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Fischer Black

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Interest Rates as Options

FISCHER BLACK*

ABSTRACT

Since people can hold currency at a zero nominal interest rate, the nominal short rate cannot be negative. The real interest rate can be and has been negative, since low risk real investment opportunities like filling in the Mississippi delta do not guarantee positive returns. The inflation rate can be and has been negative, most recently (in the United States) during the Great Depression. The nominal short rate is the “shadow real interest rate” (as defined by the investment opportunity set) plus the inflation rate, or zero, whichever is greater. Thus the nominal short rate is an option. Longer term interest rates are always positive, since the future short rate may be positive even when the current short rate is zero. We can easily build this option element into our interest rate trees for backward induction or Monte Carlo simulation: just create a distribution that allows negative nominal rates, and then replace each negative rate with zero.

I. Interest Rate Processes

WHEN VALUING INTEREST RATE derivatives, we commonly model the nominal interest rate process, though investors ultimately care more about the real interest rate. When we model the nominal interest rate, we ignore the supply of and demand for capital. We assume a simple random process, without worrying about the forces that influence the interest rate.

We generally choose a normal process, a lognormal process, or a square root process. With the normal process, the volatility of the change in the interest rate does not depend on the rate, though it may depend on time. With the lognormal, the volatility of the *fractional* change in the interest rate does not depend on the rate. With the square root process, the ratio of the variance of the change in the interest rate to the rate does not depend on the rate, so the volatility of the change in the rate is proportional to the square root of the rate at a given time. We can put mean reversion into any of these processes.

The normal process assumes that the interest rate volatility does not decline as the rate approaches zero. This seems a little strange since volatility does seem to decline as the rate declines, but it might be a good first approximation.

* Goldman, Sachs & Co. I am grateful to Chi-Fu Huang and Wei-Tong Shu for discussions of these issues, and to Wei-Tong Shu for producing the graphs. [Note from the Managing Editor: Fischer Black submitted this paper on May 1, 1995. His submission letter stated: “I would like to publish this, though I may not be around to make any changes the referee may suggest. If I’m not, and if it seems roughly acceptable, could you publish it as is with a note explaining the circumstances?” Fischer received a revise and resubmit letter on May 22 with a detailed referee’s report. He worked on the paper during the Summer and had started to think about how to address the comments of the referee. He died on August 31 without completing the revision.]

More seriously, though, the normal process allows the nominal interest rate to become negative.

So long as people can hold currency, we know that the nominal short rate cannot be negative. People would rather keep currency in their mattresses than hold instruments bearing negative interest rates. In practice, the chance of a negative interest rate in a process with mean reversion may be small, so the values of derivatives may not depend much on cases where the rate is negative. But the idea of a negative nominal rate is jarring.

The lognormal process implies that the nominal rate can never reach zero. This seems a little strange, too, since the nominal short rate has sometimes fallen to zero. In the United States, this happened most recently in the 1930s. More generally, the lognormal implies that the volatility falls very rapidly as the rate approaches zero. In most periods, while the volatility does seem to fall, it does not seem to fall this rapidly.

The square root process is a compromise between the normal and the lognormal, but is a little more complex than either of those processes. With the square root process, when mean reversion is strong enough (or the drift in the short rate is constant but large), the rate cannot reach zero. In other cases, the rate can reach zero, and we must decide whether zero is a reflecting barrier or an absorbing barrier. When zero is a reflecting barrier, the rate "bounces off" zero, which seems strange. I can't think of any economic process that might cause the short rate to bounce in this way. When zero is an absorbing barrier, we must decide what determines the chance that the rate will become positive again, and the model can become quite complex. Of the three choices, though, the absorbing barrier seems most realistic.

Let us look more closely at why the short rate cannot be negative. It is because currency is an option: when an instrument has a negative short rate, we can choose currency instead. Thus, we can treat the short rate itself as an option: we can choose a process that allows negative rates and can simply replace all the negative rates with zeros. We still have a process with a single number describing the state of the world: either the short rate (when it is positive or zero) or what the short rate would be without the currency option (when it is negative). We can call this number the "shadow short rate."

I find the behavior of a model like this realistic: the short rate can be stuck at zero for a time, but its escape from zero is not random. We can follow the "shadow short rate," and we know that whenever that becomes positive, the short rate will be positive too.

We can use any of the three processes above as a base, but if we use a lognormal or square root process, we will start it from a negative point. In other words, we can use a "shifted" lognormal or square root process. Thus all these processes can allow negative values for the shadow short rate, and nonnegative values for the actual short rate once we add the option element.

