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Asset Price Dynamics and Infrequent Feedback Trades

PIERLUIGI BALDUZZI, GIUSEPPE BERTOLA, and SILVERIO FORESI*

ABSTRACT

This article combines the continuous arrival of information with the infrequency of trades, and investigates the effects on asset price dynamics of positive and negative-feedback trading. Specifically, we model an economy where stocks and bonds are traded by two types of agents: speculators who maximize expected utility, and feedback traders who mechanically respond to price changes and infrequently submit market orders. We show that positive-feedback strategies increase the volatility of stock returns, and the response of stock prices to dividend news. Conversely, the presence of negative-feedback traders makes stock returns less volatile, and prices less responsive to dividends.

INVESTORS ROUTINELY REBALANCE THEIR portfolios in response to changes in prices. In order to hold portfolio weights constant, for example, investors would buy losers and sell winners, a typical negative-feedback or contrarian strategy.¹ Alternatively, investors may implement positive-feedback strategies: buy stocks when their prices rise, and sell them when their prices fall. These strategies include portfolio choice based on extrapolative expectations (trend-chasing), the use of stop-loss orders, and portfolio insurance. Positive-feedback strategies may be optimal for investors who do not tolerate falls in wealth below a given fraction of the original wealth (Grossman and Zhou (1994)) or declines in consumption (Dybvig (1995)), or for rational investors who, in turn, exploit the positive-feedback trading of noise traders (De Long, Shleifer, Summers, and Waldmann (1990)).²

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¹ Chan (1988), and more recently Lo and McKinley (1990) investigate the origin and extent of profits deriving from such strategies.

² The positive or negative response of traders to price changes may take elaborate forms, when it is the past pattern of price changes and transactions volume to determine the direction of

This article combines the continuous arrival of information with the infrequency of trades, and investigates the effects on asset price dynamics of positive and negative-feedback trading. In the presence of transactions costs, investors may find it optimal to rebalance their portfolios only occasionally, and by discrete amounts. In fact, if transactions costs are proportional, trades would occur only when the portfolio is sufficiently far "out of line," as shown in Constantinides (1986) and Davis and Norman (1991). And if part of the transactions costs is fixed, as in Duffie and Sun (1990), it becomes optimal to trade discrete amounts of assets. While some investors may trade infrequently and only in response to price changes, other investors are always ready to take the opposite side of such trades. Examples of these are "value" investors who purchase assets when offered to them at the "right" prices. Also, the market makers of an exchange have an institutional responsibility to provide liquidity by absorbing sudden changes in supply and demand.

We examine the interaction of the two types of investors described above and its implications for asset price dynamics. We model an economy where stocks and bonds are exchanged by speculators, defined as expected-utility maximizers, who are always present in the market, and feedback traders, who trade infrequently and in discrete amounts. The infrequency of trades is not only realistic in light of the existence of transactions costs, but it is also analytically quite convenient, in that it allows us to isolate effects of trade that would be much harder to investigate in more complicated settings.

Our analysis focuses on the relation between stock prices and "fundamentals," i.e., dividends. We show that the presence of positive-feedback trading increases stocks' price responses to dividend news, while negative-feedback trading reduces such responses relative to the no-trade case. Across scenarios, speculators' expectations of stock sales *depress* stock prices, while expectations of stock purchases tend to *inflate* them. Both the depression and the inflation of stock prices induce heteroskedasticity and predictability of stock returns, even though dividend growth rates are assumed i.i.d. We also find that the presence of positive-feedback traders *increases* the volatility of stock returns, while negative-feedback trading *reduces* stock-return volatility relative to the no-trade case.

To see the intuition for these results, consider the effects of *positive* feedback traders who sell stocks when stock prices reach a *lower* threshold. Rational anticipation of such trade on the part of speculators leads to prices being depressed relative to the no-trade case. In fact, the trade is welfare-improving from the speculators' point of view, and their expectations of higher future utility reduce their incentives to save; this translates into a "softer" demand for assets, and lower stock prices. This effect also makes stock prices more sensitive to dividend news and more volatile: when dividends fall, the intrinsic

trade. This is the case of trading based on technical analysis. This type of strategy can be rationalized in the context of a model with informational asymmetries, as shown by Blume, Easley, and O'Hara (1994). Recent articles that investigate the empirical performance of technical analysis in the stock market are Neftci (1991) and Brock, Lakonishok, and LeBaron (1992).

value of a stock decreases, and a wave of sales on the part of feedback traders becomes more likely.

Our article is related to a substantial stream of research that studies price reactions in the presence of portfolio insurance and of other positive-feedback strategies, and *extends* this literature by explicitly considering negative-feedback strategies. Our model resembles the treatment of portfolio insurance by Brennan and Schwartz (1989) in that we also identify the economy's pricing kernel with the speculators' marginal utility; and we find that the presence of portfolio insurers depresses stock prices and increases their volatility. Unlike their portfolio insurers, though, our feedback traders' are only allowed to trade *infrequently*, and the horizon of our economy is *infinite*, rather than finite. Basak (1993) develops a model where portfolio insurance schemes are derived endogenously from a constraint on wealth that may bind prior to the last date of trading. Since both speculators' and portfolio insurers' marginal rates of substitution determine prices, he finds the implications of portfolio insurance to be the opposite of Brennan and Schwartz's (1989): stock prices are *higher* and *less* volatile relative to prices in an economy with no portfolio insurers. In a similar spirit, Grossman and Zhou (1994) investigate the equilibrium implications of portfolio insurance, where the portfolio insurers' behavior is also derived endogenously from a "floor" on the value of their wealth at any time. Similar to positive-feedback traders in our model, Grossman and Zhou's (1994) insurers sell stocks when prices fall, and this depresses stock prices, increases their volatility, and causes negative serial correlation of returns.

A related model introduced by Campbell and Kyle (1993) shares with ours the formal decomposition of asset prices into fundamental value and "noise," where the noise component is induced by current and future expected trade on the part of noise traders. Their empirical findings show that in order to contribute to stock price behavior, noise trading must be correlated with the dividend process. This feature is captured by the examples in our article that assume dividends to trigger trades, through prices.

Section I presents the model. We identify a component of asset prices that reflects the value of having the opportunity to trade the asset at some point in the future. This decomposition of prices becomes the basis of a solution technique for equilibrium prices developed in Section II, in the context of a first example of positive-feedback trading. Section III discusses the relevance of the example for actual stock price behavior. Section IV looks at alternative positive and negative-feedback trading strategies, and at the role of risk aversion. Section V concludes, suggesting extensions of our framework to more realistic trade scenarios.

I. Trade and Prices

We begin our analysis with a description of the structure of the economy.

