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Are Momentum Profits Robust to Trading Costs?

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ABSTRACT

We test whether momentum strategies remain profitable after considering market frictions induced by trading. Intraday data are used to estimate alternative measures of proportional and non-proportional (price impact) trading costs. The price impact models imply that abnormal returns to portfolio strategies decline with portfolio size. We calculate break-even fund sizes that lead to zero abnormal returns. In addition to equal- and value-weighted momentum strategies, we derive a liquidity-weighted strategy designed to reduce the cost of trades. Equal-weighted strategies perform the best before trading costs and the worst after trading costs. Liquidity-weighted and hybrid liquidity/value-weighted strategies have the largest break-even fund sizes: \$5 billion or more (relative to December 1999 market capitalization) may be invested in these momentum strategies before the apparent profit opportunities vanish.

THERE IS A GROWING LITERATURE on the predictability of stock returns based on the information contained in past returns. At very short horizons, such as a week or a month, returns are shown to have negative serial correlation (reversal), while at 3 to 12 month horizons, they exhibit positive serial correlation (momentum). During longer horizons, such as 3 to 5 years, stock returns again exhibit reversals.[1] The momentum of individual stocks is extensively examined by Jegadeesh and Titman (1993, 2001). They show that one can obtain superior returns by holding a zero-cost portfolio that consists of long positions in stocks that have outperformed in the past (*winners*), and short positions in stocks that have underperformed during the same period (*losers*).

To date, no measures of risk have been found that completely explain momentum returns. A number of authors have found that the long-term reversals are not robust to risk adjustment (Fama and French (1996), Lee and Swaminathan (2000), and Grinblatt and Moskowitz (2003)). However, the intermediate return continuation has been a more resilient anomaly. Fama and

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[1] For evidence on short horizon reversal, see Poterba and Summers (1988), and Jegadeesh (1990); for momentum and long run reversal, see De Bondt and Thaler (1985), Jegadeesh and Titman (1993, 2001), and Grinblatt and Moskowitz (2003).

French find that a three-factor asset pricing model cannot explain the returns of the intermediate-term momentum portfolios. Grundy and Martin (2001) study the risk of momentum strategies and conclude that while factor models can explain most of the variability of momentum returns, they fail to explain their mean returns (also see Jegadeesh and Titman (2001)). Lee and Swaminathan (2000) study the interaction between momentum and turnover and find that there is a link between momentum and value strategies. Like Fama and French (1996), they find that momentum returns are not explained by the Fama and French (1993) three-factor model. Momentum has also been shown to be robust across national financial markets (see, e.g., Rouwenhorst (1998), Chui, Titman, and Wei (2000), and Griffin, Ji, and Martin (2002)). Some view this unexplained persistence of intermediate-term momentum returns throughout the last several decades as one of the most serious challenges to the asset-pricing literature (Fama and French (1996)).

In the absence of a risk premium-based explanation for momentum profits, an important question is whether there are significant limits to arbitrage (Shleifer and Vishny (1997)) that prevent investors from trading sufficiently to drive away the apparent profits. While limits to arbitrage do not explain the underlying causes for the *existence* of seemingly profitable momentum strategies, they may be sufficient for their *persistence.*

We investigate the effect of trading costs, including price impact, on the profitability of particular momentum strategies. In particular, we estimate the size of a momentum-based fund that could be achieved before abnormal returns are either statistically insignificant or driven to zero. We investigate several trading cost models and momentum portfolio strategies and find that the estimated excess returns of some momentum strategies disappear after an initial investment of $4.5 to over $5.0 billion[2] is engaged (by a single fund) in such strategies. The statistical significance of these excess returns disappears after $1.1–$2.0 billion is engaged in such strategies. Therefore, transaction costs, in the form of spreads and price impacts of trades, do not fully explain the return persistence of past winner stocks exhibited in the data. This anomaly remains an important puzzle.

These break-even fund sizes represent marginal investments over and above those already implemented by traders in this market. Thus, as in all anomaly-based trading strategies, we are unable to assess infra-marginal profits earned by existing traders.

There are several components of trading costs that differ dramatically in size and in ease of measurement. The components that can be measured with the least error are the explicit trading costs of commissions and bid/ask spreads. When trading an institutional-size portfolio, these proportional costs can be swamped by the additional nonproportional cost of price impact and the

[2] The dollar amounts reported throughout the paper are expressed relative to market capitalization at the end of December 1999. That is, we report the dollar amount at the end of 1999 that constitutes the same fraction of total market capitalization as the initial investment in February 1967.

"invisible costs" of post-trade adverse price movement (Treynor (1994, p. 71)). The nature of the price impact of trades has been the subject of extensive theoretical and empirical studies (e.g. Kyle (1985), Easley and O'Hara (1987), Glosten and Harris (1988), Hasbrouck (1991a, 1991b), Huberman and Stanzl (2000), and Breen, Hodrick, and Korajczyk (2002)). The economic importance of price impact is demonstrated by Loeb (1983), Keim and Madhavan (1996, 1997), and Knez and Ready (1996), who show that transaction costs increase substantially as the size of an order increases.

Incorporating the explicit trading costs (commissions and spreads) into portfolio returns has occurred in the literature for some time. For example, Schultz (1983) and Stoll and Whaley (1983) investigate the effect of commissions and spreads on size-based trading strategies. A number of studies investigate the effects of trading costs on prior-return-based (momentum and contrarian) trading strategies. Ball, Kothari, and Shanken (1995) show that microstructure effects, such as bid/ask spreads, significantly reduce the profitability of a contrarian strategy. Grundy and Martin (2001) calculate that at round-trip transactions costs of 1.5%, the profits on a long/short momentum strategy become statistically insignificant. At round-trip transactions costs of 1.77%, they find that the profits on the long/short momentum strategy are driven to zero.

Incorporating nonproportional price impacts of trades into trading strategies has only recently received significant attention. Knez and Ready (1996) study the effects of price impact on the profitability of a trading strategy based on the weekly autocorrelation and cross-autocorrelation of large-firm and small-firm portfolios. They find that the trading costs swamp the abnormal returns to the strategy. Mitchell and Pulvino (2001) incorporate commissions and price-impact costs into a merger arbitrage portfolio strategy. They find that the trading costs reduce the profits of the strategy by 300 basis points per year.

There is a pronounced reversal of momentum around the turn of the year that is caused by the turn of the year size effect (Jegadeesh and Titman (1993) and Grundy and Martin (2001)). Keim (1989) finds that this pattern is due largely to microstructure effects, since there are distinct seasonal patterns in the probability that the closing price is a bid price or an ask price. Sadka (2001) examines single-month past-return-based strategies at the turn of the year, since these strategies exhibit the highest excess returns during December and January, incorporating, as we do here, the costs of price impact. He concludes that only a small amount can be invested before the apparent profit opportunities vanish. We do not attempt to exploit the turn of the year reversals in the trading strategies studied here.

Chen, Stanzl, and Watanabe (CSW)(2002) estimate the maximal fund size attainable before price impacts eliminate profits on size, book-to-market, and momentum strategies. They find that maximal fund sizes are small for all strategies. Lesmond, Schill, and Zhou (2003) find that trading costs eliminate the profits on the strategies they study. While our results are broadly consistent with these studies for the strategies they examine, we find that there are alternative strategies that provide greater profits. We discuss the differences between the results in these papers and our results later in the paper.

We study the profitability of long positions in winner-based momentum strategies after accounting for the cost of trading. We incorporate several models of trading costs, including proportional and nonproportional costs. Two proportional cost models are based on quoted and effective spreads. We study two alternative price-impact models (nonproportional costs): one based on Glosten and Harris (1988), and the other based on Breen et al. (2002). In addition to value-weighted and equal-weighted trading strategies commonly found in the literature, we derive a liquidity-weighted portfolio rule that maximizes, under simplifying assumptions, post-price-impact expected return on the portfolio. We also study strategies that combine liquidity-weighted and value-weighted (buy and hold) strategies. The liquidity-weighted portfolio is derived through a static optimization problem, rather than a fully dynamic portfolio setting. For the price-impact models, trading costs are nonproportional, and therefore costs, as a percentage of trade size, grow with the size of the portfolio being traded. We calculate the size of the portfolio that (1) eliminates the statistical significance of the portfolio abnormal return, (2) drives the level of abnormal return to zero, and (3) drives the portfolio Sharpe ratio to that of the maximal Sharpe ratio obtained from combinations of the Fama and French (1993) market, size, and book-to-market portfolios.

In Section I, we discuss the momentum literature and the particular portfolio strategies that we investigate. In Section II, we introduce measures of proportional and nonproportional (price-impact) trading costs. A trading model that incorporates price impacts is developed and an optimal trading strategy with forecastable price impacts is derived in Section III. The performance of various momentum strategies is evaluated in Section IV. We analyze the sensitivity of the results to alternative samples, trading rules, and assumptions in Section V. Concluding remarks are presented in Section VI.

I. Momentum Trading Strategies

Following Jegadeesh and Titman (1993), we define momentum-based strategies by the length of the period over which past returns are calculated, J, and the length of time the position is held, K. This paper, and much of the literature, uses monthly data, so J and K are measured in months. Some studies assume that the momentum trading strategy is implemented at the end of ranking period and held for K months. Others, in order to avoid microstructure effects, wait a certain period of time before implementing a trading strategy. We call this waiting period a "skip" period and denote its length S. The triplet (J, S, K) describes the momentum strategies. For example, with $J = 12, S = 0$, and $K = 3$, the strategy would rank stocks at time t by the cumulative return from the end of month $t - 12$ to the end of month t, while the investment period would be from the end of month t to the end of month $t + K$ (if $S = 1$, then the investment period would be from the end of month $t + 1$ to the end of month $t + K + 1$).

"Winners" are those firms with the highest ranking-period returns and "losers" are those stocks with the lowest ranking-period returns. In much of the literature, stocks with the top 10% ranking-period returns are defined as "winners" and stocks with the lowest 10% ranking-period returns are defined as "losers," and we follow this convention.

Jegadeesh and Titman (1993) implement strategies with $J = \{3, 6, 9, 12\}$, $S = \{0, 0.25\}$ (i.e., no skip period, and a skip period of one week), and $K = \{3, 6, 9, 12\}$. Jegadeesh and Titman (1993, Table I) report the returns on the losers' decile, on the winners' decile, and on the zero-cost strategy of taking a long position in the winners' decile and a short position in the losers' decile. They report that all of the zero-cost momentum portfolios have positive returns; all, except one, have statistically significant returns; and the most profitable long/short strategy is the $J = 12/S = 0.25/K = 3$ strategy. Fama and French (1996) find significant abnormal returns for a $J = 11/S = 1/K = 1$ strategy. Grundy and Martin (2001) study a $J = 6/S = 1/K = 1$ strategy and find that it yields significant abnormal returns.

Our sample consists of all stocks included in the Center for Research in Security Prices (CRSP) monthly data files from February 1967 to December 1999. From 1967 to 1972, the CRSP data files include New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) stocks; after 1972, Nasdaq stocks are added to the sample. Table I contains average returns, in excess of the one-month Treasury-bill return, of portfolios of past winners (top decile) and losers (bottom decile). The strategies include ranking periods (J) of 2, 5, and 11 months, skip periods (S) of one month, and holding periods (K) of 1, 3, 6, and 12 months. With a holding period of K, the return on the portfolio strategies consists of equal-weighted average returns from the strategies implemented at the end of the previous K months.[3] The previous literature typically uses equal weights (EW) or value (measured by market capitalization) weights (VW) to form portfolios. In Table I, we use the same EW and VW strategies. We discuss alternative weighting schemes below.

We conduct the analysis first using only NYSE-listed stocks and subsequently using the entire universe of stocks (NYSE, AMEX, and Nasdaq) available on CRSP. The results for EW strategies are reported in Panel A of Table I, separately for winners and losers. Similar to Jegadeesh and Titman (1993), we conclude that, ignoring price impacts, the most profitable strategies for equal-weighted long positions in winners and short positions in losers are 11/1/1 and 11/1/3. The 5/1/6 trading strategy also exhibits high mean return.

While the momentum anomaly is the existence of significant returns to winners in excess of losers, some past research has found that most of the return to a long/short momentum trading strategy is due to the short position in losers rather than to the long position in winners. For example, Hong, Lim, and Stein (2000, Table III) find that between 73 and 100% of the long winners/short losers momentum portfolio excess return is determined by the return difference between the loser portfolio (bottom 30% of past returns) and middle return portfolio (middle 40% of past returns) for size deciles two to nine.

[3] Alternatively, one might consider strategies that require rebalancing only once, at the end of the non-overlapping K-period investment period, instead of rebalancing a fraction of the portfolio every month. We have analyzed such strategies and found them to underperform the strategies above after including price impact costs.

Table I

Average Excess Returns to Momentum Strategies

A momentum strategy is defined by the triplet (J, S, K), where J is the ranking period (according to past J-month cumulative return), S is a skip period (set to one month in all the strategies below), and K is the holding period. Every month stocks are sorted according to the chosen ranking period (J). After skipping one month (S), portfolios are formed using stocks in the top decile (winners) and in the lower decile (losers). The portfolios are held for K months. This process is repeated every month, while a $1/K$ fraction of each portfolio is rebalanced. The time-series means of momentum portfolio monthly returns (excess of the risk-free rate), as well as the associated t-statistics (two-digit numbers), are presented below for various ranking and holding periods. The analysis is performed separately using NYSE-listed stocks, and using all NYSE, AMEX, and Nasdaq stocks. Panel A uses equal weights for each stock while forming the portfolios, and Panel B uses value (market capitalization) weights. The average monthly excess returns of the NYSE-composite and the NYSE/AMEX/Nasdaq-composite are 0.0061 and 0.0072 (equal-weighted), and 0.0053 and 0.0056 (value-weighted), respectively. The analysis uses data for the period February 1967 to December 1999 (395 months).

NYSE					NYSE + AMEX + Nasdaq				
	K					K			
J	1	3	6	12	J	1	3	6	12
Panel A: Equal-Weighted Strategies									
Winners									
2	0.0098	0.0101	0.0108	0.0112	2	0.0077	0.0084	0.0089	0.0094
	3.13	3.24	3.45	3.53		2.27	2.53	2.69	2.80
5	0.0128	0.0132	0.0139	0.0124	5	0.0108	0.0111	0.0114	0.0097
	4.04	4.15	4.36	3.89		3.23	3.34	3.43	2.93
11	0.0170	0.0160	0.0146	0.0117	11	0.0147	0.0134	0.0116	0.0085
	5.13	4.80	4.41	3.59		4.26	3.92	3.44	2.54
Losers									
2	0.0055	0.0050	0.0048	0.0051	2	0.0056	0.0042	0.0043	0.0052
	1.44	1.35	1.33	1.44		1.38	1.03	1.07	1.34
5	0.0029	0.0028	0.0029	0.0044	5	0.0046	0.0037	0.0038	0.0052
	0.74	0.71	0.76	1.18		1.07	0.85	0.90	1.27
11	0.0012	0.0014	0.0026	0.0057	11	0.0028	0.0029	0.0042	0.0075
	0.29	0.34	0.66	1.45		0.63	0.67	0.95	1.73
Panel B: Value-Weighted Strategies									
Winners									
2	0.0070	0.0074	0.0072	0.0077	2	0.0085	0.0091	0.0083	0.0084
	2.44	2.70	2.65	2.82		2.66	2.97	2.78	2.81
5	0.0081	0.0087	0.0096	0.0085	5	0.0099	0.0100	0.0103	0.0088
	2.80	3.09	3.43	3.04		3.09	3.23	3.34	2.89
11	0.0117	0.0117	0.0106	0.0087	11	0.0130	0.0128	0.0111	0.0084
	3.84	3.90	3.51	2.93		3.92	3.91	3.42	2.66
Losers									
2	0.0035	0.0034	0.0034	0.0025	2	−0.0024	−0.0012	−0.0002	−0.0001
	1.06	1.11	1.12	0.86		−0.65	−0.35	−0.06	−0.03
5	0.0018	0.0023	0.0014	0.0019	5	−0.0054	−0.0039	−0.0025	−0.0010
	0.53	0.69	0.42	0.61		−1.42	−1.05	−0.70	−0.27
11	−0.0025	−0.0015	−0.0002	0.0022	11	−0.0083	−0.0065	−0.0040	0.0006
	−0.72	−0.41	−0.06	0.65		−2.10	−1.65	−1.02	0.16

Grinblatt and Moskowitz (2003, Table II) find a stronger relation between returns and past returns (for a $J = 12/S = 1/K = 1$ strategy) for losers than for winners. Jegadeesh and Titman (2001, Table IV) find larger abnormal returns (in absolute value) for loser portfolios than for winner portfolios. Lesmond et al. (2003) find that between 53% and 70% of the profits on long/short strategies come from the short side.

