

## Does Net Buying Pressure Affect the Shape of Implied Volatility Functions?

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### ABSTRACT

This paper examines the relation between net buying pressure and the shape of the implied volatility function (IVF) for index and individual stock options. We find that changes in implied volatility are directly related to net buying pressure from public order flow. We also find that changes in implied volatility of S&P 500 options are most strongly affected by buying pressure for index puts, while changes in implied volatility of stock options are dominated by call option demand. Simulated delta-neutral option-writing trading strategies generate abnormal returns that match the deviations of the IVFs above realized historical return volatilities.

If people are willing to pay foolish prices for insurance, why shouldn't we sell it to them? (Lowenstein (2000)).

ONE OF THE MOST INTRIGUING ANOMALIES REPORTED in the derivatives literature is the "implied volatility smile." The name arose from the fact that, prior to the October 1987 market crash, the relation between the Black and Scholes (1973) implied volatility of S&P 500 index options and exercise price gave the appearance of a smile. Since October 1987, however, the index implied volatility function (hereafter, IVF), as we refer to it, decreases monotonically across exercise prices. Under the assumptions of the Black–Scholes model, the IVF should be flat and constant through time.

Most attempts to explain the shape of the IVF focus on relaxing the Black–Scholes assumption of constant volatility by allowing the local volatility rate of underlying security returns to evolve either deterministically or stochastically through time. Emanuel and MacBeth (1982) examine the power of the deterministic Cox and Ross (1976) constant elasticity of variance (CEV) model to explain the cross-sectional distribution of stock option prices. With its additional degree of freedom, the CEV model (necessarily) fits the observed structure of option prices better than the Black–Scholes constant volatility model. Out of sample, however, Emanuel and MacBeth conclude that the CEV model does

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no better than the Black–Scholes model. Similarly, the implied binomial tree framework of Dupire (1994), Derman and Kani (1994), and Rubinstein (1994) offers a deterministic local volatility structure so flexible that, in sample, it can describe the cross-section of options prices exactly at any point in time.<sup>1</sup> Empirical tests by Dumas, Fleming, and Whaley (1998), however, show that a model based on a simple deterministic volatility structure has parameters that are highly unstable through time. Taken together, this evidence suggests that deterministic volatility models cannot explain the time-series variation in option prices or, equivalently, in the shape of the IVF.

Option valuation models based on stochastic volatility assumptions also have the potential to explain the shape of the IVF. In particular, a stochastic volatility model can generate the observed downward sloping IVF if innovations to volatility are negatively correlated with underlying asset returns. A negative relation between volatility and returns has been documented empirically by Black (1976) for individual stocks and Nelson (1991) for the index. Chernov et al. (2003) study a two-factor stochastic volatility model and find that it achieves a good fit to daily Dow Jones Industrial Average returns. Studies by Jorion (1989) and Anderson, Benzoni, and Lund (2002) report that randomly arriving jumps in security price in addition to stochastic volatility are required to capture the time-series dynamics of index returns.

Recent examinations of the performance of stochastic volatility option valuation models indicate that, at best, they can provide only a partial explanation of the shape of the index IVF.<sup>2</sup> Bakshi, Cao, and Chen (1997), for example, advocate the use of a stochastic volatility model with jumps for valuing S&P 500 index options. While their model appears to perform better than the Black–Scholes formula, some of the implied parameter estimates, including the volatility of volatility coefficient, differ significantly from the ones estimated directly from returns. Similarly, Bates (2000) examines the ability of a stochastic volatility model, with and without jumps, to generate the negative skewness consistent with a steep IVF. He finds that the inclusion of a jump process can improve the model's ability to generate IVFs consistent with market prices, but in order to do so parameters must be set to unreasonable values.<sup>3</sup> Along a

<sup>1</sup> Deterministic volatility models assume that the local volatility rate is a general function of security price and time. The CEV model is a special case of the deterministic volatility model in which the local volatility rate has the form  $\sigma S^{a-1}$ , where  $a$  falls in the range  $0 \leq a \leq 1$ . The Black–Scholes model, in turn, is a special case of the CEV model where  $a = 1$ .

<sup>2</sup> In related work on the time-series properties of stock indexes, Eraker, Johannes, and Polson (2003) investigate models with jumps in returns and volatility and conclude that models without jumps are misspecified for the S&P 500 and Nasdaq 100 indexes. They also illustrate the effect that volatility jumps may have on option prices and note that parameter uncertainty implies wide bands on option values. Similarly, David and Veronesi (2000) construct a theoretical model in which two processes with different drifts govern asset dividends, with an unobservable regime-switching process determining which dividend process is in effect. In such a model, stochastic volatility in asset returns is an equilibrium result, and IVFs will be negatively sloped.

<sup>3</sup> Pan (2002) studies a stochastic volatility model with jumps and argues that the presence of a jump-risk premium correlated with volatility can capture the structure of index option prices.

similar line, Jackwerth (2000) attempts to recover risk aversion functions from S&P 500 index option prices and concludes that they are “irreconcilable with a representative investor” (p. 450).<sup>4</sup>

Another avenue of investigation that may lead to a better understanding of the IVF is the study of option market participants’ supply and demand for different option series<sup>5</sup> in different option markets. One way to think of the IVF is as a series of market clearing option prices quoted in terms of Black–Scholes implied volatilities. Under dynamic replication, the supply curve for each option series is a horizontal line. No matter how large the demand for buying a particular option, its price and implied volatility are unaffected. In reality, however, there are limits to arbitrage. Shleifer and Vishny (1997) describe how the ability of professional arbitrageurs to exploit mispriced securities is limited by the responsiveness of investors to intermediate losses. Liu and Longstaff (2000) show that it is often optimal for a risk-averse investor to take a smaller position in a profitable arbitrage than his margin constraints allow, since intermediate mark-to-market losses may force liquidation of his position prior to convergence. In the same way, a market maker will not stand ready to sell an unlimited number of contracts in a particular option series. As his position grows large and imbalanced, his hedging costs and/or volatility-risk exposure also increase, and he is forced to charge a higher price. With an upward sloping supply curve, differently shaped IVFs in different markets can be expected. The result of these limits to arbitrage is that market prices can diverge from model values, and that the divergence can persist. In effect, the no-arbitrage band within which prices can fluctuate can be quite wide, allowing price to be affected by supply and demand considerations.

Interacting with the market maker’s willingness to supply options is investor demand. The level of implied volatility will be higher or lower depending upon whether net public demand for a particular option series is to buy or to sell. In the S&P 500 index option market, for example, it is well known that institutional investors buy index puts as portfolio insurance. Unfortunately, there are no natural counter-parties to these trades, and market makers must step in to absorb the imbalance. With an upward sloping supply curve, implied volatility will exceed actual return volatility, with the difference being the market maker’s compensation for hedging costs and/or exposure to volatility risk.<sup>6</sup> If institutional demand tends to be focused in a particular option series, such as out-of-the-money puts, the IVF will be downward sloping.

Our paper investigates the role of supply and demand in the options market by assessing the relation between net buying pressure and the movement and shape of the IVF of S&P 500 index options and options on 20 individual

<sup>4</sup> Using a new trading strategy test methodology, Bondarenko (2003) examines prices of out-of-the-money puts written on the S&P 500 futures during the period 1988 through 2000 and concludes the market is inefficient.

<sup>5</sup> An option series is defined by three attributes—call or put, exercise price, and expiration date.

<sup>6</sup> In contrast, the ability to dynamically replicate option positions in the idealized (frictionless) Black–Scholes world ensures that the market maker earns the risk-free rate of return.

stocks. We define *net buying pressure* as the difference between the number of buyer-motivated contracts traded each day and the number of seller-motivated contracts traded. Trades executed at a price above (below) the prevailing bid/ask midpoint are categorized as buyer-motivated (seller-motivated). The difference is computed on a series-by-series basis, and is multiplied by the absolute value of the option's delta to express demand in stock/index equivalent units.

The empirical test design is motivated by the different demands for index options and stock options. We document that most trading in index options involves puts, whereas most trading in stock options involves calls. If net buying pressure plays an important role in the options market, the different demands for index options and stock options implies that the shape of the index IVF should differ from the shape of the typical stock IVF. We characterize the shape of the index and individual stock IVFs by calculating the implied volatilities of options in five moneyness categories. Consistent with the results of Bakshi, Kapadia, and Madan (2003), and Dennis and Mayhew (2002), we find that the index IVF is significantly more negatively sloped than individual equity option IVFs. Bakshi, Kapadia, and Madan characterize the structure of the IVF by estimating the risk-neutral skewness of the return distributions of the index and of individual stocks implicit in option prices. They then explain the difference between the risk-neutral skewness of the index and that of individual stocks in the context of an asset-pricing model. In contrast, we ascribe the difference between the index IVF and individual equity option IVFs to differential demands for index options vis-à-vis stock options.

Our empirical evidence supporting the net buying pressure hypothesis has two parts. First, we assess the time-series relation between net buying pressure and the shape of the IVF. We find that changes in the level of an option's implied volatility are positively related to time variation in demand for the option, and that these changes are transitory. Second, we simulate the abnormal returns of a delta-neutral trading strategy that systematically sells options. For index options, we find significantly positive abnormal returns when selling options across the range of exercise prices, with the lowest exercise prices (e.g., out-of-the-money puts) being most profitable. In contrast, abnormal returns from selling stock options are smaller and symmetric across exercise prices. Interestingly, these patterns of profitability are consistent with the respective deviations of the IVFs from realized volatility and with known demands of investors for different options. Overall, the results suggest that net buying pressure plays an important role in determining the shape of IVFs, particularly for options on the S&P 500 index where public order imbalances are greatest. The results support the hypothesis that the IVF reflects a series of supply and demand equilibria.

The paper is organized as follows. Section I describes the sample data and the basic empirical methodology used in the analyses, and illustrates the nature of the IVFs during the sample period. Section II explores the link between demand for different options and movements in implied volatility. Section III presents a simulated trading strategy. Section IV summarizes the main results of the paper.

## **I. Sample Description**

The purpose of this section is to describe the data as well as the methods used to generate implied volatilities, and to provide a general characterization of the index and individual stock IVFs during the sample period.

### *A. Data*

The data used in the tests that follow were drawn from a variety of sources. Our index option sample contains trades and quotes of S&P 500 index options traded on the Chicago Board Options Exchange (CBOE)<sup>7</sup> over the period June 1988 through December 2000.<sup>8</sup> S&P 500 options are European-style and expire on the third Friday of the contract month. Originally, these options expired only at the market close and were denoted by the ticker symbol SPX. In June 1987, when the Chicago Mercantile Exchange (CME) changed its S&P 500 futures expiration from the close to the open, the CBOE introduced a second set of options with the ticker symbol NSX that expired at the open. Over time, the trading volume of this “open-expiry” series grew to surpass that of the “close-expiry” series, and on August 24, 1992, the CBOE reversed the ticker symbols of the two series. Our sample contains SPX options throughout: Close-expiry until August 24, 1992, and open-expiry thereafter. During the first subperiod, the option’s time to expiration is measured as the number of calendar days between the trade date and the expiration date. During the second, we use the number of calendar days remaining less one.

Our stock option sample contains trades and quotes of CBOE options on 20 individual stocks over the period January 1995 through December 2000. This particular set of stock option classes had continuous listing on the CBOE during the sample period and were the most actively traded. The 20 underlying stocks are listed in Table I. Individual stock options are American-style and expire on the Saturday following the third Friday of the contract month. Time to expiration is, therefore, measured as the number of calendar days between the trade date and the expiration date.

Estimating implied volatilities requires estimates of the risk-free interest rate and the expected dividends paid during an option’s life. We proxy for the risk-free interest rate using Eurodollar spot rates. The 1-day, 7-day, 1-month, 3-month, 6-month, and 1-year nominal rates were downloaded from Datas-tream and were converted into continuous rates. The interest rate for a particular maturity  $t$  is computed by linearly interpolating between the two continuous rates whose maturities straddle  $t$ . We proxy for expected dividends using

<sup>7</sup> The data, from the CBOE’s proprietary Master Data Retrieval (MDR) files, include not only the time-stamped option trades and quotes but also the contemporaneous price of the underlying index/stock.

<sup>8</sup> The sample begins in June 1988 because it was the first month for which Standard and Poors’ began reporting daily cash dividends for the S&P 500 index portfolio. See Harvey and Whaley (1992) regarding the importance of incorporating discrete daily cash dividends in index option valuation.

**Table I**  
**Twenty Stocks Underlying the Individual Stock Options**

Options on these stocks had continuous listing during the sample period January 1995 through December 2000, and were the most active.

Ticker	Company Name
AIG	AMERICAN INTERNATIONAL GROUP INC
AOL	AMERICA ONLINE INC
BMJ	BRISTOL MYERS SQUIBB CO
CL	COLGATE PALMOLIVE CO
CSC	COMPUTER SCIENCES CORP
CSCO	CISCO SYSTEMS INC
DAL	DELTA AIR LINES INC
DOW	DOW CHEMICAL CO
GE	GENERAL ELECTRIC CO
HWP	HEWLETT PACKARD CO
IBM	INTERNATIONAL BUSINESS MACHS COR
JNJ	JOHNSON & JOHNSON
MER	MERRILL LYNCH & CO INC
MMM	MINNESOTA MINING & MFG CO
MRK	MERCK & CO INC
SLB	SCHLUMBERGER LTD
TXN	TEXAS INSTRUMENTS INC
UAL	U A L CORP
XOM	EXXON MOBIL CORP
XRX	XEROX CORP

the actual dividends paid over an option's life. The daily cash dividends for the S&P 500 index portfolio were collected from the *S&P 500 Information Bulletin*. The cash dividends for the individual stocks were drawn from the CRSP daily data file. Many stocks in the sample experienced stock splits during the sample period. Information regarding the size of the split and the ex-split date were also drawn from CRSP.

Finally, in the trading strategy simulations, it was necessary to develop a proxy for the trading cost of the S&P 500 index portfolio. To do so, we used the bid/ask spread of the nearby S&P 500 futures contract traded on the CME. Index option market makers prefer using the S&P 500 index futures to hedge the delta-risk of their aggregate option positions.<sup>9</sup> Historical records of the bid/ask spread in the S&P 500 futures market are not kept, as the trading pit is an oral market. The bid/ask spread was estimated each day using times and sales data obtained from the Futures Industry Institute (FII) in Washington, D.C. Time and sales data are a form of censored transaction data recorded by futures exchanges throughout the trading day. Instead of recording the time and price of each trade, the exchange records only the time and price of a transaction

<sup>9</sup> Two other hedging alternatives are to (a) basket trade the index or (b) use the American Exchange's SPDRs. After scaling to an equivalent contract size, however, these alternatives prove to have higher bid/ask spreads.

if the price is different from the previously recorded price. Bid and ask quotes appear in this file only if the bid quote exceeds (or if the ask quote is below) the previously recorded transaction price. The bid/ask spread for the S&P 500 futures contract is estimated using the Smith and Whaley (1994) method of moments procedure.

