Compensation, Incentives, and the Duality of Risk Aversion and Riskiness

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ABSTRACT

The common folklore that giving options to agents will make them more willing to take risks is false. In fact, no incentive schedule will make all expected utility maximizers more or less risk averse. This paper finds simple, intuitive, necessary and sufficient conditions under which incentive schedules make agents more or less risk averse. The paper uses these to examine the incentive effects of some common structures such as puts and calls, and it briefly explores the duality between a fee schedule that makes an agent more or less risk averse, and gambles that increase or decrease risk.

WITH THE GROWING INTEREST in executive compensation and agency problems, there is a folklore about the relation between the shape of the fee schedule received by an agent and the agent's attitudes toward risk that deserves further study. As an illustration, many authors take it for granted that giving options to executives makes them more willing to take risks. DeFusco, Johnson, and Zorn (1990, p. 618), for example, note that "The asymmetric payoffs of call options make it more attractive for managers to undertake risky projects." In fact, contrary to their intuition, my intuition, and that of most observers, without further conditions on utility functions beyond monotonicity and risk aversion, this is not correct. Surprisingly, it is not the case that a convex compensation schedule makes an agent more willing to take risks, that is, less risk averse; nor does a concave compensation schedule make an agent more risk averse.

The common folklore clearly has its genesis in the observation from option pricing theory that an increase in the volatility of an option makes it more valuable (see, e.g., Haugen and Senbet (1981), Smith and Watts (1982), and Smith and Stultz (1985)). This is, however, not the same as making the option more desirable to a risk-averse investor. One clear problem with the intuition of folklore is that compensation schedules move the evaluation of any given gamble to a different part of the domain of the original utility function where the utility function can have greater or lesser risk aversion. For example, suppose that an option grant is part of an incentive package that raises base compensation. With such an incentive compensation package, the agent assesses risk from the vantage point of being wealthier, and an agent can have a very different

attitude toward risk at a higher level of wealth than at a lower level. But this is not the only difficulty.

Several others have also noted that an agent's risk aversion can significantly affect their view of compensation. In a model with risk aversion, Lambert, Larcker, and Verrecchia (1991) derive a number of comparative statics results describing the sensitivity of the agent's valuation of a compensation package to variables such as wealth and the degree of risk aversion. Closer to our analysis, in an intertemporal model where a portfolio manager adjusts to a convex incentive structure, Carpenter (2000) observes that the manager may behave in a counterintuitive fashion. More recently, Lewellen (2001) makes a similar observation in the context of executive compensation schedules. Both these papers present examples where convex incentive structures do not imply that the manager is more willing to take risks, but the general question of why and under what conditions this might occur remains somewhat mysterious.

Of course, agency theory has long intensively studied the functional relation between the optimal (efficient) contract and the utility function of the agent (see, e.g., Ross (1973) or Holmstrom (1979)). Unfortunately, the effort to characterize optimality—often in highly specific and parametric models—has crowded out the study of the behavior of the agent given the specific contract forms of the sort that are commonly observed in practice. In particular, little is known about the derived risk preferences of agents given common types of incentive structures. In this paper we will take some first steps toward such an analysis by finding necessary and sufficient conditions under which the folk intuition is valid; that is, we will answer the question of when option-like incentive schedules lead to increased risk taking. Perhaps more important, we will develop a notion of compensating variation that disentangles three separable effects that a fee schedule has on an agent's attitudes towards risk. This result will allow us to draw important distinctions between, say, the incentive impact of put options and those of call options. Perhaps contrary to our intuitions, we will show that for the usually assumed preferences, put options make individuals less risk averse, while call options do not. In addition, along the way we will make some observations on the equivalence between making gambles riskier and making agents behave as though they were more risk averse.

Section I looks in detail at the effect on risk-taking in two special cases; first, when the fee schedule is a simple call option and, second, when it is equivalent to a bond position with a short put option. Here we introduce some examples to illuminate the basic intuitions of the problem and set the stage for the analysis of Section II. Section II develops the general theory and finds the necessary and sufficient conditions for a fee schedule to make an agent more or less risk averse, that is, to concavify or convexify the original utility function. The general theory orders total compensation schedules by their risk-inducing behavior. In Section III, we examine how to modify an existing total compensation schedule so as to make the agent more or less willing to take a risk. Section IV separates the influence of the fee schedule into three locally independent components: The

convexity effect, the translation effect, and the magnification effect. Roughly, the convexity effect is really the statement of the original intuition of folklore, and the translation and magnification effects are the remaining influences of an incentive schedule that have to be traded off against convexity to produce the net effect. Section V briefly explores the duality between a fee schedule that concavifies a utility function and a schedule that makes the underlying payoff riskier. The final section, Section VI, concludes the paper with a brief summary and some suggestions for future research.

I. Some Simple Examples: Put and Call Option Fee Schedules

Consider an executive whose compensation consists of a fixed fee together with some options on the company's stock. We will assume, as is usual, that the options are not fungible and that the executive must hold them to maturity. For simplicity, we will also ignore the complications of time and model the issue in a single period world. In many settings—such as a complete market—the qualitative results we obtain will hold intertemporally as well.