Despite the evidence that greater momentum profits are obtained from past losers versus past winners, we limit our analysis to winners alone. The reason stems from the potential asymmetry of trading costs between engaging in a long position and short-selling. The nature of short-selling execution, especially large positions, involves additional costs not fully captured by our measure of price impact. For example, losers are stocks that have extreme past underperformance, and as such they are biased to small firms, which may be difficult to short-sell. We show below that losers are much less liquid than winners, as shown by their higher price impact coefficients. In addition, implementing the short side of momentum strategies may violate the up-tick rule. Although there is evidence that the costs of short-selling are not sufficient to eliminate momentum profits (Geczy, Musto, and Reed (2002)), we choose the more conservative approach of studying past winner-based portfolio strategies.[4] Additionally, the strategy is conservative to the extent that we ignore potential income the long strategy could earn through securities lending. The persistence of winners is an important anomaly on its own, since the excess returns of winners exhibited in the data are statistically significant. Although restricting the analysis to winners and to long strategies would potentially bias toward not finding significant post-transactions costs return, we do in fact find significant returns.

Since the 11/1/3 and 5/1/6 strategies are profitable and similar to those extensively studied in the literature, we will focus on these strategies. We will do this for winners only. Without considering price concessions and using only NYSE-listed stocks, these winners-based strategies earn excess returns of 1.17 and 1.60% (raw returns of 1.71 and 2.13%) per month for 11/1/3 VW and EW, respectively, and excess returns of 0.96 and 1.39% (raw returns of 1.49 and 1.93%) per month for 5/1/6 VW and EW, respectively. Their Sharpe ratios (not reported in the table) are 0.20, 0.24, 0.17, and 0.22, respectively. For comparison, the mean excess return of the Standard & Poors (S&P) 500 portfolio over the sample period is 0.61% per month with a Sharpe ratio of 0.13.[5]

[4] The existing literature indicates that the winners-only strategy is conservative relative to the long/short strategy before trading costs. Given that losers are less liquid, it might be the case that the strategy is not conservative on an after-trading cost basis.

[5] Since momentum arbitrage strategies exhibit a reversal during January, one might consider altering our investment strategies accordingly. We note that the January reversal is mainly a loser phenomenon (see, e.g., Sadka (2001)), and has little effect on winners. The average returns during January are as follows: Equal-weighted strategies earn 3.87% (11/1/3) and 4.05% (5/1/6) for winners and 8.56% (11/1/3) and 8.08% (5/1/6) for losers. Value-weighted strategies earn 1.99% (11/1/3 winners), 2.03% (5/1/6 winners), 3.64% (11/1/3 losers), and 3.32% (5/1/6 losers). We proceed to investigate strategies based on long winners throughout the entire year.

II. Measures of Trading Costs

We study the effects on the profitability of the past winner-based momentum strategies implied by four alternative measures of trading costs. Two of the measures are proportional trading cost models, and are therefore independent of the size of the portfolio traded. These are based on quoted and effective spreads. The remaining two measures are nonproportional trading cost models and reflect the fact that the price impact of trading increases in the size of the position traded. The price-impact measures are based on Glosten and Harris (1988) and Breen et al. (2002). All of the liquidity measures are estimated using the transaction data from the Trade and Quotation (TAQ) data supplied by the NYSE. Our momentum strategies cover a much longer time period than that covered by the TAQ data. We first describe the in-sample estimation of the different trading cost models and then introduce a method of estimating them outside the initial estimation period.

A. In-Sample Estimation

A.1. Proportional Cost Models: Effective and Quoted Spreads

For each trade in the TAQ data for our sample firms, the effective percentage half-spread is the absolute value of the transaction price and midpoint of quoted bid and ask, divided by the bid/ask midpoint. Quoted percentage half-spreads are measured minute by minute as the ratio of half the quoted bid–ask spread and the bid/ask midpoint. Monthly estimates of these two measures are obtained as their simple average throughout the month. We denote k_t^{E} and k_t^{Q} as the average effective and quoted half-spreads for month t, respectively.

A.2. Nonproportional Cost Model I: Breen et al. (2002)

For nonproportional trading costs we use two alternative specifications of the price-impact function. One is the price impact estimated in Breen et al. (2002). This (Breen–Hodrick–Korajczyk, BHK) measure posits a proportional relation between percentage returns and net share turnover over 30-minute duration time periods:

$$\frac{\Delta p_{i,t}}{p_{i,t-1}} = \lambda_i^{\mathrm{BHK}} \times \mathit{Turnover}_{i,t}, \tag{1}$$

where $p_{i,t}$ is the last transaction price of asset i in time period t, $\Delta p_{i,t} = p_{i,t} - p_{i,t-1}$ is the price impact associated with the transactions in period t, λ_i^{BHK} is asset i's price impact coefficient, and $\mathit{Turnover}_{i,t}$ is the net number of shares traded (multiplied by 1,000) divided by the number of shares outstanding for firm i. Trades are signed according to the price relative to the quote midpoint (see Lee and Ready (1991)). Buyer-initiated trades correspond to positive values of $\mathit{Turnover}_{i,t}$ and seller-initiated trades correspond to negative values. This specification is motivated by the linear pricing rule of Kyle (1985), which expresses price changes as a linear function of net volume. Breen et al. (2002) use scaled measures (i.e., net turnover rather than net volume, and returns rather than price changes) in order to obtain more meaningful cross-sectional and time-series comparisons of price impact. Using returns rather than price changes does induce convexity in the price impact, which we discuss later. Hasbrouck (1991b) finds that the convex versus linear specification does not affect his results significantly.

A.3. Nonproportional Cost Model II: Glosten and Harris (1988)

Our second specification for the price impact function is from Glosten and Harris (1988, equation (5)). The Glosten and Harris (GH) specification allows a decomposition of the price impact into fixed and variable components. The regression model is

$$\Delta p_{i,t} = \alpha_i + \lambda_i^{\mathrm{GH}} q_{i,t} + \Psi_i \Delta d_{i,t} + \varepsilon_{i,t}, \tag{2}$$

where $\Delta p_{i,t}$ is the price change of stock i from trade $t-1$ to trade t as a consequence of a (signed) trade of $q_{i,t}$ shares of the stock. As before, every trade is classified as a buy or a sell according to the transaction price relative to the bid/ask midpoint. The sign of a trade is denoted $d_{i,t}$ and is assigned a value of $+1$ for a buy and -1 for a sell. The difference between the sign of a current trade and the previous trade is denoted $\Delta d_{i,t}$. The regression coefficient λ_i^{GH} represents the variable cost of trading, while Ψ_i represents the fixed costs.

A.4. Shape of the Price Impact Function

Theoretically, the permanent component of the price impact function should be linear (e.g., Kyle (1985) and Huberman and Stanzl (2000)). Empirical studies often find concave price impact functions (see, e.g., Hasbrouck (1991a), Hausman, Lo, and MacKinlay (1992), and Keim and Madhavan (1996)). Our cost functions are either convex (BHK) or linear (GH). (For an illustration of the different trading cost functions see Figure 1.) We believe that the use of linear and convex price impact functions is reasonable in our case for several reasons. First, the choice of trade size is endogenous. Those large trades that researchers observe in the data are likely to be ones for which the price impact is low (i.e., due to credible signaling that the trader is uninformed). Otherwise, the trade would be broken into smaller trades (Bertsimas and Lo (1998)). It is not plausible to assume that the naive momentum trading strategies discussed in the literature could be executed under these favorable conditions. Second, concave empirical price-impact functions may be observed in the data due to leakage of information while a block trade is being "shopped" (see, e.g., Nelling (1996)). That is, the measured price impact for a block underestimates the true price impact, thus leading to unattainable concavity in the measured price

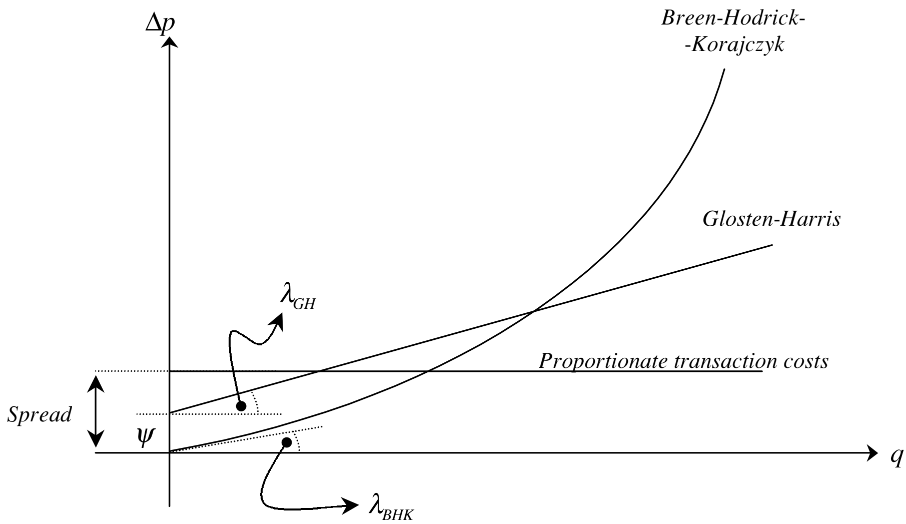


Figure 1. Transaction cost functions. In this paper we consider four different measures of transaction costs: Two nonproportionate costs, the Breen-Hodrick-Korajczyk (2002) measure, and the Glosten-Harris (1988) measure; and two proportionate costs, effective spreads and quoted spreads. The Breen-Hodrick-Korajczyk measure is based on the model $\Delta p_{i,t}/p_{i,t} = \lambda_{\mathrm{BHK},i}\Delta q_{i,t}$, where $\Delta p_{i,t}/p_{i,t}$ is the relative price change of stock i as a result of trading a net total of $q_{i,t}$ (signed) shares in a 30-minute interval (t). The Glosten-Harris measure is based on the model $\Delta p_{i,t} = \lambda_{\mathrm{GH},i}\Delta q_{i,t} + \psi_i \Delta d_{i,t}$, where $\Delta p_{i,t}$ is the absolute price change as a result of trading $q_{i,t}$ (signed) shares at time t (here t represents event time), and $d_{i,t}$ is an indicator for buyer-initiated (+1) or seller-initiated (−1) trade. Effective spreads are measured as the absolute price change relative to the midpoint of quoted bid and ask. Quoted spread is measured as the ratio between the quoted bid–ask spread and the midpoint (half the quoted spread is considered as cost). The figure above illustrates these different functions.

impact function. Last, if the true price-impact functions are concave, then our results are conservative, since we overestimate the costs of trading for large trades.

A.5. Assumed Trading Interval

The measure of time differs across the two price impact specifications. In the BHK formulation, equation (1), trades are aggregated over 30-minute intervals so that $\Delta p_{i,t}$ is the change in the last transaction price from time interval $t-1$ to time interval t, and $Turnover_{i,t}$ is the signed (net) turnover in time interval t. In the GH formulation, equation (2), time is defined in terms of trades. That is, $q_{i,t}$ is the signed size of trade t, and $\Delta p_{i,t}$ is the price change of stock i from trade $t-1$ to trade t.

A.6. Time Series of Trading Costs

We use intraday data to estimate the price impact coefficient each month, $\tau, (\tau = 1, \ldots, T)$, for our cross-section of firms. This provides a time series of coefficients, $\lambda_{i,\tau}^{\mathrm{BHK}}$, $\lambda_{i,\tau}^{\mathrm{GH}}$, and $\Psi_{i,\tau}$. We estimate the time series of monthly coefficients using the TAQ data over the period January 1993 (the beginning date of TAQ) to May 1997. The quoted and effective half-spreads, $k_{i,\tau}^{\mathrm{Q}}$ and $k_{i,\tau}^{\mathrm{E}}$, are estimated using the same sample. The resulting sample consists of 6,513 firms, not all of which have data for each month. For the average month there are 3,699 firms with data. Approximately two-thirds of the firms trade on the NYSE and AMEX, while one-third of the firms trade on Nasdaq. We estimate $\lambda_{i,\tau}^{\mathrm{BHK}}$ separately for NYSE/AMEX and Nasdaq firms. For computational reasons we estimate $\lambda_{i,\tau}^{\mathrm{GH}}$ and $\Psi_{i,\tau}$ using NYSE firms only.

B. Out-of-Sample Estimation

Since our momentum strategies cover a much longer time period than that covered by the TAQ data, we need a method of estimating the coefficients outside the initial estimation period. We do this by estimating the cross-sectional relation (over January 1993 to May 1997) between the trading cost estimates ($\lambda_{i,\tau}^{\mathrm{BHK}}$, $\lambda_{i,\tau}^{\mathrm{GH}}$, $\Psi_{i,\tau}$, effective spreads, $k_{i,\tau}^{\mathrm{E}}$, and quoted spreads, $k_{i,\tau}^{\mathrm{Q}}$) and a set of predetermined firm-specific variables meant to be proxies for market-making costs (due to adverse selection and carrying costs). We use this cross-sectional relation to estimate price impact in the out-of-sample period using the firm-specific predetermined variables that are observable in the out-of-sample period.

For example, for the BHK specification, equation (1), let $\hat{\Gamma}_\tau$ be the estimated vector of coefficients from the cross-sectional relation:

$$\hat{\lambda}_{\tau}^{\mathrm{BHK}} = X_{\tau-1}\Gamma_\tau + \upsilon_\tau, \tag{3}$$

where $\hat{\lambda}_{\tau}^{\mathrm{BHK}}$ is the $N_\tau \times 1$ vector of price-impact coefficients of N_τ firms estimated for month τ, and $X_{\tau-1}$ is the $N_\tau \times k$ matrix of predetermined variables for the cross-section of firms with $X_{i,\tau-1} = (1, X_{1,i,\tau-1}, \ldots, X_{9,i,\tau-1})$. The predetermined variables consist of (1) the market cap of firm i at the end of month $\tau - 1$ divided by the average market cap of CRSP firms, minus one; (2) total volume for firm i from month $\tau - 3$ to month $\tau - 1$ divided by the total volume, over the same period, for the average NYSE firm, minus one; (3) firm i's stock price at the end of month $\tau - 1$ divided by the price at the end of month $\tau - 7$, minus one; (4) the absolute value of variable 3; (5) a dummy variable equal to unity if the firm is included in the S&P 500 index; (6) the stock's dividend yield; (7) the R^2 of firm i's returns regressed on returns of the NYSE index over the preceding 36 months; (8) a dummy variable equal to unity if the firm is traded on NYSE; and (9) the inverse of stock price of the previous month.

