Multilevel Models for Applied Social Research

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0930-1500hrs, 18/Jun/2025

Github resources:
https://github.com/paul-lambert/SGSSS-2025/

0930-1100	Talk 1: Classical perspectives on multilevel modelling
	Talk 2: Realistic complexity
1115-1230	Lab 1: Implementing selected popular multilevel models
	Talk 3: Case study on effect scores from random effects residuals
1330-1500	Talk 4: MLMs meet econometrics
	Lab 2: Responding to complex data and to complex analytical options

Multilevel Models for Applied Social Research – course overview

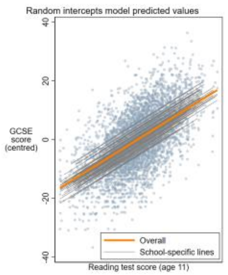
Workshop abstract: In this session we introduce and reflect upon the role of multilevel models in applied social research, and provide practical training materials which demonstrate ways of running multilevel models in survey data analysis scenarios.

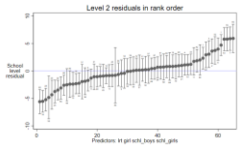
Multilevel models are popular in the social sciences as statistical analytical devices which can be useful in a variety of scenarios where data has a complex or 'clustered' structure. The most popular formats for running multilevel models are outlined in session materials and illustrated in multiple software environments (using Stata, SPSS and R). There remain plenty of situations, however, when the added value of using a multilevel model is ambiguous, and there are different views on the best strategies to use activities such as specifying, estimating, and interpreting suitable models. As well as giving introductory accounts, lecture and workshop materials also provide critical reflections on the place of multilevel models as statistical analytical procedures in applied social research, and describe and explore enduring debates about these methods.

Lectures will introduce and reflect upon multilevel models in social research, with opportunities for questions and answers and discussion. Workshops will involve computer-based exercises which open existing datasets and run multilevel models on them, guided by software example files that are provided to participants.

The session ought to be helpful for participants who have only a little statistical background, as it will use an introductory style to describe the features of multilevel models, and improve participants' confidence in this approach. The session should also be useful to people who already have a more extensive statistical methodology background, as materials also provide critical reflections on strengths and limitations of the approach and explore selected advanced issues.

Github files: https://github.com/paul-lambert/SGSSS-2025/





0930-1100	Talk 1: Classical perspectives on multilevel modelling Talk 2: Realistic complexity
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Multilevel Models for Applied Social Research – course overview

Talk 1: Classical perspectives on multilevel modelling

- (1a) Responding to multilevel and complex data
- (1b) Statistical models can help us understand societies
- (1c) Multilevel models are easy & popular ways to adapt statistical models to common features of social research data
- (1d) It's never been easier to use multilevel models in applied social research, though they are not without complexities

Talk 2: Realistic complexity

- (2a) Parameters for random slopes and intercepts
- (2b) Higher level residuals
- (2c) Multilevel models with more than two levels
- (2d) Multilevel models with non-linear outcome variables
- (2e) Realistic complexity in software tools

Talk 3: Case study on effect scores from random effects residuals

- (3a) Recap: The standard random effects multilevel model
- (3b) Things we could do with categorical data…
- (3c) Example: Education scores..
- (3d) Example: Using ESRES in CAMSIS scaling
- (3e) Summary: When might 'effect scores from random effects residuals' (ESRES's) be useful?

Talk 4: MLMs meet econometrics

- (4a) Do you just mean it's a random effects model?
- (4b) Why not just use robust standard errors?
- (4c) Why not just use the fixed effects model?
- (4d) What if I want to do something that's not a GLM?

