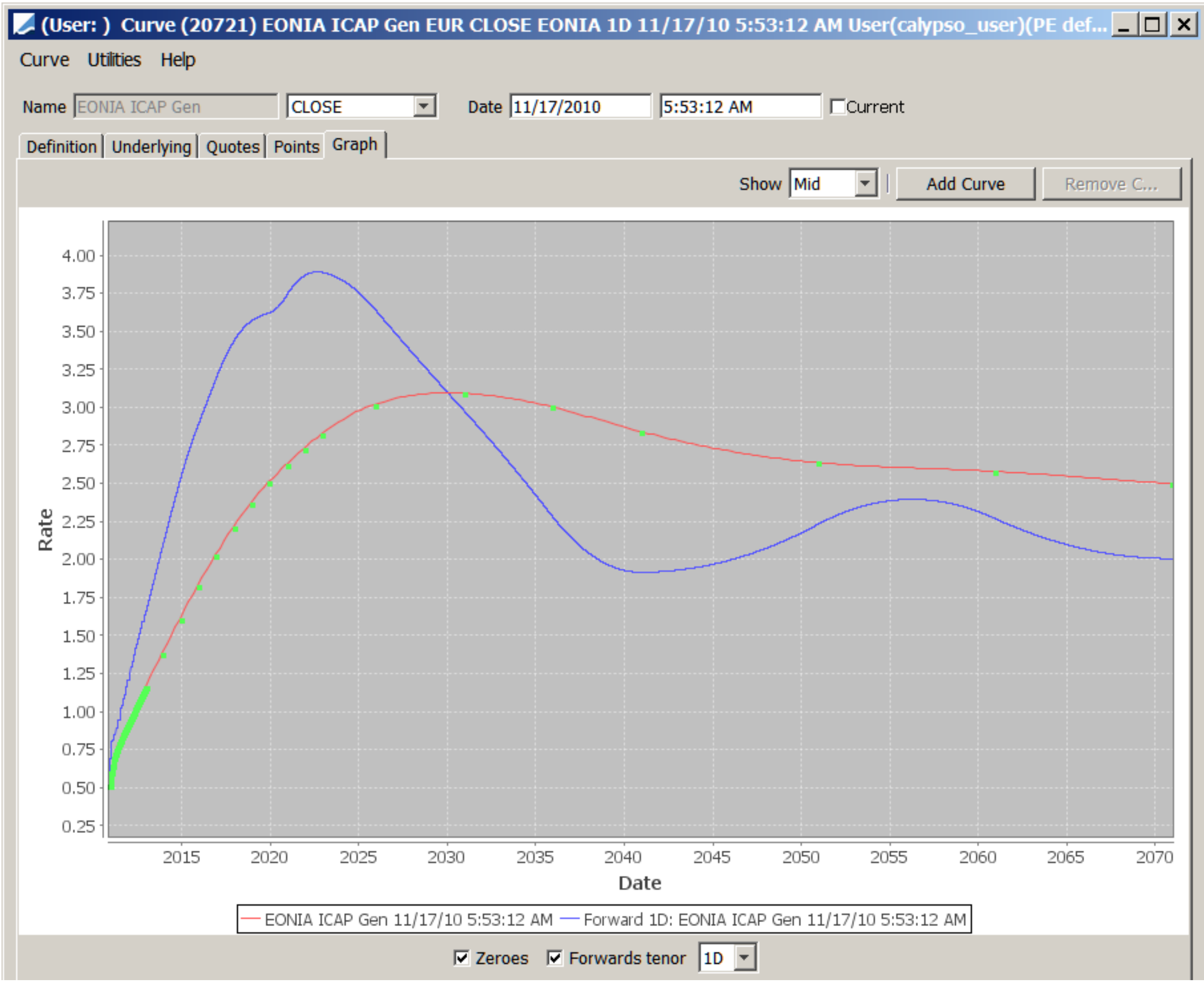
The following depicts the quotes for the EONIA curve:



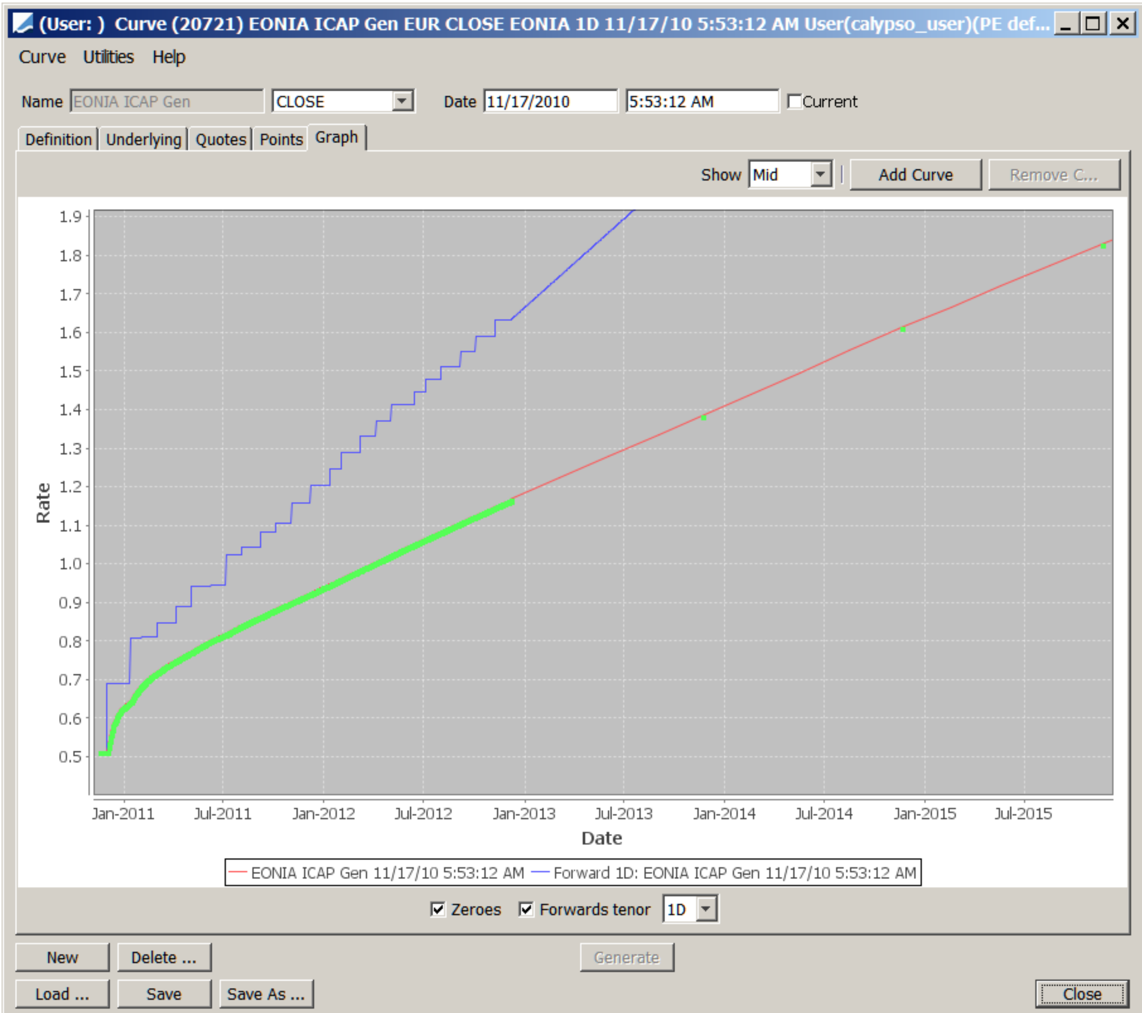
Quote Name	Type	+/-(bps)	CLOSE	Future Convexity [bp]
FRA.EUR.EONIA.0Dx1D	Yield		0.74500000	0.00000000
Swap.1M.EUR.EONIA.1D/1Y.T247	Yield		0.61400000	0.00000000
Swap.2M.EUR.EONIA.1D/1Y.T247	Yield		0.66500000	0.00000000
Swap.3M.EUR.EONIA.1D/1Y.T247	Yield		0.71500000	0.00000000
Swap.4M.EUR.EONIA.1D/1Y.T247	Yield		0.74300000	0.00000000
Swap.5M.EUR.EONIA.1D/1Y.T247	Yield		0.76600000	0.00000000
Swap.6M.EUR.EONIA.1D/1Y.T247	Yield		0.79100000	0.00000000
Swap.7M.EUR.EONIA.1D/1Y.T247	Yield		0.81400000	0.00000000
Swap.8M.EUR.EONIA.1D/1Y.T247	Yield		0.83400000	0.00000000
Swap.9M.EUR.EONIA.1D/1Y.T247	Yield		0.85700000	0.00000000
Swap.10M.EUR.EONIA.1D/1Y.T247	Yield		0.87800000	0.00000000
Swap.11M.EUR.EONIA.1D/1Y.T247	Yield		0.89800000	0.00000000
Swap.1Y.EUR.EONIA.1D/1Y.T247	Yield		0.92000000	0.00000000
Swap.15M.EUR.EONIA.1D/1Y.T247	Yield		0.98000000	0.00000000
Swap.18M.EUR.EONIA.1D/1Y.T247	Yield		1.04200000	0.00000000
Swap.21M.EUR.EONIA.1D/1Y.T247	Yield		1.10200000	0.00000000
Swap.2Y.EUR.EONIA.1D/1Y.T247	Yield		1.16100000	0.00000000
Swap.3Y.EUR.EONIA.1D/1Y.T247	Yield		1.38800000	0.00000000
Swap.4Y.EUR.EONIA.1D/1Y.T247	Yield		1.61300000	0.00000000
Swap.5Y.EUR.EONIA.1D/1Y.T247	Yield		1.82600000	0.00000000
Swap.6Y.EUR.EONIA.1D/1Y.T247	Yield		2.02000000	0.00000000
Swap.7Y.EUR.EONIA.1D/1Y.T247	Yield		2.19400000	0.00000000
Swap.8Y.EUR.EONIA.1D/1Y.T247	Yield		2.34600000	0.00000000
Swap.9Y.EUR.EONIA.1D/1Y.T247	Yield		2.47300000	0.00000000
Swap.10Y.EUR.EONIA.1D/1Y.T247	Yield		2.58000000	0.00000000
Swap.11Y.EUR.EONIA.1D/1Y.T247	Yield		2.67800000	0.00000000
Swap.12Y.EUR.EONIA.1D/1Y.T247	Yield		2.76500000	0.00000000
Swap.15Y.EUR.EONIA.1D/1Y.T247	Yield		2.93900000	0.00000000
Swap.20Y.EUR.EONIA.1D/1Y.T247	Yield		3.01900000	0.00000000
Swap.25Y.EUR.EONIA.1D/1Y.T247	Yield		2.97200000	0.00000000
Swap.30Y.EUR.EONIA.1D/1Y.T247	Yield		2.87100000	0.00000000
Swap.40Y.EUR.EONIA.1D/1Y.T247	Yield		2.73600000	0.00000000
Swap.50Y.EUR.EONIA.1D/1Y.T247	Yield		2.69400000	0.00000000
Swap.60Y.EUR.EONIA.1D/1Y.T247	Yield		2.64600000	0.00000000

Parameter	
Extrapolation	Flat o
Use Future Convexity	
Use Manual Future Convexity	
Use month end tenors for MM	
Generate on all flow points	
Use MMkt up to first future	
Roll Method	Rol
Future Rolling Type	
Future Rolling Lag	
LAST Generates MID Only	
Central Bank Calendar	
Central Bank Last Tenor	
Jump Constraint Method	
Jump Stability	

The resulting curve looks like this (Global with log spline discount factor interpolation):



The following is a close-up of the short end showing the jumps on the central bank meeting dates:



The central bank dates extend to either the end of the meeting calendar or to a user-specified time horizon. Daily points are placed on the curve during the central bank period; after that, the interpolator specified by the user defines the look of the forward curve (in this case, spline on the log of discount factors). The Global generator has parameters that control the construction of the central bank portion, that identify the calendar to use and the time horizon for its use, and two parameters that adjust the numerical method used in the solving. Recommended settings for EONIA swaps are as follows:

Central Bank Calendar	**ECB**	A Calypso Calendar Definition holding central bank meeting dates.
Central Bank Last Tenor	**2y**	The amount of the calendar to use.
Jump Constraint Method	**2**	Method for handling case of multiple meeting dates between instrument maturities.

Jump Stability	**100**	A stabilization factor to reduce effect of noisy or inconsistent input quotes.

It is important to note that the central bank 'stair-step' portion of the curve is highly sensitive to the input quotes, and these quotes are often not completely reliable. This can cause the solving to become unstable, with widely oscillating rates. To obtain a stable curve one may have to surrender exact calibration to market quotes. Calypso allows the user to select the trade-off between stability and calibration exactness through the Jump Stability parameter. A value of 100 creates a stable curve that might not calibrate exactly, while a value of 0 will exactly calibrate the curve but can create extremely unacceptable rates if the quotes are not consistent. The goodness of calibration fit can be seen (as described previously) using the log category GenGlobal.

For a detailed example of the swap deltas obtained from an EONIA curve versus a LIBOR curve, see the Calypso spreadsheet examples "Curve Construction Effect on Deltas.xls" and "Curve Construction Effect on Deltas Using EONIA Curves.xls".

7.2.1 Generator parameter: JUMP_CONSTRAINT_METHOD

When using a central bank calendar, the solution for the constant rate between meeting dates might not be unique, depending on the relation between meeting dates and underlying instruments. The ideal case is to have one meeting date between each pair of instrument end dates. This allows a one-to-one substitution of meeting dates for instrument dates, so the usual curve solving method is applicable. But two other cases can occur:

- Several meeting dates between instrument points. For example, the instruments may be money markets and swaps at 6M, 9M, 1Y, 18M and 2Y, while the meeting dates are monthly. Then there will be three to six meeting dates for each swap end date.
- More than one instrument date between meeting dates. For example, if using money market instruments at 1M, 2M, 3M, etc., a meeting date may fall just before the 2M maturity point and the next meeting may fall just after the 3M maturity, giving two instrument points between them.

The second case is handled in Calypso through a least-mean-squares fit. In that event the instruments cannot be guaranteed to price exactly back to their input quotes. However in practice this has not been seen to be a problem, as the market tries to price instruments consistently with the constant forward assumption. That is, if two instruments fall within one meeting period the market tends to use the single forward rate to price both instruments.

The first case creates greater ambiguities. At each meeting date there is a jump in the forward rate, and in this case there are multiple jumps between two instruments. What is to determine the size of these jumps?

The parameter JUMP_CONSTRAINT_METHOD provides a choice of constraints to place on the jump sizes so that they are completely determined and a unique solution can be obtained.

There are currently three choices, numbered 0, 1 and 2. The numbering serves as a mnemonic for their meanings.

- Method 0: Extra meetings that fall between instrument dates are simply dropped. Only the last meeting is retained. Thus there is only one jump to solve for between any two instrument end dates.

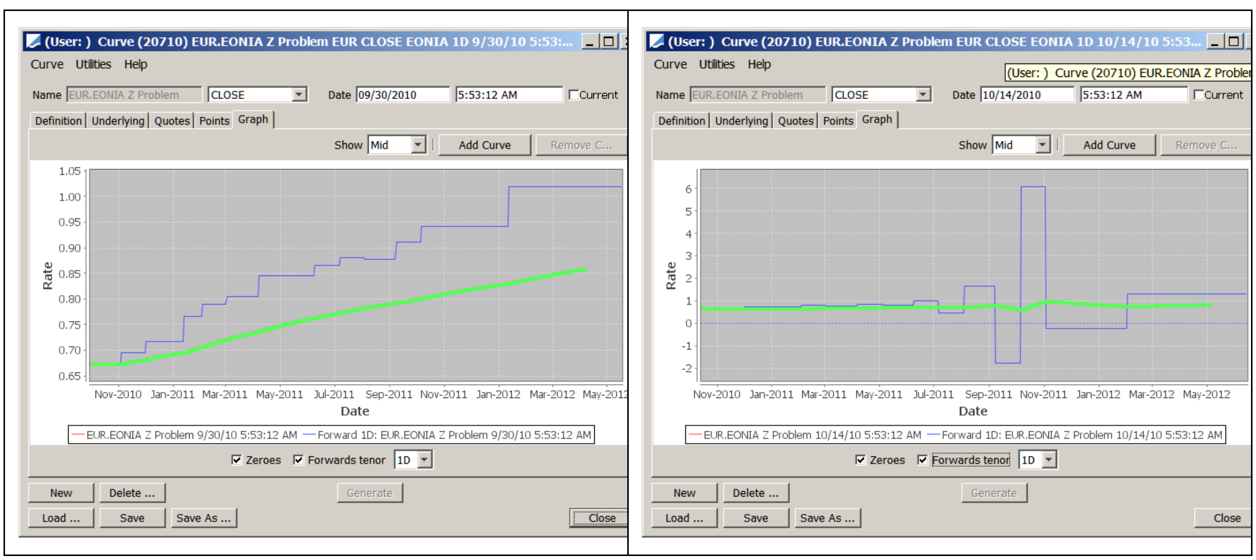
- Method 1: Linear jumps – the size of successive jumps fall along a line drawn between the first jump in each instrument period.
- Method 2: Equal jumps – jumps that fall between two instrument end dates are constrained to be of equal size.

The default is Method 2.

7.2.2 Generator parameter: JUMP_STABILITY

Due to timing differences, the solution for the constant forward rates between meeting dates is extremely sensitive to the incoming market quotes for swaps and money market instruments. If the meeting dates all exactly aligned with instrument dates there would be no difficulties. But consider what happens if each meeting date fell one day before each instrument date. The constant forward rate for that day is required to have a certain value in order to properly price the instrument maturing the next day, but then that same forward rate will extend into the next instrument period to the day before that next maturity date – and therefore will almost entirely determine the discount factor for that maturity date. This leaves just one day left for the next forward rate to correct the discount factor to price that swap correctly, which can be a substantial correction. This rate then applies to most of the next instrument period, which would require afterward a substantial one-day correction in the opposite direction , and so on, producing a wildly oscillating behavior.

This oscillation can occur because of inconsistent or noisy market quotes. It can also come from misdated quotes: a given curve may generate perfectly well with a set of quotes, but if the same quotes are rolled forward a week or two the curve can show oscillations. The following screenshots show this effect. The curve on the left was generated Sep 30 2010; the curve on the right uses the identical quotes but the generation date was changed to Oct 14 2010. The jumps are at European Central Bank meeting dates, which are absolute dates and so the same in both curves; when the generation date was changed, the maturity dates of the spot-starting underlying instruments changed with respect to the bank meeting dates, setting up the unstable quote condition. Note the different scale of the forward rates in the second curve, from -2% to 6%. (The instability is not related to the choice of JUMP_CONSTRAINT_METHOD, as it occurs even in the ideal case of one meeting date per instrument.)



In order to reduce or eliminate the effect of noisy data, Calypso employs a stability parameter which the user can set to improve the curve. As the shape of the curve is a subjective matter, Calypso does not attempt to automatically detect oscillations in order to determine an optimal parameter. The user can set a default value in the pricer configuration model parameters, and then override these for each curve after a visual inspection of the resulting curve.

The first step in the algorithm for the construction of the constant forward rates is the generation of a smooth curve from the instrument quotes, in the usual fashion. Then on this curve the central bank meeting dates are located and the discount factors at those points found using the curve's interpolator (as defined by the user). These discount factors are retained, the other points within the central bank portion of the curve are discarded (up to the "Last Central Bank Tenor"), and the constant forward rates between these discount factors are solved for. This is equivalent to using log-linear interpolation on these discount factors.

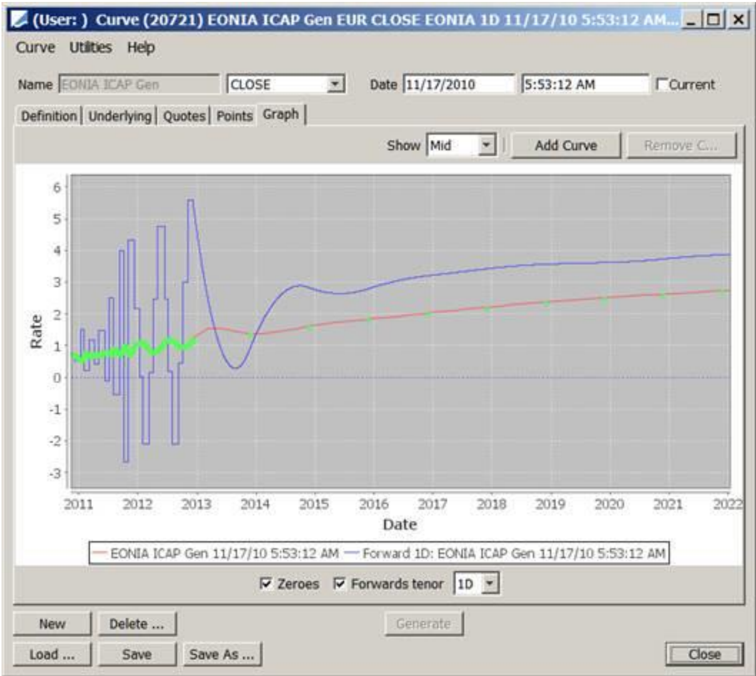
The resulting curve is a very nice "stairstep" curve for the central bank short end, which transitions into a smooth curve for the long end. While this is a good looking curve, it no longer calibrates exactly to the input market quotes, as this process has changed the interpolation. The calibration can still be very good; it is just not exact.

The next step then is to vary the discount factors all along the curve to improve the calibration. This is where the oscillation problem can occur. An exact calibration can produce a wildly oscillating curve; a rational curve can be found instead, but then the calibration is inexact.

