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Backwardation in Oil Futures Markets: Theory and Empirical Evidence

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ABSTRACT

Oil futures prices are often below spot prices. This phenomenon, known as strong backwardation, is inconsistent with Hotelling's theory under certainty that the net price of an exhaustible resource rises over time at the rate of interest. We introduce uncertainty and characterize oil wells as call options. We show that (1) production occurs only if discounted futures are below spot prices, (2) production is non-increasing in the riskiness of future prices, and (3) strong backwardation emerges if the riskiness of future prices is sufficiently high. The empirical analysis indicates that U.S. oil production is inversely related and backwardation is directly related to implied volatility.

OIL FUTURES PRICES FREQUENTLY exhibit strong backwardation in which futures prices are below the current spot price. Even more frequently they exhibit weak backwardation which means that *discounted* futures prices are below the current spot price. Between February 1984 and April 1992 the nine months futures price was strongly backwardated 77 percent of the time and weakly backwardated 94 percent of the time.[1,2] Hotelling's (1931) theory, developed under certainty, suggests that in an interior equilibrium the net price of an exhaustible resource will rise over time at the rate of interest.[3] This implies that oil futures prices will not exhibit weak backwardation unless extraction costs rise by less than the interest rate. For strong backwardation to exist, extraction costs would have to decline over time.[4] This paper introduces uncertainty into the theory of commodity pricing and production and shows that it can account for weak and strong backwardation, even when extraction

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[1] Table I presents summary statistics of weak and strong backwardation for the West Texas Intermediate (WTI) crude oil contracts (traded on the NYMEX). It also shows the fraction of time futures prices were weakly or strongly backwardated. Figure 1 presents the futures price of the nearest to maturity contract and the weak backwardation for the nine months contract.

[2] The persistence of backwardation in the oil futures market was a key factor in Metallgesellschaft's strategy of selling long-term heating oil forwards and hedging by rolling over purchases of nearer term crude and heating oil futures (see Culp and Miller (1995)).

[3] Net price, as used by Hotelling, means the price minus the extraction cost per unit, assuming constant returns to scale.

[4] Elaborated discussion follows in Section II.

Table I

Descriptive Statistics for Backwardation

The descriptive statistics are computed for the period of February 1984 through April 1992 using daily data. Weak backwardation is defined by $B_\tau^w = S - e^{-r_\tau \tau} F_\tau$, and strong backwardation is defined by $B_\tau^s = S - F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract, and r_τ is the LIBOR rate for the corresponding maturity.

Futures Contract	2nd	3rd	4th	5th	6th	7th	8th	9th
Panel A: Weak Backwardation (in Dollars)								
Average	0.29	0.49	0.64	0.76	0.87	0.96	1.04	1.11
Median	0.21	0.35	0.44	0.53	0.59	0.64	0.68	0.71
Standard deviation	0.48	0.79	1.04	1.25	1.43	1.57	1.70	1.82
Minimum	−2.05	−2.14	−2.67	−3.04	−3.34	−3.52	−3.66	−3.76
Maximum	3.53	4.70	5.79	6.82	7.80	8.70	9.82	10.80
Panel B: Strong Backwardation (in Dollars)								
Average	0.24	0.43	0.58	0.70	0.81	0.90	0.98	1.05
Median	0.15	0.29	0.38	0.47	0.53	0.58	0.61	0.65
Standard deviation	0.48	0.78	1.03	1.24	1.42	1.57	1.70	1.82
Minimum	−2.09	−2.19	−2.72	−3.09	−3.39	−3.58	−3.72	−3.82
Maximum	3.43	4.60	5.70	6.75	7.71	8.61	9.73	10.70
Panel C: Fraction of the Time in Backwardation (in %)								
Weak	81.80	85.08	87.04	89.38	90.31	91.63	93.00	93.88
Strong	71.09	71.87	72.41	73.09	74.51	75.00	76.13	76.91

costs increase at the rate of interest. The empirical evidence on oil production and futures prices is consistent with the predictions of the theory.

Under uncertainty, ownership of oil reserves may be viewed as holding a call option whose exercise price corresponds to the extraction cost.[5] Backwardation arises from the equilibrium tradeoff between exercising the option (i.e., producing the oil) and keeping it alive (i.e., leaving the oil in the ground). If discounted futures prices were higher than the spot price and if extraction costs grew by no more than the interest rate, all producers would rationally choose to defer production. Therefore, weak backwardation is a necessary condition for current production. If the uncertainty about futures prices is substantial, strong backwardation may be required in order to induce current production, even if extraction costs rise at the rate of interest.

The basic results are derived in a two-period economy in which oil reserves are owned by a continuum of price taking oil producers with heterogeneous extraction costs that rise at the rate of interest.[6] It is assumed that there is a spot market for oil, as well as financial markets for futures and options

[5] See Tourinho (1979).

[6] We choose to focus on extraction cost rising at the rate of interest since it implies a situation of no backwardation under Hotelling's theory and hence, allows us to isolate the option effect as a source of backwardation.

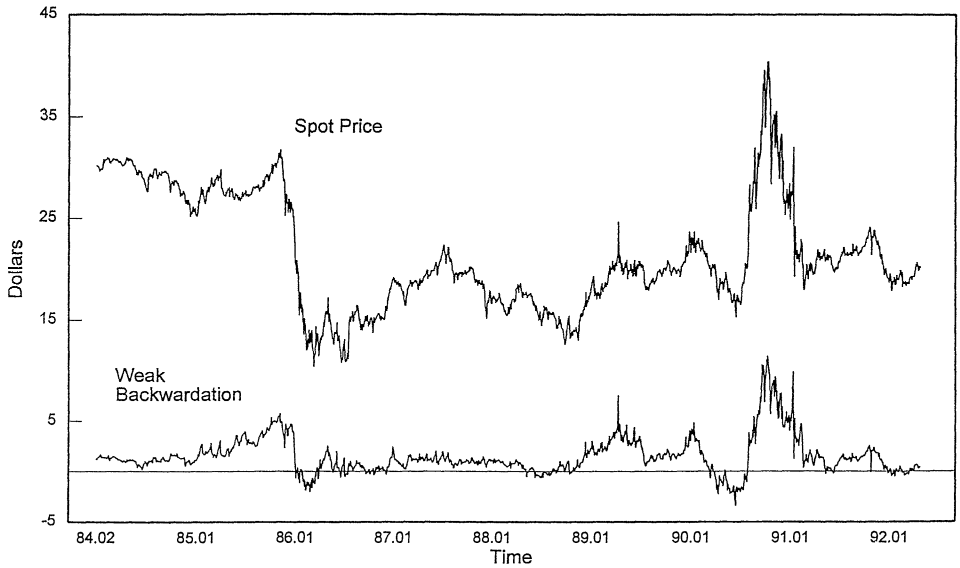

Figure 1. Spot price and weak backwardation for WTI crude oil. The figure presents the futures price of the nearest to maturity contract (which is used as a proxy for the spot price) and the weak backwardation of the ninth nearby futures price versus the nearest to maturity futures price. Weak backwardation is defined by $B_\tau^w = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity.

contracts on oil. The price of oil in the second period is stochastic as a result of a random demand shock. Oil reserves that are not extracted in the first period will be extracted in the second period only if the realized price is higher than the extraction cost. In this framework, we prove the existence and uniqueness of equilibrium in the market and present sufficient conditions for an interior equilibrium in which some but not all oil reserves are produced in the first period. We then demonstrate that in an interior equilibrium the market always exhibits weak backwardation and may exhibit strong backwardation. The weak backwardation is equal to the value of a put option with an exercise price equal to the second period extraction cost of the first period marginal producer. Strong backwardation is required to yield production in the first period if the value of the put option is sufficiently large. The Hotelling result is obtained as a special case of our model in which the distribution of the demand shock is degenerate so that the put option has no value.

The model shows that equilibrium production decreases (or remains unchanged) as the demand shock becomes more risky, because the value of the option to extract the oil is increasing in the riskiness of the future oil price.[7] The equilibrium level of weak and strong backwardation is shown to be

[7] "Riskiness" of the demand shock is in the sense of mean preserving spread (Rothschild and Stiglitz (1970)).

non-decreasing in the riskiness (in the sense of mean preserving spread) of the demand shock.

We extend our main result to a multiperiod framework and show that in any interior equilibrium the market is weakly backwardated with respect to any future time t. We also show that weak backwardation for time t is bounded from below by the value of a European futures put option with an exercise price equal to the time t extraction cost of the first period marginal producer.

We then report the results of empirical tests of some of the model's predictions using data on U.S. oil production and reserves as well as WTI futures and option prices. We first examine how production relates to the expected volatility of future prices. Consistent with the model's prediction we find that the oil production rate (production as a fraction of reserves) is significantly inversely related to the implied volatility of at-the-money options on oil futures.

The theory predicts that weak backwardation is equal to the value of a put option with a strike price equal to the extraction cost of the marginal producer. Since the extraction cost of the marginal producer is unobservable we regress weak backwardation on the at-the-money-futures put option price and find a highly significant positive association. Similar results are obtained when backwardation is regressed on implied volatility calculated from at-the-money oil futures options: backwardation is increasing in implied volatility.

The paper proceeds as follows: In Section I the existing literature on the price behavior of exhaustible resources is discussed. In Section II we present the model and derive the results for equilibrium production, prices, and backwardation in the market. Empirical tests of the model's predictions are presented in Section III. A summary and some concluding remarks are included in Section IV.

I. Previous Literature

The Hotelling Principle (1931), developed under the conditions of perfect competition and certainty, states that the net price of an exhaustible resource, such as crude oil, should rise over time at the rate of interest.[8] Denoting the spot price of oil at time t by S_t and the extraction cost per unit at time t by x_t, this may be expressed as:

$$S_t - x_t = (S_0 - x_0)e^{rt}. \tag{1}$$

This principle is based on the condition that in an interior equilibrium (in which oil is produced in every period), each producer must be indifferent between present and future production of the oil.

