Noise Trading in Small Markets

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ABSTRACT

Considering noise traders as agents with unpredictable beliefs, we show that in an imperfectly competitive market with risk averse investors, noise traders may earn higher expected utility than rational investors. This happens when, by deviating from the Nash equilibrium strategy, noise traders hurt rational investors more than themselves. It follows that the willingness of arbitrageurs to exploit noise traders' misperceptions is lower relative to a perfectly competitive economy. This result reinforces the theory that noise trading may explain closed-end fund discounts and small firms' returns, since these markets are less competitive than the market for large firms' stock.

DE LONG, SHLEIFER, SUMMERS, AND WALDMANN [DSSW henceforth] (1989, 1990, 1991) define noise traders as those who "falsely believe that they have special information about the future price of risky assets."[1] A consequence of noise trading is that, if investors have short horizons and noise traders' misperceptions cannot be forecasted by arbitrageurs, then the fundamental risk is not the only source of risk in the market. Investors care about the resale price of the assets they hold, and not merely about the present value of dividends. Given such an assumption, the unpredictability of noise traders' sentiments brings an additional risk into the market: the risk that noise traders' beliefs will not revert to their mean for a long time and might become more extreme in the meantime.

To give an example of profitable destabilizing speculation (when rational speculation is price-stabilizing), DSSW (1990) studied arbitrage dedicated to exploiting noise traders' misperceptions. They show that in a perfectly competitive economy with risk-averse agents, noise traders, bearing a larger amount of risk relative to rational investors, may earn higher expected returns. They conclude that speculation based on irrational beliefs may be profitable and that, if relative success breeds imitation, noise traders are not driven out of the market and influence prices in the long run.

[1] An alternative definition is usually found in the literature on informed speculation. Noise trading is modeled by way of a random, exogenous, inelastic demand for an asset as in Kyle (1984, 1985, 1989). For a general discussion on noise in finance, see Black (1986, pp. 530–534).

It is odd to assert that noise traders perform better when risk-averse agents are considered and the amount of risk undertaken by traders is not taken into account in evaluating relative success. When agents maximize welfare, an agent can be said to perform better than other agents only if he reaches a higher welfare level. In DSSW's model, first, noise traders earn a lower expected utility than rational investors and, second, noise traders' expected utility is unbounded below as a function of their beliefs. As a consequence, an outsider observing both rational investors' and noise traders' performances, not only concludes that rational investors always perform better but also that noise traders would sometime be better off not trading. Hence, we think that in a perfectly competitive market with risk-averse agents, the noise traders' strategy should not attract many investors and should be driven out of the market in the long run.

In this article, we show that, when considering imperfectly competitive markets[2] with risk-averse agents, bets against rational speculators, based on irrational beliefs, may be profitable: noise traders may earn higher returns and obtain *higher expected utility* than rational investors. As a consequence, if relative success (i.e., higher utility) breeds imitation, noise traders will dominate the market in the long run.

Imperfect competition influences speculative markets in two ways. First, quantities traded are smaller than in a perfectly competitive economy because of agents' market power. By lowering quantities traded, arbitrageurs and noise traders limit their price stabilizing and price destabilizing effects. Second, irrational behavior can impose higher costs on rational competitors than on the irrational agent himself. For example, consider a Cournot oligopoly, in which all firms have the same cost function and face a downward sloping inverse demand curve. Assume that some irrational firms produce a larger quantity than the Cournot equilibrium quantity. Then all firms earn lower profits relative to the Cournot equilibrium profits. However, as all firms sell their production at the same price, irrational firms have higher revenues than rational firms. If the difference in total costs is lower than the difference in revenues, irrational firms make larger profits relative to rational firms (see Hansen and Samuelson (1988), Schaffer (1989), Palomino (1995), Palomino and Waldmann (1995)). In financial markets, if we consider expected returns as revenues and risks borne as total costs, the same result is obtained: irrational traders perform better than rational investors.

The organization of this article is as follows. In Section I, the model is presented and the difference in expected utility is calculated. Section II analyzes the persistence of noise traders in an extended model in which agents earning higher utility are imitated.

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[2] Even though financial markets are usually seen as the best example for a perfectly competitive market, not all markets have this property, and the study of imperfectly competitive markets is far from being useless. As Lakonishok, Shleifer, and Vishny (1991) demonstrate, in small markets, i.e., stocks with small market capitalization, institutional investors influence prices. Therefore, at the scale of the New York Stock Exchange, imperfect competition may be neglected, but at the scale of a European stock exchange, it may be an important issue.