The main argument for giving the executive options is that it will align interests with those of the owners of the firm. Giving options rather than stock alone may also have important tax benefits. A consequence of an option position, though, is that it may appear to make the executive more willing to take risks, and the argument for this is that obviously raising the volatility raises the market value of the options. But the executive who cannot simply sell the options to pocket the increased value must instead evaluate them not with the linear valuation of the market but, rather, through the filter of their own personal preferences and trade-off between risk and return. The result, as we will show, may not be that the manager wants more risk.

If the intuition that a convex fee schedule will make an agent less risk averse has any force, it should certainly hold when the fee schedule is a simple option. Suppose, then, that the fee schedule is a fixed wage plus a package of call options on a number of shares with a total value of x and with a total exercise price of a. Ignoring the manager's fixed wage, the variable payoff, f, is the familiar call option function on the value of the underlying shares,

$$f(x) = \max\{x - a, 0\}. \tag{1}$$

Assuming that the agent's utility function, U, is monotone and concave, then the derived utility function is given by

$$U(f(x)) = U(\max\{x - a, 0\}) = U(x - a), x \geq a, \text{and} = U(0), x \leq a. \tag{2}$$

For $x \leq a$, the utility function is fixed at $U(0)$. For $x \geq a$, it rises as the original utility function (see Figure 1). For bets that are near a, then, the induced utility function may well be more risk loving, but for bets in the range where $x > a$, the result depends on whether $U(x - a)$ is less risk averse than $U(x)$. This, in turn, depends on whether or not U has increasing or decreasing risk aversion.
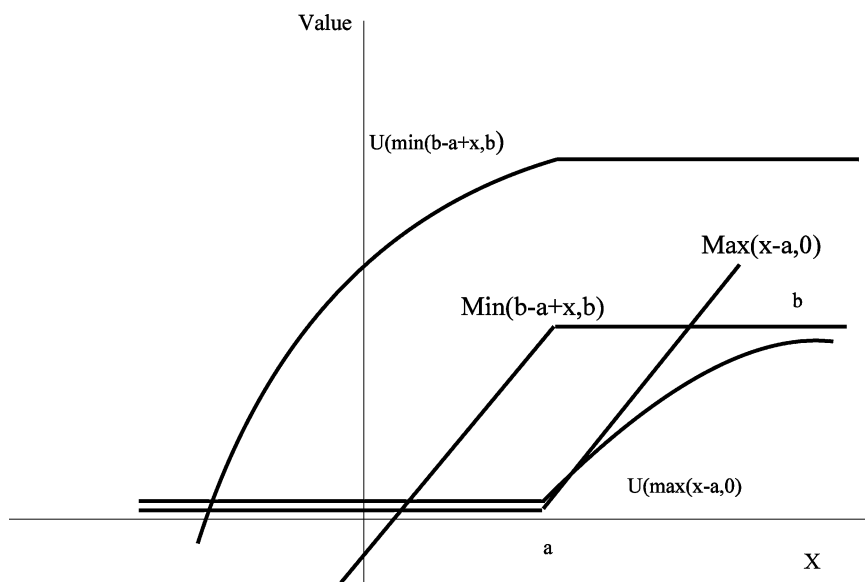
Figure 1. The utility of put and call incentive schedules.

If $a > 0$ and U has decreasing risk aversion, then $U(x - a)$ will be more risk averse than $U(x)$.[1]

Despite the fact, then, that the value of a call option is an increasing function of the risk (volatility) of the underlying payoff, nevertheless, the derived utility function, $U(f(x))$, is not uniformly less risk averse than $U(x)$. In the case of the call option, it is less risk averse in some parts of the domain and may well be more so in others. In Section IV, we will present an example where the fee schedule is convex, but the derived utility is everywhere more risk averse.

The paradox is resolved by observing that any fee schedule has several effects—two of which, for shorthand, we will label the convexity effect and the translation effect. On one hand, the convexity of a schedule like a call option clearly makes risky bets more desirable. On the other, the fee schedule also shifts or translates the evaluation of any bet to a different portion of the domain of the agent's utility function. These two effects can be enhancing or offsetting with an a priori ambiguous result.

As another example, typically a manager will receive a bonus payout that is proportional to some indicator of performance in their division, such as some measure of accounting earnings using transfer pricing to value inputs and outputs. Such bonus schemes almost always have both floors and ceilings, but to make our point simply, suppose that the relevant range is near the ceiling and that the manager is making decisions that will affect whether the payout hits the ceiling or is in a range that is proportional to the performance indicator.

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[1] It is assumed that the individual is not allowed to make use of a market for fair gambles or a complete market to concavify the utility function (see Ross (1974)).

Will the manager become more risk averse toward earnings in that case and be unwilling to risk falling below the ceiling? Certainly if the market value of the manager's payout option was proportional to the performance indicator, then the market would treat risk just as for a junior bond in the range near its principal payout and the value would decline with increases in volatility. But, as before, the behavior of the agent who cannot sell the payout is ambiguous.

