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Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test?

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ABSTRACT

In a duopoly model of informed speculation, we show that overconfidence may strictly dominate rationality since an overconfident trader may not only generate higher expected profit and utility than his rational opponent, but also higher than if he were also rational. This occurs because overconfidence acts like a commitment device in a standard Cournot duopoly. As a result, for some parameter values the Nash equilibrium of a two-fund game is a Prisoner's Dilemma in which both funds hire overconfident managers. Thus, overconfidence can persist and survive in the long run.

THE RATIONAL EXPECTATIONS HYPOTHESIS implies that economic agents make decisions as though they know a correct probability distribution of the underlying uncertainty. According to the traditional view (Alchian (1950) and Friedman (1953)), the rational expectations hypothesis is empirically plausible because rational beliefs are better able to survive the market test than irrational beliefs. Yet, the empirical literature on judgment under uncertainty provides extensive evidence that people tend to exhibit *overconfidence* in judgment, i.e., people's subjective probability distributions are too *tight* (Alpert and Raiffa (1959), and Lichtenstein, Fischhoff, and Phillips (1982)).

This article examines the survival of overconfident beliefs in a model of speculative trading with asymmetric information. The model is based on a duopoly version of the trading model in Kyle (1984, 1985). Two risk-neutral informed traders have different noisy signals of the unobserved liquidation value. We relax the rational expectations assumption that traders have common priors on the joint probability distribution of each private signal and the liquidation value. We generalize this approach by allowing traders to have different distributions of the private signals. The different distributions may be due to cognitive errors (Kahneman, Slovic, and Tversky (1982)). In this view, a trader is "overconfident" if his distribution is too tight and "underconfident" if his distribution is too loose. Alternatively, firms may institute some incentive schemes to shift their rational traders' distributions towards greater

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or lesser aggression as if the traders were "overconfident" or "underconfident" (Fershtman and Judd (1987)).

Our model has a unique linear equilibrium, in which each informed trader's strategy is characterized by a trading intensity parameter. Since one trader's intensity parameter is a linear function of the other trader's intensity parameter, these intensity parameters are analogous to quantity choices in a standard Cournot duopoly model with linear reaction functions. Overconfidence acts like a commitment device in the duopoly model, giving the overconfident trader a reputation for trading so aggressively that his rational opponent (recognizing this) trades less aggressively. As a result, the overconfident trader may make a higher expected profit and utility than his rational opponent. This result is similar to De Long, Shleifer, Summers, and Waldmann (1991) and Blume and Easley (1992). But unlike their models, the survival of overconfidence in our model is not due to a tradeoff between beliefs and attitudes toward risk.

In addition, an overconfident trader facing a rational opponent may make a higher profit and utility than if he were also rational. This result has a strategic implication for delegated fund management between two rival funds. For some parameter specifications, there exists a unique Prisoner's Dilemma Nash equilibrium, in which both funds hire overconfident managers and yet both make lower expected profits than if they hire rational managers. This equilibrium is an evolutionarily stable strategy (ESS) in the evolutionary process of market selection (Schaffer (1988)). Thus, overconfidence not only may dominate rationality, but also may persist and survive in the long run. Importantly, this result is a much stronger result than previous results about the survival of irrationality, obtained through irrational agents' "spiteful" behavior. With spiteful behavior, "irrationality" is a strictly dominated strategy, but performs relatively better than the dominant "rationality" strategy in a polymorphic, finite population (Schaffer (1989), Palomino (1996), Rhode and Stegeman (1996), and Vega-Redondo (1996)).

Since the survival of overconfidence is due to the fact that overconfidence acts like a commitment device to aggressive trading, a fund can also promote its long-run survival by instituting an incentive scheme to shift its rational manager's probability distribution toward more aggressive trading as if he were overconfident. The survival of overconfidence may explain the empirical evidence that experts tend to be more prone to overconfidence than novices (Griffin and Tversky (1992)) in the sense that overconfident experts succeed and keep their reputation for expertise in the process of market selection. It is worth noting that the survival of overconfidence in our model cannot be attributed to inefficient prices, because prices are efficiently set by rational market makers. Also, our result is not due to the noise trader's risk such as in De Long, Shleifer, Summers, and Waldmann (1990), since in our model there is no uncertainty about the mistake made by overconfident agents.

In the past few years, several finance working papers featuring overconfidence have appeared. Benos (1992) develops a model in which a trader misperceives the precision of his opponent's signal but not his own. The first working paper version of our model (1993) is a duopoly in which overconfident traders misperceive the quality of their signals. This approach combines the game theoretic notion that traders have agreed to disagree (about priors) with Black's (1986) idea that traders trade on noise as if it were information. Subsequently, Odean (1995) considers a variant of our model along with some alternative settings, and Benos (1995) adopts the special case of our model in which overconfident traders believe their own signals have no errors. By contrast, Caballé and Sákovics (1995) consider an inconsistent beliefs structure in which all informed traders' beliefs are erroneous, and yet common knowledge. Daniel, Hirshleifer, and Subrahmanyam (1996) model overconfidence to explain returns anomalies in security markets.

Overconfidence, as a well-known cognitive error, may be due to an "anchoring and adjustment" process described in Slovic and Lichtenstein (1971) and Tversky and Kahneman (1974). When asked about an uncertain quantity, one naturally starts with a point estimate such as median, serving as an anchor. Then, one adjusts downward and upward from the anchor to assess the entire distribution. But the anchor has such a dominating influence that the adjustment is typically insufficient, yielding a tight distribution. Such an overconfidence bias has been noticed in a large variety of professional fields including clinical psychologists (Oskamp (1965)), lawyers (Wagenaar and Keren (1986)), entrepreneurs (Cooper, Woo, and Dunkelberg (1988)), managers (Russo and Schoemaker (1992)), security analysts, and economic forecasters (see e.g., Staël von Holstein (1972), Ahler and Lakonishok (1983), Elton *et al.* (1984), Froot and Frankel (1989), De Bondt and Thaler (1990), and De Bondt (1991)). Odean (1995) contains a good summary of psychology literature on overconfidence.

The next section develops the duopoly trading model of informed speculation. Section II examines the survival of overconfidence. Section III concludes.

I. The Model

Based on the trading mechanism of Kyle (1984, 1985), a single risky asset with an ex post liquidation value $\tilde{v}$ is traded in a one-shot market with three kinds of risk-neutral traders. Two informed traders submit their market orders $\tilde{x}_1$ and $\tilde{x}_2$, respectively, to maximize their expected profits conditional on their private signals $\tilde{s}_1$ and $\tilde{s}_2$. Noise traders together submit an exogenous random quantity $\tilde{z}$. Market makers (also called trader 0) then clear the market and set prices $\tilde{p}$ efficiently (in the semi-strong form) conditional on the market order imbalance $\tilde{y} = \tilde{x}_1 + \tilde{x}_2 + \tilde{z}$.

