

Does Idiosyncratic Risk Really Matter?

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ABSTRACT

Goyal and Santa-Clara (2003) find a significantly positive relation between the equal-weighted average stock volatility and the value-weighted portfolio returns on the NYSE/AMEX/Nasdaq stocks for the period of 1963:08 to 1999:12. We show that this result is driven by small stocks traded on the Nasdaq, and is in part due to a liquidity premium. In addition, their result does not hold for the extended sample of 1963:08 to 2001:12 and for the NYSE/AMEX and NYSE stocks. More importantly, we find no evidence of a significant link between the value-weighted portfolio returns and the median and value-weighted average stock volatility.

THE INTERTEMPORAL RELATION BETWEEN RISK AND RETURN has long been an important topic in asset pricing literature. Most asset pricing models postulate a positive relationship between a stock portfolio's expected returns and risk, which is often modeled by the variance or standard deviation of the portfolio's returns. However, there is no agreement about the existence of such a trade-off for stock market indices.¹ While return volatility is an intuitively appealing measure of risk, different approaches used by previous researchers suggest that no clear consensus has emerged regarding its relevance.² Whether or not investors require a larger risk premium on average for investing in a security during times when the security is more risky remains an open question.

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¹ See, for example, Baillie and DeGennaro (1990), Campbell (1987), Campbell and Hentschel (1992), Chan, Karolyi, and Stulz (1992), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993), Merton (1973, 1980), Nelson (1991), Pindyck (1984), Schwert (1989), Turner, Startz, and Nelson (1989), and Whitelaw (1994).

² In this paper, “volatility” represents the variance, standard deviation, or log-variance of stock returns.

Campbell et al. (2001) (hereafter CLMX) use a disaggregated approach to study the volatility of common stocks at the market, industry, and firm levels. They use the firm-level return data to examine the volatility of the value-weighted NYSE/AMEX/Nasdaq composite index and the value-weighted average stock volatility. CLMX measure average stock risk in each month as the cross-sectional value-weighted average of the variances of all the stocks traded in that month. To obtain a consistent measure of risk, CLMX compute the market return and the average stock variance using the same market value weights. CLMX do not assess the predictive power of alternative risk measures for the excess return on the market.

Goyal and Santa-Clara (2003) (henceforth GS) investigate the predictability of stock market returns and propose a new approach to test the presence and significance of a time-series relationship between risk and return for the aggregate stock market. Their major contribution is to use average stock risk in predictive regressions. They compute average stock risk in each month as the cross-sectional equal-weighted average of the variances of all the stocks traded in that month. GS find a positive relation between the equal-weighted average stock volatility and the value-weighted portfolio returns on the NYSE/AMEX/Nasdaq stocks without including market capitalization weights in their volatility measures. Consistent with some previous studies, they show that the lagged volatility of market returns has no predictive power for the expected return on the market.

This paper finds that GS's empirical results based on the equal-weighted average stock risk are not robust across different stock portfolios and sample periods,³ and are driven in part by a liquidity premium. In addition, the predictive regressions in GS relate a value-weighted portfolio return to an equal-weighted average stock volatility. Their conclusions disappear when the more natural value-weighted measure of average stock risk or the more robust median stock volatility is used in predictive regressions. Finally, their results do not hold for measures of idiosyncratic volatility that are free of systematic volatility.

The parameter estimates in GS indicate a significantly positive relation between the equal-weighted average stock variance and the excess return on the value-weighted market index. We replicate their result for the sample period from 1963:08 to 1999:12 but show that this positive trade-off does not exist for the extended sample from 1963:08 to 2001:12. We also find no significant relation between the equal-weighted average stock volatility and the value-weighted portfolio returns on the NYSE/AMEX or NYSE stocks. In other words, the GS result is driven by small stocks traded on the Nasdaq, and holds for the 1963:08 to 1999:12 period, but disappears for the extended sample.

We hypothesize that the ability of the equal-weighted average stock variance to predict future market returns for the period from 1963:08 to 1999:12 is partly

³ GS's data include all New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and Nasdaq firms from the Center for Research in Security Prices (CRSP) for the period from July 1962 to December 1999. Their regression analyses are based on the August 1963 to December 1999 period.

driven by a liquidity premium. Amihud (2002) and Jones (2000) show that, over time, expected market excess returns are positively related to expected market illiquidity. When liquidity is poor, bid-ask spreads are high and trading is less active, which in turn leads to a greater component of spurious volatility in the equal-weighted measure. Thus, the positive relation between the equal-weighted average volatility and the excess market return might simply reflect the documented liquidity premium. To test this possibility, we construct an illiquidity measure similar to that in Amihud. After controlling for expected and unexpected market illiquidity, we indeed find that the predictive power of the equal-weighted average stock variance disappears.

The above results demonstrate that the equal-weighted average stock variance is not a robust predictor of future market returns. More importantly, we contend that it is more natural to use a value-weighted measure of idiosyncratic volatility for the following reasons. First, the value-weighted volatility measures are less affected by microstructure issues such as the bid-ask bounce problem than the equal-weighted measures. It is well known that the bid-ask bounce can inflate volatility, and this effect is most pronounced for small and illiquid stocks.

Second, the market index used by GS includes firms traded on the NYSE, AMEX, and Nasdaq. Since the firms traded on these exchanges have diverse market capitalizations, and the index is value-weighted, it seems more natural to use the same market value weights in constructing the average stock variance. Since the value-weighted market index is weighted toward large stocks, and is clearly affected more by the fluctuations in returns on large stocks, we think that the value-weighted average stock volatility provides a more natural way of examining the relation between idiosyncratic risk and return on the value-weighted market portfolio.

Third, one explanation given by GS for the impact of average stock volatility on stock returns considers equity and debt as contingent claims on the assets of the company. Following Black and Scholes (1973) and Merton (1974), GS view the firm's equity as a call option on its total assets. An increase in the assets' variance would increase the option value, and hence the value of the stocks. Since the market index used is the value-weighted average of individual stocks, the value-weighted average stock volatility more accurately captures the value of a portfolio of options.⁴

Given the above rationale, we rerun the earlier predictive regressions using the value-weighted average stock volatility measures. The results provide no evidence of a significant link between the value-weighted portfolio returns

⁴ Our objective is not to provide an exhaustive list of theoretical justifications or a rigorous theoretical model for the value-weighted average stock volatility. Although we feel that it is more natural to use a value-weighted measure of idiosyncratic volatility when predicting returns on the value-weighted market portfolio, if one finds that the average stock volatility predicts the value-weighted portfolio returns, researchers will find it interesting regardless of how the average is computed. In any case, we present strong empirical evidence that there is no robust relation between the equal-weighted volatility and the value-weighted portfolio returns, and various measures of total or idiosyncratic risk cannot explain the excess return on the market.

and various measures of the value-weighted and median stock volatility. This result holds for both sample periods and portfolios of stocks traded on the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq.

To further check the robustness of GS's findings, we introduce an alternative measure of volatility that involves a screen for size, liquidity, and price level. After excluding the smallest, least liquid, and lowest-priced stocks traded on the NYSE/AMEX/Nasdaq, we find no evidence of a significant link between any of the risk measures and the future market returns for both the shorter and the extended sample periods.

The paper is organized as follows. Section I introduces alternative risk measures. Section II presents the empirical results from time-series regressions. Section III concludes the paper.

I. Alternative Risk Measures

A. Equal-Weighted Average Stock Variance

Following GS, we calculate the monthly variance of stock i using the within-month daily return data as

$$V_{i,t} = \sum_{d=1}^{D_t} r_{i,d}^2 + 2 \sum_{d=2}^{D_t} r_{i,d} r_{i,d-1}, \quad (1)$$

where D_t is the number of trading days in month t and $r_{i,d}$ is the return of stock i on day d . The second term on the right-hand side adjusts for the autocorrelation in daily returns using the approach of French, Schwert, and Stambaugh (1987).

GS compute the average stock variance as the arithmetic average of the monthly variance of each stock's returns

$$VAR_{ew,t} = \frac{1}{N_t} \sum_{i=1}^{N_t} V_{i,t}, \quad (2)$$

where N_t is the number of stocks that exist in month t .⁵ GS use the equal-weighted average stock variance given in equation (2) to forecast the next period's excess return on the value-weighted market index.

