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Circuit Breakers and Market Volatility: A Theoretical Perspective

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ABSTRACT

This paper examines ex ante effects of "circuit breakers" (mandated trading halts). We show that circuit breakers, by causing agents to suboptimally advance trades in time, may have the perverse effect of increasing price variability and exacerbating price movements. We next consider a situation in which a circuit breaker causes trading to be halted in both a "dominant" (more liquid) and a "satellite" market. As agents switch from the dominant market to the satellite market, price variability and market liquidity decline on the dominant market and increase on the satellite market.

TO ADDRESS GROWING CONCERNS among market participants about excessive short-run price swings, the New York Stock Exchange (NYSE) has recently introduced "circuit breakers," which mandate trading halts for a stipulated period of time if the Dow Jones Industrial Average (DJIA) moves by more than a certain amount over the previous day's close.[1] In its report to the NYSE, the Market Volatility and Investor Confidence Panel recommended the adoption of more stringent circuit breakers than those currently in place.[2] Circuit breakers have also affected trading regulations on regional markets (e.g., the Philadelphia, Midwest, and Pacific stock exchanges) and on markets for derivative securities. For example, whenever the NYSE halts trading in a particular stock or on the entire exchange, trading in the corresponding regional markets is also halted (though the regional exchanges do not have independently triggered circuit breakers). Furthermore, when trading is halted on the NYSE, it is also halted in the S & P 500 futures market.[3] In

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[1]The current rule involves halting trade in all stocks for one or two hours if the DJIA moves by more than 250 or 400 points respectively over the previous day's close. See Mann and Sofianos (1990) for a detailed description of this and other proposed circuit breakers. In this paper, a circuit breaker is interpreted as a mandated trading halt triggered on extreme market movements, as opposed to the "price limits" common on various futures markets. However, the model may also be construed as applying to trading halts initiated by extreme order flow imbalances in individual stocks, provided market participants have a "good idea" of the degree of order flow fluctuations required to trigger a halt.

[2]See *Market Volatility and Investor Confidence* (1990), pp. 3–10.

[3]At this time, there are no trading halts triggered by price movements in the S & P 500 futures contract; instead, there are price limits. We do not address price limits in our analysis.

addition, trading halts in individual stocks or on the NYSE as a whole entail trading halts in the corresponding stock or index options on the Chicago Board Options Exchange.[4]

A significant issue is the effect of these impediments to trade on the ex ante trading decisions of market participants and consequently on market liquidity and price variability. This article addresses the above question by using two distinct frameworks that are designed to capture some institutional features of circuit breakers and trading halts in financial markets. The central theme of our analysis is that the existence of circuit breakers may distort optimal trading decisions of large institutions that have the flexibility to choose the time-periods and/or markets in which to execute their trades, and this distortion may have some perverse effects on price variability and market liquidity.

This paper is organized as follows. A model with strategic informed traders, informationless traders with exogenous demands, and risk-averse market makers is provided in Section I. Sections II and III respectively address intertemporal and mutimarket issues related to circuit breakers, and Section IV concludes.

I. The Model

In this section, we illustrate our basic model in a one-period, one-market setting; this scenario is later generalized to two periods (Section II) and two markets (Section III). In the model, one risky security is traded by three types of traders: a competitive market maker, who absorbs the net demand of the other traders and then sets the price, risk-neutral traders who possess private information about the fundamental value of the risky security, and informationless liquidity traders whose share demands are exogenous. Within our context, informationless traders may be interpreted as the trades executed by index funds or pension funds to replicate broad market indexes or synthesize derivative securities (portfolio insurance).

We depart from the standard paradigms of Kyle (1985) and Glosten and Milgrom (1985) by assuming that market makers are risk averse and possess negative exponential utility with a risk aversion coefficient R. One might argue that since risk aversion is a "cost" of providing services, the market maker who actually operates would be the least cost (i.e., least risk averse) agent. We feel that modeling market makers as being risk averse is justified, and indeed, reasonable in our context because circuit breakers are triggered on extreme price movements which are also likely to correspond to conditions with extreme trading volume. Under such conditions, market makers are very likely to act as risk-averse agents.

[4]Rules governing circuit breakers across markets were verified by personal communication with Brian Murray and Paul Adair, both from the marketing departments of the Pacific and Philadelphia Stock Exchanges, respectively, and with Paul Munos of the Research and Planning division of the NYSE, and Thomas Knorring of the trading Operations department at the Chicago Board Options Exchange.

Trading takes place at time 0 and the security is liquidated at time 1. The liquidation value of the security at time 1 (denoted by F) is given by

$$F = \overline{F} + \varepsilon \tag{1}$$

where $\overline{F}$ is known to all agents at time 0 and informed traders observe perfectly the realization of the random variable ε. The number of informed traders is denoted by n and the total random demand of the liquidity traders is denoted by z. The random variables ε and z are assumed to be mutually independent and normally distributed with mean zero. We denote $\text{var}(z) \equiv \sigma^2$ and $\text{var}(\varepsilon) \equiv \sigma_\varepsilon^2$.

In the analysis that follows, we consider the unique linear equilibrium of our model. Let Q denote the total order flow. The expected utility which a market maker obtains by taking the order flow can be written in the mean-variance form

$$E[Q(P - F)|Q] - \frac{R}{2}\text{var}[Q(P - F)|Q]. \tag{2}$$

Due to competition, the market maker who makes the market earns the utility he would obtain by not making the market, which is normalized to zero for convenience. It is easy to show that the unique linear equilibrium is characterized by the market maker using a linear rule and the informed traders following symmetric linear strategies. Substituting a linear pricing rule $P = \overline{F} + \lambda Q$ in equation (2) and setting the right-hand side of this equation to zero yields

$$\lambda = \frac{\text{cov}(\varepsilon, Q)}{\text{var}(Q)} + \frac{R}{2}\text{var}[\varepsilon|Q] \tag{3}$$

The closed-form solution for λ is given in the following lemma.[5,6]

LEMMA 1: *The equilibrium slope of the linear pricing function is given by*

$$\lambda = \frac{R\sigma_\varepsilon^2}{4} + \left[\left(\frac{R\sigma_\varepsilon^2}{4}\right)^2 + \frac{n\sigma_\varepsilon^2}{(n+1)^2\sigma^2}\right]^{\frac{1}{2}}. \tag{4}$$

When $R = 0$, the above expression reduces to the standard Admati and Pfleiderer (1988) expression (see their Lemma 1). From (4), it can be seen that the liquidity parameter λ declines in σ^2. Let us now examine price variability. The variance of the price is easily calculated to be

$$\text{var}(P) = \frac{n\sigma_\varepsilon^2}{n+1} + \frac{R^2\sigma_\varepsilon^4}{8}\sigma^2 + \frac{R\sigma_\varepsilon^2}{2}\left[\frac{R^2\sigma_\varepsilon^4}{16}\sigma^4 + \frac{n\sigma_\varepsilon^2}{(n+1)^2}\sigma^2\right]^{\frac{1}{2}}. \tag{5}$$

[5]All lemmas and propositions, unless otherwise stated, are proved in the Appendix.

