THE JOURNAL OF FINANCE • VOL. LX, NO. 3 • JUNE 2005

A Comparison of Centralized and Fragmented Markets with Costly Search

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ABSTRACT

How does quotation transparency affect financial market performance? Biais's irrelevance proposition in 1993 shows that centralized markets yield the same expected bid–ask spreads as fragmented markets, other things equal. However, de Frutos and Manzano demonstrated in 2002 that expected spreads in fragmented markets are smaller and market participants prefer to trade in fragmented markets. This paper introduces liquidity traders' costs of searching for a better quote into the Biais model and derives opposite conclusions to these previous studies: expected spreads in centralized markets are smaller and liquidity traders prefer centralized markets, while market makers prefer fragmented markets.

THE TRADING MECHANISM OF A MARKET is pivotal to its performance. Biais (1993) groups securities markets into two categories according to their trading features. "Telephone dealer markets such as NASDAQ, SEAQ, the foreign exchange market, and the Treasury bonds market are fragmented" because "deals are the outcome of bilateral negotiations that other market participants cannot observe" (page 157). On the other hand, the markets of "the stock and future exchanges, such as the NYSE or the CBOT" are centralized since "all the orders are addressed to the same location so that market participants can observe all the quotes and trades and take them into account in their strategies" (page 157). The core of this market classification is the transparency of markets. Centralized markets resemble multilateral negotiations or open auctions, so that as soon as an agent quotes a price other agents in the market can observe it and supply a competing offer. In fragmented markets, trade and quote information displayed on screens is neither instantaneous nor sufficient for trading. Transactions through bilateral negotiations on the phone or electronic network are often executed at prices within the screen quotes. Therefore, rather than observe competitors' quotes and trades instantaneously, dealers have to estimate competitors' quotes and inventory positions.[1] Although centralized

*Xiangkang Yin is at the Department of Economics and Finance, La Trobe University. I benefited greatly from the discussion with Ming Fang on an earlier draft of the paper. I would also like to thank an anonymous referee and associate editor, Buly Cardak, David Prentice, Robert Stambaugh (Editor), and seminar participants at La Trobe University for their valuable comments on the paper.

[1] Specialists, market makers, proprietary limit order traders in centralized markets and dealers in fragmented markets stand ready to buy and sell at their bid and ask quotes. They play an identical role in terms of supplying liquidity, and so market makers and dealers are used interchangeably in the paper. The other type of agents in markets is outside investors or market order traders who demand liquidity. They are referred in the paper as the public or liquidity traders.

markets are more transparent than fragmented markets, a surprising finding by Biais (1993) is that market structure does not affect market performance to a certain extent. Particularly, Biais concludes that expected bid–ask spreads in centralized and fragmented markets are equal.

Following the same model and approach as Biais (1993), de Frutos and Manzano (2002) (dFM hereafter) show that fragmented markets dominate centralized markets in the sense that expected bid–ask spreads in fragmented markets are smaller and liquidity traders prefer to trade in fragmented markets, if other things are equal, while dealers are indifferent between these two market structures. The main reason for the difference between dFM and Biais (1993) is that dFM fully accommodate risk aversion, while Biais adopts a linear approximation in computing dealers' optimal quoting strategies so that dealers in fragmented markets are virtually risk neutral. Combining the findings of Biais and dFM, it is easy to draw the conclusion that the information transparency of centralized markets does not improve market liquidity. In contrast, the more opaque and fragmented markets generate a Pareto improvement over centralized markets, although the difference is in the second order.

A notable feature of the Biais model is that liquidity traders have superior information over dealers in fragmented markets. A liquidity trader can shop around dealers asking for quotes and obtain complete knowledge of prices, while competing dealers are unable to obtain their competitors' quotes. This asymmetric distribution of information allows the liquidity traders to select the best quote to execute a deal, but forces the dealers to compete severely to win a deal. It is clear that the results in Biais (1993) and dFM rely on this information advantage of liquidity traders. If they have incomplete price information as the dealers do, the competition among the dealers would not be so severe because a quote less attractive (to the public) than competing quotes may still win the order. Therefore, the execution ask (bid) price in a fragmented market is likely to be higher (lower) than its counterpart in a centralized market.

This paper modifies Biais's (1993) and dFM's implicit assumption that liquidity traders in fragmented markets can observe all quotes without any costs. Rather, it is assumed that the public has the same information as dealers if they do not search for the best quote. However, unlike the dealers, the public can obtain price information by visiting dealers and obtaining quotes from them. Although dealers provide their own quotes to the public for free, liquidity traders still bear some costs in searching for better prices. These costs include the time spent on communicating with dealers and other direct costs or opportunity costs related to communication and negotiation with dealers. This quoting process and accompanied search costs are realistic in fragmented markets such as telephone dealer markets or electronic quoting systems (see Lyons (1995) for a interesting description of the Reuters Dealing 2000-1 system for FX markets). Search costs are very small relative to traders' other operation costs, but they can significantly change the behavior of all market participants. The key argument of this paper is that no matter how small search costs are, they can reverse the results of Biais (1993) and dFM; that is, the expected bid–ask spread in a fragmented market is greater than the expected bid–ask spread in a centralized market if other things are equal. It is also found that dealers definitely prefer fragmented markets to centralized markets, but liquidity traders are generally worse off in fragmented markets, even if the expenses of searching are neglected. Moreover, fragmented markets are inefficient in the sense that they lead to inferior risk sharing among dealers relative to centralized markets. These predictions of the model are consistent with empirical findings, in particular, the empirical comparison between the NYSE and Nasdaq. The details of these empirical studies will be discussed in Section V.

The debate about the effects of market transparency has recently attracted wide attention. Pagano and Röell (1996) compare auction markets with dealer markets and they find that the transparency of market mechanism generally enhances liquidity and reduces trading costs for uninformed traders. However, Madhavan (1995) shows that a more opaque market in terms of fewer dealers disclosing their trade information may have a narrower bid–ask spread. Moreover, dealers and large traders with multiple orders can benefit from a fragmented market with less price competition than a unified market. Recent experimental studies have also drawn somewhat contradictory conclusions in the effects of market transparency. From their experiments with university students, Bloomfield and O'Hara (1999) find that trade disclosure increases bid–ask spreads and the informational efficiency of transaction prices, but quote disclosure has little effect on market performance. In contrast, Flood et al. (1999) conclude from their experiments with market practitioners that market transparency in the form of pretrade quote disclosure reduces spreads and increases trade volume. In an empirical study, Simaan, Weaver, and Whitcomb (2003) find that market makers are more likely to quote odd ticks (so that the probabilities of odd ticks and even ticks are close to equal) when they can do so anonymously on Nasdaq. This finding leads them to conclude that the more opaque market in the form of allowing anonymous quotes can reduce market makers' fear of retaliation from competitors and, in turn, enhance price competition and narrow spreads.

The diverse and even contradictory results of these studies point out that the effects of market transparency do not seem obvious and unambiguous, and information dispersion affects different market participants differently. Whether increased transparency can improve market efficiency and liquidity depends on whether it enhances competition among market makers and/or whether it transfers market power from market makers to liquidity traders. This paper argues that the opaqueness of fragmented markets makes prices more rigid and bid–ask spreads greater because liquidity traders have to conduct costly searches for price quotes. It is also worthwhile to notice that there is an important difference between models mentioned in the previous paragraph and the model in this paper. In these models, informed traders know the true value of risky assets and dealers have to set bid–ask spreads to protect themselves from the losses they incur by trading with insiders. The model in this paper, as in Biais (1993), is an inventory model, depicting where dealers set bid–ask spreads to cover their inventory holding costs.

The microeconomics literature of costly search is large. However, the applications to financial economics are relatively few. Williams (1995) considers costly sequential search in real assets markets. Sirri and Tufano's (1998) empirical evidence demonstrates that search costs are an important determinant of mutual fund flows. Flood et al. (1999) conjecture that dealers' search costs may make them adopt more aggressive pricing strategies. More closely related to our work is Yavaş (1993), which investigates the effects of search costs on bid–ask spreads. He finds that bid–ask spreads decline as search intensities rise (due to lower search costs or higher efficiency of search, etc.). However, in his model a liquidity trader has to determine whether she searches for a trade partner or goes directly to stand-by market makers. Market makers therefore provide services to avoid search and facilitate trade. In contrast, following Biais (1993), trade is restricted between liquidity traders and market makers in our model. Moreover, liquidity traders search for a better quote among market makers rather than search for a potential trading partner among themselves.