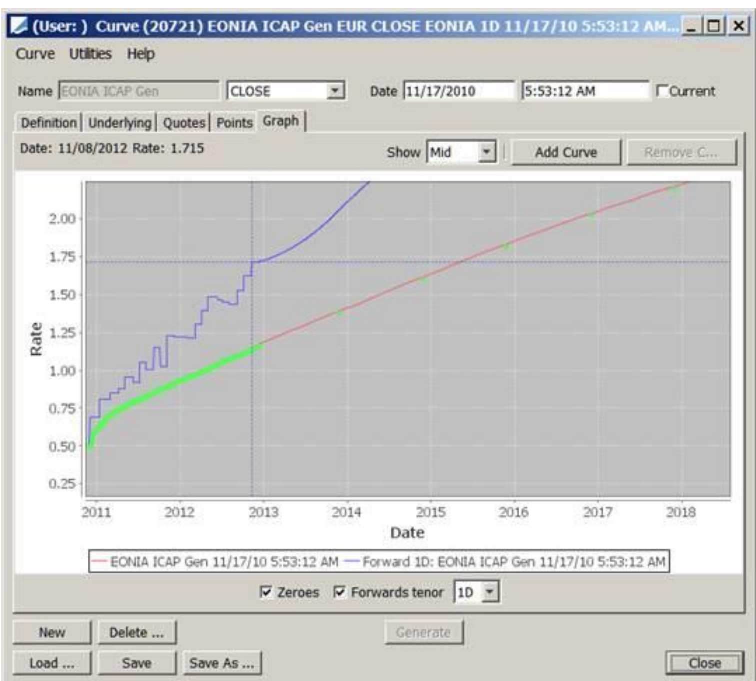
The stability parameter allows the user to interpolate between these two extremes. The parameter takes on the values of 0 to 100. A value of 0 means to apply no stabilization: it will perform the exact solution, even if an irrational curve results. A value of 100 means to stay with the completely stable but possibly inexact curve of the first construction step. Intermediate values will be a mixture of the two: that is, in solving for the discount factors some weight is given to the original, stable solution while trying to find an improved calibration.

The following example shows the effect of the stability parameter for a curve that has 'noisy' EONIA swap quotes. Each screenshot is preceded by the value of the stability parameter and the corresponding calibration error, given as the maximum difference, of all instruments, between the market quote and the breakeven rate found from the curve. Calibration is considered acceptable if the error is less than a basis point, and "exact" if the error is on the order of a tenth of a basis point or less. The first screenshot is the "exactly" calibrated curve, using no stabilization.

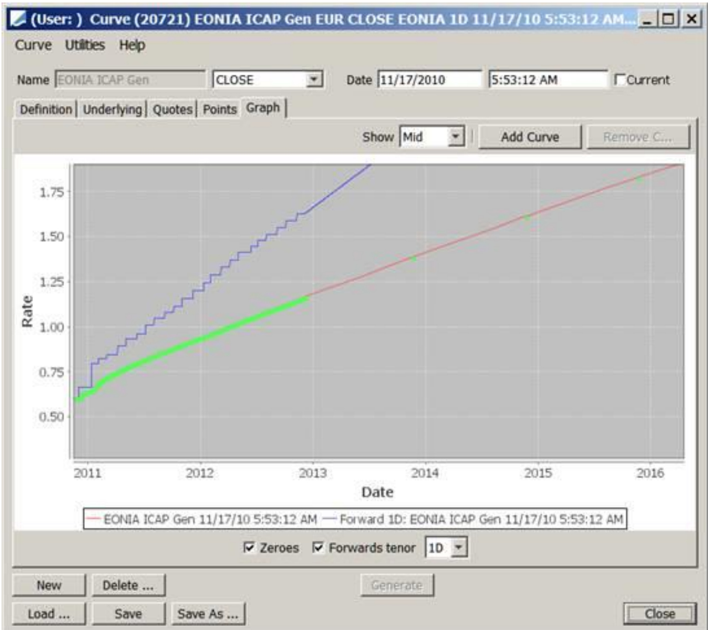
JUMP_STABILITY = 0. Calibration error (max) = 0.0002 bp



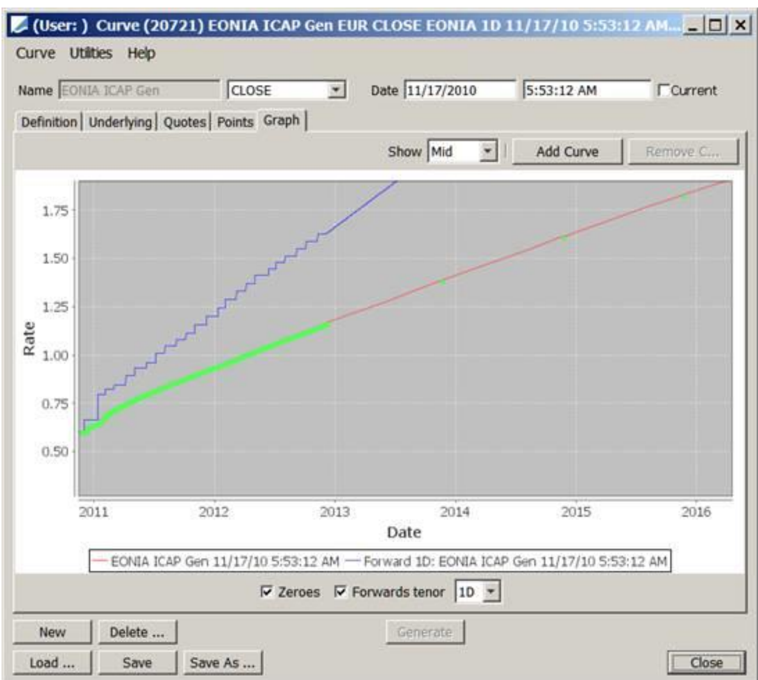
JUMP_STABILITY = 1. Calibration error <= 0.1 bp



JUMP_STABILITY = 10, Calibration error <= 0.25 bp



JUMP_STABILITY = 100. Calibration error <= 2.5 bp



The specifics of the curve are as followed.

Curve definition:

Global Generator

Interpolator Log Spline on Discount Factors

Generated on Nov 17 2010

Quotes:

FRA.EUR.EONIA.0Dx1D	Yield	0.74500000
Swap.1M.EUR.EONIA.1D/1Y.T247	Yield	0.61400000
Swap.2M.EUR.EONIA.1D/1Y.T247	Yield	0.66500000
Swap.3M.EUR.EONIA.1D/1Y.T247	Yield	0.71500000
Swap.4M.EUR.EONIA.1D/1Y.T247	Yield	0.74300000
Swap.5M.EUR.EONIA.1D/1Y.T247	Yield	0.76600000
Swap.6M.EUR.EONIA.1D/1Y.T247	Yield	0.79100000
Swap.7M.EUR.EONIA.1D/1Y.T247	Yield	0.81400000
Swap.8M.EUR.EONIA.1D/1Y.T247	Yield	0.83400000
Swap.9M.EUR.EONIA.1D/1Y.T247	Yield	0.85700000
Swap.10M.EUR.EONIA.1D/1Y.T247	Yield	0.87800000
Swap.11M.EUR.EONIA.1D/1Y.T247	Yield	0.89800000
Swap.1Y.EUR.EONIA.1D/1Y.T247	Yield	0.92000000
Swap.15M.EUR.EONIA.1D/1Y.T247	Yield	0.98000000
Swap.18M.EUR.EONIA.1D/1Y.T247	Yield	1.04200000
Swap.21M.EUR.EONIA.1D/1Y.T247	Yield	1.10200000
Swap.2Y.EUR.EONIA.1D/1Y.T247	Yield	1.16100000
Swap.3Y.EUR.EONIA.1D/1Y.T247	Yield	1.38800000
Swap.4Y.EUR.EONIA.1D/1Y.T247	Yield	1.61300000
Swap.5Y.EUR.EONIA.1D/1Y.T247	Yield	1.82600000
Swap.6Y.EUR.EONIA.1D/1Y.T247	Yield	2.02000000
Swap.7Y.EUR.EONIA.1D/1Y.T247	Yield	2.19400000
Swap.8Y.EUR.EONIA.1D/1Y.T247	Yield	2.34600000
Swap.9Y.EUR.EONIA.1D/1Y.T247	Yield	2.47300000
Swap.10Y.EUR.EONIA.1D/1Y.T247	Yield	2.58000000
Swap.11Y.EUR.EONIA.1D/1Y.T247	Yield	2.67800000
Swap.12Y.EUR.EONIA.1D/1Y.T247	Yield	2.76500000
Swap.15Y.EUR.EONIA.1D/1Y.T247	Yield	2.93900000
Swap.20Y.EUR.EONIA.1D/1Y.T247	Yield	3.01900000
Swap.25Y.EUR.EONIA.1D/1Y.T247	Yield	2.97200000
Swap.30Y.EUR.EONIA.1D/1Y.T247	Yield	2.87100000
Swap.40Y.EUR.EONIA.1D/1Y.T247	Yield	2.73600000
Swap.50Y.EUR.EONIA.1D/1Y.T247	Yield	2.69400000
Swap.60Y.EUR.EONIA.1D/1Y.T247	Yield	2.64600000

An important observation is that one cause of noisy data is that not all instruments are liquidly traded. ***One can avoid many cases of unstable curves by limiting the instruments to those most frequently traded: 6M, 9M, 1Y, 18M, and 2Y.***

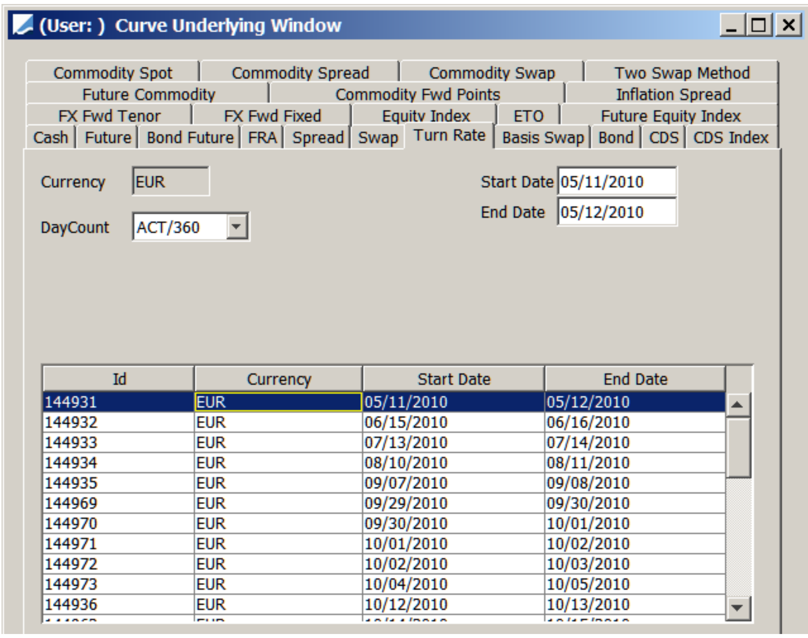
In the drop down menu for the JUMP_STABILITY parameter a useful selection of values is given. These are 100, 75, 50, 25, 10, 5, 1, .75, .25, .1, and 0. These are based on an exponential scale; as seen in the example, a factor of 10 creates a significant change in the curve if there are noisy quotes. Often, however, the quotes are not noisy, and a rational curve can be constructed even at a 0 stability parameter.

If no parameter is chosen, the default value is 100 (maximum stability).

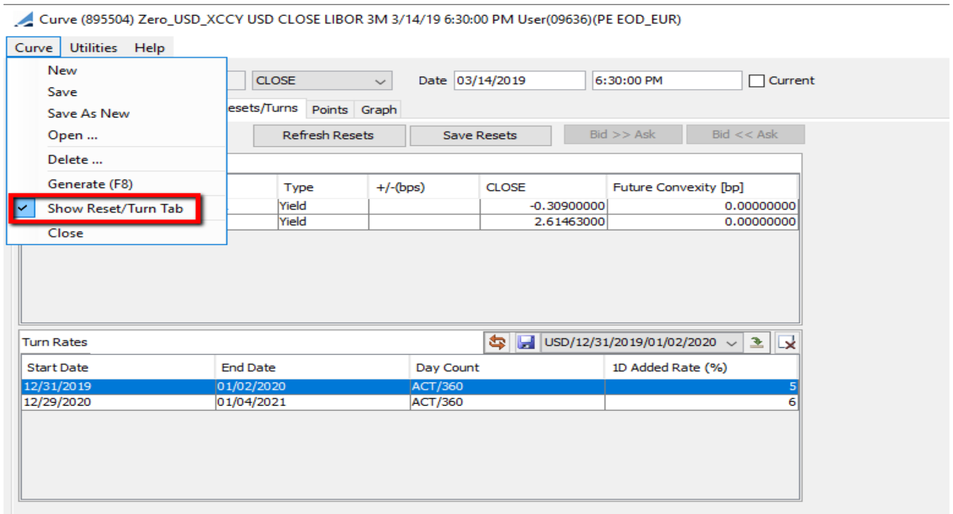
Section 8. Turn Rates

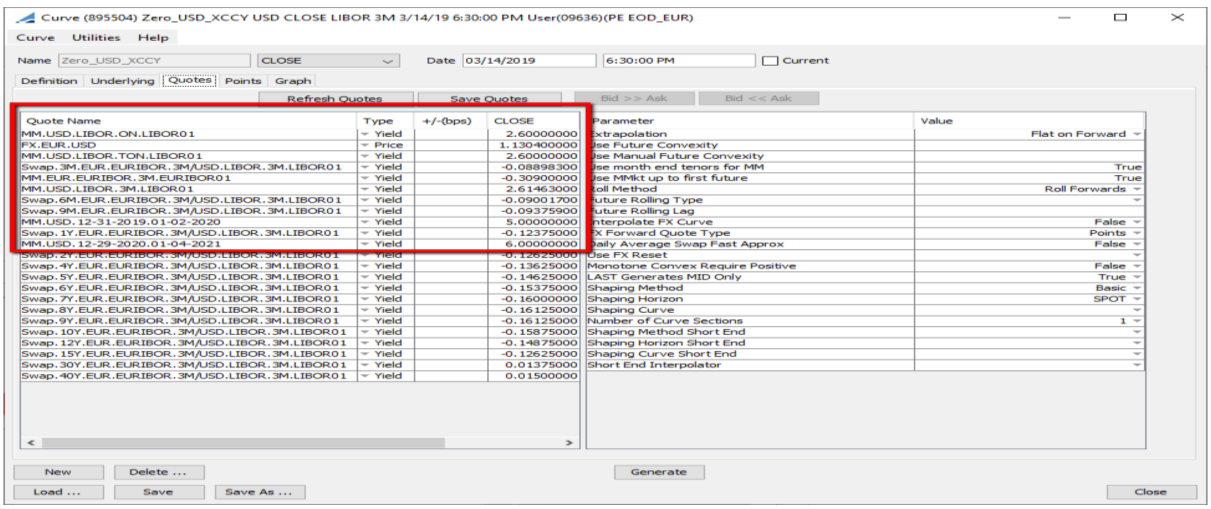
On certain dates the market has expectations of temporarily heightened overnight rates due to sharp changes in demand. These expectations need to be included in forward curve construction in order to properly price instruments that span these dates. The most common examples are the turn of the year and the end of central bank reserve periods.

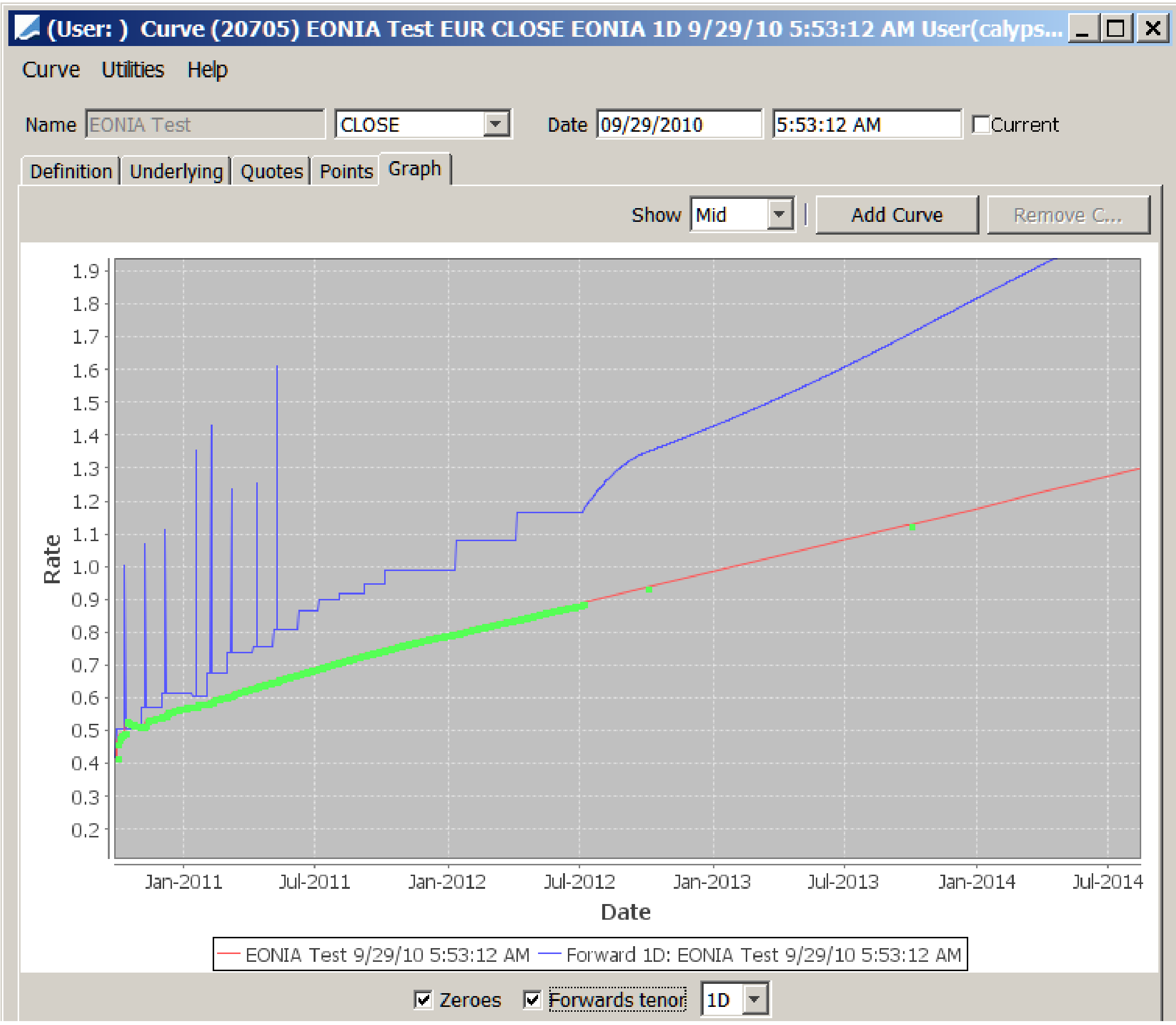
Calypso incorporates these expectations through user-defined turn rates. As shown in the following screen shot, a "Turn Rate" period is specified by means of two dates: the period will be *inclusive* of the Start Date and *exclusive* of the End Date, in the usual manner of accrual periods. The Day Count of the Turn Rate quote is specified.



Then in the Resets/Turns panel of the curve, you can select a Turn Rate and specify the quote which is the amount of yield enhancement to the overnight rate that would otherwise have been expected in the absence of the turn rate. If you hide the Resets/Turns panel, the reset/turn rate quotes are displayed in the Underlying panel, along with the quotes of the underlying instruments. For example, a Turn Rate quote of 0.50 means that there is expected to be a short term bump of 0.50% annualized in the rates.


Curve (895504) Zero_USD_XCCY USD CLOSE LIBOR 3M 3/14/19 6:30:00 PM User(09636)(PE EOD_EUR)
Curve
Utilities
Help
New
Save
Save As New
Open ...
Delete ...
Generate (F8)
Show Reset/Turn Tab
Close
CLOSE
Date
03/14/2019
6:30:00 PM
Current
Points
Graph
Refresh Resets
Save Resets
Bid >> Ask
Bid << Ask
Type
+/-(bps)
CLOSE
Future Convexity [bp]
Yield
-0.30900000
0.00000000
Yield
2.61463000
0.00000000
Turn Rates
USD/12/31/2019/01/02/2020
Start Date
End Date
Day Count
1D Added Rate (%)
12/31/2019
01/02/2020
ACT/360
5
12/29/2020
01/04/2021
ACT/360
6


Curve (895504) Zero_USD_XCCY USD CLOSE LIBOR 3M 3/14/19 6:30:00 PM User(09636)(PE EOD_EUR)
Curve
Utilities
Help
Name
Zero_USD_XCCY
CLOSE
Date
03/14/2019
6:30:00 PM
Current
Definition
Underlying
Quotes
Points
Graph
Refresh Quotes
Save Quotes
Bid >> Ask
Bid << Ask
Quote Name
Type
+/-(bps)
CLOSE
Parameter
Value
New
Delete ...
Load ...
Save
Save As ...
Generate
Close



Turn Rates are only supported by the Global generator. They can be applied either with or without central bank meeting dates.

For detailed information on the algorithm please see the later section "Turn Rates Algorithm in Yield Curve Construction."

Section 9. Simultaneous Curve Generation

As described in the section "Chain of Curves," if the instruments available in the market all require two or more different curves for pricing then the curves will have to be generated simultaneously rather than sequentially.

9.1 Multicurve Package

Calypso provides the ability to build two or three curves simultaneously. These are linked together in a "Multicurve Package." Curves in a package can only be generated simultaneously. When one curve is regenerated, as for example during a risk calculation, the other curves in the package are generated at the same time.

The Multicurve Package Window provides the means to define the curves. Each curve is specified individually with its own underlyings, quotes and parameters, and refers to other curves in the package and, optionally, to curves outside the package. When all curves have been specified the generator solves for them at the same time. When saving the package all the curves are saved at the same time.

Once saved the curves can be independently assigned for use in the PricerConfig. There is no need for their use to be connected, although for consistency one would want to assign them to the same uses they were employed for pricing during their generation.

