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Housing Collateral, Consumption Insurance, and Risk Premia: An Empirical Perspective

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ABSTRACT

In a model with housing collateral, the ratio of housing wealth to human wealth shifts the conditional distribution of asset prices and consumption growth. A decrease in house prices reduces the collateral value of housing, increases household exposure to idiosyncratic risk, and increases the conditional market price of risk. Using aggregate data for the United States, we find that a decrease in the ratio of housing wealth to human wealth predicts higher returns on stocks. *Conditional* on this ratio, the covariance of returns with aggregate risk factors explains 80% of the cross-sectional variation in annual size and book-to-market portfolio returns.

HOUSE PRICE FLUCTUATIONS PLAY AN IMPORTANT ROLE in explaining the time-series and the cross-sectional variation in asset returns. Given the magnitude of the housing market, this is unsurprising. This paper shows that the way in which housing affects asset returns is through the role of housing as a collateral asset.

We identify a novel collateral channel that transmits shocks in the housing market to risk premia. In a model with collateralized borrowing and lending, the ratio of housing wealth to human wealth, the housing collateral ratio, changes the conditional distribution of consumption growth across households. When the collateral ratio is low, the dispersion of consumption growth across households is more sensitive to aggregate consumption growth shocks, and this raises the market price of aggregate risk.

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This paper focuses on connecting the model to asset price data. The predictions of the model are confirmed by U.S. equity return data over time and in the cross-section. U.S. investors seem to demand a larger risk compensation in times when the housing collateral ratio is low, because the housing collateral ratio predicts aggregate stock returns. In the cross-section, the model predicts that assets with returns that are more tightly correlated with aggregate consumption growth shocks when collateral is scarce trade at a discount. Conditional on the housing collateral ratio, the covariance of returns with aggregate consumption growth shocks explains about 80% of the cross-sectional variation in U.S. stock returns, because the returns of value stocks are more closely correlated with aggregate consumption growth shocks during low collateral periods than are growth stocks.

In the model, there are two main channels that transmit shocks originating in the housing market to the risk premia in asset markets. First and foremost, a drop in the housing collateral ratio adversely affects risk-sharing that enables households to insulate consumption from labor income shocks. The distribution of consumption growth fans out as this ratio decreases. When housing prices decrease, collateral is destroyed and households are more exposed to idiosyncratic labor income risk. The risk associated with these collateral constraints contributes a risk factor to the stochastic discount factor.

Second, households want to hedge against rental price shocks or consumption basket composition shocks when the utility function is nonseparable in nondurable consumption and housing services. This introduces a new risk factor which is the focus of recent work by Piazzesi, Schneider, and Tuzel (2004) and Yogo (2003). In particular, if housing services and consumption are complements, then households command a larger risk premium if returns and rental price growth are positively correlated. Dunn and Singleton (1986) and Eichenbaum and Hansen (1990) report substantial evidence against the null of separability in a representative agent model with nondurable consumption and durables, but they conclude that introducing durables does not help in reducing the pricing errors for stocks.

The collateral effect does not hinge on the nonseparability of preferences. Nevertheless, we incorporate nonseparability in the model to isolate the nonseparability effect from the collateral mechanism. Instead, it relies on imperfect consumption insurance among households induced by occasionally binding collateral constraints. Without these collateral constraints, our model collapses to the standard consumption-based capital-asset-pricing model of Lucas (1978) and Breeden (1979). That model prices only aggregate consumption growth risk and it has been rejected by the data (e.g., Hansen and Singleton (1983)). Our paper addresses two empirical failures of the consumption-based capital-asset-pricing model (CCAPM).

First, because U.S. aggregate consumption growth is approximately i.i.d., the CCAPM implies a market price of risk that is approximately constant. However, in the data, stock market returns are predictable, and this suggests that the market price of aggregate risk varies over time (e.g., see Fama and French (1988), Campbell and Shiller (1988), Ferson, Kandel, and Stambaugh (1987), Whitelaw (1997), Lamont (1998), Campbell (2000) and Lettau and Ludvigson (2001a) for an overview). Our model delivers time variation in the market price endogenously through the housing market. As the housing collateral ratio decreases, the conditional volatility of the liquidity factor increases. In the data, the housing collateral ratio *does* predict the aggregate U.S. stock market return, mainly at lower frequencies.

Second, the covariance of asset returns with consumption growth explains only a small fraction of the variation in the cross-section of stock returns of firms sorted in portfolios according to size (market capitalization) and value (book-value to market-value ratio) characteristics (Fama and French (1992)).[1] The collateral model explains 80% of the variability in annual returns of the Fama–French size and book-to-market portfolios. For annual returns, this matches the empirical success of the Fama and French (1993) three-factor model and recent conditional consumption-based asset-pricing models (e.g., Lettau and Ludvigson (2001b)). The estimated market price of consumption growth risk is positive and significant, while an increase in the collateral ratio lowers the estimated price of consumption growth risk, as predicted by the theory.

We measure the aggregate stock of housing collateral in three different ways: using the value of outstanding mortgages, the value of residential real estate (structures and land), and the value of residential fixed assets (structures). The housing collateral ratio, which we label *my*, is measured as the deviation from the cointegration relationship between the value of the aggregate housing stock and the aggregate labor income. Housing is by far the most important collateral asset for households. In the United States, two-thirds of households own their houses. For the median-wealth homeowner, home equity represents 70% of household net worth (Survey of Consumer Finance (1998)). Residential real estate wealth accounts for 28% of total household net worth and 68% of non-financial assets, while home mortgages make up 64% of household liabilities (Flow of Funds, Federal Reserve, averages for 1952 to 2002). Currently, the value of residential wealth exceeds the total household stock market wealth ($13.6 trillion) and the mortgage market is the largest credit market in the United States ($6.1 trillion).

Our model contains the following essential ingredients. It is an endowment economy with a continuum of agents that are subject to labor income shocks. As in Lustig (2003), we allow households to forget their debts. The new feature of our model is that each household owns part of the housing stock. Housing provides utility services and collateral services. When a household chooses to forget its debts, it loses all its housing wealth, but its labor income is protected from creditors, and the household is not excluded from trading. This gives rise to collateral constraints whose tightness depends on the abundance of housing

[1] In response to this failure, Fama and French (1993) directly specify the stochastic discount factor as a linear function of the market return, the return on a small minus big firm portfolio, and a high minus low book-to-market firm portfolio. The empirical success of this three-factor model has motivated recent research on the underlying macroeconomic sources of risk for which their factors proxy (e.g., see Bansal, Dittmar, and Lundblad (2002), Lettau and Ludvigson (2001b), Santos and Veronesi (2001), and Cochrane (2001) for an overview).

collateral. We measure this by the housing collateral ratio: the ratio of collateralizable housing wealth to noncollateralizable human wealth.

The stochastic discount factor contains a new component that we label as the aggregate liquidity shock. It is the growth rate of a cross-sectional moment of the consumption share distribution. The household's consumption share of the total endowment, both for nondurables and for housing services, increases whenever the household switches to a state with a binding constraint. When a large fraction of households is constrained, this growth rate is high. We call this a liquidity shock. The housing collateral ratio changes the conditional moments of the aggregate liquidity shock. When the housing collateral ratio is low, households run into binding collateral constraints more frequently. This increases the conditional standard deviation of the aggregate weight shock, and by the same token, the market price of risk. Thus, endogenous movements in the housing collateral ratio turn the liquidity shocks in the stochastic discount factor on and off, and this induces heteroskedasticity and counter-cyclicality in the Sharpe ratio. This collateral mechanism is a novel feature of the model.

In Lustig and Van Nieuwerburgh (2004a), we fully calibrate and solve the model. The equilibrium aggregate liquidity shock is a function of the primitives of the model: the preferences, the household endowment process, the aggregate endowment process, and the aggregate nondurable expenditure ratio process. The model generates large, time-varying risk premia and a value premium.

Our empirical strategy is to estimate the stochastic process for the aggregate weight shocks. In a first step, we allow this process to depend in a nonlinear fashion on the aggregate pricing factors. We estimate the parameters from the moment conditions implied by the Euler equations for the aggregate market return, the risk-free rate, a long-term bond, and a limited number of size and book-to-market portfolios. The Euler inequalities for the representative agent allow us to precisely estimate the coefficient of risk-aversion. The estimated coefficients of relative risk-aversion are plausible (between 2 and 5) and much lower than those for the other models we consider, as shown by Figure 1. In addition, the parameters in the aggregate weight shock specification are estimated precisely and have the sign predicted by the collateral channel. The pricing errors are small and the model cannot be rejected.

The linear specification for the aggregate weight shock fits the data best and allows us to make contact with the linear factor models in empirical finance. This specification delivers a conditional version of the CCAPM with the housing collateral ratio as the conditioning variable. The housing collateral ratio summarizes the investor's time-varying information set. This model prices the 25 Fama–French size and book-to-market portfolios surprisingly well by imputing the value premium to risk compensation for a higher consumption growth beta of value returns in times when collateral is scarce. We provide evidence about dividend dynamics that potentially explains why value returns respond differently to aggregate shocks when the collateral ratio is small.

We organize the paper as follows. In Section I, we briefly discuss other related literature. Section II describes the essence of the model. Section III discusses the composition channel and collateral channel in more depth.

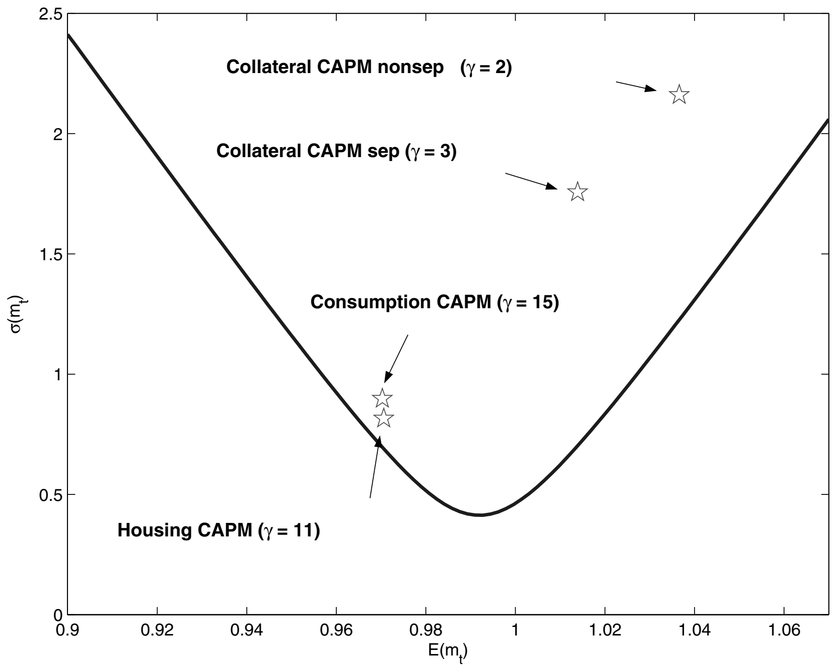

Figure 1. First and second moments of the stochastic discount factor. The Hansen–Jagannathan bounds are computed using annual data from 1926 to 2002 for the real value-weighted market return, the risk-free rate, a 10-year bond and R^{HML}, the return on the high value minus low value stock portfolio. The model parameters for the consumption-CAPM, the Housing-CAPM, the Collateral-CAPM under separable preferences, and the Collateral-CAPM under nonseparable preferences are estimated by GMM for these four test assets. The plot shows the moments of the SDF, evaluated at the parameter estimates.

Section IV contains the time-series predictability results, and Sections V and VI contain the empirical results for the cross-section. Section VII concludes. The Appendix contains a detailed description of the data.

I. Related Literature

Our paper is related most closely to the work of Lettau and Ludvigson (2001b). We also develop a scaled version of the CCAPM. Our state variable *my* summarizes information about future returns on housing relative to human capital, while the scaling variable in Lettau and Ludvigson is the consumption–wealth ratio, which summarizes household expectations about future returns on the entire market portfolio.

In a different class of models, Cogley (2002) and Brav, Constantinides, and Geczy (2002) find that including higher moments, such as the standard deviation and skewness of the consumption growth distribution, reduces the size of the Euler equation errors for stock returns. This evidence is consistent with our model. In case of a large aggregate weight shock, the dispersion of the consumption growth distribution increases, while its skewness decreases. This provides indirect evidence for the consumption growth distribution shocks that drive our results. We provide a theory of what governs these shocks.

Cochrane (1996) explores the explanatory power of residential and nonresidential investment for equity returns in the context of his production-based asset-pricing framework (Cochrane (1991a)). Li, Vassalou, and Xing (2002) find that investment growth, including household sector investment that is largely residential, can help account for a large fraction of the cross-sectional variation in equity returns. Similarly, Kullmann and Siegel (2003) use returns on residential and commercial real estate to improve the performance of the capital asset pricing model. Finally, life-cycle and portfolio choice models with housing, such as in Fernandez–Villaverde and Krueger (2001), Cocco (2000), Yao and Zhang (2004), and Flavin and Yamashita (2002) posit an exogenous price process for housing and consider a limited menu of traded assets. We endogenize the price of the asset, but we abstract from life-cycle considerations.

II. Model

Our economy's risk-sharing technology is subject to shocks originating in the housing market, and these shocks determine the size of the wedge between the market's valuation of payoffs and the representative agent's intertemporal marginal rate of substitution (IMRS). The stochastic discount factor (SDF) in our model is

$$m_{t+1} = m^a_{t+1} g^{\gamma}_{t+1},$$

where m^a_{t+1} is the IMRS of a representative agent who consumes nondurable consumption and housing services, and g^{γ}_{t+1} is the liquidity factor contributed by the solvency constraints. This factor can be interpreted as the aggregate cost of the solvency constraints. When these solvency constraints do not bind, the liquidity factor disappears and payoffs can be priced off the representative agent's IMRS m^a_{t+1}. We show that this liquidity factor can explain some of the variation in U.S. stock returns over time and in the cross-section.

A. Endowments and Preferences

A continuum of agents are endowed with claims to stochastic labor income streams. These agents consume nondurable consumption and housing services. We use s^t to denote the history of events $s^t = (y^t, z^t)$, where y^t denotes the history of idiosyncratic events and z^t denotes the history of aggregate events. The notation $\{c^a_t(z^t)\}$ denotes the aggregate endowment stream of nondurable consumption and $\{h^a_t(z^t)\}$ denotes the aggregate endowment of housing services. The evolution of the non-housing/housing expenditure ratio is defined as

$$r_t(z^t) = \frac{c^a_t(z^t)}{\rho_t(z^t) h^a_t(z^t)}, \tag{1}$$

where ρ_t denotes the rental price of housing services and governs the relative supply of housing services in this economy.

The endowment processes are Markovian. The growth rate $\lambda_t(z^t)$ of aggregate nondurable consumption $\{c^a_t(z^t)\}$ only depends on the current aggregate state z_t and the expenditure ratio is Markov in z_t and r_{t-1}. Finally, the household's labor endowment share $\hat{\eta}_t(y_t, z_t)$, as a fraction of the aggregate nondurable endowment $c^a_t(z^t)$, depends only on the current realization of the idiosyncratic shock y_t and the aggregate shock z_t.

Preferences are standard. The households rank stochastic consumption streams according to the usual criterion:

$$U(\{c\}, \{h\}) = \sum_{s^t \mid s_0} \sum_{t=0}^{\infty} \delta^t \pi\left(s^t \mid s_0\right) u\left(c_t(s^t), h_t(s^t)\right), \tag{2}$$

where δ is the time discount factor and the power utility kernel is defined over a CES-composite consumption good:

$$u(c_t, h_t) = \frac{1}{1-\gamma}\left[c_t^{\frac{\varepsilon-1}{\varepsilon}} + \psi h_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}},$$

where $\psi > 0$ converts the housing stock into a service flow and ε is the intratemporal elasticity of substitution between nondurable consumption and housing services.[2]

B. Markets

The markets for housing services are frictionless, in that the ownership of the asset and the use of its services are completely distinct. Households can purchase housing services $\{h_t(s^t)\}$ in the spot markets at spot prices $\{\rho_t(z^t)\}$ as well as nondurable consumption $\{c_t(s^t)\}$.

Although households cannot sell claims to their labor income stream $\{\eta_t(s^t)\}$, they can trade a complete set of contingent claims to insure against idiosyncratic labor income risk, but these trades are subject to solvency constraints. The solvency constraints can be stated as restrictions on the value of a household's consumption claim, net of its labor income claim:

$$\Pi_{s^t}\left[\left\{c_t(s^t) + \rho_t(z^t)h_t(s^t)\right\}\right] \geq \Pi_{s^t}\left[\left\{\eta_t(s^t)\right\}\right],$$

where $\Pi_{s^t}[\{d_t(s^t)\}]$ denotes the price of a claim to $\{d_t(s^t)\}$.

The supply of housing wealth relative to human wealth governs the tightness of the solvency constraints. We call this ratio the housing collateral ratio my:

$$my_t(z^t) = \frac{\Pi_{z^t}[\{\rho h^a\}]}{\Pi_{z^t}[\{c^a\}]}.$$

[2] The preferences belong to the class of homothetic power utility functions found in Eichenbaum and Hansen (1990). Special cases are separability ($1-\gamma = \frac{\epsilon-1}{\epsilon}$) and Cobb-Douglas preferences ($\epsilon = 1$).

The effectiveness of the risk-sharing technology depends on the ratio of housing wealth to total wealth. Suppose the households in this economy derive no utility from housing services; then there is no collateral in this economy and *my* is zero. All the solvency constraints necessarily bind at all nodes and households are in autarchy. As *my* increases, perfect risk-sharing becomes feasible.

Shocks to *my* change the conditional distribution of consumption across households and asset prices, but this mechanism is only quantitatively interesting if the expenditure ratio r or rental prices ρ are subject to large, persistent shocks that significantly change *my*:

$$my_t(z^t) = \frac{\Pi_{z^t}\left[\left\{\frac{c^a}{r}\right\}\right]}{\Pi_{z^t}\left[\{c^a\}\right]},$$

and this is borne out by the data. All of these processes are highly autocorrelated. Table I reports the AR(1) coefficients for the log expenditure ratio and the log rental price of housing services.

