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Stock Volatility and the Levels of the Basis and Open Interest in Futures Contracts

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ABSTRACT

This article tests a theoretical model of the basis and open interest of stock index futures. The model is based on the differences between stock and futures in terms of investors' ability to customize stock portfolios and liquidity. Empirical evidence confirms the model's prediction that increased volatility decreases the basis and increases open interest.

STOCK INDEX FUTURES HAVE become a popular means of achieving short-run adjustments in asset allocations for pension funds and other institutional investors. Differences between stocks and futures influence their relative prices and the degree to which they are employed in reallocating positions. Direct stock investments differ from futures contracts in ways that may be well suited for some investors' objectives. First, for exogenous hedging reasons, individual investors may prefer to hold tailored portfolios (e.g., inflation-sensitive or interest rate-sensitive stocks) rather than a stock index. Of course, in aggregate, investors must hold the market index. Second, based on private information, investors may believe they can distinguish between stocks of positive or negative value (relative to the index). Third, tax considerations may generate a preference for a tailored portfolio. Taxable investors who have accrued gains and losses on certain segments of their stock portfolio may prefer to tailor their trading by taking losses and deferring the realization of gains. Therefore, stocks have certain advantages relative to futures.

In contrast to stock positions, futures contracts offer the advantage of better market liquidity. This advantage of futures over stocks creates a tradeoff for investors. For a given investor, a stock position may have advantages that exceed those of a futures position. We call the net advantage of the stock position its customization value (CV). Depending on the relative

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attractiveness of portfolio tailoring and market liquidity to an investor, CV may be positive or negative.

When making asset allocation adjustments, stocks can be traded selectively. Because of this, the CV associated with these trades can have a significant influence on the choice between trading stocks or futures, as well as on which stocks to trade. Diversified investors are likely to have an inventory of stock positions with differential and nonzero CVs. Suppose they choose to reduce their exposure in equities. By selling futures instead of stocks, they can keep the CV of their inventory intact as they reduce their equity exposure. Should the spread between the futures and cash prices be sufficient to justify stock trades, investors will maximize the CV of their remaining portfolio by selling the shares with the smallest CVs first. Therefore, as equity exposure is reduced, the marginal CV of portfolios increases.[1]

The marginal CV of portfolios should be reflected in the relative prices of stocks and index futures (up to arbitrage bound constraints). As the underlying volatility changes, risk-averse investors respond by adjusting their portfolios, affecting relative prices. In particular, as volatility increases, investors holding equity positions respond by selling stocks and futures. The CV of the marginal stocks in their portfolios increases; this is reflected in a higher equilibrium price of stocks relative to futures. New investors, entering the market to help bear the increased risk, purchase both stocks and futures; this increases the number of futures contracts outstanding (open interest). Both a formal model and empirical evidence in support of these hypotheses are presented in this article.

Section I reviews the relevant literature. Section II presents the theoretical model and its predictions. Section III presents the empirical results, while Section IV concludes.

I. Background

This article examines how volatility affects the basis and open interest of stock index futures. The basis is defined as the market futures price minus the "fair" futures price (as given by the current spot price of the cash index grossed up by the interest rate and adjusted for expected dividends). The theoretical and empirical differences between forward prices and futures prices have been examined extensively (see, for example, Black (1976), Rendelman and Carabini (1979), Cornell and Reinganum (1981), Cox, Ingersoll, and Ross (1981), and French (1983)) and it is generally agreed that in practice these differences are not large. Thus, the cost-of-carry forward price

[1] Consider also those investors who invest through agents (money managers). Principals may be reluctant to take money from managers who are perceived to be adding CV to their portfolios, breaking a mutually beneficial principal-agent relation. Given this, a short-term adjustment in asset allocation made by liquidating cash may not be fully reversible—imposing an additional cost associated with cash adjustments.

can be used as the fair futures price, based on the current cash index, interest rates, and expected dividends.

Deviations of the stock index futures price from its cost-of-carry value are bounded by index arbitrage. Recent studies on index arbitrage bounds and the costs and profitability of index arbitrage include Cornell and French (1983), Figlewski (1984), MacKinlay and Ramaswamy (1988), Brennan and Schwartz (1990), Sofianos (1990), and Stoll and Whaley (1990). The evidence indicates that observed futures prices and cost-of-carry values are close approximations.[2] This may be due to index arbitrage operations in the market, or simply a reflection of the market equilibrium. This study explores why observed futures prices may deviate from cost-of-carry forward prices.[3] In particular, it examines how different levels of stock return volatility induce different degrees of risk sharing, which in turn generate stock-futures price differentials reflecting different marginal CVs. Although price differentials are limited by index arbitrage, the data still show a strong negative relation between the futures basis and the stock return volatility. These results also reinforce the finding of MacKinlay and Ramaswamy (1988), who document that the arbitrage bounds decrease with time-to-expiration. The size of this volatility effect is bounded by arbitrage, and this article finds that the impact of volatility decreases with time-to-expiration.

This article also explores how the level of stock return volatility affects open interest,[4] through risk sharing in the stock and futures markets. The data show a positive relation between open interest and the level of volatility.

II. Theoretical Results

This article presents a formal model in which agents have the choice of investing in stocks, futures, or simply holding cash. Investors receive an idiosyncratic benefit from stock, relative to futures. The expected marginal value of this benefit may be positive or negative; the value varies across investors and decreases with the size of an investor's stock position. There is a fixed cost of entering the stock and futures market. Finally, except for the idiosyncratic benefit, stocks and futures are perfect substitutes.

These assumptions create three types of investors. Those with the largest expected idiosyncratic benefits (call them type 1) hold large amounts of stock

[2] The days on and after the October 1987 crash and the day of October 13, 1989 are exceptions. See, for example, Harris (1989).

