

Payments for Order Flow on Nasdaq

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ABSTRACT

We present a model of Nasdaq that includes the two ways in which marketmakers compete for order flow: quotes and direct payments. Brokers in our model can execute small trades through a computerized system, preferencing arrangements with marketmakers, or vertical integration into market making. The comparative statics in our model differ from those of the traditional model of dealer markets, which does not capture important institutional features of Nasdaq. We also show that the empirical evidence is inconsistent with the traditional model, which suggests that preferencing and vertical integration are important components in understanding Nasdaq.

THE TRADITIONAL MODEL OF DEALER MARKETS assumes a competitive market among marketmakers who actively adjust their quotes to compete for order flow. Although such a model is still implicitly assumed in most of the empirical literature, it ignores important institutional features of actual dealer markets such as Nasdaq. In particular, it does not take into account the variety of alternatives available to brokers to achieve execution of their order flow, or the ability of marketmakers to compete in ways other than through their quotes. For example, Nasdaq marketmakers can purchase orders from brokers under preferencing arrangements, which involve explicit payments by dealers to brokers.¹ Brokers can vertically integrate into market making and handle execution of their orders internally; this practice is referred to as internalization. Our goal in this paper is to develop a more realistic model of a dealer market and to contrast its predictions with those of a traditional model that assumes competition only through quotes.

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¹ Payments in 1995 were as high as three cents per share. Contracts specify the nature of orders the dealer must accept—small orders for stocks that trade above a certain price and have a quoted spread above some minimum amount. Additional restrictions typically require that brokers send all qualified orders to the same dealer, that there be at least a minimum order flow per month, and that there be no limit orders, professional orders, or program orders. Stocks with a tick size of one-sixteenth or less are frequently excluded (see NASD Report (1991)).

In our model, brokers choose whether to execute their trades through a computer matching system such as Nasdaq's Small Order Execution System (SOES), through preferencing arrangements, or through vertical integration into market making. The three venues have different cost structures: SOES provides almost costless execution and does not involve side payments; preferencing brings in a per-share payment, but a system of exclusive dealing with a marketmaker is costly to set up and maintain; and vertical integration requires even larger capital outlays, but has higher profit margins. We allow marketmakers to choose whether to make market in a stock (entry and exit) and whether to enter preferencing arrangements with brokers. Brokers and marketmakers interact anonymously for nonpreferenced trades, but they precommit to preferred trades. The model does not deal with large trades, which are typically negotiated over the phone or executed through venues not accessible to retail traders, such as Nasdaq's Selectnet or Reuter's Instinet.

The model assumed in most of the existing literature ignores preferencing and vertical integration. In such a model, an exogenous increase in the profit margin causes marketmakers to enter, and a decrease causes them to exit. Our model predicts a more complex response because the payment for order flow is an endogenous variable in our model. In our model, exogenous changes that decrease profit margins, such as a decrease in the tick size, can cause an increase in the number of marketmakers. The reason for this is that a decrease in the profit margin for marketmakers lowers the equilibrium payment for order flow, which shifts brokerage volume away from preferencing marketmakers and toward SOES marketmakers. The decrease in the number of preferencing marketmakers and vertically integrated brokers can be more than offset by an increase in the number of SOES-only marketmakers. A decrease in the profit margin can even result in the elimination of preferencing and vertical integration. Exogenous changes that increase profit margins (such as a change in the effective tick size, which could be caused by odd-eighth quote avoidance) can cause marketmakers with no preferred trades to exit the market, leading to a decrease in the total number of marketmakers.

Though an increase in volume generally leads to an increase in the number of marketmakers, as is true in a model without preferencing or vertical integration, in our model an increase in volume can also result in a decrease in the number of marketmakers. This can occur if the increase in volume causes a shift away from SOES trading and toward preferencing arrangements and vertical integration. We also show that in a market in which all types of marketmakers are present, a regulatory prohibition on preferencing arrangements causes the total number of marketmakers to increase.

We show that the empirical evidence is inconsistent with a model that only allows competition through quotes, but is consistent with our model. Christie and Schultz (1997) show that large changes in average spreads, which are driven by the exogenous changes in the effective tick size, do not lead to a decrease in the number of marketmakers. Barclay et al. (1998) also show that the significant decrease in profit margins associated with the rules changes on Nasdaq in January 1997 does not lead to a decrease in the number of marketmakers. In fact, they show that the number of market-

makers per stock increases after the rules changes. These results cannot be explained by a model that does not allow preferencing or vertical integration, but they are consistent with our model. We obtain similar results using data on IPOs on Nasdaq for the period 1990 to 1994.

Our paper contributes to a growing literature on payments for order flow. Recent contributions include Glosten (1989), Hasbrouck (1995), Chordia and Subrahmanyam (1995), Battalio and Holden (1996), Bloomfield and O'Hara (1996), Easley, Kiefer, and O'Hara (1996), Godek (1996), Macey and O'Hara (1996), and Battalio (1997). Most of these papers study payments for order flow designed to steer order flow from the NYSE to the regional exchanges (Godek (1996) is an exception). We focus on agreements and payments designed to steer order flow for a Nasdaq stock to a particular marketmaker for that stock.²

Marketmakers are willing to pay for order flow because of rents that exist in market making; bid-ask spreads typically exceed marginal cost of market making.³ Glosten (1989) attributes rents on the NYSE to the unique position of the specialist; Chordia and Subrahmanyam (1995) attribute them to the minimum tick size, which constrains spreads to be larger than the marginal cost of market making. Godek (1996) views the rents as being caused primarily by the existence of preferencing arrangements, which he takes as a primitive.⁴ Our approach differs from that of Godek (1996), but is similar to that of Glosten (1989) and Chordia and Subrahmanyam (1995) in that we view preferencing and vertical integration as behavior that occurs in equilibrium because of rents in market making, which are caused by the relatively large effective tick size and the NASD rules that allow quoted prices to satisfy a broker's best execution requirement.⁵ In our model, average spreads, preferencing, and vertical integration are derived as equilibrium outcomes; we take only the market structure as given.

The model is described in Section I. Readers not interested in the technical details of the model can skip to Section II, which contains our analytic and simulation results. Most of the results and conclusions in Section II can be understood without reading Section I. Section III discusses empirical evidence, and Section IV concludes.

² Payments for NYSE order flow can create a cream-skimming effect, which can increase spreads. See Hasbrouck (1995), Battalio, Greene, and Jennings (1996), Battalio and Holden (1996), Easley et al. (1996), and Battalio (1997) for a discussion of cream skimming. The cream-skimming effect is not of great importance on Nasdaq because the price discovery process is already fragmented.

³ See Christie and Schultz (1994, 1997), Huang and Stoll (1996), Barclay (1997), and Bessembinder (1997) on the existence of economic rents in the market making activity on Nasdaq. Dutta and Madhavan (1997) and Kandel and Marx (1997) provide a theoretical basis for the existence of these rents. The presence of momentum traders on Nasdaq may also cause spreads, and thus rents, to increase.

⁴ Godek (1996) also argues that large spreads cause marketmakers to avoid odd-eighth quotes. Results in Bloomfield and O'Hara (1996), Kandel and Marx (1997), and the current paper contradict this argument.

⁵ Macey and O'Hara (1996) discuss the implications of this NASD rule and other legal aspects of payments for order flow.

I. Model

We consider an environment in which the decision makers are brokers and marketmakers. Brokers, who are endowed with some volume of customer orders, choose the optimal venue for the execution of their trades: SOES, preferencing arrangement, or vertical integration.⁶ For the purposes of the model, we ignore other options such as Selectnet, Instinet, or direct negotiations with a marketmaker. These assumptions are reasonable because we focus on the execution of retail orders not exceeding 1,000 shares. For these orders, preferencing, internalization, and SOES execution are the most likely choices.⁷ We avoid issues of discreteness by assuming real numbers of brokers and marketmakers and real numbers of trades.⁸

In the model, each marketmaker can receive a proportional share of the SOES trades and receives all the order flow from the brokers with which he has preferencing arrangements. Marketmakers pay brokers for each preferred trade they receive on a per-share basis. We assume that there is a mass m of brokers in a single stock with volume levels distributed according to probability density function $g(v)$ with support $[0, \infty)$ and finite moments. We assume that the market-making operation in a vertically integrated firm executes trades preferred to it from its own brokerage operation as well as trades it receives through SOES. Brokers who are not vertically integrated execute their trades through either preferencing arrangements or SOES.

We let n_s be the number of marketmakers handling only SOES trades, n_p be the number of marketmakers handling both preferred and SOES trades, and n_v be the number of vertically integrated brokers. We denote the total number of marketmakers, all of whom participate in SOES, by n , where $n \equiv n_s + n_p + n_v$. We denote the marginal per-trade cost of making market by C . Thus, the marginal cost of a round-trip trade is $2C$, which we assume is small relative to the effective tick size, which we denote by Δ . Throughout the paper we maintain the assumption that $2C < \Delta$. This assumption is for convenience only—it guarantees that spreads of one effective tick are sufficient to cover trading costs, but the results continue to hold without it.

A. Determination of Spreads

Throughout the paper, we assume that the effective tick size is given exogenously and does not depend on the other variables in the model. In particular, we assume that the effective tick size does not depend on the number of marketmakers. However, we do not assume any particular source for the effective tick size—it could be set by the exchange (the minimum tick size)

⁶ We do not model competition among brokers for clients and take the distribution of volume for the brokers as given. Competition among brokers is affected by many features of the market (for example, some payments received by brokers for their order flow may be passed through to their clients), but for simplicity we ignore these issues in our model.

