Trading Volume and Transaction Costs in Specialist Markets

Thomas J. George; Gautam Kaul; M. Nimalendran

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THOMAS J. GEORGE, GAUTAM KAUL,
and M. NIMALENDRAN*

ABSTRACT

Prior work with competitive rational expectations equilibrium models indicates that there should be a *positive* relation between trading volume and differences in beliefs or information among traders. We show that this result is sensitive to whether and how transaction costs are modeled. In a specialist market with endogenous transaction costs we show that trading volume can be *negatively* related to the degree of informational asymmetry in the market. Our analysis highlights the dependence of volume on market structure, and our results suggest that the "volume effects" of corporate or macroeconomic events reflect a decrease, rather than an increase, in heterogeneity of beliefs or asymmetry of information.

PRIOR RESEARCH ON RATIONAL expectations equilibrium models of competitive markets indicates that trading volume is generated "primarily" by differences in beliefs which arise because agents possess different information and noise in the market prevents prices from perfectly aggregating private information (see, for example, Milgrom and Stokey (1982), Pfleiderer (1984), Grundy and McNichols (1989), Holthausen and Verrecchia (1990), and Kim and Verrecchia (1991a, 1991b)). The common conclusion of previous studies is that volume is (a) *directly* related to (some measure of) asymmetric information or differences in beliefs; and (b) this relationship is *unambiguously* positive. For example, Kim and Verrecchia (1991b) state that; "The presence of differential precision [of private information] thus causes *differential belief revisions* among investors which, in turn, creates volume. When there are no differences in precision across investors... there is no volume."[1]

[1] In related research, Harris and Raviv (1991) show that even if all traders are homogeneously informed, *differences in opinion* (i.e., differences in priors and/or the models used to assess the implications of new information) between traders is the sole determinant of trading volume (see also Varian (1985)).

One critical feature of most models of volume is that transacting is costless. Consequently, the distribution of liquidity trading is unaffected by the informational aspects of the market. Prior work therefore focuses on the quality (or precision) and heterogeneity of private information among informed agents as the primary economic determinants of trading volume, and liquidity trading simply determines the amount of noise in the market. Motivated by this theoretical research, numerous empirical researchers use volume of trading as a measure of differences in beliefs, information and/or opinion among market participants (see, for example, Beaver (1968), Morse (1981), Bamber (1986, 1987), Richardson, Sefcik, and Thompson (1986), Comisky, Walkling, and Weeks (1987), Jain (1988), Ziebart (1990), and Lang, Litzenberger, and Madrigal (1992)). Nevertheless, explicit measures of differences in beliefs do not explain a large proportion of the cross-sectional variation in volume (e.g., Comisky, Walkling, and Weeks (1987) and Lang, Litzenberger, and Madrigal (1992)). Similarly, in studying volume reactions to firms' earnings announcements Bamber (1986) finds that aggregate volume is *less* for securities whose volume is positively correlated with the magnitude of unexpected earnings. This evidence suggests that asymmetric information or differences in beliefs may not be the primary determinant of trading volume.

In this article, we study a model of a specialist market with endogenous transaction costs in which liquidity trading is transaction-cost elastic. We show that the relationship between trading volume and the degree of heterogeneity in beliefs and informational asymmetry is both *indirect* and *ambiguous*. As in most existing models, disagreement in our model arises from private information. In the absence of transaction costs, and hence transaction-cost-elastic liquidity trading, anything that increases disagreement among traders in equilibrium creates volume. However, in our model this also implies an adverse-selection problem that increases transaction costs and decreases agents' willingness to trade. We show that whether volume increases or decreases with an increase in informational asymmetry or divergence in beliefs depends on whether liquidity trading decreases in transaction costs at an increasing or decreasing rate. These findings provide an alternative interpretation for volume reactions to macroeconomic or corporate announcements that are typically attributed to heterogeneous beliefs or asymmetric information. Namely, since announcements *decrease* adverse selection faced by specialists, transaction costs decrease resulting in greater trading by the uninformed.

The article is organized as follows. Section I presents the basic model and some preliminary results. Section II contains a characterization of equilibrium expected volume of trading and our main results, examines how the predictions of our model relate to existing empirical evidence, and compares our results to those obtained in markets with competitive market makers. Section III concludes the article with a brief summary. Proofs of propositions are collected in the Appendix.

I. The Basic Model

We consider a pure-exchange economy in which there is a riskless bond (the numeraire) and a single risky asset; each pays off in units of a single consumption good. The economy is composed of informed agents, liquidity traders, and a monopolist specialist who sets bid and ask prices at which shares of the risky asset are traded. All market participants take the specialist's quoted prices as given. Information is assumed to be short lived, and, for tractability, we rule out the possibility of agents being informed for consecutive periods.

To make our analysis comparable to previous models, we use a simple mean-variance framework in which the informed agents have exponential utility of consumption of the single good. At time t, the risky asset's value, V_t, is publicly announced. Old informed agents redeem their claims for V_t, consume, and return to the pool of uninformed agents. After the redemption, the new informed agents observe their private information regarding V_{t+1} and are given the opportunity to trade with the specialist. At time $t + 1$ these agents redeem their shares of the risky asset for V_{t+1} units of the consumption good.[2] Suppose θ_{t+1} is the component of the change in the risky asset's value that informed agent i observes at time t with idiosyncratic error η_t^i, and ϵ_{t+1} is the component that is unknown at t. The risky asset's value at time $t + 1$ is

$$\tilde{V}_{t+1} = V_t + \tilde{\theta}_{t+1} + \tilde{\epsilon}_{t+1}. \tag{1}$$

For simplicity, we assume that the θs, ϵs, and ηs are mutually independent, normally distributed random variables with zero means and variances σ_θ, σ_ϵ, and σ_η, respectively. Furthermore, we assume that the random variables $\{\tilde{\eta}_t^i\}_i$ are mutually independent. The return on the riskless bond is normalized to zero so that its value is always unity.