Under certainty the future spot price in period t is known and is equal to the current t-period futures price. If the extraction cost per unit is proportional to the price of oil, the principle implies that the discounted futures price will

[8] "Net price," as used by Hotelling, means the price minus the extraction cost per unit, assuming constant returns to scale.

equal the spot price.[9] If the extraction cost per unit has a fixed component, then discounted futures prices will equal spot and there will be no backwardation only if the fixed extraction cost rises at the rate of interest. In fact, whenever total per unit extraction costs rise by less than the interest rate, the market will exhibit weak backwardation. Strong backwardation, however, will occur only if extraction costs decline sufficiently fast over time.[10] It follows that unless extraction costs were anticipated to decline sufficiently fast 77 percent of the time between February 1984 and April 1992, the persistent strong backwardation could not be explained by the Hotelling theory.

A binding production capacity constraint can also account for backwardation, since such a constraint would prevent the supply response that underlies the Hotelling Principle. However, for capacity constraints to explain the persistence of backwardation, they would have to have been binding 77 percent of the time between February 1984 and April 1992. Moreover, neither theories based on capacity constraints, nor variations of Hotelling's model under certainty predict any association between output and volatility or between backwardation and volatility. The theory developed in this paper, which is based on the call option characteristic of oil reserves, predicts a negative association between production and volatility and a positive association between backwardation and volatility. It is useful to note that the alternative explanations of backwardation are not mutually exclusive.

The notion of "convenience yield" (Kaldor (1939), Working (1948)) is often used as an explanation for backwardation in futures prices of storable commodities. Convenience yield, as defined by Brennan and Schwartz (1985), is *"the flow of services that accrues to an owner of the physical commodity but not to an owner of a contract for future delivery of the commodity."* The nature of these services and benefits depends on the type of commodity, the identity of the holder and the location of storage (Brennan (1958)). Brennan and Schwartz (1985) as well as Gibson and Schwartz (1990) argue that backwardation is equal to the present value of the marginal convenience yield of the commodity inventory. However, in these papers convenience yield is exogenously determined. In this paper backwardation is determined endogenously and is associated with the option feature of reserves.[11] Since oil reserves are spanned by a forward contract and an at-the-money-forward put option, convenience yield can be interpreted as the expected present value of the price protection ser-

[9] Extraction cost would be proportional to the price of oil, if oil by itself were the only productive input in extraction.

[10] Denoting the rate of change in extraction cost by g (i.e., $x_t = x_0 e^{gt}$), we have $S_t < S_0$ if and only if $g < \ln[e^{rt} - (S_0/x_0)(e^{rt}-1)]^{1/t} < 0$.

[11] Convenience yield from storing oil *above* ground is complementary to our notion of backwardation. However, this paper does not analyze this type of convenience yield. Storage of oil above ground is minimal because the physical storage cost per unit value is extremely high. Most of the above ground storage stocks are in a state of transition (in pipelines and tankers) from oil fields to refineries. The rest is held on refinery sites due to discreteness of oil shipments and to allow for smooth refining process. Imported oil carried in tankers to the United States may have a speculative aspect to it as the tankers can change their speed of sailing or even their final destination in order to take advantage of locational basis.

vices associated with storing oil in the ground rather than holding a forward contract.

Other papers analyze the producers' extraction problem within an equilibrium framework under uncertainty. Sundaresan (1984) considers monopolistic as well as price taking producers. In contrast with our main result, his model suggests that discounted futures prices are always strictly *higher* than the spot price. This result, however, is driven by his assumption of zero extraction cost, which implies that the value of oil reserves is simply equal to its current value if extracted and there is no option premium.

Pindyck (1980) also investigates the price behavior of exhaustible resources under uncertainty. He allows for both demand and reserves uncertainty and does not assume zero extraction cost. He finds that the expected future spot prices rise at the rate of interest. This outcome, however, is driven by the fact that producers in his model employ a non-optimal stopping rule. Pindyck assumes that production permanently stops as soon as the price falls below extraction cost. When this occurs, the reserves which are not yet extracted are lost and the producers do not have the option to resume production in the future.

For some oil wells a complete cessation of production can reduce the total recovery. In the extreme, as in the case of stripper wells, an oil well that is closed cannot be reopened. Therefore, even if the spot price of oil is below the extraction cost, it may be worth maintaining a minimal level of production in order to preserve the option of producing in the future (See Brennan and Schwartz (1985)). This consideration could account for the short periods of time when the oil market did not exhibit weak backwardation. However, this is beyond the scope of our model and is left for future research.

Related empirical papers by Miller and Upton (1985a, 1985b) have examined what they refer to as the Hotelling Valuation Principle. This principle predicts that the market value of an oil producing firm divided by its total reserves should equal the current price of oil, net of extraction costs. The results obtained in the first paper provide some support for the Hotelling Valuation Principle, but the results in the follow-up paper are inconclusive. Miller and Upton note that volatility has changed during their extended sample period. Since under uncertainty an exhaustible resource can be characterized as a call option whose value depends on price volatility, the regression analysis of Miller and Upton can be refined by accounting for the call option premium using call option prices or estimated price volatility.

II. The Equilibrium Model

In this section we present a two period model and show that the market is always weakly backwardated in an interior equilibrium. The weak backwardation is equal to the value of a put option with a strike price equal to the extraction cost of the marginal producer. If the value of the put is sufficiently large, strong backwardation emerges. Oil production is shown to be non-increasing in the riskiness of future prices. Finally, the result of weak backwardation in an interior equilibrium is extended to a multiperiod framework, and a lower bound for the backwardation is provided.

A. The Producer's Problem

Consider a two period economy with finite reserves of oil, Q. Of the total reserves, Q, Q_0 is available at time 0 and can be extracted immediately or later; the remainder, Q_1, is available for production at time 1 only.[12] There is a continuum of oil producers, each of whom owns an equal share of reserves. They differ with respect to their time 1 extraction costs, x, which are uniformly distributed between 0 and $\bar{x}$. Extraction costs for all producers are assumed to increase at the interest rate, r, from time 0 to time 1, so that extraction costs at time 0 range from 0 to $e^{-r}\bar{x}$.[13]

All producers with an extraction cost below that of the marginal producer extract their oil at time 0, and those with a higher extraction cost do not. Thus, denoting the time 1 extraction cost of the marginal producer by x^e and aggregate production by q, we obtain the following relation between aggregate production and the extraction cost of the marginal producer (see also Figure 2):

$$q(x^e) = \begin{cases} (x^e/\bar{x})Q_0, & \text{for} \quad 0 \le x^e < \bar{x} \\ Q_0, & \text{for} \quad \bar{x} \le x^e. \end{cases} \tag{2}$$

Assume that there is a continuum of put and call options on the time 1 price of oil traded at time 0, and let P_x (C_x) denote the prices at time 0 of a put (call) option with an exercise price of x. Then, the optimal extraction policy of a producer whose time 1 extraction cost is x can be written as,

$$S_0 - e^{-r}x < C_x \Rightarrow \text{Leave oil in the ground} \tag{3}$$

$$S_0 - e^{-r}x = C_x \Rightarrow \text{Indifferent} \tag{4}$$

$$S_0 - e^{-r}x > C_x \Rightarrow \text{Extract oil now} \tag{5}$$

The optimal production policy is determined by comparing the net value of oil above the ground (spot price minus extraction cost) with its value in the ground (the price of the call option). There will be no extraction of oil for storage above the ground, since underground storage involves no physical storage costs.

It is shown in the Appendix that $J(S_0, x) \equiv S_0 - e^{-r}x - C_x$ is strictly decreasing in x. Therefore, abstracting from corner solutions, there exists a

[12] Q_1 is introduced in order to avoid the case of zero supply at time 1. With $Q_1 = 0$ there is zero supply for low prices at time 1. If, in addition, the demand realization is low (demand intersects supply at zero quantity), the equilibrium price is not well defined. For this reason we let Q_1 be positive, though it can be arbitrarily small.

[13] We choose to focus on this case since it would imply a situation of no backwardation under Hotelling's conditions. In this way we are able to isolate the option effect as a source of backwardation.

