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Rumors

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ABSTRACT

A Kyle (1985) model with private information diffusion is used to examine the motivation to spread stock tips. An informed investor with limited investment capacity spreads imprecise rumors to an audience of followers. Followers trade on the advice and move the price. Due to the imprecision of the rumor, the price overshoots with positive probability. This gives the rumormonger the opportunity to trade twice: First when she receives information, then when she knows the price to be overshooting. In equilibrium, rumors are informative and both rumormongers and followers increase their profits at the expense of uninformed liquidity traders.

IT IS DIFFICULT TO DISPUTE that rumors have an important impact on stock prices. Phrases like "Stock X soared amidst rumors of … " can be read or heard almost daily in the popular media. Yet an economic theory about the phenomenon is conspicuously absent. This article argues that the sources of rumors are small informed investors who manipulate prices to increase their information-based profits. Rumormongers can be skillful amateur analysts, investors with access to serendipitous information such as suppliers or clients, or individuals with access to inside information. They have in common that their trading capacity is too small to fully exploit their information by trading in the stock market. A rational expectations model shows that informed investors can increase their profits by *giving* informative but imprecise trading advice to a chain of followers to manipulate market prices.

In recent years, the Internet has proven a productive incubator of rumors. Investors exchange information in chatrooms, newsgroups, and message boards. Rumors can also spread through word of mouth or newsletters. This article considers those privately communicated messages *that contain information* and hence induce rational profit-maximizing investors to trade. The presented analysis

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abstracts from adverse selection problems between followers and rumormongers and assumes that only traders with access to valuable information send rumors. The problem of identifying credible information sources is left for future research.

The paper's analysis uses a special case of a Crawford and Sobel (1982) signaling game in which an informed trader occasionally receives private information about a security's true value. Upon receiving the information, she trades in a Kyle (1985) auction that represents a short period of trading activity. Because her trade is swamped by the total trading volume, her price impact is negligible so that she has information left after trading. To enhance her profits, the informed trader spreads vague yet informative rumors to followers who, through their collective trading, move the price. By spreading a binary (either a bullish or a bearish) rumor whenever she knows a security's future value, the rumormonger can steer the price to one of two rumor prices. Because the liquidation value follows a continuous distribution, the rumor drives the market price beyond the true value with positive probability. This potential overshooting gives the rumormonger the opportunity to benefit from her information twice: First she trades in the direction of her information, then she trades in the opposite direction when she knows the rumor to be overshooting. In the presented model, the rumormonger increases her expected profits from being informed by more than 50 percent.

The suggested rumor strategy is different from the fraudulent practice of scalping, a practice where an uninformed manipulator buys a thinly traded stock and then convinces others, often using false arguments and fake evidence, to buy the stock while unloading his position at a profit.[1] The equilibrium presented in this paper is based on rational expectations and is played within the law. To meet regulations, rumormongers will send buy and sell rumors by privately telling followers either "buy, I bought" or "sell, I sold".

The strategies of followers and rumormongers are derived endogenously from their profit-maximizing behavior. To find the optimal rumor mongering strategy, I first consider the simplified one-shot game where rumormongers are restricted to send honest buy and sell advice only. I then let informed traders spread rumors when they have no information (they bluff), and finally I give them the freedom to spread false rumors (they cheat). I find that the opportunity to cheat constitutes a moral hazard problem for the protagonist. Opportunistic rumormongers would like their followers to believe that they only send imprecise but honest rumors, but cannot credibly commit to this behavior, because in a one-shot game, cheating is more profitable.

This finding contrasts with Benabou and Laroque (1992), in whose model journalists or gurus who receive noisy signals about a stock's future value occasionally cheat by revealing bad news in case the information is good to buy the stock at a low price. Opportunistic manipulators get away with this deceitful behavior

[1] This practice, also known as "pump and dump," becomes illegal when the sender of information fails to disclose his interest in the stock or communicates untrue information.

because the public cannot distinguish between fraudulent manipulation and honest mistakes.

Our bluffing variant of the rumor game is akin to the strategy suggested by Fishman and Hagerty (1995) for corporate insiders who are required to disclose their trades. Mandatory trade disclosure decreases insiders' profits due to an increased price impact, but, at the same time, gives them the opportunity to bluff that they are trading on information when, in fact, they are not. The authors show that the insider's gains from occasionally bluffing can be greater than the disclosure-induced decrease in profits from information-based trading. John and Narayanan (1997) present an extension of this model and show that if good events and bad events are asymmetric, insiders subject to trade disclosure also manipulate when informed. Huddart, Hughes, and Levine (2000) study the problem of the corporate insider in a Kyle (1985) setting and find that insiders add noise in the form of random trades so as to hide their intentions. Similarly, Allen and Gale (1992) show that an uninformed manipulator can mimic an informed trader and occasionally fool the public into believing that there is good news about a stock. In Brunnermeier (2000), informed traders add trade-based noise when they know that there are other informed players in the market. Trading aggressively on their imprecise information gives them the opportunity to unravel the information of the other informed traders from the past order flow so that they maintain an informational advantage even when their own information is revealed.

Like the above-mentioned studies, this paper hinges on the fact that noise is the informed trader's friend: When regulations or competing informed traders reduce uncertainty, insiders attempt to confuse the market with trades and announcements. In this paper, informed traders add noise to the system to overcome their wealth constraint. Most research in market microstructure postulates informed agents of unlimited wealth who face the problem that their trades have a price impact. Kyle (1985) suggested that informed traders ration their trades so as to minimize the price impact. However, given today's large daily trading volumes, few non-floor traders will be concerned with the price elasticity of demand. In this paper, the informed traders are relatively small and cannot fully exploit their informational advantage by playing the Kyle game. To overcome their wealth constraint, they play a rumor game.

Alternative vehicles through which informed investors can monetize excess information are studied by Admati and Pfleiderer (1986, 1990). In their 1986 paper, the authors suggest that informed investors *sell* information through newsletters with personalized noisy signals.[2] Admati and Pfleiderer (1990) suggest that informed traders start a mutual fund to maximize their information-based profits. They analyze the fund managers' problem and suggest they charge a two-tier management fee. Whereas the newsletter and mutual fund strategies provide the informed trader with more profits than the rumor strategy, the barriers of

[2] Selling news comes with several complicating externalities. Buyers can resell purchased information, moral hazard problems between agents exist, and information leaks into the market price due to trading of information buyers. Admati and Pfleiderer (1986) focus on the information leakage problem: Information buyers reveal information by trading on it.

entry of the money management industry are too high for the investors considered in this paper.

The paper is organized as follows. Section I describes the model and the players' equilibrium strategies without rumor mongering. In Section II, the model is adjusted to accommodate private information diffusion. In the adjusted model, three rumor strategies—"honest," "bluffing," and "cheating"—are analyzed. In Section III, the message space is made endogenous. This is important because unlike a limited wealth, a restricted message space is unrealistic. With unrestricted communication, informed rumormongers will use messages in the form of {"stock goes up a lot or down a bit," "stock goes down a lot or up a bit"} to maximize their profits. The intuition behind these informative but imprecise messages is that vagueness is the key to the rumormonger's profits. Section IV explores how repeated interaction and security regulations affect the suggested equilibrium. A discussion of how the model can explain puzzling but pervasive empirical observations is contained in Section V. Section VI summarizes and concludes. Proofs are in the Appendix.