[3] In Hemler and Longstaff (1991), a Cox-Ingersoll-Ross-type general equilibrium model of stock and futures prices, the market volatility level affects the basis. In their model, the volatility level affects movements of both stock prices and interest rates; this affects the basis because futures contracts are marked-to-market. The direction in which volatility affects the basis is undetermined in their model; it may even switch signs as time-to-expiration changes. In contrast, our model predicts that increased volatility decreases the basis; the magnitude of this effect increases with the time-to-expiration.

[4] See also Bessembinder and Seguin (1992), who use open interest as a proxy for market depth in examining the causes of market volatility.

and reduce their market exposure by shorting futures. Those with intermediate idiosyncratic benefits (type 2) buy some stock, but the futures price induces them to also buy futures. In equilibrium, the supply of futures from type 1 equals the demand for futures from type 2. Those investors with the smallest idiosyncratic benefits (type 3) are better off not paying the fixed cost of entering the market, so they hold neither stock nor futures.

The model focuses on the effect of volatility on the futures basis and open interest. An increase in volatility increases the type 1 investors' demand to hedge their stock positions, while it reduces type 2 investors' willingness to take long futures positions. As both types of investors reduce their stock positions, the higher marginal idiosyncratic benefit of stock is reflected in an increase in the stock price relative to the futures price. The increase in volatility also increases the expected return on stocks. With the increased expected stock return (and even larger increase in the expected return on futures), some type 3 investors switch to type 2 investors: they choose to enter the stock and futures market. The net effect is a decrease in the basis and an increase in the open interest.[5]

A. The Formal Model

There are two risky securities (stock and futures), a continuum of traders (indexed by $i \geq 0$), and a single trading period.[6] Stock is in positive net supply X, and futures are in zero net supply. The time-to-expiration (when all securities are liquidated) is τ. Futures have an uncertain final payoff distributed normally as $N(\mu\tau, \sigma^2\tau)$, where σ is the underlying volatility. For simplicity, there is no discounting.

Stock (representing a stock index) has the same basic final payoff as futures; however, a stock position[7] may be tailored by a particular investor, generating additional customization value. For trader i, stock generates a CV distributed normally as $N(\alpha_i\tau, \beta^2\tau)$, independent of the futures payoff, with α_i positive. Traders are ordered so that α_i is weakly decreasing in i; α_i has lower bound $\alpha_\infty > 0$. The value $\alpha_i\tau$ represents trader i's CV associated with the "first" or "favorite" stock in his portfolio; thus, α_i reflects i's preference for customization. The value $\beta^2\tau$ represents additional risk incurred due to diversification given up in portfolio customization. As the investor's stock position increases, diversification becomes an increasingly important concern, as reflected by the variance.

[5] Other models considered were unsatisfactory in generating both of these empirically observed relationships. Such models included (1) analyzing the difference between forward and futures prices under stochastic interest rates, and (2) a nonzero basis reflecting differential direct transaction costs in the cash and futures markets.

[6] This model can be extended in a straightforward way to a τ-period dynamic model, with qualitatively similar results for the basis and open interest.

[7] To abstract away from details not central to the story and to keep the mathematics tractable, the individual securities making up the index are not explicitly modeled here. In the formal model, it is useful to think of the aggregate trades of all agents with preference for customization α_i (bundled together) to be a multiple of a typical index share.

Trader i is a representative trader for individual investors sharing characteristic α_i. Individual investors each receive customization value from their own tailored stock position. Although no individual holds the stock index, it is held by the investors in aggregate (and thus by the representative trader). Therefore, each representative trader i in the model trades the stock index, yet still receives customization value for stock.

Traders have exponential utility with risk-aversion γ, and act as price takers. Traders may have various nonzero endowments; this does not affect the results. Traders who hold nonzero stock and futures positions incur a cost of entry $k\tau$, representing the opportunity cost of active trading. Since traders have exponential utility and uncertainty is normally distributed, traders face mean-variance optimization. Trader i's optimization problem (assuming entry), expressed in certainty equivalent form, is

$$\max_{\theta_{Si},\,\theta_{Fi}} \left[\mu\tau(\theta_{Si} + \theta_{Fi}) - \frac{\gamma\sigma^2\tau}{2}(\theta_{Si} + \theta_{Fi})^2 \right] + \left[\alpha_i \tau \theta_{Si} - \frac{\gamma\beta^2\tau}{2}\theta_{Si}^2 \right]$$
$$- \left[P_S\left(\theta_{Si} - \bar{\theta}_{Si}\right) + P_F\left(\theta_{Fi} - \bar{\theta}_{Fi}\right) \right] - k\tau, \tag{1}$$

where θ_{Si} and θ_{Fi} are trader i's final stock and futures positions, respectively, $\bar{\theta}_{Si}$ and $\bar{\theta}_{Fi}$ are trader i's initial endowments of stock and futures, respectively, and P_S and P_F are the stock and futures market prices, respectively. Entering traders face a standard quadratic mean-variance optimization problem, including entry and CV considerations. Nonentering traders liquidate their portfolios and simply receive their endowment's value $(P_S\bar{\theta}_{Si} + P_F\bar{\theta}_{Fi})$. Trader i's marginal stock CV, $(\alpha_i\tau - \gamma\beta^2\tau\theta_{si})$, is decreasing in his stock position and may be either positive or negative.

Two parametric assumptions are made.

$$\gamma\beta^2 X > \int_0^\infty (\alpha_i - \alpha_\infty)\, di \tag{2}$$

$$k > \frac{(\alpha_0)^2}{2\gamma\beta^2} \tag{3}$$

The first assumption guarantees that the traders with the highest preference for customization are the ones that enter in equilibrium. The second places a lower bound on entry cost.

