THE JOURNAL OF FINANCE • VOL. LI, NO. 2 • JUNE 1996

Non-Fundamental Speculation

VICENTE MADRIGAL*

ABSTRACT

We study an intertemporal asset market where insiders coexist with "non-fundamental" speculators. Non-fundamental speculators possess no private information on fundamental values of assets, but have superior knowledge about some aspect of the market environment. We show that the entry of these (rational) speculators can lead to reductions in market liquidity and in the information content of prices, even in an efficient market. Also, equilibrium trades display patterns of empirical interest. For example, speculators appear to chase trends and lose money after market "overreactions," while insiders trade as contrarians and profit after such overreactions.

ARE INSIDERS WITH PRIVATE fundamental information the only investors who can profitably trade? If not, what will be the effect of "non-insider" speculators on asset prices and market liquidity?

In this article, we study a market in which insiders coexist with traders who do not have information on fundamental values, but who possess superior knowledge of the market trading process or environment. Such knowledge allows these traders to estimate fundamental information from public data more accurately than the market at large.

As an example, firms who possess brokerage services may have more detailed information than the public on the components of order flows.[1] These components could partly reveal the motives behind different trades. Another example is that some investors may have better knowledge of insiders' trading strategies, because of information concerning some parameter on which this strategy depends, such as trading costs or private values.

Our point is that this form of speculation is different in important ways from speculation based on inside fundamental information. In particular, the entry of non-fundamental speculators in certain instances actually reduces liquidity and the information content of prices, even if all speculators are risk neutral, rational, and trading profitably. This phenomenon is, of course, contrary to the traditional view. According to this view, speculation can only be profitable if

* New York University and Hoover Institution. I would like to thank Yakov Amihud, Darrell Duffie, Jennie France, Charlie Kahn, Hedi Kallal, Jose Scheinkman, Michael Woodford, and seminar participants at New York University, Chicago, and Northwestern for helpful comments. I would especially like to thank a referee and the editor, René Stulz, whose comments substantially improved the style and content of the paper.

[1] Another application might be dual trading. See, for example, the *Wall Street Journal,* February 7, 1992 article. For analyses of dual trading see Grossman (1989), Sarkar (1993), and the references therein. "Front-running" in, for example, currencies markets is another relevant example.

one buys an undervalued asset or sells an overvalued one (e.g., Friedman (1953)). Price therefore should be expected to move in the right direction, and price movements convey the "correct" information. This conclusion is essentially sound (with some caveats, such as those in Hart and Kreps (1986)), if one only considers trading based on the fundamental value of an asset. What this argument ignores are the externalities that one type of speculation might impose on another.

We consider a two period market, along the lines of Kyle (1985), in which the insider trades in the first and second periods, and the non-fundamental speculator trades only in the last period. Noise traders trade every period. The speculator, prior to trading, observes the realization of one component of first period noise trades. This observation allows the speculator to make better-than-market inferences about the asset's value from past prices. This advantage enables him to profitably trade.

The speculator[2] profits because he can free ride on the insider's trades—note that if the insider doesn't trade, then the speculator's observation is worthless. As a result, the insider modifies his first period trade to convey less information. Price efficiency is therefore lower before the speculator enters the market. Another externality imposed by the speculator is that the information from his trades also allows the market to make better inferences about the insider's private information. Thus, the effect of the speculator's entry in the second period is to reduce market liquidity and increase price volatility. This externality also implies that, although the joint trades of the insider and the speculator cause more inside information to be incorporated into price in the second period, this occurs at the expense of insider profits. Thus, there is less incentive to gather information. The overall informativeness of price may actually fall.

Specifically, we show that the insider's first period trade can be decomposed into two parts. The first component exploits the difference between the insider's private signal and price, and depends on the intertemporal structure of market liquidity. This first component, however, does not directly account for the influence (through prices) of the insider's trade on the speculator's beliefs. The second component is positively related to the gap between the speculator's (expected) belief and the insider's information. Thus, this component is designed to increase *future* profits by moving the speculator's belief away from the asset's true value.

In the absence of a speculator, the second component will be zero. The effect of the speculator's presence is to increase the (absolute) magnitudes of both components of the insider's trade, relative to the case with no speculator. The first component increases because of the anticipated fall in second period liquidity. However, we show that as long as the precision of the speculator's estimate is not too high, the manipulative component will dominate the first component. Insider trading in the first period is therefore lower than the

[2] From now on, we will use the term "speculator" to exclusively mean a non-fundamental speculator.

monopoly level. As a result, liquidity is higher, but price efficiency is decreased in the period(s) prior to the speculator's entry.

In the second period, the speculator forms an estimate of the insider's information based on his noise trading observation and past prices. The second period trading decisions of the insider and the speculator resemble "Cournot" models of competition. Thus, the insider continues to trade less than he would as a monopolist, but the aggregate informed trade is larger in the sense that the induced price movement is on average larger. As a result, second period market liquidity is lower and price volatility is higher than in the absence of a speculator. Because of the greater aggregate informed trading, price efficiency given a fixed amount of inside information increases. However, insider profits are lower than in a market with no speculator. Thus, the incentive to acquire information also falls, which generally leads to an overall decline in the information content of price.

Certain empirically interesting trading patterns can arise with non-fundamental speculation. First, speculators' trades are positively correlated with the previous period's price changes. Thus, speculators appear to be chasing trends. Second, the speculator and insider will frequently trade in opposite directions. In addition, if the noise trading component that is unobserved by the speculator is sufficiently large and of the same sign as the insider's first period trade, then the insider will reverse his trade between the first and second periods. In this case, first period price "overreacts," and the insider will profit from his contrarian trade, while the speculator loses by chasing trends.

The possibility that profit-maximizing investors might trade in opposite directions has been previously noted by Hirshleifer, Subrahmanyam, and Titman (1994), in a model in which investors receive inside information at different times. In their model, informed investors reverse trades to reduce risk associated with future price movements. We extend this result by showing that reversal can occur even with risk-neutral traders, if some traders base their decisions only on order flow and price movements. In this case, the trading asymmetry means that one group of traders loses money on average.

We also examine how equilibrium variables change with the quality of the speculator's observation. We show that the volume of the insider's first period trade is nonmonotonic in the quality of the observation. This produces a corresponding nonmonotonicity in market liquidity and efficiency. Initially, as the precision of the insider's estimate improves, the intensity of the manipulative part of the insider's trade dominates the first ("information trading") component. Thus, the volume of insider trades falls, liquidity rises, and efficiency falls in the first period, as the quality of the observation increases. Beyond a particular threshold of quality, however, second period liquidity becomes low enough that the benefits of increased trade in the first period outweigh the gains from manipulation. When this happens the insider trades more in the first period than he does as a monopolist. First period efficiency then rises, but liquidity falls and price volatility increases. However, insider profits also fall with the quality of the speculator's observation. Thus, information acquisition is even further diminished.

The remainder of the article is organized as follows. The model is presented in the next section, and market equilibrium is defined. Equilibrium prices and trading strategies with and without non-fundamental speculation are then derived in Section I. Comparative static results concerning the quality of the speculator's observations are studied as well. Section II introduces endogenous information acquisition to the model. Section III studies the trading patterns of the model, and relates our results to recent research on herding and manipulation in financial markets. Sections IV and V discuss extensions of the basic model, Section VI concludes.

I. Order Flow Based Speculation

We consider a two period market for a risky asset along the lines of Kyle (1985). The risky asset has a liquidation value given by a random variable $\tilde{v} \sim N(p_0, \Sigma_0)$. The market is populated by four types of agents: an insider, a (non-fundamental) speculator, a market-maker, and noise traders. The insider trades in periods 1 and 2, while the non-fundamental speculator trades only in period 2. Noise traders trade every period. The insider knows the realization v of $\tilde{v}$. Noise traders trade a net aggregate quantity $\tilde{u}_t + \tilde{w}_t$ in each period $t = 1, 2$. For all t, $\tilde{u}_t$ and $\tilde{w}_t$ are each independent and normally distributed, $\tilde{u}_t \sim N(0, \sigma_u^2)$, $\tilde{w}_t \sim N(0, \sigma_w^2)$. Prior to trading in period 2, the non-fundamental speculator observes the first period order flow of noise trades $\tilde{w}_1$ (but *not* $\tilde{u}_1$). At the end of each period, after observing the total order flow ω_t for that period, the market-maker sets a price at which the orders are transacted.