### B. Implied Volatility Computation

With the data in hand, we compute implied volatilities. One implied volatility is computed for each option series each day. That volatility is based on the midpoint of the last pair of bid/ask price quotes prior to 3:00 PM (CST). We use the Black and Scholes (1973) formulae to compute implied volatilities for the European-style S&P 500 index options. The call ( $c$ ) and put ( $p$ ) option valuation formulae are

$$\begin{aligned} c &= (S - PVD)N(d_1) - Xe^{-rT}N(d_2) \quad \text{and} \\ p &= Xe^{-rT}N(-d_2) - (S - PVD)N(-d_1), \end{aligned} \quad (1)$$

where

$$d_1 = \frac{\ln((S - PVD)e^{rT}/X) + 0.5\sigma^2T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

The notation in (1) is as follows:  $S$  is the current index level,  $X$  is the option's exercise price,  $T$  is the option's time to expiration,  $r$  is the risk-free rate of interest, and  $N(\cdot)$  is the normal cumulative density function. To compute the present value of the dividends paid during the option's life,  $PVD$ , the daily dividends are discounted at the rates corresponding to the ex-dividend dates and summed over the life of the option, that is,

$$PVD = \sum_{t=1}^n D_t e^{-r_t t}, \quad (2)$$

where  $D_t$  is the  $t^{\text{th}}$  cash dividend payment,  $t$  is the time to ex-dividend from the current date,  $r_t$  is the  $t$ -period risk-free interest rate, and  $n$  is the number of dividend payments during the option's life. To compute implied volatilities for the American-style stock options, the dividend-adjusted binomial method is used.<sup>10</sup>

### C. Implied Volatility Functions

To characterize the shape of the IVF, we first group options into five different moneyness categories, as described below, and then compute an average implied volatility for each of the categories. In essence, an option's "moneyness"

<sup>10</sup> For a description of this valuation method, see Harvey and Whaley (1992).

is intended to reflect its likelihood of being in the money at expiration. Typically, it is measured as the relative difference between the forward price of the underlying asset and the option's exercise price,<sup>11</sup> that is,

$$\text{Moneyness} = \frac{(S - PVD)e^{rT}}{X} - 1. \quad (3)$$

The greater (lower) the level of moneyness, the more likely a call (put) will be exercised at expiration. Unfortunately, expression (3) fails to account for the fact that the likelihood that the option will be in the money at expiration also depends heavily on the volatility rate of the underlying asset and the time remaining to expiration of the option. This makes comparisons of IVFs across stocks and the index problematic. To account for these effects, we measure moneyness using the option's delta. Delta is sensitive to the volatility of the underlying asset as well as the option's time to expiration, as the delta of a European-style call option,

$$\Delta_C = N \left[ \frac{\ln((S - PVD)e^{rT}/X) + 0.5\sigma^2 T}{\sigma \sqrt{T}} \right] \quad (4)$$

shows. It ranges in value from zero to one, and can be loosely interpreted as the risk-neutral probability that the option will be in the money at expiration.<sup>12</sup>

Deltas are computed for each option series each day using the valuation methodologies and parameter assumptions described earlier. The proxy for the volatility rate is the realized return volatility of the underlying stock/index over the most recent sixty trading days. Based on their deltas, options are then placed into five moneyness categories. The upper and lower bounds of the moneyness categories are listed in Table II. Note that all of the delta pairings for the calls and puts reflect the fact that buying (selling) a call and selling (buying) a put is tantamount to buying (selling) the underlying asset. A put option with a delta of  $-0.125$  should have the same implied volatility as a call option with a delta of  $0.875$  by virtue of put-call parity.<sup>13</sup> Options with absolute deltas below  $0.02$  or above  $0.98$  are excluded due to the distortions caused by price discreteness.<sup>14</sup>

<sup>11</sup> Moneyness is also often written with the stock price net of the present value of the escrowed dividends in the numerator and the present value of the exercise price in the denominator. Numerically, of course, this is the same as the ratio in (3). In other instances, the reciprocal of the ratio in (3) is used.

<sup>12</sup> Technically it would be more correct to use the  $N(d_2)$  from the Black and Scholes (1973) valuation formula (1). We adopt the use of delta to conform with the industry practice of quoting Black-Scholes volatilities by option delta.

<sup>13</sup> Put-call parity is derived in Stoll (1969). Unlike the dynamic replication of the Black-Scholes model, the put and call prices are held into alignment by static arbitrage strategies called *conversion* and *reverse conversion*.

<sup>14</sup> To illustrate the magnitude of the possible distortions, consider a call option with an exercise price of 65 and a time to expiration of 30 days. If the interest rate is 5 percent and the underlying stock has a price of 52.10, has a volatility rate of 36 percent, and pays no dividends, the put's Black-Scholes value is 0.038 and its delta is 0.02. If, for reporting purposes, the option's price is rounded up to, say, the nearest one-eighth, the implied volatility of the option is 43.9 percent, 790 basis points higher than its actual level.



**Table II**  
**Moneyneess Category Definitions**

Listed are category numbers, labels, and corresponding delta ranges of options in our sample. Options with absolute deltas below 0.02 and above 0.98 are excluded.

Category	Labels	Range
1	Deep in-the-money (DITM) call	$0.875 < \Delta_C \leq 0.98$
	Deep out-of-the-money (DOTM) put	$-0.125 < \Delta_P \leq -0.02$
2	In-the-money (ITM) call	$0.625 < \Delta_C \leq 0.875$
	Out-of-the-money (OTM) put	$-0.375 < \Delta_P \leq -0.125$
3	At-the-money (ATM) call	$0.375 < \Delta_C \leq 0.625$
	At-the-money (ATM) put	$-0.625 < \Delta_P \leq -0.375$
4	Out-of-the-money (OTM) call	$0.125 < \Delta_C \leq 0.375$
	In-the-money (ITM) put	$-0.875 < \Delta_P \leq -0.625$
5	Deep out-of-the-money (DOTM) call	$0.02 < \Delta_C \leq 0.125$
	Deep in-the-money (DITM) put	$-0.98 < \Delta_P \leq -0.875$

#### *D. Empirical Properties of IVFs*

Figure 1 illustrates the time-series properties of the index IVF. Shown are the “level” of the index IVF, defined as the average implied volatility of ATM or category 3 options, as well as the “slope” of the index IVF, defined as the percentage difference between the implied volatility of category 2 options and the implied volatility of category 3 options. In Figure 1A, the level and slope of the index IVF, as well as the level of the S&P 500 index, are plotted over the full sample. A salient feature of this plot is that while the level of the index IVF evolved relatively smoothly over time, the slope varied dramatically from month to month. Figure 1B makes this point clear by plotting the first difference of the level and slope of the index IVF. Even though the two variables have similar ranges over the sample period, the first difference of the slope is much more volatile than the first difference of the level.

Table III contains the average implied volatilities of the S&P 500 index options as well as of the 20 individual stock options over the period January 1995 to December 2000. As the results in the table show, the index IVF is monotonically decreasing across the delta categories. The average implied volatility of the category 1 options (DOTM puts and DITM calls) is 26.25 percent, about 55 percent higher than the average implied volatility of category 5 options (DITM puts and DOTM calls), 16.94 percent.

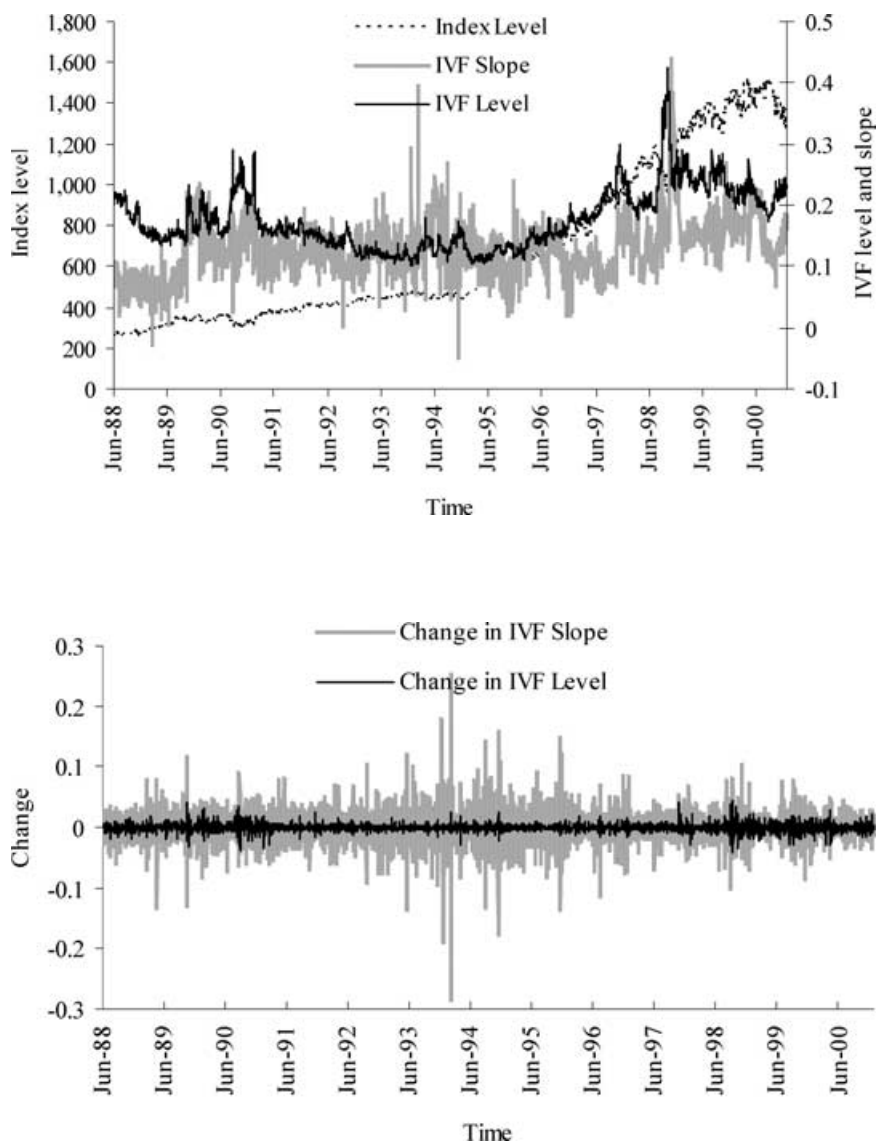
The average stock option IVF differs remarkably from the index option IVF, as is shown in Figure 2.<sup>15</sup> In place of the monotonically declining index IVF, stock

<sup>15</sup> It is worthwhile to note that IVFs such as those shown in Figure 2 usually appear to be quite smooth and not jagged. In part, this may be due to a no-arbitrage convexity condition. The convexity relation, as it applies to call options, is  $C(X_2) \leq qC(X_1) + (1 - q)C(X_3)$ , where the option exercise prices have the order  $X_1 < X_2 < X_3$  and  $q$  is defined by  $X_2 = qX_1 + (1 - q)X_3$ . In the event the convexity relation is violated, a costless arbitrage profit may be earned by engaging in a “butterfly spread,” that is, by selling the call with exercise price  $X_2$ , and buying  $q$  and  $1 - q$  units of the calls with exercise prices  $X_1$  and  $X_3$ , respectively.

option IVFs “smile,” with the implied volatility of the ATM options generally being lowest and symmetrically increasing with movement in either direction. In addition, the stock IVFs are much flatter on average than the shape of the index IVF. The average implied volatility of category 1 options, 38.33 percent, is less than 8 percent higher than the average implied volatility of ATM options. On the other end of the spectrum, the average implied volatility of category 5 options, 39.45 percent, is less than 11 percent higher than the implied volatility of ATM options.

One possible explanation for the difference in the shapes of the index option and stock option IVFs is that the stochastic process governing index returns is different from the typical stochastic process governing the returns of an individual stock. Bakshi, Kapadia, and Madan (2003) study the risk-neutral skewness implicit in the prices of index options and individual stock options. As noted by Dennis and Mayhew (2002), risk-neutral skewness is isomorphic to the slope of the IVF (i.e., a negatively sloped IVF corresponds to negative implicit risk-neutral skewness). Bakshi, Kapadia, and Madan show how the market value of portfolios of options can be used to approximate the higher order moments of the implied risk-neutral density. They then show how risk-neutral skewness is related to the higher order moments of the underlying asset's physical return distribution and to the coefficient of relative risk aversion. They develop a market model of skewness and show that under certain assumptions, the risk-neutral skewness implicit in individual stock options will be less negative than the risk-neutral skewness of the index. Thus, Bakshi, Kapadia, and Madan propose an explanation for the shapes of the IVFs based on the underlying assets' stochastic processes without having to specify a particular functional form.

Bakshi, Kapadia, and Madan (2003) note that as long as the idiosyncratic returns of individual stocks are less negatively skewed than the market, one can expect to find a difference in the risk-neutral skewness of stocks versus the index. Consistent with this view, Duffee (1995) finds evidence that firm-level idiosyncratic returns are positively skewed. But, if different underlying stochastic processes were the reason for the differences between the IVFs of index options and stock options, we should be able to see differences in the underlying asset return distributions. Figure 3 compares the empirical cumulative distribution function (CDF) of the S&P 500 index to the CDF of the pooled stock returns calculated using 1,515 daily observations of standardized returns over the period January 1995 to December 2000. Standardized returns are constructed in two steps. First, each day's standardized return is computed as the difference between the raw return and the sample mean, scaled by the sample standard deviation measured over the prior 21 trading days. Second, the standard deviation of the standardized returns, measured over the full sample, is used to whiten the standardized returns to ensure that they have an unconditional volatility of 1.0. Figure 3A shows the complete distributions, and Figure 3B shows only the left tail. There is little apparent difference between the stock and the index CDFs in Figure 3A. Both CDFs are steeper than the normal's, indicating the presence of leptokurtosis (i.e., fat tails). In Figure 3B, the index

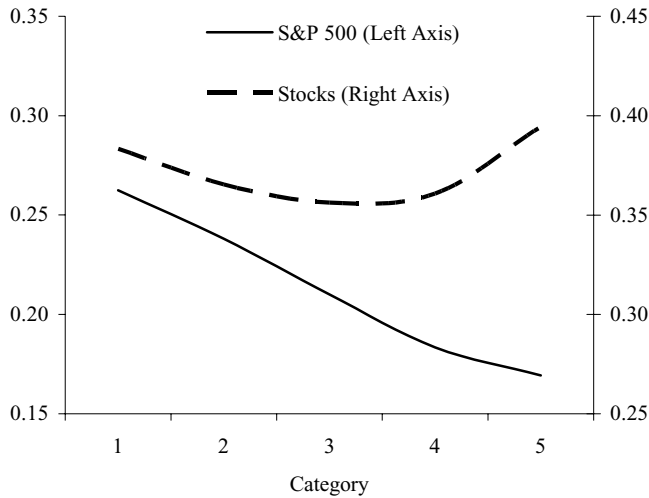


**Figure 1. Level and slope of the S&P 500 implied volatility function (IVF) from June 1988 through December 2000.** The IVF is the average implied volatility of options in five moneyness categories based on option delta. For puts, the five delta ( $\Delta$ ) categories are  $-0.02 \geq \Delta > -0.125$ ,  $-0.125 \geq \Delta > -0.375$ ,  $-0.375 \geq \Delta > -0.625$ ,  $-0.625 \geq \Delta > -0.875$ , and  $-0.875 \geq \Delta \geq -0.98$ . The corresponding call categories are:  $0.875 \leq \Delta \leq 0.98$ ,  $0.625 \leq \Delta < 0.875$ ,  $0.375 \leq \Delta < 0.625$ ,  $0.125 \leq \Delta < 0.375$ , and  $0.02 \leq \Delta < 0.125$ . Implied volatilities are computed daily based on the midpoint of the bid/ask quotes as of 3 PM (CST). All volatilities are annualized. “Level” is the average implied volatility of category three options. “Slope” is the percentage difference between the average implied volatility of category two options and the average implied volatility of category three options. “Index Level” is the closing S&P 500 index level on the dates the IVFs are estimated.