Equilibrium is defined as prices (P_S, P_F) and portfolios $\{(\theta_{Si}, \theta_{Fi}) \mid i \geq 0\}$ such that all traders make optimal portfolio and entry decisions, and markets clear.

B. Results

The equilibrium has the following properties. (The proof of these results is in Appendix A.) The number of entering traders, N, is positive and increasing in volatility σ. (Increased volatility leads to increased expected returns, so some type 3 investors switch to type 2.) The traders entering in equilibrium (types 1 and 2) are those with the highest preference for customization, as measured by α_i.

Trader i's optimal stock position θ_{Si} and equity exposure $\theta_{Si} + \theta_{Fi}$ are positive if he enters the market (for all i). His optimal stock position θ_{Si}, futures position θ_{Fi}, and equity exposure $\theta_{Si} + \theta_{Fi}$ are decreasing in σ. (Increased volatility leads to increased hedging demand from investor types 1 and 2.)

The basis equals

$$P_F - P_S = \left(\gamma\beta^2 \frac{X}{N} - \bar{\alpha}\right)\tau, \tag{4}$$

where

$$\bar{\alpha} = \frac{1}{N}\int_0^N \alpha_i \, di$$

is the mean α_i characteristic of entering traders. The basis is decreasing in volatility σ, while its magnitude is increasing in time-to-expiration τ.

For small σ, entry N is low, and stock positions are large. The marginal tailoring advantage of stock is low and is outweighed by liquidity considerations. Thus, marginal CV is negative, and the basis (the negative of marginal CV) is positive. For large σ, the situation is reversed: entry N is high, stock positions are small, marginal CV is positive (the marginal tailoring value of stocks outweighs liquidity considerations), and the basis is negative.

As volatility increases, each share of stock is riskier, and the total risk in the market increases. More traders enter to help bear the risk. Since the supply of securities is fixed (X for stock and 0 for futures), individuals' stock positions and equity exposures decrease. Investors selling stock part with progressively greater marginal CVs. This is reflected by an increase in the price of stock relative to futures, a decrease in the basis. Note that although α_i is positive, the CV of the marginal stock (and thus the basis) may be either positive or negative, depending on the size of the stock position.

Futures open interest equals

$$Open\ Interest = \frac{1}{2}\int_0^N |\theta_{Fi}| \, di = \frac{1}{2\gamma\beta^2}\int_0^N |\bar{\alpha} - \alpha_i| \, di. \tag{5}$$

As volatility increases, investors sell futures to lower equity exposure while preserving CV. Short futures positions become larger, and some long positions become short, in order to keep valued stock. Some (type 3) investors enter to take the long side of these trades. Open interest therefore increases with volatility.

C. Arbitrage Bounds

It is important to note that the effects of our model implicitly take place within an arbitrage band, within which no profitable arbitrage between the stock and futures markets are possible. To see this, note that the nature of equilibrium is as follows. Entered traders hold positive stock positions. The equilibrium price of stock relative to futures equals the marginal CV, the marginal tailoring advantage of stock (*TA*), less the liquidity (or transactions) costs of stock (*LC*). Thus, at the margin for each trader i,

$$P_S - P_F = TA_i - LC. \tag{6}$$

When is arbitrage possible? An investor can profitably buy the stock index (with zero tailoring advantage) and sell futures if

$$P_F > P_S + LC. \tag{7}$$

Note that no individual investor holds the entire index (only representative traders do). An investor can therefore only sell the stock index by shorting stock, incurring shorting costs (*SC*), in so doing. An investor can profitably buy futures and short sell the stock index if

$$P_S - SC > P_F. \tag{8}$$

Therefore, if the basis falls within the band

$$-SC \leq P_F - P_S \leq LC, \tag{9}$$

then no profitable arbitrage is possible. Since the equilibrium basis equals $LC - TA_i$, the basis normally falls within this band.

It is important to note that both this theoretical band and the predicted effect of volatility are of reasonable magnitude. Appendix B provides a calibration of the model, and it shows that this no-arbitrage band is of "reasonable" size and that the model provides "reasonable" predictions of the effect of volatility on the basis.

III. Empirical Results

We test the two main predictions of our model with the S&P 500 Index. The model predicts that (i) the S&P 500 basis decreases as volatility of the index increases, and (ii) open interest of the S&P 500 Index futures increases as the volatility of the index increases.

A. Data

Synchronized daily observations at 4 P.M. (Eastern Time) of the S&P 500 Cash Index, S&P 500 futures, and the basis are provided to us by Paine-Webber for the period January 1986 through May 1990. The daily basis is computed as the market futures price minus the "fair" futures price. In making their estimates of the fair futures price, Paine-Webber compounds expected dividends to the expiration date of the futures contract using the London Interbank Offered Rate. For each firm in the S&P 500 Index, the anticipated ex-dividend date is the business day closest to the ex-dividend date in the preceding year. For smaller firms in the index, the expected dividend is taken to be that paid in the preceding quarter. For larger firms, Paine-Webber attempts to estimate the magnitude of the dividend using current information available as of the date for which the fair futures price is estimated.

The implied volatility of the S&P 500 Cash Index is also provided to us by Paine-Webber. It is calculated as a weighted average of the implied volatilities of the closest-to-expiration at-the-money put and call options on the S&P 500 Index with expected dividend adjustments. Implied volatility is plotted in Figure 1. We also use realized volatility estimated from the daily S&P 500 Cash Index to corroborate some of the findings. We obtain the S&P futures open interest data from Commodity Systems Inc.

B. Volatility and the Basis

Our theoretical model suggests that when the return volatility of the underlying asset (the S&P 500 Cash Index) increases, the basis decreases. The intuition is that when volatility increases, investors spread risk by unloading some stock positions and selling some futures. Investors retain stock positions that are most "valuable" to them and the perceived relative "value" of futures to stock decreases.