ASSUMPTION 1: *The economy is populated by two groups of investors: speculators and feedback traders. Speculators are identical, maximize their utility from*

consumption, and have rational expectations about the dynamics of future cash flows generated by available assets, and about the structure of possible trades. Feedback traders submit infrequent and discrete market orders. The determinants of these trades are not modeled.

ASSUMPTION 2: Two assets exist in positive net supply: stocks, which entitle their owner to a stochastic flow of dividends $\{\xi\}$, and bonds, which entitle their owner to a certain and constant coupon flow $\{r\}$. The time horizon is infinite. We abstract from issues of physical investment and economic growth, and assume that no storage technology is available.

ASSUMPTION 3: The state of the economy is summarized by a vector of state variables $[\xi, S, B, Y]$, where ξ is the dividend flow on stocks, and S and B denote per-capita holdings of stocks and bonds, respectively, among speculators. The probabilistic structure of trade is summarized by the state vector Y , which contains information relevant to the speculators' assessment of the likelihood and size of market orders at every point in the economy's state space.

Speculators' optimum. Let c denote the speculators' consumption flow, $U(\cdot)$ an increasing and concave utility function, and ρ the rate of time preference. Also, let P_s and P_b denote the prices of stocks and bonds. The following first-order conditions link optimal speculator's consumption to asset prices (these conditions follow directly from the Bellman equation, as shown in Appendix A):

$$\rho U'(c)P_s = U'(c)\xi + \frac{1}{dt} E_t\{d[U'(c)P_s]\}, \quad (1)$$

$$\rho U'(c)P_b = U'(c)r + \frac{1}{dt} E_t\{d[U'(c)P_b]\}. \quad (2)$$

After adjusting prices and payoffs by the marginal utility of consumption, all assets should yield the same expected rate of return, ρ . Also, speculators' consumption need satisfy the following transversality conditions

$$\lim_{\tau \rightarrow \infty} E_t[U'(c(\tau))P_i(\tau)]e^{-\rho(t-\tau)} = 0, \quad i = s, b. \quad (3)$$

Equilibrium. As we have assumed away storage or physical investment, speculators consume dividends and coupons generated by the stocks and bonds they own:

$$c = S\xi + Br.$$

Consider next the prices supporting the equilibrium. We define risk-adjusted asset prices p_i and cash-flows f_i ,

$$p_s \equiv U'(c)P_s, \quad f_s \equiv U'(c)\xi, \quad p_b \equiv U'(c)P_b, \quad f_b \equiv U'(c)r,$$

and rewrite equations (1) and (2) as:

$$\rho p_i = f_i + \frac{1}{dt} E_t(dp_i), \quad i = s, b. \quad (4)$$

The differential equations (4) must be satisfied at all points in the economy's state space, regardless of the likelihood of trade.

No-expected-jump conditions. Given equation (4), $E_t(dp_i)$ cannot be of order larger than dt , and this rules out expected jumps in risk-adjusted price paths. It follows that

$$E_t(\Delta p_i) = 0, \quad i = s, b, \quad (5)$$

at trading times. Hence, when trade takes place, the discrete change in asset prices is "compensated" by a discrete change in the marginal utility of consumption so that expected risk-adjusted price paths remain continuous.

Trade and prices. Note that unlike in Lucas (1978), consumption and risk-adjusted payoffs $f_i(\tau)$ are affected by trade as well as by dividends. Hence it is useful to decompose risk-adjusted prices in the form

$$p_i(\xi, S, B, Y) = g_i(\xi, S, B) + h_i(\xi, S, B, Y), \quad i = s, b, \quad (6)$$

for

$$g_i = \int_t^{\infty} E_t[f_i(\tau) \mid \text{no trade}] e^{-\rho(t-\tau)} d\tau, \quad i = s, b, \quad (7)$$

where $[f_s(\tau) \mid \text{no trade}] \equiv U'(S_t \xi_\tau + B_t r) \xi_\tau$, and $[f_b(\tau) \mid \text{no trade}] \equiv U'(S_t \xi_\tau + B_t r) r$. The function g_i takes the speculators' portfolio composition as given, and would be the asset's equilibrium value if trade could be disregarded. Hence, h_i reflects the effects of trade on the asset's price, and can be viewed as the value of trading the asset at some point in the future.

Campbell and Kyle (1993) obtain a similar decomposition of asset prices. They identify the "fundamental value," which depends on current and expected future cash flows only, and the "noise," which is induced by current and future expected trade on the part of noise traders.

II. A Specialized Framework

We now specialize our framework in terms of preferences, cash-flow dynamics, and structure of trade. We confine ourselves to situations where the stock price is a monotonic function of the dividend flow.

ASSUMPTION 4: *The speculators' instantaneous utility function has the form*

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the relative risk aversion parameter.

ASSUMPTION 5: *Dividends follow a geometric random walk*

$$d\xi = \mu\xi dt + \sigma\xi dw,$$

with μ and σ positive constants.

Expanding $E_t(dp_i)$ in equation (4) by the usual stochastic calculus arguments, we find that in the interior of no-trade regions the p_i functions satisfy the valuation equations

$$\rho p_i = f_i + \mu\xi p'_i + \frac{\sigma^2\xi^2}{2} p''_i, \quad i = s, b. \quad (8)$$

No-trade solutions. When either $B = 0$ or $S = 0$, the solution for the no-trade component g_i in equation (6) is obtained from evaluation of the integral in equation (7). We have

$$B = 0 \Rightarrow g_s = \frac{\xi}{(S\xi)^{\gamma}d_s}, \quad g_b = \frac{r}{(S\xi)^{\gamma}d_b}, \quad (9)$$

where $d_s \equiv [\rho + (\gamma - 1)\mu - (\gamma - 1)\gamma\sigma^2/2]$, $d_b \equiv [\rho + \gamma\mu - \gamma(\gamma + 1)\sigma^2/2]$; and

$$S = 0 \Rightarrow g_s = \frac{\xi}{(\rho - \mu)(Br)^{\gamma}}, \quad g_b = \frac{r}{\rho(Br)^{\gamma}}. \quad (10)$$

Goods-denominated prices P_i are computed from the risk-adjusted prices p_i by the relation $P_i = p_i c^{\gamma}$, with $c = S\xi$ in the $B = 0$ case, and $c = Br$ in the $S = 0$ case. To ensure that prices are positive and finite, parameter values must be such that $d_s > 0$, $d_b > 0$, and $\rho > \mu$.

Homogeneous solutions. The second component of asset prices in equation (6), h_i , can be found as the homogeneous solution of equation (8). The solution has the form

$$h_i = Q_i(S, B, Y)\xi^{\lambda_1} + N_i(S, B, Y)\xi^{\lambda_2}, \quad i = s, b, \quad (11)$$

where λ_1 and λ_2 solve the characteristic equation associated with equation (8) (see Appendix B), and Q_i and N_i are constants of integration.