The rest of the paper is organized as follows. Section I is a brief review of Biais (1993) and dFM, which serves as a background of our analysis. Section II develops a model of fragmented markets with costly search. The pure-strategy Bayesian–Nash equilibrium is shown in Section III. Then, Section IV compares bid–ask spreads and the welfare implications of fragmented markets with those of centralized markets. Section V discusses the empirical and policy implications of the model. Section VI concludes the paper. The proofs of lemmas and propositions are given in the Appendix.

I. A Review of Biais (1993) and de Frutos and Manzano (2002)

The model developed by Biais (1993) features a five-stage game.

Stage 1: N agents out of a population of agents decide to become a dealer of a particular risky security at a given lump-sum cost.

Stage 2: Dealer i is endowed with cash and a random inventory position of the risky security I_i with cumulative distribution function (cdf) $F(\cdot)$ and probability density function (pdf) $f(\cdot)$ on $[-M, M]$.[2]

Stage 3: With a given probability, a liquidity shock occurs. If it occurs, an outside investor is, equally likely, endowed with either a long position $+L$ or a short position $-L$ of the security and places a market order to a dealer to buy or sell Q units of the asset. It is assumed that $L > M$ to ensure trading between the public and the dealers.

Stage 4: Dealer i posts a selling and a buying price, denoted by A_i and B_i, and supplies liquidity to the public. All market participants are assumed to have identical expectations about the final value of the risky asset $E(P)$, which is normalized to 1. Thus, A_i and B_i can be rewritten as $A_i \equiv (1 + a_i)E(P) = (1 + a_i)$ and $B_i \equiv (1 - b_i)E(P) = (1 - b_i)$, where a_i and b_i can be interpreted as ask and bid premiums, respectively.[3]

[2] To facilitate comparison, I adopt the same notations as dFM when it is possible.

[3] Since the computations of a_i and b_i are similar, I will give only formulas relevant to the ask premium and omit those of the bid premium below to reduce lengthy equations.

Stage 5: The final value of the risky asset, P, is realized, where P is normally distributed with mean 0 and variance σ^2.

In this model, all participants have the common CARA utility that

$$U(W) = -\exp(-RW), \tag{1}$$

where R is the risk aversion coefficient and W is a player's terminal wealth. All random variables—including the inventory positions of the dealers and the public, liquidity shock, and the final value of the risky security—are independent and their distributions are common knowledge. A dealer's pricing strategy is a function of his inventory position, since the only dealer heterogeneity affecting his security valuations and price decisions is his endowments of the risky asset. The distinction between the market structures is that, in the centralized market, all participants can observe all pretrade quotes; but, in the fragmented market, the dealers cannot observe competing dealers' quotes, while the public can observe all pretrade quotes.

Because of risk aversion, every market maker has a pair of reservation selling and buying prices. More specifically, Ho and Stoll (1983) show that dealer i with inventory I_i has a reservation selling price $1 + a_{r,i}$, where

$$a_{r,i} \equiv a_r(I_i) = R\sigma^2(Q - 2I_i)/2. \tag{2}$$

As Ho and Stoll demonstrated, the process of setting bid–ask quotes in the centralized market resembles a progressive open (English) auction. Thus, price competition among dealers induces the agent who can offer the most attractive prices to undercut all his rivals. In other words, the execution price of selling the asset to the public in the centralized market is the second-lowest reservation price; that is,

$$A^C = 1 + a_r\left(I^*_{N-1}\right), \tag{3}$$

where I^*_j is the jth order statistic associated with a sample of N random inventories drawn from $F(\cdot)$. Thus, I^*_N refers to the largest out of N random inventory draws, I^*_{N-1} the second largest, and so on.

Turning to the fragmented market, Biais (1993), by linearizing the utility function given in (1), shows that the equilibrium execution price with a given profile of inventory positions $\{I_i\}$ and order quantity Q is $A^B = 1 + a^B(I^*_N)$, where Biais pricing strategy $a^B(I)$ is defined as

$$a^B(I) \equiv a_r(I) + R\sigma^2 F^{1-N}(I) \int_{-M}^{I} F^{N-1}(x)\,dx. \tag{4}$$

More importantly, Biais proves that $E(a_r(I^*_{N-1})) = E(a^B(I^*_N))$, where E is an expectation operator, and concludes that the expected bid–ask spreads in the centralized and fragmented markets are equal.

dFM argue that Biais's irrelevance proposition of market structures and spreads relies on the linear approximation of dealers' utility function. Using

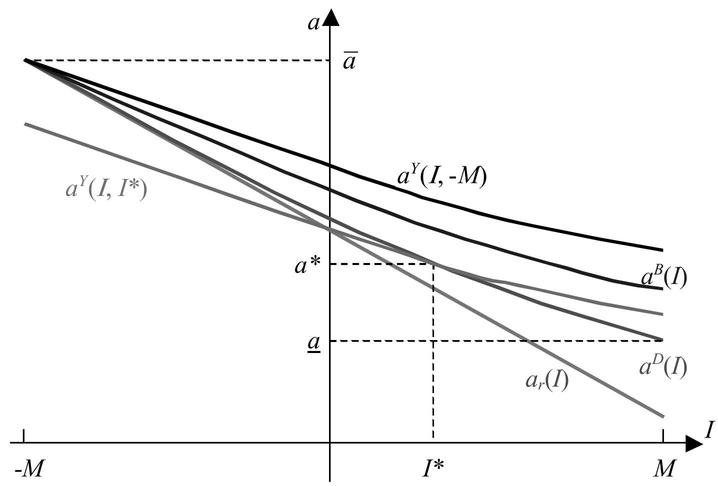

Figure 1. Reservation ask premium and pricing strategy curves. A dealer with an inventory position I has a reservation ask premium $a_r(I)$. In a fragmented market without search costs, a dealer adopts pricing strategy $a^B(I)$ if he is risk neutral or $a^D(I)$ if he is risk averse. In a fragmented market with costly search, a risk-averse dealer adopts pricing strategy $a^Y(I, -M)$ if $a^* > \bar{a}$ or $a^D(I)$ on $[-M, I^*]$ and $a^Y(I, I^*)$ on $[I^*, M]$ if $a^* \in [\underline{a}, \bar{a}]$.

the utility function in (1), they find that the equilibrium ask price in the fragmented market is $A^D = 1 + a^D(I^*_N)$, where dFM pricing strategy $a^D(I)$ is defined as

$$a^D(I) \equiv \frac{-1}{RQ} \ln \left(F^{1-N}(I) \int_{-M}^{I} e^{-RQa_r(x)}\, dF^{N-1}(x) \right). \tag{5}$$

Furthermore, they show that for a given order quantity Q the expected spread in the fragmented market is smaller than that in the centralized market. The dealers are indifferent between the two market structures, but the public strictly prefers the fragmented market. Obviously, the difference between the conclusions drawn from Biais (1993) and dFM stems from the difference in pricing strategies (4) and (5). However, $a^B(I)$ and $a^D(I)$ intersect at $I = -M$ (i.e., $a^B(-M) = a^D(-M) = a_r(-M)$), as illustrated by Figure 1.

II. A Model of Fragmented Dealer Markets with Costly Search

Our model is a modification of Biais (1993). First, it abstracts the first stage of the Biais model so that the number of dealers, N, in the market is exogenously given. Second, the liquidity traders in our model are assumed to be risk neutral. Thus, if a liquidity shock occurs in stage 3, a liquidity trader buys or sells a fixed amount of the risky security, Q, independent of price. This can be interpreted as the liquidity trader having a vertical demand or supply curve up to her reservation price and her reservation buying (selling) price is higher (lower) than the dealers' quotes. This modification does not change the market makers' behavior, since the market makers in Biais (1993) and dFM ignore the effects of their prices on the quantity demanded or supplied by the public when they determine their price quotes.[4]

Since a distinguishing feature of centralized markets is market transparency and all participants can observe the quote process, there is no need for liquidity traders to search for the best price. These two modifications do not change the game in centralized markets and the equilibrium price is still determined by (2) and (3).