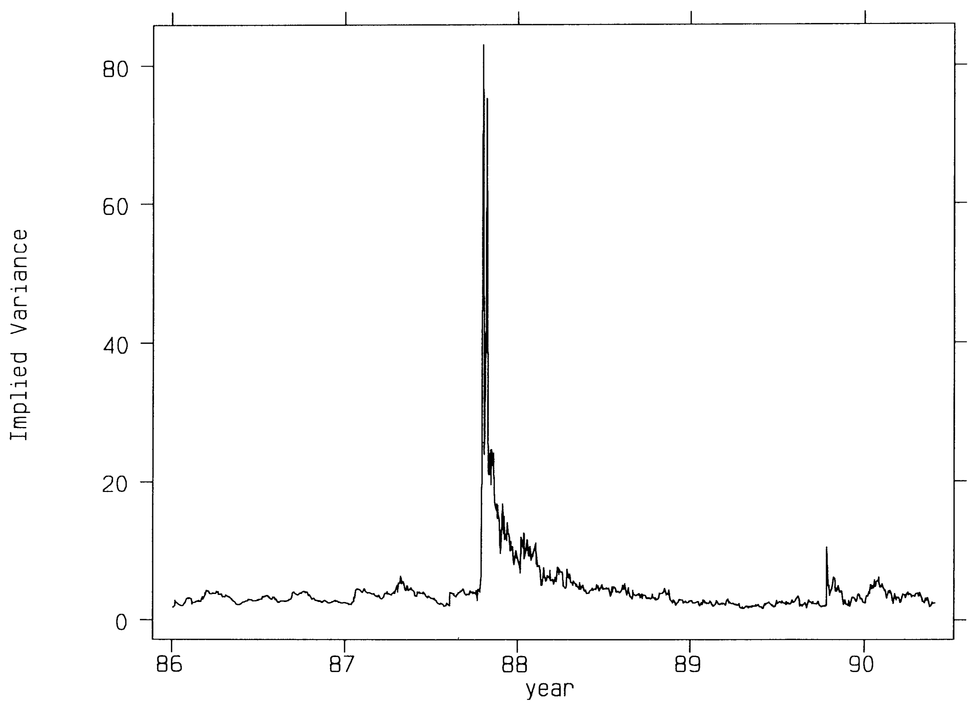

Figure 1. The variance of the S&P 500 Index implied from the S&P 500 options. Implied variance is a weighted average of the implied volatilities of the closest-to-expiration at-the-money put and call options on the S&P 500 Index with dividend adjustments.

Descriptive statistics of the basis in our sample period January 1986 through May 1990 are reported in Table I. With the week of the October 1987 crash and the minicrash of 1989 included, the sample would have contained some large negative observations. However, the events around those days suggest that these observations may be heavily influenced by non-synchronized trading in the futures and cash markets. The negative relation between volatility and basis would be exaggerated if these observations were included in our regressions. Therefore, observations from the week of the October 1987 crash as well as October 13, 1989 are excluded from those tests directed at explaining the magnitude of the basis. We also exclude observations on the futures expiration days.

The time series of the basis exhibits some autocorrelations at the daily level. This may arise because of the nontrading or thin trading in the cash market of the component stocks in the S&P 500 Index. Since there is no reliable autocorrelation pattern beyond the third lag, we include the first three lags of the basis in the basis regressions. Existing evidence also suggests that prices in the futures market tend to lead the cash market. This may be due to the lower trading costs in the futures market which tend to allow information to be reflected in the futures prices first.[8] Thus an increase in volatility may have two separate depressing effects on the basis. In addition to the effect suggested by our model, there is a possible indirect effect caused by the downward reevaluation of (stock) prices in response to an increase in the volatility (see, e.g., French, Schwert, and Stambaugh (1987), and Haugen, Talmor, and Torous (1991)) if the cash market lags behind the futures market. Of course, a lagging cash market response will always create an exaggerated movement in the basis whenever the market moves. In order to control for this effect, we include the contemporaneous as well as the next day's S&P 500 Cash Index return as independent variables in our basis regressions.

Table II reports the daily basis regressions using the closest-to-expiration futures contracts. The first three lagged bases all have reliable and positive slope coefficients, indicating that the observed basis has a tendency to persist for a few days. The two S&P 500 Cash Index returns also have strong positive slope coefficients, consistent with the tendency for the cash market to lag behind the futures market when prices move. In regression (1), the slope coefficient for the implied variance (σ^2) is negative with a t-statistic of -3.25. This is consistent with the prediction that an increase in volatility leads to a decrease in the basis. Regression (2) shows that the time-to-expiration variable (τ) has some explanatory power.[9] Since our model suggests that the basis response to a change in volatility should be scaled by time-to-

[8] See, example, Kawaller, Koch, and Koch (1987), Stoll and Whaley (1990), Chan, Chan, and Karolyi (1991), and Chan (1992). Miller, Muthuswamy, and Whaley (1994) attribute most of the lead/lag relation to the staleness of the cash index.

[9] The basis is on average negative. The negative slope coefficient of the time-to-expiration variable suggests that the basis has a tendency to increase (back to the zero level) as time-to-expiration decreases.

Table I

Descriptive Statistics of Basis

S&P 500 futures basis is defined as the market futures price minus the "fair" futures price based on the cash index. The observation unit is the S&P 500 index point. The sample period is from January 2, 1986 to May 29, 1990. Data corresponding to the October 1987 crash (October 19 to 23) and October 1989 crash (October 13) are excluded from the summary statistics.