In the following we consider a positive-feedback trade scenario: feedback traders sell off stocks, in exchange for all the bonds held by the speculators, when stock prices reach a *lower* threshold.

ASSUMPTION 6: *Initially speculators do not hold stocks: $B = B_0 > 0$, $S = 0$. Feedback traders buy all of the speculators' bonds, selling S_1 shares of stocks when P_s hits a lower threshold P_{sl} . After the barrier is hit, it disappears and no more trades take place at P_{sl} or any other price.*

Similar positive-feedback strategies may be optimal for the investors described in Grossman and Zhou (1994), who do not tolerate falls in wealth below

a given fraction of the original wealth, or for the consumers of Dybvig (1995), who do not tolerate declines in consumption.

We do not model feedback traders' motives and constraints, but we take as given their demand of stocks (and bonds) as a function of stock prices. The infrequent and discrete trades postulated in Assumption 6 can be rationalized by considering realistic frictions. With transactions costs, for example, it is not optimal to trade unless the portfolio composition is sufficiently far out of line. The optimal no-trade region when transactions costs are proportional to the amounts traded has been studied by Constantinides (1986) and Davis and Norman (1991) in a continuous-time setting similar to ours. When trade can take place continuously, however, traded amounts will not be discrete unless part of the costs is fixed, as in the framework considered by Duffie and Sun (1990).

While the initial and final asset allocations assumed here are extreme, this allows us to use the formulas in equations (9) and (10) to compute explicit solutions for before and after-trade prices. Also, trade occurs only once. This need not be taken literally: we may think that after one trade takes place, other trades are expected in the future, but that the likelihood of their occurring soon is *low*.

In equilibrium there is a unique level of dividends ξ_l at which trade takes place. This level is implicitly defined by the condition $P_{sl} \equiv P_s(\xi_l, 0, B_0, Y_0)$. The state variable $Y = Y_0$ summarizes the information in Assumption 6 on the possible future trade at ξ_l , and $Y = Y_1$ indicates that no trade is expected to ever take place after the barrier is hit.

At $\xi = \xi_l$ the no-jump conditions (5) reduce to

$$p_i(\xi_l, 0, B_0, Y_0) = p_i(\xi_l, S_1, 0, Y_1), \quad i = s, b. \quad (12)$$

As trade occurs with certainty at ξ_l , (12) implies that the risk-adjusted price of each asset cannot jump at the time of trade.

Trade and asset allocation. Since no trade takes place after the barrier is hit, the amount of stocks held by speculators after the trade is endogenously determined by the budget constraint:

$$\begin{aligned} S_1 &= \frac{P_b(\xi_l, S_1, 0, Y_1)}{P_s(\xi_l, S_1, 0, Y_1)} B_0 = \frac{p_b(\xi_l, S_1, 0, Y_1) c^{-\gamma}}{p_s(\xi_l, S_1, 0, Y_1) c^{-\gamma}} B_0 \\ &= \frac{p_b(\xi_l, S_1, 0, Y_1)}{p_s(\xi_l, S_1, 0, Y_1)} B_0 = \frac{r d_s}{\xi_l d_b} B_0. \end{aligned} \quad (13)$$

Note that because of the boundary conditions in equation (12), the relative price at which trade takes place can be obtained as the ratio between after-trade prices. Equation (13) illustrates how traded quantities are endogenous, and how trade events are not the same as endowment shocks in a modified Lucas (1978) "tree" economy: the number of stocks received by speculators in exchange for bonds is determined by endogenous prices.

As shown in Appendix C, the transversality conditions (3) require $Q_i = N_i = 0$ after trade takes place, and $Q_i = 0$ before trade takes place. Moreover, the constants N_i in the before-trade price functions must satisfy the following version of equation (12)

$$\frac{\xi_l}{(\rho - \mu)(B_0 r)^\gamma} + N_s \xi_l^{\lambda_2} = \frac{\xi_l}{d_s(S_1 \xi_l)^\gamma}, \quad \frac{r}{\rho(B_0 r)^\gamma} + N_b \xi_l^{\lambda_2} = \frac{r}{d_b(S_1 \xi_l)^\gamma} \quad (14)$$

where the after-trade allocation S_1 can be obtained from equation (13).

In the rest of the article we shall focus our discussion on the behavior of stock prices for explicit choices of parameter values. The behavior of bond prices is similar, but less interesting, due to the absence of fluctuations in the coupon flow.

Risk-adjusted versus goods-denominated prices. Figure 1 illustrates the before-trade, after-trade, and no-trade risk-adjusted stock pricing functions p_s , for the choice of parameter values: $\gamma = 1.5$, $\rho = .075$, $\mu = .018$, and $\sigma^2 = 0.015$. We further set $\xi_l = 1$ and $B_0 = 1$, and normalize $r = 1$.

Figure 1 shows that trade cannot induce discrete changes in the risk-adjusted price p_s . On the other hand, Figure 2 shows that actual prices jump because the speculators' marginal utility of consumption changes at trade times.

For our choice of parameter values, stock prices *rebound* promptly after the trade.

The next section further discusses the implications of the example above, and relates them to actual price behavior.

III. Implications for Stock Price Behavior

A. About Crashes and Rebounds

Our Assumption 6 postulates a demand function for stocks on the part of feedback traders that is positively related to stock prices; this demand resembles that of the "hedgers, rebalancers, and others who use dynamic strategies akin to portfolio insurance" considered by Gennotte and Leland (1990) (p. 1003). The trade situation described in the previous section can be viewed as a stop-loss sale by which feedback traders try to protect the value of their portfolio.

The strategy implemented by feedback traders may not ensure that their portfolio value is a "convex function of some underlying [...] reference portfolio," as Brennan and Schwartz (1989) define portfolio insurance, pp. 455–456, since they sell stocks in exchange for risky bonds, and bond prices may decline after the trade. For the parameter values of Figure 1, however, the bond price does not fall.