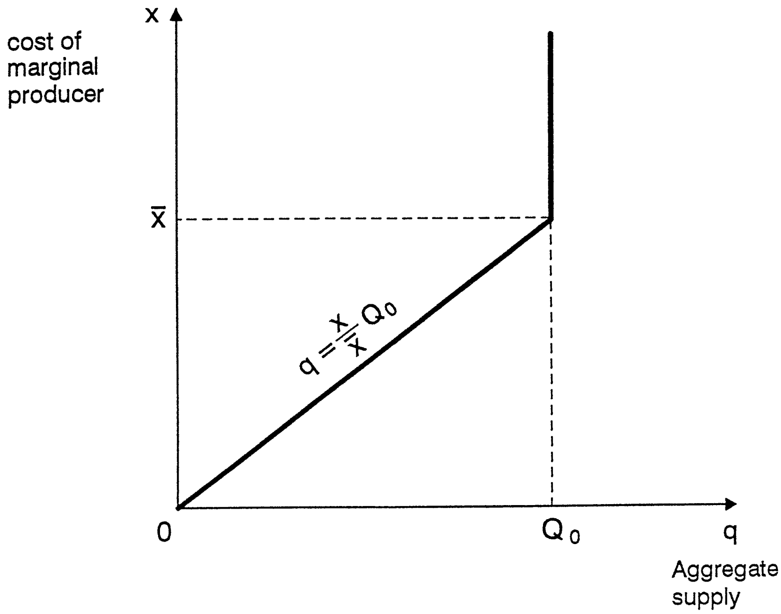

Figure 2. Production technology at time 0. The figure presents aggregate production at time 0 as a function of the (time 1) extraction cost of the marginal producer (equation (2)). The total amount of oil available for extraction at time 0 is Q_0. There is a continuum of heterogeneous oil producers, each of whom owns an equal share of reserves. They are uniformly distributed with respect to their time 1 extraction cost from 0 to $\bar{x}$.

single value of x, x^e, such that $S_0 - e^{-r}x^e = C_{x^e}$.[14] It follows that given the uniform distribution of extraction costs, the aggregate oil supply at time 0 is a well defined function of the spot price of oil and the call price schedule:

$$q\left(S_0,\ \{C_x\}_{x\in[0,\bar{x}]}\right).$$

Since $\partial x^e/\partial S_0 > 0$ it follows that $\partial q/\partial S_0 > 0$; i.e., ceteris paribus, an increase in the spot price of oil will lead the marginal producer to extract its oil at time 0 instead of leaving it in the ground. The partial derivative of the aggregate supply with respect to a change in the call price, C_x, is negative when evaluated at x^e (extraction cost of the marginal producer), and zero elsewhere: $\partial q/\partial\{C_x\}_{x=x^e} < 0$, $\partial q/\partial\{C_x\}_{x\neq x^e} = 0$. Other things equal, an increase in the value of the call options for every exercise price will lead marginal producers to defer extraction.

[14] The corner cases are: (a) No production of oil at time 0 if $S_0 \le e^{-r}F$. In this case the zero cost producer does not produce since the value of his oil above the ground, S_0, is lower than its value in the ground, C_0 (notice that $C_0 \equiv e^{-r}F$). All other producers, whose extraction costs are higher, do not produce as well. (b) Maximum production of oil at time 0, Q_0, if $S_0 - e^{-r}\bar{x} \ge C_{\bar{x}}$. In this case the highest cost producer, and hence all other producers, extract their oil.

B. Consumers' Demand

The demand functions for oil at time 0 and time 1 are assumed to be of the form,[15]

$$D_0 = a - bS_0 \tag{6}$$

and

$$D_1 = a + \tilde{\epsilon} - bS_1, \tag{7}$$

where a and b are positive and $\tilde{\epsilon}$ is a random shock with zero mean. To rule out the trivial case where the first period demand curve lies entirely below the highest extraction costs, we require that $a/b > \bar{x}$.[16] Thus, the parameter space and the random shock space are given by:

$$\Theta \equiv \left\{ \theta \equiv (a,\, b,\, \bar{x},\, r,\, Q_0,\, Q_1) \;\in\; \mathbb{R}_+^6 \backslash \{0\} \,\middle|\, \frac{a}{b} > \bar{x} \right\}$$

$$\mathrm{E} \equiv \{\tilde{\epsilon} \,|\, \tilde{\epsilon} \;\in\; \mathscr{L}^1 \text{ and } \mathrm{E}(\tilde{\epsilon}) = 0\}.$$

C. Equilibrium and Backwardation

The equilibrium at time 0 can be characterized by the time 1 extraction cost of the marginal producer, x^e, which depends on the parameters of the model, θ, and on the probability distribution function of the demand shock, denoted by $d_{\tilde{\epsilon}}$.[17] We let $x^e(d_{\tilde{\epsilon}},\, \theta)$ denote the extraction cost of the marginal producer, $q^e(d_{\tilde{\epsilon}},\, \theta)$ denote the equilibrium production at time 0 and $S_0(q^e(d_{\tilde{\epsilon}},\, \theta),\, \theta)$ and $S_1(q^e(d_{\tilde{\epsilon}},\, \theta),\, \theta;\, \tilde{\epsilon})$ denote the equilibrium prices at time 0 and time 1, respectively (the abbreviated notation of x^e, q^e, S_0^e and $\tilde{S}_1^e$ is used henceforth).

A unit of oil from the marginal producer's reserve pays off $\max[\tilde{S}_1^e - x^e, 0]$ at time 1. Its value at time 0, which is now determined endogenously, is the same as the value of a call option with the exercise price of x^e, and is denoted by C_{x^e}. In an interior equilibrium the marginal producer is just indifferent between extracting his oil at present and leaving it in the ground,

$$S_0^e - e^{-r}x^e = C_{x^e}. \tag{8}$$

Without further restrictions on the parameters and on the demand shock, boundary equilibria are possible as well. However, we restrict our attention here to the case of an interior equilibrium.[18] Let F^e denote the equilibrium

[15] This demand structure assumes no temporal substitution of consumption between the two periods. However, the result of backwardation in equilibrium is not driven by this assumption. Any structure of less than perfect temporal substitution will do in our model; we choose the extreme case for reasons of simplicity.

[16] In this way we allow for the possibility that all initial reserves will be extracted at time 0.

[17] An illustration of these curves at time 1 is given in Figure 3.

[18] Theorem A1 in appendix A provides necessary and sufficient conditions for boundary and for interior equilibria. The uniqueness of the equilibrium is proven there as well. These proofs assume risk neutrality to obtain a unique option pricing relation in equilibrium.

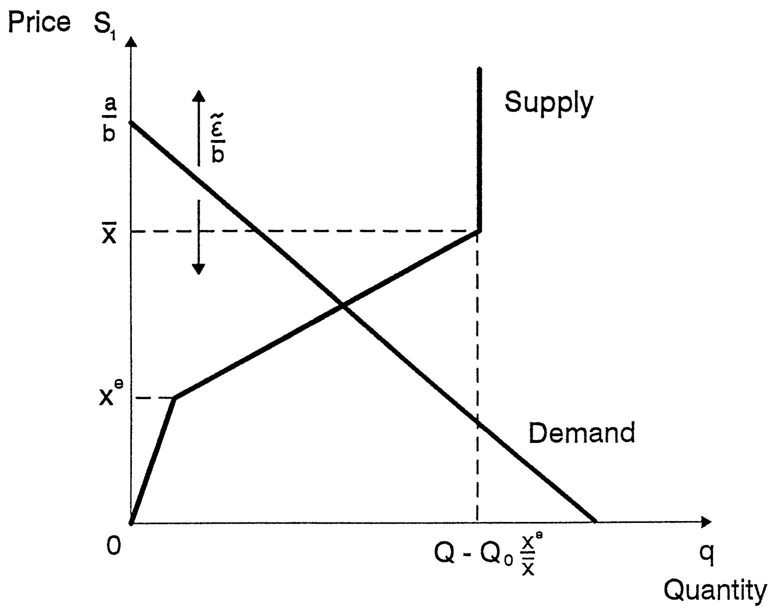

Figure 3. Supply and demand at time 1. The figure presents the supply function and the average demand function at time 1. Q is the total reserves of oil in the economy, of which Q_0 is available for extraction at time 0. The extraction cost of the highest cost producer is $\bar{x}$ and that of the marginal producer is x^e. a and b are the demand function parameters (equations (6) and (7)) and $\tilde{\varepsilon}$ is the demand shock.

futures price of oil. Then, the equilibrium levels of weak and strong backwardation are given by,

$$B_w(q^e(d_{\tilde{\varepsilon}}, \theta), \theta) \equiv [S_0^e - e^{-r}F^e]^+ \tag{9}$$

$$B_s(q^e(d_{\tilde{\varepsilon}}, \theta), \theta) \equiv [S_0^e - F^e]^+, \tag{10}$$

(the abbreviated notation of B_w^e and B_s^e is used henceforth).

We now present the following result with respect to the backwardation in equilibrium:

THEOREM 1: *In an interior equilibrium in which extraction costs rise at the rate of interest the market is weakly backwardated. The weak backwardation is equal to the value of a put option on time 1 oil with an exercise price of x^e, the extraction cost of the marginal producer. Strong backwardation, if it exists, is equal to the put option value minus the difference between the undiscounted and the discounted futures price. That is, for all $(\tilde{\varepsilon}, \theta) \in (\mathrm{E} \times \Theta)$ such that $q^e(d_{\tilde{\varepsilon}}, \theta) \in (0, Q_0)$, we have*

$$B_w^e = [P_{x^e}]^+, \tag{11}$$

and

$$B_s^e = [P_{x^e} - (1 - e^{-r})F^e]^+. \tag{12}$$

Proof: As stated in equation (8), in case of an interior equilibrium the marginal producer is indifferent between producing his oil at time 0 or leaving it in the ground. Its value in the ground is the call option value C_{x^e}. Using put-call parity, it follows that:

$$C_{x^e} = e^{-r}(F^e - x^e) + P_{x^e}. \tag{13}$$

In this way the value of the reserves can be viewed as the sum of the discounted difference between the futures price and the extraction cost, $F^e - x^e$, plus the value of the option to forgo production at time 1—the put option price. To obtain the results of the theorem, combine the equilibrium condition (equation (8)) with the put-call parity (equation (13)). □

Theorem 1 provides an understanding of the conditions under which weak and strong backwardation exist in equilibrium. Weak backwardation is equal to the put option value with an exercise price of x^e. The put option value is always non-negative. In fact, it is strictly positive in an interior equilibrium, where there is production at time 0. In this sense the theorem states that weak backwardation is necessary for the existence of an interior equilibrium in the market. From equation (12) it follows that the market is strongly backwardated if the put option value is larger than $(1 - e^{-r})F^e$. Hence, the market will exhibit strong backwardation if the uncertainty about future prices is sufficiently large. As oil prices are known to be volatile, the above result can explain the persistence of strong backwardation in this market. In the case of a degenerate demand shock (i.e., under certainty), all producers are indifferent between present and future production and the backwardation is zero. Hence, the futures price is equal to the spot price compounded by the interest rate, which is Hotelling's original result under certainty.

