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The Interaction between Nonexpected Utility and Asymmetric Market Fundamentals

MAO-WEI HUNG*

ABSTRACT

This paper studies a nonexpected utility, general equilibrium asset pricing model in which market fundamentals follow a bivariate Markov switching process. The results show that nonexpected utility is capable of exactly matching the means of the risk-free rate and the risk premium. Asymmetric market fundamentals are capable of generating a negative sample correlation between the risk-free rate and the risk premium. Moreover, an equilibrium asset pricing model endowed with asymmetric market fundamentals is consistent with all five first and second moments of the risk-free rate and the risk premium in the U.S. data.

IN THEIR SEMINAL WORK, Mehra and Prescott (1985) argue that the observed historical average for the equity risk premium is too large, and the average risk-free rate is too small, to be rationalized by a complete and frictionless asset pricing model. The result of Mehra and Prescott, which is known as the equity premium puzzle, has triggered a great deal of research. Much of this research has attempted to resolve the puzzle by relaxing the assumptions of their analysis: the agent has von Neumann-Morgenstern time additive expected utility, the return to the stock market is approximately equal to the return on the market portfolio, and the consumption growth rate follows a symmetric Markov chain.

One branch of research focuses on nonexpected utility functions of the Kreps and Porteus (1978, 1979) type as a means of resolving the puzzle. It is well known that von Neumann-Morgenstern time additive expected utility has an intrinsic restriction that the reciprocal of the coefficient of relative risk aversion equals the elasticity of intertemporal substitution. Kreps and Porteus establish a nonexpected utility framework which is capable of distinguishing the coefficient of relative risk aversion from the elasticity of intertemporal substitution. However, Kocherlakota (1990a) and Weil (1989) demonstrate that relaxing the restriction between the coefficient of relative risk aversion and the elasticity of intertemporal substitution is not capable of resolving the puzzle.

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A second direction of research focuses on modifications of market fundamentals. Kocherlakota (1990b) correctly argues that consumption and dividend growth rates are less than perfectly correlated, and thus the return to the market portfolio and the return to the stock market portfolio are different. Consequently, instead of using a univariate degenerate process to govern consumption, Kocherlakota adopts a bivariate vector autoregressive process for consumption and dividends. He finds, however, that an asset pricing model governed by a bivariate linear process is not capable of resolving the puzzle. Along the same lines, Hung (1989) and Cecchetti, Lam, and Mark (1990b) model the laws of motion for the consumption growth rate and the dividend growth rate as a Markov switching process. Although this process is nonlinear and is able to capture asymmetry over the business cycle, they show that the solution to the equity premium puzzle cannot be found by employing asymmetric market fundamentals.

In the analysis presented in this article, elements of both branches of research are integrated. The results show that the combination of these two factors is able to resolve the equity premium puzzle. The resolution of the puzzle is due to the following forces: First, using aggregate dividends rather than aggregate consumption as the payoff to the stock market decreases the covariance between the equity return and the marginal rate of substitution and hence increases the equity return. Second, nonexpected utility allows a separation between the elasticity of intertemporal substitution and the inverse of the coefficient of relative risk aversion. This additional flexibility in the preference parameters helps resolve the puzzle.

This article also investigates the ability of the equilibrium asset pricing model to match the variances and covariance of the risk premium and the risk-free rate. It is shown that asymmetric market fundamentals are capable of matching the negative sample correlation between the risk premium and the risk-free rate. Furthermore, an equilibrium asset pricing model endowed with asymmetric market fundamentals is consistent with all five first and second moments if the uncertainty in the estimation of the sample moments and the market fundamentals is taken into account. The smallest parameter value for the coefficient of relative risk aversion for which the asset pricing model is not rejected is around seven.

In an independent paper, Kandel and Stambaugh (1991) also attempt to match the means and variances of these two rates of returns. While the Kreps and Porteus utility is adopted in both papers, the method to estimate the market fundamentals is different. In Kandel and Stambaugh and this article, a bivariate Markov switching process for the consumption and dividend growth rates is estimated. Kandel and Stambaugh follow Mehra and Prescott's approach to use the method of moments to estimate a four-state symmetric Markov chain for the consumption process. It is shown in this article that the implied second moments for the bivariate Markov switching process differ from those for the symmetric Markov chain. Particularly, employing a symmetric Markov chain as the market fundamentals is incapable of generating the negative sample correlation between the risk premium and the risk-free rate.

The remainder of the article proceeds as follows: In Section I, the asset pricing model with nonexpected utility is constructed and discussed. The bivariate Markov switching model is developed in Section II. The equilibrium asset prices and returns are derived in the next section. Section IV gives the empirical results for this bivariate Markov switching process. The results are given in Section V. Section VI concludes the paper.

I. Asset Pricing with Nonexpected Utility Preference

Consider a pure exchange economy in which a single, infinitely lived representative agent chooses consumption and portfolio composition to maximize utility. There is one good and N real assets in the economy. The agent in this economy is assumed not to be indifferent to the timing of the resolution of uncertainty over temporal lotteries. The agent's preferences are assumed to be represented recursively by

$$V_t = U[C_t, E_t V_{t+1}],$$

where $U(.,.)$ is the aggregator function, C_t is the consumption level at time t, and E_t is the mathematical expectation conditional on the information set at time t.[1] As shown by Kreps and Porteus, agents prefer early resolution of uncertainty over temporal lotteries if $U(.,.)$ is convex in its second argument. Alternatively, if $U(.,.)$ is concave in its second argument, agents will prefer late resolution of uncertainty over temporal lotteries.