I. The Model

In a Kyle (1985) auctions market, one informed trader with limited wealth and a continuum of noise traders submit orders to competitive market makers who execute the orders at a price that earns them zero expected profit. The informed trader knows the asset's liquidation value, $\tilde{v}$, which is a random variable from a uniform probability distribution between -2 and 2: $\tilde{v} \sim \mathrm{U}[-2, 2]$.[3]

Before $\tilde{v}$ is exogenously revealed, N sequential auctions take place. At auction n, the noise traders' aggregate demand, $\tilde{u}_n$, follows the Standard Normal probability distribution. Noise demand is independent over the N auctions and is uncorrelated with $\tilde{v}$: $\tilde{\boldsymbol{u}} \sim \mathrm{N}(\mathbf{0}, I)$, $\mathrm{cov}(\tilde{\boldsymbol{u}}, \tilde{v}) = 0$. The noise demand $\tilde{\boldsymbol{u}}$ models the liquidity needs of uninformed market participants and does not depend on the presence of informed traders or on the equilibrium played. As in other models in this literature, the noise demand is the source of the informed traders' profits.

One risk-neutral informed trader, the leader, L, sees $\tilde{v}$ before the first auction. She has a trading capacity of x_L, which is very small relative to the standard deviation of the noise demand: $0 < \mathrm{x}_L \ll 1$. L conditions her trade in auction n, $\tilde{x}_n$, on $\tilde{v}$ and on the demand history to date, $H_{n-1} \equiv \{\tilde{y}_i, i = 1, \ldots, n-1\}$ where $\tilde{y}_i \equiv \tilde{x}_i + \tilde{u}_i$. Or, at auction n, L's trading rule is $\tilde{x}_n = \mathrm{X}_n(\tilde{v}, H_{n-1})$. L's strategy is a series of trading rules $\mathrm{X} = \langle \mathrm{X}_1, \ldots, \mathrm{X}_N \rangle$.

At auction n, competitive market makers, M, set the auction price, $\tilde{p}_n$, equal to the expected $\tilde{v}$ given the demand history H_n: $\tilde{p}_n = \mathrm{P}_n(H_n)$. The strategy of M is a

[3] Using this Uniform distribution instead of a Normal distribution simplifies the exposition. Any distribution can be used to demonstrate the intuition of this paper. For tractability, $\tilde{v}$ is standardized around zero. Using a liquidation value distributed U[\$9, \$102] provides the same insights.

series of pricing rules $\mathrm{P} = \langle \mathrm{P}_1, \ldots, \mathrm{P}_N \rangle$. The discount rate between auctions is zero.[4]

Kyle (1985) showed that an informed investor with unlimited wealth maximizes her profits by rationing her trades. The important difference with Kyle's analysis is that L has very small trading capacity relative to the standard deviation of the noise demand. Solving the equilibrium (X, P) with limited wealth is an interesting challenge. One can however conjecture that a solution will still tell L to ration her trade over the N auctions so as to minimize information leakage. To keep the analysis tractable, I avoid a rationing equilibrium by assuming a small round trip transaction cost $c \ll \mathrm{x}_L$ that makes rationing unprofitable for L. Hence, in the first auction, L buys x_L units whenever she sees $\tilde{v} > c$ and sells x_L when she sees $\tilde{v} < -c$. The following proposition characterizes the equilibrium price process.

PROPOSITION 1: *In a trading game described above,*

(i) *the price process is given by*

$$\tilde{p}_1 = \mathrm{P}_1(H_1) = \frac{\frac{2+c}{2}\varphi(y_1 - \mathrm{x}_L) - \frac{2+c}{2}\varphi(y_1 + \mathrm{x}_L)}{\varphi(y_1 - \mathrm{x}_L) + \varphi(y_1 + \mathrm{x}_L) + \frac{2c}{2-c}\varphi(y_1)} \approx 0,$$

where φ is the pdf of the Standard Normal, and $\tilde{p}_n = \mathrm{P}_n(H_n) = \tilde{p}_1 \forall n \geq 2$;

(ii) *L's expected gross profit, $E[\Pi_L]_{silent} \approx \mathrm{x}_L$.*

The pricing rule $\mathrm{P}_1(H_1) = P_1(\tilde{y}_1)$ follows from competitive, Bayesian-updating market makers. Upon observing the total demand $\tilde{y}_1$, M infer that L either sold x_L, did not trade, or bought x_L. M multiply the likelihoods of these events with the expected values of $\tilde{v}$ conditional on the events. Only at the first auction does the price depend on the demand. At the following auctions, the price is perfectly inelastic because M know that the demand carries no information. Since the informed demand x_L is very small compared to the standard deviation of the noise demand, L's price impact from the assumed strategy is very small so that her *ex ante* expected profit approaches x_L. The Appendix contains a proof that, for small x_L, the transaction cost c that is necessary to keep L from rationing is small compared to x_L. The proof is based on the fact that the *additional* profit from rationing (and hence minimum c that keeps L from rationing) is, at most, the expected price impact from trading x_L, and that the expected price impact goes to zero faster than x_L.

II. Rumors

The above analysis shows that a small informed trader has a negligible price impact and therefore has information left after trading. This section examines

[4] The protagonist of the model is female, followers (introduced in Section II) male, and market makers are referred to with plural pronouns.

how L can increase her profits by spreading rumors. The basic idea is that L manipulates the price by sending imprecise rumors to an audience of followers who trade on them. If the price happens to overshoot, L will reverse her position before exogenous revelation. To model the proliferating rumor, profit-maximizing followers are added to the game. To accommodate the bluffing strategy, L is assumed to receive a perfect signal only occasionally.

ASSUMPTION: *L, whose maximum market position is x_L units long or short, sees $\tilde{v}$ with probability q before the first auction. In the first auction, L trades conditional on her signal: $\tilde{x}_1 = \mathrm{X}_1(\tilde{v}) \in \{\mathrm{x}_L, -\mathrm{x}_L\}$. After each auction, L gives advice, $A \in \{$"buy," "sell," "…"$\}$ to m followers, F. When receiving advice, each follower trades x_F units in the recommended position. After each auction, all informed followers pass the advice to m uninformed followers. At the final auction, L trades $\mathrm{X}_N(\tilde{v}) \in [-2\mathrm{x}_L, 0]$ whenever $\tilde{x}_1 = \mathrm{x}_L$, $\mathrm{X}_N(\tilde{v}) \in [0, 2\mathrm{x}_L]$ whenever $\tilde{x}_1 = -\mathrm{x}_L$, and $\mathrm{X}_N(\tilde{v}) \in [-\mathrm{x}_L, \mathrm{x}_L]$ whenever $\tilde{x}_1 = 0$.*[5]

DEFINITION: *$x(n)$ is the absolute value of the potential informed demand at the nth auction: $\mathrm{x}(1) \equiv \mathrm{x}_L$, $\mathrm{x}(n) \equiv m^{(n-1)}\mathrm{x}_F \forall n > 1$. Or, the total informed demand at the nth auction is $\tilde{x}_n = \mathrm{x}(n)$ if $A =$*"buy," *$\tilde{x}_n = -\mathrm{x}(n)$* if *$A =$*"sell," and *$\tilde{x}_n = 0$ if $A =$*"…".

The role of the followers is to reveal the rumor gradually to the market makers. The proliferation of the rumor is modeled as an exponential diffusion process. Through the followers' demand, M will eventually assess the nature of the rumor. The trading capacity of a follower, x_F, is in the same size-order as x_L. Like L, followers do not ration their trades: The presence of competing informed followers gives followers the incentive to trade aggressively immediately after receiving the rumor.[6] Although followers' behavior is assumed to be exogenous, it will become clear that followers expect to profit from trading on rumors. The message space {"buy","sell","…"} is, for the time being, exogenous. In Section III, I look for the message space that maximizes the rumormonger's profit. In the final auction, when the rumor is fully incorporated in the price, L can "flip" her position by trading $2\mathrm{x}_L$ or $-2\mathrm{x}_L$.

L reverses her position whenever her rumor leads the price to overshoot the true value. Because the price can never be larger (smaller) than the rumor price, L will wait to reverse her position until the rumor is fully incorporated in the price and the overshooting (if any) is at its maximum. Without loss of generality, I assume that L's second trade takes place in the final auction.[7]

[5] These constraints follow from the fact that L cannot go more *long* or *short* than x_L. It neglects (for tractability) L's profits on the trade at the first auction.