We discussed previously that the bid-ask bounce can inflate the volatility. Note that the cross product term in equation (1) cannot completely remove the impact of the bid-ask bounce. To see this, consider the following simple example: Assume that the bid-ask prices \$5–\$5 $\frac{1}{4}$ do not change, but that closing prices bounce between the bid price and the ask price. Suppose we observe the following closing prices for four consecutive days: \$5, \$5 $\frac{1}{4}$, \$5 $\frac{1}{4}$, and \$5. The daily returns are then approximately 5%, 0%, and –5%. The return volatility is

⁵ As discussed in GS, equation (2) is not strictly speaking a variance measure because it does not demean returns before taking the expectation. However, for short holding periods, the impact of subtracting the means is trivial. Using daily data, French, Schwert, and Stambaugh (1987) and Scruggs (1998) also find that the squared mean term is irrelevant to variance calculations.

approximately 4% per day, but it should be zero. In this case, the cross-product term does not pick up the bid-ask bounce.

B. Value-Weighted Average Stock Variance

We calculate the value-weighted average stock variance using market capitalization weights

$$\text{VAR}_{vw,t} = \sum_{i=1}^{N_t} w_{i,t} V_{i,t}, \quad (3)$$

where for weights in period t , $w_{i,t}$, we use the market capitalization of firm i in period $t - 1$ and assume that the weights are constant within period t .⁶ To the extent that the bid-ask bounce problem is most pronounced for small stocks, using $\text{VAR}_{vw,t}$ can mitigate this problem because it gives greater weights to larger stocks.

C. Median Stock Variance

In addition to the equal-weighted and value-weighted average stock variance, we use the median stock variance, $\text{VAR}_{m,t}$, as an alternative measure of risk. The measure $\text{VAR}_{m,t}$ is the middle value (or average of the two middle values) of the monthly stock variances when they are ordered from the smallest to the largest. The median is a robust measure of the center of the distribution that is less sensitive to outliers than the mean. We do not consider the median stock variance to be a proper measure of risk for the value-weighted index returns, but $\text{VAR}_{m,t}$ can be viewed as a more robust measure of risk than $\text{VAR}_{ew,t}$ because it reduces the impact of outliers generated by small stocks.

D. Low-Frequency Measures of Average Stock Variance

GS also construct a low-frequency measure of average stock variance using monthly stock returns. For ease of comparison, we construct the low-frequency equal-weighted average stock variance ($\text{VAR}_{ew,t}^{LF}$) in exactly the same way as in GS

$$\text{VAR}_{ew,t}^{LF} = \frac{1}{N_t} \sum_{i=1}^{N_t} r_{i,t}^2 - \left(\frac{1}{N_t} \sum_{i=1}^{N_t} r_{i,t} \right)^2, \quad (4)$$

where N_t is the number of stocks in month t , and $r_{i,t}$ is the monthly return of stock i in month t . The first term in equation (4) is the average monthly squared return and the second term is the squared average return.

⁶The same approach is used by Campbell et al. (2001) to calculate the market-level, industry-level, and firm-level volatilities of stocks traded on the NYSE, AMEX, and Nasdaq.

We construct the low-frequency value-weighted average stock variance ($VAR_{vw,t}^{LF}$) similarly as $VAR_{ew,t}^{LF}$, except that each stock is weighted by its lagged market capitalization

$$VAR_{vw,t}^{LF} = \sum_{i=1}^{N_t} w_{i,t} r_{i,t}^2 - \left(\sum_{i=1}^{N_t} w_{i,t} r_{i,t} \right)^2. \quad (5)$$

E. Idiosyncratic Risk

The measures $VAR_{ew,t}$, $VAR_{vw,t}$, and $VAR_{m,t}$ approximate the variance of a stock by its squared return. These are alternative measures of total risk, including both systematic and idiosyncratic components. To better understand the contribution of idiosyncratic risk to the prediction of excess market returns, we assume that the return of each stock i is driven by a common factor and firm-specific shock ε_i . To be concrete, assume a single factor in the return generating equation

$$R_{i,t} - r_{f,t} = \beta_i(R_{m,t} - r_{f,t}) + \varepsilon_{i,t}, \quad (6)$$

where $R_{i,t}$ is the return on stock i , $R_{m,t}$ is the market return, $r_{f,t}$ is the risk-free rate, and $\varepsilon_{i,t}$ is the idiosyncratic return. Similar to total risk, idiosyncratic risk is measured using the equal-weighted, value-weighted, and median variance of $\varepsilon_{i,t}$.⁷

The capital asset pricing model (CAPM) implies that investors can earn r_f by investing in the risk-free asset, and that $\beta_i [E(R_m) - r_f]$ is the required risk premium for asset i . Because $E(R_m) - r_f$ is common to all assets, β_i is the only factor that is specific to the i^{th} asset in determining the expected rates of return and, hence, the required risk premium. The CAPM does not account for the component $\sigma_{\varepsilon_i}^2$, and implies that this idiosyncratic risk is irrelevant because it can be eliminated by holding a well-diversified portfolio. However, as discussed by GS, there are several asset pricing models in the literature that take idiosyncratic risk into account. Levy (1978), Merton (1987), and Malkiel and Xu (2002) introduce extensions of the CAPM in which investors, for some exogenous reasons, hold undiversified portfolios. The resulting pricing equation relates the returns of stocks to their betas with the market and their beta with respect to a market-wide measure of idiosyncratic risk.

Assuming that the idiosyncratic shocks are i.i.d. across stocks, GS show that the effect of idiosyncratic risk is diversified away in the equal-weighted portfolio variance measure, but it makes up almost 85% of the equal-weighted average stock variance. They take $VAR_{ew,t}$ as a measure of idiosyncratic risk and do not use the variance or standard deviation of $\varepsilon_{i,t}$ in their predictive regressions. In this section, we directly compute idiosyncratic volatility from the market model

⁷ Note that the total variance, $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$, can be broken down into two terms. The first term, $\beta_i^2 \sigma_m^2$, is the firm's systematic risk component, which represents the part of a stock's variance that is attributable to overall market volatility. The second term, $\sigma_{\varepsilon_i}^2$, is the firm's unsystematic risk component, which represents the part of a stock's variance that is not attributable to overall market volatility. The unsystematic risk component is related to the firm's specific volatility.

and check whether it helps explain the time-series variation in excess return on the market. Specifically, the equal-weighted average idiosyncratic variance is defined as

$$VAR_{ew,t}(\varepsilon) = \frac{1}{N_t} \sum_{i=1}^{N_t} VAR(\varepsilon_{i,t}), \quad (7)$$

where the monthly idiosyncratic variance of stock i , $VAR(\varepsilon_{i,t})$, is calculated using the within-month daily return data on $R_{i,t}$ and $R_{m,t}$.⁸ Similar to $VAR_{vw,t}$, the value-weighted average idiosyncratic variance is computed using market capitalization weights

$$VAR_{vw,t}(\varepsilon) = \sum_{i=1}^{N_t} w_{i,t} VAR(\varepsilon_{i,t}). \quad (8)$$

In addition to using $VAR_{ew,t}(\varepsilon)$ and $VAR_{vw,t}(\varepsilon)$, we use the median idiosyncratic variance, $VAR_{m,t}(\varepsilon)$, that reduces the impact of outliers on $VAR_{ew,t}(\varepsilon)$.

F. The Idiosyncratic Volatility Measure of Campbell et al. (2001)

Besides the above idiosyncratic volatility measures, we follow CLMX to construct a measure of aggregate firm-level volatility (FIRM) as follows: Let r_{jit} denote the simple excess return of firm j that belongs to industry i in month t . Let $\lambda_{j,it}$ be the portfolio weight of firm j in industry i . The firm-level variance is then given by

$$\begin{aligned} FIRM_t &= \sum \lambda_{j,it} \sum_{s \in t} \eta_{jis}^2, \quad \text{where} \\ \eta_{j,it} &= r_{j,it} - r_{it} \quad \text{and} \quad r_{it} = \sum_{j \in i} \lambda_{j,it} r_{j,it}. \end{aligned} \quad (9)$$

Basically, individual stock returns are decomposed into three components: the market component, the industry component, and the firm-specific component. The measure FIRM is simply the weighted average of firm-level variance across all firms, and can be viewed as an alternative measure of idiosyncratic risk given in equation (8). Like $VAR_{vw,t}(\varepsilon)$, FIRM is a value-weighted measure of idiosyncratic volatility. Moreover, FIRM is different from $VAR_{ew,t}$, $VAR_{vw,t}$, and $VAR_{m,t}$ in that it is free of industry and market volatility.