⁷ Other arrangements, such as negotiations between brokers and marketmakers, are clearly important when considering large trades, but are not practical for small trades.

⁸ We also compute the equilibrium of the model with discreteness restrictions, and the qualitative results are unchanged.

or by coordination among marketmakers to avoid odd-eighth quotes. Later in the paper we provide some justification for our assumption by providing evidence that the degree of odd-eighths avoidance is uncorrelated with the number of marketmakers. Given this assumption, we show elsewhere (Kandel and Marx (1997)) that there exist at least two Nash equilibrium spreads. The discussion below provides the intuition, but we refer the reader to the earlier paper for details.⁹

Suppose that every period each of n marketmakers chooses bid and ask prices, which must be integer multiples of the effective tick size. Then κ buy and sell orders arrive in the market. Sell orders are split equally among the marketmakers with the highest bid, denoted by b , and buy orders are split equally among the marketmakers with the lowest ask, denoted by a . For intuition, suppose that inventory held at the end of the period is disposed of at the bid-ask midpoint price $(a + b)/2$. It is clear that the equilibrium spread must be greater than or equal to the round-trip trading cost $2C$. Now consider the upper bound on the equilibrium spread. If the other $n - 1$ marketmakers offer bid and ask prices b and a , then a single marketmaker will have an incentive to deviate by offering a higher bid or lower ask (i.e., reducing the spread) if the additional quantity received by that marketmaker provides increased profits despite the smaller spread. Thus, a marketmaker will deviate by offering prices $b + \Delta$ and $a - \Delta$ if and only if the profit from deviating is greater than the profit from offering prices b and a , that is,

$$\kappa((a - \Delta) - (b + \Delta) - 2C) > \frac{\kappa}{n}(a - b - 2C). \quad (1)$$

Rearranging, this implies that prices b and a are an equilibrium only if

$$a - b \leq 2C + 2\Delta \frac{n}{n - 1}. \quad (2)$$

Of course, other possible deviations must be considered as well, but one can show that condition (2) gives the upper bound on the spread $a - b$ such that no marketmaker has an incentive to deviate.

This result holds for any inventory disposal price, d , that is not too far from the bid-ask midpoint price, $(a + b)/2$. In particular, it holds for inventory disposal prices, d , that are within C of the bid-ask midpoint. The following proposition is proved in Kandel and Marx (1997):¹⁰

PROPOSITION 1: *If $|d - (a + b)/2| \leq C$, then all market makers choosing prices (a, b) is a Nash equilibrium if and only if $2C \leq a - b \leq 2C + 2\Delta(n/(n - 1))$.*

⁹ We do not expect that allowing the probability of avoidance to depend on the number of marketmakers would greatly change the results of our model, although it would make equilibria with an effective tick size of one-eighth relatively less likely.

¹⁰ In the notation of Kandel and Marx (1997), $|d - ((a + b)/2)| = \varepsilon_s = \varepsilon_b$.

Since we assume that $2C < \Delta$ and because a and b must satisfy $a - b \geq \Delta$, Proposition 1 implies that (a, b) are Nash equilibrium prices if and only if

$$\Delta \leq a - b \leq 2C + 2\Delta \frac{n}{n-1}. \quad (3)$$

Since the rightmost expression in equation (3) is greater than 2Δ , there always exist two Nash equilibria: one with a spread of one effective tick and another with a spread of two effective ticks. As long as the inventory disposal price, d , is not too different from $(a + b)/2$, no matter how large the number of marketmakers, there always exist at least two equilibria. Moreover, using equation (3), we can show that there exists n sufficiently large such that there are exactly two equilibria.

To understand the intuition for Proposition 1, note that price competition is usually expected to drive prices (here spreads) down to the level of marginal cost. This occurs because each competitor has an incentive to undercut the others by a small amount in order to gain a large increase in volume. However, when prices are discrete, the smallest price reduction can be as large as 50 percent. The associated increase in volume may not be sufficiently large to outweigh the cost of such a large reduction in price. Thus, with discrete prices, there can be equilibria with prices above the level of marginal cost. Discrete price effects, together with the fact that marketmakers compete on both the buy and sell sides of the market, lead to multiple equilibria.

In the remainder of the paper we assume that the parameter values are such that there exist exactly two equilibria—the number of marketmakers is sufficiently large not to allow more equilibria. In our numerical examples, we verify that the condition for having only two equilibrium spreads holds. A more elaborate model could allow the number of equilibrium spreads to depend on the number of marketmakers, but the discontinuities this introduces add significantly to the complexity of the model. In such a model, the range of possible spreads would be decreasing in the number of marketmakers, which would reinforce our results.¹¹

Since spreads typically must change when prices move, we further assume that spreads oscillate between the two equilibrium spreads over time. In order to adjust prices, marketmakers typically must increase or decrease the spread, and we expect that the adjustment process can terminate with a spread equal to either of the two equilibrium spreads. We let $\theta \in [0, 1]$ be the

¹¹ We would expect the profit on a round-trip trade to be a decreasing function of the number of marketmakers if a greater degree of coordination were required for a spread of two effective ticks to obtain. Kandel and Marx (1997) argue that the probability that price transitions occur through smaller spreads is increasing in the number of marketmakers. This is consistent with Wahal (1997), who shows that average per-share profit declines as the number of marketmakers increases, holding constant whether or not marketmakers avoid odd-eighth quotes. The order of magnitude of this effect is consistent with its being a result of changes in the frequency of spreads of one and two effective ticks. In a different setting, Dutta and Madhavan (1997) find that average spreads may be increasing in the number of marketmakers.

proportion of time that the spread is one effective tick and $1 - \theta$ be the proportion of time that the spread is two effective ticks (in our numerical examples, we assume a “neutral” value of $\theta = 0.5$, so spreads are equally likely to be either of the two equilibrium spreads). Our main qualitative results continue to hold if θ is an increasing function of the number of marketmakers, which we would expect if spreads are more likely to be small when there are more marketmakers.¹²

B. Marketmaker Profits

Using our assumption about the proportion of time that spreads are one effective tick or two effective ticks, the profit of a marketmaker on a round-trip SOES trade is $\Delta - 2C$ with probability θ and $2\Delta - 2C$ with probability $1 - \theta$. We let π denote the per-share profit on a round-trip trade, so

$$\begin{aligned}\pi &\equiv \theta(\Delta - 2C) + (1 - \theta)(2\Delta - 2C) \\ &= 2\Delta - 2C - \theta\Delta.\end{aligned}\tag{4}$$

The cost to a marketmaker of SOES trades is just the market-making cost of $2C$ for a round-trip trade. With preferred trades, a marketmaker has less control over the trades he has to execute because he cannot withdraw from the market by adjusting quotes as can be done in the SOES market. This increases the inventory holding costs associated with preferred trades. Thus, we assume that each marketmaker with preferred trades must bear an additional cost $f_m(\cdot)$, which is a function of the volume of preferred trades he handles. The magnitude of these costs can vary if increased volume provides significantly better information or if economies of scope are important for marketmakers with multiple stocks. We assume a particular functional form for the marketmakers’ cost for preferred trades, $f_m(x) = (\alpha/8)x^2$, where $\alpha > 0$.¹³

C. Demand for Preferred Trades

The cost structure faced by marketmakers results in a downward sloping demand curve for preferred trades. A marketmaker’s profit depends on the marketmaker’s volume of preferred round-trip trades x (resulting in $2x$ total trades) and the payment p that must be made to brokers for pref-

¹² If we allow θ to be an increasing function of the number of marketmakers, that is, if more intense competition causes the one-tick equilibrium to occur relatively more frequently, then average spreads, and hence profits on round-trip trades, are decreasing in the number of marketmakers (instead of being independent of the number of marketmakers). Also, the equilibrium payment for order flow is decreasing in the fixed cost associated with preferred trades, decreasing in the number of brokers, and increasing as the distribution of broker volume shifts to the right. The results of Panel B of Table I imply that average spreads would decrease if preferencing agreements and vertical integration were eliminated.

¹³ It is sufficient for our qualitative results that $f_m(0) = 0$ and that the function f_m be continuous, increasing, and convex.

erenced order flow. If the volume of unpreferred round-trip trades is K (endogenously determined), then in a symmetric equilibrium, each market-maker processes K/n unpreferred round-trip trades. Thus, the profit for a marketmaker handling volume x of preferred trades is

$$\begin{aligned}\Pi(n, p, x, K) &\equiv \left(x + \frac{K}{n}\right)\pi - 2px - f_m(2x) \\ &= \left(x + \frac{K}{n}\right)\pi - 2px - \frac{\alpha}{2}x^2.\end{aligned}\tag{5}$$

Fixing the payment for order flow, if a marketmaker's profit-maximizing volume of preferred trades x^* is positive, it satisfies the following first-order condition:

$$\pi - 2p = 2f'_m(2x^*).\tag{6}$$

If $\pi < 2p$, then profits are insufficient to cover the payments to the broker. In this case, the demand for preferred order flow is zero. Thus, the demand for preferred order flow by a single marketmaker, $d(p)$, is

$$d(p) = \max\left\{0, \frac{\pi - 2p}{\alpha}\right\}.\tag{7}$$

For $\pi - 2p > 0$, the demand for preferred trades by each marketmaker is linear and decreasing in p . Note that $d(p)$ is the demand for preferred trades by an individual marketmaker, so the total demand for preferred order flow is $n_p d(p)$.