9.2 Simultaneous Curve Algorithm

There generators for simultaneous curves are the DoubleGlobal for two curves and the TripleGlobal for three curves. There is also a generator DoubleGlobalSingle which makes only a single curve, not a Multicurve Package, and simply discards the other curve. This generator is used in the CurveZero window, which must be given all of the instruments for both curves. Examples of these generators are shown in the next sections.

The algorithm for simultaneous curve generation is just a repetition of the Global generator. These are the steps:

1. Make a trial guess for each curve by solving a fast bootstrap assuming the discount and forward curves are all the same in order to price the curve's instruments.
2. Starting with the trial guess, price all the trades in both curves. Compare with the quoted prices.
3. Improve the guess for both curves.
4. Repeat the pricing and improvement until a termination condition is met.

Like the Global generator the Levenberg-Marquardt multidimensional solver is used to make the iterative improvements.

9.3 DoubleGlobal

The DoubleGlobal generator will solve for two curves simultaneously and save them in a Multicurve package. Any regeneration of one of the curves will cause, via the link through the package, the simultaneous regeneration of the other. (In prior versions of Calypso this generator was called DoubleGlobalM; this name can still be used.)

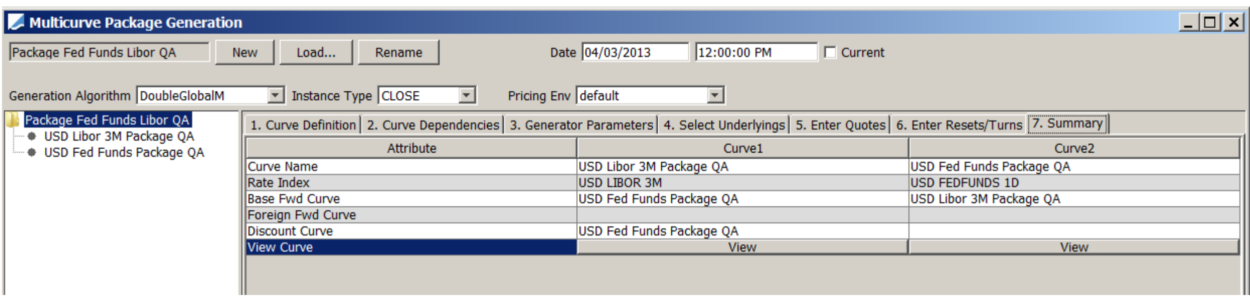
9.3.1 USD OIS and LIBOR Curves

As an example, consider the generation of the **USD OIS** curve. Because Libor swaps are currently quoted on an OIS discounting basis the swaps require both the forward Libor curve and the discount curve for pricing. The discount curve usually is created with Fed Fund instruments, but unlike the EONIA example in the “Chain of Curves” section, pure Fed Fund instruments are not presently liquid out beyond a few years. For longer maturities the most liquid instruments are Libor versus Daily Averaging Fed Fund basis swaps. As these also require two curves for pricing, the Libor and the Fed Fund curves must be generated simultaneously.

The standard instruments are described in Appendix 2, “The USD OIS Curve.”

The curves are created in the Multicurve Package window. As a first step a name is given to each curve in order that the other curve(s) can refer to it in the window. Then the curve parameters and underlyings are chosen and their references to each other defined.

The references among the curves are seen in the following screenshot of the Summary tab. One sees that the Libor curve has been defined with the Fed Fund curve as a both the discount and the base forward curve, that is, the forward curve for the non-Libor leg. The Fed Fund curve has been defined with the Libor curve as its base forward curve (non-Fed Fund curve). If the discount curve is not specified then the behavior is as in the single curve window, that is, the generated curve is used for discounting. In this example the Fed Fund curve does not specify its discount curve, so the Fed Fund curve itself will be used for discounting the instruments assigned to it.



The instruments for each curve are summarized in the following table:

Libor Curve	Fed Fund Curve
MM.USD.LIBOR.ON.T3750	Swap.USD.3M.FEDFUNDS.1D/LIBOR.3M.T3750
MM.USD.LIBOR.TON.T3750	Swap.USD.6M.FEDFUNDS.1D/LIBOR.3M.T3750
FRA.USD.LIBOR.1Dx91D.T3750	Swap.USD.9M.FEDFUNDS.1D/LIBOR.3M.T3750
FRA.USD.LIBOR.3Mx6M.T3750	Swap.USD.1Y.FEDFUNDS.1D/LIBOR.3M.T3750
FRA.USD.LIBOR.6Mx9M.T3750	Swap.USD.18M.FEDFUNDS.1D/LIBOR.3M.T3750
FRA.USD.LIBOR.9Mx1Y.T3750	Swap.USD.2Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.2Y.USD.LIBOR.3M/6M.T3750	Swap.USD.3Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.3Y.USD.LIBOR.3M/6M.T3750	Swap.USD.4Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.4Y.USD.LIBOR.3M/6M.T3750	Swap.USD.5Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.5Y.USD.LIBOR.3M/6M.T3750	Swap.USD.7Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.6Y.USD.LIBOR.3M/6M.T3750	Swap.USD.10Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.7Y.USD.LIBOR.3M/6M.T3750	Swap.USD.12Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.8Y.USD.LIBOR.3M/6M.T3750	Swap.USD.15Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.9Y.USD.LIBOR.3M/6M.T3750	Swap.USD.20Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.10Y.USD.LIBOR.3M/6M.T3750	Swap.USD.25Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.12Y.USD.LIBOR.3M/6M.T3750	Swap.USD.30Y.FEDFUNDS.1D/LIBOR.3M.T3750
Swap.15Y.USD.LIBOR.3M/6M.T3750	

Swap.20Y.USD.LIBOR.3M/6M.T3750	
Swap.25Y.USD.LIBOR.3M/6M.T3750	
Swap.30Y.USD.LIBOR.3M/6M.T3750	
Swap.40Y.USD.LIBOR.3M/6M.T3750	

9.3.2 AUD 3M and 6M Curves

An example is the AUD Libor 3M and 6M curves. In the Australian market the most liquid swaps are vanilla 3M swaps out to 3 years maturity, after which 6M vanilla swaps become the preferred instrument. The 3M x 6M basis swaps are traded in all maturities.

This means to build the 3M curve one would use the vanilla swaps to 3 years, but then one would use the 3M x 6M basis swaps and the 6M forward curve to create the 3M curve beyond 3 years. Similarly, one would build the 6M curve by starting with 3M x 6M basis swaps and the 3M forward curve to build the 6M curve out to 3 years, followed by 6M vanilla swaps for longer maturities.

So the

- 3M curve: Bill futures, then 3M vanilla swaps to 3 years, then 3M x 6M Basis swaps against 6M curve
- 6M curve: 3M x 6M basis swaps to 3 years against 3M curve, then 6M vanilla swaps

If one has a discount curve already generated it can be referenced by these curves for generation purposes. The discount curve will remain outside the package, that is, it is an "external" curve that will not be regenerated automatically with the curves in the package.

The references then will look like this:

Curve Usage	AUD 3M Curve	AUD 6M Curve
Base Forward Curve	AUD 6M Curve	AUD 3M Curve
Discount Curve	AUD Discount	AUD Discount

The following table summarizes the underlying instruments.

AUD 3M Curve	AUD 6M Curve
MM.AUD.BBR.ON.BBSW	Swap.AUD.1Y.BBR.3M/BBR.6M.BBSW
MM.AUD.BBR.1M.BBSW	Swap.AUD.2Y.BBR.3M/BBR.6M.BBSW
MM.AUD.BBR.2M.BBSW	Swap.AUD.3Y.BBR.3M/BBR.6M.BBSW
Future.AUD.SFE.IR.JUN.13	Swap.4Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.SEP.13	Swap.5Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.DEC.13	Swap.7Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.MAR.14	Swap.10Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.JUN.14	Swap.15Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.SEP.14	Swap.20Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.DEC.14	Swap.30Y.AUD.BBR.6M/6M.BBSW
Future.AUD.SFE.IR.MAR.15	
Swap.3Y.AUD.BBR.3M/3M.BBSW	

Swap.AUD.4Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.5Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.6Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.7Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.8Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.9Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.10Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.12Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.15Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.20Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.25Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.30Y.BBR.3M/BBR.6M.BBSW	
Swap.AUD.40Y.BBR.3M/BBR.6M.BBSW	

9.4 TripleGlobal

The TripleGlobal generator will solve for three curves simultaneously and save them in a Multicurve package. Any regeneration of one of the curves will cause, via the link through the package, the simultaneous regeneration of the others.

The principle use of three-curve generation is a market whose instruments are collateralized in another currency. This can require generating not only two forward curves but also a cross-currency discount curve.

Details of this construction are given in the section “Simultaneous Cross-Currency Curve Generation.”

9.5 Spread Curve type: CurveBasis in Multicurve

The type of a single curve in the Multicurve package can be chose to be either a discount factor type – “CurveZero” – or a spread type – “CurveBasis.”

Select the “Curve Type” field in the Multicurve window to choose the type. This is only enabled for newly created packages: one can’t change the curve type of an existing curve.

The CurveBasis acts as with a single curve, that is, it has one base curve over its entire length. So cannot define basis over only portions of the curve: in particular, you cannot switch roles of base/basis between two curves at different tenors, which one might have wanted to do for AUD 3m/6m curves.

One cannot have two basis curves where each has the other as base curve. There must always be a discount factor curve to spread from. One will receive an error message if this is attempted. However, one can have two basis curves as long as they refer to some other curve(s) as base. One can also have a basis curve use another basis curve as base, and so on, as long as the chain does not circle around.

9.6 DoubleGlobalSingle

The DoubleGlobalSingle generator creates only one curve rather than a package. It is used in the ordinary single curve window rather than the Multicurve Package window. It uses the same simultaneous curve generation procedure as the other generators, but it discards one of the curves at the completion of the generation. The remaining curve is displayed in the window and can be saved by itself.

The curve being generated is the *discount* curve. It is also the ‘basis’ curve for basis swaps. The curve that is discarded prices the base leg of the basis swaps.

In prior versions of Calypso this generator was called DoubleGlobal and was the first version of a simultaneous generator. Curves generated with the previous version need to have the name of their generator mutated in the database to “DoubleGlobalSingle” in order to be used in the single curve window. The name DoubleGlobal is now reserved for use in the Multicurve Package window.

Section 10. Collateralized Discount Curves

10.1 Pricing Collateralized Trades: Discount Curve for CSA

The pricing of collateralized trades relies on the choice of the correct curve for discounting cashflows. Standard practice is to discount a collateralized trade with a curve representing the cost of collateral, while an uncollateralized trade is discounted with a curve reflecting counterparty credit risk. In Calypso (since Version 13) the user can associate a discount curve to a trade and a combination of these parameters:

– Cashflow Currency
– Collateral Policy

The Collateral Policy is a name (a String type). It is usually the name of the currency to be delivered as collateral.

The assignments by Trade and Collateral Policy are made using the Pricer Config window. There are no restrictions on the curve that is assigned, and can be a Calypso generated curve, a user-generated curve, or an externally acquired curve.

A CSA agreement can be set on a trade to define the collateral currencies. The CSA agreement can have a good deal of other information, but for pricing purposes only the Collateral Policy is used. *Important: In order to use the policy in pricing the environment parameter COLLATERALIZED_PRICING must be set to "ON."*

When pricing a trade, the pricer will determine whether there is a CSA attached to that trade. If there is then the Collateral Policy is obtained. The default behavior assumes this policy is the name of a single currency. In that case the appropriate discount curve is found from the Pricer Config for that policy currency and cashflow currency.

To put into effect more complicated mappings of the Collateral Policy requires the creation of a pricer that implements the desired logic.

10.2 Collateralized Cross-Currency Discount Curves

When a trade is collateralized with a currency that differs from its settlement currency the discount curve must reflect the cost of translation between the currencies. The following describes a standard market approach to building the collateralized cross-currency curve.

10.2.1 Cross-Currency Curve Building

Before considering collateralization, let us review how a standard cross-currency curve is built in Calypso. The goal is to build a 'domestic currency' discount curve that correctly prices cross-currency basis swaps. The inputs to the calculation are:

– Foreign currency forward curve
– Foreign currency discount curve (which can be the same as the foreign forward curve)
– Domestic currency forward curve
– Cross-currency floating versus floating basis swaps
– (Optional) FX Forwards

The generator solves for the domestic currency discount curve that retrieves the market quotes for the basis swaps.

The curve can be built in Calypso either in a ‘Basis Curve’ window, or in the ‘Zero Curve’ window when a BasisBootStrap or BasisGlobal generator is chosen.

Some variations in the procedure include building a forward curve rather than a discount curve, and to use cross-currency swaps that are fixed versus floating or fixed versus fixed.

10.2.2 Collateralized Cross-Currency Curve

References:

M. Fujii and A. Takahashi, “Choice of Collateral Currency” (Univ. of Tokyo, Dec 2010)

If a single-currency trade is collateralized in a second currency, how should its discount curve be generated? After some debate the market has gravitated to the method described the works of Fujii, Shimada and Takahashi.

The argument can be summarized as follows.

When collateral is posted it will earn the compounded daily overnight interest for the firm at the agreed-upon collateral rate, typically the OIS rate. Denote the annual compounded OIS rate in currency 1 to be R_1 and in currency 2 to be R_2.

Consider the case where a firm borrows an amount X of Currency 1, and is required to post collateral Y in Currency 2. The amount to post is Y = S_{12} * X, where S_{12} is the spot exchange rate.

To obtain this amount, the firm can borrow Y by lending out its X amount by means of an OIS cross-currency swap with spread b. The firm will earn R_1+ b on X and pay out R_2 on the borrowed Y amount. At the same time, the borrowed Y can be deposited into the collateral account and earn R_2. Thus the firm will receive a total rate of R_1 + b on the amount X in currency 1.

Because there is no other credit risk on the overnight rate, R_1 + b must be the discount rate for currency 1. One concludes the correct rate is the overnight discount rate of Currency 1 modified by the cross-currency basis swap spread into currency 2.

The paper of Fujii and Takahashi is a rigorous treatment of this argument. Here we summarize their result, for the case where correlations between FX and interest rates are ignored. To create a curve for discounting cashflows paid in Currency 1 and collateralized in Currency 2:

- Start with the single-currency collateralized discount curve for Currency 2, usually the OIS curve.
- Obtain quotes for cross-currency basis swaps between Currency 1 and 2.
- Make use of the forward curves for the rate indexes of the two legs of the basis swaps. These are assumed to have been previously generated.
- Generate the Currency 1 discount curve using the classic cross-currency curve generation.

There are two differences between this method and uncollateralized cross-currency curve generation:

- The assumption that the swaps that price to par (zero NPV) and quoted by brokers are *collateralized* mark-to-market cross-currency swaps, rather than uncollateralized.
- The foreign discount curve is the collateralized curve (OIS curve) in that currency.

In the simplest deterministic model, Equation 2.5 of Fujii-Takahashi can be written

$$D^{i,j}(t) = D^{i}(t)\exp\left(-\int_{0}^{t} y^{ij}(t)dt\right)$$

where:

$D^{i}(t)$ = Discount factor for cashflows in currency *i* collateralized in native currency

$D^{i,j}(t)$ = Discount factor for cashflows in currency *i* collateralized in currency *j*

$D^{j}(t)$ = Discount factor for cashflows in currency *j* collateralized in native currency

$D^{j,i}(t)$ = Discount factor for cashflows in currency *j* collateralized in currency *i*

And the instantaneous cross-currency basis is

$$y^{ij}(t) = y^{i}(t) - y^{j}(t)$$

where y^i and y^j are the instantaneous credit spreads in each currency,

$$y^{i}(t) = r^{i}(t) - c^{i}(t)$$

for money market rate r^i and default-free collateral rate c^i. In terms of the collateral rates,

$$D_{i}(t) = \mathrm{e}^{-\int_0^t \mathrm{c_i}(s)ds}$$

$$D_{j}(t) = \mathrm{e}^{-\int_0^t \mathrm{c}_j(s)\mathrm{ds}}$$

Reversing the roles of i and j in the j-collateralization equation, and noting $y^{ij} = -y^{ji}$, one has

$$D^{j,i}(t) = D^{j}(t)\exp\left(\int_{0}^{t} y^{ij}(t)dt\right)$$

Multiplying this form of the equation by the original gives

$$D^{i,j}(t)D^{j,i}(t) = D^{i}(t)D^{j}(t)$$

or

$$D^{i,j}(t) = D^{i}(t)\frac{D^{j}(t)}{D^{j,i}(t)}$$

This converts a discount curve in one collateralization to that in another.

10.2.3 Simultaneous Cross-Currency Curve Generation

Single-currency swaps require two curves for pricing, a discount curve and a forward curve. Swaps can be quoted on either a collateralized or uncollateralized basis. If uncollateralized the discount curve could be taken to be the same as the forward curve to take into account credit risk. If collateralized in the swap currency the discount curve is generally taken to be the collateral rate curve, typically the OIS curve, which requires the discount and forward curves be generated in either a two-step or simultaneous approach.

What if the market only quotes swaps that are collateralized in a currency other than the swap currency? The discount curve would need to be the cross-currency collateralized discount curve described above, depending on the foreign collateral curve and built using cross-currency swaps. Since the forward curve is needed to create the cross-currency discount curve from cross-currency basis swaps, and that discount curve is needed to generate the forward curve from single-currency swaps collateralized in the foreign currency, these two curves must be generated in a simultaneous process that involves both cross-currency and single-currency swaps.

An even more complex example is the AUD market, where the liquid swaps not only are collateralized in USD but also switch tenors after three years. Then one must solve *three* curves simultaneously: the 3M forward curve, the 6M forward curve, and the cross-currency discount curve. The instruments used are as follows:

- 3M curve: Bill futures, then 3M swaps to 3 years, then 3M x 6M Basis swaps against 6M curve
- 6M curve: 3M x 6M basis swaps to 3 years against 3M curve, then 6M swaps
- XCCY discount curve: AUD vs USD 3M Libor XCCY swaps against the 3M AUD curve.

In order to price the USD side of the cross-currency swaps one makes use of the already-constructed USD Fed Funds curve (playing the role of the collateral curve) and the USD 3M Libor Forward curve.

10.3 Cheapest-to-Deliver Collateral Curves

A collateralized contract may have a CSA agreement that allows collateral to be delivered in any one of several currencies. This gives the option to choose the collateral currency that provides the most benefit to the deliverer. How does one value this option? The simplest approach is to discount the contract with a curve that blends the cross-currency collateral curves, choosing over each time segment the discounting that provides the most benefit according to the current estimation of the market.

This section describes the algorithm and the **CTDCollateral** generator for creating this blended discount curve under the assumption of deterministic interest rates. For the more complex case of probabilistic rates, see the later section on the Cheapest-to-Deliver Collateral Option Generator.

10.3.1 The Cheapest-to-Deliver Collateral Curve Algorithm

Consider a contract which makes payments in currency P and which, according to the CSA agreement, can be collateralized in either Currency A or Currency B. The collateral currencies A and B can be the same or different from the payment currency P. What discount curve should be used to value the contract?

We will assume that the discount curves for payments under each of the collateral currencies have been separately generated. That is, there are two curves of discount factors:

D(P,A; t) = discount factor for payment made at time t in currency P collateralized in currency A

D(P,**B; t**) = discount factor for payment made at time t in currency P collateralized in currency B

The construction of these curves is described in other documents and follows the Fujii-Takahashi procedure. That is, if currency A is not the same as P, then one first finds a collateralized discount curve for payments in currency A, then use cross-currency swaps from A to P to transform this curve into a discount curve for currency P. This may require a simultaneous curve generation procedure to create forward curves in currency P at the same time as the cross-currency discount curve.

Given these two curves, the goal is to derive a curve to be used throughout the life of the contract. The following assumptions are made:

- Deterministic interest rates: No stochastic interest rate model will be used.
- Bilateral CSA agreement: Both parties to the contract will deliver the cheapest collateral when required

- Cash collateral
- No thresholds, no haircuts.
- Bid/ask spreads will be neglected

Besides these calculation assumptions, there are also some practical requirements:

- The curve is to be created on demand for a particular contract, not built and stored in a database. This avoids the complication of having to maintain many different curves for all CSA currency combinations.
- Retain policy choices, that is, have the ability to choose to use the blended curve or a specific collateral currency.

These considerations require a simple and fast algorithm to find the blended cheapest to deliver curve. The following is the proposed construction.

1. Select a set of points from the current date to the maturity date of the swap, for example, every three months.
2. At each point t_i, find the three-month forward rates for each curve using the rate index associated to the curve. This will give a set of two forward rates for each point:

$$f_i(A) = \left(\frac{D(P,A;t_i)}{D(P,A;t_{i+1})} - 1\right)\frac{1}{(t_{i+1} - t_i)}$$

$$f_i(B) = \left(\frac{D(P,B;t_i)}{D(P,B;t_{i+1})} - 1\right)\frac{1}{(t_{i+1} - t_i)}$$

Type equation here.

The time differences written in shorthand as $t_{i+1} - t_i$ will depend on the day counts of each rate index.

3. At each point t_i, one wants to find the greater of the two forward rates, in order for the poster of collateral to earn the greatest return. However, as these are in different day counts, instead compare the wealth factors that indicate the interest that would accrue over the three-month period. Take the maximum of the wealth factors.

$$W_i = \max\left((1 + f_i(A))(t_{i+1} - t_i), (1 + f_i(B)(t_{i+1} - t_i)\right)$$

In terms of discount factors, this is

$$W_i = \max\left(\frac{D(P,A;t_i)}{D(P,A;t_{i+1})}, \frac{D(P,B;t_i)}{D(P,B;t_{i+1})}\right)$$

4. Compound the wealth factors to find discount factors at each point:

$$D(P;t_i) = \prod_{j=0}^{i} \frac{1}{W_j}$$

5. The set of discount factors $D(P; t_i)$ define the blended cheapest-to-deliver curve.

The procedure is readily extended to more than two collateral currencies.

This method can be seen as an implementation of the collateral choice equation in Fujii-Takahashi (see reference above). This equation states that the factor for discounting a cashflow that can be collateralized in several different currencies is

$$E_t^{Q^j}\left[\exp\left(-\int_t^T \max_{i\in C}\{y^{ji}(s)\}ds\right)\right]$$

For deterministic discount curves, and approximating the integral, one arrives at the described method.

