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Potential Biases from Using Only Trade Prices of Related Securities on Different Exchanges: A Comment

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IN THEIR SEPTEMBER 1982 *Journal of Finance* article, Manaster and Rendleman [5] (hereafter MR) investigate the role of call option prices as predictors of equilibrium prices of the underlying stocks using daily (trade) prices. This paper points out that a bias associated with the bid-ask spread and nonsynchronous trading may create the impression that option prices contain information not reflected in the contemporaneous stock prices even during times when the two prices are in equilibrium. The MR methodology is described in Section I, and the potential biases arising due to the bid-ask bounce and the non-synchroneity of stock and option prices are illustrated in Section II. Following an observation in their paper that the stock prices implied by the option prices are smaller than the corresponding stock prices only thirty-seven percent of the time, Section III looks at the distribution of trade prices relative to the last quoted bid-ask prices for both stocks and options. Many more during-the-day option trades occur at the ask than at the bid, whereas the distribution is approximately symmetric for stocks. A likely implication is that the option trade prices, and therefore the implied stock prices, are upward-biased estimates of the corresponding *true prices*.[1] This bias increases at day end for both stocks and options.

I. Manaster and Rendleman Methodology and Results

MR test the null hypothesis, "There is no information contained in implied stock prices (calculated from option prices) regarding equilibrium stock prices that is not contained in observed stock prices," using *closing trade prices* of options and stocks over the period April 1973 to June 1976. The implied stock price, S^*_{jt}, and the standard deviation, V^*_{jt}, for stock j on day t are calculated by minimizing the sum of squared deviations between observed and calculated option prices:

$$(S^*_{jt}, V^*_{jt}) = \text{argmin}[Q(S_{jt}, V_{jt}) = \textstyle\sum_{i=1}^{N_{jt}} [W^i - W^i(S_{jt}, V_{jt})]^2],$$

where W^i is the closing price of option i on stock j on day t, $W^i(S_{jt}, V_{jt})$ is the

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[1] The trade price consists of the marketmakers' estimates of the *true price* plus transaction costs.

Black and Scholes price corresponding to S_{jt} and V_{jt}, and N_{jt} is the number of options traded on the stock that day.[2]

The proportional error, Δ_{jt}, is defined to measure the information lag as the percent difference between the implied price S^*_{jt} and the observed stock price S_{jt}:

$$\Delta_{jt} = (S^*_{jt} - S_{jt})/S_{jt}.$$

Under the assumption that the null hypothesis is not true and that the implied prices do contain information regarding equilibrium stock prices that will be reflected in the stock exchange prices by the end of the next trading day, the stock return over the next day, R_{jt+1}, will be positively correlated with Δ_{jt}. This gives the key predictive equation (7) in their paper, an ex post time-series regression, as follows:

$$R_{jt+1} = \alpha_j + \beta_j R_{mt+1} + \gamma_j \Delta_{jt} + \varepsilon_{jt+1}, \tag{MR7}$$

where R_{mt+1} is the return on the market portfolio on day $t + 1$. Under the null hypothesis, $\gamma_j = 0$ (i.e., no correlation exists between Δ_{jt} and R_{jt+1}).

If, however, closing option trades generally occur after the closing stock trades, then γ_j may be positive simply because option prices contain *more recent* information, known to the stock market traders but not observed due to the lack of trading. Due to availability of only day-end prices, MR cannot directly ascertain whether the closing stock trade or the closing option trade occurs later. The fact that $\hat{c}_j$ in equation (6) below is significantly negative in sixty cases and significantly positive only once (out of a total of 172 cases) is interpreted by them as evidence that the stock trade occurs later in time, as would also be expected due to the thinner market in options during the earlier years of the CBOE.

$$R_{jt} = a_j + b_j R_{mt} + c_j \Delta_{jt} + e_{jt}. \tag{MR6}$$

Therefore, $\hat{\gamma}_j$ in equation (7) being positive ninety-six times and negative seventy-six times (significant at the five percent level thirteen and seven times, respectively) suggests that closing option prices usually, but not always, contain information not contained in the contemporaneous stock prices.