[6]An expression similar to that in this lemma appears in Subrahmanyam (1991). The above expression is a more general form of the expression in that paper, which is derived for the special case of $\sigma_\varepsilon = 1$.

This expression, together with (4), directly leads to the following proposition, which is therefore stated without proof.

PROPOSITION 1: 1) *When market makers are risk neutral* ($R = 0$), *an increase in the variance of liquidity trades*, σ^2, *increases market liquidity and has no effect on the variance of the price.*

2) *When market makers are risk averse, an increase in the variance of liquidity trades increases market liquidity and increases the variance of the price.*

Thus, price variability depends on informationless liquidity trading as well as informed trading when market makers are risk averse. The intuition is simply that the extra compensation demanded by the risk-averse market maker for taking on order flow risk makes price variability sensitive to the variance of liquidity trades.[7]

Next, we discuss price efficiency. The measure of price efficiency (denoted by Γ) is defined, as is standard in the literature, to be the precision of the security's terminal value conditional on the price (or equivalently, the order flow), i.e., $\Gamma = [\mathrm{var}(\varepsilon|Q)]^{-1}$. For a given variance of liquidity trading σ^2, it can easily be shown that Γ in terms of λ is given by

$$\Gamma = \frac{\dfrac{n^2\sigma_\varepsilon^2}{(n+1)^2\lambda^2} + \sigma^2}{\sigma_\varepsilon^2\sigma^2}. \tag{6}$$

Substituting for λ from (4), the following lemma obtains as a result of straightforward calculations:[8]

LEMMA 2: 1) Γ *is invariant to the variance of liquidity trading when market makers are risk neutral.*

2) Γ *decreases in the variance of liquidity trading when market makers are risk averse.*

Finally, we consider how trading volume varies with the variance of uninformed trading. Let std(.) denote the standard deviation operator, I denote the total informed trade, and z denote the quantity of liquidity trade. Our measure of expected trading volume, denoted by V, is defined as $V \equiv V_I + V_L + V_M$, where $V_I \equiv \mathrm{std}(I)$, $V_L \equiv \mathrm{std}(z)$, and $V_M \equiv \mathrm{std}(Q)$. The quantities V_I, V_L, and V_M thus measure volume due to informed traders, liquidity traders, and market makers respectively; thus the total trading volume measure V takes into account the trades crossed between traders as well as trading done with the market maker. The following lemma provides expressions for the volume measures V_I and V_M. (Note from the definition of V_L that under our notation, $V_L = \sigma$.)

[7] This result is analogous to the result in Stoll (1978) that the spread set by a risk-averse market maker increases in the inventory he holds.

[8] A result similar to that in this lemma is derived in Subrahmanyam (1991).

LEMMA 3: *For a given variance of liquidity trading* σ^2 *in a market, the volume measures* V_I *and* V_M *are given by*

$$V_I = \frac{n}{n+1}\frac{\sigma_\varepsilon}{\frac{R\sigma_\varepsilon^2}{4} + \left[\left(\frac{R\sigma_\varepsilon^2}{4}\right)^2 + \frac{n\sigma_\varepsilon^2}{(n+1)^2\sigma^2}\right]^{\frac{1}{2}}}, \tag{7}$$

$$\begin{aligned} V_M &= \sqrt{V_I^2 + V_L^2} \\ &= \sqrt{\left(\frac{n}{n+1}\frac{\sigma_\varepsilon}{\frac{R\sigma_\varepsilon^2}{4} + \left[\left(\frac{R\sigma_\varepsilon^2}{4}\right)^2 + \frac{n\sigma_\varepsilon^2}{(n+1)^2\sigma^2}\right]^{\frac{1}{2}}}\right)^2 + \sigma^2}. \end{aligned} \tag{8}$$

Note that an increase in liquidity trading increases V_M both directly (through an increase in V_L) and indirectly (through an increase in V_I). From (7) it can be seen that if σ^2 increases, V_I increases because the informed traders trade more aggressively in response to an increase in the variance of liquidity trading. Since V_L also increases, it follows that V increases.

Thus, an increase in the variance of liquidity trading increases the variability of the price, decreases price efficiency, but increases market liquidity (decreases λ) and expected trading volume. Moreover, it can easily be shown that an increase in liquidity trading leads to increased expected profits for the informed traders. Increased variability due to an increase in the variance of liquidity trades, therefore, does not necessarily accompany a decrease in the well-being of all market participants, though it does destabilize prices if market makers are risk averse.

II. Circuit Breakers: An Intertemporal Model

In this section we apply the basic model of the previous section to a two-period scenario, in order to consider the impact of circuit breakers on price variability and efficiency, market liquidity, and trading volume. We analyze the strategic trading decision of informationless traders with exogenous needs to trade. In Subsections II.*A*, II.*B*, and II.*C*, respectively, we present the basic two-period model, introduce discretionary liquidity trading, and analyze the case of circuit breakers.

A. The Basic Two-Period Model

The security trades in two periods and its liquidation payoffs are given by (1).[9] All informed traders have symmetric access to both markets. For

[9] In the context of our model, a "period" can be construed as any interval of time less than a trading day.

tractability, we assume that informed traders may trade only in period 1, and that they know precisely the realization of ε. In period 2, there is no further informed trading and the innovation ε is revealed following trade in period 2. We further assume that there are two pools of competing market makers and that market makers in the first pool bid for the period 1 order flow, while those in the second pool bid for the period 2 order flow. The assumption ensures that in each period, the price is set so that the expected market maker payoff equals half the risk-aversion times the variance of the payoff, conditional on all public information (as per equation (2)), and allows us to tractably focus on the intertemporal problem of informationless liquidity traders. It will become clear below that no part of our intuition is sensitive to the above assumptions.