Theorem 1 relies on the assumption that extraction costs rise from time 0 to time 1 at the rate of interest. In the general case where extraction costs change at the rate g, the corresponding expressions for the equilibrium level of weak and strong backwardation are given by,

$$B_w^e = [P_{x^e} + (e^{-g} - e^{-r})x^e]^+, \tag{14}$$

and

$$B_s^e = [P_{x^e} + (e^{-g} - e^{-r})x^e - (1 - e^{-r})F^e]^+. \tag{15}$$

Examining these expressions we note the following: First, if $g < r$, weak backwardation may be explained in part by the slow rate of increase (or by the decrease) of the extraction cost. Second, unless g is sufficiently negative (smaller than $\ln[e^{-r} - (F^e/x^e)(e^{-r} - 1)] < 0$), a positive put option value is necessary in order to support strong backwardation. Third, if $g \geq r$, a positive put option value is necessary in order to support both weak and strong backwardation in equilibrium.

We present now the following corollary concerning the storage of oil above ground:

COROLLARY 1: *In an interior equilibrium there is no storage of oil above the ground.*

By Theorem 1, the market is always weakly backwardated in an interior equilibrium. Under weak backwardation above ground storage is unprofitable (even in the absence of physical storage costs) and will not take place.

Next, we examine the effect of increased uncertainty about future prices on equilibrium production using comparative statics analysis. More specifically, we let the demand shock in the second period become "riskier" in the sense of a mean preserving spread (MPS, henceforth) (Rothschild and Stiglitz (1970)), and obtain the following result:

THEOREM 2: *The equilibrium production of oil at time 0 is non-increasing with the riskiness (in the sense of MPS) of the demand shock. That is, if* $\tilde{\epsilon}_1, \tilde{\epsilon}_2 \in \mathrm{E}$ and $\xi \in \mathrm{E}$, where[19] $\tilde{\epsilon}_2 \overset{d}{\sim} \tilde{\epsilon}_1 + \tilde{\xi}$ *and* $\mathrm{E}(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$, *then,*

$$q^e(d_{\tilde{\epsilon}_2}, \theta) \leq q^e(d_{\tilde{\epsilon}_1}, \theta), \tag{16}$$

for all $\theta \in \Theta$.

Proof: See Appendix. □

The result is intuitive—as uncertainty about the second period realization increases, the value of oil reserves, like the value of a call option, increases or remains unchanged for any level of extraction cost. Hence, with an increase in riskiness of the demand shock no fewer producers will choose to leave their oil in the ground.[20] If uncertainty about future prices becomes extremely high, all producers will choose not to extract and the boundary equilibrium of no production will be reached. A sufficient condition for positive production at time 0 is presented in Appendix A, Theorem A2. More specifically, we show that if the random demand shock is bounded (or alternatively if its variance is bounded) as described in the theorem, positive production at time 0 is guaranteed.

The next result relates to the effect of an increase in riskiness of the demand shock on the level of weak and strong backwardation in equilibrium. Consider the demand shock, $\tilde{\epsilon}_1$, and the range of the demand shock realizations for which the oil price at time 1, $S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \tilde{\epsilon}_1)$, falls between the extraction cost of the marginal producer, $x^e(d_{\tilde{\epsilon}_1}, \theta)$, and the highest extraction cost, $\bar{x}$ (a range of linear supply function). This range is denoted by $[\epsilon_1^*, \epsilon_1^{**}]$, where ϵ_1^* and ϵ_1^{**} are the solutions for $S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \epsilon_1^*) = x^e(d_{\tilde{\epsilon}_1}, \theta)$ and $S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \epsilon_1^{**}) = \bar{x}$, respectively. The following result is obtained:

[19] "$\overset{d}{\sim}$" means "is equal in distribution as."

[20] Merton (1973) has previously demonstrated that option prices are non-decreasing in the riskiness of a stock price. Jagannathan (1984) notes that Merton's analysis implicitly assumes that the stock price is not affected by the change in riskiness. In contrast to Merton's analysis, the proof of Theorem 2 analyzes the effect of an increase in the riskiness of the demand shock while allowing spot and futures prices to vary endogenously.

THEOREM 3: *The equilibrium level of weak and strong backwardation is non-decreasing with an increase in the riskiness of the demand shock resulting from a series of mean preserving spreads within the range* $[\epsilon_1^*, \epsilon_1^{**}]$. *That is, if* $\tilde{\epsilon}_1, \tilde{\epsilon}_2 \in \mathrm{E}$ *and* $\tilde{\xi} \in \mathrm{E}^*$, *where* $\tilde{\epsilon}_2 \overset{d}{\sim} \tilde{\epsilon}_1 + \tilde{\xi}$, $\mathrm{E}(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$ *and* $\mathrm{E}^* \equiv \{\tilde{\xi} \in \mathrm{E} \mid \epsilon_1^* \le \epsilon_1 + \xi \le \epsilon_1^{**}\ \forall \epsilon_1 \in [\epsilon_1^*, \epsilon_1^{**}]$, *and* $\xi = 0$ *otherwise*$\}$ *then,*

$$B_w(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta) \ge B_w(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta), \tag{17}$$

and

$$B_s(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta) \ge B_s(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta), \tag{18}$$

for all $\theta \in \Theta$.

Proof: See Appendix. □

From Theorem 2 it follows that an increase in the riskiness of the demand shock (in the sense of MPS) leads to a decrease in time 0 production and to an increase in time 1 supply ("decrease"/"increase" are used here in a weak sense meaning non-increasing/non-decreasing). Consequently, the oil price at time 0 increases. If the supply function at time 1 were unchanged under the new demand shock, $\tilde{\epsilon}_2$, the expected price of oil would remain unchanged since $\tilde{\epsilon}_2$ is a mean preserving spread of $\tilde{\epsilon}_1$, in a range where both demand and supply functions are linear (see Figure 3). However, since the supply function at time 1 uniformly increases under $\tilde{\epsilon}_2$, the expected price decreases.

D. Multiperiod Analysis

The above results are obtained in a two period framework. Next, we show that our main result, the existence of weak backwardation for any interior equilibrium, carries over to a multiperiod framework.

Assume a T-period economy with total reserves of oil Q. Further assume a continuum of oil producers who are heterogeneous with respect to their extraction cost. As before, the producers' extraction costs rise over time at the rate of interest and at time T they range from 0 to $\bar{x}$. Each one of the producers can extract the oil at any time throughout time T.

Consider an oil producer with time T extraction cost of x. Let x_t be the producer's extraction cost at time t.[21] The producer decides whether or not to produce at time 0 by comparing the net price of oil above ground, $S_0 - x_0$, with its value in the ground. The value of oil in the ground, denoted by $C^T_{\{x_t\}}$, is nothing but a T-period American call option on the spot with an exercise price of $\{x_t\}_{t \le T}$ increasing over time. In case of an interior equilibrium, the time 0 marginal producer is just indifferent between extracting the oil and leaving it in the ground; i.e.,

$$S_0^{e0} - x_0^{e0} = C^T_{\{x_t^{e0}\}}, \tag{19}$$

[21] $x_t \equiv e^{-r(T-t)}x$.

where S_0^{e0} is time 0 equilibrium spot price of oil and x_t^{e0} is the time t extraction cost of the time 0 marginal producer.

The next theorem generalizes the existence of weak backwardation in an interior equilibrium to a multiperiod framework. We adopt the following notation: F_t^{e0} denotes the equilibrium t-period futures price as of time 0, $p^t_{x_t^{e0}}$ denotes the price at time 0 of a t-period European put option on oil futures with a strike price of x_t^{e0} and finally,

$$B_{w,t}^{e0} \equiv [S_0^{e0} - e^{-rt}F_t^{e0}]^+ \tag{20}$$

$$B_{s,t}^{e0} \equiv [S_0^{e0} - F_t^{e0}]^+ \tag{21}$$

denote the equilibrium level of the t-period weak and strong backwardation as of time 0.[22]

THEOREM 4: *In an interior equilibrium the market is weakly backwardated with respect to any future time t, $t \leq T$. The t-period weak backwardation for time t is no lower than the value of a t-period European put option on oil futures with an exercise price of x_t^{e0}. Strong backwardation, if it exists, is no lower than the put option value minus the difference between the undiscounted and the discounted t-period futures price. That is, for any interior equilibrium we have,*

$$B_{w,t}^{e0} \geq [p^t_{x_t^{e0}}]^+ \qquad \textit{for all} \quad t \leq T, \tag{22}$$

and

$$B_{s,t}^{e0} \geq [p^t_{x_t^{e0}} - (1 - e^{-rt})F_t^{e0}]^+ \qquad \textit{for all} \quad t \leq T. \tag{23}$$

Proof: See Appendix. □

Consider the first result. The zero cost producer will choose to defer extraction whenever $S_0^{e0} \leq e^{-rt} F_t^{e0}$ for some $t \leq T$. All other producers, who have higher extraction costs, will choose to defer extraction as well. Hence, a necessary condition for positive production at time 0 is the existence of weak backwardation with respect to each of the future periods. Moreover, not only must weak backwardation exist, it has to be higher than the price of a t-period European put option with an exercise price of x_t^{e0}. The outcome is obtained by combining the result that a T-period American call option on the spot is more valuable than any t-period European call option on the futures (for $t \leq T$) together with the equilibrium condition (19) and with put-call parity. Similarly, a lower bound is obtained for the equilibrium level of strong backwardation.

[22] In the two period setting there was no difference between futures and forward prices of oil. However, in the multiperiod framework these prices differ if interest rate is correlated with prices (see Cox, Ingersoll, and Ross (1981)). While the analysis refers to forward prices, we continue to use the terminology of futures since the interest rate is assumed to be constant and in order to maintain continuity from previous sections.