Table III  
Average Implied Volatilities by Option Delta for S&P 500 Index Options and Twenty Stock Options Traded on the Chicago Board Options Exchange during the Period January 1995 through December 2000

Stock option classes are the 20 most active that traded continuously throughout the sample period. Implied volatilities are computed daily based on the midpoint of the bid/ask quotes as of 3 PM (CST). The analytical European-style formula is used to compute implied volatilities for S&P 500 index options, and the dividend-adjusted binomial method is used to compute implied volatilities for the American-style stock options. The delta value of each option series is computed using the closing stock/index price, the actual dividends paid during the option's life, the Eurodollar rate matching the option's time to expiration, and the realized volatility over the most recent 60 trading days. All volatilities are annualized. The "mean" row reported at the bottom of the table contains the average values across the twenty stock options.

Category	Average Implied Volatility					Average Diff. Between Implied and Realized Volatility				
	1	2	3	4	5	1	2	3	4	5
SPX	0.2625	0.2381	0.2100	0.1834	0.1694	0.0958	0.0627	0.0317	0.0107	0.0079
AIG	0.3174	0.3133	0.3049	0.3022	0.3184	0.0177	-0.0112	-0.0201	-0.0135	0.0105
AOL	0.6881	0.6195	0.5992	0.6034	0.6492	0.0235	-0.0490	-0.0583	-0.0476	0.0025
BMJ	0.3285	0.3093	0.3007	0.2902	0.3114	0.0146	-0.0380	-0.0598	-0.0402	0.0067
CL	0.3256	0.3217	0.3108	0.3029	0.3278	0.0124	-0.0249	-0.0325	-0.0183	0.0118
CSC	0.3845	0.3765	0.3739	0.3764	0.4178	0.0269	-0.0102	-0.0150	-0.0033	0.0271
CSCO	0.5463	0.4827	0.4550	0.4601	0.5043	0.0744	-0.0135	-0.0427	-0.0441	0.0001
DAL	0.3519	0.3548	0.3560	0.3712	0.3986	0.0247	-0.0051	-0.0173	0.0045	0.0365
DOW	0.2971	0.2910	0.2866	0.2832	0.2913	0.0428	-0.0016	-0.0127	-0.0003	0.0253
GE	0.3284	0.3086	0.2910	0.2816	0.3006	0.0563	0.0243	0.0038	-0.0002	0.0165
HWP	0.4621	0.4320	0.4249	0.4368	0.4956	0.0177	-0.0375	-0.0491	-0.0379	0.0093
IBM	0.3851	0.3503	0.3345	0.3400	0.3698	0.0313	-0.0189	-0.0384	-0.0288	-0.0016
JNJ	0.2935	0.2789	0.2686	0.2712	0.2896	0.0312	0.0020	-0.0121	-0.0154	0.0058
MER	0.4743	0.4435	0.4206	0.4246	0.4772	0.0119	-0.0460	-0.0641	-0.0404	0.0046
MMM	0.2790	0.2688	0.2655	0.2636	0.2820	0.0322	-0.0022	-0.0188	-0.0179	0.0101
MRK	0.3173	0.3037	0.2914	0.2888	0.3046	0.0370	0.0016	-0.0151	-0.0124	0.0101
SLB	0.3722	0.3670	0.3698	0.3761	0.4123	0.0020	-0.0184	-0.0242	-0.0211	0.0063
TXN	0.5261	0.4964	0.4898	0.5101	0.5726	-0.0201	-0.0675	-0.0799	-0.0622	-0.0027
UAL	0.3685	0.3653	0.3716	0.3874	0.4152	0.0381	0.0084	-0.0045	0.0045	0.0294
XOM	0.2596	0.2475	0.2403	0.2346	0.2442	0.0210	-0.0117	-0.0214	-0.0173	0.0082
XRX	0.3611	0.3756	0.3684	0.4103	0.5072	0.0217	-0.0360	-0.0479	-0.0424	-0.0046
Mean across stocks	0.3833	0.3653	0.3562	0.3607	0.3945	0.0259	-0.0178	-0.0315	-0.0227	0.0106

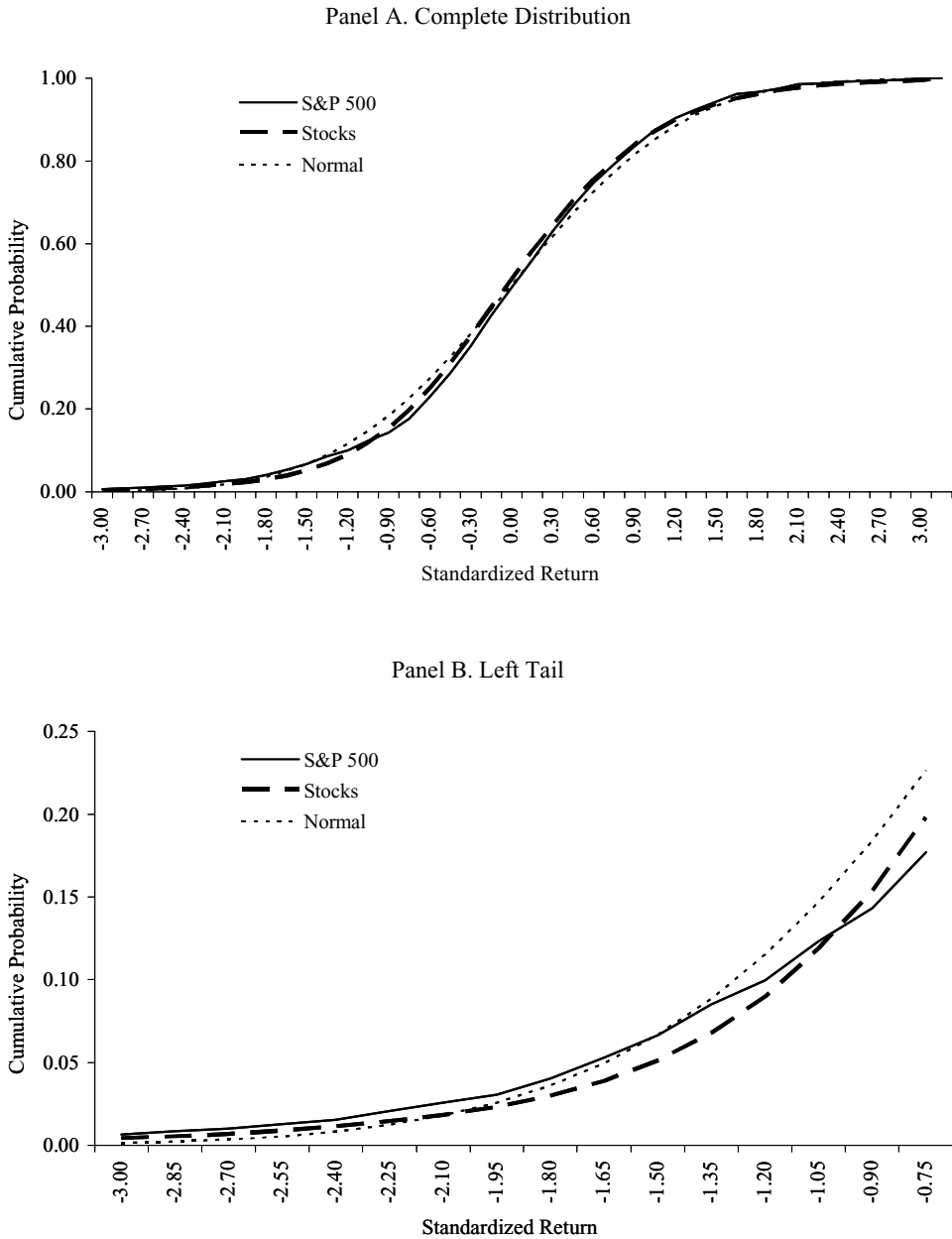


**Figure 2. Estimated implied volatility functions (IVFs) of S&P 500 index options and of options on 20 different individual stocks from January 1995 to December 2000.** The IVF is the average implied volatility of options in five moneyness categories based on option delta. For puts, the five delta ( $\Delta$ ) categories are  $-0.02 \geq \Delta > -0.125$ ,  $-0.125 \geq \Delta > -0.375$ ,  $-0.375 \geq \Delta > -0.625$ ,  $-0.625 \geq \Delta > -0.875$ , and  $-0.875 \geq \Delta \geq -0.98$ . The corresponding call categories are  $0.875 \leq \Delta \leq 0.98$ ,  $0.625 \leq \Delta < 0.875$ ,  $0.375 \leq \Delta < 0.625$ ,  $0.125 \leq \Delta < 0.375$ , and  $0.02 \leq \Delta < 0.125$ . Implied volatilities are computed daily based on the midpoint of the bid/ask quotes as of 3 PM (CST). The analytical European-style formula is used to compute implied volatilities for S&P 500 index options, and the dividend-adjusted binomial method is used to compute implied volatilities for the American-style stock options. The delta value of each option series is computed using the closing stock/index price, the actual dividends paid during the option's life, the Eurodollar rate matching the option's time to expiration, and the realized volatility over the most recent 60 trading days. All volatilities are annualized.

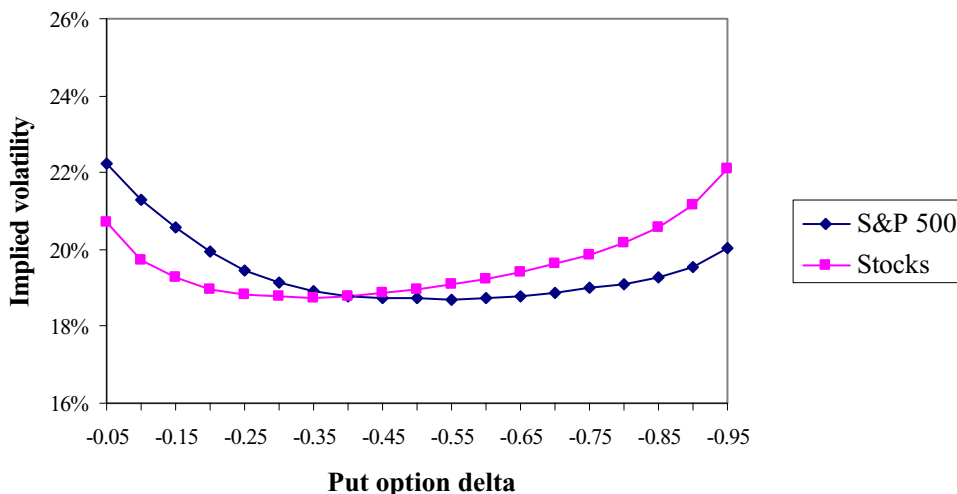
CDF lies slightly above the average stock, consistent with the results of Duffee (1995); however, the difference appears fairly small.

The subtle differences shown in Figure 3 can be translated into implied volatilities by (1) computing hypothetical risk-neutral put option prices based on the empirical distribution and then (2) using the hypothetical put prices to compute implied volatilities. For convenience, the stock and index levels are both set equal to 100, the standardized returns are scaled to reflect a volatility of 20 percent, and the interest rate is set equal to 5 percent. Figure 4 shows the resulting IVFs. The figure clearly shows that the leptokurtosis in the empirical distributions translates into smile-shaped IVFs. The slight difference in the skewness of the empirical distributions also appears: OTM index puts are more valuable than OTM stock puts. The thicker left tail of the index CDF, which leads to slightly higher implied volatilities for OTM puts than OTM stock puts, may be caused by asymmetric correlation among the stocks.<sup>16</sup>

<sup>16</sup> See Longin and Solnik (2001) for a study of this phenomenon in international equity markets.



**Figure 3. Empirical cumulative distribution functions (CDF) of standardized daily returns for the S&P 500 index, average empirical CDF of the standardized returns of 20 stocks, and the analytical CDF of a standard normal.** Data are from January 1995 through December 2000. Panel A shows the complete distribution. Panel B focuses on the left tail. On the vertical axis in each plot is the probability of drawing an observation at or below the value indicated on the horizontal axis.

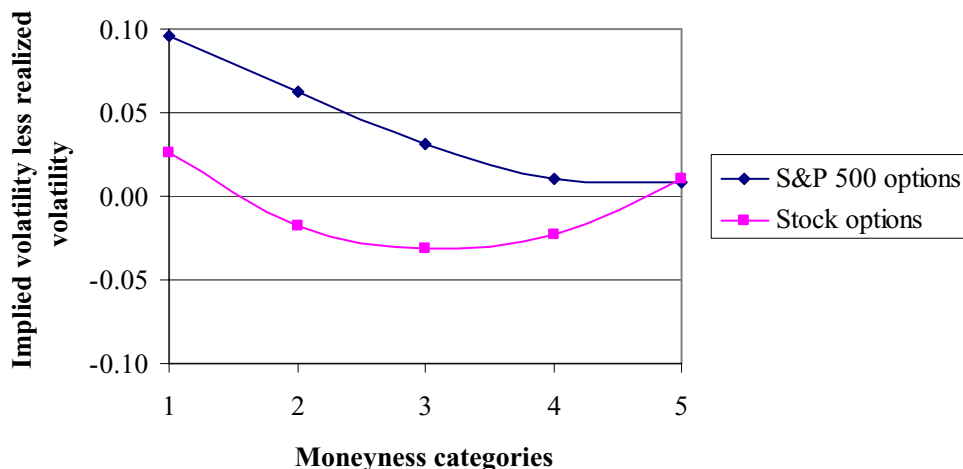


**Figure 4. Hypothetical implied volatilities of one-month European-style put options based on an asset price of 100, a volatility rate of 20 percent, and a risk-free rate of interest of 5 percent.** The underlying empirical distributions of the daily standardized returns of the S&P 500 index and the individual stocks are tabulated over the period January 1995 through December 2000.

Overall, however, the similarity between these hypothetical IVFs suggests that the difference between the actual IVFs of index options and stock options cannot be explained solely by the underlying asset return distributions. Furthermore, while the hypothetical and actual stock option IVFs are quite similar in appearance and range (recall Figure 2), the hypothetical and actual index IVFs differ dramatically.<sup>17</sup> This suggests that explanations for the IVF based on the stochastic processes governing underlying asset returns will have much less success when applied to index options than to individual stock options.

Another reason to question whether underlying stochastic processes can explain the IVF is revealed in Table III, which reports the average difference between the implied volatility of each delta category and the 60 trading day realized volatility prior to each measurement of implied volatility. Figure 5 highlights the average differences of the S&P 500 index options and the averages across the 20 stock options. Even though the CDFs of standardized index and stock returns are quite similar, the levels of implied volatility show significantly different degrees of bias. For the index options, the difference is monotonically decreasing—from 9.58 percentage points for category 1, representing a 50 percent bias over historical volatility, to less than 1 percentage point for category 5. The ATM index options have an average difference of 3.17 percentage points,

<sup>17</sup> One possible explanation for the difference in the IVFs is that index option prices reflect a large jump premium, as in Pan (2002), and options on equities with low correlation with the market do not.