In work related to ours, Gennotte and Leland (1990) and Donaldson and Uhlig (1991) develop one-period models which generate discontinuities in prices (crashes) when exogenous traders sell stocks in response to price de-

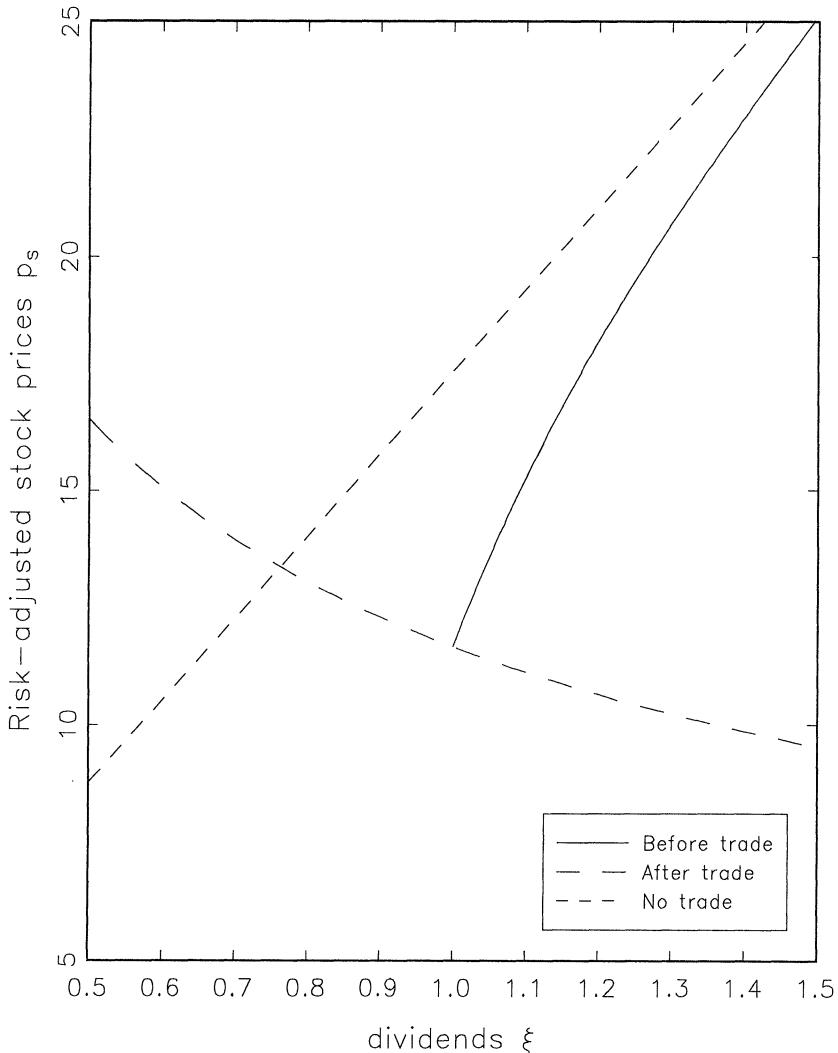


Figure 1. Risk-Adjusted Stock Prices. This figure shows the risk-adjusted pricing function for stocks p_s as a function of dividends ξ , before and after infrequent traders sell stocks at low prices, and compares it to the pricing function without trade opportunities. The speculators' initial endowment of bonds is $B_0 = 1$ and of stocks is $S_0 = 0$; we set the relative risk aversion parameter $\gamma = 1.5$, the rate of time preference $\rho = 0.075$, the instantaneous expected rate of growth of dividends $\mu = 0.018$, and the instantaneous variance of the rate of growth of dividends $\sigma^2 = 0.015$.

creases. Gennette and Leland (1990) show that the uncertainty about price-triggered trades plays a crucial role in reducing market liquidity and argue that even mildly negative news could have been sufficient to cause a stock-market crash like that of October 1987. They find that a rebound, which in their context is a “melt-up” to precrash levels, is conceivable, but unlikely.

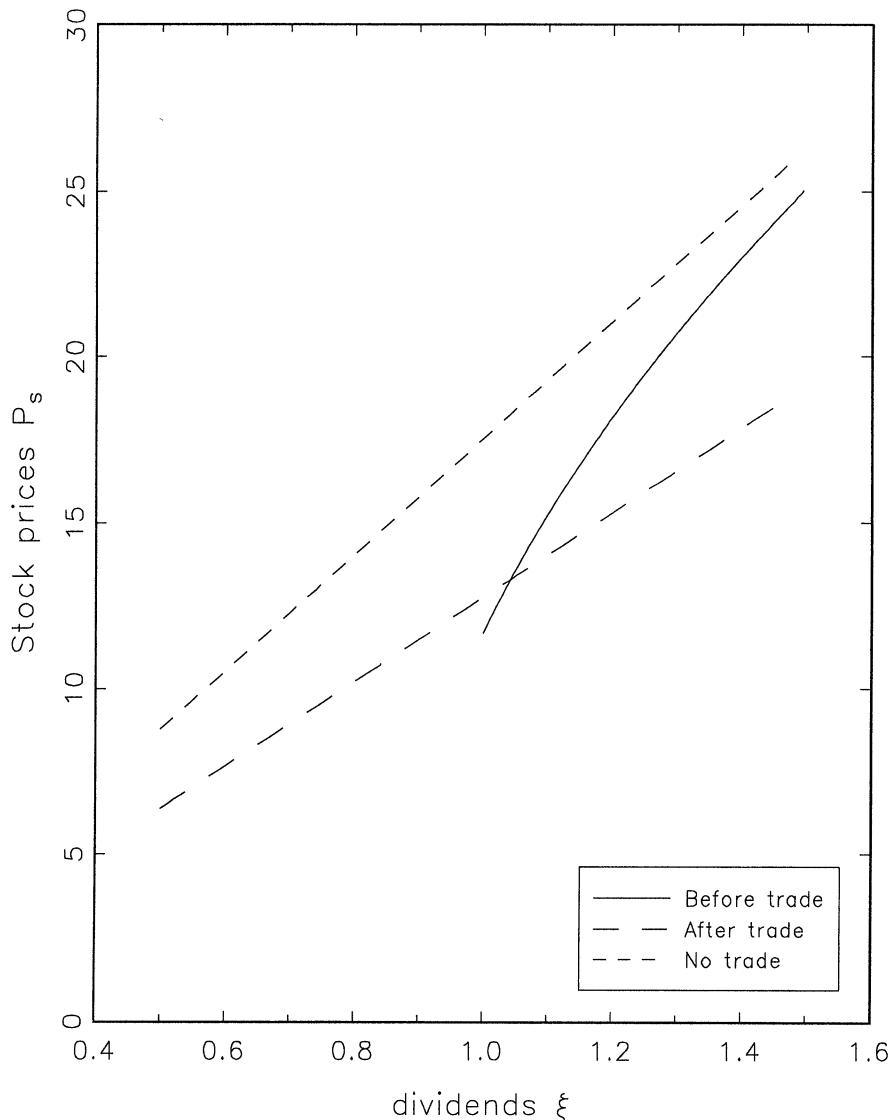


Figure 2. Stock Prices: Anticipated Stock Sales (Positive Feedback). This figure shows the pricing function for stocks P_s as a function of dividends ξ , before and after infrequent traders sell stocks at low prices, and compares it to the pricing function without trade opportunities. The speculators' initial endowment of bonds is $B_0 = 1$ and of stocks is $S_0 = 0$; we set the relative risk aversion parameter $\gamma = 1.5$, the rate of time preference $\rho = 0.075$, the instantaneous expected rate of growth of dividends $\mu = 0.018$, and the instantaneous variance of the rate of growth of dividends $\sigma^2 = 0.015$.