A. The Distribution Assumptions

The private signals $\tilde{s}_1$ and $\tilde{s}_2$ are noisy signals of the unobserved liquidation value $\tilde{v}$ such that $\tilde{s}_j = \tilde{v} + \tilde{e}_j, j = 1, 2$. The liquidation value $\tilde{v}$, the errors $\tilde{e}_1$ and $\tilde{e}_2$, and noise trade $\tilde{z}$ are normally and independently distributed with zero means and variances σ_v^2, σ_e^2, σ_e^2 and σ_z^2, respectively. Under the rational

Table I

Distribution Assumptions of the Two Private Signals

This table shows each trader's distribution assumptions about the private signals, $\tilde{s}_1$ and $\tilde{s}_2$. The risky asset's liquidation value $\tilde{v}$ and the error terms $\tilde{e}_1$ and $\tilde{e}_2$ are normally and independently distributed with zero means and variances σ_v^2, σ_e^2 and σ_e^2. Market makers' beliefs reflect the correct distributions of the two signals, i.e., $\tilde{s}_1 = \tilde{v} + \tilde{e}_1$ and $\tilde{s}_2 = \tilde{v} + \tilde{e}_2$. In contrast, informed traders' subjective beliefs may not reflect the correct distributions. Specifically, informed trader 1 thinks that the two signals' distributions are given by $\tilde{s}_1 = \tilde{v} + K_{11}\tilde{e}_1$ and $\tilde{s}_2 = \tilde{v} + K_{12}\tilde{e}_2$, respectively, where K_{11} and K_{12} are two exogenous constants. Similarly, informed trader 2 thinks that the two signals' distributions are given by $\tilde{s}_1 = \tilde{v} + K_{21}\tilde{e}_1$ and $\tilde{s}_2 = \tilde{v} + K_{22}\tilde{e}_2$, respectively, with the associated exogenous constants K_{21} and K_{22}.

Private Signals	$\tilde{s}_1$	$\tilde{s}_2$
Market makers' distributions	$\tilde{v} + \tilde{e}_1$	$\tilde{v} + \tilde{e}_2$
Informed trader 1's distributions	$\tilde{v} + K_{11}\tilde{e}_1$	$\tilde{v} + K_{12}\tilde{e}_2$
Informed trader 2's distributions	$\tilde{v} + K_{21}\tilde{e}_1$	$\tilde{v} + K_{22}\tilde{e}_2$

expectations paradigm, all traders have the same correct distributions. In this article, we relax the rational expectations assumption by allowing traders to have different distributions. In particular, we focus on the differences in the precision of the signals. Hence, trader i's belief about the signal $\tilde{s}_j$'s distribution can be characterized by $\tilde{s}_j = \tilde{v} + K_{ij}\tilde{e}_j$, where $K_{ij} \geq 0$, for $i = 0, 1, 2; j = 1, 2$. Thus, the differences in beliefs are captured completely by the "precision" parameter K_{ij}. The efficient markets concept that price is the conditional expectation of liquidation value given order flow is based on the assumption that the market makers' distributions are correct, i.e., $K_{01} = K_{02} = 1$. All traders' distributions are common knowledge, as described in Table I.

The different distributions may be due to some well-known cognitive errors documented in psychology literature (Alpert and Raiffa (1959) and Lichtenstein, Fischhoff, and Phillips (1982)). Under this view, informed traders misperceive the precision of the two signals. Using market makers' correct belief $K_{0j} = 1$ as the benchmark "rational" belief, informed trader i is "overconfident" if his distribution is too tight (i.e., $0 \leq K_{ii} < 1$) and "underconfident" if it is too loose (i.e., $K_{ii} > 1$). Alternatively, one can think of K_{ii} as a parameter in trader i's incentive scheme, by which his firm influences the trader's aggressiveness of trading (Fershtman and Judd (1987)). Under this view, $K_{ii} = 1$ represents the benchmark incentive scheme for profit maximization. Trader i is "overcompensated" for his trading volume if $0 \leq K_{ii} < 1$ and is "undercompensated" for the volume if $K_{ii} > 1$. In both interpretations, $K_{ii} \neq 1$ represents some form of irrationality because of the deviation from rational profit maximization behavior. But the deviation under the first interpretation is due to irrational prior beliefs, whereas under the second interpretation it comes from the nonprofit-maximization behavior. Regardless of the differences in interpretation, both views yield the same effect that the parameter K_{ii} acts like a commitment device in a standard Cournot duopoly model. For the ease of exposition, we will focus on the first interpretation and draw its analogy in the second interpretation at the end of our analysis.

B. Definition and Characterization of Equilibrium

Given the above trading mechanism and belief structure, traders make optimal decisions based on their own information and beliefs, while taking into account other traders' decision rules. The optimal trading strategy of informed trader i, conditional on his private signal $\tilde{s}_i$, is denoted by $\tilde{x}_i = X_i(\tilde{s}_i)$. The optimal pricing rule of the market makers, conditional on the order imbalance $\tilde{y}$, is denoted by $\tilde{p} = P(\tilde{y})$. The profit of informed trader i is thus given by $\tilde{\Pi}_i = (\tilde{v} - \tilde{p})\tilde{x}_i$. Let $E_i[\ldots]$ refer to the expectation operator of traders for $i = 0, 1, 2$. Following Kyle (1984, 1985), an equilibrium is a triple (X_1, X_2, P) such that the following two conditions hold:

(i) Profit Maximization: for any alternative trading strategies X_1', X_2' and for any s_1, s_2

$$E_1[\tilde{\Pi}_1(X_1, X_2, P)|\tilde{s}_1 = s_1] \geq E_1[\tilde{\Pi}_1(X_1', X_2, P)|\tilde{s}_1 = s_1],$$

$$E_2[\tilde{\Pi}_2(X_1, X_2, P)|\tilde{s}_2 = s_2] \geq E_2[\tilde{\Pi}_2(X_1, X_2', P)|\tilde{s}_2 = s_2].$$

(ii) Market Efficiency: the pricing rule $\tilde{p}$ satisfies

$$\tilde{p}(X_1, X_2, P) = E_0[\tilde{v}|\tilde{y} = \tilde{x}_1 + \tilde{x}_2 + \tilde{z}].$$

Our model has a unique linear equilibrium (X_1, X_2, P), as shown below:

THEOREM 1: *Given the noise-to-signal ratio $\theta \equiv \sigma_e/\sigma_v$ and the noise trading ratio $\phi \equiv \sigma_z/\sigma_v$, there exists a unique linear equilibrium in which the trading strategies and the price are of the form:*

$$X_1(\tilde{s}_1) = \gamma_1\tilde{s}_1 \tag{1}$$

$$X_2(\tilde{s}_2) = \gamma_2\tilde{s}_2 \tag{2}$$

$$P(\tilde{y}) = \lambda\tilde{y} \tag{3}$$

where the trading intensity parameters γ_1 and γ_2, and liquidity parameter λ are given by

$$\gamma_1 = \frac{(1 + 2K_{22}^2\theta^2)\phi}{\sqrt{b}} \tag{4}$$

$$\gamma_2 = \frac{(1 + 2K_{11}^2\theta^2)\phi}{\sqrt{b}} \tag{5}$$

$$\lambda = \frac{\sqrt{b}}{g\phi} \tag{6}$$

and the constants b and g are defined explicitly in the Appendix. The second order condition is $\lambda > 0$.