We first present the liquidity parameters in periods 1 and 2 for given variances of liquidity trading in each period in the following lemma. The lemma follows from the rules governing conditional expectations and variances of multivariate normal random variables.

LEMMA 4: *For given variances of liquidity trading σ_1^2 and σ_2^2 in periods* 1 *and* 2, *the prices in the two periods are given by*

$$P_1 = \bar{F} + \lambda_1 Q_1 \tag{9}$$

$$P_2 = \bar{F} + \lambda_{12} Q_1 + \lambda_2 Q_2 \tag{10}$$

where Q_1 and Q_2 denote the order flows in periods 1 *and* 2, *λ_1 is given by* (4), *with σ^2 replaced by σ_1^2,*

$$\lambda_{12} = \frac{t\sigma_\varepsilon^2\sigma_2^2}{(t^2\sigma_\varepsilon^2 + \sigma_1^2)\sigma_2^2 - \sigma_{12}^2} \tag{11}$$

and

$$\lambda_2 = \frac{\frac{R}{2}\sigma_\varepsilon^2\left[\sigma_1^2\sigma_2^2 - \sigma_{12}^2\right] - t\sigma_\varepsilon^2\sigma_{12}}{(t^2\sigma_\varepsilon^2 + \sigma_1^2)\sigma_2^2 - \sigma_{12}^2}, \tag{12}$$

where

$$t = \frac{n}{(n+1)\lambda_1}$$

and where σ_{12} is the covariance between the liquidity trades in periods 1 *and* 2.

Note that the period 2 price is a linear function of both the first- and the second-period order flow. This is because the first period's order flow conveys information which is relevant to setting the period 2 price.

The liquidity parameters in Lemma 4 reflect combinations of adverse selection costs and inventory risks borne by the market makers. To obtain additional intuition, it is useful to consider special cases. Suppose first that $\sigma_{12} = 0$. In this case, $\lambda_{12} = t\sigma_\varepsilon^2/(t^2\sigma_\varepsilon^2 + \sigma_1^2)$ and is simply the regression coefficient of ε on the period 1 order flow Q_1, as in the familiar risk-neutral Kyle market. Further, $\lambda_2 = (R/2)\sigma_\varepsilon^2\sigma_1^2/(t^2\sigma_\varepsilon^2 + \sigma_1^2)$, and is a pure risk premium. Thus, when the liquidity trades are uncorrelated across periods, the period 2 adverse selection and inventory effects separate themselves into λ_{12} and λ_2 respectively.

Next, suppose that $R = 0$. In this situation, note that if the liquidity trades in periods 1 and 2 are positively correlated, λ_2 is negative. To better understand this, note that λ_2 is the coefficient of the period 2 order flow in a multiple regression of the security value on both the period 1 and period 2 order flows; this partial regression coefficient is negative because of the positive correlation between the liquidity trades and the fact that there is no informed trading in period 2.

B. Discretionary Liquidity Trading

It is reasonable to expect large financial institutions (who usually have low fixed costs per submitted order) to possess the capacity to break up their total order into portions over time. We therefore model a single discretionary liquidity trader, say a large financial institution, with an exogenous demand $2l$, who may choose to either split his trade equally across two periods or concentrate his trading in a single period.[10] The random variable l is assumed to be normally distributed with mean zero and variance σ_l^2 and uncorrelated with ε. There is also a certain amount of nondiscretionary liquidity trading in each period. The net nondiscretionary trade in period i is denoted by z_i. The random variables z_i, $i = 1, 2$ are identically and independently normally distributed with mean zero and variance σ_z^2 and are each uncorrelated with ε and l.

Note that under our structure, all agents except the discretionary trader have a one-period strategy. In deriving equilibria throughout the paper, we assume that agents change their strategies in a consistent fashion in response to a deviation by a particular agent. That is, we assume that when a discretionary liquidity trader changes his strategy, informed traders and market makers respond by changing their strategies to conform to the new variance of liquidity trading in the particular periods. An equilibrium is obtained simply when the discretionary trader uses the lowest expected cost strategy and the market liquidity parameters reflect the total (discretionary and nondiscretionary) variances of liquidity trading in the different periods under the lowest expected cost strategy.

We now consider the strategic trading decision of the discretionary trader. When the discretionary trader splits his trades across periods 1 and 2, his expected trading cost is given by

$$E[(P_{1s} - F)l + (P_{2s} - F)l] = (\lambda_{1s} + \lambda_{12s} + \lambda_{2s})\sigma_l^2$$

where the subscript s represents a price or a liquidity parameter when the discretionary trader splits his trade across periods. Expressions for λ_{1s}, λ_{12s}, and λ_{2s} are obtained from (4), (11), and (12) respectively, with $\sigma = \sigma_1^2 = \sigma_2^2$

[10] Note that we have assumed the existence of only a single discretionary trader and that the discretionary order can only be split equally across periods. Generalization of the model to relax these assumptions does not lead to any substantial additional insight.

$= \sigma_l^2 + \sigma_z^2$, and $\sigma_{12} = \sigma_l^2$. If the discretionary trader chooses to concentrate his trading in periods 1 or 2, his expected costs are given by $E[(P_{1f} - F)2l] = 4\lambda_{1f}\sigma_l^2$ and $E[(P_{2d} - F)2l] = 4(\lambda_{12d} + \lambda_{2d})\sigma_l^2$ respectively, where the subscripts f and d denote liquidity parameters and prices when the discretionary trader concentrates his trading in periods 1 and 2 respectively. Again λ_{1f} is given by (4), with $\sigma^2 = 4\sigma_l^2 + \sigma_z^2$, while λ_{12d} and λ_{2d} are given by (11) and (12) respectively with $\sigma_1^2 = \sigma_z^2$, $\sigma_2^2 = 4\sigma_l^2 + \sigma_z^2$, and $\sigma_{12} = 0$.

The optimal strategy of the discretionary trader is found by comparing the expected trading costs incurred by the trader in each of his three possible strategies. The Appendix shows that the unique equilibrium involves the discretionary trader splitting his trades across periods.