D. Supply of Preferred Trades

Since brokers choose among three venues for executing trades, we must consider the profits associated with each option. For simplicity, we assume that brokers do not incur per-share transactions costs associated with routing orders to SOES or submitting preferred trades. However, brokers must incur fixed costs $F_p \geq 0$ or $F_v \geq 0$ to maintain a preferencing arrangement or a vertically integrated market-making division, respectively. We assume that $F_v > F_p$ because the fixed costs associated with vertical integration are typically much larger than those associated with preferencing arrangements. We also assume that a vertically integrated market-making division bears cost $f_b(\cdot)$, which is a function of the volume of trades the division handles. We assume a particular functional form for the brokers' cost for preferred trades, $f_b(v) \equiv (\beta/2)v$, where $0 < \beta < \Delta - 2C$.¹⁴

¹⁴ It is sufficient for our qualitative results that the function f_b is continuous and increasing and that $f_b(0) = 0$.

If a broker with volume v preferences its trades and obtains a payment for order flow of p per share, then the broker receives $2vp$ but incurs the cost F_p , for a total profit of $2vp - F_p$. If a broker with volume v vertically integrates, establishing its own market-making operation, the broker's profits are

$$v\pi - f_b(2v) - F_v = v\pi - bv - F_v. \quad (8)$$

Thus, the profit to a broker with volume v from preferencing are greater than those from vertical integration if

$$2vp - F_p > v\pi - bv - F_v. \quad (9)$$

A broker's profits from preferencing are greater than those from SOES if

$$2vp - F_p > 0. \quad (10)$$

And, a broker's profits from vertically integrating are greater than those from SOES if

$$v\pi - bv - F_v > 0. \quad (11)$$

In what follows, it will be convenient to let

$$\tilde{v} \equiv \frac{F_v}{\pi - \beta}, \quad (12)$$

which is the volume for which a broker is indifferent between vertical integration and SOES, and

$$\tilde{p} \equiv \frac{F_p}{2F_v} (\pi - \beta), \quad (13)$$

which is the price at which a broker with volume \tilde{v} is indifferent between SOES, preferencing, and vertical integration.

Using equations (9)–(13), we prove the following proposition, which describes how the trade-offs between the costs (both fixed and variable) and benefits of the brokers' different options affect their decisions.

PROPOSITION 2: *If p is the payment for order flow, then a broker with volume v maximizes profit by vertically integrating if $v > \phi_1(p)$, using SOES if $v < \phi_2(p)$, and preferencing otherwise, where ϕ_1 and ϕ_2 are given by:*

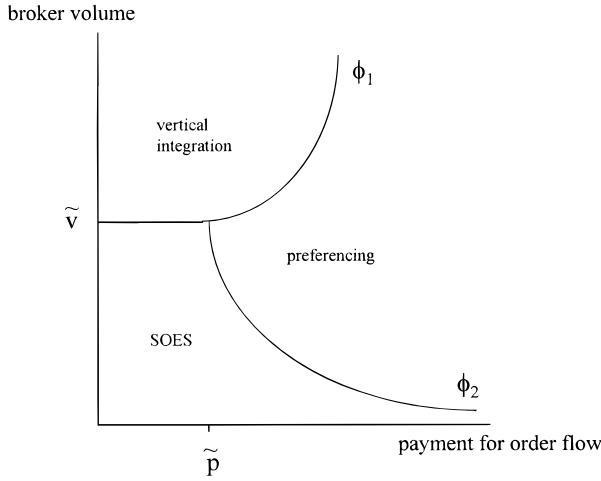


Figure 1. Broker choices by payment for order flow and broker volume. Regions in which brokers use SOES, preferencing, and vertical integration for different payments for order flow (horizontal axis) and the broker's volume (vertical axis).

$$\phi_1(p) = \begin{cases} \infty, & \frac{\pi - \beta}{2} \leq p \\ \frac{F_v - F_p}{\pi - 2p - \beta}, & \tilde{p} \leq p < \frac{\pi - \beta}{2} \\ \tilde{v}, & p < \tilde{p} \end{cases}$$

and

$$\phi_2(p) = \begin{cases} \frac{F_p}{2p}, & p \geq \tilde{p} \\ \tilde{v}, & p < \tilde{p}. \end{cases}$$

Proof: If $v > \phi_1(p)$, then equation (9) is not satisfied and equation (11) is satisfied, so a broker's profits are higher with vertical integration than with preferencing or SOES. If $v < \phi_2(p)$, then neither equation (10) nor equation (11) is satisfied, so a broker's profits are higher with SOES than with preferencing or vertical integration. If $v < \phi_1(p)$ and $v > \phi_2(p)$, then equations (9) and (10) are satisfied, so a broker's profits are higher with preferencing than with vertical integration or SOES. Using our definitions of \tilde{v} and \tilde{p} in equations (12) and (13), the result follows. Q.E.D.

The functions ϕ_1 and ϕ_2 are depicted in Figure 1. The figure shows that if $p < \tilde{p}$, then, regardless of volume, brokers do not use preferencing arrangements. Brokers with low volume use SOES and brokers with high volume

vertically integrate. If $p > \tilde{p}$, then brokers with low volume use SOES, broker with medium volume use preferencing arrangements, and brokers with high volume vertically integrate.

Using Proposition 2, we can derive the following corollary.

COROLLARY 1: *The number of vertically integrated brokers is $n_v = m \int_{\phi_1(p)}^{\infty} g(v) dv$; the supply of SOES trades is $K(p) \equiv m \int_0^{\phi_2(p)} v g(v) dv$; and the supply of preferred trades is*

$$S(p) \equiv \begin{cases} 0, & p \leq \tilde{p} \\ m \int_{\phi_2(p)}^{\phi_1(p)} v g(v) dv, & p > \tilde{p}. \end{cases} \quad (14)$$

The supply of preferred trades is continuous because $\phi_1(\tilde{p}) = \phi_2(\tilde{p})$. Since ϕ_2 is decreasing in p and ϕ_1 is increasing in p for $p \geq \tilde{p}$, Corollary 1 implies that, as the payment for preferred order flow increases, the supply of preferred order flow increases. This occurs because as the payment for order flow increases, brokers with a greater range of volumes preference their order flow.

Proposition 3 states that the supply curve is upward sloping. Its proof is in Appendix A.

PROPOSITION 3: *The supply of preferred trades is nondecreasing in p and strictly increasing in p for $p > \tilde{p}$.*

In equilibrium, the per-marketmaker preferred volume is $d(p)$ and total unpreferred volume is $K(p)$, so we can define the equilibrium profits of a marketmaker who handles preferred trades as a function of the number of SOES marketmakers, n , and the payment for order flow, p :

$$\begin{aligned} \Pi^*(n, p) &\equiv \Pi(n, p, d(p), K(p)) \\ &= \begin{cases} \pi \frac{K(p)}{n}, & \text{if } \pi \leq 2p \\ \frac{(\pi - 2p)^2}{2\alpha} + \pi \frac{K(p)}{n}, & \text{otherwise,} \end{cases} \end{aligned} \quad (15)$$

where the equality uses equations (5) and (7).

E. Equilibrium

The per-share payment for order flow and the number of marketmakers are determined in equilibrium and satisfy market clearing for preferred order flow, individual rationality for brokers (embedded in the supply curve for preferred trades), and individual rationality and no-entry conditions for the marketmakers.

We assume that marketmakers who are active only in SOES have reservation value $\bar{\Pi}_1 > 0$, and that marketmakers with both SOES and preferred trades have reservation value $\bar{\Pi}_2 > 0$. The reservation values represent fixed costs that must be covered, including outside options and entry costs. Based on our knowledge of the market, we postulate that $\bar{\Pi}_2 \geq F_v > \bar{\Pi}_1 \geq F_p$. This condition holds in the numerical examples used in the next section.

We can now define an equilibrium in terms of the payment for order flow, p^* , the number of marketmakers handling only SOES trades, n_s^* , the number of marketmakers handling preferred trades, n_p^* , and the number of vertically integrated brokers, n_v^* .¹⁵

Definition 1: $(p^*, n_s^*, n_p^*, n_v^*) \in \mathbb{R}_+^4$ is an equilibrium if the following four conditions are satisfied, where $n^* \equiv n_s^* + n_p^* + n_v^*$:

1. $S(p^*) = n_p^* d(p^*)$
2. if $n_s^* > 0$ then $\frac{K(p^*)}{n^*} \pi = \bar{\Pi}_1$,
and if $n_s^* = 0$ then $\frac{K(p^*)}{n^*} \pi \leq \bar{\Pi}_1$
3. if $n_p^* > 0$ then $\Pi^*(n^*, p^*) = \bar{\Pi}_2$,
and if $n_p^* = 0$ then $\Pi^*(n^*, p^*) \leq \bar{\Pi}_2$
4. $n_v^* = m \int_{\phi_1(p^*)}^{\infty} g(v) dv$.

The first condition is a market clearing condition and requires that supply be equal to demand for preferred trades. The second and third conditions are individual rationality and no-entry conditions for the two types of marketmakers. The fourth condition follows from Corollary 1 and specifies the number of brokers who vertically integrate. In our model, vertically integrated marketmakers are able to earn abnormal rents because of their captive order flow.

Given our assumptions on the parameter values, we can prove the following theorem.¹⁶ (The proof is in Appendix A.)

THEOREM 1: *For $\bar{\Pi}_2$ sufficiently large, there exist finite n_s^* , n_p^* , n_v^* , and p^* satisfying equilibrium conditions 1–4.*

¹⁵ In specifying the equilibrium conditions above, we have ignored possible economies of scope in preferencing. In reality a large number of brokers sell their order flow to a small number of large dealers whose investment in the system is amortized over a large number of stocks (3000–4000). Thus, marketmakers have some degree of monopsonistic power, and so the price paid for preferred order flow may be lower than that implied by the model. However, this does not change the intuition for the model.