The aggregator function is further parameterized by

$$U[C_t, E_t V_{t+1}] = \left[(1-\beta)C_t^{1-\rho} + \beta(E_t V_{t+1})^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\gamma}{1-\rho}}. \tag{1}$$

The parameter β is the agent's subjective time discount factor and γ can be interpreted as the Arrow-Pratt coefficient of relative risk aversion. It can also be shown that $1/\rho$ measures the elasticity of intertemporal substitution. In this parameterization, the agent's preference over the timing of the resolution of uncertainty is determined by the parameters γ and ρ. For instance, if the agent's coefficient of relative risk aversion (γ) is greater than the reciprocal of his elasticity of intertemporal substitution (ρ), then he prefers early resolution of uncertainty. Conversely, if the reciprocal of the agent's elasticity of intertemporal substitution is larger than his coefficient of relative risk aversion, he prefers late resolution of uncertainty. If γ is equal to ρ, the agent's utility becomes an isoelastic, von Neumann-Morgenstern utility and he is indifferent to the timing of the resolution of uncertainty.

[1] The preferences satisfy axioms 2.1, 2.2, 2.3, and 3.1 in Kreps and Porteus (1978).

The Euler equations are

$$1 = E_t\left[U_{2,t}\frac{U_{1,t+1}}{U_{1,t}}R_{i,t+1}\right], \quad i = 1,\ldots,N, \tag{2}$$

where $R_{i,t+1}$ is the return to the ith asset, $U_{1,t}$ and $U_{2,t}$ represent the partial derivatives of the aggregator function U at time t with respect to its first and second arguments, respectively. Following the solution techniques employed in Epstein and Zin (1989) and Weil (1989), the Euler equations (2) can be rewritten as

$$1 = E_t\left\{\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}(R^m_{t+1})^{\frac{1-\gamma}{1-\rho}-1}R_{i,t+1}\right\}, \quad i = 1,\ldots,N, \tag{3}$$

where R^m_{t+1} is the return to the market portfolio.

Notice that $\left\{\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}(R^m_{t+1})^{\frac{1-\gamma}{1-\rho}-1}\right\}$ is the marginal rate of substitution, which depends on both the consumption growth rate and the return to the market portfolio. When $\gamma = \rho$, the agent has a von Neumann-Morgenstern expected utility and the marginal rate of substitution depends only on the consumption growth rate.

II. Asymmetric Market Fundamentals

In order to capture the asymmetric characteristics of market fundamentals, a bivariate Markov switching model is employed as the stochastic process governing consumption and dividends.[2] The bivariate Markov switching process is a nonlinear process where the consumption growth rate and the dividend growth rate are characterized by an occasional shift between a positive growth state and a negative growth state. The switching process between states is assumed to follow a first-order Markov chain. The representative agent is assumed to recognize the possibility of recession and to assign a probability to this event. The econometrician does not observe the state variables, but rather estimates the parameters of the process based on observations of the time series of the observable variables.

To describe the bivariate Markov switching process, let c_t denote the logarithm of per capita consumption and d_t the logarithm of the real dividend. In this bivariate Markov switching model, it is assumed that the logarithmic consumption growth rate $(c_t - c_{t-1})$ is the sum of a Markov switching process, S_t, and a noise term, e_t. Similarly, let the logarithmic dividend growth rate $(d_t - d_{t-1})$ be the sum of the Markov switching process,

[2] Hamilton (1989) has modelled the gross national product as a Markov switching process and successfully identified the boom and recession periods of U.S. business cycles.

S_t, and a different noise term, v_t, i.e.,

$$c_t - c_{t-1} = a_0(1 - S_t) + a_1 S_t + e_t \tag{4}$$

$$d_t - d_{t-1} = b_0(1 - S_t) + b_1 S_t + v_t \tag{5}$$

where $\begin{pmatrix} e_t \\ v_t \end{pmatrix}$ is an i.i.d. conditional normal distribution $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} \sigma_c^2 & \sigma_{cd} \\ \sigma_{cd} & \sigma_d^2 \end{pmatrix}$ and S_t ($= 0$ or 1) denotes the unobserved states of the system. Transition between states is assumed to be a first-order Markov chain. Parameters a_1 and b_1 are normalized to be less than zero. Then, $S_t = 0$ denotes that both average consumption growth and dividend growth are in a boom state and $S_t = 1$ denotes that average consumption growth and dividend growth are in a depression state. Define

$$\text{Prob}(S_t = j | S_{t-1} = i) = \phi_{ij} \quad i, j = 0, 1. \tag{6}$$

then $\phi = \{\phi_{ij}\}$ is a 2×2 transition matrix for the bivariate Markov switching model. Each element in the transition matrix is between zero and one, and the sum of each row is equal to one.

III. Equilibrium Prices and Returns

In this section the closed form solutions for equilibrium asset prices and returns in the Markov switching environment are derived. Basically the equilibrium asset prices and returns can be obtained by repeatedly applying the law of iterated conditional expectations. To see this, begin with the pricing function for the market portfolio. The market portfolio pays off C_t in period t. Denote its price and return by P_t^m and R_t^m respectively.

As in Mehra and Prescott (1985) and Weil (1989), a stationary equilibrium is assumed such that

$$P_t^m(C_t, i) = \eta_i C_t, \tag{7}$$

where C_t is the consumption level at time t, i is the state of nature at time t, and η_i is an undetermined coefficient. Let $G_t = C_t / C_{t-1}$ be the consumption growth rate at time t. Then the one-period rate of return for the market portfolio, if the current state is i and the next period state is j, is

$$R^m(i, j) = \left(\frac{\eta_j + 1}{\eta_i} \right) G_j, \tag{8}$$

where G_j is the consumption growth rate of state j.