Price in period t, p_t, is required to satisfy the zero-profit (or efficiency) condition.

$$p_t = \mathrm{E}(\tilde{v} \mid M_t) \tag{1}$$

where M_t is the market-maker's information set, $M_t = \{\omega_s: s \leq t\}$. As in Kyle's (1985) discrete time model, the market-maker sets price depending on the order flow; other traders know the previous period's price and the price rule (1), but do not choose orders conditional on the current (end-of-period) price. The insider's information sets I_t are $I_1 = \{v\}$ and $I_2 = \{v, w_1, p_1\}$, while the speculator's (period 2) information set is $U = \{w_1, p_1\}$.

Note that we assume the insider also observes w_1. This is not a strong assumption in our context, because the insider does not trade after the speculator. There is no information, other than p_1, which depends on w_1. In particular, the insider does not condition trades on any observation that is affected by the speculator's trade. Let m denote the speculator's estimate of v. To study the case in which the insider does not observe w_1, we only need to substitute $\mathrm{E}(m \mid p_1, v) = \mathrm{E}(m \mid u_1 + w_1)$ for m in the insider's second period objective function. Since u_1 and w_1 are not choice variables, the method of analysis does not change. Our general results also do not change. (The only quantitative change is that, at the margin, the insider trades more in the first period, and less in the second period.)

The information $\{w_1\}$ on the first period order flow allows the speculator to form an estimate m of v, based on the price observation p_1, which is superior to the market-maker's:

$$m = \mathrm{E}(v \mid w_1, p_1) = p_0 + \hat{\beta}(p_1 - \mathrm{E}(p_1 \mid w_1)) \tag{2}$$

where $\hat{\beta}$ is the "regression coefficient" on the price variable. Note that $m \neq p_1$, since p_1 represents the market-maker's estimate of v prior to the speculator's trade. The speculator can then profit on the difference $m - p_1$, because the noise trades in period 2, $\tilde{u}_2 + \tilde{w}_2$, will keep the order flow from revealing the speculator's information. Thus, the speculator has an incentive to enter the market.

The response of the insider to the speculator's presence is as follows. In the next section, we show that the insider's first period trade can be written in the form

$$a_1(v - p_0) + b_1(\mathrm{E}(m \mid I_1) - v),$$

where $\mathrm{E}(m \mid I_1)$ is the estimate of the speculator's belief based on the insider's first period information. The insider therefore not only trades on the difference between his information v and the beginning of period market estimate p_0, but also modifies his trade according to the anticipated divergence of the speculator's belief from the true value v. We show that $b_1 > 0$. Thus, if the insider expects the speculator to overestimate the asset's value, he increases his trade in an attempt to keep the speculator's belief away from v. This strategy has implications for the behavior of market prices.

We now turn to the construction of equilibrium.

A. *Construction and Analysis of Equilibrium*

The insider wishes to choose x_1 to maximize total expected profits

$$\mathrm{E}((\tilde{v} - p_1(\omega_1))x_1 + \mathrm{E}(\tilde{v} - p_2(\omega_1, \omega_2))x_2 \mid v) \tag{3}$$

given that x_2 is the trade that maximizes second period profits

$$\mathrm{E}((\tilde{v} - p_2(\omega_1, \omega_2))x_2 \mid v, p_1), \tag{4}$$

where $\omega_1 = x_1 + w_1 + u_1$, and $\omega_2 = x_2 + y + w_2 + u_2$.

In the second period, the speculator chooses the trade y to maximize

$$\mathrm{E}((\tilde{v} - p_2(\omega_1, \omega_2))y \mid w_1, p_1) \tag{5}$$

In addition, prices in equilibrium must satisfy

$$p_1 = \mathrm{E}(\tilde{v} \mid x_1 + w_1 + u_1) \tag{6}$$

$$p_2 = \mathrm{E}(\tilde{v} \mid x_2 + y + w_2 + u_2). \tag{7}$$

We can then show that there exists a (unique) linear equilibrium.

PROPOSITION 1: *There exists an equilibrium in which trading strategies and prices are of the form:*

$$x_1 = a_1(v - p_0) + b_1(\mathrm{E}(m \mid v) - v) \tag{8}$$

$$x_2 = a_2(v - p_1) - (1/2)A(m - p_1) \tag{9}$$

$$y = A(m - p_1) \tag{10}$$

where $m = \mathrm{E}(\tilde{v} \mid U)$, *the speculator's belief conditional on* $U = \{w_1, p_1\}$. *Prices follow*

$$p_t = p_{t-1} + \lambda_t \omega_t \qquad t = 1, 2 \tag{11}$$

The coefficients a_1, b_1, a_2, A, λ_1, λ_2, *and the conditional variances*

$$\Sigma_1 = \mathrm{var}(\tilde{v} \mid p_1) \tag{12}$$

$$\Sigma_2 = \mathrm{var}(\tilde{v} \mid p_1, p_2) \tag{13}$$

are defined explicitly in Appendix A.

Proof: See Appendix

The variances (12) and (13) measure how much of the insider's information is conveyed by price. (12) and (13), as well as the coefficients of (8)–(11), will of course generally depend on the quality of the speculator's information, which is defined by the parameter $k = (\sigma_w^2/\sigma_u^2)$. Table I gives the values of (8)–(13) for different levels of k. However, before further discussing the interpretation of this equilibrium, let us first consider the incentives of the speculator to actively trade in the market, and the effect of this activity on insider profits.

COROLLARY 1: *Let* π^s *denote the (unconditional) expected profit to the speculator. Let* π_t^i *denote the (unconditional) expected profit to the insider in period t, and let* $\pi^i = \pi_1^i + \pi_2^i$. *Analogous to* π^i, *let* π^m *denote the (monopoly) profit to an insider in the absence of a speculator. Then,* $\pi^m > \pi^i$ *and*

$$\pi^s = 2/9\, a_2 \mathrm{E}((v - p_1)^2 - (m - v)^2) > 0$$

Explicit expressions for π_1^i, π_2^i, *and* π^s *are derived in the Appendix.*

Proof: See Appendix.

Thus, the speculator expects to profit from trading, and therefore has an incentive to enter the market. Also, the speculator's presence decreases the gain from having inside information. This last fact, although expected, should not be completely obvious, since the speculator's activity might change price behavior (i.e., liquidity) in such a way that noise trading losses increase in the second period, which would generate additional surplus for the profit-maximizing traders.

Table I

Equilibrium Values and the Quality of the Speculator's Information

This table provides the values of equilibrium variables for different levels of the quality of the speculator's observations $k = \sigma_w^2/\sigma_u^2$. a_1, b_1 are the coefficients giving the intensities of the first and second components of the insider's first period trading strategy, while a_2 gives the intensity of the insider's second period trade. λ_t measures (the inverse of) market liquidity in period t. A gives the intensity of the speculator's trade. Σ_t denotes the variance of the asset's value conditional on price observations up to period t. π^i, π^s denote the insider's and speculator's unconditional expected profits. The superscript N denotes the corresponding variables for the model with no speculator. a_2, A, λ_t, π^i, and π^s are normalized to remove the effects of the increase in total noise trading variance. This normalization isolates the effects of the improvement in the precision of the speculator's estimate. The normalization also facilitates the comparison with the case of no speculator.

$(\sigma_w^2/\sigma_u^2) = k$	0.01	0.25	1	5.6808	20	No Speculator
			Panel A: *First Period Variables*			
a_1	0.79963	0.87456	1.22959	1.70492	2.79859	0.667115
b_1	0.24747	0.28569	0.39320	0.85891	1.63977	—
$\lambda_1/\lambda_1 N$	0.99993	0.99849	0.99624	1	1.00870	1
			Panel B: *Second Period Variables*			
$a_2/\sqrt{1+k}$	1.20101	1.17761	1.12775	1.03326	0.99528	1.20210
$A/\sqrt{1+k}$	0.80067	0.78507	0.75183	0.68884	0.66352	—
$\lambda_2/\lambda_2 N$	1.00091	1.02079	1.06593	1.16340	1.20780	1
			Panel C: *Conditional Variances*			
Σ_1	0.692099	0.69368	0.69615	0.69202	0.68207	0.69202
Σ_2	0.345695	0.33862	0.32103	0.27262	0.24281	0.34601
			Panel D: *Profits*			
π^i/π^N	0.99859	0.97210	0.91328	0.78927	0.73158	1
$\pi^s/\sqrt{1+\text{k}}$	0.00057	0.01291	0.04065	0.10111	0.13035	—

Interpretation

As we discussed previously, the insider's strategy can be decomposed into two parts. The first part is designed to profit from the insider's informational advantage, and exploits the difference between the inside information v and the market's initial valuation p_0 of the asset. The second part takes into account the impact of the insider's trade on the speculator's belief. Since the insider will be "competing" with the speculator in the next period, this component increases future profits by keeping m away from v. We will see later that the coefficient b_1, which measures the intensity with which the insider attempts to manipulate the speculator's belief, increases with the quality of the speculator's observation.