Adding the nonnegative option to a process may kill any chance of having analytic solutions for the prices of derivatives, but it hardly complicates numerical solutions at all. If we use a tree, we simply start with an ordinary tree

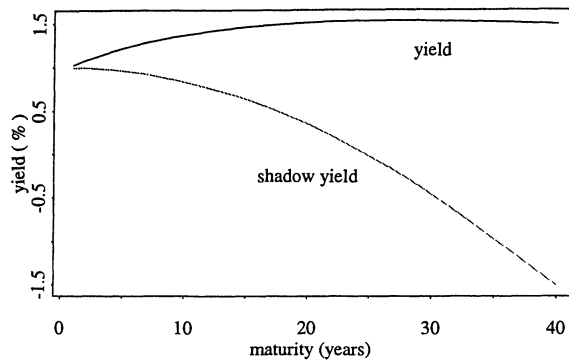


Figure 1. Zero-coupon yield.

for the shadow short rate, and replace any negative values with zeros before valuing long-term securities or derivatives.

II. Yield Curves

Although the short rate can be zero while the shadow short rate is negative, no longer term rate can be zero so long as there is a chance that the short rate can become positive again. When we are valuing securities on a tree with finite interval size, longer term rates may be zero on parts of the tree, but each rate eventually becomes positive as we shrink the interval size. In the world, where interval sizes are not relevant, all longer term rates should be positive in equilibrium.

In effect, a forward rate is an option. The short rate at a future time is the maximum of zero and the shadow rate, so it has an option-like value. The usual factors affect the option value: increasing volatility or the current shadow rate increases the forward rate. The effect of increasing maturity is ambiguous, since the drift matters, and since “convexity” pushes the forward rate below the expected short rate.

When the shadow rate is well above zero, especially when the interest rate shows strong mean reversion, none of this matters much. But when the shadow rate is near zero or negative, this causes the yield curve to slope upward at first. As we increase maturity, the effective volatility increases, so the forward rate increases (relative to what it would otherwise be).

To illustrate this, I have run a simple example with a normal distribution for the shadow rate and no mean reversion. In Figure 1, we have the zero-coupon yield curves for two cases: one where we allow the short rate to be negative, and the other where we cut off the short rate distribution at zero. The shadow short rate starts at 1 percent. It has zero drift in “risk-neutral space,” and its volatility is one percentage point. (Its variance rate is constant at .0001 per year.)

Because of convexity, the yield curve tends to be concave downward. When we put no floor on the short rate, the yield curve crosses zero and becomes

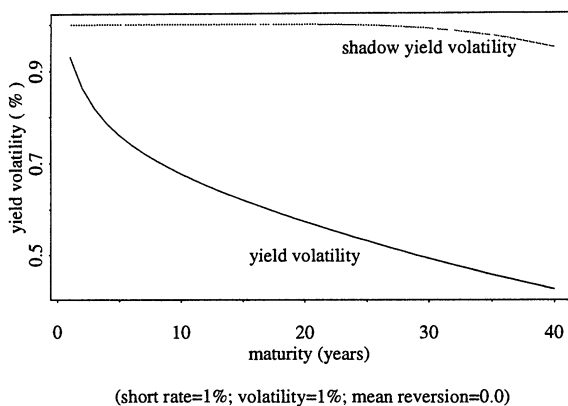


Figure 2. Zero-coupon yield volatility.

negative at a maturity of 26 years. Putting a floor of zero on the short rate causes the yield curve to rise at first. It reaches a maximum of 1.5 percent at year 26 and then starts declining.

In Figure 2, we have the volatility curves for the same two cases. These are yield volatilities in points, rather than fractional yield volatilities, which make little sense when yields can be negative. Without the floor, yield volatilities stay near one percentage point for all maturities. With the floor, the yield volatility falls to about half a point at a maturity of 30 years.

Thus when the short rate is at or near zero, the floor has a big effect on both the yield curve and the volatility curve.

III. Real Rates and Inflation

We define the real rate as the short-term nominal rate less the expected inflation rate. We can define a “shadow real rate” as the shadow short-term nominal rate less the expected inflation rate. When the nominal rate is zero, the shadow real rate has an economic meaning independent of any model. It is what the real rate would be if we faced no floor on the nominal rate. For the rest of this section, we will assume no floor on the nominal rate, so the real rate and the shadow real rate are identical.