Recall that the Euler equation for the market portfolio can be written as

$$1 = E_t \left\{ \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} (R_{t+1}^m)^{\frac{1-\gamma}{1-\rho}} \right\}. \tag{9}$$

Substituting equation (8) into equation (9), the following equation is obtained:

$$1 = \sum_{j=0}^{1} \phi_{ij} E_t \left\{ \beta^{\frac{1-\gamma}{1-\rho}} \left(\frac{\eta_j + 1}{\eta_i} \right)^{\frac{1-\gamma}{1-\rho}} G_j^{1-\gamma} \right\}. \tag{10}$$

By using the lognormal moment generator, the following equations are obtained:

$$1 = \beta^{\frac{1-\gamma}{1-\rho}} \exp\{0.5(1-\gamma)^2 \sigma_c^2\} \left\{ \phi_{i0} \left(\frac{\eta_0 + 1}{\eta_i} \right)^{\frac{1-\gamma}{1-\rho}} \exp[a_0(1-\gamma)] \right.$$
$$\left. + \phi_{i1} \left(\frac{\eta_1 + 1}{\eta_i} \right)^{\frac{1-\gamma}{1-\rho}} \exp[a_1(1-\gamma)] \right\}, \quad i = 0, 1. \tag{11}$$

The equations in (11) form a nonlinear system in which the η_i ($i = 0$ and 1) are positive solutions if the equilibrium exists.[3] It is possible to solve η_i as a function of the estimated parameters in the bivariate Markov switching process, the coefficient of relative risk aversion, the discount factor, and the elasticity of intertemporal substitution.

After obtaining the solutions for η_i, it is possible to calculate the return for the stock market. The stock market pays D_t in period t. Denote its price and return by P_t^{sm} and R_t^{sm}, respectively.

In a stationary equilibrium, the price for the stock market can be expressed as

$$P_t^{sm}(D_t, i) = \kappa_i D_t, \tag{12}$$

where D_t is the dividend level at time t, i is the state of nature at time t and κ_i is an undetermined coefficient. Let $H_t = D_t/D_{t-1}$ be the dividend growth rate at time t. Then the one-period rate of return for the stock market, if the current state is i and the next period state is j, can be written as

$$R^{sm}(i,j) = \left(\frac{\kappa_j + 1}{\kappa_i} \right) H_j, \tag{13}$$

where H_j is the dividend growth rate of state j.

Substituting equation (13) into equation (3) implies

$$1 = \sum_{j=0}^{1} \phi_{ij} E_t \left\{ \beta^{\frac{1-\gamma}{1-\rho}} \left(\frac{\eta_j + 1}{\eta_i} \right)^{\frac{1-\gamma}{1-\rho} - 1} G_j^{-\gamma} \left(\frac{\kappa_j + 1}{\kappa_i} \right) H_j \right\}. \tag{14}$$

[3] The nonlinear system is solved by calling the application module NLSYS in GAUSS. The NLSYS employs a quasi Newton algorithm with Broyden's secant update method.

By using the lognormal moment generator, the following equations are obtained:

$$\kappa_i = \beta^{\frac{1-\gamma}{1-\rho}} \exp(0.5\gamma^2\sigma_c^2 + 0.5\sigma_d^2 - \gamma\sigma_{cd}) \times \left\{ \phi_{i0}\left(\frac{\eta_0 + 1}{\eta_i}\right)^{\frac{1-\gamma}{1-\rho}-1} (\kappa_0 + 1)\exp(-\gamma a_0 + b_0) + \phi_{i1}\left(\frac{\eta_1 + 1}{\eta_i}\right)^{\frac{1-\gamma}{1-\rho}-1} (\kappa_1 + 1)\exp(-\gamma a_1 + b_1) \right\}, \quad i = 0, 1. \tag{15}$$

Equations (15) form a system of linear equations. The κ_i ($i = 0$ and 1) are positive solutions to equations (15) if the equilibrium exists. The κ_i are functions of the estimated parameters in the bivariate Markov switching process, the coefficient of relative risk aversion, the discount factor, and the elasticity of intertemporal substitution.

If the current state is i, the risk-free rate RF_i can be calculated as follows. Notice that the Euler equation (3) implies that the price of the risk-free asset in state i, $1/RF_i$, is

$$\frac{1}{RF_i} = \beta^{\frac{1-\gamma}{1-\rho}} E_t \left\{ \sum_{j=0}^{1} \phi_{ij}\left(\frac{\eta_j + 1}{\eta_i}\right)^{\frac{1-\gamma}{1-\rho}-1} G_j^{-\gamma} \right\}, \quad i = 0, 1. \tag{16}$$

By using the lognormal moment generator, the price of the risk-free asset is

$$1/RF_i = \beta^{\frac{1-\gamma}{1-\rho}} \exp(0.5\gamma^2\sigma_c^2) \left\{ \phi_{i0}\left(\frac{\eta_0 + 1}{\eta_i}\right)^{\left(\frac{1-\gamma}{1-\rho}^{-1}\right)} \exp(-\gamma a_0) + \phi_{i1}\left(\frac{\eta_1 + 1}{\eta_i}\right)^{\left(\frac{1-\gamma}{1-\rho}^{-1}\right)} \exp(-\gamma a_1) \right\}, \quad i = 0, 1. \tag{17}$$

Let ψ_i be the stationary probability of state i. The vector $\psi = [\psi_0, \psi_1]'$ is the solution to the system of equations

$$(I - \phi')\psi = 0, \tag{18}$$

where I is a 2×2 identity matrix and ϕ' is the transpose of ϕ.[4] Then, the unconditional expected rate of return on equity is

$$\mu^{sm} = \sum_{i=0}^{1} \sum_{j=0}^{1} \psi_i \phi_{ij} E_t(H_j)\left(\frac{\kappa_j + 1}{\kappa_i}\right). \tag{19}$$

[4] It can be shown that $\psi_0 = (1 - \phi_{11})/(2 - \phi_{00} - \phi_{11})$ and $\psi_1 = (1 - \phi_{00})/(2 - \phi_{00} - \phi_{11})$ in the two state model.