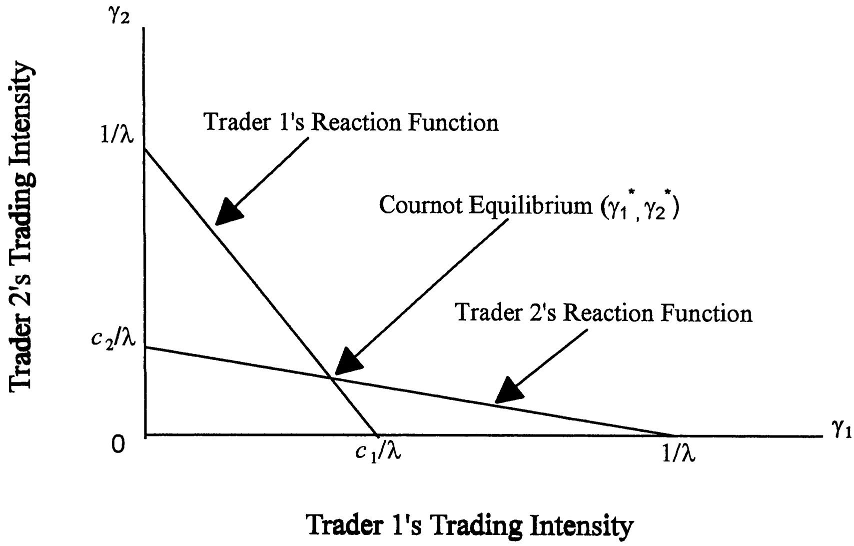

Figure 1. Cournot equilibrium of the duopoly trading game. This figure illustrates how the unique Cournot equilibrium, (γ_1^*, γ_2^*), of the duopoly trading game is determined, holding the market liquidity parameter, λ, constant. The two traders' Cournot reaction functions are given by equations (A3) and (A4). The two constants, c_1 and c_2, are defined by $c_1 = 1/2(1 + K_{11}^2\theta^2)$ and $c_2 = 1/2(1 + K_{22}^2\theta^2)$, respectively, where K_{11} and K_{22} are the two traders' own precision parameters and $\theta \equiv \sigma_e/\sigma_v$ is the noise-to-signal ratio.

Proof. See Appendix.

The equilibrium depends on the noise-to-signal ratio, θ, the noise trading ratio, ϕ, and the traders' *own* precision parameters, K_{11} and K_{22}, but not on the *other* precision parameters, K_{12} and K_{21}. Equation (A3) in the Appendix gives trader 1's trading intensity parameter γ_1 as a linear Cournot reaction function to trader 2's intensity parameter γ_2. Equation (A4) gives trader 2's reaction function analogously. As shown in Figure 1, if informed trader j increases trading intensity by one unit, the other informed trader i's best response is to reduce his trading intensity by $1/[2(1 + K_{ii}^2\theta^2)]$, holding λ constant. Thus, the parameter K_{ii} can be viewed as a commitment device in the standard Cournot duopoly game.

Properties of the Equilibrium. Overconfident traders tend to trade a great deal while underconfident traders do not trade much. For finite parameter values, trading volume may be arbitrarily large for extremely overconfident traders (i.e., $0 \le K_{ii} \ll 1$). Market depth is great and the price is very informative. In such an equilibrium, overconfident informed traders trade very aggressively, because they think they have a lot of information and because market depth is high. Market makers do not think that informed traders are as informed as they think, and hence are willing to provide more depth than they otherwise would. These results depend on parameter choices for K_{ii} as well as θ. (To see this, consider the extreme case $K_{ii} = 0$, i.e., informed traders think their signals have no error. In this case, $g = 3$ and $b = 2(1 - \theta^2)$. Hence, equilibrium exists if and only if $0 \leq \theta < 1$, in which case $\lambda = \sqrt{2(1 - \theta^2)}/3\phi$ and $\gamma_1 = \gamma_2 = \phi/\sqrt{2(1 - \theta^2)}$. If θ is very close to one, then λ is very small and γ_1 and γ_2 are very large.) The case where equilibrium does not exist corresponds to trading volume and liquidity being infinite, not to market shutting down. For example, for any symmetric choices of trading intensities parameters $\gamma_1 = \gamma_2 = \gamma$ conjectured by market makers, the market makers would provide so much depth that the overconfident traders would want to trade more aggressively than γ. Extremely underconfident informed traders (i.e., $K_{ii} \gg 1$) trade little, because they do not realize how much information they have. The resulting equilibrium has great liquidity (because adverse selection is low) but a relatively uninformed price. (To see this, model extreme underconfidence by letting $K_{ii} \to \infty$. Holding other exogenous parameters constant, it is easy to show that γ_1, γ_2, and λ all converge to zero.)

II. Survival of Overconfidence

In this section, we examine the model's implications for the survival of irrational traders in the market in terms of their profits (and utility). In particular, we want to see whether or not irrational traders, i.e., $K_{ii} \neq 1$, can outperform rational ones, i.e., $K_{ii} = 1$, by making higher expected profits (and utility).

A. Profit Comparison between Rational and Irrational Traders

Both traders' expected profits depend on their precision parameters K_{11} and K_{22}. To emphasize this dependence, write $E_0(\tilde{\Pi}_i)(K_{11}, K_{22})$ as trader i's expected profit, where the subscript 0 denotes the correct expectations. Theorem 1 implies that an irrational trader makes greater expected profits than his rational opponent if and only if he is overconfident, but not too overconfident when the true forecasting ability of traders is not too good (i.e., a large noise-to-signal ratio θ), as shown in Proposition 1.

