UNIVERSITY OF THE PHILIPPINES DILIMAN
ELECTRICAL AND ELECTRONICS ENGINEERING INSTITUTE

EE 274 / COE 197E: Discrete Time Signals & Systems (DSP1)
Programming Exercise 03
DEADLINE: 23-OCT-2020

INSTRUCTIONS. This is an open-notes, open-books exercise, and can be done solo or by pair (For those under EE 274, you are required to do this individually). Write your name/s and student number/s on the topmost part of the lab report. The exercises are designed to be done in MATLAB or Octave. For convenience, use the **Publish** to **DOC/PDF** feature of MATLAB. Comment your answers to the questions.

Submit copies of your well-commented codes and automatically generated lab report in a compressed folder via UVLE no later than the designated deadline. Use the filename EE274_ProgEx03.zip or CoE197E_ProgEx03.zip. Anything submitted beyond the deadline will get a 0.6 multiplier penalty to the final score.

A. THE BILATERAL Z-TRANSFORM

The bilateral z-transform of a sequence x(n) is defined as:

$$X(z) = \mathrm{Z}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

where z is a complex variable. For any given sequence x(n), the set of values of z for which X(z) exists is called the region of convergence (ROC) and is given by:

$$R_{x^-} < |z| < R_{x^+}$$

for some positive numbers R_{x^-} and R_{x^+}.

<u>Z-Transform of Sequences</u>

1. Determine the z-transform of the following sequences, using the definition for $\mathrm{Z}[x(n)]$. Indicate the region of convergence for each sequence. Express $X(z)$ as a rational function in z^{-1}. You may comment your solutions in the script.

   (a) $x(n) = \left(\frac{4}{3}\right)^n u(1-n)$ (b) $x(n) = 2^{-|n|} + \left(\frac{1}{3}\right)^{|n|}$

2. Verify the z-transform expression by using Octave/Matlab. Use **deconv()** to generate the coefficients of the power-series expansion of $X(z)$. Note that computation of power-series expansion depends on causality of the signal. List the first 8 coefficients (for each letter).

3. Determine the z-transform of the sequence using the z-transform properties and table of common transform pairs.

   $$x(n) = \left(\frac{1}{3}\right)^n u(n-2) + (0.9)^{n-3}u(n)$$

   Express X(z) as a rational function in z^{-1}. Indicate the region of convergence. Use **impz()** (or **filter()** with input x(n) = δ(n)) to verify your results. Plot the first 20 samples.

B. THE INVERSE Z-TRANSFORM

The inverse z-transform of a complex function X(z) is given by the contour integral:

$$x(n) = Z^{-1}[X(z)] = \frac{1}{j2\pi}\oint_C X(z)z^{n-1}dz$$

where C is a counter-clockwise closed contour encircling the origin and lying in the ROC.

<u>Inverse z-Transform</u>

4. Determine the inverse z-Transform of

$$X(z) = \frac{1 - z^{-1} - 4z^{-2} + 4z^{-3}}{1 - \frac{11}{4}z^{-1} + \frac{13}{8}z^{-2} - \frac{1}{4}z^{-3}}$$

Using partial-fraction expansion method. The sequence is absolutely summable. In Matlab®, the function **residuez()** can be used to solve partial fraction expansion of X(z).

C. FOURIER TRANSFORM OF DT SIGNALS

<u>Synthesis equation</u>

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$$

<u>Analysis equation</u>

$$c_k = \frac{1}{N}\sum_{k=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

<u>Analysis of Signal frequency components</u>

5. Generate the periodic even symmetric square pulse signal x(n) from [0,1]. The period of the pulse is 1 second and a pulse width of 250 milliseconds with a sampling frequency of 8 KHz. Plot one period of the x(n) and verify if you have the correct waveform.

   a) How many samples in one period?
   b) How many samples with a value of 1?
   c) How many zeros?

6. Using the analysis equation of the Fourier series, write a program that will compute the Fourier series coefficients (complex) of the periodic square pulse signal. Plot the magnitude and phase of the first 10 Fourier series coefficients (c_k).
   a) What is the fundamental frequency of the square pulse?
   b) Enumerate the magnitude and phase of the first 10 coefficients of the Fourier series (c_0, c_1, …, c_{10})

7. Using the synthesis equation for the Fourier series, synthesize the original square pulse using the first 10 Fourier coefficients. Generate a plot of the original square pulse and the synthesized square pulse.
    a) What is the average mean square error (MSE) of the original square pulse and the synthesized pulse?
    b) If you use 20 Fourier coefficients, what will be the average MSE?
    c) What is the effect on the fundamental frequency if I increase the pulse width to 300 ms? Explain.
    d) What is the effect on the Fourier series coefficients if I change the pulse width?
    e) What is the effect on the Fourier series coefficients if I change the period?

References:

https://www.mathworks.com/