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The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets

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ABSTRACT

This article develops an equilibrium framework for strategic option exercise games. I focus on a particular example: the timing of real estate development. An analysis of the equilibrium exercise policies of developers provides insights into the forces that shape market behavior. The model isolates the factors that make some markets prone to bursts of concentrated development. The model also provides an explanation for why some markets may experience building booms in the face of declining demand and property values. While such behavior is often regarded as irrational overbuilding, the model provides a rational foundation for such exercise patterns.

IN THE CASE OF TRADITIONAL financial options, optimal exercise strategies can be derived without consideration of the strategic interactions across option holders. Most financial options represent widely held side-bets between agents external to the firm, and therefore their exercise does not influence the characteristics of the underlying security or the options themselves. A notable exception is the case of warrants or convertible securities. In the case of warrants, exercise results in the firm issuing new shares of common stock, thereby influencing the underlying stock value as well as the value of the remaining warrants.

In this article, I demonstrate that a game-theoretic approach to option exercise can be very useful in explaining real-world investment decisions. I develop an equilibrium framework for solving option exercise strategies. In order to emphasize the applicability of such an approach, I focus on a particular real-world example: the behavior of real estate markets. This analysis of the strategic equilibrium exercise policies of real estate developers (i.e., the exercise of the "option to build") provides a potential explanation for several puzzling real estate market phenomena. For example, some real estate markets have been prone to pronounced bursts of development activity, while others have been characterized by smooth patterns of development over time. Thus, one can use the model to examine the conditions that influence the time between construction starts (i.e., exercise). Similarly, some real estate markets have been prone to lengthy periods of overbuilding, where new development

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increases in the face of significant declines in demand and building values. The model provides a potential rational explanation for this phenomenon. Developers, fearing preemption by a competitor, proceed into a "panic" equilibrium in which all development occurs during a market downturn. The model is also extended to the case of a perfectly competitive equilibrium, and the impact of market structure on equilibrium exercise strategies is examined.

Research on the strategic exercise of warrants and convertible securities provides an important background for the present analysis. Emanuel (1983) demonstrates that if a warrant holder has a sufficient degree of monopoly power, it is optimal to exercise warrants sequentially, rather than simultaneously. Extending the analysis to a competitive market structure, Constantinides (1984) analyzes the case in which a continuum of competitive warrant holders choose their equilibrium exercise strategies. Spatt and Sterbenz (1988) expand on these results in several directions. First, they demonstrate that for oligopolistic markets, the equilibrium set of exercise strategies may lead to warrant valuations that may be even lower than those of the competitive case. Essentially, fear of preemptive exercise by competitors can lead to a simultaneous rush to early exercise. Second, they demonstrate that optimal exercise strategies depend on the issuing firm's policy regarding the use of the warrant-exercise proceeds. For example, when firms reinvest the proceeds in their underlying assets, the underlying risk of the equity shares increases, leading to an increase in the value of the unexercised warrants. This can then lead to sequential exercise. Alternatively, if the proceeds are invested in riskless bonds, the underlying equity risk is not increased, thus leading to a simultaneous exercise equilibrium.

This article's strategic option exercise framework has numerous applications in real asset markets. This area of research has been relatively untapped. Williams (1993) is a significant exception. Williams derives an equilibrium set of exercise strategies for real estate developers, where equilibrium development is symmetric and simultaneous. In Williams' equilibrium, all developers build at the maximum feasible rate whenever income rises above a critical value. In contrast, equilibrium development in this model may arise endogenously as either simultaneous or sequential. In addition, the present model allows for the possibility of preemptive equilibria in which development bursts occur following a fall in demand.[1] The framework provided in this paper also has implications for corporate investment policy. For example, expenditures on research and development are analogous to exercising the option to invest in future growth opportunities. With a limited number of competitors, each firm's timing and quantity of research effort must take into consideration the equi-

Several key assumptions account for the differing nature of the Williams equilibrium from the present model's. The Williams model may be seen as an approximation of a steady-state equilibrium, in that the supply of undeveloped assets is permitted to increase at the same rate as the supply of developed assets. In addition, in the Williams model development can be done in infinitesimal increments and with no construction lag. In the present model, development is done in finite increments, there is time-to-build, and there is no automatic increase in land holdings following development.

librium exercise strategies of its rivals. Related applications to real options have included the study of capital budgeting in settings with preemption and learning (Spatt and Sterbenz (1985)), investment with strategic competition (Kulatilaka and Perotti (1995)), and the adoption of new technology (Dybvig and Spatt (1983)). Porter and Spence (1978) provide an interesting actual example of a real option in which exercise was simultaneous due to preemption in a study of capacity additions in the corn wet milling industry. An early application of option pricing analysis to real estate development is Titman (1985). Grenadier (1995a) uses an option pricing approach to provide a rational explanation for boom-and-bust cycles in real estate markets.

In this article I develop an equilibrium model for the case of real estate development. Two building owners lease their existing properties in a local real estate market. In addition, each holds the option to develop a new, superior building. The option to develop is a call option with an exercise price equal to the cost of construction, and the underlying security is a new building. However, the exercise of the development option by one building owner has repercussions on the value of both options. The first to build (the Leader) will pay the construction cost earlier, but benefit by being able to lease the superior space without competition. The other developer (the Follower) will see the value of its existing building affected: it is rendered (relatively) obsolete by the presence of a new building. If and when the Follower exercises his development option, he will gain the value from renting a new building. The Leader will see his monopoly rentals discontinued, and must now compete in a duopoly. The exercise strategy is complicated, yet made more realistic by the inclusion of a time-to-build feature.

I identify and analyze subgame perfect equilibria for this strategic development option game. Depending on the initial conditions and parameter values, either simultaneous or sequential exercise may take place. The methodology is that of stochastic stopping-time games; in particular, the results of Dutta and Rustichini (1991) underlie the approach. The approach in this article, however, is more on a heuristic level. For a more rigorous development of the theory of equilibrium in stochastic stopping-time games, see Dutta and Rustichini (1991) or Huang and Li (1991). Smets (1993) provides an application in the area of international finance.

The article is organized as follows. Section I provides an overview of the model. Section II derives the value of pursuing either the Leader or Follower strategy. Section III determines the equilibrium set of exercise strategies for both developers. Section IV considers the model's implications on the propensity for development cascades. Section V applies the model to analyzing building booms in the face of declining demand. Section VI generalizes the model to the case of a perfectly competitive market setting. Section VII concludes.

I. A Summary of the Model

The strategic exercise of options is illustrated through a specific case: the option to develop real estate. At the outset of the model, a local real estate market consists of two buildings, owned by distinct individuals. Each building is identical and rents at a rate of R per unit time. Both owners have an opportunity to redevelop their properties into new, superior buildings. These buildings can earn potentially greater rentals. Thus, each owner holds an option to develop. The option to develop has an exercise price equal to I, the cost of construction.

A new building will yield a rental rate according to a downward-sloping inverse demand function. The demand function will be subject to continuous shocks. The inverse demand function is of the following form:[2]

$$P(t) = X(t) \cdot D[Q(t)] \tag{1}$$

where $P(t)$ is rent flow at time t, $Q(t)$ is the supply of improved buildings at time t, and $D(\cdot)$ is a differentiable function with $D' < 0$. This market is characterized by evolving uncertainty in the state of demand. Even demand at the next instant is uncertain today. $X(t)$ represents a multiplicative demand shock, and evolves as a geometric Brownian motion:

$$dX = \mu X \, dt + \sigma X \, dz \tag{2}$$

where dz is an increment of a standard Wiener process. The constant μ is the instantaneous conditional expected percentage change in X per unit time and the constant σ is the instantaneous conditional standard deviation per unit time. One can easily envision potential real-world candidates for such market shocks. For office development, such shocks may take the form of business profitability. For apartment development, the shocks may be due to job growth. For industrial markets, the shocks may be due to industrial production.

The exercise of the development option does not yield immediate payoffs. An important aspect of real estate development is the time-to-build. The construction period for real estate varies widely across property types. For office buildings, the average length of time between the initiation and completion of construction is 2½ years (Wheaton (1987)). For some very large office projects in highly regulated cities, the construction lags could extend to 5 years or more. In the case of industrial buildings, the lag is much shorter, perhaps on the order of 6 months (Wheaton and Torto (1990)). Developers must use current information on demand and supply in order to determine the optimal time to begin construction. I model this aspect of real estate development by assuming a time-to-build of δ years. Thus, if an owner begins construction (exercises the option) at time τ, the building will not begin receiving rentals until time $\tau + \delta$.