Lab 1: Implementing selected popular multilevel models

Lab 2: Responding to complex data and to complex analytical options

(1a) Responding to multilevel and complex data

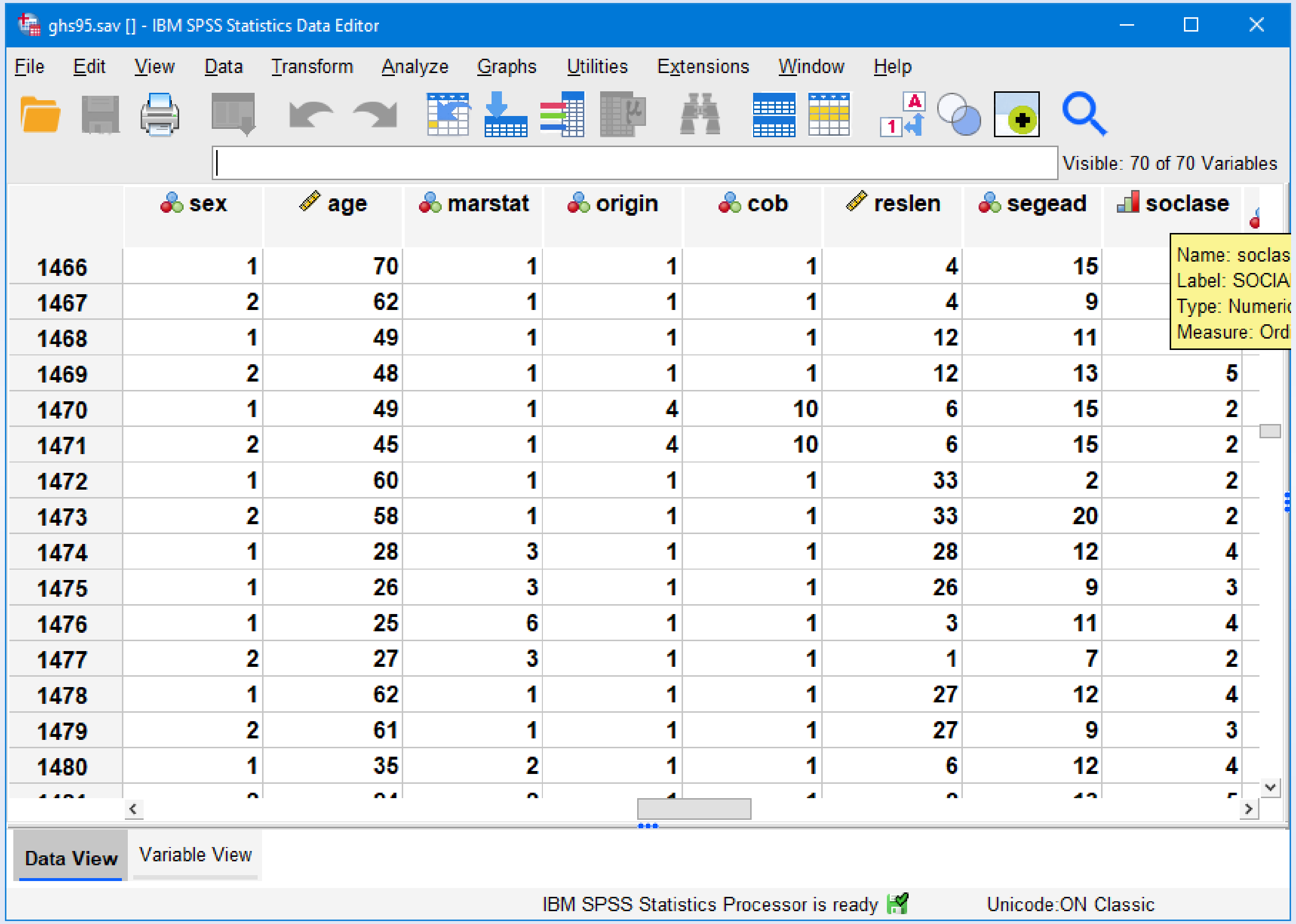
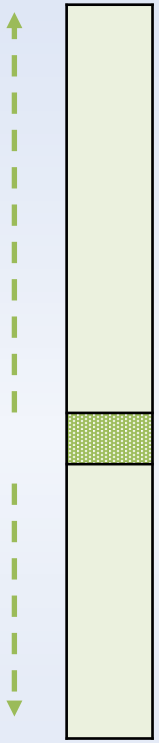