For fragmented markets, our model has another important difference from the Biais model. As argued in the introduction, it seems plausible to assume that a liquidity trader has the same price information as the dealers when she just enters the market. However, the advantage the liquidity trader possesses is that she can, or through her broker, visit the dealers by making a phone call or internet inquiry, etc., to obtain price quotes. Therefore, our model of fragmented markets has an extra stage between stages 3 and 4 of the Biais model: namely, a search stage. In this stage, the public shops around to find a better price. It is important to note that when a search is costly, the public does not necessarily visit all dealers. Rather, if she finds the expected cost of another visit is greater than the expected benefit, she will stop searching and cut a deal with the dealer who posts the best quote among all visited dealers.[5]

It is well known that the introduction of homogenous search cost in a Bertrand oligopoly leads to a monopoly price or nonexistence of the market (see Stiglitz (1979)). In our context, if the dealers know the public's search cost, they will charge an identical price, resulting in nonexistence of pure-strategy equilibrium. On the other hand, heterogeneous search costs are more realistic; for instance, the liquidity traders are likely to have different opportunity costs of time. So, it is assumed that a liquidity trader's search cost is determined by the state of the nature. Although the liquidity traders can perfectly observe their own search costs, the dealers only know the distribution of search costs: With probability p a liquidity trader has a low search cost and with probability $1 - p$ she has a high search cost. To simplify analysis and facilitate comparison with the previous studies, it is further assumed that the low search cost is equal to 0, while the high search cost of visiting a dealer is $c > 0$.[6]

III. The Equilibrium of Fragmented Market

To determine the equilibrium in the fragmented market, we first need to know the liquidity traders' search behavior. It is obvious that a zero-cost liquidity

[4] Biais (1993) justifies the ignorance of the price effect on quantity by arguing that "at Stage 4, the dealers worry only about the quotes of their competitors and not about the reaction of their customers to increases in the bid–ask spread."

[5] Here free recall is implicitly assumed; that is, the public can go back to a previously visited dealer and trade with him at his quote without extra costs.

[6] This is not an innocuous assumption. If it is assumed that both low and high search costs are strictly positive, our result can be strengthened. For instance, the condition $p \to 1$ in Proposition 5 can be removed.

trader visits all dealers and trades with the dealer who quotes the lowest ask/bid premium. However, a positive-cost liquidity trader has to trade off the benefit of finding a better quote with the cost of an extra search. The searching rule can be summarized in the following lemma.

LEMMA 1: *Assume a liquidity trader with search cost c knows that the ask premium is an i.i.d. random variable a with cdf H(a). Let a_i be the ask premium quoted by dealer i in the liquidity trader's last search. Then she searches for another quote if and only if $a_i \geq a^*$, where a^* is determined by*[7]

$$\left[a^* H(a^*) - \int_{-\infty}^{a^*} a\, dH(a)\right] Q = c. \tag{6}$$

The left-hand side of (6) is the expected benefit of doing one more search when a liquidity trader knows she can buy the risky security from a dealer at the price of $1 + a^*$ while the right-hand side is the cost of an extra search. So, equation (6) generates a threshold of observed ask premiums at which an additional search breaks even. If the liquidity trader observes an ask premium greater than this threshold, she should continue her sampling. However, if she finds a dealer charges an ask premium below the threshold she should buy the security from that dealer.

The analysis of the paper assumes that liquidity traders directly buy from or sell to dealers. If the security is traded through brokers (i.e., a liquidity trader transmits her order to her broker without knowing the best price and the broker finds the quotes and transacts the prescribed quantity on her behalf), then there is an agency problem. It arises because brokers cannot fully capture the benefit of better prices, but have to pay the full costs of a search. Assuming a broker only captures a fraction, η, of the liquidity trader's gain, the left-hand side of (6) should be multiplied by η. In other words, when liquidity traders delegate the search and transaction tasks to a broker, the search cost c in the model should be replaced by c/η. Thus, the agency problem generally amplifies the effects of search costs.

Since some liquidity traders have to pay search costs to find a better price, it is not necessary for a dealer to set the lowest ask premium to win a deal. The lemma below calculates the probability of dealer i winning an order.

LEMMA 2: *Conditional on the public having a positive search cost and adopting the searching rule in Lemma* 1, *the probability of dealer i winning the deal by setting ask premium $a_i < a^*$ is*[8]

$$\varphi = \frac{1 - F^N(a^{-1}(a^*))}{[1 - F(a^{-1}(a^*))]N}. \tag{7}$$

[7] At this stage, no restriction is imposed on $H(a)$, and it is assumed that $H(a)$ is distributed on $(-\infty, \infty)$.

[8] In equations (7) and (8), it is assumed that other dealers adopt the same downward-sloping pricing strategy $a(I)$.

If the public has a zero search cost or the dealer sets an ask premium $a_i \geq a^*$, then the probability of winning the deal is

$$\pi_i = F^{N-1}\big(a^{-1}(a_i)\big). \tag{8}$$

Lemma 2 considers two scenarios of a dealer succeeding in the trade. The first is the case where the dealer does not post the lowest ask price. The necessary conditions for him to be successful are setting an ask premium below a^* and the public having a positive search cost. Thus, equation (7) is virtually the probability that the public meets dealer i before she meets other dealers who charge an ask premium below a^*. It is worthwhile to notice that this probability is independent of the particular value of the ask premium dealer i sets as long as it is below a^*. The other scenario is that the winning dealer has to post the best price to the public. This happens when the public samples all dealers, which requires in turn that either the public has no search costs or all dealers set prices greater than $1 + a^*$. Thus, the probability in (8) is the same as that in Biais (1993) and dFM.

Since the dealers' pricing strategies are determined by maximizing their expected utilities, we need to determine the effects of the ask premium on the utility function. Let $W_i(0)$ and $W_i(a_i)$ be dealer i's wealth levels if he does not trade with the public and if he sells Q units of the risky asset at ask premium a_i, respectively. dFM show that the expected utilities of trading and not trading have the following relationship

$$E(U(W_i(a_i)) \mid I_i) = E(U(W_i(0)) \mid I_i)e^{-RQ(a_i - a_{r,i})}.$$

Recalling that dealer i can only win the order if his ask premium is the lowest when he sets $a_i \geq a^*$, the expected utility in this case is

$$U_i = E(U(W_i(0)) \mid I_i)\big[\pi_i\big(e^{-RQ(a_i - a_{r,i})} - 1\big) + 1\big]. \tag{9}$$

On the other hand, the dealer has two scenarios to consider if he sets an ask premium $a_i < a^*$. With probability p, he meets a positive-cost liquidity trader. In this situation, his probability of winning the deal is φ. With probability $1 - p$, he has to beat all other dealers to win the deal because the liquidity trader has a zero search cost. Then the expected utility is

$$\begin{aligned} U_i = E(U(W_i(0)) \mid I_i)\big\{&p\big[\varphi e^{-RQ(a_i - a_{r,i})} + (1 - \varphi)\big] \\ &+ (1 - p)\big[\pi_i e^{-RQ(a_i - a_{r,i})} + (1 - \pi_i)\big]\big\}. \end{aligned} \tag{10}$$

It is obvious that if $p = 0$, (10) collapses to (9); that is, the expected utility in our model is the same as that of Biais (1993) and dFM. Since p and φ are positive constants and $E(U(W_i(0) \mid I_i))$ is a negative constant, the dealer's utility-maximization problem can be written as

$$\max_{a_i} \pi_i\big(1 - e^{-RQ(a_i - a_{r,i})}\big) \quad \text{when} \quad a_i \geq a^*, \tag{11}$$

$$\max_{a_i} (1 - p)\pi_i\big(1 - e^{-RQ(a_i - a_{r,i})}\big) - p\varphi e^{-RQ(a_i - a_{r,i})} \quad \text{when} \quad a_i < a^*. \tag{12}$$

Following Biais (1993) and dFM, it is assumed that dealers' pricing strategies are continuous functions of their own inventory positions and the ask premium decreases in the level of inventory. Moreover, these strategies are symmetric in equilibrium in the sense that they have the same functional form; that is, $a_i = a(I_i)$. Define

$$a^Y(I, y) \equiv \frac{-1}{RQ} \ln \left[\frac{(1-p)\int_y^I e^{-RQa_r(x)} dF^{N-1}(x) + ((1-p)F^{N-1}(y) + p\varphi)e^{-RQa^D(y)}}{(1-p)F^{N-1}(I) + p\varphi} \right].$$