I. The Model

A. *Presentation of the Model*

We consider a model similar to DSSW (1990), i.e., an overlapping generation model with two-period-lived agents. There is no consumption in the first period, and the only decision agents make is to choose a portfolio when young.

The economy contains two assets. The first asset b is a risk free bond available in perfectly elastic supply and pays a fixed coupon r. In each period, one unit of b can be changed into one unit of consumption good. With consumption each period taken as numeraire, the price of b is always fixed to one. The second asset u is a risky stock. It pays a random dividend δ_t, independently and identically normally distributed with mean $\delta^* > r$ and variance σ_δ^2. The stock supply is a fixed and unchangeable quantity: Z units. The price of u in period t is denoted p_t.

There are two types of agents: sophisticated investors (denoted s) who have rational expectations and noise traders (denoted n). We assume that J sophisticated investors and M noise traders are present in the model and that all agents of a given type are identical. Both types of agents choose their portfolio when young to maximize perceived expected utility given their own beliefs about the ex-ante distribution of δ_{t+1} and p_{t+1}. Both noise traders and sophisticated investors who are young in period t accurately perceive the distribution of p_{t+1}.[3] The representative sophisticated investor accurately perceives the distribution of δ_{t+1}. The representative noise trader misperceives the expected dividend (δ^*) by an independent random variable ρ_t identically normally distributed with mean ρ^* and variance σ_ρ^2. Each agent's utility is a constant absolute risk aversion function of wealth when old:

$$U = -e^{-(\gamma w)} \qquad (1)$$

where γ is the coefficient of absolute risk aversion. At time t ($t = 1, \ldots, \infty$), each sophisticated investor j ($j = 1, \ldots, J$) chooses a demand schedule $X_{j,t}(\cdot)$ and each noise trader m ($m = 1, \ldots, M$) chooses a demand schedule $Y_{m,t}(\cdot)$. Given a market clearing price p_t, the quantities traded by both sophisticated investors and noise traders can be written

$$x_{j,t} = X_{j,t}(p_t), j = 1, \ldots, J; \quad y_{m,t} = Y_{m,t}(p_t), m = 1, \ldots, M. \qquad (2)$$

When old, agents convert their holdings of b to the consumption good, sell their holdings of u for price p_{t+1} to the new youngs, and consume all their wealth.

B. *Existence of a Nash Equilibrium in Demand Schedules*

Since utility function maximization is equivalent to a quadratic function maximization, we know from Kyle (1989), that if all agents conjecture linear

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[3] In DSSW (1990), noise traders accurately perceive the distribution of the dividend but misperceive the distribution of the price.

residual supply curves (i.e., for all $j = 1, \ldots, J$, $\tilde{p}_t = \tilde{\mu}_{s,t} + \lambda_{s,t} x_{j,t}$ and for all $m = 1, \ldots, M$, $\tilde{p}_t = \tilde{\mu}_{n,t} + \lambda_{n,t} y_{m,t}$) then there exists a symmetric linear Nash equilibrium in demand schedules that is also a Rational Expectation Equilibrium. Let E_t denote the expectation operator as of time t and N be the number of youngs in the economy (i.e., $N = J + M$). Then, demand schedules are given by the following proposition:

PROPOSITION 1: *Assume $N > 2$. There exists a symmetric linear Nash equilibrium in demand schedules such that (for all $j = 1, \ldots, J$ and for all $m = 1, \ldots, M$)*

$$X_{j,t}(p_t) = \frac{(N-2)}{(N-1)} \left[\frac{E_t(\tilde{\delta}_{t+1} + \tilde{p}_{t+1}) - p_t(1+r)}{\gamma \mathrm{Var}_t(\tilde{\delta}_{t+1} + \tilde{p}_{t+1})} \right] \tag{3}$$

$$Y_{m,t}(p_t) = \frac{(N-2)}{(N-1)} \left[\frac{E_t(\tilde{\delta}_{t+1} + \tilde{p}_{t+1}) + \rho_t - p_t(1+r)}{\gamma \mathrm{Var}_t(\tilde{\delta}_{t+1} + \tilde{p}_{t+1})} \right] \tag{4}$$

Proof: see Appendix.

This equilibrium may not be unique. However it is the only one such that both sophisticated investors and noise traders consider the risky asset a normal good. Furthermore, as the only random variable in $\tilde{\mu}_{s,t}$ is $\tilde{p}_t$, it follows that rational investors have a perfect knowledge of noise traders' beliefs when they compute their demand schedule.