[6] Admati and Pfleiderer (1988) and Holden and Subrahmanyam (1992) show that competing informed traders trade more aggressively on their information than a single informed trader.

[7] We could give L the opportunity to reverse her position before the final auction (e.g., as soon as the price reaches the rumor price), but this would not change her profits from rumormongering nor the intuition of the paper. Note also that once it is common knowledge that there is a "buy" or "sell" rumor in the market, trades no longer have a price impact. Hence there is no reason for L to ration her flipping trades.

In the following, the optimal strategies of three L types are derived. First, I consider honest informed traders who send a "buy" rumor whenever they know $\tilde{v}>0$, a "sell" rumor whenever they knows $\tilde{v}<0$, and announce "…" when they do not see $\tilde{v}$. I then consider bluffing L types who may spread "buy" or "sell" when they do not see $\tilde{v}$. Finally, I consider opportunistic types who are free to spread *false* rumors: Advising "buy" ("sell") when they know that $\tilde{v}<(>)0$. For the time being, the three types are constrained by their codes of conduct. In this section, we will see that intrinsic honesty pays, and in Section IV it is shown that apart from moral codes, reputation concerns and regulations can motivate opportunistic types to refrain from bluffing or cheating.

A. *Honest Rumors*

This subsection discusses how an honest L can increase her expected profits from spreading rumors. An honest L incurs a large nonpecuniary punishment from spreading false or bluffing rumors. L's type is common knowledge.

The crux of the rumor strategy is that L manipulates the price by steering her followers' demand. Although L's own trade is practically invisible to M, an exponentially increasing followers' demand is not. When the nature of the rumor is —through the aggregate demand—revealed to M, the price stabilizes at the rumor price 1 or -1.[8] A profit maximizing L waits until $\tilde{v}$ is exogenously revealed whenever she sees $|\tilde{v}|>1$, but reverses her position before exogenous revelation when she knows $|\tilde{v}|<1$.

PROPOSITION 2: *In equilibrium of a rumor game with an "honest" L and F,*

(i) *L plays:*

$$\begin{array}{ll} (-x_L, \text{``}sell\text{''}, 0) & \textit{when she sees} \quad -2 \le \tilde{v} < -1 \\ (-x_L, \text{``}sell\text{''}, 2\mathrm{x}_L) & \textit{when she sees} \quad -1 \le \tilde{v} < 0 \\ (x_L, \text{``}buy\text{''}, -2\mathrm{x}_L) & \textit{when she sees} \quad 0 \le \tilde{v} < 1 \\ (x_L, \text{``}buy\text{''}, 0) & \textit{when she sees} \quad 1 \le \tilde{v} \le 2 \\ (0, \text{``}\ldots\text{''}, 0) & \textit{when she does not see } \tilde{v} \end{array}$$

and M play

$$\mathrm{P}_n(H_n) = E[\tilde{v}|H_n] = \frac{1 - e^{-2\Sigma_n y_i \mathrm{x}(i)}}{2\,\frac{1-q}{q}\, e^{\Sigma_n \frac{1}{2}\mathrm{x}(i)^2 - y_i \mathrm{x}(i)} + 1 + e^{-2\Sigma_n y_i \mathrm{x}(i)}};$$

(ii) $\dfrac{\partial E[\tilde{p}_n|\text{``}buy\text{''}]}{\partial n} > 0, \dfrac{\partial E[\tilde{p}_n|\text{``}sell\text{''}]}{\partial n} < 0, \dfrac{\partial E[\tilde{p}_n| = \text{``''}]}{\partial n} > 0;$

(iii) $\lim_{N\to\infty} E[\tilde{p}_N|\text{``}buy\text{''}] = 1 \quad \lim_{N\to\infty} E[\tilde{p}_N|\text{``}sell\text{''}] = -1 \quad \lim_{N\to\infty} E[\tilde{p}_N|\text{``''}] = 0;$

(iv) $\lim_{N\to\infty} \mathrm{Var}[\tilde{p}_N] = 0;$

[8] Alternatively, the price also reaches 1 or -1 when the rumor reaches one of the market makers. To keep the model tractable, this variant is ruled out.

(v) $$E[\Pi_L]_{honest} = \lim_{\substack{x_L \to 0 \\ N \to \infty}} \frac{1}{2} q\mathrm{x}_L \left(\int_0^1 (E[\tilde{p}_N|\mathrm{x}_L] - E[\tilde{p}_1|\mathrm{x}_L] + (E[\tilde{p}_N|\mathrm{x}_L] - v))\mathrm{d}v + \int_1^2 (v - E[\tilde{p}_1|\mathrm{x}_L])\mathrm{d}v \right)$$
$$\approx \frac{3}{2} q\mathrm{x}_L \approx \frac{3}{2} E[\Pi_L]_{silent}.$$

Proposition 2(i) gives the equilibrium strategies of L and M if the informed trader is known to be of the honest type. In contrast to the equilibrium of the silent game L always trades when she sees v because the trading payoff is greater than the (small) transaction cost c for all $v \in [-2, 2]$. Propositions 2(ii) and 2(iii) state that if L spreads a rumor, the *expected* price increases monotonically to 1 or -1. In the absence of a rumor, the expected price will be zero. Proposition 2(iv) implies that convergence to 1, -1, or 0 is in expectation and in probability. Proposition 2(v) states that the expected gross profit of an honest L, $\mathrm{E}[\Pi_L]_{honest}$, is 50 percent greater than the expected profit of a silent L.

The equilibrium is of the perfect Bayesian kind: All players' strategies are profit maximizing given information and beliefs, and beliefs are correct.[9] The intuition behind the optimal (nonlinear) price function is that at every auction, M use the aggregate demand y_n to update their beliefs about the nature of the demand. Notice that a sufficient statistic for the conditional expectation of $\tilde{v}$ is $\Sigma_n y_i \mathrm{x}(i)$, which means that the significance of each observed demand y_i is weighted by the magnitude of the total informed demand that might be present in the ith auction.[10]

The first integral in Proposition 2(v) denotes L's profits from flipping in case L sees $|\tilde{v}| < 1$. In this case, she makes a profit of x_L from liquidating a long (short) position before revelation, plus $(1 - v)\mathrm{x}_L$ from liquidating a short (long) position upon revelation. The second integral denotes L's profits if she sees $|\tilde{v}| > 1$. Since the expected gross profit of a small L who does not spread rumors is approximately $q\mathrm{E}[\Pi_L]_{silent} = q\mathrm{x}_L$, an honest L can increase her profits by 50 percent using the rumor strategy.[11]

[9] Note that in equilibrium L does not profit from sending a rumor if she sees $|v| > 1$. However, neglecting to send a rumor when seeing $|v| > 1$ does not support the equilibrium. Hence, if rumormongering bears a small cost, the suggested equilibrium (*in the one-shot game*) breaks down.

[10] Two remarks are in order: (1) When L flips a long (short) position, she can go short (long) more than x_L units because she made a profit on the trade made in the first auction. I implicitly assume that profits are small relative to the investment and ignore the increase in trading capacity to avoid unnecessary complexity. (2) I also ignore the profits that L can make from knowing the absence of a rumor. When no rumor is sent, the price follows an erratic path to zero. In practice, L could exploit this information by buying (selling) when she sees a negative (positive) price. A simulation using $\tilde{v} \sim \mathrm{U}[98, 102]$ shows that the (small) effects of (1) and (2) that increase L's profits from rumormongering are significantly larger than the negative effect of the price impact at the first auction.

[11] L's *net* profit is slightly smaller than $\frac{3}{2}\, q\mathrm{x}_L$ due to the transaction cost c. Note however that L's profit from playing the honest rumor game, $\mathrm{E}[\Pi_L]_{honest}$, is slightly higher than $\mathrm{E}[\Pi_L]_{silent}$, because in the rumor game, she trades always, even for v near zero.