The unconditional expected rate of return on the risk-free security is

$$\mu^F = \sum_{i=0}^{1} \psi_i RF_i, \tag{20}$$

and the unconditional expected risk premium is

$$\mu^{RP} = \mu^{sm} - \mu^F. \tag{21}$$

The variance-covariance matrix for the risk premium and the risk-free rate can be calculated as follows:

$$VAR = \sum_{i=0}^{1} \sum_{j=0}^{1} \psi_i \phi_{ij} \begin{bmatrix} \left(\frac{\kappa_j + 1}{\kappa_i}\right) E_t(H_j) - RF_i - \mu^{RP} \\ RF_i - \mu^{RF} \end{bmatrix} \cdot \begin{bmatrix} \left(\frac{\kappa_j + 1}{\kappa_i}\right) E_t(H_j) - RF_i - \mu^{RP} \\ RF_i - \mu^{RF} \end{bmatrix}'. \tag{22}$$

IV. The Estimates of the Markov Switching Model

To estimate the bivariate Markov switching process, the Expectation and Maximization (EM) algorithm is employed.[5] The EM algorithm is applied to annual real dividends and real per capita consumption from 1889 to 1985. Nominal stock prices are from January Standard & Poor's Composite Stock Price Index, which are extended back to 1889 by using data in Cowles (1939). Nominal dividends are total dividends per share accruing to the index. The real dividends are the nominal Standard & Poor's data deflated by the Consumer Price Index from Wilson and Jones (1988). The data on real consumption start with the Hendrick data reported in Balke and Gordon (1986) and continue in 1929 with the National Income and Product Accounts series. Aggregate consumption in each year is divided by the population in that year to obtain per capita consumption. The data are the same as in Cecchetti, Lam, and Mark (1990a).

The estimates for the Markov switching model are shown in Table I. The average dividend growth rate is estimated to be 3.26 percent per year in positive growth states and -31.01 percent in negative growth states. The consumption growth rates are 2.04 and -1.69 percent per year for states with positive and negative growth, respectively. The estimates of the standard deviations of the consumption growth rate and dividend growth rate are 3.61 and 8.89 percent, respectively. The estimate of the covariance between the consumption growth rate and dividend growth rate is 12.45 (percent).[2]

The transition matrix is also shown in Table I and is obviously not symmetric. The probability that consumption and dividends will be both in

[5] See Hamilton (1990) for the details regarding the EM algorithm in this setting.

Table I

Estimates of the Markov Switching Model

Annual nominal stock prices are from the January S & P Composite Stock Price Index, which are extended back to 1889 by using data in Cowles (1939). Annual nominal dividends are total dividends per share accruing to the index. The real dividends are the nominal Standard & Poor's data deflated by the Consumer Price Index. The data on real consumption start with the Hendrick data reported in Balke and Gordon (1986) and continue in 1929 with the National Income and Produce Accounts series. Aggregate consumption in each year is divided by population in that year to obtain per capita consumption. The sample period is from 1889 to 1985. The EM algorithm described in the Appendix is employed to estimate the following Markov switching process:

$$c_t - c_{t-1} = a_0(1 - S_t) + a_1 S_t + e_t \tag{4}$$

$$d_t - d_{t-1} = b_0(1 - S_t) + b_1 S_t + v_t \tag{5}$$

$$\mathrm{Prob}(S_t = j | S_{t-1} = i) = \phi_{ij} \quad i, j = 0, 1. \tag{6}$$

where: $\begin{pmatrix} e_t \\ v_t \end{pmatrix}$ are error terms with an i.i.d. normal distribution $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & \sigma_{cd} \\ \sigma_{cd} & \sigma_d^2 \end{pmatrix}$, where σ_c^2 and σ_d^2 denote the variances, and σ_{cd} denotes the covariance; S_t (= 0 or 1) denotes the unobserved states of the system, where 0 denotes a boom state and 1 denotes a depression state; c_t and d_t are the logarithms of per capita consumption and real dividends at time period t, respectively; a_0 and b_0 are the consumption and dividend growth rates in a boom state, respectively; a_1 and b_1 are the consumption and dividend growth rates in a depression state, respectively; and ϕ_{ij} is the probability that consumption and dividends will be in state j next period, given that they are in state i this period.

Parameter	Estimate	Standard Error
a_0	2.04%	0.38%
b_0	3.26%	0.94%
a_1	−1.69%	1.61%
b_1	−31.01%	4.16%
ϕ_{00}	0.956	0.020
ϕ_{11}	0.300	0.200
σ_c	3.61 (%)	0.52 (%)
σ_d	8.89 (%)	1.32 (%)
σ_{cd}	12.45 $(\%)^2$	3.59 $(\%)^2$

high-growth state next period, given that they are in high-growth state this period, is estimated to be 0.956. If dividends and consumption are presently depressed, there is a probability of 0.300 that both will continue to be depressed. This transition matrix implies that the unconditional probabilities of both consumption and dividends being in either a boom state or a depression state are 0.941 and 0.059 respectively.

Since the implied rates of returns may depend on the specification of the driving process for the market fundamentals, a two-state symmetric Markov chain as in Mehra and Prescott (1985) is adopted as an alternative to the Markov switching process. The two-state symmetric Markov chain is estimated by the method of moments. Five sample moments are used to pin down five parameters values. These five moments are the means and variances of both consumption and dividend growth rates and first-order autocorrelation of the consumption growth rate.[6] The results are reported in Table II. The standard errors are calculated by using a just identified Generalized Method of Moments (GMM) procedure as described in the appendix.

V. The Results

A. The Equity Premium Puzzle

In this section the implied asset returns are calculated using the derivations in Section III. As stated earlier, both the implied equilibrium risk premium and the implied risk-free rate are functions of the transition probabilities; the means and variances of consumption and dividend growth rates; the discount factor β; the coefficient of relative risk aversion, γ; and the elasticity of intertemporal substitution, $1/\rho$.

The sample averages for the risk premium and the risk-free rate for U.S. data, which the above asset pricing model attempts to match, are 6.18 and 0.80 percent respectively. To understand the influence of the preference parameters on both the implied risk premium and the implied risk-free rate in this economic environment, the two implied asset returns are calculated,

[6] This procedure restricts $\sigma_c^2 = \sigma_d^2 = \sigma_{cd} = 0$ and $\phi_{00} = \phi_{11}$ in the Markov switching process. See Mehra and Prescott (1985) for details.