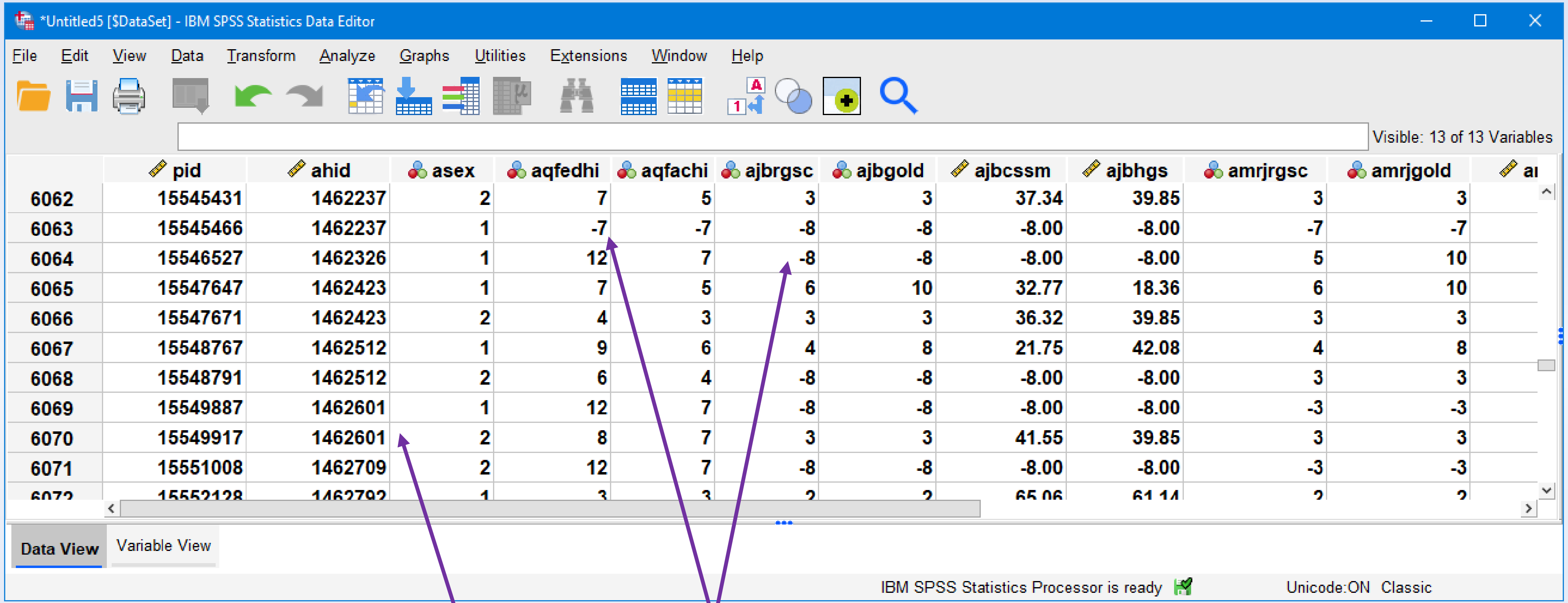
Image of 'ghs95.sav' in SPSS: 70 variables, 4633 cases, we behave as if the sample that have been selected are a simple random sample, with perfect measures, of independent cases with no missing values. Under those assumptions, sample patterns will, on average, be the same as those of the wider population ('representative')



Many statistics textbooks assume a 'simple' dataset, such as a simple random sample of independent cases with complete data on a smallish number of clearly defined variables

(1a) Responding to multilevel and complex data

Image of a small selection of measures from the 10264 cases on the first wave of the UK BHPS.