III. Empirical Evidence

In this section we test some empirical implications of the model developed above. Nonregulated oil production within the United States is found to be inversely related to implied volatility from at-the-money option prices. Weak backwardation is shown to have a significant positive relation with at-the-money put option prices and their implied volatilities.

A. Production and Volatility

An oil reserve is characterized as a call option where the extraction cost corresponds to the exercise price. In deciding whether or not to extract oil in the current period, producers compare the call option value with the net price of oil above the ground. In Theorem 2 it is shown that equilibrium production is non-increasing in the riskiness (in the sense of mean preserving spread) of the demand shock. The result is intuitive—as uncertainty about the future price of oil increases, the value of oil reserves, like the value of a call option, increases or remains unchanged for any level of extraction cost. Hence, with an increase in riskiness of the demand shock no fewer producers will choose to leave their oil in the ground.

This idea carries over to a multiperiod framework. In order to define the notion of uncertainty in the multiperiod context, consider the Gibson and Schwartz (1990) partial equilibrium model where the spot price of oil is assumed to follow a geometric Brownian motion and the slope of the oil price term structure follows an arithmetic Brownian motion. Under this model the futures price for every maturity is lognormally distributed and its volatility is monotonically increasing in the volatility of both the spot price and the oil term structure slope. Oil in the ground, like the value of an American call option, is decreasing in the volatility of any futures price. Hence, current oil production is non-increasing in the volatility of the futures price.

We note that in practice, changing oil prices affect the intensity of field development and exploration. These, in turn, lead to fluctuations in extraction costs and reserves. Therefore, when examining the time series data, it is more appropriate to use relative variables for the test as opposed to absolute variables. Furthermore, based on a comparative statics analysis Theorem 2 predicts a change in production in response to a change in riskiness of the demand shock. Therefore, the change in the production rate (production as fraction of reserves) is taken to be the dependent variable, and the change in implied volatility from an at-the-money-futures call option price is taken to be the explanatory variable.[23] Following the above discussion, an inverse relation between the production rate and implied volatility is expected.

The U.S. oil production and reserves data, as well as the WTI crude oil futures and option prices, are used to construct the aforementioned vari-

[23] Exact definitions of the variables follow.

Table II

Production and Volatility $\Delta q_t = \alpha_\tau + \beta_\tau \Delta\bar{\sigma}_{\tau,t-1} + \varepsilon_{\tau,t}$

Changes in production rate (monthly production divided by reserves), q, are regressed on lagged changes in the monthly averaged implied volatility, $\bar{\sigma}_\tau$. The implied volatility, σ_τ, is computed from the τ-th month at-the-money-futures call option price using the Black (1976) formula. Production rate and volatility are expressed in percentage annual terms.

	$\tau = 2$		$\tau = 3$		$\tau = 4$	
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$
Panel A: December 1986–December 1991 (60 obs.)						
Estimate	−0.024	−0.002	−0.025	−0.003	−0.025	−0.004
t-statistic	−1.740	−1.190	−1.775	−1.543	−1.853	−2.069
p-value	0.044	0.119	0.041	0.064	0.034	0.021
Panel B: Excluding the Gulf War, August 1990–February 1991 (53 obs.)						
Estimate	−0.029	−0.007	−0.030	−0.008	−0.030	−0.009
t-statistic	−1.876	−2.351	−1.925	−2.318	−1.982	−2.267
p-value	0.033	0.011	0.030	0.012	0.026	0.014

ables.[24] Since some of the states regulate production and others do not, we consider only non-regulated states where producers are able to vary their production in response to market conditions. The implied volatility is computed using the Black (1976) formula from at-the-money-futures call options written on the second, third and fourth nearby futures contracts.[25] The longest horizon examined is four months since the options are not actively traded beyond the fourth nearby contract.

The change in production rate (monthly production divided by reserves), Δq, is regressed on the change in lagged monthly averaged implied volatility, $\Delta\bar{\sigma}_\tau$:

$$\Delta q_t = \alpha_\tau + \beta_\tau \Delta\bar{\sigma}_{\tau,t-1} + \epsilon_{\tau,t}. \tag{24}$$

The results of this regression for the period of December 1986 through December 1991 are presented in Panel A of Table II. Consistent with our prediction, the coefficients on the implied volatility are negative and are usually significant at the 10 percent level.

The Gulf War period, August 1990 through February 1991, was characterized by extreme backwardation and volatility. Regression (24) is repeated excluding the Gulf War period. The results are presented in Panel B of Table II. As can be seen in the table, the volatility coefficient estimates are significantly negative at the 1 percent level and are higher in absolute value than

[24] The monthly data on production and reserves were obtained from the Energy Information Administration (Department of Energy). The daily data on WTI crude oil futures and options contracts (listed on the NYMEX) were obtained from DRI McGraw Hill.

[25] The first nearby futures contract is the nearest to maturity contract and its time to expiration ranges from one day to thirty one days. The second nearby contract has one additional month to expiration. The third nearby has two additional months, etc.

Table III

Production, Volatility and Spot Price

$$\Delta q_t = \alpha_\tau + \beta_\tau \Delta\bar{\sigma}_{\tau,t-1} + \gamma_\tau \Delta\bar{S}_{t-1} + \varepsilon_{\tau,t}$$

Changes in production rate (monthly production divided by reserves), q, are regressed on lagged changes in the monthly averaged implied volatility, $\bar{\sigma}_\tau$, and on lagged changes in the monthly averaged spot price (first nearby WTI futures), $\bar{S}$. The implied volatility, σ_τ, is computed from the τ-th month at-the-money-futures call option price using the Black (1976) formula. Production rate and volatility are expressed in percentage annual terms.

	$\tau = 2$			$\tau = 3$			$\tau = 4$		
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\gamma}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\gamma}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$	$\hat{\gamma}_4$
Panel A: December 1986–December 1991 (60 obs.)									
Estimate	−0.025	−0.002	0.010	−0.026	−0.004	0.012	−0.027	−0.006	0.013
t-statistic	−1.819	−1.500	1.423	−1.881	−1.933	1.593	−2.001	−2.560	1.848
p-value	0.037	0.069	0.080	0.032	0.029	0.058	0.025	0.007	0.034
Panel B: Excluding the Gulf War, August 1990–February 1991 (53 obs.)									
Estimate	−0.029	−0.007	0.004	−0.030	−0.007	0.006	−0.031	−0.008	0.006
t-statistic	−1.873	−1.895	0.245	−1.930	−1.876	0.329	−1.983	−1.820	0.385
p-value	0.032	0.032	0.404	0.023	0.034	0.372	0.026	0.037	0.371

those obtained in the full sample regression. This is consistent with the fact that during the war period U.S. production has increased and volatility has increased as well.

It has been suggested by the reviewer that the implied volatility of oil prices is directly related to the price level. This implies that the observed correlation between changes in production and changes in volatility may be non-causal and may be due to their common association to the omitted price variable. As can be seen in Table III, when the change in oil price is introduced into the regression, its coefficients are positive and insignificant while the volatility coefficients remain significant.

B. Backwardation and Put Option Prices

As suggested by Theorem 1 in case of an interior equilibrium, weak backwardation is equal to the value of a put option on the second period futures price with a strike price equal to the extraction cost of the marginal producer. Theorem 4 provides a lower bound for weak backwardation in a multiperiod framework—weak backwardation is no lower than the value of a European futures put option with a strike price equal to the time t extraction cost of the marginal producer. Thus, the theory predicts a positive association between backwardation and the price of a put option with a strike price equal to the marginal producer's extraction cost. However, the extraction cost of the marginal producer at each point in time is unknown. Therefore, at-the-money-futures put options are being used as surrogates (discussion follows).

Weak backwardation is computed for the second, third, and fourth nearby futures contracts relative to the first nearby contract, where the latter serves as a proxy for the spot price.[26] The weak backwardation for the τ-th contract is defined by $B_{\tau}^{w} = S - e^{-r_{\tau}\tau}F_{\tau}$, where S is the price of the nearest to maturity contract, F_{τ} is the price of the τ-th nearby futures contract and r_{τ} is the LIBOR rate for the corresponding maturity.

The put option prices are "at-the-money-futures" in the sense that the strike price is equal to the futures price.[27] "At-the-money-futures" is written in quotations since the strike price of available options do not usually match the underlying futures price, though they are the closest match available. Nevertheless, it is possible to construct put option prices which are *exactly* at-the-money-futures. In order to do that, the available at-the-money-futures put prices are utilized to compute implied volatilities based on the Black (1976) formula. Then, on the basis of the same formula, the implied volatilities are used to construct new put option prices, where each strike price is set to equal the underlying future price. The exactly at-the-money-futures put price for the τ-th nearby contract is denoted by P_{τ}.

Using the computed weak backwardation and put option prices, the following regression is estimated,

$$B_{\tau,t}^{w} = \alpha_{\tau} + \beta_{\tau}P_{\tau,t} + \epsilon_{\tau,t}. \tag{25}$$

The estimation is based on the Cochrane-Orcutt procedure, which corrects for serial correlation in the error term. The results of this regression for the period of December 1986 through December 1991 are presented in Panel A of Table IV. Consistent with our theory, the estimated put option coefficients are positive and highly significant.

Theorem 4 indicates that the value of a European put option struck at the extraction cost of the marginal producer is a lower bound for the level of weak backwardation. If this put price were used in the regression, the intercept would be greater than zero and/or the slope coefficient would be greater than unity. However, as this put option price is not observable, an at-the-money-futures put price is used in the regression. The resulting intercept is insignificantly different from zero and the slope coefficient, although significantly positive, is significantly less than unity. These results are consistent with the fact that in an interior equilibrium the extraction cost of the marginal producer is lower than the prevailing futures price, i.e., these options are more valuable than those suggested by the model.