¹⁶ As a reminder, the following is a summary of the assumptions being maintained: $\alpha, \bar{\Pi}_1, \bar{\Pi}_2$, and m are positive; $0 < \beta < \Delta - 2C$; $0 < 2C < \Delta$; $0 \leq \theta \leq 1$; $0 \leq F_p < F_v$; and the pdf $g(v)$ has support $[0, \infty)$ and finite moments.

As $\bar{\Pi}_2$ approaches zero, the number of marketmakers grows arbitrarily large, so Theorem 1 requires a lower bound on $\bar{\Pi}_2$; however, the lower bound constructed in the proof of Theorem 1 is not tight and the conditions for existence are satisfied for most values in our numerical examples. Furthermore, as is clear from the proof of Theorem 1 (see Lemma 3), if there exist n_s^* , n_p^* , n_v^* , and p^* satisfying equilibrium conditions 1–4 for a given $\bar{\Pi}_2$, then equilibria also exist for all larger values of $\bar{\Pi}_2$.

II. Results

In the previous section we define our model and show conditions under which an equilibrium exists. An equilibrium of the model consists of the number of marketmakers handling only SOES trades, n_s , the number of marketmakers handling preferenced trades, n_p , the number of vertically integrated brokers, n_v , and the payment for order flow, p . Given these values, we can calculate the average equilibrium bid-ask spread.

We can characterize the equilibria in our model by the different types of marketmakers that are present. Three types of equilibria are of interest: (i) equilibria with preferencing arrangements, marketmakers handling only SOES trades, and vertically integrated brokers all at the same time; (ii) equilibria with no preferencing; and (iii) equilibria with no marketmakers handling only SOES trades. For comparison, we also solve for the equilibrium under the assumption that preferencing and vertical integration are not allowed. This allows us to compare our results with a traditional model that does not have preferencing arrangements or vertical integration. Throughout, we maintain the assumption that $2C \leq \Delta$ and that there are only two Nash equilibrium spreads.

For the three types of equilibria, we obtain some analytic comparative static results. We also use numerical examples to obtain numerical comparative statics, analyze changes across types of equilibria, and provide illustrations.

A. A Model without Preferencing or Vertical Integration

Most of the existing literature on dealer markets assumes (either implicitly or explicitly) a model in which preferencing and vertical integration are not allowed. Applied to our model, this would mean that a broker's entire order flow is submitted to SOES. If we impose this restriction on our model, we can easily calculate the equilibrium number of marketmakers and derive comparative static results. We do this now so that we can contrast the results of a traditional model with the results of our model, which allows preferencing and vertical integration.

When there is no preferencing or vertical integration, the supply of SOES trades is $m \int_0^\infty v g(v) dv$ and the equilibrium number of SOES marketmakers,

n_s^{**} , is strictly positive (we use two asterisks to indicate equilibrium values in a model without preferencing or vertical integration). Thus, equilibrium condition 2 implies that

$$\frac{m \int_0^\infty vg(v) dv}{n_s^{**}} \pi = \bar{\Pi}_1. \quad (16)$$

The comparative static results in this case are unambiguous. The number of marketmakers is increasing in tick size and volume (either through an increase in the number of brokers or an increase in mean broker volume) and decreasing in marketmakers' reservation value, $\bar{\Pi}_1$.

Moreover, Proposition 4 states that, if preferencing and vertical integration are allowed and the equilibrium has some SOES-only marketmakers, the abolition of preferencing and vertical integration causes the total number of marketmakers to increase. When preferencing and vertical integration are eliminated, those trades must be executed by SOES marketmakers, and because SOES marketmakers have lower costs, in equilibrium the trades support more marketmakers than they did before.

PROPOSITION 4: *Suppose that, when preferencing and vertical integration are allowed, there are a positive number of SOES marketmakers, that is, $n_s^* > 0$. Then the total number of marketmakers is at least as large when preferencing and vertical integration are not allowed than when they are allowed, that is, $n_s^{**} \geq n^*$.*

Proof: If $n_s^* > 0$, then equilibrium condition 2 implies

$$\frac{m \int_0^{\phi_2(p^*)} vg(v) dv}{n^*} \pi = \bar{\Pi}_1,$$

which implies

$$\frac{m \int_0^\infty vg(v) dv}{n^*} \pi \geq \bar{\Pi}_1.$$

Then equation (16) implies that $n_s^{**} \geq n^*$. Q.E.D.

The numerical example that we discuss below gives results consistent with Proposition 5—the total number of marketmakers increases when we do not allow preferencing or vertical integration.

Table I
Equilibrium Values in the Numerical Example

Equilibrium values are shown for three different tick sizes. The case with a tick size of one-eighth is used as the base case in our analysis. Panel A shows the equilibrium numbers of marketmakers, payments for order flow, and average spreads. Panel B shows how the equilibrium number of marketmakers changes when preferencing and vertical integration are not allowed. One can verify that in all cases the condition for having only two equilibrium spreads is satisfied; that is, $2C + 2\Delta(n^*/(n^* - 1)) < 3\Delta$, when C is the marginal cost of market making and Δ is the tick size. We assume the proportion of time that the spread is one effective tick is one-half (i.e., $\theta = 0.5$).

Panel A						
Tick Size	Total Market-makers, n^*	SOES Market-makers, n_s^*	Preferencing Market-makers, n_p^*	Vertically Integrated Brokers, n_v^*	Payment for Order Flow, p^*	Average Spread
One-eighth	15.05	10.27	3.71	1.08	0.0442	0.1875
One-quarter	5.91	0.00	4.54	1.36	0.1364	0.3750
One-sixteenth	22.22	22.11	0.00	0.11	0.0068	0.0938

Panel B		
Tick Size	Total Market-makers (as in Panel A), n^*	Total Marketmakers without Preferencing or Vertical Integration, n_s^{**}
One-eighth	15.05	61.46
One-quarter	5.91	139.58
One-sixteenth	22.22	22.40

B. Equilibria with All Types of Marketmakers

For some parameter values, the equilibrium in our model has SOES-only marketmakers, preferencing arrangements, *and* vertical integration: $n_s^*, n_p^*, n_v^* > 0$. In this case, brokers with high volume internalize their order flow, brokers with medium volume preference their order flow, and brokers with low volume use SOES. There are some marketmakers handling only SOES trades and some handling SOES and preferred trades, which is true for some Nasdaq stocks.

To obtain additional comparative static results and illustrate this type of equilibrium, we consider a numerical example with an effective tick size $\Delta = 1/8$ and other parameters as follows: $m = 100$, $C = 0.02$, $\theta = 0.5$, $\alpha = 0.00005$, $\beta = 0.0001$, $F_p = 5$, $F_v = 20$, $\bar{\Pi}_1 = 15$, $\bar{\Pi}_2 = 50$, and g is the Gamma distribution with parameters (1.25, 50). An effective tick size of one-eighth is typical for Nasdaq stocks when marketmakers do not avoid odd-eighth quotes. In what follows, we also consider tick sizes of one-quarter and one-sixteenth, which correspond to the other effective tick sizes commonly observed on Nasdaq. The equilibria for the different tick sizes are shown in Panel A of Table I. We choose

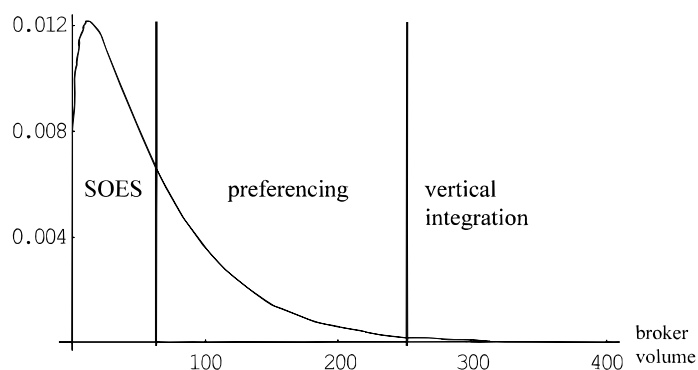


Figure 2. Distribution of broker volume. Brokers with low volume choose to use SOES, brokers with intermediate volume enter preferencing arrangements, and brokers with high volume vertically integrate. (The vertical lines indicate the partition for the numerical example with tick size of one-eighth.)

fairly round numbers for our parameters rather than trying to calibrate the model precisely, but the equilibrium values are of the correct order of magnitude. One can verify that in all cases the condition for having only two equilibrium spreads is satisfied, that is, $2C + 2\Delta(n/(n-1)) < 3\Delta$. Panel B of Table I reports the equilibrium number of marketmakers in a model without preferencing or vertical integration.

For the numerical example with a tick size of one-eighth, Figure 2 shows that there is a region of low volume for which brokers use SOES, a region of intermediate volume for which brokers use preferencing arrangements, and a region of high volume for which brokers internalize their order flow.

