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Underreaction, Overreaction, and Increasing Misreaction to Information in the Options Market

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ABSTRACT

This paper investigates options market reaction to changes in the instantaneous variance of the underlying asset. There are three main findings. First, options market investors underreact to individual daily changes in instantaneous variance. Second, these same investors overreact to periods of mostly increasing or mostly decreasing daily changes in instantaneous variance. Third, they tend to underreact (overreact) to current daily changes in instantaneous variance that are preceded mostly by daily changes of the opposite (same) sign. The third finding can reconcile the first two and is also consistent with well-established cognitive biases.

THE EXISTENCE AND NATURE of predictable investor misreaction to information is a central concern of financial economists. Recently, one focus of this concern has been a body of stock market studies that can be interpreted as indicating that stock market investors underreact to information over short horizons and overreact to information over long horizons (see Chapters 1 and 5 of Shleifer (2000) for a review of the stock market studies that advocates this interpretation). Some researchers have accepted this interpretation and embraced the challenge of building theoretical models that provide a unified explanation of short-horizon underreaction and long-horizon overreaction (Barberis, Shleifer, and Vishny (1998) (henceforth, BSV), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999)). Other re-

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searchers, however, have argued that the stock market studies do not warrant the conclusion that investors underreact or overreact at all (Fama (1998), Fama and French (1996)).

Although Stein (1989) documents long-horizon overreaction to information in the S&P 100 index (OEX) options market in the mid-1980s, there is currently no evidence on short-horizon underreaction in the options market. The first goal of the present paper is to contribute to the current debate over investor misreaction by investigating whether the options market shows evidence of the type of misreaction under discussion in the stock market. In particular, the first aim is to determine whether options market investors underreact to the information contained in daily changes in instantaneous variance over short horizons and whether the long-horizon overreaction documented by Stein (1989) in the OEX options market is present in the S&P 500 index (SPX) options market in a later period.

The BSV paper reconciles short-horizon underreaction and long-horizon overreaction by positing a representative investor who is subject to two well-established cognitive biases: conservatism and the representativeness heuristic. An investor who exhibits conservatism clings too strongly to prior beliefs, and, consequently, tends to underreact to individual pieces of information entering the market. An investor who displays the representativeness heuristic finds patterns in data too readily, and, as a result, tends to overreact to periods of mostly similar information. Although conservatism causes unconditional underreaction to individual pieces of information, the interaction of conservatism with the representativeness heuristic makes the investor's reaction to current information conditional on past information more complex. The investor tends to underreact to information that is preceded by a small quantity of similar information and to overreact to information that is preceded by a large quantity of similar information. This pattern of conditional misreaction to information is the mechanism that reconciles short-horizon underreaction with long-horizon overreaction in the BSV model. The second goal of this paper is to determine whether the BSV mechanism is at work in the options market. More precisely, the second aim is to test whether options market investors tend to underreact (overreact) to current daily changes in instantaneous variance that are preceded mostly by daily changes in instantaneous variance of the opposite (same) sign.

There are several reasons to believe that options market tests will be especially helpful in moving the debate forward. First, the instantaneous variance of the underlying asset can be identified as the stochastic variable of primary importance for option pricing. This stands in stark contrast to the post-CAPM situation in stock pricing where the number, identity, and relative importance of salient stochastic variables is unknown. As a result, there can be greater confidence that the model of market equilibrium employed in options market tests encompasses the proper set of variables. Second, the hypotheses that options market investors underreact to information in the short run and that they tend to underreact (overreact) to a current piece of information that is preceded by a small (large) quantity of similar information have not been examined in the options market. In the stock market, on the other hand, there have already been a large number of studies of investor misreaction—many of which use the same data. Third, the SPX options data that are employed comes from a highly liquid market dominated by professional investors. Some of the stock return results, by contrast, are driven by small capitalization, illiquid firms that are less likely to be traded by professional money managers.

The tests in this paper assume that SPX options investors subscribe to a standard stochastic variance model. The model is used to determine total daily changes in instantaneous variance and to separate these changes into expected and unexpected parts. Investors respond to the unexpected part of each daily change in instantaneous variance when they set option prices. As in Stein (1989), investors are considered to have underreacted (overreacted) to this information if they underproject (overproject) the unexpected daily change in instantaneous variance into the far future relative to its projection into the near future. This relative projection is measured from daily price changes of long-maturity and short-maturity options in conjunction with the assumed option pricing model. Using this approach, this paper provides evidence to support three main findings. First, SPX options market investors underreact to individual daily changes in instantaneous variance. Second, these same investors overreact to periods of mostly increasing or mostly decreasing daily changes in instantaneous variance. Third, they tend to underreact (overreact) to current daily changes in instantaneous variance that are preceded mostly by daily changes of the opposite (same) sign.

An alternative to misreaction as an explanation of the paper's results is that the option pricing model employed is misestimated or misspecified. Although it is impossible to rule out this alternative, it appears unlikely for a number of reasons. First, the results are robust to several different methods for choosing the parameters of the assumed option pricing model even though the different methods yield substantially different parameter estimates. Second, the results are largely unchanged when the model of market equilibrium is switched to a simpler or a more general stochastic variance model or to a model that includes jumps in the returns process. Finally, eliminating the misreaction found in this paper by employing a different model of market equilibrium would seem to require a model in which the variance process is specified to be non-Markovian in a particular way. Specifically, it would seem to require that *ceteris paribus* changes in instantaneous variance are projected more into the far future when they are preceded by more similar changes in instantaneous variance. Direct tests on the underlying SPX index, however, indicate that the instantaneous variance process does not possess this non-Markovian property.[1]

The remainder of this paper is organized as follows. Section I specifies the assumed stochastic variance model. The data and the procedure for estimating the stochastic variance model are described in Section II. Section III

[1] All of the tests in this paragraph are provided in Section V except for the final test, which is contained in the working paper version of this article and is available from the author upon request.

presents tests for short-horizon underreaction and long-horizon overreaction to information, and Section IV conducts tests for misreaction to information conditional on the quantity of past similar information. The potential of model misestimation or misspecification to provide an alternative explanation for the results is addressed in Section V. Section VI concludes. The Appendix contains some details on computing underlying index levels from futures prices.

I. Specification of the Presumed Stochastic Variance Model

A number of different option pricing models are employed as the model of market equilibrium for the paper's behavioral tests: (1) a simple stochastic variance model that produces investor expectations that are equivalent to the Black–Scholes model for at-the-money options, (2) the Heston (1993) model that incorporates a nonzero market price of variance risk, (3) the nonparametric stochastic variance model of Poteshman (1998), and (4) the stochastic variance with jumps model of Pan (2000). All of these models produce qualitatively similar results. For ease of exposition, the paper proceeds by presenting results from the Heston model. Section V, however, presents some robustness results from all of these models. The Heston model has the advantages of familiarity, relative parsimony, and comparable performance to more sophisticated alternatives. Bakshi, Cao, and Chen (1997) systematically investigate in the SPX options market the empirical gains to models that add stochastic variance, returns process jumps, and stochastic interest rates to the Black–Scholes framework. They find that the Heston model's addition of "stochastic volatility alone achieves the first-order pricing improvement" and that with respect to hedging the Heston model "outperforms all the others."