To illustrate this ambiguity we form a fee schedule as a fixed fee with a short position in a put option (see Figure 1),

$$f(x) = b - \max\{a - x, 0\} = \min\{b - a + x, b\}. \tag{3}$$

where b is the fixed fee and a is the exercise price of the put option. In this example, x represents the performance indicator, b is the maximum payout and a is the performance level at which the payout "maxes out."

In this case,

$$U(f(x)) = U(\min\{b - a + x, b\}) = U(b - a + x), x < a, \text{and } = U(b), x \geq a. \tag{4}$$

Even though the fee schedule is concave, whether $U(f(x))$ is more risk averse than $U(x)$ depends on whether the shift in the domain moves U to a more or less risk averse region. If, for example, $b - a > 0$, then the domain is shifted to the right. If U has increasing risk aversion, then in this portion it will be more risk averse than $U(x)$, which will augment the increase in risk aversion at the exercise price, $x = a$. On the other hand, if U has decreasing risk aversion, then the rightward shift moves it into a region of lower risk aversion, and for $x > b - a$, $U(f(x))$ is locally less risk averse than $U(x)$, despite the concavity of f.

The special fee schedules of this section hone our intuition and make us wary of jumping to quick conclusions about the impact of a fee schedule on an agent's willingness to accept risk. In the next section we will derive some general results that will enable us to determine when a fee schedule makes an agent more or less risk averse.

II. The General Theory

The intention of this section is to develop a general theory of how compensation schedules affect decision making without any particular specification of the problems the agent will face. For example, if we were willing to say that the CEO of a firm would only be choosing the allocation between stocks and bonds, then we could force any particular allocation by simply choosing as an incentive schedule one that pays off only when the desired allocation is chosen. Our intention, instead, is to make broader statements about behavior independent of the particular choice problem faced by the agent. The rationale for doing so is twofold. First, quite commonly the schedule must be set before the actual decision-making context is known. Second, our aim is to examine the properties of particular incentive schedules, and insofar as we can say that the

schedule is risk inducing or risk averting independent of the particular problem the agent faces, we will have learned something about its influence in a decentralized decision making environment. Setting the schedule permits the board or the top executives of a company to give simple rules to their employees that will induce them to behave in certain ways without requiring a detailed knowledge of their environment. Throughout, the reader can imagine the argument of the compensation schedule as the future value of a set number of shares in the firm and can think of the agent as an executive in the firm who is making decisions that will change the character of the distribution of future value.

We will begin by analyzing the entire schedule, and in Section III we will consider the problem of modifying an existing schedule. A bit of nomenclature and some definitions will simplify the exposition. We will say that a compensation schedule, f, concavifies a utility function, U, if the derived utility of the schedule is more risk averse that the original. If the derived utility is less risk averse than the original utility function, we will say that f convexifies U. Unless specifically specified otherwise, all functions are assumed to be arbitrarily differentiable, fee schedules are strictly monotone increasing and utility functions are monotone increasing and concave—the classes of such functions are denoted by M and MC, respectively. In addition, individual preferences are assumed to still be monotone and concave after the application of the fee schedule.

Formally speaking, these conditions rule out, for example, a call option fee since it is not strictly monotone increasing, it is nondifferentiable at the strike price, and since it makes any differentiable concave utility function convex in the neighborhood of the strike price. In general, the requirement that the schedule be strictly monotone is not a very restrictive condition; it is just a limiting case of a strictly monotone fee and we will occasionally point this out in the formal results derived below. For example, simply increasing the time to maturity smooths the option value and renders it strictly monotone, and in our examples we will continue to use simple options while keeping this interpretation in mind. By contrast, allowing the fee schedule to create a convex region for the derived utility of the fee alters the problem in interesting ways. For an analysis of such nonconcavities, see Basak, Pavlova, and Shapiro (2002).

We will adopt the following definition to characterize fee schedules that increase or decrease risk aversion.

DEFINITION 1: *A fee schedule, f, concavifies a utility function, U, if and only if there exists a monotone concave function, G, such that*

$$U(f) = G(U). \tag{5}$$

For notational ease we will omit the argument of a function when that function depends directly on wealth, w. For example, U is understood to be the function $U(w)$. All functions will be assumed to be differentiable as needed. (Strictly speaking this rules out simple piecewise linear options, but a simple closure argument extends all our results to these cases.)

From the Arrow–Pratt theorem (Arrow (1965), Pratt (1964)), we know that the existence of a concave function, G, as defined above is equivalent to the induced utility function $U(f)$ having a greater coefficient of risk aversion than U and is also equivalent to the local risk premium for gambles being greater. Similarly, we will employ the following definition.

DEFINITION 2: *A fee schedule, f, convexifies a utility function, U, if and only if there exists a monotone convex function, G, such that*

$$U(f) = G(U). \tag{6}$$

Note that since G^{-1} is concave if and only if G is convex, f convexifies U if and only if there exists a concave H such that

$$U = H(U(f)). \tag{7}$$

We will let A denote the coefficient of absolute risk aversion,

$$A = -\frac{U''(x)}{U'(x)}. \tag{8}$$

Since U is monotone, for any f there always exists a function G such that,

$$U(f) = G(U). \tag{9}$$

Furthermore, f is monotone if and only if G is monotone, since

$$U'(f)f' = G'(U)U'. \tag{10}$$

Note that in what follows we use the assumption that f is strictly monotone increasing. Differentiating again we have

$$U''(f)(f')^2 + U'(f)f'' = G''(u)(U')^2 + G'(u)U'', \tag{11}$$

which rearranges to

$$G''(u)(U')^2 = U'(f)f'\left[A + \frac{f''}{f'} - A(f)f'\right]. \tag{12}$$

Hence, we have the following result.

