

# Insertion Sort Example

A = 12, 9, 3, 7, 14, 11

*i* = 0, 1, 2, 3, 4, 5

for *i* = 1 to *i* = 5:

set the key to A[*i*] ( A[1] = 9 ). Hold the key to create an empty space.

9

A = 12, , 3, 7, 14, 11

working with items at indices *i*-1 → 0,

while the key is less than each item, move each up by 1 and then put the key in the empty space.

A = 9, 12, 3, 7, 14, 11

increment *i* (now is *i* = 2)

---

set the key to A[*i*] ( A[2] = 3 ). Hold the key to create an empty space.

3

A = 9, 12, , 7, 14, 11

working with items at indices *i*-1 → 0,

while the key is less than each item, move each up by 1 and then put the key in the empty space.

A = 3, 9, 12, 7, 14, 11

increment *i* (now is *i* = 3)

---

set the key to A[*i*] ( A[3] = 7 ). Hold the key to create an empty space.

7

A = 3, 9, 12, , 14, 11

working with items at indices *i*-1 → 0,

while the key is less than each item, move each up by 1 and then put the key in the empty space.

A = 3, 7, 9, 12, 14, 11

increment *i* (now is *i* = 4)

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set the key to A[*i*] ( A[4] = 14 ). Hold the key to create an empty space.

14

A = 3, 9, 7, 12, , 11

working with items at indices *i*-1 → 0,

while the key is less than each item, move each up by 1 and then put the key in the empty space.

A = 3, 7, 9, 12, 14, 11

increment *i* (now is *i* = 5)

---

set the key to A[*i*] ( A[5] = 11 ). Hold the key to create an empty space.

11

A = 3, 9, 7, 12, 14, ,

working with items at indices *i*-1 → 0,

while the key is less than each item, move each up by 1 and then put the key in the empty space.

A = 3, 7, 9, 11, 12, 14

---

we have reached the end of the array and the sort is complete

# Selection Sort Example

A = 12, 9, 3, 7, 14, 11

$i = 0, 1, 2, 3, 4, 5$

for  $i = 0$  to  $i = 4$ :

$i = 0$

find the smallest item by checking all items between  $i = 0$  and  $i = 5$  (inclusive)

and swap the smallest item with the item at  $i = 0$

A = 12, 9, 3, 7, 14, 11

A = 3, 9, 12, 7, 14, 11



increment  $i$  (now is  $i = 1$ )

---

find the smallest item by checking all items between  $i = 1$  and  $i = 5$  (inclusive)

and swap the smallest item with the item at  $i = 1$

A = 3, 9, 12, 7, 14, 11

A = 3, 7, 12, 9, 14, 11



increment  $i$  (now is  $i = 2$ )

---

find the smallest item by checking all items between  $i = 2$  and  $i = 5$  (inclusive)

and swap the smallest item with the item at  $i = 2$

A = 3, 7, 12, 9, 14, 11

A = 3, 7, 9, 12, 14, 11



increment  $i$  (now is  $i = 3$ )

---

find the smallest item by checking all items between  $i = 3$  and  $i = 5$  (inclusive)

and swap the smallest item with the item at  $i = 3$

A = 3, 7, 9, 12, 14, 11

A = 3, 7, 9, 11, 14, 12



increment  $i$  (now is  $i = 4$ )

---

find the smallest item by checking all items between  $i = 4$  and  $i = 5$  (inclusive)

and swap the smallest item with the item at  $i = 4$

A = 3, 7, 9, 11, 14, 12

A = 3, 7, 9, 11, 12, 14



increment  $i$  (now is  $i = 5$ )

---

the second last item is now sorted and this means that the last item is also sorted.  
So the sort is complete

# Bubble Sort Example

$A = 12, 9, 3, 7, 14, 11$

$i = 0, 1, 2, 3, 4, 5$

for  $i = 0$  to  $i = 4$ :

if  $A[i+1]$  is less than  $A[i]$  swap the two items

$i = 0$   
 $A = \underline{12}, 9, 3, 7, 14, 11$

$i = 1$   
 $A = 9, \underline{12}, 3, 7, 14, 11$

$i = 2$   
 $A = 9, 3, \underline{12}, 7, 14, 11$

$i = 3$   
 $A = 9, 3, 7, \underline{12}, 14, 11$

$i = 4$   
 $A = 9, 3, 7, 12, \underline{14}, 11$

$A = 9, 3, 7, 12, 11, 14$       end of first pass (the last item is certainly in the right place)

---

for  $i = 0$  to  $i = 3$ :

if  $A[i+1]$  is less than  $A[i]$  swap the two items

$i = 0$   
 $A = \underline{9}, 3, 7, 12, 11, 14$

$i = 1$   
 $A = 3, \underline{9}, 7, 12, 11, 14$

$i = 2$   
 $A = 3, 7, \underline{9}, 12, 11, 14$

$i = 3$   
 $A = 3, 7, 9, \underline{12}, 11, 14$       end of the second pass ( the last and second last items are certainly in the right place).