Finally, tests of two stock-trading strategies based on the information content of closing option prices are carried out. Individual stocks are ranked on Δ_{jt} and assigned to five portfolios every day.[3] In the ex post study, stocks are bought at the closing prices on day t, the same prices used to calculate Δ_{jt}, and sold at the closing prices on day $t + 1$. An F-statistic rejects the null hypothesis that average returns are equal for the five portfolios at a significance level of 0.0001. A nonparametric χ^2 test also rejects the null hypothesis that the return rankings of the five portfolios are independent of their delta rankings. Assuming that the five portfolios contain otherwise similar stocks, a positive correlation between the average delta ranking and the ex post portfolio returns indicates that closing option prices predict the direction of equilibrium stock prices on the following

[2] Analysis showed that the bias arising due to nonlinearities in the option-pricing formula is likely to be very small unless the implied price is calculated using very few options.

[3] Forming portfolios increases the power of the tests.

day. In the ex ante study, the stocks constituting the portfolios formed by ranking Δ_{jt}'s are bought at the end of day $t+1$ (next day) and sold one day later, i.e., at the closing prices on day $t+2$. The results for the ex ante strategy are much weaker. For example, the difference between the highest and lowest ranking portfolio returns is positive on only 414 days of the 800 days.

The next section demonstrates that, given the limitations of (day-end) trade prices, results of both of the time-series regressions and the ex post portfolio strategies are biased and that unbiased inferences are possible only with ex ante strategies.

II. Potential Biases due to Bid-Ask Bounce and Nonsynchroneity

A. Overview and Intuitive Explanation

This section illustrates that the bid-ask bounce and the nonsynchroneity of closing stock and option trade prices introduce biases into the ex post methodology. These biases, however, will arise whenever only trade prices are used, whether or not they are the last prices for the day.

Briefly speaking, a positive Δ_{jt} may result when the closing stock price is a bid price and therefore too low compared with the true stock price, which should have been used to calculate Δ_{jt}. Alternatively, due to nonsynchroneity of closing stock and options trades, a positive Δ_{jt} may result because the closing stock price is lower than the price prevalent in the stock market at the time the options traded. Due to either influence, Δ_{jt} and R_{jt+1} will be overstated and R_{jt} will be understated. Conversely, when the closing stock price is too high, because it is an ask price or because of nonsynchroneity, both Δ_{jt} and R_{jt+1} will be understated and R_{jt} will be overstated. This will induce negative correlation between Δ_{jt} and R_{jt} and positive correlation between Δ_{jt} and R_{jt+1} and will bias the portfolio return for the ex post strategy in the direction of the portfolio delta, even when the true contemporaneous stock and option prices contain the same information.[4] A detailed analysis follows below.

B. Analysis of the Bias

To illustrate the bias, let us start with just one option on the stock. Assume that the true contemporaneous stock and option prices contain the same information, so that Δ_{jt} calculated with the true stock and option prices ("true" prices ignore the bid-ask and other measurement errors) is uncorrelated with R_{jt} and R_{jt+1}, similarly calculated. The true values of c_j and γ_j in equations (6) and (7) will therefore be zero. Now, suppose that the stock price and the implied stock price are measured with error:

$$\hat{S}^*_{jt} = S^*_{jt}(1 + \xi_{jt}) \quad \text{and} \quad \hat{S}_{jt} = S_{jt}(1 + \eta_{jt}). \tag{1}$$

ξ_{jt} arises due to the bid-ask spread on the option, and η_{jt} represents the bid-ask spread on the stock plus the nonsynchroneity between the stock price and the

[4] MR recognize that stock and option prices may not be simultaneous. However, they interpret that negative c_js indicate that the stock trade is more recent than the option trade, and positive γ_js result when the option trade is the more recent one. As shown below, negative c_js and positive γ_js will result irrespective of which trade occurs later in time.