The terms in square brackets in equations (3) and (4) are demands of perfect competition. As in the competitive model, when noise traders are bullish ($\rho_t > 0$), they demand larger quantities than sophisticated investors; in periods they are bearish ($\rho_t < 0$), they demand lower quantities. The coefficient $(N-2)/(N-1)$ represents the degree of imperfect competition in the economy. The lower the degree of competition in the economy, the higher the impact of an individual agent's trade on price, and so the larger the difference in quantities traded between a perfectly competitive economy and an imperfectly competitive one.

C. The Pricing Function

From the demand functions, we can compute the equilibrium price. As the olds sell their holdings, the demand of the youngs must be equal to Z in equilibrium. We deduce that

$$p_t = \frac{1}{(1+r)} \left[\delta^* + E_t(\tilde{p}_{t+1}) + \frac{M}{N} \rho_t - \frac{Z}{N} \frac{(N-1)}{(N-2)} \gamma \mathrm{Var}_t(\tilde{\delta}_{t+1} + \tilde{p}_{t+1}) \right] \tag{5}$$

We can see that p_{t+1} is independent of δ_{t+1}, therefore

$$\mathrm{Var}_t(\tilde{\delta}_{t+1} + \tilde{p}_{t+1}) = \mathrm{Var}_t(\tilde{\delta}_{t+1}) + \mathrm{Var}_t(\tilde{p}_{t+1}) \tag{6}$$

As DSSW (1990), we consider only steady state equilibria by requiring that the unconditional distribution of $\tilde{p}_{t+1}$ is equal to the unconditional distribution of $\tilde{p}_t$. Then, we obtain

$$p_t = \frac{\delta^*}{r} + \frac{M}{N}\frac{(\rho_t - \rho^*)}{(1+r)} + \frac{M}{N}\frac{\rho^*}{r} - \frac{Z}{N}\frac{(N-1)}{(N-2)}\frac{\gamma}{r}[\sigma_\delta^2 + \text{Var}_t(\tilde{p}_{t+1})] \quad (7)$$

It follows that

$$\text{Var}(\tilde{p}_{t+1}) = \text{Var}_t(\tilde{p}_{t+1}) = \frac{\sigma_\rho^2}{(1+r)^2}\frac{M^2}{N^2} \quad (8)$$

Therefore, market volatility is only the result of noise traders' misperception of the dividend's distribution function. We can now compute the equilibrium price:

$$p_t = \frac{\delta^*}{r} + \frac{M}{N}\frac{(\rho_t - \rho^*)}{(1+r)} + \frac{M}{N}\frac{\rho^*}{r}\frac{Z}{N}\frac{(N-1)}{(N-2)}\frac{\gamma}{r}\left[\sigma_\delta^2 + \frac{M^2}{N^2}\frac{\sigma_\rho^2}{(1+r)^2}\right] \quad (9)$$

The first three terms of equation (9) are identical to those of the model of perfect competition: The first term represents the fundamental value of the asset: the present value of expected future dividends. The second term captures the price variations due to fluctuations of noise traders' beliefs and the third term represents the average impact of noise traders on price. The term in square brackets represents the risks borne by traders. The coefficient (Z/N) is the number of shares supplied per trader, while the coefficient $(N - 1)/(N - 2)$ indicates that the market rewards the lack of competition. Therefore, for given fundamental value, risks borne by traders and noise traders' misperception, the less competitive the economy the lower the price of the risky asset.

Lee, Shleifer, and Thaler (henceforth LST) (1991) review the standard explanation of the closed-end fund puzzle, i.e., closed-end fund shares are sold at prices that are, on average, lower than the per share market value of assets that the fund holds. They suggest that noise trading may explain closed-end fund discounts. Closed-end fund shares are predominantly traded by individual investors and so the noise trader risk is higher in closed-end fund share markets than in stock markets. Equation (9) proposes an additional explanation: part of the discount can be explained by a market size effect. Given the share of noise traders in the market (M/N), their misperceptions (ρ_t) and the number of stocks per trader (Z/N), the smaller the size of the market, the lower the price. Closed-end fund share markets are much smaller than markets for the stocks held by the funds. Therefore, if closed-end fund share markets are seen as imperfectly competitive, the lack of competition is rewarded; additionally, the thinner the market, the larger the discount, on average. Our explanation is complementary to LST's theory. The thinner a market, the larger the relative share of individual investors. As a result, noise trader risk and the reward for the lack of competition are larger. It follows that, on average, closed-end fund shares are traded at a discounted price.