Panel A. Basis Statistics	
No. Observations	1095
Mean	−0.1527
Std. Dev.	0.8924
Variance	0.7964
Skewness	−0.9047
Kurtosis	14.0921

Panel B. Percentiles	
1%	−2.4829
5%	−1.5146
10%	−1.1261
25%	−0.5781
50%	−0.0900
75%	0.3600
90%	0.7200
95%	1.0133
99%	1.6100

Panel C. Observations
Smallest 4 observations
−8.1129
−5.4253
−4.1436
−3.7989
Largest 4 observations
1.8700
2.3616
2.5300
6.7112

Panel D. Basis During Two Crashes	
Around October 1987	
October 19	−24.64
October 20	−21.72
October 21	−1.43
October 22	−4.84
October 23	−8.20
Around October 1989	
October 13	−7.74

Table II

Daily Regressions of Basis on Implied Volatility

The dependent variable is the computed basis at time t. *SP500* is the contemporaneous return of the S&P 500 Cash Index, τ is the time-to-maturity (in days) of the futures contract, and σ^2 is the implied volatility. t-statistics are in parentheses. Ljung-Box statistics' degrees of freedom are also in parentheses. Data corresponding to the October 1987 crash (October 19 to 23) and October 1989 crash (October 13) are excluded. The sample period is from January 2, 1986 to May 29, 1990.

	Regressions				
	1	2	3	4	5
$Basis_{t-1}$	0.164	0.158	0.151	0.151	0.151
	(6.42)	(6.17)	(5.85)	(5.86)	(5.85)
$Basis_{t-2}$	0.109	0.103	0.095	0.096	0.095
	(4.22)	(3.99)	(3.69)	(3.69)	(3.67)
$Basis_{t-3}$	0.089	0.084	0.077	0.078	0.077
	(3.54)	(3.31)	(3.06)	(3.07)	(3.06)
$SP500_t$	0.272	0.270	0.269	0.269	0.268
	(12.5)	(12.45)	(12.43)	(12.40)	(12.40)
$SP500_{t+1}$	0.085	0.083	0.082	0.083	0.084
	(4.76)	(4.64)	(4.62)	(4.67)	(4.71)
$\sigma^2 \cdot \tau \cdot 10^{-4}$			−8.011	−5.515	−5.794
			(−3.94)	(−4.25)	(−4.94)
$\sigma^2 \cdot 10^{-2}$	−1.982	−2.119	1.404		
	(−3.25)	(−3.47)	(1.34)		
$\tau \cdot 10^{-3}$		−2.511		−0.488	
		(−2.83)		(−0.50)	
Constant	−0.034	0.080	−0.034	0.0005	−0.016
	(−1.04)	(1.55)	(−1.05)	(0.01)	(−0.55)
No. of observations	1078	1078	1078	1078	1078
Adj. R^2	0.270	0.275	0.280	0.279	0.279
Ljung-Box (4)	0.694	0.583	0.417	0.578	0.694

expiration, the variable in the regression should be the product of implied variance with time-to-expiration ($\sigma^2\tau$). Regressions (3) and (4) show that $\sigma^2\tau$ subsumes the effect of σ^2 and the effect of τ. Regression (5) summarizes the result. When we regress the basis on $\sigma^2\tau$, the slope coefficient is negative, with a t-statistic of -4.94. The evidence indicates that an increase in volatility leads to a decrease in the basis.

Table III repeats the experiments reported in Table II, replacing the implied variance by an estimate of the realized variance. For each day t, the estimated variance s^2 is the return variance of the S&P 500 Cash Index of the 5 days before t ($t-5$ to $t-1$) and the 5 days after t ($t+1$ to $t+5$), excluding the day the basis is measured (Day t). The purpose of Table III is to offer an independent corroboration of the results in Table II without using the implied variance. The reason is that the S&P 500 Cash Index price enters

Table III
Daily Regressions of Basis on Realized Volatility

The dependent variable is the computed basis at time t. *SP500* is the contemporaneous return of the S&P 500 Cash Index, τ is the time-to-maturity (in days) of the futures contract, and s^2 is the realized volatility computed using the S&P 500 Cash Index returns from $t-5$ to $t-1$ and $t+1$ to $t+5$. t-statistics are in parentheses. Ljung-Box statistics' degrees of freedom are also in parentheses. Data corresponding to the October 1987 crash (October 19 to 23) and October 1989 crash (October 13) are excluded. The sample period is from January 2, 1986 to May 29, 1990.

	Regressions				
	1	2	3	4	5
$Basis_{t-1}$	0.164	0.160	0.164	0.162	0.165
	(6.38)	(6.20)	(6.37)	(6.30)	(6.41)
$Basis_{t-2}$	0.112	0.107	0.112	0.110	0.113
	(4.33)	(4.16)	(4.33)	(4.27)	(4.38)
$Basis_{t-3}$	0.093	0.088	0.093	0.090	0.093
	(3.67)	(3.49)	(3.67)	(3.57)	(3.70)
$SP500_t$	0.262	0.261	0.262	0.263	0.263
	(11.82)	(11.80)	(11.80)	(11.91)	(11.89)
$SP500_{t+1}$	0.065	0.062	0.064	0.063	0.064
	(3.42)	(3.30)	(3.38)	(3.32)	(3.35)
$s^2 \cdot \tau \cdot 10^{-4}$			−0.536	−2.714	−2.892
			(−0.10)	(−3.00)	(−3.21)
$s^2 \cdot 10^{-2}$	−1.815	−2.828	−1.487		
	(−3.24)	(−3.27)	(−0.46)		
$\tau \cdot 10^{-3}$		−2.257		−1.995	
		(−2.54)		(−2.24)	
Constant	−0.092	0.006	−0.093	−0.011	−0.097
	(−3.89)	(0.13)	(−3.75)	(−0.25)	(−4.12)
No. of observations	1071	1071	1071	1071	1071
Adj. R^2	0.269	0.273	0.268	0.272	0.269
Ljung-Box (4)	1.476	1.609	1.193	1.930	1.676

both in the calculation of the implied variance and the calculation of the basis. Although there is no apparent reason why this would create a bias in the results, it is useful to examine the evidence with another measure of the variance.