THEOREM 1: *The compensation schedule, f, concavifies (convexifies) U if and only if*

$$\frac{f''}{f'} \leq (\geq)A(f)f' - A. \tag{13}$$

Proof: See above.   Q.E.D.

The following lemma verifies what is certainly true about the folk result.

COROLLARY 1: *The compensation schedule, f, concavifies (convexifies) U for all U only if f is concave (convex).*

Proof: Taking U to be risk neutral, $A = 0$, then the result follows from Theorem 1 and the monotonicity of f.   Q.E.D.

Indeed, the concavity of f is necessary for $U(f)$ to even be risk averse for all concave U. However, while the concavity (convexity) of f is necessary for the derived utility function to be concavified (convexified), it is not sufficient. To see this, observe that for any point x such that $f(x) \neq x$ and for any values $f'(x)$ and $f''(x)$, we can set $A(x)$ sufficiently large while holding $A(f(x))$ fixed and construct a violation of the condition of Theorem 1 for f to concavify U.

This verifies the following important corollary.

COROLLARY 2: *There is no compensation schedule that concavifies (convexifies) all U.*

Proof: See above argument.   Q.E.D.

By restricting the class of utility functions, though, it is possible to obtain a complete result that is close in spirit to the folk result. We will use the acronym DARA to denote the class of utility functions that exhibit decreasing absolute risk aversion and IARA for the class with increasing absolute risk aversion.

THEOREM 2: *The compensation schedule, f, concavifies all $U \in$ DARA if and only if f is concave, $f \leq x$, and $f' \geq 1$, and it convexifies all $U \in$ DARA if and only if f is convex, $f \geq x$, and $f' \leq 1$. The compensation schedule f concavifies all $U \in$ IARA if and only if f is concave, $f \geq x$, and $f' \geq 1$, and f convexifies all $U \in$ IARA if and only if f is convex, $f \leq x$, and $f' \leq 1$.*

Proof: Suppose $U \in$ DARA. If $f \leq x$, then $A(f) \geq A$, and if $f' \geq 1$, then $A(f)f' - A \geq 0$. Since, by concavity, $f'' \leq 0$, we have

$$\frac{f''}{f'} \leq A(f)f' - A. \tag{14}$$

and, by Theorem 1, f concavifies U. To prove necessity observe that if A is constant, then we must have

$$A(f)f' - A = A[f' - 1] \geq \frac{f''}{f'}. \tag{15}$$

Picking U risk neutral, that is, $A = 0$, verifies that $f'' \leq 0$. Picking A arbitrarily large reverses the inequality unless $f' \geq 1$. Now, suppose that $f(x) > x$ for some x. We can set $A(f)f' - A$ arbitrarily small by setting $A(x)$ as large as desired relative to $A(f)$, and this also reverses the inequality. The proofs for convexity and for IARA are similar.   Q.E.D.
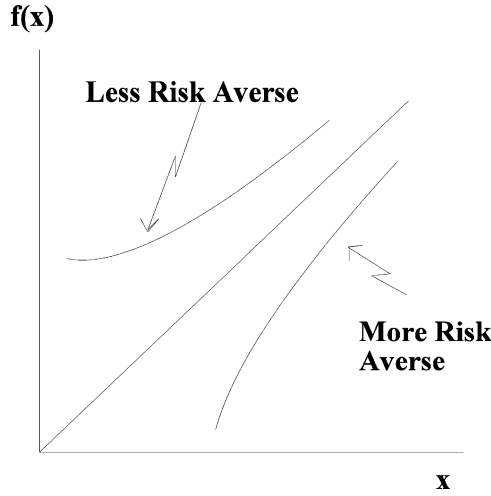
f(x)

Figure 2. Risk-inducing and risk-averting fee schedules for DARA utility.

Theorem 2 tells us the necessary and sufficient conditions on the fee schedule for it to concavify or convexify utility functions with decreasing and increasing risk aversion. In other words, for a broad class of utility functions it characterizes the fee schedules that increase and decrease risk aversion. Figure 2 illustrates the implied shape of the compensation schedules for the DARA case. Corollary 2 simply says that there is no fee schedule that concavifies or convexifies all monotone concave utility functions. The natural converse question to ask, then, is whether the classes of utility functions specified in Theorem 2 are the broadest classes of utility functions that these fee schedules concavify and convexify. To address this we first define four classes of fee schedules as described in Theorem 2.