II. Empirical Results from Time-Series Regressions

We compute the risk measures using the CRSP data from July 1962 to December 2001. This is an extended sample of GS and corresponds to the

⁸ We use the standard market model, $R_{i,t} = \alpha_i + \beta_i R_{m,t} + \varepsilon_{i,t}$, to compute the variance of $\varepsilon_{i,t}$. Note that alternative models that exclude the intercept term from the market model, $R_{i,t} = \beta_i R_{m,t} + \varepsilon_{i,t}$, or that use the lagged market return as an additional factor, $R_{i,t} = \alpha_i + \beta_{1,i} R_{m,t} + \beta_{2,i} R_{m,t-1} + \varepsilon_{i,t}$, do not affect the average idiosyncratic risk measures. The results are robust across different measures of $VAR(\varepsilon_{i,t})$.

availability of daily stock return data in CRSP. Following GS, each month we use all the stocks that have a valid return for that month and a valid market capitalization at the end of the previous month.⁹

The existence and significance of a time-series relationship between average stock risk and the market return is determined by regressing the realized excess returns on the lagged volatility measures. The fitted value of this regression gives the expected excess return conditional on the lagged volatility. The predictive regressions with alternative measures of average stock risk are

$$r_{vw,t+1} = \alpha + \beta VAR_{ew,t} + e_{t+1} \quad (10a)$$

$$r_{vw,t+1} = \alpha + \beta VAR_{vw,t} + e_{t+1} \quad (10b)$$

$$r_{vw,t+1} = \alpha + \beta VAR_{m,t} + e_{t+1}, \quad (10c)$$

where $r_{vw,t+1}$ is the simple (not log) excess return on the value-weighted market index, and $VAR_{ew,t}$, $VAR_{vw,t}$, and $VAR_{m,t}$ are the equal-weighted, value-weighted, and median stock variances, respectively. Following GS, we also run regressions of market returns on the lagged standard deviations ($STD_{ew,t}$, $STD_{vw,t}$, and $STD_{m,t}$) and log-variances ($LNVAR_{ew,t}$, $LNVAR_{vw,t}$, and $LNVAR_{m,t}$).¹⁰

A. Forecasting Value-Weighted Portfolio Returns: NYSE/AMEX/Nasdaq

The first panel in Table I presents results from the one-month-ahead predictive regressions of $r_{vw,t+1}$ on $VAR_{ew,t}$, $STD_{ew,t}$, and $LNVAR_{ew,t}$. In order to compare our estimates with those in GS, we replicate their results for the sample period from 1963:08 to 1999:12. In their Table II, GS present the estimated slope coefficients and the Newey-West (1987) adjusted t -statistics in parentheses: 0.336 (2.57), 0.129 (2.88), and 0.011 (2.92) for the equal-weighted variance, standard deviation, and log-variance, respectively. Our Table I reports the corresponding estimates: 0.339 (2.62), 0.131 (2.91), and 0.011 (2.94). The equal-weighted average stock volatility is significantly positive in predicting market

⁹ Following GS, stocks with fewer than 5 trading days in a month are excluded from calculations for that month. Note also that if the autocorrelation of returns is less than -0.5 , then the second term in the monthly stock variance dominates and makes the total variance estimate negative. Although we do not observe this problem for the volatility of value-weighted index returns, it sometimes occurs for individual stocks. When the autocorrelation is less than -0.5 , following GS, we ignore the second term and compute the stock variance as the sum of squared returns only.

¹⁰ GS show that the time-series of the variance measures exhibit large and statistically significant kurtosis and skewness. This can potentially affect the distribution of standard errors. Like Andersen et al. (2001), they find that the square-root and log transformation of the average stock variance measures are more likely to follow a normal distribution than the variances themselves.

Table I
Forecasts of Value-Weighted Portfolio Returns
on the NYSE/AMEX/Nasdaq Stocks

This table presents results from the one-month-ahead predictive regressions of the excess value-weighted portfolio returns on the lagged total volatility measures: VAR_{ew} , STD_{ew} , and $LNVAR_{ew}$ are the equal-weighted average stock variance, standard deviation, and log-variance. VAR_{vw} , STD_{vw} , and $LNVAR_{vw}$ are the value-weighted average stock variance, standard deviation, and log-variance. VAR_m , STD_m , and $LNVAR_m$ are the median stock variance, standard deviation, and log-variance. The first three rows in each regression show the intercepts (α), the Newey-West adjusted t -statistics in parentheses, and the two-sided p -values in square brackets. The second three rows present the slope coefficients (β) on the lagged volatility measures, the Newey-West adjusted t -statistics in parentheses, and the two-sided p -values in square brackets. The last row reports the adjusted R^2 values. The regressions are run for the sample period of Goyal and Santa-Clara (1963:08 to 1999:12) and for the extended sample (1963:08 to 2001:12).

	1963:08 to 1999:12			1963:08 to 2001:12		
	VAR_{ew}	STD_{ew}	$LNVAR_{ew}$	VAR_{ew}	STD_{ew}	$LNVAR_{ew}$
Equal-Weighted Volatility						
α	-0.0043	-0.0161	0.0460	0.0003	-0.0066	0.0278
t -statistic (α)	(-1.0052)	(-2.0872)	(3.3124)	(0.0632)	(-0.7977)	(1.8842)
p -value [α]	[0.3154]	[0.0374]	[0.0010]	[0.9496]	[0.4254]	[0.0602]
β	0.3398	0.1312	0.0111	0.1430	0.0663	0.0064
t -statistic (β)	(2.6163)	(2.9194)	(2.9399)	(1.0564)	(1.3715)	(1.5259)
p -value [β]	[0.0092]	[0.0037]	[0.0035]	[0.2913]	[0.1709]	[0.1277]
Adjusted R^2	1.25%	1.32%	1.23%	0.12%	0.25%	0.26%
	1963:08 to 1999:12			1963:08 to 2001:12		
	VAR_{vw}	STD_{vw}	$LNVAR_{vw}$	VAR_{vw}	STD_{vw}	$LNVAR_{vw}$
Value-Weighted Volatility						
α	0.0063	0.0028	0.0179	0.0083	0.0105	-0.0047
t -statistic (α)	(1.6061)	(0.3099)	(0.7662)	(2.9366)	(1.6488)	(-0.2467)
p -value [α]	[0.1090]	[0.7567]	[0.4440]	[0.0035]	[0.0999]	[0.8052]
β	-0.0711	0.0317	0.0025	-0.3648	-0.0614	-0.0020
t -statistic (β)	(-0.1559)	(0.2978)	(0.5470)	(-1.4980)	(-0.8687)	(-0.5202)
p -value [β]	[0.8762]	[0.7660]	[0.5847]	[0.1348]	[0.3854]	[0.6032]
Adjusted R^2	-0.22%	-0.20%	-0.16%	0.23%	-0.04%	-0.16%
	1963:08 to 1999:12			1963:08 to 2001:12		
	VAR_m	STD_m	$LNVAR_m$	VAR_m	STD_m	$LNVAR_m$
Median Volatility						
α	0.0026	-0.0071	0.0424	0.0045	0.0001	0.0214
t -statistic (α)	(0.4673)	(-0.6591)	(1.7063)	(0.9865)	(0.0089)	(0.9147)
p -value [α]	[0.6405]	[0.5102]	[0.0887]	[0.3244]	[0.9929]	[0.3608]
β	0.2849	0.1258	0.0080	0.0169	0.0447	0.0036
t -statistic (β)	(0.5379)	(1.1304)	(1.5243)	(0.0407)	(0.4599)	(0.7362)
p -value [β]	[0.5909]	[0.2589]	[0.1282]	[0.9676]	[0.6457]	[0.4620]
Adjusted R^2	-0.08%	0.19%	0.30%	-0.22%	-0.16%	-0.10%

Table II
**Forecasts of Value-Weighted Portfolio Returns on the NYSE/AMEX,
NYSE, and Nasdaq Stocks**

This table presents results from the one-month-ahead predictive regressions of the excess value-weighted portfolio returns on the lagged total volatility measures: VAR_{ew} , VAR_{vw} , and VAR_m are the equal-weighted, value-weighted, and median stock variance. The first two rows in each regression show the intercepts (α) and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The second two rows present the slope coefficients (β) on the lagged volatility measures and the two-sided p -values in square brackets, based on the Newey-West adjusted t -statistics. The last row reports the adjusted R^2 values. The regressions are run for the sample period used by Goyal and Santa-Clara and for the extended sample.