The version of Heston's model that is employed in this paper is described by the following five equations:

$$\frac{dS_t}{S_t} = \mu(S_t, V_t, t)\,dt + \sqrt{V_t}\,dW_t^S \tag{1}$$

$$dV_t = k(\theta - V_t)\,dt + \eta\sqrt{V_t}\,dW_t^V \tag{2}$$

$$\mathrm{Corr}(dW_t^S, dW_t^V) = \rho \tag{3}$$

$$\lambda(S_t, V_t, t) = \lambda V_t \tag{4}$$

$$r = \delta = \text{constant}, \tag{5}$$

where $k, \theta, \eta, \rho, \lambda, r$, and δ are constants. The time t level and instantaneous variance of the underlying asset are denoted, respectively, by S_t and V_t. The system is driven by standard Weiner processes with increments dW_t^S and dW_t^V that have correlation ρ. The market price of variance risk is represented by $\lambda(S_t, V_t, t), r$ is the riskless borrowing and lending rate, and δ is the dividend payout rate of the underlying asset. The next section first summarizes the SPX options data that are used in this paper and then describes the estimation of the Heston model from those option prices.

II. Data and Model Estimation

SPX options are studied because they constitute the most liquid European options market. These options trade at the Chicago Board Options Exchange (CBOE) with expiration dates in the three near-term months along with the following three months from the March expiration cycle (March, June, September, December). The options expire on the third Friday of the contract month. Strike price intervals are five points for near months and 25 points for far months. The minimum tick for options trading below three dollars is 1/16 and 1/8 for options trading at higher prices.

SPX bid-ask price quotes that are time-stamped to the nearest second were obtained directly from the CBOE. Data from the period June 1, 1988, through August 29, 1997, are used in this paper. Data after August 1997 are not available, and data from before June 1988 are not used because of the evidence presented in Jackwerth and Rubinstein (1996) that there is a structural break in the SPX options market data at the time of the October 1987 stock market crash. In addition, the market was considerably less liquid during its earlier years, so earlier data may not be as reliable. The options trade on the CBOE from 8:30 a.m. to 3:15 p.m. Central Standard Time. Following Bakshi et al. (1997), on each trade date, for each strike price and time to expiration, information is extracted on the last bid-ask quote reported prior to 3:00 p.m. Central Standard Time. For each of these bid-ask quotes, the bid-ask midpoint, the time to expiration, the exercise price, and the type of option (i.e., call or put) are recorded. The time-to-expiration is computed following the convention spelled out in Dumas, Fleming, and Whaley (1998). The risk-free rate of interest associated with each option is the one-month LIBOR rate on the day the option bid-ask quote is observed.

SPX options, unlike OEX options, are European and do not have a wildcard feature. These facts considerably simplify empirical work with SPX options. Nonetheless, two serious challenges remain. The first challenge is to match observations on option prices with observations on the underlying index level. Even if a quote on the underlying SPX index can be exactly matched temporally to an option price, the quoted index level will not be the proper underlying value for the option, because all 500 component stock prices will not correspond to trades that occurred at the quoted time. The second challenge is to determine the *expected* future rate of dividend payments by the stocks that compose the index until the expiration of an option. Of course, dividend rates can always be calculated after the fact from the actual dividends paid out by the SPX stocks. The *ex post* rate, however, may not match the *ex ante* expectation at the time the option is priced.

When addressing these two challenges, it is not necessary to determine the contemporaneous underlying value and dividend rate separately. It is sufficient to determine the quantity $Se^{-\delta T}$, where δ is the dividend rate paid by the index and T is the time to expiration of the option. If an accurate value for $Se^{-\delta T}$ can be determined, then the option can be priced under the assumption that the underlying value is equal to $Se^{-\delta T}$ and that the dividend rate is zero. The quantity $Se^{-\delta T}$ is determined via spot-futures parity from transactions data on SPX futures that trade on the Chicago Mercantile Exchange (CME). Data on these futures were obtained from the Futures Industry Institute, which is the official data supplier for the CME. The method used to compute $Se^{-\delta T}$ from the futures prices is described in the Appendix. In the empirical work below, it is *always* assumed that the quantity $Se^{-\delta T}$ computed from the futures prices is the level of the underlying index for an option and that the index pays out no dividends.

There are a number of possible approaches to estimating the Heston model from option prices, data on the underlying index, or both (see Chernov and Ghysels (1999) for a review). This paper estimates the model by finding parameter values that minimize the sum of squared SPX option pricing errors. This simple and intuitive method has been widely applied by both academics and practitioners (see Bakshi et al. (1997) for references).[2]

Because the model is estimated from option prices, the estimated parameters correspond to the "risk-neutral" process described by:

$$\frac{dS_t}{S_t} = (r - \delta)dt + \sqrt{V_t}\,dW_t^S \tag{6}$$

$$dV_t = k^*(\theta^* - V_t)dt + \eta\sqrt{V_t}\,dW_t^V \tag{7}$$

$$\mathrm{Corr}(dW_t^S, dW_t^V) = \rho, \tag{8}$$

where

$$k^* = k + \lambda \tag{9}$$

and

$$\theta^* = k\theta/(k + \lambda). \tag{10}$$

The estimation procedure first extracts the risk-neutral parameters k^*, θ^*, η, and ρ from the option prices and then equations (9) and (10) are inverted to derive the k and θ parameters under the statistical measure to conduct the behavioral tests. Setting λ, the market price of variance risk parameter, to any reasonable value has little impact on the test results.

[2] It will be seen in Section V that the results are robust to estimating the parameters using the more sophisticated techniques developed in Benzoni (1999), Chernov and Ghysels (2000), and Pan (2000).

The parameters of the model that need to be estimated are k^*, θ^*, η, and ρ. Over the period June 1, 1988, through August 29, 1997, there are 476 Wednesday trade dates. The parameters are estimated by minimizing over the four model parameters and the 476 Wednesday instantaneous variances (i.e., the minimization is over 480 variables) the sum of the squared dollar pricing errors of all SPX options that trade on one of those Wednesdays and that meet the following criteria:

- (C1) The time to expiration is greater than or equal to six calendar days and less than or equal to 7/12 of a year.
- (C2) The bid price for the option is greater than or equal to \$0.375 and the bid-ask spread is less than or equal to \$1.
- (C3) The Black–Scholes implied volatility is greater than 0 and less than or equal to 0.7.
- (C4) The option is out of the money (OTM). For this criterion (and only for this criterion) *at the money* is defined as the strike price being equal to the CRSP closing value for the SPX index.

These criteria are similar to those imposed in Aït-Sahalia and Lo (1998) and are designed to exclude the use of options that are illiquid (e.g., ITM options), have extreme characteristics (e.g., fewer than six calendar days to expiration), or are likely to correspond to data errors (e.g., Black–Scholes implied volatility less than zero). Table I provides descriptive statistics on the options that conform to (C1) through (C4) for the period June 1, 1988, through August 29, 1997. The resulting parameter estimates are $k^* = 1.16$, $\theta^* = 0.046$, $\eta = 0.52$, and $\rho = -0.61$. These estimates and the behavioral tests reported below are insensitive to reasonable changes in criteria (C1) through (C4). Likewise, the estimation and behavioral test results are largely unchanged if the estimation is carried out on Tuesday or Thursday rather than Wednesday options data.

III. Short-horizon Underreaction and Long-horizon Overreaction

A. Short-horizon Underreaction

To construct the necessary variables, let $\{V_t\}$, $t = 1, \ldots, T$ be a daily time series of implied instantaneous variances where T is the number of trade dates from June 1, 1988, through August 29, 1997. Each V_t is defined as the number that minimizes the sum of squared dollar pricing errors under the Heston model of the options observed on the tth trade date that conform to criteria (C1) through (C4). When performing the minimization, the values estimated for the parameters of the Heston model in the previous section are taken as given.