LEMMA 3: *If the positive-cost liquidity traders follow the searching rule in Lemma* 1, *then dealer i's optimal ask premium is the following:*

(i) If $a^* \in [\underline{a}, \bar{a}]$, where $\underline{a} = a^D(M)$ and $\bar{a} = a_r(-M)$, then

$$a_i \equiv a(I_i) = \begin{cases} a^D(I_i) & \text{when } I_i \in [-M, I^*] \\ a^Y(I_i, I^*) & \text{when } I_i \in [I^*, M], \end{cases} \tag{13}$$

where I^* is implicitly determined by

$$\ln\left(F^{1-N}(I^*) \int_{-M}^{I^*} e^{-RQa_r(x)} dF^{N-1}(x)\right) = -a^* RQ. \tag{14}$$

(ii) If $a^* > \bar{a}$, the pricing rule is

$$a_i = a^Y(I_i, -M), \quad \forall I_i \in [-M, M]. \tag{15}$$

(iii) If $a^* < \underline{a}$, the optimal ask premium is $a_i = a^D(I_i)$ for all $I_i \in [-M, M]$.

In the fragmented market where all liquidity traders have no search costs, the liquidity traders are passive agents if the quantity of trading, Q, is exogenously given. The market's operation resembles a sealed auction, and the dealers compete with each other based on their inventory positions. But, when search costs are introduced into the model, the game is no longer a simple sealed Dutch auction. In contrast, the dealers have to consider the client's search decision in addition to reacting to the rivals' competing strategies. Roughly speaking, dealer i knows that his expected utility function is (9) (i.e., must be the lowest-price dealer to win the order) if the public's threshold a^* is smaller than $\underline{a}$ or if $a^* \in [\underline{a}, \bar{a}]$ and his ask premium is greater than a^*. Thus, the corresponding utility-maximization problem (11) is identical to that of dFM and the pricing strategy is the dFM strategy (i.e., $a^D(I)$). On the other hand, dealer i's expected utility becomes (10) (i.e., a quote above the lowest price can also win the deal) if the public's a^* is greater than $\bar{a}$ or if $a^* \in [\underline{a}, \bar{a}]$ and the dealer's own inventory position is longer than I^*. Then, the corresponding utility-maximization problem (12) leads to a pricing strategy of $a^Y(I_i, -M)$ or $a^Y(I_i, I^*)$.

The equilibrium we consider in this model is the pure-strategy Bayesian–Nash equilibrium. In particular, the public has the knowledge of dealers' distributions of inventory positions $F(I)$. Knowing dealers adopt optimal pricing strategies as specified in Lemma 3, the public can infer the distribution of the ask premium, $H(a)$, and, in turn, can calculate a^*. On the other hand, when dealers' expectations of a^* are self-fulfilling, they can immediately determine their equilibrium ask premiums based on their own inventory positions. Therefore, with the aid of Lemmas 1 and 3, the equilibrium can be easily determined.

PROPOSITION 1: *If* $\bar{a} \geq \int_{-M}^{M} a^Y(x, -M)\,dF(x) + c/Q$, *there is an equilibrium where the pricing strategy is given by* (13) *and*

$$a^* = \left[c/Q + \int_{I^*}^{M} a^Y(x, I^*)\,dF(x)\right] \bigg/ [1 - F(I^*)], \tag{16}$$

with I^* *determined by* (14). *If*

$$\bar{a} < \int_{-M}^{M} a^Y(x, -M)\,dF(x) + c/Q,$$

there is an equilibrium with $a_i = a^Y(I_i, -M)$ *and*

$$a^* = \int_{-M}^{M} a^Y(x, -M)\,dF(x) + c/Q.$$

From the definition of $a^Y(I, y)$, it is not hard to see that $a^Y(-M, -M) = a_r(-M) \equiv \bar{a}$. Since $a^Y(I, -M)$ is a decreasing function of I, we have

$$\int_{-M}^{M} a^Y(x, -M)\,dF(x) < \bar{a}.$$

So, whether $\int_{-M}^{M} a^Y(x, -M)\,dF(x) + c/Q$ is greater or smaller than $\bar{a}$ depends on the public's cost of visiting a dealer. Although the searching and pricing rules are deterministic, the execution price is uncertain. In addition to the uncertainty caused by random dealer inventory positions, as identified by Biais (1993) and dFM, costly search also induces the uncertainty of which dealer wins the order. When search is free of cost, the successful dealer is always the one with the longest position. But, when search is costly, the public stops searching if she finds an offer's ask premium below a^* and purchases the asset from the offering dealer. Since all dealers whose inventory positions are longer than I^* charge an ask premium below a^*, all of them are likely to win the deal. Thus, when

$$\bar{a} \geq \int_{-M}^{M} a^Y(x, -M)\,dF(x) + c/Q,$$

the equilibrium execution price of the asset is

$$A^Y = \begin{cases} 1 + a^Y(I_g, I^*) & \text{if the public has a positive search cost and } I_N^* > I^* \\ 1 + a^Y(I_N^*, I^*) & \text{if the public has a zero search cost and } I_N^* > I^* \\ 1 + a^D(I_N^*) & \text{if } I_N^* \le I^*, \end{cases} \tag{17}$$

where I_g is randomly drawn from a pool of inventories greater than I^*. The first row on the right-hand side of (17) indicates that the positive-cost liquidity trader continues searching until she meets a dealer who charges an ask premium smaller than a^* and buys the security from him; the second row indicates that the zero-cost liquidity trader samples all dealers and trades at the lowest premium (below a^*); the last row indicates that the public, independent of her search cost, searches across all dealers (since all of them quote a price higher than $1 + a^*$) and trades with the dealer whose inventory position is I_N^*.

When the positive search cost is high, we have

$$\bar{a} < \int_{-M}^{M} a^Y(x, -M)\, dF(x) + c/Q,$$

which implies $a^* > a^Y(I, -M)$ for all $I \in [-M, M]$. Thus, all dealers set their ask premiums smaller than a^*. The positive-cost public purchases the asset from the dealer she meets first while the zero-cost trader searches all dealers. Proposition 1 shows that the equilibrium execution price in this case is

$$A^Y = \begin{cases} 1 + a^Y(I_N^*, -M) & \text{if the public has a zero search cost} \\ 1 + a^Y(I_f, -M) & \text{otherwise,} \end{cases} \tag{18}$$

where I_f is the inventory position of the dealer who meets the public first.

IV. A Comparison between Centralized and Fragmented Markets

To facilitate comparison, I illustrate the relative positions of pricing strategy curves $a^D(I)$, $a^Y(I, I^*)$, and $a^Y(I, -M)$ in Figure 1 accompanied by curves $a^B(I)$ and $a_r(I)$.

PROPOSITION 2: *The risky security in the fragmented market with search costs is traded at a higher ask price than in the fragmented market without search costs, ceteris paribus;*[9] *that is,* $A^Y \ge A^D$.

The intuition behind Proposition 2 is obvious. If all liquidity traders have no search costs, the dealers adopt pricing strategy $a^D(I)$ as shown by dFM. Since the public definitely visits all dealers, only the dealer possessing the longest inventory, who sets the lowest price, $1 + a^D(I_N^*)$, can win the deal. When

[9] In our model, some liquidity traders have search costs while the others do not, but I still label the market as a market with search costs. The fragmented market without search costs is the one studied by Biais (1993) and dFM.

some of the liquidity traders have a positive search cost, the dealers compete with each other less aggressively, knowing that it is possible to win an order even when they do not quote the lowest price. This is illustrated in Figure 1 where the curves of their pricing strategies, $a^Y(I, -M)$ or $a^Y(I, I^*)$ (for $I > I^*$) are above curve $a^D(I)$. Moreover, a dealer with an inventory position shorter than the longest position quotes a higher price than the dealer endowed with the longest position. So, there are two reasons to make $A^Y \geq A^D$. First, given an inventory position, a dealer tends to quote a higher price. Second, the winning dealer is not necessarily the one that quotes the lowest price among all dealers.

dFM demonstrate that the expected ask price in the fragmented market without search costs is lower than its counterpart in the centralized market. However, in the centralized market, the dealers face more price uncertainty because the execution price is not determined by the order winner's inventory position (the longest position), but by the second longest position. An interesting finding by dFM is that the extra expected return generated by the higher ask premium in the centralized market is exactly offset by its extra risk. Thus, the dealers are indifferent between two market structures. Applying Proposition 2 and dFM's results, it is obvious that the centralized market's higher return (if it exists) to the dealers over the fragmented market *with costly search* cannot compensate for the cost of additional risk. This leads to the following proposition.