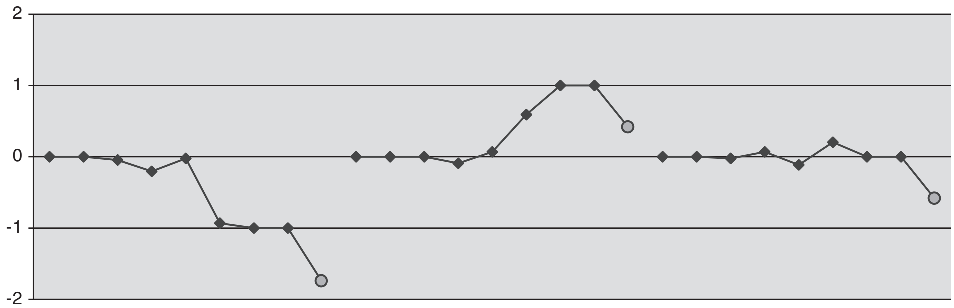

Figure 1. Simulation of the honest equilibrium. The graph displays the price paths of three trading rounds with $x_L = x_F = 0.01$, $m = 2$, and $N = 8$.

Followers too expect to make a profit from the rumor game. Early followers expect a profit of almost $q x_F$, whereas late followers' expected profits go to zero. The fact that following is profitable justifies the assumption that followers follow the leader in both trading and rumormongering.

Although the analysis focuses on informed traders of modest wealth, the rumor strategy can also be used by wealthy informed traders. *All* informed traders of finite wealth can increase their information-based profit using the rumor strategy, albeit by less than 50 percent. A simulation shows that the requirements of a small L and many auctions to attain the 50 percent increase in profit are easily met.

To illustrate the model, I simulate three rounds of eight auctions with $x_L = x_F = 0.01$ and $m = 2$. Figure 1 shows two rounds where L is informed and spreads a rumor. In the first round, L knows that $v = -1.75$. She sells x_L and spreads a "sell" rumor. Her profit is $1.75 x_L$. In the second round, she knows $v = 0.4$. She buys x_L, spreads a "buy" rumor, and, when the price stabilizes at rumor price 1, she flips her position by selling $2x_L$. Her profit in this round is $1.6x_L$, four times the silent profit of $0.4x_L$. In the final round, L does not see v, does not trade, and does not spread a rumor. Still, the price fluctuates before stabilizing at zero because M conjecture that the erratic demand may contain trades of L or F.

B. Bluffing

This subsection considers an L of the bluffing type. This type faces a private nonpecuniary punishment for spreading false rumors, but may bluff a rumor when she does not see $\tilde{v}$. L's type is assumed to be common knowledge.

Also a profit-maximizing bluffing L can use rumors to manipulate the price to her advantage. When informed, L spreads honest rumors and thereby maintains a reputation of being a valuable source of information. When uninformed, L invests her entire endowment in a random market position and subsequently spreads a rumor corresponding to the position taken. Because her followers believe that the rumor may contain information about $\tilde{v}$, they follow L's rumors and bid the price in the, for L, desirable direction. The next proposition characterizes the price process when L is known to be of the bluffing type.

PROPOSITION 3: *In the equilibrium of a rumor game with a bluffing L,*

(i) *L plays honest whenever see sees $\tilde{v}$, and plays* $(\mathrm{x}_L, \text{“}buy\text{”}, -2\mathrm{x}_L)$ or $(-\mathrm{x}_L, \text{“}sell\text{”}, 2\mathrm{x}_L)$ *with equal probabilities when she does not see $\tilde{v}$. M play*

$$\mathrm{P}_n(H_n) = E[\tilde{v}|H_n] = q\frac{1 - e^{-2\Sigma_n y_i \mathrm{x}(i)}}{1 + e^{-2\Sigma_n y_i \mathrm{x}(i)}};$$

(ii) $$\frac{\partial E[\tilde{p}_n|\text{“}buy\text{”}]}{\partial n} = -\frac{\partial E[\tilde{p}_n|\text{“}sell\text{”}]}{\partial n} > 0;$$

$$\lim_{N\to\infty} E[\tilde{p}_N|\text{“}buy\text{”}] = -\lim_{N\to\infty} E[\tilde{p}_N|\text{“}sell\text{”}] = q;\ \lim_{N\to\infty} \mathrm{Var}[\tilde{p}_N] = 0;$$

(iii)

$$E[\Pi_L]_{bluff} \approx q\mathrm{x}_L\left(3 - 2q + \frac{1}{2}q^2\right) > E[\Pi_L]_{honest}\forall q.$$

Proposition 3(i) gives the equilibrium strategies of L and M. The pricing rule of M again follows from Bayesian updating. Its quotient assesses whether the rumor is bearish or bullish. A bullish rumor results in predominantly positive demands $\tilde{y}_i$, letting the quotient converge to one. If the rumor is bearish, the quotient converges to negative one. Because the rumor is bluffing with probability $1-q$, the rumor prices are $-q$ and q. Proposition 3(ii) implies that in the presence of a “buy” (“sell”) rumor, the market price converges to q $(-q)$ in expectation and probability. Proposition 3(iii) shows that the profits of a bluffing L, $E[\Pi_L]_{bluff}$, are higher than those of an honest L.

C. Cheating

In this subsection, an opportunistic L is considered. An opportunistic L, who has no scruples against spreading false or bluffing rumors, can cheat by acquiring a market position opposite to the information she has. When cheating, L sends a false rumor (a rumor that induces a price with the opposite sign of $\tilde{v}$), and flips her position at the final auction before revelation.

The problem for an opportunistic L is that she always has an incentive to cheat on any informative equilibrium. Therefore an opportunistic L cannot credibly transmit information to the public.

PROPOSITION 4: *In the equilibrium of a rumor game with opportunistic L,*

$$\mathrm{P}_1(H_1) = \frac{\varphi(\tilde{y}_1 - \mathrm{x}_L) - \varphi(\tilde{y}_1 + \mathrm{x}_L)}{\varphi(\tilde{y}_1 - \mathrm{x}_L) + \varphi(\tilde{y}_1 + \mathrm{x}_L)},$$

$\tilde{p}_n = \mathrm{P}_n(H_n) = \tilde{p}_1 \forall n \geq 2$*, there is no information transmission after the first auction, and L's profits reduce to* $qE[\Pi_L]_{silent}$

Proof: Through the trades of followers, M can always infer L's rumor from the observed demand. Hence, in equilibrium, the price eventually stabilizes at the expected value of $\tilde{v}$ given advice A *and the beliefs about* L's equilibrium strategy σ_L.[12] Or, $\tilde{p}_N = E[\tilde{v}|A, \sigma_L]$. If in any equilibrium, advice A' of any strategy σ_L' leads to a rumor price $E[\tilde{v}|A', \sigma_L'] > (<) \tilde{p}_1$, the profit-maximizing L will *always* cheat on this equilibrium by spreading A' whenever she sees $\tilde{v} < (>) 0$, and reverse her position at the final auction. But then $E[\tilde{v}|A', \sigma_L']$ cannot be $>$ $(<)$ 0, a contradiction.

Proposition 4 says that due to the availability of the cheating strategy, no rumors will ever be believed and an opportunistic L loses all rumor profits. The cheating strategy thus burdens an opportunistic L with a moral hazard problem. She would like to commit not to cheat, but she cannot credibly promise such behavior.

III. Equilibrium with Endogenous Message Space

In the preceding section, it was shown that informed traders whose trading capacity limits them to fully capitalize on their information advantage spread rumors to enhance their profits. Whereas many traders are subject to a wealth constraint, exogenous restrictions on the rumor messages are less realistic. To analyze the rumor strategy rigorously, it is important to investigate the rumor game with an endogenous message space. As in the signaling model of Crawford and Sobel (1982), the *ex ante* expected profits for the strategic players depends on the messages employed by the sender. Because in the rumor game the message space is chosen by L, only the message space that maximizes L's profits meets the intuitive criterion of Cho and Kreps (1987).