option price. Both ξ_{jt} and η_{jt} are white-noise terms and are uncorrelated with leading, lagged, or contemporaneous values of any other variables.[5] Ignoring higher order terms, the following expressions result:

$$\hat{R}_{jt} = (\hat{S}_{jt} - \hat{S}_{jt-1})/\hat{S}_{jt-1} = R_{jt} + \eta_{jt} - \eta_{jt-1},$$

$$\hat{\Delta}_{jt} = (\hat{S}^*_{jt} - \hat{S}_{jt})/\hat{S}_{jt} = \Delta_{jt} + \xi_{jt} - \eta_{jt}. \tag{2}$$

It follows that $\hat{c}_j$ will be negative and $\hat{\gamma}_j$ will be positive, although their true values are zero.

$$\hat{c}_j = \frac{\operatorname{cov}(\hat{R}_{jt}, \hat{\Delta}_{jt})}{\operatorname{var}(\hat{\Delta}_{jt})} = \frac{-\operatorname{var}(\eta_{jt})}{\operatorname{var}(\eta_{jt}) + \operatorname{var}(\xi_{jt}) + \operatorname{var}(\Delta_{jt})}, \tag{3}$$

$$\hat{\gamma}_j = \frac{\operatorname{cov}(\hat{R}_{jt+1}, \hat{\Delta}_{jt})}{\operatorname{var}(\hat{\Delta}_{jt})} = \frac{\operatorname{var}(\eta_{jt})}{\operatorname{var}(\eta_{jt}) + \operatorname{var}(\xi_{jt}) + \operatorname{var}(\Delta_{jt})}. \tag{4}$$

When there are multiple options on the stock, measures of the implied price become more precise; i.e., $\operatorname{var}(\xi_{jt})$ and $\operatorname{var}(\Delta_{jt})$ become smaller, which increases the bias.

Finally, the pth portfolio in the ex post strategy invested equally in each of the n_{pt} stocks will have a return of $\hat{R}_{pt+1} = \sum_{j=1}^{n_{pt}} \hat{R}_{jt+1}/n_{pt}$ and an average proportional error of $\hat{\Delta}_{pt} = \sum_{j=1}^{n_{pt}} \hat{\Delta}_{jt}/n_{pt}$. In view of the preceding discussion, the cross-sectional correlation (across p for any given t) between the portfolio returns and the average proportional errors will be positive if the stocks included in the portfolios are otherwise similar.[6]

Therefore, tests of whether option prices contain information not yet contained in stock prices must be ex ante in nature. The bid-ask bounce and the nonsynchroneity do not bias the correlation between Δ_{jt} and R_{jt+2}. Since only day-end prices were available, MR could only test for the ex ante strategy after a gap of twenty-four hours. It being unlikely that lags in information persist for a day, their results were expectedly weak.[7]

III. Are Trade Prices Unbiased Estimators of True Prices?

A. *Overview*

An important observation in the MR study is that the implied stock prices in options are smaller than the corresponding stock prices only thirty-seven percent

[5] Although the measurement errors are assumed to be proportional to the stock price, for simplicity, the results will be very similar for any other functional form.

[6] If the biases are not accounted for, then the answer to the reverse question, "Do stock prices predict option prices," can simultaneosly be in the affirmative. In this case, the ex post strategy would prescribe buying options on stock j when $(\hat{S}_{jt} - \hat{S}^*_{jt})/\hat{S}^*_{jt}$ is positive, which would be positively correlated with the return on the option.