As in Fama and MacBeth (1973), we use the time-series average of the monthly estimates, $\hat{\Gamma}_\tau$, to estimate the average cross-sectional coefficient vector, $\hat{\Gamma} = (\hat{\Gamma}_1 + \hat{\Gamma}_2 + \cdots + \hat{\Gamma}_T)/T$. To estimate the price impact for firm i over month τ, we calculate the product of $\hat{\Gamma}$ and $X_{i,\tau-1.}$

$$\hat{\lambda}_{i,\tau}^{\mathrm{BHK}} = X_{i,\tau-1}\hat{\Gamma}. \tag{4}$$

While the coefficient $\hat{\Gamma}$ is estimated over the 1993 to 1997 time period, the predetermined variables are observable before the momentum trading strategy is implemented. The predetermined variables are constructed to avoid scale differences across the time period. For example, while the market capitalization of a large firm in 1967 is very different from the market capitalization of a large firm in 1997, a large firm will always have a high relative market capitalization. The same type of cross-sectional regression approach is taken to estimate the coefficients for the GH model, $\lambda^{\mathrm{GH}}_{i,\tau}$ and $\Psi_{i,\tau}$, and effective and quoted spreads, $k^{\mathrm{E}}_{i,\tau}$ and $k^{\mathrm{Q}}_{i,\tau}$. The results of the cross-sectional regressions, equation (3), are reported in Table II. In general, the t-statistics for the cross-sectional coefficients are quite large. Table III presents details of the distribution of the predicted spread and price-impact measures obtained from the cross-sectional regressions, such as equation (4) for $\hat{\lambda}^{\mathrm{BHK}}_{i,\tau}$. Panel A of Table III compares the parameters for the winner decile and loser decile for the 11/1/1 strategy. Panel B presents an equivalent comparison of winners and losers for the 5/1/1 strategy. By every metric, the loser stocks are less liquid, on average, than the winner stocks.

III. Trading Models with Price Impacts

The typical momentum strategies investigated in the literature are not optimized to take into account the price impact costs of trading. To incorporate transaction costs of trades, we first develop the formulation of the total cost of a trade.

A. Cost of a Trade

We start the discussion of the cost of execution of trades with a general derivation. Denote the prevailing market price of an asset by p. A purchase of q units of this asset would cost a total of x as follows:

$$pq + \int_0^q f(p,q)\,\mathrm{d}q = x, \tag{5}$$

where $f(p,q)$ is the price impact cost function and the price acts as a state variable that could influence the cost function. This formulation implicitly assumes that the trade of q shares is divided into many infinitesimal trades (as in Bertsimas and Lo (1998)) and that over the trading period there is no price reversion.[6]

The BHK specification for price impact generates an exponential price-impact function. In the context of equation (5), the price impact cost function is expressed as $f(p,q) = p(e^{\bar{\lambda}q} - 1)$ where $\bar{\lambda}$ is defined as λ^{BHK} scaled by the number

[6] The assumption of no price reversion throughout the trading process somewhat relaxes the need to define the time horizon of the trade, as long as the time horizon for expected return begins after the trade is fully executed. This assumption is plausible for market orders and especially for situations in which a trade must be executed as soon as possible.

Table II

Transaction Costs and Firm Characteristics

Estimates of the average cross-sectional relation between different transaction costs and firm-specific pre-determined variables are provided below (these relations are estimates of Γ, as explained in the paper). Two nonproportionate transaction costs are considered. The Breen-Hodrick-Korajczyk (2002) measure is based on the model $\Delta p_{i,t}/p_{i,t} = \lambda_{\mathrm{BHK},i}\Delta q_{i,t}$, where $\Delta p_{i,t}/p_{i,t}$ is the relative price movement of stock i as a result of trading a net total of $q_{i,t}$ (signed) shares in a 30-minute interval (t). The Glosten-Harris (1988) measure is based on the model $\Delta p_{i,t} = \lambda_{\mathrm{GH},i}\Delta q_{i,t} + \psi_i \Delta d_{i,t}$, where $\Delta p_{i,t}$ is the relative price improvement as a result of trading $q_{i,t}$ (signed) shares at time t (here t represents event time), and $d_{i,t}$ is an indicator for buyer-initiated (+1) or seller-initiated (−1) trade ($\lambda_{\mathrm{GH},i}$ and ψ_i are scaled by the beginning-of-month price of stock i). For proportionate costs, we consider effective and quoted spreads. Effective spreads are measured as the absolute price improvement relative to midpoint of quoted bid-and-ask. Quoted spread is measured as the ratio between the quoted bid-ask spread and the midpoint. All transaction costs are estimated on a monthly basis. The analysis is separated for NYSE/AMEX and Nasdaq for the BHK measure, and includes only NYSE for all other measures. The analysis uses data for the period January 1993 to May 1997.

	Nonproportionate Costs								Proportionate Costs			
	Breen-Hodrick-Korajczyk				Glosten-Harris				Effective Spread		Quoted Spread	
	NYSE/AMEX		Nasdaq		NYSE				NYSE		NYSE	
Variable	$\lambda^{\mathrm{BHK}} \times 10^5$	t-statistic	$\lambda^{\mathrm{BHK}} \times 10^5$	t-statistic	$\lambda^{\mathrm{GH}} \times 10^6$	t-statistic	$\psi \times 10^4$	t-statistic	$k^{\mathrm{E}} \times 10^4$	t-statistic	$k^{\mathrm{Q}} \times 10^4$	t-statistic
Intercept	2.48	28.40	1.38	30.20	0.454	12.55	26.60	20.72	35.47	22.41	108.29	24.75
X_1	0.25	13.60	0.19	10.80	0.005	5.14	−0.14	−8.13	−0.03	−1.50	−0.07	−1.47
X_2	−0.61	−15.50	−0.15	−12.70	−0.069	−12.93	−0.71	−7.85	−1.93	−15.04	−6.01	−19.33
X_3	−0.63	−3.99	−0.14	−2.04	−0.060	−0.68	−54.13	−12.78	−57.00	−14.44	−158.17	−16.76
X_4	0.34	2.13	−0.21	−2.51	0.250	1.46	59.83	12.30	70.74	13.10	192.43	17.79
X_5	−1.68	−20.90	−0.47	−10.20	−0.201	−4.89	−14.13	−26.24	−18.44	−28.13	−48.09	−28.57
X_6	4.15	4.84	17.70	12.80	−0.796	−2.47	19.13	1.52	−51.56	−3.60	−315.34	−8.45
X_7	−3.42	−9.23	−2.46	−16.00	−0.540	−2.81	−27.91	−7.89	−36.08	−7.32	−86.14	−7.11
X_8	−0.26	−5.23										
X_9	3.23	15.70	3.06	15.00	0.566	5.56	121.92	14.07	168.64	14.38	363.96	12.46

X_1 = Market cap at the end of last month divided by the average market cap of CRSP, minus one; X_2 = Total volume during the last three months divided by the average firm volume on NYSE, minus one; X_3 = Stock price at the end of last month divided by the price six month prior, minus one; X_4 = Absolute value of X_3; X_5 = Dummy variable equal to unity if the firm is included in the S&P 500 index; X_6 = Dividend yield; $X_7 = R^2$ of returns regressed on NYSE index, (monthly returns over the last 36 months); X_8 = Dummy variable equal to unity if the firm is traded on NYSE; X_9 = Inverse of stock price of the previous month.

Table III

Estimated Measures of Liquidity

Time-series means of cross-sectional diagnostics of different liquidity measures are presented below. Two nonproportionate transaction costs are considered. The Breen-Hodrick-Korajczyk (2002) measure is based on the model $\Delta p_{i,t}/p_{i,t} = \lambda_{\mathrm{BHK},i}\Delta q_{i,t}$, where $\Delta p_{i,t}/p_{i,t}$ is the relative price movement of stock i as a result of trading a net total of $q_{i,t}$ (signed) shares in a 30-minute interval (t). The Glosten-Harris (1988) measure is based on the model $\Delta p_{i,t} = \lambda_{\mathrm{GH},i}\Delta q_{i,t} + \psi_i \Delta d_{i,t}$, where $\Delta p_{i,t}$ is the relative price movement as a result of trading $q_{i,t}$ (signed) shares at time t (here t represents event time), and $d_{i,t}$ is an indicator for buyer-initiated (+1) or seller-initiated (−1) trade ($\lambda_{\mathrm{GH},i}$ and ψ_i are scaled by the beginning-of-month price of stock i). For proportionate costs, we consider effective and quoted spreads. Effective spreads are measured as the absolute price movement relative to midpoint of quoted bid and ask. Quoted spread is measured as the ratio between the quoted bid–ask spread and the midpoint (half the quoted spread is provided below). All transaction costs are estimated on a monthly basis. The estimation analysis includes NYSE-listed stocks for the period January 1993 to May 1997. Using cross-sectional relations between the different liquidity measures and pre-determined firm characteristics, the liquidity measures are re-estimated for the entire sample period, February 1967 to December 1999. Panels A and B include only stocks in the top (winners) and bottom (losers) deciles according to 11/1/1 and 5/1/1 equal-weighted momentum strategies, respectively (see Table I for a description of momentum strategies). The analysis uses data for the period February 1967 to December 1999.

Winners						Losers					
Variable	Mean	Standard Deviation	Minimum	Median	Maximum	Variable	Mean	Standard Deviation	Minimum	Median	Maximum
Panel A: 11/1/1 Momentum Strategy											
$\lambda^{\mathrm{BHK}} \times 10^5$	2.13	1.30	0.02	2.22	10.68	$\lambda^{\mathrm{BHK}} \times 10^5$	3.81	3.77	0.09	3.07	26.80
$\lambda^{\mathrm{GH}} \times 10^{6\mathrm{a}}$	0.51	0.19	0.02	0.53	1.39	$\lambda^{\mathrm{GH}} \times 10^{6\mathrm{a}}$	0.77	0.57	0.08	0.64	3.88
$\psi \times 10^{3\mathrm{a}}$	3.62	2.04	0.69	3.36	19.29	$\psi \times 10^{3\mathrm{a}}$	11.62	12.01	2.65	8.33	82.68
$k^{\mathrm{E}} \times 10^3$	5.01	2.82	0.78	4.67	26.54	$k^{\mathrm{E}} \times 10^3$	14.97	16.07	3.11	10.37	108.01
$k^{\mathrm{Q}} \times 10^3$	6.92	3.31	1.15	6.60	30.91	$k^{\mathrm{Q}} \times 10^3$	18.58	17.64	4.16	13.72	118.53
Panel B: 5/1/1 Momentum Strategy											
$\lambda^{\mathrm{BHK}} \times 10^5$	2.11	1.40	0.02	2.18	11.83	$\lambda^{\mathrm{BHK}} \times 10^5$	3.63	3.58	0.07	3.02	26.51
$\lambda^{\mathrm{GH}} \times 10^{6\mathrm{a}}$	0.55	0.21	0.02	0.57	1.60	$\lambda^{\mathrm{GH}} \times 10^{6\mathrm{a}}$	0.73	0.53	0.06	0.63	3.72
$\psi \times 10^{3\mathrm{a}}$	3.87	2.55	0.83	3.49	23.80	$\psi \times 10^{3\mathrm{a}}$	11.47	10.99	3.77	8.46	79.87
$k^{\mathrm{E}} \times 10^3$	5.49	3.58	0.98	4.95	33.05	$k^{\mathrm{E}} \times 10^3$	14.50	14.69	4.16	10.36	103.88
$k^{\mathrm{Q}} \times 10^3$	7.41	4.06	1.38	6.90	37.50	$k^{\mathrm{Q}} \times 10^3$	18.31	16.09	5.62	13.89	114.26

[a] For each stock i, $\lambda_{\mathrm{GH},i}$ and ψ_i are scaled by the beginning-of-month price.

of shares outstanding. For the GH specification, the trading costs may be described by $f(p, q) = \lambda^{\mathrm{GH}} q + \Psi p$. Similar to the fixed costs in the GH model, proportional trading costs may be expressed as $f(p, q) = kp$, where k is a constant proportional cost (in our study, k^E and k^Q are the effective and quoted half-spreads, respectively).

B. Trading Strategies with Price Impact

Once a specific momentum strategy and initial investment amount are chosen, we calculate the monthly returns net of trading costs, assuming that the strategy is self-financed. For brevity, we only include here a description of the general methodology. The explicit trading model may be found in Appendix A.

The trading strategy determines which stocks are included in the portfolio every month and the weight of each of these stocks in the portfolio. The actual number of shares traded while rebalancing the portfolio at the beginning of every month is determined by satisfying a generalized portfolio version of equation (5), given total value of the investment portfolio at the end of the previous month and the required weights of each stock in the portfolio. The price impact costs result in the total investment amount being lower after rebalancing. We assume that the monthly returns observed on CRSP are earned only on the amount invested after the costs of rebalancing. Therefore, the net monthly returns, calculated as the ratio between the monthly values of the investment portfolio just before rebalancing, are lower than the observed returns on CRSP (see Figure 2 for an illustration of the portfolio value process).

Since the nonproportional price-impact costs increase with the amount of investment, the average monthly returns of any given momentum strategy decrease with the amount of initial investment. The proportional price-impacts

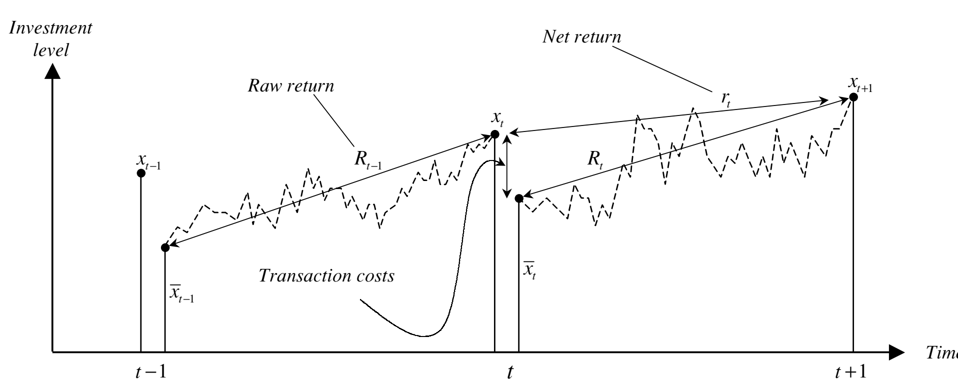


Figure 2. The process of investment level. The figure above illustrates the innovation of level of investment according to the trading model assumed in this paper. At time t, just before rebalancing, the total amount invested in the portfolio is x_t. Due to transaction costs induced by rebalancing, the actual amount invested after rebalancing drops to $\bar{x}_t$. Consequently, the expected returns, denoted by R_t, drop to r_t.

(i.e., effective and quoted spreads) induce a fixed decrease in portfolio returns independent of the amount of initial investment.

As mentioned earlier, standard momentum strategies are not optimized to take into account the price impact costs of trading. It is conceivable that liquidity-conscious portfolios, which attribute more weight to more liquid stocks, would potentially earn higher net average returns. Therefore, we also investigate the performance of liquidity-weighted momentum portfolios, i.e., the weight of each stock in the portfolio is proportional to its market value and inversely proportional to its liquidity measure. This trading rule is optimal for the BHK specification, under some fairly restrictive conditions (see Appendix B). We apply a similar liquidity-weighting strategy under the GH specification, realizing that doing so is somewhat ad hoc.

IV. Performance Evaluation of Momentum Strategies

We wish to evaluate the performance of various momentum-based trading strategies. For proportional transactions cost models, a trading strategy's performance is independent of the size of the portfolio. For nonproportional price impact transactions costs, the performance of the trading strategy declines with the size of the portfolio. Therefore, we are interested in determining the amount that a single portfolio manager could invest before the performance of momentum strategies breaks even with that of the benchmark.

A. Benchmark Asset Pricing Model

We compute Sharpe ratios and abnormal returns (α) relative to the three-factor model of Fama and French (1993) for different initial investment levels. Using the Fama-French (1993) three-factor model, we estimate the time-series regression

$$R_{W,t} - R_{f,t} = \alpha_W + \beta_{W,t} R_{M,t} + s_{W,t} SMB_t + h_{W,t} HML_t + \varepsilon_{W,t}, \tag{6}$$

where $R_{W,t} - R_{f,t}$ is the monthly return of the past-winner momentum portfolio ($W \equiv (J, S, K)$), in excess of the one-month Treasury bill return ($R_{f,t}$); $R_{M,t}$, SMB_t, and HML_t are the Fama–French factors.[7] The conditional exposures of the momentum portfolio to the three factors are denoted by $\beta_{W,t}$, $s_{W,t}$, and $h_{W,t}$.