Table I

Expenditure Share and Rental Price Regression Results

Panel A reports regression results for $\log(z_{t+1}) = \theta\log(z_t) + \lambda\Delta\log(c_{t+1}) + \epsilon_{t+1}$, where z is the expenditure share of nondurable consumption. Panel B reports results for the regression $\log(\rho_{t+1}) = \theta\log(\rho_t) + \lambda\Delta\log(c_{t+1}) + \epsilon_{t+1}$, where ρ is the rental price. Below the OLS point estimates are HAC Newey–West standard errors. The left panel reports the results for the entire sample, while the right panel reports the results for the post-war sample. The variables with superscript 1 are available for 1926–2002. The variables with superscript 2 are only available for 1929–2002. The data appendix contains detailed definitions and data sources for these variables.

	1926/1929–2002		1945–2002	
Expl. Var.	θ	λ	θ	λ
Panel A: Expenditure Share				
$\log(z^1)$	0.925		0.950	
	(0.039)		(0.033)	
$\log(z^1)$	0.890	0.824	0.957	0.824
	(0.033)	(0.141)	(0.033)	(0.180)
$\log(z^2)$	0.940		0.936	
	(0.037)		(0.026)	
$\log(z^2)$	0.940	0.816	0.952	0.816
	(0.032)	(0.159)	(0.027)	(0.181)
Panel B: Rental Price				
$\log(\rho^1)$	0.953		0.851	
	(0.027)		(0.056)	
$\log(\rho^1)$	0.955	0.102	0.817	0.261
	(0.027)	(0.181)	(0.054)	(0.240)
$\log(\rho^2)$	0.941		0.911	
	(0.023)		(0.046)	
$\log(\rho^2)$	0.932	−0.321	0.896	0.259
	(0.023)	(0.158)	(0.047)	(0.172)

The housing collateral ratio is high today when the expenditure ratio r is low today, or if ϵ is smaller than one, when the rental price ρ is high, and the effect on my grows as the persistence of r increases. In the model, the aggregate housing collateral ratio my, the current expenditure ratio, and the rental price are all quasi-sufficient statistics for the risk-sharing capacity of this economy. In equilibrium, the stock of collateral is allocated efficiently across households in a stationary equilibrium and this absolves us from the need to track the entire distribution of collateral across households. The next section explains exactly how shocks to my impinge on allocations and prices.

C. Equilibrium Consumption

In equilibrium, household consumption follows a strikingly simple pattern: As long as a household does not switch to a state in which the solvency constraint binds, or equivalently, in which the net wealth is zero, the household's consumption as a fraction of total consumption decreases. However, when the household enters a state with zero net wealth, its consumption share jumps up. The size of these jumps for the latter and the rate of consumption share decline for the former class of households depends on the housing collateral ratio my. When the ratio is low, the cutoff consumption shares at which at the household solvency constraint binds increase. As a result, other households experience steeper declines in their consumption shares.

We use consumption weights ω to characterize the equilibrium prices and allocations. The new consumption weight of a household that enters the period with consumption weight ω is $\tilde{\omega}_t(\omega, s^t)$. These consumption weights are constant as long as the agent does not switch to a state with a binding constraint, but when it does, its new weight $\tilde{\omega}_t(\omega, s^t)$ is set to a cutoff weight $\underline{\omega}_t(y_t, z^t)$. To compute the aggregate consumption weight, we integrate over the new household weights at aggregate node z^t: $\xi^a_t(z^t) = \int \tilde{\omega}_t(\omega, s^t) d\,\Phi_t(\omega; z^t)$. The notation $\Phi_t(\cdot; z^t)$ is the distribution over weights at the start of period t, and this distribution depends on the entire aggregate history z^t. The actual consumption share of an agent equals the ratio of his individual stochastic consumption weight to the aggregate consumption weight:

$$c_t(\omega, s^t) = \frac{\tilde{\omega}_t(\omega, s^t)}{\xi^a_t(z^t)} c^a_t(z^t) \quad \text{and} \quad h_t(\omega, s^t) = \frac{\tilde{\omega}_t(\omega, s^t)}{\xi^a_t(z^t)} h^a_t(z^t), \tag{3}$$

where $\xi^a_t(z^t)$ is a nondecreasing stochastic process. These risk-sharing rules clear the markets for nondurable consumption and housing services by construction. At the end of the period, we store the household's consumption share $\omega = \frac{\tilde{\omega}_t(\omega, s^t)}{\xi^a_t(z^t)}$ as its identifying label. The cutoff levels for the consumption weights increase as the housing collateral ratio decreases. When there is no housing collateral at all, the cutoff level for the consumption share equals the household's labor income share:

$$\frac{\underline{\omega}_t(y_t, z^t)}{\xi^a_t(z^t)} =\nearrow \widehat{\eta}(y_t, z_t) \text{ as } my \searrow,$$

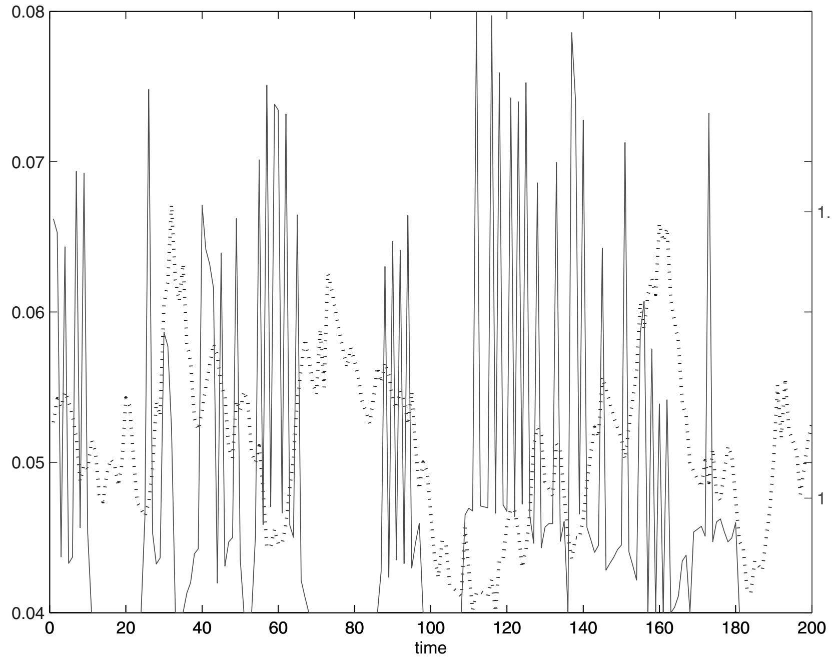

Figure 2. The consumption share cutoff of one household and the housing collateral ratio. The dotted line is the housing collateral ratio and the full line is the cutoff $\underline{\omega}_t$ that determines the optimal consumption share. The graph shows a 200-period simulation of the model at an annual frequency. The parameters are $\delta = 0.95$, $\gamma = 8$, and $\varepsilon = 0.15$. The calibration is discussed in detail in Lustig and Van Nieuwerburgh (2004a).

where $\hat{\eta}(y_t, z_t)$ is the labor income share relative to the total nondurable endowment. The lower the collateral ratio, the larger the increase in its consumption share when it switches to a state with a binding solvency constraint. Household consumption becomes increasingly sensitive to income shocks as the housing collateral ratio decreases, as illustrated in Figure 2.

In a stationary equilibrium, each household's consumption share is drifting downward as long as it does not switch to a state with a binding constraint. The rate at which these shares decrease is $\log(g_t) \equiv \Delta\log(\xi_t^a)$. This rate depends on the housing collateral ratio. When this ratio is low, the solvency constraints are tight, many households are highly constrained, and the remainder experience large consumption share drops. The risk-free rate is low, inducing households to decrease assets at a high rate. When this ratio is high enough, none of the households are constrained and interest rates are high. The growth rate of the aggregate weight process $\log(g_t)$ determines the consumption growth of the unconstrained households, and these households price payoffs in each state of the world. That is the focus of the next section.

III. The Market Price of Aggregate Risk

Using the risk-sharing rules in (3), the following expression for the IMRS of the unconstrained household emerges: $m_{t+1} = m^a_{t+1} g^{\gamma}_{t+1}$. This is the SDF in the sense of Hansen and Jagannathan (1991) that prices payoffs. The first section focuses on m^a and examines the data through the lens of a representative agent model. The new risk factor contributed by the nonseparability in the utility function is referred to as composition risk. We show that the market price of composition risk is likely to be small. The second section focuses on g_{t+1} and examines more carefully the liquidity risk factor contributed by the collateral channel.

A. Composition Effect

Without the collateral constraints, ours is a representative agent economy. If utility is nonseparable, the housing market introduces a novel risk factor: shocks to the nonhousing expenditure share. The representative agent's marginal utility growth is determined by aggregate consumption growth and nonhousing expenditure share growth:

$$m^a_{t+1} = \delta \left(\frac{c^a_{t+1}}{c^a_t} \right)^{-\gamma} \left(\frac{\alpha_{t+1}}{\alpha_t} \right)^{\frac{\varepsilon - \frac{1}{\gamma}}{\frac{1}{\gamma}(\varepsilon - 1)}},$$

where α_t is the nonhousing expenditure share, γ is the coefficient of relative risk-aversion, and ε is the intratemporal elasticity of substitution between housing services and nondurable consumption. Composition risk is small. Piazzesi et al. (2004) show that only values for the elasticity of intratemporal substitution ϵ that are slightly larger than one, and low values for the intertemporal elasticity deliver a volatile SDF. However, this comes at the cost of overstating the volatility of rental price growth by a factor ranging from 3 when ϵ is 1.05 to 15 when ε is 1.01. In addition, this composition effect generates little or no variation in the conditional Sharpe ratios on risky assets.

B. Liquidity Effect and the Collateral Channel

We now focus on the case of separable utility. The representative agent's IMRS m^a is the aggregate consumption growth rate raised to the power $-\gamma$ and the SDF reduced to

$$m_{t+1} = \delta \left(\frac{c^a_{t+1}}{c^a_t} \right)^{-\gamma} g^{\gamma}_{t+1}.$$

For the liquidity effect to increase the volatility of the SDF, the liquidity factor needs to be negatively correlated with aggregate consumption growth. There are two features that deliver a negative correlation between aggregate consumption growth and the aggregate liquidity shock: (1) An increase in the cross-sectional dispersion of labor income shocks, and (2) a decrease in the amount of collateral, both when aggregate consumption growth is low. The first one relies on the time series properties of labor income in the United States. The second one relies on the time series properties of rental prices in the housing market. Both of these channels amplify the effect of aggregate consumption growth shocks on the SDF.

B.1. Dispersion of Labor Income Shocks

Constantinides and Duffie (1996) build a negative correlation between the dispersion of consumption growth across households and aggregate stock returns in their model to generate large risk premia, drawing on earlier work by Mankiw (1986). The first channel in our model is a different version of this (see Lustig (2003)). It delivers a negative correlation between the standard deviation of the consumption growth distribution and stock returns, but this correlation is the equilibrium outcome of the interaction between the solvency constraints and the time series properties of the labor income process. A larger fraction of agents draws higher labor income shares $\hat{\eta}(y, z)$ when aggregate consumption growth is low, and as a result of the persistence of labor income shocks, higher cutoff levels $\tilde{\omega}_t(\omega, s^t)$. This increases the size of the aggregate weight shock in low aggregate consumption growth states. There is some empirical support for this channel. Storesletten, Telmer, and Yaron (2004) conclude that the volatility of idiosyncratic labor income shocks in the United States more than doubles during recessions.

B.2. Collateral Supply Shocks

If the rental price of housing services declines in response to negative aggregate consumption growth shocks, liquidity shocks will tend to be larger when aggregate consumption growth is low, because the destruction of collateral tightens the solvency constraints. Table I lists the results of regressing log rental prices on aggregate consumption growth and lagged rental prices. In the United States, rental prices do increase in response to a positive aggregate consumption growth shock in the post-war sample. Over the entire sample, the evidence is mixed.

B.3. Time-Varying Market Price of Risk

The housing collateral ratio governs the amount of risk-sharing that can be sustained, and variations in the ratio endogenously generate heteroskedasticity in the SDF. Low housing collateral ratios coincide with a high conditional volatility of the SDF, because a large fraction of households will be severely constrained in the case of an adverse aggregate consumption growth shock. This mechanism leaves a huge footprint in the cross-sectional standard deviation of consumption growth, plotted in Figure 3.

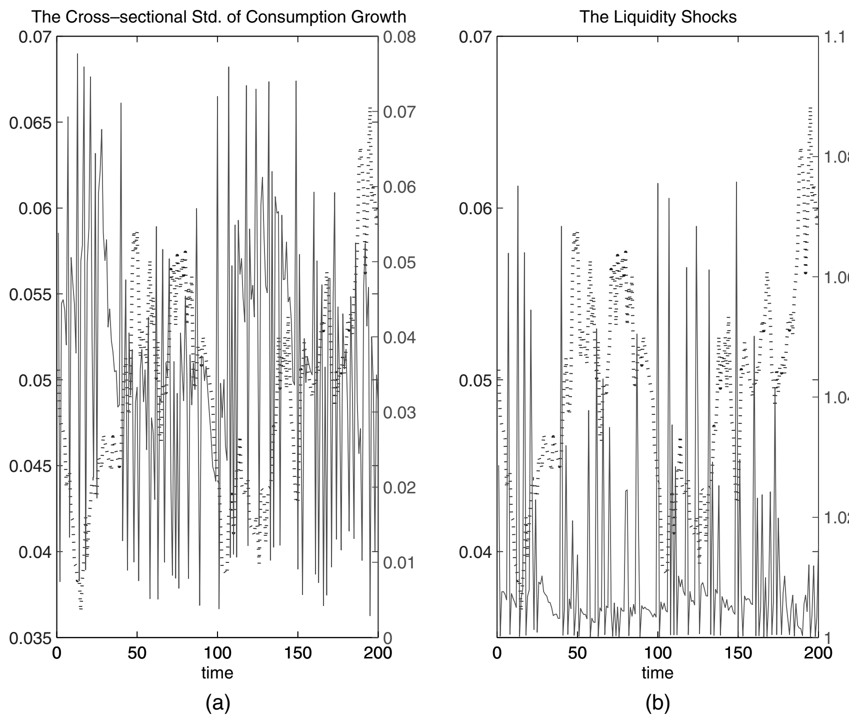

Figure 3. The standard deviation of consumption growth, liquidity shocks, and the housing collateral ratio. The dotted line is the housing collateral ratio *my*. The left panel plots *my* against the cross-sectional standard deviation of consumption growth (right axis). The right panel plots *my* against the aggregate liquidity shock g (right axis). The graph shows a 200-period simulation of the model at an annual frequency. The parameters are $\delta = 0.95$, $\gamma = 8$, and $\varepsilon = 0.15$. The calibration is discussed in detail in Lustig and Van Nieuwerburgh (2004a).

The next section concentrates on directly measuring the U.S. housing collateral ratio. Our measure reveals a surprising amount of historical variation that is consistent with the variation in U.S. stock returns.

IV. Time Series Evidence

In testing the model, we chose to measure the housing collateral ratio *my* directly, simply because forces outside our model probably influence housing prices. In the model, both the housing collateral ratio and the rental price or the expenditure share are valid state variables. Section IV.A concentrates on its measurement. Section IV.B studies the predictive power of our measure for stock returns.

A. Measuring the Housing Collateral Ratio

The measure *my* is defined as the ratio of collateralizable housing wealth to non-collateralizable human wealth. In the model, only the total supply of collateral matters, not the precise way in which it is allocated across households, because the available collateral is used when the demand for insurance is highest.[3] We can safely abstract from the distribution of collateral across households. Still, human wealth is unobserved. Following Lettau and Ludvigson (2001a), we assume that the nonstationary component of human wealth H is well approximated by the nonstationary component of labor income Y. In particular, $\log(H_t) = \log(Y_t) + \epsilon_t$, where ϵ_t is a stationary random process. The assumption is valid in a model in which the expected return on human capital is stationary (see Campbell (1996), Jagannathan and Wang (1996)).

A.1. Housing Collateral

We use three distinct measures of the housing collateral stock HV: the value of outstanding home mortgages (mo), the market value of residential real estate wealth (rw), and the net stock current cost value of owner-occupied and tenant-occupied residential fixed assets (fa). The first two time series are from the historical statistics for the United States (Bureau of the Census) for the period 1889–1945 and from the Flow of Funds data (Federal Board of Governors) for 1945–2001. The last series is from the Fixed Asset Tables (Bureau of Economic Analysis) for 1925–2001.

We use both the value of mortgages HV^{mo} and the total value of residential fixed assets HV^{rw} to allow for changes in the extent to which housing can be used as a collateral asset, and we use both HV^{rw}, which is a measure of the value of housing owned by households, and HV^{fa}, which is a measure of owner-occupied and tenant-occupied housing, to allow for changes in the homeownership rate over time. The Appendix provides detailed sources. Real per household variables are denoted by lowercase letters. The real per household housing collateral series hv^{mo}, hv^{rw}, and Hv^{fa} are constructed using the all items CPI from the BLS, p^a, and the total number of households, N, from the Bureau of the Census.

A.2. Income

Aggregate income is labor income plus net transfer income. Nominal data are from the Historical Statistics of the United States for 1926–1930 and from the National Income and Product Accounts for 1930–2001. Consumption and income are deflated by p^c and p^a, and divided by the number of households N.

A.3. Cointegration

Log, real, per household real estate wealth ($\log hv$), and labor income plus transfers ($\log y$) are nonstationary. According to an augmented Dickey–Fuller

[3] In reality, any given household may be constrained in terms of access to consumption insurance by the remaining equity in its own house, but that is not a feature of our model, as was pointed by a referee.

test, the null hypothesis of a unit root cannot be rejected at the 1% level. This is true for all the three measures of housing wealth ($hv = mo, rw, fa$).