PROPOSITION 1. *In an equilibrium of informed speculation with irrational trader 1 (i.e., $K_{11} = K \neq 1$) and rational trader 2 (i.e., $K_{22} = 1$),*

$$(a) \quad \text{if} \quad 0 \leq \theta^2 < \frac{1}{\sqrt{2}}, \qquad \text{then} \quad E_0(\tilde{\Pi}_1)(K, 1) > E_0(\tilde{\Pi}_2)(K, 1) \Leftrightarrow 0 \leq K < 1;$$

$$(b) \quad \text{if} \quad \theta^2 \geq \frac{1}{\sqrt{2}}, \qquad \text{then} \quad E_0(\tilde{\Pi}_1)(K, 1) > E_0(\tilde{\Pi}_2)(K, 1) \Leftrightarrow \sqrt{\frac{2\theta^4 - 1}{2\theta^4 + 2\theta^2}} < K < 1.$$

Proof. See Appendix.

Proposition 1 indicates that an overconfident trader outperforms his rational opponent either if the true forecasting ability is good (i.e., a small θ) or if the degree of overconfidence is modest (i.e., a large K given a large θ). This occurs because overconfidence, i.e., $K < 1$, acts like a commitment device in a stan-

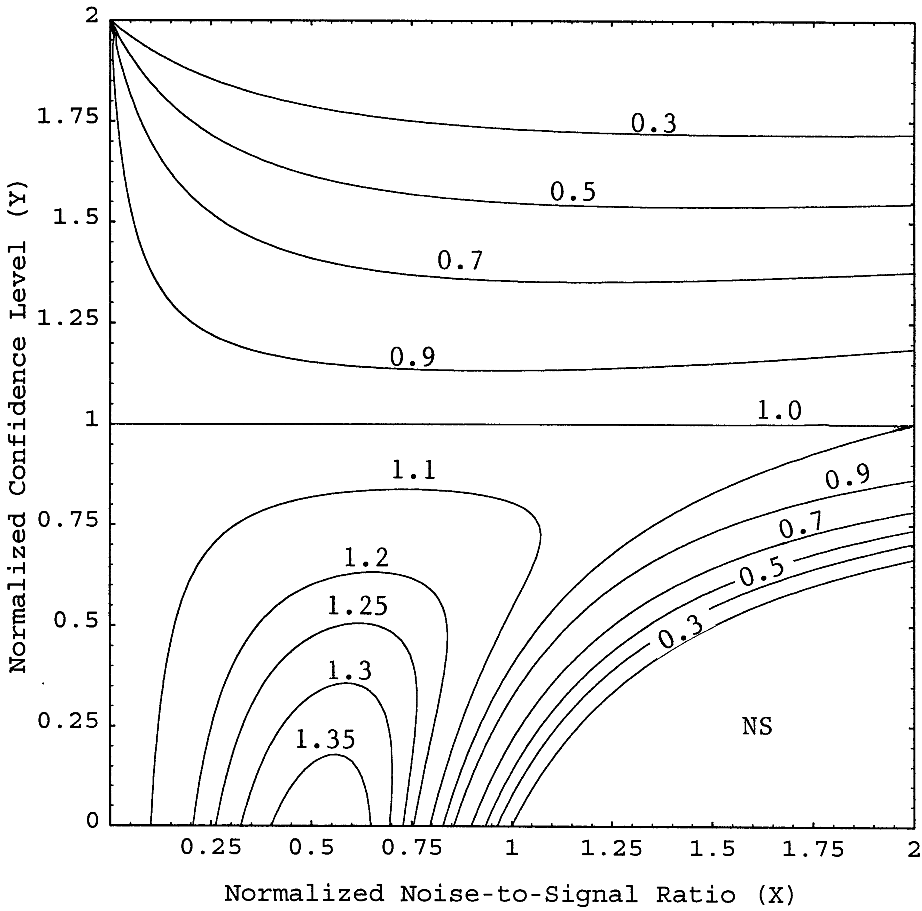

Figure 2. Ratio of an irrational trader's expected profit over a rational trader's $[E_0(\tilde{\Pi}_1)(K, 1)]/[E_0(\tilde{\Pi}_2)(K, 1)]$. This figure gives representative level curves of the ratio of irrational trader 1's expected profit over rational trader 2's on the normalized X-Y space. The normalized noise-to-signal ratio X and the normalized confidence level Y are defined by $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$, where $\theta \equiv \sigma_e/\sigma_v$ is the original noise-to-signal ratio and the constant K is irrational trader 1's own precision parameter. Region NS corresponds to the set of parameter combinations (θ, K), or equivalently (X, Y), such that no symmetric linear equilibrium exists.

dard Cournot duopoly model, giving the overconfident trader a reputation for trading so aggressively that his rational opponent (recognizing this) trades less aggressively. As a result, the overconfident trader may make a higher expected profit and utility than his rational opponent. This result is in spirit similar to De Long *et al.* (1991) and Blume and Easley (1992). But, unlike their results, the dominance of overconfidence in our model is not due to a tradeoff between beliefs and attitudes toward risk. Proposition 1 implies also that an underconfident trader, i.e., $K > 1$, always underperforms. The result of Proposition 1 is shown graphically in Figure 2 by plotting the ratio of the two traders' expected profits, $[E_0(\tilde{\Pi}_1)(K, 1)]/[E_0(\tilde{\Pi}_2)(K, 1)]$, on the *normalized* X–Y space, where the

Table II

Payoff Bimatrix of the Two Funds

This table provides the payoff bimatrix of the two-fund game, in which each fund i can choose either a rational manager (denoted by $K_{ii} = 1$) or an irrational manager (denoted by $K_{ii} = K$), where $i = 1, 2$ and K is an exogenous, nonnegative constant. For example, if both funds choose rational managers (i.e., $K_{11} = K_{22} = 1$) then both funds receive the same payoff, denoted by $E_0(\tilde{\Pi})(1, 1)$; if fund 1 chooses an irrational manager (i.e., $K_{11} = K$) while fund 2 chooses a rational manager (i.e., $K_{22} = 1$), then fund 1's payoff is $E_0(\tilde{\Pi}_1)(K, 1)$ while fund 2's payoff is $E_0(\tilde{\Pi}_1)(1, K)$. The payoffs for the two other strategy combinations (i.e., $(K_{11}, K_{22}) \epsilon \{(1, K), (K, K)\}$) are denoted analogously.

	$K_{22} = 1$	$K_{22} = K$
$K_{11} = 1$	$(E_0(\tilde{\Pi})(1, 1), E_0(\tilde{\Pi})(1, 1))$	$(E_0(\tilde{\Pi}_1)(1, K), E_0(\tilde{\Pi}_1)(K, 1))$
$K_{11} = K$	$(E_0(\tilde{\Pi}_1)(K, 1), E_0(\tilde{\Pi}_1)(1, K))$	$(E_0(\tilde{\Pi})(K, K), E_0(\tilde{\Pi})(K, K))$

normalized noise-to-signal ratio, X, and the normalized confidence level, Y, are defined by $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$.