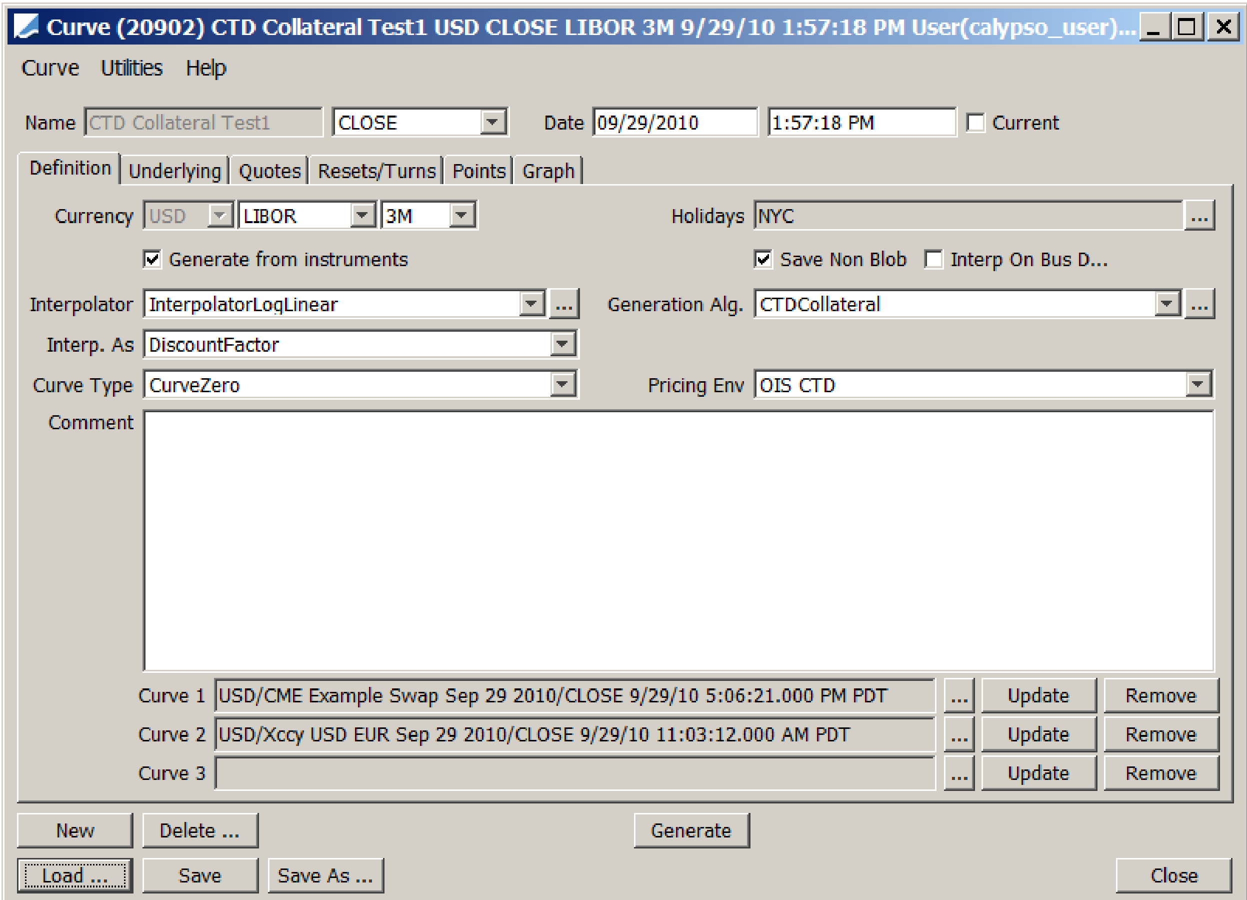
10.3.2 Generating the Cheapest-to-Deliver Curve

The generator "CTDCollateral" builds a cheapest-to-deliver collateral curve by blending up to three other collateral discount curves. To generate the blended curve, perform the following steps.

- Open the CurveZero window.
- Select the CTDCollateral generation algorithm.
- Select an interpolator and specify if it is on Discount Factors or Rates.
- In the fields at the bottom of the screen, select Curve1, Curve2 and, optionally, Curve3. These should be the discount curves for the curve currency for delivery of collateral in the various CSA deliverable currencies.
- Select "Generate."
- Save the curve under any desired name.

The dates on the curve will include the union of the dates on all the underlying curves, in order to preserve their shape information as much as possible. In addition, dates are created every three months. The dates from the curve tends to capture the short end shape, while the three-month points are most useful for the long end.

The following screenshot shows the curve setup with the CTDCollateral generator.

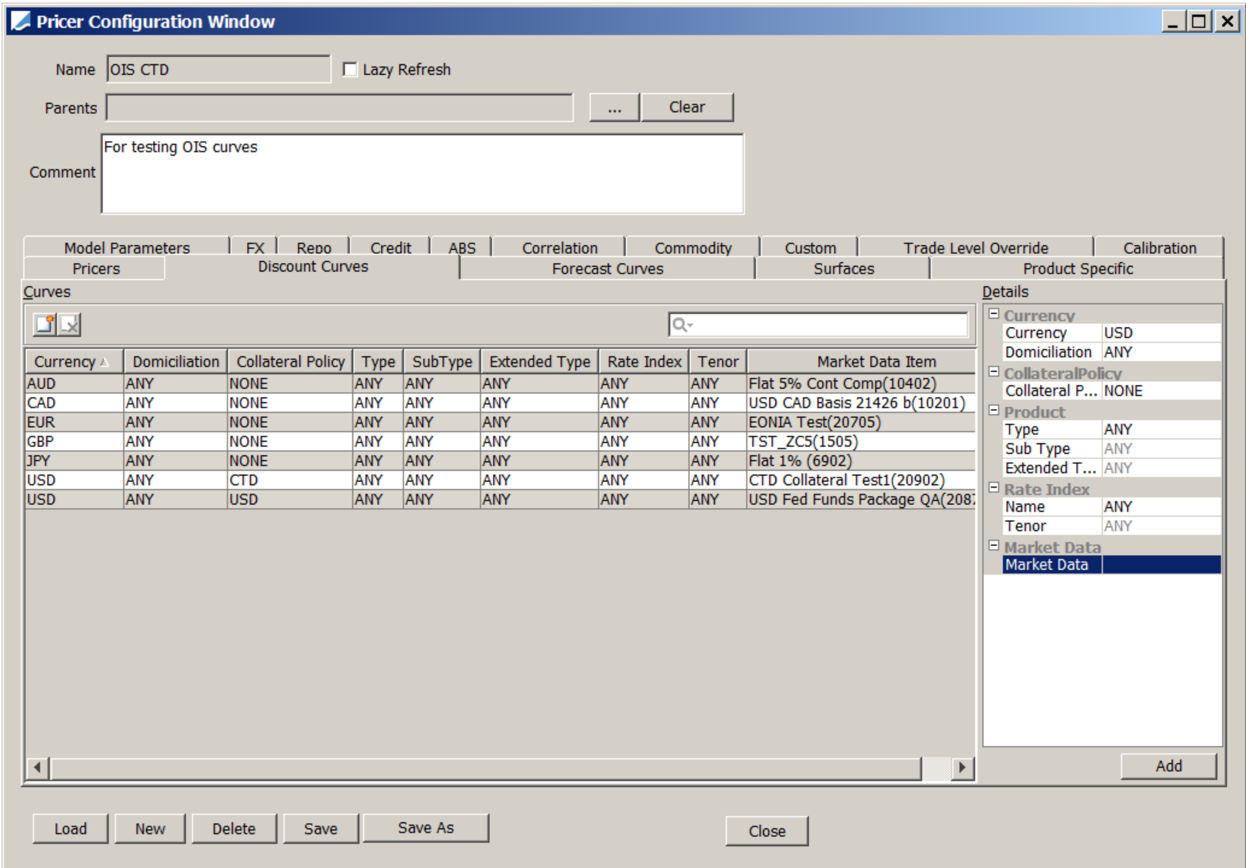


10.3.3 Pricing with the Blended Curve

The cheapest-to-deliver curve is used in the same manner as any other discount curve. All that is required is that the trades that are desired to be discounted with this curve are properly identified. To do this:

- Assign the cheapest-to-deliver curve to the desired currency and Collateral Policy in the Pricer Config. Any name for the Collateral Policy may be chosen according to the user's protocols.
- Trades that are to use this curve should be given a CSA agreement with the same Collateral Policy defined. The pricer will look at the trade's CSA and find the correct discount curve with the trade's currency and that CSA's policy name in the Pricer Config.

Below is a simple example of a Pricer Config with two discount curves for USD. One curve is used with the Collateral Policy named "USD", which may be associated to agreements that only allow delivery of USD collateral. The other curve is associated with a Policy named "CTD", which may indicate the agreement allows several currencies to be delivered as collateral. The curve associated with the CTD policy is the blended cheapest-to-deliver curve that was shown in the previous section.



10.3.4 Risk Analysis Tips

The sensitivity analysis report will show sensitivity to *all of the curves* used to generate the cheapest-to-deliver curve, if:

- the analysis is performed by bumping underlying quotes (not zero rate points), and
- **"Generate Dependents"** is selected in the analysis parameters.

An important caution: A curve that is an underlying of the underlying curves will *only* appear in the risk analysis report if it is *also* in the Pricer Config under which the report is run. For example, if one of the curves in the cheapest-to-deliver curve was itself made with another discount curve, that latter discount curve will not appear in the risk report unless it is in the Pricer Config.

"FX Adjust Curve Not Found" error: If the ADJUST_FX parameter of the Pricing Environment is set to True and risk is being run on a single-currency portfolio, it is possible that an error can be shown during the risk run stating the FX adjust curve was not found. This is avoided by making sure that a discount curve for a currency appears both in the Pricer Config and in one of the curves the cheapest-to-deliver curves depends upon. The reason is, an FX conversion is done from the trade currency into the currency of the instrument whose quote was bumped during sensitivity analysis. If a discount curve for this currency is in the Pricing Environment then this will be used for adjustment of the exchange rate from spot FX. But if that environment discount curve was not one of the curves that will be regenerated in the risk analysis, the risk process will remove that curve from the environment, and it won't be available for the ADJUST_FX procedure.

10.4 CTD Curve Generator with Optionality Valuation

The discount curve based on delivering the cheapest collateral is very simply derived for deterministic interest rates using the CTDCollateral generator described in the preceding sections. If instead stochastic interest rates are assumed, the curve generation in general requires Monte Carlo simulation, which is quite slow, especially for the repeated curve generation needed for risk analyis.

Sankovich and Zhu [Ref 1] proposed an attractive alternative to Monte Carlo simulation. They derived a semi-analytical approximation of the discount factors that incorporate the collateral option value, consistent with the assumed dynamics and partially based on calibration to historical data. Calypso uses this approximation in the **CTDCollateralOption** generator to calculate an appropriate blended curve for a given basket of currencies.

10.4.1 CTDCollateralOption Curve Generator

The CTDCollateralOption generator is used a similar way to the CDTCollateral generator described in the preceding section. After choosing the generator name, one can select two or three curves of different currencies to blend. The additional requirement is that a set of parameters describing the interest rate model must also be entered.

The Hull-White short rate model is used to model each of the currencies. It is assumed the user has already calibrated these using the Calypso LGM model calibration or another system. These are entered as generator parameters as values separated by commas, as follows:

"Short Rate Volatilities"	Vol(1),Vol(2)[,Vol(3)]
"Short Rate Mean Reversions"	MR(1),MR(2)[,MR(3)]
"Short Rate Correlations"	R(12)[,R(13),R(23)]

For the model volatility in each currency, the user substitutes the desired values for Vol(1) and Vol(2) and, if there is a third curve, for Vol(3). These are given as decimal values, not percentages; that is, a value 0.2 is entered to represent a 20% volatility. The mean reversions are similarly entered for the two or three currencies. If there are only two currencies only a single correlation value is entered, but three currencies require the three pairwise correlations. These, too, are entered as decimals rather than percentages.

10.4.2 Outline of the Algorithm for Blending with Stochastic Rates

Suppose the CSA allows the collateral poster to choose from a basket of *N* currencies. Assume that the funding short rate $r_i(t)$ of each individual currency follows a Hull-White process, so that *N* correlated one-factor Hull-White processes are involved. The goal is then to find the discount factors of a blended cheapest-to-delivery (CTD) curve for any time t given by:

$$DF(t) = E_0\left[\exp\left(-\int_0^t \max_{i=1,\dots,N}\big(r_i(s)\big)ds\right)\right] \qquad 1$$

To avoid the use of Monte Carlo simulation, Sankovich and Zhu find a semi-analytical approximation. The calculation proceeds through the following steps in succession.

The short rate process

The short rate $r_i(t)$ of each one of the N currencies ($i = 1, \dots, N$) is taken to follow a Hull-White process:

$$dr_i(t) = [\theta_i(t) - a_i r_i(t)]dt + \sigma_i dW_i(t) \qquad 2$$

where the $W_i(t)$ are correlated standard Brownian motions, with correlation $\rho_{i,j}$ between each pair (W_i, W_j). As stated in the description of the generator parameters, it is assumed the mean reversions a_i, volatilities σ_i, and correlations $\rho_{i,j}$ are given for all i, j.

Estimating the time-dependent Hull-White parameters

The first step is to calculate the time-dependent functions $\theta_i(t)$. To do this, Calypso uses the term structures of the curves to be blended:

$$\theta_i(t) = \left.\frac{\partial F_i}{\partial t}\right|_{0,t} + aF_i(0,t) + \frac{{\sigma_i}^2}{2a_i}(1 - e^{-2a_i t}) \qquad 3$$

where $F_i(0, t)$ is the instantaneous forward rate for maturity t as seen at time 0 for currency i.

Approximation of the maximum short rate

Under the proposed Hull-White dynamics, the short rates $r_i(t)$ follow a normal distribiution. Then the expression for the blended discount factors involves the expected maximum rate from among all N short rates. This raises the question of evaluating the distribution of the maximum of N Gaussian random variables.

When $N = 2$, the answer is known exactly, as shown by Clark [Ref 3]. When $N > 2$ there is no exact form for the distribution of the max of N normal random variables, so we need to use an approximation. Sankovich and Zhu propose a combination of Clark's approximation (which is based on only the first two moments) with the Gram-Charlier adjustments (which take into account the third moment effects). This is an iterative procedure described in detail in their paper; see also [Ref 2]. A Calypso white paper is also available.

When the procedure has been completed, the result is the first three moments $(\mu_{MAX,N}, \sigma^2_{MAX,N}, k_{MAX,N})$ of the distribution of the maximum of the N short rates.

Calculating the integral of the maximum

The next step is to integrate the estimated distribution of the maximum, so that one can then take the expected value and get the desired discount factors $DF(t)$. The integral to evaluate is:

$$Y(t) = \int_0^t \max_{i=1,\dots,N} \big(r_i(s)\big) ds$$

The method of Sankovich and Zhu uses the calculated moments of the maximum to estimate the first three moments of $Y(t)$. Through their procedure, they arrive at the moments $\mu_Y(t)$, $\sigma_Y(t)$, $k_Y(t)$ for $Y(t)$.

Next one $Y(t)$ with a quadratic function of a standard normal distribution

$$Y(t) \approx a(t)z^2 + b(t)z + c(t), \quad z \sim \mathcal{N}(0,1)$$

It can be shown that the best fit to the estimated moments for Y is obtained when the coefficients a, b and c satisfy:

$$4a^3 - 6\sigma_Y^2 a + k_Y = 0, \quad b^2 = \sigma_Y^2 - 2a^2, \quad c = \mu_Y - a$$

Finally, having solved for a, b and c, the discount factors are given by:

$$DF(t) = E[\exp(-Y(t)] = \frac{\exp\left(\frac{b^2(t)}{2(1+2a(t))} - c(t)\right)}{\sqrt{1+2a(t)}}$$

This is the generated discount factor from today to time T that incorporates the value of the option to switch collateral currency at any time up to time T.

References

1. Sankovich, Vladimir and Zhu, Qinghua, "Collateral Option Valuation Made Easy", *Risk,* October, 2015

2. Sinha, Debjit, Zhou, Hai and Shenoyy, Narendra V., "Advances in Computation of the Maximum of a Set of Random Variables", IEEE, 2006, *7th International Symposium on Quality Electronic Design*.

3. Clark, Charles E., "The greatest of a finite Set of Random Variables", *Operations Research,* Vol. 9, No. 2 (1961), pp. 145-162.

10.5 Triangulation of Collateralized Discount Curves

Collateralization of trades under a CSA agreement can be allowed to take place in several different currencies. However, market quotes may only be available for trades in some of the allowed collateral currencies. To price trades in any collateral currency, a triangulation method can be used to convert discount factors from one collateral currency to another.

10.5.1 The Goal

The situation is that the bank has trades in currency A that are collateralized in currency B. There are no liquid quotes for any vanilla instruments in currency A with B collateralization, so a collateralized discount curve cannot be built directly.

If there were cross-currency basis swaps in A versus B that are collateralized in A, then one could generated the appropriate discount curve by combining the basis spread with the A-currency OIS curve. The problem arises when there are no such cross-currency swaps.

The problem can be resolved if there are liquid trades in both A and B currency that are collateralized in a third currency, C. By going through C one can “triangulate” the valuation of the A trades collateralized in B.

$$D_{A/B}(T_0, T) = \frac{D_{A/C}(T_0, T)}{D_{B/C}(T_0, T)} D_{B/B}(T_0, T).$$

Each term of the form $D_{X/Y}(T_0, T)$ is a discount factor for currency X collateralized in currency Y. This formula states that one can divide the two discount curves for C collateralization and multiply by the B OIS curve to obtain the curve for B collateralization in currency A.

This relation is proved in the following sections.

The idea would be to transact in liquid instruments in A with C collateral, convert A to B using the FX markets to create C-collateralized B flows, convert these to B-collateralized B currency flows, and then use FX again to convert these to B-collateralized A currency flows.

Denote the curves by:

$$X/Y = Curve\ for\ discounting\ currency\ X\ cashflows\ collateralized\ by\ currency\ Y$$

The following curves are assumed to be able to found from liquid market quotes:

B/B	Currency B curve collateralized in B
A/C	Currency A curve collateralized in C
B/C	Currency B curve collateralized in C

The goal is to switch the collateralization of the A curve from currency C to currency B, which is an illiquid market. That is, find the curve:

A/B	Currency A curve collateralized in B

10.5.2Cross-Currency Calculations

Denote the curves by:

$$X/Y = Curve\ for\ discounting\ currency\ X\ cashflows\ collateralized\ by\ currency\ Y$$

The situation of section 1.1. is that market quotes allow direct computation of the following curves:

B/B	Currency B curve collateralized in B
A/C	Currency A curve collateralized in C
B/C	Currency B curve collateralized in C

The goal is to switch the collateralization of the A curve from currency C to currency B. That is, find:

A/B	Currency A curve collateralized in B

The primary assumption is this: Two cashflows in currency A and B should have present related to FX forward rates by the interest rate parity calculations, as long as they are collateralized in the same currency.

The interest rate parity equation for the forward FX rate $R_{fwd}(T)$ given the spot rate R_{SPOT} is (using MID rates):

If T >= T_{spot} (forward projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_{\text{Primary}}(T_{SPOT},T)^{MID}}{D_{\text{Secondary}}(T_{SPOT},T)^{MID}}, \quad T > T_{SPOT}$$

If $T < T_{spot}$ ("backward" projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_{\text{Primary}}(T,T_{SPOT})^{MID}}{D_{\text{Secondary}}(T,T_{SPOT})^{MID}}, \quad T < T_{SPOT}$$

The discount factors are for the primary and secondary currencies respectively, where the exchange rates are the amount of secondary currency per unit of primary currency.

In the present application, the discount factors for the A and B currencies are both collateralized in C. This gives, for the A to B exchange rate:

$$\frac{R_{AB,SPOT}}{R_{AB,fwd}(T)} = \frac{D_{A/C}(T_{SPOT},T)}{D_{B/C}(T_{SPOT},T)}$$

Similarly, for collateralization in B, the parity equation is

$$\frac{R_{AB,SPOT}}{R_{AB,fwd}(T)} = \frac{D_{A/B}(T_{SPOT},T)}{D_{B/B}(T_{SPOT},T)}$$

The assumption is the exchange rates are independent of the collateralization. So these two expressions can be equated to give:

$$D_{A/B}(T_{SPOT},T) = \frac{D_{A/C}(T_{SPOT},T)}{D_{B/C}(T_{SPOT},T)} D_{B/B}(T_{SPOT},T), \quad T > T_{SPOT}\,.$$

The same form of the equation applies to times before the spot date, for, as seen above, at these times the ratio of discount factors is inverted, and as this occurs in both equations the result is to have no change in the form of the combined equation. That is:

$$D_{A/B}(T,T_{SPOT}) = \frac{D_{A/C}(T,T_{SPOT})}{D_{B/C}(T,T_{SPOT})} D_{B/B}(T,T_{SPOT}), \quad T < T_{SPOT}\,.$$

The discount factors from the curve date T_0 to date T is then obtained by multiplying these two equations together, to obtain:

$$D_{A/B}(T_0,T) = \frac{D_{A/C}(T_0,T)}{D_{B/C}(T_0,T)} D_{B/B}(T_0,T)\,.$$

This is the formula used by the Triangulation generator.

Bid/Ask Relationships

It is usually advisable to use Mid rates in interest rate parity calculations, as the parity arguments using Bid and Ask rates are ambiguous: the derivations produce inequalities rather than equalities. The results using Bid and Ask can then be inconsistent, either producing spreads that are unnaturally wide or in which the Bid and Ask are the reverse of the normal relationships and can be arbitraged.