In equilibrium, one owner will begin construction first and become the Leader. The other will build later and become the Follower. Of course, it may be the case that each wishes to enter simultaneously and become the Leader. A slight technical assumption is imposed. It is assumed that if each tries to

[2] This demand structure is similar to that in Williams (1993) and Grenadier (1995b).

build first, one will randomly (i.e., through a toss of a coin) win the race.[3] The loser of the race may then, if he wishes, enter a split second later. This will be indistinguishable from simultaneous development.

The strategic nature of the dual option exercise problem will now be made clear. The exercise of the development option will result in repercussions on both the option exerciser as well as the other building owner. First, consider the exercise of the development option by the Leader. The Leader pays an initial construction cost today, and loses current rentals on the existing building. In return, at the end of construction in δ years, the Leader begins receiving monopoly rents of $X(t) \cdot D(1)$ on the newly developed building. This option exercise also affects the fortunes of the Follower. The competitor's construction of an improved building lessens the demand for the existing building. For example, the existing tenants in the older building may demand rent concessions or improvements to their space, in return for not moving to the newer building at the end of the lease. I account for the degree of "harm" to the old building's operations caused by the Leader's exercise by assuming that the old building's rentals fall to $(1 - \gamma) \cdot R$, with $\gamma \in (0, 1)$.[4]

Now, consider the impact of the Follower's exercise of the development option on both owners. The Follower will pay the cost of construction, lose current rent, and at the end of construction begin receiving rent on the new, improved building. The Leader is also affected because he now faces competition in the leasing of improved space. Each building will lease at a duopoly rental of $X(t) \cdot D(2)$.

In the following sections I derive a set of equilibrium exercise strategies. Specifically, I derive a pair of symmetric, Markovian exercise strategies that form a subgame perfect equilibrium. Simply put, at each point of the game, each owner's exercise strategy is optimal conditional on the other's exercise strategy. Depending on the initial conditions, some equilibria will be characterized by sequential development, while others will be characterized by simultaneous development. The starting point of the game can arise in various ways. One possibility is that an exogenous shock, such as a zoning change allowing commercial development in an area, instigates developer interest. Another possibility is that the game has an infinite history and developers merely wait for the local market to grow to a sufficient size before beginning development.

A brief comment about the option valuation framework is in order. In traditional financial option pricing models, the approach to valuation is based on arbitrage arguments, where one uses the fact that one can instantaneously trade the underlying asset and a riskless asset so that the option is precisely replicated. In the case of real estate, where one cannot short sell the building,

[3] A potential rationale for this assumption is that development requires approval from the local government. Approval may depend on who is first in line or arbitrary considerations.

[4] It would be more realistic to assume that the deterioration in the rent to the old building depends on the stage of the Leader's completion. The assumption that the harm is immediate is done for tractability. However, none of the basic results of the model are altered under a time-varying harm function.

transactions costs are high, and assets cannot be traded in infinitesimal increments, such an arbitrage technique is highly unrealistic. The alternative valuation technique used here is an equilibrium approach. This approach is now standard in much of the literature on real options. For simplicity, I assume risk neutrality.[5] This seemingly restrictive assumption can easily be relaxed by adjusting the drift rate μ to account for a risk premium in the manner of Cox and Ross (1976).

II. The Value of the Leader and Follower

In the model, one individual will develop first and become the Leader, and the other will become the Follower. In this section, I derive the value of pursuing each strategy. For the moment, I simply take the roles as given. In the next section, I derive the equilibrium set of exercise strategies that determine the identity of the Leader and Follower.

First, I value the payoff of choosing to be the Follower. As is standard in solving dynamic games, I work backward in a dynamic-programming fashion. Thus, in this section it is assumed that one owner has already begun (or completed) construction. Since the Leader has already exercised his option, the Follower can construct his exercise strategy without fear of preemption. Therefore, this is a standard option pricing problem in which the equilibrium Follower strategy is to maximize his option's value.

The value of the Follower can be characterized as a portfolio containing the existing building, plus an option to exchange the existing building for a new building.[6] The existing building, conditional on the Leader having begun development, yields a rent flow of $(1 - \gamma) \cdot R$. Since this payment is certain, its value is $(1 - \gamma) \cdot R/r$. The value of the Follower also emanates from the value of the option to exchange the old building for a new building. Upon exercise, the Follower pays a construction cost of I, and exchanges the old building for a new building δ years from completion. Denote the value of this option to exchange as $W(X)$. The total value of the Follower, $\mathrm{F}(X)$, will be the portfolio value: $\mathrm{F}(X) = (1 - \gamma) \cdot R/r + \mathrm{W}(X)$.

Consider the dynamics of $\mathrm{W}(X)$ in the region where the Follower does not exercise. By Itô's Lemma, the instantaneous change in $\mathrm{W}(X)$ can be written as:

$$d\mathrm{W}(X) = [\tfrac{1}{2}\, \sigma^2 X^2 \mathrm{W}''(X) + \mu X \mathrm{W}'(X)]dt + \sigma X \mathrm{W}'(X) dz. \tag{3}$$

Since this option yields no current cash flow, the total instantaneous return comes in the form of capital gains, $d\mathrm{W}(X)$. Let α_{F} denote the expected return per unit time on the Follower's option value. Applying the expectations operator to equation (3) and dividing by W, the expected return can be written as:

$$\alpha_{\mathrm{F}} = \frac{1}{\mathrm{W}(X)} \cdot \left[\frac{1}{2}\, \sigma^2 X^2 \mathrm{W}''(X) + \mu X \mathrm{W}'(X) \right]. \tag{4}$$

[5] Or, equivalently, that the process dz is uncorrelated with aggregate wealth.

[6] See Margrabe (1978) for an analysis of the option to exchange one risky asset for another.

In equilibrium, the expected return on the Follower's option must equal the risk-free rate r. Setting α_F equal to r yields the following differential equation, which must be satisfied in equilibrium:

$$0 = \tfrac{1}{2}\,\sigma^2 X^2 \mathrm{W}''(X) + \mu X \mathrm{W}'(X) - r\mathrm{W}(X). \tag{5}$$

Equation (5) must be solved subject to specific boundary conditions. Upon exercise of the development option, the Follower exchanges the cash flows on the old building, $(1 - \gamma)R$, for the rentals on a new building, $XD(2)$, which begin in δ years. As in American option pricing problems of this sort, there will be a trigger value at which it will be optimal to exercise. Specifically, there will exist a trigger value, X_F, such that the Follower will exercise the option to build the first time that $X(t)$ equals or exceeds X_F.[7] Mathematically, one can write the optimal entry time of the Follower as $T_F \equiv \inf\,[t \geq 0\colon X(t) \geq X_F]$. Therefore, I impose two boundary conditions which serve to define the optimal exercise policy:

$$\begin{aligned} \mathrm{W}(X_F) &= \left[\frac{D(2)}{r-\mu}\,e^{-(r-\mu)\delta}\right]\cdot X_F - \frac{(1-\gamma)R}{r} - I, \\ \mathrm{W}'(X_F) &= \left[\frac{D(2)}{r-\mu}\,e^{-(r-\mu)\delta}\right]. \end{aligned} \tag{6}$$

The first boundary condition is commonly termed the "value-matching" condition. It simply reflects the fact that, upon exercise, the payoff of the option is the exchange of a new building for the older building (δ years into the future), minus the cost of construction.[8] The second boundary condition is known as the "high-contact" or "smooth-pasting" condition.[9] Essentially, this condition ensures that X_F is the trigger that maximizes the value of the Follower's option.

A final boundary condition is imposed: $\mathrm{W}(0) = 0$. This is the result of the fact that zero is an absorbing barrier of the rent process; if $X(t)$ ever reaches zero, it will remain there forever. Thus, if X hits zero, the option will never be exercised and will become worthless.

Solving the equilibrium differential equation in (5) subject to the above boundary conditions results in the following value of the Follower's option (conditional upon the Leader having entered):

$$\mathrm{W}(X) = \begin{cases} \left(\dfrac{I + (1-\gamma)R/r}{\beta - 1}\right)\cdot\left(\dfrac{X}{X_F}\right)^{\beta}, & \text{if } X < X_F; \\[2ex] \left[\dfrac{D(2)}{r-\mu}\,e^{-(r-\mu)\delta}\right]\cdot X - \dfrac{(1-\gamma)R}{r} - I, & \text{if } X \geq X_F; \end{cases} \tag{7}$$

[7] See the appendix to Chapter 4 in Dixit and Pindyck (1994) for a discussion of optimal stopping regions.