The results in Table III are largely the same as in Table II. Indeed, it is reassuring to know that the negative relation between basis and volatility is robust in regard to this alternative measure of the variance.[10]

We also examine the robustness of the results using only Wednesday observations. This avoids the serial correlation problems in the daily basis. The results are reported in Table IV. The first two regressions use the

[10] We have also examined the robustness of the results when we exclude other outlying observations with large bases (in addition to dropping observations around the 1987 and 1989 crashes). The reliable negative relation between the basis and the volatility remains.

Table IV

Wednesday Regressions of Basis on Volatility

Regressions are run using Wednesday observations only. The dependent variable is the computed basis at time t. *SP500* is the contemporaneous return of the S&P 500 Cash Index, τ is the time-to-maturity (in days) of the futures contract, σ^2 is the implied volatility, and s^2 is the realized volatility computed using the S&P 500 Cash Index returns from $t-5$ to $t-1$ and $t+1$ to $t+5$. t-statistics are in parentheses. Ljung-Box statistics' degrees of freedom are also in parentheses. Data corresponding to the October 1987 crash (October 19 to 23) and October 1989 crash (October 13) are excluded. The sample period is from January 2, 1986 to May 29, 1990.

	Regressions			
	1	2	3	4
$Basis_{t-7}$	0.053		0.085	
	(0.080)		(1.31)	
$SP500_t$	0.197	0.199	0.149	0.144
	(3.74)	(3.88)	(2.74)	(2.65)
$SP500_{t+1}$	0.105	0.106	0.042	0.037
	(2.37)	(2.40)	(0.98)	(0.87)
$\sigma^2 \cdot \tau \cdot 10^{-3}$	−5.998	−6.320		
	(−4.15)	(−4.56)		
$s^2 \cdot \tau \cdot 10^{-3}$			−4.587	−4.812
			(−4.31)	(−4.58)
Constant	0.070	0.075	−0.049	−0.052
	(1.14)	(1.23)	(−1.03)	(−1.10)
No. of observations	223	224	222	222
Adj. R^2	0.133	0.140	0.139	0.136
Ljung-Box (4)	5.027	6.294	9.603	11.395

implied variance and the last two regressions use the realized variance. With only Wednesdays, the observations are not autocorrelated. The slope coefficients for $\sigma^2\tau$ remain negative and at least three standard deviations from zero. Thus, the Wednesday results are consistent with the daily results in that the basis is negatively related to volatility.[11]

Finally, we split all the observations into 4 groups arranged in increasing time-to-expiration. The first group contains only observations from the week of the contract expiration. The second group contains observations from 2 to 5 weeks from expiration, the third 6 to 9 weeks, and the fourth 10 to 13 weeks. We regress the basis on the implied variance σ^2 rather than the product $\sigma^2\tau$. The results are reported in Table V. Although the t-statistics are smaller than those when we pool the observations together, the slope coefficients for σ^2 are all negative. Furthermore, the slope coefficients are increasing in

[11] We have also rerun the regression with only every four Wednesdays (similar to a monthly regression). The slope coefficient for the volatility is still significant (-1.544, $t = -2.74$ with implied volatility; -0.526, $t = -2.97$ with realized volatility) even though only 1/20 of the data are used in the regression.

Table V

Grouped Daily Regressions of Basis on Implied Volatility

Observations are divided into four groups based on time-to-maturity of the futures contract. The first group contains only observations from the week of the expiration. The second, third, and fourth groups contain observations two to five weeks, six to nine weeks, and more than nine weeks before expiration, respectively. The dependent variable is the computed basis at time t. *SP500* is the contemporaneous return of the S&P 500 Cash Index, τ is the time-to-maturity (in days) of the futures contract, and σ^2 is the implied volatility. t-statistics are in parentheses. Ljung-Box statistics' degrees of freedom are also in parentheses. Data corresponding to the October 1987 crash (October 19 to 23) and October 1989 crash (October 13) are excluded. The sample period is from January 2, 1986 to May 29, 1990.

	Regressions			
	1	2	3	4
	τ (weeks)			
	1	2–5	6–9	10–13
$Basis_{t-1}$	0.248	0.038	0.226	0.116
	(2.28)	(0.77)	(5.21)	(2.64)
$Basis_{t-2}$	0.114	0.129	0.077	0.102
	(1.03)	(2.48)	(1.74)	(2.31)
$Basis_{t-3}$	0.274	0.094	0.017	0.154
	(2.43)	(1.83)	(0.42)	(3.43)
$SP500_t$	0.262	0.145	0.328	0.323
	(2.78)	(4.05)	(8.13)	(8.31)
$SP500_{t+1}$	0.190	0.130	0.114	0.035
	(2.43)	(3.78)	(2.56)	(1.40)
$\sigma^2 \cdot 10^{-2}$	−1.187	−1.959	−2.435	−5.383
	(−0.32)	(−1.48)	(−2.95)	(−2.15)
Constant	−0.005	0.031	0.024	−0.037
	(−0.04)	(0.50)	(0.44)	(−0.38)
No. of observations	68	332	354	324
Adj. R^2	0.343	0.118	0.336	0.303
Ljung-Box (4)	1.108	3.007	5.835	1.502

absolute value from group 1 to group 4; the relation between basis and volatility becomes more negative as time-to-expiration increases.

C. Volatility and Open Interest

Our model suggests that when volatility goes up, investors would like to entice more people into the market to share the risk. Since investors are unwilling to reduce their market risk exposure solely through selling stocks, they share risk by selling the S&P 500 futures, and the open interest increases. Since investors can sell futures of any expiration date to spread the risk, we use the total open interest of the S&P 500 futures in the following analysis.

