New INM RAS Earth ionosphere F region global dynamical model

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Formulation of the problem; vector equation

- Developing the dynamical 3-D Earth ionosphere model (F layer);
- Providing the coupled numerical ionosphere and thermosphere model, combining with the existing INM RAS thermosphere model.

Equation, describing the electron concentration evolution (follows from the continuity equation):

$$\frac{\partial n_i}{\partial t} = -div(n_i \vec{u}_{\parallel}) - div\left(n_i \frac{1}{B^2}[\vec{E} \times \vec{B}]\right) +$$
$$+div\left(D\left[\nabla_{\parallel} n_i + n_i \frac{1}{T_p} \nabla_{\parallel} T_p - \frac{n_i m_i}{2kT_p} \vec{g}_{\parallel}\right]\right) + [P - k_i n_i].$$

Taken assumptions:

- The ambipolar diffusion is the dominating dynamic process;
- Taking into account only the F layer;
- Plasma is taken quasineutral;
- The only considered ion is O^+;
- The earth magnetic field is taken dipole;
- Geographical and magnetic poles are considered to coinside.

Equation in spherical coordinates in the assumption of a thin spherical layer

$$\frac{\partial n_i}{\partial t} = DYZ(n_i) + DTr(n_i) + [P - kn_i].$$

$$DYZ(n_i) = \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\left(D\cos\varphi\left[\frac{1}{a}\frac{\partial n_i}{\partial\varphi}\cos^2 I - \frac{\partial n_i}{\partial z}\cos I\sin I\right]\right) + \frac{\partial}{\partial z}\left(D\left[\frac{\partial n_i}{\partial z}\sin^2 I -\right.\right.$$

$$\left.\left. - + Tr(n_i) + Tr(n_i)\frac{1}{a}\frac{\partial n_i}{\partial\varphi}\cos I\sin I\right]\right);$$

$$DTr(n_i) = \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\left[\left(\frac{1}{a}\frac{1}{T_p}\frac{\partial T_p}{\partial\varphi}\cos^2 I - \frac{1}{T_p}\frac{\partial T_p}{\partial z}\cos I\sin I - \frac{1}{H}\sin I\cos I\right)Dn_i\cos\varphi\right] +$$

$$+\frac{\partial}{\partial z}\left[\left(-\frac{1}{a}\frac{1}{T_p}\frac{\partial T_p}{\partial\varphi}\cos I\sin I + \frac{1}{T_p}\frac{\partial T_p}{\partial z}\sin^2 I + \frac{1}{H}\sin^2 I\right)Dn_i\right].$$

At the upper boundary the electron flux is considered to be known and close to zero. In this work it is taken 0. The corresponding upper boundary condidion:

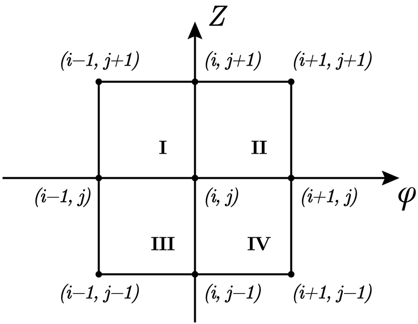
$$D\left(\frac{\partial n}{\partial z}\sin^2 I - \frac{1}{a}\frac{\partial n}{\partial\varphi}\cos I\sin I + \frac{1}{H}n\cdot\sin^2 I\right) = F_{ub} \approx 0.$$

Spatial approximation

- For the diffusion components standard conservative approximation with central difference is used:

$$\frac{\partial}{\partial z} D \frac{\partial n}{\partial z} \approx \frac{1}{h_{i+1/2}} \left(\frac{D_{i+1/2}(n_{i+1} - n_i)}{h_i} - \frac{D_{i-1/2}(n_i - n_{i-1})}{h_{i-1}} \right);$$

- Transfer components are also approximated with central difference formula;
- Key issue and difficulty: existence of mixed derivatives. Second order approximation formula, depending on the sign of $\sin I$ is used.
- Upper boundary condition is approximated with directed difference, consistent with the main equation approximation.