The unexpected change in the instantaneous variance from trade date $t-1$ to trade date t is the total change in instantaneous variance minus the expected change in instantaneous variance:

$$\Delta V_t^{\text{Unexpected}} = (V_t - V_{t-1}) - \Delta V_t^{\text{Expected}}, \tag{11}$$

Table I

Descriptive Statistics for SPX Index Options, June 1, 1988, through August 29, 1997

Sample description for daily observations on Chicago Board Options Exchange traded SPX index options for the period June 1, 1988, through August 29, 1997. The call and put options described in Panels A and B meet four conditions: (C1) The time to expiration, T, is greater than or equal to six calendar days and less than or equal to 7/12 year; (C2) The bid price for the option is greater than or equal to \$0.375, and the bid-ask spread is less than or equal to \$1; (C3) The Black–Scholes implied volatility (BSIVol) is greater than 0 and less than or equal to 0.7; (C4) The option is out of the money, which is determined by comparing the strike price, X, of the option to the Center for Research in Security Prices closing value for the SPX index on the trade date. The price information for each option on each trade date is the last bid-ask quote prior to 3:00 p.m. CST, and the option prices are defined as the bid-ask midpoint. The interest rate, r, is the one-month LIBOR rate on the day that the option is observed. The reported moneyness of an option is the strike price divided by the futures prices.

					Percentiles					
Variable	Obs.	Mean	S.D.	Min	1%	10%	50%	90%	99%	Max
Panel A: Calls										
Call price (\$)	53,644	7.038	7.146	0.406	0.438	0.875	4.813	15.875	35.008	71.250
Spread (\$)	53,644	0.410	0.231	0.063	0.063	0.125	0.375	0.750	1.000	1.000
BSIVol (%)	53,644	0.141	0.036	0.023	0.085	0.099	0.137	0.189	0.237	0.696
T (Years)	53,644	0.245	0.152	0.019	0.022	0.066	0.211	0.482	0.573	0.581
X (Ind. pnts.)	53,644	515	175	260	275	330	465	795	970	995
r (%)	53,644	0.058	0.018	0.029	0.030	0.032	0.056	0.084	0.100	0.103
Moneyness	53,644	1.054	0.041	0.975	1.004	1.013	1.044	1.105	1.206	1.330
Panel B: Puts										
Put price (\$)	94,152	5.061	4.972	0.406	0.438	0.750	3.500	11.375	23.375	55.500
Spread (\$)	94,152	0.345	0.200	0.063	0.063	0.125	0.250	0.625	1.000	1.000
BSIVol (%)	94,152	0.195	0.054	0.080	0.110	0.136	0.187	0.260	0.369	0.700
T (Years)	94,152	0.243	0.152	0.019	0.022	0.066	0.208	0.482	0.575	0.581
X (Ind. pnts.)	94,152	480	159	175	235	300	435	725	885	960
r (%)	94,152	0.056	0.017	0.029	0.030	0.031	0.055	0.083	0.099	0.103
Moneyness	94,152	0.927	0.063	0.544	0.717	0.845	0.941	0.993	1.008	1.028

where

$$\Delta V_t^{\text{Expected}} = V_{t-1} e^{-k\tau} + \theta(1 - e^{-k\tau}) - V_{t-1} \tag{12}$$

and τ is the number of years (i.e., 1/252) between trade dates. Given the specification of the instantaneous variance process in equation (2), the expression for $\Delta V_t^{\text{Expected}}$ follows from Equation (19) in Cox, Ingersoll, and Ross (1985). Once a value is chosen for λ, k and θ follow directly from equations (9) and (10) and the estimated values of k^* and θ^*. If λ is greater than zero, then investors seek variance risk, whereas if λ is less than $-k^*$, then the risk-neutral variance process becomes explosive. Hence, it seems that reasonable values for λ lie between zero and $-k^*$. All of the tests reported below set $\lambda = -k^*/2$. The results do not change substantively if λ is set to 0 or $-k^*$ instead.

A second variable called $FarMisProj_t$ measures the extent to which the unexpected change in instantaneous variance from trade $t - 1$ to trade date t is overprojected into the far future. This variable compares the price changes from trade date $t - 1$ to trade date t of short and long maturity options. There is, of course, no reason that the price changes of options with different maturities should be the same when measured in dollar terms. However, when the metric for measuring price changes is converted from dollars to implied instantaneous variances, then the price changes should be exactly the same if the option pricing model that is used to imply instantaneous variances is correctly specified and estimated.

Intuitively, when the current unexpected change in instantaneous variance is positive, it is projected too much into the far future when the current change in implied instantaneous variance from long maturity options is greater than the current change in implied instantaneous variance from short maturity options. Similarly, when the current unexpected change in instantaneous variance is negative, it is projected too much into the far future when the current change in implied instantaneous variance from long maturity options is less than the current change in implied instantaneous variance from short maturity options. These considerations motivate the following definition of $FarMisProj_t$:

$$FarMisProj_t \equiv \mathrm{sign}(\Delta V_t^{\mathrm{Unexpected}})(\Delta V_t^{\mathrm{Long}} - \Delta V_t^{\mathrm{Short}}). \tag{13}$$

In this expression, $\Delta V_t^{\mathrm{Short}}$ is the change in the instantaneous variance implied from short maturity options from trade date $t - 1$ to trade date t and $\Delta V_t^{\mathrm{Long}}$ is the change in the instantaneous variance implied from long maturity options from trade date $t - 1$ to trade date t. The $\Delta V_t^{\mathrm{Unexpected}}$ variable is still defined by equation (11). The $FarMisProj_t$ variable measures at time t the extent to which an unexpected change in instantaneous variance from trade date $t - 1$ to trade date t is misprojected too much into the far future given how much it is projected into the near future.[3]

The estimated Heston model parameters are used to compute $FarMisProj_t$. At any time t, the short maturity options are taken to be those with the shortest maturity greater than or equal to 10 calendar days that meet criteria (C2) through (C4), and the long maturity options are taken to be those that mature one month after the short maturity options and meet criteria (C2) through (C4). The instantaneous variances are defined to be the numbers that minimize the sum of squared dollar pricing errors for the options of the appropriate maturity.

The $FarMisProj_t$ variable is increasing in the extent to which the unexpected part of daily changes in instantaneous variance is misprojected into the far future. Consequently, if investors underreact to the unexpected

[3] See the working paper version of this article for a somewhat more formal justification of the definition of $FarMisProj_t$ in terms of Proposition 8H from Duffie (1996).

part of daily changes in instantaneous variance, $FarMisProj_t$ will be decreasing in $|\Delta V_t^{\text{Unexpected}}|$. Hence, to ascertain whether there is short-horizon underreaction to information in the options market, it must be determined whether $FarMisProj_t$ is decreasing in $|\Delta V_t^{\text{Unexpected}}|$.

Since the functional form of the relationship between $FarMisProj_t$ and $|\Delta V_t^{\text{Unexpected}}|$ is not known, a nonparametric test for monotonicity is appropriate. The Kendall tau test, which is effectively a nonparametric analog to a correlation coefficient, is used. The Kendall tau test statistic and its properties are detailed in Sheskin (1997). The Kendall tau statistic for $FarMisProj_t$ and $|\Delta V_t^{\text{Unexpected}}|$ is -0.1310 with a one-sided p-value of 0.0000. Because the statistic has a significantly negative value, this test suggests that options market investors underproject the unexpected part of daily changes in instantaneous variance into the far future.

A regression test for the relationship between $FarMisProj_t$ and $|\Delta V_t^{\text{Unexpected}}|$ is performed as well. The drawback of the regression test is that it imposes without justification a linear relationship between $FarMisProj_t$ and $|\Delta V_t^{\text{Unexpected}}|$. The advantage of the test, however, is that it is possible to control for serial correlation and heteroskedasticity. The following regression is estimated:

$$FarMisProj_t = \alpha + \beta|\Delta V_t^{\text{Unexpected}}| + \varepsilon_t.$$

The OLS estimate for $\hat{\beta}$ in this equation is -0.1251 with a t-statistic (adjusted for serial correlation and heteroskedasticity using the Newey–West procedure) of -1.81. The negative sign on the coefficient estimate is consistent with the underreaction hypothesis. Using a one-tailed test (because the hypothesis is one-sided), the negative coefficient is significantly different from zero at conventional levels. It should be noted, however, that the significance of the $\hat{\beta}$ coefficient is marginal.