$A = 3, 7, 9, 11, 12, 14$       We can note that the array is in fact sorted but the algorithm will need another pass to "know" that it has been sorted.

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for  $i = 0$  to  $i = 2$ :

if  $A[i+1]$  is less than  $A[i]$  swap the two items

$i = 0$   
 $A = \underline{3}, 7, 9, 11, 12, 14$

$i = 1$   
 $A = 3, \underline{7}, 9, 11, 12, 14$

$i = 2$   
 $A = 3, 7, \underline{9}, 11, 12, 14$

$A = 3, 7, 9, 11, 12, 14$       end of the third pass.  
No swaps have occurred so the array must be sorted

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# Concept of QuickSort Example

A = 12,9,3,7,14,11

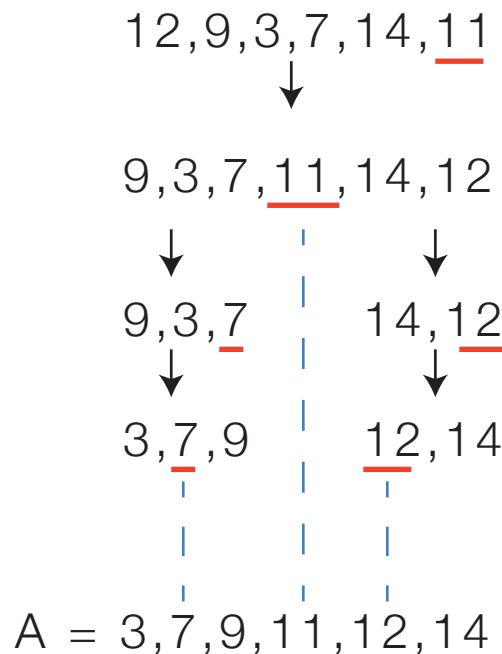
i = 0 , 1 , 2 , 3 , 4 , 5

choose the last item of the array as the "pivot".

arrange the list so that all items to the left of the pivot are less than (or =) the pivot and all items to the right of the pivot are greater than the pivot.

partition the array into  
sub-divisions above and  
below the pivot

for each sub-division, choose the  
last item as the "pivot".  
arrange each sub-divided array so  
that all items to the left of the  
pivot are less than (or =) the pivot  
and all items to the right of the  
pivot are greater than the pivot.



the process is continued  
unless the subdivision is  
size = 1

# Binary Search Examples

A = 26, 27, 29, 37, 45, 58, 68, 69, 73

i = 0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8

Search for the item 58 in the list

Find the item at the middle index in the array (if there are two middle values choose the lower)

26, 27, 29, 37, 45, 58, 68, 69, 73  
index 4

58 is more than 45 so go right and use the sub-array above 45.

Find the item at the middle index in the sub-array (if there are two middle values choose the lower)

58, 68, 69, 73  
index 6

58 is less than 68 so go left and use the sub-array below 68

Find the item at the middle index in the sub-array (if there are two middle values choose the lower)

58,  
index 5

58 is equal to 58

return "index is 5"

---

A = 26, 27, 29, 37, 45, 58, 68, 69, 73

i = 0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8

Search for the item 11 in the list

Find the item at the middle index in the array (if there are two middle values choose the lower)

26, 27, 29, 37, 45, 58, 68, 69, 73  
index 4

11 is less than 45 so go left and use the sub-array below 45.

Find the item at the middle index in the sub-array (if there are two middle values choose the lower)

26, 27, 29, 37  
index 1

11 is less than 27 so go left and use the sub-array below 27.

26  
index 0

11 is less than 26 so go left and use the sub-array below 26.

There is nothing there because my sub-array is now of length 1

return "11 is not in the array"

# Binary Search Algorithm

For a sorted array of length  $n$  (with index numbers 0 to  $n-1$ ) and with search value  $x$ .