PROPOSITION 3: *If some liquidity traders have positive search costs, the dealers prefer the fragmented market to the central market, ceteris paribus.*

Although the dealers prefer the fragmented market, it does not lead to more efficient risk sharing among the dealers than the centralized market. The reason is that, in the centralized market, it is always the dealer with the longest (shortest) inventory who sells to (or buys from) the public, while, in the fragmented market, other dealers are also likely to sell or buy. Thus, the fragmented market results in a more uneven posttrade distribution of inventories if the dealers in the two markets have identical pretrade inventory distributions; consequently, the dealers in the fragmented market have to bear greater aggregate security risk. This result can be summarized in the following proposition.

PROPOSITION 4: *The fragmented market generates a less efficient outcome in risk sharing among dealers than the centralized market in the sense that dealers bear greater aggregate security risk.*

Although Proposition 2 demonstrates that search costs can increase the ask price in the fragmented market from our model, we still cannot immediately draw the conclusion that the fragmented market definitely results in a higher expected ask price than the centralized market. This conclusion requires a moderate condition.

PROPOSITION 5: *If the probability that the public has a positive search cost is sufficiently close to* 1*, then the fragmented market results in a higher expected ask price than the central market.*

The reason that we need the condition of p being close to one can be seen from the effect of this probability. As the probability rises, the dealers quote less aggressively because a dealer who quotes a price higher than the lowest quote has more chance of winning the order. This is captured by $a^Y(I, I^*)$ and $a^Y(I, -M)$ curves in Figure 1 becoming flatter as p rises. Expecting the dealers to quote less aggressively, the public anticipates a higher a^*; that is, the threshold for the public to stop searching becomes higher.[10] This expectation in turn induces the dealers to quote even higher prices. When the probability is sufficiently close to 1, all quotes are within a very narrow band below $1 + \bar{a}$, which is equal to the highest possible price in the centralized market.

Because the liquidity traders in our model are risk neutral, Proposition 5 implies that they prefer the centralized market to the fragmented market.

It is important to notice that the result in Proposition 5 is independent of the magnitude of the positive search cost. In other words, no matter how small the search cost is, Proposition 5 holds as long as the cost is positive. Moreover, the condition for Proposition 5 is sufficient but not necessary. All we need is $a^Y(I, -M)$ above $a^B(I)$.

It is obvious that, in the case of dealers buying the risky asset (rather than selling), results similar to Propositions 1–5 hold. Consequently, we can conclude that the fragmented market yields greater expected bid–ask spreads than the centralized market, ceteris paribus, when almost all liquidity traders have positive search costs.

V. Discussion

As indicated in the introduction, the NYSE and AMEX are close to our model of centralized markets, while the Nasdaq (before the 1997 market reform in particular) and London Stock Exchange resemble the features of fragmented markets. According to Proposition 5, the bid–ask spreads in the NYSE or AMEX are, on average, smaller than their comparable counterparts in the Nasdaq market. This prediction is consistent with a large body of empirical comparisons between the NYSE/AMEX and Nasdaq. For example, Christie and Huang (1994), and Barclay (1997) examine securities that were traded on Nasdaq and then moved to the NYSE or AMEX, finding that the bid–ask spreads declined significantly after the move.[11] Huang and Stoll (1996), and Weston (2000) compare

[10] Also note, I^* declines as p increases. This means that $a^Y(I, I^*)$ occupies a larger segment in pricing strategy (equation (13)).

[11] Barclay (1997), for example, reports that the average quoted half spread of 472 sample securities declined 10 cents (from 23 cents to 13 cents) and the average effective half-spread declined 11 cents (from 18 cents to 7 cents).

Nasdaq securities to the matched samples of NYSE stocks. Both studies find that quoted and effective spreads of the Nasdaq samples are substantially greater than their matches.[12] Of course, differences between the NYSE and Nasdaq are not limited to the fragmentation and opaqueness of these markets. The literature has documented other institutional factors that can explain greater bid–ask spreads on Nasdaq, including commissions, order-processing costs (Affleck-Graves, Hagde, and Miller (1994)), implicit collusion (Christie and Schultz (1994) and Dutta and Madhavan (1997)), payment for order flow (Parlour and Rajan (2001)), and preference trading (Godek (1996)). Our model provides an alternative explanation for the empirically confirmed performance difference of the NYSE and Nasdaq; that is, the fragmentation and opaqueness of Nasdaq require customers to engage in costly searching for a better price, which leads to bid–ask spreads wider than observed comparable counterparts in the NYSE.

A testable prediction, which can be drawn from our model and those of Biais (1993) and dFM, is that the pattern of trade execution can identify whether search costs exist or not because the dealers with the most extreme inventory position should execute all trades in a zero-search-cost fragmented market, but other dealers are also likely to execute trades when positive search costs exist. While the existence of search costs seems obvious, there is empirical literature further verifying the existence of such costs and consequences. For example, evidence documented by Hansch, Naik, and Viswanathan (1998) supports our hypothesis of positive search costs. They find that in the fragmented London Stock Exchange market only about 50% of the trades are executed by either dealers with extreme inventory or dealers whose inventory is within one standard deviation from the extreme inventory position. Hansch et al. argue that long-term trading relationships and the noise in inventory measurement are likely to be the reasons for the trades not being solely executed by the dealers with extreme inventory. Our analysis offers search costs as an alternative and possibly more plausible explanation.

Proposition 1 implies that search costs have a positive effect on trading costs in fragmented markets. Since search costs are likely to decrease as technology improves, we would expect bid–ask spreads to decline through time. Thus, another empirical test of the model is to run a regression of a time series of search costs (say, using the time spent on acquiring a quote as a proxy) against a time series of bid–ask spreads in a fragmented market. Moreover, because the model predicts spreads in a centralized market are not affected by search costs, we should see different time series patterns for spreads in fragmented and centralized markets, after controlling for other factors. Similarly, as we have noticed, traders may have different search costs at the same point of time. For example, investors using an electronic quotation system are more efficient than those relying on the telephone. Therefore, we can use panel data to test

[12] Weston (2000), for instance, compares spreads on Nasdaq securities to three matched samples of NYSE securities and finds that quoted (effective) spreads for his Nasdaq sample are about 15% (25%) greater than spreads for their matches (see Table IV of Weston (2000)).

bid–ask spreads across investors in a fragmented market against their costs spent on quotation and negotiation.[13]

The evolution of the Nasdaq market also vividly reveals the effects of market fragmentation and search costs.[14] After the accusation of implicit collusion among Nasdaq dealers by the pioneering work of Christie and Schultz (1994), the Securities and Exchange Commission (SEC) launched market reforms in 1997. The most significant changes were the mandatory display of customer limit orders and the dissemination of superior prices placed in proprietary trading systems. Before the reforms, public limit orders falling inside a dealer's posted quotes were not publicly available. Under the new trading rules, when a Nasdaq dealer receives a limit order, it may be executed against his own inventory, posted as a new public quote, or sent to another market maker. Thus, the public can observe the best limit order on a market maker's book and does not need to contact the market maker to obtain the information. This improvement in market transparency obviously saves the public's search costs. On the other hand, prior to the reforms, quotes (actually limit orders) placed on Electronic Communication Networks (ECNs) by Nasdaq market makers were not included in the Nasdaq National Best Bid and Offer quote montage. But the change in trading rules gives the public access to superior prices posted on ECNs; that is, if a dealer places a limit order into an ECN, the price and quantity are displayed on Nasdaq montage if it improves the quote. The incorporation of ECN quotes into Nasdaq integrates previously partially separated markets. Although we do not formally model such a change of market environment, the results of our analysis imply that the market reforms, which promote market integration and reduce search and search costs, should enhance competition among the dealers and reduce bid–ask spreads. Empirical investigations support this conjecture, as Barclay et al. (1999) and Weston (2000) report that the reforms substantially reduce the trading costs on Nasdaq.[15]

As mentioned above, the introduction of new Order Handling Rules partially eased the fragmentation between ECN and Nasdaq because the new rules require market makers and specialists to reflect in their quotes the price of any order they placed in an ECN if the price is better than their own public quotation.[16] However, the new rules "did not require all market participants to report to the public quotation stream the orders they placed in ECNs. Thus, in many cases institutional orders and non-market maker orders remained undisclosed to the public" (Division of Market Regulation (2000)). This means that the best price in an ECN is likely to be better than the Nasdaq best price. Consequently, to spot the universal best price of a security the public has to compare the best price in public Nasdaq quotes with the best prices of all private ECNs.