DEFINITION 3:

$$A(dc) \equiv \{f \mid f \text{ is concave, } f \leq x, \text{ and } f' \geq 1\}$$
$$A(dx) \equiv \{f \mid f \text{ is convex, } f \geq x, \text{ and } f' \leq 1\}$$
$$A(ic) \equiv \{f \mid f \text{ is concave, } f \geq x, \text{ and } f' \geq 1\}$$
$$A(ix) \equiv \{f \mid f \text{ is convex, } f \leq x, \text{ and } f' \leq 1\}.$$

The next theorem shows that if a utility function is concavified or convexified for all of the members of one of the classes defined above, then it must be DARA or IARA, depending on the class that is used.

THEOREM 3: *If $f \in A(dc)$ or $f \in A(ic)$ implies that f concavifies U, then U is DARA or IARA, respectively. If $f \in A(dx)$ or $f \in A(ix)$ implies that f convexifies U, then U is DARA or IARA, respectively.*

Proof: The proofs are all similar so we will only do the first. Assume, then, that $f \in A(dc)$ implies that f concavifies U. From Theorem 1 a necessary

condition for f to concavify U is that

$$\frac{f''}{f'} \leq A(f)f' - A. \tag{16}$$

If $A(f)f' - A < 0$, then for $f'' < 0$ sufficiently close to 0 we can violate this condition for some $f \in A(dc)$; hence we must have $A(f)f' - A \geq 0$ for all $f \in A(dc)$. Since, aside from concavity, the only other conditions on f are $f \leq x$, and $f' \geq 1$, we must also have $A(f) \geq A(x)$ for all $f \leq x$. Hence, U must belong to *DARA*.   Q.E.D.

III. Risk-Inducing and Risk-Averting Modifications of an Existing Schedule

Theorems 2 and 3 may appear to be about an odd case in which the comparison is made between the agent receiving the entire payoff, x, or some schedule, $f(x)$. With this interpretation in mind, a result that requires $f(x) > x$ may seem difficult to interpret in a real-world setting. For example, if x represents the nominal value of the stocks on which a CEO's compensation is based, then setting $f(x) > x$ would require that the executive be paid more and presumably that would raise the cost of the compensation beyond the company's opportunity cost. Typically, instead, the question is what changes should be made in a compensation structure with the total value of the payout constrained to be at some level determined by market conditions. In other words, the typical problem is to alter the incentive schedule by, say, an option grant, but not to replace it in its entirety. Fortunately, the results we have obtained are easily extended to any potential alteration. For simplicity we will only extend Theorem 2; the extensions of Theorem 3 are obvious.

To start, we might think of amending an existing schedule, $f(\cdot)$, by transforming it to $g(f(\cdot))$. This could, for example, be a wholesale alteration of the schedule where both f and g are required to satisfy some budget constraint. Whether this concavifies or convexifies the derived utility function, $U(f(\cdot))$, depends on the shape of the transformation, H, defined by

$$U(g(f(x))) = H(U(f(x))). \tag{17}$$

Letting

$$z = f(x), \tag{18}$$

this becomes

$$U(g(z)) = H(U(z)), \tag{19}$$

and we have the following result.

COROLLARY 3: *Altering an existing fee schedule, $f(\cdot)$, to $g(f(\cdot))$, concavifies the derived utility function for all $U \in DARA$ if and only if g is concave, $g \leq x$, and*

$g' \geq 1$, *and it convexifies it for all $U \in DARA$ if and only if g is convex, $g \geq x$, and $g' \leq 1$. The alteration g concavifies the derived utility function for all $U \in IARA$ if and only if g is concave, $g \geq x$, and $g' \geq 1$, and g convexifies it for all $U \in IARA$ if and only if g is convex, $g \leq x$, and $g' \leq 1$.*

Proof: As described above, simply set $z = f(x)$ and apply Theorem 2.   Q.E.D.

Interestingly, in this case, the character of the alteration, g, is independent of the original fee schedule, f.

As another extension, often we think of adding options to an existing fee schedule. In this case the options might be interpreted as an alternative to simply raising the base pay for the CEO. Formally, if $f(\cdot)$ is the current payoff, what is the implication of adding $g(\cdot)$ to this? Now the question of how this addition influences the agent's attitudes toward risk hinges on the shape of $H(\cdot)$ defined by

$$U(f(x) + g(x)) = H(U(f(x))). \tag{20}$$

Letting

$$z = f(x), \tag{21}$$

and assuming that f is strictly monotone, this becomes

$$U(z + g(f^{-1}(z))) = H(U(z)). \tag{22}$$

Letting

$$q(z) = z + g(f^{-1}(z)), \tag{23}$$

we have

$$U(q(z)) = H(U(z)). \tag{24}$$

and the following result.