**Figure 5. Average difference between implied volatility and realized volatility for S&P 500 index options and average of options on 20 different individual stocks.** The sample period is January 1995 through December 2000. The IVF is the average implied volatility of options in five moneyness categories based on option delta. For puts, the five delta ( $\Delta$ ) categories are  $-0.02 \geq \Delta > -0.125$ ,  $-0.125 \geq \Delta > -0.375$ ,  $-0.375 \geq \Delta > -0.625$ ,  $-0.625 \geq \Delta > -0.875$ , and  $-0.875 \geq \Delta \geq -0.98$ . The corresponding call categories are  $0.875 \leq \Delta \leq 0.98$ ,  $0.625 \leq \Delta < 0.875$ ,  $0.375 \leq \Delta < 0.625$ ,  $0.125 \leq \Delta < 0.375$ , and  $0.02 \leq \Delta < 0.125$ . Implied volatilities are computed daily based on the midpoint of the bid/ask quotes as of 3 PM (CST). The analytical European-style formula is used to compute implied volatilities for S&P 500 index options, and the dividend-adjusted binomial method is used to compute implied volatilities for the American-style stock options. The delta value of each option series is computed using the closing stock/index price, the actual dividends paid during the option's life, and the Eurodollar rate matching the option's time to expiration. The realized volatility is computed using the most recent sixty trading days. All volatilities are annualized.

which is similar to typical comparisons between implied and subsequently realized volatility in the literature on the information content of implied volatility. This result suggests that the category 1 options are the most overpriced relative to the Black and Scholes (1973) assumptions, as we will test in a simulated trading strategy in Section III. For the individual stock options, the average difference between historical and implied volatility is substantially smaller, especially as a percentage of historical volatility. The category 1 options, for example, feature an average difference of 2.59 percentage points, which is only a 7 percent bias. If this volatility bias is indicative of a volatility markup, then we can expect much lower profitability when selling stock options in our simulated trading strategy.

Earlier we discussed the fact that while option valuation models based on elaborate deterministic and stochastic volatility processes can explain the observed cross-sectional structure of index option prices, they can do so only with implausible parameter values. The results of our preliminary analyses recast the same message, albeit in a different way. First, although the empirical return



distributions for the S&P 500 index and the 20 individual stock options in the sample are remarkably similar, the IVFs for index options and the stock options are dramatically different. The index IVF is steeply downward sloping, whereas the stock IVFs are on average symmetric and much flatter. Second, option-implied volatilities deviate from historical estimates of volatility. For index options, the difference is largest for DOTM puts/DITM calls and declines monotonically across exercise prices. But, even for DITM puts/DOTM calls, the implied volatility is higher than historical volatility on average. For stock options, the average difference between implied volatility across options is less than 1 percentage point, with the implied volatility above realized volatility for the DOTM/DITM categories and below realized volatility for ATM options. Taken together, these results suggest that explanations for the IVF based on the underlying asset's stochastic process and parameters estimated from time-series data will have difficulty reconciling the difference between the index and individual stock IVFs.

## **II. Net Buying Pressure and Movements in Implied Volatility**

This section explores the linkage between net buying pressure and movements in implied volatility. Under the Black–Scholes assumption of frictionless markets, suppliers of option market liquidity can perfectly and costlessly hedge their inventories, so supply curves will be flat. Neither time variation in the demands to buy or sell options nor public order imbalances for particular option series will affect option price and, hence, implied volatility. The null hypothesis, therefore, is that no relation exists between demand for options and corresponding implied volatilities.

Two alternative hypotheses support a positive relation between demand for options and corresponding implied volatilities. The first alternative hypothesis is based on the widespread belief that, in practice, limits to arbitrage exist. Shleifer and Vishny (1997) describe how the ability of professional arbitrageurs to exploit mispriced securities is limited by the ability of investors to absorb intermediate losses. Liu and Longstaff (2000) show that margin requirements can also limit arbitrage effectiveness. Figlewski (1989) and Green and Figlewski (1999) discuss and measure empirically various sources of risk faced by arbitrageurs when hedging their positions, including model misspecification, biased parameter estimation, and discretely rebalanced portfolios. As suppliers of liquidity are required to absorb larger positions in particular options series, their hedging costs and/or desired compensation for risk increase. Consequently, option price and implied volatility increase as well. With supply curves upward sloping, an excess of buyer-motivated trades will cause price and implied volatility to rise, and an excess of seller-motivated trades will cause implied volatility to fall.

The second alternative hypothesis is based on the view that the stochastic process governing an option's underlying asset returns is potentially complex, unobservable, and time-varying. In this context, a positive relation between demand for options and corresponding implied volatilities would be observed if

the order imbalance merely reflected a change in investor expectations about future volatility. In other words, the trading activity of investors provides information to the market maker, who continually learns about the underlying asset dynamics and updates prices as a result.

We structure our empirical tests in order to differentiate between these alternative hypotheses. In particular, there are two instances where the alternative hypotheses generate different predictions. First, we include the lagged change in implied volatility as an independent variable in a regression that assesses the relation between changes in implied volatility and option demand. Under the “learning hypothesis,” as argued below, there should be no serial correlation in changes in implied volatility. In contrast, the “limits to arbitrage hypothesis” predicts that changes in implied volatility will reverse, at least in part, as the market maker has the opportunity to rebalance his portfolio. Second, under the learning hypothesis, demand for ATM options should be the dominant factor determining the implied volatility of all options, since ATM options are most informative about future volatility. Thus, the implied volatilities of all option series in a class should move in concert. In contrast, the limits to arbitrage hypothesis predicts that implied volatilities of different option series need not move together as they are primarily affected by option series’ own demand.

Most studies that have attempted to explain movements in the IVF focus on changes in *level*, usually measured from ATM option prices.<sup>18</sup> Volatility is known to have a strong contemporaneous relation with movements in stock price and trading volume. Black (1976), for example, argues that stock return volatility is inversely related to stock returns due to a leverage effect—the higher the firm’s market value of equity to debt, the lower the firm’s leverage, and the lower the stock’s return volatility. Using stock return data, he provides empirical support for his claim. Using index option data, Fleming, Ostdiek, and Whaley (1995) offer corroborating evidence. They find that the change in the level of the CBOE’s VIX (i.e., the ATM implied volatility of S&P 100 index options) is significantly negatively correlated with the contemporaneous S&P 100 index return. Changes in volatility are also contemporaneously related to trading volume. Under the mixture of distributions hypothesis, trading volume and volatility are jointly dependent on information flow—the more new information flowing into the market, the greater the trading volume and return volatility. Empirical support for this hypothesis is provided in studies by Clark (1973), Epps and Epps (1976), and Tauchen and Pitts (1983), to mention only a few.

The only study to focus on the determinants of the *slope* of the IVF (i.e., differential implied volatility effects) is by Dennis and Mayhew (2002). They attempt to explain the level of risk-neutral skewness implied by stock option prices (a variable that is tantamount to using the slope of the IVF) using firm-specific

<sup>18</sup> A closely related study is that of Longstaff (1995), who simultaneously estimates the implied index level and volatility from S&P 100 call option prices on a daily basis and then regresses the percentage difference between the implied index level and the observed index level on various determinants. Among other things, he finds that the coefficients on open interest and trading volume are significantly negative.

variables such as leverage and trading volume. To assess whether trading pressure from public order flow affects the slope, they use the ratio of average daily put volume to average daily call volume. They find no robust relation between implied risk-neutral skewness and the put-to-call volume ratio, and conclude that net buying pressure does not affect the relative prices of options on individual stocks.

One possible reason that Dennis and Mayhew fail to detect a relation between the slope of the IVF and trading pressure is that their proxy for net buying pressure is imprecise. Volume and net buying pressure need not be highly correlated. On days with significant information flow, for example, trading volume may be high, but with as many public orders to buy as to sell. In this case, net buying pressure is zero. Moreover, the aggregate call and put option volumes fail to distinguish between moneyness characteristics of options. This means a deep out-of-the-money put is counted in the same way as a deep in-the-money put. If both have positive net buying pressure, we should expect the level of the IVF to rise but the slope to be unchanged. In an effort to more accurately assess the impact of investor demand on the shape of the IVF, we tabulate trading volume and net buying pressure by option moneyness category.

We now turn to our investigations of the determinants of changes in the IVFs for S&P 500 and stock options. We begin by describing our test methodology, and then follow-up with a discussion of our results.

### *A. Empirical Methodology*

Our empirical methodology is designed to uncover the role net buying pressure plays in determining changes in the level of implied volatility for options with different exercise prices. It differs from past studies in three primary ways. First, rather than using aggregate trading volume across all call and all put option series, we use trading volume on a series-by-series basis. In this way, we can tie changes in volatility to demands created from specific option trading strategies. Second, in addition to examining trading volume, we examine the net buying pressure for each option series. As defined previously, net buying pressure equals the difference between the number of contracts traded during the day at prices higher than the prevailing bid/ask quote midpoint (i.e., buyer-motivated trades) and the number of contracts traded during the day at prices below the prevailing bid/ask quote midpoint (i.e., seller-motivated trades) times the absolute value of the option's delta. We then scale this difference by the total trading volume across all option series in the class in that day. Third, to analyze the time-series dynamics of the IVF function, we consider separately the levels of implied volatility in the five different moneyness categories. An alternative approach would be to estimate some arbitrarily specified function for implied volatility; however, it is difficult to find a single structural form that is flexible enough to handle the cross-sectional differences in the shape of the IVFs for index and stock options as well as the variation in the IVFs through time.

To assess the relation between the shape of the IVF and net buying pressure, we regress the daily change in the average implied volatility of options

in a particular moneyness category on contemporaneous measures of security return, security trading volume, and net buying pressure. The contemporaneous return of the underlying security and its trading volume are included as control variables for leverage and information flow effects. Recall that stock return volatility may be inversely related to stock returns due to a leverage effect, and that trading volume and volatility depend jointly on information flow. Since trading volume in financial markets has generally increased over time, nonstationarity may be an issue. Lo and Wang (2000), for example, study equity trading volume from July 1962 to December 1996 and reject the null hypothesis of stationarity. They try six methods of detrending trading volume and find all fail to adequately remove serial correlation. For this reason, Lo and Wang advocate using shorter measurement intervals (e.g., 5 years) when analyzing trading volume. Since our estimation interval is only 6 years, we report results from regressions in which trading volume is not detrended. To test the robustness of the results, regressions using the natural logarithm of trading volume were also estimated (but not reported) with no meaningful change in the results other than the magnitude of the trading volume regression coefficient estimate.<sup>19</sup>

The regression also includes the lagged change in implied volatility as an explanatory variable. The null hypothesis predicts that its coefficient is not different from zero. Recall that there are two competing alternative hypotheses. If changes in implied volatility are driven by shifts in investor expectations regarding volatility, changes in implied volatility should be permanent and uncorrelated through time, hence the “learning” hypothesis also predicts an insignificant coefficient on the lagged change in implied volatility. This view is consistent with past research, which concludes that, although volatility tends to be mean-reverting over long periods of time, it tends to be highly persistent at short intervals. Engle and Mustafa (1992), for example, find that in a GARCH(1,1) framework, the sum of the coefficients on the lagged squared residual and the lagged variance are close to unity for both individual stocks and the S&P 500 index using daily return data. Based on this evidence, they conclude that shocks to volatility of major stocks are permanent.

In contrast to the null hypothesis and the learning hypothesis, the limits to arbitrage hypothesis predicts that the coefficient of the lagged change in implied volatility is negative. If net buying pressure has a price impact because of limits to arbitrage, part of the impact is likely to be transitory. As market makers gradually rebalance their portfolios, prices should return at least partly to their previous levels. These price reversals are market-impact costs and are akin to the reversals observed in the market at the time of large block trades or when stocks are added or deleted from the S&P 500 index.<sup>20</sup>

<sup>19</sup> Copies of the results are available from the authors.

<sup>20</sup> Empirical assessments of the price impact of block trades are provided in Scholes (1972), Kraus and Stoll (1972), and Holthausen, Leftwich, and Mayers (1987). Empirical assessments of the price impact of stocks added to the S&P 500 index portfolio are contained in Shleifer (1986), Harris and Gurel (1986), and Beneish and Whaley (1996).

The regression model specification is

$$\Delta\sigma_t = \alpha_0 + \alpha_1 RS_t + \alpha_2 VS_t + \alpha_3 D_{1,t} + \alpha_4 D_{2,t} + \alpha_5 \Delta\sigma_{t-1} + \varepsilon_t, \quad (5)$$

where  $\Delta\sigma_t$  is the change in the average implied volatility in a moneyness category from the close on day  $t - 1$  to the close on day  $t$ ,  $RS_t$  is the underlying security return from the close on day  $t - 1$  to the close on day  $t$ ,  $VS_t$  is the trading volume of the underlying stock (index) on day  $t$  expressed in millions (billions) of dollars,<sup>21</sup> and  $D_{1,t}$  and  $D_{2,t}$  are the net buying pressure variables (whose definitions vary in the regression tests that follow). For an observation to be included in the regression analysis, at least one option series must appear in an option moneyness category on *three* consecutive trading days, thereby permitting the measurement of both  $\Delta\sigma_t$  and  $\Delta\sigma_{t-1}$ .

### B. Empirical Results

To begin the empirical analyses, we summarize the trading activity in S&P 500 index options and options on individual stocks over the January 1995 through December 2000 sample period. The total number of contracts traded in each moneyness category is reported in panel A of Table IV, and net purchases of contracts in panel B. The summaries show that index option trading activity is different from stock option trading activity in a number of important respects. First, the ratio of call option volume to put option volume is much greater for stock options than for index options. For stock options, 66.6 percent of all contracts traded were call options, with only 33.4 percent being puts. For index options, on the other hand, only 45.0 percent were calls and 55.0 percent were puts. These results suggest that investors trade index options and stock options for different reasons.

Second, comparing across moneyness categories, trading volume for calls on stocks is heaviest for ATM options (category 3) and relatively symmetric. For index calls, the ATM and OTM options (i.e., categories 3 and 4) are the most active. On the other hand, for puts on stocks, OTM options (category 2) have the heaviest trading volume, followed by ATM puts, then DOTM puts (category 1). For index options, the OTM put options are also the largest category of puts traded, but the DOTM puts (category 1) are about as heavily traded as the ATM puts (category 3). This evidence is consistent with the use of S&P 500 index puts as portfolio insurance by equity portfolio managers.

The net purchases summarized in panel B lead to similar interpretations. For stock options, the results show that investors are net buyers of DOTM, OTM, and ATM calls but only DOTM and OTM puts. For index options, on the other hand, the results show that investors are net buyers of only DOTM calls, but for DOTM, OTM, and ATM puts. The large net buying pressure of OTM index puts in particular suggests that portfolio insurers prefer this moneyness category.

<sup>21</sup> We use the dollar trading volume of NYSE stocks as a proxy for the trading volume of the S&P 500 index portfolio.