Table II

Estimates of the Mehra-Prescott Model

Annual nominal stock prices are from the January S & P Composite Stock Price Index, which are extended back to 1889 by using data in Cowles (1939). Annual nominal dividends are total dividends per share accruing to the index. The real dividends are the nominal Standard & Poor's data deflated by the Consumer Price Index. The data on real consumption start with the Hendrick data reported in Balke and Gordon (1986) and continue in 1929 with the National Income and Product Accounts series. Aggregate consumption in each year is divided by population in that year to obtain per capita consumption. The sample period is from 1889 to 1985. The method of moments is used to estimate the Mehra and Prescott model. There are five moments in the system: the means and variances of both consumption and dividend growth rates and the first order autocorrelation of consumption growth rate. The standard errors are calculated by using a just identified GMM procedure with the constant being the only instrumental variable. Parameters a_0 and b_0 are the consumption and dividend growth rates in a boom state, respectively. Parameters a_1 and b_1 are the consumption and dividend growth rates in a depression state, respectively. Parameter ϕ_{00} is the probability that consumption and dividends will continue to be in a boom state next period, given that they are in a boom state this period.

Parameter	Estimate	Standard Error
a_0	5.54 (%)	0.45 (%)
b_0	13.25 (%)	2.01 (%)
a_1	−1.90 (%)	0.69 (%)
b_1	−10.81 (%)	2.45 (%)
ϕ_{00}	0.466	0.024

for each combination of β, γ, and $1/\rho$, from the following set:

$$\beta \in \{0.98\}, \qquad \gamma \in \{0.5, 2, 5, 10, 20\}, \qquad 1/\rho \in \{0.05, 0.1, 0.2, 0.5, 2\}.$$

Columns three and four in Table III show the implied risk-free rate and the implied risk premium, respectively. There are some noticeable patterns in this table. First, holding $1/\rho$ fixed, increasing γ increases the implied risk premium and decreases the implied risk-free rate. Second, holding γ fixed, the implied risk-free rate increases and the implied risk premium decreases if $1/\rho$ is decreased. The results in Table III show the satisfactory performance of an asset pricing model with nonexpected utility to match the sample

Table III

Implied Moments for Asymmetric Market Fundamentals

This table reports the implied moments of the risk-free rate and the risk premium for the asset pricing model endowed with the Markov switching process. These implied moments are calculated according to the methodology described in Section III. $1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and Corr. denotes the correlation between the implied risk-free rate and the implied risk premium. H is the test statistic for the null hypothesis that the asset pricing model matches all five first and second moments of the risk-free rate and the risk premium. The 5 percent critical value for the H test is 11.07.

Parameters		Mean (%)		Standard Deviation (%)			
$1/\rho$	γ	Risk-Free Rate	Equity Premium	Risk-Free Rate	Equity Premium	Corr.	H
2	0.5	2.96	0.10	0.12	8.61	0.00	88.58
2	2	2.80	0.45	0.13	8.72	−0.02	88.86
2	5	2.46	1.20	0.15	9.00	−0.04	86.65
2	10	1.90	2.66	0.18	9.58	−0.10	83.94
2	20	0.70	6.87	0.26	11.28	−0.24	77.44
0.5	0.5	5.86	0.07	0.47	8.46	−0.00	122.59
0.5	2	5.53	0.41	0.48	8.55	−0.01	116.13
0.5	5	4.87	1.15	0.51	8.78	−0.04	104.88
0.5	10	3.76	2.56	0.56	9.25	−0.10	91.15
0.5	20	1.52	6.54	0.66	10.73	−0.22	75.13
0.2	0.5	11.92	0.01	1.23	8.18	0.00	134.93
0.2	2	11.23	0.35	1.25	8.23	0.00	124.36
0.2	5	9.85	1.07	1.29	8.35	−0.03	108.42
0.2	10	7.58	2.39	1.35	8.63	−0.08	92.64
0.2	20	3.16	5.93	1.47	9.66	−0.20	77.35
0.1	0.5	22.80	−0.06	2.65	7.79	0.01	45.60
0.1	2	21.41	0.27	2.67	7.75	0.00	34.19
0.1	5	18.68	0.98	2.71	7.69	−0.02	17.73
0.1	10	14.25	2.19	2.77	7.65	−0.06	5.94
0.1	20	5.87	5.08	2.84	7.93	−0.16	18.40
0.05	0.5	47.86	−0.13	6.13	7.25	0.02	103.73
0.05	2	44.69	0.24	6.12	7.02	0.01	94.55
0.05	5	38.53	0.93	6.07	6.57	0.00	75.65
0.05	10	28.80	1.99	5.97	5.84	−0.03	50.32
0.05	20	11.28	3.84	5.68	4.51	−0.08	29.76

averages of the risk-free rate and the risk premium. For instance, the implied risk premium is 6.54 percent and the implied risk-free rate is 1.52 percent when β is equal to 0.98, γ is equal to 20, and $1/\rho$ is equal to 0.5. These implied asset returns correspond to the same magnitudes of their sample counterparts in the U.S. data.[7] As previously shown, when γ is equal to ρ the agent has a von Neumann-Morgenstern time additive expected utility. It can be seen in Table III that when γ is equal to ρ the implied risk-free rate is too high and the implied risk premium is too low to match their sample averages.[8] As noted earlier, the Mehra and Prescott (MP) specification is a "restricted" version of the Markov switching process. The implied risk-free rate and the implied risk premium are also calculated for the MP specification. These two implied asset returns are shown in column three and column four in Table IV. The results in the MP specification are similar to those in the Markov switching process, although the MP specification generates a higher risk premium.