To facilitate the search for the profit maximizing messages, I analyze a game with an honest L who always sees $\tilde{v}$ $(q = 1)$ and invoke the revelation principle (Myerson (1979)) so that I only need to consider messages that are complete sets on $[-2, 2]$. Honesty then implies that L says "(a, b]" if and only if she sees $a < \tilde{v} \leq b$. Given that competitive market makers eventually deduce the rumor from the demand, with each value v a message $A(v)$ is associated as well as a rumor price $\rho(v) = E[\tilde{v}|\tilde{v} \in A(v)$. Within this framework, I can calculate L's expected profits for a given message space. I only need to calculate the rumor price $\rho(v)$ for each v, deduce L's optimal trading tactic given v, and integrate over all possible v's.

The trading tactic for a given v can be of two types: (1) only trade at the first auction, which is optimal if v lies beyond its rumor price, or (2) trade at the first auction and flip before exogenous revelation. The latter tactic is optimal if v is smaller than its positive rumor price or greater than its negative rumor price.

Only the second strategy exploits the rumor's imprecision and generates *extra* profits (vis-à-vis playing the silent game), and the extra profits are increasing in

[12] L could use a mixed strategy. Hence $\sigma(\tilde{v})$ is a probability distribution over the strategy space $\{x_L, -x_L, 0\} \times \{$"buy," "sell," " "$\} \times \{2x_L, -2x_L, 0\}$.

the distance between v and $\rho(v)$. Because the ex ante expected profits are calculated by integrating the v-contingent profits over $[-2, 2]$, it is optimal to have one rumor price at each side of zero. Using the above observations, I look for symmetric message spaces of the following kind: {"$[-\xi, 0] \cup (\xi, 2]$", "$[-2, -\xi) \cup (0, \xi]$"}. The first message leads to rumor price $\rho = \frac{1}{2}(2 - \xi^2)$, the second message leads to $-\rho$. I conjecture that the ξ that maximizes L's profits, ξ^*, is larger than ρ. Then any $\tilde{v} \in (-\xi, \xi)$ calls for "flipping," while any $\tilde{v} \in [-2, -\xi) \cup (\xi, 2]$ calls for the "only trade at the first auction" tactic, so that the expected profit for an honest trader is given by

$$E[\Pi] = \frac{1}{2}\int_0^{\xi} \big((2 - \xi^2) + v\big)\mathrm{d}v + \frac{1}{2}\int_0^{\xi} v\mathrm{d}v = 1 + \xi - \frac{1}{2}\xi^3 \tag{1}$$

The expected profit is maximized at $\xi^* = \sqrt{2/3} = 0.8165$. With this strategy, $\rho = 2/3$, which confirms the conjecture that $\xi^* > \rho$. With $\xi^* = \sqrt{2/3}$, L's expected profit is 1.544, an increase of 54.4 percent with respect to spreading no rumors.

Announcements like "$\tilde{v}$ is higher than 0.8165 or lies between -0.8165 and 0" and "$\tilde{v}$ is lower than -0.8165 or lies between 0 and 0.8165" may not be very practical. Still, from the revelation principle, we know that any set of messages can replace the full information messages. This section therefore shows that in the Pareto optimal and Cho and Kreps (1987) intuitive equilibrium, any bullish rumor (e.g. "the stock is hot," "If I were you I'd buy this stock") means that stock can either go down a bit or up a lot.

IV. Supporting the Informative Equilibrium

In Section II, it was shown that if opportunistic informed traders spread rumors, they are not believed. In this section, I investigate how also in a game with opportunistic types, the honest equilibrium can be supported. First, I direct my analysis to the repeated rumor game. Studying the repeated game is important because informed investors can only earn and maintain a reputation through repeated interaction. Then I discuss how SEC regulations support the informative equilibrium.

A. Repeated Games and Reputation

Clearly, a repetition of the equilibrium derived in Section II is a Nash equilibrium for the repeated variants. In the rumor game with a knowingly opportunistic L, the repeated equilibrium of the stage game predicts that rumors never transmit information. The reason behind this sad equilibrium is the moral hazard problem of the rumormonger: her incentive to send false rumors.

Using a standard reputation model it can be shown however that with a discount factor δ greater than $\frac{8}{q+8}$, the informative honest equilibrium can be sustained, even if L is opportunistic. To see this, consider an opportunistic informed L in the honest equilibrium with "buy" and "sell" rumors. The value of her game is $\mathrm{x}_L(\max(\tilde{v}, 2 - \tilde{v}) + \frac{\delta}{1-\delta}\frac{3}{2}q$. If she cheats she makes a quick $\mathrm{x}_L(2 + \tilde{v})$, but only an expected $q\mathrm{x}_L$ per round thereafter. Hence, to support honesty we need: $\max(\tilde{v}, 2-\tilde{v})+\frac{\delta}{1-\delta}\frac{3}{2}\ q > 2+\tilde{v}+\frac{\delta}{1-\delta}q\ \forall\tilde{v}$, or (after some algebra) $\delta > \frac{4}{q+4}$. Now consider an *un*informed L in the honest equilibrium. The value of her game is $\frac{\delta}{1-\delta}\frac{3}{2}q\mathrm{x}_L$ if she does not spread a rumor (she spreads " ") or $2\mathrm{x}_L+\frac{\delta}{1-\delta}\frac{5}{4}q\mathrm{x}_L$ if she deviates and bluffs.[13] To keep her from bluffing we need: $\frac{\delta}{1-\delta}\frac{3}{2}q > 2+\frac{\delta}{1-\delta}\frac{5}{4}q$ or $\delta > \frac{8}{q+8}$.[14,15] To gauge the importance of this restriction, consider trading rounds of one month and a monthly risk-adjusted discount rate of one percent ($\delta = 0.99$). In this case, L must see $\tilde{v}$ with a probability of at least eight percent every month (about once a year) to support the informative equilibrium.

In the above, reputation is measured by the value of the continuing game. Still, in reality, a stock-pick reputation will vary over time, and its value is likely to be higher than simply the continuation value of the rumor game discussed here. Moreover, sufficient reputation capital may give rumormongers access to more profitable games such as the newsletter- or fund-manager business (Admati and Pfleiderer (1986, 1990)). Access to more profitable information liquidation vehicles provides skilled analysts with even greater incentives to stick to the "honest" rumor game.

B. U.S. Securities Laws

Probably the most powerful deterrent from "cheating" or "bluffing" is provided by U.S. securities laws. According to Section 17(b) of the 1933 Securities Act, it is illegal to give trading advice to other investors without disclosing one's own interest. Section 10(b) of the 1934 Securities Act regulates "the use of manipulative and deceptive devices." According to this section and rule 10b-5 thereunder, it is unlawful "[for any person] to make untrue statements about material facts or to omit to state a material fact necessary in order to make the statements made, in the light of the circumstances under which they were made, not misleading [in connection with the purchase or sale of any security]." In other words, current regulations forbid lying about relevant facts in order to manipulate the price. Hence, the regulations make the cheating and bluffing strategies illegal, but do not prevent the informative equilibrium suggested in this paper.

The regulations concerning fraudulent manipulation are vigorously enforced by the SEC. Because the Internet is particularly susceptible to illegal manipulation practices, former SEC chairman Arthur Levitt created the Office of Internet Enforcement to enforce securities law in cyberspace. The office warns investors against fraud through a variety of programs and employs more than 250 "cybercops" to search the Internet for scams and scalping schemes.