It is possible that strong backwardation as well as high implied volatility come about in periods of temporary shortage in the spot market. However, strong backwardation occurs in 77 percent of the sample period, which sug-

[26] There is no organized trading in standardized spot contracts for oil, and therefore, quoted spot prices will vary by dealer, quality and place of delivery. To this extent, the use of the first nearby futures price is preferable.

[27] We use "at-the-money-futures" because the options are written on futures contracts. This is equivalent to the common terminology on the street—"at-the-money-forward."

Table IV

Backwardation and the Put Option Price $B^w_{\tau,t} = \alpha_\tau + \beta_\tau P_{\tau,t} + \varepsilon_{\tau,t}$

Weak backwardation for the τ-th month futures contract, B^w_τ, is regressed on the corresponding at-the-money-futures put option price, P_τ. Weak backwardation is defined by $B^w_\tau = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR for the corresponding maturity. The Cochrane-Orcutt procedure is used for estimation ($\hat{\rho}$ is the autocorrelation coefficient estimate). The data used is daily.

	$\tau = 2$		$\tau = 3$		$\tau = 4$	
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$
Panel A: December 1986–April 1992 (1222 obs.)						
Estimate	0.147	0.139	0.018	0.450	−0.049	0.633
t-statistic	4.339	5.645	0.339	13.209	−0.682	15.129
p-value	0.000	0.000	0.367	0.000	0.248	0.000
$\hat{\rho}$	0.652		0.733		0.780	
Panel B: Excluding the Gulf War, August 1990–February 1991 (1083 obs.)						
Estimate	−0.047	0.356	0.047	0.359	0.093	0.407
t-statistic	−1.001	5.965	0.675	4.975	0.997	4.433
p-value	0.158	0.000	0.250	0.000	0.159	0.000
$\hat{\rho}$	0.766		0.799		0.805	

gests that it is not attributable to temporary shortages. Nevertheless, the Gulf War period (August 1990–February 1991) did result in some temporary shortage and was associated with extreme backwardation and high volatility. To examine whether the significant association between backwardation and the put option price stems solely from this period, we exclude it from our sample and repeat regression (25). The put price coefficients are still significant when the Gulf War is excluded as can be seen in Panel B of Table IV.

C. Backwardation and Volatility

Theorem 3 suggests that the level of backwardation is non-decreasing with an increase in the riskiness (in the sense of MPS) of the demand shock. Since an increase in riskiness implies an increase in volatility, a positive association is expected between the level of backwardation and the expected volatility of future oil prices. In order to examine this prediction, daily weak backwardation for the second, third, and fourth nearby futures contract, B^w_τ, is regressed on the implied volatility from the corresponding at-the-money-futures put option price, σ_τ:

$$B^w_{\tau,t} = \alpha_\tau + \beta_\tau \sigma_{\tau,t} + \epsilon_{\tau,t}. \tag{26}$$

Weak backwardation and implied volatility are defined above. The sample is unchanged as well. As can be seen in Table V, the coefficients are significantly positive. This provides further reinforcement for the previous result. As can be

Table V

Backwardation and Volatility $B^w_{\tau,t} = \alpha_\tau + \beta_\tau \sigma_{\tau,t} + \varepsilon_{\tau,t}$

Weak backwardation for the τ-th month futures contract, B^w_τ, is regressed on the implied volatility from the corresponding at-the-money put option price, σ_τ. Weak backwardation is defined by $B^w_\tau = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The implied volatility, σ_τ, (in percentage annual terms) is computed using the Black (1976) formula. The Cochrane-Orcutt procedure is used for estimation ($\hat{\rho}$ is the autocorrelation coefficient estimate). The data used is daily.

	$\tau = 2$		$\tau = 3$		$\tau = 4$	
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$
Panel A: December 1986–April 1992 (1222 obs.)						
Estimate	0.087	0.006	−0.042	0.019	0.130	0.021
t-statistic	1.931	5.132	−0.580	9.517	1.349	8.027
p-value	0.027	0.000	0.281	0.000	0.089	0.000
$\hat{\rho}$	0.652		0.743		0.810	
Panel B: Excluding the Gulf War, August 1990–February 1991 (1083 obs.)						
Estimate	−0.103	0.011	0.116	0.009	0.386	0.004
t-statistic	−1.846	5.952	1.512	3.766	5.040	1.703
p-value	0.032	0.000	0.065	0.000	0.000	0.044
$\hat{\rho}$	0.771		0.863		0.870	

seen in Panel B, the coefficients on volatility are also significantly positive when the Gulf War period is excluded.

As discussed in Section I, a binding production capacity constraint suggests a positive association between the spot price of oil and backwardation. In order to examine this prediction, the spot price of oil is introduced into the regression of backwardation on volatility. The results of the new regression are presented in Table VI. As can be seen in the table, the spot price coefficient is positive and highly significant. This provides support to the above argument of binding capacity constraint. Yet, the observed correlation between backwardation and the spot price may be spurious due to measurement errors, given that the spot price appears in both sides of the regression. It can be also seen in the table that the implied volatility coefficients remain significantly positive when the spot price is introduced. This suggests that the significance of the implied volatility in regression (26) cannot be attributed to an omitted price variable.

The tests conducted above also serve to distinguish between the proposed theory of backwardation and some other potential explanations. As suggested previously, slowly increasing extraction costs (slower than the interest rate) can support weak backwardation. Extraction costs that decrease sufficiently fast can support strong backwardation. Production at full capacity may explain backwardation as well. However, while these explanations predict no systematic relation between production and implied volatility and between backwardation and implied volatility, the proposed theory predicts such a relation.

Table VI

Backwardation, Volatility and Spot Price

$$B^{w}_{\tau,t} = \alpha_\tau + \beta_\tau \sigma_{\tau,t} + \gamma_\tau S_t + \varepsilon_{\tau,t}$$

Weak backwardation for the τ-th month futures contract, B^{w}_{τ}, is regressed on the implied volatility from the corresponding at-the-money put option price, σ_τ, and on the spot price, S. Weak backwardation is defined by $B^{w}_{\tau} = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The implied volatility, σ_τ, (in percentage annual terms) is computed using the Black (1976) formula. The Cochrane-Orcutt procedure is used for estimation ($\hat{\rho}$ is the autocorrelation coefficient estimate). The data used is daily.

	$\tau = 2$			$\tau = 3$			$\tau = 4$		
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\gamma}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\gamma}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$	$\hat{\gamma}_4$
Panel A: December 1986–April 1992 (1222 obs.)									
Estimate	−0.926	0.004	0.038	−2.151	0.011	0.087	−3.482	0.014	0.137
t-statistic	−12.931	3.672	12.201	−20.469	7.251	19.135	−27.694	7.235	26.773
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\hat{\rho}$		0.811			0.879			0.935	
Panel B: Excluding the Gulf War, August 1990–February 1991 (1083 obs.)									
Estimate	−3.054	0.013	0.096	−5.101	0.012	0.182	−6.628	0.009	0.248
t-statistic	−16.077	7.702	16.386	−21.168	5.913	22.710	−24.636	2.037	28.356
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.000
$\hat{\rho}$		0.839			0.902			0.949	

The above results suggest that the persistence of backwardation in the oil market results from rational extraction decisions of producers. This raises the question whether the small percentage of observations when the oil market was not in weak backwardation is prima facie evidence of irrational behavior. Earlier we briefly mentioned that many oil wells, such as stripper wells, must have a minimum level of production at any given time to be able to continue production in the future. If these high cost producers cease current production, future production would be reduced. When the price of oil is low and marginal producers are losing money at time 0 in order to maintain their options to produce in the future, it is possible for the oil market to be in contango (spot price lower than the discounted futures price). Figure 1 provides support for this conjecture. As can be seen in the figure, periods of negative backwardation are associated with relatively low oil prices.

IV. Conclusions

This paper presents a theory of backwardation in oil futures markets. The paper focuses on the producer's behavior and demonstrates that the call option feature of oil in the ground is a significant source of backwardation.

In a two period framework it is shown that the market is always weakly backwardated in an interior equilibrium. The weak backwardation is equal to the value of a put option with an exercise price equaling the extraction cost of the marginal producer. If the value of the put is sufficiently large, strong backwardation emerges. Equilibrium production is shown to be non-increasing in the riskiness of future prices. The existence of weak backwardation in an interior equilibrium is generalized to a multiperiod framework—the market is weakly backwardated with respect to any future time t. Weak backwardation is bounded from below by the value of a t-period European futures put option with a strike price equal to the time t extraction cost of the marginal producer.

The empirical evidence is consistent with the predictions of the model. Non-regulated oil production within the United States is found to be inversely related to the implied volatility from at-the-money-futures option prices. Weak backwardation is shown to have a significant positive association with at-the-money-futures put option prices and their implied volatility.