The robustness of the underreaction evidence is checked in a number of ways. The assumption on the market price of variance risk was changed from $\lambda = -k^*/2$ to $\lambda = 0$ and $\lambda = -k^*$. Estimates for the Heston model were taken from the papers by Benzoni (1999), Chernov and Ghysels (2000), and Pan (2000), which extract the parameters from SPX data using more sophisticated econometric procedures. The sample was also divided in half temporally. The Kendall tau statistics and $\hat{\beta}$ coefficients remain negative under all of these variations in the testing procedure. The one-sided value for the Kendall tau test is always 0.0005 or smaller and the t-statistics on the $\hat{\beta}$ coefficient range from -0.93 to -2.21.[4] Taken together, the tests indicate that SPX options market investors unconditionally underreact to the information in unexpected daily changes in instantaneous variance.

[4] The t-statistic of -0.93 is for the subsample covering the June 1, 1988, through September 15, 1992, data period. Aside from this test, the t-statistics ranged from -1.39 to -2.21.

B. Long-horizon Overreaction

Stein (1989) presents evidence that in the OEX options market in the mid-1980s, the prices of long-maturity options are too high when the overall level of instantaneous variance is high and the prices of long-maturity options are too low when the overall level of instantaneous variance is low. Stein interprets these results as the options market overreacting to a long period of mostly positive or mostly negative unexpected daily changes in instantaneous variance by overprojecting into the far future extended periods of mostly similar instantaneous variance shocks.

This subsection of the paper examines whether there is similar long-horizon overreaction in the SPX options market over the period June 1, 1988, through August 29, 1997. If such long-horizon overreaction is present, then the difference between the instantaneous variance implied from long-maturity options and the instantaneous variance implied from short-maturity options will be increasing in the level of instantaneous variance. Accordingly, the hypothesis that $V_t^{\text{Long}} - V_t^{\text{Short}}$ is increasing in V_t is tested.

The Kendall tau statistic for the relationship between $V_t^{\text{Long}} - V_t^{\text{Short}}$ and V_t is 0.2687 with a one-sided p-value of 0.0000. Hence, this test provides statistically significant evidence that SPX option investors overproject long periods of mostly similar shocks to instantaneous variance into the far future. Estimating the following equation,

$$V_t^{\text{Long}} - V_t^{\text{Short}} = \alpha + \beta V_t + \varepsilon_t \tag{14}$$

yields an OLS $\hat{\beta}$ value of 0.088 with a t-statistic (adjusted for serial correlation and heteroskedasticity) of 5.83. This test also provides evidence suggesting that there is long-horizon overreaction to innovations in instantaneous variance in the options market. These two tests establish that the results of Stein (1989) extend to the SPX options market over the June 1988 through August 1997 period.[5]

[5] Stein (1989) tests for long-horizon overreaction under a stochastic variance model that assumes that the correlation between the Weiner processes is zero and the market price of variance risk is zero. The tests in his paper that make these assumptions were also conducted on the SPX options data from June 1988 through August 1997. Following Stein's notation, let i_t^n and i_t^d be times series of Black–Scholes implied volatilities from, respectively, a nearby ATM call and ATM put and a distant ATM call and ATM put. Then the coefficient from fitting an AR(1) process to the i_t^n series is $\rho = 0.838$. This implies a linear endpoint approximation for Stein's coefficient of elasticity of $\beta = 0.747$. Regressing i_t^d onto i_t^n produces a coefficient estimate of 0.939, and regressing $[(i_{t+4}{}^n - i_t^n) - 2(i_t^d - i_t^n)]$ onto i_t^n yields a coefficient estimate of -0.181. In the Stein paper $\rho = 0.906$, $\beta = 0.837$, the first regression coefficient is 0.929, and the second regression coefficient is -0.166. Because the first regression coefficient is greater than the β estimate and the second regression coefficient is less than zero, long-horizon overreaction is indicated. Thus, the evidence from the present paper's SPX options data on this set of tests for long-horizon overreaction is similar to, but a bit stronger, than that in Stein's OEX options data.

The empirical results in this section of the paper provide evidence that SPX options market investors unconditionally underreact to the unexpected part of daily changes in instantaneous variance and overreact to long periods of mostly similar innovations to instantaneous variance.

IV. Increasing Misreaction

Barberis et al. (1998) propose a behavioral model in which investors tend to underreact to information that is preceded by a small quantity of similar information and to overreact to information that is preceded by a large quantity of similar information. More precisely, in the BSV model, investor misreaction to information is increasing (along a scale that ascends from underreaction to overreaction) in the quantity of previous similar information. Barberis et al. (1998) put forward this model to reconcile short-horizon underreaction and long-horizon overreaction in the stock market. This section of the paper tests whether the BSV mechanism is present in the options market to serve as a candidate for reconciling the short-horizon options market underreaction and the long-horizon options market overreaction documented above. In particular, it tests whether the extent to which investors misproject the unexpected part of a current daily change in instantaneous variance into the far future is an increasing function of the quantity of previous similar unexpected daily changes in instantaneous variance.

A variable called $QPrevSim_t^w$ is constructed to measure, for the unexpected part of the change in instantaneous variance from trade date $t-1$ to trade date t, the quantity of similar unexpected changes in instantaneous variance over a window of the previous w trade dates. Recall that $\Delta V_t^{\text{Unexpected}}$, which is given by equation (11) measures the unexpected part of the change in instantaneous variance from trade date $t-1$ to trade date t. The $QPrevSim_t^w$ variable then is defined by

$$QPrevSim_t^w \equiv \text{sign}(\Delta V_t^{\text{Unexpected}}) \sum_{i=1}^{w} \text{sign}(\Delta V_{t-i}^{\text{Unexpected}}). \tag{15}$$

According to this definition, the $QPrevSim_t^w$ value when the current daily unexpected change in instantaneous variance is positive is equal to the number of positive minus the number of negative daily unexpected changes in instantaneous variance over the previous w trade dates. Conversely, when the current daily unexpected change in instantaneous variance is negative, the $QPrevSim_t^w$ value is equal to the number of negative minus the number of positive daily unexpected changes in instantaneous variance over the previous w trade dates.

The *FarMisProj* variable is increasing in the extent to which the unexpected part of a current change in instantaneous variance is misprojected too much into the far future relative to how much it is projected into the near future. The *QPrevSim* variable measures for a current unexpected change in instantaneous variance, the quantity of previous similar unexpected changes in instantaneous variance. Consequently, if *FarMisProj* is increasing in *QPrevSim*, then the extent to which a current unexpected change in instantaneous variance is projected too much into the far future relative to how much it is projected into the near future increases as a function of the quantity of previous similar unexpected changes in instantaneous variance. Hence, to determine whether options market investors exhibit increasing misreaction to information as a function of the quantity of previous similar information, tests are conducted to determine whether *FarMisProj* is increasing in *QPrevSim*.

Panel A of Table II contains the Kendall tau test for the monotonicity of $FarMisProj_t$ and $QPrevSim_t^w$ for w values of one through five previous trade dates. The Kendall tau statistic is positive for all five w values, which suggests an increasing relationship between the two variables. The one-sided p-values of 0.0000 indicate that it is highly improbable that the statistics would be as positive as they are by chance in the absence of a positive relationship between the two variables. This remains true when the value of w is increased at least through 10. (It should be noted, however, that the tests are not independent for different values of w.)