	pid	ahid	asex	aqfedhi	aqfachi	ajbrgsc	ajbgold	ajbcssm	ajbhgs	amrjrgsc	amrjgold	ai
6062	15545431	1462237	2	7	5	3	3	37.34	39.85	3	3	
6063	15545466	1462237	1	-7	-7	-8	-8	-8.00	-8.00	-7	-7	
6064	15546527	1462326	1	12	7	-8	-8	-8.00	-8.00	5	10	
6065	15547647	1462423	1	7	5	6	10	32.77	18.36	6	10	
6066	15547671	1462423	2	4	3	3	3	36.32	39.85	3	3	
6067	15548767	1462512	1	9	6	4	8	21.75	42.08	4	8	
6068	15548791	1462512	2	6	4	-8	-8	-8.00	-8.00	3	3	
6069	15549887	1462601	1	12	7	-8	-8	-8.00	-8.00	-3	-3	
6070	15549917	1462601	2	8	7	3	3	41.55	39.85	3	3	
6071	15551008	1462709	2	12	7	-8	-8	-8.00	-8.00	-3	-3	
6072	15552128	1462792	1	3	3	2	2	65.06	61.14	2	2	

Cases with the same 'ahid' are different adult respondents resident in the same household

Negative values usually mean the measure is missing for some reason

The extract here features several different plausible measures of educational qualifications and occupation-based indicators

In applied social research, data might be a non-random sample of cases, that aren't independent, with lots of missing data & ambiguous measurement quality

(1a) Responding to multilevel and complex data

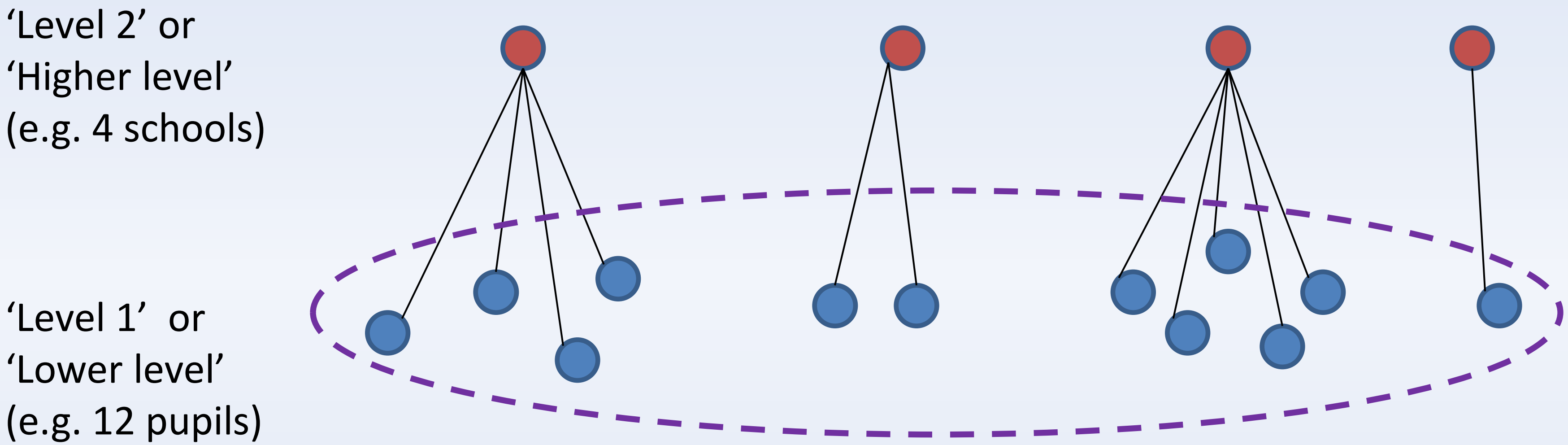
One form of complex data is a ‘**multilevel**’ or ‘**clustered**’ or ‘**hierarchical**’ data structure…

- ‘Clustered data’ arises when individual records in the data can be located in one or more subgroups
  - Cluster coding information is preserved in data
  - The membership of categories involves one or more ‘hierarchies’ (which might not overlap)

  – *It would be wrong for an analysis to ignore the clustering*
    - Statistically: results could be wrong
    - Substantively: the clusters might matter/be interesting

This image is a logo used for the UK’s ‘Centre for Multilevel Modelling’ (www.bristol.ac.uk/cmm/)

‘Multilevel data’ with a two-level hierarchical structure



- Most commonly, analysis proceeds at the lower level, but it would seem wrong to ignore the ‘clustering’ or ‘nesting’ into higher level units.
- We might be really interested in the higher level units, deliberately designing study for them, or we might just want to control for them
- **‘Multilevel modelling’** can be thought of as any adjustment to a model that is designed to take appropriate account of clustering