The most common suits filed by the SEC concern "pump and dump" schemes, where offenders spread deceitful messages, sometimes using fabricated press

[13] Bluffing brings L a quick profit of $2\mathrm{x}_L$. With equal probabilities, L bluffs either correctly and can continue the "always honest" equilibrium worth $\frac{\delta}{1-\delta}\frac{3}{2}q\mathrm{x}_L$, or incorrectly, in which case she descends to the bad equilibrium worth $\frac{\delta}{1-\delta}q\mathrm{x}_L$.

[14] If the Pareto optimal game of Section III is played this requirement reduces to $\delta > 7.35/(7.35+q)$.

[15] Notice that a rumormonger in the honest equilibrium is more inclined to bluff than to cheat: The binding restriction derives from the requirement that bluffing is unprofitable.

releases, with the objective to manipulate prices. Another common infliction is deliberately misrepresenting one's identity or track record. The tendency to engage in these practices suggests that equilibria with reputed honest rumormongers and informative rumors exist alongside unscrupulous individuals who try to free ride on the informative equilibria.

V. Empirical Implications

If a rumor game can survive the moral hazard of cheating, rumors are informative. This means that followers, especially early followers, profit from trading on rumors. This prediction is corroborated by several event studies on public and nonpublic announcements.

Whereas recommendation event studies disagree on the price impact of published recommendations, most studies agree that *before* the publication of recommendations, significant price buildups occur. Representative studies are those of Lloyd-Davies and Canes (1978) and Liu, Smith, and Syed (1990), who study the impact of recommendations in *The Wall Street Journal*'s "Heard on the Street" (HOTS) column. The article reports positive abnormal returns during the 10 days prior to publication and significant positive abnormal returns on the event day and the two days preceding to the event. The authors suggest that publication in the HOTS column is the latest step in an information diffusion process. Similarly, Mathur and Waheed (1995) report significant excess returns preceding the publication of recommendations in *Business Week*'s "Inside Wall Street." Pound and Zeckhauser (1990) and Zivney, Bertin, and Torabzadeh (1996) find price buildups of seven percent before publication of takeover rumors in *The Wall Street Journal*. They also find that approximately half the takeover rumors led the price to overshoot. This is nicely consistent with the models presented in this paper.

The rumor theory is also corroborated by event studies on *unpublished* tips. Bjerring, Lakonishok, and Vermaelen (1983) assess the value of the privately communicated advice of a Canadian stockbroker. Their study shows significant gradual abnormal price increases during periods that a stock was a (privately circulated) recommended buy. Dimson and Marsh (1984) *ex post* analyze the usefulness of the advice from 35 U.K. brokers. Although some brokers gave better advice than others, they find listening to the privately communicated advice to be profitable.

Another platform for the diffusion of stock tips is the Internet. Many Internet discussion forums record thousands of messages per day. The most popular Web sites have rules and enforcement mechanisms to support an honest exchange of information that is based on reputation.[16] Several recent studies report evidence on the effect of Internet postings on security prices. Wysocki (1999) studies the predictive power of posting volume on Yahoo! message boards for 50 stocks and

[16] Discussion forums such as siliconinvestor.com, ragingbull.com, and the motley fool (fool.com) restrict posters to a single username and employ webmasters to police the site for illegal activity.

finds that overnight posting volume is a significant predictor of next day trading volume, return variance, and to a lesser extent, abnormal returns. He concludes that "posting activity is not just noise, but associated with real information flows." Similarly, Bagnoli, Beneish and Watts (1999) find that whisper numbers, unofficial earnings forecasts that appear on Internet message boards, predict stock returns. On the other hand, Dewally (2001), Tumarkin and Whitelaw (2001), and Antweiler and Frank (2001) cannot find any predictive power of Internet postings. The latter two papers however find a significant explanatory power of posting activity for next day volume and volatility, while the former two papers report conspicuous abnormal returns *prior* to posting events.[17] The above-mentioned findings combined do not reject semi-strong market efficiency, but do cast doubt on the strong form of the EMH: Whereas econometric analysis of Internet message board postings do not predict stock returns, electronic grapevines may offer valuable information about future stock prices.

VI. Summary and Conclusion

After having made an informed investment in the stock market, investors are inclined to tell friends and acquaintances to follow their actions. This behavior is most common among small investors who try to gain a reputation. This article shows that the propensity of rumormongering in financial markets is more than a symptom of the psychological trait that man is prone to justify and defend his own past actions. A dynamic model with rational profit-maximizing traders shows that spreading rumors makes economic sense, as it increases demand for a security and can drive its price beyond the price that the rumormonger privately knows.

Three rumor strategies are discussed. First, rumormongers spread honest rumors only. A rumor will drive the price to a rumor price, at which informed traders will reverse their position whenever they know the rumor price to be overshooting. Then, informed traders may bluff rumors when they have no information and trade on them. Finally, rumormongers can cheat by spreading false rumors and trade on them. It is shown that this last strategy imposes a moral hazard cost on rumormongers: If followers understand that an informed trader has an incentive to cheat, they will no longer take notice of rumors.

An analysis of repeated games shows that under relatively weak conditions, the honest equilibrium can be supported. Even opportunistic rumormongers will

[17] Dewally (2001) uses isolated postings on Internet newsgroups misc.invest.stocks and alt.invest.penny-stocks. He finds that, on average, postings do not predict future performance but are preceded by significant positive drifts. If however pre-posting performance is bad, post-posting performance is significantly positive. Tumarkin and Whitelaw (2001) study message volume and opinion changes on ragingbull.com. Although messages do not predict stock returns, messages (quantity and opinion) are predicted by stock returns, volumes, and lagged message volume. Antweiler and Frank (2001) study both Yahoo! and ragingbull.com postings. They use intraday data and find significant predictability between message volume, bullishness, trading volume, and volatility.

refrain from bluffing or cheating because they may lose their reputation and hence the ability to manipulate prices.

Appendix

A. *Proof of Proposition 1*

The competing *M* set the price after the first auction so that $P_1(H_1) = E[\tilde{v}|y_1]$. To find $E[\tilde{v}|y_1]$, I integrate over the conditional distribution of $\tilde{v}$ given $\tilde{y}_1$, $f(v|y_1)$, which can be found by dividing the joint pdf of $(\tilde{v}, \tilde{y}_1)$ by the marginal pdf of $\tilde{y}_1$. $f(v,y_1)$ follows from the joint pdf of $(\tilde{v}, \tilde{\boldsymbol{u}}\})$, *L*'s strategy, and the fact that $\tilde{y}_1 \equiv \tilde{x}_1 + \tilde{u}_1$:

$$\begin{aligned} f(v, y_1) &= \frac{1}{4}\varphi(y_1 + \mathrm{x}_L) \quad \text{for } v \in [-2,\ -c) \\ &= \frac{1}{4}\varphi(y_1) \qquad\quad\ \text{for } v \in [-c, c] \\ &= \frac{1}{4}\varphi(y_1 - \mathrm{x}_L) \quad \text{for } v \in (c, 2]. \end{aligned} \tag{A1}$$

The marginal distribution of $\tilde{y}_1$ distribution is then

$$f_{y_1}(y_1) = \int_v f(v, y_1) = \frac{2-c}{4}\varphi(y_1 + \mathrm{x}_L) + \frac{c}{2}\varphi(y_1) + \frac{2-c}{4}\varphi(y_1 - \mathrm{x}_L). \tag{A2}$$

So that is

$$\begin{aligned} f(v|y_1) = \frac{f(v, y_1)}{f_{y_1}(y_1)} &= \frac{\varphi(y_1 + \mathrm{x}_L)}{(2-c)(\varphi(y_1 - \mathrm{x}_L) + \varphi(y_1 + \mathrm{x}_L)) + 2c\varphi(y_1)} \ \text{for } v \in [-2,\ -c) \\ &= \frac{\varphi(y_1)}{(2-c)(\varphi(y_1 - \mathrm{x}_L) + \varphi(y_1 + \mathrm{x}_L)) + 2c\varphi(y_1)} \ \text{for } v \in [-c, c] \\ &= \frac{\varphi(y_1 - \mathrm{x}_L)}{(2-c)(\varphi(y_1 - \mathrm{x}_L) + \varphi(y_1 + \mathrm{x}_L)) + 2c\varphi(y_1)} \ \text{for } v \in (c, 2]. \end{aligned} \tag{A3}$$