If a linear combination of $\log hv$ and $\log y$, $\log(hv_t) + \varpi \log(y_t) + \chi$, is trend stationary, the components $\log hv$ and $\log y$ are said to be stochastically cointegrated with the cointegrating vector $[1, \varpi, \chi]$. We additionally impose the restriction that the cointegrating vector eliminates the deterministic trends, so that $\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi$ is stationary. A likelihood-ratio test (Johansen and Juselius (1990)) shows that the null hypothesis of no-cointegration relationship can be rejected, whereas the null hypothesis of one cointegration relationship cannot. This is evident for one cointegration relationship between housing collateral and labor income plus transfers. Table II reports the results of this test and of the vector error correction estimation of the cointegration coefficients:

$$\begin{bmatrix} \Delta \log(hv_t) \\ \Delta \log(y_t) \end{bmatrix} = \alpha[\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi] + \sum_{k=1}^{K} D_k \begin{bmatrix} \Delta \log(hv_{t-k}) \\ \Delta \log(y_{t-k}) \end{bmatrix} + \varepsilon_t. \tag{4}$$

The K error correction terms are included to eliminate the effect of regressor endogeneity on the distribution of the least-squares estimators of $[1, \varpi, \vartheta, \chi]$. The housing collateral ratio *my* is measured as the deviation from the cointegration relationship

$$my_t = \log(hv_t) + \hat{\varpi} \log(y_t) + \hat{\vartheta} t + \hat{\chi}.$$

The OLS estimators of the cointegration parameters are superconsistent: They converge to their true value at rate $1/T$ (rather than at $1/\sqrt{T}$). The superconsistency allows us to use the housing collateral ratio *my* as a regressor without needing an errors-in-variables standard error correction.

We also estimate the constant and trend in the cointegrating relationship while imposing the restriction $\varpi = -1$. This is the second block of each panel in Table II. For *mo* and *fa*, we find strong evidence for one cointegrating relationship. The coefficient on $\log y_t$ is precisely estimated (significant at the 1% level, not reported), varies little between subperiods, and the 95% confidence interval contains -1. The resulting time series are stationary. The null hypothesis of a unit root is rejected for *mymo* and *myfa*. For each subperiod, the correlation between the residual estimated assuming $\varpi = -1$ and the one with ϖ freely estimated is higher than 0.95. For *rw*, the evidence for a cointegrating relationship is weaker, except for the 1925–2002 period. Furthermore, the slope coefficient in the cointegration relationship varies considerably between subperiods and does not contain -1 in its 95% confidence interval. The correlation between the residual estimated assuming $\varpi = -1$, and the one with ϖ freely estimated is 0.81 for the entire sample, 0.88 for 1925 to 2002, and 0.89 for the post-war period.

For consistency, we impose $\varpi = -1$ on all the three of these series. The housing collateral ratios are labeled *mymo, myrw*, and *myfa*. For the common sample period 1925–2001, the correlation between *mymo* and *myrw* is 0.89, 0.76

Table II

Cointegration Analysis

The vector error-correction model of housing collateral measure hv and labor income plus transfers y_t is estimated for 1889–2002, 1925–2002, and 1945–2002 for the mortgages ($hv = mo$) in Panel A and real estate wealth ($hv = rw$) in Panel B, and for 1925–2001 and 1945–2001 for residential fixed assets ($hv = fa$) in Panel C. The second through fourth columns show cointegration coefficient estimates: ϖ for labor income y_t, ϑ for the time trend, and χ is a constant. These are the coefficients in a regression of log hv on a constant, a time trend t, and labor income y_t. Coefficient estimates for autoregressive terms (8 lags) are not reported. The fifth column shows the likelihood ratio statistic of the Johansen cointegration test (constant and a trend in the cointegration relationship). The last column shows the value of the ADF test statistic (8 lags) of the null hypothesis of a unit root in the resulting cointegration series. For both tests, significance at the 10% level is denoted by *, significance at the 5% level by **, and at the 1% level by ***. The second subpanel of each panel, labeled *Restricted*, imposes the restriction that $\varpi = 1$.

Sample Period	ϖ	ϑ	χ	*LHR*	*ADF*
Panel A: Housing Wealth Measure: Mortgages *mo*					
			Unrestricted		
1889–2002	−1.5164	−0.0066	1.8010	21.07*	−3.46**
1925–2002	−1.2064	−0.0164	2.3546	35.04***	−5.38***
1945–2002	−1.2987	−0.0176	2.6511	30.77***	−3.06**
			Restricted		
1889–2002	−1	−0.0102	1.6974		−3.08**
1925–2002	−1	−0.0148	2.0624		−4.16***
1945–2002	−1	−0.0233	2.8302		−2.89*
Panel B: Housing Wealth Measure: Residential Wealth *rw*					
			Unrestricted		
1889–2002	−1.8255	0.0084	−0.3659	15.16	−3.46**
1925–2002	−0.5480	−0.0120	0.1895	34.00***	−4.01***
1945–2002	−0.4108	−0.0147	0.3311	25.00*	−3.32**
			Restricted		
1889–2002	−1	0.0011	−0.4434		−2.29
1925–2002	−1	−0.0023	−0.1720		−3.42**
1945–2002	−1	−0.0083	0.3784		−3.51**
Panel C: Housing Wealth Measure: Fixed Assets *fa*					
			Unrestricted		
1925–2001	−1.0137	−0.0004	−0.2257	52.01***	−4.70***
1945–2001	−1.0055	−0.0011	−0.1624	28.45**	−3.41**
			Restricted		
1925–2001	−1	−0.0005	−0.2254		−4.65***
1945–2001	−1	−0.0026	−0.0365		−2.88*

between *mymo* and *myfa*, and 0.86 between *myrw* and *myfa*. Figure 4 displays *my* between 1889 and 2002. All the three series exhibit large persistent swings. They reach a maximum deviation from 1932 to 1933. Residential wealth and residential fixed assets are 30% and 34% above their respective joint trends

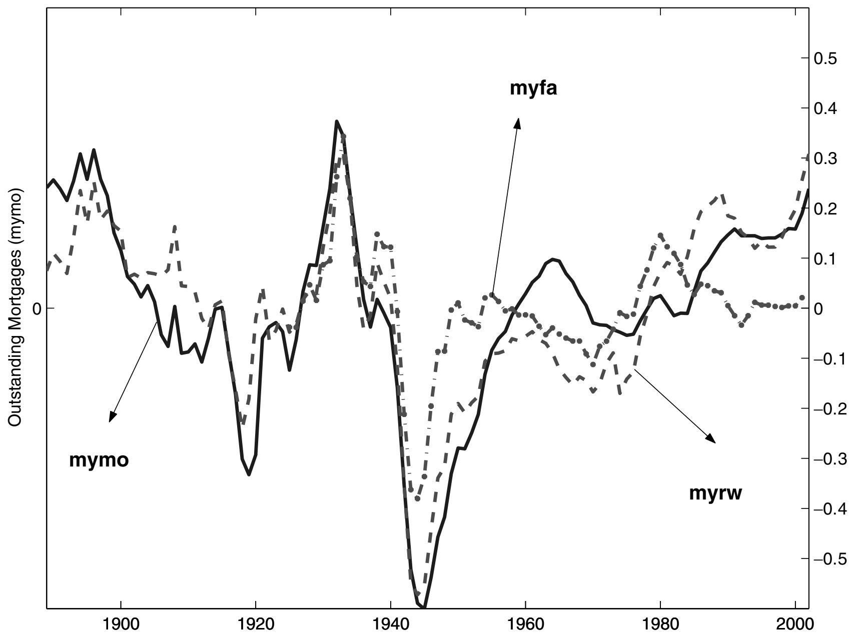

Figure 4. Estimated housing collateral ratio, 1889–2002. Deviation from the cointegration relationship between human wealth (y) and outstanding home mortgages (mo, full line), nonfarm residential wealth (rw, dashed line) and residential fixed asset wealth (fa, dash-dotted line). Data are for 1889–2002.

with human wealth. Mortgage debt is 53% above its trend. The series reach a minimum from 1944 to 1945, when *mymo* is −0.92, *myrw* is −0.57, and *myfa* is −0.38. The measures *mymo* and *myrw* have increased considerably since the year 2000: from 0.24 to 0.36 and from 0.19 to 0.30, respectively. Figure 5 shows the cointegration residuals *my* for that post-war period. Housing collateral wealth fluctuates within 30% below and above the long-run trend with human wealth.

In the model, *my* is the ratio of two asset prices and hence is always positive. Our empirical measure is the deviation from a cointegration relationship and therefore is occasionally negative. To put the model and the data on the same footing, we rescale the housing collateral ratio: $\widetilde{my}_t = my^{\max} - my_t$. The rescaled housing collateral measures collateral scarcity and is always positive. In the remainder of the paper, $my^{\max}$ is the sample maximum in 1925–2002 (which coincides with the sample maximum in the 1889–2002 sample). The only exception is when we explicitly ask the question: If post-war data only were available, what would the housing collateral model predict for the time series and cross-section of returns?

The next section provides some evidence that the housing collateral ratio predicts stock returns before we look at the cross-section of stock returns.

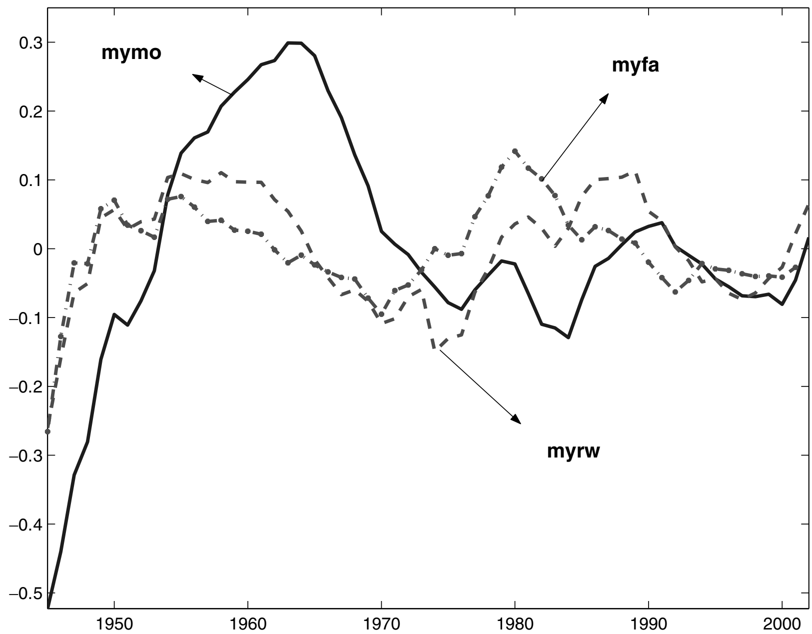

Figure 5. Estimated housing collateral ratio, 1945–2002. Deviation from the cointegration relationship between human wealth (y) and outstanding home mortgages (mo, full line), nonfarm residential wealth (rw, dashed line) and residential fixed asset wealth (fa, dash-dotted line). Data are for 1945–2002.

B. Time-Series Predictability

In the model, the market price for aggregate risk increases when housing collateral is scarce. We document this positive relationship for the U.S. market return at longer horizons. In addition, we show that an increase in collateral scarcity predicts higher spreads on future value minus growth returns, especially at horizons of less than 5 years: These stocks carry a higher risk premium in bad times with little housing collateral. The cross-sectional results shed light on why this is so.

B.1. Market Return and Fama–French Benchmark Portfolios

We use the cum-dividend return on Standard and Poor's composite stock price index, denoted by R_t^{vw}. The market return is expressed in excess of a risk-free rate, the return on 6-month prime commercial paper. The returns are available for the period 1889–2001 from Robert Shiller's web site. In addition, we create spreads on the basis of the six Fama–French benchmark portfolios, and we look at the predictability of these spreads. The Fama–French benchmark portfolios are rebalanced annually using two independent sorts, on size (market equity) and book-to-market (the ratio of book equity to market equity).

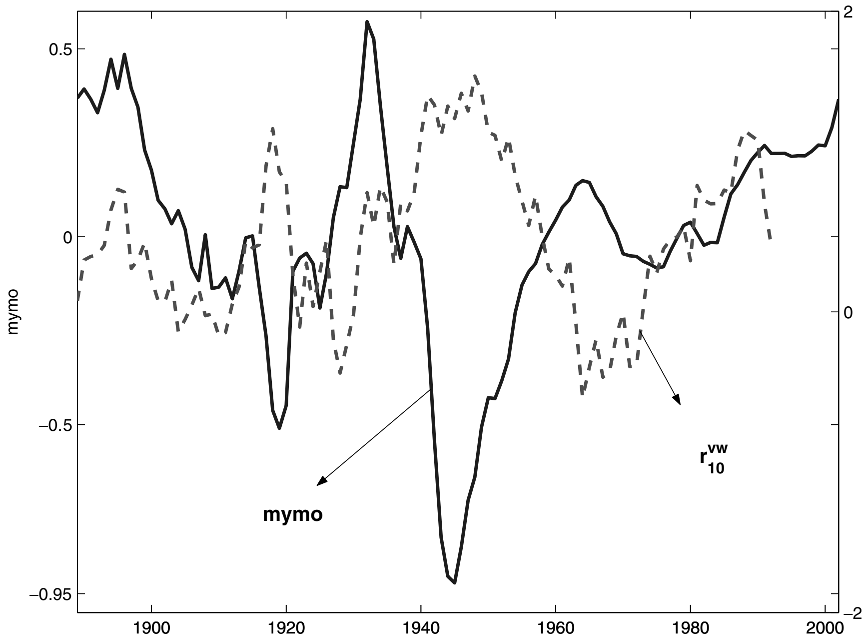

Figure 6. Ten-year excess market return and the housing collateral ratio. The housing collateral ratio is *mymo*, the measure based on outstanding mortgages. We use the cum-dividend return on Standard and Poor's composite stock price index, denoted R_t^{vw}. The market return is expressed in excess of a risk-free rate, the return on 6-month prime commercial paper.

The returns are available for the period 1926–2002 from Kenneth French's website.[4]

B.2. Long-Horizon Predictability

The K-year continuously compounded log return on the market is defined as $r^K_{t+K,vw} = (r^1_{t+1,vw} + \ldots + r^1_{t+K,vw})$, where $r^1_{t,vw}$ equals $\log(1 + R_t^{vw})$ and $r^{e,K}_{t+K,vw}$ is the log return on the market less the log return on the riskless asset over the same holding period K. Figure 6 shows the housing collateral ratio ($mymo$) and the annualized 10-year excess return. The series exhibit a negative correlation of -0.52.

In addition, we project long-horizon excess returns on the rescaled housing collateral ratio $\widetilde{my}$. Row 1 of Table III reports the least squares coefficient estimate on the housing collateral ratio in the regression of holding period returns on the collateral ratio:

$$r^{e,K}_{t+K,vw} = b_0 + b_{my}\widetilde{my}_t + e_{t+1}. \tag{5}$$

[4] http://www.econ.yale.edu/shiller/data.htm and http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table III

Long-Horizon Predictability Regressions

The results are for the regression $r^{e,K}_{t+K,vw} = b_0 + b_{my}\widetilde{my}_t + \epsilon_{t+K}$, where $r^{e,K}_{t+K,vw}$ are cumulative (log) excess returns on the S & P Composite Index over a K-year horizon. Panel A reports results for the full sample 1889–2002; the sample size decreases from 113 observations for $K = 1$–104 years for $K = 10$. Panel B reports the results for 1926–2002; the sample size decreases from 77 observations for $K = 1$–68 years for $K = 10$. The subpanels report the results for the different collateral measures. Subpanel 1 reports results for the mortgage-based collateral measure *mymo*, subpanel 2 for the measure based on residential wealth *myrw*, and subpanel 3 for the measure based on fixed assets *myfa*. The housing collateral ratio is rescaled so that it lies between 0 and 1 and measures collateral scarcity: $\widetilde{my}_t = (\frac{my^{\max} - my_t}{my^{\max} - my^{\min}})$, where $my^{\max}$ and $my^{\min}$ are the maximum and minimum observation in the respective samples. The first row of each subpanel reports least squares estimates for $b = b_{my}$. Newey–West HAC standard errors σ^{nw} are reported in the second row of each subpanel. The standard errors correct for serial correlation of order K, where K is the holding period. The third row reports the R^2 for this OLS regression. The fourth row of each subpanel reports the p-value of the null hypothesis of no predictability, obtained by bootstrap.