‘Multilevel’ data

Regions (Level 3)	k	**1**										**2**			**3**	**.**
Households (Level 2)	j	**1** 1			**2** 2		**3** 3	**4** 4				**5** 1	**6** 2		**7** 1	**.**
Individuals (Level 1)	i	**1** 1	**2** 2	**3** 3	**4** 1	**5** 2	**6** 1	**7** 1	**8** 2	**9** 3	**10** 4	**11** 1	**12** 1	**13** 2	**14** 1	**.** **.**

This data has a three level hierarchy, but we could denote an individual response in the data as:

Y_i (one level model) $(y_1 , y_2 ,\ldots, y_{14})$

Y_{ij} (two level model) $(y_{1,1} , y_{2,1} ,\ldots, y_{14,7})$ $(y_{1,1} , y_{2,1} ,\ldots, y_{1,7})$

Y_{ijk} (three level model) $(y_{1,1,1} , y_{2,1,1} ,\ldots, y_{14,7,3})$ $(y_{1,1,1} , y_{2,1,1} ,\ldots, y_{1,1,3})$

(1a) Responding to multilevel and complex data

‘Multilevel’ or ‘clustered’ or ‘nested’ data

- ‘Clustered data’ refers to categorical coding information preserved in the data that places units into different subgroups
- *Random effects multilevel models are mostly used when there are many subgroups, which don’t sustain independent analysis of each, but are expected to influence the process we’re studying. Typical examples:*
  - Respondents in households (e.g. N = 5000, k=3000, $\bar{k}$ = 1.7)
  - Sample survey respondents clustered into PSUs or interviewer subgroups (e.g. N = 10000, k = 200, $\bar{k}$ = 50)
  - Students in classes or schools (e.g. N = 2000, k = 100, $\bar{k}$ = 20)
  - Subjects in companies / institutions (e.g. N = 500, k = 50, $\bar{k}$ = 10)
  - Respondents in countries in cross-national studies (e.g. N = 16000, k = 16, $\bar{k}$ = 1000)
  - N = sample size; k = N_g = number of clusters; k-bar = mean cases per cluster
  - Random effects models are generally productive when both N and k and large (we often design a study to maximise k rather than $\bar{k}$);
  - **they are not inappropriate, but may be suboptimal, when k or $\bar{k}$ is small.**

(1b) Statistical models can help us understand societies

- ‘**Statistical models**’ are ‘**parametric analogies**’ that describe data patterns usefully [cf. Tarling 2009]

Data = Model fit + errors

- We use models to try to find an effective, parsimonious description of moderately complex social processes
- Statistical modelling involves finding parameters to describe empirical patterns in a model, assessed against whether they give a reasonably good description of the actual data
  - ‘Parametric analogies’ – numeric values are used to describe, in simplified form, the average statistical relationship between one thing and another
  - See e.g. Gilbert (1993), Treiman (2009), Angrist & Pischke (2015): Models as flexible tools for describing the relationships been variables in the context of each other

(1b) Statistical models can help us understand societies

Model statements in form of (algebraic expressions of) relations between variables

Cases ↓	←	Variables		→
varname	sex	age	height	
algebra	x1	x2	y1	
1	1	17	1.73	.
2	1	18	1.85	.
3	0	17	1.60	.
4	0	18	1.69	.
.	.	.	.	.
N				

- **Example model:** Men are taller than women
- **Arithmetic Formulation:**

  ⇒ Height = A constant population height + function of gender (increase if male) + error
- **Algebra:**