Panel B of Table II reports the results of the tests when the $QPrevSim_t^w$ variable is modified to take account of the magnitude as well as the sign of the previous unexpected daily changes in instantaneous variance. For Panel B, $QPrevSim_t^w$ is redefined by:

$$QPrevSim_t^{2w} \equiv \text{sign}(\Delta V_t^{\text{Unexpected}}) \sum_{i=1}^{w} \Delta V_{t-i}^{\text{Unexpected}}. \tag{16}$$

The results for this second version of the variable are much the same. All of the Kendall tau statistics are positive with one-sided p-values of 0.0000. The remaining tests in the paper use the original definition of the *QPrevSim* variable given in equation (15), because it is neutral between investors responding to unexpected changes in instantaneous volatility or unexpected changes in instantaneous variance.

The tests that have been presented assume that SPX options market participants attend to changes in instantaneous variance at a daily frequency. This is a reasonable interval, as option traders are likely to consider the level of instantaneous variance when assessing their positions after the market has closed or when preparing for the day's trading before the market opens. The working paper version of this article checks the robustness of the results presented in Table II to this assumption by changing the observation interval from one to two, three, or four trade dates. The increasing misreaction effect is present at these other observation intervals as well. In addition, tests reported in the working paper indicate that results similar to those in Table II are present in temporal subsamples of the data.

Table II

Tests for Increasing Investor Misreaction to a Current Unexpected Change in Instantaneous Variance, June 1, 1988, through August 29, 1997

The tests reported in this table make use of two variables. The $FarMisProj_t$ variable measures the extent to which the current unexpected daily change in instantaneous variance is overprojected into the far future. The $QPrevSim_t^w$ variable measures for the unexpected part of a current daily change in instantaneous variance the quantity of previous similar unexpected daily changes in instantaneous variance. The w superscript indicates the number of previous unexpected daily changes in instantaneous variance that are included when constructing $QPrevSim_t^w$. The Heston model is used as the model of market equilibrium. It is estimated by minimizing the sum of the squared dollar pricing errors of SPX options that trade on Wednesdays over the period June 1, 1988, through August 29, 1997. If $FarMisProj_t$ is increasing in $QPrevSim_t^w$, then investor misreaction to the current daily unexpected change in instantaneous variance increases as a function of the quantity of previous similar unexpected changes in instantaneous variance. To determine whether $FarMisProj_t$ is increasing in $QPrevSim_t^w$, a τ_{Kendall} nonparametric correlation coefficient along with its one-sided p-value is calculated for these two variables. Panels A and B contain the results of tests that, respectively, construct the $QPrevSim_t^w$ or $QPrevSim_t^{2w}$ variable from only the sign or both the sign and the magnitude of previous unexpected changes in instantaneous variance. The market price of variance risk is set to $\lambda = -k^*/2$. Each test covers the 2,009 trade dates from June 1, 1988, through August 27, 1997 for which the $FarMisProj_t$ and $QPrevSim_t^w$ variables can be constructed.

Window Size, w	τ_{Kendall}	p-Value (One-sided)
Panel A: $QPrevSim_t^w$ Constructed Only from Signs of Previous Unexpected Changes in Instantaneous Variance		
1	0.0866	0.0000
2	0.0871	0.0000
3	0.0894	0.0000
4	0.0958	0.0000
5	0.0753	0.0000
Panel B: $QPrevSim_t^{2w}$ Constructed from Signs and Magnitudes of Previous Unexpected Changes in Instantaneous Variance		
1	0.1091	0.0000
2	0.0938	0.0000
3	0.0949	0.0000
4	0.1109	0.0000
5	0.0873	0.0000

Next a series of regressions is run to explore further the relationship between $FarMisProj_t$ and $QPrevSim_t^w$. The first set of regressions examines the impact of controlling for the current level of instantaneous variance by estimating the following two equations:

$$FarMisProj_t = \alpha + \beta QPrevSim_t^w + \varepsilon_t \tag{17}$$

$$FarMisProj_t = \alpha + \beta QPrevSim_t^w + \gamma V_t + \varepsilon_t. \tag{18}$$

If the estimate of $\hat{\beta}$ in the second regression is significantly greater than zero, then, after controlling for the current level of instantaneous variance, there is evidence for increasing investor misreaction to information. The first regression is run to allow comparison with the previous nonparametric tests and so that the impact of controlling for the overall level of instantaneous variance can be assessed. The results of estimating these equations are presented in Table III. The $\hat{\beta}$ estimates reported in Panel A of Table III are positive for all window sizes regardless of whether there is a control in the regression for the overall level of instantaneous variance. The t-statistics, which are adjusted for serial correlation and heteroskedasticity, range from 4.77 to 5.99, and, therefore, indicate that the positive $\hat{\beta}$ estimates are highly significant. Furthermore, the size and significance of the $\hat{\beta}$ estimates are virtually unchanged by the inclusion of the control for the level of instantaneous variance. Like the nonparametric tests, these tests provide evidence that SPX index options market investors' misreaction to information increases as a function of previous similar information.

A final pair of regressions control for the current unexpected shock to instantaneous variance. This can be thought of as controlling for underreaction or overreaction to the current shock. The two regressions are specified as follows:

$$FarMisProj_t = \alpha + \beta QPrevSim_t^w + \delta|\Delta V_t^{\text{Unexpected}}| + \varepsilon_t \tag{19}$$

$$FarMisProj_t = \alpha + \beta QPrevSim_t^w + \gamma V_t + \delta|\Delta V_t^{\text{Unexpected}}| + \varepsilon_t. \tag{20}$$

The results of estimating these two equations are also presented in Table III. The $\hat{\beta}$ estimates are positive with t-statistics close to five for all window sizes of both regressions. Accordingly, after controlling for the current unexpected change to instantaneous variance, there is still statistically significant evidence of increasing investor misreaction to unexpected changes in instantaneous variance. The negative $\hat{\delta}$ estimates in Panel C are consistent with the short-horizon underreaction to information documented above.

The regression results presented in Table III can be used to form a rough estimate of the impact of the investor extrapolation phenomenon on the price of options. Consider the case of a window size of five previous trading days that produces a regression estimate of $\hat{\beta} = 0.00007$, regardless of whatever other controls are present in the regression. Because the $QPrevSim_t^5$ variable takes on values from -5 to 5, its range is 10. Multiplying these two numbers together shows that if the relationship between $FarMisProj_t$ and $QPrevSim_t^5$ is linear, different values of $QPrevSim_t^5$ are associated with changes in $FarMisProj_t$ of 0.0007. Suppose an ATM call has parameter values that would be typical for those used in this study, for example, $S = 500$ points, $X = 502.09$ points, $T = 30$ days, $r = 0.05$/year, and $V_0 = 0.01$. Under the Heston model with the estimates obtained in

Table III

Regression Tests for Increasing Investor Misreaction to a Current Unexpected Change in Instantaneous Variance, June 1, 1988, through August 29, 1997

Regression 1: $FarMisProj_t = \alpha + \beta QPrevSim_t^w + \varepsilon_t$

Regression 2: $FarMisProj_t = \alpha + \beta QPrevSim_t^w + \gamma V_t + \varepsilon_t$

Regression 3: $FarMisProj_t = \alpha + \beta QPrevSim_t^w + \delta|\Delta V_t^{\text{Unexpected}}| + \varepsilon_t$

Regression 4: $FarMisProj_t = \alpha + \beta QPrevSim_t^w + \gamma V_t + \delta|\Delta V_t^{\text{Unexpected}}| + \varepsilon_t$

The $FarMisProj_t$ variable measures the extent to which the current unexpected daily change in instantaneous variance is overprojected into the far future. The $QPrevSim_t^w$ variable measures for the unexpected part of a current daily change in instantaneous variance the quantity of previous similar unexpected daily changes in instantaneous variance. The w superscript indicates the number of previous unexpected daily changes in instantaneous variance that are included when constructing $QPrevSim_t^w$. The level of instantaneous variance on trade date t is denoted by V_t, and $\Delta V_t^{\text{Unexpected}}$ is the unexpected part of the change in instantaneous variance from trade date $t-1$ to trade date t. All instantaneous variances are implied from the Heston model. Estimates of $\hat{\beta}$ that are significantly greater than zero indicate that investor misreaction to the current daily unexpected change in instantaneous variance increases as a function of the quantity of previous similar unexpected changes in instantaneous variance. Coefficients are estimated via OLS and the t-statistics given in parentheses are calculated from heteroskedasticity and autocorrelation consistent standard errors computed using the procedure developed in Newey and West (1987). The market price of variance risk is set to $\lambda = -k^*/2$.