	1963:08 to 1999:12			1963:08 to 2001:12		
	VAR_{ew}	VAR_{vw}	VAR_m	VAR_{ew}	VAR_{vw}	VAR_m
NYSE/AMEX						
α	0.0009	0.0078	0.0072	0.0019	0.0079	0.0067
p -value [α]	[0.8761]	[0.0148]	[0.0483]	[0.6998]	[0.0054]	[0.0634]
β	0.2274	-0.3164	-0.2144	0.1394	-0.3698	-0.2194
p -value [β]	[0.4208]	[0.4295]	[0.6234]	[0.5677]	[0.2150]	[0.6022]
Adjusted R^2	0.04%	-0.05%	-0.15%	-0.11%	0.09%	-0.13%
NYSE						
α	0.0037	0.0077	0.0075	0.0036	0.0078	0.0070
p -value [α]	[0.4299]	[0.0145]	[0.0223]	[0.4115]	[0.0051]	[0.0297]
β	0.1143	-0.3124	-0.2821	0.0770	-0.3686	-0.2899
p -value [β]	[0.7228]	[0.4442]	[0.5260]	[0.7910]	[0.2216]	[0.4929]
Adjusted R^2	-0.18%	-0.05%	-0.10%	-0.19%	0.09%	-0.08%
Nasdaq						
α	-0.0221	-0.0019	-0.0146	-0.0072	0.0106	0.0028
p -value [α]	[0.0087]	[0.8230]	[0.1361]	[0.5033]	[0.0848]	[0.7784]
β	0.6702	0.5892	1.4273	0.2704	-0.2219	0.1811
p -value [β]	[0.0001]	[0.2375]	[0.0210]	[0.2578]	[0.4618]	[0.7657]
Adjusted R^2	5.89%	0.83%	3.29%	0.81%	0.03%	-0.21%

returns. The slope coefficients (β) have Newey-West adjusted t -statistics in the range of 2.62–2.94, and the two-sided p -values are less than 1%.¹¹ The adjusted R^2 values are in the range of 1.23–1.32%.

¹¹ Following GS, the standard errors of the regression coefficients are calculated using the procedure of Newey and West (1987), who proposed a more general variance–covariance matrix estimator that is consistent in the presence of both heteroskedasticity and autocorrelation of the residuals. We also calculated the bootstrap p -values. They are generally very similar to Newey-West p -values and are not reported. However, the results of the bootstrap analysis are available on request.

For the extended sample period from 1963:08 to 2001:12, the slope coefficients are found to be positive, but statistically insignificant. The t -statistics are 1.06, 1.37, and 1.53 for β 's attached to $VAR_{ew,t}$, $STD_{ew,t}$, and $LNVAR_{ew,t}$, respectively. The p -values are in the range of 13–29%. The results in Table I confirm the findings of GS for their sample period from 1963:08 to 1999:12, but the positive relation between the equal-weighted average stock volatility and the excess return on the value-weighted market index does not exist for the extended sample.

The second panel in Table I reports the regression results from the value-weighted average stock variance, standard deviation, and log-variance. For both the shorter and the extended sample, there is no significant relation between the value-weighted index return and the value-weighted average stock volatility. For the sample period from 1963:08 to 1999:12, the t -statistics are -0.16, 0.30, and 0.55 for the slope coefficients on $VAR_{vw,t}$, $STD_{vw,t}$, and $LNVAR_{vw,t}$, respectively. The corresponding p -values are in the range of 58–88%. The adjusted R^2 values are all negative. The results are almost the same for the extended sample. The intertemporal relation between return and various measures of volatility is found to be negative and statistically insignificant. The t -statistics are -1.50, -0.87, and -0.52 for β 's attached to $VAR_{vw,t}$, $STD_{vw,t}$, and $LNVAR_{vw,t}$, respectively. The two-sided p -values are in the range of 13–60%. The adjusted R^2 values are negative or very close to zero. These results indicate that when predictive regressions of the value-weighted portfolio returns are run on the value-weighted average stock volatility, there is no evidence of a positive or negative link between risk and return.

The third panel in Table I displays the parameter estimates and the related statistics from the regressions of excess market returns on the median stock variance, standard deviation, and log-variance. The t -statistics and the corresponding p -values imply that $VAR_{m,t}$, $STD_{m,t}$, and $LNVAR_{m,t}$ do not help explain the time-series variation in $r_{vw,t+1}$, that is, the median stock volatility has no predictive power for the market return. For the shorter sample period, the t -statistics are 0.54, 1.13, and 1.52 for β 's attached to $VAR_{m,t}$, $STD_{m,t}$, and $LNVAR_{m,t}$, respectively. For the longer sample, the corresponding t -statistics are about 0.04, 0.46, and 0.74. These results confirm our findings from the value-weighted average stock volatility, and imply that the positive and significant relation between the equal-weighted average stock volatility and the market return is caused by the outliers generated by small stocks. We use the median stock volatility to reduce the impact of outliers on $VAR_{ew,t}$, $STD_{ew,t}$, and $LNVAR_{ew,t}$, and find no evidence of a significant link between average stock risk and the excess return on the market.¹²

¹² For the extended sample of 1963:08 to 2001:12, the sample means of the equal-weighted, value-weighted, and median stock volatility (standard deviation) are 17.06%, 9.47%, and 10.40%, respectively. Obviously the equal-weighted average stock volatility is affected more by small stocks that are mostly traded on the Nasdaq and in general have greater volatility than big stocks. We also calculated the median values, which are about 16.52%, 8.87%, and 9.97%, respectively.

B. Forecasting Value-Weighted Portfolio Returns: NYSE/AMEX, NYSE, and Nasdaq

To determine whether the results in GS are driven by small stocks traded on the Nasdaq, we run the predictive regressions of the excess market return on the lagged volatility measures for the NYSE/AMEX, NYSE, and Nasdaq stocks separately.

The first panel in Table II indicates that the equal-weighted, value-weighted, and median stock volatility cannot predict the excess return on the value-weighted NYSE/AMEX stock portfolios.¹³ For the sample period of GS and for the extended sample, the slope coefficients are found to be statistically insignificant. Although not presented in the paper, the *t*-statistics of the slope coefficients are in the range of -0.49 to 0.81 for the short sample, and in the range of -0.52 to -1.24 for the extended sample.¹⁴ The *p*-values are in the range of 42–62% for the shorter sample, and range from 22% to 60% for the extended sample. The adjusted R^2 values are negative or very close to zero.

The second panel in Table II presents the one-month-ahead predictive regression results for the value-weighted portfolios returns on the NYSE stocks. None of the slope coefficients are significant for the shorter or extended sample. The results provide no evidence for a significant link between any of the risk measures and the expected excess return on the NYSE stocks. We conclude that the positive and statistically significant relation between the equal-weighted average stock volatility and the value-weighted NYSE/AMEX/Nasdaq index returns for the shorter sample is driven by small stocks traded on the Nasdaq.

The third panel in Table II shows that the equal-weighted average stock risk is significantly positive in predicting the value-weighted portfolio returns on the Nasdaq stocks for the shorter sample. The *t*-statistic of the slope coefficient on $VAR_{ew,t}$ is about 3.93 with a *p*-value of 0.01%. However, the positive relation between the equal-weighted average stock volatility and the excess return on the Nasdaq does not exist for the extended sample: The *t*-statistic of β is 1.13 with a *p*-value of 26%.

Note that the third panel in Table II shows no evidence of a positive or negative link between the value-weighted average stock volatility $VAR_{vw,t}$ and the excess return on the value-weighted Nasdaq stock portfolios. For both sample periods, the slope coefficients are found to be statistically insignificant, and the adjusted R^2 values are very close to zero. Similar to our findings from the equal-weighted average stock risk, for the shorter sample there is a positive and statistically significant relation between the median stock volatility $VAR_{m,t}$ and the value-weighted portfolio returns on the Nasdaq stocks. The *t*-statistic of β is about 2.32, and the adjusted R^2 is above 3%. However, for the extended

¹³ The empirical findings from the median, equal-weighted, and value-weighted average standard deviation and log-variance turn out to be very similar to those reported in Table II. To save space, we do not present results based on STD_{ew} , STD_{vw} , STD_m , $LNVAR_{ew}$, $LNVAR_{vw}$, and $LNVAR_m$ for the NYSE/AMEX, NYSE, and Nasdaq stock portfolios. They are available on request.