B. Nash Equilibrium of Delegated Fund Management

The dominance of overconfidence has a strategic implication for delegated fund management. Consider two rival funds and a fixed degree of irrationality K (i.e., $K \neq 1$) such that each fund can hire either an irrational fund manager with $K_{ii} = K$ or a rational fund manager with $K_{ii} = 1$. The competition between the two funds can be modeled as a two-fund game, in which each fund's payoff is its manager's correct expected profit. For a wide range of parameter values $K < 1$, this game is a Prisoner's Dilemma in which both funds choose overconfident managers. To see this, note that if both funds choose identical strategies, then they obtain identical payoffs. In this case, we drop the trader's index i and write $E_0(\tilde{\Pi})(c, c) \equiv E_0(\tilde{\Pi}_i)(c, c)$, for $c \epsilon \{1, K\}$ and $i = 1, 2$. Theorem 1 implies that fund 2's payoff matrix is the transpose of fund 1's payoff matrix. Thus, the two-fund game is a symmetric two-player game with the payoff bimatrix, as described in Table II.

It is clear that if $[E_0(\tilde{\Pi}_1)(K, 1)]/[E_0(\tilde{\Pi})(1, 1)] > 1$ and $[E_0(\tilde{\Pi})(K, K)]/[E_0(\tilde{\Pi}_1)(1, K)] > 1$, then the two-fund game has a unique Nash equilibrium that both funds choose irrational managers. It is easy to show that both profit ratios depend only on the two key parameters, θ and K. Hence, we can evaluate the above two inequalities for the entire universe of parameter combinations $(\theta, K) \epsilon [0, \infty) \times [0, \infty)$ by plotting the two ratios on the normalized X–Y space, where $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$. Figures 3 and 4 show some representative level curves of the two ratios, respectively. The two figures clearly indicate that for the case of overconfidence, i.e., $K < 1$, there exists a large set of parameter combinations (θ, K) such that both profit ratios are greater than one. In this set of parameters, the two-fund game has a unique Nash equilibrium that both funds choose overconfident managers. Furthermore, the Nash equilibrium is a Prisoner's Dilemma, because both funds make less expected

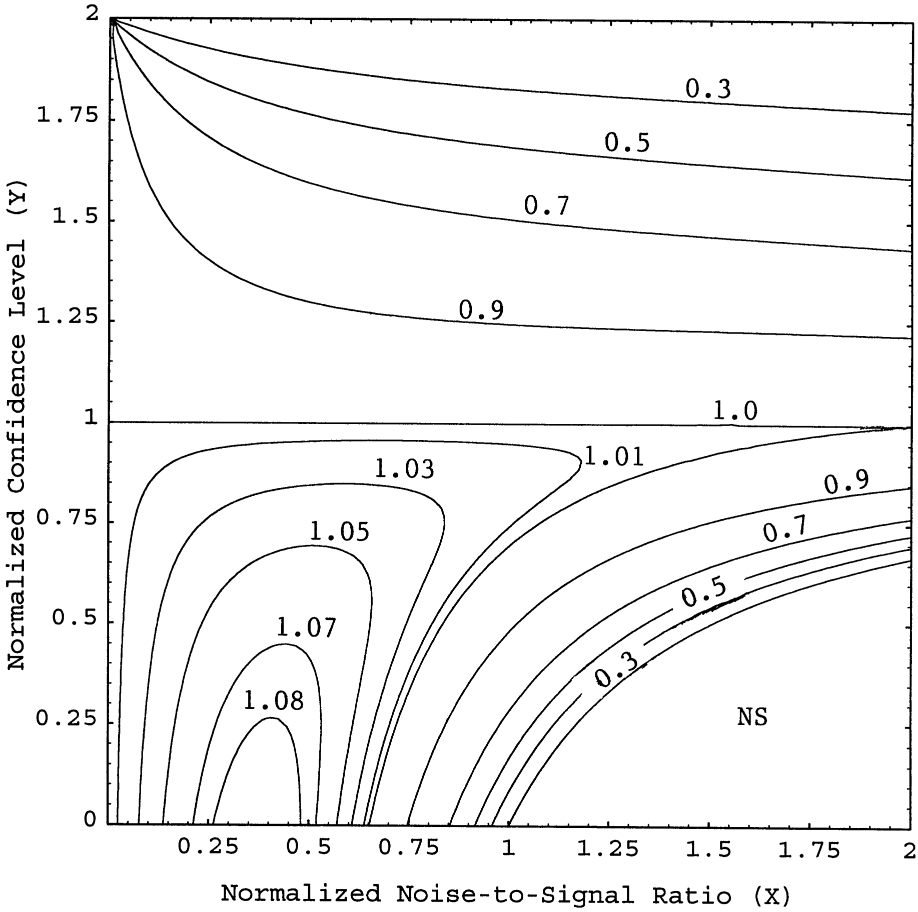

Figure 3. Ratio of an irrational trade's expected profit over a rational trader's, given a rational opponent, $[E_0(\tilde{\Pi}_1)(K, 1)]/[E_0(\tilde{\Pi})(1, 1)]$. This figure gives representative level curves of the ratio of irrational trader 1's expected profit over rational trader 1's, given rational trader 2, on the normalized X–Y space. The normalized noise-to-signal ratio X and the normalized confidence level Y are defined by $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$, where $\theta \equiv \sigma_e/\sigma_v$ is the original noise-to-signal ratio and the constant K is irrational trader 1's own precision parameter. Region NS corresponds to the set of parameter combinations (θ, K), or equivalently (X, Y), such that no symmetric linear equilibrium exists.

profits than if they both choose rational managers, i.e., $E_0(\tilde{\Pi})(K, K) < E_0(\tilde{\Pi})(1, 1)$ for $K < 1$, as shown in Figure 5. These results are formalized in Proposition 2.

PROPOSITION 2. *For all parameters* $(\theta, K)\epsilon\{[0, \infty) \times [0, 1)|[E_0(\tilde{\Pi}_1)(K, 1)]/[E_0(\tilde{\Pi})(1, 1)] > 1$ *and* $[E_0(\tilde{\Pi})(K, K)]/[E_0(\tilde{\Pi}_1)(1, K)] > 1\}$, *the two-fund game has a unique Nash equilibrium in which both funds choose overconfident managers. Furthermore, the Nash equilibrium is a Prisoner's Dilemma, in which both funds make less expected profits than if they both choose rational managers, i.e.,* $E_0(\tilde{\Pi})(K, K) < E_0(\tilde{\Pi})(1, 1)$ *for* $K < 1$.