Horizon K	1	2	3	4	5	6	7	8	9	10
					Panel A: 1889–2002					
					Subpanel 1: Collateral Measure: Mortgages *mymo*					
b	0.08	0.19	0.29	0.32	0.48	0.74	1.12	1.65	2.15	2.54
σ^{nw}	0.04	0.07	0.11	0.15	0.19	0.24	0.29	0.33	0.36	0.40
R^2	0.01	0.02	0.02	0.02	0.03	0.06	0.09	0.15	0.22	0.26
$p - val$	0.15	0.12	0.13	0.16	0.11	0.06	0.03	0.01	0.00	0.00
					Subpanel 2: Collateral Measure: Residential Wealth *myrw*					
b	0.06	0.09	0.08	0.02	0.08	0.24	0.43	0.71	1.08	1.45
σ^{nw}	0.04	0.08	0.12	0.18	0.24	0.30	0.38	0.46	0.53	0.61
R^2	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.05	0.08
$p - val$	0.21	0.27	0.34	0.44	0.38	0.27	0.19	0.11	0.05	0.02
					Panel B: 1926–2002					
					Subpanel 1: Collateral Measure: Mortgages *mymo*					
b	0.14	0.28	0.39	0.47	0.71	1.05	1.50	2.00	2.51	3.04
σ^{nw}	0.04	0.07	0.11	0.15	0.19	0.24	0.29	0.34	0.39	0.44
R^2	0.02	0.03	0.04	0.03	0.06	0.12	0.18	0.24	0.30	0.33
$p - val$	0.11	0.11	0.11	0.13	0.09	0.06	0.03	0.02	0.01	0.00
					Subpanel 2: Collateral Measure: Residential Wealth *myrw*					
b	0.09	0.13	0.11	0.03	0.08	0.24	0.44	0.68	0.91	1.15
σ^{nw}	0.04	0.07	0.12	0.17	0.23	0.31	0.40	0.49	0.59	0.70
R^2	0.01	0.01	0.00	0.00	0.00	0.01	0.02	0.03	0.04	0.05
$p - val$	0.20	0.26	0.34	0.44	0.41	0.33	0.25	0.17	0.13	0.09
					Subpanel 3: Collateral Measure: Fixed Assets *myfa*					
b	0.06	0.04	−0.08	−0.17	−0.09	0.11	0.41	0.69	0.86	1.02
σ^{nw}	0.04	0.08	0.13	0.18	0.24	0.31	0.39	0.49	0.59	0.72
R^2	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.02	0.03
$p - val$	0.34	0.43	0.57	0.62	0.55	0.43	0.29	0.20	0.17	0.14

We consider two samples. The first panel reports the 1889–2002 results. The second panel reports the results for the 1926–2002 sample, the longest sample for which all three collateral measures are available. The standard errors on b_{my} are the HAC Newey–West standard errors with lag length K.[5] We also report the p-value of a two-sided test of the no-predictability null computed by bootstrapping. The procedure consists of estimating the slope coefficients in this predictability regression on simulated returns and collateral ratios under the null of no predictability. The p-value is computed as the fraction of slope estimates above our least squares estimate.[6]

All of the slope coefficients b_{my} are positive, except for the coefficients obtained using the fixed assets measure of housing collateral *myfa* at short horizons. A low housing collateral ratio (high $\widetilde{my}$) predicts a high future risk premium, as predicted by the model. In the case of *mymo*, the mortgage-based housing collateral measure, the R^2 of the least-squares regression increases to 26% for the entire sample (row 3, first panel) and to 33% for the shorter sample (row 3, second panel). Over the post-war sample, only the mortgage-based collateral measure *mymo* and the expenditure share α, also a valid state variable, predict returns on the market.[7]

According to the theory, our collateral measures should also predict the spreads on value minus growth portfolios. Table IV looks at the predictability of longer holding period spreads between value and growth portfolios. We run the same regression as before but now $r^{sp_{i,j}}_{t+K}$ is the spread in log K-holding-period returns for two extremum portfolios i and j formed on the basis of the six Fama–French benchmark portfolios. The coefficients have the right sign for all collateral measures: the scarcer housing collateral becomes, the larger the spread on value minus growth portfolios. This is consistent with our findings in the cross-sectional pricing exercise: value stocks are riskier in times characterized by a low housing collateral measure. As a result, an increase in $\widetilde{my}$ should predict a larger spread in the longer horizon returns on value stocks. At short horizons of less than 5 years, the null of no predictability is rejected at the 10% level for all three collateral measures, both for small and big firms. The results become less significant at longer horizons. This provides evidence that the collateral ratio predicts returns even at shorter horizons of less than 5 years.

[5] A VAR analysis shows that the innovations in *my* and unpredicted returns have correlation close to zero. Therefore, there is no persistent regressor bias (Stambaugh (1999)).

[6] We estimate an AR(2) for the housing collateral ratio. The least squares coefficients for the long-horizon predictability regression are computed off simulated cumulative returns and collateral ratios. Under the null of no predictability, the 1-year return is unpredictable and the K-period returns follow an MA(K) process with a mean equal to the K-period average return. We sample i.i.d. errors for the collateral ratio and the 1-year return processes and recursively build up the cumulative return and collateral ratio series. We estimate 20,000 long-horizon regressions and count how many of the bootstrap slope estimates are above the least squares slope estimate. This is the p-value for the null of no predictability.

[7] Results are not reported, but are available upon request.

Table IV

Long-Horizon Predictability Regressions

The results are for the regression $r^{sp_{i,j}}_{t+K} = b_0 + b_{my}\widetilde{my}_t + \epsilon_{t+K}$, where $r^{sp_{i,j},K}_{t+K}$ is the spread in log K-period holding returns for two extremum portfolios i and j formed on the basis of the six Fama–French benchmark portfolios (two independent sorts, two by size and three by book-to-market). Panel A reports results for the mortgage-based collateral measure *mymo*, Panel B for the measure based on residential wealth *myrw* and Panel C for the measure based on fixed assets *myfa*. The housing collateral ratio is rescaled so that it lies between 0 and 1 and measures collateral scarcity. The first row of each panel reports least-squares estimates for $b = b_{my}$. Newey–West HAC standard errors σ^{nw} are reported in the second row of each panel. The standard errors correct for serial correlation of order K, where K is the holding period. The third row reports the R^2 for this OLS regression. The fourth row of each panel reports the p-value of the null hypothesis of no predictability, obtained by bootstrap. The estimation results are for 1927–2002.

Horizon K	1	2	3	4	5	6	7	8	9	10
Panel A: Collateral Measure: Mortgages *mymo*										
Spread: Small Value Minus Small Growth										
b	0.32	0.56	0.75	0.93	1.04	1.10	1.07	0.99	0.84	0.69
σ^{nw}	0.03	0.06	0.09	0.13	0.16	0.19	0.22	0.24	0.26	0.28
R^2	0.12	0.14	0.14	0.18	0.16	0.14	0.12	0.09	0.06	0.04
$p-val$	0.00	0.01	0.02	0.02	0.03	0.04	0.06	0.08	0.14	0.19
Spread: Big Value Minus Big Growth										
b	0.21	0.42	0.61	0.73	0.79	0.86	0.93	0.90	0.77	0.57
σ^{nw}	0.03	0.06	0.10	0.14	0.17	0.21	0.25	0.29	0.33	0.37
R^2	0.05	0.09	0.12	0.11	0.10	0.10	0.10	0.08	0.05	0.02
$p-val$	0.03	0.02	0.03	0.04	0.06	0.06	0.08	0.10	0.15	0.22
Panel B: Collateral Measure: Residential Wealth *myrw*										
Spread: Small Value Minus Small Growth										
b	0.28	0.46	0.60	0.73	0.79	0.82	0.78	0.70	0.54	0.41
σ^{nw}	0.03	0.06	0.09	0.12	0.15	0.17	0.19	0.22	0.24	0.25
R^2	0.10	0.10	0.10	0.12	0.10	0.08	0.07	0.05	0.03	0.01
$p-val$	0.01	0.01	0.02	0.03	0.04	0.06	0.09	0.13	0.20	0.26
Spread: Big Value Minus Big Growth										
b	0.23	0.43	0.60	0.70	0.77	0.85	0.98	1.02	0.98	0.88
σ^{nw}	0.03	0.06	0.10	0.13	0.15	0.18	0.20	0.23	0.26	0.29
R^2	0.06	0.10	0.13	0.11	0.10	0.10	0.12	0.11	0.09	0.06
$p-val$	0.02	0.02	0.03	0.04	0.06	0.07	0.06	0.08	0.10	0.13
Panel C: Collateral Measure: Fixed Assets *myfa*										
Spread: Small Value Minus Small Growth										
b	0.37	0.54	0.59	0.66	0.74	0.76	0.68	0.52	0.34	0.17
σ^{nw}	0.04	0.07	0.10	0.14	0.17	0.20	0.23	0.26	0.28	0.30
R^2	0.11	0.09	0.06	0.06	0.06	0.05	0.03	0.02	0.01	0.00
$p-val$	0.00	0.02	0.05	0.07	0.08	0.10	0.14	0.22	0.30	0.38
Spread: Big Value Minus Big Growth										
b	0.20	0.43	0.52	0.58	0.65	0.77	0.90	0.93	0.87	0.77
σ^{nw}	0.04	0.07	0.11	0.15	0.19	0.22	0.25	0.29	0.33	0.36
R^2	0.03	0.07	0.06	0.05	0.05	0.06	0.06	0.06	0.05	0.03
$p-val$	0.08	0.06	0.09	0.12	0.13	0.12	0.11	0.12	0.15	0.18

V. Cross-Sectional Evidence: Nonlinear Factor Model

We use returns on stock portfolios sorted by size and value characteristics, bond returns, and the return on a risk-free asset to test our model. Size and book-to-market value are asset characteristics that challenge the standard CCAPM. Historically, small firm stocks and high book-to-market firm stocks have had much higher returns. In the post-war period, the size premium has largely disappeared, but the value premium is still prominent. The value premium—the average return difference between the lowest and the highest book-to-market decile—is 5.7% over the entire sample.

A total of 25 portfolios of NYSE, AMEX, and nasdaq stocks are grouped each year into five size bins and five book-to-market bins. Size is market capitalization at the end of June. Book-to-market is book equity at the end of the prior fiscal year, divided by the market value of equity in December of the prior year. Portfolio returns are value-weighted. The stock returns are available for the period 1927–2002 from Kenneth French's web site and are described in more detail in Fama and French (1992). We also include the market return R^{vw}, the value-weighted return on all NYSE, AMEX, and nasdaq stocks. The bond return is the annual holding period return on a 10-year government bond (from CRSP). All returns are expressed in excess of an annual return on a 1-month Treasury bill rate (from Ibbotson Associates). The first column of Table V shows mean and standard deviation for the 26 excess returns. For comparison, the table also lists the mean and standard deviation of equally weighted returns and the book-to-market ratio of each portfolio.

A. Measuring the Liquidity Factor

In the model, the aggregate weight shock depends on the entire history of aggregate shocks z^∞ and my_0. To solve the model numerically, we rely on an approximation of $g(z^\infty, my_0)$, the growth rate of the aggregate weight process using a truncated history of aggregate shocks and the current my_t.[8]

To bring the model to the data, we take a similar approach and use a flexible, nonlinear function of the relevant state variables to parameterize the investor's forecast of aggregate weight growth:

$$\log\left(g_t\left(z_t^\infty, my_0\right)\right) \simeq \phi\left(F_t^a, F_{t-1}^a, \ldots, F_{t-k}^a; my_t\right),$$

where F_t^a denotes the vector of aggregate factors $F_{t+1}^a = (\Delta\log(c_{t+1}^a), \Delta\log(\alpha_{t+1}))'$, consisting of aggregate consumption growth and expenditure

[8] This is discussed in Lustig and Van Nieuwerburgh (2004a). These approximations work well because the supply of collateral is distributed efficiently across households. The percentage forecast error has a natural interpretation: It equals the percentage deviation between aggregate consumption and the aggregate endowment.

Table V

Annual Portfolio Returns, 1927–2002

Time-series mean and standard deviation of gross portfolio returns. All returns are in excess of a 1-month T-bill return. The first two columns are value-weighted portfolios, the next two are equally weighted portfolios, and the last column denotes the value-weighted portfolio average book-to-market ratio. The first two symbols of the asset identifier denote the size quintile (in increasing order), and the next two symbols denote the book-to-market quintile (again in increasing order). All data are from Kenneth–French for 1927–2002.

Asset	Mean	*SD*	Mean	*SD*	*B/M*
R^{vw}	7.9	20.9			
S1B1	3.8	38.0	7.3	40.4	0.35
S1B2	9.7	37.4	15.6	45.2	0.70
S1B3	13.8	35.9	17.6	40.1	1.03
S1B4	17.8	44.6	22.1	53.4	1.55
S1B5	18.2	37.6	26.2	48.6	5.52
S2B1	6.9	32.3	7.1	35.5	0.38
S2B2	11.8	30.3	12.6	32.6	0.70
S2B3	13.7	30.5	15.0	33.6	1.03
S2B4	14.7	32.8	15.3	35.1	1.52
S2B5	15.1	33.0	16.5	36.2	3.76
S3B1	8.5	30.5	8.0	30.2	0.38
S3B2	11.4	28.0	11.7	29.8	0.69
S3B3	12.3	27.2	12.8	28.2	1.02
S3B4	13.1	27.8	13.7	28.1	1.51
S3B5	13.9	32.6	14.9	32.8	3.40
S4B1	8.4	24.0	8.4	24.5	0.37
S4B2	9.2	25.6	9.4	26.2	0.69
S4B3	11.1	25.9	11.4	26.9	1.01
S4B4	12.1	27.0	12.4	27.8	1.49
S4B5	13.6	34.5	14.3	36.6	3.35
S5B1	7.6	21.6	6.9	21.1	0.33
S5B2	7.2	19.5	8.4	20.3	0.68
S5B3	8.8	22.1	9.5	23.7	1.00
S5B4	9.5	25.4	10.6	27.3	1.50
S5B5	11.0	33.7	11.5	34.4	1.59

share growth. We use GMM to identify the function ϕ, in addition to the structural parameters from the moment conditions:

$$E_t\left[m^a_{t+1}\exp\left(\gamma * \phi\left(F^a_{t+1}, F^a_t, \ldots, F^a_{t-k}; my_{t+1}\right)\right)R^j_{t+1}\right] = 1,$$

where $R^j_{t+1}, j = 1, \ldots, n$ are the returns on the test assets. In addition, the theory imposes two kinds of inequality constraints: (1) $\phi(F^a_{t+1}, F^a_t, \ldots, F^a_{t-k}; my_{t+1}) \geq 0$ and (2) $E_t[m^a_{t+1}R^j_{t+1}] \leq 1$ that further restrict the set of feasible parameters. (1) follows from the fact that $\xi^a_t(z^t)$ is a nondecreasing process and this immediately implies (2). The second set of inequality constraints will prove useful in identifying the structural parameter γ. Luttmer (1991) exploits such restrictions to derive Hansen-Jagannathan bounds in an environment with solvency constraints.

B. The Nonlinear Factor Model and GMM

We use the Tchebychev orthogonal polynomials to approximate the nonlinear ϕ-function. The advantage of this is that the different terms of the polynomial have the usual interpretation as linear pricing factors (Chapman (1997)).

B.1. Moment Restrictions

The first moments are the average pricing errors for the test asset returns and the risk-free rate

$$E\left[m_{t+1}R^{j}_{t+1} - 1\right] = 0.$$

The theory tells us that the aggregate weight shock is exactly equal to one when the constraints do not bind and strictly greater than one in all other periods. First, we impose parametric restrictions on the polynomial, such that $\phi(F^a_t, F^a_{t-1}, \ldots, F^a_{t-k}; my_t) = 0$ when my equals $my^{\max}$. In particular, we restrict ourselves to functions of the form $\phi(\cdot) = (my^{\max} - my_t) * \tilde{\phi}(F^a_t, F^a_{t-1}, \cdots, F^a_{t-k})$.[9]

Second, we impose the inequality restrictions $E[m^a_{t+1}R^i_{t+1} - 1] \leq 0$ on the representative agent's Euler equations by adding the Kuhn–Tucker moment conditions to the standard moment conditions:

$$\lambda(\theta)E\left[m^a_{t+1}R^{j}_{t+1} - 1\right] = 0.$$

We adopt the penalty function approach by parameterizing the Lagrangian multiplier λ as $exp(cE[m^a{}_{t+1}R^j{}_{t+1} - 1])$ for a positive penalty parameter c. To solve saddle point problems numerically, the shadow price of a binding constraint is usually approximated by the product of the penalty parameter and the constraint violation (see Judd (1998), theorem 4.7.1.). The algorithm prescribes increasing the penalty parameter until convergence is achieved. The parameter γ is the exponent on both components of the SDF. Without these inequality constraints, γ is not separately identified.

In a first pass, we estimate the model using four test assets. In a second stage, we add size and value portfolios to the set of test assets.

B.2. Results with Four Test Assets

First, we estimate our model using only four test assets: the risk-free rate, the value-weighted market return, the 10-year bond return, and the return on a portfolio that goes long in value and short in growth (the Fama–French benchmark portfolio R^{HML}). We use annual real, gross holding period returns from 1926 until 2002 (77 observations). Results for quarterly data for 1952.1–2002.4 are discussed in Section VI. There are four standard moment conditions

[9] We cannot allow the aggregate weight shock to be a function of aggregate consumption growth in isolation, because that would preclude identification of γ.

and four Kuhn–Tucker moment conditions, adding up to a total of eight moment conditions. We use the identity matrix as a weighting matrix.

The Tchebychev polynomial $\tilde{\phi}(\cdot)$ is restricted to be first-order: It contains a constant and the aggregate pricing factor, $\phi(\cdot) = \theta^1(my^{\max} - my_t) + \theta^2(my^{\max} - my_t)F_t^a$. This restriction is tested in the next subsection. We estimate the model for separable preferences: F^a is aggregate consumption growth. The coefficient estimates in Table VI lend support to the collateral channel. The positive estimate for $\hat{\theta}_1$ implies that periods with less collateral coincide with a high value for the aggregate weight shock and SDF. The negative estimate for $\hat{\theta}_2$ implies that when aggregate consumption growth is below average (the rescaled consumption growth is negative), the aggregate weight shock is large. This effect increases as collateral becomes scarcer (higher $my^{\max} - my_t$). This is the collateral effect predicted by the model.

Table VI also shows how the coefficient estimates converge as the penalty parameter c is increased (reading from left to right in the table). Interestingly, for high penalty parameters, the inequality restrictions are satisfied and we obtain low risk-aversion estimates. The representative agent's Euler inequalities rule out high γ estimates. Finally, when the penalty parameter is increased, θ_2 is estimated more precisely. The average pricing errors are small (the J-stat decreases) and the null hypothesis that all pricing errors are zero cannot be rejected (p-value on last line).

B.3. Results with Seven Test Assets

Next, we use a more extensive set of seven test assets: a 3-month T-bill, a 10-year government bond, the value-weighted aggregate stock market, and the four extreme size and value portfolios S1B1, S1B5, S5B1, and S5B5. With the corresponding inequality conditions, this adds up to a total of 14 moment conditions.

We estimate the collateral model under separable and nonseparable preferences. To investigate the effect of nonlinearities, we vary the order of the Tchebychev polynomial from 1 to 3. The left panel of Table VII reports the estimates for the collateral model with separable preferences. In the first-order specification of the aggregate weight shock (column 1), all coefficients are estimated precisely and the point estimates are not very different from the ones we reported in the case of four test assets. This model dominates the ones with higher-order polynomial terms (columns 2 and 3) on the basis of a likelihood ratio test, a Wald test, and a Lagrange multiplier test. The null hypothesis that all higher-order polynomial coefficients are zero cannot be rejected.[10] The right panel of Table VII shows the results for nonseparable preferences. The growth rate of the nondurable expenditure share $\Delta \log \alpha_{t+1}$ is an additional aggregate

[10] When the unrestricted model is the second-order and the restricted model is the first-order polynomial model, the p-values are: polynomial order 1 versus 2: LHR 1.00, Wald 0.93, LM 0.91, and polynomial order 1 versus 3: LHR 1.00, Wald 0.70, LM 0.76. We conclude that the first-order polynomial specification for the aggregate weight shock is the best fitting one.