¹⁴ To save space in our tables, we present the two-sided *p*-values instead of both the *p*-values and the Newey-West adjusted *t*-statistics.

sample, the relation between the median stock risk and the excess return on the Nasdaq is flat. The t -statistic of β is 0.30 with a p -value of 77%.

C. Forecasting Value-Weighted Portfolio Returns: Using Control Variables

Following GS, we consider five control variables known to predict market returns: lagged excess market returns ($VWRET_{t-1}$), dividend yields (DP_{t-1}), relative T-bill rates ($RREL_{t-1}$), default spreads (DEF_{t-1}), and term spreads ($TERM_{t-1}$).¹⁵ In this section, we examine whether the average stock variance measured by $VAR_{ew,t}$ and $VAR_{vw,t}$ can forecast future excess market return after controlling for $VWRET_{t-1}$, DP_{t-1} , $RREL_{t-1}$, DEF_{t-1} , and $TERM_{t-1}$. As in GS, the dependent variable is the one-month-ahead CRSP value-weighted index excess return.

Panel A of Table III reports the regression results for the equal-weighted average stock variance for the sample period from 1963:08 to 1999:12. As discussed earlier, when used alone for the shorter sample, $VAR_{ew,t}$ is a significant predictor of future market excess returns. The measure $VAR_{ew,t}$ remains significant after controlling for the lagged market excess returns, dividend yields, and term spreads. When default spreads or relative T-bill rates are included in the regression, although $VAR_{ew,t}$ is still positively related to the future market excess returns, it is no longer statistically significant even for the shorter sample. For the extended sample, $VAR_{ew,t}$ is not a significant predictor of market returns with or without the macroeconomic variables.

Panel B of Table III shows that $VAR_{vw,t}$ is not significantly related to the future market excess returns, whether used alone or together with other predictive variables. In fact, $VAR_{vw,t}$ is negatively correlated with the market excess return in four out of six regressions. These results are in stark contrast to those of $VAR_{ew,t}$.

D. Idiosyncratic Risk and Value-Weighted Portfolio Returns: NYSE/AMEX/Nasdaq

We have so far provided evidence about the relation between total risk measures and the excess return on the market. We now explore the link between average idiosyncratic risk and the value-weighted portfolio returns on the NYSE/AMEX/Nasdaq stocks.

The first panel in Table IV presents results from the one-month-ahead predictive regressions of $r_{vw,t+1}$ on $VAR_{ew,t}(\varepsilon)$, $STD_{ew,t}(\varepsilon)$, and $LNVAR_{ew,t}(\varepsilon)$. For the sample period from 1963:08 to 1999:12, we obtain similar results to those reported in Table I. The equal-weighted average idiosyncratic volatility is significantly positive in predicting market returns. Although not presented in Table IV, the slope coefficients have t -statistics in the range of 3.00–3.37, and

¹⁵ We obtain dividend yields and earnings yields of the S&P 500 index from Robert Shiller's website: <http://aida.econ.yale.edu/~shiller/>. Daily 3-month T-bill rates, 10-year Treasury bond yields, and Moody's Baa corporate bond yields are from the Federal Reserve Bank of St. Louis.

Table III
Forecasting Value-Weighted Portfolio Returns Using
Five Control Variables

The sample period is from 1963:08 to 1999:12. All variables are monthly. VAR_{ew} and VAR_{vw} are the equal-weighted and value-weighted average stock variance. $VWRET$ is the CRSP value-weighted index excess return. DP is the dividend yields of S&P 500 index. $RREL$ is the relative 3-month T-bill rate calculated as the difference between the current T-bill rate and its 12-month backward moving average. DEF is the default yield calculated as the difference between Moody's Baa corporate bond yields and 10-year Treasury bond yields. $TERM$ is the term premium calculated as the difference between 10-year Treasury bond yields and three-month T-bill rates. The dependent variable is the one-month-ahead CRSP value-weighted excess returns. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *p*-values.

Panel A. Equal-Weighted Average Stock Variance							
Constant	$VWRET_{t-1}$	$VAR_{ew,t-1}$	DP_{t-1}	DEF_{t-1}	$TERM_{t-1}$	$RREL_{t-1}$	$Adj R^2$
-0.4257	0.0459	0.3310					1.23%
[0.2915]	[0.4183]	[0.0085]					
-1.4722	0.0481	0.3817	0.2582				1.35%
[0.0832]	[0.4044]	[0.0038]	[0.2041]				
-1.4338	0.0280	0.2173		0.7631			2.03%
[0.0223]	[0.6209]	[0.1188]		[0.0410]			
-0.6397	0.0384	0.2859			0.2535		1.52%
[0.1672]	[0.5018]	[0.0322]			[0.1392]		
-0.1719	0.0155	0.2542				-0.5767	3.36%
[0.6915]	[0.7869]	[0.0590]				[0.0047]	
-1.1617	0.0152	0.2580	0.1519	0.2685	-0.0186	-0.4970	2.31%
[0.1992]	[0.7932]	[0.0635]	[0.4314]	[0.4638]	[0.9351]	[0.0669]	

Panel B. Value-Weighted Average Stock Variance							
Constant	$VWRET_{t-1}$	$VAR_{vw,t-1}$	DP_{t-1}	DEF_{t-1}	$TERM_{t-1}$	$RREL_{t-1}$	$Adj R^2$
0.5346	0.0541	0.0018					-0.17%
[0.1466]	[0.2634]	[0.9966]					
0.2121	0.0555	0.0098	0.0906				-0.35%
[0.7792]	[0.2522]	[0.9812]	[0.6688]				
-1.0463	0.0128	-0.4100		1.1098			1.79%
[0.0825]	[0.7899]	[0.2548]		[0.0027]			
0.0860	0.0429	-0.0009			0.3310		0.52%
[0.8646]	[0.3758]	[0.9985]			[0.0477]		
0.6779	0.0135	-0.1197				-0.6667	1.92%
[0.0681]	[0.7858]	[0.7638]				[0.0009]	
-0.2575	-0.0014	-0.3613	-0.0351	0.7419	-0.0294	-0.4658	1.91%
[0.7637]	[0.9770]	[0.3308]	[0.8626]	[0.0553]	[0.8997]	[0.0843]	

the *p*-values are less than 1%. The adjusted R^2 values are in the range of 1.30–1.61%. For the extended sample, the slope coefficients are found to be positive, but statistically insignificant. The *t*-statistics are 1.34, 1.55, and 1.61 for β 's attached to $VAR_{ew,t}(\varepsilon)$, $STD_{ew,t}(\varepsilon)$, and $LNVAR_{ew,t}(\varepsilon)$, respectively. The *p*-values are in the range of 11–18%. That is, the positive relation between the equal-weighted average idiosyncratic risk and the excess return on the value-weighted market index does not exist for the extended sample.

Table IV
Forecasts of Value-Weighted Portfolio Returns with Idiosyncratic Risk for NYSE/AMEX/Nasdaq

This table presents results from the one-month-ahead predictive regressions of the excess value-weighted portfolio returns on the lagged idiosyncratic volatility measures: $VAR_{ew}(\varepsilon)$, $STD_{ew}(\varepsilon)$, and $LNVAR_{ew}(\varepsilon)$ are the equal-weighted average idiosyncratic variance, standard deviation, and log-variance. $VAR_{vw}(\varepsilon)$, $STD_{vw}(\varepsilon)$, and $LNVAR_{vw}(\varepsilon)$ are the value-weighted average idiosyncratic variance, standard deviation, and log-variance. $VAR_m(\varepsilon)$, $STD_m(\varepsilon)$, and $LNVAR_m(\varepsilon)$ are the median idiosyncratic variance, standard deviation, and log-variance. The first two rows in each regression show the intercepts (α) and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The second two rows present the slope coefficients (β) on the lagged volatility measures and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The last row reports the adjusted R^2 values. The regressions are run for the sample used by Goyal and Santa-Clara and for the extended sample.