Qualitative comparative static results based on the numerical calculations are shown in Table II. For more details on the calculation of the comparative static results, see Appendix B. Rows 1 and 4 of Table II show that a small increase in the cost of market making, C , or the reservation value for marketmakers with preferred trades, $\bar{\Pi}_2$, causes the equilibrium payment per share to decrease and the total number of marketmakers to increase. These changes also imply a decrease in preferred volume. Rows 2 and 3 show that a small increase in the tick size, Δ , or the reservation value for marketmakers with only SOES trades, $\bar{\Pi}_1$, causes the equilibrium payment per share to increase and the total number of marketmakers to decrease. These changes also imply an increase in preferred volume. Row 5 shows that a small increase in the brokers' fixed cost for preferencing, F_p , causes exit from preferencing and entry into SOES and vertical integration, and row 6 shows that a small increase in brokers' fixed cost for vertical integration, F_v , causes exit from vertical integration and entry into SOES and preferencing. Row 7 of Table II shows that a small increase in the number of brokers causes the total number of marketmakers to increase, but row 8 shows that

Table II
Simulated Comparative Static Results

Results are for the case with a tick size of one-eighth. The table shows the directions of change in the equilibrium values of the endogenous variables for small increases in individual exogenous variables. A \dagger indicates comparative static results that differ from those for a model without preferencing or vertical integration. We assume the proportion of time that the spread is one effective tick is one-half (i.e., $\theta = 0.5$).

	Exogenous Variables	Endogenous Variables				
		Total Market-makers, n^*	SOES Market-makers, n_s^*	Preferencing Market-makers, n_p^*	Vertically Integrated Brokers, n_v^*	Payment for Order Flow, p^*
1	Marginal cost of market making, C	$+\dagger$	+	−	0	−
2	Tick size, Δ	$-\dagger$	−	+	0	+
3	Reservation value for SOES, $\bar{\Pi}_1$	−	−	+	−	+
4	Reservation value for preferencing, $\bar{\Pi}_2$	+	+	−	+	−
5	Preferencing fixed cost, F_p	+	+	−	+	0
6	Vertical integration fixed cost, F_v	0	+	+	−	0
7	Number of brokers, m	+	+	+	+	0
8	Distribution of broker volume, g (shift right)	$-\dagger$	−	+	+	0

a small shift to the right in the distribution of broker volume (an increase in the mean and variance of broker volume) causes the total number of marketmakers to decrease.

Although many of our comparative static results are the same as those for a model without preferencing or vertical integration, some important results are not. In a model with no preferencing or vertical integration, a change that reduces marketmakers' profit margins (e.g., a reduction in the tick size) also reduces the number of marketmakers. This argument regularly appears in industry publications. We show that when preferencing and vertical integration are added to the model, the effect of a reduction in the profit margin on the number of marketmakers is ambiguous. In equilibrium, the lower

trading profits imply lower payments for order flow, and these changes can cause brokers to switch from preferencing and vertical integration to SOES. The increase in SOES trades can more than compensate for the decreases in the other types of marketmakers, so the decrease in profit margin can actually cause the number of marketmakers to increase. Comparative static results that differ for models with and without preferencing and vertical integration are marked with a dagger in Table II. These somewhat counter-intuitive results are more likely under parameter configurations that result in the equilibrium payment for order p^* being close to the threshold payment \tilde{p} (see Figure 1).

When the cost of market making increases or the tick size decreases, a model without preferencing or vertical integration predicts that the number of marketmakers will decrease. We get the opposite results in our model (see rows 1 and 2 in Table II). Although an increase in the cost of market making or a decrease in the tick size causes the number of preferencing marketmakers to decrease, in our model this decrease is more than offset by an increase in the number of SOES-only marketmakers.

When volume increases, a model without preferencing or vertical integration predicts that the number of marketmakers will increase. Our numerical simulation produces the opposite results when the increase in volume is due to a shift in the distribution of broker volume (see row 8 in Table II). In this case, the increase in preferencing marketmakers and vertically integrated brokers is not sufficient to compensate for the decrease in SOES-only marketmakers.

The differences in the comparative static results for models with and without preferencing and vertical integration highlight the importance of including alternative venues for execution in models of dealer markets.

C. Equilibria with No Preferencing

For some parameter values, the equilibrium in our model has no preferencing arrangements but does have SOES-only marketmakers and vertically integrated brokers: $n_p^* = 0$ and $n_s^*, n_v^* > 0$. This type of equilibrium occurs in our numerical example when the tick size is one-sixteenth (see Table I, Panel A). Brokers with high volume internalize their order flow, and the remainder use SOES.

In the case with no preferencing, Proposition 2 implies that $\phi_1(p^*) = \phi_2(p^*) = \tilde{v}$. Since $\tilde{v} = F_v/(\pi - \beta)$ by (12), equilibrium condition 4 implies that

$$n_v^* = \int_{\frac{F_v}{\pi - \beta}}^{\infty} g(v) dv. \quad (17)$$

Since an increase in the tick size or decrease in the cost of market making causes the lower bound of integration in equation (17) to decrease, such changes cause the number of vertically integrated brokers to increase. However, the effect of such changes on the total number of marketmakers is ambiguous.

D. Equilibria with No Marketmakers Handling Only SOES Trades

For still other parameter values, the equilibrium in our model has all trades executed either through preferencing arrangements or through vertically integrated market-making divisions: $n_s^* = 0$ and $n_p^*, n_v^* > 0$. This type of equilibrium occurs in our numerical example when the tick size is one-quarter (see Table I, Panel A), which corresponds to the case in which marketmakers avoid odd-eighth quotes (see Christie and Schultz (1994)). In equilibrium, most brokers preference or internalize their order flow, leaving too little SOES volume to support marketmakers handling only SOES trades.

In this case, Proposition 2 implies that $\bar{p} \leq p^* < (\pi - \beta)/2$ (if p^* were smaller, brokers with volume less than $\bar{v} > 0$ would use SOES, and if p^* were larger, no brokers would vertically integrate). Thus, Proposition 2 implies

$$\phi_1(p^*) = \frac{F_v - F_p}{\pi - 2p^* - \beta}$$

and equilibrium condition 4 implies

$$n_v^* = \int_{\frac{F_v - F_p}{\pi - 2p^* - \beta}}^{\infty} g(v) dv. \quad (18)$$

As in the previous case, an increase in the tick size or decrease in the cost of market making causes the number of vertically integrated brokers to increase. And, as before, the effect of such changes on the total number of marketmakers is ambiguous.

E. Effects of Changes in the Tick Size

In our numerical example, a decrease in the tick size causes the total number of marketmakers to decrease, even though the number of preferencing marketmakers increases, because of the large decrease in the number of SOES-only marketmakers. (The number of vertically integrated brokers does not change; see line 2 of Table II.)

We now consider the effect of larger changes in the tick size. In our numerical example, an increase in the tick size from one-eighth to one-quarter causes the SOES-only marketmakers to disappear because most of the order flow is either preferenced or internalized. Consequently, the total number of marketmakers decreases and the payment for order flow increases.¹⁷ As the

¹⁷ In our numerical examples, $\theta = 0.5$, so if the tick size increases from one-eighth to one-quarter (e.g., due to the avoidance of odd-eighth quotes) then the average spread increases by 18.75 cents. This is consistent with the actual increase in the bid-ask spreads due to odd-eighth quote avoidance as reported by Barclay (1997), Bessembinder (1997), and Kandel and Marx (1997).

tick size increases, the total cost of the system, excluding market-making costs, increases as well because preferenced and vertically integrated trades are more costly than SOES trades. This suggests that a system with a tick size of one-quarter is potentially more expensive than one with a smaller tick size.

If marketmakers adopt a policy of avoiding certain quotes, such as odd-eighth quotes, they effectively increase the tick size. This change increases preferenced trading and vertical integration and increases the total cost of the system. Our example illustrates the rather counterintuitive result that a change, such as the avoidance of odd-eighth quotes, that increases the per-share profit can actually result in a decrease in the number of marketmakers.

If the tick size is reduced from one-eighth to one-sixteenth, as was recently done on Nasdaq, then, in the numerical example, preferenced trades disappear and vertical integration almost disappears, but the total number of marketmakers increases due to the increase in the number of SOES-only marketmakers. (We can also construct an example in which the total number of marketmakers decreases as a result of a decrease in the tick size.)

It is interesting that, in a model without preferencing or vertical integration, increases and decreases in the tick size have symmetric effects, but in a model with preferencing and vertical integration, increases and decreases in the tick size can have different (asymmetric) effects on the endogenous variables due to changes in the type of equilibrium.

F. Effects of Changes in Volume

There are two ways in which volume can change in our model. First, the number of brokers, m , can change, and second, the probability distribution for an individual broker's volume can shift. An increase in the number of brokers simply causes the numbers of all types of marketmakers to increase, but an increase in volume due to a shift in the distribution of broker volume can cause the total number of marketmakers to decrease. For example, if we increase the mean and variance of the distribution of broker volume in our numerical example by moving from distribution $\Gamma(1.25, 50)$ to distribution $\Gamma(1.3, 50)$, the total number of marketmakers decreases. There are increases in the number of preferencing marketmakers and in the number of vertically integrated brokers, but these increases are not enough to make up for the large decrease in the number of SOES-only marketmakers (see row 8 of Table II). This type of decrease in the number of marketmakers as a result of an increase in volume is not possible in a model without preferencing or vertical integration. Since a model without preferencing or vertical integration predicts that the total number of marketmakers would increase with the increase in volume, our model offers different predictions on the relation between volume and the number of marketmakers. Moreover, a model without preferencing or vertical integration cannot predict the shifts between the different types of marketmakers that can occur as a result of an increase in volume.

III. Empirical Evidence

The goal of this section is to present empirical evidence pertaining to the theoretical predictions derived in the paper. Because of the lack of reliable data on preferencing and payments for order flow, we cannot perform direct tests of the model. Instead, we show that the available data are consistent with the indirect implications of the theory.