After observing $\theta_{t+1} + \eta_t^i$, informed agent i maximizes his expected utility by choosing to hold x_t^i shares of the risky asset. If a^i is the agent's coefficient of constant absolute risk aversion,

$$x_t^i = \frac{\left(\hat{V}_{t+1}^i - \hat{P}_t\right)}{a^i \hat{\sigma}} \tag{2}$$

where $\hat{P}_t$ is the price (viewed by the agent as exogenously given) at which x_t^i units of the risky asset can be purchased (or sold short if $x_t^i < 0$),

$$\hat{V}_{t+1}^i \equiv E\left[\tilde{V}_{t+1} \mid \theta_{t+1} + \eta_t^i\right] = V_t + \frac{\sigma_\theta}{\sigma_\theta + \sigma_\eta}(\theta_{t+1} + \eta_t^i), \tag{3}$$

[2] The assumption that information is short lived motivates the informed agent to trade with the specialist rather than waiting until the redemption period when information is revealed.

and

$$\hat{\sigma} \equiv \text{Var}\big[\tilde{V}_{t+1} \mid \theta_{t+1} + \eta_t^i \big] = (\sigma_\theta + \sigma_\epsilon) - \frac{\sigma_\theta^2}{(\sigma_\theta + \sigma_\eta)}. \tag{4}$$

The liquidity traders submit random trades in the risky asset. This liquidity trading is independent of $\tilde{V}_{t+1}$. Aggregate demand at t by liquidity traders is assumed to be the realization of the random variable

$$\tilde{Y}_t = \tilde{Y}_t^+ + \tilde{Y}_t^- \tag{5}$$

where $\tilde{Y}_t^+ > 0$ and $\tilde{Y}_t^- < 0$ (with probability one). The volume of trading by the liquidity traders is $|\tilde{Y}_t^+| + |\tilde{Y}_t^-|$; and the mean of its distribution is denoted by μ. The price of the risky asset in this market is set by a risk-neutral (uninformed) specialist who quotes bid and ask prices. The liquidity traders perceive the midpoint of the bid. and ask prices to be a semi-strong-form efficient assessment of security value, and that the bid-ask spread is a transaction cost (these beliefs are correct in equilibrium). Specifically, we assume that their willingness to trade is unaffected by the level of the midpoint of bid and ask prices, but that they are less willing to trade when the spread is large than when it is small. this implies that μ is a decreasing function of the bid-ask spread.[3]

Since the specialist and his behavior are critical elements of our model, in the remainder of this section we characterize his optimal choices of bid and ask prices. We also provide a graphical representation of the specialist's problem and present some preliminary results that simplify our characterization of equilibrium trading volume presented in Section II.

A. The Specialist's Choice of Bid and Ask Prices

The specialist knows the structure of the economy—the composition of agents in the market and the parameters of the distributions describing uncertainty. He chooses bid and ask prices in each reading round to maximize his expected profits. Let m_t denote one-half the bid-ask spread, and let P_t be the midpoint of the bid and ask prices quoted by the specialist. Given the quoted prices, informed agents' aggregate demand is positive (negative) if their conditional expected value of the risky asset exceeds (is less than) the

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[3]We do not explicitly model liquidity trading to avoid placing restrictions on μ. However, μ can be endogenized by including in our model a class of exponential-utility-maximizing uninformed traders who are endowed with a risk that is correlated with the value of the risky asset at time $t + 1$. If these endowments are i.i.d. and normal, the wider the spread, the less these traders hedge, and the less is the volume of their trading. In this case, μ will be a decreasing *convex* function of m.

ask (bid) price,

$$
x_t^i = \begin{cases}
\dfrac{\hat{V}_{t+1}^i - P_t - m_t}{a^i \hat{\sigma}} & \text{if } \hat{V}_{t+1}^i > P_t + m_t \\[4mm]
\dfrac{\hat{V}_{t+1}^i - P_t + m_t}{a^i \hat{\sigma}} & \text{if } \hat{V}_{t+1}^i < P_t - m_t \\[4mm]
0 & \text{otherwise.}
\end{cases} \tag{6}
$$

For each share of the risky asset bought or sold, the specialist earns one-half the spread. If the asset is overpriced relative to its true value, the specialist makes money if individuals buy the asset and loses money if individuals sell the asset

$$
\tilde{\pi}_t = m_t \left(|\tilde{Y}_t^+| + |\tilde{Y}_t^-| \right) + m_t \left(\sum_i |\tilde{x}_t^i| \right) - \left(\tilde{V}_{t+1} - P_t \right) \left(\sum_i \tilde{x}_t^i + \tilde{Y}_t \right). \tag{7}
$$

The tildes are used to emphasize values that are unknown to the specialist. He chooses P_t and m_t to maximize his expectation of $\tilde{\pi}_t$,

$$
E[\tilde{\pi}_t] = m_t \mu(m_t) + m_t \phi(P_t, m_t) - \psi(P_t, m_t) \tag{8}
$$

where $\phi(P_t, m_t) \equiv \sum_i E[|\tilde{x}_t^i|]$ and $\psi(P_t, m_t) \equiv \sum_i E[(\tilde{V}_{t+1} - P_t)\tilde{x}_t^i]$. Derivations of expressions for ϕ and ψ are contained in the Appendix as equations (A1) and (A2).