The effect of the manipulative part of the insider's strategy is to decrease his first period trade. At the same time, however, the first part of the insider's strategy $a_1(v - p_0)$ will tend to have a higher intensity a_1 than if he were a monopolist because of the anticipated lower liquidity in the second period caused by the speculator's entry. As our comparative static analysis will later show, if the accuracy of the speculator's estimate (which can be measured by k) is not too high, then the manipulative effect will dominate the increase in a_1. Thus, the insider's first period trade will be lower with the existence of a speculator than without. As a result of this diminished trade, price efficiency will be lower in the period prior to the speculator's entry.

Will the insider ever end up trading in the opposite direction of $(v - p_0)$? (That is, can the second component be opposite in sign and larger in magnitude than the first component?) In equilibrium, the answer is no. If the insider should trade in the opposite direction, speculators (and market-makers) would anticipate this and take it into account when forming their estimates. Thus, the trade would be just as informative as (but obviously less profitable than) a trade with the same intensity, but in the right direction. For this reason the optimal strategy of an insider would be to diminish the informativeness of his trade, but not to trade in the "wrong" direction.

In the second period, the insider and the speculator make trading decisions in a way similar to "Cournot-Nash" competitors. That is, they form optimal trades based on the residual difference after accounting for the other investor's trade. Of course, since the traders are asymmetrically informed, the residual differences they perceive are not identical. Thus, the insider subtracts the quantity $A(m - p_1)$ from the price equation, while the speculator subtracts his expectation of the insider's trade.

Note that the speculator is able to trade only because he can free ride on the insider's information. For example, if there were no insider trading, the speculator's observations would be useless. The effect of the speculator's activity is to *decrease* liquidity $(1/\lambda_2)$. Consequently, price volatility (which can be measured by $\lambda_2^2((A^2 + 1)\sigma_u^2 + \sigma_w^2))$ is higher than in the absence of a speculator.

Another result of the Cournot type of competition in the second period is that, although the insider trades less than he would as a monopolist, aggregate speculative trading increases. More information is conveyed in the second period with a speculator than in a monopolist market. Thus, given a fixed level of insider information, the efficiency of price is lower in periods prior to the speculator's entry, but higher afterwards. Nonetheless, as Corollary 1 shows, expected insider profit is lower. Thus, the incentives to acquire information are diminished. As we show in the next section, this implies that the overall informativeness of price generally *falls*.

To understand more completely the impact of the speculator's presence on market activity, we now compare the equilibrium of Proposition 1 to the equilibrium without a speculator, and then consider actual equilibrium levels of parameters for different values of $k = \sigma_w^2/\sigma_u^2$. σ_w^2/σ_u^2 measures the precision of the speculator's information, relative to the residual market noise. The higher σ_w^2/σ_u^2 is, the greater is the speculator's inferential ability, relative to the market as a whole, and the greater is his scope for free riding.

PROPOSITION 2: *In the absence of a non-fundamental speculator, an equilibrium of the form (8)–(11) exists in which (we use the superscript N to denote corresponding strategy parameters for the "no speculator" model, thus, we have* a_1^N *instead of* a_1, λ_1^N *instead of* λ_1, *etc.):*

$$a_1^N = c\,\frac{\sigma_u\sqrt{1+k}}{\Sigma_0} \tag{14}$$

$$b_1^N = 0$$

$$a_2^N = \frac{\sigma_u\sqrt{(1+k)(1+c^2)}}{\Sigma_0} \tag{15}$$

$$A^N = 0$$

$$\lambda_1^N = \frac{c}{(1+c^2)\sqrt{1+k}}\,\frac{\Sigma_0}{\sigma_u} \tag{16}$$

$$\lambda_2^N = \frac{\Sigma_0}{2\sigma_u\sqrt{(1+k)\,(1+c^2)}} \tag{17}$$

where c solves the equation

$$\frac{c^4}{\sqrt{1+c^2}} - \frac{1}{\sqrt{1+c^2}} + c = 0 \tag{18}$$

(so that $c \approx 0.66711458$).

Proof: See Appendix.

Similarly, we can obtain values for traders' profits and the variance of $\tilde{v}$ conditional on prices.

COROLLARY 2: *With no fundamental speculator, the insider's expected profit is*

$$\begin{aligned}\pi^{iN} &= \left(c\left(1 - \frac{c^2}{1+c^2}\right) + \left(1 - \frac{2c^2}{1+c^2}\right)\frac{\sqrt{1+c^2}}{2}\right)\sqrt{1+k}\,\Sigma_0\sigma_u \\ &\approx .877597\sqrt{1+k}\,\Sigma_0\sigma_u\end{aligned} \tag{19}$$

Conditional variances are

$$\Sigma_1 = \frac{\sigma_v^2}{1+k^2} \approx .69202\sigma_v^2 \qquad \Sigma_2 = \frac{\Sigma_1}{2} \approx .34601\sigma_v^2 \tag{20}$$

The values of equilibrium variables depend on the level of $k = \sigma_w/\sigma_u$. These values for $k = 0.01, 0.25, 1, 5.68$, and 20 are shown in Table I, both for a market with a speculator and one without. If $k = 1$, then the variance of the noise trading component observed by the speculator is half of the noise trading variance faced by the market-maker. As k falls, the precision of the speculator's estimate also falls. In contrast, for large ks, the speculator is able to infer the insider's information with a high degree of accuracy.

As expected, insider profits fall as the quality of the speculator's observation improves. The greater and more accurate the speculator's trade, the larger the externality he imposes on the insider.

It is also interesting to note that the better the speculator's information, the more intensely the insider tries to manipulate the speculator's belief. That is, the greater is the coefficient b_1 of the manipulative component $b_1(m - v)$ of the insider's first period strategy. The coefficient a_1 of the component $a_1(v - p_0)$, which trades on the private information, also increases (because of the anticipated increase in second period liquidity). The net effect of the trade can be measured by its impact on end of period price efficiency, Σ_1. For values of k which are not too large, Σ_1 falls with the quality of the speculator's information. Thus, the "disinformation" caused by the manipulative component dominates the increase in the first component.

One should notice, however, that the insider's first period trading strategy is not monotonic in k. At $k = 0$, the speculator does not observe any nonpublic data, so that the model collapses to one of a monopolist insider. As k increases from 0, the insider decreases his overall trading intensity because of the impact of the manipulative part of his strategy. However, after k passes a certain threshold (about $k = 2.5$), the speculator's inferences become precise enough (and second period liquidity becomes low enough), that it becomes advantageous to the insider to start increasing his trade to take advantage of the higher liquidity during the first period. a_1, the coefficient of the first component of the insider's first period trade, starts to increase at a relatively faster rate than b_1. At $k \approx 5.6807$, the insider trades the same net amount in the first period as he would as a monopolist, which can be seen from the fact that Σ_1 is the same in both cases. For $k > 5.6807$, the insider's first period trade is greater than a monopolist's. Thus, first period liquidity is *lower* and Σ_1 is lower than in a monopoly market when k is large (see, for example, the case $k = 20$ in Table I).

However, it seems that $k < 2.5$ should be considered the "normal" case. Dealers and other agents who observe nonpublic order flow data tend to have special access to only a (relatively small) part of the market (while $k \geq 2.5$ implies that more than 70 percent of market noise trading is observed). In this "normal" case, the effect of speculation is to decrease first-period efficiency, while first-period liquidity increases slightly.

Note that, after factoring out the effect of increased noise trading σ_w^2, the second period trading intensities a_2 and A both decrease with k. This occurs because second period liquidity declines with k (whether or not changes in σ_w^2 are included), due to the improved inferential ability of the speculator.

In summary, liquidity falls and efficiency (given a fixed amount of inside information) increases after the entry of the speculator. Liquidity also falls and efficiency increases in periods prior to the speculator's entry, if the quality of the speculator's observation is sufficiently high. However, first period efficiency is lower and liquidity is higher than in a monopoly market if $k = \sigma_w^2/\sigma_u^2$ is not very large.