The results show that the equity premium puzzle can be resolved by adopting Kreps and Porteus's class of nonexpected utility. The resolution of the equity risk premium puzzle is due to the following reasons. First, using aggregrate dividends rather than aggregate consumption as the payoff to the stock market decreases the covariance between the equity return and the marginal rate of substitution and hence increases the equity return. Second, freeing the elasticity of intertemporal substitution from being restricted to the inverse of the coefficient of relative risk aversion widens the parameter space for the elasticity of intertemporal substitution. The coefficient of relative risk aversion is around 20 to match the sample average of the risk premium. If the elasticity of intertemporal substitution is allowed to be higher than 0.05 (which is the inverse of 20), this helps reduce the implied risk-free rate to its sample average and thus resolves the puzzle.[9]

B. Five First and Second Moments

In this subsection, the ability of the model to simultaneously match the sample means and sample variances of the risk-free rate and the risk premium, as well as the sample covariance between these two rates of return is investigated. The sample standard deviations of the risk-free rate and the risk premium are 5.67 and 16.67 percent in the U.S. data, respectively. The

[7] There are lots of combination of the parameter values which are capable of exactly matching the sample averages of the risk premium and the risk-free rate. One such combination is $\beta = 0.99$, $\gamma = 19.46$ and $1/\rho = 0.43$.

[8] Moreover, restricting ρ being equal to γ, and searching over the ranges of γ between 0 and 30 and β between 0 and 1, it is impossible to find any combination of γ and β which is capable of matching the sample averages of the risk-free rate and the risk premium.

[9] Several other studies also offer resolutions to the equity premium puzzle within the framework of complete and frictionless markets. For example, Kocherlakota (1990b) resolves the puzzle by adopting a discount factor larger than one. Constantinides (1990) uses habit formation.

Table IV

Implied Moments for the Mehra-Prescott Model

This table reports the implied moments of the risk-free rate and the risk premium for the asset pricing model endowed with the symmetric Markov chain. These implied moments are calculated according to the methodology described in Section III. $1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and Corr. denotes the correlation between the implied risk-free rate and the implied risk premium. H is the test statistic for the null hypothesis that the asset pricing model matches all five first and second moments of the risk-free rate and the risk premium. The 5 percent critical value for the H test is 11.07.

Parameters		Mean (%)		Standard Deviation (%)			
$1/\rho$	γ	Risk-Free Rate	Equity Premium	Risk-Free Rate	Equity Premium	Corr.	H
2	0.5	2.91	0.21	0.13	11.60	0.00	66.71
2	2	2.77	0.81	0.13	11.65	0.00	66.61
2	5	2.49	2.00	0.12	11.76	0.00	65.77
2	10	2.06	3.96	0.11	11.97	0.01	64.13
2	20	1.35	7.43	0.06	12.43	0.02	64.25
0.5	0.5	5.67	0.26	0.52	12.27	0.00	92.47
0.5	2	5.37	0.90	0.52	12.31	0.00	89.91
0.5	5	4.77	2.15	0.51	12.39	0.00	84.58
0.5	10	3.82	4.19	0.48	12.55	0.01	74.13
0.5	20	2.20	7.73	0.38	12.87	0.02	57.58
0.2	0.5	11.41	0.38	1.38	13.66	0.00	85.86
0.2	2	10.76	1.08	1.37	13.66	0.00	76.44
0.2	5	9.48	2.47	1.34	13.68	0.00	57.70
0.2	10	7.45	4.66	1.26	13.71	0.01	39.07
0.2	20	3.94	8.32	1.06	13.76	0.03	26.02
0.1	0.5	21.68	0.59	3.00	16.07	0.00	79.17
0.1	2	20.37	1.41	2.97	16.01	0.00	61.32
0.1	5	17.81	3.02	2.88	15.89	0.01	50.50
0.1	10	13.78	5.48	2.69	15.68	0.01	37.06
0.1	20	6.90	9.33	2.23	15.27	0.03	31.85
0.05	0.5	45.24	1.07	7.16	21.29	0.00	64.51
0.05	2	42.25	2.16	7.00	21.06	0.00	55.55
0.05	5	36.49	4.22	6.67	20.60	0.01	43.36
0.05	10	27.63	7.23	6.05	19.83	0.02	34.54
0.05	20	13.11	11.42	4.77	18.33	0.05	32.90

sample correlation between these two rates of returns is -0.06.[10] The corresponding implied second moments from the model are calculated for the same set of the parameters values as in the previous subsection. Columns five to seven in Tables III report these results. There are some noticeable patterns regarding the effects of the change of parameters values on the second moments. When $1/\rho$ is increased, holding γ fixed, the standard deviation of

[10] The numbers for the sample means and standard deviations of the risk-free rate and the risk premium are taken from Mehra and Prescott (1985). The sample covariances are calculated by using the equity returns and the risk-free rate in the data set of Cecchetti, Lam, and Mark (1990a). The qualitative results do not change much when all five sample moments are calculated from the data set.

the implied risk-free rate decreases and the standard deviation of the implied risk premium increases. Holding $1/\rho$ fixed, increasing γ increases the standard deviation of both the risk-free rate and the risk premium when $1/\rho$ is bigger than 0.1. However, when $1/\rho$ is less than 0.1, the influences are reversed. Increasing either $1/\rho$ or γ will decrease the correlation betwen the implied risk-free rate and the implied risk premium. In general, the model generates a negative correlation between the risk premium and the risk-free rate.

There is no set of parameter values for which the model can exactly match all five first and second moments simultaneously. Following Cecchetti, Lam, and Mark (1990b), a chi-square test is employed to test whether the implied moments from the model is able to match their population counterparts. Let Λ be the 5×1 vector of the population moments which the asset pricing model attempts to match. Let Θ be the vector of parameters of the Markov switching process. Recall that each individual implied moment is a function of the preference parameters and the estimates of the parameters of the Markov switching process. Let $f(\Theta, \beta, \gamma, 1/\rho)$ be the corresponding 5×1 vector of the implied moments from the model. Then, the null hypothesis in this calibration exercise is

$$\mathrm{H}_0: f\left(\Theta, \beta, \gamma, \frac{1}{\rho}\right) = \Lambda.$$

There are two sources of uncertainty in the calibration exercise. One is from the estimation of the sample moments. The other one is from the calculation of the implied sample moments where the uncertainty is transformed from the estimation of the parameters of the Markov switching process.