D. Relative Utility Levels of Noise Traders and Sophisticated Investors

Since agents are risk-averse, we choose the difference in expected utility level as an index of relative performance between noise traders and sophisticated investors. From quantities traded and the equilibrium price, we can compute expected utilities. Define coefficients a, b, c, d, and g as follows:

$$a = M \frac{Z}{N}$$

$$b = -\frac{(N-2)}{2(N-1)^2} \frac{M^2}{N\gamma \mathrm{Var}(R)}$$

$$c = \frac{Z}{2N(N-1)}$$

$$d = \frac{N-2}{2(N-1)^2} \frac{[(N-2) + 2(M/N)]}{\gamma \mathrm{Var}(R)}$$

$$g = -\frac{Z^2 \gamma \mathrm{Var}(R)}{N(N-2)}$$

(10)

where $\mathrm{Var}(R)$ is the unconditional variance of the return per unit of risky asset, i.e.,

$$\mathrm{Var}(R) = \sigma_\delta^2 + \frac{M^2}{N^2} \frac{\sigma_\rho^2}{(1+r)^2}$$

(11)

It can be shown that

$$E_t(U_s) = -\exp(g + a\rho_t + b\rho_t^2)$$

(12)

$$E_t(U_n) = -\exp[g + (a - c)\rho_t + (b + d)\rho_t^2]$$

(13)

Call $\Delta U_{n-s,t}$ the difference of utility between noise traders and sophisticated investors who are young in t. Then, from the assumption that ρ_t is normally distributed, it follows that

$$E(\Delta U_{n-s,\,t}) > 0 \text{ is equivalent to}[4]$$

(14)

$$\frac{(a-c)^2 \sigma_\rho^2 + 2(a-c)\rho^* + 2(b+d)\rho^{*2}}{2[1 - 2(b+d)\sigma_\rho^2]} - \frac{a^2\sigma_\rho^2 + 2a\rho^* + 2b\rho^{*2}}{2(1 - 2b\sigma_\rho^2)}$$

$$< \frac{1}{2} \ln\left(\frac{1 - 2(b+d)\sigma_\rho^2}{1 - 2b\sigma_\rho^2}\right)$$

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[4] If X is a random variable normally distributed with mean m and variance v, then, for all α and $\beta < (2v)^{-1}$, $E[\exp(\alpha X + \beta X^2)] = \exp\{(\alpha^2 v + 2\alpha m + 2m^2\beta)/[2(1 - 2\beta v)]\}(1 - 2\beta v)^{-1/2}$. A sufficient condition to have $2(b + d)\sigma_\rho^2 < 1$ is that $\sigma_\delta > \sigma_\rho$.

For any finite market size and fundamental risk, there exists parameters ρ^* and σ_ρ^2 such that $E(\Delta U_{n-s}) > 0$. This happens when noise traders are, on average, moderately over-optimistic and the variance of their misperception is small relative to the variance of the dividend.[5] In a perfectly competitive economy, $c = 0$ and $d = 1$. Then, the left-hand-side of equation (14) is always positive, and the right-hand-side is always negative. Therefore, the inequality never holds, and sophisticated investors always earn higher expected utility than noise traders.

The difference in expected utility is increased relative to perfect competition because (*i*) the difference in expected returns is increased and (*ii*) imperfect competition decreases quantities traded and so decreases the difference in risk borne between noise traders and sophisticated investors. But the crucial point is that the difference in expected utility can be positive. Such a result is a direct consequence of agents' market power, which leads to market prices that are higher than marginal costs. Consider a Cournot oligopoly, in which firms have the same production costs and face a downward sloping inverse demand curve. If some irrational firms choose to produce a larger quantity than the Cournot equilibrium quantity, they act as Stackelberg leaders. Rational firms take these quantities as given and so reduce their production. Since all firms sell their production at the same price, irrational firms earn higher revenues. However, they also face higher total production costs. As the Cournot equilibrium market price is higher than the marginal cost, the difference in revenues can be larger than the difference in costs, and so irrational firms can earn higher profits than rational firms.

Such a result is impossible under the assumption of perfect competition since, when all firms produce the equilibrium quantity, the market price is equal to the marginal cost. Irrational firms over-producing always earn lower profits than rational firms (even if an infinite number of firms are irrational and, as a group, have an impact on price).

In financial markets, interpreting expected returns and the amount of risk borne as revenues and the cost from trading activities respectively, we have the same result as in any oligopolistic market: the lack of competition is rewarded. For a given amount of risk borne, expected returns are larger in an imperfectly competitive market. Noise traders trading larger quantities relative to sophisticated investors earn higher expected returns and the extra cost (i.e., the extra amount of risk borne) does not offset the higher revenues. As a result, noise traders obtain a higher expected utility than do rational investors.