  $\Rightarrow y_1 = \beta_0 + \beta_1[\text{sex=male}] + \varepsilon_1$

  $\Rightarrow \mathbf{y_1 = \beta_0 + \beta_1 x_1 + \varepsilon_1}$

  $\{ \Rightarrow \hat{y}_1 = \beta_0 + \beta_1 x_1 \}$

(1b) Statistical models can help us understand societies

…Models generate expected values & deviance…

Cases ↓	← Variables →					
	_cons	Sex	Height	Age	Pred	E
	x_{0i}	x_{1i}	y_{1i}	x_{2i}	$\hat{y}_{1i}$	e_i
1	1	1	1.73	17	1.79	-.06
2	1	1	1.85	18	1.79	.06
3	1	0	1.60	17	1.65	-.05
4	1	0	1.69	18	1.65	.04
	Intercept: β_0=1.65 (to multiply by '_cons')			β_1=0.14 (to multiply by 'sex')		

$$y_1 = \beta_0 + \beta_1 x_1 + \varepsilon_1$$

…The canonical model is the linear regression model…

$$Y = \beta X + \varepsilon$$

- **Y** = vector of numeric values of an outcome variable
- **X** = matrix of explanatory variables
- **β** = vector of parameters to describe the influences of X
- **ε** = vector of error terms for each response (assumed to be randomly distributed)

- We ‘**estimate’ parameters for a model** using a statistical routine
- This **calculates the parameter coefficients** (numerical values) which give **lowest total errors** for the relationship being investigated.
- We **interpret the parameters** as evidence about the relationships between variables

(1b) Statistical models can help us understand societies

- Models help us to describe patterns in data
  - Models can be adapted to appropriately account for multilevel and complex features
    - Doing so lets us embed ‘realistic complexity’ in our descriptions of the patterns in data

- **Y = βX + ε** (the linear model)
- ***Generalised linear models*** *(esp. McCullagh & Nelder 1989)*
  - *Ways of reformulating the linear model to allow for a non-linear outcome measure*
- ***Generalised linear latent and mixed models*** *(esp. Rabe-Hesketh and Skrondal 2022)*
  - *Adding complexity to the LM, such as by allowing Y, X and/or ε to have a different format or structure (include ‘multilevel models’ as a component)*

(1c) Multilevel models are easy & popular ways to adapt statistical models to common features of social research data

- ‘**Statistical models**’ are ‘**parametric analogies**’ that describe data patterns usefully [cf. Tarling 2009]
- ‘**Multilevel models**’ (MLM’s) are versions of statistical models that are designed to address data with a clustered/hierarchical/nested character