[7] The referee pointed out the existence of two recent papers on the area. Snelling [6] uses the ex post portfolio strategy using intraday trade prices for stocks and options. The issues raised here will influence her results. Bhattacharya [1] includes bid-ask spread for stocks and options and identifies underpriced stocks as $\hat{\Delta}_{jt} = (\hat{S}^{*bid}_{jt} - \hat{S}^{ask}_{jt})/(0.5 \times (\hat{S}^{bid}_{jt} + \hat{S}^{ask}_{jt})) >$ twenty-seven cents and the ex post stock return as $(\hat{S}^{bid}_{jt+1} - \hat{S}^{ask}_{jt})/(0.5 \times (\hat{S}^{bid}_{jt} + \hat{S}^{ask}_{jt}))$. The measurement-errors-induced bias in his study, arising because he estimates the stock bid-ask as the narrowest band from a series of last traded stock price observations recorded only when there is an option quote or trade, will be smaller but may explain why his average ex post return of minus 12 cents per share is higher than the average return under the null hypothesis equal to minus the average spread of 17 cents per share.

Table I

Distribution of Trades over Last Quoted Bid-Ask Spread: Includes All Trades for Which Preceding Quote Was on the Same Day

SN	Description	Options (Sep–Oct 76)	Options (Mar–Apr 85)	Stocks (Mar–Apr 85)
	Number of Trades		(Percent of Total Trades)	
1.	Above the Quote Midpoint	43.80	48.13	35.78
	a. Higher Than Ask	2.41	2.57	0.25
	b. At the Ask	38.52	41.17	31.09
	c. Between Ask and Midpoint	2.87	4.39	4.44
2.	At the Quote Midpoint	21.02	17.38	29.85
3.	Below the Quote Midpoint	35.18	34.49	34.37
	e. Between Midpoint and Bid	2.59	3.71	4.40
	f. At the Bid	30.84	28.98	29.61
	g. Lower Than Bid	1.75	1.80	0.36
	Size of Trades	(Contracts)	(Contracts)	(Round Lots)
4.	Average Size of Trade in Upper Half of Spread	4.81	9.39	17.31
5.	Average Size of Trade at Midpoint	5.77	14.55	27.50
6.	Average Size of Trade in Lower Half of Spread	5.83	13.48	18.44

of the time. They suggest that it may be due to the exclusion of options that violate the parity-value condition of option pricing or, alternatively, due to systematic biases in the Black and Scholes formula employed for calculating the implied prices. Whereas both of these explanations are plausible, I ask the more direct question: could it simply be that option trade prices, consisting of the marketmakers' estimates of true prices plus the transaction costs, are upward-biased estimates of the (unobserved) true prices?

It appears on the basis of empirical evidence presented below that, since during-the-day option trade prices are more frequently at (or near) the ask than at (or near) the bid, they may be upward-biased estimates of the true option prices. During-the-day trade prices for stocks, however, are nearly symmetric with respect to the last quote and are therefore likely to be unbiased estimates of the true stock prices. Further, the day-end trade prices for both stocks and options are more often at (or near) the ask, as compared with during-the-day prices. Thus, day-end trade prices may contain an additional bias.

My database for the following discussion consists of every reported quote and trade for equity options traded on the floor of the CBOE during September and October 1976 and for options on the CBOE and the corresponding stocks on the NYSE during March and April 1985.

B. Distribution of Trades over the Last Quoted Bid-Ask Spread

Table I presents the distribution of trade prices with respect to the last quoted bid-ask spread for every trade when the preceding quote occurred on the same day.[8] The distribution is approximately symmetric about the quote midpoint for stocks but is skewed toward the ask for options. Of the total option trades during September and October 1976, 43.80, 21.02, and 35.18 percent are above, at, and below the quote midpoint, respectively. These figures change to 48.13, 17.38, and 34.49, respectively, for options during March and April 1985. Corresponding figures for stocks during March and April 1985, at 35.78, 29.85, and 34.37, show that the distribution is approximately symmetric. The significantly higher percentage of trades in the upper half of the spread in the case of options shows that a larger number of trades on the options exchange are likely to be buyer-initiated than seller-initiated.[9]