[13] I would like to thank an anonymous referee who suggested these tests.

[14] Nasdaq was, in fact, created in 1971 to ease the fragmentation of over-the-counter markets.

[15] Barclay et al. (1999) find that the quoted and effective spreads for Nasdaq stocks declined approximately 30% while Weston (2000) reports that realized spread declined by 0.085 percentage points on average.

[16] Market makers and specialists here are narrowly defined while in the formal model of the previous sections they include all liquidity suppliers (limit order traders in particular).

As of the year 2000 when there were nine ECNs operating (Division of Market Regulation (2000)), a comprehensive search for the best quote did cost a substantial amount of time. The results of our analysis imply that to incorporate these private trading venues into the national market system is important to the reduction of trading costs. The best quotes of all market participants should be included in the national public quotation system rather than just the best prices offered by market makers.

To form a centralized market without search costs, two elements are required. The first is that pretrade firm quotes and posttrade information are promptly accessible by all market participants. But more importantly, the displayed firm quotes must be nonnegotiable. If the quotes are negotiable and deals are executed inside the quotes, the displayed prices cannot be truly the best and the public has to conduct costly search or negotiation to find better prices. In this sense, the Nasdaq (even post reform) quote montage and London Stock Exchange's Stock Exchange Automatic Quotation system do not eliminate search because, although quotes in these systems are firm, they are still negotiable. As a result, orders are often executed inside the quotes for large tables. However, the market environment is changing. In 2002, Nasdaq launched SuperMontage, which fully integrates order display and execution systems for all Nasdaq National Market and Nasdaq SamllCap Market securities. Moreover, it allows all market participants to view the five most attractive prices rather than just the best price. These features have virtually converted Nasdaq into an automated centralized market. According to our theoretical analysis, it would significantly reduce trading costs on Nasdaq.

Our analysis shows that market markers prefer more opaque and fragmented markets to more transparent and centralized markets. Then, why would market makers form or participate in centralized markets, and why do traditionally fragmented markets such as Nasdaq increasingly become more integrated? The answer to this question lies behind the other side of the market—outside investors. As our model shows, the public prefers centralized markets for lower trading costs. This creates three forces for the transparency and centralization of markets. First, regulatory authorities promote market transparency and centralization through legislation to protect outside investors. Typical examples are the creation of Nasdaq in 1971, which established a nation-wide automated quoting system for a large number of over-the-counter market makers, and new SEC Order Handling Rules introduced in 1997. Second, outside investors' trading preferences imply that they would choose the centralized market rather than the fragmented market if there are two venues trading the same security. Thus, the competition between markets coerces fragmented markets to become more transparent and centralized. This is evidenced by the London Stock Exchange's move to electronic order book trading for the FT100 in 1997 and Nasdaq's introduction of the Small Order Execution System in 1984 and SuperMontage in 2002. Related to the second factor is the third factor—trading volume. High trading costs inevitably reduce liquidity demand and trading volume and, consequently, the profitability of dealers. Therefore, unlike the first force, the second and third forces drive market makers into more transparent and centralized markets. Limited by the scope, this paper does not formally analyze these two factors. Incorporating them into a formal model to study the endogenous choice of market type is an interesting topic for further investigation.

VI. Concluding Remarks

Prices in financial markets change much more frequently than in other markets. Therefore, market transparency facilitating market participants instantaneously acquiring pre- and post-trade information has a more significant role in the determination of participants' behavior and market performance. This paper develops a model based on Biais (1993) by integrating an important feature of fragmented dealership markets—information search costs. It is found that an insignificant search cost borne by liquidity traders in searching for a better price can substantially change market makers' pricing strategies. More specifically, search costs are likely to make liquidity traders stop searching when they find a satisfactory (but not necessarily the best) price. Noting this possibility, dealers in the market quote less aggressively in comparison with their counterparts in a fragmented market with costless search. Therefore, the model predicts that fragmented markets yield higher expected bid–ask spreads than centralized markets ceteris paribus, and that dealers (liquidity traders) prefer fragmented (centralized) markets to centralized (fragmented) markets. These predictions are in contrast with Biais (1993) and dFM, who assume no search costs in fragmented markets.

Fragmented markets, where search is either costly or costless, are more opaque than centralized markets. Why are their performances relative to centralized markets so different? The key point is how market transparency affects the behavior of agents in markets. As indicated by Ho and Stoll (1983), the centralized market in our model acts as an open English (upward) auction where all participants have complete bidding information. So, the liquidity trader acting as the auctioneer has the market power to suppress the price to the second lowest ask (or the second highest bid) reservation price of market makers. The fragmented market without search costs as modeled by Biais (1993) resembles a sealed (Dutch) auction. While dealers can take advantage of the lack of visibility to post ask (bid) prices higher (lower) than their reservations, the liquidity trader has the market power to pick the lowest ask (highest bid) among all quotes. Thus, the change in market structure from a centralized market to a zero-search-cost fragmented market results in two variations: (i) It enforces the liquidity trader's market power by allowing her to pick the best rather than the second best price; (ii) It enables dealers to post higher ask (lower bid) prices for a given inventory position. Biais (1993) shows these effects are approximately offset on average while dFM demonstrates the former effect is greater on average. However, when liquidity traders have to pay search costs, sampling all dealers is no longer a credible strategy. Thus, a liquidity trader is likely to execute a trade at the second best or further inferior price (to the liquidity trader) in the fragmented market. The liquidity trader in such a market does not necessarily have more market power than her counterpart in the centralized market. In contrast, the dealers in the positive-search-cost fragment market quotes ask (bid) prices not only higher (lower) than their reservation prices, but also higher (lower) than corresponding quotes in the zero-search-cost fragmented market. Consequently, very small search costs can make a substantial difference in market performance.

Appendix

Proof of Lemma 1: Mathematic induction is used to prove the lemma.

(i) Suppose that dealer i is sampled by the public in the first-round search and the quote is a_i. The gain of doing the second-round search is

$$Q\int_{-\infty}^{\infty}\max(a_i-a,0)\,dH(a)=Q\int_{-\infty}^{a_i}(a_i-a)\,dH(a)$$
$$=Q\left[a_iH(a_i)-\int_{-\infty}^{a_i}a\,dH(a)\right].$$

Noting the right-hand side is increasing in a_i, the second-round search produces nonnegative net gain (net of search cost c) if and only if $a_i \geq a^*$.

(ii) Assume the searching rule is true for the case where k dealers have been sampled. Now, consider that $k+1$ dealers have been sampled and dealer i is sampled in the $(k+1)$th round. Let $a_{\min}$ be the minimum of ask premiums obtained from the first k rounds of sampling. Since the public does not stop searching in the kth round, it implies that $a_{\min} \geq a^*$ and $Q\int_{-\infty}^{a_{\min}}(a_{\min}-a)\,dH(a) > c$. The gain of doing the $(k+2)$th round search is

$$Q\int_{-\infty}^{\min(a_i,a_{\min})}[\min(a_i,a_{\min})-a]\,dH(a). \tag{A1}$$

If $a_i > a_{\min} \geq a^*$, we have

$$(\mathrm{A1})=Q\int_{-\infty}^{a_{\min}}(a_{\min}-a)\,dH(a)>c.$$

So the public should do the $(k+2)$th round search. On the other hand, if $a_i \leq a_{\min}$, we have

$$(\mathrm{A1})=Q\int_{-\infty}^{a_i}(a_i-a)\,dH(a)=Q\left[a_iH(a_i)-\int_{-\infty}^{a_i}a\,dH(a)\right].$$

By the same argument as in (i), we find that the public should continue the $(k+2)$th round search if and only if $a_i \geq a^*$. Q.E.D.