	Window Size				
Regression	1	2	3	4	5
			Panel A: $\hat{\beta}$		
1	0.00019	0.00011	0.00009	0.00009	0.00007
	(5.45)	(4.86)	(4.90)	(5.99)	(5.32)
2	0.00018	0.00011	0.00009	0.00009	0.00007
	(5.37)	(4.80)	(4.77)	(5.76)	(4.96)
3	0.00019	0.00011	0.00009	0.00009	0.00007
	(5.68)	(5.27)	(4.81)	(5.45)	(4.79)
4	0.00019	0.00011	0.00009	0.00009	0.00007
	(5.68)	(5.26)	(4.82)	(5.48)	(4.88)
			Panel B: $\hat{\gamma}$		
2	−0.0098	−0.0099	−0.0098	−0.0097	−0.0095
	(−2.18)	(−2.20)	(−2.17)	(−2.14)	(−2.11)
4	−0.0021	−0.0022	−0.0023	−0.0022	−0.0020
	(−0.39)	(−0.41)	(−0.43)	(−0.42)	(−0.38)
			Panel C: $\hat{\delta}$		
3	−0.1257	−0.1259	−0.1235	−0.1221	−0.1223
	(−1.82)	(−1.83)	(−1.80)	(−1.78)	(−1.77)
4	−0.1181	−0.1179	−0.1150	−0.1139	−0.1149
	(−1.47)	(−1.47)	(−1.44)	(−1.43)	(−1.43)

Section II, the price of this option is \$5.80. Raising the current variance to V_0 = 0.0107 increases the call price by \$0.18, or about three percent, to \$5.98.

A three percent price impact on close to ATM options is sizeable when compared to the price impact of other deviations from the Black–Scholes assumptions. For example, one of the most important extensions of the Black–Scholes framework is the inclusion of stochastic volatility. In the Stein and Stein (1991) model, the introduction of stochastic volatility never changes the price of a one-month ATM option by more than one percent for any of the choices of parameters of the underlying volatility process that they find it reasonable to consider. Nonetheless, the considerable bid-ask spreads in the SPX index options market makes it uncertain whether a trading strategy could be devised that would profit from the documented pattern of investor projection after transaction costs have been taken into account.

V. Model Misestimation and Misspecification

The previous section reported evidence that was interpreted as indicating that investor misreaction to a current unexpected daily change in instantaneous variance increases as a function of the quantity of previous similar unexpected daily changes in instantaneous variance. This section investigates the potential of either model misestimation or misspecification, rather than increasing misreaction, to account for this evidence.

A. Misestimation

Although minimizing the sum of squared option pricing errors is a widely used method for estimating the parameters of the Heston model, there are a number of other ways that the estimation can be performed. Three alternative procedures that use data on both the SPX options and the SPX index level are developed in the papers by Benzoni (1999), Chernov and Ghysels (2000), and Pan (2000). These papers' estimates of the Heston model are reported in Table IV. It can be seen that the estimates of some of the parameters differ widely from one another and also from the estimates used above (i.e., $k = 1.74$, $\theta = 0.031$, $\eta = 0.52$, $\rho = -0.61$, and $\lambda = -0.58$). Panels A–C of Table V report the results of Kendall tau tests that use the parameter estimates from, respectively, Pan (2000), Chernov and Ghysels (2000), and Benzoni (1999). For all of the tests in each of the panels, the Kendall tau statistic is significantly positive, which indicates that the test results reported above are not sensitive to the method of estimating the Heston model, even though different methods yield substantially different estimates.

The results that have been presented so far may be subject to a look-ahead bias insofar as model parameters are estimated only once using data over the entire sample period. This estimation strategy, in effect, has investors forming forecasts of future variance at date t using data that was not available until after date t. To determine whether the results presented above might be driven by look-ahead bias, the Kendall tau tests are rerun with the following two mod-

Table IV

Heston Model Estimates from Data on Both SPX Index Level and SPX Option Prices

This table reports Heston model parameter estimates from various papers. The Heston model is specified as follows:

$$\frac{dS_t}{S_t} = \mu(S_t, V_t, t)dt + \sqrt{V_t}\, dW_t^S$$

$$dV_t = k(\theta - V_t)dt + \eta\sqrt{V_t}\, dW_t^V$$

$$\mathrm{Corr}(dW_t^S, dW_t^V) = \rho$$

$$\lambda(S_t, V_t, t) = \lambda V_t$$

$$r = \delta = \text{constant},$$

where k, θ, η, ρ, λ, r, and δ are constants. The time t level and instantaneous variance of the underlying asset are denoted, respectively, by S_t and V_t. The system is driven by standard Weiner processes with increments dW_t^S and dW_t^V that have correlation ρ. The market price of variance risk is represented by $\lambda(S_t, V_t, t), r$ is the riskless borrowing and lending rate, and δ is the dividend payout rate of the underlying asset. Pan (2000) develops an implied-state generalized method of moments procedure to estimate the parameters. Chernov and Ghysels (2000) and Benzoni (1999) develop extensions of the efficient method of moments procedure to estimate the parameters. All of these papers use data from both the SPX index level and from SPX options prices in their estimations.

Reference	SPX Index Level Data Dates	SPX Options Data Dates	k	θ	η	ρ	λ
Pan (2000)	1/89–12/96	1/89–12/26	7.1	0.0137	0.38	−0.57	−7.6
Chernov and Ghysels (2000)	11/85–10/93	11/85–10/93	0.93	0.0154	0.06	−0.02	−0.24
Benzoni (1999)	1953–1996	1/96–3/97	3.23	0.0134	0.18	−0.59	−4.64

ifications. On any given trade date t, the $FarMisProj_t$ and $QPrevSim_t^w$ variables are constructed using Heston model estimates obtained by minimizing the sum of squared dollar pricing errors of the options that traded on the previous 20 trade dates and that meet criteria (C1) through (C4) above. In addition, the data period does not begin until July 1, 1993, which corresponds to the time the Heston model was published. As can be seen from Panel D of Table V, the test results are insensitive to these changes.

B. Misspecification

This subsection tests whether any of three alternative models of market equilibrium has the potential to explain the results presented in Section IV. The first alternative model generalizes the variance process given in equation (2) to

$$dV_t = m(V_t)dt + \eta(V_t)dW_t^V, \tag{21}$$

Table V

Tests for Increasing Investor Misreaction to a Current Unexpected Change in Instantaneous Variance Varying Heston Model Estimates

For Panels A through C, the sample period is June 1, 1988, through August 29, 1997, the Heston model estimates are obtained from the indicated papers, and the tests cover the 2,009 trade dates for which $FarMisProj_t$ and $QPrevSim_t^w$ variables can be constructed. The $FarMisProj_t$ variable measures the extent to which the current unexpected daily change in instantaneous variance is overprojected into the far future. The $QPrevSim_t^w$ variable measures for the unexpected part of a current daily change in instantaneous variance the quantity of previous similar unexpected daily changes in instantaneous variance. The w superscript indicates the number of previous unexpected daily changes in instantaneous variance that are included when constructing $QPrevSim_t^w$. If $FarMisProj_t$ is increasing in $QPrevSim_t^w$, then investor misreaction to the current daily unexpected change in instantaneous variance increases as a function of the quantity of previous similar unexpected changes in instantaneous variance. To determine whether $FarMisProj_t$ is increasing in $QPrevSim_t^w$, a τ_{Kendall} nonparametric correlation coefficient along with its one-sided p-value is calculated for these two variables. In Panel D, the test period is July 1, 1993, through August 29, 1997, the model is estimated at each trade date by minimizing the squared-pricing errors of options from the 20 previous trade dates, the market price of variance risk is set to $\lambda = -k^*/2$, and the tests cover the 1,052 trade dates for which the $FarMisProj_t$ and $QPrevSim_t^w$ can be constructed.