[8] The term $\{[D(2)/(r-\mu)]e^{-(r-\mu)\delta}\}X_F$ equals the present value of a perpetual rent flow of $X(t)\cdot D(2)$, beginning in δ years, and conditional on the current level of X being X_F.

[9] See Merton (1973) for a derivation of the high-contact condition.

where:

$$\beta = \frac{-(\mu - \tfrac{1}{2}\,\sigma^2) + \sqrt{(\mu - \tfrac{1}{2}\,\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} > 1,$$

$$X_F = \left(\frac{\beta}{\beta - 1}\right)\left(\frac{r - \mu}{D(2)}\right) e^{(r-\mu)\delta}\left[I + \frac{(1-\gamma)\cdot R}{r}\right].$$

Finally, the value of being the Follower equals the sum of the value of the existing building plus the value of the option to exchange. Thus,

$$\mathrm{F}(X) = \frac{(1-\gamma)\cdot R}{r} + \mathrm{W}(X). \tag{8}$$

Proposition 1 summarizes the optimal Follower strategy:

PROPOSITION 1: *Conditional on the Leader having begun construction, the optimal Follower strategy is to begin construction the first moment that* $X(t)$ *equals or exceeds the trigger value* X_F*, as defined in equation* (7). *That is, the optimal entry time of the Follower,* T_F*, can be written as:*

$$T_F = \inf\left\{t \geq 0\colon X(t) \geq \left(\frac{\beta}{\beta - 1}\right)\left(\frac{r - \mu}{D(2)}\right) e^{(r-\mu)\delta}\left[I + \frac{(1-\gamma)\cdot R}{r}\right]\right\}.$$

Now, consider the value of the Leader, conditional on the Follower pursuing the optimal Follower strategy. Suppose the Leader has already exercised his development option and now has τ years until completion, where $\tau \in [0, \delta]$. Over the next τ years, the Leader receives no rentals. Once completed, the Leader earns a monopoly rental rate of $X(t) \cdot D(1)$. However, since the Follower enters at the stopping time T_F, the Leader will begin receiving the duopoly cash flow, $X(t) \cdot D(1)$, at time $T_F + \delta$. It is important to note that it is the Follower's exercise strategy, T_F, which impacts the Leader's valuation.

The value of the Leader can be replicated by the following portfolio of options:

1. Purchase a call option on an asset which pays a perpetual dividend rate of $X(t) \cdot D(1)$. This call option has a zero exercise price, and a fixed expiration date of τ.
2. Purchase a call option on an asset which pays a perpetual dividend rate of $X(t) \cdot [D(2) - D(1)]$. This call option has a zero exercise price, and a stochastic expiration date of $T_F + \delta$.

The value of this portfolio can be derived using the same approach as in the case of the Follower (except the smooth-pasting condition is no longer needed). Denote the value of the Leader, prior to the exercise of the Follower, $\mathrm{L}(X, \tau)$.

X is the current level of demand, and τ is the remaining time before completion, $\tau \in [0, \delta]$. The solution for $\mathrm{L}(X, \tau)$ can be expressed as:

$$\mathrm{L}(X, \tau) \tag{9}$$

$$= \begin{cases} \dfrac{e^{-(r-\mu)\tau}}{r-\mu} XD(1) + \dfrac{\beta}{\beta-1} \dfrac{D(2)-D(1)}{D(2)} \left(I + \dfrac{(1-\gamma)R}{r}\right)\left(\dfrac{X}{X_{\mathrm{F}}}\right)^{\beta}, & \text{if } X < X_{\mathrm{F}}; \\ \dfrac{e^{-(r-\mu)\tau}}{r-\mu} XD(1) + \dfrac{Xe^{-(r-\mu)\delta}}{r-\mu}[D(2)-D(1)], & \text{if } X \geq X_{\mathrm{F}}. \end{cases}$$

Finally, consider the relative values of the Leader and Follower at the moment the Leader begins construction. The Leader receives a payoff of $\mathrm{L}(X, \delta) - I$ and the Follower receives $\mathrm{F}(X)$. Depending on the initial entry time (which is determined by equilibrium considerations and derived in the next section), the Leader's payoff may be greater or less than that of the Follower. The following proposition describes the relative valuations.

PROPOSITION 2: *There exists a unique point, $X_L \in (0, X_F)$, with the following properties:*

$$\begin{aligned} \mathrm{L}(X, \delta) - I &< \mathrm{F}(X) && \text{for } X < X_{\mathrm{L}}, \\ \mathrm{L}(X, \delta) - I &= \mathrm{F}(X) && \text{for } X = X_{\mathrm{L}}, \\ \mathrm{L}(X, \delta) - I &> \mathrm{F}(X) && \text{for } X_{\mathrm{L}} < X < X_{\mathrm{F}}, \\ \mathrm{L}(X, \delta) - I &= \mathrm{F}(X) && \text{for } X \geq X_{\mathrm{F}}. \end{aligned}$$

Proof: See Appendix.

Proposition 2 demonstrates that there is a unique value of $X \in (0, X_{\mathrm{F}})$, called X_{L}, at which the payoffs to both Leader and Follower are equal. At any point below X_{L}, each developer would prefer being the Follower. At any point above X_{L}, each developer would prefer (or be indifferent to) being the Leader. At X_{L}, the benefits of a potentially temporary monopoly just equal the costs of paying the construction costs earlier.

III. The Equilibrium Exercise Strategies

Given the value of being the Leader or Follower, I now work back to the beginning of the game, where the two developers choose their equilibrium entry strategies. Each is currently leasing the previous generation of buildings at the rent flow of R. I construct a pair of symmetric, subgame perfect equilibrium entry strategies in which each developer's exercise strategy, conditional upon the other's exercise strategy, is value-maximizing.

I focus on two classes of equilibria that occur depending on the initial state of demand, $X(0)$. If $X(0) < X_{\mathrm{F}}$, equilibrium exercise will be sequential; once one agent begins development, a period of time passes before the other begins development. If $X(0) \geq X_F$, equilibrium exercise will be simultaneous; one exercises instantly after the other.

A. Equilibrium With Sequential Exercise: $X(0) < X_F$

If the development game begins with an initial level of X that is lower than X_F, then we will observe a sequential equilibrium: the Leader will begin development the first moment that $X(t)$ equals or exceeds X_L, and the Follower will wait and begin development the first moment that $X(t)$ rises to the level X_F. I state this more formally in the following definition.

Sequential Exercise Equilibrium: Suppose the initial level of X is below X_F. A pair of symmetric, subgame perfect equilibrium exercise strategies is for each agent to act as follows:

> *If your competitor has not yet begun construction, then initiate development the first moment that $X(t)$ equals or exceeds X_L. If your competitor has already begun construction, then wait until $X(t)$ rises to X_F before beginning construction.*

Given the above strategies, the sequential equilibrium will appear as follows. If $X(0) < X_L$, one developer will wait until the trigger X_L is reached, and the other will wait until the trigger X_F is reached. The developers will be indifferent between leading or following. If $X(0) \in [X_L, X_F)$, each will race to build immediately. The (random) winner of the race will then build, and the loser will wait until the trigger X_F is reached.

It is simple to demonstrate that this set of strategies forms an equilibrium. From Proposition 2, entry before X hits X_L leads to a value strictly less than that of following. Thus, if your competitor follows the equilibrium strategy, the best that you can do is also to pursue the equilibrium strategy by attempting to enter the first moment that X equals or exceeds X_L. In addition, Proposition 1 demonstrates that the optimal strategy for whomever builds second is to enter when the trigger X_F is reached.

B. Equilibrium With Simultaneous Exercise: $X(0) \geq X_F$

Now, consider the range where $X(0) \geq X_F$. In this range, any equilibrium will be characterized by simultaneous exercise. If either developer begins construction at a level of X greater than X_F, the other will enter immediately thereafter. This is due to Proposition 1, which depicts the optimal Follower strategy of entering the first moment that X_F is equaled or exceeded. Thus for any exercise strategy over this region, the equilibrium will be characterized by simultaneous entry: one enters an instant after the other.

There will be an infinite number of equilibrium strategies over this region. I focus on a class of equilibria that can be characterized by a band of values, $(X_F, \hat{X})$, where both agents begin development the first time $X(t)$ exits this range. It is simple to demonstrate that such a set of strategies forms an equilibrium. First, we know that if X ever falls to the level X_F, each will be forced to enter immediately. This is due to the threat of preemption. Should the value of X fall below X_F, then the firm that enters first will have a value strictly greater than the laggard's value, as in Proposition 2. Thus, it is in each firm's interest to enter immediately. Second, for any $\hat{X} > X_F$, it is an equilibrium for both to enter when $X(t)$ first equals or exceeds $\hat{X}$. That is, given that one firm will wait until $\hat{X}$ is reached, it is optimal for the other firm to do likewise. This follows from Propositions 1 and 2: the value of leading is strictly greater than following, and the Follower's optimal response to losing the race is to enter immediately.