Table VI

Cross-Sectional Results Collateral Model: Four Assets

Shown here are GMM parameter estimates for the collateral model under separable preferences for 1926–2002. The estimates minimize the pricing errors on the market return, the T-bill return, a 10-year bond, and a portfolio that goes long in value and short in growth (hml). GMM estimates also impose four inequality conditions. The Tchebychev polynomial is first order: $\tilde{\phi}(\cdot) = \theta_1 \widetilde{my} + \theta_2 \widetilde{my}\, \Delta \log c_{t+1}$. The coefficient of relative risk-aversion is γ. The time discount factor δ is fixed at 0.95 in the estimation. The identity weighting matrix is used in the first stage. The Newey–West matrix with lag length 3 is used to compute the standard errors. The penalty parameter c is varied between 10 and 500. The last line reports the J-statistic and the p-value of the null that all pricing errors are zero.

	Penalty Parameter c										
Parameters	10	15	20	30	40	60	80	100	150	300	500
$\hat{\gamma}$	23.48	13.23	8.83	5.62	4.13	2.84	2.23	1.84	1.24	1.24	1.24
SE	(16.32)	(9.34)	(5.79)	(4.40)	(3.46)	(2.67)	(2.52)	(5.52)	(0.78)	(0.75)	(0.72)
$\hat{\theta}_1$	0.13	0.18	0.24	0.34	0.44	0.62	0.77	0.92	1.34	1.34	1.34
SE	(0.05)	(0.09)	(0.14)	(0.25)	(0.36)	(0.59)	(0.880	(2.66)	(0.74)	(0.71)	(0.68)
$\hat{\theta}_2$	−0.34	−0.60	−0.90	−1.42	−1.93	−2.80	−3.56	−4.30	−6.41	−6.40	−6.39
SE	(0.39)	(0.74)	(1.09)	(1.89)	(2.68)	(4.20)	(6.00)	(15.16)	(4.14)	(4.05)	(3.96)
J	31.56	9.68	3.83	1.61	1.16	0.94	0.89	0.89	0.79	0.40	0.27
p	0.000	0.0847	0.5748	0.8999	0.9487	0.9671	0.9729	0.9708	0.9779	0.9953	0.9982

Table VII

Cross-Sectional Results Collateral Model: Seven Assets

Shown here are GMM parameter estimates for the collateral model for 1926–2002. The estimates minimize the pricing errors on the market return, the T-bill return, a 10-year bond, and the four extremum size and value portfolios. The estimation also imposes seven inequality conditions. The left panel imposes separability by fixing ϵ at 0, while the left panel fixes ϵ at 0.85. In columns 1 and 4, $\tilde{\phi}(\cdot)$ is a first-order, in columns 2 and 5, it is a second-order, and in columns 3 and 6, it is a third-order Tchebychev polynomial. The penalty parameter c is 100 (columns 1), 12 (column 2), 11 (column 3), 300 (column 4), 2 (columns 5 and 6). The time discount factor δ is fixed at 0.95 in the estimation. The identity weighting matrix is used in the first stage, and the Newey–West matrix with lag length 3 is used to compute standard errors.

	Separability			Nonseparability		
	Polynomial Order			Polynomial Order		
Parameters	1	2	3	1	2	3
$\hat{\gamma}$	4.78	4.96	4.67	2.07	1.35	1.35
SE	[1.72]	[1.68]	[1.58]	[0.32]	[0.19]	[0.20]
$\hat{\theta}_1$	0.37	0.63	−0.31	−0.15	7.31	0.44
SE	[0.31]	[3.15]	[2.77]	[2.43]	[3.96]	[6.78]
$\hat{\theta}_2$	−2.00	−1.93	−6.61	−5.16	−8.14	6.44
SE	[0.92]	[0.85]	[13.46]	[1.78]	[4.87]	[7.40]
$\hat{\theta}_3$		0.33	−0.77	−3.67	3.63	0.23
SE		[4.05]	[3.16]	[8.31]	[28.37]	[10.14]
$\hat{\theta}_4$			−1.69		−0.92	−6.59
SE			[5.26]		[19.73]	[5.83]
$\hat{\theta}_5$					7.56	6.53
SE					[14.21]	[3.13]
$\hat{\theta}_6$						7.53
SE						[7.95]
$\hat{\theta}_7$						−0.70
SE						[13.37]
J	33.68	110.37	56.31	14.18	60.36	54.15
p	0.0000	0.0000	0.0000	0.1671	0.0000	0.0000

factor in the aggregate weight shock with loading θ^3. The evidence for nonseparability is weak: θ^3 is measured imprecisely in columns 4–6. The first-order specification (column 4) fits the data best: all parameter estimates have the right sign and θ^2 is significant. Furthermore, the null that all pricing errors are zero cannot be rejected for this specification. The p-value is 0.17 in the last row.

B.4. Comparing Pricing Errors

It is informative to examine in more detail the pricing errors on the seven test assets implied by the nonlinear collateral model. Table VIII contrasts the collateral model in columns 3–5 with the standard CCAPM in column 1 and with the representative agent model with nonseparable

Table VIII

Average Pricing Errors, Nonlinear Collateral Model

Shown here are average pricing errors implied by the GMM estimations with seven assets (see Table VII). GMM t-stats are in parentheses. The Coll-CAPM pricing errors pertain to the model with first-order polynomials only. Coll-CAPM 1 is the model with separability; Coll-CAPM 2 is under nonseparability, while Coll-CAPM 3 adds a lagged consumption growth term as a factor in $\tilde{\phi}(\cdot)$. The coefficient estimates for 2 and 3 are not reported. The errors for the inequality restrictions are not reported. The penalty parameter c is 100, 300, and 12 in columns 3, 4, and 5, respectively. The first two symbols of the asset identifier denote the size quintile (in increasing order), while the next two symbols denote the book-to-market quintile (again in increasing order).

Test Asset	CCAPM	H-CAPM	Coll-CAPM 1	Coll-CAPM 2	Coll-CAPM 3
R^f	−0.069	−0.057	0.016	0.006	0.015
$(t-stat)$	(−3.11)	(−3.78)	(0.68)	(0.30)	(3.38)
R^{vw}	−0.016	−0.000	−0.010	−0.002	−0.010
$(t-stat)$	(−0.85)	(−0.04)	(−0.73)	(−0.25)	(−1.38)
R^{S1B1}	−0.060	−0.042	−0.017	0.009	−0.011
$(t-stat)$	(−1.53)	(−1.68)	(−0.19)	(0.23)	(−0.78)
R^{S1B5}	0.070	0.078	0.042	0.024	0.043
$(t-stat)$	(1.83)	(−3.44)	(1.71)	(0.39)	(1.42)
R^{S5B1}	−0.021	−0.001	−0.021	−0.007	−0.021
$(t-stat)$	(−1.00)	(−0.09)	(−1.58)	(−0.25)	(−1.56)
R^{S5B5}	0.071	0.070	0.001	−0.003	−0.003
$(t-stat)$	(2.20)	(3.76)	(0.01)	(−0.14)	(−0.18)
R^{bond}	−0.059	−0.045	−0.013	−0.029	−0.017
$(t-stat)$	(−2.72)	(−2.73)	(−0.25)	(−0.88)	(−2.98)

preferences (HCAPM) in column 2. The pricing errors for the collateral CAPM are much smaller for all test assets. Only the pricing error on the small value portfolio (S1B5) is still significant for the collateral model with separability (Coll-CAPM 1). Under the collateral model with nonseparability (Coll-CAPM 2), this error is reduced to 2.5% and is no longer statistically different from zero. Coll-CAPM 3 in the last column allows for limited history dependence in the aggregate weight shock. It contains lagged consumption growth in $\tilde{\phi}(\cdot)$. This specification does not produce significant improvements over the corresponding model without history dependence in column Coll-CAPM 1.

By contrast, the CCAPM and HCAPM do a poor job at pricing the risk-free rate and the long-term bond, while massively overpricing growth portfolios and underpricing value portfolios. In addition, the estimated relative risk-aversion for the CCAPM is high at 10.7. The null hypothesis that all pricing errors are zero for the CCAPM and HCAPM is strongly rejected.

Since the higher order terms in the polynomials do not significantly improve the fit of the collateral model, we henceforth impose linearity on $\tilde{\phi}$. This allows us to connect our work with the linear factor model literature, and it also allows us to increase the number of test assets in the estimation stage.

VI. Cross-Sectional Evidence: The Linear Factor Model

First, we assume that $\tilde{\phi}$ is linear in $(F^a_t, F^a_{t-1}, \ldots, F^a_{t-k})$, and, second, we assume that the housing collateral ratio follows an autoregressive process $my_{t+1}(my_t, F^a_{t+1})$ that interacts linearly with the aggregate factors. The innovations to the aggregate factors are the structural innovations in our model.

We propose a linear expression for $\phi(\cdot)$:

$$\phi\big(F^a_t, F^a_{t-1}, \ldots, F^a_{t-k}; my_t\big) = \big(my^{\max} - my_t\big)B(L)\big(F^a_t - \Upsilon\big), \tag{6}$$

where $B(L)$ is a polynomial of order k in the lag operator and Υ is the unconditional mean of the aggregate factors F^a_t. For now, we set $k = 0$, but we test for additional history dependence in the estimation exercise by including up to four lags of the factors, F^a_{t-k} for $k = 1, 2, 3, 4$, in $B(L)$. The coefficient on factor i in lag j is denoted by B_{ij}.

A. The Linear Factor Model and Fama–MacBeth

The factor model for the weight shocks and the autoregressive process for my provide a complete description of the pricing model. By combining $\phi_{t+1}(my_{t+1}, F^a_{t+1})$ and $my_{t+1}(my_t, F^a_{t+1})$, the stochastic discount factor can be stated in terms of the aggregate factors F^a_{t+1} and the state variable my_t. A first-order Taylor approximation of this expression provides our linear factor model

$$m_{t+1} \approx \tilde{\delta}\big(const - \theta^a F^a_{t+1} - \theta^c F^c_{t+1} + \gamma\varepsilon_{t+1}\big), \tag{7}$$

where the constraint factors ${F^c}_{t+1}$ are:[11]

$$F^c_{t+1} = \big(my^{\max} - my_t\big)\big(1, F^a_{t+1}\big).$$

When the utility kernel is separable, the equity risk premium is determined by the conditional covariance of its returns with consumption growth and a state-varying market price of risk

$$E_t\big[R^{e,j}_{t+1}\big] \approx \tilde{\delta}R^f_t\,\gamma\big[(1+\Upsilon_c)^{-1} - B_{10}\gamma(1-\rho)my^{\max} - B_{10}\rho\big(my^{\max} - my_t\big)\big] \\ Cov_t\big(\Delta \log c^a_{t+1}, R^{e,j}_{t+1}\big),$$

where R^f_t is the risk-free rate at time t. If B_{10} is zero, the expression collapses to the standard CCAPM of Lucas (1978) and Breeden (1979). The market price of consumption risk is determined by the coefficient of relative risk-aversion γ.

[11] The associated factor loadings are:

$$\theta^a = \left(\gamma - \gamma(1-\rho)B_{10}my^{\max}, \frac{-\varepsilon + \frac{1}{\gamma}}{\frac{1}{\gamma}(\varepsilon - 1)} - \gamma(1-\rho)B_{20}my^{\max}\right)$$

$$\theta^c = \big(\gamma\rho(B_{10}\Upsilon_c + B_{20}\Upsilon_\rho), -\gamma\rho B_{10}, -\gamma\rho B_{20}\big).$$

In contrast, our theory predicts an increase in the size of the aggregate weight shock when aggregate consumption growth is low, driven by an increase in idiosyncratic risk. Consumption growth has an effect on the liquidity shock: $B_{10} < 0$. When housing collateral is scarce (when $my^{\max} - my_t$ is large), the market price of consumption risk is high.

Nonseparability introduces a second covariance in the risk premium equation: the covariance with expenditure share changes. If B_{20} is zero, the market price of composition risk is constant. In contrast, if $B_{20} < 0$, the market price of composition risk is high when housing collateral is scarce (when $my^{\max} - my_t$ is large).

A.1. Unconditional β-Representation

The discount factor consists of a representative agent and a constraint component:

$$m_{t+1} = -\theta F_{t+1}, \tag{8}$$

where θ is a vector of constants, $\theta = (const, \tilde{\theta})$ and $\tilde{\theta} = (\theta^a, \theta^c)$ and $F_{t+1} = (1, \tilde{F}_{t+1})$. The vector $\tilde{F}_{t+1} = (F^{a\prime}_{t+1}, F^{c\prime}_{t+1})'$ is a vector of representative agent and constraint risk-pricing factors. The conditioning information is embedded in the scaled constraint factors, while θ itself is constant.

These constraint factors contain the original aggregate factors scaled by the housing collateral ratio my_t. The conditioning variable my_t summarizes the investor's information set. The model can be tested using the unconditional orthogonality conditions of the discount factor and excess asset returns *j*:

$$E\left[m_{t,t+1} R^{e,j}_{t+1}\right] = 0. \tag{9}$$

Using the definition of the risk-free rate and the covariance, the unconditional factor model in (8) implies an unconditional β-representation:

$$E\left[R^{e,j}_{t+1}\right] = \tilde{\delta}\bar{R}^f\tilde{\theta} \quad \mathrm{Cov}\left(\tilde{F}_{t+1}, R^{e,j}_{t+1}\right) = \tilde{\lambda}\tilde{\beta}^j,$$

where $\bar{R}^f$ is the average risk-free rate, $\tilde{\beta}^j$ is asset *j*'s risk exposure, and $\tilde{\lambda}$ is a transformation of the parameter vector $\tilde{\theta}$:[12]

$$\begin{aligned} \tilde{\beta}^j &= \mathrm{Cov}(\tilde{F}, \tilde{F}')^{-1}\mathrm{Cov}(\tilde{F}, R^{e,j}) \\ \tilde{\lambda} &= \tilde{\delta}\bar{R}^f\tilde{\theta}\mathrm{Cov}(\tilde{F}, \tilde{F}'). \end{aligned}$$

This unconditional β-representation is the equation we estimate using the Fama–MacBeth procedure.

[12] Lettau and Ludvigson (2001b) point out that $\tilde{\lambda}$ does not have a straightforward interpretation as the vector of market prices of risk. The market prices of risk λ depend on the conditional covariance matrix of factors that is unobserved.

A.2. Computational Procedure

We apply the two-stage Fama–MacBeth procedure and estimate the unconditional β-representation $E[R^{e,j}_{t+1}] = \tilde{\lambda}\tilde{\beta}^j$. In a first time-series stage, for each asset j separately, excess returns are regressed on factors to uncover the $\tilde{\beta}$'s. In a second cross-sectional stage, average excess returns are regressed on the $\tilde{\beta}$'s from the first stage to obtain the market prices of risk $\tilde{\lambda}$. Cochrane (2001), chapter 12, describes the procedure in more detail.

B. Results for the Collateral Model

We use all 25 size and book-to-market portfolios and the value-weighted market return as test assets. Table IX reports the estimates for the market price of

Table IX

Cross-Sectional Results, Linear Factor Model

Shown here is the estimation of the market prices of risk $\tilde{\lambda}$, using the Fama-MacBeth procedure for 1926–2002. The asset-pricing factors are $\Delta\log(c_{t+1})$ in row 1, $\Delta\log(c_{t+1})$ and $\Delta\log(\alpha_{t+1})$ in row 2, $\Delta\log(c_{t+1})$, $\widetilde{my}_t$, $\widetilde{my}_t\Delta\log(c_{t+1})$ in rows 3–5, and $\Delta\log(c_{t+1})$, $\Delta\log(\alpha_{t+1})$, $\widetilde{my}_t$, $\widetilde{my}_t\Delta\log(c_{t+1})$, and $\widetilde{my}_t\Delta\log(\alpha_{t+1})$ in rows 6–8. The housing collateral variable is the mortgage-based *mymo* in rows 3 and 6, the residential-wealth-based *myrw* in row 4 and 7, and the fixed-assets-based *myfa* in row 5 and 8. *my* is estimated with data from 1925 to 2002. OLS standard errors are in parentheses, and Shanken-corrected standard errors are in brackets. The last column reports the R^2 and the adjusted R^2 just below it.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\alpha$	$\tilde{\lambda}_{my}$	$\tilde{\lambda}_{my.c}$	$\tilde{\lambda}_{my.\alpha}$	R^2
1	8.87	1.61					**9.4**
CCAPM	(2.55)	(1.01)					5.6
	[2.77]	[1.18]					
2	6.90	0.51	0.55				**37.5**
HCAPM	(2.31)	(0.88)	(0.24)				32.0
	[2.60]	[1.08]	[0.30]				
3	4.22	1.94		−0.03	2.23		**86.5**
Separable Prefs.	(2.29)	(1.05)		(0.06)	(0.79)		84.6
mymo	[3.31]	[1.58]		[0.09]	[1.17]		
4	3.52	2.12		−0.03	1.36		**87.8**
Separable Prefs.	(2.25)	(1.02)		(0.03)	(0.47)		86.1
myrw	[3.33]	[1.58]		[0.05]	[0.72]		
5	2.81	0.97		−0.00	0.66		**73.3**
Separable Prefs.	(2.27)	(0.94)		(0.02)	(0.35)		69.7
myfa	[2.93]	[1.28]		[0.03]	[0.47]		
6	2.87	2.59	0.11	−0.02	2.77	0.05	**87.4**
Nonsep. Prefs.	(2.73)	(0.81)	(0.26)	(0.06)	(0.64)	(0.18)	84.3
mymo	[4.45]	[1.38]	[0.44]	[0.10]	[1.09]	[0.30]	
7	3.62	2.30	0.25	−0.03	1.45	0.11	**88.1**
Nonsep. Prefs.	(2.48)	(0.92)	(0.20)	(0.03)	(0.41)	(0.08)	85.1
myrw	[3.81]	[1.47]	[0.33]	[0.06]	[0.65]	[0.13]	
8	3.20	1.56	−0.05	−0.02	0.94	0.05	**85.4**
Nonsep. Prefs.	(2.44)	(1.01)	(0.21)	(0.02)	(0.38)	(0.06)	81.7
myfa	[4.11]	[1.75]	[0.38]	[0.04]	[0.66]	[0.11]	

risk $\tilde{\lambda}$ obtained from the second stage of the Fama–MacBeth procedure. Below the estimates for $\tilde{\lambda}$, we report conventional standard errors and Shanken (1992) standard errors, which correct for the fact that the $\tilde{\beta}$'s are generated regressors from the first time-series step.