Appendix

Define the function $H(x(d_{\tilde{\epsilon}}, \theta); \tilde{\epsilon}, \theta)$ as follows (suppressing the arguments of x):

$$H(x;\ \tilde{\epsilon},\ \theta) \equiv S_0 - e^{-r}x - C(\tilde{S}_1,\ x,\ r) \tag{27}$$

Also define the function $c_i(\tilde{\epsilon}, k_i, \theta)$, $i = 1, 2$, where

$$k_1 = bx - a + Q_1 \frac{x}{\bar{x}} \tag{28}$$

$$k_2 = b\bar{x} - a + Q - Q_0 \frac{x}{\bar{x}}, \tag{29}$$

which is the value of a call option written on $\tilde{\epsilon}$ with an the exercise price of k_i. The option $C(\tilde{S}_1, x, r)$ pays off at time 1 $\max[\tilde{S}_1 - x, 0]$. It is easy to verify that,

$$\max[\tilde{S}_1 - x,\ 0] \equiv \frac{\bar{x}}{b\bar{x} + Q} \max[\tilde{\epsilon} - k_1,\ 0] + \frac{Q}{b(b\bar{x} + Q)} \max[\tilde{\epsilon} - k_2,\ 0], \tag{30}$$

which implies that

$$C(\tilde{S}_1,\ x,\ r) \equiv \frac{\bar{x}}{b\bar{x} + Q} c_1(\tilde{\epsilon},\ k_1,\ \theta) + \frac{Q}{b(b\bar{x} + Q)} c_2(\tilde{\epsilon},\ k_2,\ \theta). \tag{31}$$

Hence, the function $H(x; \tilde{\epsilon}, \theta)$ can be written as:

$$H(x; \tilde{\epsilon}, \theta) = \frac{a}{b} - \frac{Q_0}{b\bar{x}}x - e^{-r}x - \frac{\bar{x}}{b\bar{x} + Q}c_1(\tilde{\epsilon}, k_1, \theta) - \frac{Q}{b(b\bar{x} + Q)}c_2(\tilde{\epsilon}, k_2, \theta). \tag{32}$$

We now establish the following results which are used in the proofs to follow:

LEMMA 1: *$H(x; \tilde{\epsilon}, \theta)$ is decreasing in x; i.e.,*

$$\frac{\partial H(x; \tilde{\epsilon}, \theta)}{\partial x} < 0. \tag{33}$$

Proof of Lemma 1: By the standard option result that $-e^{-r} \leq \partial c_i/\partial k_i \leq 0$ we obtain from (32)

$$\begin{aligned}\frac{\partial H}{\partial x} &= -\frac{Q_0}{b\bar{x}} - e^{-r} - \frac{b\bar{x} + Q_1/\bar{x}}{b\bar{x} + Q}\frac{\partial c_1}{\partial k_1} + \frac{QQ_0}{b\bar{x}(b\bar{x} + Q)}\frac{\partial c_2}{\partial k_2} \\ &\leq -\frac{Q_0}{b\bar{x}} - e^{-r} - \frac{b\bar{x} + Q_1/\bar{x}}{b\bar{x} + Q}\frac{\partial c_1}{\partial k_1} \\ &\leq -\frac{Q_0}{b\bar{x}} - \frac{e^{-r}Q_0}{\bar{x} + Q} \\ &< 0.\end{aligned}$$

□

LEMMA 2: *Let $J(x) = S_0 - e^{-r}x - C(\tilde{S}_1, x, r)$. Then, $J(x)$ is non-increasing in x; i.e.,*

$$\frac{\partial J(x)}{\partial x} \leq 0.$$

(Note: In the function $J(\cdot)$, $\tilde{S}_1$ is taken as given, whereas in the function $H(\cdot)$, $\tilde{S}_1$ is endogenous.)

Proof of Lemma 2: By the standard option result that $-e^{-r} \leq \partial C/\partial x \leq 0$ we obtain:

$$\frac{\partial J}{\partial x} = -e^{-r} - \frac{\partial C}{\partial x} \leq 0.$$

□

LEMMA 3: *Let $\tilde{\epsilon}_1, \tilde{\epsilon}_2 \in E$ and $\tilde{\xi} \in E$, where*[28] $\tilde{\epsilon}_2 \overset{d}{\sim} \tilde{\epsilon}_1 + \tilde{\xi}$ *and* $E(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$*, Then;*

$$H(x; \tilde{\epsilon}_1, \theta) \geq H(x; \tilde{\epsilon}_2, \theta), \tag{34}$$

for all $x \in [0, \tilde{x}]$ and for all $\theta \in \Theta$.

[28] "$\overset{d}{\sim}$" means "is equal in distribution as."

Proof of Lemma 3: The value of the European call option is non-decreasing in the riskiness of the demand shock (in the sense of mean preserving spread). Thus, we have

$$c_i(\tilde{\epsilon}_1, k, \theta) \leq c_i(\tilde{\epsilon}_2, k, \theta).$$

The desired result follows immediately from the definition of $H(x, \tilde{\epsilon}, \theta)$ and the use of Jensen's inequality. □

THEOREM A1: *Existence and uniqueness of equilibrium (necessary and sufficient conditions): Let $q^e(d_{\tilde{\epsilon}}, \theta)$ be the equilibrium production. Then,*

1. *For all $(\tilde{\epsilon}, \theta) \in (E \times \Theta)$ such that*

$$\frac{a}{b} \leq e^{-r} F^e, \tag{35}$$

we have $q^e(d_{\tilde{\epsilon}}, \theta) = 0$.

2. *For all $(\tilde{\epsilon}, \theta) \in (E \times \Theta)$ such that*

$$\frac{a - Q_0}{b} - e^{-r}\bar{x} \geq C(\tilde{S}_1, \bar{x}, r), \tag{36}$$

we have $q^e(d_{\tilde{\epsilon}}, \theta) = Q_0$.

3. *For all $(\tilde{\epsilon}, \theta) \in (E \times \Theta)$ such that*

$$\frac{a}{b} > e^{-r} F^e, \tag{37}$$

and

$$\frac{a - Q_0}{b} - e^{-r}\bar{x} < C(\tilde{S}_1, \bar{x}, r), \tag{38}$$

there exists a unique $q^e(d_{\tilde{\epsilon}}, \theta) \in (0, Q_0)$.

Proof of Theorem A1:

1. Necessity:
$q^e(d_{\tilde{\epsilon}}, \theta) = 0$ implies $H(0; \tilde{\epsilon}, \theta) \leq 0$. Hence,

$$H(0; \tilde{\epsilon}, \theta) = \frac{a}{b} - C(\tilde{S}_1, 0, r) \leq 0.$$

The result (equation (35)) follows immediately since $e^{-r}F^e = C(\tilde{S}_1, 0, r)$.

Sufficiency:
Let $a/b \leq e^{-r}F^e$ and suppose that $q^e(d_{\tilde{\epsilon}}, \theta) > 0$. Using Lemma 1 for the

strict inequality, it follows that

$$\begin{aligned} 0 &\leq \mathrm{H}(x^e;\, \tilde{\epsilon},\, \theta) \\ &< \mathrm{H}(0;\, \tilde{\epsilon},\, \theta) \\ &= \frac{a}{b} - e^{-r} F^e \\ &< 0. \quad \rightarrow \leftarrow \end{aligned}$$

Hence, $q^e(\tilde{\epsilon}, \theta) = 0$.

2. Necessity:
$q^e(d_{\tilde{\epsilon}}, \theta) = Q_0$ implies $\mathrm{H}(\bar{x}; \tilde{\epsilon}, \theta) \geq 0$. Hence,

$$\mathrm{H}(\bar{x};\, \tilde{\epsilon},\, \theta) = \frac{a - Q_0}{b} - e^{-r}\bar{x} - \mathrm{C}(\tilde{S}_1,\, \bar{x},\, r) \geq 0.$$

The result (equation (36)) follows immediately.

Sufficiency:
Let $(a - Q)/b - e^{-r}\bar{x} \geq C(\bar{S}_1, \bar{x}, r)$ and suppose that $q^e(d_{\tilde{\epsilon}}, \theta) < Q_0$. It follows that (using Lemma 1 for the strict inequality),

$$\begin{aligned} 0 &\geq \mathrm{H}(x^e;\, \tilde{\epsilon},\, \theta) \\ &> \mathrm{H}(\bar{x};\, \tilde{\epsilon},\, \theta) \\ &= \frac{a - Q_0}{b} - e^{-r}\bar{x} - \mathrm{C}(\tilde{S},\, \bar{x},\, r) \\ &\geq 0. \quad \rightarrow \leftarrow \end{aligned}$$

Hence, $q^e(d_{\tilde{\epsilon}}, \theta) = Q_0$.

3. To prove the necessity and sufficiency of the conditions for the existence of an interior equilibrium (equations (37) and (38)), as well as its uniqueness, just combine the first and second results in this theorem together with Lemma 1. The third result follows then by the intermediate value theorem. □

THEOREM A2: *Sufficient conditions for interior equilibrium:*

1. If $(\tilde{\epsilon}, \theta) \in (\mathrm{E} \times \Theta)$ *and*

$$\mathrm{Var}(\tilde{\epsilon}) < Q, \tag{39}$$

and

$$\frac{a - Q_0}{b} \leq e^{-r}\bar{x}, \tag{40}$$

then $q^e(d_{\tilde{\epsilon}}, \theta) \in (0, Q_0)$.

2. *If* $(\tilde{\epsilon}, \theta) \in (\mathrm{E} \times \Theta)$ *and*

$$\tilde{\epsilon} < Q, \tag{41}$$

and

$$\frac{a - Q_0}{b} \leq e^{-r}\bar{x}, \tag{42}$$

then $q^e(d_{\tilde{\epsilon}}, \theta) \in (0, Q_0)$.