PROPOSITION 2: *Under the above assumptions, in the unique equilibrium, the discretionary trader splits his trades across periods, rather than concentrating his trades in periods* 1 *or* 2.

This proposition obtains because the discretionary trader can reduce the price impact of his trade by trading in a dispersed fashion across periods. The result is similar to that in Chowdhry and Nanda (1991), where discretionary uninformed traders find it optimal to split their trades across markets for the same security rather than concentrating their trading in a particular market.

C. The Circuit Breaker

Let us now examine the consequences of introducing a rule that stipulates that if the price in period 1 falls outside either a lower bound ρ_1 or an upper bound ρ_2, trading in the market will be halted in period 2 (while the period 1 trade will go through), and the market will remain closed for a period of time until ε is revealed. This captures the feature that circuit breakers are not activated until the realized value of the stock index (following the execution of an extreme order flow) crosses a certain bound.

In the presence of the circuit breaker, the problem of the discretionary trader is one of calculating which trading strategy to use after factoring in the probability that the period 2 trade may not go through. We parameterize the cost of not being able to trade in a period as a number c; this number can typically be expected to be much larger than the expected losses suffered to informed traders. The decision of the discretionary trader in the presence of a circuit breaker is governed by the following proposition.

PROPOSITION 3: *In the presence of a circuit breaker, the unique equilibrium is characterized by the discretionary trader concentrating his trades in period* 1 *if and only if*

$$\lambda_{1s}\sigma_l^2 + (\lambda_{2s} + \lambda_{12s})\sigma_l^2\left[1 - N\left(\frac{\rho_1 - \bar{F}}{\text{std}(P_{1s})}\right) - N\left(\frac{\bar{F} - \rho_2}{\text{std}(P_{1s})}\right)\right] + \left[N\left(\frac{\rho_1 - \bar{F}}{\text{std}(P_{1s})}\right) + N\left(\frac{\bar{F} - \rho_2}{\text{std}(P_{1s})}\right)\right]c > 4\lambda_{1f}\sigma_l^2 \quad (13)$$

where $N(.)$ *represents the cumulative standard normal distribution function.*

In intuitive terms, the proposition implies the following: If the expected cost of trading in period 1 is less than that of splitting trades across securities, taking into account the possibility of a circuit breaker bound being crossed, then in the unique equilibrium, the discretionary trader trades in period 1.[11] If c is large and if the public value of the security $\bar{F}$ is close to one of the breaker limits, condition (13) holds under a wide range of parameter values.

The following Proposition, which describes how price variability, market liquidity, and price efficiency changes as a consequence of the discretionary trader changing his strategy, follows directly from equations (4) and (5) and Lemma 2 and is therefore stated without proof.

PROPOSITION 4: *If condition* (13) *holds, then, in the equilibrium following the introduction of the circuit breaker,*

1) *price variability, market liquidity, and V, the measure of trading volume increase and price efficiency decreases in period* 1, *and*

2) *the probability of the period* 1 *price crossing either circuit breaker bound increases.*

In our model, therefore, if the price is close to the breaker limit, the breaker can actually *increase* ex ante price variability and the probability of the price hitting the circuit breaker bounds, because the discretionary trader suboptimally trades in a concentrated fashion, rather than splitting his trades across time.[12,13] Also, contrary to popular notion, Proposition 4 shows that an increase in price variability is not necessarily accompanied by a decrease in market liquidity. In our model, although price variability increases, market liquidity also increases if the public value of the security is close to the breaker limit.

Thus, the breaker has a perverse effect that is exactly the opposite of what regulation intended it to accomplish; it increases price variability and exacer-

[11] The Appendix shows that in the absence of a circuit breaker, the expected cost of concentrating trades in period 2 is always higher than the expected cost of concentrating trades in period 1. (See the proof of Proposition 2). Since the circuit breaker further increases the expected cost of concentrating trades in period 2, a comparison of the expected costs of concentrating trades in periods 1 and 2 is superfluous.

[12] Note here that under condition (13), provided the circuit breaker is not triggered in period 1, it has effects *opposite* to those in Proposition 4 on the *period* 2 price variability and market liquidity. We focus on period 1 because from a policy perspective it is of more relevance to examine the ex ante effect of the circuit breaker on market parameters under conditions of extreme market movements (i.e., conditions in which the price is close to the breaker limit). This situation is captured by period 1 under parameter spaces in which $\bar{F}$ is close to either ρ_1 or ρ_2.

[13] If the circuit breaker is defined such that if the price that yields zero-expected utility to the market maker falls outside some exogenous bound, the *contemporaneous* (i.e., period 1) order will not be executed, then the analysis becomes much more complicated, as in this case agents will scale back their trades to reduce the probability of the bounds being crossed. This leads to linear strategies being nonoptimal for the informed traders and, therefore, one loses tractability. Note, however, that in this case, the result (2) of Proposition 4 could be reversed. The definition of stock market circuit breakers is consistent with the definition used in the analysis; the price limits on futures markets, however, bear some similarity to the alternative definition. I thank the referee for mentioning this point.

bates price fluctuations, though it increases ex ante trading volume and market liquidity. An empirical implication of Proposition 4 is that for price levels near the circuit breaker bounds, prices should be more volatile relative to their normal levels. At this point, there may be a shortage of data points to test this implication, because the levels of the DJIA have not often approached the trigger points since the circuit breakers were put in place.

III. A Multimarket Model

In this section we apply the basic two-period model of Section II.*A* to a two-market scenario. The situation we have in mind is that of a highly liquid "dominant" market and a less liquid "satellite" market.[14] In keeping with institutional reality (see the introduction), we assume that the circuit breaker is triggered based on extreme price movements in the dominant market, and if triggered, results in trading on both markets being halted.

A. Discretionary Liquidity Trading without a Circuit Breaker

We analyze the strategic trading decision of an informationless trader with exogenous needs to trade across *markets* for the same security. There are two markets for the same security, whose liquidation payoffs are given by (1) and which is traded in two periods. All informed traders have symmetric access to both markets. The link between the two markets is therefore provided by the informed traders, rather than by agents arbitraging on price differences. There is a single discretionary trader, who has exogenous demands l_1 and l_2 in periods 1 and 2 respectively, and who may choose to realize each of these demands either in market 1 or market 2. The demands l_1 and l_2 are assumed to be mutually uncorrelated, normally distributed, each with mean zero and variance σ_l^2, and uncorrelated with ε. Further, l_2 is realized following period 1 and prior to period 2. Thus, the issue of the discretionary trader allocating trades across *periods* does not arise in this section.