In our example, stock prices decline *before* the trade as dividends fall. The decline in price due to news on dividends is more pronounced than without feedback traders (see the no-trade curve of Figure 2), but it is too smooth to

have the flavor of a stock-market crash. The price increase in Figure 2, instead, is reminiscent of the increase of stock prices following the 1987 stock-market crash, when, by Wednesday, October 21st, stock prices had recovered close to half of the previous “Black Monday” loss. Our analysis suggests that stock prices may rebound after a wave of price-triggered stock sales if speculators try to increase their stock holdings when portfolio insurers stop selling stocks.

B. Volatility, Heteroskedasticity, and Predictability of Stock Returns

The pricing function in Figure 2 is steeper everywhere than its no-trade counterpart, and becomes steeper as dividends approach the trade-trigger point: expectations of trade increase the sensitivity of stock prices to current dividends, and hence the volatility of stock returns. If no trade is expected in the future, the sensitivity of stock prices to dividends is $\partial P_s / \partial \xi = 1/(\rho - \mu)$. When dividends carry information on the likelihood of trade in the future we have

$$\frac{\partial P_s}{\partial \xi} = \frac{1}{\rho - \mu} + N_s \lambda_2 (B_0 r)^\gamma \xi^{\lambda_2 - 1}.$$

In fact, one can decompose the elasticity of prices with respect to dividends as in Lucas (1978),

$$\frac{\xi \partial P_s / \partial \xi}{P_s} \equiv - \frac{\xi \partial U' / \partial \xi}{U'} + \frac{\xi \partial p_s / \partial \xi}{p_s},$$

thus making it explicit that a larger dividend has both an income and an “information” effect. The income effect (the first term) is always positive, as rational investors attempt to pass part of a positive windfall over to future dates through purchases of securities. This effect drives securities prices up. In our example, however, before-trade utility does not depend on dividends ($S_0 = 0$) and the income effect is nil. We can then focus exclusively on the second term, which reflects *two* types of information carried by decreasing dividends. First, since dividends are autocorrelated, lower dividends signal lower future cash flows, lowering stock prices. Second, the current size of dividends relative to the trigger level affects the likelihood and timing of future trade.

Moreover, in the no-trade case the rate of return on stocks is given by

$$\frac{dP_s + \xi dt}{P_s} = \rho dt + \sigma dw.$$

Absent trade, rates of return on stocks are i.i.d. normal variates, with mean $\rho\tau$ and variance $\sigma^2\tau$ over nonoverlapping time-segments of length τ . Thus, rates of return on stocks are as volatile as dividend growth rates and homoskedastic, and current dividends provide no information as to future rates of return.

Conversely, the possibility of trade increases the volatility of rates of return on stocks, and makes them heteroskedastic and predictable. Since $P_s = \xi/(\rho - \mu) + N_s(B_0 r)^\gamma \xi^{\lambda_2}$, the instantaneous return on stocks is

$$dP_s + \xi dt = \left[\frac{\mu \xi}{\rho - \mu} + \mu N_s \lambda_2 (B_0 r)^\gamma \xi^{\lambda_2} + \xi + \frac{1}{2} N_s \lambda_2 (\lambda_2 - 1) (B_0 r)^\gamma \xi^{\lambda_2} \sigma^2 \right] dt \\ + \left[\frac{\sigma \xi}{\rho - \mu} + \sigma N_s \lambda_2 (B_0 r)^\gamma \xi^{\lambda_2} \right] dw.$$

The diffusion term for the instantaneous before-trade return on stocks (the term in the second square brackets) exceeds its no-trade counterpart by the quantity

$$\sigma N_s \lambda_2 (B_0 r)^\gamma \xi^{\lambda_2}$$

whose sign is positive, since $N_s < 0$ (see equation (14)) and $\lambda_2 < 0$ (see Appendix B). Also, the before-trade stock price is lower than its no-trade counterpart (again, $N_s < 0$). As in Brennan and Schwartz (1989), the conditional volatility of rates of return on stocks is *higher* than in the no-trade case. The conditional volatility also *changes over time*, since it depends on the level of dividends.

Also, unlike in the no-trade case, the conditional expectation of rates of return on stocks is a function of the dividend level. Hence, stock returns are *predictable*, since dividends are serially correlated. These results are similar to those of Grossman and Zhou (1994), who find that stock prices are more volatile, and stock returns are serially correlated, in the presence of portfolio insurers.

The price depression and increased volatility of stock prices are apparent in Figure 2, where the before-trade pricing function is everywhere lower and steeper than the no-trade one.

IV. Alternative Trade Scenarios

The example of Section II suggests that the presence of positive-feedback traders may lead to a *depression* of stock prices and an increase in their *volatility* relative to the no-trade case. Here, we discuss the implications of alternative positive and negative-feedback strategies, and the role of risk aversion.

A. Positive and Negative-Feedback Trading

For concreteness, we frame our discussion in the context of the four specific trade scenarios illustrated in Figure 3.

We assume that feedback traders buy all the bonds in the hands of the speculators in exchange for stocks (top two panels), or buy all the stocks in the hands of the speculators in exchange for bonds (bottom two panels). Also, trade may occur at a *lower* price threshold (left two panels), or at an *upper* price