When the nominal rate has no floor, the real rate plays two roles: it helps balance willingness to save and invest against opportunities to save and invest, and it helps balance willingness to take risk across people. In an unrestricted market, the cost of capital consists of the real interest rate and the price of risk. (The price of risk is the extra expected return per unit of market risk.) We cannot add these numbers, since they are not comparable. Thus the cost of capital is always at least two numbers. In a restricted market, we need even more numbers to describe the cost of capital. (A restricted market is one where governments use quotas or taxes to regulate the flow of capital into or out of specific countries or sectors.)

Thus the real rate is at best only a part of the cost of capital. In an actual economy, we have a term structure of real rates for each of several markets and a term structure of prices of risk for each market.

Other things equal, when people become more willing to save and invest, the real rate and the price of risk tend to fall. When saving and investment opportunities become more attractive, the real rate and the price of risk tend to rise. So long as people dislike risk, the price of risk must be positive. But there is no similar reason why the real rate must be positive.

People will hold assets even when all expected returns are negative. They may dissave and disinvest on balance, but they will continue to hold assets. Tastes do not force the real rate to be positive.

Investment opportunities generally offer positive expected returns, which is consistent with positive prices for risk, but need not imply a positive real interest rate. If riskless real investment opportunities exist, they put a floor on the real rate, but I don't know of any such opportunities. Martin Bailey once claimed that filling in the Mississippi delta to create new farmland is a nearly riskless investment with a positive return, but he failed to persuade me. That investment looks risky to me, and I suspect its expected return is negative, especially in today's environmentally sensitive climate. In fact, in most parts of the delta, the investment would violate our "wetlands" laws.

People borrow and lend at the real rate to shift risk. Those who are more tolerant of risk borrow, and those who are less tolerant lend. People who are very averse to risk will lend even when the real rate is negative, so this does not put a floor on the real rate either.

Moreover, in the years around 1980 the U.S. real rate was clearly and consistently negative. Neither theory nor data suggest that the real rate must be positive.

The inflation rate is a wholly different beast. The forces affecting the inflation rate need not even overlap much with the forces affecting the aggregate behavior of the real economy. Some see a significant tie between inflation and phenomena like unemployment, but others see no tie at all.

In any case, the inflation rate need not be positive. Indeed, some have argued that we should set the inflation rate to keep the nominal rate at zero, to reduce the costs of economizing on currency balances. When the real rate is positive, this means a negative inflation rate. Moreover, the expected inflation rate has been negative at different times in U.S. history: most recently in the 1930s.

Since both the shadow real rate and the inflation rate can be negative, the shadow nominal rate can be negative too. This means the floor at zero can be binding.

IV. Equilibrium

What happens when the shadow nominal rate is negative? How does the economy reach equilibrium?

When the government fixes a price, the exact method it uses affects the resulting equilibrium. Buying unlimited amounts of corn at a fixed price gives

one equilibrium; maintaining the price by paying farmers not to plant corn gives another.

When the government mandates that all trades must occur at a given price, or above a certain minimum price, the economy must find a way to adapt to the resulting excess supply. The minimum wage, for example, causes workers to drop out of the “official” workforce. Many join the underground economy.

Meanwhile, employers ration jobs. They rearrange their production processes to make less use of low skill labor. Speed in responding to a new job opening becomes crucial among those with few skills. Employers may even use the equivalent of lotteries to hire at these levels.

Similar issues arise when the shadow short rate is negative. By making currency available, the government fixes the nominal short rate at zero. The real rate is higher than the shadow real rate, so the government is “losing money” on its short term liabilities. Seignorage has turned from a source of profits to a source of losses.

At the artificially high real rate, the private sector wants to lend a lot but borrow only a little. It has an excess supply of lending. The government makes up the difference by issuing currency, bills, notes, and bonds.

Investment opportunities that would be attractive at the shadow real rate become unattractive at the actual real rate. The private sector disinvests, perhaps excessively, unless the government steps in to subsidize these investments. Or the government may simply buy the businesses from the private sector and operate them at a loss. The taxpayers make up the difference.

Since governments are usually less efficient than the private sector in operating businesses, and since tax rates must be higher than normal in this scenario, the economy may face substantial welfare losses. If governments fail to see what’s happening and don’t subsidize or operate the businesses, the economy may face massive unemployment and welfare losses.

Low-risk investments are displaced more than high-risk investments, since the primary distortion is to the real rate rather than the price of risk. The average risk of the private sector’s investments goes up. This makes the economy less stable.

The last time we faced this situation in the United States was in the 1930s. The nominal short rate was near zero, and longer term rates were higher than their shadow values because of the option provided by currency. (This is closely related to what some macroeconomists call the “liquidity trap.”) I claim this contributed, perhaps a lot, to the severity of the Great Depression.