Table IV  
**Summary of Number of Stock options and S&P 500 Index Options Traded during the Period January 1995 through December 2000**

Stock options include all trades for the 20 most active option classes traded continuously on the Chicago Board Options Exchange during the sample period. Index options include all trades of the S&P 500 index options. The delta value of each option series is computed using the closing stock/index price, the actual dividends paid during the option's life, the Eurodollar rate matching the option's time to expiration, and the realized volatility over the most recent 60 trading days. The net purchases of an option contracts in Panel B are defined as the number of contracts traded above the prevailing bid/ask midpoint less the number of contracts traded below the prevailing midpoint times the absolute value of the option's delta.

Delta Value Category	Stock Options			Index Options				
	Calls		Puts	Calls		Puts		
	No. of Contracts	Prop. of Total		No. of Contracts	Prop. of Total			
	Panel A. Number of Contracts Traded							
1	6,021,537	0.0378	8,124,981	0.0510	2,205,279	0.0210	14,560,770	0.1390
2	22,140,393	0.1389	23,344,704	0.1465	6,396,646	0.0610	21,722,475	0.2073
3	41,185,829	0.2584	15,258,972	0.0958	16,685,024	0.1592	16,469,569	0.1572
4	29,974,015	0.1881	5,317,562	0.0334	14,807,015	0.1413	4,020,688	0.0384
5	6,797,064	0.0427	1,196,973	0.0075	7,029,690	0.0671	889,658	0.0085
Totals	106,118,838	0.6659	53,243,192	0.3341	47,123,654	0.4497	57,663,160	0.5503
Panel B. Net Purchases of Contracts								
1	-164,229		148,703		-17,430		225,027	
2	-144,430		70,735		-57,408		464,688	
3	912,707		-359,895		-56,440		64,716	
4	130,558		-241,970		-62,067		-56,223	
5	408,842		5,609		86,191		-37,027	
Totals	1,143,448		-376,818		-107,154		661,181	

Another oddity is that, for stock options, net buying pressure is positive for ATM calls and negative for ATM puts, yet for index options, the reverse is true.

### B.1. Changes in ATM Implied Volatility

In all, three pairs of regression tests are performed. In the first pair, we assess the degree to which the variables in (5) explain changes in the volatility of ATM options (category 3). The regression is estimated for calls and puts separately, and its specification is

$$\Delta\sigma_t = \alpha_0 + \alpha_1 RS_t + \alpha_2 VS_t + \alpha_3 ATMC_t + \alpha_4 ATMP_t + \alpha_5 \Delta\sigma_{t-1} + \varepsilon_t, \quad (6)$$

where  $ATMC_t$  ( $ATMP_t$ ) is the net buying pressure for ATM calls (puts). The coefficients  $\alpha_3$  and  $\alpha_4$  should be informative regarding investor trading motivation. If trading is motivated by changes in expected future volatility, the coefficient values should be indistinguishable from one another. ATM calls and ATM puts are equally responsive to shifts in volatility, so there is no reason for traders to prefer one type of option over the other. Indeed, the most effective way to trade given an impending upward revision in volatility is to buy straddles.<sup>22</sup> On the other hand, if ATM calls and puts are used in strategies unrelated to volatility changes and if the attendant buying pressure moves prices as a result, the coefficients will differ.

Table V contains a summary of the regression results of (6) for S&P 500 index options as well as 20 individual stock options. Panel A shows the results for changes in the implied volatility of call options. Note first that virtually all coefficients of the control variables  $RS_t$  and  $VS_t$  have their expected signs, and most are significant in a statistical sense. This evidence corroborates the results of past studies using a different sample period.

The coefficients of the net buying pressure variables in panel A of Table V offer some intriguing insights. For the index, the coefficient on  $ATMC$  is negative and insignificant, while the coefficient on  $ATMP$  is significantly positive. In addition, the coefficients are significantly different from one another. Apparently, the net buying pressure on ATM puts has a greater influence on the change in the level of the ATM call volatility than does the net buying pressure of ATM calls. To some degree, this should not be surprising. We have already documented the fact that put option trading in the S&P 500 index option dominates call option trading. If there is excess demand to buy ATM index puts, ATM index put implied volatility increases, and ATM index call implied volatility gets dragged along as a result of reverse conversion arbitrage.

The evidence for stock options provides a striking contrast. The coefficient on  $ATMC$  is significantly positive for 17 of the 20 option classes, while the coefficient on  $ATMP$  is significantly positive for only 1 of 20. This, too, is consistent with the trading volume evidence reported in Table IV; that is, trading

<sup>22</sup> Of the available option series, buying ATM calls and puts maximizes portfolio vega while holding the portfolio approximately delta-neutral.

Table V  
Summary of Regression Results of Change in at-the-Money Implied Volatility for S&P 500 Index Options and 20 Stock Options Traded on the Chicago Board Options Exchange during the Period January 1995 through December 2000

Stock option classes are the 20 most active that traded continuously throughout the sample period. The regression specification underlying the results reported in this table is

$$\Delta\sigma_t = \alpha_0 + \alpha_1RS_t + \alpha_2VS_t + \alpha_3ATMC_t + \alpha_4ATMP_t + \alpha_5\Delta\sigma_{t-1} + \varepsilon_t,$$

where  $\Delta\sigma_t$  is the change in the average ATM option-implied volatility from the close on day  $t - 1$  to the close on day  $t$ ,  $RS_t$  is the index/stock return from the close on day  $t - 1$  to the close on day  $t$ ,  $VS_t$  is the stock volume on day  $t$  expressed in millions of dollars, and  $ATMC_t$  and  $ATMP_t$  are the net buying pressure on ATM calls and ATM puts, respectively. For the index option regression,  $VS_t$  is the dollar volume of shares traded on the NYSE expressed in billions of dollars. Panel A contains the results for the change ATM call volatility, and panel B contains the results for the change in ATM put volatility. The asterisk denotes that the coefficient is significantly different from zero at the 5 percent probability level. The second asterisk field beside the coefficient  $\alpha_4$  tests the null hypothesis that  $\alpha_3 = \alpha_4$ .

Ticker	No. of Obs.	$R^2$	Adj. $R^2$	Panel A. Changes in ATM Call Volatility as a Function of $ATMC$ and $ATMP$					
				Parameter Estimates					
				$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$
SPX	1,507	0.4925	0.4908	0.0017*	-0.7299*	-0.0016*	-0.0027	0.1074**	-0.1727*
AIG	1,354	0.1371	0.1339	-0.0009	-0.1060*	0.0063*	0.0111*	0.0059	-0.2567*
AOL	1,503	0.0990	0.0960	-0.0009	-0.1447*	0.0016*	0.0951*	-0.0157*	-0.0566*
BMJ	1,212	0.2246	0.2214	-0.0028*	-0.1809*	0.0153*	0.0260*	0.0051	-0.2891*
CL	1,158	0.1144	0.1106	-0.0021*	-0.1450*	0.0359*	0.0186*	0.0117	-0.1448*
CSC	1,325	0.0775	0.0740	-0.0008	-0.0985*	0.0228	0.0183*	-0.0098*	-0.2209*
CSCO	1,445	0.2243	0.2216	-0.0006	-0.3212*	0.0006	0.2415*	-0.0036*	-0.2659*
DAL	1,353	0.1329	0.1297	-0.0017*	-0.1544*	0.0422*	0.0239*	0.0086	-0.2289*
DOW	1,113	0.0778	0.0737	-0.0011	-0.0686*	0.0164*	0.0101*	0.0068	-0.2013*
GE	1,139	0.2167	0.2133	-0.0004	-0.2440*	0.0018	0.0641*	0.0143*	-0.1902*
HWP	1,477	0.1303	0.1273	0.0011	-0.1403*	-0.0020	0.0583*	0.0126	-0.2120*
IBM	1,465	0.1814	0.1786	-0.0008	-0.2406*	0.0017	0.0478	-0.0474*	-0.1104*
JNJ	1,228	0.1890	0.1857	-0.0015*	-0.2148*	0.0078*	0.0294*	0.0258*	-0.2808*
MER	1,334	0.0999	0.0965	-0.0030*	-0.1790*	0.0191*	0.0073	0.0002	-0.0657*
MMM	1,240	0.2647	0.2617	-0.0016*	-0.2589*	0.0234*	0.0140*	-0.0001	-0.3761*



MRK	1,229	0.2385	0.2354	-0.0030*	-0.2077*	0.0094*	0.0496*	0.0021*	-0.2696*
SLB	1,331	0.1094	0.1060	-0.0014*	-0.0954*	0.0101*	0.0107*	0.0113	-0.2218*
TXN	1,476	0.1736	0.1708	-0.0019*	-0.1591*	0.0073*	0.0683*	0.0319	-0.2164*
UAL	1,456	0.1323	0.1293	-0.0018*	-0.0697*	0.0580*	0.0259*	-0.0048*	-0.3081*
XOM	1,087	0.1568	0.1529	-0.0011	-0.1589*	0.0039	0.0169*	0.0119	-0.2607*
XRX	1,316	0.1178	0.1144	-0.0048*	-0.2418*	0.0614*	0.0195	0.0128	-0.1105*
<i>Mean across stocks</i>									
Panel B. Changes in ATM Put Volatility as a Function of ATMC and ATMP									
SPX	1,507	0.5663	0.5648	0.0010	-0.8283*	-0.0005	0.0138	0.0989**	-0.1334*
AIG	1,358	0.1142	0.1109	-0.0007	-0.0873*	0.0057*	0.0045	0.0037	-0.2430*
AOL	1,503	0.0832	0.0801	-0.0011	-0.1146*	0.0018*	0.1033*	0.0288	-0.1234*
BMJ	1,206	0.1638	0.1603	-0.0029*	-0.1228*	0.0148*	0.0153*	0.0038	-0.2737*
CL	1,161	0.0828	0.0788	-0.0018*	-0.1010*	0.0305*	0.0100*	0.0155*	-0.1790*
CSC	1,323	0.0822	0.0787	-0.0010	-0.1354*	0.0252*	0.0106	-0.0086	-0.2128*
CSCO	1,448	0.2617	0.2591	-0.0006	-0.2951*	0.0005	0.1879*	0.0711*	-0.1014*
DAL	1,355	0.0791	0.0757	-0.0014	-0.0991*	0.0293*	0.0112*	0.0102	-0.1982*
DOW	1,107	0.1125	0.1085	-0.0014*	-0.0892*	0.0174*	0.0059	0.0056	-0.2207*
GE	1,146	0.1900	0.1865	-0.0009	-0.1942*	0.0024	0.0637*	0.0375*	-0.1854*
HWP	1,474	0.1320	0.1290	0.0008	-0.1197*	-0.0013	0.0392*	0.0321	-0.2726*
IBM	1,465	0.1566	0.1537	-0.0004	-0.2175*	0.0011	0.0425	-0.0240	-0.0875*
JNJ	1,215	0.1804	0.1770	-0.0019*	-0.1352*	0.0100*	0.0183*	0.0283*	-0.3374*
MER	1,344	0.1057	0.1024	-0.0029*	-0.1831*	0.0187*	0.0061	-0.0087	-0.0830*
MMM	1,244	0.1298	0.1263	-0.0019*	-0.1104*	0.0244*	0.0106*	0.0084	-0.2993*
MRK	1,229	0.1246	0.1210	-0.0029*	-0.1077*	0.0088*	0.0262*	-0.0042*	-0.2526*
SLB	1,334	0.1117	0.1083	-0.0011	-0.0655*	0.0087*	0.0079	-0.0016	-0.2948*
TXN	1,480	0.1443	0.1414	-0.0017*	-0.1348*	0.0067*	0.0411*	0.0019	-0.2195*
UAL	1,462	0.1447	0.1418	-0.0015*	-0.0714*	0.0513*	0.0145*	0.0297*	-0.3267*
XOM	1,079	0.1320	0.1279	-0.0011	-0.0657*	0.0041*	0.0079	0.0243*	-0.3157*
XRX	1,308	0.0375	0.0338	-0.0073*	-0.2124*	0.0753*	0.0305	0.0634	-0.2319*
<i>Mean across stocks</i>									
					0.1331	0.0168	0.0329	0.0158	-0.2229

in the stock option market predominantly involves calls, at least during our sample period. Buying pressure on ATM puts has little impact on the level of the implied volatility of the call.

Panel B shows the results for changes in the implied volatility of *ATM* put options. The inferences that can be drawn from the results are remarkably similar to the results for the calls. For the index, the coefficient on *ATMP* is significantly positive, while the coefficient on *ATMC* is insignificant. For the stock options, 12 of the 20 exhibit a significantly positive coefficient on *ATMC* and only five feature a significant coefficient on *ATMP*. These results indicate that the level of ATM implied volatility of index options is driven largely by the demand for ATM index puts, while for options on individual stocks, the level of ATM implied volatility is driven by the demand for ATM calls.

Perhaps, the most interesting results reported in Table V are the consistency in sign, magnitude, and significance of the lagged implied volatility variable. Under the null hypothesis and the learning hypothesis, the coefficient should not be different from zero. Instead, it hovers around a value of about  $-0.15$  for ATM index calls and puts and  $-0.21$  for ATM stock options. Apparently, prices reverse. One possible explanation for this result is measurement error. The implied volatility on day  $t - 1$  appears in the computation of both  $\Delta\sigma_t$  and  $\Delta\sigma_{t-1}$  but with opposite sign. To the degree that there is measurement error in  $\sigma_{t-1}$ , there will be negative serial correlation in observed changes in implied volatility.

Before testing the robustness of the results to measurement error, it is important to recognize how measurement error may have crept into the analysis and what steps have already been taken to mitigate its effects. The two most common types of measurement error in the measurement of implied volatility are (1) bid/ask bounce in both the option and stock/index prices, (2) price discreteness, and/or (3) nonsimultaneity of prices of the option and its underlying stock/index. The option price quote record in the database includes the stock price or the index level at the time the option quote was provided. For the option, the bid/ask quote midpoint is used and should be a reasonably accurate measure of true option price.<sup>23</sup> For the underlying stock, however, the price is from the last trade prior to the option price quote and will tend to be at either a bid or an ask depending on the motivation for the last stock trade. To mitigate the effects of bid/ask price bounce and price discreteness, the implied volatilities of all option series in a given option class are averaged within each moneyness category each day.<sup>24</sup> For the underlying index, the index level is an average of the last trade stock prices, so the bid/ask and price discreteness effects are less of a concern (and whatever concern there may be is also mitigated by the averaging procedure).

Nonsimultaneity of prices is more of a concern for S&P 500 index options than for the 20 individual stock options. The reason is that the underlying stocks in our sample are highly actively traded. Moreover, trading activity at

<sup>23</sup> Of course, the effects of price discreteness will still be present (see footnote 14).

<sup>24</sup> End-of-day price quotes occur at different times for different option series in the same class.

the close of the market tends to be higher than at any time during the day other than at the open. The observed S&P 500 index level, on the other hand, is an amalgam of last trade prices of 500 stocks, some of which may not have traded for several minutes, perhaps much longer. To the extent the observed index level lags the true index level, there will be measurement error in the average implied volatilities of each moneyness category, and the effects will be opposite (and approximately equal) for calls versus puts in a particular category.