Let Λ_T be the 5×1 sample moment vector, then Λ_T has the following distribution:

$$T^{1/2}(\Lambda_T - \Lambda) \to N(0, \Omega_\Lambda), \tag{23}$$

where $\to$ denotes convergence in distribution and Ω_Λ is the covariance matrix of Λ.

The estimator of the parameters of the Markov switching process, Θ_T, has the following distribution:

$$T^{1/2}(\Theta_T - \Theta) \to N(0, \Omega_\Theta), \tag{24}$$

where Ω_Θ is the covariance matrix of Θ.

It can be shown that under the null hypothesis, the test statistic

$$H = T\left[\Lambda_T - f\left(\Theta_T, \beta, \gamma, \frac{1}{\rho}\right)\right]' \Omega^{-1} \left[\Lambda_T - f\left(\Theta_T, \beta, \gamma, \frac{1}{\rho}\right)\right], \tag{25}$$

is a χ^2 distribution with five degrees of freedom and where[11]

$$\Omega = \Omega_\Lambda + \frac{\partial f}{\partial \Theta'}\Omega_\Theta \frac{\partial f}{\partial \Theta} - T\left\{ \frac{\partial f}{\partial \Theta'} E[(\Theta_T - \Theta)(\Lambda_T - \Lambda)'] + E[(\Theta_T - \Theta)'(\Lambda_T - \Lambda)] \frac{\partial f}{\partial \Theta} \right\}.$$

The last column in Table III reports the test statistic H. The critical value for 95 percent confidence interval of a $\chi^2(5)$ is 11.07. For the parameter values considered in Table III, the model is not rejected when $\gamma = 10$ and $1/\rho = 0.1$. There is a wide range of values for the preference parameters for which the model is not rejected. For $\beta = 0.98$, the parameter values for $1/\rho$ which are capable of statistically matching all five first and second moments are between 0.08 and 0.11. The lower and upper bounds for the parameter γ for which the model is not rejected are 7 and 20.2 respectively.

In order to compare the Markov switching process to the symmetric Markov chain, the implied second moments and the test statistic H for the MP specification are also calculated for the same set of parameter values as in the previous subsection.[12] The results are also shown in Table IV. It can be seen that the MP specification is incapable of matching five first and second moments. The comparative statics of the implied second moments for the MP specification are somewhat different from those for the Markov switching process. Holding γ fixed, the implied standard deviation for both the risk premium and the risk-free rate decrease when $1/\rho$ is increased. Holding $1/\rho$ fixed, increasing γ decreases the standard deviation of the risk-free rate, but has little influence on the standard deviation of the implied risk premium.[13]

Unlike the Markov switching process, the MP specification generates a positive correlation between the implied risk premium and the risk-free rate for the parameter values investigated here. Since the investor demands a larger compensation when he faces the risk of recession, the risk premium is higher in a depressed state than in a boom state. Moreover, because of the symmetric Markov chain and the probability of continuing to stay at the same state next period is estimated to be less than 0.5 in the MP specification ($\phi_{00} = \phi_{11} = 0.466$ in the model), the risk-free rate in a boom state is always lower than that in a depressed state. Hence, the correlation between the

[11] The calculation of Ω is described in the Appendix.

[12] Notice that in the case of the MP specification, Θ is a 5×1 vector instead of a 9×1 vector as in the case of the Markov switching process.

[13] Although the magnitudes of the means and variances of the implied risk premium and the risk-free rate are different, the comparative static results in an independent paper by Kandel and Stambaugh (1991) are similar to those for the MP specification (but not to those for the Markov switching process). Kandel and Stambaugh employ a univariate consumption process as the driving process for the market fundamentals. They use a leverage ratio of 0.44 for the levered equity. They follow Mehra and Prescott's approach to use the method of moments to estimate a four-state symmetric Markov chain for the monthly growth rate of consumption. Kandel and Stambaugh show that the means of the risk premium and the risk-free rate as well as the standard deviation of the risk premium can be matched when $\gamma = \rho = 29$. But they do not investigate the correlation between the risk premium and the risk-free rate.

implied risk premium and the implied risk-free rate are positive in the MP specification. But, in the Markov switching process, the implied risk-free rate is positive in the MP specification. But, in the Markov switching process, the implied risk-free rate in a boom state may be higher than that in a depressed state. This results in the negative correlation between the risk premium and the risk-free rate when the market fundamentals follow the Markov switching process.[14]

The test statistic H in Table IV shows that an equilibrium asset pricing model is always rejected when the market fundamentals follow the MP specification.[15] In terms of matching all five first and second moments of the risk premium and the risk-free rate simultaneously, asymmetric market fundamentals are superior to the MP specification. One reason is that asymmetric market fundamentals are capable of matching the sample correlation (-0.06) between the risk premium and the risk-free rate, while the MP specification is incapable of producing a negative correlation. Another reason is that the uncertainty of the parameter estimation of the Markov switching process may result in a larger variance-covariance matrix of the implied moments than that which results from the uncertainty of the estimation of the symmetric Markov chain. This helps reduce the normalized distance between the implied moments and their population counterparts and hence decrease the test statistic H.

VI. Conclusion

This paper studies a general equilibrium asset pricing model with nonexpected utility and asymmetric market fundamentals. The results show that the separation between the coefficient of relative risk aversion and the elasticity of intertemporal substitution is capable of resolving the equity premium puzzle. It is shown that the implied second moments for the bivariate Markov switching process are different from those for the symmetric Markov chain. Particularly, an asset pricing model endowed with the symmetric Markov chain is incapable of generating the negative sample correlation between the risk premium and the risk-free rate. Although it is not capable of exactly matching all five first and second moments of the risk premium and the risk-free rate, an equilibrium asset pricing model endowed with asymmetric market fundamentals cannot be rejected by the U.S. data. The parameter value for the elasticity of intertemporal substitution for which the asset pricing model is not rejected is around 0.1, while the parameter value for the coefficient of relative risk aversion ranges from 7 to 20.