Given that the composition of momentum-based portfolio strategies, by definition, is based on past returns, it is also based partially on conditional factor risk. For example, if the return on the market is high over the ranking period, our winner portfolio will tend to include high market risk assets. Conversely, if the return on the market is low over the ranking period, our winner portfolio will tend to include low market risk assets. This time variation in conditional

[7] See Fama and French (1993) for a description of the construction of the factor portfolio returns. A description of the factor construction and the return series are available from Ken French at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, February 5, 2004.

systematic risk is discussed in a number of papers (e.g., Chopra, Lakonishok, and Ritter (1992), Jones (1993), and Grundy and Martin (2001)). Grundy and Martin derive a model in which momentum-based portfolios have conditional factor risk exposures that are linear functions of the ranking-period factor portfolio returns. While other effects, such as leverage effects, may make the relation more complex (Chopra et al.), we rely on the results of Grundy and Martin and model the momentum portfolio's conditional factor risk as a linear function of the ranking-period factor returns. That is

$$\begin{aligned} \beta_{W,t} &= a_\beta + b_\beta R_{M,W,t} + c_\beta SMB_{W,t} + d_\beta HML_{W,t}, \\ s_{W,t} &= a_s + b_s R_{M,W,t} + c_s SMB_{W,t} + d_s HML_{W,t}, \\ h_{W,t} &= a_h + b_h R_{M,W,t} + c_h SMB_{W,t} + d_h HML_{W,t} \end{aligned} \tag{7}$$

where $R_{M,W,t}$, $SMB_{W,t}$, and $HML_{W,t}$ are the average cumulative (excess) returns of the factors over the K overlapping ranking periods of length J used to define the momentum strategy. Plugging the formulation of the conditional factor loadings from equation (7) into equation (6), we have the following regression model

$$\begin{aligned} R_{W,t} - R_{f,t} = \alpha_W &+ a_\beta R_{M,t} + b_\beta R_{M,t} \times R_{M,W,t} + c_\beta R_{M,t} \times SMB_{W,t} \\ &+ d_\beta R_{M,t} \times HML_{W,t} + a_s SMB_t + b_s SMB_t \times R_{M,W,t} \\ &+ c_s SMB_t \times SMB_{W,t} + d_s SMB_t \times HML_{W,t} + a_h HML_t \\ &+ b_h HML_t \times R_{M,W,t} + c_h HML_t \times SMB_{W,t} \\ &+ d_h HML_t \times HML_{W,t} + \varepsilon_{W,t}. \end{aligned}$$

Figure 3 plots the estimated time-varying factor risk exposures, $\hat{\beta}_{W,t}$, for the 11/1/3 winner portfolio, along with the unconditional factor sensitivity (figures for $\hat{s}_{W,t}$ and $\hat{h}_{W,t}$ are available from the authors). The figure also includes the ranking-period market factor return, $R_{M,W,t}$. As predicted by the analysis of Grundy and Martin (2001), there is significant time variation in risk that is related to ranking-period factor returns, as in equation (7). Although we estimate $\hat{\beta}_{W,t}$, $\hat{s}_{W,t}$, and $\hat{h}_{W,t}$ as functions of $R_{M,W,t}$, $SMB_{W,t}$, and $HML_{W,t}$, the figure only plots the own-factor ranking-period return. The 11/1/3 equal-weighted winner portfolio has estimated factor loadings that range from 0.73 to 1.48 (time series average of 1.06) for the market factor, from 0.19 to 2.13 (average of 1.01) for the size factor, and from −0.68 to 0.47 (average of −0.07) for the book-to-market factor. For comparison, the unconditional factor loadings are 1.05, 0.97, and −0.09, respectively. The unconditional factor loadings are similar to the values of 1.13, 0.68, and 0.04 found for a 11/1/1 strategy by Fama and French (1996, Table VII).

For comparison purposes we also estimate an unconditional, one-factor CAPM specification. The market risk, β_M, is 1.23 and 1.20 (1.26 and 1.22) for VW (EW) 11/1/3 and 5/1/6 strategies, respectively. The pretrading cost CAPM abnormal returns, α_W, are similar to, but generally smaller than those reported for the conditional three-factor model reported in Table IV. The CAPM alphas are

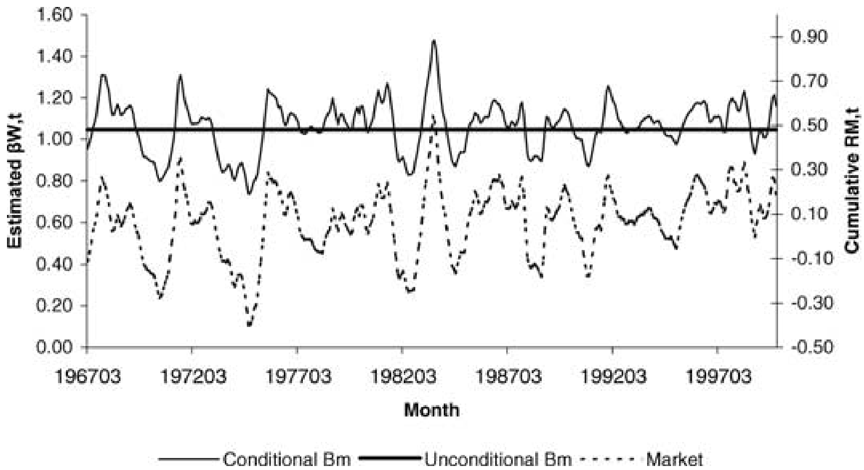


Figure 3. Conditional factor loadings of momentum (11/1/3 equal-weighted strategy). Factor loadings are estimated through the time-series regression

$$\begin{aligned}R_{W,t} - R_{f,t} &= \alpha + a_\beta R_{M,t} + b_\beta R_{M,t} R_{M,W,t} + c_\beta R_{M,t} SMB_{W,t} + d_\beta R_{M,t} HML_{W,t} \\ &\quad + a_s SMB_t + b_s SMB_t R_{M,W,t} + c_s SMB_t SMB_{W,t} + d_s SMB_t HML_{W,t} \\ &\quad + a_h HML_t + b_h HML_t R_{M,W,t} + c_h HML_t SMB_{W,t} + d_h HML_t HML_{W,t} + \varepsilon_t\end{aligned}$$

where $R_{W,t} - R_{f,t}$ is the monthly excess return of the 11/1/3 equally weighted momentum portfolio, $R_{M,t}$, SMB_t, and HML_t are the Fama and French (1993) factors, and $R_{M,W,t}$, $SMB_{W,t}$, and $HML_{W,t}$ are the corresponding cumulative (excess) returns of the factors. Conditional factor loadings $\beta_{W,t}$, $s_{W,t}$, and $h_{W,t}$ are then calculated through

$$\begin{aligned}\beta_{W,t} &= a_\beta + b_\beta R_{M,W,t} + c_\beta SMB_{W,t} + d_\beta HML_{W,t} \\ s_{W,t} &= a_s + b_s R_{M,W,t} + c_s SMB_{W,t} + d_s HML_{W,t} \\ h_{W,t} &= a_h + b_h R_{M,W,t} + c_h SMB_{W,t} + d_h HML_{W,t}\end{aligned}$$

The time-series of the conditional factor loading $\beta_{W,t}$, as well as the cumulative (excess) return of the market portfolio, are plotted above (similar plots for the other factor loadings, $s_{W,t}$ and $h_{W,t}$, are available from the authors upon request). Unconditional loadings of the momentum strategy are obtained via a standard Fama and French time-series regression, i.e., constraining all the coefficients above, except for α, a_β, a_s, and a_h, at zero; they are also plotted above. The analysis uses monthly returns of all NYSE, AMEX, and Nasdaq stocks available on CRSP for the period March 1967 until December 1999.

statistically significant with t-statistics in the range of 2.5 to 2.7 (compared to 3.5 to 8.9 for the conditional three-factor model reported in Table IV).

B. Abnormal Momentum Profits with Proportional Costs

Our analysis is restricted to 11/1/3 and 5/1/6 strategies, since they exhibit significant performance before price impacts (see Table I) and are similar to

Table IV

Performance under Proportionate Transaction Costs

We evaluate the performance of momentum trading strategies according to the trading model developed here, using proportionate transaction costs. We form a portfolio in the beginning of February 1967 according to a chosen momentum strategy with a certain initial amount of investment. The portfolio is rebalanced on a monthly basis, following the trading rule of the chosen strategy, until the end of December 1999. The proportionate costs considered here include effective and quoted spreads. Effective spreads are measured as the absolute price relative to midpoint of quoted bid and ask. Quoted spread is measured as the ratio between the quoted bid-ask spread and the midpoint (half the quoted spread is considered as trading cost). Transaction costs are estimated on a monthly basis, using NYSE-listed stocks for the period January 1993 to May 1997. Then, using cross-sectional relations between the different liquidity measures and pre-determined firm characteristics (see Table II), the spreads are re-estimated for the entire sample period, February 1967 to December 1999. Assuming that the estimated price spreads are perfectly foreseeable, we rebalance the portfolio every month while keeping it self-financing, after considering the execution costs of trades. For every momentum-based trading strategy we calculate the time series of monthly returns, net of transaction costs. Three performance measures are reported: (1) The intercept (alpha) of the conditional Fama and French (1993) regressions; (2) The t-statistic associated with alpha; (3) The Sharpe ratio of the portfolio; and (4) The slope of the investment frontier of a set consisting of four assets: the three Fama and French (1993) portfolios and the momentum portfolio (this is calculated as the maximum attainable Sharpe ratio of a combination of the four assets). The maximum attainable Sharpe ratio of the universe containing only the Fama-French three factors is 0.23. Since we use proportionate transaction costs, all performance measures are invariant to the initial investment. The analysis uses monthly returns of all NYSE stocks available on CRSP.

	Alpha	t-Stat of Alpha	Sharpe Ratio of Momentum	Max Sharpe Ratio of Momentum and FF Factors
Panel A: 11/1/3 Momentum Strategy				
			Equal-Weighted	
Raw return	0.0080	8.92	0.24	0.44
Return net of effective	0.0061	6.86	0.21	0.38
Return net of quoted	0.0054	6.08	0.20	0.35
			Value-Weighted	
Raw return	0.0057	4.54	0.19	0.32
Return net of effective	0.0045	3.59	0.17	0.29
Return net of quoted	0.0040	3.17	0.16	0.28
Panel B: 5/1/6 Momentum Strategy				
			Equal-Weighted	
Raw return	0.0059	8.07	0.22	0.41
Return net of effective	0.0041	5.60	0.19	0.34
Return net of quoted	0.0035	4.72	0.18	0.32
			Value-Weighted	
Raw return	0.0033	3.46	0.17	0.29
Return net of effective	0.0022	2.31	0.15	0.26
Return net of quoted	0.0017	1.82	0.14	0.25

trading strategies that are extensively studied in the literature. The results for VW and EW momentum portfolios with proportional transactions costs are shown in Table IV for NYSE-listed stocks.

The estimated abnormal returns, $\hat{\alpha}$, ignoring transactions costs, are 80 and 57 basis points per month for the EW and VW 11/1/3 momentum strategies, respectively. The value for the EW strategy is higher than the 59 basis points found with an unconditional three-factor model by Fama and French (1996, Table VII) for a 11/1/1 strategy. For the 5/1/6 strategy, the abnormal returns are 59 and 33 basis points per month for the EW and VW strategies, respectively. These are smaller than the 148 basis point abnormal return found by Grundy and Martin (2001, Table I (panel B)) for an EW 6/1/1 strategy; smaller than the 70 basis point abnormal return (relative to an unconditional one-factor model) found by Jegadeesh and Titman (1993, Table III (panel B)) for an EW 6/0/6 strategy; and similar to the 12 to 47 basis point abnormal return (relative to an unconditional three-factor model) found by Lee and Swaminathan (2000, Table VA) for an EW 6/0.25/6 strategy. All four abnormal returns (EW and VW for 11/1/3 and 5/1/6) are statistically significant.

With proportional transactions costs equal to the effective spread, $\hat{\alpha}$ is 61 and 45 basis points with t-statistics of 6.86 and 3.59 for EW and VW 11/1/3 momentum strategies, respectively. For the 5/1/6 strategy, the abnormal returns are 41 and 22 basis points per month for the EW and VW strategies, with t-statistics of 5.60 and 2.31.

For proportional transactions costs implied by the quoted spread, $\hat{\alpha}$ is 54 and 40 basis points with t-statistics of 6.08 and 3.17 for EW and VW 11/1/3 momentum strategies, respectively. For the 5/1/6 strategy the abnormal returns are 35 and 17 basis points per month for the EW and VW strategies, with t-statistics of 4.72 and 1.82. The results indicate that proportional spread costs do not eliminate the statistical significance of momentum profits (with the exception of using quoted spreads for the 5/1/6 VW strategy).

We also calculate the improvement in the Sharpe ratio when the momentum strategies are added to the three Fama-French factor portfolios. This is done by calculating the maximal slope of the tangency portfolio, with and without momentum strategies. In our sample, an investment frontier spanned by the three Fama-French factors has a maximum attainable Sharpe ratio slope of 0.23. The last column in Table IV shows the maximal Sharpe ratio obtainable from the momentum portfolio and the three Fama-French factors. Ignoring transactions costs, adding the 11/1/3 EW momentum strategy to the Fama-French factors increases the attainable slope to 0.44. When effective and quoted spreads are considered as proportional trading costs, the maximal Sharpe ratios are 0.38 and 0.35, respectively. Both 11/1/3 and 5/1/6 (EW and VW) strategies improve the investment frontier, even after considering proportionate spread costs.

C. Abnormal Momentum Profits with Price Impact Costs

We now turn to the nonproportional-cost, price impact models. In addition to calculating the performance of value-weighted (VW) and equal-weighted (EW) momentum portfolios, we also investigate liquidity-weighted (LW) momentum portfolios. The LW portfolios are constructed using the simplifying assumption of Corollary 1 (in Appendix B) that all assets in the winner portfolio have the same expected return.[8] Additionally, we investigate the performance of portfolios whose weights are convex combinations of the VW and LW weights. We study the performance of these strategies as we vary the initial amount invested at the end of January 1967. We report a December 1999 equivalent to this 1967 dollar amount by computing the 1999 value that constitutes the same fraction of total market capitalization as the initial investment in January 1967. The translation ratio between 1967 and 1999 is 29.7. Every month, the portfolios are rebalanced according to the rules dictated by the trading strategies. These rules define both the stocks to be included in the portfolio (according to the different ranking and holding periods) and their weight in the portfolio. The portfolios are self-financing, since no additional funds are added to or removed from the portfolios during the entire investment period. The net returns are calculated using the trading model discussed in Section III. Since the set of firm characteristics used to predict price impact, X_{t-1}, is predetermined at time t, the strategies are adapted to the information set available at the time of each trade, and therefore these strategies are admissible. However, for much of the sample, Γ is estimated with future data.

C.1. Breen et al. (2002) Price Impact Specification

We first investigate the performance after price impacts implied by the BHK specification in equation (1). The results for the 11/1/3 strategy applied to NYSE-traded firms are given in Figure 4. In Figure 4a we plot the estimated portfolio abnormal returns, $\hat{\alpha}$, for several weighting strategies as a function of the level of initial investment (expressed in terms of December 1999 market capitalization). Price impact quickly drives away the profitability of equal-weighted strategies. Abnormal returns are driven to zero with investment portfolios larger than $2 billion for value-weighted strategies. However, for the liquidity-weighted (LW) strategy, or the 50/50 weighting of the LW and VW strategies, $\hat{\alpha}$ is driven to zero only after approximately $5 billion is invested.