One way to examine whether infrequent trading of index stocks is a potential problem is to examine the serial correlation in the daily returns of the S&P 500 index during the sample period. Over the period January 1995 through December 2000, the first-order serial correlation of the daily returns of the S&P 500 index was 0.0007, a trivial level by any standard. Nonetheless, to double-check that price reversals of the index were not being driven by measurement error, we replaced the ATM volatility change series for the S&P 500 index used in the regression that generated the results of Table V with the changes in the CBOE's Market Volatility Index (VIX). Since VIX averages call and put volatilities at the same exercise price, it is relatively immune to the effects of infrequent trading.<sup>25</sup> Interestingly, where the lagged implied volatility change of the ATM call and the ATM put regressions had coefficient estimates of  $-0.17$  and  $-0.13$  in Table V, the coefficient estimate is  $-0.12$  in the regression using VIX. In other words, measurement error is not what drives the price reversals. About 15 percent of the index option-implied volatility change observed today gets reversed tomorrow, perhaps as a result of market makers rebalancing their portfolios. On face appearance, the evidence supports the hypothesis that limits to arbitrage permit a relation between the demand for options and corresponding implied volatility. The price reversals of stock option-implied volatilities are about 21 percent.

### B.2. Changes in OTM Implied Volatilities

The next set of tests examines changes in implied volatility of OTM calls and OTM puts, respectively. These are category 4 options for calls and category 2 options for puts. We focus on these options since they, together with the ATM options, have the largest trading volume (recall Table IV). The first pair of regression tests focuses on changes in the implied volatility of OTM calls. The regression specification is

$$\Delta\sigma_t = \alpha_0 + \alpha_1 RS_t + \alpha_2 VS_t + \alpha_3 OTMC_t + \alpha_4 ATMC_t + \alpha_5 \Delta\sigma_{t-1} + \varepsilon_t \quad (7)$$

for the results reported in panel A of Table VI. For panel B, the net buying pressure for the ATM puts,  $ATMP$ , replaces  $ATMC$  in (7). In essence, the regressions attempt to assess whether net buying pressure of OTM calls affects the implied volatility of OTM calls after controlling for the effects of

<sup>25</sup> The construction of the CBOE's Market Volatility Index (VIX) is described in Whaley (1993).

Table VI

Summary of Time-Series Regression Results of Change in Out-of-the-Money Call Option-Implied Volatility for S&P 500 Index Options and 20 Stock Options Traded on the Chicago Board Options Exchange during the Period January 1995 through December 2000

Stock option classes are the 20 most active that traded continuously throughout the sample period. The regression specifications underlying the results reported in Panels A and B are

$$\Delta\sigma_t = \alpha_0 + \alpha_1RS_t + \alpha_2VS_t + \alpha_3OTMC_t + \alpha_4ATMC_t + \alpha_5\Delta\sigma_{t-1} + \varepsilon_t$$

and

$$\Delta\sigma_t = \alpha_0 + \alpha_1RS_t + \alpha_2VS_t + \alpha_3OTMC_t + \alpha_4ATMP_t + \alpha_5\Delta\sigma_{t-1} + \varepsilon_t,$$

where  $\Delta\sigma_t$  is the change in the average OTC call option-implied volatility from the close on day  $t - 1$  to the close on day  $t$ ,  $RS_t$  is the index/stock return from the close on day  $t - 1$  to the close on day  $t$ ,  $VS_t$  is the stock volume on day  $t$  expressed in millions of dollars, and  $OTMC_t$ ,  $ATMC_t$  and  $ATMP_t$  are the net buying pressures of OTM calls, ATM calls, and ATM puts, respectively. For the index option regression,  $VS_t$  is the dollar volume of shares traded on the NYSE expressed in billions of dollars. The asterisk denotes that the coefficient is significantly different from zero at the five percent probability level. The second asterisk field beside the coefficient  $\alpha_4$  tests the null hypothesis that  $\alpha_3 = \alpha_4$ .

Panel A. Changes in OTM Call Volatility as a Function of OTMC and ATMC

Ticker	No. of Obs.	R <sup>2</sup>	Adj. R <sup>2</sup>	Parameter Estimates					
				$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$
SPX	1,507	0.4878	0.4861	0.0015*	-0.6779*	-0.0015*	0.0041	0.0010	-0.1228*
AIG	1,240	0.0719	0.0681	-0.0009	-0.0708*	0.0053*	0.0221*	0.0010*	-0.1776*
AOL	1,372	0.0857	0.0823	-0.0015	-0.1256*	0.0010	0.1619*	0.0896*	-0.0650*
BMJ	1,264	0.1922	0.1890	-0.0025*	-0.1499*	0.0166*	0.0417*	0.0235*	-0.2822*
CL	1,327	0.1359	0.1326	-0.0016*	-0.1770*	0.0306*	0.0208*	0.0106*	-0.1956*
CSC	1,300	0.0762	0.0726	-0.0007	-0.1589*	0.0225	0.0181	0.0046	-0.2016*
CSCO	1,423	0.2528	0.2502	-0.0011	-0.3077*	0.0008	0.3677*	0.1823**	-0.1911*
DAL	1,308	0.1141	0.1107	-0.0039*	-0.1543*	0.0771*	0.0352*	0.0090	-0.2353*
DOW	1,231	0.1137	0.1101	-0.0010	-0.0879*	0.0163*	0.0052	0.0040	-0.3208*
GE	1,334	0.2148	0.2118	-0.0003	-0.2408*	0.0013	0.0658*	0.0611*	-0.1684*
HWP	1,421	0.1533	0.1503	0.0012	-0.1677*	-0.0029	0.0678*	0.0261*	-0.1705*
IBM	1,473	0.1614	0.1585	-0.0010	-0.2146*	0.0015	0.1515*	0.0488**	-0.0871*
JNJ	1,304	0.2109	0.2078	-0.0010	-0.2388*	0.0081*	0.0431*	0.0206*	-0.2751*
MER	1,394	0.0659	0.0626	-0.0024*	-0.1314*	0.0164*	0.0237*	0.0106*	-0.0629*
MMM	1,327	0.2114	0.2084	-0.0012*	-0.2575*	0.0194*	0.0171*	0.0088	-0.2971*

MRK	1,257	0.1740	0.1707	-0.0024*	-0.1834*	0.0083*	0.0434*	0.0326*	-0.2044*
SLB	1,328	0.1144	0.1111	-0.0006	-0.1169*	0.0069*	0.0339*	0.0093**	-0.2178*
TXN	1,382	0.1491	0.1461	-0.0021*	-0.1448*	0.0074*	0.1248*	0.0594*	-0.1452*
UAL	1,372	0.0603	0.0569	-0.0015*	-0.0768*	0.0508*	0.0172	0.0092	-0.1673*
XOM	1,231	0.1976	0.1943	-0.0008	-0.1705*	0.0041	0.0261*	0.0110	-0.3380*
XRX	1,379	0.1468	0.1437	-0.0058*	-0.2081*	0.0705*	0.0331	0.0193	-0.1747*
<i>Mean across stocks</i>									
					-0.1692	0.0180	0.0660	0.0321	-0.1989

Panel B. Changes in OTM Call Volatility as a Function of OTMC and ATMP									
SPX	1,507	0.4905	0.4888	0.0015*	-0.6812*	-0.0013	-0.0009	0.0798*	-0.1250*
AIG	1,240	0.0722	0.0684	-0.0009	-0.0718*	0.0053*	0.0222*	0.0040*	-0.1772*
AOL	1,372	0.0751	0.0717	-0.0018	-0.1361*	0.0014	0.1708*	-0.0168*	-0.0599*
BMJ	1,264	0.1852	0.1819	-0.0028*	-0.1552*	0.0169*	0.0456*	-0.0066*	-0.2776*
CL	1,327	0.1313	0.1280	-0.0016*	-0.1817*	0.0288*	0.0219*	0.0043	-0.1935*
CSC	1,300	0.0767	0.0732	-0.0008	-0.1602*	0.0224	0.0184	-0.0105*	-0.2029*
CSCO	1,423	0.2322	0.2295	-0.0006	-0.3395*	0.0009	0.4830*	-0.0563*	-0.1765*
DAL	1,308	0.1129	0.1095	-0.0040*	-0.1585*	0.0782*	0.0364*	0.0017*	-0.2355*
DOW	1,231	0.1133	0.1097	-0.0011	-0.0894*	0.0165*	0.0055	0.0006	-0.3206*
GE	1,334	0.2022	0.1992	-0.0004	-0.2511*	0.0015	0.0773*	0.0039*	-0.1595*
HWP	1,421	0.1518	0.1488	0.0011	-0.1717*	-0.0032	0.0748*	-0.0247*	-0.1678*
IBM	1,473	0.1607	0.1578	-0.0008	-0.2157*	0.0015	0.1596*	-0.0499*	-0.0872*
JNJ	1,304	0.2060	0.2029	-0.0009	-0.2443*	0.0075*	0.0471*	0.0036*	-0.2714*
MER	1,394	0.0627	0.0593	-0.0025*	-0.1344*	0.0166*	0.0238*	0.0038	-0.0641*
MMM	1,327	0.2101	0.2071	-0.0013*	-0.2612*	0.0190*	0.0183*	-0.0085*	-0.2982*
MRK	1,257	0.1654	0.1621	-0.0023*	-0.1887*	0.0080*	0.0591*	-0.0090*	-0.2005*
SLB	1,328	0.1118	0.1085	-0.0007	-0.1206*	0.0073*	0.0352*	0.0020*	-0.2152*
TXN	1,382	0.1409	0.1378	-0.0019*	-0.1548*	0.0077*	0.1504*	-0.0194*	-0.1416*
UAL	1,372	0.0589	0.0555	-0.0015*	-0.0796*	0.0524*	0.0180	0.0072	-0.1673*
XOM	1,231	0.1955	0.1923	-0.0008	-0.1748*	0.0040	0.0277*	0.0029	-0.3344*
XRX	1,379	0.1467	0.1436	-0.0059*	-0.2117*	0.0724*	0.0354	0.0350	-0.1733*
<i>Mean across stocks</i>									
					-0.1750	0.0182	0.0765	-0.0066	-0.1962

net buying pressure of ATM options. If the learning story is correct and buying pressure arises from a revision to investor expectations regarding future volatility, the buying pressure of ATM options (i.e., the options most informative about future volatility expectations) is more likely to drive changes in OTM implied volatility than OTM buying pressure. The reason for this is that ATM options have the highest sensitivity to volatility, hence they are the natural vehicle to exploit new information. On the other hand, if the limits to arbitrage story is correct, we should expect the OTM buying pressure (i.e., the option series' own buying pressure) to be more important to that of other series. Thus, equation (7) can be viewed as an attempt to see whether differential buying pressure affects the slope of the IVF controlling for a change in level.

The results in panels A and B of Table VI offer a number of interesting insights. First, for the index options reported in panel A, the coefficients on *OTMC* and *ATMC* are negative and insignificant. In panel B, however, the coefficient on *ATMP* is significantly positive. This evidence suggests that put trading is more influential than call trading for the index options. Second, holding constant the net buying pressure of index puts, the net buying pressure of OTM index calls has no discernible effect on the change in OTM call volatility. Like the evidence reported in panel A of Table V, the evidence in Table VI indicates that the put option trading in the S&P 500 index option market drives the changes in call option-implied volatility.

The stock option results reported in Table VI support just the opposite conclusion. For stock options, the coefficients on both *OTMC* and *ATMC* are all positive, and in most cases, statistically significant. On the other hand, when ATM put net buying pressure replaces ATM call net buying pressure in the regression, none of the stock option classes has significant coefficients on ATM option net buying pressure. In the stock option market, call option trading appears to drive movements in the level and the slope of the call option IVF. It is also worth noting that in panel A, the coefficient estimates of *OTMC* are greater on average than the coefficients of *ATMC* (0.0660 vs. 0.0321). This implies that the option's own demand is more important than ATM call option demand in determining changes in OTM call option-implied volatility.

Finally, the coefficient of the lagged implied volatility variable in the results of Table VI is again consistently negative and significant and about the same order of magnitude as in Table V. Approximately 20 percent of the change in the OTM call option volatility for stock options gets reversed on the following day. For S&P 500 index calls, the reversal is about 12 percent.

Table VII shows the results for changes in the implied volatility of OTM put options. In panel A, the regression specification is

$$\Delta\sigma_t = \alpha_0 + \alpha_1 RS_t + \alpha_2 VS_t + \alpha_3 OTMP_t + \alpha_4 ATMP_t + \alpha_5 \Delta\sigma_{t-1} + \varepsilon_t. \quad (8)$$

For index options, the coefficients on both *OTMP* and *ATMP* are significantly positive. In panel B, where *ATMC* replaces *ATMP* in the regression, on the other hand, the coefficient on *OTMP* is significantly positive while the coefficient on

Table VII

Summary of Time-Series Regression Results of Change in Out-of-the-Money Put Option-Implied Volatility for S&P 500 Index Options and 20 Stock Options Traded on the Chicago Board Options Exchange during the Period January 1995 through December 2000

Stock option classes are the 20 most active that traded continuously throughout the sample period. The regression specifications underlying the results reported in Panels A and B are

$$\Delta\sigma_t = \alpha_0 + \alpha_1RS_t + \alpha_2VS_t + \alpha_3OTMP_t + \alpha_4ATMP_t + \alpha_5\Delta\sigma_{t-1} + \varepsilon_t$$

and

$$\Delta\sigma_t = \alpha_0 + \alpha_1RS_t + \alpha_2VS_t + \alpha_3OTMP_t + \alpha_4ATMC_t + \alpha_5\Delta\sigma_{t-1} + \varepsilon_t,$$

where  $\Delta\sigma_t$  is the change in the average OTM put option-implied volatility from the close on day  $t - 1$  to the close on day  $t$ ,  $RS_t$  is the index/stock return from the close on day  $t - 1$  to the close on day  $t$ ,  $VS_t$  is the stock volume on day  $t$  expressed in billions of dollars, and  $OTMP_t$ ,  $ATMC_t$  and  $ATMP_t$  are the net buying pressures of OTM puts, ATM calls, and ATM puts, respectively. For the index option regression,  $VS_t$  is the dollar volume of shares traded on the NYSE expressed in billions of dollars. The asterisk denotes that the coefficient is significantly different from zero at the five percent probability level. The second asterisk field beside the coefficient  $\alpha_4$  tests the null hypothesis that  $\alpha_3 = \alpha_4$ .