Proof of Lemma 2: If dealer i sets $a_i < a^*$ and the positive-cost liquidity trader adopts the searching rule in Lemma 1, he wins the deal as long as he is sampled, independent of the competing prices. Thus, the probability of the public purchasing the risky security from dealer i in her first round search is $1/N$. With probability $(N-1)/N$, the public does not visit dealer i in the first search. But, the probability that the public does not purchase from dealer $j \neq i$ and continues searching after she samples dealer j is $\Pr(a_j > a^*)$. In the second search, the probability of dealer i being sampled is $1/(N-1)$. Thus, the probability that dealer i sells the asset to the public in the second round of sampling is $[(N-1)/N]\Pr(a_j \geq a^*)[1/(N-1)] = \Pr(a_j > a^*)/N$. Since there are N dealers, the probability of the public purchasing from dealer i is the sum of a series of probabilities that the public purchases from dealer i in the first, second, third, ..., round of search. That is,

$$\varphi = \frac{1}{N} + \frac{\Pr\left(a_j \geq a^*\right)}{N} + \cdots + \frac{\left[\Pr\left(a_j \geq a^*\right)\right]^{N-1}}{N}, \quad \forall j \neq i.$$

Recalling $a_j = a(I_j)$ and $\Pr(a_j \geq a^*) = F(a^{-1}(a^*))$, the above equation yields (7).

On the other hand, if the public has a zero searching cost or dealer i sets an ask premium $a_i \geq a^*$, the public buys the security from dealer i only if she has visited all dealers and dealer i quotes the lowest ask premium. So, the probability of winning the deal is

$$\pi_i = \prod\nolimits_{j \neq i} \Pr(a_i < a_j) = F^{N-1}\left(a^{-1}(a_i)\right).$$

Q.E.D.

Proof of Lemma 3: Consider three cases separately.

(i) $a^* \in [\underline{a}, \bar{a}]$. If dealer i sets $a_i \geq a^*$, the optimization problem is the same as (10) in dFM, and $a^D(I)$ in (5) is their (A5). But, when the dealer sets $a_i < a^*$, the FOC of (12) is

$$(1-p)\frac{d\pi_i}{da_i}\left(1 - e^{-RQ(a_i - a_{r,i})}\right) + (1-p)\pi_i e^{-RQ(a_i - a_{r,i})} RQ$$
$$+ p\varphi e^{-RQ(a_i - a_{r,i})} RQ = 0.$$

Applying (8) and multiplying both sides by $e^{-RQa_{r,i}} \frac{da_i}{dI_i}$:

$$(1-p)\frac{dF^{N-1}(I_i)}{dI_i}\left(e^{-RQa_{r,i}} - e^{-RQa_i}\right)$$
$$+ \left[(1-p)F^{N-1}(I_i) + p\varphi\right] e^{-RQa_i} RQ \frac{da_i}{dI_i} = 0. \quad \text{(A2)}$$

Because $a^* \in [\underline{a}, \bar{a}]$ and the left-hand side of (14) is downward sloping, equation (14) has a unique solution of I^* on $[-M, M]$ for any given a^*, and I^* is implicitly determined by $a^* = a^D(I^*)$. By integrating (A2) and routine calculation, we obtain

$$(1-p)\int_{I^*}^{I_i} e^{-RQa_r(x)}dF^{N-1}(x) - \big[(1-p)F^{N-1}(I_i) + p\varphi\big]e^{-RQa(I_i)}$$
$$+\big[(1-p)F^{N-1}(I^*) + p\varphi\big]e^{-RQa(I^*)} = 0.$$

Rearranging the terms and using the continuity of $a(I)$ at I^* yield $a_i = a^Y(I_i, I^*)$.

(ii) $a^* > \bar{a}$. In this case, the dealer never chooses $a_i \geq a^*$. This is because if the dealer sets $a_i \geq a^*$, the optimal ask premium is $a_i = a^D(I_i) < \bar{a}$. This is in contradiction with $a_i \geq a^* > \bar{a}$. For $a_i < a^*$, extending the integration in (i) from $[I^*, I_i]$ to $[-M, I_i]$ yields the pricing strategy that $a_i = \tilde{a}^Y(I_i)$, where

$$\tilde{a}^Y(I) \equiv \frac{-1}{RQ}\ln\left[\frac{(1-p)\displaystyle\int_{-M}^{I} e^{-RQa_r(x)}dF^{N-1}(x) + p\varphi e^{-RQ\tilde{a}^Y(-M)}}{(1-p)F^{N-1}(I) + p\varphi}\right].$$

Since $a^Y(I, -M) = \tilde{a}^Y(I)$ if $\tilde{a}^Y(-M) = a_r(-M)$ and $\lim_{a^*\to\bar{a}-0} a(I) = a^Y(I, -M)$, we choose $\tilde{a}^Y(-M) = a_r(-M)$ to ensure that the pricing strategies are identical when $a^* \to \bar{a} - 0$ and $a^* \to \bar{a} + 0$.

(iii) $a^* < \underline{a}$. In this case, the dealer never chooses $a_i < a^*$. I show this by making a contradiction. Suppose the dealer sets $a_i < a^*$ on $I_i \in [I', I'']$ but sets $a_i \geq a^*$ when $I_i < I'$. If $I' > -M$, then the optimal ask premium for $I_i < I'$ is $a^D(I_i)$ and we have $a(I' - 0) = a^D(I') \geq \underline{a} > a^* > a(I' + 0)$, which is in contradiction with the continuity of price strategy. If $I' = -M$, then $a(-M) < a^* < \underline{a} < a_r(-M)$. But, $1 + a_r(-M)$ is the reservation price of a dealer with inventory $-M$, and he is not willing to sell the asset below that price. So, the dealer must set $a_i \geq a^*$, and the pricing rule is $a_i = a^D(I_i)$ for all $I_i \in [-M, M]$. Q.E.D.

Proof of Proposition 1: According to Lemma 3, the dealers' optimal pricing strategy is given by equation (13) if $a^* \in [\underline{a}, \bar{a}]$. So, substituting (13) into (6) yields (16). After substituting a^* in (16) into (14), the difference between the left-hand side and right-hand side of (14) is equal to

$$L(I^*) \equiv \ln\left(F^{1-N}(I^*)\int_{-M}^{I^*} e^{-RQa_r(x)}dF^{N-1}(x)\right)$$
$$+RQ\left[c/Q + \int_{I^*}^{M} a^Y(x, I^*)\,dF(x)\right]\Big/[1 - F(I^*)].$$

Thus,

$$\lim_{I^*\to M} L(I^*) = +\infty,$$

$$L(-M) = RQ\left[c/Q + \int_{-M}^{M} a^Y(x, -M)\,dF(x) - a_r(-M)\right].$$

Because $L(-M) \leq 0$ when

$$\bar{a} \geq \int_{-M}^{M} a^Y(x, -M)\, dF(x) + c/Q,$$

$L(I^*)$ has a zero point on $[-M, M]$; that is, there exists an I^* satisfying (14).

On the other hand, if

$$\bar{a} < \int_{-M}^{M} a^Y(x, -M)\, dF(x) + c/Q,$$

the public chooses the searching rule in Lemma 1 with

$$a^* = \int_{-M}^{M} a^Y(x, -M)\, dF(x) + c/Q > \bar{a},$$

and the dealers follow equilibrium pricing rule $a(I) = a^Y(I, -M)$. Q.E.D.

Proof of Proposition 2: First, I prove the following lemma.

LEMMA 4: *For an equilibrium $I^* \in [-M, M)$, there is $a^D(I) \leq a^Y(I, I^*)$ on $I \in [I^*, M]$.*

Proof: By definition, $a^D(I^*) = a^Y(I^*, I^*)$. The proof of Lemma 3 shows that

$$\frac{\partial a^Y(I, I^*)}{\partial I} = -\frac{(1-p)\left(e^{RQa^Y(I,I^*)-RQa_r(I)} - 1\right)}{RQ[(1-p)F^{N-1}(I) + p\varphi]} \frac{dF^{N-1}(I)}{dI}. \tag{A3}$$

Similarly,

$$\frac{da^D(I)}{dI} = -\frac{e^{RQa^D(I)-RQa_r(I)} - 1}{RQF^{N-1}(I)} \frac{dF^{N-1}(I)}{dI}. \tag{A4}$$

Now, I show that for any I' it is impossible that $a^D(I) > a^Y(I, I^*)$ on $[I^*, I']$. If it is true, (A3) and (A4) imply $\frac{da^D(I)}{\partial I} < \frac{\partial a^Y(I, I^*)}{\partial I}$ on $[I^*, I']$. Since $a^D(I^*) = a^Y(I^*, I^*)$, the last inequality implies that $a^D(I) < a^Y(I, I^*)$ on $[I^*, I']$, and we obtain a contradiction.