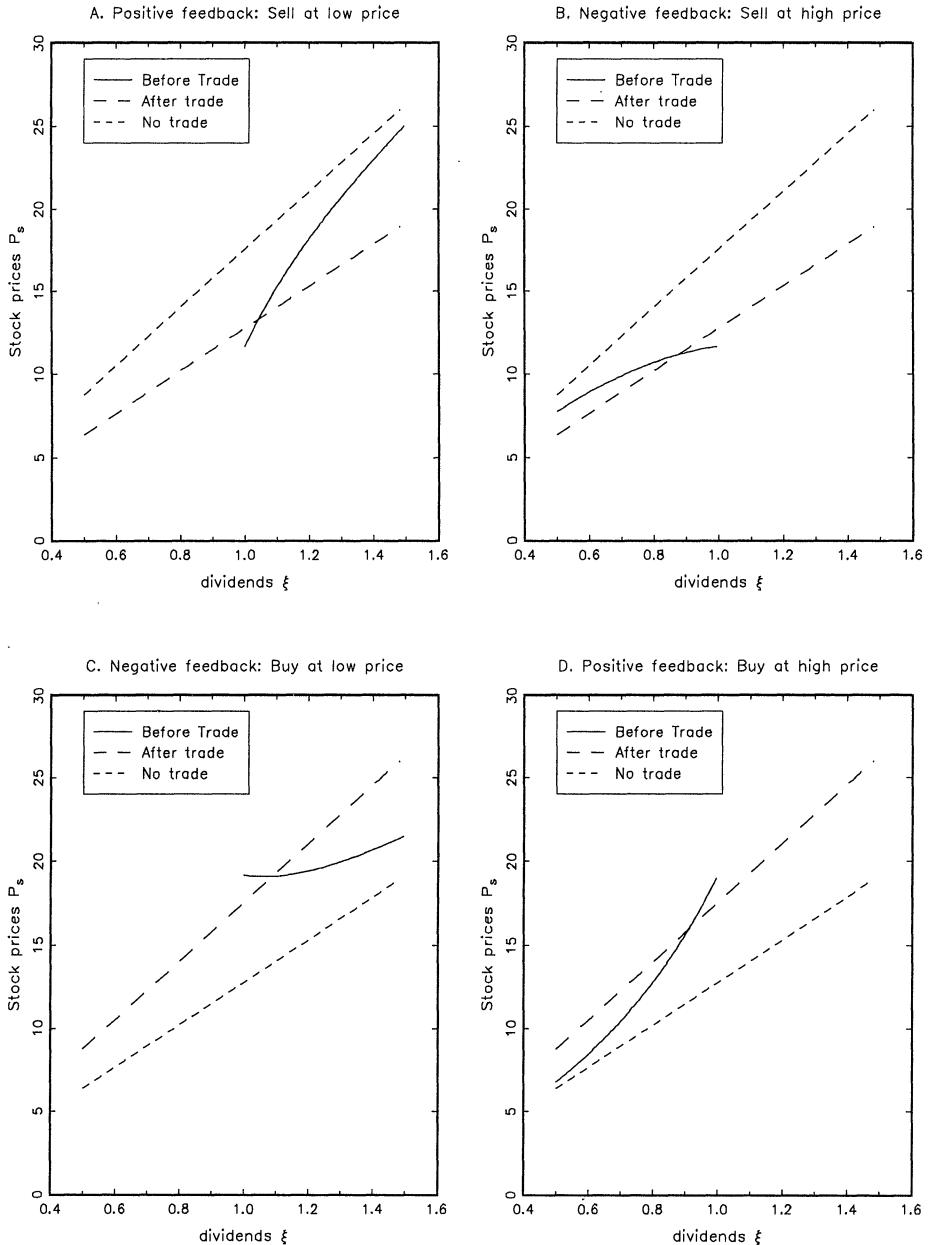


Figure 3. Positive- and Negative-Feedback Trade. For different trade scenarios, this figure shows the stock price P_s as a function of dividends ξ , before and after trade, and compares it to the pricing function without trade opportunities. The speculators' initial endowment of bonds and stocks is $B_0 = 1$ and $S_0 = 0$ (top two panels), and $B_0 = 0$ and $S_0 = 1$ (bottom two panels). The price threshold is either lower (left two panels), or higher (right two panels) than the before-trade price. We set the relative risk aversion parameter $\gamma = 1.5$, the rate of time preference $\rho = 0.075$, the instantaneous expected rate of growth of dividends $\mu = 0.018$, and the instantaneous variance of the rate of growth of dividends $\sigma^2 = 0.015$.

threshold (right two panels). The solution technique is analogous to that of Section III, and parameter values are the same as in Figures 1 and 2. The constant of integration Q_s has been set to satisfy the transversality condition (left two panels), or to ensure no jumps in risk adjusted prices (right two panels). Similarly, the constant of integration N_s has been set to satisfy the no-jump conditions in equation (5) (left two panels), or to ensure that stock prices do not diverge to $\pm\infty$ as dividends tend to zero (right two panels).

The main message from Figure 3 is that stock prices are *depressed*, relative to the no-trade case, when a *sale* of stocks is anticipated; they are *inflated* when a *purchase* of stocks is anticipated. Both price depression and price inflation become stronger as the market gets closer to the price threshold. Hence, both the conditional volatility and the conditional mean of stock returns depend on the dividend flow, and stock returns are heteroskedastic and predictable.

Moreover, when before-trade stock prices are depressed relative to their no-trade counterparts, they are likely to jump *upwards* after the occurrence of trade, as in the top two panels of Figure 3. On the other hand, when before-trade stock prices are inflated, they are likely to jump *downwards* after the occurrence of trade, as in the bottom two panels of Figure 3.

Finally, volatility can be either higher (Panels A and D) or lower (Panels B and C) than in the no-trade case, depending on whether the trade can be characterized as a positive or a negative-feedback strategy.

In the following, we further discuss each of the four trading scenarios of Figure 3:

Panel A. For the sake of comparison, we have replicated the situation of Figure 2. Feedback traders purchase all the bonds in the hands of the speculators in exchange for stocks, when the stock price hits a *lower* threshold. Stock prices rebound after the trade.

Panel B. The initial and final asset allocations are the same as in Panel A, but trade takes place at an *upper* threshold. Similar to Panel A, the stock price is depressed in anticipation of the trade, but this translates into a *lower* volatility relative to the no-trade case. In fact, we can think of feedback traders as implementing a *negative-feedback* strategy, where stocks are sold when their price is increasing. This strategy is rationally anticipated by speculators, and hence the stock price does not increase much with dividends: a wave of sales is expected soon. Again, stock prices rebound after the trade, as no other sell orders are expected.

Panel C. The initial and final asset allocations are the *opposite* of Panels A and B: feedback traders purchase all the stocks in the hands of the speculators in exchange for bonds; trade takes place at a *lower* threshold. Again, feedback traders' behavior is a *negative-feedback* strategy: stocks are purchased when their price is falling. The price depression of the previous two scenarios is here *reversed*: as dividends decrease a wave of purchases is expected soon, and the fall in stock prices reduced. As a consequence, volatility is also reduced. Stock prices fall after the trade, since no other buy orders are expected.

Panel D. Initial and final asset allocations are the same as in Panel C, while trade takes place at an *upper* threshold. In this scenario feedback traders implement a *positive-feedback* strategy: like trend-chasers, they purchase stocks as their price is already increasing. Stock prices increase further in anticipation of a buy order, and this increases stock-price volatility in the proximity of the threshold. Again, stock prices fall after the trade.

In summary, the effects of trade on prices and price volatility depend on the specific trade scenario under scrutiny. Stock prices are depressed if stock sales are expected soon, whereas they are inflated if stock purchases are expected. The implications for volatility crucially depend on the relative position of the price threshold, that is, on the type of strategy followed by feedback traders. Stop-loss and trend-chasing strategies exacerbate the trend in stock prices, leading to an increase in volatility. Contrarian strategies, on the other hand, dampen the trend in prices and reduce volatility.