Therefore, there is an infinite set of Markovian, subgame perfect equilibrium exercise strategies characterized by immediate construction upon the first moment that $X(t)$ falls outside the range $(X_F, \hat{X})$, $\forall \hat{X} \geq X_F$. From this set of simultaneous equilibria, I focus on one in particular: the Pareto optimal one.[10] That is, given that the two developers will simultaneously enter the first moment that $X(t)$ reaches $\hat{X}$ (provided $X(t)$ does not first fall to X_F), then the Pareto optimal equilibrium will set $\hat{X}$ such that both values are maximized.

I now solve for the Pareto optimal set of simultaneous exercise strategies. Denote the value of each developer as $\mathrm{G}(X)$. Using an analogous line of reasoning to that of Section II, over the range at which each developer is idle, their values will satisfy the following equilibrium differential equation:

$$0 = \tfrac{1}{2}\,\sigma^2 X^2 \mathrm{G}''(X) + \mu X \mathrm{G}'(X) - r\mathrm{G}(X) + R. \tag{10}$$

The boundary conditions will determine the optimal simultaneous exercise region. Denote the optimal joint exercise trigger as X_J. Thus, if X rises to the trigger X_J, each will exercise their development option, and their compound option values will be maximized. However, if X falls to the level X_F, each will exercise in order to avoid preemption. These boundary conditions are:

$$\begin{aligned} \mathrm{G}(X_F) &= \frac{1}{r-\mu}\, e^{-(r-\mu)\delta} X_F D(2) - I, \\ \mathrm{G}(X_J) &= \frac{1}{r-\mu}\, e^{-(r-\mu)\delta} X_J D(2) - I, \\ \mathrm{G}'(X_J) &= \frac{1}{r-\mu}\, e^{-(r-\mu)\delta} D(2). \end{aligned} \tag{11}$$

The first boundary condition reflects the fact that if X falls to the level of X_F, each will enter immediately. Even though one firm will win the race, the other will find it optimal to enter in the next instant. The second boundary condition is a "value-matching" condition: at the optimal joint-entry trigger, each firm enters immediately. The final boundary condition is a "smooth-pasting" condition which ensures that the joint-entry trigger X_J is indeed the one that maximizes value.

[10] Fudenberg and Tirole (1985) argue that if one equilibrium Pareto dominates all others, then it is the most reasonable to expect.

The solution to differential equation (10) can be written as:

$$G(X) = \begin{cases} A_1X^{\alpha} + A_2X^{\beta} + \dfrac{R}{r} & \text{for } X \in (X_F, X_J), \\[2ex] \dfrac{e^{-(r-\mu)\delta}XD(2)}{r-\mu} - I & \text{for } X \notin (X_F, X_J), \end{cases} \tag{12}$$

where:

$$\alpha = \frac{-(\mu - {}^1\!/_2\,\sigma^2) - \sqrt{(\mu - {}^1\!/_2\,\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} < 0,$$

$$\beta = \frac{-(\mu - {}^1\!/_2\,\sigma^2) + \sqrt{(\mu - {}^1\!/_2\,\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} > 1,$$

and A_1, A_2, and X_J are the solutions to the following system of three nonlinear equations:

$$\begin{aligned} A_1X_F^{\alpha} + A_2X_F^{\beta} + \frac{R}{r} &= \frac{e^{-(r-\mu)\delta}X_FD(2)}{r-\mu} - I, \\ A_1X_J^{\alpha} + A_2X_J^{\beta} + \frac{R}{r} &= \frac{e^{-(r-\mu)\delta}X_JD(2)}{r-\mu} - I, \\ \alpha A_1X_J^{\alpha-1} + \beta A_2X_J^{\beta-1} &= \frac{e^{-(r-\mu)\delta}D(2)}{r-\mu}. \end{aligned} \tag{13}$$

While this system does not simplify to a closed-form solution, it is easily solved numerically.

I now summarize the simultaneous equilibrium in the following definition:

Simultaneous Exercise Equilibrium: Suppose the initial level of X is at or above X_F. A symmetric, subgame perfect equilibrium set of exercise strategies is for each owner to act as follows:

> *Begin development the first moment that $X(t)$ falls to X_F, or equals or exceeds X_J. If your competitor wins the race to build first, then enter instantaneously thereafter.*

Given the above strategies, the sequential equilibrium will appear as follows. If $X(0) \in (X_F, X_J)$, then there will be an initial period of inactivity. Both agents will begin construction the first moment that either X_F or X_J is reached. If $X(0) = X_F$, or $X(0) \geq X_J$, then both begin construction immediately.

IV. Development Cascades: The Rapidity of Construction Starts

In this section, I analyze the amount of time between the exercise of options. The goal of this analysis is to determine the average time between construction starts. That is, once development occurs in a market, when is it likely to be followed by additional development? Here I focus on the sequential exercise equilibrium, where development options are exercised at distinct stopping times. Depending on the underlying conditions of the market, equilibrium construction may occur in short bursts or more evenly over time. I refer to a rapid succession of exercise strategies as a "development cascade."[11]

A. A Measure of the Rapidity of Construction Starts

I focus on the case in which the market begins with a period of inactivity followed by a sequence of construction starts. Thus, in this section I analyze the case in which the market begins with $X(0) < X_L$.[12]

Briefly, recall the nature of the sequential equilibrium setting. The real estate market is dormant until demand rises to the trigger level X_L. At that time, one developer begins construction. The other developer will hold off on construction until demand rises to the second trigger level, X_F. Therefore, the time between construction starts is the first passage time of the stochastic process $X(t)$ reaching the level X_F, when starting at the level X_L.

Define the time between construction starts as T_s. Thus, T_s can be expressed as:

$$T_s \equiv \inf\{t \geq 0: X(t) = X_F,\ X(0) = X_L\}. \tag{14}$$

Using equation (1.11) in Harrison (1985), and the application of a simple change in variables, the cumulative distribution function of the first passage time T_s can be written as:

$$P[T_s \leq t] = \Phi\left[\frac{-\ln(X_F/X_L) + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right] + \left(\frac{X_F}{X_L}\right)^{2(\mu-(1/2)\sigma^2)/\sigma^2} \Phi\left[\frac{-\ln(X_F/X_L) - (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right] \tag{15}$$

To provide a measure of the average time between construction starts, one might first consider the expected value of T_s. Unfortunately, for the case in which $\mu < \frac{1}{2}\sigma^2$, the expectation is undefined. Essentially, if the variance in demand is sufficiently high, the probability that the upper trigger is never

[11] Caplin and Leahy (1994) provide an example of a development cascade in a specific location: Lower Sixth Avenue in New York City. Caplin and Leahy posit an information-spillover explanation for the rapid growth of a previously deserted retail area.

[12] Of course, a similar analysis could be performed for other ranges of initial starting points. However, this is the only case in which the time between starts is independent of the (arbitrary) initial starting level of demand.

reached is large enough to make the expectation infinite. To overcome this problem, I choose to measure the average time between starts by the median of T_s. The median time between construction starts, M_s, is the solution to the following nonlinear equation:

$$\Phi\left[\frac{-\ln(X_F/X_L) + (\mu - \tfrac{1}{2}\sigma^2)M_s}{\sigma\sqrt{M_s}}\right] + \left(\frac{X_F}{X_L}\right)^{2(\mu-(1/2)\sigma^2)/\sigma^2}\Phi\left[\frac{-\ln(X_F/X_L) - (\mu - \tfrac{1}{2}\sigma^2)M_s}{\sigma\sqrt{M_s}}\right] = \frac{1}{2} \quad (16)$$

B. Comparative Statics

I now examine the effect of changes in underlying parameters on the median time between construction starts. In particular, I consider three potential influences on M_s: the volatility of demand (σ), the time-to-build (δ), and the degree of current obsolescence (γ).

Consider the effect of market volatility on the time between construction starts. There are two competing forces at work here. First, a well-established result from the theory of real options is that increasing uncertainty leads to an increased willingness to postpone investment.[13] Thus, beginning from any initial level of demand, this result would tend to delay the exercise of any future development option. However, a second force at work here is that increasing volatility may also make it more likely that a substantial increase in demand is reached in a shorter period of time. Therefore, changing σ alters the optimal exercise strategies as well as the stochastic properties of the underlying demand.