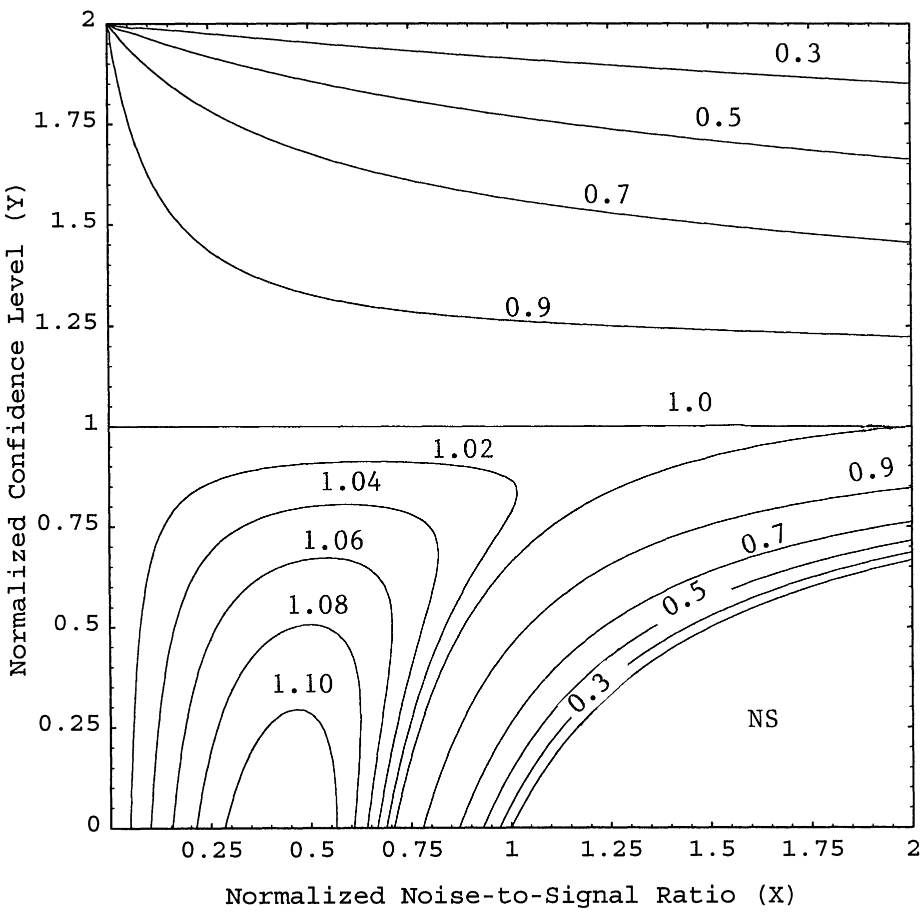

Figure 4. Ratio of an irrational trader's expected profit over a rational trader's, given an irrational opponent, $[E_0(\tilde{\Pi})(K, K)]/[E_0(\tilde{\Pi}_1)(1, K)]$. This figure gives representative level curves of the ratio of irrational trader 1's expected profit over rational trader 1's, given irrational trader 2, on the normalized X–Y space. The normalized noise-to-signal ratio X and the normalized confidence level Y are defined by $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$, where $\theta \equiv \sigma_e/\sigma_v$ is the original noise-to-signal ratio and the constant K is irrational trader 1's own precision parameter. Region NS corresponds to the set of parameter combinations (θ, K), or equivalently (X, Y), such that no symmetric linear equilibrium exists.

In Figure 6, the set of parameter combinations (θ, K) that yields the Prisoner's Dilemma Nash equilibrium is identified as region PD. In region H, an overconfident manager makes higher expected profits than his rational opponent, but makes less than if he were also rational. This region corresponds to "spiteful" behavior of the overconfident manager in the sense that his overconfidence hurts his rational opponent more than himself. In region L, an irrational manager (overconfident or underconfident) makes less expected profits than his rational opponent. Proposition 2 and Figure 6 together indicate that overconfidence may strictly dominate rationality. This is so because in

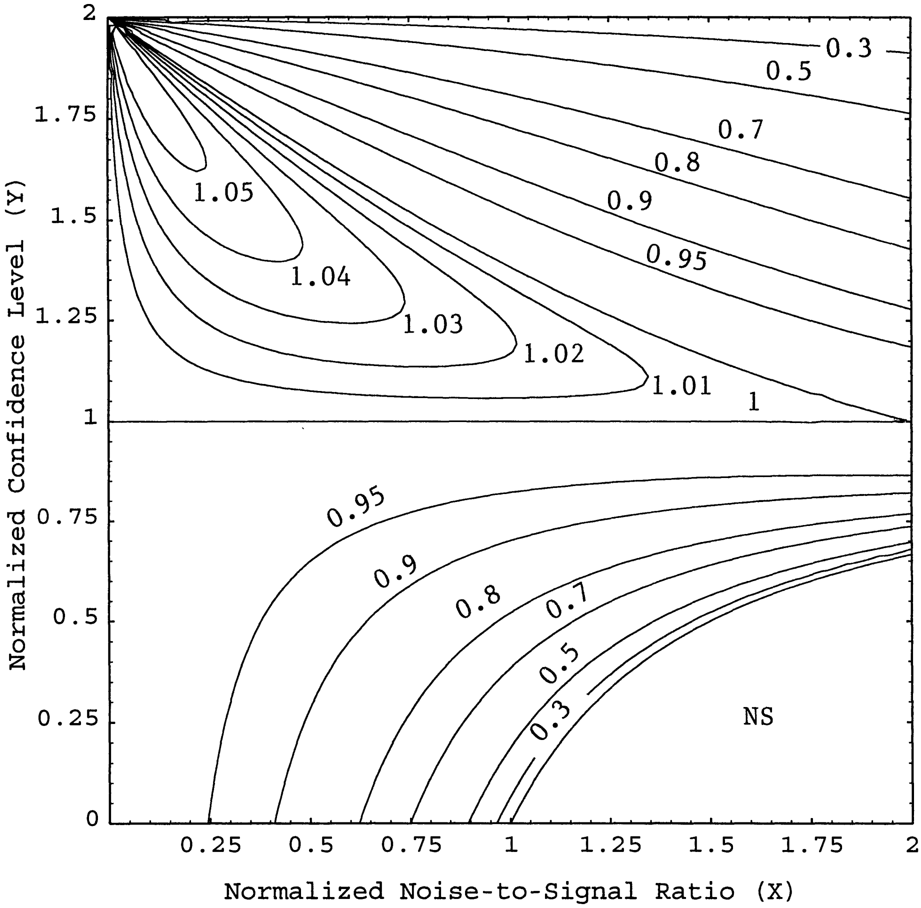

Figure 5. Ratio of an irrational trader's expected profit over a rational trader's, given symmetric beliefs, $[E_0(\tilde{\Pi})(K, K)]/[E_0(\tilde{\Pi})(1, 1)]$. This figure gives representative level curves of the ratio of an irrational trader's expected profit over a rational trader's, given symmetric (identical) beliefs of traders 1 and 2, on the normalized X–Y space. The normalized noise-to-signal ratio X and the normalized confidence level Y are defined by $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$, where $\theta \equiv \sigma_e/\sigma_v$ is the original noise-to-signal ratio and the constant K is irrational trader 1's own precision parameter. Region *NS* corresponds to the set of parameter combinations (θ, K), or equivalently (X, Y), such that no symmetric linear equilibrium exists.