There is also a certain amount of nondiscretionary liquidity trading in each market. The net nondiscretionary trade in market i and period j is denoted by z_{ij}. The random variables z_{ij}, $i = 1, 2$ are normally distributed with mean zero and variance σ_{zi}^2 and are each uncorrelated with ε, l_1, and l_2. Thus, the parameterization imposes the same variance of nondiscretionary liquidity trade across periods in a particular market (a reasonable restriction) but allows the variance of such trading to differ across markets. We finally assume that neither the period 1 or period 2 orders in market 1 can be observed in market 2 and vice versa. While this assumption is made primarily for tractability, it strikes us as reasonable upon observing that in times of extreme price movements (when our analysis becomes particularly relevant), heavy trading would probably cause market makers to focus on execution of

[14]Chowdhry and Nanda (1991) provide a model in which a dominant market arises endogenously.

orders and would therefore prevent them from paying much attention to activity on the trading floors of other exchanges. The risk aversion coefficients of the market makers in the two markets are given by R_1 and R_2 respectively.

The discretionary trader has four possible strategies: submit the period 1 order in market 1 and the period 2 order in market 1 (denoted by p1m1, p2m1), submit the period 1 order in market 1 and the period 2 order in market 2 (denoted by p1m1, p2m2), and so on. We use λ to denote a liquidity parameter in market 1 and γ to denote a liquidity parameter in market 2. In addition, we use the subscript l to denote a liquidity parameter in a particular market/period when the discretionary trader trades in that market/period. Thus, if the discretionary trader submits his period 1 order in market 1, the period 1 liquidity parameter in market 1 is denoted by λ_{1l}, if he submits his period 1 order in market 2, the period 1 liquidity parameter in market 1 is denoted by γ_{1l}, and so on. Expressions for the liquidity parameters are obtained from (4) and (12) by replacing R with the appropriate coefficients of risk aversion, σ_1^2 and σ_2^2 by the appropriate variance of the total liquidity trading in the relevant market, and σ_{12} with zero.[15] Thus, λ_{1l} is obtained by replacing R with R_1 and σ^2 with $\sigma_l^2 + \sigma_{z1}^2$ in (4), λ_{2l} by replacing R with R_1, σ_1^2 and σ_2^2 with $\sigma_l^2 + \sigma_{z1}^2$, and σ_{12} with zero in (12), and γ_{1l} and γ_{2l} are obtained analogously.

The expected costs associated with each of the discretionary trader's possible strategies are provided below.

$$\text{p1m1 p2m1: } (\lambda_{1l} + \lambda_{2l})\sigma_l^2$$

$$\text{p1m1 p2m2: } (\lambda_{1l} + \gamma_{2l})\sigma_l^2$$

$$\text{p1m2 p2m1: } (\gamma_{1l} + \lambda_{2l})\sigma_l^2$$

$$\text{p1m2 p2m2: } (\gamma_{1l} + \gamma_{2l})\sigma_l^2.$$

Note that the coefficients λ_{12} and γ_{12} (see Lemma 4) do not appear in the cost expressions. This is because the intertemporal trades of the discretionary traders are uncorrelated, causing the price impact in period 2 to arise only from the coefficient of the period 2 order flow λ_2.

We will now discuss reasonable parameter restrictions that enable us to identify the optimal trading strategy of the discretionary trader. Given the dominance of the NYSE over all the regional exchanges,[16] it is reasonable to suppose that if market 1 is the NYSE, then $\sigma_{z1}^2 > \sigma_{z2}^2$, and since market making is typically performed by large financial institutions on the NYSE floor, which have high risk-bearing capacities, the condition $R_1 < R_2$ will also have a strong tendency to hold. Now, from (4) it is evident that if $\sigma_{z1}^2 > \sigma_{z2}^2$ and $R_1 \leq R_2$, then $\gamma_{1l} > \lambda_{1l}$.

[15] Recall that the discretionary orders are uncorrelated across periods, which, in turn causes the total liquidity trading to also be uncorrelated across periods.

[16] In 1989, about 75 percent of all program trades executed by U.S. securities firms were on the NYSE. See Quinn, Sofianos, and Tschirhart (1990).

Turning now to period 2, note that λ_{2l} is simply half the risk aversion times the variance of the security value conditional on the period 1 order flow (i.e., half the risk aversion times the inverse of the price efficiency measure) —see the discussion following Lemma 4. Now, the conditional variance of the security value increases in the variance of the liquidity trade (from Proposition 2), so that if $\sigma_{z1}^2 > \sigma_{z2}^2$, λ_{2l} tends to be *higher* than γ_{2l}. From (12), however, it follows that $\gamma_{2l} > \lambda_{2l}$ as long as

$$\frac{R_2}{R_1} > \frac{\dfrac{n^2\sigma_\varepsilon^{\,2}}{(n+1)^2\gamma_{1l}^2(\sigma_l^{\,2}+\sigma_{z2}^2)}+1}{\dfrac{n^2\sigma_\varepsilon^{\,2}}{(n+1)^2\lambda_{1l}^2(\sigma_l^{\,2}+\sigma_{z1}^2)}+1} \tag{14}$$

If R_2 is large relative to R_1, the above condition will have a strong tendency to hold because the left-hand side tends to be large and the right-hand side tends to be small in this case. (Note from (4) that γ_{1l} and λ_{1l} are increasing in R_2 and R_1 respectively, which implies that as R_2 increases and R_1 decreases, the numerator on the right-hand side decreases and the denominator increases; both effects increase the tendency for (14) to hold.)

Thus, if $\sigma_{z1}^2 > \sigma_{z2}^2$ and R_2 is sufficiently large so that $R_2 > R_1$ and condition (14) holds, then $\lambda_{1l} < \gamma_{1l}$ and $\lambda_{2l} < \gamma_{2l}$. Under these conditions, the discretionary trader will find submitting both period 1 and 2 orders in market 1 to be his optimal strategy. This captures the notion that most large financial institutions, primarily for reasons of market liquidity, prefer to submit their orders to the dominant market (the NYSE) rather than to satellite markets.