Nonetheless, the Triangulation generator will compute using Bid and Ask discount curves if required. The relations it uses are:

$$D_{A/B}(T_{SPOT},T)^{BID} = \frac{D_{A/C}(T_{SPOT},T)^{BID}}{D_{B/C}(T_{SPOT},T)^{ASK}} D_{B/B}(T_{SPOT},T)^{ASK}$$

and

$$D_{A/B}(T_{SPOT},T)^{ASK} = \frac{D_{A/C}(T_{SPOT},T)^{ASK}}{D_{B/C}(T_{SPOT},T)^{BID}} D_{B/B}(T_{SPOT},T)^{BID}.$$

10.5.3Relation to the Two-Currency case

For the case where currency C is the same as currency A, the triangulation equation reduces to the case described in an earlier section, relating one collateralization to another:

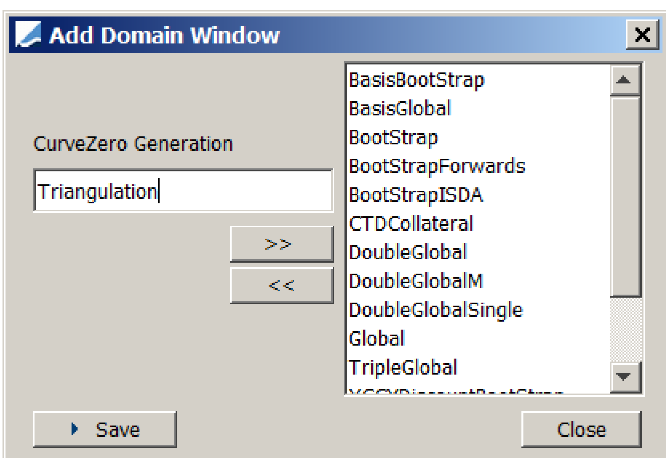
$$D_{A/B}(T_0,T) = \frac{D_A(T_0,T)}{D_{B/A}(T_0,T)} D_B(T_0,T)$$

This is the equation of Fujii et al.

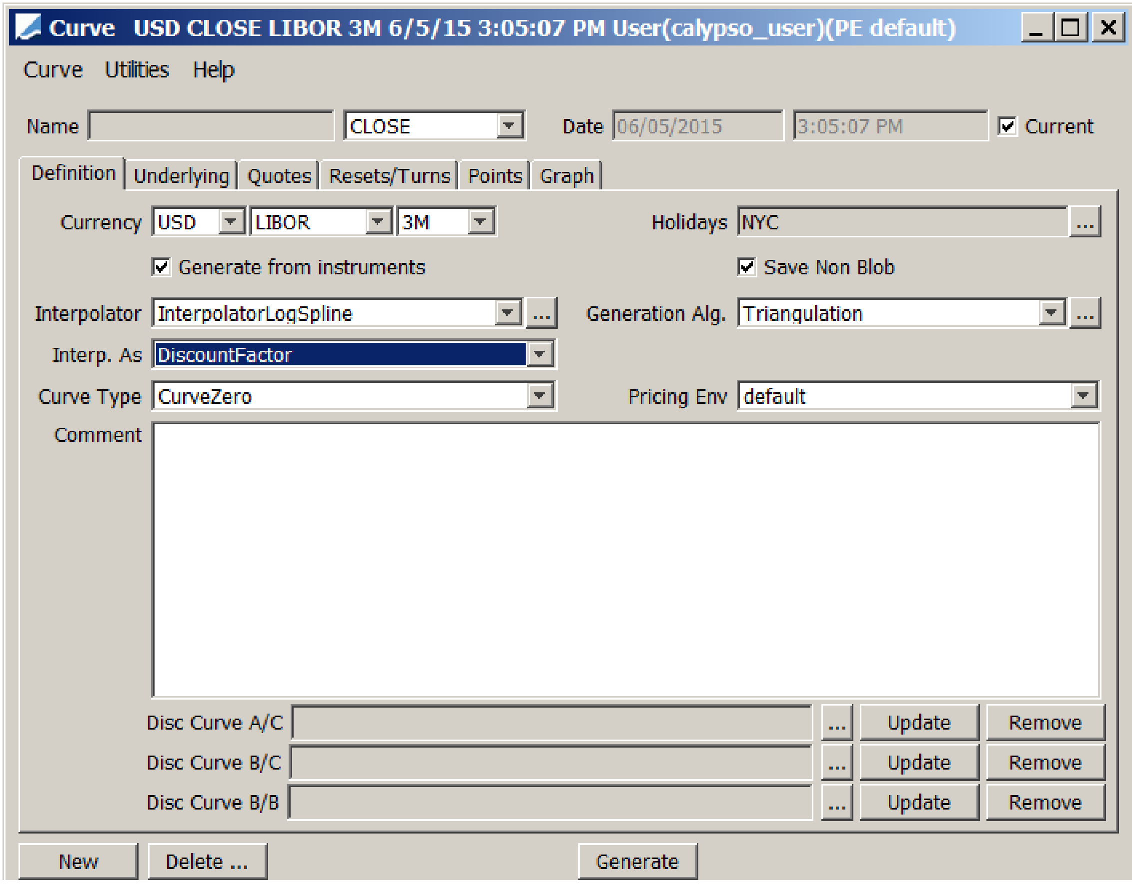
10.5.4 How to Use the Calypso Triangulation Generator

The Calypso Triangulation generator is available in the CurveZero window.

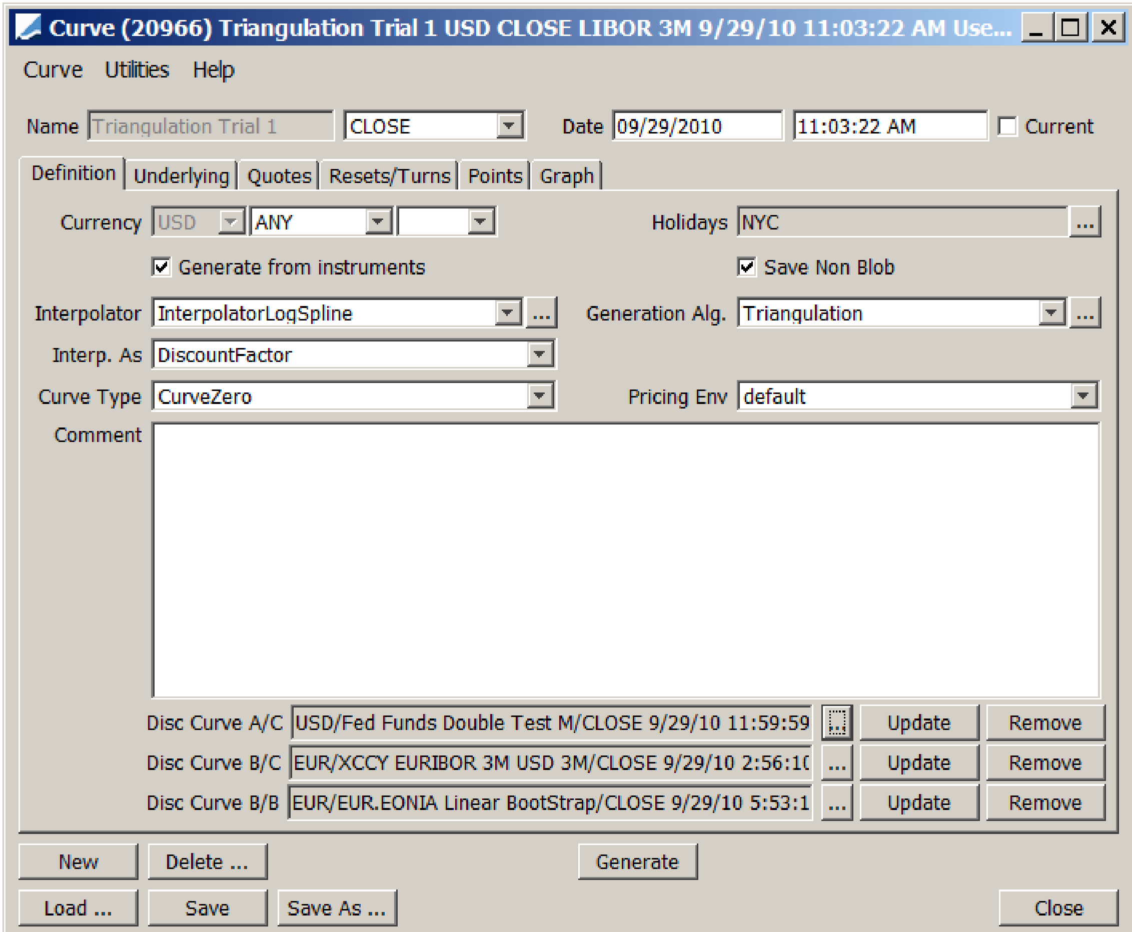
- Open the CurveZero window.
- In the generation algorithm field "Gen. Alg.", select the "Triangulation" generation from the menu. If it does not appear in the menu, it can be added by selecting the button labeled "…" and typing in "Triangulation" in the menu dialog, as shown below. Select the ">>" button to move the name onto the menu. Make sure to SAVE this dialog before closing.



- After selecting the Triangulation generator algorithm, the fields of the window will change as shown below. The labels of the fields follow the definitions used in Section 1.2 of this document, that is, "Disc A/C" indicates the curve for discounting cashflows in currency A that are collateralized in currency C. **The curve that will be generated is in currency A for trades collateralized by currency B.**



- Select the three input curves. Note that in the simplest case currency C can be the same as currency A. The following example shows the USD / EUR case of Section 1, where A = C = USD, B = EUR.



- Click the Generate button to produce the curve using the formulas given in the first part of this document.
- Save the curve. Use in pricing and in risk in the same manner as any other discount factor curve.

Section 11. Country-Specific Curves

11.1 Brazilian Clean and Dirty Cupom Curves

The clean cupom curve used in the Brazilian markets is a USD onshore curve generated from the following curve underlyings:

1. Spot USD/BRL FX rate
2. The first maturing Dollar FX Future
3. One or more FRAs on DI x US Dollar Spread (FRCs)

In addition to the curve underlyings, Brazil's local currency discount curve (CDI Curve) is also used to build the clean cupom curve.

To calculate the first point of the curve the following interest-rate parity relationship is used:

$$Df^{USB}(SD, FD) = \frac{FP}{X} Df^{CDI}(SD, FD) \qquad (1)$$

where

- *SD* is the Spot Date
- *FD* is the next Future Dollar delivery date, usually the first business day of the next month;
- *X* is the Spot FX rate;
- *FP* is the Future Dollar Price, i.e. the exchange rate at *FD*;
- $Df^{CDI}(t_1, t_2)$ is the forward discount factor from t_1 to t_2 for the CDI curve, where $t_1 < t_2$;
- $Df^{USB}(t_1, t_2)$ is the forward discount factor from t_1 to t_2 for the USD clean cupom curve;

Using *t* to denote the curve date, the spot discount factor $Df^{USB}(t, SD)$ is extrapolated from $Df^{USB}(SD, FD)$ by assuming a constant forward rate from *SD* to *FD*. The resulting first point of the curve is then:

$$Df^{USB}(t, FD) = Df^{USB}(t, SD)\frac{FP}{X} Df^{CDI}(SD, FD) \qquad (2)$$

Succeeding points on the cupom curve come from FRA contracts on DI x US Dollar Spread, also known as Cupom Cambial FRAs, or FRCs, which are traded and quoted on the BM&F Brazilian exchange. The start date of these FRA contracts is the next Future Dollar date (*FD*) and for each FRA contract, the delivery date T_i is the first business day of the contract month. In a consistent manner with US money markets, the day count is Actual/360. The forward discount factor $Df^{USB}(FD, T_i)$ is thus calculated as:

$$Df^{USB}(FD, T_i) = \frac{1}{1 + \left(F(t, FD, T_i) * \left(\frac{T_i - FD}{360} \right) \right)} \qquad (3)$$

where $F(t, FD, T_i)$ is the market quote of the i^{th} FRA contract, which covers the time period from *FD* to T_i.

Multiplying discount factors (2) and (3), the FRA-derived points of the curve are:

$$Df^{USB}(t,T_i) = \frac{Df^{USB}(t,FD)}{1+\left(F(t,FD,T_i)*\left(\frac{T_i-FD}{360}\right)\right)} \qquad (4)$$

A common setup is to use a USD/BRL FX rate with zero spot days, in which case in formula (2), t = SD, $Df^{USB}(t,SD)=1$, and the first point of the curve becomes:

$$Df^{USB}(t,FD) = \frac{FP}{X} Df^{CDI}(t,FD) \qquad (5)$$

In this case, the clean cupom curve is constructed from formulas (4) and (5).

The dirty cupom curve is constructed in the same way as the clean cupom curve. The only difference between the two curves is that the dirty cupom curve is generated with the PTAX USD/BRL fixing, instead of the Spot FX rate. The PTAX fixing used for the dirty cupom curve is published one business day before the curve date *t*.

11.2 BU252 Interpolation on business days

This is a new checkbox in the curve window. When checked, only business days are used to interpolate between curve points, even if the Day Count in the curve window is other than BU 252. For example, the point interpolated on Friday is the average of the points on Thursday and the following Monday.

11.3 LTN and NTN-F Brazilian Government Bond Curves

The LTN and NTN-F curves are generated from zero-rate spreads over the DI curve. There is a new generator which performs this calculation, the **ZeroRateSpread** generator.

LTN bonds (Letras do Tesouro Nacional) are zero coupon bonds that pay a single principal payment at maturity. The principal amount is fixed at R$1000.

Because quotes are given as spreads over DI zero rates, a basis curve generation will be done.

NTN-F Bonds (Notas de Tesouro Nacional, F serie) are fixed coupon bonds paying semi-annual compound interest and a fixed principal payment at maturity. Current coupons are 10% and principal amounts are $R1000. They are the interest-paying analog of the zero-coupon LTNs.

ZeroRateSpread Curve Generator

The ZeroRateSpread generator interprets spread quotes on bonds as zero rate spreads and calculates discount factor points on the curve using compounded business day discounting. This accommodates the conventions for Brazilian government bonds.

Bond Spread Quote

Quotes for LTNs and NTN-Fs are most commonly given as spreads over the curve of overnight Central Bank rates, the DI curve. To generate the bond curve, the DI curve must be given. The curve underlyings are of type Bond and the quote type must be specified as Spread. The units of the bond spread quote are basis points. The curve generator ZeroRateSpread will interpret these as zero rate spreads, that is, they will be added to the DI curve zero rates.

The only difference between LTN and NTN-F bond underlyings is the reference bond, which will be correspondingly be either an LTN or NTN-F.

Overnight point: The curve also requires an overnight instrument to represent the Brazilian Central Bank SELIC overnight average rate. The SELIC quote is the standard Money Market quote, not a spread. In the case of a LTN matures in the same date of this overnight rate, Calypso uses the overnight rate.

Yield Calculation

During curve generation, the Base curve on the Curve Definition tab of the curve window will be used to calculate the yield of the bond. The user will choose the Base curve to be the DI curve. The zero rate for the bond maturity will be found from the curve. The day count for the zero rate will be BU252 and the compounding will be exponential annual. Then the LTN or NTN-F bond yield will be found by adding the spread quote to the DI zero rate:

$$Yield_{LTN} = ZeroRate_{DI} + SpreadQuote/10000$$

In this expression the Yield and Zero Rates are decimals (not percentages), the SpreadQuote in basis points.

LTN Curve Generation

As LTNs are zero-coupon bonds, the generated curve will produce a price for the bond by simple discounting of the final principal payment. That is, the discount factor at maturity will be the value *D(T)* satisfying

$$\frac{D(T)}{D_0} = \frac{1}{(1 + Yield_{LTN})^{\frac{W}{252}}}$$

where D_0 is the discount factor on the spot date (value date). The spot date is by default the curve date, in which case D_0 is 1.

NTN-F Curve Generation

The generic bond pricer is used to find the price of the semi-annual NTN-F bond given a BU252 yield. Then standard Calypso curve optimization routines will solve for the curve such that the price obtained from the curve by discounting and summing each bond cashflow will equal the price obtained from the yield.

11.4 LFT Brazilian Government Inflation Indexed Curves

LFT bonds accrue at the compounded overnight SELIC rate and pay a single principal payment at maturity. The principal amount is adjusted by the accumulated rate daily. The SELIC Accumulate Rate that indexes the notional amount is published by the Brazilian Central Bank.

The Gross Unitary Price quote type is supported for LFT bonds in curve generation. This reflects the fact that LFTs are indexed notional bonds, with indexing according to the SELIC accumulated rate. The curve underlying type is the normal Bond type.

The pricer for LFT bonds can price from the conventional yield quote and also price from the curve, i.e., discount cashflows to give a present value including the adjustment for the notional. Normal curve bootstrap optimization is performed by finding the curve so that these two prices are equal.

11.5 NTN-B Brazilian Government Inflation Indexed Zero Curve

The NTN-B zero curve represents "real" rates of return after inflation is taken into account. It is generated from inflation-adjusted instruments.

NTN-B is a fixed coupon bond paying semi-annual interest and a principal payment at maturity, where the payments are adjusted for inflation. Inflation is measured by reference to the IPCA level of the consumer price index (absolute value). The payments are adjustment by multiplying the initial notional by the ratio of the current value of the IPCA level to its value on a reference date associated with the bond. Pricing between IPCA release dates is adjusted using the pro-rated one-month IPCA inflation estimate.

This zero curve also requires a point on the next business day after the curve date, representing the CDI overnight deposit rate, inflation adjusted using the one-month IPCA estimate.

InflationIndexed Curve Generator

The curve generatior named InflationIndexed takes into account the first point for the index level. This is described in the following sections.

Bond Quote Type

The Gross Unitary Price quote type is supported for NTN-B bonds in curve generation. This reflects the fact that NTN-Bs are indexed notional bonds, with indexing according to the IPCA absolute level.

The IPCA index level will not be an input value on the curve. It is expected to be in the quote database if required by the pricer.

Curve Underlying Types – Inflation-Adjusted Money Market Point

The curve requires a one-day point on the curve obtained from the inflation-adjusted overnight money market rate. The user specifies this on the curve through the use of two underlyings:

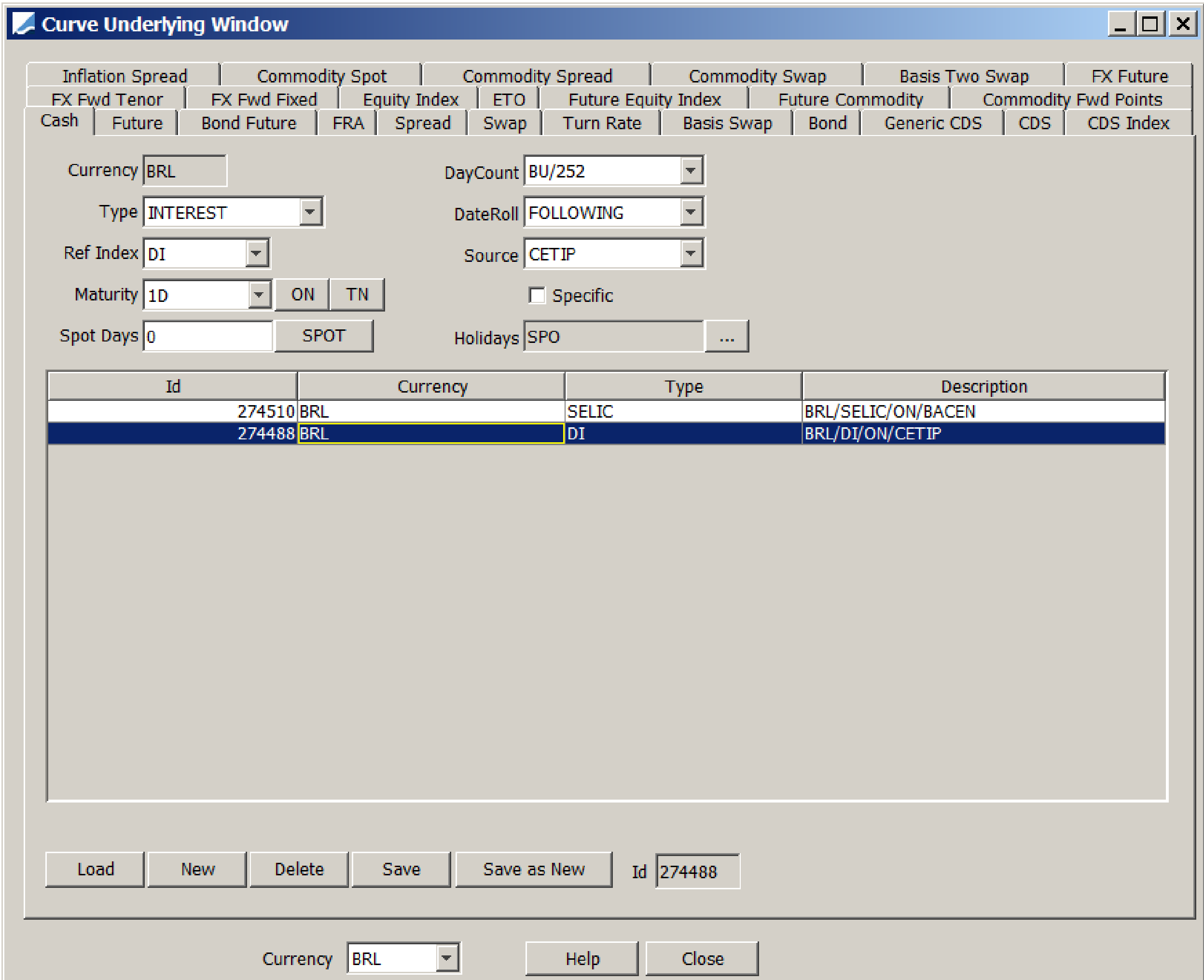
- Nominal SELIC rate
- ICPA one-month inflation percentage.

There are two reasons for having the ICPA rate as a separate underlying. First, it tells the curve generator that the SELIC rate needs to be inflation adjustment. Without the rate present another method would need to be developed, such as adding a new inflation-adjusted underlying type, which would be a more complex solution. Second, and of more significance, is that risk can be calculated with respect to the IPCA rate. It's quote can be bumped on the curve as part of the usual sensitivity analysis, producing an "IPCA01" delta.

These underlyings are described below.