Figure 1 demonstrates that the net effect of increasing demand volatility is a decrease in the median time between starts.[14] For example, given two markets differing only in the degree of demand volatility, the one with the greater volatility will be the one more prone to development cascades. Notice that for $\sigma = 0.025$, the median time between starts is almost 9.5 years. For $\sigma = 0.15$, the median time between starts falls to 3.5 years. For $\sigma = 0.35$, the median time between starts falls to less than one year. That is, there is a 50 percent likelihood that the time between starts will be less than one year.

The intuition for this result is as follows. As volatility increases, the option to wait increases. Of course, this option to wait affects both the Leader and Follower. Thus, even though the Leader will delay entering, so too will the Follower. Therefore, the "option to wait" factor will have little effect on the time between starts. On the other hand, the increased volatility makes it more

[13] In particular, see McDonald and Siegel (1986).

[14] In this case, an analytical solution to the comparative static result is not obtainable. However, equation (16) is easily solved using numerical procedures. While the results in Figure 1 were derived under a set of initial parameter values, the same qualitative results hold under a wide range of assumed parameter values.

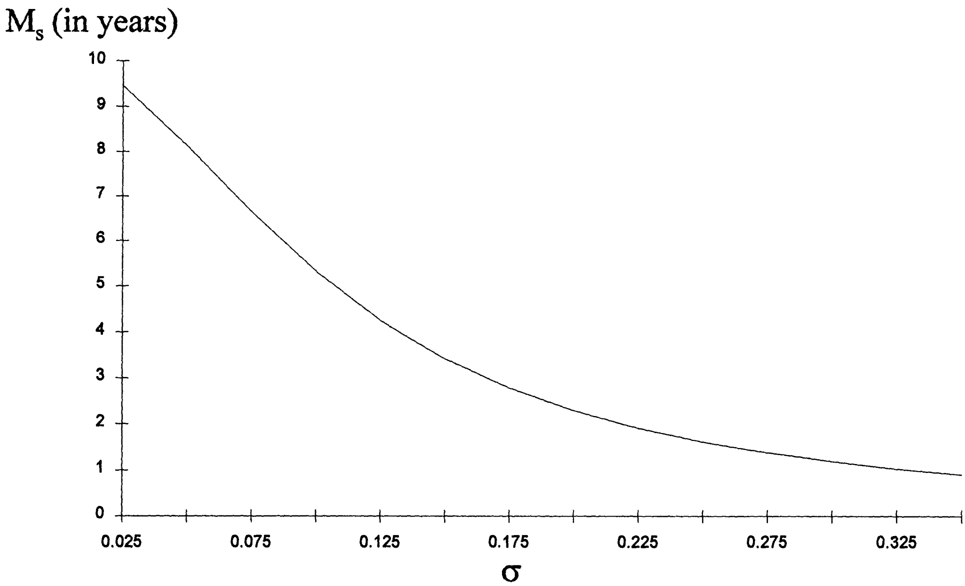


Figure 1. The effect of demand volatility on the median time between construction starts. This figure plots the effect of demand volatility, σ, on the median time between construction starts, M_s. From the instant that the first development option is exercised, M_s is the median number of years before the second development option is exercised. The greater the market's volatility, the greater the propensity for development to occur in bursts. An existing building, earning a rent of R, can be redeveloped into a new building earning a stochastic rent flow of $X(t)D[Q(t)]$, where the demand state variable X is a geometric Brownian motion with drift μ and volatility σ, and $D(\cdot)$ is a downward-sloping inverse demand function. γ is the fractional loss in rent to an older building when a new building is constructed. The cost of construction is I, and development requires a construction period of δ years. The risk-free rate is r. For this figure, the following parameters are fixed: $R = 0.1$, $\mu = 0.02$, $D(1) = 1$, $D(2) = 0.9$, $\gamma = 0.8$, $I = 1$, $\delta = 1$, and $r = 0.04$.

likely that the time it takes for demand to rise from X_L to X_F will fall. Therefore, it is this effect which dominates.

I now consider the effects of changes in the time-to-build and degree of current obsolescence on the propensity for development cascades. Proposition 3 provides a mathematical result that leads to a striking comparative static.

PROPOSITION 3: *The ratio of the two trigger values, X_F/X_L, is independent of the time-to-build parameter δ and the degree of current obsolescence γ:*

$$\frac{\partial(X_F/X_L)}{\partial\delta} = \frac{\partial(X_F/X_L)}{\partial\gamma} = 0.$$

Proof. Proposition 2 defines X_L implicitly as the solution to $L(X_L, \delta) - I - F(X_L) = 0$. Using this identity, and the implicit function theorem, one may obtain an expression for $\partial X_L/\partial\delta$ and $\partial X_L/\partial\gamma$. In addition, one can directly differentiate the expression for X_F given in equation (7). Finally, simple algebraic manipulation yields the stated result. Q.E.D.

Proposition 3 leads directly to the following result: changes in the time-to-build or degree of current obsolescence have no effect on the time between construction starts. In fact, not only will such changes have no effect on the median time between starts, but they will also have no effect on the entire probability distribution of the time between starts. That is, not only is the median time between starts unaffected, but so to is the mean, variance, skewness, and other moments of the distribution. This is apparent by considering the cumulative distribution function of the time between starts given in equation (15). The distribution function for the time between starts, $P[T_s \leq t]$, depends on the parameters δ and γ only through the ratio (X_F/X_L). Proposition 3 demonstrates that this ratio is independent of these two parameters.

The reason for this result is that both the Leader and Follower adjust their equilibrium exercise triggers proportionately to the changes in the underlying parameters. Unlike the case of changes in the underlying volatility, these parameters have no effect on the underlying state variable process. Therefore, changes in δ and γ do not affect the underlying stochastic environment, but simply the consequences of competitive exercise. Since the exercise strategy response is symmetric and proportional, there will be no change in the distribution of the time between exercise.

C. Empirical Implications

The model predicts that demand volatility increases the likelihood of construction cascades. This implies that both regions and property types with more volatile demand would be the most prone to concentrated bursts of development.

One implication of this result is that cities with diversified economies would be less prone to development cascades than cities with significant dependence on a single economic factor.[15] Consider the cases of the Denver and Houston office markets. These cities have historically been heavily dependent on the oil industry. Over the thirty-year period from 1960 through 1990, over half of all office construction was completed in a four-year interval: 1982–1985.[16] Contrast these two markets' experiences with those of Chicago and Minneapolis, both historically more diversified than the oil-price dependent cities. Over no four-year period did Chicago have more than one-quarter of its completions, and over no four-year period did Minneapolis have more than one-third of its completions. Relatively speaking, Chicago and Minneapolis saw development occur more gradually, rather than through short bursts. Thus, we do find that

[15] An additional factor that may influence the timing of development in a city (or submarket) is zoning restrictions. Such restrictions can slow down or speed up development and are subject to changes over time (e.g., changes in the zoning commission).

[16] This data was provided by William Wheaton. This proprietary database contains square-footage office completions, by city, semi-annually over the period from 1960–1991.

less diversified economies may be more prone to development concentrated over a short span of time.[17]

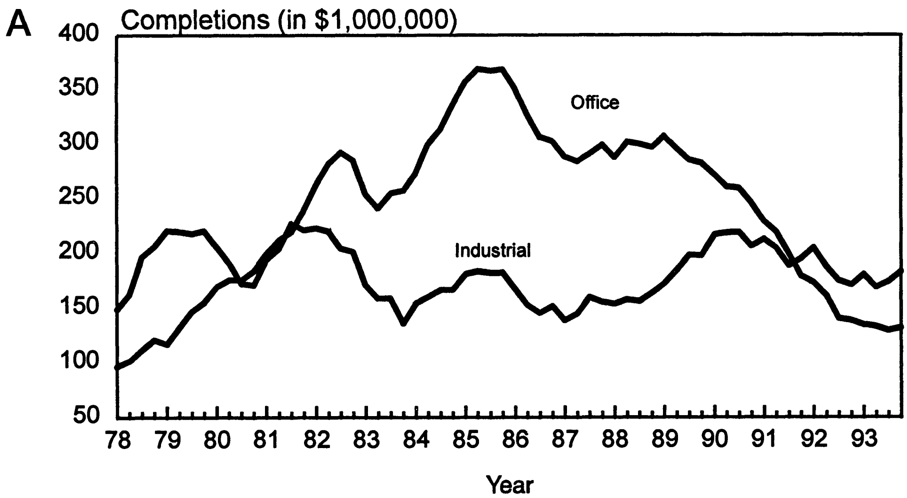
In addition to providing predictions about the cross-sectional differences across cities, the model also points to timing distinctions across property types. For example, office markets appear more volatile than industrial space markets. The Russell-NCREIF database provides a time-series of net operating income by property type. Over the period from 1978 through the first-half of 1994, the annualized standard deviation of the growth rate in office earnings was 0.1587. For warehouse space the number was 0.0892. Thus, office earnings were twice as volatile. Figure 2*A* displays the patterns of development for the office and industrial property markets. Note that industrial development occurred fairly evenly throughout the 1978–1993 period, while office development was more erratic. In terms of square footage, Birch (1990) estimates that 43 percent of all office space ever built in the U.S. was built during the 1980s, and 60 percent was built in the 1970s and 1980s.