B. Risk Aversion, Price Effects, and Welfare

Experimenting with different values of the speculators' relative risk aversion parameter γ , we found both the price-depression and the price-inflation effect to be stronger the higher γ . In fact the two effects can be so strong that in the scenarios of Panels B and C, the monotonicity of the before-trade pricing function can be lost. This is not a problem in general, but it prevents the inversion of the price-dividend correspondence, and the intuition of a *price* threshold (rather than a dividend threshold) triggering the trade would be lost.

The mechanisms driving the price effects above go as follows: Consider, for example, the trade scenario of Panel A in Figure 3, which replicates the scenario discussed in Section III. Before trade we have $p_s = g_s + N_s \xi^{\lambda_2}$, whereas, if trade were ruled out, we would have $p_s = g_s$. Since $P_s = p_s U'(c)$, the difference between before- and no-trade stock prices is given by the quantity

$$N_s \xi^{\lambda_2} (B_0 r)^\gamma. \quad (15)$$

It can be shown that N_s is always negative and decreasing in γ , and hence the quantity in equation (15) is more negative the higher γ . A similar argument holds for the trade scenario of Panel B. Here, the difference between before- and no-trade stock prices is given by $Q_s \xi^{\lambda_1} (B_0 r)^\gamma$, which is also negative and decreasing in γ . Hence, the higher γ , the stronger the price-depression effect.

In Panels C and D, the difference between before- and no-trade stock prices is given by $N_s \xi^{\lambda_2} (S_0 \xi)^\gamma$ and $Q_s \xi^{\lambda_1} (S_0 \xi)^\gamma$, respectively. It can be shown that both N_s and Q_s are here positive, and increasing in γ . Hence, the higher γ , the stronger the price-inflation effect.

The economics behind the price effects of anticipated trade has to do with welfare effects. In Appendix D we show how to calculate the speculators' expected lifetime utility before trade, after trade, and in the no-trade case, for the four trade scenarios of Figure 3. We find that when feedback traders buy bonds in exchange for stocks (top two panels) speculators' expected lifetime

utility is *higher* than in the absence of trade, and more so the closer the trade. Hence, stock prices are depressed because speculators look forward to higher welfare, are less willing to save, and have weaker demand for stocks. When feedback traders buy stocks in exchange for bonds (bottom two panels) the opposite holds true: speculators' expected lifetime utility is *lower* than in the absence of trade, and this effect is more pronounced the closer the trade. Stock prices are inflated because speculators expect lower welfare, are more willing to save, and have stronger demand for stocks.

Whether trade has a positive or a negative effect on welfare depends, in turn, on the relative price at which stocks and bonds are exchanged. Risk-adjusted prices at the time of trade must equal (in expectation) their after-trade counterparts (see equation (5)): equilibrium conditions fix the rate at which the two assets are exchanged. A welfare-decreasing trade, at these prices, will be accepted by speculators only if they are somehow "committed" to trade, much like market makers on the floor of an exchange.

The parameter γ regulates the degree of intertemporal substitution as well as risk aversion. Hence, the higher γ , the more speculators try to hedge future changes in welfare and the stronger are the price effects discussed above.

V. Conclusions

This article investigates the effects on asset price dynamics of positive and negative-feedback trading. Specifically, we model an economy where stocks and bonds are traded by two types of agents: speculators who maximize expected utility, and feedback traders who mechanically respond to price changes and infrequently submit market orders. We find that stock prices are depressed in anticipation of a wave of price-triggered sales, while they are inflated in anticipation of a wave of price-triggered purchases. We show that positive-feedback strategies increase the volatility of stock returns, and the response of stock prices to dividend news. Conversely, the presence of negative-feedback traders makes stock returns less volatile, and prices less responsive to dividends. Both positive- and negative-feedback strategies bring about heteroskedasticity and predictability of stock returns, even though dividend growth rates are assumed independent and identically distributed (i.i.d.).

Appendix

A. First-Order Conditions

Let $W \equiv SP_s + BP_b$ be the speculator's wealth, and $I \equiv I(W, \xi, S, B, Y)$ denote the value function:

$$Ie^{-\rho t} = \max_{c(\tau), s(\tau), b(\tau)} \int_t^\infty E_t\{U[c(\tau)]\}e^{-\rho\tau} d\tau.$$

The speculator's optimal program obeys the relations

$$0 = \max_{c(\tau), s(\tau), b(\tau)} \left\{ e^{-\rho t} U(c) + E_t \left[\frac{1}{dt} d(I e^{-\rho t}) \right] \right\} \quad (16)$$

s.t. $-cdt = \Delta s(P_s + \Delta P_s) + \Delta b(P_b + \Delta P_b) - (s\xi + br)dt$.

The first order condition for optimal equity holdings is given by

$$e^{-\rho t} U'(c) \xi + \frac{\partial}{\partial s} E_t \left[\frac{1}{dt} d(I e^{-\rho t}) \right] = 0.$$

Hence, we obtain

$$U'(c) \xi + \frac{\partial}{\partial s} E_t \left[\frac{1}{dt} (-\rho I dt + dI) \right] = 0,$$

and thus

$$U'(c) \xi - \rho \frac{\partial I}{\partial W} \frac{\partial W}{\partial s} + E_t \left[\frac{1}{dt} d \left(\frac{\partial I}{\partial W} \frac{\partial W}{\partial s} \right) \right] = 0.$$

Using the envelope condition $\partial I / \partial W = U'(c)$ and the definition of wealth, yields equation (1). A similar argument yields equation (2).

B. Homogeneous Solutions

The homogeneous solutions of equation (8) have the form

$$h_i(\xi) = Q_i \xi^{\lambda_1} + N_i \xi^{\lambda_2},$$

where Q_i and N_i are constants of integration and λ_1, λ_2 are the roots of the characteristic equation associated with (8):

$$\lambda_1 = \frac{-(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2\rho\sigma^2}}{\sigma^2} > 0,$$

$$\lambda_2 = \frac{-(\mu - \sigma^2/2) - \sqrt{(\mu - \sigma^2/2)^2 + 2\rho\sigma^2}}{\sigma^2} < 0.$$

C. Prices and Transversality Conditions

Consider again the transversality conditions

$$\lim_{\tau \rightarrow \infty} E_t [p_i(\xi, s, b, Y)] e^{-\rho(\tau-t)} = 0, \quad i = s, b.$$

Writing $p_i = g_i + h_i$ as in equation (6), we note that the particular solution satisfies the transversality condition by construction. As to h_i , we compute

$$E_t(\xi^\lambda) e^{-\rho(\tau-t)} = \xi_t^\lambda e^{[(\sigma^2/2)\lambda^2 + (\mu - \sigma^2/2)\lambda - \rho](\tau-t)},$$

and since λ_1 and λ_2 solve $(\sigma^2/2)\lambda^2 + (\mu - \sigma^2/2)\lambda - \rho = 0$, we have

$$\lim_{\tau \rightarrow \infty} E_t[h_i(\xi_\tau)] e^{-\rho(\tau-t)} = Q_i \xi_t^{\lambda_1} + N_i \xi_t^{\lambda_2}, \quad i = s, b.$$

Hence, homogeneous solutions h_i satisfy the transversality conditions only if $Q_i = N_i = 0$.