Row 1 shows the standard CCAPM. It explains 9% of the cross-sectional variation in excess returns of the size and book-to-market portfolios between 1926 and 2002. Unsurprisingly, the coefficient of relative risk-aversion γ implied by the market price of consumption risk $\tilde{\lambda_c}$ is very high (22, not reported). With nonseparable preferences but perfect commitment, the change in the nondurable expenditure share is an additional asset-pricing factor. This is the HCAPM of Piazzesi et al. (2004). The nonseparability effect increases the R^2 to 50% (row 2). Rows 3 through 8 investigate the collateral model. With separable preferences, the new asset-pricing factors are the housing collateral ratio *my* and consumption growth scaled by *my*. The fit improves to 73–88% for the respective measures of the housing collateral ratio (rows 3–5). The coefficients on the interaction terms $\lambda_{my.c}$ are positive and significant (column 5). With nonseparable preferences, the interaction term of *my* with expenditure share growth is an additional asset-pricing factor (rows 6–8). The new interaction term has a positive factor loading, but is not statistically significant. Except for the conditioning variable *myfa*, nonseparability does not add much to the explanatory power of the collateral CAPM.

The intercept in the cross-sectional regression, $\tilde{\lambda}_0$, should be zero. Its estimate is positive and significant in rows 1 and 2, but becomes insignificant for the collateral CCAPM.

The coefficient estimates for $\tilde{\lambda}$ can be related to the structural parameters of the model, and we can infer that a decrease in the housing collateral ratio my_t increases the market price of consumption risk. This follows because the estimated $\tilde{\lambda}_{my.c}$ is positive. The loadings $\tilde{\theta}$ can be backed out of the $\tilde{\lambda}$ estimates using $\tilde{\lambda} = \tilde{\theta}[\tilde{\delta}\bar{R}^f \mathrm{Cov}(\tilde{F}, \tilde{F}')]$. These factor loadings θ are listed in Table X. The loadings on the constraint factors θ_1^c and θ_2^c are positive. Assuming the persistence coefficient ρ for *my* is positive, this implies that B_{10}, the coefficient on aggregate consumption growth in the aggregate weight growth function, is negative. An adverse aggregate consumption growth shock increases the aggregate weight shock and hence the risk premium. This is exactly the effect predicted by the theory.[13] The implied B_{20} estimates are negative as well (right panel of Table X). The conclusion is the same.

[13] A negative consumption growth shock has two effects. First, a recession decreases *my*, which makes the risk-sharing bounds narrower. Second, a recession coincides with an increase in the income dispersion, which makes the bounds narrower as well. In either case, the extent to which a recession narrows the bounds depends on the level of the housing collateral ratio *my*. When the risk-sharing bounds are narrower, agents run more frequently into them and the aggregate weight growth is high. When housing collateral is scarce, $my^{\max} - my_{t+1}$ is large. A negative consumption growth shock increases ϕ_{t+1} for $B_{10} < 0$. When $my_{t+1} = my^{\max}$, there is no effect of aggregate consumption and rental price growth innovations on the expression for the aggregate weights: ϕ_{t+1} is 1.

Table X
Factor Loadings in Collateral CAPM

Shown here are factor loadings θ implied by the Fama–MacBeth coefficient estimates for λ reported in Table IX. The left panel reports the θ estimates for the separable collateral model, and the right panel reports for the nonseparable model. The first row inverts the λ estimates from row 3 in Table IX (separable collateral model using *mymo*) and row 6 (nonseparable collateral model using *mymo*). The row labeled *myrw* (*myfa*) does the same for rows 4 and 7 (5 and 8) in Table IX.

	Separability			Nonseparability				
Collateral Measure	θ_1^a	θ_1^c	θ_2^c	θ_1^a	θ_2^a	θ_1^c	θ_2^c	θ_3^c
Mortgages *mymo*	44.17	0.91	29.42	57.67	8.10	1.17	38.09	4.43
Residential Wealth *myrw*	35.59	41	12.68	39.04	5.91	0.44	13.85	1.66
Fixed Assets *myfa*	16.19	0.11	5.02	25.39	3.74	0.17	7.85	1.06

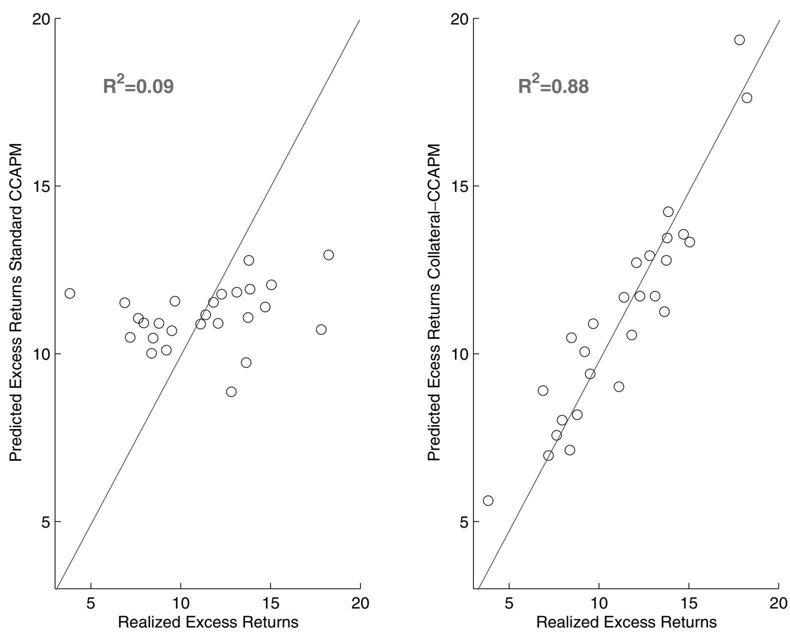

Figure 7. Realized versus predicted returns: The consumption-CAPM and collateral-CAPM. Left panel: Realized average excess returns on 25 Fama–French portfolios and the value-weighted market return against predicted excess returns by standard consumption-CAPM. Right panel: Against predicted returns by collateral-CAPM (under separability).

Figure 7 compares the CCAPM and the collateral-CAPM under separability. The left panel plots the sample average excess return on each of the 26 portfolios against the return predicted by the standard CCAPM. The CCAPM hardly explains any of the variation in excess returns across portfolios. The right panel, which corresponds to the estimates in row 4 of Table IX, shows the returns

Table XI

Average Pricing Errors, Linear Collateral Model

Shown here are pricing errors from the cross-sectional Fama–MacBeth regressions. The set of returns is the value-weighted market return R^{vw} and the 25 size and book-to-market portfolio returns. The second column reports errors from the consumption CAPM, the third reports those from the three-factor Fama–French model, and the last column reports average errors from the collateral CAPM with scaling variable *myrw* (residential-wealth-based collateral measure) and separability in preferences (line 4 in Table IX). The last two rows report the square root of the average squared pricing errors (RMSE) and the χ^2 statistic for the null hypothesis that all pricing errors are zero. The degrees of freedom are 25, 23, and 23, respectively. Three stars denote rejection of the null hypothesis at the 1% level, 2 stars at the 5% level, and 1 star at the 10% level. The sample is 1926–2002. The first two symbols of the asset identifier denote the size quintile (in increasing order), and the next two symbols denote the book-to-market quintile (again in increasing order).

Test Asset	CCAPM	Fama–French	Coll-CCAPM
R^{vw}	2.97	−0.20	0.07
S1B1	7.97	3.96	1.79
S1B2	1.89	2.51	1.23
S1B3	−1.01	−1.08	−0.34
S1B4	−7.10	−2.07	1.53
S1B5	−5.29	−2.78	−0.62
S2B1	4.64	1.94	2.03
S2B2	−0.30	−1.15	−1.27
S2B3	−2.65	−1.23	−0.96
S2B4	−3.31	−0.79	−1.14
S2B5	−3.00	−0.08	−1.72
S3B1	1.99	−1.41	2.00
S3B2	−0.24	−1.12	0.28
S3B3	−0.50	−0.62	−0.55
S3B4	−1.28	0.03	−1.40
S3B5	−1.92	1.58	0.38
S4B1	1.65	−2.54	−1.25
S4B2	0.90	0.70	0.85
S4B3	−0.22	0.24	−2.08
S4B4	−1.18	0.64	0.62
S4B5	−3.90	−0.11	−2.38
S5B1	3.41	−2.52	−0.07
S5B2	3.31	0.73	−0.21
S5B3	2.13	0.46	−0.59
S5B4	1.19	1.96	−0.10
S5B5	−3.95	−0.86	0.10
RMSE	3.27	1.61	1.21
χ^2	72.1***	61.1***	35.1*

predicted by the collateral-CAPM. Most of the size and value portfolios line up along the 45-degree line.

Table XI reports the sample average pricing errors on each of the 26 portfolios. Relative to the CCAPM, the collateral-CAPM largely eliminates the overpricing of growth stocks and the underpricing of value stocks. The root mean squared error (RMSE) across portfolios is 3.27% per annum for the CCAPM (first column, second to last row) but less than half as large for the collateral-CAPM (1.21%, last column). The errors are comparable in size and sign to the Fama and French (1993) three-factor model (second column of Table XI; see also section C). However, the pricing errors on the small growth firms (S1B1 and S1B2) and large growth and value firms (S5B1, S5B4, and S5B5) are lower for the collateral model than for the three-factor model. The last row of the table shows a χ^2-distributed test statistic for the null hypothesis that all pricing errors are zero. The collateral-CAPM is the only model for which the hypothesis of zero pricing errors cannot be rejected at the 5% level.[14]

B.1. Sensitivity Analysis

As a first robustness check, we relax the Markov assumption that we imposed on the aggregate weight shock by including k additional lags of the aggregate factors (consumption growth and expenditure share growth) in the empirical specification of the aggregate weight process. We vary k from 1 to 4 in equation (6). This introduces additional asset-pricing factors in the unconditional β-representation. Table XII reports the estimation results for the sample 1930–2002 and collateral measure *myrw*. Lines 1 and 5 repeat the results for the case of no history dependence. This corresponds to lines 4 and 7 in Table IX, but the sample period is slightly different. Lines 2–4 add the interaction of the housing collateral ratio with lagged consumption growth to the set of factors for the separable model. Lines 6–8 do the same for the nonseparable model. The fit of the cross-sectional estimation does not improve significantly by adding more lagged aggregate factors. The extra factors enter mostly insignificantly, while leaving the estimates on the factors from the model with no history dependence largely unchanged. Only for $k = 4$ is there some additional explanatory power. We conclude that the Markov assumption in the linear collateral model fits the data rather well. This is consistent with our results for the nonlinear model (column 5 in Table VIII).

Second, the theory implies that the expenditure share α_t and the relative rental price ρ_t are valid state variables that measure the capacity of risk-sharing in the economy. We estimate the collateral models with the expenditure share and the relative rental price as conditioning variables, instead of the housing collateral ratio *my*. The fit, as measured by the cross-sectional R^2 or by pricing errors, is very close to the results reported in the paper for *my*. Again, the differences between the separable and nonseparable model are small (results are available upon request).

[14] Because of the sampling error in the regressors, the Shanken correction for the χ^2 test statistics is large. This is because the macro-economic factors have a low sample variance and the size of the standard-error correction is inversely related to this variability. While increasing the standard errors on the estimated market prices of risk, this correction reduces the χ^2 test statistic. The result that the collateral-CAPM fails to reject the null hypothesis of zero pricing errors should be interpreted in this light.

Table XII

Cross-Sectional Results with Lagged Aggregate Pricing Factors

Shown here is the estimation of the market prices of risk $\tilde{\lambda}$, using the Fama–MacBeth procedure for 1930–2002. The asset-pricing factors are $\widetilde{my}_t, \widetilde{my}_t \Delta \log(c_{t+1})$ in row 1; row 2–4 contain in addition the one through four period lagged interaction terms $\widetilde{my}_t \Delta \log(c_{t+1-k}),\ k = 1-4$. Row 5 contains the factors $\Delta \log(c_{t+1})$, $\Delta \log(\alpha_{t+1})$, $\widetilde{my}_t$, $\widetilde{my}_t \Delta \log(c_{t+1})$, and $\widetilde{my}_t \Delta \log(\alpha_{t+1})$. Rows 6–8 contain in addition the one through four period lagged interaction terms $\widetilde{my}_t \Delta \log(c_{t+1-k}),\ k = 1-4$ and $\widetilde{my}_t \Delta \log(\alpha_{t+1-k}),\ k = 1-4$. The housing collateral variable is *myrw* in all rows. OLS standard errors are in parentheses, and Shanken-corrected standard errors are in brackets. The last column reports the R^2 and the adjusted R^2 just below it.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{my}$	$\tilde{\lambda}_{my.c}$	$\tilde{\lambda}_{my.\rho}$	$\tilde{\lambda}_{my.c_{-1}}$	$\tilde{\lambda}_{my.\rho_{-1}}$	$\tilde{\lambda}_{my.c_{-2}}$	$\tilde{\lambda}_{my.\rho_{-2}}$	$\tilde{\lambda}_{my.c_{-3}}$	$\tilde{\lambda}_{my.\rho_{-3}}$	R^2
1	3.28	2.51		−0.03	1.52								**90.3**
Separable Prefs.	(2.31)	(1.00)		(0.03)	(0.47)								89.0
myrw	[3.55]	[1.60]		[0.06]	[0.75]								
2	3.47	2.39		−0.02	1.48		0.13						**90.7**
Separable Prefs.	(2.20)	(1.12)		(0.03)	(0.51)		(0.81)						88.9
myrw	[3.37]	[1.77]		[0.05]	[0.80]		[1.25]						
3	3.48	2.38		−0.02	1.48		0.13		−0.65				**90.7**
Separable Prefs.	(2.26)	(0.98)		(0.04)	(0.46)		(0.81)		(0.98)				88.3
myrw	[3.45]	[1.55]		[0.06]	[0.72]		[1.25]		[1.51]				
4	3.42	1.44		−0.04	0.79		−0.17		−0.62		−1.42		**94.9**
Separable Prefs.	(2.26)	(1.10)		(0.04)	(0.52)		(0.88)		(0.97)		(0.60)		93.2
myrw	[3.45]	[1.72]		[0.06]	[0.81]		[1.36]		[1.49]		[0.94]		
5	1.80	3.11	0.25	−0.03	1.82	0.06							**90.8**
Non-Sep. Prefs.	(2.69)	(0.84)	(0.23)	(0.03)	(0.43)	(0.10)							88.5
myrw	[4.63]	[1.51]	[0.41]	[0.06]	[0.76]	[0.17]							
6	1.94	2.91	0.20	−0.02	1.74	0.04	0.13	−0.04					**91.1**
Non-Sep. Prefs.	(2.56)	(0.82)	(0.21)	(0.05)	(0.43)	(0.09)	(0.45)	(0.17)					87.7
myrw	[4.36]	[1.46]	[0.39]	[0.08]	[0.76]	[0.15]	[0.78]	[0.29]					
7	1.94	2.83	0.13	−0.01	1.75	0.04	0.17	−0.03	−0.66	0.05			**91.4**
Non-Sep. Prefs.	(2.69)	(0.89)	(0.19)	(0.05)	(0.48)	(0.08)	(0.44)	(0.15)	(0.53)	(0.18)			86.5
myrw	[4.60]	[1.58]	[0.36]	[0.08]	[0.83]	[0.15]	[0.78]	[0.26]	[0.92]	[0.31]			
8	2.57	1.46	−0.05	−0.01	0.88	−0.05	−0.75	−0.20	−0.81	−0.01	−1.33	0.03	**95.9**
Non-Sep. Prefs.	(2.66)	(0.80)	(0.21)	(0.05)	(0.40)	(0.08)	(0.64)	(0.18)	(0.55)	(0.19)	(0.60)	(0.20)	92.8
myrw	[5.05]	[1.57]	[0.42]	[0.09]	[0.79]	[0.16]	[1.22]	[0.35]	[1.05]	[0.37]	[1.15]	[0.38]	

Table XIII

Cross-Sectional Results, Fama–MacBeth, and Quarterly Data

Shown here is the estimation of the market prices of risk $\tilde{\lambda}$, using the Fama–MacBeth procedure for 1952.1–2002.4. The asset-pricing factors are $\Delta\log(c_{t+1})$ in row 1, $\Delta\log(c_{t+1})$ and $\Delta\log(\alpha_{t+1})$ in row 2, $\Delta\log(c_{t+1})$, $\widetilde{my}_t$, $\widetilde{my}_t\Delta\log(c_{t+1})$ in rows 3–4, and $\Delta\log(c_{t+1})$, $\Delta\log(\alpha_{t+1})$, $\widetilde{my}_t$, $\widetilde{my}_t\Delta\log(c_{t+1})$, and $\widetilde{my}_t\Delta\log(\alpha_{t+1})$ in rows 6–7. The housing collateral variable is *mymo* in rows 3 and 6 and *myfa* in row 4 and 7. *my* is estimated with data from 1925 to 2002. The conditioning variable is the nondurable expenditure share α_t in rows 5 and 8. In row 9, the asset-pricing factors are $\Delta\log(c_{t+1})$, cay_t, and $cay_t\Delta\log(c_{t+1})$. The measure *cay* is rescaled to ensure positivity: we use $cay/std(cay)+3.5$. In row 10, the asset-pricing factors are R^{vw}_{t+1}, lc_t, and $lc_tR^{vw}_{t+1}$. In line 11, the asset-pricing factors are $R^{vw}_{t+1}, R^{SMB}_{t+1}$, and R^{HML}_{t+1}. The OLS standard errors are in parentheses. The last column reports the R^2 and the adjusted R^2 just below it.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	$\tilde{\lambda}_5$	R^2
CCAPM	−0.09	1.18					**48.2**
	(0.79)	(0.41)					[46.0]
HCAPM	−0.60	1.49	−0.004				**50.1**
	(1.06)	(0.45)	(0.04)				[45.7]
Separable Prefs.	−0.58	1.32		0.16	0.51		**67.7**
mymo	(0.81)	(0.43)		(0.06)	(0.11)		[63.3]
Separable Prefs.	0.43	1.36		−0.001	0.11		**62.2**
myfa	(0.85)	(0.40)		(0.01)	(0.04)		[57.0]
Separable Prefs.	0.51	1.09		0.006	0.90		**70.0**
α	(0.84)	(0.44)		(0.002)	(0.36)		[65.9]
Nonsep. Prefs.	0.52	0.86	0.01	0.13	0.33	−0.02	**78.8**
mymo	(0.74)	(0.29)	(0.03)	(0.05)	(0.09)	(0.01)	[73.5]
Nonsep. Prefs.	1.38	0.66	0.01	−0.02	0.06	0.007	**72.6**
myfa	(0.69)	(0.27)	(0.03)	(0.01)	(0.03)	(0.003)	[65.7]
$\alpha_t - HCAPM$	0.52	1.03	−0.01	0.006	0.85	−0.01	**71.7**
	(0.67)	(0.34)	(0.03)	(0.002)	(0.28)	(0.02)	[64.6]
$cay_t - CCAPM$	2.35	0.79		−0.71	2.58		**66.6**
	(0.97)	(0.30)		(0.41)	(1.19)		[62.1]
$lc_t - CAPM$	2.88	−1.17		0.01	−1.14		**64.3**
	(1.04)	(1.17)		(0.02)	(1.69)		[59.5]
FF	3.78	−2.07		0.46	1.27		**74.0**
	(1.13)	(1.27)		(0.41)	(0.43)		[70.5]

Third, the collateral model performs well when estimated on quarterly data. The flow of funds data needed to construct our collateral measures start in 1952.1. The value of residential fixed assets is only available through 2001.4; all other data are through 2002.4. Rows 3–8 of Table XIII show the estimated market prices of risk for collateral model. Rows 1 and 2 show the standard CAPM and HCAPM. The CCAPM explains more of the cross-sectional variation in stock returns in the post-war sample ($R^2 = 0.48$). Irrespective of whether the collateral effect is captured using the housing collateral ratio using outstanding mortgages, the housing collateral ratio using the value of residential wealth, or the expenditure share, the data support the collateral effect. The parameters have the right sign: scarcer collateral is associated with a higher market price of risk.