B. The Circuit Breaker

Let us now examine the consequences of introducing a circuit breaker in market 1. As discussed earlier, the circuit breaker is a rule that stipulates that if the price in market 1 falls outside either a lower bound ρ_1 or an upper bound ρ_2, trading in both markets will be halted in period 2, and the markets will remain closed for a period of time until ε is revealed.

Suppose that conditions that cause the discretionary trader to trade in market 1 in both periods (in the absence of a circuit breaker) i.e., condition (14) and the conditions $R_2 > R_1$ and $\sigma_{z1}^2 > \sigma_{z2}^2$ hold. We assume, as in Section II, that the cost of a trade not being executed in period 2 is given by a number c. Then, in the presence of the circuit breaker, the problem of the discretionary trader is one of calculating which market to trade in after factoring in the probability that the trade may not go through in period 2 in the first market. Let q_l and q_w denote the probabilities that the price in market 1 will cross a circuit breaker bound if the discretionary trader submits his period 1 order in markets 1 and 2 respectively. Thus, $q_l = N\left(\frac{\rho_1 - \overline{F}}{s_l}\right) + N\left(\frac{\overline{F} - \rho_2}{s_l}\right)$ and $q_w = N\left(\frac{\rho_1 - \overline{F}}{s_w}\right) + N\left(\frac{\overline{F} - \rho_2}{s_w}\right)$ where s_l and s_w are given by the square root of (5), with σ^2 replaced by $\sigma_l^2 + \sigma_{z1}^2$ and σ_{z1}^2, respectively.

The decision of the discretionary trader in the presence of a circuit breaker is governed by the following proposition.

PROPOSITION 5: *Suppose that the conditions* $\sigma_{z1}^2 > \sigma_{z2}^2$, $R_2 > R_1$, *and* (14) *hold, so that in the unique equilibrium, the discretionary trader trades in market* 1 *in both periods. Then, upon introduction of a circuit breaker, in the new equilibrium, the discretionary trader switches to market* 2 *in period* 1 *if and only if*

$$c > \left[\frac{\gamma_{1l} - \lambda_{1l}}{q_l - q_w} + \lambda_{2l}\right]\sigma_l^2. \tag{15}$$

Again, (15) holds if c is sufficiently high. Notice that since $s_w < s_l$, $q_w < q_l$, that is, by not trading in market 1, the discretionary trader can reduce the chance that the price in market 1 crosses the breaker bounds; this provides him an incentive to switch to market 2.

The following proposition describes how the period 1 market liquidity, price variability, and price efficiency change as a consequence of the discretionary trader switching to market 2.[17]

PROPOSITION 6: *If the conditions* $\sigma_{z1}^2 > \sigma_{z2}^2$, $R_2 > R_1$, (14), *and* (15) *hold, then the period* 1 *price variability, market liquidity, and expected trading volume V decrease in market* 1 *and increase in market* 2 *after the circuit breaker is introduced, while the effect of the circuit breaker on period* 1 *price efficiency (conditional on both market* 1 *and market* 2 *order flows) is ambiguous.*

Focusing on the unambiguous results, Proposition 6 shows that circuit breakers can decrease price variability, but at the cost of decreasing market liquidity and volume in the market in which they are introduced. Our analysis implies that it is critical from the perspective of dominant exchange liquidity to ensure that the triggering is price coordinated, in the sense that breakers should be triggered whenever the price in *any* market (and not just the dominant market) crosses exogenous levels.

Note that Section II indicates that liquidity traders with discretion across time periods may suboptimally concentrate their trades in earlier periods and *increase* ex ante price variability (as well as market liquidity) as the price approaches the circuit breaker bounds, whereas Section III indicates that price variability and market liquidity may decrease on the dominant market as agents switch to the satellite market to reduce the chance that the price on

[17] The price efficiency measure is computed conditional on both market 1 and market 2 prices in view of the fact that prices across markets are generally observable simultaneously.

the dominant market crosses the circuit breaker bound. While which of the two effects dominates is ultimately an empirical issue, we conjecture that in periods of extreme price movements, it may be costly for many financial institutions to "shop around" for trading opportunities on other exchanges, as institutions would be required to pay an abnormally large amount of attention to monitoring prices on the dominant exchange. Based on this observation, the intertemporal effect in Section II may be expected to dominate.

IV. Summary and Concluding Remarks

This article has examined the impact of a relatively recent regulatory tool, the circuit breaker, in intertemporal and multimarket contexts.

In an intertemporal, one-market model, we showed that the circuit breaker may actually increase price variability and the probability of the price crossing the circuit breaker bounds if the price is very close to the breaker limit and if agents place a high value on their desire to trade. These effects can occur because strategic traders may suboptimally advance their trades to assure themselves of their ability to trade. The circuit breaker thus may yield results that are exactly the opposite of what regulation intended it to accomplish.

The paper next considered a two-market situation in which a circuit breaker was assumed to be triggered on extreme price movements in a relatively liquid "dominant" market. We showed that agents with intertemporal needs to trade may migrate out of dominant markets on whose price movements the circuit breakers are triggered. This phenomenon has the effect of transferring price variability from the dominant market to the "satellite" market. The transfer of price variability, however, is also found to be accompanied by a transfer of market liquidity and expected trading volume in the same direction. As a general point, our analysis shows that policies designed to reduce price variability are not necessarily consistent with the objective of maximizing market liquidity.

It is often argued that circuit breakers may reduce panic-driven selling by allowing investors to "calmly assess their position in times of market stress" (Market Volatility and Investor Confidence (1990), p. 3), and also may slow down large index-arbitrage–related trades.[18] Our analysis suggests that the benefits, if any, from the implementation of circuit breakers are likely to entail a perverse cost in terms of increased price variability or migration of trading volume. A noteworthy feature of our results is that they obtain under a large parameter set regardless of whether the actual realization of the price crosses the exogenous bound, while the claimed benefits of circuit breakers can obtain only if the circuit breakers are actually triggered. It therefore

[18] Miller (1990a, 1990b) questions the role of circuit breakers in reducing index arbitrage volume and argues that regulators should instead consider eliminating "price continuity" rules imposed on specialists, in order to prevent arbitrageurs from "picking off" the posted quotes.

seems important that policymakers keep the effects described by this paper in mind when implementing such impediments to trade.