Window Size, w	τ_{Kendall}	p-Value (One-Sided)
Panel A: Pan (2000) Heston Model Estimates		
1	0.1014	0.0000
2	0.1102	0.0000
3	0.1150	0.0000
4	0.1246	0.0000
5	0.1125	0.0000
Panel B: Chernov and Ghysels (2000) Heston Model Estimates		
1	0.0668	0.0000
2	0.0714	0.0000
3	0.0689	0.0000
4	0.0835	0.0000
5	0.0692	0.0000
Panel C: Benzoni (1999) Heston Model Estimates		
1	0.0678	0.0000
2	0.0791	0.0000
3	0.0761	0.0000
4	0.0909	0.0000
5	0.0806	0.0000
Panel D: Rolling Estimation Heston Model, July 1, 1993, through August 29, 1997		
1	0.1166	0.0000
2	0.1035	0.0000
3	0.1106	0.0000
4	0.1073	0.0000
5	0.0812	0.0000

where $m(V_t)$ and $\eta(V_t)$ are flexible nonparametric functions while restricting ρ and λ in equations (3) and (4) to zero. Under this model, the Black–Scholes implied variance of an ATM option is almost exactly equal to the average variance expected over the life of the option (Feinstein (1988)). As a result, this model can be used to construct tests that are free from both specification and estimation error in the drift and diffusion coefficients of the variance process.

These tests are similar in nature to the term-structure forecasting exercise presented in Section III of Stein (1989). At each trade date *t,* a short maturity close to ATM option that expires at a time T_1 in the future and a long maturity close to ATM option that expires at a time T_2 in the future are used to construct a forecast of the forward variance that is observed at time $t + T_1$ over the period $t + T_1$ to $t + T_2$. A forecast error is then constructed by subtracting the forward variance predicted at time t from the forward variance observed at time $t + T_1$. Under the null hypothesis of rationality, the forecast errors will be white noise. Under the alternative hypothesis of increasing misreaction to information, the forecast error will be negatively related to the quantity of previous unexpected changes in instantaneous variance.

The test is conducted over the period June 1, 1988, through August 29, 1997. On each trade date *t,* T_1 is the time to expiration of the shortest maturity options that expire in more than nine calendar days. T_2 is the time to expiration of the options that expire in the month after the short maturity options. The average expected variance at time t over the period t to $t + T_1$, $E_t[\overline{V}_t^{t+T_1}]$, is taken to be the Black–Scholes implied variance of the closest to ATM call that matures at time $t + T_1$. The average expected variance at time t over the period t to $t + T_2$, $E_t[\overline{V}_t^{t+T_2}]$, is taken to be the Black–Scholes implied variance of the closest to ATM call that matures at time $t + T_2$. Here an ATM call is one for which the strike price is equal to the futures price (i.e., $X = Se^{(r-\delta)T}$).[6] The average variance expected at time t over the period $t + T_1$ to $t + T_2$, $E_t[\overline{V}_{t+T_1}{}^{t+T_2}]$, is computed by

$$E_t[\overline{V}_{t+T_1}{}^{t+T_2}] = \frac{T_2}{T_2 - T_1} E_t[\overline{V}_t^{t+T_2}] - \frac{T_1}{T_2 - T_1} E_t[\overline{V}_t^{t+T_1}]. \tag{22}$$

The forecast error at time t, $ForecastError_t$, is then the Black–Scholes implied variance of the closest to ATM call observed at time $t + T_1$ that expires at time $t + T_2$, $E_{t+T_1}[\overline{V}_{t+T_1}{}^{t+T_2}]$, minus the average variance expected over this period at time t:

$$ForecastError_t \equiv E_{t+T1}[\overline{V}_{t+T1}{}^{t+T_2}] - E_t[\overline{V}_{t+T1}{}^{t+T_2}]. \tag{23}$$

[6] The exact procedure for choosing the closest to ATM call for a given time to expiration is as follows: First, all calls of a given expiration are identified and the quantity $Se^{-\delta T}$, which is computed as described in the Appendix, is multiplied by e^{rT} to obtain futures prices. Second, the median of these futures prices is taken to be the target strike price, and the call with strike price closest to the target is taken to be the closest to ATM call.

Because the drift and diffusion coefficients of the variance process have been neither specified nor estimated, it is impossible to back out either the total or unexpected changes to instantaneous variance from option prices. Nonetheless, the unexpected changes in instantaneous variance can be proxied for by the innovations to the Black–Scholes implied variance of the short maturity closest to ATM call on each trade date. Consequently, for this forecasting exercise, a variable, *QPrevBSIVar*, is constructed that on a given trade date t records the quantity of previous changes in Black–Scholes implied variance. The *QPrevBSIVar* variable is defined by

$$QPrevBSIVar_t^w \equiv \sum_{i=1}^{w} \text{sign}(\Delta BSIVar_{t-i}^{\text{Short}}), \tag{24}$$

where $\Delta BSIVar_t^{\text{Short}}$ is the change in the Black–Scholes implied variance of the short maturity closest to ATM call from trade date $t-1$ to trade date t.

Kendall tau tests for the relationship between $ForecastError_t$ and $QPrevBSIVar_t$ produce significantly negative τ_{Kendall} values. These results are not reported, however, because the overlap in the $ForecastError_t$ variables spuriously increases the statistical significance. Instead, the results of running the following regression are presented in Table VI:

$$ForecastError_t = \alpha + \beta QPrevBSIVar_t + \varepsilon_t. \tag{25}$$

All of the OLS $\hat{\beta}$ estimates are negative, which is the correct sign to reject market rationality in favor of increasing investor misreaction in the options market. Furthermore, the t-statistics (which are adjusted for serial correlation and heteroskedasticity) indicate that the negative signs on the coefficient are significant at conventional levels. The reduction in significance relative to the results reported in Section IV is to be expected, because the *QPrevBSIVar* variable was constructed from changes in Black–Scholes implied variances rather than unexpected changes in instantaneous variances. In addition, the $ForecastError_t$ variable is constructed from only one option at each time, rather than all options that meet criteria (C2) through (C4). Because both of these factors add noise to the tests, the significance levels of the $\hat{\beta}$ estimates appear to be consistent with the results reported in Section IV.

The second alternative to the Heston model that is considered also replaces the variance specification in equation (2) with that in equation (21). This alternative, however, keeps the nonzero correlation between the Weiner processes described by equation (3) and generalizes the parametric specification of the market price of variance risk given in equation (4) to the nonparametric specification $\lambda(S_t, V_t, t) = \lambda_V(V_t)$. This model is estimated using the procedure developed in Poteshman (1998). The results of the Kendall tau tests are presented in Panel A of Table VII. Once again, all of the Kendall tau statistics are significantly positive, which indicates that relative to this model investors exhibit increasing misreaction to information.

