The sum of positive integers upto and including n is equal to:

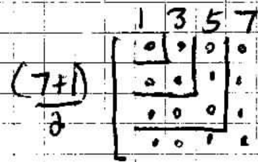
$$\frac{n(n+1)}{2}$$

or concisely written:

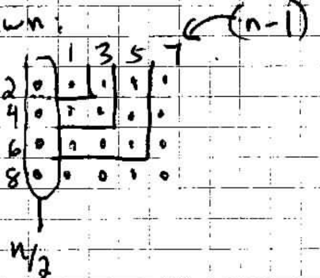
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

PROOF:

The sum of positive integers upto n equals the sum of even integers plus the sum of odd integers upto n.
The sum of odd integers equals $\left(\frac{(n+1)}{2}\right)^2$ as shown below:



The sum of even integers equals the sum of odd integers plus $n/2$ as shown:



or equivalently: $\left(\frac{((n-1)+1)}{2}\right)^2 + \frac{n}{2} = \left(\frac{n}{2}\right)^2 + \frac{n}{2}$

if n is odd then the sum equals the sum of odds to n and sum of evens to $n-1$ or:

$$\left(\frac{(n+1)}{2}\right)^2 + \left(\frac{(n-1)}{2}\right)^2 + \frac{(n-1)}{2}$$

$$= \frac{1}{4}\left(n^2+2n+1+n^2-2n+1+2n-2\right)$$

$$= \frac{1}{4}\left(2n^2+2n\right)$$

$$= \frac{n(n+1)}{2}$$

if n is even then the sum equals the sum of evens to n and the sum of odds to $n-1$ or:

$$\left(\frac{n}{2}\right)^2 + \frac{n}{2} + \left(\frac{((n-1)+1)}{2}\right)^2$$

$$= \left(\frac{n}{2}\right)^2 + \frac{n}{2} + \left(\frac{n}{2}\right)^2$$

$$= \frac{n}{2}\left(\frac{n}{2} + 1 + \frac{n}{2}\right)$$

$$= \frac{n}{2}(n+1)$$

Therefore if the number n is even or odd they both result in the sum equalling $\frac{n}{2}(n+1)$

QED.