A. Comparison with the “Naive” Model

We contrast the hypotheses derived from our model with the hypotheses from the “naive” competitive model, which is often implicitly assumed in the literature (e.g., Kleidon and Willig (1995), Furbush et al. (1996), and Grossman et al. (1997)). The “naive” model ignores brokers’ decisions regarding the choice of execution venue for their trades and assumes instead that all trades are executed in a competitive market. The “naive” model corresponds to the case in our model in which payment for order flow and internalization are not permitted. In what follows, we refer to the naive model, without quotation marks, to mean a model with no allowance for preferencing or internalization.

Both the naive model and our model are consistent with either positive or negative cross-sectional correlations between the number of marketmakers and the degree of odd-eighths avoidance in the population of stocks. The reasons for these predictions are different, but empirically we cannot distinguish between the models based on cross-sectional correlations. Both models are consistent with the evidence presented in Barclay (1997) and Wahal (1997) that the number of marketmakers in stocks that did not avoid odd-eighth quotes is more than 50 percent higher than in the stocks that avoid odd-eighth quotes.¹⁸

Although the cross-sectional correlations do not allow us to distinguish between the two models, the time-series predictions do. The naive model implies that an exogenous increase in the profit margin causes marketmakers to enter and that a decrease causes them to exit. Our model predicts a more complex response. A change in the profit margin affects the profitability of all the marketmakers, just as in the naive model. However, the magnitude of the change in profit varies across different types of marketmakers. A decrease in the profit margin can cause the number of preferencing marketmakers and vertically integrated brokers to decrease, but cause the number of SOES-only marketmakers to increase. The change in the total number of marketmakers is ambiguous. For example, if the SOES volume is sufficiently low that entry is unprofitable, then, contrary to the naive model, a small increase in the profit margin will not affect the number of market-

¹⁸ Barclay (1997, Table 1) considers stocks that moved from Nasdaq to the NYSE or AMEX during the period 1983 to 1992. Before changing exchanges, 239 stocks avoided odd-eighth quotes and 233 did not avoid odd-eighth quotes; the average number of marketmakers was 10.27 and 15.64, respectively. The two groups of stocks had similar volume and their spreads were almost the same after moving to a different exchange.

makers. In fact, an increase in the profit margin can cause a decrease in the number of marketmakers if the increase in profit margin causes an increase in the payment for order flow, which causes more brokers to preference. This reduces the SOES order flow, causing some of the independent marketmakers to exit and the total number of marketmakers to decrease. Thus, an increase in the number of marketmakers in response to an exogenous increase in average spreads is consistent with both models, but our model is also consistent with no change or a decrease in the number of marketmakers.

To summarize the testable hypotheses, the naive model predicts that the number of marketmakers increases with volume and profit margin, and that the average spread decreases with volume. Our model is consistent with either an increase or decrease in the number of marketmakers when volume or profit margin increase.

B. Empirical Evidence in the Literature

A recent paper by Christie and Schultz (1997) studies events in which the use of odd-eighth quotes is either initiated or withdrawn. They show that the changes typically take place within one day and are difficult to predict using the observable changes in stock fundamentals or microstructure variables. Although the withdrawal of odd-eighth quotes causes average spreads to increase by about \$0.20, and the initiation of the use of odd-eighths causes a similar decline in average spreads, these events do not seem to affect other determinants of stock-trading patterns. Thus, the withdrawals and initiations of odd-eighth quotes appear to be largely attributable to coordination issues that are unrelated to stock fundamentals. Consequently, we treat them as exogenous to the decisions of individual marketmakers.

Since the withdrawal of odd-eighth quotes causes marketmakers' profit margins to increase, the naive model predicts that the number of marketmakers will increase after such an event; similarly, it predicts a decrease in the number of marketmakers after the initiation of odd-eighth quotes. Our model, however, allows for either an increase, no change, or a decrease in the number of marketmakers, depending on the parameter values. In fact, evidence in Christie and Schultz (1997) shows that the number of marketmakers barely rises in response to the withdrawal of odd-eighth quotes (one more marketmaker after a year). The number of marketmakers increases slightly after the initiation of odd-eighth quotes. These results are inconsistent with the predictions of the naive model.

Barclay et al. (1998) study the impact of January 1997 Nasdaq rules changes on the market. They show that the average quoted and effective spreads fall dramatically (30 percent on average) for most affected stocks in response to this exogenous event. The naive model predicts mass exit;¹⁹ however, empirical results indicate no exit (some entry by small marketmakers takes place) from the affected stocks.

¹⁹ Numerous industry reports prior to the rule change predicted a drastic decline in the number of marketmakers following the implementation of the rules.

Our model predicts that the payment for order flow decreases after the exogenous reduction in the profit margin. There is anecdotal evidence suggesting that in 1994–1995 the payment for order flow for Nasdaq stocks was as high as 3.5 cents per share. A *Bloomberg News* article (April 20, 1997) indicates that since the January 1997 rules changes the payment for order flow has fallen to 2 cents per share; furthermore, the article indicates that large buyers of order flow are planning to lower the payment to 1 cent and that some marketmakers have stopped paying for order flow altogether.²⁰ A *Wall Street Journal* article (September 9, 1997, p. C1, “New Rules on Nasdaq Pinch Firms”) indicates that the move to a minimum tick size of one-sixteenth, which took place on June 1, 1997, further reduced the price per share and, in many cases, eliminated payments for order flow. The article also states that Merrill Lynch, one of the largest vertically integrated firms, reduced the number of stocks in which it makes market by 40 percent following a decrease in profit margins. This evidence is consistent with the predictions of our model.

Evidence consistent with these result, but inconsistent with the naive model, is also provided on the NASD home page (www.nasd.com, 1/12/98). The NASD reports that, following a change in Nasdaq’s minimum tick size from one-eighth to one-sixteenth, even though marketmakers’ profit margins decreased, the number of marketmakers per stock increased in all categories of Nasdaq stocks. In some active stocks, the increase was by more than two marketmakers.

C. Evidence from IPOs

Below we present additional empirical evidence to illustrate our results using data from IPOs. Focusing on IPOs allows us to consider a short-run equilibrium in this market and compare it to a medium-run equilibrium (three months later). Though the IPO is not a representative period for a stock, it is illuminating for analyzing the behavior of marketmakers. When an IPO is held, we observe the creation of the market for a stock and observe entry and exit by marketmakers as the market for the stock matures.

We use data from the Institute for the Study of Securities Markets (ISSM) (1990–92), Trades and Quotes (TAQ) (1993–94), and CRSP (for the number of marketmakers) databases to examine IPOs on Nasdaq (NMS) taking place in 1990–1994 with at least three months of data, 1074 observations altogether.²¹ Date 1 for each firm is defined to be the first trading day after the IPO. We focus on the first day, first trading week, and the trading week 60 days after the IPO.

²⁰ The decline in the price per share from 3.5 to 2 cents can be attributed to a reduction in profit margins due to significant decline in the percentage of firms avoiding odd-eighth quotes from 1994 to 1996 (see Barclay et al. (1998)).

²¹ We omit the data from 10/1/92 through 12/31/92 (58 observations) because of difficulties merging data from the ISSM and TAQ databases. Because of a limitation of the database we use (some firms that held IPOs at the end of 1992 are not included), we are only able to extend our data to the 1990–1992 time period if we omit firms holding IPOs in the last three months of the period. The qualitative results continue to hold if we use only the TAQ data.

Table III
Correlations between Odd-Eighth Avoidance and Spreads
and the Number of Marketmakers

Panel A shows the correlations between the average proportion of even-eighth quotes and the number of marketmakers for three time periods: the first day of the IPO, the first 5 days after the IPO, and the 61st to 65th days after the IPO. Panel B shows the correlations between the average quoted time-weighted dollar spread and the number of marketmakers for the same three time periods. MM1DAY is the number of marketmakers on the first trading day. MM5DAY is the average number of marketmakers on the first five trading days. MM3MON is the average number of marketmakers on trading days 61–65. AV1DAY is a dummy equal to one if the fraction of even-eighth quotes on the first trading day is greater than 0.9 and zero otherwise. AV5DAY is a dummy equal to one if the average fraction of even-eighth quotes on the first five trading days is greater than 0.9 and zero otherwise. AV3MON is a dummy equal to one if the average fraction of even-eighth quotes on trading days 61–65 is greater than 0.9 and zero otherwise. QS1DAY is the quoted time-weighted dollar spread on the first trading day. QS5DAY is the average quoted time-weighted dollar spread on first five trading days. QS3MON is the average quoted time-weighted dollar spread on trading days 61–65. The *p*-values are in parentheses. The data are from the TAQ database on Nasdaq (NMS) IPOs and includes stocks holding IPOs in 1990–1994 with at least three months of data. The sample size is 1074. Date 1 for each firm is defined to be the day of the IPO.

	MM1DAY	MM5DAY	MM3MON
Panel A			
AV1DAY	−0.035 (0.25)		
AV5DAY		−0.039 (0.20)	
AV3MON			0.01 (0.74)
Panel B			
QS1DAY	−0.081 (0.01)		
QS5DAY		−0.082 (0.01)	
QS3MON			−0.07 (0.01)

We first consider the short-term relation between the number of marketmakers and odd-eighth avoidance. According to Kandel and Marx (1997), the decision of marketmakers in a stock to avoid odd-eighths should not depend on the number of marketmakers in that stock. To test this hypothesis, we calculate the correlations among the number of marketmakers and the degree of odd-eighth avoidance in three time periods: the first day of the IPO, the first 5 days after the IPO (AV5DAY and MM5DAY), and the 61st to 65th days after the IPO (AV3MON and MM3MON). Results are in Panel A of Table III. The data indicate that avoidance of odd-eighths is not related to the number of marketmakers.