The open interest tends to increase as time-to-expiration for the nearby contract decreases, and this time trend is different for each 3-month period. To allow for the influence of this pattern on the open interest, the regression analysis includes a level dummy variable and a time-to-expiration variable for each quarter. We also include lags of the open interest variable to take into account the time-series behavior of open interest. The result of the regression is (with t-statistics in parentheses):

$$
\begin{aligned}
OI_t = {} & \underset{(28.12)}{0.96\, OI_{t-1}} - \underset{(-0.86)}{0.04\, OI_{t-2}} - \underset{(-2.79)}{0.09\, OI_{t-3}} + \underset{(8.34)}{158.84\, \sigma^2} + \sum_i a_i D_i \\
& + \sum_i b_i (D_i \tau) + \epsilon_t \qquad (10)
\end{aligned}
$$

where OI is the open interest, σ^2 is the implied variance, τ is the time-to-expiration, and D_i is the dummy variable ($D_i = 1$ for observations in the ith quarter, and 0 otherwise) for the ith quarter (quarters are defined by the expiration dates) in our sample period. Almost all the slope coefficients for "time-to-expiration" are significantly negative, indicating that the open interest has a pattern that tends to peak around the expiration dates. The first lag of the open interest is highly significant, with a coefficient close to 1. Thus, much of today's open interest comes from the "carry over" from yesterday's open interest.[12]

After all the "trend" effects are taken into account, we find a statistically reliable positive relation between open interest and the implied variance. The slope coefficient is 158.84, with a t-statistic of 8.34. Thus, after taking into account the level, trend, and autocorrelations, the open interest is positively correlated with volatility of the index.

We include observation points coincident with large volatility shifts, because they are arguably the most relevant evidence relating the risk-sharing behavior in terms of open interest to an increase in the perceived risk in the market. This is important because open interest is subject to noise due to the multiplicity of factors motivating trading, and we expect investors to engage in significant restructuring in asset allocation only in response to meaningful shifts in volatility. Fortunately, open interest does not have the nonsynchronized observation problems of the basis. While data points corresponding to other days in this sample period may be obscured by other factors affecting the open interest, the few points corresponding to the days after the crash clearly show a strong positive relation between volatility and open interest. If we drop these informative data points with large volatility increases (all the data around the 1987 and 1989 crashes), the slope coefficient for the volatility

[12] Since the first lag of the open interest in the above regression has a slope coefficient that is close to 1, we report the first difference regression (with t-statistics in parentheses):

$$OI_t - OI_{t-1} = \underset{(8.11)}{562.05} + \underset{(9.49)}{195.90\,(\sigma_t^2 - \sigma_{t-1}^2)} + e_t.$$

becomes 45.22 ($t = 1.80$). Thus, we can still observe a positive, although weaker, relation between open interest and volatility without this informative data in spite of the noise in the open interest. This highlights the evidence in regression (10), which includes these data points. More relevant to our story, when there is a large positive shift in volatility, we can definitely observe a strong positive relation between volatility and open interest.

IV. Summary

The empirical results presented are consistent with the predictions of the model. The basis, defined as the futures price minus the "fair" futures price (implied by stock prices), decreases as the volatility of the S&P 500 Cash Index increases. The open interest of S&P 500 futures increases as the volatility of the S&P 500 Cash Index increases.

This is consistent with the following scenario. As the market volatility increases, investors lower equity exposure by selling stocks and futures; the increased risk is shared among a larger pool of market participants. The new participants have lower value attached to being in the market than the existing participants, but are enticed to enter the market when benefits outweigh the entry costs. The new equilibrium is characterized by higher open interest and a lower futures price relative to the "fair" futures price. On the other hand, as the market volatility decreases, those market participants with higher value attached to being in the market remain in the market and hold a larger equity position, while those with lower value find the benefits of being in the market less than the costs and, therefore, exit. The new equilibrium is characterized by lower open interest and a higher futures price relative to the "fair" futures price.

Appendix A: Proof of Results

From equation (1), the first-order conditions for an entering trader are

$$\mu\tau - \gamma\sigma^2\tau(\theta_{Si} + \theta_{Fi}) - P_F = 0,$$
$$\mu\tau - \gamma\sigma^2\tau(\theta_{Si} + \theta_{Fi}) - P_S + \alpha_i\tau - \gamma\beta^2\tau\theta_{Si} = 0. \tag{A1}$$

Optimal positions thus satisfy

$$\theta_{Si} + \theta_{Fi} = \frac{\mu\tau - P_F}{\gamma\sigma^2\tau},$$
$$\theta_{Si} = \frac{P_F - P_S + \alpha_i\tau}{\gamma\beta^2\tau}. \tag{A2}$$

Taking the mean of equation (A2) over entering traders yields price equations

$$\frac{X}{N} = \frac{\mu\tau - P_F}{\gamma\sigma^2\tau},$$
$$\frac{X}{N} = \frac{P_F - P_S + \bar{\alpha}\tau}{\gamma\beta^2\tau}, \tag{A3}$$

where $\bar{\alpha}$ is the mean of α_i over the entered traders. The second of these yields equation (4). Equilibrium positions are

$$\theta_{Si} + \theta_{Fi} = \frac{X}{N},$$
$$\theta_{Si} = \frac{X}{N} + \frac{\alpha_i - \bar{\alpha}}{\gamma\beta^2}, \tag{A4}$$
$$\theta_{Fi} = \frac{\bar{\alpha} - \alpha_i}{\gamma\beta^2}.$$