SELIC Deposit Underlying and Quote

The SELIC deposit quoted rate is represented in the curve by a standard Cash deposit curve underlying. The user can instead choose the CDI rate. To make a Cash deposit curve underlying, a Rate Index of the desired name first needs to be created, and then the Cash underlying is made that refers to this Rate Index and gives other characteristics. An example is shown in the screenshot below.



IPCA Monthly Inflation Rate and Quote

The inflation factor to adjust the SELIC rate is represented by a separate **Cash Inflation** underlying. It will be associated with an inflation index such as “IPCA-MONTHLY%” representing for the user the inflation estimate for a one-month change. The quote will be a percentage representing a one-month change in inflation; a typical value currently is 0.5%.

Calculation of Overnight Point

The presence of an Inflation underlying signals to the InflatinIndexed generator that money-market quotes are nominal quotes that need to be adjusted before being placed on the curve. This is needed as the zero curve represents “real” rates of return after inflation is taken into account.

The inflation adjustment of daily compounded rates is calculated in general as follows. If C is the nominal annual interest rate and I the annual inflation rate, then the annualized real rate of return R is computed as:

$$1 + R = \frac{1 + C}{1 + I}$$

In the present case, I, the annualized inflation rate, needs to be annualized from the published IPCA one-month rate of change. To annualize, one first finds the daily rate by taking the number of working days in the BRL calendar in the IPCA reporting month, from the 15th before the calculation date to the next 15th , and then compounds that for 252 business days in the BU252 convention. That is:

$$1 + I = (1 + IPCA)^{\frac{252}{WD}}$$

Therefore, one has

$$1 + R = \frac{1 + SELIC}{(1 + IPCA)^{\frac{252}{WD}}}$$

or

$$(1 + R)^{\frac{1}{252}} = \frac{(1 + SELIC)^{1/252}}{(1 + IPCA)^{\frac{1}{WD}}}$$

This gives the real overnight rate, R, adjusted for inflation. The real one-day discount factor which will be placed on the curve at one business day after the curve date is:

$$DF_{1day} = \frac{1}{(1 + R)^{\frac{1}{252}}}$$

The rate R will be visible in the Points table of the curve application in the Zero Rate column, if the user selects the Zero Rate display to be in the BU252 day count convention.

Section 12. Frequently Asked Questions

Why don't the dates generated for the curve match the maturity dates of the swap?

The date placed on the curve is the last date for which the trade needs a discount factor for pricing. In the case of the swap this is either the last cashflow date or the end date of the last forward period (needed to project the last forward reset rate). Because of the difference between money market conventions and swap conventions the forward deposit periods do not always line up with swap coupon periods, so the last forward end date can fall after the swap maturity date.

Why do the futures produce dates that don't align with the futures settlement dates?

Deposit futures contracts are derivatives of underlying forward money market term deposits. The deposit period typically starts on the spot date after the futures settlement date and ends three months later, according to the money market convention. So neither the start nor end date of the deposit period need fall on the settlement date. It is these dates for which the forward rate is given by the future's settlement price. In the BootStrap generator these dates are especially evident as both the start and end date of the deposit are placed on the curve, requiring special handling when deposit periods have gaps between them (sometimes of one or two days) or overlaps. In the Global generator this is not a problem for simple shaping methods, as only the end date of the deposit period is placed on the curve; the discount factor for the other date is found by interpolation. However, if the shaping method results in generation of more points, such as daily points, then the convexity value will show on each point.

In the Multicurve window, the convexity display has been improved over the single curve window so that multiple rows do not appear for one convexity (Version 14.4). The convexity is shown on only one date at the expiry of the futures contract. More precisely, it will be two business days after the futures settlement date, representing the START date of the futures forward deposit period.

The CurveBasis window does not display convexity on the Points tab. The Log category described at the end of the previous section should be used instead.

I created a swap trade to match a swap in the curve, but when I price with the generated curve the breakeven rate does not match the market quote used by the curve. What is wrong with this round-trip pricing check?

The procedure for validating curves is described in Chapter 1 in the section "How to Validate Curve Results." Some particular points to bear in mind:

- Make sure the pricing environment used in validation is actually using the curve that was generated and all the curves that were used during that curve's generation. Especially check the dates on the curves, as some may have been rolled before being used in the trade window.
- Check the fixing rates on the cashflow tab in the trade window to make sure they are the same as the fixing rates used in the curve. You made need to prevent the use of a fixing quote by selecting the RESET_FROM_CURVE pricing parameter in the trade window, if that is available.

The shape of the curve doesn't look right. How can it be fixed?

The shape of the curve raises questions when it displays unexpected dips or jumps, or unaccountably low or high zero rates. These are never (with notably rare exceptions) due to a failure of the solving algorithm. The problem will be with the information sent into the solving algorithm. If one encounters a strange curve, the following checklist can help resolve the problem.

- **Inconsistency of the curve underlying definitions.** The list of swaps in the underlyings list may look the same judging by their names, but closer examination can reveal a sudden change in definition. To verify there are no problems, each swap should be examined in the Curve Underlying Definition window. There can be changes from swap to swap in such parameters as the "Manual First Reset" checkbox or the "Compounding" specification. The change in definition can cause a jump in the curve.
- **Peculiar underlying definitions**. Look for unintended settings on swap and basis swap underlyings, such as a mismatch of the payment frequency and the rate index tenor if there is no compounding or averaging involved, an unconventional Day Count or Date Roll, or a mis-specification of the "End Period" or "Beg Period" rate reset timing.
- **Underlyings ending on the same or nearly the same date**. It may not be obvious, but two underlyings may end on the same maturity date, leading the generator to discard one of them. Because swap end dates shift in time while futures maturity dates do not, a curve that was fine one day may on the next have a Futures contract whose forward period ends on the same date as a swap. Even if the maturities are not on the same date, they may be so close as to cause sensitivity to slight differences between the futures rate and the subsequent swap. Also, make sure that there aren't two swaps with the same maturity. This can happen unexpectedly with short-dated swaps, for example, due to the presence of holidays, month ends, and the peculiarity of date roll conventions.
- **Incorrect quotes**. Examine the quotes, both the par quotes and the reset fixings, for peculiar values. Check the **quote type**: for example, BasisSwaps may have a quote type of Yield, but the user may have entered them as basis points.
- **Swaps with no free parameters**. For example, if a swap has a maturity of 6 months and its reset frequency is 6 months then it has only a single floating payment. If it is given a fixing quote, there are no free floating rates and therefore no free parameters to fit to the swap quote. (This will usually result in an exceptions being thrown.)
- **Use of an old curve as a discount, foreign or base curve**. The curve generation window does NOT roll curves. It takes the user's choice of curve very literally: if the user takes a curve generated on a past date, the generation will use the discount factors associated with each date on that curve without rolling them in any fashion to the current curve generation date. This will produce unusual results if the user has not intended this behavior.
- **Past resets when generating on holidays**. Sometimes a curve is generated on a day that is a holiday in one of the calendars: an example a USD Libor swap generating on Easter Monday, which is a London but not a US hoiday. In this case, the swaps in the curve will need a reset that was published in the past in order to value the first cashflow. If the reset is missing or has an unexpected value the curve generation may throw an exception, or produce an unexpected result through having projected the past reset. Make sure any past resets are present in the quote sets, as they cannot be picked up in the curve: resets in the curve only apply to the curve generation date, not to past dates.
- **Using a custom or unexpected index calculator**. Index calculators are specified on the Attributes window of a Rate Index. Generally there is no reason to set an index calculator unless a customized pricer is used. An unexpected index calculator could lead the curve generation to misprice an instrument.
- **Use of approximations**. A parameter such as the "Daily Average Swap Fast Approx" employs approximations instead of exact pricing. This is unlikely to prouce an odd curve shape, but it can affect the check of round-trip pricing. Try turning off any approximations to determine their effect on the curve.

We want to reproduce the Calypso curves in our own spreadsheets for our validation project. Can we have details of the optimization algorithms?

The techniques are too complex to attempt to reproduce in another environment. The BootStrap can be manually reproduced in a spreadsheet and Calypso can provide these examples. But the Global generator is too involved. To validate the curves the best approach is to perform the round-trip pricing checks. Again, the Log category “GenGlobal” provides a fast way to check the curves.

Section 13. Appendix 1: The USD OIS Curve

The USD OIS curve represents the market expectation of the United States Federal Reserve target rate for overnight interbank lending of Fed Funds, i.e., balances on deposit at the Federal Reserve. The rate is published between 07:30-08:00am EST on each business day for the **prior** business day. The Federal Reserve releases this under the code H.15, and the latest release can be found on the web at http://www.federalreserve.gov/releases/h15/update/. The data are published on Reuters page FEDFUNDS1 (formerly Telerate 118) and Bloomberg page FEDL 01. As Credit Suisse notes: "The fixing is released with a one-day lag, and thus to avoid confusion, the fixing has an attached correct reference date. For example, brokered trades in the federal funds market for value date 2-Mar-09, are averaged and published on 3-Mar-09, but carry an associated date label of 2-Mar-09."

The one-day ACT/360 forward rate obtained from the USD OIS curve for each future date corresponds to the market-expected target fixing. This rate is typically constant between Federal Reserve meeting dates; on those dates, changes in the target can be announced.

13.1 The Market Instruments

13.1.1 Fed Fund Futures

Fed Fund Futures are traded on the CME and represent the interest rate on a 30-day money market deposit. The interest rate is determined to be the arithmetic average of the published Fed Funds overnight rate for the contract month. The last trading day of the contract is the last business day of the contract month (not the IMM date), so the averaging period exactly corresponds to the calendar month. Typically Fed Fund futures are liquidly traded out to one year or more. For more information, see:

http://www.cmegroup.com/trading/interest-rates/stir/30-day-federal-fund_quotes_globex.html

Data is released by the Federal Reserve between 07:30-08:00am EST for the prior business day. The index represents the volume-weighted average of interest rates at which depository institutions lend balances at the Federal Reserve to other depository institutions overnight. This is established by trading government securities. The weekly Fed Funds average is weighted and runs from Thursday to Wednesday. For proper calculation of the weekly average, Friday's rate is also used for both Saturday and Sunday. This index uses a 360 day count.

13.1.2 OIS Swaps

USD OIS Swaps are over-the-counter fixed-versus-floating interest rate swaps where the floating leg pays out at the daily compounded Fed Funds rate. They are the USD equivalent of the EUR EONIA swaps. While EONIA swaps trade liquidly for maturities as long as 60 years, the USD OIS swaps are typically only liquid out to two years, although 5 year quotes are available. (This may change in future as the market for longer-dated OIS grows.)

Although OIS swaps cover the same period as Fed Fund futures, many market participants prefer to build curves from the swaps rather than the futures. This may be due to the daily *compounding* of OIS, which more convenient mathematical properties than the *arithmetic averaging* of the futures.

Because of the one-day lag of the Fed Funds effective rate, one will not know what rate accrues on a given interest accrual day until the next business day. Thus the payment date has a lag after maturity. The following are the specifications of the swap.

Reference Rate: Fed Funds Effective

Start Date ("Value Date"): Spot (T + 2)

Payment Date: Maturity + 2

Fixed Leg: Single payment or, if over 1 year, annual payments. Daycount ACT/360.

Floating Leg: Pays the daily compounded Fed Fund rate over the interest period. Payment is on the same date as the fixed leg payment ("annual" payment). Daycount is ACT/360. On non-business days simple interest accrues at the latest fixing rate until the next business day. For example, Friday's rate applies at simple interest for three days, Friday, Saturday, and Sunday; Monday's rate will again accrue by daily compounding.

The following is an example of an OIS trade. Note the payment lag due to the one-day delay in the Fed Fund published rate.

Example of a hypothetical one-week USD OIS trade (spot starting)

Trade Date: Mon 04-Jan-10

Value Date: Wed 06-Jan-10

End Date: Tue 12-Jan-10

Payment Date: Thu 14-Jan-10

	Wed 06-Jan-10	Thu 07-Jan-10	Fri 08-Jan-10	Mon 11-Jan-10	Tue 12-Jan-10
Fed Funds Effective Rate	0.12%	0.13%	0.14%	0.13%	0.10%
Day Count Fraction (Act/360)	1/360	1/360	3/360	1/360	1/360
1 + Fed Effective x DCF	1.0000033	1.0000036	1.0000117	1.0000036	1.0000028
Cumulative Product	1.0000033	1.0000069	1.0000186	1.0000222	1.0000250
Breakeven Fixed Rate = (1.00002500 -1)*(360/7) = 0.1286%					
Source: Credit Suiss					

The following example gives broker quotes for OIS swaps at the commonly traded maturities. These would be used as input to curve generation.

Name		Bid	Ask	Mid
USD SWAP OIS	1 WK	0.096	0.102	0.099
USD SWAP OIS	2 WK	0.096	0.102	0.099
USD SWAP OIS	3 WK	0.092	0.112	0.102
USD SWAP OIS	1 MO	0.088	0.118	0.103
USD SWAP OIS	2 MO	0.098	0.105	0.1015
USD SWAP OIS	3 MO	0.105	0.112	0.1085
USD SWAP OIS	4 MO	0.113	0.12	0.1165
USD SWAP OIS	5 MO	0.12	0.127	0.1235

USD SWAP OIS	6 MO	0.126	0.133	0.1295
USD SWAP OIS	7 MO	0.131	0.138	0.1345
USD SWAP OIS	8 MO	0.138	0.145	0.1415
USD SWAP OIS	9 MO	0.142	0.152	0.147
USD SWAP OIS	10 MO	0.154	0.161	0.1575
USD SWAP OIS	11 MO	0.163	0.17	0.1665
USD SWAP OIS	1 YR	0.173	0.181	0.177
USD SWAP OIS	15 MO	0.196	0.226	0.211
USD SWAP OIS	18 MO	0.256	0.266	0.261
USD SWAP OIS	21 MO	0.297	0.346	0.3215
USD SWAP OIS	2 YR	0.389	0.414	0.4015
USD SWAP OIS	3 YR	0.756	0.794	0.775
USD SWAP OIS	4 YR	1.17	1.215	1.1925
USD SWAP OIS	5 YR	1.584	1.613	1.5985

13.1.3 Fed Fund/Libor Basis Swaps

Popular swaps for longer-dated rates are the Fed Fund versus Libor floating/floating basis swaps. Institutions are often funded at the Fed Fund rate but have risk to Libor, so the basis swaps can be used to hedge the spread risk. Because of the credit risk of Libor versus the nearly riskless Fed Funds, the Fed Fund/Libor basis spread serves as a credit spread.

The following describes the legs of the swap. **Note that the basis spread is added to the Fed Funds leg.** Note also that the Fed Fund leg uses **arithmetic averaging** (analogous to Fed Fund Futures), in contrast to the OIS swap which uses **daily compounding**.

Floating Leg 1:

Reference Rate: 3M Libor, Reset at beginning of period, paid at end

Payment Frequency: Quarterly

Day count: ACT/360

Floating Leg 2:

Reference Rate: Fed Funds Effective, daily weighted arithmetic average, calculated and reset and paid at the end of the period, PLUS a fixed spread (the basis spread). There is a two-day rate cutoff (usually) due to the one-day publication lag, so the final Fed fixing is applied to the last two fixing days.

Payment Frequency: Quarterly

Day count: ACT/360

The following example shows broker quotes for commonly traded maturities.

Name	Bid	Ask	Mid
USD BASIS SWAP FFv3 3 MO	12.25	16.25	14.25
USD BASIS SWAP FFv3 6 MO	14.671	16.515	15.593
USD BASIS SWAP FFv3 9MO	15	19	17
USD BASIS SWAP FFv3 1 YR	15.75	19.75	17.75
USD BASIS SWAP FFv3 18MO	17.38	21.38	19.38
USD BASIS SWAP FFv3 2 YR	19	23	21
USD BASIS SWAP FFv3 3 YR	21	25	23
USD BASIS SWAP FFv3 4 YR	21	25	23
USD BASIS SWAP FFv3 5 YR	20	24	22
USD BASIS SWAP FFv3 7 YR	20	24	22
USD BASIS SWAP FFv3 10YR	19.5	23.5	21.5
USD BASIS SWAP FFv3 12YR	18.13	23.13	20.63
USD BASIS SWAP FFv3 15YR	16.88	21.88	19.38
USD BASIS SWAP FFv3 20YR	15.38	20.38	17.88
USD BASIS SWAP FFv3 25YR	14.5	19.5	17
USD BASIS SWAP FFv3 30YR	15	19	17

A detailed reference to the market instruments is Credit Suisse Basis Points, "A Guide to the Front-End and Basis Swap Markets" (18 February 2010).

13.1.4 ISDA Definition of OIS Swaps

The interest rate formula is a standard daily compounding formula, with one modification: on non-business days simple interest is applied – this appears in the formula as the n_i factor, the number of days at which a published rate does not change due to lack of activity.

Source: Supplement number 6 to the 2000 ISDA® Definitions and Annex to the 2000 ISDA

Definitions (June 2000 Version)

"USD-Federal Funds-H.15-OIS-COMPOUND" will be calculated as follows, and the resulting percentage will be rounded, if necessary, in accordance with the method set forth in Section 8.1(a) of the Definitions:

$$\left[\prod_{i=1}^{d_0}\left(1+\frac{\text{FEDFUND}_i \text{ x } n_i}{360}\right)-1\right] \text{x } \frac{360}{d}$$

where:

"d_0" for any Calculation Period is the number of New York Banking Days in the relevant Calculation Period;

"i" is a series of whole numbers from one to d_0, each representing the relevant New York Banking Days in chronological order from, and including, the first New York Banking Day in the relevant Calculation Period;

"FEDFUND_i", for any day "i" in the relevant Calculation Period, is a reference rate equal to the rate set forth in H.15(519) in respect of that day opposite the caption "Federal funds (effective)", as such rate is displayed on Telerate Page 118. If such rate does not appear on Telerate Page 118 in respect of any day "i", the rate for that day will be as agreed between the parties, acting in good faith and in a commercially reasonable manner. If the parties cannot agree, the rate for that day will be the rate displayed on Telerate Page 118 in respect of the first preceding New York Banking Day;

"n_i" is the number of calendar days in the relevant Calculation Period on which the rate is FEDFUND_i; and

"d" is the number of calendar days in the relevant Calculation Period.

Section 14. Appendix 2: Turn Rates Algorithm in Yield Curve Construction

A *turn rate* is any rate that applies over a short future time period that is not continuous with the rates outside of the period.

14.1 Specification of Turn Rates

As an input to yield curve construction, turn rates require specification of dates, day count, and value.

Dates

The period over which a turn rate is effective is defined by two dates specifying the start and end of the interest accrual that incorporates the rate. The following definitions will be used:

s1 : Start date of the turn period (inclusive)

s2 : End date of the turn period (exclusive)

For example, if Dec 31 and Jan 1 are the dates on which the rate is in effect for interest accrual, then

s1 is Dec 31 and s2 is Jan 2 – the latter date is the first date on which the turn rate no longer has a daily accrual effect.

Day Count

The day count of the rate must also be specified. This will always be taken to be a simple interest rate.

14.2 Methods of Specifying Turn Rate Values

Spread to an Instantaneous Rate

A direct way to specify the rate is as an add-on spread to an instantaneous or one-day forward rate.

Absolute Rate

As an absolute rate, the specified turn rate will substitute for the rate that would otherwise be applied over the turn period.

Forward Rate Adjustment

Market quotes can be used to specify the turn rate as a difference of forward rates. From the quotes one has the forward rate for a FRA or convexity-adjusted futures contract over a period of time that covers the turn period. The forward rate can also be estimated by interpolation on quoted FRAs and futures that fall to either side of the turn period. The difference in forwards is the quoted turn adjustment over the entire forward rate period; this adjustment can then be transformed into an instantaneous rate adjustment that applies only to the turn period. Define:

f^{MKT} : Market forward rate

f^0 : Turnless forward rate (e.g., interpolated from neighboring rates)

a : Turn adjustment

where the turn adjustment is defined as

$a = f^{MKT} - f^0$.

So the adjustment is in the day count of the forward rates.

The necessary information for this specification is at least:

- Turn Start Date
- Turn End Date
- Forward Period Start Date (e.g., start of the FRA contract accrual period)
- Forward Period End Date (e.g., end of the FRA period, exclusive)
- Day Count
- Forward Rate Adjustment due to the Turn

For purposes of software input, rather than directly specify the forward start and end dates an instrument (FRA or future) can be specified instead. Or the appropriate contract can be searched for automatically by the software: given the turn period, another instrument in the curve construction whose forward period covers (at least partially) the turn period can be found, and its forward start and end dates used.

For present purposes it will be assumed the turn rate is given in the form of the forward rate adjustment.

14.3 Calculation of Turn Rate from Forward Adjustment

Given a forward rate adjustment, which applies to forward rates of perhaps 90 days or more, the task is to concentrate this adjustment in the turn period, which can be as little as a single day. This section describes the calculation.

Suppose one has created a "turnless" forward curve, that is, a curve which can be used to project rates that would correspond to forward contracts if the turn rates were zero. Such a curve could be constructed through use only of contracts that do not cover the turn period, or one can just propose a trial curve as a guess for a turnless curve and improve the guess subsequently.

The curve is specified by a discount factor $D^0(t)$ at each date, taken with respect to the curve start date, i.e., D(curve start date) = 1. Dates of particular interest are

t1 : Forward period start date

t2 : Forward period end date

$D^0(t1)$: Discount factor of turnless curve at t1

$D^0(t2)$: Discount factor of turnless curve at t2

As before one has the turn dates

s1 : Turn start date

s2 : Turn end date

where it is assumed for present purposes

$$t1 < s1 < s2 < t2$$

This need not be the case: the turn period could overlap one end of the forward period. But the latter case is readily derived from the former.