region PD an overconfident manager not only makes higher expected profits (and utility) than his rational opponent, but also higher than if he were also rational. Hence, both funds should hire overconfident managers as the unique Nash equilibrium.

Our result that overconfidence may strictly dominate rationality is much stronger than previous results about the survival of irrationality, obtained through irrational agents' "spiteful" behavior. With spiteful behavior, "irrationality" is a strictly dominated strategy, but performs relatively better than the dominant "rationality" strategy in a polymorphic, finite population (see, e.g.,

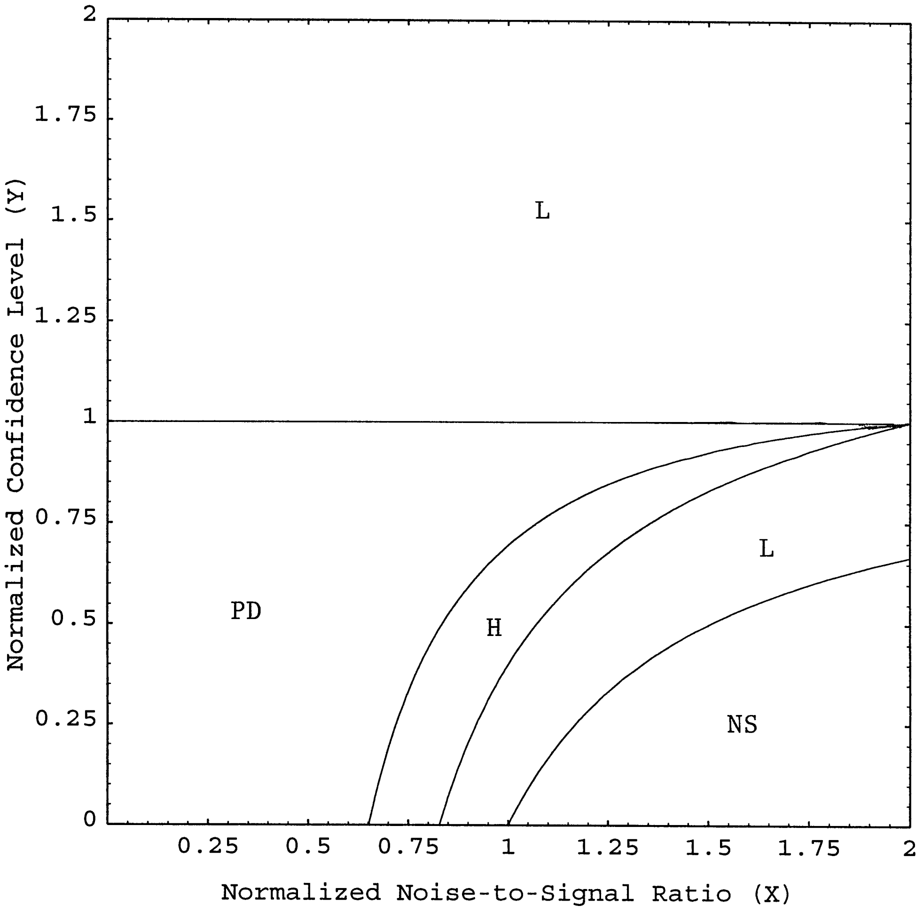

Figure 6. Characterization of profit comparison. This figure classifies profit comparison for traders. Region *PD* corresponds to the set of parameter combinations (θ, K) such that an overconfident trader not only makes greater expected profits than his rational opponent, but also higher than if he were also rational. Region *H* corresponds to the set of parameter combinations (θ, K) such that an overconfident trader makes greater expected profits than his rational opponent, but less than if he were also rational. Region *L* corresponds to the set of parameter combinations (θ, K) such that an irrational trader makes less expected profits than his rational opponent. The normalized noise-to-signal ratio X and the normalized confidence level Y are defined by $X = 2\theta^2/(1 + \theta^2)$ and $Y = 2K^2/(1 + K^2)$, where $\theta \equiv \sigma_e/\sigma_v$ is the original noise-to-signal ratio and the constant K is the irrational trader's own precision parameter. Region *NS* corresponds to the set parameter combinations (θ, K) such that no symmetric linear equilibrium exists.

Schaffer (1989), Palomino (1996), Rhode and Stegeman (1996), and Vega-Redondo (1996)). Furthermore, Proposition 2 indicates that the unique Prisoner's Dilemma Nash equilibrium is strict and symmetric. This result implies that hiring overconfident managers is an evolutionarily stable strategy (ESS). An ESS is robust to evolutionary selection pressures in the long run (Schaffer (1988)). Hence, for all parameters (θ, K) in region PD, overconfidence not only strictly dominates rationality, but also persists and survives in the long run.

In essence, the survival of overconfidence, $K_{ii} = K < 1$, is due to the effect that overconfidence acts like a commitment device in a standard Cournot duopoly model. Thus, a fund facing a major rival in the market can promote its survival either by hiring overconfident managers or by instituting an incentive scheme to make its rational managers trade as aggressively as if they were overconfident. While the incentive scheme does not evoke irrational prior beliefs, it is not useful when owners are themselves managers. In this case, an overconfident owner-manager may dominate his rational opponent only if he is "born" overconfident, since the owner needs no incentive.

III. Conclusion

Our model shows that overconfidence may strictly dominate rationality and survive in the long run. This occurs because overconfidence acts like a commitment device in a standard Cournot duopoly model. This result has a strategic implication for delegated fund management: for some parameter values, a fund facing a major rival in an efficient market should hire overconfident managers. Alternatively, a fund can promote its long-run survival by instituting an incentive scheme to make its rational managers trade as aggressively as if they were overconfident. The survival of overconfidence is consistent with the empirical evidence that experts tend to be more prone to overconfidence than novices (Griffin and Tversky (1992)) in the sense that overconfident experts succeed and keep their reputation for expertise in the process of market selection.