Figure 4b provides an estimate of the monthly dollar value creation ($\hat{\alpha}$ times the level of investment) for different levels of investment. For the LW and the combined LW/VW portfolios, value creation is maximized with an initial investment of approximately $2.5 billion. In Figure 4c, we plot the maximal Sharpe ratio attainable through combinations of Treasury bills, the three Fama-French factor portfolios, and long positions in the winner momentum portfolio. A horizontal line (at a value of 0.23) is drawn at the maximal Sharpe ratio attainable through combinations of Treasury bills and the three Fama-French factor portfolios only. These results mirror those in Figure 4a: the EW Sharpe ratio drops

[8] The definition of LW differs across the BHK and GH price-impact models. Corollary 1 directly addresses the BHK case, and therefore we use weights proportional to $MVE_i/\lambda_i^{\mathrm{BHK}}$ (where MVE_i is the market value of equity for asset i). For the GH case we use a weighting scheme that is similar in spirit. Since there are fixed and variable costs in that model, LW are calculated as the average between weights $p_i^2/\lambda_i^{\mathrm{GH}}$ and $1/\bar{\Psi}_i$ (see Appendix A, equation (A17)).

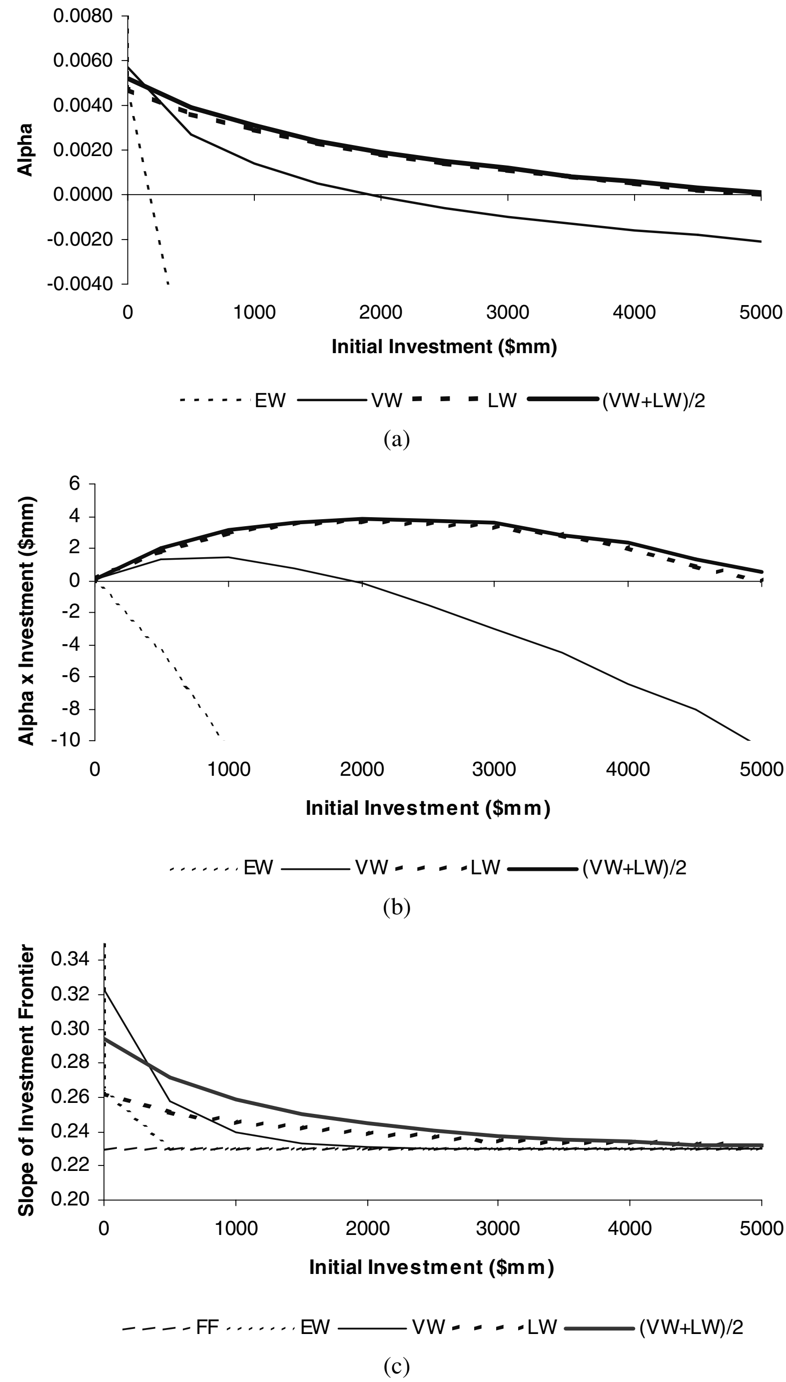


Figure 4. Performance evaluation of momentum strategies (NYSE, Breen-Hodrick-Korajczyk). We evaluate the performance of momentum trading strategies using the trading

to that of the factor portfolios for low levels of investment; the VW Sharpe ratio drops to that of the factor portfolios for a level of investment around \$2 billion; and the LW and LW/VW Sharpe ratios drop to that of the factor portfolios for a level of investment around \$5 billion. The performances of the 5/1/6 strategies are similar to those of the 11/1/3 strategies, with the exception that the 5/1/6 strategies exhibit lower break-even levels. For brevity, these results are not included in Figure 4 and are available from the authors upon request.

C.2. Glosten and Harris (1988) Price-impact Specification

We now turn to performance, assuming price impacts implied by the GH specification, equation (2). The results for the 11/1/3 strategy applied to NYSE-traded firms are given in Figure 5. The basic patterns are similar to those in Figure 4. In Figure 5a we plot the estimated portfolio abnormal returns, $\hat{\alpha}$, for momentum strategies as a function of the level of initial investment. As with the previous specification, price impact quickly drives away the profitability of equal-weighted strategies. Abnormal returns are driven to zero with investment portfolios larger than \$3 billion for value-weighted strategies. However, for the liquidity-weighted (LW) strategy, $\hat{\alpha}$ is driven to zero only after over \$5 billion is invested. For the 50/50 weighting of the LW and VW strategies, $\hat{\alpha}$ is driven to zero after approximately \$4.5 billion is invested. Figure 5b plots $\hat{\alpha}$ times the level of investment for different levels of investment. As before, for the LW and the combined LW/VW portfolios, value creation is maximized with portfolios investing approximately \$2.5 billion. In Figure 5c we plot the maximal Sharpe ratio attainable through combinations of Treasury bills, the

Figure 4— *Continued*

model developed here. Specifically, we implement the 11/1/3 strategy using various weighting schemes (equal weights (EW), value weights (VW), liquidity weights (LW), and a convex combination (VW + LW)/2). We form a portfolio in the beginning of February 1967 according to a chosen momentum strategy with a certain initial monetary amount of investment. We rebalance the portfolio on a monthly basis, following the trading rule of that strategy, until the end of December 1999. The execution costs of trading any stock i are assumed to follow the model $\Delta p_i/p_i = \lambda_i \Delta q_i$, where $\Delta p_i/p_i$ is the relative price improvement as a result of trading Δq_i (signed) shares. The price impact coefficients λ_i are calculated as the fitted values of cross-sectional regressions of measured price impacts on firm characteristics. These regressions used the Trades and Quotes data for the period January 1993 until May 1997. Assuming that the estimated price impacts are perfectly foreseeable, we rebalance the portfolio every month while keeping it self-financing, after considering the price impact of trades. For every momentum-based trading strategy and initial investment, we calculate the time series of monthly returns, net of price impacts. Four performance measures are reported: (a) the intercept (alpha) of the conditional Fama and French (1993) regressions, (b) alpha multiplied by the amount of investment, and (c) the slope of the investment frontier of a set consisting of four assets: the three Fama and French (1993) portfolios and the momentum portfolio (this is calculated as the maximum attainable Sharpe ratio of a combination of the four assets). The initial investment is quoted relative to market capitalization of December 1999. The analysis uses monthly returns of all NYSE stocks available on CRSP.

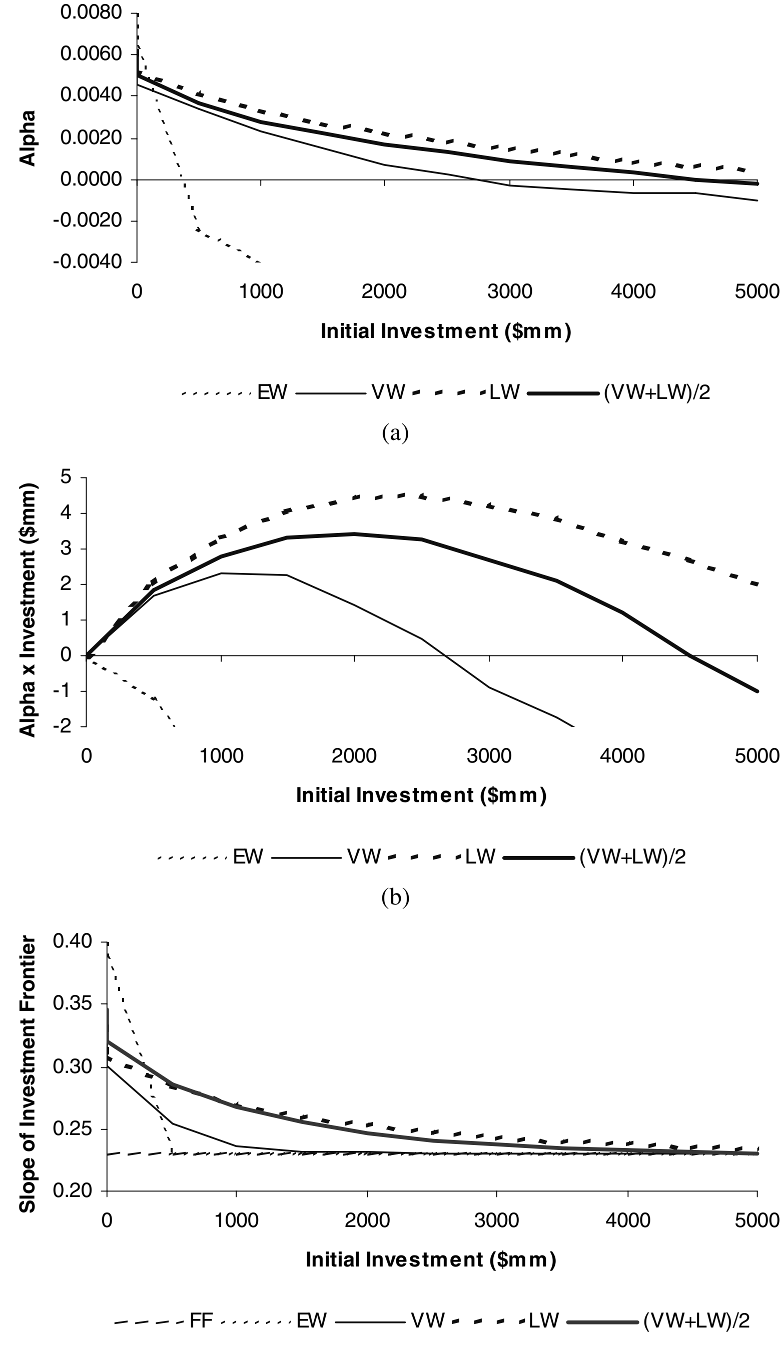


Figure 5. Performance evaluation of momentum strategies (NYSE, Glosten-Harris). We evaluate the performance of momentum trading strategies using the trading model developed here.

three Fama-French factor portfolios, and the winner momentum portfolio. A horizontal line is drawn at the maximal Sharpe ratio attainable through combinations of Treasury bills and the three Fama/French factor portfolios. As in Figure 4, the Sharpe ratios mirror the values of $\hat{\alpha}$: the EW Sharpe ratio drops to that of the factor portfolios for low levels of investment; the VW Sharpe ratio drops to that of the factor portfolios for a level of investment around \$2 billion; and the LW and LW/VW Sharpe ratios drop to that of the factor portfolios for a level of investment around \$4.5 to \$5 billion.

V. Robustness of the Results

We check for the robustness of the results in several dimensions. We begin (in Section V.A) by extending the cross-sectional sample to include AMEX and Nasdaq stocks in addition to the NYSE stocks previously examined. This has two possible offsetting effects. The added stocks are less liquid, on average, than NYSE stocks, suggesting lower break-even fund size. However, with more stocks held in the strategy, a fund of a given size has a smaller position in any given stock, and therefore should have lower price impact. The second effect dominates. In Section V.B we augment the momentum strategies with a momentum/volume strategy based on the findings of Lee and Swaminathan (2000). Since the augmented momentum strategy tends to invest in less liquid stocks, it underperforms pure momentum strategy (after trading costs).

Our results seem to be at odds with some recent studies. In Section V.C, we compare our approach to two papers. In some dimensions the results are not

Figure 5— *Continued*

Specifically, we implement the 11/1/3 strategy using various weighting schemes (equal weights (EW), value weights (VW), liquidity weights (LW), and a convex combination (VW + LW)/2). We form a portfolio in the beginning of February 1967 according to a chosen momentum strategy with a certain initial monetary amount of investment. We rebalance the portfolio on a monthly basis, following the trading rule of that strategy, until the end of December 1999. The execution costs of trading any stock i are assumed to follow the model $\Delta p_{i,t} = \lambda_i \Delta q_{i,t} + \psi_i \Delta d_{i,t}$, where $\Delta p_{i,t}$ is the relative price improvement as a result of trading $q_{i,t}$ (signed) shares at time t (here t represents event time), and $d_{i,t}$ is an indicator for buyer-initiated (+1) or seller-initiated (−1) trade. The price impact coefficients λ_i and ψ_i are calculated as the fitted values of cross-sectional regressions of measured price impacts on firm characteristics. These regressions used the Trades and Quotes data for the period January 1993 until May 1997. Assuming that the estimated price impacts are perfectly foreseeable, we rebalance the portfolio every month while keeping it self-financing, after considering the price impact of trades. For every momentum-based trading strategy and initial investment, we calculate the time series of monthly returns, net of price impacts. Four performance measures are reported: (a) the intercept (alpha) of the conditional Fama and French (1993) regressions; (b) alpha multiplied by the amount of investment; and (c) the slope of the investment frontier of a set consisting of four assets: the three Fama and French (1993) portfolios and the momentum portfolio (this is calculated as the maximum attainable Sharpe ratio of a combination of the four assets). The initial investment is quoted relative to market capitalization of December 1999. The analysis uses monthly returns of all NYSE stocks available on CRSP.