The model also predicts that the tendency toward development booms should be unrelated to time-to-build or obsolescence. For example, high-rise office towers will take considerably longer to build than small, one-story office buildings. If one assumes that the demand volatility is relatively equal between these two building types, the model predicts no difference in their tendencies toward development cascades. As another example, the model would predict that, ceteris paribus, markets with dilapidated buildings would be just as prone to development cascades as those with relatively modern buildings. Unfortunately, a test of such propositions cannot be accomplished until appropriate data become available.

V. A Rational Underpinning for "Irrational" Construction: Recession-Induced Construction Booms

In this section, I examine the properties of the simultaneous exercise equilibrium. In this equilibrium, both options are immediately exercised the first moment that $X(t)$ either *rises* to an upper trigger, or *falls* to a lower trigger. Obviously, both developers would prefer exercising after a run-up in demand, but it is still rational for each of them to rush to exercise should demand fall. I refer to this phenomenon, simultaneous development following a decline in demand, as a "recession-induced construction boom" (RCB).

[17] One might possibly explain such behavior by looking for bursts in office demand. However, this does not appear to be the case. Office employment comes predominantly from two sectors of the economy: the Finance-Insurance-Real Estate and the Services sectors (DiPasquale and Wheaton (1996)). The growth rate in these two sectors of employment over the period from 1982 through 1985 was the opposite of what one would expect, given the rates of construction growth. Denver grew by 12 percent, Houston grew by 9.7 percent, Chicago grew by 14.3 percent, and Minneapolis grew by 16.1 percent. Of course, since real estate is a very long-lived asset, development reflects expectations of demand growth over a longer period. However, even demand growth over the period from 1985 through 1990 fails to explain the differing development rates.



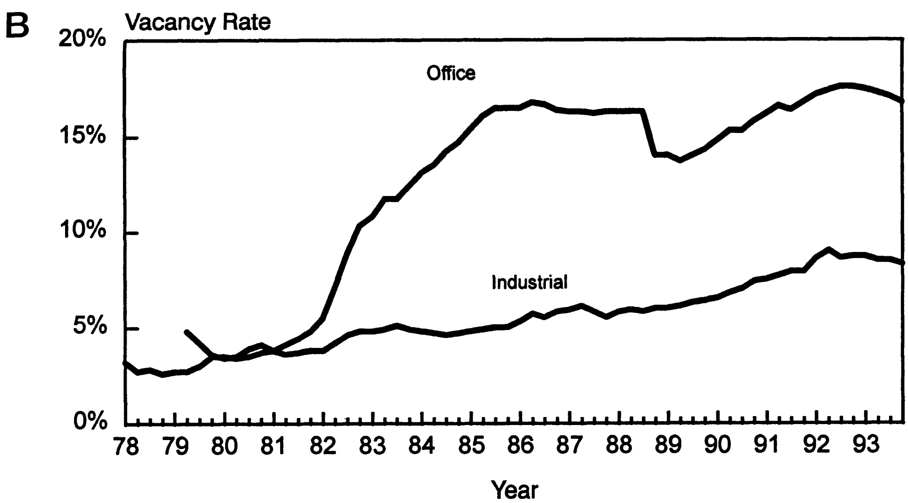


Figure 2. Office and industrial property market behavior (1978–1993). Figure 2*A* plots the real value of office and industrial property completions over time. The completion series are obtained by deflating the Commerce Dept. Construction put in place series by the implicit construction cost. Thus, all completions are in constant (1993) dollars. Figure 2*B* plots the office and industrial property vacancy rates. The source of the vacancy rate series is CB Commercial.

Periods of construction booms in the face of rising vacancies and plummeting demand are a recurring phenomenon in real estate markets. This is often termed "overbuilding" and almost universally attributed to some form of irrationality on the part of developers, lenders, or both.[18] The model in this article

[18] See Wheaton and Torto (1986) for a discussion of the popular explanations for overbuilding.

presents an alternative, rational explanation for RCBs. Developers see the market begin to erode. They realize that if it erodes any further, and if any of their competitors begins to build, they will be shut out of the market. Before this comes to fruition, each builds simultaneously in an attempt to avoid preemption. While building in a downturn is harmful to developers, it is less harmful than the alternative of becoming a Follower in a down market.

Another potential rational explanation for building in the face of high and rising vacancies is a Tobin's q type of argument: even though the values of new buildings were falling, the building costs were falling even faster. On a national level, this does not appear to be the case. During the past decades, national indices of construction costs (e.g., the Department of Commerce Implicit Price Deflator for Construction) were consistently rising, even during periods when commercial property values were falling dramatically. However, such figures ignore the circumstances of local markets. Labor and material market conditions in different cities may have led to rational building during periods of declining rents and prices.

A. Definition of a Recession-Induced Construction Boom

Recall from the construction of an equilibrium in Section III that there are conditions under which rational, value-maximizing developers will simultaneously build in the presence of declining demand. This is true if the initial level of demand satisfies $X_F \leq X(0) \leq X_J$. The equilibrium set of strategies is for both owners to commence construction simultaneously the first moment that $X(t)$ reaches either X_F or X_J. I now analyze the likelihood that the lower trigger is reached first.

Let t denote the current time and suppose that neither owner has started construction. Thus, $X(t) \in [X_F, X_J]$. The following definitions will be useful:

$$\begin{aligned} T_1(x) &= \inf\{\omega \geq 0: X(t+\omega) = X_F,\ X(t) = x,\ x \in [X_F, X_J]\}, \\ T_2(x) &= \inf\{\omega \geq 0: X(t+\omega) = X_J,\ X(t) = x,\ x \in [X_F, X_J]\}, \qquad (17) \\ T(x) &= \min[T_1(x),\ T_2(x)]. \end{aligned}$$

Thus, $T_1(x)$ is the first passage time for X hitting X_F starting from the level x, and $T_2(x)$ is the first passage time for X hitting X_J starting from the level x. In equilibrium, both will begin construction at the minimum of these two stopping times, $T(x)$.

Mathematically, a RCB occurs if $T_1(x) < T_2(x)$. Denote the probability of a RCB, conditional on starting at demand level x, as $\Psi(x)$, where $\Psi(x) = \Pr[T_1(x) < T_2(x)]$.

Using the techniques in Harrison (1985), Chapter 3, this probability can be expressed as follows:

$$\Psi(x) = \frac{x^{-2(\mu-(1/2)\,\sigma^2)/\sigma^2} - X_J^{-2(\mu-(1/2)\,\sigma^2)/\sigma^2}}{X_F^{-2(\mu-(1/2)\,\sigma^2)/\sigma^2} - X_J^{-2(\mu-(1/2)\,\sigma^2)/\sigma^2}} \quad \text{when} \quad \mu \neq \frac{1}{2}\sigma^2, \tag{18}$$

and

$$\Psi(x) = \frac{\ln(x) - \ln(X_J)}{\ln(X_F) - \ln(X_J)} \quad \text{when} \quad \mu = \frac{1}{2}\sigma^2.$$

B. Comparative Statics

I now investigate two factors that influence a market's propensity for RCBs. First, consider the influence of time-to-build on the propensity for RCBs. For a given level of demand, x, I vary δ and investigate the effect on $\Psi(x)$. Clearly, $\Psi(x)$ will always be decreasing in x; the higher the demand, the less likely it will be for demand to hit the lower trigger prior to the upper trigger. Simulation results are presented in Figure 3a, for three different levels of demand. Notice that for each level of demand, the likelihood of a RCB is monotonically increasing in the time-to-build parameter. For example, consider the curve with $x = 0.10$. Suppose two real estate markets were exactly equivalent, except one took 6 months to build and the other took 6.5 years to build. They will display substantially different propensities for RCBs. The market with the shorter lag will have a 50 percent probability of experiencing a RCB; the market with the longer lag will have a 75 percent probability of experiencing a RCB.