II. Imitation of Beliefs

As we have observed, noise traders earn higher expected utility relative to rational investors; therefore, their success is not a consequence of pure luck, hence we may think their importance does not diminish over time. Unfortunately, as in

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[5] From equation (10), it follows that $\sigma_\rho^2 d < [N(N-2)\sigma_\rho^2]/[(N-1)^2\sigma_\delta^2]$. When $\sigma_\rho/\sigma_\delta$ is small, the right-hand-side of equation (14) is almost equal to 0, and the two denominators of the left-hand-side are almost equal. Furthermore, for ρ^* small enough, ρ^*d is negligible relative to c. In such a case, inequality (14) holds.

DSSW, the two-period model does not permit the study of the accumulation of wealth of noise traders relative to rational investors. An alternative approach is to consider the emulative behavior of new generations of traders.

A. The Imitation Rule

We assume that each generation of investors earns exogenous labor income when young and consumes all its wealth when old. Each generation has the same number of investors following noise traders and sophisticated investors strategies as the previous one, except a few investors in each generation change type on the basis of relative past performance.

Since we are employing a model with an integer number of traders, we cannot apply the imitation rule used in DSSW.[6] The one chosen is along the lines of that proposed by Kirman (1993): in each generation, all but one noise trader and all but one rational investor mimic their predecessors' strategy. The last two agents investigate which strategies were played, how they performed, and then choose the better one. Therefore, if noise traders have recently performed better, one young who would otherwise have been a sophisticated investor becomes a noise trader, and vice versa if sophisticated investors have performed better. Call $\Delta PERF_{n-s}$ the relative performance index chosen by a new generation of investors, and let M_t be the number of noise traders in the economy at time t. The imitation rule is the following:

$$M_{t-1} \in \{1, \ldots, N-1\} \quad \text{If } \Delta PERF_{n-s,t-1} > 0 \text{ and } M_t < N \text{ then } M_{t+1} = M_t + 1$$

$$\text{If } \Delta PERF_{n-s,t-1} > 0 \text{ and } M_t = N \text{ then } M_{t+1} = N$$

$$\text{If } \Delta PERF_{n-s,t-1} < 0 \text{ and } M_t > 0 \text{ then } M_{t+1} = M_t - 1$$

$$\text{If } \Delta PERF_{n-s,t-1} < 0 \text{ and } M_t = 0 \text{ then } M_{t+1} = 0$$

$$M_{t-1} = 0 \qquad \text{If } M_t = 0 \qquad \text{then } \text{Prob}(M_{t+1} = 1) = \alpha$$

$$\text{then } \text{Prob}(M_{t+1} = 0) = 1 - \alpha$$

$$\text{If } M_t = 1 \qquad \text{then } M_{t+1} = 1$$

$$M_{t-1} = N \qquad \text{If } M_t = N \qquad \text{then } \text{Prob}(M_{t+1} = N - 1) = \alpha$$

$$\text{Prob}(M_{t+1} = N) = 1 - \alpha$$

$$\text{If } M_t = N - 1 \qquad \text{then } M_{t+1} = N - 1$$

$$(15)$$

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[6] DSSW's model considers a continuum of length θ_t of noise traders and a continuum of length $(1 - \theta_t)$ of rational investors. The evolution rule is $\theta_{t+1} = \text{Max}[0, \text{Min}(\theta_t + \varepsilon \Delta R_{n-s,t}, 1)]$ where $\Delta R_{n-s,t}$ is the difference in returns between noise traders and rational investors who are young in t, and ε is the rate at which additional new investors become noise traders per unit difference in realized returns.

where $(\Delta \text{PERF}_{n-s,t-1})$ is the relative performance of traders who are young in period $t - 1$ and $\text{Prob}(M_{t+1} = 1) = \alpha$ means that there is a probability α that $(M_{t+1} = 1)$.

The imitation rule says that success breeds imitation whenever imitation is possible, that is, when both noise traders and sophisticated investors have participated in the market recently (i.e., M_{t-1} and M_t belong to $\{1, \dots, N-1\}$). The specific cases $M_{t-1} = 0$ and $M_{t-1} = N$ are necessary conditions to avoid the process getting stuck at 0 or N. However the larger N, the smaller α can be chosen (see Kirman (1993)). The case $M_{t-1} = 0$ and $M_t = 1$ ($M_{t-1} = N$ and $M_t = N - 1$) means that if imitation is impossible because only sophisticated investors (noise traders) were participating in the market at time $t - 1$, but both categories of speculators are in the market at time t, then no imitation occurs and $M_{t+1} = M_t$.

As in DSSW, the strategy that performed better recently will attract new investors. The main difference with DSSW is that, here, whatever the extent of the success, imitation is constant. Moreover, if one of the two categories of agents is driven out of the market, it will almost surely reappear in the long run: one agent will play a different strategy from the single one he could observe in the market.