II. Information Acquisition and the Information Content of Price

The simplest way to introduce information acquisition into our model is to assume that a potential insider who spends the amount c learns the true realization $\tilde{v}$ with probability $\psi(c)$, where ψ is an increasing function of c. That is, if an investor spends c, then with probability $\psi(c)$, he learns v, and with probability $1 - \psi(c)$, he learns nothing. This follows Admati and Pfleiderer (1989). We assume that ψ is concave and twice continuously differentiable.[3]

One can think of the potential insider as spending resources in research to discover potentially valuable information about a certain firm or project. The more resources he spends, the more likely he is to discover valuable nonpublic information. For clarity, we assume that in the beginning of period 1, other agents in the market know whether the potential insider has managed to acquire the privileged information v. This last assumption is not really necessary.[4]

Thus, the potential insider who expects to earn profit π from private information will spend c^* to acquire information, where c^* is given by the first-order condition

$$\psi'(c^*)\pi \leq 1 \tag{21}$$

with strict equality if $\psi'(\infty)\pi < 1$ and $\psi'(0)\pi \geq 1$. Thus, c^* (and $\psi(c^*)$) falls with π since

$$\frac{dc}{d\pi} = \frac{-\psi'(c)}{\pi\psi''(c)} > 0 \tag{22}$$

The informational content of price at the end of period 2 can be measured by (the inverse of)

$$\mathrm{IC} = \psi(c)\Sigma_2 + (1 - \psi(c))\Sigma_0, \tag{23}$$

where Σ_2 is the end of second period conditional variance of $\tilde{v}$, $\Sigma_2 = \mathrm{var}(\tilde{v}\ p_1, p_2)$. IC_2 measures the expected conditional variance at the end of the trading horizon. The *lower* IC_2 is, the *greater* is the information content of last period price.

[3] Papers that study information acquisition in financial markets include Grossman and Stiglitz (1980), Verrecchia (1982), Admati and Pfleiderer (1988, 1989), Holden and Subrahmanyam (1992), and Hirshleifer, Subrahmanyam, and Titman (1994).

[4] If we don't make this assumption, we need to multiply the speculator's objective function by the equilibrium probability that an insider exists $\psi(c)$. Since ψ depends only on c, this modification does not change the speculator's decision. We also need to change equations (5) and (6) so that the total order flow is $z/2$ with probability $(1 - \psi(c))$. The details of the analysis and all qualitative results are the same.

Equation (23) implies that resources used to acquire inside information fall as expected insider profits fall. Thus, as long as Σ_2 is not much lower than Σ_2^N, the second period conditional variance without a speculator, the information content of price will be lower with a speculator than without. Moreover, the end of the trading horizon may not be the most appropriate time to measure the informativeness of price. Presumably, what defines the *end* of the possibility of trading is that the asset's value becomes publicly known. In this case, there are no welfare gains from improving the informativeness of price near the end.

If we use the first period expected conditional variance

$$\mathrm{IC}_1 = \psi(c)\Sigma_1 + (1 - \psi(c))\Sigma_0$$

as our measure of price informativeness, then as long as $\sigma_w^2/\sigma_u^2 < 5.68$ price informativeness is lower with the existence of a speculator, because $\Sigma_1 > \Sigma_1^N$ for this case.[5]

III. Trading Patterns, Herding, and Manipulation

Recent research has studied the phenomenon of herding in financial markets (e.g., Welch (1992), Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994)). While our model does not consider how investors might herd on a particular asset, the equilibrium of Proposition 2 has two interesting characteristics often associated with herding in general and with institutional investors in particular.

The trade of the speculator, equation (10), is positively correlated with the previous period's price change:

$$\mathrm{cov}(y, \Delta p_1) = A(\beta - \lambda_1)\lambda_1 (\alpha\Sigma_0 + \sigma_u^2) > 0$$

since $\beta - \lambda_1 > 0$. Thus, speculators' trades appear to chase trends.

Another consequence of the speculator's lack of fundamental information is that the direction of his trade may be different from the insider's. Also, it is possible for the insider to reverse the direction of his first period trade in the second period. Thus, an insider may buy (sell) in the first period and sell (buy) in the second period, while the speculator at the same time buys (sells) in the second period.

These patterns of trades have several implications. For example, if the insider and speculator trade in opposite directions, the speculator's trend chasing (or "feedback") strategy will ex-post appear to be a money-losing

[5] Of course, in a more general model, the speculator might trade for more than one period. Nevertheless, IC_t will generally be larger than in a market without a speculator, as long as the number of periods in which the speculator is active does not significantly exceed those in which he is not. This is because, roughly speaking, Σ_t (and consequently IC_t) will be larger as long as the speculator has been active for less than $t/2$ periods. Moreover, if one thinks of the speculator as reacting to other investors' orders, so that he continually trades in response to insider and noise trades, then one can show that IC_t always falls for all t (see Section V).

strategy, when in fact it is an optimal strategy and the speculator is behaving rationally. Before developing these implications further, however, let us first show that these patterns of trades can occur quite frequently.

PROPOSITION 3: *i.) For any fixed v and w_1, the mispricing estimated by the insider, $v - p_1$, is decreasing, while the mispricing estimated by the speculator, $m - p_1$, is increasing in u_1, the noise trading component that is unobserved by the speculator. ii.) For any given v and w_1, if the volume of the observed noise trades $|u_1|$ is large enough, then the insider and speculator will trade in opposite directions in the second period, regardless of the sign of u_1. iii.) If, in addition, u_1 has the same sign as the insider's first period trade x_1, then the insider will reverse his trade between the first and second periods.*

Proof: See Appendix.

(i.) indicates the tendency for the speculator's and the insider's beliefs concerning how the asset is mispriced to diverge. Thus, as long as the volume of unobserved noise trades is large enough, regardless of its sign, they will trade in different directions, as shown in (ii.) If u_1 has the same sign as the insider's trade, the market will overreact. Price overshoots the asset's value and the insider profits by making a "contrarian" trade in the second period. In the process, he reverses his first period trade.

Hirshleifer, Subrahmanyam, and Titman (1994) find similar patterns of trades in a model in which traders receive inside (fundamental) information at different times. In their model, insiders reverse their trades to reduce risk. Our result shows that reversal can occur even if traders are *risk-neutral,* if one group of traders base their decisions primarily on price movements. Of course, one difference is that if the insider and speculator trade in opposing ways, then the speculator loses money on average. If both traders had fundamental information, then each trader would expect the other to earn positive profits.

It is also useful to connect our work with recent papers on market manipulation (e.g., Kumar and Seppi (1992), Froot, Scharfstein, and Stein (1992), Allen and Gale (1992)), in which an uninformed investor profits because his trades are confused with those of an insider. In our model, it is an insider who has a natural incentive to manipulate other profit maximizing traders.[6] Also, note how the volume of noise trades may cause an ex post observer to exaggerate the insider's ability to manipulate the market. The empirical pattern we observe above, in which an insider reverses his trade and both the speculator and first period noise traders lose money, appears to be manipulative. However, although the insider's attempt to manipulate the speculator's belief contributes to this pattern, it is primarily the speculator's action of trading on the basis of price movements that leads to his losses.

[6] A model that studies informed manipulation is Kyle (1984).

IV. Extensions of the Model

In a previous version of this article (Madrigal (1994)), we analyze a continuous time version of the model of Section II, in which the speculator trades every period, and each noise trading component follows a Brownian motion. The speculator's equilibrium strategy turns out to be of the same form as equation (10). Similarly, the insider's strategy is of the form of equation (8), so that there are also informative and manipulative components. We show that, in equilibrium, the speculator's observations are completely revealed by prices at all points in time (this is possible because equilibrium strategies have infinite variation). The stochastic process of price is then the same as in a Kyle (1985) continuous auction, but with the noise reduced by the amount of the speculator's observation. Liquidity at all points in time is therefore lower than in the absence of a speculator. However, equilibrium conditional variance does not depend on order flow noise (this is also the case in Kyle (1985)). Thus, price efficiency given a fixed amount of inside information is unchanged. If information acquisition is endogenous, then the information content of price unambiguously falls.

The results of Sections II and III are therefore supported, if not strengthened, by the extension to continuous time.