Two time approximation methods and model solution

In this work two different approaches to solving the equation are compared:

I. 2D implicit scheme: all components of spatial approximation are taken implicitly (linear system is solved with BiCG Stabilized iteration method);

II. Splitting method:

- z-diffusion, mixed derivatives and photochemistry — first splitting step;
- diffusion along longitude — second splitting step;

(linear system is solved with tridiagonal matrix algorithm);

Scheme properties and convergence nuances are tested on the stationary model solution:

$$n_{mod}(z,\varphi) = A\cdot e^{-B(z-C)}\cdot(z-C)\cos^2\frac{\varphi}{2}$$

with proper photoionization and upper flux functions $P(z,\varphi)$ and $F_{ub}(\varphi)$.

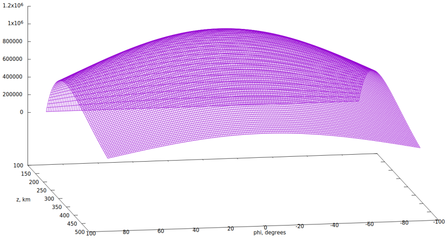


Figure 1. Model solution $n_{mod}(z,\varphi)$ from 100 km to 500 km and φ from -90° to 90°.

Time approximation comparison

I. Implicit scheme is significantly more precise;
II. Splitting method is 4 times faster (with respect to tridiagonal matrix algorithm), but additional time approximation error gives sizeable error;

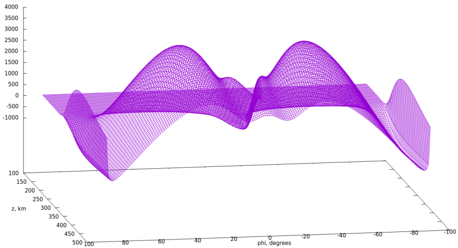


Figure 2. Error in the stationary solution for the implicit method, implemented for model solution.

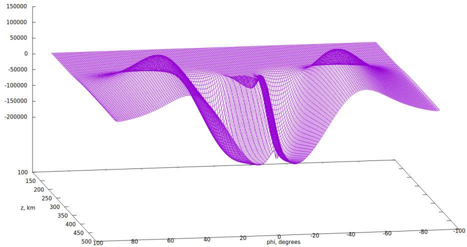


Figure 3. Error in the stationary solution for the splitting method, implemented for model solution, with time step $\tau = 100$.

The relative error for the splitting method with different time steps is presented in a table below:

τ, sec	100	10	1
Relative error, %	17	4	0,5

Daytime stationary solution and diurnal evolution

During the diurnal evolution modelling the photoionization function P is changed in time according the zenith angle χ evolution:

$$P(z,t) = \begin{cases} P_0(z) \cdot e^{\tau_{\mathbf{0}}(z)(1-\sec\chi)}, |\chi| \le \dfrac{\pi}{2}; \\ 0, |\chi| \ge \dfrac{\pi}{2}. \end{cases}$$

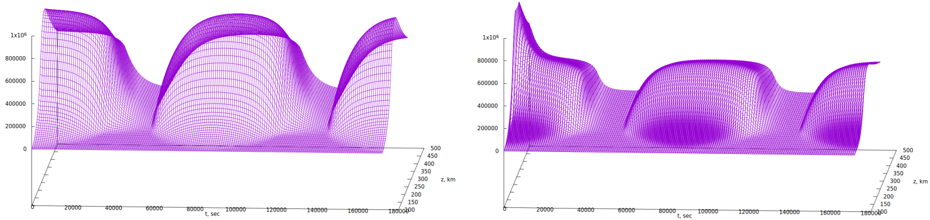


Figure 4. The diurnal evolution of the ion concentration at the latitude 60°, calculated with the implicit scheme (left) and results obtained from splitted scheme (right).