Substitute equations (A3) and (A4) into optimization (1) to get trader i's entry condition:

$$\pi_i = \left[\frac{\gamma\sigma^2 X^2}{2N^2} + \frac{\left(\dfrac{\gamma\beta^2 X}{N} - \bar{\alpha} + \alpha_i \right)^2}{2\gamma\beta^2} - k \right] \tau \geq 0 \tag{A5}$$

if trader i enters. (Certainty equivalent) profit π_i decreases in i (increases in α_i) if

$$\frac{\partial \pi_i}{\partial \alpha_i} = \left(\frac{X}{N} + \frac{\alpha_i - \bar{\alpha}}{\gamma\beta^2} \right) \tau > 0. \tag{A6}$$

Since

$$\bar{\alpha} - \alpha_i = \frac{1}{N}\int_0^N (\alpha_j - \alpha_i)\, dj \leq \frac{1}{N}\int_0^\infty (\alpha_j - \alpha_\infty)\, dj < \frac{\gamma\beta^2 X}{N}, \tag{A7}$$

expected profit decreases in i. Traders therefore enter in order of increasing i (decreasing α_i). Marginal entrant N's break-even condition $\pi_N = 0$ can be written, using equation (A5), as

$$2\gamma\beta^2 k N^2 - \left(\gamma\beta^2 X - \int_0^N \alpha_i\, di + N\alpha_N \right)^2 = \gamma^2\beta^2 X^2 \sigma^2. \tag{A8}$$

Denote the left-hand side of equation (A8) by $L(N)$. Differentiating, and applying equation (A7) to the term in parentheses,

$$\frac{dL}{dN} = 4\gamma\beta^2 k N - 2N^2 \left(\frac{\gamma\beta^2 X}{N} - \bar{\alpha} + \alpha_N \right) \frac{d\alpha_N}{dN} > 0. \tag{A9}$$

Thus, L is increasing in N. Note that $L(0) = -\gamma^2\beta^4X^2 < 0$, and that L increases without bound. Therefore, $N(\sigma)$ exists for all σ, is independent of τ, and is increasing in σ.

Since N increases in σ, $\bar{\alpha}$ decreases in σ. By equation (A4), an entered trader's futures position θ_{Fi} decreases in σ. By equations (A4) and (A6), his stock position θ_{Si} is positive. By equation (A4), his equity exposure $\theta_{Si} + \theta_{Fi}$ is positive and decreasing in σ.

From equation (A3), the basis can be written

$$P_F - P_S = \left(\frac{\gamma\beta^2X}{N} - \bar{\alpha}\right)\tau = \left[\frac{\gamma\beta^2X - \int_0^N(\alpha_i - \alpha_\infty)\,di}{N} - \alpha_\infty\right]\tau. \quad \text{(A10)}$$

This is decreasing in N (and σ), since $[\gamma\beta^2X - \int_0^N(\alpha_i - \alpha_\infty)\,di]$ is positive by equation (2) and decreasing in N. From equation (A2), an entered trader's stock position θ_{Si} is decreasing in σ.

From equation (A8), as $\sigma \to \infty$, $N \to \infty$, and $\gamma\beta^2X/N \to 0$. Since $\bar{\alpha}$ is bounded away from zero, (A10) implies the basis is negative for sufficiently large σ. At the other volatility extreme, at $\sigma = 0$, $N > 0$. Using (A8) and (3),

$$\frac{\gamma\beta^2X}{N} - \bar{\alpha} = \sqrt{2\gamma\beta^2k} - \alpha_N \geq \sqrt{2\gamma\beta^2k} - \alpha_0 > 0, \quad \text{(A11)}$$

implying a positive basis for sufficiently small σ.

Open interest equals

$$\frac{1}{2}\int_0^N|\theta_{Fi}|\,di = \frac{1}{2\gamma\beta^2}\int_0^N|\bar{\alpha} - \alpha_i|\,di = \frac{1}{\gamma\beta^2}\int_0^{N^*}(\alpha_i - \bar{\alpha})\,di, \quad \text{(A12)}$$

where N^* is the entering trader holding a zero futures position (so $\alpha_{N^*} = \bar{\alpha}$). N^* increases in N since $\bar{\alpha}$ decreases in N. Therefore, open interest increases in N (and σ).

Appendix B: Calibration of the Model

From Appendix A, the basis B equals

$$B = \left(\gamma\beta^2\frac{X}{N} - \frac{1}{N}\int_0^N\alpha_i\,di\right)\tau, \quad \text{(B1)}$$

where N satisfies equation (A8). Therefore,

$$2\gamma\beta^2k - (B + \alpha_N)^2 = \gamma^2\beta^2X^2\left(\frac{\sigma}{N}\right)^2. \quad \text{(B2)}$$

To find the effect of a change in volatility, differentiate equation (B1) with respect to N.

$$\frac{\partial B}{\partial N} = -\left(\frac{B + \alpha_N}{N}\right)\tau. \quad \text{(B3)}$$

For a basis near zero,

$$\Delta B \approx -\alpha_N \tau \frac{\Delta N}{N}. \tag{B4}$$

For small changes in the basis and α_N, σ and N are approximately proportional by equation (B2). Thus,

$$\frac{\Delta N}{N} \approx \frac{\Delta \sigma}{\sigma}. \tag{B5}$$

For example, taking $\Delta\sigma/\sigma = +25$ percent (a 25 percent volatility increase), $\alpha_N = 2$ percent (the marginal entrant's customization value of his "best" or "favorite" stock is 2 percent annual return), and $\tau = 1.5$ months (mean time-to-expiration for nearest contract) yields a basis change of $-1/16$ percent. At an S&P level of 400 points, this is -0.25 index points. For $\tau = 3$ months, the basis change is -0.50 index points.

With these reasonable parameters, the data are consistent with this model. Sofianos (1990) estimates the transaction costs associated with index arbitrage trading. His estimate, which provides the narrowest empirical band that we know of, is that transaction costs are approximately 0.70 index points. Our theoretical model of the basis is easily contained inside, and the data mostly fall within his estimate of the index arbitrage band.

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