Finally for annual post-war data, the collateral-CAPM with separable preferences explains between 70 and 83% and the collateral-CAPM with nonseparable preferences explains between 76 and 84% of the cross-sectional variation in the 26 portfolios (results are available upon request).

B.2. Time-Varying Betas

Why does the collateral-CAPM help explain the value premium? In the model, a stock's riskiness is determined by the covariance of its returns with aggregate risk factors conditional on the state variable my. The conditional covariance reflects time-variation in risk premia. If time variation in risk premia is important for explaining the value premium, then stocks with high book-to-market ratios should have a larger covariance with aggregate risk factors in risky times, when my is low ($my^{\max} - my_t$ is high), than in less risky times, when my is high ($my^{\max} - my_t$ is low). This is the pattern we find in the data.

We estimate the risk exposure (the β's) for each of the 25 size and book-to-market portfolios and the value-weighted market return. This is the first step of the Fama–MacBeth two-step procedure. To make the point more forcefully, we impose separability on the preferences over housing and nondurable consumption:

$$R_{t+1}^{e,j} = \tilde{\beta}_0^j + \tilde{\beta}_c^j \Delta \log c_{t+1} + \tilde{\beta}_{my}^j \left(my^{\max} - my_t\right) + \tilde{\beta}_{my.c}^j \left(my^{\max} - my_t\right) \Delta \log c_{t+1}. \tag{10}$$

Equation (10) allows the covariance of returns with consumption growth to vary with my. For each asset j, we define the conditional consumption beta as $\beta_t^j = \tilde{\beta}_c^j + (my^{\max} - my_t)\tilde{\beta}_{my.c}^j$. We estimate equation (10) and compute the average consumption beta in good states, defined as times in which my is 1 SD above zero, and in bad states (risky times) when my is 1 SD below zero. Table IV shows that the high book-to-market portfolios (B4 and B5) have a consumption β that is large when housing collateral is scarce and is small in times of collateral abundance. The opposite is true for growth portfolios (B1 and B2). Moreover, the value stocks have higher consumption betas than the growth stocks in bad states, and vice versa for the good states. This is the sense in which value portfolios are riskier than growth portfolios.

The left panel of Figure 8 shows that the value portfolios (B4, B5) have a high return and that the growth portfolios (B1, B2) have a low return. The right panel plots realized excess returns against $\tilde{\beta}_{my.c}^j$, the exposure to the interaction term of the housing collateral ratio with aggregate consumption growth. Growth stocks in the lower left corner have a low exposure to collateral constraint risk whereas value stocks have a large exposure. So, value stocks are riskier than growth stocks because their returns are more highly correlated with the aggregate factors when risk is high ($my^{\max} - my_t$ is high) than when risk is low ($my^{\max} - my_t$ is low). Furthermore, there is a substantial cross-sectional variation in these betas. Because both the estimates of $\tilde{\lambda}_{my.c}$ and of

Table XIV

Consumption Betas

Consumption betas are computed as $\beta_t = \beta_c + \beta_{c.my}\widetilde{my}_t$. Good states are states in which the residential-wealth-based collateral measure *myrw* is 1 SD below zero and bad states are times when *myrw* is 1 SD above zero (11 observations each). The second column reports the average consumption beta. The third and fourth columns report the average consumption betas in good states and bad states, respectively. The first two symbols of the asset identifier denote the size quintile (in increasing order), and the next two symbols denote the book-to-market quintile (again in increasing order). The sample is 1926–2002.

Test Asset	All States	Good States	Bad States
R^{vw}	1.56	0.89	2.34
S1B1	1.48	1.27	1.76
S1B2	2.28	0.47	4.34
S1B3	3.20	1.70	4.95
S1B4	3.39	−0.70	8.00
S1B5	3.96	0.85	7.50
S2B1	1.94	0.98	3.04
S2B2	2.23	1.25	3.37
S2B3	2.42	0.33	4.80
S2B4	2.68	0.49	5.17
S2B5	2.89	0.58	5.51
S3B1	1.80	0.33	3.46
S3B2	2.27	0.61	4.17
S3B3	2.52	1.16	4.08
S3B4	2.53	0.91	4.39
S3B5	2.99	0.53	5.79
S4B1	1.05	0.37	1.84
S4B2	1.58	0.04	3.33
S4B3	1.71	0.65	2.93
S4B4	2.34	0.04	4.93
S4B5	1.62	−1.10	4.67
S5B1	1.54	1.20	1.95
S5B2	1.21	0.76	1.74
S5B3	1.57	0.77	2.50
S5B4	1.69	0.32	3.25
S5B5	1.57	−1.66	5.18

$\tilde{\beta}^j_{my.c}$ are positive, value stocks are predicted to have higher risk premia. The value premium is the compensation for the fact that high book-to-market firms pay low returns when housing collateral is scarce and when constraints bind more frequently.

C. Comparison across Models

The cross-sectional explanatory power of the collateral-CAPM for size and value portfolios compares favorably to other asset-pricing models. We discuss model comparison in two ways. The first one presents the Fama–MacBeth

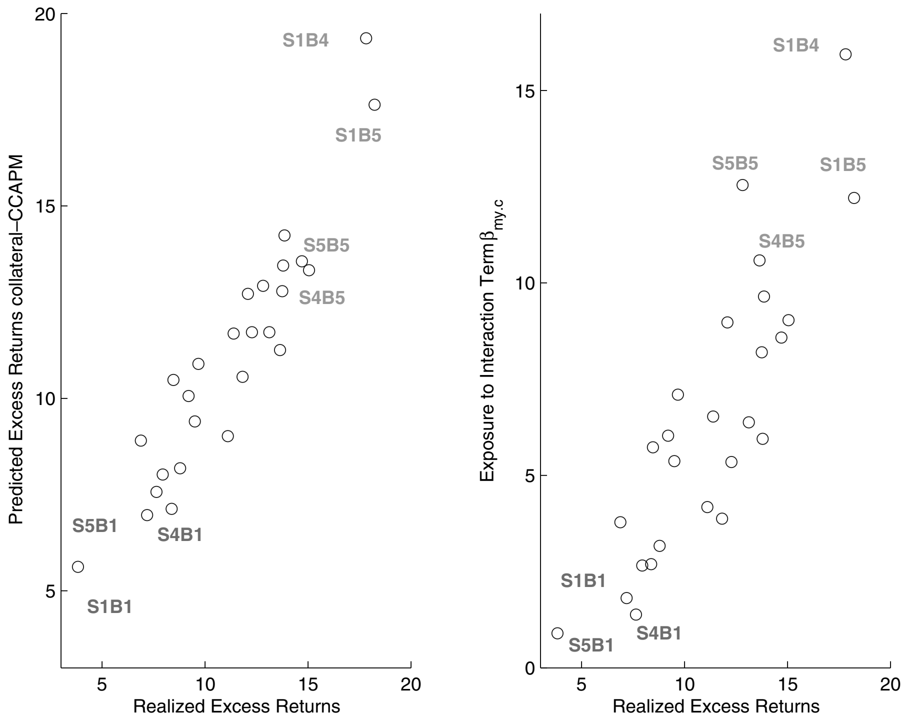

Figure 8. The collateral CAPM and the value premium. Left panel: Realized average excess returns on 25 Fama–French portfolios and the value-weighted market return against excess returns predicted by the collateral-CAPM with *myrw*. Right panel: Realized average excess returns against $\tilde{\beta}_{my.c}$, the exposure to interaction term of $my^{\max} - my_t$ and $\Delta \log c_{t+1}$, estimated in the first-stage of the Fama–MacBeth regression.

regressions for other models, in the same fashion as we discussed the results for the collateral model. The advantage of this approach is that we can infer the structural parameters from the estimated market prices of risk. A drawback of comparing models by the cross-sectional R^2 is that the mimicking portfolio of asset-pricing factors may be mean-variance inefficient (Kandel and Stambaugh (1995)). The second model comparison looks at spreads of portfolios that are correctly priced by our model and are predicted to have a high expected return. It asks whether the alternative models also price these spreads. And conversely, what are the high return spreads that alternative models price correctly but fail to be priced by our model? Here we focus on size and value premia. In Section VI.D, we look at other dimensions of cross-sectional variation in stock returns.

C.1. Fama–MacBeth Regression

Table XV compares return-based asset-pricing models in rows 1–3 with consumption-based models in rows 4–6.

Table XV

Comparison of Seven Linear Factor Models, 1926 to 2002

Reported are the market prices of risk $\tilde{\lambda}$, estimated from the second-stage Fama–MacBeth procedure. Row 1 is the CAPM with the value-weighted market return $R^{vw,e}_{t+1}$ as the asset-pricing factor. Row 2 is the human capital augmented CAPM with factors $R^{vw,e}_{t+1}$ and $R^{hc,e}_{t+1}$, the return on human wealth. Row 3 is the scaled CAPM model, where the scaling variable is the labor income to consumption ratio lc_t. The other factors are $R^{vw,e}_{t+1}$ and $lc_t R^{vw,e}_{t+1}$. Row 4 is the consumption-CAPM with the aggregate consumption growth as factor $\Delta\log(c_{t+1})$. Row 5 is the scaled consumption CAPM with the consumption wealth ratio *cay* as the scaling variable. The factors are $\Delta\log(c_{t+1})$, cay_t, and $cay_t\Delta\log(c_{t+1})$. Row 6 is the collateral model under separability, with factors $\Delta\log(c_{t+1})$, $myrw^{\max} - myfa_t$, and $(myrw^{\max} - myfa_t)\Delta\log(c_{t+1})$. Row 7 is the three-factor model with factors $R^{vw,e}_{t+1}$, the excess return associated with size $R^{smb,e}_{t+1}$, and the excess return associated with value $R^{hml,e}_{t+1}$. The second column gives the zero-β return $\tilde{\lambda}_0$. OLS standard errors are in parentheses, and Shanken-corrected standard errors are in brackets.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	R^2
1	−0.30	9.35			28.3
Static CAPM	(4.02)	(4.68)			25.3
Sharpe–Lintner	[4.40]	[5.65]			
2	2.24	7.13	3.91		37.2
Human Capital-CAPM	(3.93)	(4.58)	(1.28)		31.7
Jagannathan–Wang	[4.87]	[6.16]	[1.71]		
3	1.47	7.48	−0.00	5.57	49.3
lc-conditional CAPM	(4.17)	(4.82)	(0.02)	(4.99)	42.4
Santos–Veronesi	[5.49]	[6.42]	[0.03]	[6.62]	
4	8.88	1.61			9.4
Static CCAPM	(2.55)	(1.01)			5.6
Breeden–Lucas	[2.77]	[1.18]			
5	3.88	2.68	0.02	0.42	86.2
cay-conditional CCAPM	(2.32)	(1.05)	(0.02)	(0.30)	84.3
Lettau–Ludvigson	[3.84]	[1.80]	[0.03]	[0.53]	
6	3.52	2.12	−0.03	1.36	87.8
Collateral-CAPM	(2.25)	(1.02)	(0.03)	(0.47)	86.1
This paper	[3.33]	[1.58]	[0.05]	[0.71]	
7	10.21	−2.46	2.71	6.30	78.1
Three-factor model	(4.63)	(5.17)	(1.68)	(1.74)	75.1
Fama–French	[5.24]	[6.32]	[2.52]	[2.56]	

The capital asset-pricing model relates the returns on stocks to their correlation with the return on the wealth portfolio. In the standard CAPM of Lintner (1965), the return on the wealth portfolio is proxied by the market return R^{vw} (row 1). It explains 28% of annual returns. Because stock market wealth is an incomplete total wealth measure, Jagannathan and Wang (1996) include the return on human wealth in the return on the wealth portfolio. That return is measured by the growth rate in labor income (plus transfers). The R^2 in row 2 increases slightly to 37%. Kullmann and Siegel (2003) investigate the improvements to the CAPM when residential housing wealth is incorporated into the definition of wealth. In our model, housing wealth affects returns only through the collateral ratio. In the economy of Santos and Veronesi (2001), times in which investors finance a large fraction of consumption out of labor income (*lc* is low) are less risky. Their conditional CAPM explains 50% of the annual returns (row 3).[15]

The Fama and French (1993) three-factor model adds a size and a book-to-market factor to the standard CAPM. The size factor is the return on a hedge portfolio that goes long in small firms and short in big firms (*smb*). The value factor is the return on a hedge portfolio that goes long in high book-to-market firms and short in low book-to-market firms (*hml*). This model accounts for 78% of the cross-sectional variation in annual returns (row 7). Given its good fit, this model serves as the empirical benchmark.

In contrast to the previous models, consumption-based asset-pricing models measure the riskiness of an asset directly off the covariance with marginal utility growth. One of the objectives of this literature has been to identify macroeconomic sources of risk that can explain the empirical success of the Fama and French (1993) size and book-to-market factors. The fourth row reports the standard CCAPM of Breeden (1979). Lettau and Ludvigson (2001b) explore a conditional version of the CCAPM with the consumption–wealth ratio as scaling variable. The market price of consumption risk increases in times with low *cay* (recessions). The Lettau–Ludvigson model explains 86% of the annual cross-sectional variation.

Model 6 is our collateral-CCAPM under separability (as previously reported in row 4 of Table IX). The model goes a long way in accounting for the cross-sectional differences in returns on the 25 Fama–French portfolios and the market return. The R^2 of 88% improves upon the fit of the Fama–French model.

For quarterly data, Table XIII shows that the collateral model with separability prices the cross-section of size and book-to-market portfolios as well as the Lettau–Ludvigson (row 9) and Santos–Veronesi models (row 10).

As a robustness exercise, we investigate the residual explanatory power of the idiosyncratic portfolio characteristics, size (log market capitalization), and value (log value-weighted book-to-market ratio of the portfolio) for each of these seven models. In contrast to the return-based models and the static CAPM, there are no residual size nor value effects in the collateral-CCAPM. The same conclusion is found for the post-war sample (results are available on request).

C.2. Spreads

We construct size and value spreads from the 25 size and book-to-market portfolios. For each size quintile, the value spread is defined as the difference between the highest book-to-market quintile return and the lowest book-to-market quintile return. Similarly, for each book-to-market quintile, the size spread is defined as the difference between the highest-size quintile return and the lowest-size quintile return.

[15] The authors also investigate a scaled version of the CCAPM, as we do, but their results for the scaled CCAPM are not as strong as for the scaled CAPM.

Table XVI

Model Comparison: Value and Size Spreads

This table reports the realized spreads and the model predicted spreads on value quintile portfolios (panel A) and size quintile portfolios (panel B), implied by the Fama–MacBeth estimates of the market prices of risk. The first column reports the realized value spreads and size spreads, computed as the difference in sample means between the constituent portfolios. The second through fourth columns are the predicted spreads by the collateral CAPM, the standard consumption CAPM, and the *cay*-CCAPM. The fifth through eighth column are the predicted spreads by the CAPM, human capital augmented CAPM, *lc*-conditional CAPM, and the Fama–French three-factor model. We use annual data for 1926–2002.

		Predicted Spreads						
Quintile	Realized Spreads	Coll-CCAPM	CCAPM	*cay*-CCAPM	CAPM	HC-CAPM	*lc*-CAPM	FF
Value		Panel A: Value Spreads						
V1	14.41	12.00	1.14	11.54	0.75	3.80	4.06	7.88
V2	8.18	4.43	0.54	4.81	0.15	0.29	2.97	6.15
V3	5.38	3.75	1.46	5.17	−0.18	2.75	0.20	8.37
V4	5.26	4.12	−0.28	4.05	3.02	2.96	3.14	7.70
V5	5.17	5.34	−2.18	6.79	0.31	−1.76	−1.74	6.83
Size		Panel B: Size Spreads						
S1	−3.81	−1.95	0.75	−1.91	4.02	2.17	0.25	2.66
S2	2.49	3.93	1.08	2.32	5.26	4.11	2.64	4.27
S3	5.02	5.27	1.88	5.88	4.49	4.29	3.49	3.48
S4	8.33	9.95	0.03	8.19	5.21	4.15	7.70	4.30
S5	5.43	4.70	4.08	2.85	4.47	7.72	6.05	3.51

Table XVI displays the observed value and size spreads (first column) and compares them to the spreads predicted by the various consumption-based (left panel) and return-based models (right panel).