The average size of the option trades in the upper half of the spread, however, is much smaller than in the lower half. Consequently, the total number of contracts traded (one contract represents the right to buy one hundred shares) that are likely to be buyer-initiated (upper half) is just a little larger than the number of contracts that are likely to be seller-initiated (lower half). The ratio of the number of option contracts traded above and below the quote midpoint is 1.024 in 1976. For 1985 stocks and options, the corresponding figures are 1.023 and 1.029, respectively. The near equality of contracts traded in the upper and lower halves of the spread suggests that the average of the bid and ask prices may be an unbiased estimator of the true price for both stocks and options. The greater frequency of (smaller) option trades at the ask in that case may cause the difference between the trade price and the true price to be positive on average.

The most commonly used price (also employed in the MR study), however, is the last trade price for a security for the day. The fact that the day-end price is widely quoted and forms the benchmark for the daily performance evaluation of many market participants may be a reason for it to be biased. Cox and Rubinstein [2] report that, in the earlier years of the CBOE, marketmakers sometimes used to introduce fictitious quotes because their margin requirements were based on the closing ask prices. To lend credibility to an otherwise fictitious quote, the next logical step would appear to be to trade a few contracts at the closing ask. If marketmaker inventories are as likely to be positive as negative, the closing ask prices may be too high or too low equally often. The higher frequency of trades at the ask, therefore, will cause the closing trade prices to be further upward-biased estimates of the true prices. Similarly, Harris [3] finds that the trade prices for NYSE stocks rise sharply in the last fifteen minutes and by 0.05 percent on the very last trade. It may be that the day-end stock trades are also more likely to be at the ask.[10]

[8] The CBOE is a multiple-dealer market, and a question arises as to what is the best estimate of the "market-spread," defined as the lowest ask and highest bid available at any point in time. Vijh [7] shows that the last quoted dealer spread is a significantly better estimator of market spread than two other estimators: the minimum quoted spread during a constant stock-price interval and the lowest ask-maximum bid over the interval.

[9] The skewness increases as the option price decreases. In the extreme case, very deep out-of-the-money options can only trade at the ask because no marketmaker may be willing to quote a bid price of a sixteenth, the minimum allowed. However, even for options priced $5.00 or more during March to April 1985, there are 1.30 times as many trades above the quote midpoint as below.

[10] Using Fitch types (intraday trade prices) Harris [4] also finds a higher frequency of ask prices at day end.

The distribution of day-end trade prices for both stocks and options does seem to be different. A study of day-end trades (if occurring in the last half-hour of trading) for 1976 options shows that there are 2,933 trades in the upper half and 2,238 in the lower half, with 1,588 exactly at the midpoint. The ratio of trades in the upper and lower halves at 1.310 is higher than 1.245 for all option trades. For stocks underlying CBOE options during March to April 1985, there are 2,310 day-end trades in the upper half, 1,968 in the lower half, and 1,638 at the middle. The ratio at 1.174 is again higher than 1.041 for all stock trades.

C. Implications of Using Only (Day-End) Trade Prices

Trade prices are sometimes used as proxies for true prices in studies of option pricing and of information lags between the stock and options markets. Earlier discussion indicates that these proxies may often be biased. Trade prices in general, and day-end trade prices in particular, may not be sufficient for investigation of whether option prices contain information not yet contained in stock prices if the price differentials arising due to the lags in information are small.

IV. Conclusions

A study to explore whether the option prices predict the stock prices would be more conclusive if a time-stamped history of bid and ask prices for *both* securities, besides the trade prices and trading volume, were used. Unfortunately, such data bases were unavailable to researchers until very recently. Not accounting for the bid-ask bounce and the nonsynchroneity between stock and option prices in an ex post study can give the impression that the option prices lead the stock prices even when the two are in equilibrium. Also, this paper finds that many more trades in the options market occur at the ask, as opposed to the bid, which may cause option trade prices to be upward-biased estimates of the corresponding true prices.

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