In Figure 3b, the impact of demand volatility, σ, is displayed. For the most part, the probability of a RCB is increasing in the volatility of demand. For example, consider the curve with $x = 0.14$. Suppose two real estate markets were exactly equivalent, except one had little volatility ($\sigma = 0.05$) and the other had substantial volatility ($\sigma = 0.16$). The market with low volatility will have virtually no chance of a RCB occurring. The more volatile market will have a 50 percent chance of a RCB occurring. However, for higher levels of initial demand, the probability begins to decline as σ rises to higher levels. This is seen in the curve for which $x = 0.16$. This effect appears quite modest.

C. Empirical Implications

The model predicts that markets with greater time-to-build will display a greater propensity toward RCB. This has implications on the difference between the office and industrial property markets. For office space, an estimate of the average length of time between the initiation and completion of construction is 2.5 years. The construction time for industrial buildings (which are predominately one story, and basically shells of buildings) is significantly less, perhaps on the order of six months. The model would predict a greater tendency for RCBs in the office market than in the construction market. Figure 2b

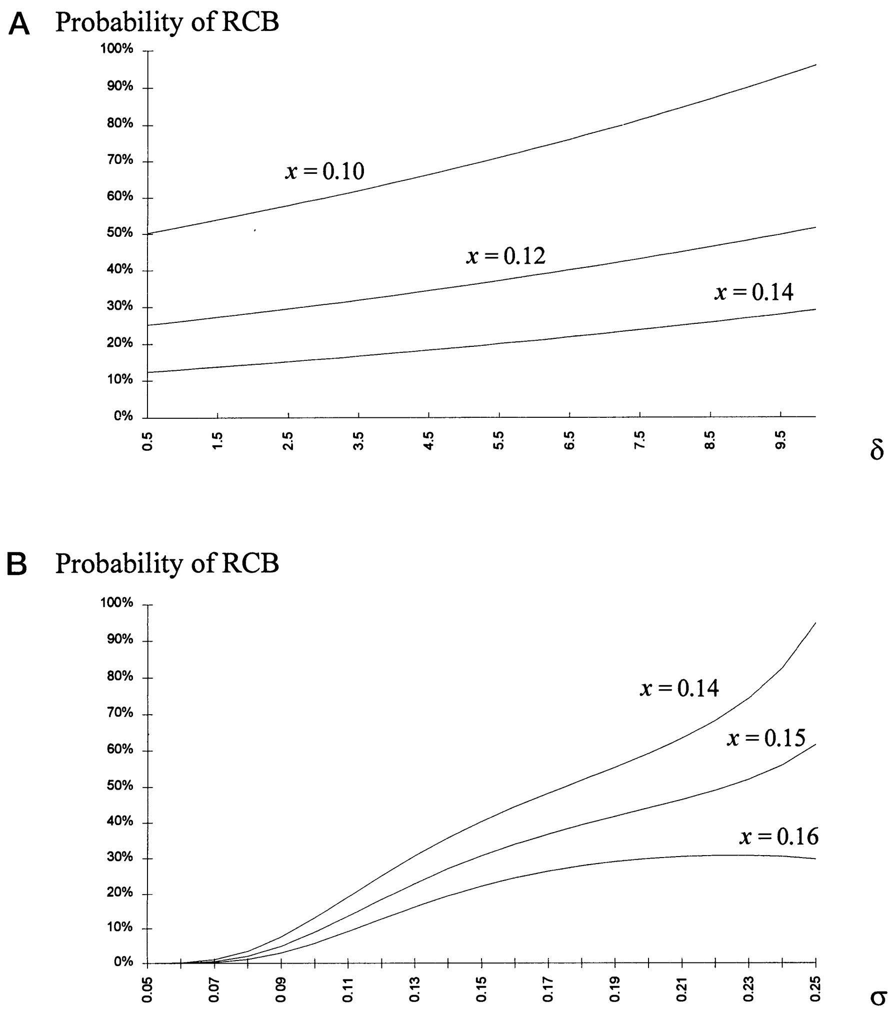


Figure 3. The probability of a recession-induced construction boom (RCB). A RCB is the event in which both owners exercise their options to build simultaneously, following a downturn in demand. Such a simultaneous exercise equilibrium results from a rational fear of preemption. Each of the two graphs displays the sensitivity of the probability of a RCB to changing parameter values, for three levels of initial demand, x. Figure 3a displays the effect of the time-to-build, δ, on the probability of a RCB. The effect is monotonically increasing; markets in which there are large construction lags will be more prone to overbuilding. Figure 3b displays the effect of demand volatility, σ, on the probability of a RCB. For most ranges of demand, the effect is monotonically increasing; volatile real estate markets will be more prone to building booms in the face of declining demand. An existing building, earning a rent of R, can be redeveloped into a new building earning a stochastic rent flow of $X(t)D[Q(t)]$, where the demand state variable X is a geometric Brownian motion with drift μ and volatility σ, and $D(\cdot)$ is a downward-sloping inverse demand function. γ is the fractional loss in rent to an older building when a new building is constructed. The cost of construction is I, and r is the risk-free rate. For this figure, the following parameters are fixed (unless varied along the horizontal axis): $R = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, $D(1) = 1$, $D(2) = 0.9$, $\gamma = 0.8$, $I = 1$, $\delta = 1$, and $r = 0.04$.

displays the vacancy rate patterns for the office and industrial property markets. When combined with the construction patterns displayed in Figure 2a, the tendency for office markets to experience RCBs is highlighted. Even as office markets experienced dramatically high and rising vacancy rates, new development continued to arrive and be initiated.

The model also predicts that greater volatility will contribute to the propensity toward RCBs. On a locational basis, this would imply that cities which are less economically diversified should be more prone to overbuilding. This would help explain the cases of the oil-patch cities, such as Denver and Houston, described earlier. During the time of their concentrated burst of construction in 1982–1985, each had vacancy rates near 30 percent. In fact, by 1983, vacancy rates were already above 27 percent in each of these cities. In terms of property types, the volatility effect would also make office markets more prone to overbuilding than industrial markets. Recall from Section IV.C that the standard deviation of the growth rate in office earnings was twice that of industrial earnings. This would tend to reinforce the time-to-build effect which implies that office markets should be especially prone to overbuilding.

VI. Analysis of Competitive Equilibrium Exercise Strategies

The model characterizes the equilibrium set of exercise strategies in the case of a symmetric duopoly. In this section I derive the equilibrium exercise strategies of developers in a competitive equilibrium formulation. This is analogous to Constantinides' (1984) construction of competitive equilibrium warrant exercise strategies. In this case, developers ignore the impact of their entry on the underlying rent process. In addition, competitive pressures ensure that the value of development, at any point of entry, equals the cost of construction. I show that the exercise strategies are very similar to those derived in the duopoly case. The one significant difference is in the case for RCBs. Under perfect competition, there can be no such phenomenon.

The presentation of the competitive equilibrium approach will be mostly heuristic. A more detailed description of the technique can be found in Grenadier (1995b) and Leahy (1993). For simplicity of presentation, I derive the equilibrium development strategies by assuming zero construction lags.[19]

A. Derivation of Competitive Equilibrium Exercise Triggers

Consider a competitive real estate market where a large number of owners lease their buildings. Some owners have exercised their development option and earn a rent flow of $P(t)$, while others have not exercised their development option and earn a rent flow of $R \cdot (1 - \gamma)$. At any time, an owner can exercise his development option by paying a construction cost of I. Assume that the number of redeveloped buildings can be represented as a continuum whose mass at time t is $Q(t)$. Therefore, the setup is the same as in the duopoly

[19] In order to accommodate a time-to-build feature, the supply of completed units Q must be modified to include construction in progress, as in Bar-Ilan, Sulem, and Zanello (1992).

model, with the demand function, $P(t) = X(t) \cdot D[Q(t)]$. However, Q should now be considered a continuous variable rather than only taking the values 0, 1, or 2.

Using Itô's lemma, the evolution of the equilibrium rent process can be written as:[20]

$$dP = \mu P\ dt + \sigma P\ dz + \frac{D'[Q(t)]}{D[Q(t)]} P dQ. \tag{19}$$

Over the range in which no new development occurs ($dQ = 0$), equilibrium rents follow a geometric Brownian motion process. Note that while developers are idle, the instantaneous mean and variance of the rent process are independent of the level of supply. At moments at which new development occurs, the level of the rent processes will shift downward.

