EE 274 / CoE 197E Lab Exercise: Properties of the DFT
Rhandley D. Cajote, PhD

Reference: Digital Signal Processing, Sanjit K. Mitra

A. Computation of the DFT

The Matlab function freqz() also computers for the DFT given the numerator and denominator coefficients of H(jw). For faster computation, the length L of the DFT should be multiples of powers of two. The Matlab code below shows how freqz() is used to compute the DFT of a given range of frequencies, and demonstrates the periodic nature of the DFT.

```
% Program P3_1
% Evaluation of the DTFT
clf;
% Compute the frequency samples of the DTFT
w = -4*pi:8*pi/511:4*pi;
num = [2 1];den = [1 -0.6];
h = freqz(num, den, w);
% Plot the DTFT
subplot(2,1,1)
plot(w/pi,real(h));grid
title('Real part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,imag(h));grid
title('Imaginary part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
pause
subplot(2,1,1)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum |H(e^{j\omega})|')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,angle(h));grid
title('Phase Spectrum arg[H(e^{j\omega})]')
xlabel('\omega /\pi');
ylabel('Phase in radians');
```

1. Compare the output of the code above with the output of >> figure; freqz(num,den). What is the basic difference between the two?
2. What can you say about the symmetries in real and imaginary parts of the DFT?
3. Modify the above program to evaluate in the range $0 \leq \boldsymbol{\omega} \leq$ □ the following DFT using freqz.

$$U(e^{j\omega}) = \frac{0.7 - 0.5e^{-j\omega} + 0.3e^{-j2\omega} + e^{-j3\omega}}{1 + 0.3e^{-j\omega} - 0.5e^{-j2\omega} + 0.7e^{-j3\omega}},$$

4. Again modify the given code to evaluate the DFT of the following finite point sequence, g[n] = {1 3 5 7 9 11 13 15 17}. Use two functions freqz() and fft() to compute the DFT. You should get the same magnitude and phase response. Compare the two methods of computing for the DFT.

B. Time shift property of the DFT. The code below demonstrates the time-shift property of the DFT. Please explain the time shift property and how the given figures illustrate this property.

EE 274 / CoE 197E Lab Exercise: Properties of the DFT
Rhandley D. Cajote, PhD

```
% Program P3_2
% Time-Shifting Properties of DTFT
clf;
w = -pi:2*pi/255:pi; wo = 0.4*pi; D = 10;
num = [1 2 3 4 5 6 7 8 9];
h1 = freqz(num, 1, w);
h2 = freqz([zeros(1,D) num], 1, w);
subplot(2,2,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of Original Sequence')
subplot(2,2,2)
plot(w/pi,abs(h2));grid
title('Magnitude Spectrum of Time-Shifted Sequence')
subplot(2,2,3)
plot(w/pi,angle(h1));grid
title('Phase Spectrum of Original Sequence')
subplot(2,2,4)
plot(w/pi,angle(h2));grid
title('Phase Spectrum of Time-Shifted Sequence')
```

C. Frequency-Shift Property of the DFT. The code below demonstrates the frequency shift property of the DFT. Modify the code to include labels on the x-axis of the generated plots. Which parameter controls the amount of shift?

```
% Program P3_3
% Frequency-Shifting Properties of DTFT
clf;
w = -pi:2*pi/255:pi; wo = 0.4*pi;
num1 = [1 3 5 7 9 11 13 15 17];
L = length(num1);
h1 = freqz(num1, 1, w);
n = 0:L-1;
num2 = exp(wo*i*n).*num1;
h2 = freqz(num2, 1, w);
subplot(2,2,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of Original Sequence')
subplot(2,2,2)
plot(w/pi,abs(h2));grid
title('Magnitude Spectrum of Frequency-Shifted Sequence')
subplot(2,2,3)
plot(w/pi,angle(h1));grid
title('Phase Spectrum of Original Sequence')
subplot(2,2,4)
plot(w/pi,angle(h2));grid
title('Phase Spectrum of Frequency-Shifted Sequence')
```

D. Convolution Property of the DFT. The code below demonstrates the convolution property of the DFT. Explain the convolution property and how it is demonstrated by the figures.

```
% Program P3_4
% Convolution Property of DTFT
clf;
w = -pi:2*pi/255:pi;
x1 = [1 3 5 7 9 11 13 15 17];
x2 = [1 -2 3 -2 1];
y = conv(x1,x2);
h1 = freqz(x1, 1, w);
h2 = freqz(x2, 1, w);
hp = h1.*h2;
h3 = freqz(y,1,w);
subplot(2,2,1)
```

```
plot(w/pi,abs(hp));grid
title('Product of Magnitude Spectra')
subplot(2,2,2)
plot(w/pi,abs(h3));grid
title('Magnitude Spectrum of Convolved Sequence')
subplot(2,2,3)
plot(w/pi,angle(hp));grid
title('Sum of Phase Spectra')
subplot(2,2,4)
plot(w/pi,angle(h3));grid
title('Phase Spectrum of Convolved Sequence')
```