V. Other Forms of Non-Fundamental Speculation

Suppose now that the speculator observes w_2, instead of w_1, in the beginning of period 2. That is, he observes a fraction of the period's noise trades ahead of the rest of the market. In this case, the speculator can profit by trading in the opposite direction of his observation (his optimal trade is $-w_2/2$). The results are similar. Insider profits and second period liquidity fall because of the speculator's activity. The main difference is that the speculator's profits do not come from making inferences about the insider's information. He profits by being able to estimate the direction of the current period mispricing of the asset: the asset will (on average) be overpriced, if w_2 is positive, or underpriced, if w_2 is negative. Since the speculator is not free riding on the insider's information, the insider has no incentive to try to manipulate him. The insider then increases his first period trade, in anticipation of the second period decrease in liquidity (the second component of the insider's trade in Proposition 1 is zero). As a result, first period liquidity also falls.

The externality imposed by the speculator is solely of the second sort: the information conveyed by his trades reduces the noise in the second period order flow, and so allows the market to form improved inferences from prices.

What is interesting about this form of non-fundamental speculation is that the speculator can profitably trade because his observation is uncorrelated with fundamentals. Taken together with the model of Section I, this reflects a more general point: any special information on order flows enables profitable trading. If the observation is correlated with past order flows, then, as in our main model, the speculator can profit by making superior estimates of the asset's value. If no such correlation holds, however, then the trader knows that the price change associated with this order flow observation will be an overreaction. Thus, he can profit by trading in the opposite direction.

Other types of non-fundamental speculation can be studied using the analysis in Section I. For example, the speculator might have special information on the insider's trading strategy. This can occur for several reasons. The speculator might have better information than the market about the precision of the insider's private signal. Alternatively, the insider might possess an objective function that does not depend only on trading revenues. For instance, the insider may incur trading costs or may have a private (nonmarket) value for the asset. If the speculator has special information concerning a parameter of the insider's objective function, he would be able to profitably trade. The results for many of these examples, will be qualitatively the same as those obtained in Section I (see Madrigal (1994)).

VI. Conclusion

In this article, we study the effect on prices of traders without privileged fundamental information, who nevertheless can speculate profitably in a market in which prices are set efficiently. Since these traders' information do not involve fundamentals, they can obtain profits only by free riding on the insider's trades. This reduces insider profits. Another externality imposed by non-fundamental speculators is that the information revealed by their trades enables the market at large to make better inferences from prices. This lowers market liquidity, raises price volatility, and increases noise trader losses in periods when the speculator is active. In periods before the speculator's entry, the insider generally cuts back on his trade, thus revealing less information to the market. This decrease in trading intensity occurs despite the anticipated fall in liquidity accompanying the speculator's entry, because of the insider's attempt to manipulate the speculator's beliefs.

This last property differentiates the effects of non-fundamental speculation from trading with fundamental information.[7] If all profit-maximizing traders possess fundamental information, price efficiency would improve at all times. With non-fundamental speculation, the effect of the reduced insider trading is to decrease price efficiency. If the speculator subsequently trades for a long enough time, then price efficiency might eventually improve, because of the effect of the joint trades of the two types of investors. However, note that the longer the speculator trades, the greater is the externality imposed on the insider, so that the incentive of the insider to acquire information is even

[7] Market-maker models in which more than one rational trader exists are Admati and Pfleiderer (1988, 1989) and Holden and Subrahmanyam (1992), which have multiple insiders. Madrigal and Scheinkman (forthcoming) study a one period model with heterogeneously informed insiders. Duffie and Huang (1986) study a competitive multiperiod securities market in which one trader's filtration is finer than another trader's. Our article is the only article we are aware of that includes rational speculators that do not possess inside fundamental information.

lower. (Moreover, in a continuous time market in which the insider and the speculator trade every period, price efficiency is always lower).

Another property that differentiates our model from one with multiple informed traders is that the speculator will often "chase trends." Also, the insider and speculator can trade in opposite directions.

Taken together, these results confirm the following conjectures about the presence of non-fundamental speculators in a market. Since these speculators are trading based on imperfect inferences, they make prices more volatile. Because their trades can reduce order flow noise, their activity does not always strictly reduce price efficiency (although they don't necessarily increase efficiency either), given a fixed amount of undisseminated information in the market. What their presence does is to affect adversely the incentives of other traders to spend resources on gathering useful information. It is this externality that diminishes the information content of price.

Appendix

PROPOSITION 1: *There exists an equilibrium in which trading strategies and prices are of the form:*

$$x_1 = a_1(v - p_0) + b_1(\mathrm{E}(m \mid v) - v) \tag{A1}$$

$$x_2 = a_2(v - p_1) - (1/2)A(m - p_1) \tag{A2}$$

$$y = A(m - p_1) \tag{A3}$$

where $m = \mathrm{E}(\tilde{v} \mid U)$, *the speculator's belief conditional on* $U = \{w_1, p_1\}$. *Prices follow*

$$p_t = p_{t-1} + \lambda_t \omega_t \qquad t = 1,\ 2 \tag{A4}$$

Let $k = \sigma_w^2 / \sigma_u^2$, *equilibrium parameters are given by*

$$\lambda_1 = \frac{q}{\sqrt{1 + k(1 + q^2)}} \frac{\Sigma_0^{1/2}}{\sigma_u} \tag{A5}$$

$$a_1 = D\left(1 - \frac{5\lambda_1}{18}(2\lambda_1 - \beta)a_2^2 - \frac{\beta A}{6}\right) \tag{A6}$$

$$b_1 = \frac{DA}{2}\left(\frac{2}{3}\lambda_1 + \frac{\beta}{3}\right) \tag{A7}$$

where

$$D = \lambda_1^{-1}\left(2 - \frac{\lambda_1}{2\lambda_2} + \frac{5}{12}A(2\lambda_1 - \beta) - \frac{\beta A}{3}\right)^{-1} \tag{A8}$$

$$a_2 = \left(\frac{(1+k)\sigma_u^2}{(\Sigma_1 + 2/3\Sigma_{vm} - 1/9\Sigma_{mm})}\right)^{1/2} \tag{A9}$$

$$\lambda_2 = \frac{1}{2a_2} \tag{A10}$$

$$m = \beta(x_1 + u), \qquad \beta = \frac{q\sqrt{1+k}}{1 + (1+k)q^2}\frac{\Sigma_0^{1/2}}{\sigma_u} \tag{A11}$$

$$A = \frac{2}{3}a_2 \tag{A12}$$

$$\Sigma_1 = \mathrm{var}(\tilde{v}\,|\,p_1) = \frac{\Sigma_0}{1+q^2} \tag{A13}$$

$$\Sigma_{vm} = \mathrm{cov}(\tilde{v}, \tilde{m}\,|\,p_1) = \frac{q^2 k\Sigma_0}{(1 + (1+k)q^2)(q^2+1)} \tag{A14}$$

$$\Sigma_{mm} = \mathrm{var}(\tilde{m}\,|\,p_1) = \Sigma_{vm} \tag{A15}$$

$$\begin{aligned}\Sigma_2 &= \mathrm{var}(\tilde{v}\,|\,p_1, p_2)\\ &= \frac{A^2}{4}\frac{(\Sigma_{mm}\Sigma_1 - \Sigma_{vm}^2) + \Sigma_1(\sigma_u^2(1+k) - 3/2\,A^2\Sigma_{vm})}{a_2^2\Sigma_1 + (A^2/4)\Sigma_{mm} + \sigma_u^2(1+k)}\end{aligned} \tag{A16}$$

The constant q *is defined by*

$$q = (d\sqrt{1+k})^{-1}\left(1 - \frac{a_2}{3}(\lambda_1 + \beta)\right)\frac{\Sigma_0^{1/2}}{\sigma_u} \tag{A17}$$

and

$$d = \lambda_1\left(2 - \lambda_1 a_2 - \frac{2A\lambda_1}{3}\right) - \frac{\beta}{3}\left(2\lambda_1 A + \frac{A\beta}{2}\right) \tag{A18}$$

Note that λ_1, β, Σ_1, Σ_{vm}, and Σ_{mm} depend only on q and k. Thus, a_2 depends only on q and k. Also, A and λ_2 can be written as functions only of a_2. Once these variables are defined, so are a_1, b_1, and Σ_2. Thus, all these variables can be written as functions of q and k. Moreover, the RHS of equation (A17), which defines q, depends only on q and k. This means that for any given value of $k = \sigma_w^2/\sigma_u^2$, all the equilibrium quantities (A1)–(A16) can be derived. Examples of values of these variables are presented in Table I. The parameter q

measures the overall intensity of the insider's first period trade (8), once we net out the effects of its two components. Since only the insider trades in the first period, the (net) intensity of his trade can be measured by the amount of information it conveys. This informativeness is proportional to q, as can be seen by the fact that Σ_1, which determines (the inverse of) first period price efficiency, is strictly negatively related to q.