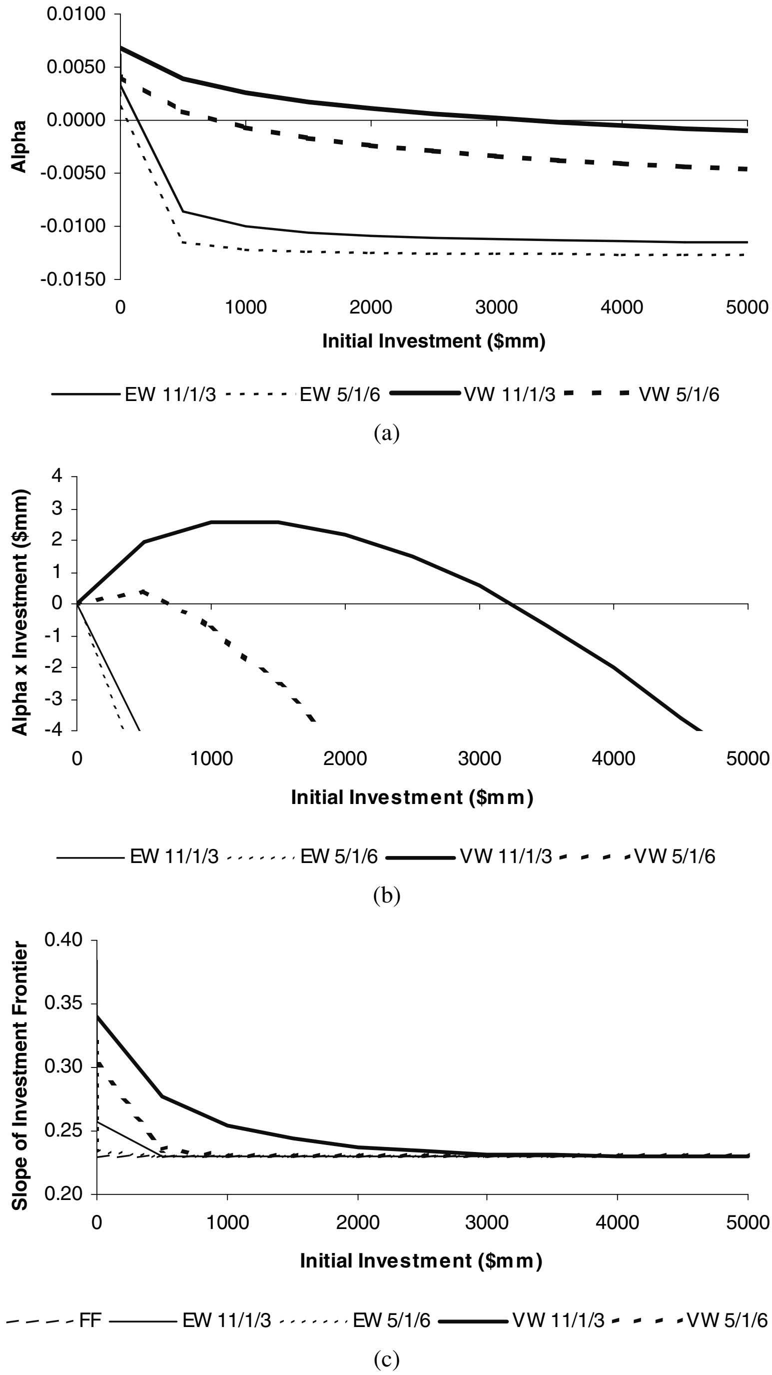


Figure 6. Performance evaluation of momentum strategies (NYSE, AMEX, and Nasdaq, Breen-Hodrick-Korajczyk). We evaluate the performance of momentum trading strategies

as different as they appear at first glance. However, there remain important differences in approaches and results. Finally, in Section V.D, we argue that the assumptions used here are, on balance, conservative in the sense that true break-even portfolio sizes are likely to be larger than those reported here.

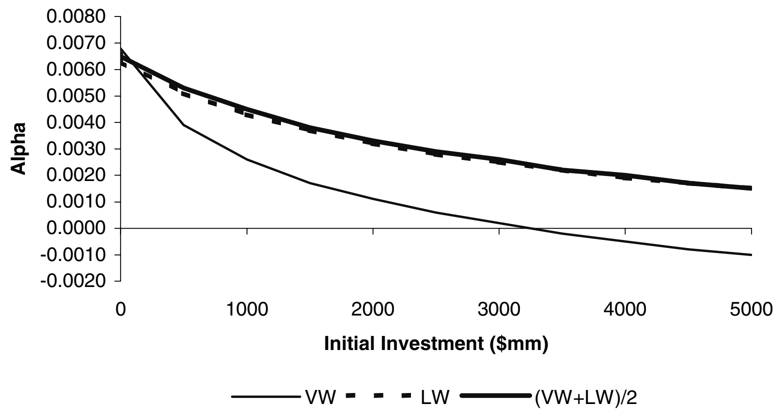
A. Extending the Admissible Set of Assets

We apply the BHK price-impact model, (equation (1)), to a strategy that invests in AMEX and Nasdaq stocks, in addition to NYSE stocks. Expanding the sample in this manner has two offsetting effects. First, the newly included firms are likely to be smaller and less liquid, on average, than the NYSE stocks. This would tend to reduce the break-even investment amounts for the expanded sample. Second, with a larger sample of firms, a portfolio of a given size can spread those funds across a greater number of firms. Since the trading required in any single stock is lower, the price impact cost is lower. This would tend to increase the break-even investment amounts for the expanded sample. Figure 6 compares the performance of EW and VW weighting of 11/1/3 and 5/1/6 strategies. After price impacts, VW strategies dominate EW strategies and the 11/1/3 strategy dominates the 5/1/6 strategy.

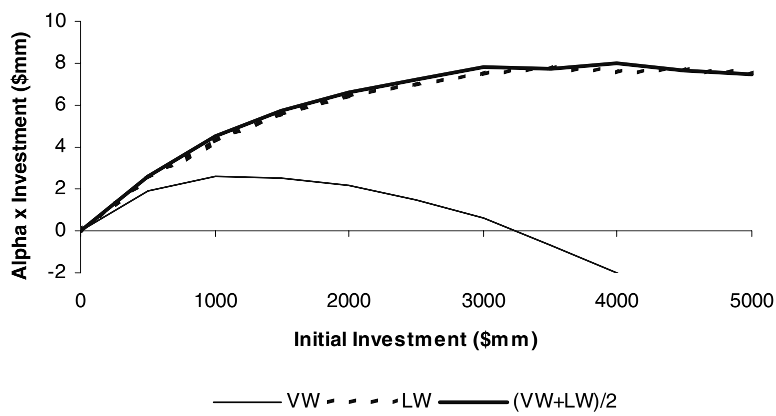
Because of the dominance of VW 11/1/3 strategies in Figure 6, in Figure 7 we only look at the performance of VW, LW, and LW/VW 11/1/3 strategies. In comparing Figures 7a to c to Figures 4a to c , all three strategies have larger break-even investment amounts with the expanded sample. This is true in terms of the portfolio size that drives $\hat{\alpha}$ to zero and the size that drives the maximal Sharpe ratio to that of the Fama and French factors. Turning to

Figure 6— *Continued*

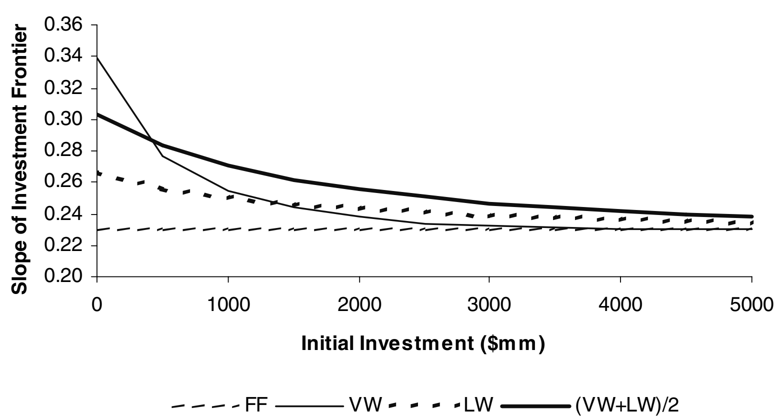
using the trading model developed here. Specifically, we implement the 11/1/3 and 5/1/6 strategies using various weighting schemes (equal weights (EW), and value weights (VW)). We form a portfolio in the beginning of February 1967 according to a chosen momentum strategy with a certain initial amount of investment. We rebalance the portfolio on a monthly basis, following the trading rule of that strategy, until the end of December 1999. The execution costs of trading any stock i are assumed to follow the model $\Delta p_i/p_i = \lambda_i \Delta q_i$, where $\Delta p_i/p_i$ is the relative price improvement as a result of trading Δq_i (signed) shares. The price-impact coefficients λ_i are calculated as the fitted values of cross-sectional regressions of measured price impacts on firm characteristics. These regressions use the Trades and Quotes data for the period January 1993 until May 1997. Assuming that the estimated price impacts are perfectly foreseeable, we rebalance the portfolio every month while keeping it self-financing, after considering the price impact of trades. For every momentum-based trading strategy and initial investment, we calculate the time series of monthly returns, net of price impacts. Four performance measures are reported: (a) the intercept (alpha) of the conditional Fama and French (1993) regressions; (b) alpha multiplied by the amount of investment; and (c) the slope of the investment frontier of a set consisting of four assets: the three Fama and French (1993) portfolios and the momentum portfolio (this is calculated as the maximum attainable Sharpe ratio of a combination of the four assets). The initial investment is quoted relative to market capitalization of December 1999. The analysis uses monthly returns of all NYSE, AMEX, and Nasdaq stocks available on CRSP.



(a)



(b)



(c)

Figure 7. Performance evaluation of momentum strategies (NYSE, AMEX, and Nasdaq, Breen-Hodrick-Korajczyk). We evaluate the performance of momentum trading strategies

Figures 4b and 7b, the fund size that creates maximal value is also larger for the expanded sample. Therefore, the advantage of spreading the investment over more assets (hence a lower price impact) outweighs the disadvantage in investing in assets with higher average price impact coefficients.

B. Volume Weighting

Lee and Swaminathan (2000) present evidence that past trading volume provides a link between momentum and "value" trading strategies. They find that low volume stocks earn higher subsequent returns, on average, than high volume stocks. In particular, they find that "high (low) volume winners (losers) experience faster momentum reversals" (p. 2018). Moreover, they find that low volume stocks display many of the characteristics of "value" stocks while high volume stocks display many of the characteristics of "glamour" stocks. This suggests that variants of the momentum strategy that tilt the portfolio toward low volume stocks might outperform a simple momentum strategy. The potential downside to such strategies is that they may lead to large trading costs, since low volume stocks might be illiquid.

We analyze two such low-volume, winner-based strategies. We restrict our analysis to the 11/1/3 strategy. In the first strategy, the stocks in the winners' decile are weighted by the inverse of their turnover over the ranking period (Lee and Swaminathan use turnover as the measure of trading volume). In the second strategy, the portfolio weights for stocks in the winners' decile are proportional to market capitalization and inversely proportional to their turnover over the ranking period. For both the NYSE sample and the NYSE/AMEX/Nasdaq

Figure 7— *Continued*

using the trading model developed here. Specifically, we implement the 11/1/3 strategy using various weighting schemes (value weights (VW), liquidity weights (LW), and a convex combination (VW + LW)/2). We form a portfolio in the beginning of February 1967 according to a chosen momentum strategy with a certain initial amount of investment. We rebalance the portfolio on a monthly basis, following the trading rule of that strategy, until the end of December 1999. The execution costs of trading any stock i are assumed to follow the model $\Delta p_i/p_i = \lambda_i \Delta q_i$, where $\Delta p_i/p_i$ is the relative price improvement as a result of trading Δq_i (signed) shares. The price impact coefficients λ_i are calculated as the fitted values of cross-sectional regressions of measured price impacts on firm characteristics. These regressions use the Trades and Quotes data for the period January 1993 until May 1997. Assuming that the estimated price impacts are perfectly foreseeable, we rebalance the portfolio every month while keeping it self-financing, after considering the price impact of trades. For every momentum-based trading strategy and initial investment, we calculate the time series of monthly returns, net of price impacts. Four performance measures are reported: (a) the intercept (alpha) of the conditional Fama and French (1993) regressions; (b) alpha multiplied by the amount of investment; and (c) the slope of the investment frontier of a set consisting of four assets: the three Fama and French (1993) portfolios and the momentum portfolio (this is calculated as the maximum attainable Sharpe ratio of a combination of the four assets). The initial investment is quoted relative to market capitalization of December 1999. The analysis uses monthly returns of all NYSE, AMEX, and Nasdaq stocks available on CRSP.

sample, the strategies have positive values of $\hat{\alpha}$, ignoring transactions costs. However, the estimated abnormal return of the strategies becomes negative before the portfolios reach an initial investment of $500 million. The first strategy applied to the full NYSE/AMEX/Nasdaq sample has a negative $\hat{\alpha}$ with an initial investment of merely $1 million. Thus, the low-volume winners' strategies seem to entail relatively large price impact costs.

C. Comparison to Other Approaches

Chen et al. (2002) analyze break-even fund sizes for several anomalies (based on size, book-to-market ratios, and momentum), and conclude that the break-even fund sizes are much smaller than those reported here. The methodology employed here differs from theirs in many dimensions. However, the difference between our reported break-even levels and theirs is due mainly to two effects, one purely mechanical and one substantive difference in the trading strategy. The mechanical difference is the fact that they report sizes in terms of 1963 values and we report them in terms of 1999 values. When similar EW and VW strategies are compared on the same basis, our break-even fund sizes are typically smaller than theirs. We chose to express break-even fund sizes at the end of the sample to facilitate comparisons to currently existing funds (discussed below). The substantive trading strategy difference is that we introduce a liquidity-weighted strategy that significantly increases the break-even investment levels versus the value-weighted and equal-weighted strategies typically studied.

A third difference is that, even before any transactions costs, the CSW returns are much lower than ours. In fact, they do not find that momentum leads to statistically significant profits in three of the four strategies studied (the significant strategy is value-weighted over the 1963 to 2000 period). Our before-transactions cost returns are closer to those found in the previous literature (e.g., Jegadeesh and Titman (1993, 2001)).[9] We find that the difference is due mainly to the use of non-overlapping holding periods (rebalancing the whole portfolio every K periods) in CSW, while Jegadeesh and Titman (1993, 2001) and we use overlapping periods (rebalancing a Kth of the portfolio every period). A very small amount of the reported difference between our numbers and those of CSW is due to the fact that we have a skip month and they do not, resulting in some return reversals in their strategy due to bid/ask bounce.

[9] Their equally weighted average annual return on the winners-losers 12/0/6 strategy is 3.6% (1963–2000). Our equally weighted average monthly return on the winners-losers 11/1/6 strategy is 1.19% (NYSE) and 0.74% (NYSE/AMEX/Nasdaq) (1967–1999). By comparison, Jegadeesh and Titman (1993) find an equally weighted average monthly return on the winners-losers 12/0/6 strategy of 1.14% (NYSE/AMEX) and an average return on the winners-losers 12/0.25/6 strategy of 1.21% (NYSE/AMEX) (1965–1989). Jegadeesh and Titman (2001) update their original analysis and find equally large returns. The CSW value-weighted average annual return on the winners-losers 12/0/6 strategy is 9.4% (1963–2000). Our value-weighted average monthly return on the winners-losers 11/1/6 strategy is 1.08% (NYSE) and 1.51% (NYSE/AMEX/Nasdaq) (1967–1999). Jegadeesh and Titman (1993, 2001) do not report value-weighted results.

Lesmond et al. (2003) find that proportional trading costs eliminate the profits on the strategies they study. They study equal-weighted strategies, which we also find to be unprofitable. However, their estimates of proportionate spread costs are higher than our estimates. Comparing our averages in Table III (panel B) to their's in Table II shows that their spreads are higher by factors ranging from 1.18 to 5.55 (i.e., from 18% higher to 455% higher). While our results are broadly consistent with theirs for the EW strategies they study, we find that there are alternative VW and LW strategies that provide greater profits.

D. Effects of Relaxing Assumptions

Any analysis is predicated on a set of underlying assumptions. We feel that, on balance, our break-even fund sizes are likely to be conservative (i.e., too small).

The empirical evidence in Breen et al. (2002) indicates that the predicted price impacts were substantially higher than the actual price impacts, on average. They compare the predicted price impact (from the cross-sectional regressions, similar to equation (3)) to the actual price impact experienced by a sample of institutional traders (using data from the Plexus Group). There are a number of potential explanations for this upward bias in the predicted price impacts. For example, our strategies are implemented at the end of each month without any attempt to "trickle out" the trades beyond the price impact estimation interval. Since the BHK price impact coefficients are measured over a 30-minute interval, we are implicitly assuming that the month-end rebalancing takes place over a 30-minute interval. The GH price impact coefficients are measured on a trade-by-trade basis, so in that case, we are implicitly assuming that the month-end rebalancing takes place in a single trade. Certainly an astute portfolio manager might choose to transact in a more patient fashion, thereby reducing price impact costs. Also, if momentum traders can signal to the market that they are not informed, they might be able to execute trades more favorably than assumed here. However, transacting in a more patient manner imposes other potential costs, such as the failure to execute the trade. Also, the results in Keim and Madhavan (1997) do not indicate that technical traders have lower trading costs.