1963:08 to 1999:12			1963:08 to 2001:12		
$VAR_{ew}(\varepsilon)$	$STD_{ew}(\varepsilon)$	$LNVAR_{ew}(\varepsilon)$	$VAR_{ew}(\varepsilon)$	$STD_{ew}(\varepsilon)$	$LNVAR_{ew}(\varepsilon)$
Equal-Weighted Volatility					
α	-0.0054	-0.0171	0.0472	-0.0008	-0.0078
p -value [α]	[0.1749]	[0.0228]	[0.0008]	[0.8566]	[0.3384]
β	0.4201	0.1444	0.0111	0.1971	0.0776
p -value [β]	[0.0008]	[0.0014]	[0.0029]	[0.1819]	[0.1213]
Adjusted R^2	1.61%	1.50%	1.30%	0.28%	0.36%
1963:08–1999:12			1963:08–2001:12		
$VAR_{vw}(\varepsilon)$	$STD_{vw}(\varepsilon)$	$LNVAR_{vw}(\varepsilon)$	$VAR_{vw}(\varepsilon)$	$STD_{vw}(\varepsilon)$	$LNVAR_{vw}(\varepsilon)$
Value-Weighted Volatility					
α	0.0029	-0.0007	0.0199	0.0086	0.0114
p -value [α]	[0.5524]	[0.9423]	[0.4394]	[0.0103]	[0.1175]
β	0.4226	0.0804	0.0028	-0.5339	-0.0809
p -value [β]	[0.5915]	[0.5305]	[0.5657]	[0.2007]	[0.3761]
Adjusted R^2	-0.14%	-0.14%	-0.16%	0.12%	-0.05%
1963:08–1999:12			1963:08–2001:12		
$VAR_m(\varepsilon)$	$STD_m(\varepsilon)$	$LNVAR_m(\varepsilon)$	$VAR_m(\varepsilon)$	$STD_m(\varepsilon)$	$LNVAR_m(\varepsilon)$
Median Volatility					
α	-0.0006	-0.0107	0.0469	0.0034	-0.0009
p -value [α]	[0.9163]	[0.3158]	[0.0705]	[0.5048]	[0.9268]
β	0.6739	0.1736	0.0087	0.1374	0.0587
p -value [β]	[0.2935]	[0.1401]	[0.1343]	[0.7977]	[0.5922]
Adjusted R^2	0.19%	0.34%	0.34%	-0.20%	-0.14%
					-0.11%

The second panel in Table IV reports the regression results from the value-weighted average idiosyncratic variance, standard deviation, and log-variance. Based on the t -statistics and the corresponding p -values, there is no significant relation between $r_{vw,t+1}$ and $VAR_{vw,t}(\varepsilon)$, $STD_{vw,t}(\varepsilon)$, and $LNVAR_{vw,t}(\varepsilon)$ for the shorter and for the extended sample.

The third panel in Table IV displays the parameter estimates and the related statistics from the regressions of excess market return on the median idiosyncratic variance, standard deviation, and log-variance. The *t*-statistics and the *p*-values imply that $VAR_{m,t}(\varepsilon)$, $STD_{m,t}(\varepsilon)$, and $LNVAR_{m,t}(\varepsilon)$ cannot explain the time-series variation in $r_{vw,t+1}$, that is, the median idiosyncratic volatility has no predictive power for the market return. These results support our earlier findings from the value-weighted average stock volatility, and imply that the positive and significant relation between the equal-weighted average idiosyncratic volatility and the market return is attributed to small stocks. We use the median idiosyncratic volatility to reduce the impact of outliers, and find no evidence of a significant link between average idiosyncratic risk and the excess return on the market.

Similar to $VAR_{vw,t}(\varepsilon)$, FIRM, CLMX's aggregate idiosyncratic volatility measure, is value-weighted across stocks, and is free of systematic volatility. Table V examines whether FIRM can forecast the future excess market returns for the sample period from 1963:08 to 1999:12. The dependent variable is again the one-month-ahead CRSP value-weighted index excess return. The control variables are exactly the same as in Section II.C. The measure FIRM does not forecast the excess return on the market, whether used alone in the

Table V
Forecasting Value-Weighted Portfolio Returns Using Firm-Level Volatility (FIRM)

The sample period is from 1963:08 to 1999:12. All variables are monthly. FIRM is the firm-level volatility measure constructed by Campbell et al. (2001). VWRET is the CRSP value-weighted index excess return. DP is the dividend yields of S&P 500 index. RREL is the relative 3-month T-bill rate calculated as the difference between the current T-bill rate and its 12-month backward moving average. DEF is the default yield calculated as the difference between Moody's Baa corporate bond yields and 10-year Treasury bond yields. TERM is the term premium calculated as the difference between 10-year Treasury bond yields and 3-month T-bill rates. The dependent variable is the one-month-ahead CRSP value-weighted excess returns. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *p*-values.

Constant	FIRM _{t-1}	VWRET _{t-1}	DP _{t-1}	DEF _{t-1}	TERM _{t-1}	RREL _{t-1}	Adj R ²
0.1363	0.7085						-0.02%
[0.7936]	[0.4249]						
0.0887	0.7369	0.0556					0.06%
[0.8507]	[0.3563]	[0.2932]					
-0.4475	0.8383	0.0576	0.1363				-0.07%
[0.6105]	[0.3098]	[0.2797]	[0.5094]				
-1.1047	-0.2008	0.0263		1.0051			1.52%
[0.0740]	[0.8209]	[0.6100]		[0.0100]			
-0.3267	0.6897	0.0445			0.3260		0.72%
[0.5819]	[0.4227]	[0.4124]			[0.0537]		
0.2607	0.5101	0.0191				-0.6444	2.00%
[0.5956]	[0.5294]	[0.7271]				[0.0011]	
-0.4593	0.0535	0.0124	0.0032	0.5549	0.0028	-0.4762	1.70%
[0.6121]	[0.9547]	[0.8155]	[0.9873]	[0.1949]	[0.9905]	[0.0760]	

regression or together with other predictive variables. The coefficients of FIRM are mostly positive, but none of the *p*-values is less than 10%.

E. Forecasting Value-Weighted Portfolio Returns: Controlling for Market Liquidity

Amihud (2002) and Jones (2000) show that high (low) market liquidity predicts low (high) future market excess return at monthly and yearly horizons. We hypothesize that the predictive power of equal-weighted measures of idiosyncratic volatility regarding future market excess returns is driven by a liquidity premium. The basic idea is as follows: When liquidity is poor, the bid-ask spread is high and trading is less active, which in turn leads to a bigger component of spurious volatility in $VAR_{ew,t}$ caused by the bid-ask bounce. Since low liquidity predicts high future market return, high $VAR_{ew,t}$ also predicts high market return.

To investigate whether the predictive power of $VAR_{ew,t}$ for the shorter sample is attributable to a liquidity premium, we first construct a market illiquidity measure similar to that in Amihud (2002). Specifically, for each NYSE stock in each month, we construct an illiquidity measure as follows:

$$ILLIQ_{i,m} = \frac{1}{D_{i,m}} \sum_1^{D_{i,m}} \frac{|R_{i,d}|}{VOLD_{i,d}}, \quad (11)$$

where $D_{i,m}$ is the number of trading days for stock i in month m , $|R_{i,d}|$ is the absolute return for stock i on day d , and $VOLD_{i,d}$ is the dollar trading volume of stock i on day d .¹⁶ Then we aggregate the illiquidity measure across all stocks for each month

$$ILLIQ_m = \frac{1}{N_m} \sum_1^{N_m} ILLIQ_{i,m}. \quad (12)$$

To obtain the expected and unexpected market liquidity, we follow Amihud to run the following autoregressive model:

$$\begin{aligned} \ln(ILLIQ_m) &= -0.036 + 0.973 \ln(ILLIQ_{m-1}) + \text{residual}, \\ (t=) &\quad (-2.32) \quad (88.82) \end{aligned} \quad R^2 = 0.95. \quad (13)$$

The first two terms of the right-hand side of the above equation give the expected illiquidity and the last term gives the unexpected illiquidity. According to the liquidity hypothesis, the expected stock returns are positively related to expected illiquidity and negatively related to unexpected illiquidity (see Amihud (2002)).

¹⁶ This ratio gives the absolute percentage price change per dollar of monthly trading volume. As discussed in Amihud (2002), $ILLIQ_{i,t}$ follows Kyle's (1985) concept of illiquidity, that is, the response of price to the associated order flow or trading volume. The measure of stock illiquidity can be interpreted as the price response associated with one dollar of trading volume, thus serving as a rough measure of price impact.

Table VI presents the predictive regression results for the shorter sample when controlling for the expected market illiquidity and the unexpected market illiquidity. Basically, the coefficients on $VAR_{ew,t}$ are no longer statistically significant once we control for the liquidity variables. Specifically, all the *p*-values of $VAR_{ew,t}$ are above 10%. This result suggests that the predictive power of $VAR_{ew,t}$ is at least in part attributable to a liquidity premium.