Appendix

Proof of Lemma 1: Each informed trader maximizes $E[(F - P)y|\varepsilon]$, where y is each informed trader's order. Suppose that each such trader submits an order $y = \beta\varepsilon$. Substituting for the linear pricing rule $P = \overline{F} + \lambda Q$, where $Q = n\beta\varepsilon + z$, and maximizing with respect to β yields $\beta = 1/[(n + 1)\lambda]$. Now, if u and v are two independent and normally distributed random variables, each having a mean of zero, then it is a standard result (see, e.g., DeGroot (1986)) that

$$\operatorname{var}(u|u + v) = \frac{\sigma_u^2\sigma_v^2}{\sigma_u^2 + \sigma_v^2}. \tag{16}$$

Noting again that $Q = n\beta\varepsilon + z$, substituting $\beta = 1/[(n + 1)\lambda]$ into (3), and using (16) yields a quadratic equation for λ, the unique positive root of which yields (4). □

Proof of Lemma 2*:* The relation (6) is obtained by using (16). Part (1) is a well-known property of the Kyle model; see, for example, Admati and Pfleiderer (1988). In our context, this result is obtained by substituting $R = 0$ in (4), substituting the resulting expression for λ into (6), and verifying that the resulting expression for Γ does not involve σ^2. For proving part (2), note that (6) can be rewritten as $\sigma_\varepsilon^{-2}\left[1 + \dfrac{n^2\sigma_\varepsilon^2}{(n + 1)^2\lambda^2\sigma^2}\right]$ which is decreasing in $\sigma^2\lambda^2$. Noting from (4) that $\sigma^2\lambda^2$ is increasing in σ^2 completes the proof. □

Proof of Lemma 3: The expression for V_I is obtained by using the facts that the informed trade $I = n\beta\varepsilon$, where $\beta = 1/[(n + 1)\lambda]$, and substituting for λ from (4). The proof of the expression for V_M is trivial. □

Proof of Lemma 4: From the proof of Lemma 1, the informed traders use linear strategies of the form $\beta_1\varepsilon$, where $\beta_1 = 1/[(n + 1)\lambda_1]$; the period 1 price is thus set to satisfy

$$E[(Q_1(P_1 - F))|Q_1] = (R/2)\operatorname{var}[(Q_1(P_1 - F))|Q_1]$$

which implies that $P = \overline{F} + \lambda_1 Q_1$, where λ_1 is given by (4), with σ^2 replaced by σ_1^2. Further, the period 2 price is set to satisfy

$$E[(Q_2(P_2 - F))|Q_1, Q_2] = (R/2)\operatorname{var}[(Q_2(P_2 - F))|Q_1, Q_2]$$

which implies that

$$P_2 = \overline{F} + E(\varepsilon|Q_1, Q_2) + (R/2)Q_2 \operatorname{var}(\varepsilon|Q_1, Q_2). \tag{17}$$

To calculate the conditional moments in (17), we use the well-known result (see, e.g., Anderson (1984, chapter 2)) that if there exist random vectors x_1

and x_2 such that

$$(x_1, x_2) \sim N\left[(0,0), \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]$$

then the conditional distribution of x_1 given $x_2 = X_2$ is normal with mean

$$E(x_1 | x_2 = X_2) = \Sigma_{12}\Sigma_{22}^{-1}X_2 \tag{18}$$

and variance

$$\mathrm{var}(x_1 | x_2 = X_2) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}. \tag{19}$$

To compute the conditional expectation and variance in (17), let $x_1 = \varepsilon$, $x_2 = (Q_1, Q_2)$, with $Q_1 = t\varepsilon + s_1$ and $Q_2 = s_2$, where s_1 and s_2 denote the total quantity of liquidity trading in periods 1 and 2 respectively. Substituting for the conditional moments from (18) and (19) back into (17), one obtains the expression for P_2 as given in the lemma. □

Proof of Proposition 2: The proof involves computing and comparing the expected trading costs of the discretionary traders for the various strategies in terms of exogenous parameters. In this proof, we define $N \equiv (n+1)/n$ and $n_1 \equiv (n+1)^2/n$. Let us first compare the expected trading costs of splitting the order evenly and concentrating trades in period 1. Now, by substituting $\sigma_1^2 = \sigma_2^2 = \sigma_l^2 + \sigma_z^2$ and $\sigma_{12} = \sigma_l^2$ into (11) and (12), we have

$$\lambda_{12s} + \lambda_{2s} = \frac{N\lambda_{1s}\sigma_\varepsilon^2\sigma_z^2 + (R/2)N^2\lambda_{1s}^2\sigma_z^2(\sigma_z^2 + 2\sigma_l^2)\sigma_\varepsilon^2}{\sigma_\varepsilon^2(\sigma_l^2 + \sigma_z^2) + N^2\lambda_{1s}^2\sigma_z^2(\sigma_z^2 + 2\sigma_l^2)} \tag{20}$$

while λ_{1s} and λ_{1f} are given by (4) with σ^2 replaced by $\sigma_z^2 + \sigma_l^2$ and $4\sigma_l^2 + \sigma_z^2$. We need to compare $4\lambda_{1f}$ with $\lambda_{1s} + \lambda_{12s} + \lambda_{2s}$. We will first show that $\lambda_{12s} + \lambda_{2s} < \lambda_{1s}$, and then show that $2\lambda_{1f} > \lambda_{1s}$, which will ensure that $4\lambda_{1f} > \lambda_{1s} + \lambda_{12s} + \lambda_{2s}$.