COROLLARY 4: *Let A_g and A_f denote the coefficients of absolute risk aversion for g and f, respectively. Adding g to an existing fee schedule, f, concavifies the derived utility function for all $U \in DARA$ if and only if $A_f \leq A_g$, $g \leq 0$, and $g' \geq 0$, and it convexifies it for all $U \in DARA$ if and only if $A_f \leq A_g$, $g \geq 0$, and $g' \leq 0$. The alteration g concavifies the derived utility function for all $U \in IARA$ if and only if $A_f \leq A_g, g \geq 0$, and $g' \geq 0$ and g convexifies it for all $U \in IARA$ if and only if $A_f \leq A_g, g \leq 0$, and $g' \leq 0$.*

Proof: Assuming that f is invertible, applying the transform,

$$q(z) = z + g(f^{-1}(z)) \tag{25}$$

and differentiating we obtain

$$q'(f(x)) = 1 + \frac{g'(x)}{f'(x)}, \qquad (26)$$

where

$$x = f^{-1}(z) \qquad (27)$$

and

$$q''(x) = \frac{f'(x)g''(x) - g'(x)f''(x)}{(f'(x))^3} = \frac{g'(x)}{f'(x)^2}[A_f - A_g]. \qquad (28)$$

The result now follows by applying Corollary 3. (If f is not strictly monotone, let $s(x)$ be any strictly monotone function with bounded derivatives and carry out the above analysis for $f(x) + \delta s(x)$. Since $U(f + \delta s + g)$ is more (or less) risk averse than $U(f + \delta s)$ for all $\delta > 0$, this must also hold for $\delta = 0$.)   Q.E.D.

An interesting special case of this occurs when $f(x) = x$ and we are simply adding to the entire payoff. Since $A_f = 0$, the conditions, for example, for convexifying a utility function with decreasing absolute risk aversion is that the addition g is nonnegative, convex, and has a nonpositive slope. In other words, adding a positive, monotonely declining convex function will convexify an agent with decreasing absolute risk aversion.

Note, then, that adding a call option will not convexify an agent with decreasing absolute risk aversion, but that adding a put will. This, in turn, implies that to make agents more willing to take risks there should be more of a focus on offering downside protection than on offering them upside potential.

Consider the following concrete example. A CEO currently receives a base annual pay of $1 million and has existing at the money call option grants on one million shares of stock with both a current price and an exercise price of $20 per share. In the coming year, perhaps in response to competitive pressures, the board is contemplating raising the total compensation, but union and shareholder pressure has been brought to bear and the board feels that it cannot raise the annual fixed fee and must do so with a further option grant. As a further consideration, the CEO's annual performance assessment report, while generally positive, reflects a generally held sentiment that the CEO has been too conservative in assuming the kinds of strategic risks that the board feels will be necessary in the future. It has been proposed to solve both problems with an at-the-money grant of calls on an additional one million shares. The argument advanced is that doing so will simultaneously meet market pressures and induce the CEO to be bolder. Unfortunately, though, and contrary to a rather standard intuition, assuming that the CEO displays the normal characteristic of decreasing absolute risk aversion, Corollary 4 tells us that the impact of this option grant will be ambiguous and may actually induce the CEO to be even more conservative.

By contrast, suppose, instead, that the board decided to transform the current option grant into a collar by granting puts on one million shares with an exercise price of \$10 per share. Now the manager would still gain from increases in the share price above its current level, but would be protected if the price dropped into the range below \$10 per share. Corollary 4 assures us that this would make the manager more willing to assume a riskier strategy.

The next section develops a compensating approach that allows us to separate the impact of the two effects of a fee schedule, translation and convexity. This will make the advantage of the put option more transparent.

IV. The Convexity, Translation, and Magnification Effects

In the above example, while the result follows from the analytics, it is far from clear exactly why adding call options does not necessarily induce more risk taking. Intuitively, the impact on the CEO of a further call option grant, can be broken into three distinct pieces. First, the CEO already benefits from the optionality through the existing call option grant, which is why further call options will not induce more risk taking through this effect. Second, any addition will bring compensation into a range where risk aversion is less and will induce further risk taking. But offsetting this effect is the fact that the new options will raise the delta of the total compensation package, and if the CEO is sufficiently risk averse, then the net result will be to induce a curtailment in risk-taking behavior. This intuition is developed in full below.

We begin by defining the derived utility function,

$$V(x) = U(f(x)). \tag{29}$$

From the basic relation of Theorem 1 we have that

$$
\begin{aligned}
A_V(x) - A(x) &= -\frac{U''(f)f'}{U'(f)} - \left[-\frac{U''(x)}{U'(x)} \right] + \left[-\frac{f''}{f'} \right] \\
&= A(f)f' - A(x) + A_f(x) \\
&= [A(f) - A(x)] + A(f)[f' - 1] + A_f(x),
\end{aligned}
\tag{30}
$$

where $A_f(x)$ denotes the absolute risk aversion coefficient of the fee schedule, $-f''/f'$. It is natural to define the three effects as:

$$\text{Translation Effect} = A(f) - A(x), \tag{31}$$

$$\text{Magnification Effect} = A(f)[f' - 1], \tag{32}$$

and

$$\text{Convexity Effect} = A_f(x). \tag{33}$$

Hence,

$$A_V(x) - A(x) = \text{Translation Effect} + \text{Magnification Effect}$$
$$+ \text{Convexity Effect}. \tag{34}$$

Whether the derived utility function is more or less risk averse than the original depends on whether the sum of the three effects is positive or negative.

One merit of this decomposition is that these effects are locally independent, in the sense that for a given utility function we can vary each of them without impacting the others. Varying the level of f affects the translation effect but not the convexity effect. It impacts the magnification effect through $A(f(x))$, but this is a scaling effect and does not impact the sign. It can also be undone by a scaling of f'. Similarly, changing f' has no impact on the translation effect and its impact on the convexity effect can be offset by a corresponding change in f''. Lastly, changing f'' impacts only convexity.