F. Alternative Volatility Measures with a Screen for Size, Liquidity, and Price

Given that all stocks traded on the NYSE/AMEX/Nasdaq are included in GS's equal-weighted average stock volatility, their construction of the risk measure puts a very large relative weight on the small stocks and little weight on the larger stocks. However, the value-weighted market portfolio return is largely determined by the large stocks. Therefore, including small and illiquid stocks in the computation of the idiosyncratic volatility measure does not provide a natural way of testing for the presence of a significant relation between idiosyncratic risk and future market returns. In addition, using realized volatility measures for small stocks with low prices and large bid-ask spreads may incorporate a lot of microstructure noise in the volatility measure and potentially inflate the underlying volatility. This creates an obvious problem for the equal-weighted average stock volatility, but will matter less for the value-weighted measure.

To check further the robustness of GS's findings, we introduce an alternative measure of volatility that involves a screen for size, liquidity, and price level. We exclude the smallest, least liquid, and lowest-priced stocks, and recalculate the total and idiosyncratic risk measures described in Section I. Then we run the one-month-ahead predictive regressions of the value-weighted portfolio returns on the new volatility measures. Our screening process for size, liquidity, and price can be explained as follows:

1. *Price.* We exclude stocks with a price less than \$5. Returns on low-price stocks are greatly affected by the minimum tick of \$1/8, which may add noise to the construction of average stock risk.¹⁷
2. *Size.* In each month, all NYSE stocks on CRSP are sorted by firm size to determine the NYSE decile breakpoints for the market capitalization. Then we exclude all NYSE/AMEX/Nasdaq stocks with market capitalizations that would place them in the smallest NYSE size decile.
3. *Liquidity.* Liquidity generally implies the ability to trade large quantities quickly, at low cost, and without inducing a large change in the price level.

¹⁷ Most exchanges require that quotes and transaction prices be stated as some multiple of a minimum price variation, or trading tick. For example, the minimum price variation on the NYSE was \$1/8 for stocks priced at and above \$1, \$1/16 for stocks under \$1 and at or above \$0.25, and \$1/32 for stocks under \$0.25. The benchmark of \$5 was used in 1992 by the NYSE when it reduced the minimum tick (i.e., when the minimum price variation rule was changed to sixteenths for stocks under \$5). Also, the conventional term of "penny stocks" applies to stocks whose price is below \$5. Harris (1994) discusses the minimum tick and its effects on market depth and trading volume. Jegadeesh and Titman (2001) exclude stocks priced below \$5 while evaluating various explanations of momentum strategies.

Table VI
Forecasting Value-Weighted Portfolio Returns after Controlling for Market Liquidity

The sample period is from 1963:08 to 1999:12. All variables are monthly. VAR_{ew} is the equal-weighted average stock variance. $VWRET$ is the CRSP value-weighted index excess return. DP is the dividend yields of the S&P 500 index. $RREL$ is the relative 3-month T-bill rate calculated as the difference between the current T-bill rate and its 12-month backward moving average. DEF is the default yield calculated as the difference between Moody's Baa corporate bond yields and 10-year Treasury bond yields. $TERM$ is the term premium calculated as the difference between 10-year Treasury bond yields and 3-month T-bill rates. $ILLIQ$ is expected market liquidity. $ILLIQ^U$ is the unexpected market liquidity. The liquidity variables are constructed as in equations (11) and (12). The dependent variable is the one-month-ahead CRSP value-weighted excess returns. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted p -values.

Constant	$VWRET_{t-1}$	$VAR_{ew,t-1}$	$ILLIQ_{t-1} \times 100$	$ILLIQ_t^U$	DP_{t-1}	DEF_{t-1}	$TERM_{t-1}$	$RREL_{t-1}$	$Adj R^2$
0.0417 [0.9194]	-0.1285 [0.0348]	0.2240 [0.1745]	0.0589 [0.8479]	-0.0824 [0.0000]					14.33%
-0.5440 [0.5650]	-0.1263 [0.0388]	0.2304 [0.1648]	-0.0169 [0.9595]	-0.0820 [0.0000]	0.1395 [0.5127]				14.22%
-0.9087 [0.1198]	-0.1446 [0.0179]	0.1146 [0.4936]	0.0475 [0.8723]	-0.0821 [0.0000]		0.7160 [0.0373]			15.04%
-0.1307 [0.7775]	-0.1335 [0.0307]	0.2048 [0.2354]	0.1243 [0.6735]	-0.0818 [0.0000]			0.2181 [0.1655]		14.50%
0.2690 [0.5371]	-0.1566 [0.0105]	0.1364 [0.4179]	0.0065 [0.9822]	-0.0821 [0.0000]				-0.5491 [0.0035]	15.66%
-0.2578 [0.7837]	-0.1600 [0.0091]	0.0985 [0.5454]	-0.0342 [0.9141]	-0.0821 [0.0000]	0.0266 [0.9005]	0.3486 [0.3016]	-0.0827 [0.6891]	-0.5060 [0.0440]	15.27%

Trading volume is a natural measure of stock liquidity. Hence, we first use trading volume in shares of stock to identify the least liquid stocks traded on the NYSE/AMEX/Nasdaq. In addition to the number of shares traded, following Amihud (2002), we measure stock illiquidity as the ratio of absolute stock return to its dollar volume.

Our screening process for volume and illiquidity is very similar to that for size. In each month, all NYSE stocks are sorted by the number of shares traded to determine the NYSE decile breakpoints for the trading volume. Then we exclude all NYSE/AMEX/Nasdaq stocks that belong to the smallest NYSE volume decile (i.e., 10% of the total sample of stocks with the lowest number of shares traded in that month).¹⁸

Table VII presents results from the more sensible volatility measures that involve a screen for size, liquidity (measured by trading volume), and price. The first panel in Table VII indicates that the equal-weighted average stock volatility cannot predict the excess return on the value-weighted NYSE/AMEX/Nasdaq stock portfolios. For the sample period in GS and for the extended sample, the slope coefficients are found to be statistically insignificant. Although not presented in the paper, the *t*-statistics of β 's are in the range of 0.36–1.30 for the shorter sample, and in the range of 0.10 to –0.55 for the extended sample. The corresponding *p*-values are between 19% and 72% for the shorter sample, and between 45% and 92% for the extended sample.

The second panel in Table VII shows that the time-series relation between the value-weighted average stock volatility and the value-weighted portfolio returns on the NYSE/AMEX/Nasdaq stocks is statistically insignificant. The *t*-statistics of the slope coefficients are in the range of 0.18–0.43 for the shorter sample and in the range of –0.64 to –1.51 for the extended sample. The third panel in Table VII provides no evidence for a significant link between the median stock volatility and the value-weighted portfolio returns. For both sample periods, the β coefficients turn out to be insignificant.¹⁹

¹⁸ The same procedure is followed for the illiquidity measure in Amihud (2002). In each month, all NYSE stocks are sorted by the ratio of absolute stock return to its dollar volume to determine the NYSE decile breakpoints for the illiquidity measure. Then we exclude all NYSE/AMEX/Nasdaq stocks that belong to the smallest NYSE liquidity decile (or the largest NYSE illiquidity decile).

¹⁹ At an earlier stage of the study, we compute the total and idiosyncratic volatility measures with a screen for size, liquidity, and price level. Since we identify stock liquidity with trading volume and the illiquidity measure of Amihud, two types of screen are used: one with size, volume, and price, and the other with size, illiquidity, and price. Table VII presents results from the average stock variance, standard deviation, and log-variance that involve a screen for size, trading volume, and price level. The results from the aforementioned total volatility measures with a screen for size, illiquidity, and price turn out to be very similar to those reported in Table VII. Note that we also calculate the average idiosyncratic variance, standard deviation, and log-variance after excluding the smallest, least liquid, and lowest-priced stocks. The qualitative results from the idiosyncratic volatility measures that involve the same type of screening process are very similar to those shown in Table VII. The full set of details about the alternative volatility measures is available on request.