In this economy (and also in the perfectly competitive one), agents are boundedly rational in two ways. First, agents are not aware that, for any M, sophisticated investors' strategy strictly dominates noise traders' strategy. Second, agents act as if the only risks in the economy were the fundamental risk and noise traders' misperception of the distributed dividend. They are not aware of their impact on the number of noise traders in the following generation.

B. Noise Trading in the Long Run

To analyze the persistence of noise traders in the market, we first need to choose a relative performance indicator. Since agents are risk averse, the one we propose is such that, in any period, for a given difference in returns, the difference in ex-ante risk undertaken influences the choice of the strategy imitated. Relative realized utility does not satisfy this condition since our imitation rule is such that, whatever the extent of the success, the number of converts is constant. As a consequence, if relative realized returns or relative realized utility were chosen as a relative performance index, they would lead to the same evolution. Therefore, the relative performance index chosen is the following:

$$\Delta u_{n-s,t} = (Y_t - X_t)[p_{t+1} + \delta_{t+1} - (1+r)p_t] - \frac{\gamma}{2}(Y_t^2 - X_t^2)\left[\sigma_\delta^2 + \frac{M_t^2}{N^2}\frac{\sigma_\rho^2}{(1+r)^2}\right] \tag{16}$$

$\Delta u_{n-s,t}$ exactly captures the same effects as those described in the previous section. If noise traders earn higher returns but at the cost of a much larger

amount of risk borne, then $\Delta u_{n-s,t} < 0$. It means that the new generation of investors will consider noise traders' higher return as pure luck and will prefer to mimic sophisticated investors. Furthermore, the larger investors' risk aversion, the larger the weight of the difference in risk undertaken in the relative performance index.

Given the imitation rule and the performance index chosen, the survival of noise traders in the long run can be analyzed. The following proposition gives a sufficient condition to ensure that on average noise traders dominate the market in the long run.

PROPOSITION 2: *Let the imitation rule and the performance index be respectively given by equations* (15) *and* (16). *Then, the stochastic process* (M_t) *has a stationary distribution* v. *Furthermore, if, for all* $M_1 \in \{1, \ldots, N - 1\}$ *and* $M_2 \in \{M_1 - 1, M_1, M_1 + 1\}$, $E[\Delta u_{n-s}(M_1, M_2)] > 0$, *then* v *is an increasing function of M on* $\{2, \ldots, N - 1\}$, $v(N) > v(0)$ *and* $v(N - 1) > v(1)$.

Proof: See Appendix.

Proposition 2 states that, in an imperfectly competitive economy in which imitation of investors is based on both relative return and relative risk undertaken, if noise traders are, on average, moderately over-optimistic and the variance of their misperception is small relative to the fundamental risk, then they dominate the market in the long run. Here, market domination means that, on average, they are of greater number in the market relative to sophisticated investors.

III. Conclusion

Regarding destabilizing speculation, Friedman (1953) argued that "Professional investors might on average make money while a changing body of amateurs regularly lost large sums." Our article can be seen as a response to Friedman as far as small markets are concerned. In an imperfectly competitive market where investors are risk-averse, if the opinion of amateurs is unpredictable, amateurs may, on average, obtain higher utility relative to professional investors. This means that noise traders earn higher profits and that this success cannot be attributed to pure luck. This result is a consequence of the market power of amateurs. By playing nonoptimal strategies, from a professional investor's point of view, amateurs may hurt professionals more than they hurt themselves, as well as earn higher utility.

Our results strongly reinforce theories suggesting that noise trading may explain some market anomalies, in particular LST's (1991) theory that noise trader risk explains closed-end fund discounts and small firm abnormal returns. These markets are less competitive than large firm stock markets. It has two main implications. First, the market pays a premium for the lack of competition. Secondly, arbitrageurs' willingness to bet against noise traders is low not only because noise traders impound an additional risk in the market, but also because market power can make arbitrageurs worse off in a market

with noise traders than in a market with only rational investors. As a consequence, rational investors are reluctant to trade in small markets and noise trader risk persists, hence closed-end fund discounts and small firm abnormal returns fluctuate according to noise traders' beliefs.

