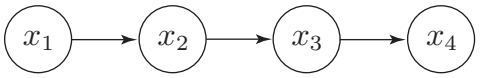
ECE 239AS.2, Spring 2025 **Project 2: Diffusion models**
Neural Networks & Deep Learning 2 Prof. J. Kao
UCLA TAs: S. Dong, J. Lee, K. Pang, X. Yan

Due Monday, May 5, 2025, by 11:59 pm to Gradescope.
50 points total.

1. (10 points) Consider the following directed graph:



   (a) (2 points) Write the joint distribution $P(x_1, x_2, x_3, x_4)$ in terms of a product of conditional distributions, one for each node in the graph.

   (b) (4 points) A possible application of this directed graph is to model a stimulus that changes over time. Let

$$x_1 \sim \mathcal{N}\left(0,\, \sigma^2\right)$$
$$x_t | x_{t-1} \sim \mathcal{N}\left(x_{t-1},\, \sigma^2\right),$$

   where $t = 2, 3, 4$. This is known as a *random walk model*, since x_t is obtained by adding a random increment (in this case, a Gaussian) to x_{t-1}. What is the 4×4 covariance matrix of the vector $[x_1\ x_2\ x_3\ x_4]^{\mathrm{T}}$ in terms of σ^2?

   (c) (2 points) Find the inverse of the covariance matrix found in part (b) in terms of σ^2. This is known as the *precision matrix*.

   (d) (2 points) Relate where zeros appear in the precision matrix to the graph structure.

2. (40 points) **Implement Conditional DDPM.** Complete **ConditionalDDPM.ipynb**, **ResUNet.py** and **DDPM.py**. Print out **all three files along with your solutions for Question 1**, and submit them as one PDF to Gradescope.