Regression analyses (not reported to conserve space) also show that for stocks with a tick size of one-eighth, the degree to which marketmakers in a stock avoid odd-eighths does not depend on the number of marketmakers in that stock. However, for stocks with a tick size of one-sixteenth, an increase in the number of marketmakers does reduce the probability of avoidance in this sample. An increase in the number of marketmakers by one standard deviation (four marketmakers) results in a decrease in avoidance of about 7 percent. This is consistent with the results in Kandel and Marx (1997), who argue that the avoidance of odd-eighths in stocks with a tick size of one-sixteenth is unlikely to be an equilibrium; thus, an increase in the number of marketmakers should negatively affect its frequency.

Avoidance after three months of trading is a function of the number of marketmakers, the price, and whether odd-eighths are avoided in the early days of trading. However, an increase in the number of marketmakers is associated with an increase (rather than the expected decline) in the probability of avoidance.

Next we show that the cross-sectional correlations between the number of marketmakers and the average quoted spread (QS1DAY, QS5DAY, QS3MON) are negative, which is consistent with the studies mentioned earlier. The results are in Panel B of Table III.

In calculating time series results, we again compare the events during the first day, first five days, and first three months after the IPO. If odd-eighth quotes are used immediately after the IPO and then avoided later, the naive model predicts an increase in the number of marketmakers due to the increase in profit margin. Our model also allows the possibility of no change in the number of marketmakers or even a reduction in the number of marketmakers due to increased preferencing. The opposite predictions hold if the use of odd-eighth quotes is initiated during the first three months.

We define the following variables: *marketmaker ratio* is the ratio of the number of marketmakers in a stock three months after the IPO to the same number five days after the IPO; *volume ratio* and *quoted spread ratio* are similar measures for volume and the quoted spread, respectively. We calculate correlation coefficients among these variables for four subsamples: stocks that avoid odd-eighths initially and initiate their use later (Initiation sample); stocks that always avoid odd-eighths (Avoidance sample); stocks that do not avoid odd-eighths initially and withdraw their use later (Withdrawal sample); and stocks that never avoid odd-eighths (Nonavoidance sample). The results are shown in Table IV.

Table IV shows that there is no significant correlation between the marketmaker ratio and the quoted spread ratio for any of the samples. Point estimates for these correlations are negative, indicating that a decrease in the number of marketmakers is (insignificantly) associated with an increase (or smaller decrease) in the quoted spread. The correlation between the marketmaker ratio and the volume ratio is only significant for the Avoidance sample. In that case it is positive, indicating that an increase in volume is associated with an increase in the number of marketmakers. There is no

Table IV
Correlation Coefficients

The table shows the correlation coefficients for the marketmaker ratio, volume ratio, and quoted spread ratio. The marketmaker ratio is the ratio of the number of marketmakers for a stock three months after the IPO to the number five days after the IPO. The volume ratio and quoted spread ratio are similar measures for volume and the quoted spread, respectively. The correlations are given for four subsamples: stocks that avoid odd-eighths initially and initiate their use later (Initiation Sample); stocks that always avoid odd-eighths (Avoidance Sample); stocks that do not avoid odd-eighths initially and withdraw their use later (Withdrawal Sample); and stocks that never avoid odd-eighths (Nonavoidance Sample). The *p*-values are in parentheses. The data are from the TAQ database on Nasdaq (NMS) IPOs and includes stocks holding IPOs in 1990–1994 with at least three months of data. The sample size is 1074. Date 1 for each firm is defined to be the day of the IPO.

	Initiation Sample (69)		Avoidance Sample (644)	
	Quoted Spread Ratio	Volume Ratio	Quoted Spread Ratio	Volume Ratio
Marketmaker ratio	−0.13 (0.29)	0.1 (0.43)	−0.04 (0.36)	0.1 (0.01)
Volume ratio	−0.35 (0.003)		−0.15 (0.0001)	
	Withdrawal Sample (148)		Non-Avoidance Sample (213)	
	Quoted Spread Ratio	Volume Ratio	Quoted Spread Ratio	Volume Ratio
Marketmaker ratio	−0.01 (0.91)	0.013 (0.87)	−0.09 (0.2)	0.009 (0.9)
Volume ratio	0.03 (0.73)		−0.12 (0.08)	

correlation between the marketmaker ratio and the quoted spread ratio. Regression analysis (not reported) confirms this finding—the marketmaker ratio has no ability to explain the quoted spread ratio. This evidence is inconsistent with the naive model, but is consistent with ours. Furthermore, since the quoted spread is the profit margin plus costs, and if we assume costs do not change in any systematic way around the IPO date, then changes in quoted spreads are highly correlated with changes in profit margins.

IV. Conclusions

We model the institutional features of the Nasdaq market and contrast our results with those of a more simplistic model that does not allow preferred trades or vertical integration. Results for our model indicate that a decrease in marketmakers' profit margin, perhaps due to a decrease in the tick size, causes the equilibrium payment for order flow to decrease, but can cause the total number of marketmakers to increase. A decrease in the tick size can also result in the elimination of the preferencing market and a

reduction in vertical integration; at the same time, the total number of marketmakers increases. Conversely, a change that increases the per-share profit, such as marketmakers avoiding odd-eighth quotes, can actually result in a decrease in the number of marketmakers. An increase in the tick size, perhaps due to the avoidance of odd-eighth quotes, can result in the elimination of SOES traders, leaving only preferenced and vertically integrated trades. In contrast, in a model without preferenced trades or vertical integration, the number of marketmakers always increases with changes that increase marketmakers' profit margin, such as an increase in the tick size. Empirical evidence presented in the paper suggests that the model that includes preferenced trades and vertical integration is more appropriate.

Our results indicate that an increase in volume can cause the total number of marketmakers to decrease. In contrast, in a model without preferenced trades or vertical integration, the number of marketmakers always increases with volume. In our model, the abolition of preferencing arrangements and vertical integration is likely to increase the total number of marketmakers.

Other considerations not dealt with in our model include issues of risk that arise when buy and sell order flow is not balanced and when marketmakers are only active on one side of the market. We do not consider the possibility that risk is reduced when order flow is large. The economies of scope for marketmakers with multiple stocks are ignored as well.

Appendix A

Proof of Proposition 3: For $p < \bar{p}$, $(\partial S(p))/\partial p = 0$. For $\bar{p} \leq p < (\pi - \beta)/2$,

$$\frac{\partial S(p)}{\partial p} = m \left[\frac{\partial \phi_1(p)}{\partial p} \phi_1(p) g(\phi_1(p)) - \frac{\partial \phi_2(p)}{\partial p} \phi_2(p) g(\phi_2(p)) \right], \quad (\text{A1})$$

which, using

$$\phi_1(p) = \frac{F_v - F_p}{\pi - 2p - \beta} \quad \text{and} \quad \phi_2(p) = \frac{F_p}{2p},$$

implies

$$\frac{\partial S(p)}{\partial p} = m \left[\frac{2(F_v - F_p)^2}{(\pi - 2p - \beta)^3} g\left(\frac{F_v - F_p}{\pi - 2p - \beta}\right) + \frac{F_p^2}{4p^3} g\left(\frac{F_p}{2p}\right) \right] > 0. \quad (\text{A2})$$

For $p \geq (\pi - \beta)/2$, $\phi_1(p) = \infty$, so

$$\frac{\partial S(p)}{\partial p} = m \frac{F_p^2}{4p^3} g\left(\frac{F_p}{2p}\right) > 0.$$

Q.E.D.

Proof of Theorem 1: We begin the proof by stating and proving four lemmas.

LEMMA 1: *There exist continuous function $\hat{p}: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ and $\bar{p} > 0$ such that $\hat{p}(n_s, n_p, n_v) \leq \bar{p}$ and $S(\hat{p}(n_s, n_p, n_v)) = n_p d(\hat{p}(n_s, n_p, n_v))$.*

Proof of Lemma 1: Define $\bar{p} \equiv (\pi/2)$ and

$$\hat{p}(n_s, n_p, n_v) \equiv \begin{cases} \check{p}(n_s, n_p, n_v), & \text{if } n_p > 0 \\ \bar{p}, & \text{otherwise,} \end{cases} \quad (\text{A3})$$

where $\check{p}: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is such that

$$S(\check{p}(n_s, n_p, n_v)) = n_p d(\check{p}(n_s, n_p, n_v)). \quad (\text{A4})$$

Note that $\bar{p} \leq \pi/2$. The function \check{p} is uniquely defined and continuous and $\check{p}(n_s, n_p, n_v) \in [\bar{p}, \pi/2]$ since, for $n_p > 0$, $S(\cdot)$ and $n_p d(\cdot)$ are continuous on $[\bar{p}, \pi/2]$, $S(\cdot)$ is increasing on $(\bar{p}, \pi/2)$, $n_p d(\cdot)$ is decreasing on $(\bar{p}, \pi/2)$, $n_p d(\bar{p}) \geq S(\bar{p})$, and $S(n, (\pi(n))/2) \geq n_p d(\pi/2)$. To establish the continuity of \hat{p} , note that as $n_p \downarrow 0$, $\check{p}(n_s, n_p, n_v) \rightarrow \bar{p}$.

LEMMA 2: *There exist continuous function $\hat{n}_s: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ and $\bar{n}_s > 0$ such that $\hat{n}_s(n_p, n_v, p) \leq \bar{n}_s$, $\hat{n}_s(n_p, n_v, p) > 0$ implies*

$$\frac{K(p)}{\hat{n}_s(n_p, n_v, p) + n_p + n_v} \pi = \bar{\Pi}_1,$$

and $\hat{n}_s(n_p, n_v, p) = 0$ implies $K(p)/(n_p + n_v) \pi \leq \bar{\Pi}_1$.