Proof of Proposition 1: Given equations (9) and (10), in the second period, the insider will choose a trade x to maximize

$$\mathrm{E}(v - p_2)x = \mathrm{E}(v - p_1 - \lambda_2(x + A(m - p_1) + u_2 + w_2))x \tag{A19}$$

or

$$x = \frac{v - p_1 - \lambda_2 A(m - p_1)}{2\lambda_2} \tag{A20}$$

Note that the insider knows m, since w_1 is in his information set. The second period profit of the insider is then

$$\begin{aligned}\pi_2 = (v - p_2)x &= \left(\frac{v - p_1}{2} - \lambda_2 \frac{A}{2}(m - p_1)\right)\left(\frac{v - p_1}{2\lambda_2} - \frac{A}{2}(m - p_1)\right) \\ &= \frac{(v - p_1)^2}{4\lambda_2} - \frac{A}{4}(m - p_1)(v - p_1) \\ &\quad - \left(\frac{v - p_1}{2\lambda_2}\right)\frac{A\lambda_2}{2}(m - p_1) + \frac{\lambda_2 A^2}{4}(m - p_1)^2\end{aligned} \tag{A21}$$

Using $(m - p_1)(v - p_1) = (m - v + v - p)(v - p_1) = (m - v)(v - p) + (v - p)^2$ and $(m - p_1)^2 = (m - v + v - p_1)^2 = (m - v)^2 + 2(v - p_1)(m - v) + (v - p_1)^2$, we can rewrite π_2 as

$$\begin{aligned}\pi_2 &= (v - p_1)^2\left(\frac{1}{4\lambda_2} - \frac{A}{2} + \frac{\lambda_2 A^2}{4}\right) \\ &\quad + (m - v)(v - p_1)\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{2}\right) + \frac{\lambda_2 A^2}{4}(m - v)^2.\end{aligned} \tag{A22}$$

Thus, in the first period, the insider will choose x_1 to maximize

$$\mathrm{E}((v - p_1)x_1 + \pi_2(x_1)) \tag{A23}$$

where $\pi_2(x)$ is defined by (A23). Under the insider's conjecture that the speculator updates by $m = \beta(x_1 + \tilde{u}) + p_0$ and that $p_1 = p_0 + \lambda_1(x_1 + u)$, we have the first order condition

$$
\begin{aligned}
&\mathrm{E}(v - p_0 - 2\lambda_1 x_1) + \mathrm{E}(v - p_0 - \lambda_1 x_1)\\
&\qquad \times \left(\frac{-\lambda_1}{2\lambda_2} + \beta\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{2}\right) - 2\lambda_1\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{4}\right)\right) \qquad \text{(A24)}\\
&\qquad\qquad + \mathrm{E}(m - v)\left(\frac{\lambda_1 A}{2} - \frac{\lambda_1\lambda_2 A^2}{2} + \frac{\lambda_2 A^2}{2}\beta\right) = 0
\end{aligned}
$$

To find an expression for x_1, in terms of exogenous variables, we need to find an expression for $\mathrm{E}(m \,|\, I_1)$, where $I_1 = \{v\}$ is the insider's period 1 information set. The complication here is that m in turn depends on the (speculator's conjecture of the form of) x. We proceed as follows. First, we conjecture then verify that $\mathrm{E}(m \,|\, v)$ is of the form:

Claim 1: $$\mathrm{E}(m \,|\, v) = \gamma(v - p_0).$$

Proof: Under this assumption,

$$x_1 = a_1(v - p_0) + b_1\mathrm{E}(m - v) = (a_1 + b_1\gamma)(v - p_0) \equiv \alpha(v - p_0) \qquad \text{(A25)}$$

Using the projection theorem, the speculator's estimate m in the second period is given by

$$
\begin{aligned}
m &= \mathrm{E}(v \,|\, \alpha(v - p_0) + u_1) = \frac{\alpha\Sigma_0}{\alpha^2\Sigma_0 + \sigma_u^2}(\alpha(v - p_0) + u_1) + p_0\\
&\equiv \beta(\alpha(v - p_0) + u_1) + p_0
\end{aligned} \qquad \text{(A26)}
$$

Thus,

$$E(m - v \,|\, v) = \beta\alpha(v - p_0) + p_0 - v = (\beta\alpha - 1)(v - p_0). \qquad \text{(A27)}$$

However, under equation (A25),

$$x_1 = (a_1 + b_1(\beta\alpha - 1))(v - p_0) = A(\alpha)(v - p_0) \qquad \text{(A28)}$$

so that if

$$A(\alpha) = \alpha = (1 - b_1\beta)^{-1}(a_1 - b_1), \qquad m = \beta(x_1 + \tilde{u}),$$

and

$$E(m \,|\, v) = \beta\alpha(v - p_0)$$

then the conjecture is verified. Q.E.D. of Claim 1

A consequence of Claim 1 is that x_1 can be written as

$$x_1 = \alpha(v - p_0) \qquad \text{(A29)}$$

Also, under the result of Claim 1, $m = \mathrm{E}(\tilde{v}|x_1 + \tilde{u}) = \beta(x_1 + \tilde{u}) + p_0$, with β given by equation (A26). Thus, substituting for m, we have $\mathrm{E}(m - v) = \beta x_1 - (v - p_0)$, and the first order condition (A24) can be written as

$$\begin{aligned}(v - p_0)\Bigg(1 - \frac{\lambda_1}{2\lambda_2} - 2\lambda_1\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{4}\right) + \beta\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{2}\right) \\ - \left(\frac{\lambda_1 A}{2} - \frac{\lambda_1\lambda_2 A^2}{2} + \frac{\lambda_2 A^2\beta}{2}\right)\Bigg) \\ + x_1\Bigg(-2\lambda_1 - \lambda_1\left(\frac{-\lambda_1}{2\lambda_2} - 2\lambda_1\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{4}\right)\right) \\ + \beta\left(\left(\frac{-A}{2} + \frac{\lambda_2 A^2}{2}\right)(-\lambda_1) + \left(\frac{\lambda_1 A}{2} - \frac{\lambda_1\lambda_2 A^2}{2} + \frac{\lambda_2 A^2\beta}{2}\right)\right)\Bigg) = 0\end{aligned} \tag{A30}$$

or

$$x_1 = d^{-1}c(v - p_0) \tag{A31}$$

where c and d are the coefficients of $v - p_0$ and x_1 in equation (A30). c and d simplify to

$$d = \lambda_1\left(2 - \frac{\lambda_1}{2\lambda_2} + A\lambda_1\left(\frac{\lambda_2 A}{2} - 1\right)\right) - \beta A\left(\lambda_1(1 + \lambda_2 A) + \frac{\lambda_2 A\beta}{2}\right) \tag{A32}$$

$$c = 1 + \frac{\lambda_1}{2}\left(A - \frac{1}{\lambda_2}\right) - \frac{\beta A}{2} \tag{A33}$$

Thus, x_1 is of the form (A29) with $\alpha = d^{-1}c$. λ_1 is given by

$$\lambda_1 = \frac{\alpha\Sigma_0}{\alpha^2\Sigma_0 + (1 + k)\sigma_u^2} \tag{A34}$$

and Σ_1 by

$$\Sigma_1 = \Sigma_0 - \frac{(\mathrm{cov}(\tilde{u}, x_1 + \tilde{u}_1 + \tilde{w}_1))^2}{\mathrm{var}(x_1 + \tilde{u}_1 + \tilde{w}_1)} = \frac{(1 + k)\sigma_u^2\Sigma_0}{\alpha^2\Sigma_0 + \sigma_u^2(1 + k)} \tag{A35}$$

We now consider the speculator's trading decision. The speculator trades the quantity y in the second period to maximize

$$\begin{aligned}&\mathrm{E}((v - p_2)y \mid S_2) \\ &= \mathrm{E}\left(v - p_1 - \lambda_2\left(a_2(v - p_1) - \frac{A}{2}(m - p_1) + y + \tilde{u}_2 + \tilde{w}_2\right)\right)y\end{aligned} \tag{A36}$$

so that

$$y = \frac{1}{2\lambda_2}\,\mathrm{E}\!\left((v - p_1)(1 - \lambda_2 a_2) + \frac{\lambda_2 A}{2}\,(m - p_1)\right)$$
$$= \left(\frac{1 - \lambda_2 a_2}{2\lambda_2} + \frac{A}{4}\right)(m - p_1) = A'(m - p_1) \tag{A37}$$