D. Welfare and Price Effects of Trade

Here, we demonstrate the correspondence between welfare and price effects of anticipated trades.

From equation (16) we have

$$\rho I = U(c) + \frac{1}{dt} E_t(dI). \quad (17)$$

Given equation (17), $E_t(dI)$ cannot be of order larger than dt , and this rules out expected jumps in the value function I ,

$$E_t(\Delta I) = 0,$$

at trading times.

It is useful to decompose the value function I , in equilibrium, in the form

$$\begin{aligned} I(\xi, S, B, Y) &= \max_{c(\tau), s(\tau), b(\tau)} \int_t^\infty E_t\{U[c(\tau)]\} e^{-\rho(t-\tau)} d\tau \\ &= \max_{c(\tau), s(\tau), b(\tau)} \int_t^\infty E_t\{U[c(\tau) \mid \text{no trade}]\} e^{-\rho(t-\tau)} d\tau + V(\xi, S, B, Y), \end{aligned}$$

where $V(\xi, S, B, Y)$ summarizes the effects of trade on expected-lifetime utility.

Expanding $E_t(I)$ in equation (17) by the usual stochastic calculus arguments, we find that in the interior of no-trade regions the value function I satisfies

$$\rho I = U(c) + \mu \xi I' + \frac{\sigma^2 \xi^2}{2} I''.$$

When either $B = 0$ or $S = 0$, the solution for the no-trade component of the value function is obtained as follows. When $B = 0$ we have

$$\begin{aligned} \max_{c(\tau), s(\tau), b(\tau)} & \int_t^\infty E_t\{U[c(\tau) \mid \text{no trade}]\} e^{-\rho(\tau-t)} d\tau \\ &= \frac{S}{1-\gamma} \int_t^\infty E_t\{[S\xi(\tau)]^{-\gamma}\xi(\tau)\} e^{-\rho(\tau-t)} d\tau = \frac{S}{1-\gamma} \frac{\xi_t}{(S\xi_t)^\gamma d_s}, \end{aligned}$$

where it is useful to recognize that the integrand function corresponds to $f_s \equiv (S\xi)^{-\gamma}\xi$. When $S = 0$, we have

$$\begin{aligned} \max_{c(\tau), s(\tau), b(\tau)} & \int_t^\infty E_t\{U[c(\tau) \mid \text{no trade}]\} e^{-\rho(\tau-t)} d\tau \\ &= \frac{B}{1-\gamma} \int_t^\infty E_t\{(Br)^{-\gamma}r\} e^{-\rho(\tau-t)} d\tau = \frac{B}{1-\gamma} \frac{r}{\rho(Br)^\gamma}, \end{aligned}$$

where the integrand function corresponds to $f_b \equiv (Br)^{-\gamma}r$.

The function V is thus analogous to the homogeneous solutions of equation (11), and has the form

$$V = Q_v(S, B, Y)\xi^{\lambda_1} + N_v(S, B, Y)\xi^{\lambda_2}.$$

The constants of integration Q_v and N_v are chosen as to ensure no-expected jumps of the value function at trading times. Moreover, depending on the trade scenario, we impose the before-trade value function to be finite as $\xi \rightarrow 0$, and to converge to its no-trade counterpart as $\xi \rightarrow \infty$.

It can be shown that the function V is always positive for the trade scenarios illustrated in the two top panels of Figure 3: feedback traders buy bonds in exchange for stocks. The opposite holds true when feedback traders buy stocks in exchange for bonds (Figure 3, bottom two panels): the function V is always negative. These findings are stronger the higher γ , and are robust to different values of ξ triggering the trade.

REFERENCES

- Basak, S., 1993, A General equilibrium model of portfolio insurance, Working paper, University of Pennsylvania.
- Blume, L., D. Easley, and M. O'Hara, 1994, Market statistics and technical analysis: The role of volume, *Journal of Finance* 49, 153–181.
- Brennan, M. J., and E. S. Schwartz, 1989, Portfolio insurance and financial market equilibrium, *Journal of Business* 62, 455–472.
- Brock, W., J. Lakonishok, and B. LeBaron, 1992, Simple technical trading rules and the stochastic properties of stock returns, *Journal of Finance* 47, 1731–1764.

- Chan, K. C., 1988, On the contrarian investment strategy, *Journal of Business* 61, 147–163.
- Campbell, J. Y., and A. S. Kyle, 1993, Smart money, noise trading and stock price behavior, *Review of Economic Studies* 60, 1–34.
- Constantinides, G. M., 1986, Capital market equilibrium with transactions costs, *Journal of Political Economy* 94, 842–862.
- Davis, M. H. A., and A. R. Norman, 1990, Portfolio selection with transactions costs, *Mathematics of Operations Research* 15, 677–713.
- De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann, 1990, Positive feedback investment strategies and destabilizing rational speculation, *Journal of Finance* 45, 379–395.
- Donaldson, G. R., and H. Uhlig, 1991, Portfolio insurance and asset prices, *Journal of Finance* 48, 1943–1955.
- Duffie, D., and T. Sun, 1990, Transactions costs and portfolio choice in a discrete-continuous-time setting, *Journal of Economic Dynamics and Control* 14, 35–41.
- Dybvig, P. H., 1995, Ratcheting, Consumption and Portfolios, *Review of Economic Studies* forthcoming.
- Gennette, G., and H. Leland (1990), Market liquidity, hedging, and crashes, *American Economic Review* 80, 999–1021.
- Grossman, S. J., and Z. Zhou, 1994, Equilibrium analysis of portfolio insurance, Working paper, University of Pennsylvania.
- Lo, A. W., and C. A. MacKinlay, 1990, When are contrarian profits due to stock market overreaction? *Review of Financial Studies* 3, 175–205.
- Lucas, R. E. Jr., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.
- Neftci, S. N., 1991, Naive trading rules in financial markets and Wiener-Kolmogorov prediction theory: A study of “technical analysis,” *Journal of Business* 64, 549–571.