To find $\mathrm{E}[\tilde{v}|y_1]$ we integrate

$$E[\tilde{v}|y_1] = \int_v v\, \mathrm{d}f(v|y_1) = \frac{\frac{2+c}{2}\varphi(y_1 - \mathrm{x}_L) - \frac{2+c}{2}\varphi(y_1 + \mathrm{x}_L)}{\varphi(y_1 - \mathrm{x}_L) + \varphi(y_1 + \mathrm{x}_L) + \frac{2+c}{2}\varphi(y_1)}. \tag{A4}$$

Because $\mathrm{x}_L \ll 1$, $\mathrm{P}_1(\tilde{y}_1) = \tilde{p}_1 \approx 0$, and $\mathrm{E}[\Pi_L]_{silent} \approx \mathrm{x}_L$.

B. *Proof that the necessary transaction cost that keeps L from rationing is very small.*

The lower bound for the transaction cost necessary to keep *L* from rationing is the *ex ante* expected price impact of trading x_L. The expected price impact (denoted $\mathrm{EPI}(\mathrm{x}_L)$), can be calculated by integrating over the noise demand distribution:

$$\mathrm{EPI}(\mathrm{x}_L) \equiv E[\tilde{p}_1|\mathrm{x}_L] = \int \frac{\frac{2+c}{2}\varphi(u_1) - \frac{2+c}{2}\varphi(u_1 + 2\mathrm{x}_L)}{\varphi(u_1) + \varphi(u_1 + 2x_L) + \frac{2c}{2-c}\varphi(u_1 + \mathrm{x}_L)}\mathrm{d}\varphi(u_1). \tag{A5}$$

Below I show that $\forall\ c \in [0, 2]$, $\mathrm{EPI}(0) = 0$, $\mathrm{EPI}'(0) = 0$, and $\mathrm{EPI}'(\mathrm{x}_L) > 0\ \forall\ \mathrm{x}_L > 0$, which proves that for small x_L, the expected price impact, and hence the necessary transaction cost to keep L from rationing, is small compared to x_L. Numerically I find that for $\mathrm{x}_L = 0.1$, the minimum c to keep L from rationing, c_{min}, is 0.01, for $\mathrm{x}_L = 0.01$, $c_{min} = 0.0001$, and for $\mathrm{x}_L = 0.001$, $c_{min} = 0.000001$.

EPI(0) can be found directly by substituting $\mathrm{x}_L = 0$ in (A5). Substituting u for u_1, x for x_L, C for $2 + \mathrm{c}/2$, K for $2\mathrm{c}/(2 - \mathrm{c})$, and recognizing that $\varphi'(\mathrm{x}) = -\mathrm{x}\varphi(\mathrm{x}))$ we find that

$$\begin{aligned}\mathrm{EPI}'(\mathrm{x}) =& C\int \frac{(u+2\mathrm{x})\varphi(u+2\mathrm{x})(\varphi(u)+\varphi(u+2\mathrm{x})+K\varphi(u+\mathrm{x}))}{(\varphi(u)+\varphi(u+2\mathrm{x})+K\varphi(u+\mathrm{x}))^2}\mathrm{d}\varphi(u)\\ &+ C\int \frac{(\varphi(u)-\varphi(u+2\mathrm{x}))(2(u+2\mathrm{x})\varphi(u+2\mathrm{x})+K(u+\mathrm{x})\varphi(u+\mathrm{x}))}{(\varphi(u)+\varphi(u+2\mathrm{x})+K\varphi(u+\mathrm{x}))^2}\mathrm{d}\varphi(u).\end{aligned} \tag{A6}$$

$\mathrm{EPI}'(0)$ can be found by substitution; $\mathrm{EPI}'(\mathrm{x}) > 0$ follows upon recognition that the integrands cross zero at and are symmetric around $u = -\mathrm{x}$, while the integrators are central around $u = 0$.

C. Proof of Proposition 2

The equilibrium strategy of M is to set the market price equal to the expected value of $\tilde{v}$ given H_n. If L plays the suggested (honest) strategy, the pricing rule of M can be found similarly to the proof of Proposition 1. The joint pdf of v and the $\boldsymbol{y}$-vector is

$$\begin{aligned} f(v,\boldsymbol{y}) =& \frac{1}{4(2\pi)^{n/2}}\left\{(1-q)e^{-\frac{1}{2}\sum_n y_i^2} + qe^{-\frac{1}{2}\sum_n (y_i-\mathrm{x}(i))^2}\right\} \text{ for } v>0\\ & \frac{1}{4(2\pi)^{n/2}}\left\{(1-q)e^{-\frac{1}{2}\sum_n y_i^2} + qe^{-\frac{1}{2}\sum_n (y_i-\mathrm{x}(i))^2}\right\} \text{ for } v\le 0. \end{aligned} \tag{A7}$$

the marginal distribution of $\boldsymbol{y}$ is

$$f(\boldsymbol{y}) - \frac{1}{(2\pi)^{n/2}}\left\{(1-q)e^{-\frac{1}{2}\sum_n y_i^2} + \frac{q}{2}e^{-\frac{1}{2}\sum_n (y_i-\mathrm{x}(i))^2} + \frac{q}{2}e^{-\frac{1}{2}\sum_n (y_i-\mathrm{x}(i))^2}\right\}, \tag{A8}$$

where $\mathrm{x}(1) = \mathrm{x}_L$ and $\mathrm{x}(n) = m(m+1)^{(n-2)}\ \mathrm{x}_F$, is the *absolute value* of the informed demand at the nth auction.

The conditional distribution of $\tilde{v}$ is

$$f(v|\boldsymbol{y}) = \frac{f(v,\boldsymbol{y})}{f(\boldsymbol{y})} = \frac{(1-q) + qe^{\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(\mathrm{i})^2}}{4\left\{(1-q) + \frac{q}{2}\left(e^{\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2} + e^{-\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2}\right)\right\}} \quad \text{for } v > 0$$

$$= \frac{(1-q) + qe^{\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(\mathrm{i})^2}}{4\left\{(1-q) + \frac{q}{2}\left(e^{\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2} + e^{-\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2}\right)\right\}} \quad \text{for } v \leq 0. \tag{A9}$$

Integrating v over above conditional distribution gives

$$E[\tilde{v}|\boldsymbol{y}] = \frac{e^{\sum\limits_n y_i \mathrm{x}(i)\frac{1}{2}\mathrm{x}(i)^2} - e^{-\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2}}{2\,\frac{1-q}{q} + e^{\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2} + e^{-\sum\limits_n y_i \mathrm{x}(i) - \frac{1}{2}\mathrm{x}(i)^2}}$$
$$= \frac{1 - e^{-2\sum\limits_n y_i \mathrm{x}(i)}}{2\,\frac{1-q}{q} + e^{\sum\limits_n \frac{1}{2}\mathrm{x}(i)^2 - y_i \mathrm{x}(i)} + 1 + e^{-2\sum\limits_n y_i \mathrm{x}(i)}}. \tag{A10}$$

Because the market price converges to 1 whenever L sees $\tilde{v} > 0$, to -1 whenever L sees $\tilde{v} < 0$, the suggested strategy maximizes L's profits.