The turnless forward rate f^0 is given by

$$\frac{1}{1+f^0 t} = \frac{D^0(t2)}{D^0(t1)}$$

with t the year difference between t1 and t2 in the day count of the forward rate. If a non-zero turn period were incorporated in the curve, the result would be a curve which can be used to project the market's forward rate,

$$\frac{1}{1+f^{MKT}t} = \frac{D^{MKT}(t2)}{D^{MKT}(t1)}$$

As t1 < s1, that is, the forward start date is prior to the turn period, the turn rate cannot affect the discount factor at t1, so

$$D^{MKT}(t1) = D^{0}(t1)$$

The factor at t2 will be affected by the extra discounting due to the turn rate period. Define this discounting as D_{TURN}, that is

$$D^{MKT}(t2) = D_{TURN}\, D^{0}(t2)$$

Combining these one has

$$\frac{1}{1+f^{MKT}t} = D_{TURN}\frac{D^{0}(t2)}{D^{0}(t1)}$$

$$\frac{1}{1+f^{MKT}t} = D_{TURN}\frac{1}{1+f^{0}t}$$

Then given the specified turn adjustment to the forward rate,

$$f^{MKT} = f^{0} + a$$

we have an expression for the discount adjustment for the turn period

$$D_{TURN} = \frac{1+f^{0}t}{1+(f^{0}+a)t}$$

Define the turn rate adjustment over the turn period (s1,s2)

$$r_{TURN} = \frac{-\ln\left(D_{TURN}\right)}{s2-s1}$$

In the turn period, the market discount factors are derived from the turnless factors by

$$D^{MKT}(t) = \exp\left(-r_{TURN}(t-s1)\right)D^{0}(t)$$

$$s1 < t \leq s2.$$

After the turn period,

$$D^{MKT}(t) = D_{TURN}\, D^{0}(t)$$

$$t > s2.$$

This preserves the one-day forward rates of the trial curve when in the construction of the turn-adjusted trial curve for all dates outside the turn rate period, and produces the desired localized change to the forward rates within the turn rate period.

14.4 Curve Construction Procedure with Turn Rates

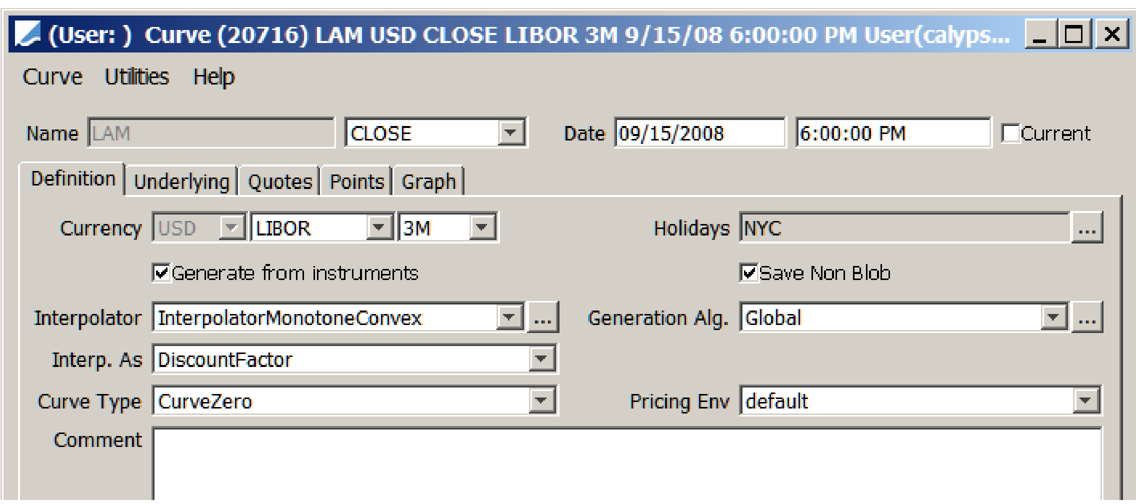
A curve with turn adjustments is solved for in the following manner:

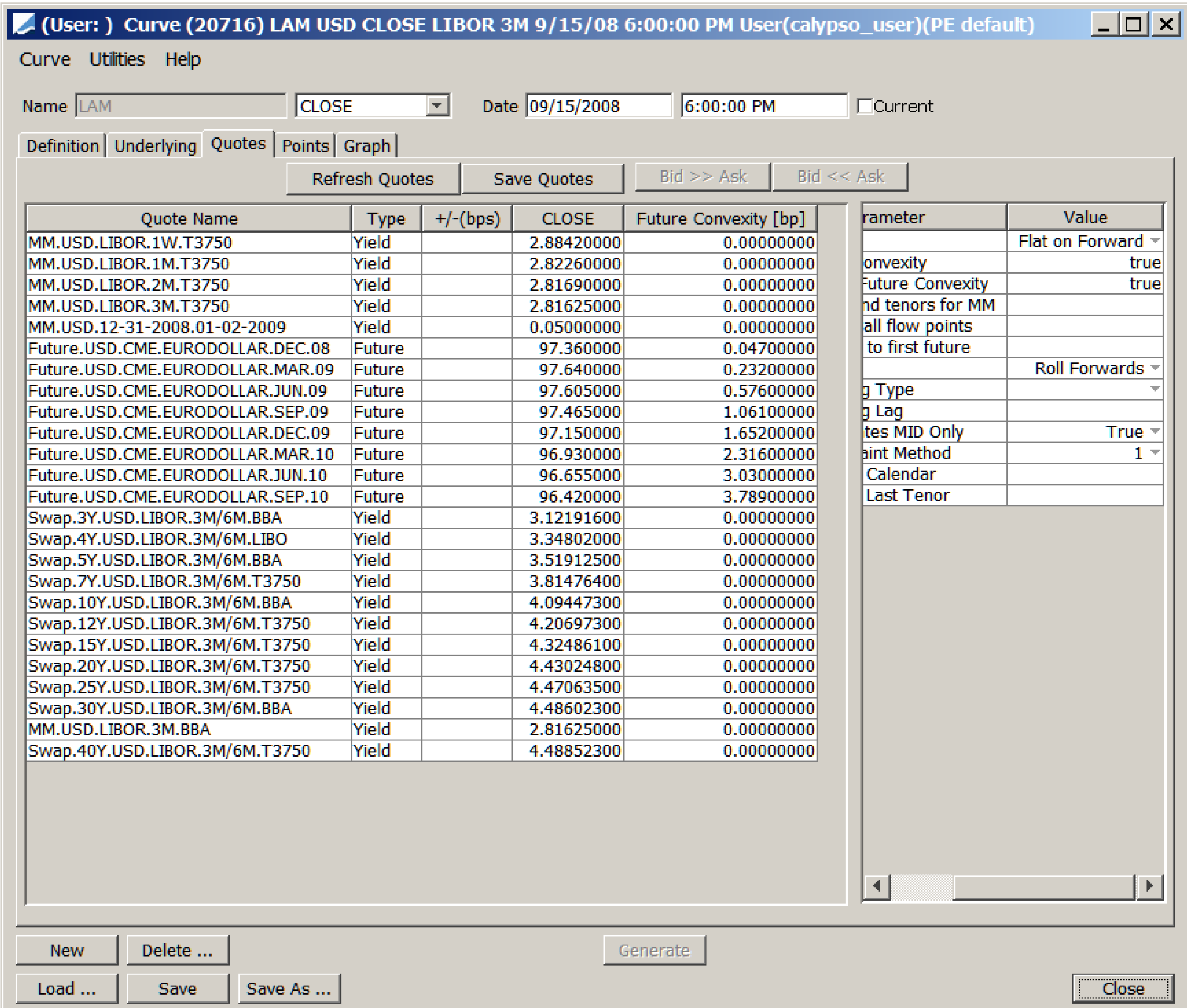
- Make a trial curve of all the turnless discount factors.
- Starting with the adjustment for the earliest turn period, adjust the turn period discount factors and all subsequent discount factors by solving for r_{TURN} and performing the required multiplication. Repeat for each given turn period, in chronological order.
- Price all the calibration instruments and compare with the market prices.
- Use the pricing results to create an improved trial curve of turnless discount factors.
- Repeat the above steps until convergence is achieved.

Concerning the trial curve, the guess of discount factors involves one factor for each of the calibration instruments. Then a discount factor for every point can be interpolated; in practice, for the purposes of doing the turn rate calculation, it is only necessary to interpolate the points covering the turn period and enough points to either side to ensure that the change to the turn period does not alter the interpolation of the other points, especially if spline interpolation is used.

14.5 Example

The following example demonstrates the implementation of this algorithm in Calypso. One turn rate is added, covering the end-of-year turn of 2008/2009. A turn adjustment of 5 basis points is shown with start date Dec 31 2008 and end date of Jan 2 2008; since the end date is not included in the turn period, there are only two days that accrue at the turn rate. One sees in the output points how daily discount factors were placed on the curve in the vicinity of the turn. The automatic technique was used to look up the instrument that covers the turn period, in this case the Dec 08 futures contract, whose deposit period extends from.





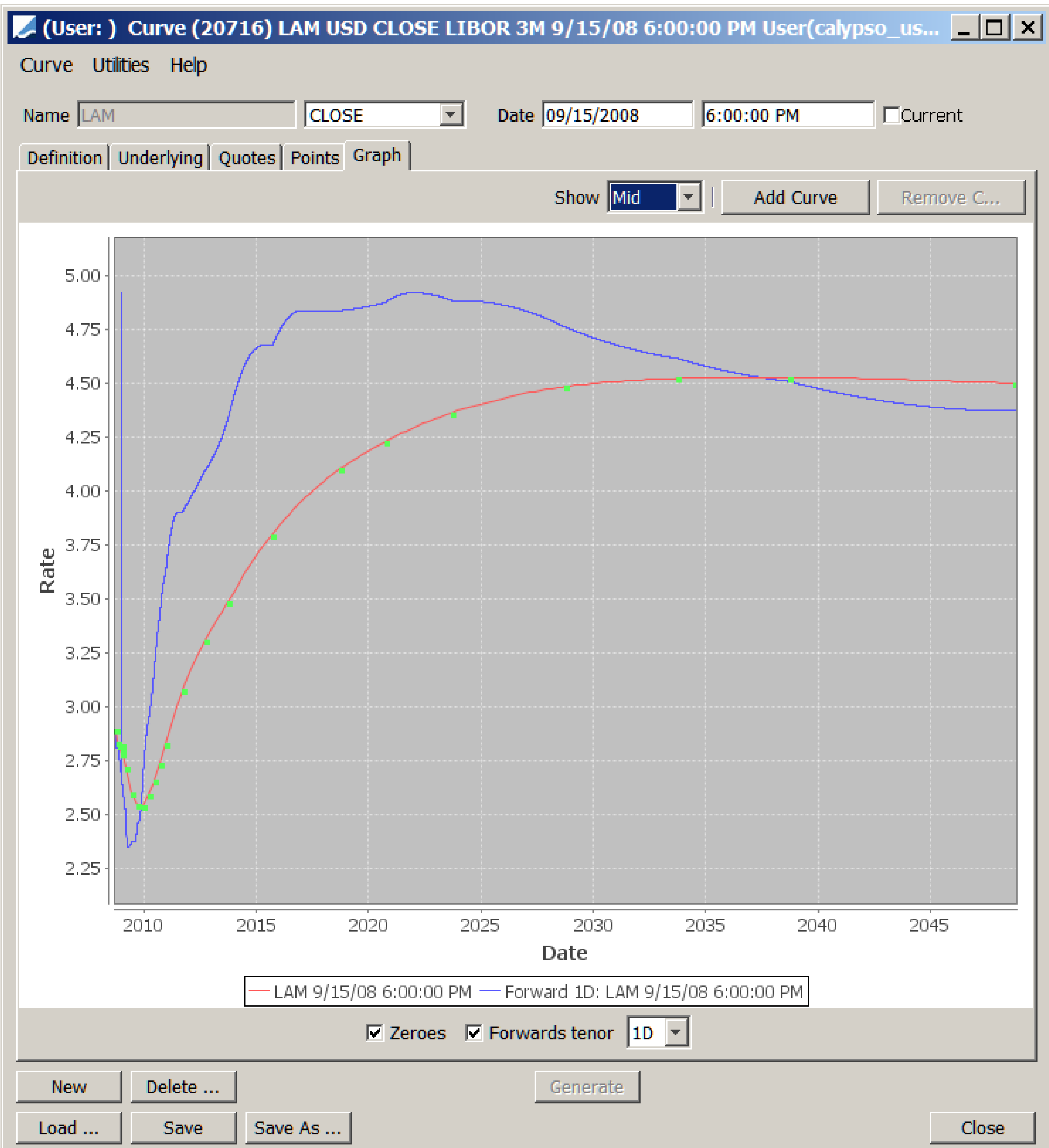
Quote Name	Type	+/-(bps)	CLOSE	Future Convexity [bp]
MM.USD.LIBOR.1W.T3750	Yield		2.88420000	0.00000000
MM.USD.LIBOR.1M.T3750	Yield		2.82260000	0.00000000
MM.USD.LIBOR.2M.T3750	Yield		2.81690000	0.00000000
MM.USD.LIBOR.3M.T3750	Yield		2.81625000	0.00000000
MM.USD.12-31-2008.01-02-2009	Yield		0.05000000	0.00000000
Future.USD.CME.EURODOLLAR.DEC.08	Future		97.360000	0.04700000
Future.USD.CME.EURODOLLAR.MAR.09	Future		97.640000	0.23200000
Future.USD.CME.EURODOLLAR.JUN.09	Future		97.605000	0.57600000
Future.USD.CME.EURODOLLAR.SEP.09	Future		97.465000	1.06100000
Future.USD.CME.EURODOLLAR.DEC.09	Future		97.150000	1.65200000
Future.USD.CME.EURODOLLAR.MAR.10	Future		96.930000	2.31600000
Future.USD.CME.EURODOLLAR.JUN.10	Future		96.655000	3.03000000
Future.USD.CME.EURODOLLAR.SEP.10	Future		96.420000	3.78900000
Swap.3Y.USD.LIBOR.3M/6M.BBA	Yield		3.12191600	0.00000000
Swap.4Y.USD.LIBOR.3M/6M.LIBO	Yield		3.34802000	0.00000000
Swap.5Y.USD.LIBOR.3M/6M.BBA	Yield		3.51912500	0.00000000
Swap.7Y.USD.LIBOR.3M/6M.T3750	Yield		3.81476400	0.00000000
Swap.10Y.USD.LIBOR.3M/6M.BBA	Yield		4.09447300	0.00000000
Swap.12Y.USD.LIBOR.3M/6M.T3750	Yield		4.20697300	0.00000000
Swap.15Y.USD.LIBOR.3M/6M.T3750	Yield		4.32486100	0.00000000
Swap.20Y.USD.LIBOR.3M/6M.T3750	Yield		4.43024800	0.00000000
Swap.25Y.USD.LIBOR.3M/6M.T3750	Yield		4.47063500	0.00000000
Swap.30Y.USD.LIBOR.3M/6M.BBA	Yield		4.48602300	0.00000000
MM.USD.LIBOR.3M.BBA	Yield		2.81625000	0.00000000
Swap.40Y.USD.LIBOR.3M/6M.T3750	Yield		4.48852300	0.00000000

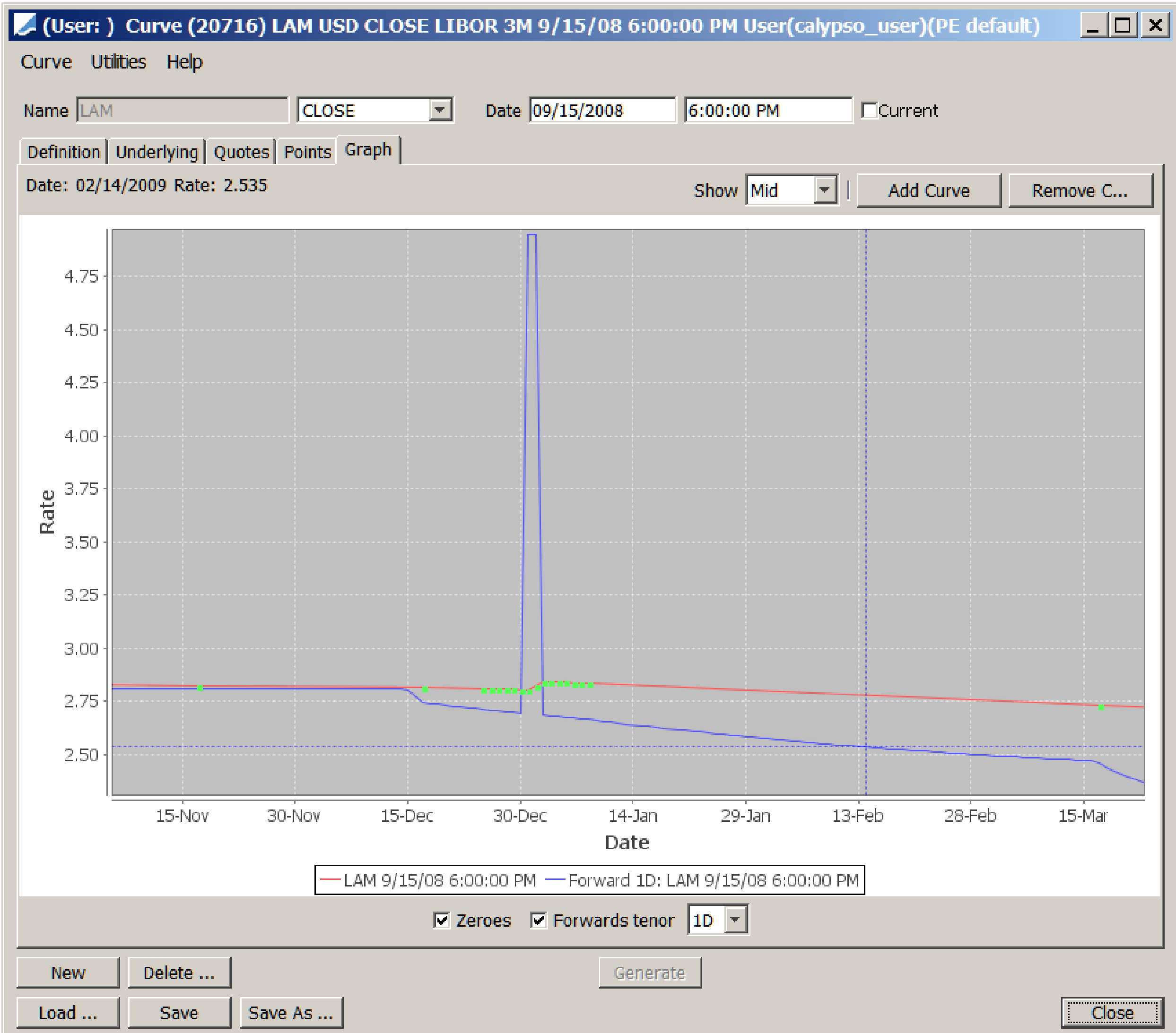
Output Points

9/17/2008	0.999839824410116
9/24/2008	0.999279411861288
10/17/2008	0.997493553323773
11/17/2008	0.995090178826535
12/17/2008	0.992772422314968
12/25/2008	0.992172516899403
12/26/2008	0.992098026235852

12/27/2008	0.992023640819362
12/28/2008	0.991949359201729
12/29/2008	0.991875179979860
12/30/2008	0.991801101793154
12/31/2008	0.991727123321104
1/1/2009	0.991591667798345
1/2/2009	0.991456322446938
1/3/2009	0.991382644715389
1/4/2009	0.991309061763996
1/5/2009	0.991235572442976
1/6/2009	0.991162175631671
1/7/2009	0.991088870237159
1/8/2009	0.991015655192960
3/17/2009	0.986264237198110
6/18/2009	0.980290609675174
9/17/2009	0.974405498493575
12/16/2009	0.968292998174611
3/16/2010	0.961482147086321
6/17/2010	0.953975946568178
9/16/2010	0.946046031789130
12/15/2010	0.937739557655773
9/19/2011	0.910589604947533
9/18/2012	0.874605303149305
9/17/2013	0.838480749209205
9/17/2015	0.764484031063810
9/18/2018	0.660784139564005

9/17/2020	0.599225966947727
9/19/2023	0.516496161770693
9/19/2028	0.404499473356651
9/19/2033	0.319515058079151
9/17/2038	0.253964923813709
9/17/2048	0.162682759325403





Close up of turn

The 1-day forward rate has a bump of about 220 bp.

This contributes to a 90-day forward rate for 2 days, so total effect on the 90-day forward is 2/90 * 220 = 5 bp approximately, as desired.

For comparison, here is the same curve with the turn instrument removed:

9/17/2008	0.999839824410116
9/24/2008	0.999279411861288
10/17/2008	0.997493553323773

11/17/2008	0.995090178826535
12/17/2008	0.992772422314968
3/17/2009	0.986264237209861
6/18/2009	0.980289950134503
9/17/2009	0.974404847092526
12/16/2009	0.968292350608017
3/16/2010	0.961481504077401
6/17/2010	0.953975308229939
9/16/2010	0.946045398847401
12/15/2010	0.937738930374723
9/19/2011	0.910589640904885
9/18/2012	0.874605339290446
9/17/2013	0.838480785940185
9/17/2015	0.764484068050129
9/18/2018	0.660784174743023
9/17/2020	0.599226000228954
9/19/2023	0.516496191896040
9/19/2028	0.404499498266983
9/19/2033	0.319515078255888
9/17/2038	0.253964940017530
9/17/2048	0.162682769713171

Section 15. Appendix 3: Relationships among FX Forward Rates and Interest Rates

Through the no-arbitrage assumption there is an interest rate relationship with the FX rates on two different dates. The basic relationship is simple, but when using bid/ask spreads the equations are not invertible - deriving interest rates from FX forwards rather than the reverse - without destroying bid/ask consistency. This document gives the formulas and explanations of these relationships.

15.1 FX Forward Rate Projection

A foreign exchange rate that applies to a currency exchange taking place on date T is said to be the "Value T" rate. Interest-rate parity relates the FX Spot rate ("Value Spot") with the FX rate for exchange on another date, which can be before or after the spot date.

15.1.1 The Basic No-Arbitrage Argument

Define the following:

- R_{spot} – The spot FX rate defined for settling FX deals on spot date T_{spot}.
- $R_{fwd}(T)$ – The forward FX rate on date T.
- D_Q (T1, T2) – The discount factor for discounting quoting currency from any date T2 to earlier date T1.
- D_B (T1, T2) – The discount factor for discounting base (primary) currency from any date T2 to earlier date T1.