*Find the value at the middle index of the array (if there are two items in the middle, choose the lower index)*

If  $x <$  than the middle value, go left and use the sub-array that is all values below the middle index

*If  $x >$  than the middle value, go right and use the sub-array that is all value above the middle index*

If  $x ==$  the middle value

*return the index number of this item (this will terminate the search)*

*If you cannot go left or right (i.e the last sub-array was of length 1 )*

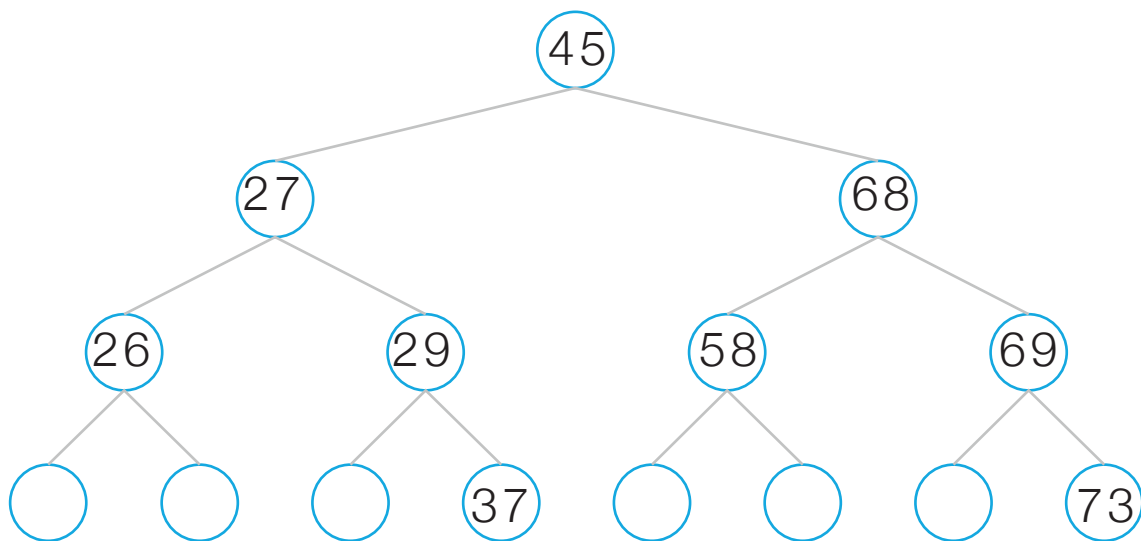
```
return x+ "is not in the array" (this will terminate the search)
```

*Repeat the process using the sub-array found above.*

# A Binary Tree

*If we examine all the paths of going left or right and choosing the middle value, we can see that we could represent the sorted array in a tree structure, this is called a binary tree*

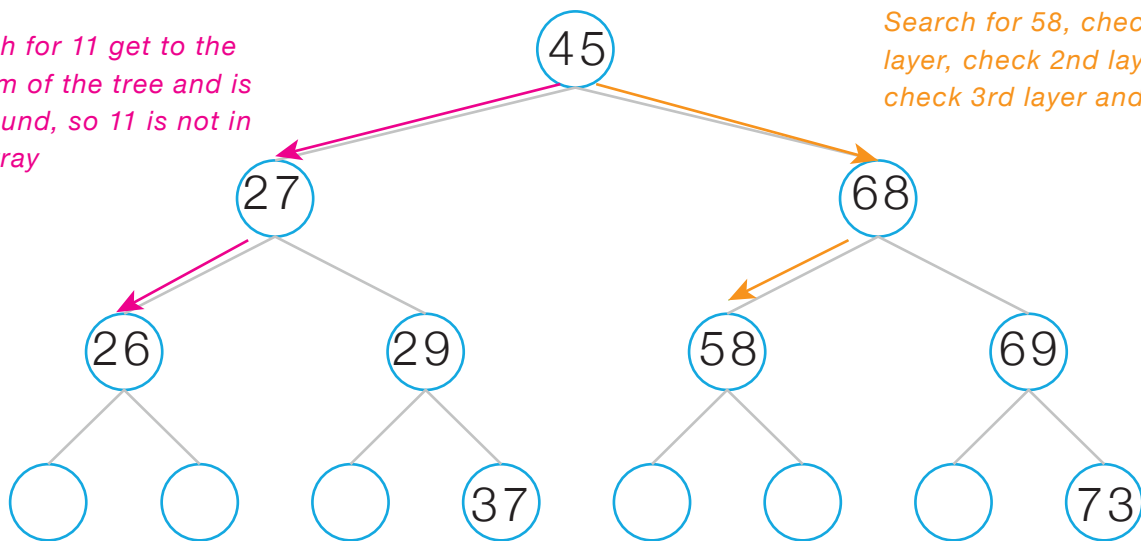
26, 27, 29, 37, 45, 58, 68, 69, 73  
26, 27, 29, 37, , 58, 68, 69, 73  
26, , 29, 37, , 58, , 69, 73  
 , 37, , 73