Appendix

Proof of Proposition 1: From Kyle (1989), we know that demand schedules are such that:

If $2\lambda_{s,t} + \gamma \mathrm{Var}_t(\tilde{R}_{t+1}) > 0$

$$X_{j,t}(p) = \frac{E_{s,t}(\tilde{R}_{t+1}) - p_t(1+r)}{\lambda_{s,t}(1+r) + \gamma \mathrm{Var}_t(\tilde{R}_{t+1})} \tag{A1}$$

If $2\lambda_{s,t} + \gamma \mathrm{Var}_t(\tilde{R}_{t+1}) = 0$, then

$$X_{j,t}(p) = \frac{2[p_t(1+r) - E_{s,t}(\tilde{R}_{t+1})]}{\gamma \mathrm{Var}_t(\tilde{R}_{t+1})} \tag{A2}$$

If $2\lambda_{n,t} + \gamma \mathrm{Var}_t(\tilde{R}_{t+1}) > 0$

$$Y_{m,t}(p) = \frac{E_{n,t}(\tilde{R}_{t+1}) - p_t(1+r)}{\lambda_{n,t}(1+r) + \gamma \mathrm{Var}_t(\tilde{R}_{t+1})} \tag{A3}$$

If $2\lambda_{n,t} + \gamma \mathrm{Var}_t(\tilde{R}_{t+1}) = 0$, then

$$Y_{m,t}(p) = \frac{2[p_t(1+r) - E_{s,t}(\tilde{R}_{t+1})]}{\gamma \mathrm{Var}_t(\tilde{R}_{t+1})} \tag{A4}$$

Assume that for all $j = 1, \ldots, J$ and for all $m = 1, \ldots, M$,

$$2\lambda_{s,t} + \gamma \mathrm{Var}_t(\tilde{R}_{t+1}) > 0 \quad 2\lambda_{n,t} + \gamma \mathrm{Var}_t(\tilde{R}_{t+1}) > 0 \tag{A5}$$

Then, for all $j = 1, \ldots, J$

$$K = (J-1)\left[\frac{E_{s,t}(\tilde{R}_{t+1}) - p_t(1+r)}{\lambda_{s,t}(1+r) + \gamma \mathrm{Var}_t(\tilde{R}_{t+1})}\right] + M\left[\frac{E_{n,t}(\tilde{R}_{t+1}) - p_t(1+r)}{\lambda_{s,t}(1+r) + \gamma \mathrm{Var}_t(\tilde{R}_{t+1})}\right] + X_{j,t} \tag{A6}$$

and, for all $m = 1, \ldots, M$

$$K = J\left[\frac{E_{s,t}(\tilde{R}_{t+1}) - p_t(1+r)}{\lambda_{s,t}(1+r) + \gamma \mathrm{Var}_t(\tilde{R}_{t+1})}\right] + (M-1)\left[\frac{E_{n,t}(\tilde{R}_{t+1}) - p_t(1+r)}{\lambda_{s,t}(1+r) + \gamma \mathrm{Var}_t(\tilde{R}_{t+1})}\right] + Y_{m,t} \tag{A7}$$

So, we have to solve the following system of equations:

$$(1 + r)\lambda_{s,t} = \left[\frac{(J - 1)}{(1 + r)\lambda_{s,t} + \gamma\text{Var}_t(\tilde{R}_{t+1})} + \frac{M}{(1 + r)\lambda_{n,t} + \gamma\text{Var}_t(\tilde{R}_{t+1})}\right]^{-1} \tag{A8}$$

$$(1 + r)\lambda_{n,t} = \left[\frac{J}{(1 + r)\lambda_{s,t} + \gamma\text{Var}_t(\tilde{R}_{t+1})} + \frac{(M - 1)}{(1 + r)\lambda_{n,t} + \gamma\text{Var}_t(\tilde{R}_{t+1})}\right]^{-1} \tag{A9}$$

Equations (A8) and (A9) imply that

$$\frac{1}{(1 + r)\lambda_{n,t}} + \frac{1}{(1 + r)\lambda_{n,t} + \gamma\text{Var}_t(\tilde{R}_{t+1})}$$
$$= \frac{1}{(1 + r)\lambda_{s,t}} + \frac{1}{(1+r)\lambda_{s,t}+\gamma\text{Var}_t(\tilde{R}_{t+1})} \tag{A10}$$

It follows that $\lambda_{s,t} = \lambda_{n,t}$. From (A8), we obtain

$$\lambda_{s,t} = \lambda_{n,t} = \frac{\gamma\text{Var}_t(\tilde{R}_{t+1})}{(N - 2)(1 + r)} \tag{A11}$$

Putting (A11) in (A1) and (A3) yields equations (3) and (4).   Q.E.D.

Proof of Proposition 2: Given the imitation rule, The process (M_{t-1}, M_t) is a Markov chain. The state of the economy at time t is represented by the pair (M_{t-1}, M_t). The transition probability from state (M_{t-1}, M_t) to (M_t, M_{t+1}) is given by $P(M_{t-1}, M_t) = \text{Prob}(\Delta u_{n-s}[M_{t-1}, M_t] > 0)$ and the transition probability from state (M_{t-1}, M_t) to (M_t, M_{t-1}) is $[1 - P(M_{t-1}, M_t)]$. As a consequence, the transition probability from state (M_{t-1}, M_t) to $(M_t, M_t + K)$, where $|K| > 1$ is equal to zero. The proof is divided in three steps.