Determination of the optimal choices of P_t and m_t by the specialist is separable. One can show that $P_t = V_t$ maximizes $E[\tilde{\pi}_t]$ regardless of the specialist's choice of m_t. Since the quotes straddle the specialist's expected value of the security, liquidity traders correctly believe that the midpoint of the bid and ask prices is a semi-strong-form efficient estimate of the security's value. Therefore, the spread is correctly regarded as a transaction cost, which is consistent with our assumption that the volume of liquidity trades is spread elastic.

The first-order condition corresponding to the choice of m is obtained by differentiating equation (8) and substituting the optimal choice of P (time subscripts can now be dropped without ambiguity)

$$
\mu(m) + m\mu'(m) + \phi(m) + m\phi'(m) - \psi'(m) = 0 \tag{9}
$$

where "$'$" denotes the partial derivative with respect to m. This expression suppresses P because at the optimal P the functions ϕ, ψ, $\phi'(m)$, and $\psi'(m)$ are independent of P.[4] Straightforward differentiation yields the expressions for $\phi'(m)$ and $\psi'(m)$ in Lemma 1.

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[4] This can be verified by a simple change of variables to "de-mean" $\hat{V}$ in equations (A1) and (A2) of the Appendix, which yields the expressions for ϕ and ψ in Lemma 1.

LEMMA 1: *At the optimal P*

$$\phi(m) = \frac{2}{a\hat{\sigma}} \int_m^\infty z f_\theta(z) \, dz - \frac{2m}{a\hat{\sigma}} \int_m^\infty f_\theta(z) \, dz$$

$$\psi(m) = \frac{2}{a\hat{\sigma}} \int_m^\infty z^2 f_\theta(z) \, dz - \frac{2m}{a\hat{\sigma}} \int_m^\infty z f_\theta(z) \, dz$$

and

$$\phi'(m) = \frac{-2}{a\hat{\sigma}} \int_m^\infty f_\theta(z) \, dz \qquad \psi'(m) = \frac{-2}{a\hat{\sigma}} \int_m^\infty z f_\theta(z) \, dz$$

where f_θ is the probability density function of $\tilde{\theta} \sim N(0, \sigma_\theta)$.

We define the contributions of liquidity and informed traders' volume to the specialist's marginal expected profits as

$$\mathscr{U}(m) \equiv \mu(m) + m\mu'(m) \qquad \text{and} \qquad \mathscr{I}(m) \equiv \phi(m) + m\phi'(m) - \psi'(m). \tag{10}$$

We now state our main lemma.

LEMMA 2: *At the optimal P*

$$\mathscr{I}(m) = 2\phi(m) \qquad \text{for all } m.$$

This result shows that the marginal benefit to the specialist of increasing the spread is proportional to the expected volume of informed trading. In other words, widening the spread is more effective in reducing expected losses of the specialist when the informed agents are expected to trade heavily. To understand this, note that (given the optimal P) informed trading is expected to be heavy only if the spread is narrow. But this is precisely when the probability of mispricing the asset can be decreased the most by a unit increase in the spread (because the density under a normal distribution from its mean to a finite limit increases slower as the limit moves farther from the mean). This is also when a unit increase in the spread increases the revenues collected from the informed agent the most. Both of these effects make small increases in the spread most beneficial to the specialist when the spread is initially small, which is when expected informed volume is the greatest.
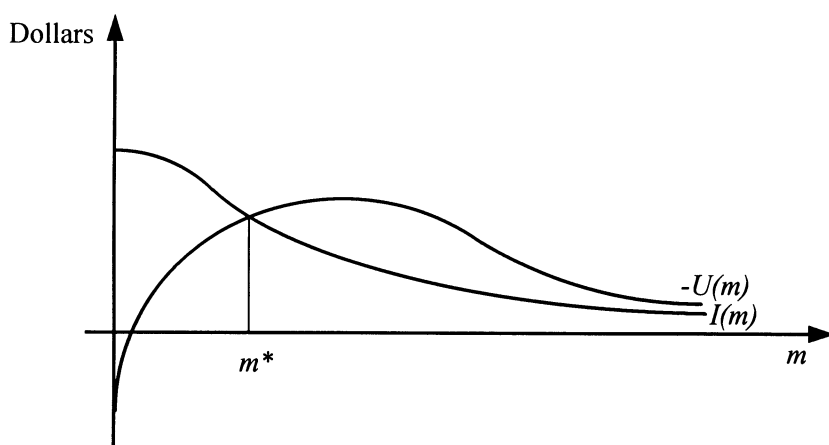
The first-order condition that specifies the optimal choice of m is

$$\mathscr{I}(m) = -\mathscr{U}(m). \tag{11}$$

Equating these is graphically represented in Figure 1 using the information in the following result.

LEMMA 3: *At the optimal P*

  (i) $\mathscr{I}(0) > 0$
  (ii) $\mathscr{I}'(m) < 0$ *for all $m > 0$.*
  (iii) $\lim_{m \to \infty} \mathscr{I}(m) = 0$.

Figure 1. Equilibrium bid-ask spread. The $\mathscr{I}(m)$ ($\mathscr{U}(m)$) schedule is the contribution of informed (uninformed liquidity) trading volume to the specialist's marginal expected profit as a function of one-half the bid-ask spread, m. The specialist's optimal choice of the bid-ask spread is the value $2m^*$, where m^* equates $\mathscr{I}$ and $\mathscr{U}$.