Panel A. Changes in OTM Put Volatility as a Function of OTMP and ATMP

Ticker	No. of Obs.	R <sup>2</sup>	Adj. R <sup>2</sup>	Parameter Estimates					
				$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$
SPX	1,507	0.5943	0.5930	0.0009	-0.8465*	-0.0004	0.0774*	0.1041*	-0.1077*
AIG	1,478	0.0891	0.0860	-0.0005	-0.0996*	0.0039	0.0075	-0.0033	-0.1759*
AOL	1,504	0.1125	0.1096	-0.0010	-0.1823*	0.0022*	0.0109	0.0460	-0.0367
BMJ	1,399	0.1456	0.1425	-0.0020*	-0.1428*	0.0121*	0.0426*	-0.0095*	-0.2271*
CL	1,398	0.0594	0.0560	-0.0007	-0.0892*	0.0128	0.0023	0.0057	-0.1678*
CSC	1,404	0.0611	0.0578	0.0006	-0.1257*	-0.0059	0.0461*	-0.0092*	-0.1793*
CSCO	1,477	0.2125	0.2098	0.0003	-0.3323*	0.0004	-0.0303	0.0633	-0.0823*
DAL	1,449	0.1106	0.1075	-0.0021*	-0.1180*	0.0432*	0.0099	0.0115	-0.2420*
DOW	1,272	0.0897	0.0861	-0.0020*	-0.0620*	0.0252*	0.0022	-0.0031	-0.2437*
GE	1,389	0.1175	0.1143	-0.0009	-0.1386*	0.0025*	0.0302	0.0281	-0.1964*
HWP	1,497	0.0656	0.0624	0.0008	-0.1105*	-0.0020	0.0264	0.0072	-0.1318*
IBM	1,501	0.1227	0.1198	-0.0005	-0.1840*	0.0017	-0.0358	-0.0187	-0.1274*
JNJ	1,408	0.1210	0.1179	-0.0010	-0.1182*	0.0069*	0.0005	0.0122	-0.2894*

Table VII—Continued

Ticker	No. of Obs.	R <sup>2</sup>	Adj. R <sup>2</sup>	Parameter Estimates					
				α <sub>0</sub>	α <sub>1</sub>	α <sub>2</sub>	α <sub>3</sub>	α <sub>4</sub>	α <sub>5</sub>
MER	1,435	0.1165	0.1134	-0.0017*	-0.1953*	0.0119*	0.0183	-0.0010	-0.1307*
MMM	1,369	0.1066	0.1033	-0.0026*	-0.1120*	0.0307*	0.0192	-0.0040	-0.2716*
MRK	1,343	0.1317	0.1285	-0.0019*	-0.1447*	0.0066*	0.0549*	-0.0084*	-0.2408*
SLB	1,436	0.1583	0.1553	-0.0013*	-0.1119*	0.0111*	0.0270	0.0020	-0.3365*
TXN	1,497	0.1894	0.1867	-0.0015	-0.1823*	0.0074*	-0.0172	-0.0181	-0.2271*
UAL	1,441	0.1216	0.1185	-0.0015*	-0.0947*	0.0515*	0.0324*	0.0111	-0.2625*
XOM	1,298	0.1123	0.1089	-0.0008	-0.0346*	0.0040*	0.0169	0.0380*	-0.3055*
XRX	1,473	0.1181	0.1151	-0.0054*	-0.1113*	0.0597*	0.0302	0.0052	-0.2077*
Mean across stocks									
Panel B. Changes in OTM Put Volatility as a Function of OTMP and ATMC									
SPX	1,507	0.5910	0.5896	0.0010	-0.8410*	-0.0006	0.0923*	0.0281	-0.1077*
AIG	1,478	0.0896	0.0865	-0.0005	-0.0984*	0.0040	0.0075	0.0032	-0.1769*
AOL	1,504	0.1308	0.1279	-0.0006	-0.1692*	0.0015*	0.0267	0.1185*	-0.0476*
BMJ	1,399	0.1505	0.1474	-0.0018*	-0.1387*	0.0118*	0.0437*	0.0178*	-0.2310*
CL	1,398	0.0641	0.0608	-0.0007	-0.0857*	0.0143	0.0033	0.0102*	-0.1704*
CSC	1,404	0.0615	0.0582	0.0007	-0.1230*	-0.0059	0.0450*	0.0075*	-0.1815*
CSCO	1,477	0.2413	0.2387	-0.0003	-0.2927*	0.0004	-0.0149	0.1698**	-0.1047*
DAL	1,449	0.1093	0.1062	-0.0021*	-0.1154*	0.0432*	0.0113	0.0026	-0.2417*
DOW	1,272	0.0908	0.0872	-0.0020*	-0.0603*	0.0250*	0.0021	0.0057	-0.2443*
GE	1,389	0.1317	0.1285	-0.0007	-0.1306*	0.0023*	0.0376	0.0609*	-0.2051*
HWP	1,497	0.0701	0.0670	0.0010	-0.1043*	-0.0020	0.0285	0.0345*	-0.1352*
IBM	1,501	0.1249	0.1220	-0.0007	-0.1816*	0.0016	-0.0317	0.0477*	-0.1275*
JNJ	1,408	0.1220	0.1189	-0.0011	-0.1144*	0.0073*	0.0022	0.0111	-0.2894*
MER	1,435	0.1175	0.1144	-0.0016*	-0.1942*	0.0119*	0.0181	0.0057	-0.1315*
MMM	1,369	0.1089	0.1056	-0.0024*	-0.1063*	0.0300*	0.0191	0.0097	-0.2714*
MRK	1,343	0.1489	0.1457	-0.0020*	-0.1345*	0.0071*	0.0543*	0.0426*	-0.2485*
SLB	1,436	0.1603	0.1574	-0.0012	-0.1083*	0.0107*	0.0266	0.0095	-0.3378*
TXN	1,497	0.1932	0.1905	-0.0016	-0.1749*	0.0072*	-0.0133	0.0425*	-0.2305*
UAL	1,441	0.1215	0.1184	-0.0015*	-0.0933*	0.0508*	0.0331*	0.0061*	-0.2627*
XOM	1,298	0.1049	0.1014	-0.0009	-0.0264	0.0047*	0.0198	0.0093	-0.3082*
XRX	1,473	0.1195	0.1165	-0.0053*	-0.1096*	0.0591*	0.0301	0.0126	-0.2082*
Mean across stocks									
					-0.1281	0.0142	0.0175	0.0314	-0.2077



*ATMC* is positive but insignificant. Again, the evidence supports the notion that changes in the demand for index puts drives the price movements of S&P 500 options.

Similarly, the stock option results reported in Table VII support the notion that for stock options, call option trading drives movements in put option-implied volatility. In panel A, only one option class has a significant coefficient on *ATMP*, while nine of the 20 have a significant coefficient on *ATMC* in panel B. In panel A, also note that the coefficients on *OTMP* are higher than those on *ATMP* on average, reaffirming the idea that options' own demand is a key driver of implied volatility movements. It is also worth noting that the coefficients on lagged change in volatility are again significantly negative, indicating price reversals.

In summary, this section documents a strong statistical relation between the change in implied volatility and net buying pressure, holding constant the effects of known determinants of volatility. In addition, the nature of the movements in implied volatility appear to be market-specific. For S&P 500 index options, net buying pressure for index puts has a more dominant role than index calls. The opposite is true for stock options. This difference in behavior in the two markets is consistent with the relative trading volume figures reported in Table IV. Our results support the limits to arbitrage hypothesis over the learning hypothesis for several reasons. Under the learning hypothesis, volatility changes are permanent and will be best reflected in the demand for ATM options. In contrast, implied volatility changes on day one are shown to reverse in part on the following day, and an option's own net buying pressure is shown to be the key buying pressure variable in explaining changes in implied volatility. Both of these results are consistent with the limits to arbitrage hypothesis.<sup>26</sup>

### III. The Profitability of Selling Options and the IVF

The results of the last section show that movements in the IVFs of index and stock options are related to net buying pressure. While this evidence sheds light on why IVFs vary through time, it does not per se explain why the IVFs of index options and stock options have distinctly different shapes on average over our sample period or why the IVFs of index options are so much higher on average than realized volatility rates. The purpose of this section is to determine whether the differences between implied and realized volatility shown in Figure 5 can generate abnormal trading opportunities. We do so by conducting a series of trading simulations.

<sup>26</sup> An alternative explanation of the documented relation between public order flow for options and implied volatility is that changes in aggregate risk aversion affects both demand for options, perhaps for hedging purposes, as well as their value. Rosenberg and Engle (2002), for example, estimate empirical risk aversion monthly using S&P 500 index option prices over the 1991 to 1995 period, and find significant monthly changes in implied aggregate risk aversion. At the daily frequency, however, changes in aggregate risk aversion are likely to be small, suggesting that market frictions, such as limits to arbitrage, are a more plausible explanation of our findings.

Simulated trading strategies have been used in past investigations of index option prices. Whaley (1986), for example, uses an American-style futures option valuation method based on the Black and Scholes (1973) assumptions to identify mispriced S&P 500 futures options during their first year of trading on the CME. He finds that abnormal profits can be earned by writing OTM puts. Similarly, Bondarenko (2001) examines prices of out-of-the-money puts written on S&P 500 futures during the period 1988 through 2000 and concludes the market is inefficient.

Some studies argue that the abnormal profits generated by option writing strategies may be driven by option buyers' willingness to pay for volatility risk. Because index returns and volatility are negatively correlated, options, which have positive vega,<sup>27</sup> can act as a hedge against falling stock prices. The higher the vega of an option, it is argued, the more effective the hedge against falling stock prices, and the higher the risk premium paid by option buyers. Option writers, on the other hand, collect these risk premia, and should, therefore, expect a positive abnormal return within the Black–Scholes framework. Consistent with this view, Fleming (1999) finds that ATM puts and calls are overpriced relative to the Black–Scholes model, though trading profits disappear after transaction costs. He finds that profits are positively related to the level of volatility as predicted by a volatility risk premium. Jackwerth (2000) finds that profitability is significant even after simulating stock market crashes, and that ATM puts are more profitable to sell than OTM puts. Similarly, Bakshi and Kapadia (2003) find that ATM calls are more overpriced than OTM calls, and argue that since ATM option values are more sensitive to changes in volatility, the profitability is evidence of a negative volatility risk premium.

Based on our analyses of the S&P 500 index option and stock option IVFs, as well as their respective trading volumes in Section II, we offer a different explanation for the abnormal simulated trading profits reported in past work. That is, in the course of supplying liquidity, option market makers often establish significant long or short positions in options. If public order flow in a particular option market is dominated by buyers, market makers, on average, must be net short. In such a market (e.g., the S&P 500 index option market), the implied volatility embedded in option prices will exceed the actual volatility rate of the underlying asset since market makers will set prices in such a way as to be compensated for their costs of operation and earn a profit. Green and Figlewski call this a “volatility markup” (1999, p. 1493).<sup>28</sup>

Volatility markups need not be constant across option series, however. The larger the short position in a particular series, the greater the market maker's hedging costs and exposure to upward movements (and possibly spikes) in volatility. Assuming the market maker is risk-averse, he has two possible

<sup>27</sup> Using the notation of the option valuation formulae (1) from Section I, the partial derivative of the option value with respect to volatility, or vega, of a call or a put written at the same exercise price and time to expiration is  $(S - PVD)n(d_1)\sqrt{T}$ , where  $n(d_1)$  is the standard normal density function. This expression is clearly positive.

<sup>28</sup> Naturally, a reverse argument would also apply. If the public order flow places market makers in a net long position, there will be a “volatility markdown.”

courses of action. First, he can self-insure by embedding a volatility risk premium in option price. Due to his risk aversion, the insurance premium per contract will grow larger the greater the number of contracts he is forced to short. Second, he can hedge the volatility risk by buying options and embed the additional cost in option price. This strategy is necessarily expensive because, as we have already shown, S&P 500 option prices are too high relative to their Black–Scholes values based on actual volatility.

This section contains the results of simulating these two types of risk management strategies for the market maker. The first assumes that the market maker sells options and hedges only his delta risk exposure. He does nothing to manage the volatility risk exposure. The second assumes that the market maker hedges both delta and vega risk. The profitability of each of these strategies is addressed in turn. Prior to doing so, we provide specific details of how the trading simulation is conducted.

### A. Trading Simulation Design

To conduct the trading simulations, we use monthly returns. Index options and stock options expire on the third Friday of the month. On each of these expiration days, we sell all calls and puts with 1 month to expiration and hold them until they expire. Since a calendar year has 52 weeks, eight of the “1-month” holding periods each year are 4 weeks in length, the others are 5. Using monthly returns in this way circumvents the confounding effects of overlapping observations. The selling of a particular option series happens only once during the sample period.

In the delta-neutral trading strategy, each option is assumed to be sold at a price equal to the midpoint of the bid and ask price quotes prevailing at 3 PM (CST). To offset the price risk of the position,  $|\Delta_t|$  units of the underlying security are purchased in the case of a call and sold in the case of a put. Each day during the life of the trade, the delta-hedge is revised by changing the number of units in the underlying asset. Any gains or losses are carried forward until the option expiration day. For a call option series, the abnormal risk-adjusted rate of return of the trading strategy over the life of the call is

$$\begin{aligned}
 & \text{ARET}_c \\
 &= \frac{\Delta_0 \left( S_T + \sum_{t=0}^T D_t e^{r(T-t)} - S_0 e^{rT} \right) - (c_T - c_0 e^{rT}) + \sum_{t=0}^{T-1} \Delta_t (S_{t+1} + D_t - S_t) e^{r(T-t)}}{\Delta_0 S_0 - c_0},
 \end{aligned} \tag{9}$$

where  $\Delta_t$  is the delta value on day  $t$  during the option’s life,<sup>29</sup>  $S_t$  is the closing price of the underlying asset on day  $t$ ,  $c_t$  is the call option price on day  $t$ , and

<sup>29</sup> The delta values are computed at the close of trading each day based upon updated values for the index level, the time to expiration, and the dividends paid during the remaining life of the option. The volatility rate and the interest rate are those prevailing in the marketplace when the position was opened.

$r$  is the risk-free rate of interest. The subscripts 0 and  $T$  represent the time when the position is opened and closed, respectively. For the S&P 500 index options,  $S_T$  is the cash settlement price of the index on expiration day, and for stock options,  $S_T$  is the stock price at the close on expiration day. The settlement price of the call is  $c_T = \max(0, S_T - X)$  in both cases.

The three terms in the numerator of the abnormal return expression (9) are as follows. The first term is the income from holding the original underlying asset position over the month net of its financing costs. The second term is the gain or loss on the call position. The sign in front of the parenthesis is negative to reflect the fact that the call is sold. Note that the call option premium collected at the outset is assumed to accrue interest over the month. Finally, the last term is the sum of the mark-to-market gains/losses from adjusting the delta each day during the holding period carried until the end of the period at the risk-free interest rate. The denominator is the cost of the position at inception. The abnormal risk-adjusted rate of return of the put is computed in a similar manner. In theory, the expected abnormal returns are zero since the strategies are risk-free and completely financed at the risk-free interest rate.

### *B. Results of Delta-Neutral Hedging Strategy*

Table VIII contains the average abnormal returns from applying the delta-neutral trading strategy to index and stock options during the period January 1995 through December 2000. The asterisks appearing in the table indicate whether the abnormal return is significantly different from zero. The underlying hypothesis test was performed using the Johnson (1978) modified  $t$ -test, which explicitly accounts for the fact that the abnormal returns are drawn from an asymmetrical distribution. The test statistic is

$$t_J = (\overline{ARET} + \sigma S/6n + \overline{ARET}^2 S/3\sigma)(\sigma^2 n)^{-0.5},$$

where  $\overline{ARET}$  is the mean abnormal return,  $\sigma$  is the standard deviation,  $S$  is the skewness, and  $n$  is the number of observations. Note that when skewness equals zero, Johnson's statistic collapses to the standard  $t$ -test.

Table VIII shows that writing S&P 500 index options appears to be remarkably profitable during the sample period. The average monthly abnormal returns are positive (and, with the exception of category 4, significantly different from zero) for all moneyness categories. They range from 6.9 percent for category 1 options to 2.0 percent for category 4 options. Interestingly, this pattern of abnormal returns corresponds to deviations between implied and realized volatility noted in Figure 5—the larger the volatility deviation, the larger the abnormal return.