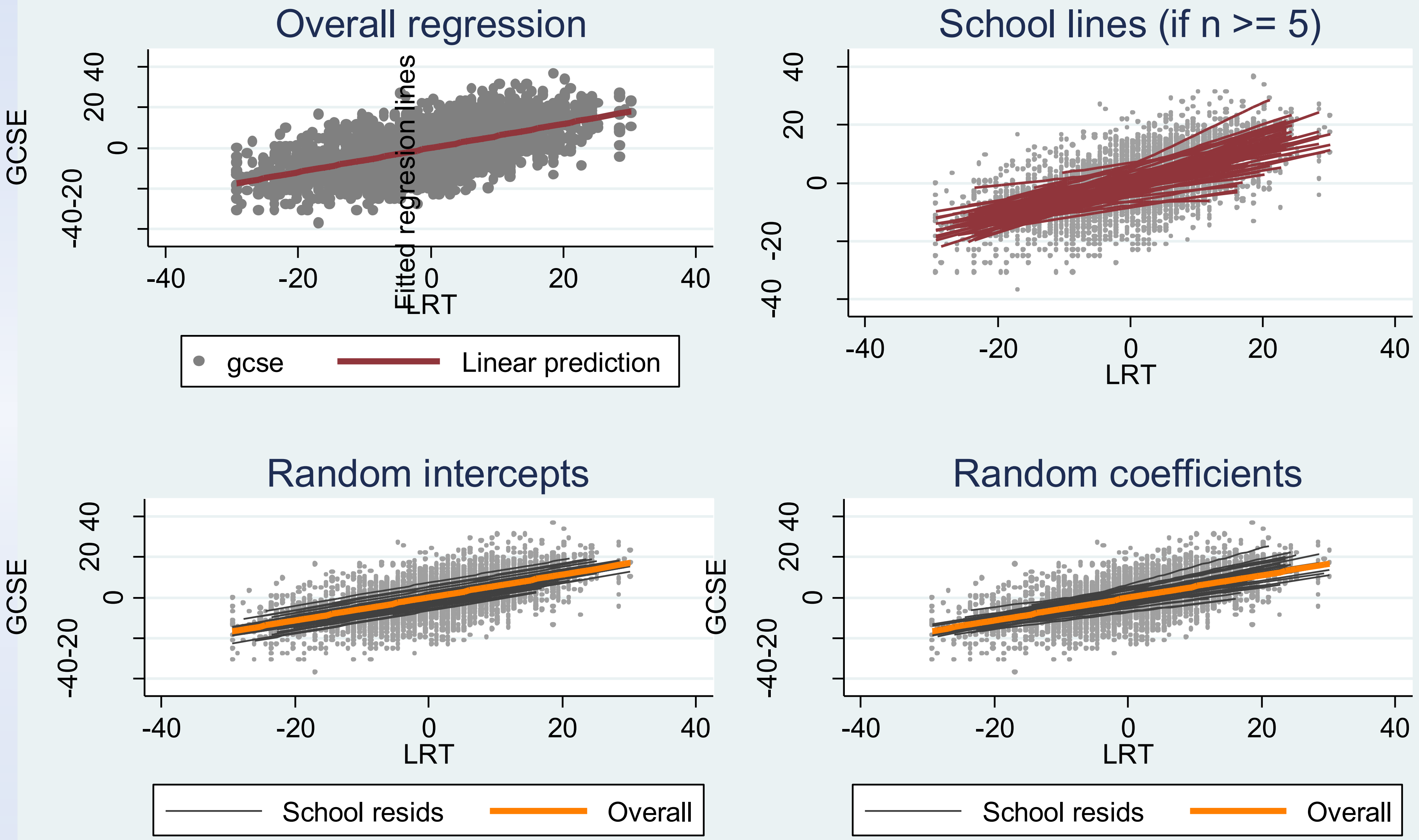
..In the social sciences the multilevel modelling tradition has developed a more specific identity…

- The use of ‘random effects’ estimators for MLMs
- An overt interest in interpreting/exploring hierarchical/clustered/nested effects

Wide-ranging texts include: Hox et al. 2017; Snijders and Bosker 2012; LEMMA online course

Easy to follow introductions include: Robson and Pevalin 2016; Luke 2020; UCLA 2021

Models for LRT effect on GCSE, with school clustering



Source: Following Rabe-Hesketh and Skrondal(2008), analsysis using 'gsce.dta'

The GCSE grades models..?

- (Used by Goldstein 2003, amongst others)
- The data is from pupils across different schools
  (65 schools, 4059 pupils)
- The overall regression finds the best line of fit
  (for all 4059 pupils)
- The ‘school lines’ fit a different regression for each school (65 different regressions)
- The two random effects multilevel models find an overall line which was informed by explicitly modelling aspects of the school clustering

We’ll reproduce this graph in one of the lab sessions

(1c) Multilevel models are easy & popular ways to adapt statistical models to common features of social research data

Arguably, a multilevel model is any statistical model that embeds extra features to take account of a multilevel data structure…..

- Fit dummy variables for each cluster unit?
- Fit explanatory variables defined at the cluster level?
- Sample or weight cases informed by the cluster structure?

Adjustments to the 'fixed part' (the βX) of the model

Adjustments to sampling assumptions about the dataset (for instance 'robust standard errors')

In the social sciences, a multilevel model usually implies a 'random effects' adjustment to take account of a multilevel data structure…..

- Make a statement about error term(s) in the model and their link to the multilevel data structure

Adjustments to the 'random part' (hitherto ε) of the model

(1c) Multilevel models are easy & popular ways to adapt statistical models to common features of social research data

- Linear model:

$$Y_i = \beta X_i + \varepsilon_i$$

- Multilevel linear model ('random intercepts' version)

$$Y_{ij} = \beta X_{ij} + \mu_j + \varepsilon_{ij}$$

… the random intercepts multilevel model involves redefining the model errors into parts associated with each level…

Bickel 2007: our focus remains on β, so multilevel models are still 'just regression'

My own take on multilevel models:

➢The core result is indeed 'just' regression

➢ There are lots of other results and issues, though, which can give extra insight and/or bring extra complications….