Given the parameters describing informed trading (i.e., for a fixed $\mathscr{I}$ schedule), existence of a finite equilibrium m depends on the shape of $\mathscr{U}(\cdot)$, which describes the responsiveness of liquidity-trading revenues to changes in the spread. In our model, an unbounded choice of m is always feasible. In the event that liquidity trading is sufficiently insensitive to changes in the spread, the specialist would increase the spread without bound because he can deter trading by the informed with little or no loss of revenue from the liquidity traders. Similarly, if the informational asymmetry between the specialist and the informed is sufficiently severe, there does not exist an equilibrium in which m is finite. To focus on instances in which markets are open, our results assume that m is finite in equilibrium.[5]

Assuming that the specialist's choice of the bid-ask spread is finite, a necessary and sufficient condition for (local) optimality is that $-\mathscr{U}'(m) > 0$ at that choice. That this is a necessary condition is easy to see from Figure 1. If $-\mathscr{U}$ were to intersect $\mathscr{I}$ from above, the specialist's expected profits at that value of m are less than the expected profits from choosing the nearest smaller value of m at which $-\mathscr{U}$ and $\mathscr{I}$ intersect. This is because the area bounded above by $-\mathscr{U}$ and below by $\mathscr{I}$ represents expected losses.[6] The sufficiency argument follows from the fact that at the optimal P, $\partial^2 E[\tilde{\pi}]/\partial m^2 = \mathscr{U}'(m) + \mathscr{I}'(m)$, and Lemma 3, Part (ii).

[5]See Glosten (1989) and Madhavan (1992) for discussions of market structure and market closures.

[6]If $-\mathscr{U}$ intersects $\mathscr{I}$ from below at multiple points there are multiple local optima, in which case the one yielding the greatest expected profit is the global optimum. Our results only require that the specialist's choice *satisfies* the first- and second-order conditions. Thus, as long as the global optimum is finite, our results present a correct characterization of it.

II. Market Equilibrium and Trading Volume

Given the specialist's optimal choices of bid and ask prices, we characterize the relationship between the bid-ask spread and informational asymmetries in the market. Then we examine equilibrium expected volume.

A. Equilibrium Bid-Ask Spreads

Using the condition that describes the optimal spread chosen by the specialist (i.e., equation (11)), we can examine the responsiveness of the spread to changes in the informational asymmetry between the specialist and the informed agents (as measured by σ_θ, holding $\sigma_v = \sigma_\theta + \sigma_\epsilon$ and σ_η fixed). This indicates the extent of the informational advantage over the specialist possessed by the informed agents, holding constant heterogeneity among their private signals and total risk associated with the asset's payoff. We also examine the relation between the bid-ask spread and changes in the *consensus* in beliefs (as measured by the correlation between $\hat{V}_{t+1}^i$ and $\hat{V}_{t+1}^j$ for $i \neq j$, holding σ_v and $\hat{\sigma}$ fixed) among the informed agents. Changes in this parameter indicate changes in the level of agreement among the informed agents, holding constant the precision in their posterior beliefs and total risk associated with the asset's payoff.[7] An increase in the degree of asymmetric information, or a decrease in consensus, increases the size of informed agents' orders (other things equal). This shifts both the ϕ schedule and (by Lemma 2) the $\mathcal{S}$ schedule upward. Therefore, we can deduce the following result from the geometry of Figure 1.

PROPOSITION 1: *Let $m(\sigma_\theta, \rho)$ denote the specialist's optimal choice.*

  (i) *Holding σ_v and σ_η fixed, $\partial m(\sigma_\theta, \rho)/\partial\sigma_\theta > 0$.*
  (ii) *Holding σ_v and $\hat{\sigma}$ fixed, $\partial m(\sigma_\theta, \rho)/\partial\rho < 0$ where $\rho = \text{Corr}[\hat{V}_{t+1}^i, \hat{V}_{t+1}^j]$ for $i \neq j$.*

The intuition for this result is clarified by Figure 1. The greater the informational asymmetry between the specialist and the informed agents, the greater are the specialist's expected losses (for all spreads) resulting from trading in the security. The specialist anticipates these losses and protects himself by setting a wider spread.[8] Also, the less is the consensus among traders (i.e., the lower is ρ), the more likely is a randomly selected informed agent's assessment of the security's value to diverge enough from a given pair of quotes so as to generate a trade. Being a monopolist, the specialist widens

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[7] This notion of consensus, however, is another measure of differences in information; it is *not* a measure of "differences of opinion" used in Harris and Raviv (1991). Specifically, while differences of opinion can never be resolved, consensus in beliefs can be increased by the exchange of information among agents.

[8] This result is, of course, much more general than the parametric setting of our model. It also holds when the specialist earns zero expected profits and quotes quantity-contingent prices (see Glosten and Milgrom (1985)).