[14] These statements regarding the conditional moments certainly depend on the specification of the stochastic process for the market fundamentals and the preference parameter values.

[15] Furthermore, searching over the following set, $\beta \in (0.95, 0.98)$, $\gamma \in (0, 25)$ and $1/\rho \in (0, 2)$, there is no set of parameter values which results in the acceptance of the model. However, if β is allowed to be larger than one, an equilibrium asset pricing model endowed with the MP specification cannot be rejected for some combinations of the parameters values. One such combination is $\beta = 1.08$, $\gamma = 10$, and $1/\rho = 0.08$, which has a test statistic H of 8.03.

Appendix

This appendix describes how the variance-covariance matrix Ω in Section V is calculated. In the EM algorithm, the estimators of the parameters of the Markov switching process are recursively weighted least squares. The weights are the full-sample smoothed probabilities $P(S_t|g_T, h_T, \ldots, g_1, h_1)$ and $P(S_t, S_{t-1}|g_T, h_T, \ldots, g_1, h_1)$ where T is the number of observations, $g_t = (c_t - c_{t-1})$ and $h_t = (d_t - d_{t-1})$ are the logarithmic consumption growth rate and the logarithmic dividend growth rate at time t, respectively. Recall that a_0 and b_0 are the consumption and dividend growth rates in a boom state, respectively. The consumption and dividend growth rates in a depression state are denoted by a_1 and b_1, respectively. Parameter ϕ_{ij} is the probability that consumption and dividends will be in state j next period, given that they are in state i this period. The estimates of the parameters can be calculated as follows:

$$a_0 = \sum_{t=1}^{T} g_t P(S_t = 0|g_T, h_T, \ldots, g_1, h_1) \div \sum_{t=1}^{T} P(S_t = 0|g_T, h_T, \ldots, g_1, h_1), \quad \text{(A1)}$$

$$b_0 = \sum_{t=1}^{T} h_t P(S_t = 0|g_T, h_T, \ldots, g_1, h_1) \div \sum_{t=1}^{T} P(S_t = 0|g_T, h_T, \ldots, g_1, h_1), \quad \text{(A2)}$$

$$a_1 = \left\{ \sum_{t=1}^{T} g_t P(S_t = 1|g_T, h_T, \ldots, g_1, h_1) \div \sum_{t=1}^{T} P(S_t = 1|g_T, h_T, \ldots, g_1, h_1) \right\}, \quad \text{(A3)}$$

$$b_1 = \left\{ \sum_{t=1}^{T} h_t P(S_t = 1|g_T, h_T, \ldots, g_1, h_1) \div \sum_{t=1}^{T} P(S_t = 1|g_T, h_T, \ldots, g_1, h_1) \right\}, \quad \text{(A4)}$$

$$\sigma_c^2 = \left\{ \sum_{t=1}^{T} (g_t - a_0)^2 P(S_t = 0|g_T, h_T, \ldots, g_1, h_1) + \sum_{t=1}^{T} (g_t - a_1)^2 P(S_t = 1|g_T, h_T, \ldots, g_1, h_1) \right\} / T, \quad \text{(A5)}$$

$$\sigma_{cd} = \left\{ \sum_{t=1}^{T} (g_t - a_0)(h_t - b_0) P(S_t = 0 | g_T, h_T, \ldots, g_1, h_1) + \sum_{t=1}^{T} (g_t - a_1)(h_t - b_1) P(S_t = 1 | g_T, h_T, \ldots, g_1, h_1) \right\} / T, \quad \text{(A6)}$$

$$\sigma_d^2 = \left\{ \sum_{t=1}^{T} (h_t - b_0)^2 P(S_t = 0 | g_T, h_T, \ldots, g_1, h_1) + \sum_{t=1}^{T} (h_t - b_1)^2 P(S_t = 1 | g_T, h_T, \ldots, g_1, h_1) \right\} / T, \quad \text{(A7)}$$

$$\phi_{00} = \left\{ \sum_{t=2}^{T} P(S_t = 0, S_{t-1} = 0 | g_T, h_T, \ldots, g_1, h_1) \right\} \div \left\{ \sum_{t=2}^{T} P(S_{t-1} = 0 | g_T, h_T, \ldots, g_1, h_1) + \psi_0 - P(S_1 = 0 | g_T, h_T, \ldots, g_1, h_1) \right\}, \quad \text{(A8)}$$

and

$$\phi_{11} = \left\{ \sum_{t=2}^{T} P(S_t = 1, S_{t-1} = 1 | g_T, h_T, \ldots, g_1, h_1) \right\} \div \left\{ \sum_{t=2}^{T} P(S_{t-1} = 1 | g_T, h_T, \ldots, g_1, h_1) - \psi_0 + P(S_1 = 0 | g_T, h_T, \ldots, g_1, h_1) \right\}. \quad \text{(A9)}$$

The smoothed probabilities depend on the estimates of parameters and change in each iteration. The smoothed probabilities employed here are those in the last iteration in which the estimates of the parameters reach the convergence criteria. These are nine moment equations. The sample estimates of the means, variances, and covariance of the risk-free rate and the risk premium are another five moment equations. A just identified GMM procedure is applied to these combined fourteen moment equations (i.e., constant is the only instrument). The Newey-West method with fourteen lags is used to calculate the weighting matrix in the GMM procedure. Ω_Λ, Ω_Θ, and $T * E[(\Theta_T - \Theta)(\Lambda_T - \Lambda)']$ are taken from the appropriate submatrices of the variance-covariance of the GMM estimators.

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