We can see that any search for a value is a pathway down the tree, and that each value check takes us one layer down.

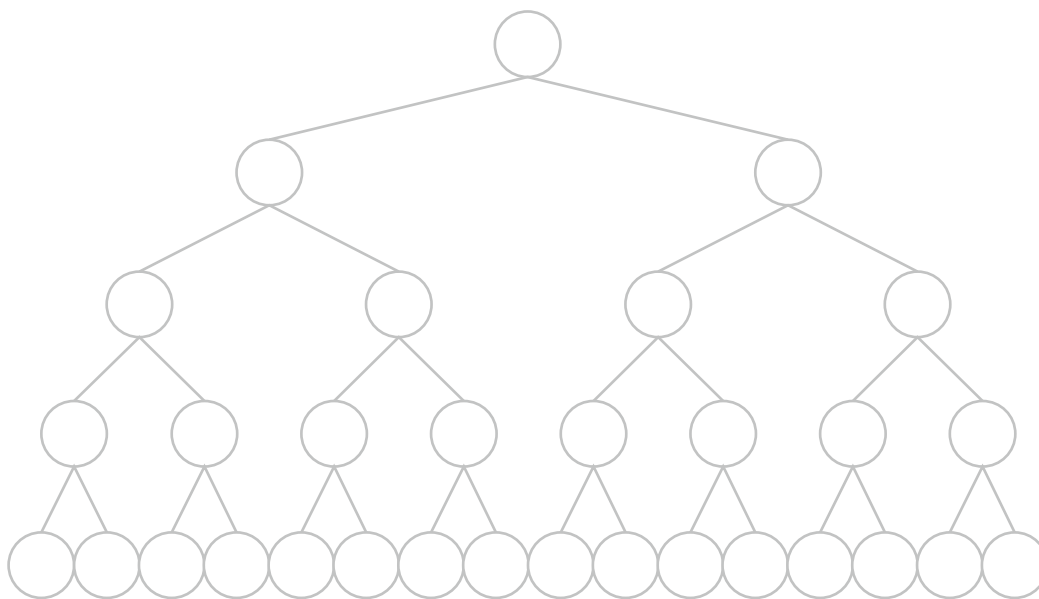
If we get to the bottom either we find the value or know that it is not in the array.

Search for 11 get to the bottom of the tree and is not found, so 11 is not in the array



Search for 58, check first layer, check 2nd layer, check 3rd layer and find it

This shows us that the binary search process is extremely efficient, and we can search through arrays of extremely large size for a very small number of checks.



Layer 1 - tree has maximum of 1 element

Layer 2 - tree has maximum of 3 elements

Layer 3 - tree has maximum of 7 elements

Layer 4 - tree has maximum of 15 elements

Layer 5 - tree has maximum of 31 elements

Layer 15 - tree has maximum of 32,767 elements

To search through a sorted list of 1 billion items, we would require a maximum of 30 checks, compared to a linear search which would require on average half a billion checks



Layer 30 - tree has maximum of 1,073,741,823 elements

# Linear Search

```
for an array of length n,  
search for x  
set i = 0  
    while i < n  
        if A[i] == x  
            return i (return the index number where x is - this will terminate the search)  
        i+=1  
  
return x+"is not in the array" (if every item has been checked and it has not been  
found return this information - this will terminate the  
search)
```

A = 51, 31, 17, 11, 8, 2, 0, 12, 36  
i = 0, 1, 2, 3, 4, 5, 6, 7, 8

Search for 17 in the array.

i = 0  
x = 17

is A[0] == 17 (i.e. does 51 = 17?)  
no so i += 1 (i is now 1)

is A[1] == 17 (i.e. does 31 = 17?)  
no so i += 1 (i is now 2)

is A[2] == 17 (i.e. does 17 = 17?)  
yes, so return 2 (the index value, and the search is now terminated)