Some differences in outlook on multilevel models..

- A bit of a hot topic in some social science disciplines
  - *"Multilevel modeling is an innovative and valuable tool for evaluating the intersectionality of health inequalities"* [Evans et al., 2018: 64]
  - *"I think you'll agree that multilevel models are pretty funky.* 'Is there anything they can't do?' *I hear you cry. Well, no, not really."* [Field, 2009: 730]

- MLM's are reasonably easy to interpret at the front end, but a little tricky behind the scenes (in terms of statistical estimation and parameter interpretation)

- Not everyone is that impressed by MLM's
  - Economics – risk of bias in random effects specification [e.g. Allison 2009]
  - Education – statistical 'methodolatry' that impedes interpretation [e.g. Gorard 2003, 2021]

(1d) It's never been easier to use multilevel models in applied social research, though they are not without complexities

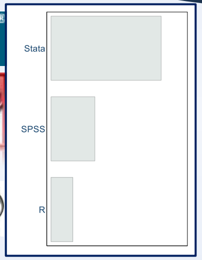
Plentiful learning resources:

- The CMM's 'LEMMA' online course and other CMM learning resources
  - [https://www.bristol.ac.uk/cmm/learning/online-course/]
  - [e.g. https://www.bristol.ac.uk/cmm/learning/support/books.html]
- Multi-day short courses on multilevel modelling (e.g. CMM, Essex Summer School, & more)
- Of the many useful textbooks, I'd nominate…

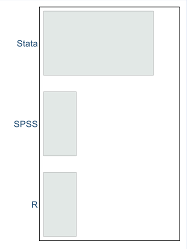
	Introductory	*Extended*
No software	Luke (2020)	Hox et al. (2017); Snijders & Bosker (2012)
Stata	Robson & Pevalin (2015); Mitchell (2020)	Rabe-Hesketh & Skrondal (2022)
SPSS	Bickel (2007)	Heck et al. (2012)
R		Gelman et al. (2009)

(1d) It’s never been easier to use multilevel models in applied social research, though they are not without complexities

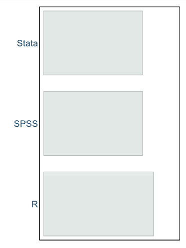
Data preparation
(understanding data, linking/defining measures)



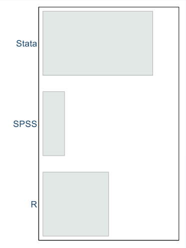
Analytical contextualisation
(descriptive stats.; exploration and comparison of plausible model formulations)



Model estimation and evaluation
(linear/non-linear outcomes, random intercepts and slopes, model residuals and checks)



Output presentation
(model-based visualisation; expression of findings and limitations; documentation)



(1d) It's never been easier to use multilevel models in applied social research, though they are not without complexities

In classical presentations, random effects multilevel models are usually good…

- We know of clustering of cases into groups, so we let that knowledge influence our model description. Conversion from single-level to multilevel model usually brings…

- ***More appropriate model specification*** *=> reduced risk of misleading conclusions*
  - ***Different standard errors (usually)*** *(important e.g. of higher level explanatory variables)*
  - ***Different parameter estimates (sometimes)*** *(important e.g. of 'within' v's 'between' effects)*
  - *Additional summarising statistics about multilevel structure*
  - *Additional insights about specific cluster units or their distribution*

Might there be any risks though…?

- ***Using a complex model specification***
  - *Inadvertent misspecification of the random effects model*
  - *Harder to take account of other complexities (e.g. of sampling weights and design effects)*
  - *The counterfactual of what other statistical features you might concentrate on…*

Summary: Classical perspectives on multilevel modelling

Responding to multilevel and complex data	Statistical models can help us understand societies
Multilevel models are easy & popular ways to adapt statistical models to common features of social research data	**It’s never been easier to use multilevel models in applied social research, though they are not without complexities**

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