We assume either linear or convex price impact functions rather than concave price impacts. If the concavity observed empirically is obtainable, rather than being due to information leakages or credible signaling by uninformed traders, then we should be able to invest larger quantities profitably, leading to larger break-even fund sizes.

In applying the GH model, we do not account for discreteness of prices when estimating $\lambda_{i,\tau}^{\mathrm{GH}}$ and $\Psi_{i,\tau}$. Glosten and Harris (1988) find that ignoring discreteness has little effect on estimates of $\lambda_{i,\tau}^{\mathrm{GH}}$, but leads to upward-biased estimates of $\Psi_{i,\tau}$. Thus, our trading costs would be lower and our break-even fund size larger if we accounted for price discreteness in the parameter estimation.

There is a pronounced momentum reversal around the turn of the year (mainly exhibited by losers rather than winners). While these do not appear to be exploitable in isolation (Sadka (2001)), one could improve the performance of the momentum strategies studied here by incorporating knowledge of the reversal into the trading strategy. We have not done so, leading to break-even investment levels that are smaller than strategies designed to take advantage of the turn-of-the-year effect.

We look at a strategy of investing in past winners only. Much of the literature also studies strategies that take long positions in past winners and short positions in past losers. Most papers find that a larger share of the abnormal returns (without trading costs) to this long/short strategy is due to the short positions in past losers. Thus, before trading costs, investing only in winners is conservative, in that it leads to lower abnormal returns. This does not necessarily carry over to the after-trading-costs case since the losers are less liquid on average (see Table III).

The liquidity-weighted strategy we study makes a number of simplifying assumptions whose relaxation might lead to more profitable trading rules and higher break-even levels of investment. The liquidity-based portfolios that we have examined are based only on partial optimization. The optimization results in a myopic trading rule, which is used in our empirical analysis. An extension of the static optimization to a dynamic setting should result in the superior performance of strategies designed to account for price impacts. We derive optimal weights for any set of expected returns, but our empirical results rely on the simplifying assumption of Corollary 1, that expected returns are the same for all assets in our "winner" momentum portfolio. Performance might be improved by a better model of expected returns.[10]

While we have incorporated spread and price impact costs of trading, we have not taken into account direct commissions. This is a case in which the assumption may lead to an overestimate of the break-even investment level. Given that the estimated price impacts are, on average, larger than the actual price impacts by more than the level of commissions (from Breen et al. (2002)), we feel that the net effect is still toward underestimating the break-even fund size.

While time variation in expected liquidity is considered in our analysis, we do not consider liquidity risk for the benchmark factor model nor for the portfolio selection problem. Momentum portfolios may experience exposure to systematic shifts in liquidity (as suggested in Pástor and Stambaugh (2003) and Sadka (2003)) and, therefore, may earn a premium associated with systematic liquidity risk. This subject should be the focus of future research.

[10] We have estimated expected returns using several momentum-based models. The strategies based on these expected returns underperform those that assume equal expected returns. This is due to the failure of the models to explain either the level of expected returns, or the cross-sectional variation of expected returns, or both. Note that not only is the cross-sectional variation an important input to our model, but also the actual level of expected returns, since the ratio of expected return over price impact is a crucial input to our model.

VI. Conclusions

This paper tests whether momentum-based strategies that previously have been shown to earn high abnormal returns remain profitable after considering price impact induced by trading. The paper develops a methodology to include liquidity in a trading model and demonstrates the importance of such measures for the performance evaluation of trading strategies. We find that when price impact is ignored, the 11/1/3 and 5/1/6 strategies earn significant abnormal returns relative to a conditional version of the Fama and French (1993) three-factor asset pricing model. The strategies remain profitable when transaction costs are proportional costs equal to the effective and quoted spreads. The 11/1/3 strategy outperforms the 5/1/6 strategy and equal-weighted strategies outperform value-weighted strategies.

In contrast to the results ignoring price impact costs, both 11/1/3 and 5/1/6 momentum strategies perform better, post-price impact, using value weights rather than equal weights. For example, the zero-α break-even point is 200 million dollars for the 11/1/3 EW strategy, while it is more than 2 billion dollars for an 11/1/3 VW strategy. This is due to the fact that value-weighting is concentrated in more liquid stocks than equal-weighting. Equal-weighted portfolios have higher price impact costs. Trading costs are crucial for equally weighted strategies, since their performance measures decrease dramatically even when a relatively small investment is considered. These results are especially important in light of recent momentum literature, which concentrates on equally weighted strategies. For example, Fama and French (1996), Grundy and Martin (2001), Yao (2001), and Lesmond et al. (2003) study 11/1/1, 6/1/1, 6/0/6, and 6/0/6 equally weighted strategies, respectively. These strategies seem to be less tractable in the context of transaction costs. The results are consistent across two alternative measures of price impact from Glosten and Harris (1988) and Breen et al. (2002), with the GH measure leading to slightly larger break-even points. We construct alternative momentum strategies by taking price impacts into account while choosing the portfolio weights. Our LW strategies provide higher post-price impact abnormal returns relative to VW strategies. Portfolio strategies that have weights that are convex combinations of the LW and VW weights often provide abnormal returns similar to the LW portfolios. The estimated excess returns of some momentum strategies disappear only after \$4.5 to over \$5.0 billion (relative to market capitalization in December 1999) is engaged in such strategies.

Whether the break-even fund sizes calculated here are large or small is somewhat in the eye of the beholder. A break-even fund size of \$4.5 to over \$5.0 billion (where $\hat{\alpha}$ is driven to zero) is small relative to the total market capitalization of the NYSE (\$11.7 trillion). Chen et al. (2002, Table XIII) report data on hedge fund sizes by investment style. From the sample of hedge funds in the TASS database, a break-even fund size of \$5.0 billion represents 2.7% of total value of hedge funds, 8.9% of total value of Arbitrage hedge funds, and 34.2% of Trend Follower hedge funds (see Chen et al. (2002), Table XIII). As noted above, there are reasons to believe that attainable break-even fund sizes are larger than those calculated here. Hence they would represent larger fractions of the hedge fund universe. The profitability of the strategies is in addition to the profits already earned by momentum-based investors in the market over the sample period.

Accounting for the price impact of trading leads to a large decline in the apparent profitability of some previously studied momentum-based strategies, particularly equally weighted strategies. However for other strategies, such as VW and LW strategies, the size of the break-even portfolios (with the likelihood that the break-even sizes are underestimated) suggests that transaction costs do not appear to fully explain the return persistence of past winner stocks exhibited in the data. This anomaly remains an important puzzle.

Appendix A: Trading Models with Price Impacts

The BHK model of price impacts generates an exponential price-impact function. $Turnover_{i,t}$ is

$$Turnover_{i,t} \equiv \frac{\Delta q_{i,t}}{(Shares\ Outstanding)_{i,t}}, \tag{A1}$$

where $\Delta q_{i,t}$ is the net number of shares bought/sold of asset i at period t in month τ. Substituting (A1) in equation (1) we have

$$\frac{\Delta p_{i,t}}{p_{i,t}} = \frac{\lambda_{i,\tau}^{\mathrm{BHK}}}{(Shares\ Outstanding)_{i,\tau}} \Delta q_{i,t}. \tag{A2}$$

By defining

$$\bar{\lambda}_{i,\tau}^{\mathrm{BHK}} \equiv \frac{\lambda_{i,\tau}^{\mathrm{BHK}}}{(Shares\ Outstanding)_{i,\tau}}, \tag{A3}$$

equation (A2) is further simplified to

$$\frac{\Delta p_{i,t}}{\Delta q_{i,t}} = \bar{\lambda}_{i,\tau}^{\mathrm{BHK}} p_{i,t}. \tag{A4}$$

Therefore, in the limit as $\Delta q_{i,t} \to 0$, the supply function is given by

$$\overline{p_{i,t}} = p_{i,t} e^{\bar{\lambda}_{i,\tau}^{\mathrm{BHK}} q_{i,t}}, \tag{A5}$$

where $\overline{p_{i,t}}$ and $p_{i,t}$ are the post- and pre-trade prices of asset i, and $q_{i,t}$ is the net traded at t. In the context of equation (A5), the price impact cost function is expressed as

$$f(p, q) = p(e^{\bar{\lambda} q} - 1). \tag{A6}$$

Therefore, the total price impact of a trade of $q_{i,t}$ shares is calculated through $\int_0^q f(p,x)\,\mathrm{d}x$:

$$\begin{aligned}\int_0^{q_{i,t}} p_{i,t}\left(e^{\bar{\lambda}_{i,\tau}^{\mathrm{BHK}}x}-1\right)\mathrm{d}x &= p_{i,t}\frac{1}{\bar{\lambda}_{i,\tau}^{\mathrm{BHK}}}\left[e^{\bar{\lambda}_{i,\tau}^{\mathrm{BHK}}q_{i,t}}-1\right]-p_{i,t}q_{i,t}\\ &= \frac{MVE_{i,t}}{\lambda_{i,\tau}^{\mathrm{BHK}}}\left[e^{\frac{\lambda_{i,\tau}^{\mathrm{BHK}}}{MVE_{i,t}}p_{i,t}q_{i,t}}-1\right]-p_{i,t}q_{i,t},\end{aligned} \tag{A7}$$

where $MVE_{i,t}$ is the market value of equity of asset i at time t. An illustration of the price impact function is provided in Figure 1.

Define x_t as the value of the portfolio at time t, before rebalancing, and $\bar{x}_t$ as the value after rebalancing. The momentum-based trading strategy, consisting of purchasing the stocks in the past winner decile, implicitly defines which stocks are included in the portfolio. The stocks that need to be traded at time t are divided into two mutually exclusive sets as follows:

$$\begin{aligned} I_{1,t} &= \left\{i : \omega_{i,t} > 0, \omega_{i,t-1} \geq 0\right\},\\ I_{2,t} &= \left\{i : \omega_{i,t} = 0, \omega_{i,t-1} > 0\right\},\end{aligned} \tag{A8}$$

where $\omega_{i,t}$ is the portfolio weight associated with asset i at time t. The expression $I_{1,t}$ consists of all stocks held at time t, which could include ones also held at time $t-1$ or those that are added to the pool of winners at time t. The expression $I_{2,t}$ consists of stocks that were held at time $t-1$ but are no longer in the winner decile at t, and therefore need to be sold. The portfolio weights are percentages of the actual investment after price impacts, $\bar{x}_t$. The purpose of defining $I_{2,t}$ is to be able to include trading rules that require liquidation of assets, as an input to an optimization problem defined later. Also, short-sale constraints are imposed, since we only consider strategies consisting of long positions. Denoting the (raw) return, without price impacts, of stock i for the period from t to $t+1$ as $R_{i,t+1}$, the following recursive relations hold:

$$\begin{aligned} x_t &= \bar{x}_{t-1}\sum_{i\in I_{1,t}\cup I_{2,t}} \omega_{i,t-1}(1+R_{i,t}),\\ E_t[x_{t+1}] &= \bar{x}_t \sum_{i\in I_{1,t}} \omega_{i,t}(1+E_t[R_{i,t+1}]).\end{aligned} \tag{A9}$$

Assume that the portfolio is rebalanced at time t. At the beginning of time t, prior to trading for rebalancing purposes, the number of shares of each stock is given by

$$q_{i,t} = \frac{\omega_{i,t-1}\bar{x}_{t-1}[1+R_{i,t}]}{p_{i,t}}. \tag{A10}$$

A trading strategy specifies the allocation of assets after rebalancing at time t by assigning the weights $\omega_{i,t}$. Therefore, the number of shares of each asset required after trading at time t is expressed as

$$\bar{q}_{i,t} = \frac{\omega_{i,t}\bar{x}_t}{p_{i,t}}. \tag{A11}$$

To solve for the post-trade portfolio value, $\bar{x}_t$, notice that the sum of the post-trade value and the total price impact must equal the pre-trade value, x_t. Explicitly, the following equality must hold:

$$\bar{x}_t + \sum_{i\in I_{1,t}\cup I_{2,t}} \left[\frac{1}{b_{i,t}}\left[e^{b_{i,t}p_{i,t}[\bar{q}_{i,t}-q_{i,t}]} - 1\right] - p_{i,t}\left[\bar{q}_{i,t} - q_{i,t}\right]\right] = x_t, \tag{A12}$$

where $b_{i,t}$ is the price impact coefficient (t is any time during month τ), adjusted for firm size

$$b_{i,t} \equiv \frac{\lambda_{i,\tau}^{\mathrm{BHK}}}{MVE_{i,\tau}}. \tag{A13}$$

Equation (A12) is a budget constraint to the investment. Notice, however, that equation (A12) holds in equality, rather than weak inequality, because of the implicit assumption that all available funds must be allocated. Therefore, the investor must plan the investment strategy so that, after considering the price impact of the trades, all the funds are allocated. To simplify the budget constraint, define

$$a_{i,t} \equiv \omega_{i,t-1}\bar{x}_{t-1}\left(1 + R_{i,t}\right), \tag{A14}$$

which is the monetary amount invested in stock i at the end of the previous investment period. The budget constraint translates to

$$\begin{aligned}\bar{x}_t &+ \sum_{i\in I_{1,t}} \left[\frac{1}{b_{i,t}}\left[e^{b_{i,t}[\omega_{i,t}\bar{x}_t - a_{i,t}]} - 1\right] - \left[\omega_{i,t}\bar{x}_t - a_{i,t}\right]\right] \\ &+ \sum_{i\in I_{2,t}} \left[\frac{1}{b_{i,t}}\left[e^{-b_{i,t}a_{i,t}} - 1\right] + a_{i,t}\right] = x_t.\end{aligned} \tag{A15}$$

Equation (A15) partitions the summation on the right-hand side so that the assets liquidated due to change in the set of feasible assets are separated from the rest of the assets. This is done because the summation associated with forced liquidation acts as a constant term. Notice that to obtain reasonable values for $\bar{x}_t$, the restriction $0 \le \bar{x}_t < x_t$ must be imposed. The constraint implies that (a) price impact costs are positive ($\bar{x}_t < x_t$) and (b) price impact costs do not exceed the amount traded ($0 \le \bar{x}_t$). However, since the total price impact is always positive, for any amount of a nonzero trade, the restriction $\bar{x}_t < x_t$ holds by construction. Thus, only $\bar{x}_t \ge 0$ need be imposed.

Given $\bar{x}_t$ from (A15), and expected returns $E_t[r_{i,t+1}]$, we use equation (A9) to find $E_t[x_{t+1}]$. Finally, the net expected return to a trading strategy, after price impacts, is found by definition

$$E_t\left[r_{p,t+1}\right] = \frac{E_t\left[x_{t+1}\right]}{x_t} - 1. \tag{A16}$$

For an illustration of the time-series process of the portfolio value see Figure 2.