the spread to profit from this increased propensity of agents to trade. This is consistent with the evidence of Chiang and Venkatesh (1988) who find a significantly positive relation between average spreads and insider holdings.
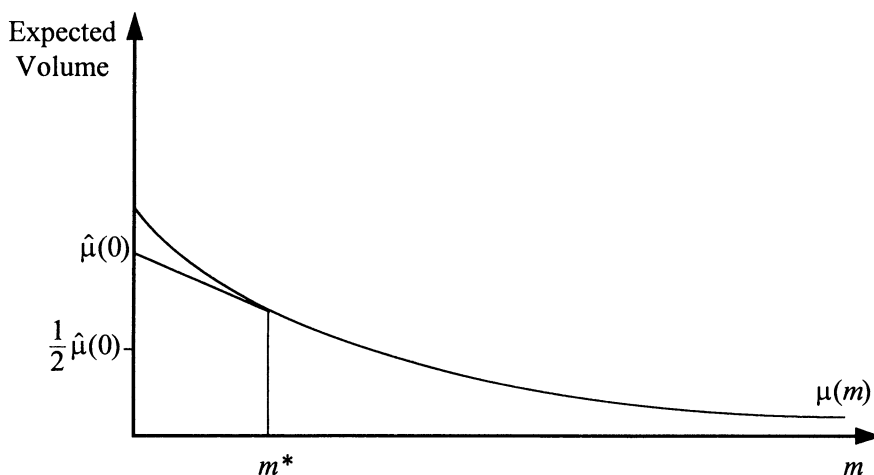
B. Equilibrium Expected Volume

Using Lemma 2 and the optimality condition for the choice of the bid-ask spread, we obtain the following result that characterizes the equilibrium relationship between expected volume and the bid-ask spread, which are both endogenous in our model.

PROPOSITION 2: *Equilibrium expected volume is*

$$\overline{\text{Vol}}(P,m) = \frac{1}{2}[\,\mu(m) - m\mu'(m)\,].$$

Proposition 2 is important for three reasons. First, it shows that expected (total) volume of trading is primarily determined by liquidity trading and transaction costs and depends on informed trading only *indirectly* through the specialist's choice of the bid-ask spread. This is because the specialist chooses the bid-ask spread to elicit the informed volume that maximizes his profits, given the schedule of expected liquidity trading. Second, it shows how equilibrium expected volume is related to the geometry of Figure 1. Given the schedule describing expected liquidity trading, and the specialist's optimal choice of the bid-ask spread, equilibrium expected volume is half of the value of the linear approximation of $\mu(0)$ around the optimal m (see Figure 2). The

Figure 2. Equilibrium expected volume. The schedule $\mu(m)$ is the expected volume of uninformed liquidity trading as a function of one-half the bid-ask spread, m, and $2m^*$ is the equilibrium bid-ask spread. By Proposition 2, the equilibrium expected (total) volume is $\frac{1}{2}\hat{\mu}(0)$, where $\hat{\mu}(0)$ is a linear approximation of $\mu(0)$ around m^*.

determination of the optimal spread and equilibrium expected volume can be seen quite clearly by "stacking" Figure 1 and Figure 2. Finally, this prediction is consistent with the empirical evidence of Glosten and Harris (1988), who document that the time-series regression relationship between volume and bid-ask spreads is positive for New York Stock Exchange (NYSE) stocks.

We now present the main results of our article that relate to the impact of liquidity trading, informational asymmetries and consensus in beliefs (exogenous variables) on equilibrium expected volume. We show that trading volume in equilibrium is positively related to the level of liquidity trading, but the relationship between total volume and informed trading is *ambiguous* (volume can be either positively or negatively related to the degree of informational asymmetry or consensus in beliefs).

To examine how liquidity trading affects total volume, we consider scale differences in liquidity trading by defining expected liquidity trading volume for security h as $\mu_h(m) \equiv s_h\,\mu(m)$, where s_h is a scale factor.[9]

PROPOSITION 3: *For small differences in scale, if $s_k > s_h$, then in equilibrium*

$$m_k < m_h \quad \text{and} \quad \overline{\text{Vol}}_k(P_k, m_k) > \overline{\text{Vol}}_h(P_h, m_h).$$

Setting a wide bid-ask spread is costly to the specialist because it deters liquidity trading that is profitable to him. The greater is the scale of liquidity trading, the greater is the marginal cost to the specialist of choosing a wider spread. Consequently, the specialist optimally chooses a smaller spread when the scale of liquidity trading is large. This encourages liquidity and informed trading, implying that equilibrium expected total volume is larger when the scale of liquidity trading is large.[10] The geometry of this result is evident in Figures 1 and 2. Since $\mu_h(m) \equiv s_h\,\mu(m)$ implies that $\mathscr{U}_h(m) = s_h\mathscr{U}(m)$, increasing the scale factor shifts the μ curve upward, while shifting the $-\mathscr{U}$ curve downward where negative and upward where positive. The result is that more liquidity trading implies greater equilibrium expected volume and a narrower bid-ask spread.

PROPOSITION 4:

(i) *Holding σ_v and σ_η fixed, $d\overline{\text{Vol}}(P, m)/d\sigma_\theta < 0$, and*
(ii) *Holding σ_v and $\hat{\sigma}$ fixed, $d\overline{\text{Vol}}(P, m)/d\rho > 0$
    if and only if $\mu''(m) > 0$.*

Proposition 4 shows that the responsiveness of equilibrium expected volume to changes in the degree of asymmetric information or consensus among

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[9] In the analytical proof of this result, we consider derivative-size changes in scale to avoid imposing structure on μ. If the first-order conditions have a unique solution, this result holds for global changes in scale as well.