The abnormal returns from selling stock options are generally insignificantly different from zero. Again, this is consistent with Figure 5, where the average deviation between implied volatility and realized volatility across moneyness categories is approximately zero. Across all option classes, the largest average

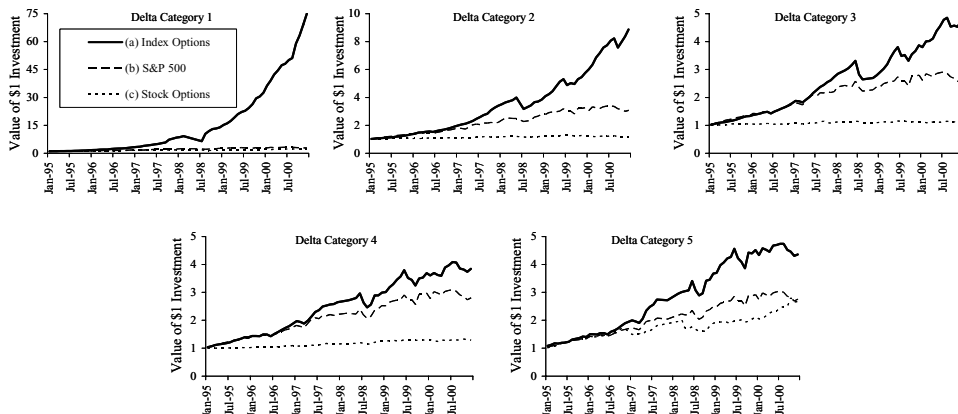
**Table VIII**  
**Average Monthly Abnormal Returns on Delta-Neutral Hedge**  
**Portfolios Formed Using S&P 500 Index Options and 20 Stock Options**  
**Traded on the Chicago Board Options Exchange during the Period**  
**January 1995 through December 2000**

Stock option classes are the 20 most active that traded continuously throughout the sample period. Implied volatilities are computed daily based on the midpoint of the bid/ask quotes as of 3 PM (CST). The analytical European-style formula is used to compute implied volatilities for S&P 500 index options, and the dividend-adjusted binomial method is used to compute implied volatilities for the American-style stock options. The delta value of each option series is computed using the closing stock/index price, the actual dividends paid during the option's life, the Eurodollar rate matching the option's time to expiration, and the realized volatility over the most recent 60 trading days. Hedge portfolios are formed by selling one option and buying/selling delta units of the underlying asset. The hedge is adjusted each day during the option's life by changing the number of units of the asset. Average returns marked with an asterisk are significant at the 5 percent level using Johnson's (1978) modified *t*-test. The "mean" row reported at the bottom of the table contains the average values across the 20 stock options.

Ticker	Delta Value Categories				
	1	2	3	4	5
	Average Abnormal Returns				
SPX	0.06873*	0.03263	0.02321*	0.02007*	0.02390*
AIG	-0.00081	0.00060	0.00061	-0.00042	0.01055
AOL	0.01652	0.00940	0.00879	0.01202	0.01737
BMJ	-0.00270	-0.00443	-0.00736	-0.00362	0.01276
CL	0.00708	0.00177	0.00385	0.00313	0.02627*
CSC	0.01932	0.00690	0.00606	0.00313	0.03221*
CSCO	0.01098	-0.00007	0.00340	0.01067*	0.02687*
DAL	0.01180	0.00543	0.00541*	0.00489	0.02638*
DOW	0.03595*	0.00322	0.00199	0.00028	0.01736
GE	0.00983	0.00464	0.00577	0.00481*	0.02278*
HWP	0.02276	0.00178	-0.00544	-0.00280	-0.03833
IBM	0.01062	-0.00086	0.00317	0.00446	0.02013
JNJ	0.01941*	0.00641	-0.00087	0.00533	0.03019*
MER	0.02143	0.00751	0.00555	0.00939*	-0.00654
MMM	0.01672	0.00029	0.00225	0.00306	0.00116
MRK	0.01817*	0.00029	-0.00204	0.00922*	0.02775*
SLB	0.00916	0.00032	0.00153	-0.00119	0.02163
TXN	0.01452	-0.00246	-0.00295	-0.00148	-0.00750
UAL	0.01457	0.00691	0.01227*	0.00317	0.02721
XOM	0.01171*	0.00127	-0.00373	0.00070	0.01211
XRJ	0.01186	-0.01703	-0.00014	0.00884	0.08455
<i>Mean across stocks</i>	0.01395	0.00159	0.00191	0.00368	0.01825

abnormal return is 1.8 percent for the category 5 options. Category 3 options have an average abnormal return of 0.2 percent.

Figure 6 contrasts the profitability of selling options on individual stocks with the profitability of selling index options. Shown is the growth of \$1 invested in the strategies over time. Each month, the rate of return is computed as an



**Figure 6. Cumulative profits from selling S&P 500 index options vis-à-vis stock options.** Trading strategy involves selling one-month call and put options on each option expiration day and then dynamically hedging the positions throughout the options' remaining lives. Shown are the cumulative growth rates of \$1 invested in (a) the strategy using S&P 500 index options, (b) a buy-and-hold strategy involving the S&P 500 index, and (c) the strategy using the 20 individual stock options. Results are categorized by option delta when the option is sold. For puts, the five delta ( $\Delta$ ) categories are  $-0.02 \geq \Delta > -0.125$ ,  $-0.125 \geq \Delta > -0.375$ ,  $-0.375 \geq \Delta > -0.625$ ,  $-0.625 \geq \Delta > -0.875$ , and  $-0.875 \geq \Delta \geq -0.98$ . The corresponding call categories are  $0.875 \leq \Delta \leq 0.98$ ,  $0.625 \leq \Delta < 0.875$ ,  $0.375 \leq \Delta < 0.625$ ,  $0.125 \leq \Delta < 0.375$ , and  $0.02 \leq \Delta < 0.125$ .

equally weighted average of the delta-neutral return from selling each option in the experiment. The index option strategy is always above the stock option strategy. The category 5 index options, which include the DOTM puts, result in a terminal balance of over \$75, reflecting a geometric mean annual return of over 105 percent.

The results of the delta-neutral trading strategy simulations indicate that systematically writing S&P 500 index options is extremely profitable, at least on a before-trading-cost basis. On the other hand, even before trading costs are imposed, systematically writing stock options does not appear to generate abnormal gains. Thus, our focus narrows to the S&P 500 index options and whether writing S&P 500 index options from the perspective of an index option market maker is a profitable activity. Also, since there is no longer a need to use the time period for which stock option data were available, we include simulation results for the sample that begins in June 1988.

### C. Results of Delta-/Vega-Neutral Hedging Strategy after Trading Costs

Table IX contains a series of trading strategy simulation results in which the benefits/costs of market making are introduced one at a time. For convenience, the abnormal returns of the trading strategy when trades are executed at bid/ask midpoints from Table VIII are presented again as hedge strategy 1. Note that the average abnormal returns for the delta-neutral hedge strategy 1

**Table IX**  
**Average Monthly Abnormal Returns on Hedge Portfolios Formed**  
**Using S&P 500 Index Options Traded on the Chicago Board Options**  
**Exchange during the Period June 1988 through December 2000**

The analytical European-style formula is used to compute implied volatilities for S&P 500 index options. The delta value of each option series is computed using the closing index level, the actual dividends paid during the option's life, the Eurodollar rate matching the option's time to expiration, and the realized volatility over the most recent sixty trading days. Average returns marked with an asterisk are significant at the five percent level using Johnson's (1978) modified *t*-test. Hedge portfolios and trading cost assumptions are as follows:

<i>Strategy</i>	<i>Description</i>
1	Delta-neutral hedge with no trading costs. Options are sold at 3 PM (CST) bid/ask midpoint. S&P 500 index position is rebalanced daily.
2	Delta-neutral hedge with options sold at 3 PM (CST) ask price. S&P 500 index position is rebalanced daily with no trading costs.
3	Delta-neutral hedge with options sold at 3 PM (CST) ask price. S&P 500 index position is rebalanced daily with trading cost equal to one-half the S&P 500 futures bid/ask spread.
4	Delta-neutral hedge with options sold at 3 PM (CST) ask price. S&P 500 futures position is rebalanced daily with trading cost equal to one-half the S&P 500 futures bid/ask spread.
5	Delta/vega-neutral hedge with trading costs on index only. Options are sold at 3 PM (CST) ask price. ATM call option is bought at the bid/ask midpoint to hedge the vega risk and is rebalanced daily. S&P 500 index is used to managed the delta risk and is rebalanced daily with a trading cost equal to one-half the S&P 500 futures bid/ask spread.
6	Delta/vega-neutral hedge with trading costs on index and vega-hedge option. Options are sold at 3 PM (CST) ask price. ATM call option is bought at the ask to hedge the vega risk and is rebalanced daily. S&P 500 index is used to managed the delta risk and is rebalanced daily with a trading cost equal to one-half the S&P 500 futures bid/ask spread.

Hedge Strategy	Delta Value Categories				
	1	2	3	4	5
	Average Abnormal Returns				
Panel A. Sample Period June 1988 through December 2000					
1	0.05423*	0.03075	0.02298*	0.01980*	0.02141*
2	0.05679*	0.03192	0.02394*	0.02087*	0.02359*
3	0.04562	0.02461	0.01713*	0.01361*	0.01407*
4	0.04576	0.02476	0.01662*	0.01262*	0.01245*
5	−0.03443	−0.01658*	−0.00112	0.00053	0.01197
6	−0.07700	−0.04441*	−0.02716*	−0.02608*	−0.02161
Panel B. Sample Period January 1995 through December 2000					
1	0.06873*	0.03263	0.02321*	0.02007*	0.02390*
2	0.07118*	0.03387	0.02430*	0.02119*	0.02609*
3	0.05643*	0.02389	0.01486*	0.01110*	0.01379*
4	0.05639*	0.02374	0.01435*	0.01041	0.01176
5	−0.03411	−0.02191*	−0.00342*	−0.00218	0.02460
6	−0.08268	−0.05298*	−0.03291*	−0.03298*	−0.01929

are lower in the period June 1988 through December 2000 than in the more subperiod January 1995 through December 2000. For category 1 options, for example, the average monthly abnormal returns are 5.4 percent and 6.9 percent, respectively. Apparently the abnormal returns are not disappearing as the market grows older.

The first adjustment to the trading strategy involves replacing trades at the midpoints with trades at the ask price. This is done to acknowledge that if public order flow is exclusively market orders to buy index options, the market maker will be selling at his ask price. Naturally, sales at the ask increase the abnormal returns of the delta-neutral option trading strategy, as the results of hedge strategy 2 show. The increased return averages about 0.2 percent a month across moneyness categories.

Hedge strategy 3 explicitly acknowledges that trading the underlying index is costly. To proxy for the effects of trading costs on the index, we use one-half the bid/ask spread of the nearby S&P 500 futures contract. Note that these trading costs are incurred when the index portfolio is purchased/sold when the position is initially taken as well as day to day when the number of units in the hedge portfolio is adjusted. Trading costs on the index reduce the abnormal returns. Category 1 options, for example, now have a monthly abnormal return of 4.6 percent per month, down a little more than 100 basis points from where trading costs on the index were ignored. It should be noted that our delta-hedge results may overstate the effects of trading costs. In providing liquidity to the marketplace, the market maker acquires short call positions, which naturally offset the delta exposure of short index puts.

Hedge strategy 4 is the same as three except that the nearby S&P 500 futures is used to delta-hedge the option position rather than the index itself. The reason for this is that S&P 500 index option market makers use the S&P 500 futures to hedge since basket trading the entire S&P 500 is costly. Potentially, the abnormal returns could go up or down as a result of this change. The reason is that the futures expires on a quarterly cycle while the options expire monthly. For the nonquarterly option expirations, the index option prices are not forced to converge to the index level, as they are for the quarterly expirations. This additional basis risk may cause average abnormal returns to become more noisy, but the direction of the bias is unclear. The results of hedge strategy 4 show that the use of the index futures rather than the index leaves the average abnormal returns virtually unchanged.

Thus far we have only accounted for the trading costs of delta-hedging a short index option position. The average abnormal returns are reduced but remain positive and abnormally large. For the category 1 options, the monthly return is 4.6 percent, although it is not significantly different from zero. For category 3 options, the average abnormal return is 1.7 percent per month and is significantly different from zero.

To this point, however, we have not addressed the issue of volatility risk. While the abnormal returns are positive, they may merely be compensation for the volatility risk that the market maker has assumed. Only the delta risk and its costs have been considered. One way to assess whether the size of the



volatility risk premium is justifiable is to also vega-hedge the option position. Since the market maker is short volatility, he must buy other index options to hedge his volatility risk. The choice of exactly which option to use is arbitrary. We choose the index call whose delta is nearest 0.5 because its vega is higher than OTM and ITM options, and therefore, fewer contracts are needed to hedge. The hedging strategy now has two steps. Like before, one option is sold. But, before the delta-hedge is put on, a number of ATM calls are purchased. The number of ATM calls equals the vega of the option sold divided by the vega of the ATM call. The net delta of the combined option position is then hedged using the index. Each day, the vega-hedge and delta-hedge are adjusted to make the overall portfolio delta and vega risk neutral. Any intermediate gains or losses on the hedge positions are carried forward until the option's expiration at the risk-free interest rate.

Hedge strategy 5 shows the abnormal returns with the vega-hedge in place. Note that these returns reflect only the cost of the hedge option, in the sense that buying the ATM call is assumed to take place at the bid/ask midpoint. But, even with only the cost of the option considered, the abnormal returns become negative for the first three moneyness categories, and are insignificantly different from zero for the remaining two. If the trading costs of the trades of the vega-hedge are incorporated, the abnormal returns are large and negative for all categories (see hedge strategy 6).

One possible explanation for the fact that all option categories have negative abnormal returns is that the market maker is not charging a high enough volatility risk premium. The most effective way that the market maker has to hedge his short volatility risk is to buy index options. If he does, he loses money.

#### **IV. Summary and Conclusions**

The intriguing relation between Black–Scholes implied volatility and the exercise prices of index options has been the focus of a number of empirical investigations. Most of the investigations examine whether the shape of the IVF and its variation through time are a consequence of inappropriate assumptions regarding the stochastic movements of asset price and volatility. It is becoming increasingly apparent, however, that none of these explanations provides a completely satisfactory explanation. In this study, we explore the possibility that market makers set option prices with a model not radically different from Black and Scholes (1973) and that the shape of the IVF is attributable to the buying pressure of specific option series and a limited ability of arbitrageurs to bring prices back into alignment. In particular, we document that daily changes in the implied volatility of an option series are significantly related to net buying pressure and that the changes are transitory, as market makers are gradually able to rebalance their portfolios. Buying pressure on index put options appears to drive the permanently downward sloping shape of the S&P 500 index option IVF, consistent with hedgers seeking portfolio insurance. In contrast, buying pressure on call options appears to drive the shape

of stock option IVFs. A simulated trading strategy that sells options, and then delta-hedges the positions using the underlying security, generates significant paper profits for the index but not for individual stocks. For index options, we find that profits are highest for the category of options that contain the OTM puts, which corresponds to the institutional demand for portfolio insurance. While the prices of these options are considerably higher than is suggested by the Black–Scholes formula and the actual level of volatility in the marketplace, they do not represent profitable arbitrage opportunities for the market maker once the costs of hedging volatility risk are considered.

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