E. Modulation property. The code below demonstrates the modulation property of the DFT. Explain the modulation property and how it is demonstrated by the figures.

```
% Program P3_5
% Modulation Property of DTFT
clf;
w = -pi:2*pi/255:pi;
x1 = [1 3 5 7 9 11 13 15 17];
x2 = [1 -1 1 -1 1 -1 1 -1 1];
y = x1.*x2;
h1 = freqz(x1, 1, w);
h2 = freqz(x2, 1, w);
h3 = freqz(y,1,w);
subplot(3,1,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of First Sequence')
subplot(3,1,2)
plot(w/pi,abs(h2));grid
title('Magnitude Spectrum of Second Sequence')
subplot(3,1,3)
plot(w/pi,abs(h3));grid
title('Magnitude Spectrum of Product Sequence')
```

E. Time-reversal property. The code below demonstrates the time-reversal property of the DFT. Explain the time-reversal property and how it is demonstrated by the figures.

```
% Program P3_6
% Time Reversal Property of DTFT
clf;
w = -pi:2*pi/255:pi;
num = [1 2 3 4];
L = length(num)-1;
h1 = freqz(num, 1, w);
h2 = freqz(fliplr(num), 1, w);
h3 = exp(w*L*i).*h2;
subplot(2,2,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of Original Sequence')
subplot(2,2,2)
plot(w/pi,abs(h3));grid
title('Magnitude Spectrum of Time-Reversed Sequence')
subplot(2,2,3)
plot(w/pi,angle(h1));grid
title('Phase Spectrum of Original Sequence')
subplot(2,2,4)
plot(w/pi,angle(h3));grid
title('Phase Spectrum of Time-Reversed Sequence')
```

EE 274 / CoE 197E Lab Exercise: Properties of the DFT
Rhandley D. Cajote, PhD

F. (For Graduate Student Only)

Circular Shift and Circular Convolution. The code below demonstrates the circular-shift and circular convolution property of the DFT, implemented as Matlab functions.

```
function y = circshift(x,M)
% Develops a sequence y obtained by
% circularly shifting a finite-length
% sequence x by M samples
if abs(M) > length(x)
   M = rem(M,length(x));
end
if M < 0
   M = M + length(x);
end
y = [x(M+1:length(x)) x(1:M)];

function y = circonv(x1,x2)
L1 = length(x1); L2 = length(x2);
if L1 ~= L2, error('Sequences of unequal lengths'), end
y = zeros(1,L1);
x2tr = [x2(1) x2(L2:-1:2)];
for k = 1:L1
   sh = circshift(x2tr,1-k);
   h = x1.*sh;
   y(k) = sum(h);
end
```

The following code demonstrates the use of these functions. Explain the circular shifting property of the DFT and how it is demonstrated in this example.

```
% Program P3_8
% Circular Time-Shifting Property of DFT
clf;
x = [0 2 4 6 8 10 12 14 16];
N = length(x)-1; n = 0:N;
y = circshift(x,5);
XF = fft(x);
YF = fft(y);
subplot(2,2,1)
stem(n,abs(XF));grid
title('Magnitude of DFT of Original Sequence');
subplot(2,2,2)
stem(n,abs(YF));grid
title('Magnitude of DFT of Circularly Shifted Sequence');
subplot(2,2,3)
stem(n,angle(XF));grid
title('Phase of DFT of Original Sequence');
subplot(2,2,4)
stem(n,angle(YF));grid
title('Phase of DFT of Circularly Shifted Sequence');
```

The following codes demonstrate the circular convolution in comparison with the linear convolution. Explain how these codes demonstrate circular convolution.

```
% Program P3_10
% Linear Convolution via Circular Convolution
g1 = [1 2 3 4 5];g2 = [2 2 0 1 1];
g1e = [g1 zeros(1,length(g2)-1)];
g2e = [g2 zeros(1,length(g1)-1)];
ylin = circonv(g1e,g2e);
disp('Linear convolution via circular convolution = ');disp(ylin);
y = conv(g1, g2);
disp('Direct linear convolution = ');disp(y)
```