For the GH specification, we state only the final results. The complete derivation of the trading model for linear price-impact costs is provided in Sadka (2002). The trading costs due to the variable cost λ^{GH} may be described by $f(p,q) = \lambda^{\mathrm{GH}} q$, and the fixed costs as $f(p,q) = \Psi p$. Thus, by redefining $b_{i,t} \equiv \lambda^{\mathrm{GH}}_{i,t}/p^2_{i,t}$, and defining $\bar{\Psi}_{i,t} = \Psi_{i,t}/p_{i,t}$, equation (A15) translates to

$$\begin{aligned} \bar{x}_t &+ \frac{1}{2}\sum_{i\in I_{1,t}} b_{i,t}\left[\omega_{i,t}\bar{x}_t - a_{i,t}\right]^2 + \sum_{i\in I_{1,t}} \bar{\Psi}_{i,t}\left|\omega_{i,t}\bar{x}_t - a_{i,t}\right| \\ &+ \frac{1}{2}\sum_{i\in I_{2,t}} b_{i,t}a^2_{i,t} + \sum_{i\in I_{2,t}} \bar{\Psi}_{i,t}a_{i,t} = x_t, \end{aligned} \tag{A17}$$

and the expected return to a trading strategy is again calculated by equation (A16).

Similar to the fixed costs in the GH model, proportional trading costs may be expressed as $f(p,q) = kp$, where k is a constant proportional cost (in our study, k^E and k^Q are the effective and quoted spreads, respectively). Under these assumptions, equation (A15) translates to

$$\bar{x}_t + \sum_{i\in I_{1,t}} k^E_{i,t}\left|\omega_{i,t}\bar{x}_t - a_{i,t}\right| + \sum_{i\in I_{2,t}} k^E_{i,t}a_{i,t} = x_t. \tag{A18}$$

Notice that the formulation in equation (A18) is effectively independent of the initial amount of investment; this can be proven through recursive induction.

Appendix B: Liquidity-Conscious Portfolios

In the framework developed above, an investment strategy at any given time t is entirely defined by the assets' weights and the actual investment amount. Therefore, the static problem of finding the strategy with the highest expected return every period, with the BHK specification of the price impact function, is[11,12]

$$\max_{\omega_t} \sum_{i\in I_{1,t}} \omega_{i,t}\bar{x}_t\left(1 + E_t\left[R_{i,t+1}\right]\right) \tag{B1}$$

[11] We focus only on the maximization of expected returns, without considering any control for second moments.

[12] Treating the optimization as a static, one-period problem does not take into account the multiperiod nature of momentum trading strategies and the consequent possibility of minimizing trading costs through a buy-and-hold policy.

$$\text{s.t.} \quad \bar{x}_t + \sum_{i \in I_{1,t}} \left[\frac{1}{b_{i,t}} \left[e^{b_{i,t}[\omega_{i,t}\bar{x}_t - a_{i,t}]} - 1 \right] - \left[\omega_{i,t}\bar{x}_t - a_{i,t} \right] \right]$$

$$+ \sum_{i \in I_{2,t}} \left[\frac{1}{b_{i,t}} \left[e^{-b_{i,t}a_{i,t}} - 1 \right] + a_{i,t} \right] = x_t \tag{B2}$$

$$\sum_{i \in I_{1,t}} \omega_{i,t} = 1 \tag{B3}$$

$$\omega_{i,t} \geq 0 \tag{B4}$$

$$\bar{x}_t \geq 0. \tag{B5}$$

To simplify the formulation of the problem, denote the following contemporaneous auxiliary variable:

$$A_t \equiv \sum_{i \in I_{1,t}} \frac{1}{b_{i,t}} + \sum_{i \in I_{2,t}} \frac{1}{b_{i,t}} \left[1 - e^{-b_{i,t}a_{i,t}} \right]. \tag{B6}$$

The budget constraint (B2) translates to

$$\sum_{i \in I_{1,t}} \frac{1}{b_{i,t}} e^{b_{i,t}[\omega_{i,t}\bar{x}_t - a_{i,t}]} = A_t, \tag{B7}$$

where $a_{i,t} = 0$ if asset i has not been included in the investment portfolio last period. Furthermore, to reduce dimensionality, it is preferable to use levels of investment rather than relative portfolio weights. For this reason, define the monetary amount $y_{i,t}$ invested in stock $i \in I_{1,t}$ as

$$y_{i,t} \equiv \omega_{i,t}\bar{x}_t. \tag{B8}$$

Notice that this definition implies that

$$\bar{x}_t = \sum_{i \in I_{1,t}} y_{i,t}. \tag{B9}$$

So far no upper bound to investment has been imposed. However, in general, such constraints may be required. Therefore, we add an upper bound, $d_{i,t}$, to the investment allowed in each asset i. In most cases, the lower bound on an investment in asset i is set to zero; however, we solve the problem for the general case where the lower bound is set to $c_{i,t}$. Suppressing the time index t, the static optimization problem translates to

$$\max_{y} \sum_{i \in I_1} y_i \left(1 + E\left[R_i \right] \right) \tag{B10}$$

$$\text{s.t.} \qquad \sum_{i \in I_1} \frac{1}{b_i} e^{b_i(y_i - a_i)} \le A \tag{B11}$$

$$c_i \le y_i \le d_i. \tag{B12}$$

Notice that the budget constraint has been changed to a weak inequality in order to formulate a convex optimization problem. Nevertheless, at the optimum, the budget constraint is binding.

The optimal solution is characterized in Theorem 1, a more general version of which is proven in Appendix C. (For a version of Theorem 1 for the GH price impact function, see Sadka (2002)).

THEOREM 1: *There exists a unique solution to the optimization problem above. Ignoring the upper and lower bounds, the optimal trading strategy is characterized by*

$$\forall i \in I_1 \qquad y_i^* = \frac{1}{b_i} \ln\left[\frac{\left(1 + E\left[R_i\right]\right) A}{\sum_{i \in I_1} \frac{1 + E[R_i]}{b_i}}\right] + a_i.$$

If our initial endowment is x_0, none of which is invested ($a_{i,0} = 0$), the optimal strategy at $t = 0$ is obtained by implementing the following specifications:

$$\begin{aligned} A_t &= x_0 + \sum_{i \in I_1} \frac{1}{b_{i,0}}, \\ a_{i,0} &= 0 \quad \forall i \in I_1. \end{aligned} \tag{B13}$$

To simplify the application of the liquidity-tilted portfolio rule of Theorem 1, we add the simplifying assumption that all assets in the trading strategy (all firms in the top past winners' decile in the empirical work below) have the same expected return.

COROLLARY 1: *Assume that all assets in the restricted set of assets chosen by the trading strategy have the same expected returns, and there are no upper bounds to investment. Then, the optimal weights at time $t = 0$ are given by*

$$\omega_i = \frac{\frac{1}{b_i}}{\sum_{i \in I_1} \frac{1}{b_i}} \qquad \forall i \in I_1. \tag{B14}$$

Adding the assumption that all assets have identical price-impact coefficients, $\lambda_i = \lambda$, yields market values as the optimal weights, since $\frac{1}{b_i} = \frac{MVE_i}{\lambda}$.

The proof of Corollary 1 is given in Appendix C. Corollary 1 shows that market values are optimal portfolio weights, under the assumption that price impacts and expected returns are equal across firms included in the trading strategy. In our empirical work below, we assume that all stocks in the winners' decile have the same expected return. However, we allow the price-impact coefficients to differ across firms.

Appendix C: Proofs

THEOREM 1: (general version): *Define the sets*

$$
\begin{aligned}
Z^c &= \left\{ \left\{ z_i^c \right\}_{i \in I_1} : z_i^c \equiv \frac{1 + E\left[R_i\right]}{e^{b_i (c_i - a_i)}} \right\}, \\
Z^d &= \left\{ \left\{ z_i^d \right\}_{i \in I_1} : z_i^d \equiv \frac{1 + E\left[R_i\right]}{e^{b_i (d_i - a_i)}} \right\}, \\
Z &= Z^c \cup Z^d.
\end{aligned}
\tag{C1}
$$

Rank all assets in $Z = \{z_{(1)}, z_{(2)}, \ldots\}$ *and for each index define the following sets of indexes:*

$$
\begin{aligned}
I_{(z)} &= \left\{ i : i \in I_1, z_i^c \geq Z_{(z)} > z_i^d \right\}, \\
J_{(z)} &= \left\{ j : j \in I_1, Z_{(z)} > z_j^c \right\}, \\
K_{(z)} &= \left\{ k : k \in I_1, z_k^d \geq Z_{(z)} \right\}.
\end{aligned}
\tag{C2}
$$

There exists a unique solution to the static optimization problem above. The optimal trading strategy is characterized by

$$
\begin{aligned}
\forall i \in I_{(z^*)} \quad & y_i^* = \frac{1}{b_i} \ln \left[\frac{1 + E\left[R_i\right]}{\lambda_{(z^*)}} \right] + a_i, \\
\forall j \in J_{(z^*)} \quad & y_j^* = c_j, \\
\forall k \in K_{(z^*)} \quad & y_k^* = d_k,
\end{aligned}
\tag{C3}
$$

where z^* *and* λ *satisfy*

$$
Z_{(z^*)} \geq \lambda_{(z^*)} > Z_{(z^* - 1)},
\tag{C4}
$$

$$
\lambda_{(z^*)} = \frac{\sum_{i \in I_{(z^*)}} \frac{1 + E[R_i]}{b_i}}{A - \sum_{j \in J_{(z^*)}} \frac{1}{b_j} e^{b_j (c_j - a_j)} - \sum_{k \in K_{(z^*)}} \frac{1}{b_k} e^{b_k (d_k - a_k)}}.
\tag{C5}
$$

Proof: The Lagrange formulation of the maximization problem is given by

$$
\begin{aligned}
\mathcal{L} = &\sum_{i \in} y_i \left(1 + E\left[R_i\right]\right) + \lambda \left(A - \sum_{i \in I_1} \frac{1}{b_i} e^{b_i (y_i - a_i)} \right) \\
&- \sum_{i \in I_1} \mu_i \left(y_i - c_i\right) - \sum_{i \in I_1} \gamma_i \left(d_i - y_i\right).
\end{aligned}
\tag{C6}
$$

The first-order conditions, along with the complementary slackness conditions, are given by

$$\frac{\partial \mathcal{L}}{\partial y_i} = 1 + E\left[R_i\right] - \lambda e^{b_i(y_i - a_i)} - \mu_i + \gamma_i = 0, \tag{C7}$$

$$\lambda\left(\sum_{i\in I_1}\frac{1}{b_i}e^{b_i(y_i-a_i)} - A\right) = 0 \qquad \lambda \geq 0, \tag{C8}$$

$$\mu_i\left(y_i - c_i\right) = 0 \qquad \mu_i \leq 0, \tag{C9}$$

$$\gamma_i\left(d_i - y_i\right) = 0 \qquad \gamma_i \leq 0. \tag{C10}$$

In general, the solution requires the division of the set of assets, I_1, into three mutually disjoint sets I, J, and K (some of which may be empty) as follows:

$$I = \left\{i : \mu_i = 0 \wedge \gamma_i = 0 \qquad (c_i \leq y_i \leq d_i)\right\}, \tag{C11}$$

$$J = \left\{j : \mu_j < 0 \wedge \gamma_j = 0 \qquad (y_j = c_j)\right\},$$

$$K = \left\{k : \mu_k = 0 \wedge \gamma_k < 0 \qquad (y_k = d_k)\right\}. \tag{C12}$$

The first-order condition (C7) implies that

$$\lambda e^{b_i(y_i-a_i)} = 1 + E\left[R_i\right] - \mu_i + \gamma_i. \tag{C13}$$

Applying equation (C13) to each of the sets above yields

$$\forall i \in I \qquad 1 + E\left[R_i\right] - \lambda e^{b_i(y_i-a_i)} = 0 \implies \lambda = \frac{1+E\left[R_i\right]}{e^{b_i(y_i-a_i)}}, \tag{C14}$$

$$\forall j \in J \qquad \mu_j = 1 + E\left[R_j\right] - \lambda e^{b_j(c_j-a_j)} < 0 \implies \lambda > \frac{1+E\left[R_j\right]}{e^{b_j(c_j-a_j)}}, \tag{C15}$$

$$\forall k \in K \qquad \gamma_k = 1 + E\left[R_k\right] - \lambda e^{b_k(d_k-a_k)} > 0 \implies \frac{1+E\left[R_k\right]}{e^{b_k(d_k-a_k)}} > \lambda. \tag{C16}$$

Also note that the upper bound and lower bound for every $i \in I_1$ satisfies $c_i \leq d_i$ by definition. This implies that

$$\forall i \in I_1 \qquad \frac{1+E\left[R_i\right]}{e^{b_i(c_i-a_i)}} \geq \frac{1+E\left[R_i\right]}{e^{b_i(d_i-a_i)}}. \tag{C17}$$

Notice that the theorem includes definitions of the sets I, J, and K using the index z^* (see equations (c2) and (c3)). One may verify that at optimum, the definitions of these sets ((c11) and (c12)) must coincide with their corresponding definitions given in the proposition. The set I contains the assets that are traded as an interior solution; the set J corresponds to the assets traded at their lower bound; and the set K corresponds to the assets traded at their upper bound.

To solve for λ, multiply the budget constraint (equation (8) with equality) by λ, and plug it in the left-hand side of the first-order condition (equation (C13)). This procedure results in

$$\lambda = \frac{\sum_{i\in I_1} \frac{1+E[R_i]-\mu_i+\gamma_i}{b_i}}{A}. \tag{C18}$$

Using the above expressions for μ_j and γ_k (equation (8)), we obtain the following equation:

$$\lambda = \frac{\sum_{i\in I} \frac{1+E[R_i]}{b_i} - \lambda \sum_{j\in J} \frac{1}{b_j} e^{b_j(c_j-a_j)} - \lambda \sum_{k\in K} \frac{1}{b_k} e^{b_k(d_k-a_k)}}{A}. \tag{C19}$$

Solving for λ from equation (C19) yields the following expression:

$$\lambda = \frac{\sum_{i\in I} \frac{1+E[R_i]}{b_i}}{A - \sum_{j\in J} \frac{1}{b_j} e^{b_j(c_j-a_j)} - \sum_{k\in K} \frac{1}{b_k} e^{b_k(d_k-a_k)}}. \tag{C20}$$

Finally, the second-order conditions for maximum must be satisfied:

$$\frac{\partial^2 \mathcal{L}}{\partial y_i^2} = -\lambda b_i e^{b_i(y_i-a_i)}. \tag{C21}$$

By construction, $\lambda > 0$. Thus, we conclude that $\partial^2\mathcal{L}/\partial y_{i,t}^2 \leq 0$, and therefore the necessary conditions for optimality are satisfied. The optimization problem is of a convex nature and thus the solution found above satisfies necessary and sufficient conditions of optimality.

The interpretation of Theorem 1 follows basic economic principles. The value z_i^c is the ratio of marginal return and marginal cost for the first dollar, above the lower bound c_i, invested in asset i. Similarly, z_i^d is the ratio of marginal return and marginal cost for the last dollar, below the upper bound, invested in asset i. The ratio of marginal return and marginal cost may be viewed as the marginal net return. Due to increasing marginal costs and constant marginal returns, the marginal net return for any asset i decreases between z_i^c and z_i^d. For this reason, the extreme marginal net returns for all assets are sorted in a descending fashion. Then, funds are allocated to the assets according to the latter ordering. The allocation is stopped when the budget constraint is met. This is controlled by the multiplier $\lambda_{(z^*)}$. Q.E.D.

Proof of Corollary 1: Assuming $A_t = x_0$ and $a_{i,0} = 0$ ($\forall i \in I_1$), and omitting the expressions associated with the lower and upper bounds, we have

$$y_i = \frac{1}{b_i} \ln\left[\frac{x_0 + \sum_{i\in I_1}(1/b_i)}{\sum_{i\in I_1}(1/b_i)}\right], \tag{C22}$$

and therefore the weights are calculated as

$$\omega_i = \frac{y_i}{\sum_{i \in I_1} y_i} = \frac{1/b_i}{\sum_{i \in I_1} (1/b_i)}. \tag{C23}$$

Since $b_i = \lambda_i / MVE_i$, assuming that all price impact coefficients are equal, produces

$$\omega_i = \frac{MVE_i}{\sum_{i \in I_1} MVE_i}. \tag{C24}$$

Q.E.D.

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