In outline, the basic relationship involves the following steps:

1. Borrow N of currency B on the spot date T_{SPOT}, with agreement to pay at time T the amount $\dfrac{N}{D_B(T_{SPOT},T)}$.

2. Enter into an FX forward to convert this amount into currency Q on date T, guaranteeing the amount

$$N\frac{R_{FWD}(T)}{D_B(T_{SPOT},T)}.$$

3. Convert the borrowed amount N on the Spot date to currency Q at the spot rate to give $R_{spot}N$.

4. Invest in currency Q rates until time T to obtain

$$N\frac{R_{SPOT}}{D_Q(T_{SPOT},T)}.$$

5. If there is no arbitrage these amounts must be equal:

$$\frac{R_{fwd}(T)}{D_B(T_{SPOT},T)} = \frac{R_{SPOT}}{D_Q(T_{SPOT},T)}.$$

15.1.2 FX Forward Mid Rates

If all the steps in the interest-rate parity argument are performed at the Mid rates then the logic is straightforward to apply these relationships to project FX forward rates using two interest rate curves and the FX spot rate.

Many market participants prefer to calculate at the Mid rates and then add on a spread. This is a more stable calculation than one in which the spreads of different curves and FX rates are all used, which can cause a build up of spreads producing an overall spread that is too wide compared to the market.

The MID forward rate for foreign exchange taking place on a date T ("for Value T") is projected from two interest rate curves using these formulas:

- If T >= T_{spot} (forward projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_B(T_{SPOT},T)^{MID}}{D_Q(T_{SPOT},T)^{MID}}, \quad T > T_{SPOT}$$

- If T < T_{spot} ("backward" projection),

$$R_{fwd}(T)^{MID} = R_{SPOT}{}^{MID} \frac{D_Q(T,T_{SPOT})^{MID}}{D_B(T,T_{SPOT})^{MID}}, \quad T < T_{SPOT}$$

15.1.3 Bid/Ask Rates in the No-Arbitrage Argument

To take the bid/ask spreads into account one needs to look at the steps in the no-arbitrage argument more carefully. There are in fact several different ways of combining trades to produce bid/ask relationships. What follows is the method Calypso has been using through Version 14. In this derivation the spot and the forward FX rates are taken to be on the same side, i.e., either both bid or both ask. In this form the equations are readily related to the usual method of quoting FX swap points in terms of the same side for spot and forward.

1. Borrow N of currency B on the spot date T_{SPOT}. Since one will be borrowing one will pay the Ask interest rate. Thus one agrees to pay at time T the amount

$$\frac{N}{D_B(T_{SPOT},T)^{ASK}}.$$

2. Enter into an FX forward to convert this amount into currency Q on date T. Because one is selling base currency this is done at the Bid FX rate. So the maturity payment in Q currency is

$$N \frac{R_{FWD}(T)^{BID}}{D_B(T_{SPOT},T)^{ASK}} \; .$$

3. Convert the borrowed amount N on the Spot date to currency Q. Since one is selling base currency this is done at the Bid FX rate, giving

$$NR_{SPOT}{}^{BID}$$

4. Invest in currency Q rates until time T. Because one is loaning one will receive the Bid interest rate. At time T one will then receive

$$N \frac{R_{SPOT}{}^{BID}}{D_Q(T_{SPOT},T)^{BID}} \; .$$

5. If there is no arbitrage the amount received cannot be greater than the amount paid:

$$\frac{R_{FWD}(T)^{BID}}{D_B(T_{SPOT},T)^{ASK}} \geq \frac{R_{SPOT}{}^{BID}}{D_Q(T_{SPOT},T)^{BID}} \; .$$

In order to use the other sides of the rates one would start by borrowing in the quoting currency Q and use the rates for buying base currency, which are the Ask FX rates. The equation can be found from the one above by reversing the bid and ask and the inequality sign.

15.1.4 FX Forward Bid/Ask Rates

The preceding argument gives limits on the FX bid/ask forward rates. In order to produce a definite number, we use the equals sign in the following. This also has the advantage of guaranteeing the correct bid/ask relationship. (Relationships that do not make these assumptions are discussed in section Triangulated FX Bid/Ask Relationships.)

Under these assumptions, bid/ask forward FX rates can be projected from interest rates as follows:

- If T >= T_{spot} (forward projection),

$$R_{fwd}(T)^{BID} = R_{SPOT}{}^{BID} \frac{D_B(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}}, \quad T > T_{SPOT}$$

$$R_{fwd}(T)^{ASK} = R_{SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}, \quad T > T_{SPOT}$$

- If T < T_{spot} (“backward” projection),

$$R_{fwd}(T)^{BID} = R_{SPOT}{}^{BID} \frac{D_Q(T,T_{SPOT})^{ASK}}{D_B(T,T_{SPOT})^{BID}}, \quad T < T_{SPOT}$$

$$R_{fwd}(T)^{ASK} = R_{SPOT}{}^{ASK} \frac{D_Q(T,T_{SPOT})^{BID}}{D_B(T,T_{SPOT})^{ASK}}, \quad T < T_{SPOT}$$

Consistency Checks

For these formulas it can be confirmed that the bid/ask definitions are consistent in that they preserve the requirement that bid rates are less than ask rates. That is, if the spot bid rate is less than the spot ask rate, and the bid interest rates are less than the ask interest rates in both currencies – so that the bid discount factors are *greater* than the ask discount factors -- then these formulas guarantee that the FX forward bid will be less than the FX forward ask.

Another check is the behavior for T very near T_{SPOT}. As T approaches T_{SPOT} the discount factors approach 1, and so the forward bid rates approach the spot bid rates, and the forward ask rates approach the spot ask rates.

Inverting the Equations

In principle these equations can be rearranged to provide formulas for discount factors given the forward rates. However, doing so can create interest rates where the bid rate is greater than the ask rate. The equations cannot be inverted and at the same time guarantee the relation of bid and ask rates. This is demonstrated in the following sections.

15.2 Deriving Interest Rates in One Currency from Interest Rates in Another Currency

15.2.1 Mid Interest Rates

One can rearrange the Mid equations to solve for one of the discount factors rather than the FX forward rate. To derive the quoting currency discount factor relative to spot, one has for forward projection

$$D_Q(T_{SPOT},T)^{MID} = \frac{R_{SPOT}^{\;\;\;MID}}{R_{fwd}(T)^{MID}} D_B(T_{SPOT},T)^{MID}, \quad T > T_{SPOT}$$

and for backward projection

$$D_Q(T,T_{SPOT})^{MID} = \frac{R_{fwd}(T)^{MID}}{R_{SPOT}^{\;\;\;MID}} D_B(T,T_{SPOT})^{MID}, \quad T < T_{SPOT}.$$

Today-based Mid Interest Rates

These give discount factors relative to the Spot date. To find the discount factors relative to Today, T_0, with $T_0 < T_{Spot}$, one can find $D_Q(T_0,T_{Spot})$ using the second equation only if the if the FX rate for Value Today is known, $R_{fwd}(T_0)^{MID}$. If this is not available one would instead need to extrapolate from the first calculable discount factors.

As an example of an extrapolation method, suppose the first given forward rate is as of T_1 which is after the Spot date. Then one can calculate $D_Q(T_{Spot.}\ T_1)$ from the first equation. Convert this to an interest rate between Spot and T_1 and then assume this rate is constant back to T_0 to derive $D_Q(T_0,T_{Spot})$.

With this factor from Today to Spot known one can transform the preceding equations to discount factors for T with respect to today:

$$D_Q(T_0,T)^{MID} = D_Q(T_0,T_{SPOT})^{MID} D_Q(T_{SPOT},T)^{MID}, \quad T > T_{SPOT}$$

$$D_Q(T_0,T)^{MID} = \frac{D_Q(T_0,T_{SPOT})^{MID}}{D_Q(T,T_{SPOT})^{MID}}, \quad T < T_{SPOT}$$

15.2.2 Bid/Ask Interest Rates

The bid/ask equations for the forward FX rates can be rearranged to give the interest rate equations. However, unlike the FX case, *the interest rate equations do not guarantee the correct bid/ask relationships*.

A rearrangement of the equations for FX forward rate projection gives

- If T >= T_{spot} (forward projection),

$$D_Q(T_{SPOT},T)^{BID} = \frac{R_{SPOT}^{\;BID}}{R_{fwd}(T)^{BID}} D_B(T_{SPOT},T)^{ASK}, \quad T > T_{SPOT}$$

$$D_Q(T_{SPOT},T)^{ASK} = \frac{R_{SPOT}^{\;ASK}}{R_{fwd}(T)^{ASK}} D_B(T_{SPOT},T)^{BID}, \quad T > T_{SPOT}$$

- If T < T_{spot} ("backward" projection),

$$D_Q(T,T_{SPOT})^{ASK} = \frac{R_{fwd}(T)^{BID}}{R_{SPOT}^{\;BID}} D_B(T,T_{SPOT})^{BID}, \quad T < T_{SPOT}$$

$$D_Q(T,T_{SPOT})^{BID} = \frac{R_{fwd}(T)^{ASK}}{R_{SPOT}^{\;ASK}} D_B(T,T_{SPOT})^{ASK}, \quad T < T_{SPOT}$$

In the next sections it is shown why this rearrangement can violate bid/ask consistency, and an alternative formula is considered.

15.2.3 Conflict Between Bid/Ask Relationships of FX and Interest Rates

The preceding equations for the interest rates do not guarantee that the bid interest rate is less than the ask rate. This is one reason why it is often more useful to simply use the Mid equations and then add in spreads to the result.

A quick way to visual the conflict is by noting that taking the ratio of the parity equations gives (for T after Spot)

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} = A\frac{D_Q(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}$$

where

$$A = \frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}} \frac{D_B(T_{SPOT},T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

In a normal market for spot FX and base currency interest rates

$$A > 1.$$

Now if quoting rates were in a normal market,

$$\frac{D_Q(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}} > 1 ,$$

which immediately shows the forward FX rates have normal bid/asks,

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > 1.$$

On the other hand, if we were given that the forward FX rates were normal, all we would know about the quoting interest rates would be

$$\frac{D_Q(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}} = \frac{1}{A} \frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > \frac{1}{A}$$

As A > 1 the left hand side can be less than 1, so the bid/ask relationship can easily be violated for the quoting interest rates.

In order to have $D_Q(T_{SPOT},T)^{BID} > D_Q(T_{SPOT},T)^{ASK}$, one must have

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > A \quad .$$

So it is not sufficient that the forward ask be greater than the bid; it must be greater by a large enough amount:

$$\left(\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}}\right) / \left(\frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}}\right) > \frac{D_B(T_{SPOT},T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

If the base discount factors correctly have the bid factor greater than the ask factor the right hand side is not less than one. So for the quoting discount factors to have the correct bid/ask relationship it must at least be true that

$$\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}} > \frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}}$$

In other words, the FX bid/ask spreads must widen as one goes out in time in order for the derived quoting discount factors to potentially have the correct bid/ask relationship. They must also widen by an amount that is governed by the primary currency discount factors.

However, this is *not* a sufficient condition for interest rate bid/ask consistency. The prior equation is more restrictive. Expressed in words, the ratio of the spot bid/ask ratio to the forward bid/ask ratio must be greater than the discount factor bid/ask ratio. If this condition does not hold, the resulting discount factor will have an inverted bid/ask relationship.

The relationship also holds for deriving primary currency discount factors, as is readily seen by repeating the argument.

This inverted relationship does happen when deriving interest rates from real market quotes. This implies either interest rates are driving FX rates rather than vice versa, or else that the market is using Mid rate relationships and adding in spreads without regard to keeping consistency among these equations.

15.2.4Alternate Formula for Interest Rates Giving Inconsistent FX

One can instead use a different no-arbitrage derivation that guarantees derived interest rates will always have the correct bid/ask relationship. But as a consequence one will find that it also is not consistently invertible. That is, inverting it to find the FX forwards can produce bid FX rates that are greater than ask rates.

Here is the alternate no-arbitrage derivation. Suppose one has an amount N of quoting currency on the Spot date. This can be invested two different ways:

Investment Method 1. Lend at the Bid interest rate to arrive at end amount

$$N / D_Q(T_{SPOT}, T)^{BID}$$

Investment Method 2. Buy base currency at the Ask rate, lend at the Bid base currency interest rate, and sell back base currency at the forward Bid rate, to arrive at

$$N \frac{R_{fwd}(T)^{BID}}{R_{SPOT}{}^{ASK} D_B(T_{SPOT}, T)^{BID}} .$$

Absence of arbitrage is guaranteed if the amount received is not greater than the amount paid. But suppose one argued these two methods of investment should be equal. Setting the two end results to be the same one would have

- **Alternate Derivation for T >= T_{spot} (forward projection),**

$$\overline{D}_Q(T_{SPOT}, T)^{BID} = \frac{R_{SPOT}{}^{ASK}}{R_{fwd}(T)^{BID}} D_B(T_{SPOT}, T)^{BID}, \quad T > T_{SPOT}$$

$$\overline{D}_Q(T_{SPOT}, T)^{ASK} = \frac{R_{SPOT}{}^{BID}}{R_{fwd}(T)^{ASK}} D_B(T_{SPOT}, T)^{ASK}, \quad T > T_{SPOT}$$

A bar has been placed over the derived discount factor in order to indicate this is a different derivation than the previous one. Note the argument was repeated for the Ask side. The difference between these equations and the previous ones are the switching of the bid and ask sides on R_{SPOT} and D_B. If this switch produces a difference in calculations then the discount factors obtained with this set of equations is not the same as with the previous set.

Dividing the bid equation by the ask equation one finds that the correct bid/ask relation is guaranteed for the discount factor $\overline{D}_Q$ if the other quotes are normal.

$$\frac{\overline{D}_Q(T_{SPOT},T)^{BID}}{\overline{D}_Q(T_{SPOT},T)^{ASK}} = A\frac{R_{fwd}(T)^{ASK}}{R_{fwd}(T)^{BID}}$$

with again

$$A = \frac{R_{SPOT}{}^{ASK}}{R_{SPOT}{}^{BID}}\frac{D_B(T_{SPOT},T)^{BID}}{D_B(T_{SPOT},T)^{ASK}}$$

One sees that the A is on the opposite side of the equation than before, showing a different dependence on the bid/ask ratios. So now the previous argument can be followed through to show that in a normal market this guarantees the correct bid/ask rates for $\overline{D}_Q$ but not for the forward rates.

15.3 Bid/Ask Limits and Widest Spreads

The formulas of sections FX Forward Bid/Ask Rates and Bid/Ask Interest Rates, on the one hand, and Alternate Formula for Interest Rates Giving Inconsistent FX on the other, made certain assumptions in order to arrive at equalities that can be used in forward rate projection. One can loosen these assumptions to find inequalities that give limits on the bid/ask spreads. The set of inequalities that gives the largest spread will provide the firm limits to arbitrage.

Neither of the formulas derived previously involved full round-trip arbitrage. To perform this, one starts with zero currency, borrows in one currency and repays in the same currency. One compares doing this with and without a currency conversion. If there is a difference beyond the bid/ask spread one has a definite arbitrage opportunity.

Borrow base currency N_B on the spot date at the ask rate. At maturity pay out

$$\frac{N_B}{D_B(T,T_{SPOT})^{ASK}}$$

At spot convert N_B to currency Q at the bid rate (selling base currency), lend at the bid rate, and enter into a forward to convert back to currency B at the ask rate (buying base currency), to receive at maturity

$$\frac{R_{SPOT}{}^{BID}N_B}{R_{fwd}(T)^{ASK}D_Q(T,T_{SPOT})^{BID}}$$

In order to prevent arbitrage, the net cashflows cannot be greater than zero:

$$\frac{R_{SPOT}{}^{BID}N_B}{R_{fwd}(T)^{ASK}D_Q(T,T_{SPOT})^{BID}} - \frac{N_B}{D_B(T,T_{SPOT})^{ASK}} \leq 0$$

This gives a lower limit on the forward FX ask rate:

$$R_{fwd}(T)^{ASK} \geq \frac{R_{SPOT}{}^{BID}D_B(T,T_{SPOT})^{ASK}}{D_Q(T,T_{SPOT})^{BID}}$$

Repeating the process but starting by borrowing in currency Q produces an upper limit on the forward FX bid rate:

$$R_{fwd}(T)^{BID} \leq \frac{R_{SPOT}{}^{ASK} D_B(T, T_{SPOT})^{BID}}{D_Q(T, T_{SPOT})^{ASK}}$$

One sees that this does not guarantee that $R_{fwd}(T)^{BID} \leq R_{fwd}(T)^{ASK}$ as the right hand side of the ask equation is less than the right hand side of the bid equation.

To be certain of bid/ask consistency one needs the bid rate to be less than the right hand side of the ask rate equation. If one makes this an equality then one arrives at the formulas of sections FX Forward Bid/Ask Rates and Bid/Ask Interest Rates, which is why those formulas guarantee FX bid/ask consistency.

Another reason to prefer the method of section Forward Bid/Ask Rates is that it provides the widest bid/ask spread for the FX rate. One can check that by rearranging the assignments of bid/ask sides there is no combination that produces a smaller bid or a larger ask.

By rearranging the inequalities one has limits on the interest rates,

$$D_Q(T, T_{SPOT})^{BID} \geq \frac{R_{SPOT}{}^{BID} D_B(T, T_{SPOT})^{ASK}}{R_{fwd}(T)^{ASK}}$$

$$D_Q(T, T_{SPOT})^{ASK} \leq \frac{R_{SPOT}{}^{ASK} D_B(T, T_{SPOT})^{BID}}{R_{fwd}(T)^{BID}}$$

This does not guarantee the bid/ask relationship for discount factors, $D_Q(T, T_{SPOT})^{BID} > D_Q(T, T_{SPOT})^{ASK}$. To be certain of the relationship the ask discount factor must be less than or equal to the right hand side of the bid equation. Turning that into an equality produces the equations of section Alternate Formula for Interest Rates Giving Inconsistent FX, which is why those equations guarantee discount factor bid/ask consistency. This also gives the widest bid/ask spread for the discount factors. But these relations are not the same as those of sections FX Forward Bid/Ask Rates and Bid/Ask Interest Rates.

Because of the inequalities one cannot guarantee both discount factor and FX forward rate consistency with a single set of equations.

15.4 Triangulated FX Bid/Ask Relationships

Consider the case of a triangulated FX rate, that is, where one converts from base (primary) currency B to quoting currency Q by first converting through the "split" currency X.

The relations between interest and FX rates can be found by writing the equations for each pair and creating the combination exchange.

If only mid rates are used then the dependence on the discount factors of currency X drop out in this process, and one is left with the original relation between the B and Q rates of section FX Forward Mid Rates.

If using bid and ask rates the discount factors of currency X do contribute to the end formulas, because in going through currency X one pays the bid ask spread on the currencies, and this cannot be ignored. Define

$R_{B/Q,SPOT}$: the spot rate for converting one unit of B into Q currency

$R_{B/X,SPOT}$: the spot rate for converting one unit of B into X currency

$R_{X/Q,SPOT}$: the spot rate for converting one unit of X into Q currency

with analogous definitions for the forward rates R_{fwd} .

Enter into a forward FX agreement to convert from currency B to X and a second agreement to convert the proceeds from X to Q. In the first trade one is selling B and so is done at the ask of rate B/X; in the second trade one is selling X so it is done at the ask of X/Q. The net of the combined trades gives the ask of a triangulated B/Q rate:

$$R_{B/Q,fwd}(T)^{ASK} = R_{B/X,fwd}(T)^{ASK} R_{X/Q,fwd}(T)^{ASK}$$

The reverse logic holds for the bid rates:

$$R_{B/Q,fwd}(T)^{BID} = R_{B/X,fwd}(T)^{BID} R_{X/Q,fwd}(T)^{BID}$$

Now the equations of section [FX Forward Bid/Ask Rates](#) for the two conversions are (for $T > T_{SPOT}$):

$$R_{B/X,fwd}(T)^{BID} = R_{B/X,SPOT}{}^{BID} \frac{D_B(T_{SPOT},T)^{ASK}}{D_X(T_{SPOT},T)^{BID}},$$

$$R_{B/X,fwd}(T)^{ASK} = R_{B/X,SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}},$$

$$R_{X/Q,fwd}(T)^{BID} = R_{X/Q,SPOT}{}^{BID} \frac{D_X(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}},$$

$$R_{X/Q,fwd}(T)^{ASK} = R_{X/Q,SPOT}{}^{ASK} \frac{D_X(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}.$$

Combining the asks to give the triangulated ask rate produces

$$R_{B/Q,fwd}(T)^{ASK} = R_{B/X,SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}} R_{X/Q,SPOT}{}^{ASK} \frac{D_X(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}}$$

which can be rewritten by combining the spot rates into a triangulated rate,

$$R_{B/Q,fwd}(T)^{ASK} = R_{B/Q,SPOT}{}^{ASK} \frac{D_B(T_{SPOT},T)^{BID}}{D_Q(T_{SPOT},T)^{ASK}} \frac{D_X(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}}$$

Similarly for the bid side,

$$R_{B/Q,fwd}(T)^{BID} = R_{B/Q,SPOT}{}^{BID} \frac{D_B(T_{SPOT},T)^{ASK}}{D_Q(T_{SPOT},T)^{BID}} \frac{D_X(T_{SPOT},T)^{ASK}}{D_X(T_{SPOT},T)^{BID}}$$

One sees these are almost the same as the original equations of Section 1.4 except for multiplication or division by the additional factor

$$\frac{D_X(T_{SPOT},T)^{BID}}{D_X(T_{SPOT},T)^{ASK}}$$

which takes into account the spread from the buy and sell of the X currency.

If using only mid rates this factor is just 1 and the discount factors of the X currency do not enter into the relations. But when using bid ask rates, if one did not include this factor then the B to Q bid/ask relationships that were guaranteed in section FX Forward Bid/Ask Rates can fail.