If we take f to be concave, then the convexity effect is positive, and the derived utility function is more risk averse if the translation effect and the magnification effects are also positive or not sufficiently negative so as to undo the convexity effect. This makes the results of Theorem 2 easy to see. If, for example, the utility function is DARA and if $f(x) \leq x$ and $f' \geq 1$, then the translation effect is positive, since the fee schedule moves the payoff to a more risk-averse region and the magnification effect is also positive. Thus, in this case, as we proved in Theorem 2, the derived utility function is more risk averse.

In some notable cases, these effects are easy to understand. For example, if the utility function has constant absolute risk aversion, then the translation effect disappears, since the utility function has the same risk aversion at all points in the domain. The magnification effect depends only on whether the fee schedule is increasing faster or more slowly than x itself. If the increase is faster, then the magnification effect is positive and if it is slower then it is negative. This makes the intuition of the magnification effect clearer. If $f'(x) > 1$, then a small gamble at x with a standard deviation of σ will be magnified to $\sigma f'(x)$. This raises the risk of the gamble and lowers the willingness of the agent to undertake it.

Note that even with a constant absolute risk aversion utility function (i.e., exponential) while there is no translation effect, the magnification effect alone can offset the convexity effect. If $f' \geq 0$ is sufficiently less than one, then the magnification effect can be negative enough to exceed the convexity effect, and the net result will be that the agent will become more risk loving even if the fee schedule is concave.

Example: Suppose that

$$U(x) = -e^{-Ax} \tag{35}$$

and that

$$f(x) = cg(x), \tag{36}$$

where $g(x)$ is a positive, monotone, and concave function.

The translation effect is zero and, differentiating, we have that the

$$\text{Convexity Effect} = A_f(x) = -\frac{g''}{g'}, \tag{37}$$

which is independent of c. The

$$\text{Magnification Effect} = A(f)[f' - 1] = A[cg' - 1]. \tag{38}$$

Hence, if the risk aversion of the fee schedule,

$$A_f(x) = -\frac{g''}{g'} < A, \tag{39}$$

then for c sufficiently close to 0, the magnification effect will dominate the convexity effect and the derived utility function will be less risk averse than the original utility function. On the other hand, if

$$A_f(x) = -\frac{g''}{g'} \geq A, \tag{40}$$

then even if $c = 0$, the derived utility function will be no less risk averse than the original.

Conversely, if g is convex, then setting c sufficiently high will make the derived utility function more risk averse than the original.

Similarly, even if at some $x, f' = 1$, while there is no magnification effect, the translation effect can obviously exceed the convexity effect. In general, then, there is no simple statement of the dominance of one effect over the others, and the impact of any fee schedule on an agent's attitudes toward risk must be analyzed in terms of all three effects.

Another way to make this point is to note that if the fee schedule is simply the total payoff, that is,

$$f(x) = x, \tag{41}$$

then all three effects are zero for all utility functions. Any alteration from this, though, has effects. For example, simply scaling the payoff to be affine,

$$f(x) = a + bx, \tag{42}$$

has no convexity effect and allows us to see the pure impact of the translation and magnification effects:

$$\text{Translation Effect} = A(f) - A(x) = A(a + bx) - A(x) \tag{43}$$

and

$$\text{Magnification Effect} = A(f)[f' - 1] = A(a + bx)[b - 1]. \tag{44}$$

From our previous discussion it is clear that these effects can take any sign and the resulting derived utility function, in general, can be more or less risk averse than the original utility function.

For example, consider the simple case discussed at the end of the previous section, namely, the advantage of a put option addition to convexify a schedule for an agent with decreasing absolute risk aversion. A put option is nonnegative, declining, and convex. Because it is nonnegative it adds to wealth and moves the agent into a less-risk-averse portion of the domain. This is the translation effect. It is convex for the convexity effect. Lastly, it has a negative slope so that its addition to the existing payoff will lower the magnification effect. In other words, when the option is in the money the agent will see even money gambles as less risky. By contrast, while adding call options to the payoff has a positive convexity effect and a positive translation effect, it has a negative magnification effect. When the call is in the money, it makes gambles appear riskier and that will lower the agent's incentive to accept them.

As a final point, it is interesting to use Corollary 2.4 and the above decomposition to interpret existing results in the literature. Carpenter (2000), for example, proves (Proposition 3) that in an intertemporal problem where a portfolio manager has a DARA or a CARA utility function and receives compensation in the form of call options, the manager would seek to reduce the volatility of the managed portfolio if the number of call options was increased. This suggests that the utility function has been concavified by the addition of the call options. From Corollary 4, though, we know that adding call options to a fee schedule will not concavify all DARA utility functions and, in particular, we must overcome the local convexity effect. For the CARA functions there is no translation effect, hence the magnification effect is larger than the convexity effect in this result. Note that in the intertemporal problem, the discontinuity of the call option is smoothed over time so it makes sense to talk of a bounded convexity effect.