Our article is different from previous works that identify irrationality with excess volatility such as Shiller (1981, 1984, 1989) or abnormal returns such as De Bondt and Thaler (1985, 1987). Campbell and Kyle (1993) show that excess volatility can result from irrational noise traders' overreaction to information in the presence of rational risk-averse informed traders. In contrast with these models, our model maintains the assumption that markets are semi-strong efficient. "Irrationality" enters into the trading strategies of informed traders, whose trading affects the information content of prices but does not lead to inefficient prices. Consequently, the survival of overconfidence in our model can not be attributed to inefficient prices.

Appendix

Proof of Theorem 1. The market efficiency condition and normal distributions imply a linear pricing rule: $P(\tilde{y}) = \lambda\tilde{y}$. With the linear pricing rule and the linear conjecture of trader 2's strategy $X_2(\tilde{s}_2) = \gamma_2\tilde{s}_2$, trader 1's expected profit conditional on his signal $\tilde{s}_1$ is given by

$$E_1[(\tilde{v} - p(\tilde{y}))x_1|\tilde{s}_1 = s_1] = E_1[(\tilde{v} - \lambda(x_1 + \gamma_2\tilde{s}_2 + \tilde{z}))x_1|\tilde{s}_1 = s_1]$$

$$= (1 - \lambda\gamma_2)E_1[\tilde{v}|s_1]x_1 - \lambda x_1^2 = \frac{1 - \lambda\gamma_2}{1 + K_{11}^2\theta^2} s_1 x_1 - \lambda x_1^2 \tag{A1}$$

The first order condition yields the optimal strategy for trader 1 as follows,

$$x_1^* = \frac{1}{2(1 + K_{11}^2\theta^2)}\left(\frac{1}{\lambda} - \gamma_2\right)s_1 \tag{A2}$$

Thus, the trading intensity parameter for trader 1 is given by

$$\gamma_1 = \frac{1}{2(1 + K_{11}^2\theta^2)}\left(\frac{1}{\lambda} - \gamma_2\right) \tag{A3}$$

Similarly, the trading intensity parameter for trader 2 is given by

$$\gamma_2 = \frac{1}{2(1 + K_{22}^2\theta^2)}\left(\frac{1}{\lambda} - \gamma_1\right) \tag{A4}$$

Given λ, the two intensity parameters γ_1, γ_2 in equations (A3) and (A4) can be solved simultaneously and the solution is unique and stable. Given γ_1 and γ_2, the market efficiency condition implies that

$$P(\tilde{y}) = E_0[\tilde{v}|\tilde{y} = \tilde{x}_1 + \tilde{x}_2 + \tilde{z}] = \frac{(\gamma_1 + \gamma_2)\sigma_v^2}{(\gamma_1 + \gamma_2)^2\sigma_v^2 + (\gamma_1^2 + \gamma_2^2)\sigma_e^2 + \sigma_z^2}\tilde{y} = \lambda\tilde{y} \tag{A5}$$

The market liquidity parameter λ is therefore given by

$$\lambda = \frac{(\gamma_1 + \gamma_2)\sigma_v^2}{(\gamma_1 + \gamma_2)^2\sigma_v^2 + (\gamma_1^2 + \gamma_2^2)\sigma_e^2 + \sigma_z^2} \tag{A6}$$

Plugging equations (A3) and (A4) into (A6) and rearranging yields a quadratic equation for λ as follows,

$$\phi^2\lambda^2 - \frac{b}{g^2} = 0 \tag{A7}$$

where the constants b and g are defined by

$$b = 2\{1 + [3(K_{11}^2 + K_{22}^2) - 1]\theta^2 + 2[(K_{11}^2 + K_{22}^2)^2 + 2(K_{11}^2K_{22}^2) - (K_{11}^2 + K_{22}^2)]\theta^4 + 2[2K_{11}^2K_{22}^2(K_{11}^2 + K_{22}^2) - (K_{11}^4 + K_{22}^4)]\theta^6\} \tag{A8}$$

$$g = 3 + 4(K_{11}^2 + K_{22}^2)\theta^2 + 4K_{11}^2K_{22}^2\theta^4 \tag{A9}$$

If $b > 0$, then equation (A7) has two nonzero real roots, in which the positive root is the relevant one, because $\lambda > 0$ is the second order condition in the informed traders' optimization problem. Q.E.D.

Proof of Proposition 1. Trader 1's conditional expected profit under the correct distributions is

$$E_0[\tilde{\Pi}_1|\tilde{s}_1 = s_1] = E_0[(\tilde{v} - p(\tilde{y}))\tilde{x}_1|\tilde{s}_1 = s_1]$$
$$= [E_0(\tilde{v}|s_1) - \lambda(\gamma_1 s_1 + \gamma_2 E_0(\tilde{s}_2|s_1))]\gamma_1 s_1$$
$$= \gamma_1\left[(1 - \lambda\gamma_2)\left(\frac{1}{1 + \theta^2}\right) - \lambda\gamma_1\right]s_1^2. \tag{A10}$$

Taking expectations on both sides of (A10) yields trader 1's unconditional expected profit

$$E_0[\tilde{\Pi}_1] = \gamma_1[1 - \lambda(\gamma_1 + \gamma_2) - \lambda\gamma_1\theta^2]\sigma_v^2 \tag{A11}$$

Similarly, Trader 2's unconditional expected profit is given by

$$E_0[\tilde{\Pi}_2] = \gamma_2[1 - \lambda(\gamma_1 + \gamma_2) - \lambda\gamma_2\theta^2]\sigma_v^2 \tag{A12}$$

Subtracting (A12) from (A11) yields

$$E_0[\tilde{\Pi}_1] - E_0[\tilde{\Pi}_2] = (\gamma_1 - \gamma_2)[1 - \lambda(\gamma_1 + \gamma_2)(1 + \theta^2)]\sigma_v^2 \tag{A13}$$

Given $(K_{11}, K_{22}) = (K, 1)$, substituting for γ_1, γ_2 and λ from Theorem 1 in equation (A13) yields

$$E_0(\tilde{\Pi}_1)(K, 1) - E_0(\tilde{\Pi}_2)(K, 1) = (\gamma_1 - \gamma_2)\frac{1 - 2\theta^4 + 2\theta^2(1 + \theta^2)K^2}{3 + 4\theta^2 + 4\theta^2(1 + \theta^2)K^2}\sigma_v^2 \tag{A14}$$

Since $\gamma_1(K, 1) > \gamma_2(K, 1) \Leftrightarrow K < 1$ by Theorem 1, equation (A14) clearly leads to the desired results. Q.E.D.

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