[10] See Admati and Pfleiderer (1989) for a similar result in a different context.

agents is generally *ambiguous* and depends on the pattern of expected liquidity trading. If the informational asymmetry increases or consensus decreases, the informed agents desire to trade more heavily. The specialist, however, increases the bid-ask spread (Proposition 1), which decreases the willingness of all agents to trade. Whether the former or the latter effect dominates depends on whether liquidity trading decreases at an increasing or a decreasing rate. If liquidity volume decreases at a decreasing rate, then relatively less liquidity trading is deterred due to an increase in the spread. This makes the specialist willing to increase the spread more in response to extreme informational asymmetries which, in turn, *decreases* equilibrium expected volume.[11] Viewing differences in exogenous parameters as characteristic of a cross-section of securities, Propositions 3 and 4 are consistent with the evidence of a negative cross-sectional relationship between average volume and average bid-ask spreads for NYSE stocks documented by Glosten and Harris (1988).[12]

The geometry of this result is as follows. Holding σ_v and σ_η fixed, increasing σ_θ increases the informational advantage of the informed. This increases the expected volume of informed trading—the ϕ schedule—which shifts the $\mathscr{I}$ schedule upward. From the way we have drawn μ (in Figure 2), and the $-\mathscr{U}$ schedule it implies, an increase in the informational asymmetry causes equilibrium expected volume to decrease. This is true if μ is a convex function of m; the opposite occurs if μ is concave in a neighborhood of the optimum. The same is true of a decrease in consensus among agents, i.e., decreasing ρ, holding σ_v and $\hat{\sigma}$ fixed.

C. Comparison to Competitive Market-Maker Structures

Models of trading with a competitive market maker based on Kyle (1985) are similar to ours because in both models an intermediary sets prices recognizing the presence of informed and uninformed traders in the market and the willingness of the informed agents to trade depends on the extent to which their beliefs about the security's value diverge from the price set by the intermediary. The models are different, however, because in our model each buyer pays the prevailing ask price and each seller receives the prevailing bid price; in Kyle's (1985) model, all trades that are executed at the same time are executed at the same price. More importantly, liquidity trading is not

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[11] In fact, a *decrease* in volume is the typical outcome in our model if liquidity trading is modeled—see footnote 3.

[12] There is some evidence to suggest that this relation holds in markets without specialists. Adopting an estimation strategy similar to the cross-sectional analysis of Glosten and Harris (1988), George and Longstaff (1993) document a negative relationship between trading frequency and bid-ask spreads for S & P 100 Index options, which are traded in open-outcry markets.

sensitive to transaction costs in Kyle (1985). Intensifying the informational advantage of the informed causes Kyle's (1985) market maker to widen the bid-ask spread for each level of net order flow. This decreases the willingness of the informed agents to trade aggressively on their private information such that the resulting equilibrium expected volume is *unchanged*.[13] In our model, however, the wider bid-ask spread *also* affects the willingness of liquidity traders to trade. Allowing liquidity trading to be cost elastic enables us to derive the broader prediction that expected volume can increase *or* decrease, depending upon the sensitivity of liquidity trading to transaction costs.

III. Conclusion

We present a microstructure model of a specialist market, and we show that the effect of informed trading on total volume is both indirect (through its effect on the specialist's choice of the bid-ask spread) and, more importantly, *ambiguous*. In fact, under reasonable conditions volume *decreases* as informational asymmetries increase and as the beliefs of agents become more divergent. These results stand in stark contrast to the predictions of existing models of volume in which transacting is costless. Existing models establish a direct relationship between volume of trading and (some measure of) information asymmetries or differences in beliefs among traders and show that this relation is unambiguously positive (i.e., volume is heavy when beliefs are heterogeneous and informational asymmetries are severe.)

Both our model and existing models of volume predict heavy volume in response to events that resolve uncertainty, like earnings announcements. However, in previous models, the source of this volume is heterogeneous beliefs among privately informed agents generated by the event. Our model, on the other hand, predicts that liquidity trading is the source of increased volume because such events decrease informational asymmetries or differences in beliefs that in turn, decrease transaction costs. Our results emphasize the importance of market structure and transaction costs for the characterization of trading volume in equilibrium and suggest an alternative interpretation for the volume effects associated with corporate and macroeconomic announcements.

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[13] The predictions are somewhat richer in models that allow the *timing* of liquidity trading to be transaction-cost-elastic (e.g., Admati and Pfleiderer (1988) and Foster and Viswanathan (1990)). In this case, equilibrium expected volume depends on the extent of asymmetric information and divergent beliefs only through their effect on liquidity trading. Both liquidity and informed trading tend to be concentrated in time. For a given number of informed agents in a trading round N (e.g., the number present in a period of concentrated trading), increasing the informational advantage of informed traders implies greater transaction costs, which could decrease liquidity trading and equilibrium expected volume in that period (depending on what happens in other periods).

Appendix

Preliminary Result

Let $\tilde{X} \sim N(\mu_x, \sigma_x)$ and let $f(\cdot)$ be the probability density function of $\tilde{X}$, then

(i) $\int_a^b (x - \mu_x) f(x)\, dx = \sigma_x[\, f(a) - f(b)]$

(ii) $\partial f(x)/\partial \sigma_x = (f(x)/2\sigma_x)[(x - \mu_x)^2/\sigma_x - 1]$

(iii) $\int_a^b \partial f(x)/\partial \sigma_x \, dr = 1/2\sigma_x[af(a) - bf(b) + \mu_x(f(b) - f(a))].$