Step 1: Stationary distribution of (M_{t-1}, M_t).

Given the imitation rule, one can show that there exists a function $\mu(\cdot, \cdot)$ such that, for all (M_i, M_j), $\mu(M_i, M_j) = \Sigma_{k,l}\mu(M_k, M_l)\text{Prob}[(M_k, M_l); (M_i, M_j)]$ where $\text{Prob}[(M_k, M_l); (M_i, M_j)]$ is the transition probability from state (M_k, M_l) to state (M_i, M_j) (See Billingsley (1986, Section 8)). Furthermore $\mu(M_i, M_j)$ can be expressed as a function of $\mu(0, 0)$. Given that we must have $\Sigma_{k,l}\mu(M_k, M_l) = 1$, we can compute $\mu(0, 0)$. Therefore, the Markov process (M_{t-1}, M_t) has a stationary distribution.

Step 2: Stationary distribution of the process (M_t)

Given the process (M_{t-1}, M_t), it is immediate that in the long run for all $M \in \{1, \ldots, N - 1\}$, the probability to have M noise traders participating in the

market is given by the probability for this state of the economy to be reached. Therefore

$$\nu(M) = \mu(M - 1, M) + \mu(M, M) + \mu(M + 1, M) \tag{A12}$$

Proceeding the same way, we have $\nu(0) = \mu(0, 0) + \mu(1, 0)$ and $\nu(N) = \mu(N, N) + \mu(N - 1, N)$.

Step 3: If, for all $M_1 \in \{1, \ldots, N - 1\}$ and $M_2 \in \{M_1 - 1, M_1, M_1 + 1\}$, $E[\Delta u_{n-s}(M_1, M_2)] > 0$ then ν is an increasing function of M on $\{2, \ldots, N - 1\}$, $\nu(N) > \nu(0)$ and $\nu(N - 1) > \nu(1)$.

Proof. From the evolution process, we deduce that

$$\mu(M - 1, M) = \mu(M, M - 1) \tag{A13}$$

$$\mu(M + 1, M) = \frac{P(M - 1, M)}{[1 - P(M + 1, M)]} \mu(M, M - 1) \tag{A14}$$

Since $E(\Delta u_{n-s})$ is always positive by assumption, it follows that for all $M = 2, \ldots, N - 2$, $P(M, M - 1) > 1/2$ and $P(M, M + 1) > 1/2$. Therefore, $\mu(M + 1, M) > \mu(M, M - 1)$. Using equation (A12), we obtain $\nu(M) - \nu(M - 1) = \mu(M, M + 1) - \mu(M - 2, M - 1) > 0$.
   Proof that $\nu(N) > \nu(0)$ and $\nu(N - 1) > \nu(1)$.

$$\nu(N) = \left(\frac{\alpha}{[1 - P(1, 0)]} \right) \tag{A15}$$

$$\times \frac{P(1, 1)P(1, 2)\ldots P(N-2, N-1)}{[1 - P(2, 1)][1 - P(3, 2)]\ldots[1 - P(N-1, N-1)]} \mu(0, 0)$$

$$\nu(0) = \left(1 + \frac{\alpha}{[1 - P(1, 0)]} \right) \mu(0, 0) \tag{A16}$$

$$\nu(N - 1) = \left(\frac{2P(N-2, N-1)}{[1 - P(N-1, N-1)]} \frac{P(1, 1)}{[1 - P(2, 1)]} + \frac{P(1, 1)}{[1 - P(2, 1)]} \right) \tag{A17}$$

$$\times \frac{P(1, 1)P(1, 2)\ldots P(N-3, N-2)}{[1 - P(1, 0)][1 - P(3, 2)]\ldots[1 - P(N-1, N-1)]} \alpha \mu(0, 0)$$

$$\nu(1) = \left(2 + \frac{P(1, 1)}{[1 - P(2, 1)]} \right) \frac{\alpha}{[1 - P(1, 0)]} \mu(0, 0) \tag{A18}$$

Since $P(1, 1) > 1/2$, $P(N - 1, N - 1) > 1/2$, and for all $M = 1, \ldots, (N - 1)$, $P(M, M + 1) > 1/2$, it is immediate that $\nu(N) > \nu(0)$ and $\nu(N - 1) > \nu(1)$.
   Q.E.D.

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