To prove Propositions 2(ii) and 2(iii), I calculate the expected price given a "buy" rumor in the market:

$$E[\tilde{p}_n|\text{``}buy\text{''}] = E[\tilde{p}_n|\tilde{v} > 0]$$
$$= \int \frac{1 - e^{-2\sum\limits_n \left(u_i \mathrm{x}(i) + \mathrm{x}^2(i)\right)}}{2\,\frac{1-q}{q} e^{-\sum\limits_n \left(u_i \mathrm{x}(i) + \frac{1}{2}\mathrm{x}(i)\right)} + 1 + e^{-2\sum\limits_n (u_i \mathrm{x}(i) + \mathrm{x}^2(i))}} \mathrm{d}f(\boldsymbol{u}). \tag{A11}$$

The integrand follows from substituting $\tilde{y}_i = \tilde{u}_i + \mathrm{x}(i)$. The integrator is the multivariate (Normal) pdf of the vector of n noise demands. Because the exponential terms decrease to zero, $E[\tilde{p}_n|\tilde{v} > 0]$ increases to 1 in a continuous way.

Similarly, the expected price when a "sell" rumor is in the market, given by

$$E[\tilde{p}_n|\text{``sell''}] = \int \frac{1 - e^{-2\sum\limits_n \left(u_i \mathrm{x}(i) \mathrm{x}^2(i)\right)}}{2\,\frac{1-q}{q}\, e^{-\sum\limits_n \left(u_i \mathrm{x}(i) - \frac{3}{2}\mathrm{x}^2(i)\right)} + 1 + e^{-2\sum_n (u_i \mathrm{x}(i) - \mathrm{x}^2(i))}} \mathrm{d}f(\boldsymbol{u}) \tag{A12}$$

decreases until $\lim\limits_{n\to\infty} E[\tilde{p}_n|\tilde{v} < 0] = -1$.

Finally, the expected price when a "…" rumor is in the market is zero:

$$E[\tilde{p}_n|\text{``}\ldots\text{''}] = \int \frac{1 - e^{-2\sum_n u_i \mathrm{x}(i)}}{2\,\frac{1-q}{q}\, e^{\sum_n \frac{1}{2}\mathrm{x}(i)^2 - u_i \mathrm{x}(i)} + 1 + e^{-2\sum_n u_i \mathrm{x}(i)}}\,\mathrm{d}f(\boldsymbol{u}) = 0 \;\forall\, n. \tag{A13}$$

Equality holds because $p_n(\boldsymbol{u}) = -p_n(-\boldsymbol{u})$ and the integrator $f(\boldsymbol{u})$ is symmetric around **0**.

Proposition 2(iv) follows from recognizing that $|\tilde{p}_n| \leq 1\;\forall\, n$, while $\lim_{n\to\infty} \mathrm{E}[\tilde{p}_n|\text{``}buy\text{''}] = 1$ and $\lim_{n\to\infty} \mathrm{E}[\tilde{p}_n|\text{``}sell\text{''}] = -1$. To see that $\mathrm{Var}(\tilde{p}_n|\text{``}\ldots\text{''})$ goes to zero with n, note that the dominating term in the integrand of (A13) is $e^{\sum_n 1/2\mathrm{x}(i)^2}$ which goes to infinity with increasing n. Hence $\tilde{p}_n|$"…" is, for large n, zero for any noise demand u_n.

To find the expected profit made by an honest L (Proposition 2(v)), we note that if L sees $|\tilde{v}| < 1$, she flips her position at the last auction and waits for exogenous revelation. This results in a profit of $1 + (1 - \tilde{v}) = 2 - \tilde{v}$. If she sees $|\tilde{v}| > 1$, she liquidates her position at $\tilde{v}$. Hence we have that her ex ante expected profit is given by

$$E[\Pi_L]_{honest} = \frac{1}{2} q \mathrm{x}_L \left(\int_0^1 (2 - v)\mathrm{d}v + \int_1^2 v\mathrm{d}v \right) = \frac{3}{2} q \mathrm{x}_L. \tag{A14}$$

D. *Proof of Proposition 3*

Given L's suggested strategy, the joint pdf of $(v, \boldsymbol{y})$ is

$$\begin{aligned} f(v, \boldsymbol{y}) &= \frac{1}{4(2\pi)^{n/2}} \left\{ (1-q) e^{-\frac{1}{2}\sum_n (y_i + \mathrm{x}(i))^2}\; e^{-\frac{1}{2}\sum_n (y_i - \mathrm{x}(i))^2} \right\} \quad \text{for } v > 0 \\ &= \frac{1}{4(2\pi)^{n/2}} \left\{ (1-q) e^{-\frac{1}{2}\sum_n (y_i - \mathrm{x}(i))^2}\; e^{-\frac{1}{2}\sum_n (y_i + \mathrm{x}(i))^2} \right\} \quad \text{for } v \leq 0. \end{aligned} \tag{A15}$$

The marginal distribution of $\boldsymbol{y}$ is then

$$f(\boldsymbol{y}) = \frac{1}{(2\pi)^{n/2}} \left\{ \frac{1}{2} e^{-\frac{1}{2}\sum_n (y_i - \mathrm{x}(i))^2} + \frac{1}{2} e^{-\frac{1}{2}\sum_n (y_i + \mathrm{x}(i))^2} \right\} \tag{A16}$$

The conditional distribution of $\tilde{v}$ is then

$$\begin{aligned} f(v|\boldsymbol{y}) = \frac{f(v, \boldsymbol{y})}{f(\boldsymbol{y})} &= \frac{(1-q) e^{-\sum_n y_i \mathrm{x}(i)} + e^{\sum_n y_i \mathrm{x}(i)}}{e^{-\sum_n y_i \mathrm{x}(i)} + e^{\sum_n y_i \mathrm{x}(i)}} \quad \text{for } v > 0 \\ &= \frac{e^{-\sum_n y_i \mathrm{x}(i)} + (1-q) e^{\sum_n y_i \mathrm{x}(i)}}{e^{-\sum_n y_i \mathrm{x}(i)} + e^{\sum_n y_i \mathrm{x}(i)}} \quad \text{for } v \leq 0. \end{aligned} \tag{A17}$$

Integrating v over the above conditional distribution gives

$$E[\tilde{v}|\boldsymbol{y}] = \frac{qe^{\sum_n y_i \mathrm{x}(i)} - qe^{-\sum_n y_i \mathrm{x}(i)}}{e^{-\sum_n y_i \mathrm{x}(i)} + e^{\sum_n y_i \mathrm{x}(i)}} = q\frac{1 - e^{-2\sum_n y_i \mathrm{x}(i)}}{1 + e^{-2\sum_n y_i \mathrm{x}(i)}}. \tag{A18}$$

Because L can immediately exploit the price impact of a rumor, an uninformed L will always spread either a "buy" or a "sell" rumor. To see that the probability of bluffing "buy" equals the probability of bluffing "sell," consider any alternative bluffing scheme. If L would bluff "buy" with probability $\theta > 1/2$, "sell" with probability $(1-\theta)$, the sell rumor price, E[$\tilde{v}$| "sell"], would be further away from zero than the buy rumor price, giving L the incentive to spread "sell" when uninformed, a contradiction.

The proof of Proposition 3(ii) is analogous to the proof of Propositions 2(ii) to 2(iv).

To find the expected profit of a bluffing L, note that L waits until exogenous revelation when she sees $|\tilde{v}| > q$. Whenever she sees $|\tilde{v}| < q$, she reverses her position in the last auction, when the price has reached q or $-q$. If L does not see $\tilde{v}$, she makes a bluffing profit of $2q\mathrm{x}_L$. Hence her ex ante expected profit is given by

$$\begin{aligned} E[\Pi_L]_{bluff} &= \tfrac{1}{2}q\mathrm{x}_L\left(\int_0^q (2q - v)\mathrm{d}v + \int_q^2 v\mathrm{d}v\right) + (1-q)2q\mathrm{x}_L \\ &= q\mathrm{x}_L(3 - 2q + 1/2q^2) > E[\Pi_L]_{honest}\forall q. \end{aligned}$$

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