Now, the right-hand side of (20) is less than λ_{1s} if and only if

$$-\sigma_\varepsilon^2(\sigma_l^2 + \sigma_z^2) + N\sigma_\varepsilon^2\sigma_z^2 + N^2\sigma_z^2(\sigma_z^2 + 2\sigma_l^2)((R/2)\sigma_\varepsilon^2\lambda_{1s} - \lambda_{1s}^2) < 0. \tag{21}$$

Substituting for λ_{1s} into this condition and simplifying shows that this condition is equivalent to $\sigma_\varepsilon^2\sigma_l^2 > 0$, which is true. Thus, $\lambda_{12s} + \lambda_{2s} < \lambda_{1s}$; therefore to prove that $4\lambda_{1f} > \lambda_{1s} + \lambda_{12s} + \lambda_{2s}$, it is sufficient to prove that $2\lambda_{1f} > \lambda_{1s}$, which is true if and only if

$$(R/2)\sigma_\varepsilon^2 + 2\sqrt{\frac{R^2\sigma_\varepsilon^4}{16} + \frac{\sigma_\varepsilon^2}{n_1(4\sigma_l^2 + \sigma_z^2)}}$$
$$> (R/4)\sigma_\varepsilon^2 + \sqrt{\frac{R^2\sigma_\varepsilon^4}{16} + \frac{\sigma_\varepsilon^2}{n_1(\sigma_l^2 + \sigma_z^2)}},$$

i.e., if $3\sigma_z^2 > 0$, which is true. Therefore, splitting trades across periods leads to a lower expected trading cost than concentrating trades in period 1.

We will now show that concentrating trades in period 1 leads to lower expected trading costs than concentrating trades in period 2. Noting that whenever the discretionary trader concentrates trading in period 2, $\sigma_{12} = 0$ and $\sigma_1^2 = \sigma_z^2$, we have

$$\lambda_{12d} + \lambda_{2d} = \frac{t_d \sigma_\varepsilon^2}{t_d^2 \sigma_\varepsilon^2 + \sigma_z^2} + \frac{R}{2} \frac{\sigma_\varepsilon^2 \sigma_1^2}{t_d^2 \sigma_\varepsilon^2 + \sigma_z^2} \tag{22}$$

where $t_d = n/(n+1)\lambda_{1d}$. The first term in the above expression is simply the regression coefficient of ε on the period 1 order flow, while the second term is half the risk aversion times the variance of the security conditional on the period 1 order flow. From (3), this implies that $\lambda_{12d} + \lambda_{2d} = \lambda_{1d}$. Now, we need to compare $4(\lambda_{12d} + \lambda_{2d})$ to $4\lambda_{1s}$. However, since from (4) and the definitions of λ_{1d} and λ_{1f}, it is evident that $\lambda_{1d} > \lambda_{1f}$. Since $\lambda_{12d} + \lambda_{2d} = \lambda_{1d}$ and $\lambda_{1d} > \lambda_{1s}$, we have $4(\lambda_{12d} + \lambda_{2d}) > 4\lambda_{1s}$. Thus, the discretionary trader prefers to concentrate his trades in period 1 rather than in period 2. Since we have shown earlier that splitting trades across periods leads to lower expected trading costs than concentrating trades in period 1, the lowest cost strategy (and therefore, the equilibrium strategy) involves the discretionary trader splitting his trades across the two periods. □

Proof of Proposition 3: The left-hand side of (13) represents the expected cost incurred by the discretionary trader if he splits his order across the two periods, while the right-hand side represents the expected cost if he concentrates his trading in period 1. Since the proof of Proposition 2 shows that the discretionary trader always prefers to concentrate trading in period 1 rather than in period 2, the proposition imposes a restriction that ensures that the lowest cost strategy, and thus the equilibrium strategy, involves the discretionary trader concentrating trades in period 1. □

Proof of Proposition 5: First note that since $\gamma_{1l} > \lambda_{1l}$ and $\gamma_{2l} > \lambda_{2l}$, the strategy (p1m1 p2m2) is dominated in terms of expected trading costs by (p1m1 p2m1) and further that (p1m2 p2m2) is dominated in terms of expected trading costs by (p1m1 p2m2). This leaves us with the strategies (p1m1 p2m1) and (p1m2 p2m1). The strategy (p1m2 p2m1) dominates (p1m1 p2m1) if and only if

$$\gamma_{1l}\sigma_l^2 + q_w c + (1 - q_w)\lambda_{2l}\sigma_l^2 > \lambda_{1l}\sigma_l^2 + q_l c + (1 - q_l)\lambda_{2l}\sigma_l^2.$$

The above condition simplifies to condition (15) in the text. □

Proof of Proposition 6: The results on market liquidity and price variability follow directly from equations (4) and (5) and Lemma 2. The ambiguous results on price efficiency are confirmed by first noting that for given variances of liquidity trading σ_1^2 and σ_2^2, price efficiency is the inverse of the posterior variance of the security value, given by

$$\operatorname{var}(\varepsilon | Q_{m1}, Q_{m2}) = \frac{\sigma_\varepsilon^2 \sigma_1^2 \sigma_2^2}{\sigma_1^2 \sigma_2^2 + \sigma_\varepsilon^2 (t_1^2 \sigma_2^2 + t_2^2 \sigma_2^2)}$$

where Q_{m1} and Q_{m2} denote the order flows in market 1 and market 2 and $t_1 = n/(n+1)\lambda_1$ and $t_2 = n/(n+1)\gamma_1$ denote the trading aggressiveness parameters associated with the informed traders in markets 1 and 2, where, in turn, λ_1 and γ_1 are given by (4), with σ^2 replaced by the appropriate quantities of liquidity trading in the two markets. When the discretionary trader switches to market 2, σ_1^2 changes from $\sigma_{z1}^2 + \sigma_l^2$ to σ_{z1}^2 and σ_2^2 changes from σ_{z2}^2 to $\sigma_l^2 + \sigma_{z2}^2$. Numerical examples can easily be generated that cause the price efficiency measure to move in either direction following the migration of discretionary trading. For example, suppose that $n = 1$, $\sigma_\varepsilon = 1$, $R_1 = 1$, $\sigma_{z1}^2 = 5$, $\sigma_{z2}^2 = \sigma_l^2 = 1$. Then, if $R_2 = 5$, condition (14) holds and $\sigma_{z1}^2 > \sigma_{z2}^2$, so that the discretionary trader trades in market 1 (in period 1) in the absence of a circuit breaker. In this case, the posterior variance given above drops from 0.859 to 0.858 when the discretionary trader switches to market 2 upon introduction of a breaker. (Note that condition (15) can be satisfied by choosing c to be appropriately large.) Now suppose instead that $R_2 = 4$. In this case condition (14) again holds; however the posterior variance now increases from 0.845 to 0.851 when the discretionary trader switches to market 2. □

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