Table VII
Forecasts of Value-Weighted Portfolio Returns on the
NYSE/AMEX/Nasdaq Stocks with a Screen for Size,
Liquidity, and Price

This table presents results from the one-month-ahead predictive regressions of the excess value-weighted portfolio returns on the lagged total volatility measures that involve a screen for size, liquidity (measured by trading volume), and price. Note that the smallest, least liquid, and lowest-priced NYSE/AMEX/Nasdaq stocks are excluded before running the predictive regressions. VAR_{ew} , STD_{ew} , and $LNVAR_{ew}$ are the equal-weighted average stock variance, standard deviation, and log-variance. VAR_{vw} , STD_{vw} , and $LNVAR_{vw}$ are the value-weighted average stock variance, standard deviation, and log-variance. VAR_m , STD_m , and $LNVAR_m$ are the median stock variance, standard deviation, and log-variance. The first two rows in each regression show the intercepts (α) and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The second two rows present the slope coefficients (β) on the lagged volatility measures and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The last row reports the adjusted R^2 values. The regressions are run for the sample period used by Goyal and Santa-Clara and for the extended sample.

1963:08 to 1999:12			1963:08 to 2001:12		
VAR_{ew}	STD_{ew}	$LNVAR_{ew}$	VAR_{ew}	STD_{ew}	$LNVAR_{ew}$
Equal-Weighted Volatility					
α	0.0038	-0.0041	0.0342	0.0072	0.0069
p -value [α]	[0.4610]	[0.6941]	[0.1328]	[0.0396]	[0.3738]
β	0.1287	0.0821	0.0066	-0.1481	-0.0173
p -value [β]	[0.7170]	[0.3683]	[0.1936]	[0.4509]	[0.7894]
Adjusted R^2	-0.16%	0.03%	0.15%	-0.07%	-0.20%
1963:08 to 1999:12			1963:08 to 2001:12		
VAR_{vw}	STD_{vw}	$LNVAR_{vw}$	VAR_{vw}	STD_{vw}	$LNVAR_{vw}$
Value-Weighted Volatility					
α	0.0067	0.0040	0.0152	0.0085	0.0111
p -value [α]	[0.0675]	[0.6450]	[0.5082]	[0.0020]	[0.0662]
β	-0.1236	0.0192	0.0019	-0.3702	-0.0698
p -value [β]	[0.7800]	[0.8548]	[0.6644]	[0.1310]	[0.3098]
Adjusted R^2	-0.20%	-0.22%	-0.19%	0.25%	0.01%
1963:08 to 1999:12			1963:08 to 2001:12		
VAR_m	STD_m	$LNVAR_m$	VAR_m	STD_m	$LNVAR_m$
Median Volatility					
α	0.0065	0.0030	0.0191	0.0075	0.0080
p -value [α]	[0.0939]	[0.7484]	[0.4500]	[0.0245]	[0.3161]
β	-0.1012	0.0293	0.0028	-0.3011	-0.0351
p -value [β]	[0.8274]	[0.7925]	[0.5801]	[0.3941]	[0.7006]
Adjusted R^2	-0.21%	-0.21%	-0.15%	-0.03%	-0.18%

G. Forecasting Value-Weighted Portfolio Returns: Low-Frequency Results

This section tests whether the low-frequency measures of average stock variance ($VAR_{ew,t}^{LF}$, $VAR_{vw,t}^{LF}$) can forecast the future market returns for the sample period in GS (1928:02 to 1999:12 and 1963:08 to 1999:12) and for the

Table VIII
Forecasting Value-Weighted Portfolio Returns Using Low-Frequency
Measures of Average Stock Variance

All variables are measured at monthly frequency. $\text{VAR}_{ew,t}^{LF}$ is the low-frequency equal-weighted average stock variance. $\text{VAR}_{vw,t}^{LF}$ is the low-frequency value-weighted average stock variance. The dependent variable is the one-month-ahead CRSP value-weighted excess returns. The first two rows in each regression show the intercepts (α) and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The second two rows present the slope coefficients (β) on the lagged volatility measures and the two-sided p -values in square brackets based on the Newey-West adjusted t -statistics. The last row reports the adjusted R^2 values. The regressions are run for the sample period used by Goyal and Santa-Clara (1928:02–1999:12 and 1963:08–1999:12) and for the extended sample period (1928:02–2001:12 and 1963:08–2001:12).

	1928:02 to 1999:12	1928:02 to 2001:12
Equal-Weighted Volatility		
α	0.0018	0.0025
p -value [α]	[0.5139]	[0.3766]
β	0.2568	0.1840
p -value [β]	[0.0225]	[0.1183]
Adjusted R^2	1.51%	0.83%
Value-Weighted Volatility		
α	−0.0005	0.0026
p -value [α]	[0.9017]	[0.4968]
β	1.4169	0.6589
p -value [β]	[0.0618]	[0.3426]
Adjusted R^2	1.49%	0.41%
	1963:08 to 1999:12	1963:08 to 2001:12
Equal-Weighted Volatility		
α	0.0003	0.0035
p -value [α]	[0.9348]	[0.3254]
β	0.2219	0.0457
p -value [β]	[0.0340]	[0.6932]
Adjusted R^2	0.71%	−0.16%
Value-Weighted Volatility		
α	0.0036	0.0075
p -value [α]	[0.3980]	[0.0192]
β	0.3437	−0.4081
p -value [β]	[0.6139]	[0.2945]
Adjusted R^2	−0.17%	0.05%

extended sample period (1928:02 to 2001:12 and 1963:08 to 2001:12). The results are reported in Table VIII.

The coefficient on $\text{VAR}_{ew,t}^{LF}$ is positive and statistically significant with a p -value of 2.25% for the period from 1928:02 to 1999:12. However, when we extend the sample to include 1928:02 to 2001:12, $\text{VAR}_{ew,t}^{LF}$ is no longer significantly related to future market returns. The relation between $\text{VAR}_{vw,t}^{LF}$ and the one-month-ahead market returns is not significant at the 5% level for the

period from 1928:02 to 1999:12. When we extend the sample period by 2 years, the coefficient on $VAR_{vw,t}^{LF}$ is statistically indistinguishable from zero with a p -value of 34.26%.

Similar to our findings from the high-frequency volatility measures, $VAR_{ew,t}^{LF}$ is significantly positive in predicting market returns for the period from 1963:08 to 1999:12. However, for the extended sample period from 1963:08 to 2001:12, the coefficient on $VAR_{ew,t}^{LF}$ is not statistically different from zero, with a p -value of 69.32%. For both sample periods—from 1963:08 to 1999:12 and from 1963:08 to 2001:12—there is no significant relation between the value-weighted index return and value-weighted average volatility.

III. Conclusion

GS investigate the predictability of stock market returns with alternative risk measures and find a significantly positive relation between the equal-weighted average stock volatility and the value-weighted portfolio returns on the NYSE/AMEX/Nasdaq stocks. We replicate their result for the sample period from 1963:08 to 1999:12 and show that this positive relation does not exist for the extended sample period from 1963:08 to 2001:12.

In addition, there is no significant relation between the equal-weighted average stock risk and the value-weighted portfolio returns on the NYSE/AMEX or NYSE stocks. Their result is driven by small stocks traded on the Nasdaq, and is in part driven by a liquidity premium. More importantly, we find no evidence for a significant link between the value-weighted portfolio returns and various measures of the value-weighted average and median stock volatility. This result holds for both sample periods and for portfolios of stocks traded on the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq.

The analysis based on the low-frequency measures of average stock volatility indicates that there is a significantly positive relation between the excess market return and the equal-weighted volatility for the period from 1928:02 to 1999:12, but this relation disappears when we extend the sample by 2 years. In addition, we find no significant link between the value-weighted portfolio returns and the value-weighted volatility for both the 1928:02 to 1999:12 and 1928:02 to 2001:12 periods. The results from the low-frequency measures of average stock variance confirm our findings from the high-frequency measures.

Idiosyncratic risk is approximated by GS with the equal-weighted average stock variance. In this study, we directly compute it from the market model and construct the firm-level volatility measure of Campbell et al. (2001) to determine its own contribution to the prediction of excess market returns. The results are very similar to our initial findings from the total volatility measures: The value-weighted average idiosyncratic volatility cannot explain the time-series variation in the value-weighted market returns. The equal-weighted average idiosyncratic risk is significantly positive in predicting the one-month-ahead market returns for the shorter sample, but this positive trade-off disappears for the extended sample.

Furthermore, we introduce an alternative measure of total and idiosyncratic risk that involves a screen for size, liquidity, and price level. After excluding the smallest, least liquid, and lowest-priced stocks traded on the NYSE/AMEX/Nasdaq, we find no evidence for a significant link between any of the risk measures and the future market returns for both the shorter and the extended sample periods.

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