Derivation of σ and ψ

From the viewpoint of the (uninformed) specialist, each informed agent's conditional expectation, $\hat{V}_{t+1}^i$, is drawn from a normal distribution having mean $E[\hat{V}_{t+1}^i] = V_t$ and variance $\text{Var}[\hat{V}_{t+1}^i] = \sigma_\theta$. Let $f(\cdot)$ denote the density of this normal distribution. Taking expectations and summing across agents yields

$$\phi(P_t, m_t) = \frac{1}{a\hat{\sigma}} \left\{ \int_{P_t + m_t}^{\infty} (v - P_t) f(v)\, dv - \int_{-\infty}^{P_t - m_t} (v - P_t) f(v)\, dv \right\}$$

$$- \frac{m_t}{a\hat{\sigma}} \left\{ \int_{P_t + m_t}^{\infty} f(v)\, dv + \int_{-\infty}^{P_t - m_t} f(v)\, dv \right\} \qquad \text{(A1)}$$

where $1/a \equiv \Sigma_i(1/a^i)$. To compute ψ, we note that

$$E\left[\left(\tilde{V}_{t+1} - P_t\right) x_t^i\right] = E\left[E\left[\left(\tilde{V}_{t+1} - P_t\right) x_t^i \mid \theta_{t+1} + \eta_{t+1}^i\right]\right] = E\left[\left(\hat{V}_{t+1}^i - P_t\right) x_t^i\right].$$

Computing this expectation and summing across agents yields

$$\psi(P_t, m_t) = \frac{1}{a\hat{\sigma}} \left\{ \int_{P_t + m_t}^{\infty} (v - P_t)^2 f(v)\, dv + \int_{-\infty}^{P_t - m_t} (v - P_t)^2 f(v)\, dv \right\}$$

$$+ \frac{m_t}{a\hat{\sigma}} \left\{ \int_{-\infty}^{P_t - m_t} (v - P_t) f(v)\, dv - \int_{P_t + m_t}^{\infty} (v - P_t) f(v)\, dv \right\}. \qquad \text{(A2)}$$

Proof of Lemma 2: Using the expressions in Lemma 1, one can easily verify that $\phi(m) = m\phi'(m) - \psi'(m)$. Therefore, $\mathscr{A}(m) \equiv \phi(m) + m\phi'(m) - \psi'(m) = 2\phi(m)$.

Proof of Lemma 3: Parts (i) and (ii): by Lemma 2, $\mathscr{A}(m) = 2\phi(m)$. Therefore, $\mathscr{A}(0) > 0$, and $\mathscr{A}'(m) < 0$ for all $m > 0$.

Part (iii):

$$\lim_{m \to \infty} \mathscr{A}(m) = 2 \lim_{m \to \infty} \phi(m) = \frac{4}{a\hat{\sigma}} \lim_{m \to \infty} \int_m^{\infty} z f_\theta(z)\, dz - \frac{4}{a\hat{\sigma}} \lim_{m \to \infty} m \int_m^{\infty} f_\theta(z)\, dz$$

provided each of the limits exists. The first limit in the last term is clearly zero. The second is also zero as can be shown by applying L'Hospital's rule

$$\lim_{m \to \infty} m \int_m^\infty f_\theta(z)\, dz = \lim_{m \to \infty} \frac{-f_\theta(m)}{-\dfrac{1}{m^2}} = \lim_{m \to \infty} m^2 f_\theta(m) = 0$$

The final equality follows since $f_\theta(m) \approx e^{-m^2}$ for large m.

Proof of Proposition 1: Implicit differentiation of equation (11) yields

$$\frac{\partial m(\sigma_\theta, \rho)}{\partial \sigma_\theta} = \frac{-\dfrac{\partial \mathscr{I}}{\partial \sigma_\theta}}{\mathscr{U}'(m) + \mathscr{I}'(m)}.$$

The denominator is the second-order condition, which is negative at the optimum. Thus

$$\mathrm{Sgn}\left\{ \frac{\partial m(\sigma_\theta, \rho)}{\partial \sigma_\theta} \right\} = \mathrm{Sgn}\left\{ \frac{\partial \mathscr{I}}{\partial \sigma_\theta} \right\} = \mathrm{Sgn}\left\{ \frac{\partial \phi}{\partial \sigma_\theta} \right\} > 0$$

where the second equality follows from Lemma 2. The last inequality follows from the fact that, holding σ_v and $\hat{\sigma}$ fixed, ϕ shifts up when σ_θ increases (see below). The argument for $\partial m(\sigma_\theta, \rho)/\partial \rho$ is identical.

It remains to show that $\partial \phi / \partial \sigma_\theta > 0$ holding σ_v and σ_η fixed; and $\partial \phi / \partial \rho < 0$ holding σ_v and $\hat{\sigma}$ fixed, where $\rho = \mathrm{Corr}[\hat{V}_{t+1}^i, \hat{V}_{t+1}^j]$. We first note that

$$\hat{\sigma} = \sigma_v - \frac{\sigma_\theta^2}{\sigma_\theta + \sigma_\eta} = \sigma_v - \frac{1}{\dfrac{1}{\sigma_\theta} + \dfrac{\sigma_\eta}{\sigma_\theta^2}}$$

which implies that, holding σ_v and σ_η fixed, $\hat{\sigma}$ decreases as σ_θ increases. Alternatively stated, $\hat{\sigma}^{-1}$ increases as σ_θ increases, holding σ_v and σ_η fixed. Second, we note that for $i \neq j$,

$$\rho = \frac{\mathrm{Cov}\left[\hat{V}_{t+1}^i, \hat{V}_{t+1}^j\right]}{\mathrm{Var}\left[\hat{V}_{t+1}^i\right]} = \left(\frac{\sigma_\theta}{\sigma_\theta + \sigma_\eta}\right)^2, \quad \text{or equivalently,} \quad (\sigma_\theta + \eta_\eta) = \frac{\sigma_\theta}{\rho^{1/2}}.$$

We can therefore write

$$\hat{\sigma} = \sigma_v - \rho(\sigma_\theta + \sigma_\eta) = \sigma_v - \rho^{1/2}\sigma_\theta$$

or

$$\sigma_\theta = \frac{(\sigma_v - \hat{\sigma})}{\rho^{1/2}}$$

which implies that, holding σ_v and $\hat{\sigma}$ fixed, decreasing ρ (consensus) is analytically equivalent to increasing σ_θ.