Second, dFM shows that the price in the fragmented market without search costs is $A^D = 1 + a^D(I_N^*)$. In the fragmented market with search costs, the price is determined by (17) or (18). Applying Lemma 4 we obtain

$$a^Y\left(I_g, I^*\right) \geq a^Y\left(I_N^*, I^*\right) \geq a^D\left(I_N^*\right),$$

$$a^Y(I_f, -M) \geq a^Y\left(I_N^*, -M\right) \geq a^D\left(I_N^*\right).$$

Thus, under all scenarios, there is $A^D \leq A^Y$. Q.E.D.

Proof of Proposition 4: Since all dealers have identical CARA preferences, the dealers' aggregate posttrade risk can be measured by the sum of the variances of their posttrade assets. For a given pretrade inventory profile, only the traded dealer's inventory is reduced by Q. So, if the dealer with inventory position $I_c (I_c < I_N^*)$ wins the deal in the fragmented market, the difference of posttrade risk between the fragmented and centralized markets is equal to

$$\left[\mathrm{Var}\left(I_c - Q\right) + \mathrm{Var}\left(I_N^*\right)\right]\sigma^2 - \left[\mathrm{Var}(I_c) + \mathrm{Var}\left(I_N^* - Q\right)\right]\sigma^2 < 0.$$

If the dealer with the longest inventory wins the deal in the fragmented market, then the aggregate posttrade risks are the same. Q.E.D.

Proof of Proposition 5: Since

$$\lim_{p\to 1} a^Y(I, -M) = (RQ)^{-1} \ln e^{-RQa^D(-M)} = \bar{a},$$

we have

$$\bar{a} < \int_{-M}^{M} a^Y(I, -M)\, dF(I) + c/Q$$

when p is sufficiently close to 1. According to equation (18), the equilibrium ask premium in this case is $a^Y(I_f, -M)$ or $a^Y(I_N^*, -M)$. On the other hand, routine calculation shows that

$$\frac{da^B(I)}{dI} = -\frac{R\sigma^2 \displaystyle\int_{-M}^{I} F^{N-1}(x)\, dx}{[F^{N-1}(I)]^2} \frac{dF^{N-1}(I)}{dI} < 0,$$

$$\frac{\partial a^Y(I, -M)}{\partial I}$$
$$= \frac{(1-p)\left\{(1-p)\displaystyle\int_{-M}^{I} e^{-RQa_r(x)} dF^{N-1}(x) + p\varphi e^{-RQa_r(-M)} - e^{-RQa_r(I)}[(1-p)F^{N-1}(I) + p\varphi]\right\}}{RQ\left[(1-p)\displaystyle\int_{-M}^{I} e^{-RQa_r(x)} dF^{N-1}(x) + p\varphi e^{-RQa_r(-M)}\right][(1-p)F^{N-1}(I) + p\varphi]}$$
$$\times \frac{dF^{N-1}(I)}{dI}.$$

Thus, $\partial a^Y(I, -M)/\partial I = 0$ when $p \to 1$. Since $a^Y(-M, -M) = \bar{a} = a^B(-M)$, we have

$$a^Y(I_f, -M) \geq a^Y\left(I_N^*, -M\right) \geq a^B\left(I_N^*\right),$$

when p is sufficiently close to 1 and

$$pE\left(a^Y(I_f, -M)\right) + \left(1-p\right)E\left(a^Y\left(I_N^*, -M\right)\right) \geq E\left(a^B\left(I_N^*\right)\right) = E\left(a_r\left(I_{N-1}^*\right)\right).$$

The last equality is proven by Proposition 4 of Biais (1993). Q.E.D.

REFERENCES

Affleck-Graves, John, Shantaram P. Hagde, and Robert E. Miller, 1994, Trading mechanism and the components of the bid-ask spread, *Journal of Finance* 49, 1471–1488.

Barclay, Michael J., 1997, Bid-ask spreads and the avoidance of odd-eighth quotes on Nasdaq: An examination of exchange listings, *Journal of Financial Economics* 45, 35–60.

Barclay, Michael J., William G. Christie, Jeffrey H. Harris, Eugene Kandel, and Paul H. Schultz, 1999, Effects of market reform on the trading costs and depths of Nasdaq stocks, *Journal of Finance* 54, 1–34.

Biais, Bruno, 1993, Price formation and equilibrium liquidity in fragmented and centralized markets, *Journal of Finance* 48, 157–185.

Bloomfield, Robert, and Maureen O'Hara, 1999, Market transparency: Who wins and who loses? *Review of Financial Studies* 12, 5–35.

Christie, William G., and Roger D. Huang, 1994, Market structures and liquidity: A transaction data study of exchange listings, *Journal of Financial Intermediation* 3, 300–326.

Christie, William G., and Paul H. Schultz, 1994, Why do Nasdaq market makers avoid odd-eighth quotes? *Journal of Finance* 49, 1813–1840.

de Frutos, M. Ángeles, and Carolina Manzano, 2002, Risk aversion, transparency, and market performance, *Journal of Finance* 57, 959–984.

Division of Market Regulation, U.S. Securities and Exchange Commission, 2000, *Special study: Electronic communication networks and after-hours trading*. Available at http://www.sec.gov/news/studies/ecnafter.htm.

Dutta, Prajit K., and Ananth Madhavan, 1997, Competition and collusion in dealer markets, *Journal of Finance* 52, 245–276.

Flood, Mark D., Ronald Huisman, Kees G. Keodijk, and Ronald J. Mahieu, 1999, Quote disclosure and price discovery in multiple-dealer financial markets, *Review of Financial Studies* 12, 37–59.

Godek, Paul E., 1996, Why Nasdaq market makers avoid odd-eighth quotes, *Journal of Financial Economics* 41, 465–474.

Hansch, Oliver, Narayan Y. Naik, and S. Viswanathan, 1998, Do inventories matter in dealership markets? Evidence from the London Stock Exchange, *Journal of Finance* 53, 1623–1656.

Ho, Thomas S. Y., and Hans R. Stoll, 1983, The dynamics of dealer markets under competition, *Journal of Finance* 38, 1053–1074.

Huang, Roger D., and Hans R. Stoll, 1996, Dealer versus auction markets: A paired comparison of execution costs on Nasdaq and the NYSE, *Journal of Financial Economics* 41, 313–357.

Lyons, Richard K., 1995, Tests of microstructural hypotheses in the foreign exchange market, *Journal of Financial Economics* 39, 321–351.

Madhavan, Ananth, 1995, Consolidation, fragmentation, and the disclosure of trading information, *Review of Financial Studies* 8, 579–603.

Pagano, Marco, and Ailsa Röell, 1996, Transparency and liquidity: A comparison of auction and dealer markets with informed trading, *Journal of Finance* 51, 579–611.

Parlour, Christine A., and Uday Rajan, 2001, Payment for order flow, Working paper, GSIA, Carnegie Mellon University.

Simaan, Yusif, Daniel G. Weaver, and David K. Whitcomb, 2003, Market maker quotation behavior and pretrade transparency, *Journal of Finance* 58, 1247–1267.

Sirri, Erik R., and Peter Tufano, 1998, Costly search and mutual fund flows, *Journal of Finance* 53, 1589–1622.

Stiglitz, Joseph E., 1979, Equilibrium in product markets with imperfect information, *American Economic Review* 69, 339–345.

Weston, James P., 2000, Competition on the Nasdaq and the impact of recent market reforms, *Journal of Finance* 55, 2565–2598.

Williams, Joseph T., 1995, Pricing real assets with costly search, *Review of Financial Studies* 8, 55–90.

Yavaş, Abdulah, 1993, A simple model of bid-ask spread and search, *Review of Financial Economics* 2, 73–84.