In equilibrium, $A = A'$ so that

$$A = \frac{2(1 - \lambda_2 a_2)}{3\lambda_2} \tag{A38}$$

Assuming that x_1 is of the form (A29), x_1 can be written as equation (A29), thus,

$$m = \mathrm{E}(v \mid \alpha(v - p_0) + \tilde{u}_1) = \beta(x_1 + \tilde{u}) + p_0 \tag{A39}$$

where

$$\beta = \frac{\alpha\Sigma_0}{\alpha^2\Sigma_0 + \sigma_u^2} \tag{A40}$$

Since price satisfies equation (1), λ_2 is given by

$$\mathrm{E}(v - p_1 \mid \omega_2)$$
$$= \mathrm{E}\!\left(v - p_1 \,\middle|\, a_2(v - p_1) - \frac{A}{2}\,(m - p_1) + A(m - p_1) + \tilde{u}_2 + \tilde{w}_2\right) = \lambda_2\omega_2$$

or

$$\lambda_2 = \frac{a_2\Sigma_1 + (A/2)\Sigma_{vm}}{a_2^2\Sigma_1 + (A^2/4)\Sigma_{mm} + \sigma_u^2(1 + k)} \tag{A41}$$

If we let the unconditional covariances be denoted by

$$S_{vm} = \mathrm{cov}(v, m) = \beta\alpha\Sigma_0 \qquad S_{v\omega} = \mathrm{cov}(v, \omega_1) = \alpha\Sigma_0$$
$$S_{\omega\omega} = \mathrm{var}(\omega_1) = \alpha^2\Sigma_0 + \sigma_u^2(1 + k) \qquad S_{m\omega} = \mathrm{cov}(m, \omega_1) = \beta(\alpha^2\Sigma_0 + \sigma_u^2)$$
$$S_{mm} = \mathrm{var}(m) = \beta^2(\alpha^2\Sigma_0 + \sigma_u^2),$$

then the covariance matrix conditional on ω_1

$$\Sigma \mid \omega_1 = \begin{pmatrix} \Sigma_1 & \Sigma_{vm} \\ \Sigma_{vm} & \Sigma_{mm} \end{pmatrix}$$

is given by

$$\begin{pmatrix} \Sigma_1 & S_{vm} \\ S_{vm} & S_{mm} \end{pmatrix} - \frac{1}{S_{\omega\omega}} \begin{pmatrix} S_{v\omega} \\ S_{m\omega} \end{pmatrix} (S_{v\omega} \quad S_{m\omega}) \tag{A42}$$

or

$$\Sigma_1 = \Sigma_0 - \frac{(S_{v\omega})^2}{S_{\omega\omega}} \tag{A43}$$

$$\Sigma_{vm} = S_{vm} - \frac{S_{v\omega}\, S_{m\omega}}{S_{\omega\omega}} \tag{A44}$$

$$\Sigma_{mm} = S_{mm} - \frac{(S_{m\omega})^2}{S_{\omega\omega}}$$

The equilibrium is defined by finding the solutions of equations (A20), (A32), (A33), (A34), (A35), (A38), A(40), A(41), and A(42). Since these equations are nonlinear, it is difficult to solve by direct substitution. We therefore proceed by first conjecturing that α in equation (A29) is of the form

$$\alpha = q\frac{\sigma_u\sqrt{1 + k}}{\Sigma_0^{1/2}} \tag{A45}$$

for some q, solving the remaining equations under this assumption and then finally solving for q. Recall that $k = \sigma_w^2/\sigma_u^2$.

1. Solving for λ_1, Σ_1.

$$\lambda_1 = \frac{\alpha\Sigma_0}{\alpha_2\Sigma_0 + (1 + k)\sigma^2} = \frac{q\Sigma_0^{1/2}\sigma\sqrt{(1 + k)}}{((1 + k)q^2 + (1 + k))\sigma^2} = \left(\frac{q}{\sqrt{(1 + k)}(q^2 + 1)}\right)\frac{\Sigma_0^{1/2}}{\sigma} \tag{A46}$$

$$\Sigma_1 = \frac{(1 + k)\sigma^2\Sigma_0}{(1 + k)q\sigma^2 + (1 + k)\sigma^2)} = \frac{\Sigma_0}{1 + q^2} \tag{A47}$$

2. Solving for β, A, a_2, λ_2,

$$\beta = \frac{\left(q\frac{\sqrt{1 + k}\sigma}{\Sigma_0^{1/2}}\right)\Sigma_0}{q^2[(1 + k)s^2/S_0]\Sigma_0 + \sigma_u^2} = \left(\frac{\sqrt{1 + k}\; q}{1 + (1 + k)q^2}\right)\frac{\Sigma_0^{1/2}}{\sigma_u} \tag{A48}$$

We now find expressions for A, a_2, λ_2, and Σ_2. Since $a_2 = 1/2\lambda_2$, equation (A38) simplifies to

$$A = \left(\frac{2}{3}\right)\left(\frac{1}{2\lambda_2}\right) = \frac{2}{3}a_2 \tag{A49}$$

Thus, using equations (A20), (A41), and (A49), a_2 is given by

$$2\left(a_2\Sigma_1 + \frac{1}{3}\,a_2\Sigma_{vm}\right)a_2 = a_2^2\Sigma_1 + \frac{a_2^2}{9}\,\Sigma_{mm} + \sigma_u^2 + \sigma_w^2$$

or

$$a_2^2 = (\sigma_u^2 + \sigma_w^2)(\Sigma_1 + {}^2\!/\!_3\Sigma_{vm} - {}^1\!/\!_9\Sigma_{mm})^{-1/2}. \tag{A50}$$

Finally, Σ_2, the variance of $\tilde{u}$ conditional on p_1 and p_2, is given by

$$\begin{aligned}\Sigma_2 &= \Sigma_1 - \frac{(\text{cov}(v\,,\ \omega_2\,|\,\omega_1)^2}{\text{var}(\omega_2\,|\,\omega_1)} \\ &= \frac{\Sigma_1\left(\dfrac{A^2}{4}\,\Sigma_{mm} + \sigma_u^2(1 + k)\right) - A\Sigma_{vm}\left(a_2\Sigma_1 + \dfrac{A}{4}\,\Sigma_{vm}\right)}{a_2^2\Sigma_1 + \dfrac{A^2}{4}\,\Sigma_{mm} + \sigma_u^2(1 + k)}\end{aligned} \tag{A51}$$

From equations (A31) and (A45), we can now find q by the substitution

$$q\frac{\sigma_u\sqrt{1 + k}}{\Sigma_0^{1/2}} = d^{-1}c \tag{A52}$$

which is possible since d^{-1}c is of the form $\delta(\sigma_u/\Sigma_0^{1/2})$ for some δ, which depends only on β, λ_1, α, λ_2, A, and a_2, which we have reduced to functions of q.

Substituting this value for q into equation (A45) allows us to obtain the equilibrium values of all variables once we are given k. In particular, first period trade $x_1 = a_1(v - p_0) + b_1(m - v)$ is given by

$$a_1 = d^{-1}\left(1 - \lambda_1 a_2 + (\beta - 2\lambda_1)\left(\frac{\lambda_2 A^2}{4} - \frac{A}{2}\right) + \frac{\beta\lambda_2 A^2}{4}\right) \tag{A53}$$

$$b_1 = d^{-1}\,\frac{A}{2}\,(\lambda_1 - \lambda_1\lambda_2 A + \lambda_2 A\beta). \tag{A54}$$

Q.E.D.

Proof of Corollary 1:

LEMMA 1: $\text{E}((m - p_1)(v - p_1)\,|\,S_2) = \text{E}((m - p_1)^2\,|\,S_2) = \text{E}((v - p)^2 - (m - v)^2\,|\,S_2)$ *where* S_2 *is the speculator's information set at time 2.*

Proof: Since $m = \text{E}(v\,|\,S_2)$, $\text{E}((m - p_1)(v - p_1)\,|\,S_2) = \text{E}((m - p_1)(v - m + m - p_1)\,|\,S_2) = \text{E}((m - p_1)^2\,|\,S_2)$, since the projection theorem implies that $v - m$ is orthogonal to all elements of S_2, so that $\text{E}((v - m)(m - p_1)\,|\,S_2) = 0$.