Table VI

Regression of $ForecastError_t$ on $QPrevBSIVar_t^w$, June 1, 1988, through August 29, 1997

$$ForecastError_t = \alpha + \beta QPrevBSIVar_t^w + \varepsilon_t$$

This table presents the results of regressing $ForecastError_t$ on $QPrevBSIVar_t^w$ over the sample period June 1, 1988, through August 29, 1997. These variables are constructed under a stochastic variance model that assumes that the Weiner processes are uncorrelated and that the market price of variance risk is zero. The $ForecastError_t$ variable measures the error in the time t forecast of the forward variance over the period $t + T_1$ to $t + T_2$ where T_1 is the time to expiration at t of the shortest maturity options with expiration at least 10 calendar days in the future, and T_2 is the time to expiration of the options that expire in the subsequent month. The $QPrevBSIVar_t^w$ variable measures for a current unexpected change in instantaneous variance the quantity of previous similar unexpected changes in instantaneous variance using the change in the Black–Scholes implied variance of a short maturity close to ATM call as a proxy for the unexpected change in instantaneous variance. Coefficients are estimated via OLS and the t-statistics given in parentheses are calculated from heteroskedasticity and autocorrelation consistent standard errors computed using the procedure developed in Newey and West (1987).

Window Size, w	$\hat{\alpha}$	$\hat{\beta}$	$\bar{R}^2$
1	−0.0040	−0.0003	0.00
	(−6.56)	(−1.91)	
2	−0.0040	−0.0004	0.00
	(−6.56)	(−1.99)	
3	−0.0040	−0.0004	0.00
	(−6.57)	(−2.21)	
4	−0.0040	−0.0005	0.01
	(−6.57)	(−2.36)	
5	−0.0040	−0.0005	0.01
	(−6.57)	(−2.38)	

The final alternative model that is considered is the SVJ model from Pan (2000). This model includes jumps in the returns process as well as stochastic interest rates and dividend yields. Pan estimates this model from data on SPX options and the SPX level over the period January 1989 through December 1996. The Kendall tau tests were performed a final time using the SVJ model with Pan's estimates. The results, which are reported in Panel B of Table VII, are very similar to those from the previous models.

The working paper version of this article contains two further tests for a model misspecification-based alternative explanation for the results that are being interpreted as evidence for increasing misreaction. The first test exams whether systematic variation in the slope of the term-structure of implied variances that is unrelated to past unexpected changes in instantaneous variance might be producing the results. The second test exams high frequency data on the SPX index level directly to see whether current changes in instantaneous variance are, in fact, projected further into the future when they are preceded by more similar past changes in instantaneous variance.

Table VII

Tests for Increasing Investor Misreaction to a Current Unexpected Change in Instantaneous Variance Varying the Model of Market Equilibrium, June 1, 1988, through August 29, 1997

The tests reported in Panel A use the nonparametric stochastic variance model of Poteshman (1998) as the model of market equilibrium, and the tests reported in Panel B use the SVJ model of Pan (2000), which includes jumps in the return process and stochastic interest rates and dividend yields as the model of market equilibrium. The first variable used to conduct the tests is denoted $FarMisProj_t$ and measures the extent to which the current unexpected daily change in instantaneous variance is overprojected into the far future. The second variable is denoted $QPrevSim_t^w$ and measures for the unexpected part of a current daily change in instantaneous variance the quantity of previous similar unexpected daily changes in instantaneous variance. The w superscript indicates the number of previous unexpected daily changes in instantaneous variance that are included when constructing $QPrevSim_t^w$. If $FarMisProj_t$ is increasing in $QPrevSim_t^w$, then investor misreaction to the current daily unexpected change in instantaneous variance increases as a function of the quantity of previous similar unexpected changes in instantaneous variance. To determine whether $FarMisProj_t$ is increasing in $QPrevSim_t^w$, a τ_{Kendall} nonparametric correlation coefficient along with its one-sided p-value is calculated for these two variables. Each test covers the 2,009 trade dates from June 1, 1988, through August 29, 1997, for which the $FarMisProj_t$ and $QPrevSim_t^w$ can be constructed.

Window Size, w	τ_{Kendall}	p-Value (One-sided)
Panel A: Poteshman (1998) Model		
1	0.0842	0.0000
2	0.0728	0.0000
3	0.0647	0.0000
4	0.0893	0.0000
5	0.0654	0.0000
Panel B: Pan (2000) SVJ Model		
1	0.0790	0.0000
2	0.0810	0.0000
3	0.0790	0.0000
4	0.0937	0.0000
5	0.0800	0.0000

The tests indicate that neither of these possibilities are promising candidates for providing a nonbehavioral interpretation for the results presented in Section IV of this paper.

VI. Conclusion

This paper examines options market investor reaction to the information contained in daily changes in instantaneous variance under the assumption that options market investors subscribe to a standard stochastic variance model. Evidence is provided that supports three findings. First, SPX options market investors underreact to individual daily changes in instantaneous variance. Second, these same investors overreact to periods of mostly increasing or mostly decreasing daily changes in instantaneous variance. Third, they tend to underreact (overreact) to current daily changes in instantaneous variance that are preceded mostly by daily changes of the opposite (same) sign.

The first two findings are consistent with the interpretation of a large body of stock market evidence that is widely accepted by behavioral theorists. This interpretation maintains that stock market investors underreact to information at short horizons and overreact to information at long horizons. Barberis et al. (1998) propose a mechanism derived from the cognitive biases of conservatism and representativeness to reconcile short-horizon underreaction and long-horizon overreaction in the stock market. The third finding is consistent with the BSV model.

The results are robust to a number of different ways of estimating the assumed stochastic variance model and also to a number of changes in the assumed model of market equilibrium. The misreaction phenomenon documented in this paper produces deviations in option prices on the order of three percent in dollar terms. This is a sizeable effect when compared to the price impact of other important deviations from the Black–Scholes assumptions, but may not be large enough to generate trading profits given the size of the bid-ask spread in the SPX options market. An interesting question for future research is whether a profitable trading strategy that realistically accounts for transactions costs in the options market can be developed.

Appendix: Computing the Quantity $Se^{-\delta T}$ from Futures Prices

This Appendix details the procedure for computing the quantity $Se^{-\delta T}$ from SPX futures prices. If a given option bid-ask quote observation corresponds to an option that expires at the time of delivery of the most liquid futures contract, then the transaction price of the futures contract that trades at the time closest to the observation of the option bid-ask quote is used to determine the quantity $Se^{-\delta T}$. In particular, spot-futures parity is used to determine $Se^{-\delta T}$ from

$$Se^{-\delta T} = Fe^{-rT} \tag{A1}$$

where F is the futures prices, r is the one-month LIBOR rate, and T is the time to expiration of the option (or time to delivery of the futures).

When, on the other hand, an option corresponding to a bid-ask quote expires on a date other than the delivery date of the most liquid futures contract, then the quantity $Se^{-\delta T}$ is computed from three futures prices as follows: Let F_1 be the transaction price of the futures contract with the most liquid delivery date that transacts closest in time to the option bid-ask quote. Let F_2 be the transaction price of the futures contract with the second most liquid delivery date that transacts closest in time to the option bid-ask quote. Let F_3 be the transaction price of the futures contract with the most liquid delivery date that transacts closest in time to F_2. Finally, let T_1, T_2, and T_3 be the times to delivery of, respectively, F_1, F_2, and F_3. Then the dividend rate is computed via spot-futures parity from F_2 and F_3 (which are typically observed within a few seconds of each other) by

$$\delta = r - \frac{\log(F_3/F_2)}{(T_3 - T_2)}. \tag{A2}$$

This dividend rate is then used in conjunction with F_1 (which is typically observed within a few seconds of the option bid-ask quote) and spot-futures parity to compute

$$Se^{-\delta T} = F_1 e^{(-(r-\delta)T_1 - \delta T)}, \tag{A3}$$

which is associated with the bid-ask quote for the option that expires at a time T in the future. (To obtain the final equation, begin with $Se^{-\delta T_1} = F_1 e^{-rT_1}$ from spot-futures parity, then multiply both sides by $e^{\delta(T_1 - T)}$.)

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