The high-return spreads with small pricing errors that are identified by our model (column 2) are the value spreads V1 (smallest firms) and V5 (largest firms) and the size spreads S1, S4, and S5. With the exception of the *cay*-CCAPM in column 4, all other models fail to price these high-return spreads. The pricing error for the other models is especially big for the value spread for small firms (V1), which is economically the most significant one. The negative size premium for growth firms (S1) is only found for the *cay*-CCAPM and our model. Even the Fama–French model, constructed to price these spreads, has a large abnormal return (alpha) associated with V1, V5, S1, and S5 relative to our model. Our model and the *cay*-CCAPM show strong similarity in the predicted pattern for the spreads. The *cay*-CCAPM has a larger error associated with V5 and S5. Our model has a relatively larger pricing error associated with V3, S2, and S4.

Conversely, we ask whether high-return spreads predicted by other models are priced by our model. As is apparent from Table XVI, the standard consumption CAPM, the standard CAPM, the human capital augmented

CAPM, and the conditional CAPM with the labor income to consumption ratio as a conditioning variable, do not generate large expected returns that are priced with small errors. The Fama–French three-factor model predicts a high return on the V2 and V3 spread. The V2 spread of 6.2% is lower than the spread in the data (8.2%), while V3 is higher. Our model predicts an even lower spread for V2 (4.4%) than the three-factor model.

D. Other Dimensions of Cross-Sectional Variation

So far we have focused on explaining cross-sectional stock return variation along the size and book-to-market dimensions. Here we briefly discuss the explanatory power of the collateral model for cross-sectional return variation in momentum portfolios and portfolios sorted by dividend-price and earnings-price ratios.

D.1. Momentum Portfolios

We use monthly data from Bansal et al. (2002) for 10 momentum decile portfolios and compound them to quarterly and annual returns. We calculate the root mean squared pricing errors on the 10 momentum decile portfolios when the market prices of risk are estimated by Fama–MacBeth on the 25 size and book-to-market portfolios and the 10 momentum portfolios. We also compute pricing errors using 10 size, 10 value, and 10 momentum portfolios as test assets. We focus on the performance of the collateral CAPM relative to the standard CCAPM and the three-factor model.

While the pricing errors on the momentum decile portfolios are large for all three models, the pricing errors are noticeably smaller for the collateral CAPM than for the consumption CAPM or for the three-factor Fama–French model. This is especially true for the quarterly data (1952.1–1999.4) where the RMSE is 0.76 per quarter, compared to 1.2% for the other two models. The pricing error on the lowest momentum decile portfolio (losers) is 2.6% per quarter for the CCAPM, 1.5% for the three-factor model, and only 0.8% for the collateral CAPM. Similarly, for the highest momentum decile portfolio (winners), the pricing errors are -1.7% (consumption CAPM), -2.2% (three-factor model), and -1.0% (collateral CAPM). For annual data on momentum portfolios (1927–1999), there still is a reduction in RMSE, but it is less pronounced (more detailed results are available on request).

D.2. Dividend-Price and Earnings-Price Portfolios

We look at five quintile portfolios formed by sorting on the dividend–price ratio and five quintile portfolios sorted on the earnings–price ratio. We add these portfolios to the market return, five book-to-market quintile portfolios, and five size quintile portfolios. The dividend–price portfolio returns are available for 1926–2002. The earnings-price portfolios are available from 1952 onward. All data are from Kenneth French's web site. For each characteristic, a spread is defined as the difference of the two extremum quintile portfolio returns.

The data indicate a spread of 2.5% per year associated with the dividend yield sort. This spread is priced well by our model (2.3%), as well as by the three-factor model (2.7%). In the post-1952 period, there is a spread of 9.3% per year associated with the portfolio sorted by earnings–price ratio. With the exception of the Fama–French model, which predicts an 8.2% expected return, other models fall short of pricing this spread. Our model underprices the spread by 3.5%, but the unexplained return is smaller than for the CCAPM and the *cay*-CCAPM. The lc-conditional CAPM does a good job of pricing the dividend-price and earnings–price spreads, but misprices the size and value spreads (more detailed results are available on request).

We conclude that, relative to other models, the collateral model does well in explaining spreads on portfolios sorted by momentum, dividend yield, and earnings–price ratios jointly with size and value spreads.

Table XVII

Dividend Share and the Housing Collateral Ratio

Shown here are regression results for $\log(d_t) - \log(y_t) = b_0 + b_1 my_t + \epsilon_t$, where d_t is the nominal dividend on each of the decile portfolios sorted on book-to-market ratios and y_t is the nominal aggregate labor income plus transfers. The regressor is *mymo* in Panel A, *myrw* in Panel B, and *myfa* in Panel C. The housing collateral ratios are estimated for the period 1889–2002. The dividend data are annual for 1952–1999, constructed from the monthly dividend yield data from Bansal, Dittmar, and Lundblad (2002). Each panel reports OLS estimates, and their standard errors as well as the R^2.

B/M Decile	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
Panel A: Collateral Measure: Mortgages *mymo*										
b_0	−5.08	−4.67	−4.59	−4.86	−3.94	−3.93	−4.32	−3.30	−3.25	−3.00
$\sigma(b_0)$	(0.12)	(0.10)	(0.09)	(0.11)	(0.08)	(0.04)	(0.04)	(0.04)	(0.06)	(0.09)
b_1	1.21	0.51	0.85	1.02	0.48	0.33	0.70	0.07	−0.63	−1.12
$\sigma(b_1)$	(0.30)	(0.26)	(0.23)	(0.29)	(0.21)	(0.10)	(0.10)	(0.12)	(0.14)	(0.23)
R^2	0.266	0.074	0.225	0.207	0.100	0.179	0.490	0.007	0.304	0.344
Panel B: Collateral Measure: Residential Wealth *myrw*										
b_0	−5.41	−4.91	−4.86	−5.13	−4.26	−4.08	−4.35	−3.23	−3.06	−2.76
$\sigma(b_0)$	(0.10)	(0.11)	(0.08)	(0.12)	(0.06)	(0.03)	(0.05)	(0.05)	(0.04)	(0.08)
b_1	1.30	0.73	0.97	1.07	0.87	0.46	0.45	−0.09	−0.71	−1.09
$\sigma(b_1)$	(0.17)	(0.17)	(0.13)	(0.18)	(0.10)	(0.05)	(0.08)	(0.08)	(0.07)	(0.13)
R^2	0.574	0.287	0.559	0.426	0.622	0.650	0.387	0.023	0.722	0.604
Panel C: Collateral Measure: Fixed Assets *myfa*										
b_0	−5.20	−4.97	−4.81	−5.05	−4.22	−4.03	−4.23	−3.05	−3.12	−3.01
$\sigma(b_0)$	(0.91)	(0.16)	(0.15)	(0.19)	(0.12)	(0.07)	(0.09)	(0.06)	(0.09)	(0.17)
b_1	0.91	0.82	0.88	0.91	0.79	0.36	0.24	−0.41	−0.59	−0.62
$\sigma(b_1)$	(0.34)	(0.26)	(0.25)	(0.32)	(0.21)	(0.11)	(0.15)	(0.11)	(0.16)	(0.29)
R^2	0.133	0.171	0.217	0.147	0.241	0.186	0.049	0.236	0.233	0.093

E. Dividends on Value Portfolios

We found that the returns of value firms are more correlated with aggregate risk in times when the housing collateral ratio is low. Here we take the next step by identifying one potential source of this pattern by examining the response of dividends to collateral shocks.

We use annual dividend data on each of the 10 portfolios sorted on book-to-market. Book-to-market is defined as book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. We follow Bansal et al. (2002) by constructing dividends from value-weighted total returns and price appreciation rates on the decile value portfolios (both from Kenneth French). We construct nominal annual dividends by summing up monthly nominal dividends. The data are for 1952–1999.

Table XVII shows how dividends, normalized by the aggregate labor income plus transfers, of the high book-to-market portfolios (B9, B10) are low when housing collateral is scarce. They are strongly positively correlated with the housing collateral ratio. The normalized dividend process for the

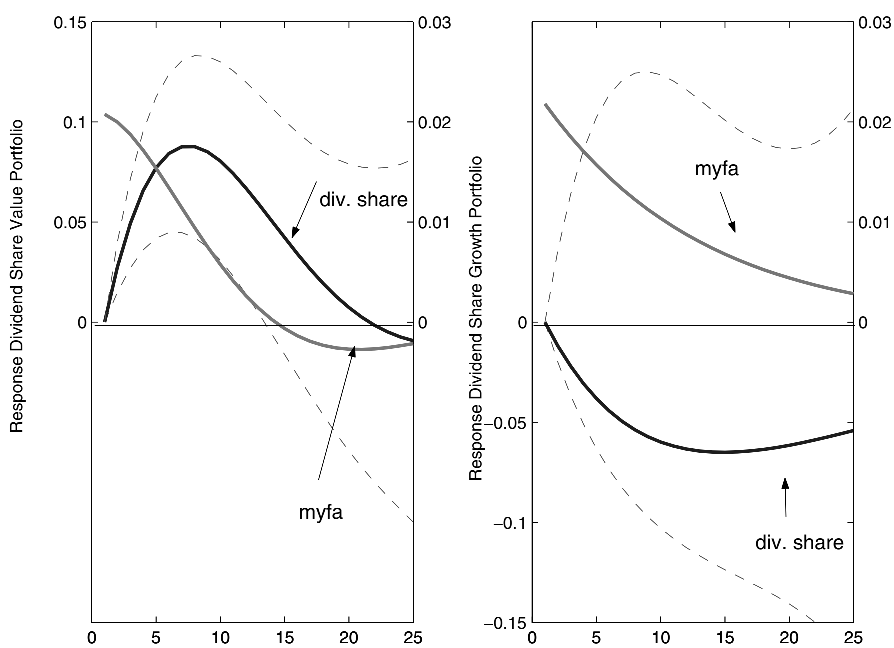

Figure 9. Response of the dividend share on value and growth portfolio to an impulse in the housing collateral ratio. Left panel: Response of the log dividend share on the highest value decile (B10) to a one standard deviation innovation in the housing collateral ratio *myfa*. Right panel: Response of the log dividend share on the lowest value decile (B1) to a one standard deviation innovation in the housing collateral ratio *myfa*. The underlying VAR contains the log dividend to labor income plus transfers ratio and the housing collateral ratio. The sample is from 1952 to 2002. The dashed lines are standard errors around the responses, computed from 5,000 Monte Carlo simulations.

low book-to-market ratio portfolios is strongly negatively correlated with *my* (B1, B2). Using a bivariate VAR, we study the response of the normalized dividend process on the growth (B1) and value (B10) portfolios to an impulse in the housing collateral ratio. Figure E shows the impulse response graphs. The responses of the dividend shares to an innovation to the housing collateral ratio *myfa* are mirror images of each other. These different cash flow responses to collateral innovations can potentially endogenously explain the joint distribution of returns and the collateral ratio in the U.S. data.

VII. Conclusion

House price fluctuations play an important role in explaining the time-series and cross-sectional variation in asset returns. Given the magnitude of the housing market, this is unsurprising. This paper shows that the way in which housing affects asset returns is through the role of housing as a collateral asset. The housing collateral mechanism endogenously generates heteroskedasticity and counter-cyclicality in the market price of risk. In Lustig and Van Nieuwerburgh (2004a), we solve for the equilibrium of the model numerically, while this paper focuses on connecting the model to the data. We specify the liquidity factor in the SDF as a semiparametric function of the housing collateral ratio and the aggregate pricing factors.

The Euler equation restrictions for the stock market return, a short-term bond, a long-term bond, and a few size and book-to-market portfolios, in addition to the Euler inequality restrictions for the representative agent, yield precise, low risk-aversion estimates. The estimated liquidity shocks are larger in times of low aggregate consumption growth when housing collateral is scarce, as predicted by the model. A linear version of this model prices the 25 size and book-to-market portfolios remarkably well.

Why does the collateral model work better than the standard CCAPM? The data suggest that the answer lies in allowing for time-variation in risk-sharing. The standard CCAPM implies that risk-sharing is always perfect. There is a wealth of empirical evidence against full consumption insurance at different levels of aggregation: at the household level (e.g., Attanasio and Davis (1996), Cochrane (1991b)), the regional level (e.g., Hess and Shin (1998)) and the international level (e.g., Backus, Kehoe, and Kydland (1992)). Blundell, Pistaferri, and Preston (2002) find evidence for time-variation in the economy's risk-sharing capacity. In a companion paper, we provide direct empirical support for the underlying time-variation in risk-sharing (Lustig and Van Nieuwerburgh (2004b)). Using U.S. metropolitan area data, we find that the degree of insurance between regions decreases when the housing collateral ratio is low. Our theory only predicts strong consumption growth correlations when housing collateral is abundant. The data seem to support this qualification; conditioning on the housing collateral ratio weakens the consumption correlation puzzle for U.S. regions.

Appendix: Data

A. Labor Income plus Transfers

For 1929–2002: Bureau of Economic Analysis, NIPA Table 2.1. Aggregate labor income is the sum of wage and salary disbursements (line 2), other labor income (line 9), and proprietors' income with inventory valuation and capital consumption adjustments (line 10). Transfers is transfer payments to persons (line 16) minus personal contributions for social insurance (line 23). Prior to 1929, labor income plus transfers is 0.65 times nominal GDP. Between 1929 and 2002, the ratio of labor income plus transfers to nominal GDP stays between 0.65 and 0.70 and equals 0.65 in 1929 and 1930. Nominal GDP for 1889–1928 is from Maddison (2001).

B. Number of Households

For 1889–1945: Census (1976), series A335, A2, and A7. Household data are for 1880, 1890, 1900, 1910, 1920, 1930, and 1940, while the population data are annual. In constructing an annual series for the number of households, we assume that the number of persons per household declines linearly in between the decade observations. For 1945–2002: U.S. Bureau of the Census, Table HH-1, Households by Type.

C. Price Indices

All Items (p^a) 1890–1912: Census (1976), Cost of Living Index (series L38). For 1913–2002: CPI (BLS), base year is 1982–1984. In parentheses are the last letters of the BLS code. All codes start with CUUR0000S. Total price index (p^a): All items (code A0). Shelter (p^h): Item rent of primary residence (code EHA). Food (p^c) 1913–2002: Item food (code AF1). Apparel (p^{app}) 1913–2002: Item apparel (code AA).

D. Aggregate Consumption

For 1909–1928: Census (1976), Total Consumption Expenditures (series G470). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929–2002: Bureau of Economic Analysis, NIPA Table 2.2. Total Consumption expenditures C is personal consumption expenditures (line 1).

For 1909–1928: Census (1976), Rent and Imputed Rent (series G477). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929–2002: Bureau of Economic Analysis, NIPA Table 2.2. Housing services consumption H is nominal consumption on housing services (line 14).

For 1909–1928: Census (1976), Food (series G471 + G472 + G473). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929–2002: Bureau of Economic Analysis, NIPA Table 2.2. Nominal consumption of food (line 7).

For 1909–1928: Census (1976), Apparel (series G474). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929–2002: Bureau of Economic Analysis, NIPA Table 2.2. Nominal consumption of clothing and shoes (line 8).

E. Housing Expenditure Share A

It is computed in two ways. The nondurable consumption share $\alpha = 1 - A$. First, for 1909–2002, the housing expenditure share A_1 is computed as rent and imputed rent divided by total consumption expenditures minus rent and imputed rent and minus apparel. The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. The cell entries for 1920, 1922, 1924, 1926, and 1928 are the average of the adjacent cells. The corresponding measure for the nondurable consumption share is $\alpha_1 = 1 - A_1$. Second, for 1929–2002: The housing expenditure share A_2 is nominal consumption on housing services (line 14) divided by nominal consumption of nondurables (line 6) and services (line 13) minus clothing and shoes (line 8). The corresponding measure for the nondurable consumption share is $\alpha_2 = 1 - A_2$.

F. Real Per Household Consumption Growth dc

It is computed in two ways. First, for 1922–2002, we construct real nondurable consumption as total consumption deflated by the all items CPI minus rent deflated by the rent component of the CPI minus clothes and shoes deflated by the apparel CPI component. Per household variables are obtained by dividing by the number of households. The missing data for 1924, 1926, and 1928 are interpolated using Mehra and Prescott (1985) real per capita consumption growth. The growth rate dc_1 is the log difference multiplied by 100. Second, for 1930–2002, we define real nondurable and services consumption (NDS) as nondurable consumption deflated by the NIPA nondurable price index plus services deflated by the NIPA services price index minus housing services deflated by the NIPA housing services price index minus clothes and shoes deflated by the NIPA clothes and shoes price index. The basis of all NIPA price deflators is 1996 = 100. They are not the same as the corresponding CPI components from the BLS. Per household variables are obtained by dividing by the number of households. The growth rate dc_2 is the log difference of *NDS* multiplied by 100.

G. Rental Price Growth $d\rho$

It is computed in two ways. First, for 1913–2002, we use the ratio of CPI rent component to the CPI food component: $\rho = \frac{p^h}{p^c}$. The growth rate $d\rho_1$ is the log difference multiplied by 100. Second, for 1930–2002, we construct nominal nondurable consumption (nondurables plus services excluding housing services and excluding clothes and shoes) and real nondurable consumption (where each item is separately deflated by its own NIPA price deflator, basis 1996 = 100). The deflator for nondurable consumption is then the ratio of the nominal to the real nondurable consumption series. The relative rental price is then the ratio of the price deflator for housing services to the price deflator for nondurable consumption. The growth rate $d\rho_2$ is the log difference multiplied by 100.

H. Financial Wealth

In order to construct the consumption wealth ratio *cay* at annual frequency (Lettau and Ludvigson (2001a)) we need a measure of financial wealth. For 1945–2002: Flow of Funds, Federal Reserve Board, balance sheet of households, and nonprofit organizations (B.100), Line 8: Financial Assets (FL154090005.Q). For 1926–1945: total deposits, all commercial banks, NBER macro-history database (series 14145). We assume that the ratio of deposits to total wealth decreases slowly from 0.205 to 0.185, its level in 1945 (FoF deposits series). The quarterly series for *cay* is from Martin Lettau's web site (http://pages.stern.nyu.edu/mlettau/).

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