Proof of Lemma 2: Define

$$\bar{n}_s \equiv \frac{m \int_0^\infty v g(v) dv}{\bar{\Pi}_1} \pi$$

and

$$\hat{n}_s(n_p, n_v, p) \equiv \begin{cases} 0, & \text{if } \frac{K(p)}{n_p + n_v} \pi \leq \bar{\Pi}_1 \\ \check{n}_s(n_p, n_v, p), & \text{otherwise,} \end{cases} \quad (\text{A5})$$

where $\check{n}_s: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is defined by

$$\check{n}_s(n_p, n_v, p) \equiv \min \left\{ n_s \geq 0 \left| \frac{K(p)}{n_s + n_p + n_v} \pi \leq \bar{\Pi}_1 \right. \right\}. \quad (\text{A6})$$

If

$$\frac{K(p)}{n_p + n_v} \pi \geq \bar{\Pi}_1,$$

the function \check{n}_s is well defined because

$$\lim_{n_s \rightarrow \infty} \frac{K(p)}{n_s + n_p + n_v} \pi = 0 < \bar{\Pi}_1, \quad (\text{A7})$$

and \check{n}_s is continuous because $((K(p))/n)\pi$ is continuous in n and p . The function \check{n}_s is bounded above by \bar{n}_s since, for all $n > \bar{n}_s$,

$$\frac{K(p)}{n} \pi \leq \frac{m \int_0^\infty v g(v) dv}{n} \pi = \frac{\bar{n}_s}{n} \bar{\Pi}_1 < \bar{\Pi}_1. \quad (\text{A8})$$

To establish the continuity of \hat{n}_s , note that if $(K(p)/(n_p + n_v))\pi = \bar{\Pi}\alpha_1$, then $\check{n}_s(n_p, n_v, p) = 0$. Q.E.D.

LEMMA 3: For

$$\bar{\Pi}_2 > \frac{\Delta - 2C}{\alpha} ((2 - \theta)\Delta - 2C),$$

there exist continuous function $\hat{n}_p: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ and $\bar{n}_p > 0$ such that $\hat{n}_p(n_s, n_v, p) \leq \bar{n}_p$, $\hat{n}_p(n_s, n_v, p) > 0$ implies

$$\Pi^*(n_s + \hat{n}_p(n_s, n_v, p) + n_v, p) = \bar{\Pi}_2,$$

and $\hat{n}_p(n_s, n_v, p) = 0$ implies $\Pi^*(n_s + n_v, p) \leq \bar{\Pi}_2$.

Proof of Lemma 3: Assume $\bar{\Pi}_2 > ((\Delta - 2C)/\alpha)((2 - \theta)\Delta - 2C)$. Define

$$\bar{n}_p \equiv \frac{m \int_0^\infty v g(v) dv}{\bar{\Pi}_2 - \frac{(2\Delta - 2C)^2}{\alpha}} (2\Delta - 2C)$$

and

$$\hat{n}_p(n_s, n_v, p) \equiv \begin{cases} 0, & \text{if } \Pi^*(n_s + n_v, p) \leq \bar{\Pi}_2 \\ \check{n}_p(n_s, n_v, p), & \text{otherwise,} \end{cases} \quad (\text{A9})$$

where $\check{n}_p: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is defined by

$$\check{n}_p(n_s, n_v, p) \equiv \min\{n_p | \Pi^*(n_s + n_p + n_v, p) \leq \bar{\Pi}_2\}. \quad (\text{A10})$$

If $\Pi^*(n_s + n_v, p) \geq \bar{\Pi}_2$, the function \check{n}_p is well defined since

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pi^*(n, p) &= \lim_{n \rightarrow \infty} \left[d(p) \left(\pi - 2p - \frac{\alpha}{2} d(p) \right) + \frac{K(p)}{n} \pi \right] \\ &= d(p) \left(\pi - 2p - \frac{\alpha}{2} d(p) \right) \\ &\leq d(p) \pi \\ &\leq \frac{\Delta - 2C}{\alpha} ((2 - \theta)\Delta - 2C) \\ &< \bar{\Pi}_2, \end{aligned} \quad (\text{A11})$$

where the second inequality uses $d(p) \leq (\Delta - 2C)/\alpha$, and the third inequality uses our assumption on $\bar{\Pi}_2$. The continuity of Π^* implies that \check{n}_p is continuous. The function \check{n}_p is bounded above by \bar{n}_p since for all $n > \bar{n}_p$,

$$\begin{aligned} \Pi^*(n, p) &\leq \frac{(2\Delta - 2C)^2}{2a} + \frac{m \int_0^\infty v g(v) dv}{n} (2\Delta - 2C) \\ &< \frac{(2\Delta - 2C)^2}{\alpha} + \frac{m \int_0^\infty v g(v) dv}{\bar{n}_p} (2\Delta - 2C) \\ &= \bar{\Pi}_2, \end{aligned} \quad (\text{A12})$$

where the first inequality uses (15), $\pi \leq 2\Delta - 2C$, and $\phi_2(p) \leq \infty$, the second inequality uses $n > \bar{n}_p$, and the equality uses the definition of \bar{n}_p . To establish the continuity of \hat{n}_p , note that if $\Pi^*(n_s + n_v, p) = \bar{\Pi}_2$, then $\check{n}_p(n_s, n_v, p) = 0$. Q.E.D.

LEMMA 4: *There exist continuous function $\hat{n}_v: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ and $\bar{n}_v > 0$ such that $\hat{n}_v(n_s, n_p, p) \leq \bar{n}_v$ and $\hat{n}_v(n_s, n_p, p) = m \int_{\phi_1(p)}^\infty g(v) dv$.*

Proof of Lemma 4: Define $\bar{n}_v \equiv m \int_0^\infty g(v) dv$ and define \hat{n}_v such that

$$\hat{n}_v(n_s, n_p, p) = m \int_{\phi_1(p)}^\infty g(v) dv. \quad (\text{A13})$$

The function \hat{n}_v is continuous by the continuity of ϕ_1 (on the extended real line), and $\hat{n}_v(n_s, n_p, p) \leq \bar{n}_v$. Q.E.D.

Using Lemmas 1–4, we can complete the proof of Theorem 1. Let $Z \equiv [0, \bar{n}_s] \times [0, \bar{n}_p] \times [0, \bar{n}_v] \times [0, \bar{p}]$, where \bar{n}_s , \bar{n}_p , \bar{n}_v , and \bar{p} are as in Lemmas 1–4, and define mapping $B: Z \rightarrow Z$ by

$$B(n_s, n_p, n_v, p) = (\hat{n}_s(n_p, n_v, p), \hat{n}_p(n_s, n_v, p), \hat{n}_v(n_s, n_p, p), \hat{p}(n_s, n_p, n_v)), \quad (\text{A14})$$

where \hat{n}_s , \hat{n}_p , \hat{n}_v , and \hat{p} are as in Lemmas 1–4. A fixed point of B satisfies equilibrium conditions 1–4, and because Z is compact and convex and B is continuous, Brouwer’s fixed-point theorem implies that an equilibrium exists.

Appendix B

Table BI

Equilibrium Values for Small Changes in the Exogenous Variables Relative to the Base Case

The table shows the equilibrium number of marketmakers and the equilibrium payment for order flow for small changes in individual exogenous variables relative to the base case with a tick size of one-eighth. One can verify that in all cases the condition for having only two equilibrium spreads is satisfied; that is, $2C + 2\Delta(n^*/(n^* - 1)) < 3\Delta$. We assume the proportion of time that the spread is one effective tick is one-half (i.e., $\theta = 0.5$). The variables in the table are as follows: n^* is the total number of marketmakers, n_s^* is the number of SOES marketmakers, n_p^* is the number of preferencing marketmakers, n_v^* is the number of vertically integrated brokers, p^* is the payment for order flow, Δ is the tick size, C is the marginal cost of market making, F_p is the preferencing fixed cost, F_v is the vertical integration fixed cost, $\bar{\Pi}_1$ is the reservation value for SOES-only marketmakers, $\bar{\Pi}_2$ is the reservation value for preferencing marketmakers, m is the number of brokers, and g is the distribution of broker volume. The base values for the exogenous variables are: $\Delta = 0.125$, $C = 0.02$, $F_p = 5$, $F_v = 20$, $\bar{\Pi}_1 = 15$, $\bar{\Pi}_2 = 50$, $m = 100$, and $g = \Gamma(1.25, 50)$.

Change in a Single Exogenous Variable Relative to the Base Case	Endogenous Variables				
	n^*	n_s^*	n_p^*	n_v^*	p^*
Base case	15.05	10.27	3.71	1.08	0.0442
$\Delta = 0.120$ (decrease)	16.33	11.73	3.53	1.08	0.0404
$C = 0.025$ (increase)	16.80	12.26	3.46	1.08	0.0392
$F_p = 6$ (increase)	19.70	14.98	3.23	1.49	0.0442
$F_v = 22$ (increase)	15.05	10.67	3.83	0.56	0.0442
$\bar{\Pi}_1 = 12$ (decrease)	19.65	14.89	3.46	1.31	0.0429
$\bar{\Pi}_2 = 55$ (increase)	16.18	11.40	3.30	1.47	0.0421
$m = 110$ (increase)	16.56	11.29	4.08	1.18	0.0442
$g = \Gamma(1.3, 50)$ (shift right)	14.88	9.78	3.91	1.19	0.0442

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