Similarly,

$$\begin{aligned}
&\mathrm{E}((m - p_1)^2 \,|\, S_2)\\
&\quad = \mathrm{E}((m - v + v - p_1)^2 \,|\, S_2)\\
&\quad = \mathrm{E}((m - v)^2 \,|\, S_2) + 2\mathrm{E}((v - m + m - p)(m - v) \,|\, S_2) + \mathrm{E}((v - p_1)^2 \,|\, S_2)\\
&\quad = \mathrm{E}((v - p)^2 \,|\, S_2) - \mathrm{E}((m - v)^2 \,|\, S_2).
\end{aligned}$$

Q.E.D. of Lemma

Note that a monopolist insider can mimic the aggregate trade of the insider and the speculator by choosing

$$x_1 = a_1(v - p_0) + b_1(m - v) \qquad x_2 = a_2(v - p_1) + \frac{A}{2}(m - p_1) \tag{A55}$$

Denote the monopoly profits from equations (A55) by π^A. Then, $\pi^M \geq \pi^A$. To obtain strict inequality, note that the speculator's expected profit is positive

$$\pi^s = \mathrm{E}(v - p_2)y = \left(\frac{1 - \lambda_2 a_2}{2} + \frac{\lambda_2 A}{4}\right)^2 \frac{1}{\lambda_2}\mathrm{E}(m - p_1)^2 > 0,$$

where Lemma 1 is used to obtain the expression on the RHS. Since the profits from equations (A55) equal the losses of noise traders under the equilibrium of Proposition 1, and the speculator shares a part of this surplus, $\pi^i < \pi^A \leq \pi^M$.

We now derive explicit expressions for π_t^i and π^s. Unconditional first period profits are

$$\begin{aligned}
\pi_1^i &= \mathrm{E}(v - p_0 - \lambda_1(\alpha(v - p_0) + \tilde{u}_1 + \tilde{w}_1))\alpha(v - p_0)\\
&= (1 - \lambda_1\alpha)\alpha\mathrm{E}(v - p_0)^2 = q\sqrt{1 + k}(1 - q\sqrt{1 + k}\lambda_1)\sigma_u\Sigma_0^{1/2}.
\end{aligned} \tag{A56}$$

From equations (A10) and (A12), we have $a = \frac{1}{2}\lambda_2$ and $A = \frac{2}{3}a_2$. Thus, applying Lemma 1 and the law of iterated expectations to equation (A22), we can write

$$\pi_2 = \mathrm{E}(v - p_1)^2\left(\frac{a_2}{2} - \frac{1}{3}a_2 + \frac{1}{18}a_2\right) - \mathrm{E}(m - v)^2\left(\frac{-3a_2}{9} + \frac{a_2}{9}\right) + \mathrm{E}(m - v)^2\left(\frac{a_2}{9}\right)$$

$$= \frac{2a_2}{9}\mathrm{E}(v - p_1)^2 + \frac{5}{18}a_2(m - v)^2$$

Since

$$\mathrm{E}(v - p_1)^2) = ((1 - \lambda_1\alpha)(v - p_0) - \lambda_1(u + w))^2 = (1 - \lambda_1\alpha)^2\Sigma_0 + \lambda_1^2(\sigma_u^2 + \sigma_w^2)$$

and

$$E(m - v)^2 = \frac{\sigma_u^2\Sigma_0}{\alpha^2\Sigma_0 + \sigma_u^2} = \frac{\Sigma_0}{(q^2(1 + k) + 1))},$$

we have

$$\pi_2^i = \frac{a_2\Sigma_0}{18}\left[4\left(1 - \frac{q^2\sqrt{1 + k}}{q^2\sqrt{1 + k} + 1}\right)^2 + \frac{4\,q^2}{(1 + k)(1 + q^2)^2} + 5(1 + q^2(1 + k)^{-1})\right]$$

Finally, using Lemma 1 and the law of iterated expectations,

$$\pi^s = \left(\frac{v - p_1}{2} - \lambda_2\frac{A}{2}(m - p_1)\right)A(m - p_1)$$

$$= \frac{2}{9}a_2E((v - p_1)^2 - (m - v)^2) \tag{A57}$$

$$= \pi_2^i - \frac{6}{18}Ea_2(m - v)^2 = \pi_2^i - \frac{a_2\Sigma_0}{2}(q^2(1 + k) + 1)^{-1}$$

Q.E.D.

Proof of Proposition 2: The equilibrium of Proposition 2 is, of course, the sequential auction equilibrium in Kyle (1985) for the case of two periods. Since we are solving for a closed form equilibrium, some additional analysis is needed. Since the additional analysis is straightforward, we only summarize the additional arguments here (Madrigal (1994) contains details). Assuming that price follows equation (11), expressions for the equilibrium values of $a_1, a_2, \lambda_1, \lambda_2$ can be derived as functions of k, σ_u^2, λ_1, λ_2, and Σ_1 (where $\Sigma_1 = \text{var}(\tilde{v}\,|\,p_1)$) following the procedure of the proof of Proposition 1 or the proof of Theorem 2 of Kyle (1985). These expressions will not be in closed form. Conjecture now that a_1 is of the form

$$a_1 = c\,\frac{\sigma_u\sqrt{1 + k}}{\Sigma_0} \tag{A59}$$

Substitution then reduces all equilibrium variables to functions of c (and known parameters k and σ_u^2). Finally, to find c, consistency of equation (A59) with the first order condition for a_1 yields equation (18). Q.E.D.

Proof of Proposition 3: (i.)

$$v - p_1 = v - p_0 - \lambda_1(x_1 + u_1 + w_1) \tag{A60}$$

$$\begin{aligned} m - p_1 &= \beta(x_1 + u_1) - p_0 - \lambda_1(x_1 + u_1 + w_1) \\ &= (\beta - \lambda_1)(x_1 + u_1) - p_0 - \lambda_1 w_1 \end{aligned} \tag{A61}$$

Thus, (i.) holds since $-\lambda_1 < 0$ and $\beta - \lambda_1 > 0$, and x_1 does not depend on u_1. (ii.) follows from equations (9), (10), (A60), and (A61). (iii.) If $x_1 > 0$, and $u_1 > 0$ is sufficiently large, then from (A60) and (A61), $v - p_1 < 0$ but $m - p_1 > 0$. From equation (9), $x_2 < 0$, reversing the first period trade. An analogous argument holds for $x_1 < 0$. Q.E.D.

REFERENCES

Admati, A., and P. Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 3–40.

Admati, A., and P. Pfleiderer, 1989, Divide and conquer: A theory of intraday and day-of-the-week mean effects. *Review of Financial Studies* 2, 189–223.

Allen, F., and D. Gale, 1992, Stock price manipulation, *Review of Financial Studies* 5, 503–529.

Duffie, D., and C. Huang, 1986a, Multiperiod security markets with differential information: Martingales and resolution times, *Journal of Mathematical Economics* 15, 283–303.

Friedman, M., 1953, *Essays in Positive Economics* (University of Chicago, Chicago, Ill.)

Froot, K., D. Scharfstein, and J. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461–1484.

Grossman, S. J., 1989, An economic analysis of dual trading, Rodney L. White Center for Financial Research Paper 33–89, The Wharton School, University of Pennsylvania.

Grossman, S. J., and J. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Hart, O., and D. Kreps, 1986, Price destabilizing speculation, *Journal of Political Economy,* 927–952.

Hirshleifer, D., A. Subrahmanyam, and S. Titman, 1994, Security analysis and trading patterns when some investors receive information before others, *Journal of Finance* 49, 1665–1698.

Holden, C., and A. Subrahmanyam, 1992, Long-lived private information and imperfect competition, *Journal of Finance* 47, 247–270.

Kumar, P., and D. Seppi, 1992, Futures manipulation with "cash settlement," *Journal of Finance* 47, 1485–1502.

Kyle, A., 1984, A theory of futures market manipulations, in R. W. Anderson, ed.: *The Industrial Organization of Futures Markets,* (Lexington Books, Lexington, Mass.).

Kyle, A., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

Madrigal, V., 1994, Non-fundamental speculation, liquidity, and efficiency, working paper.

Madrigal, V., and J. Scheinkman, Price crashes, information aggregation, and market-making, *Journal of Economic Theory,* (forthcoming).

Sarkar, A., 1993, The regulation of dual trading, BEBR Faculty Working Paper No. 93-0125, University of Illinois at Champaign-Urbana.

Verrecchia, R., 1982, Information acquisition in a noisy rational expectations economy, *Econometrica* 50, 1415–1430.

Welch, I., 1992, Sequential sales, learning, and cascades, *Journal of Finance* 47, 695–732.