V. Duality

Since a fee schedule that concavifies a utility function makes the agent more risk averse, it is tempting to conclude that it must make the underlying payoff riskier. Tempting, but untrue. Put another way, do executives, who have a cap to their option compensation, have the same incentives as executives in riskier firms but with no cap? Unfortunately, such sweeping intuitions are not true. For one thing, one random variable is riskier than another if and only if it is (weakly) inferior for all monotone, concave utility functions. If such a function existed then, by duality, it would make any agent more risk averse, but, by Corollary 2, no such schedule exists.

Usually, we say that a random variable y is less desirable than a random variable x if and only if for all U monotone and concave,

$$E[U(y)] \leq E[U(x)]. \tag{45}$$

By contrast, if we let

$$y = f(x), \tag{46}$$

then

$$E[U(y)] = E[U(f(x)] \leq E[U(x)] \tag{47}$$

for all random x if and only if

$$f(x) \leq x \tag{48}$$

for all x, a rather uninteresting criterion and one which fails to capture the spirit of the previous sections. In particular, we must contend with the fact that $f(\cdot)$ shifts the comparison for a random payoff from 0 to $f(0)$. As an alternative, we will adopt the following definition.

DEFINITION 4: *A random payoff y is said to be S-riskier by c (a constant) than a payoff x if and only if x is rejected for some U ∈ S, and, whenever x is rejected by U ∈ S, U prefers c to y, that is*

$$E[U(x)] \leq U(0) \Rightarrow E[U(y)] \leq U(c). \tag{49}$$

This is a slight generalization of the usual definition that allows the reference origin of comparison for y to be translated by a constant c. Definition 4 allows us to state a more useful duality concept.

DEFINITION 5: *A function f is a risk-inducing transform if for any random x, f(x) is S riskier by f(0) than x.*

The following result now adjusts the comparison for $f(x)$ to the origin $f(0)$.

THEOREM 4: *A function f is an S risk-inducing transform if and only if it concavifies U ∈ S.*

Proof: If f is risk inducing, then, for all x

$$E[U(x)] \leq U(0) \Rightarrow E[U(f(x))] \leq U(f(0)), \tag{50}$$

which is simply a statement that $U(f(x))$ is more concave than $U(x)$. Conversely, if f concavifies U, then there exists a monotone concave function G such that

$$U(f(x)) = G(U(x)), \tag{51}$$

which implies that if x is rejected by U, then

$$E[U(f(x))] = E[G(U(x))] \leq G(E[U(x)]) \leq G(U(0)) = U(f(0)), \tag{52}$$

the condition for f being risk inducing.   Q.E.D.

From Corollary 2, though, we know that f cannot be a risk-inducing transform for all $U \in MC$, and we must restrict the class of admissible utility functions, S. Theorem 2 provides a straightforward corollary. If we restrict S to the class of DARA utility functions, then it is immediate that f is DARA risk inducing if and only if $f \in A(dc)$. For the sake of completeness we offer the following formal statement. A parallel treatment for f being convexifying is equally straightforward.

THEOREM 5: *The compensation schedule, f, is DARA risk inducing if and only if f is concave, $f \leq x$, and $f' \geq 1$, and it is IARA risk inducing if and only if f is concave, $f \geq x$, and $f' \geq 1$.*

Proof: Follows immediately from Theorems 2 and 4.   Q.E.D.

VI. Conclusion

The folklore that a convex fee schedule makes an agent less risk averse and that a concave one induces greater risk aversion is incomplete at best. While these are necessary conditions for the result to hold for all utility functions, they are far from sufficient. The impact of the fee schedule on an agent's attitudes toward risk depends not only on the convexity of the fee schedule, but also on how it translates the domain of the utility function into more or less risk-averse portions and to the extent to which it magnifies (or contracts) any gamble at the margin. These latter two effects are as important as convexity and they can quite commonly undo the intuitive impact of convex or concave fee schedules.

These simple results have important implications for the way we think about such matters as executive compensation. It is routine for commentators to argue that call options increase the manager's willingness to take risk. We now know, though, that this also depends on the wealth effect of the options; increasing the wealth of the executive may move into more or less risk-averse portions of the utility function. In addition, depending on the amounts, options by themselves could have an important (marginal) magnification effect that could actually lead to more risk aversion.

A number of extensions of these results are desirable. In particular, we should examine how they hold up in intertemporal settings. There is reason, however, to be optimistic. In one important case, Carpenter's (2000) results for a manager with a convex fee schedule in an intertemporal portfolio problem are completely consistent with ours. Furthermore, since the convexity properties of a terminal payoff are preserved in earlier times under the martingale valuation, with some minor modifications the results we have obtained should still apply.

However, even with a successful extension to an intertemporal setting, a number of other significant questions remain. For the sake of analytic completeness, a fuller analysis of the associated concepts of duality needs to be undertaken. Of a somewhat more conjectural nature, since compensation schedules arise as equilibria in agency models, it would be interesting to further explore these results within such a setting. In particular, we should examine their implications when there is asymmetric information between agent and principal. Even

without doing so, though, it should be evident that it is important to understand the incentive implications of the compensation schedules that are actually observed.

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