Proof of Theorem A2:

1. (a) If $\mathrm{Var}(\tilde{\epsilon}) < Q$, then

$$\begin{aligned}
&\mathrm{H}(0; \tilde{\epsilon}, \theta) \\
&= \frac{a}{b} - \frac{\bar{x}}{b\bar{x} + Q} c_1(\tilde{\epsilon}, -a, \theta) - \frac{Q}{b(b\bar{x} + Q)} c_2(\tilde{\epsilon}, b\bar{x} - a + Q, \theta) \qquad (43)\\
&= \frac{a}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} \mathrm{E}(\tilde{\epsilon} | \tilde{\epsilon} \geq -a)\mathrm{P}(\tilde{\epsilon} \geq -a) + \frac{e^{-r}\bar{x}a}{b\bar{x} + Q} \mathrm{P}(\tilde{\epsilon} \geq -a) \\
&\quad - \frac{e^{-r}Q}{b(b\bar{x} + Q)} \mathrm{E}(\tilde{\epsilon} | \tilde{\epsilon} \geq b\bar{x} - a + Q)\mathrm{P}(\tilde{\epsilon} \geq b\bar{x} - a + Q) \\
&\quad + \frac{e^{-r}Q(b\bar{x} - a + Q)}{b(b\bar{x} + Q)} \mathrm{P}(\tilde{\epsilon} \geq b\bar{x} - a + Q) \qquad (44)\\
&\geq e^{-r}\frac{Q}{b} + \frac{a}{b}(1 - e^{-r}) \\
&\quad - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} \mathrm{E}(\tilde{\epsilon} 1_{\{\tilde{\epsilon} \geq -a\}}) - \frac{e^{-r}Q}{b(b\bar{x} + Q)} \mathrm{E}(\tilde{\epsilon} 1_{\{\tilde{\epsilon} \geq b\bar{x} - a + Q\}}) \qquad (45)\\
&\geq e^{-r}\frac{Q}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} \mathrm{E}|\tilde{\epsilon} 1_{\{\tilde{\epsilon} \geq -a\}}| - \frac{e^{-r}Q}{b(b\bar{x} + Q)} \mathrm{E}|\tilde{\epsilon} 1_{\{\tilde{\epsilon} \geq b\bar{x} - a + Q\}}| \qquad (46)\\
&\geq e^{-r}\frac{Q}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} (\mathrm{E}|\tilde{\epsilon}|^2)^{1/2}(\mathrm{E}|1_{\{\tilde{\epsilon} \geq -a\}}|^2)^{1/2} \\
&\quad - \frac{e^{-r}Q}{b(b\bar{x} + Q)} (\mathrm{E}|\tilde{\epsilon}|^2)^{1/2}(\mathrm{E}|1_{\{\tilde{\epsilon} \geq b\bar{x} - a + Q\}}|^2)^{1/2} \qquad (47)\\
&\geq e^{-r}\frac{Q}{b} - e^{-r}\frac{1}{b}(\mathrm{E}|\tilde{\epsilon}|^2)^{1/2} \qquad (48)\\
&> 0.
\end{aligned}$$

To pass from (44) to (45) we note that the probability takes no values larger than 1. The passage from (45) to (46) is due to the introduction of absolute values. We used the Cauchy-Schwartz inequality and the sufficient condition to pass from (46) to (47). In passing from (47) to (48) we utilize the boundedness of the indicator function by 1 and then collect terms. The last inequality is obtained from the sufficient condition. Hence, if $\mathrm{Var}(\tilde{\epsilon}) < Q$ then there is positive production at time 0.

$$
\begin{aligned}
\text{(b)} \quad \mathrm{H}(\bar{x};\, \tilde{\epsilon},\, \theta) &= \frac{a - Q_0}{b} - e^{-r}\bar{x} \\
&\quad - e^{-r}\frac{1}{b}\,\mathrm{E}(\tilde{\epsilon} - b\bar{x} + a + Q_1 | \tilde{\epsilon} - b\bar{x} + a + Q_1 \geq 0) \\
&\quad \times \mathrm{P}(\tilde{\epsilon} - b\bar{x} + a + Q_1 \geq 0) \\
&\leq \frac{a - Q_0}{b} - e^{-r}\bar{x} \\
&< 0.
\end{aligned}
$$

The first inequality holds since the second term is non-negative A sufficient condition for the second inequality is $(a - Q)/b \leq e^{-r}\bar{x}$. Hence, if this is the case, some reserves will be left for the second period.

2. (a) Assume that $\tilde{\epsilon}$ is bounded by L. Then,

$$
\begin{aligned}
\mathrm{H}(0;\, \tilde{\epsilon},\, \theta) &= \frac{a}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q}\,\mathrm{E}(\tilde{\epsilon} + a | \tilde{\epsilon} \geq -a)\mathrm{P}(\tilde{\epsilon} \geq -a) \\
&\quad - \frac{e^{-r}Q}{b(b\bar{x} + Q)}\,\mathrm{E}(\tilde{\epsilon} - b\bar{x} + a - Q | \tilde{\epsilon} \geq b\bar{x} - a + Q) \\
&\quad \times \mathrm{P}(\tilde{\epsilon} \geq b\bar{x} - a + Q) \\
&\geq \frac{a}{b} - \frac{e^{-r}\bar{x}(L + a)}{b\bar{x} + Q} - \frac{e^{-r}(L - b\bar{x} + a - Q)}{b(b\bar{x} + Q)} \\
&\geq e^{-r}\frac{Q - L}{b} \\
&> 0.
\end{aligned}
$$

The last inequality holds if $Q > L$. Hence, $\tilde{\epsilon}$ bounded by Q is a sufficient condition for positive production at time 0.

(b) Same as 1(b) above. □

Proof of Theorem 2: The proof is given for the case of an interior equilibrium; the reasoning can be exactly reproduced for the boundary equilibria. Let $\tilde{\epsilon}_1$, $\tilde{\epsilon}_2$

$\in E_{\setminus\{0\}}$ and $\tilde{\xi} \in E_{\setminus\{0\}}$, where $\tilde{\epsilon}_2 \sim \tilde{\epsilon}_1 + \tilde{\xi}$ and $E(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$. Suppose that for all $\theta \in \Theta$,

$$q^e(d_{\tilde{\epsilon}_2}, \theta) > q^e(d_{\tilde{\epsilon}_1}, \theta)$$

This in turn implies that,

$$x_2^e \equiv x^e(d_{\tilde{\epsilon}_2}, \theta) > x^e(d_{\tilde{\epsilon}_1}, \theta) \equiv x_1^e.$$

Now, using Lemma 1 for the first inequality and Lemma 3 for the second inequality, we obtain,

$$\begin{aligned} 0 &= H(x_1^e, \tilde{\epsilon}_1, \theta) \\ &> H(x_2^e, \tilde{\epsilon}_1, \theta) \\ &\geq H(x_2^e, \tilde{\epsilon}_2, \theta) \\ &= 0. \quad \rightarrow \leftarrow \end{aligned}$$

Hence, $q^e(d_{\tilde{\epsilon}_2}, \theta) \leq q^e(d_{\tilde{\epsilon}_1}, \theta)$. □

Proof of Theorem 3: To establish the results of the theorem it suffices to show that:

$$S_0(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta) \geq S_0(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta), \tag{49}$$

and

$$E[S_1(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta; \tilde{\epsilon}_2)] \leq E[S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \tilde{\epsilon}_1)]. \tag{50}$$

From Theorem 2 it follows that $q^e((d_{\tilde{\epsilon}_2}, \theta), \theta) \leq q^e((d_{\tilde{\epsilon}_1}, \theta), \theta)$ since $\tilde{\epsilon}_2$ is a mean preserving spread of $\tilde{\epsilon}_1$. Equation (49) follows since $S_0(q, \theta)$ is decreasing in q. Since $S_1(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta; \epsilon)$ is linear in ϵ for $\epsilon \in [\epsilon_1^*, \epsilon_1^{**}]$, and since $\tilde{\epsilon}_2$ is a mean preserving spread of $\tilde{\epsilon}_1$, it follows immediately that:

$$E[S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \tilde{\epsilon}_2)] = E[S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \tilde{\epsilon}_1)]. \tag{51}$$

Since $S_1(q, \theta; \tilde{\epsilon})$ is nondecreasing in q and since $q^e((d_{\tilde{\epsilon}_2}, \theta), \theta) \leq q^e((d_{\tilde{\epsilon}_1}, \theta), \theta)$, for any realization ϵ of $\tilde{\epsilon}_2$ it is true that:

$$S_1(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta; \epsilon) \leq S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \epsilon),$$

and hence,

$$E[S_1(q^e(d_{\tilde{\epsilon}_2}, \theta), \theta; \tilde{\epsilon}_2)] \leq E[S_1(q^e(d_{\tilde{\epsilon}_1}, \theta), \theta; \tilde{\epsilon}_2)]. \tag{52}$$

The result in (50) follows immediately by combining (51) and (52). □

Proof of Theorem 4: The zero cost producer will choose to defer extraction whenever $S_0^{e0} \leq e^{-rt}F_t^{e0}$ for some $t \leq T$. All other producers, who have higher extraction costs, will choose to defer extraction as well. Hence, a necessary condition for positive production at time 0 is $B_{w,t}^{e0} > 0$ for all $t \leq T$.

Now, $C^T_{\{x^{e0}_u\}}$ is a T-period American call option on the spot with a strike of $\{x^{e0}_u\}_{u<T}$ where $x^{e0}_u = e^{-r(T-u)}x^{e0}$. It is no lower in value than a t-period American call on the spot with a strike price of $\{x^{e0}_u\}_{u<t}$. This, in turn, is no lower in value than a t-period European call option on the spot with a strike price x^{e0}_t, which equals a t-period call option on the futures with the same strike price. Denoting the latter by $c^t_{x^{e0}_t}$, the above inequalities can be summarized as

$$C^T_{\{x^{e0}_u\}} \geq c^t_{x^{e0}_t} \qquad \text{for all} \quad t \leq T. \tag{53}$$

Put call parity implies

$$c^t_{x^{e0}_t} = p^t_{x^{e0}_t} + e^{-rt}(F^{e0}_t - x^{e0}_t). \tag{54}$$

Now combine the equilibrium condition (19) with (53) and (54) to obtain $B^{e0}_{w,t} \geq [p^t_{x^{e0}_t}]^+$ and $B^{e0}_{s,t} \geq [p^t_{x^{e0}_t} - (1 - e^{-rt})F^{e0}_t]^+$ for all $t \leq T$. □

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