Let $G(P)$ denote the value of a building prior to exercise of the development option, and $H(P)$ the value of a building after development. These valuations will not depend explicitly on Q. This can be seen by the following two points. First, when supply is unchanging, the rent process is independent of Q, as is clear from equation (19), with $dQ = 0$. Second, even when supply is increasing due to development, the free entry condition will ensure that the payoff from exercising the development option is simply its cost, I, independent of Q.

Using the same techniques as in Section II, the values of buildings before and after exercise must satisfy the following differential equations in equilibrium:

$$\begin{aligned} 0 &= \tfrac{1}{2}\sigma^2 P^2 G'' + \mu P G' - rG + R \cdot (1 - \gamma), \\ 0 &= \tfrac{1}{2}\sigma^2 P^2 H'' + \mu P H' - rH + P. \end{aligned} \tag{20}$$

These differential equations must be solved simultaneously. They are subject to two forms of boundary conditions. The first type are the standard individual optimization conditions. These are merely the value-matching and smooth-pasting conditions that ensure rational exercise strategies. The second type is the competitive equilibrium condition of free entry. Even though individual owners select the optimal moments at which to develop, competitive forces ensure that new development will eat away all excess profits. These boundary conditions can be written as:

$$\begin{aligned} G(P^*) &= H(P^*) - I, \\ G'(P^*) &= H'(P^*), \\ H(P^*), &= I, \end{aligned} \tag{21}$$

where P^* is the level of rent that triggers new development.

[20] There are no second order terms in Q because the supply process is variation finite. If this were not the case, the economy would be incurring infinite costs of adjustment.

The first boundary condition of equation (21) is the value-matching condition: the value of a building at the moment of development equals the value of a developed building minus the cost of construction. The second boundary condition is a smooth-pasting condition that ensures that agents exercise their development options optimally. The third condition is the free-entry condition ensuring zero net present value of development. Finally, I impose the regularity conditions, $G(0) = H(0) = 0$ to reflect the absorbing barrier at zero.

By solving differential equations (20) subject to boundary conditions (21), the solution to the optimal exercise trigger P^* can be expressed as:

$$P^* = \left(\frac{\beta}{\beta - 1}\right)(r - \mu)\left[I + \frac{(1 - \gamma)R}{r}\right]. \tag{22}$$

Using the inverse demand function (1), competitive equilibrium construction is triggered whenever $X(t)$ rises to a function of existing supply $Q(t)$. Define the level of X that triggers competitive equilibrium entry as $X^{C}(Q)$. From equations (22) and (1), we have:

$$X^{C}(Q) = \left(\frac{\beta}{\beta - 1}\right)\frac{(r - \mu)}{D(Q)}\left[I + \frac{(1 - \gamma)R}{r}\right]. \tag{23}$$

Note that this is precisely the same exercise trigger value for the Follower in the case of the duopoly (with no lags), except that it is now a function of Q.

The following proposition summarizes the competitive equilibrium development exercise strategies.

PROPOSITION 4: *In a competitive equilibrium setting, the supply of developed properties increases to the level Q the first moment that $X(t)$ equals or exceeds the trigger function $X^{C}(Q)$, as defined in equation (23). That is, the entry time of the Qth increment of new development, T_Q can be written as:*

$$T_Q = \inf\left\{t \geq 0 \colon X(t) \geq \left(\frac{\beta}{\beta - 1}\right)\left(\frac{r - \mu}{D(Q)}\right)\left[I + \frac{(1 - \gamma)\cdot R}{r}\right]\right\}. \tag{24}$$

B. Analysis

The nature of the competitive equilibrium exercise strategies is as follows. At any time other than the initial moment that the market opens, supply increases will be sequential. That is, given a supply of developed buildings $Q(t)$ at t, the next increment of supply will be developed the next instant that X rises to the level $X^{C}(Q + dQ)$. Similarly, supply will rise to the level $\bar{Q}$, for any $\bar{Q} > Q(t)$, the first moment that X reaches $X^{C}(\bar{Q})$.

This is very similar to the sequential equilibrium of the duopoly model. Suppose the supply of developed buildings has just increased to Q. Then, define $T_C(Q, \bar{Q})$ as the time it takes for the supply of developed buildings to increase from Q to $\bar{Q}$, for any $\bar{Q} > Q$. Thus, $T_C(Q, \bar{Q})$ can be expressed as:

$$T(Q, \bar{Q}) = \inf\{t \geq 0 \colon X(t) = X^{C}(\bar{Q}),\ X(0) = X^{C}(Q)\} \tag{25}$$

However, unlike the duopoly case, there will not be any chance of a recession-induced construction boom. The only time that there can be simultaneous exercise is at the moment the market opens (which could have been in the distant past), and this is not triggered by a decline in demand. When the market opens, enough new development occurs such that $Q(0)$ satisfies $X(0) = X^{C}[Q(0)]$. Following that, only incremental exercise ensues.

Therefore, the basic model's specification for conditions which lead to RCBs depends on the structure of the market. As the market structure approaches that of perfect competition, the potential for RCBs goes away.[21] The structure of the market, however, cannot be defined solely by the number of competitors, since the real estate "product" is highly differentiated by location. For example, while a major city may have a large number of potential developers and sites, there are likely to be many submarkets differentiated by such attributes as distance from the central business district, zoning restrictions, and the quality of amenities.

VII. Conclusion

Real-world situations do not always allow one to formulate an optimal exercise strategy in isolation. While one person may be solving a partial-differential equation with its associated smooth-pasting condition, a competitor may be doing likewise. In the example of real estate development, these calculations cannot be conducted separately, but must be done as part of a strategic equilibrium. The model demonstrates that development options might be exercised sequentially or simultaneously, depending on the underlying conditions of the market.

The solution to this option exercise game was not conducted for its own sake, but rather to provide an underlying theory from which one may begin to understand actual market behavior. Real estate development options are not always exercised in a smooth pattern; sometimes markets sit idle for years and then blast off in a surge of construction. The model is able to isolate the conditions that make such a phenomenon more or less likely. In addition, sometimes these bursts of construction occur when the underlying demand for space is falling. While this is often regarded as irrational overbuilding, the model provides a rational equilibrium foundation for such exercise patterns.

The model of strategic exercise can be applied to a variety of real option settings. Consider a pharmaceutical industry competing over the search for a new drug. Beginning a research program is costly and yields uncertain results. However, each firm must formulate its research strategy fully cognizant of the development strategies of its competitors. Consider also the decision of a firm to introduce a new product to consumers. The initial advertising and distribution expenses must be paid now in order to reap the uncertain benefits of sales in the future. If the firm faces competitors able to produce a similar product, then the timing of the new product option exercise is a strategic exercise game.

[21] Of course, in the case of a monopolist, there is also zero probability of preemption. Thus, a monopolist will only build when demand rises to an upper trigger, and there will be no analog to a RCB.

Appendix

Proof of Proposition 2

PROPOSITION 2: *There exists a unique point, $X_L \in (0, X_F)$, with the following properties:*

$$\begin{aligned} L(X, \delta) - I &< F(X) && \text{for} \quad X < X_L, \\ L(X, \delta) - I &= F(X) && \text{for} \quad X = X_L, \\ L(X, \delta) - I &> F(X) && \text{for} \quad X_L > X < X_F, \\ L(X, \delta) - I &= F(X) && \text{for} \quad X \geq X_F. \end{aligned}$$

Proof: Define the function $D(X) = L(X, \delta) - F(X) - I$, where $L(X, \delta)$ was derived in equation (9), by setting $\tau = \delta$, and $F(X)$ in equations (7) and (8).

First, I establish the existence of a root for $D(X)$ in the interval $(0, X_F)$. Evaluating at $X = 0$ yields $D(0) = -[I + (1 - \gamma)R/r] < 0$. Similarly, evaluating at $X = X_F$ yields $D(X_F) = 0$. Finally, some algebraic manipulation yields $\lim_{X \to X_F^-} D'(X) < 0$. Therefore, $D(X)$ must have at least one root in the interval $(0, X_F)$.

To prove uniqueness, one merely needs to demonstrate strict concavity over the interval. Differentiating $D(X)$ twice yields:

$$D''(X) = -\frac{\beta}{X^2}\left[1 + \beta\left(\frac{D(1) - D(2)}{D(2)}\right)\right]\left[1 + \frac{(1 - \gamma)R}{r}\right]\left(\frac{X}{X_F}\right)^{\beta} < 0.$$

Thus, the root is unique.

Finally, from Proposition 1, if $X \geq X_F$, the optimal response of the Follower is to begin construction immediately. Thus, both Leader and Follower will have the same value over this range. Q.E.D.

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