By definition,

$$\phi(m) = \left(\frac{2}{a}\right)\hat{\sigma}^{-1}\left\{\int_m^\infty z f_\theta(z)\,dz - m\int_m^\infty f_\theta(z)\,dz\right\} \equiv \left(\frac{2}{a}\right)\hat{\sigma}^{-1}\hat{\phi}.$$

where $\hat{\phi}$ depends on σ_θ but not on σ_ϵ or σ_η.

$$\frac{\partial\phi}{\partial\alpha} = \frac{2}{a}\left\{\frac{\partial\hat{\sigma}^{-1}}{\partial\alpha}\hat{\phi} + \hat{\sigma}^{-1}\frac{\partial\hat{\phi}}{\partial\alpha}\right\} \qquad \text{for } \alpha \in \{\sigma_\theta, \rho\}.$$

Holding σ_v and σ_η fixed, $\hat{\sigma}^{-1}$ increases in σ_θ. Therefore, it suffices to show that $\partial\hat{\phi}/\partial\sigma_\theta > 0$. To see this, use the Preliminary Result to write the following equations

$$\hat{\phi} \equiv \int_m^\infty z f_\theta(z)\,dz - m\int_m^\infty f_\theta(z)\,dz = \sigma_\theta f_\theta(m) - m\int_m^\infty f_\theta(z)\,dz, \qquad \text{and}$$

$$\frac{\partial\hat{\phi}}{\partial\sigma_\theta} = f_\theta(m) + \sigma_\theta\frac{\partial f_\theta(m)}{\partial\sigma_\theta} - m\int_m^\infty \frac{\partial f_\theta(z)}{\partial\sigma_\theta}\,dz$$

$$= f_\theta(m) + \sigma_\theta\frac{f_\theta(m)}{2\sigma_\theta}\left[\frac{m^2}{\sigma_\theta} - 1\right] - m\frac{1}{2\sigma_\theta}[mf_\theta(m)] = \frac{1}{2}f_\theta(m) > 0,$$

which is what we wanted to show. To prove the second statement, we note that $\hat{\sigma}$ is held fixed, so only the effect of ρ on $\hat{\phi}$ determines the sign of $\partial\phi/\partial\rho$. But increasing ρ, holding $\hat{\sigma}$ and σ_v fixed, is analytically equivalent to decreasing σ_θ; therefore $\partial\hat{\phi}/\partial\rho < 0$. This, in turn, implies that $\partial\phi/\partial\rho < 0$.

Proof of Proposition 2: From Lemma 2, the first-order condition for the choice of m is

$$\mu(m) + m\mu'(m) + 2\phi(P, m) = 0.$$

By definition, expected (total) volume is

$$\overline{\mathrm{Vol}}(P, m) \equiv \mu(m) + \phi(P, m)$$

Substituting for $\phi(P, m)$ from the first-order condition yields

$$\overline{\mathrm{Vol}}(P, m) = \mu(m) - \frac{1}{2}(\mu(m) + m\mu'(m)) = \frac{1}{2}[\mu(m) - m\mu'(m)].$$

as desired.

Proof of Proposition 3: In this proof we use subscripted symbols to distinguish optimal choices from arbitrary values. We define $m_k \equiv m_h + m'(s_h)$, where $m'(s_h)$ is the result of a small change from s_h to s_k. By definition

In equilibrium $-\mathscr{U}_h(m_h) = \mathscr{I}(m_h)$, or, viewing m_h as a function of s_h,

$$-s_h \mathscr{U}(m(s_h)) = \mathscr{I}(m(s_h)).$$

Differentiating yields

$$m'(s_h) = \frac{-\mathscr{U}(m_h)}{\mathscr{I}'(m_h) + \mathscr{U}'(m_h)} < 0$$

since $\mathscr{U}(m_h) < 0$, from which we conclude that

$$m_k \equiv m_h + m'(s_h) < m_h.$$

To prove the second part of the proposition, recall that, in equilibrium, $P_h = P_k \equiv P$

$$\overline{\text{Vol}}(m) \equiv \overline{\text{Vol}}(P, m) = \mu(m) + \phi(m) \qquad \text{for all } m > 0.$$

Therefore

$$\overline{\text{Vol}}'(m) < 0 \qquad \text{for all } m > 0.$$

Now, $s_k > s_h$ implies that $\mu_k(m) > \mu_h(m)$ and so

$$\overline{\text{Vol}}_k(m) > \overline{\text{Vol}}_h(m).$$

But $m_k < m_h$ and since $\overline{\text{Vol}}'(m) < 0$

$$\overline{\text{Vol}}_h(P_h, m_h) = \overline{\text{Vol}}_h(m_h) < \overline{\text{Vol}}_h(m_k) < \overline{\text{Vol}}_k(m_k) = \overline{\text{Vol}}_k(P_k, m_k)$$

as desired.

Proof of Proposition 4: For $\alpha \in \{\sigma_\theta, \rho\}$, differentiate the expression for $\overline{\text{Vol}}$ given in Proposition 3, noting that $\mu(\cdot)$ depends on α only through m, to get

$$\frac{d\overline{\text{Vol}}(P, m)}{d\alpha} = -\frac{1}{2} m \cdot \mu''(m) \cdot \frac{\partial m(\sigma_\theta, \rho)}{\partial \alpha}.$$

Proposition 4 follows from the above and from Proposition 1.

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