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2018-21366 EE274_ProgEx03

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A.1-2. The Bilateral Z-Transform

Sequence (a) $x(n) = (\frac{4}{3})^n u(1-n)$

Manual Solution

$x(n) = (\frac{4}{3})^n u(-n+1)$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^n u(-n+1) z^{-n}$

$Let \ k=-n+1 \ and \ n=1-k$

Plugging in the value of k and n, we have: $X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^{1-k} u(k) z^{k-1}$

$X(z) = \sum_{n=0}^{\infty}(\frac{4}{3}) \cdot ((\frac{4}{3})^{-1})^k \cdot ((1/z)^{-1})^k \cdot z^{-1}$

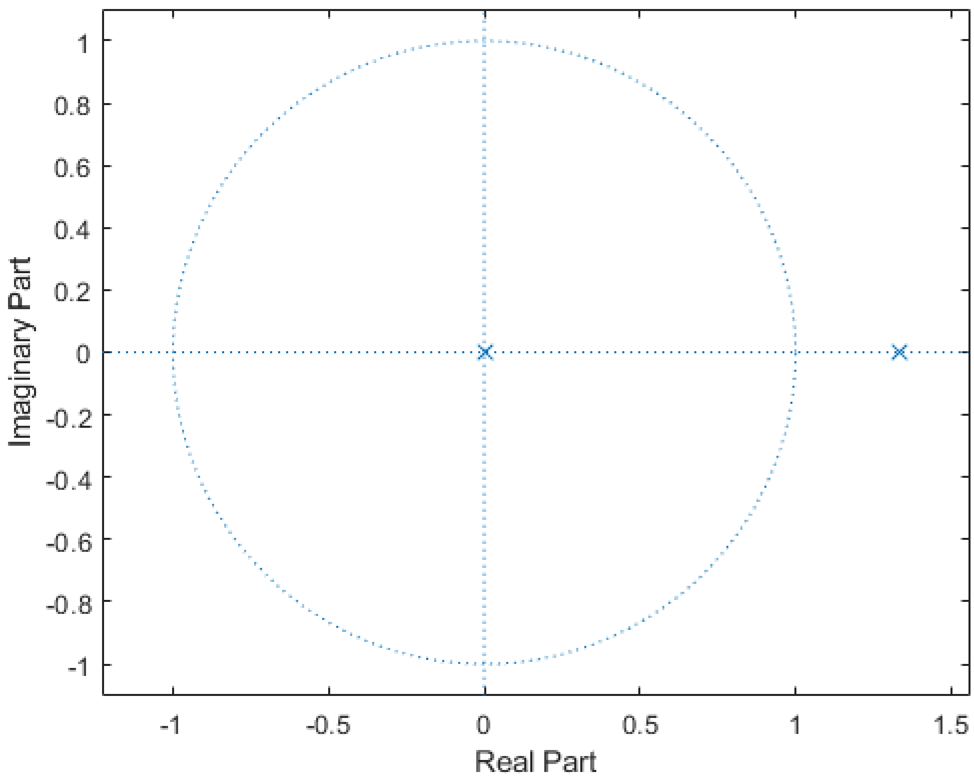
$X(z) = (\frac{4z^{-1}}{3})\ \sum_{n=0}^{\infty}(\frac{3}{4z^{-1}})^k$

$X(z) = (\frac{4z^{-1}}{3}) \cdot (\frac{1}{1-\frac{3}{4z^{-1}}}),\ 0\ < | \ z \ | < \ \frac{4}{3}$

$X(z) = \frac{16z^{-2}}{-9+12z^{-1}},\ 0\ < | \ z \ | < \ \frac{4}{3}$

z-plane for 1.(a)

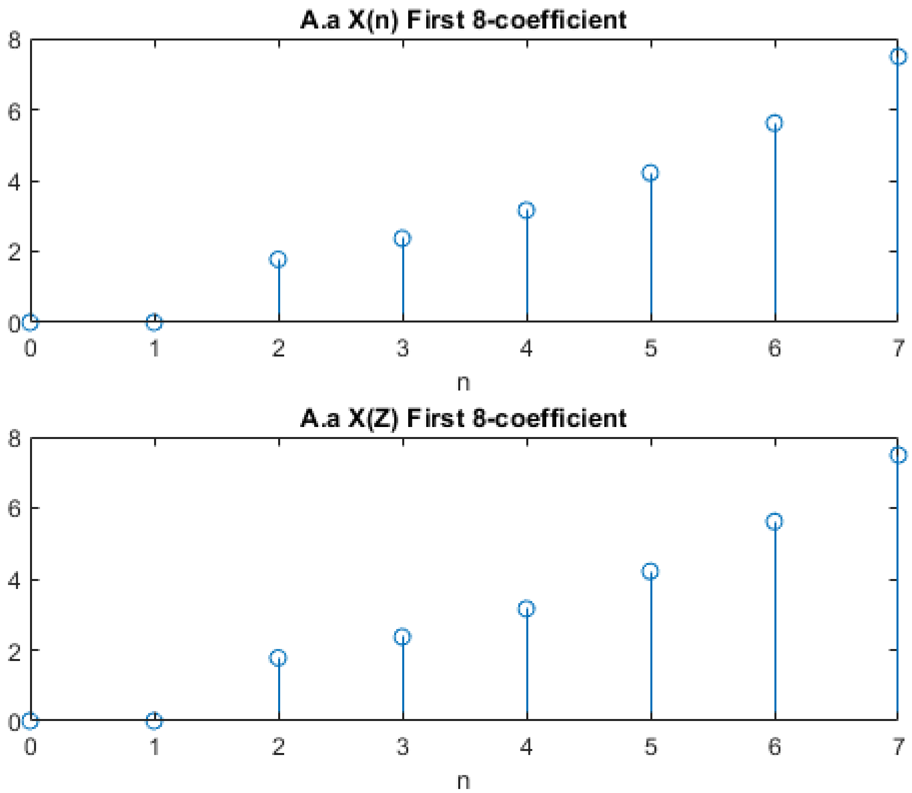
```
a_a_coef=[-9, 12, 0];
a_b_coef=[0, 0, -16];
zplane(a_b_coef,a_a_coef);
```



Verification of z-transform v. original sequence with first 8-coef.

```
[delta_n,n]= impseq(0,0,7);
A_a_Xz=filter(a_b_coef,a_a_coef,delta_n) %A_a_Xz is z-transform
 sequence
A_a_Xn=[(4/3).^n].*stepseq(1,0,7)
figure;
subplot 211
stem(n,A_a_Xz)
title('A.a X(n) First 8-coefficient')
xlabel('n')
```

```
subplot 212
stem(n,A_a_Xz)
title('A.a X(Z) First 8-coefficient')
xlabel('n')
%A_a_Xn is the original sequence.
```



*The coefficient values generated from $X(z)$ and $x(n)$ are the same. Therefore, the z-transform operation for sequence(a) is correct. *

Sequence (b) $x(n) = 2^{-|n|} + (\frac{1}{3})^{|n|}$

$$X(z) = \sum_{n=0}^{\infty} 2^{-n} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{3})^n z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (\frac{z^{-1}}{2})^n + \sum_{n=0}^{\infty} (\frac{z^{-1}}{3})^n$$

$$X(z) = \frac{1}{1-\frac{z^{-1}}{2}} + \frac{1}{1-\frac{z^{-1}}{3}}$$

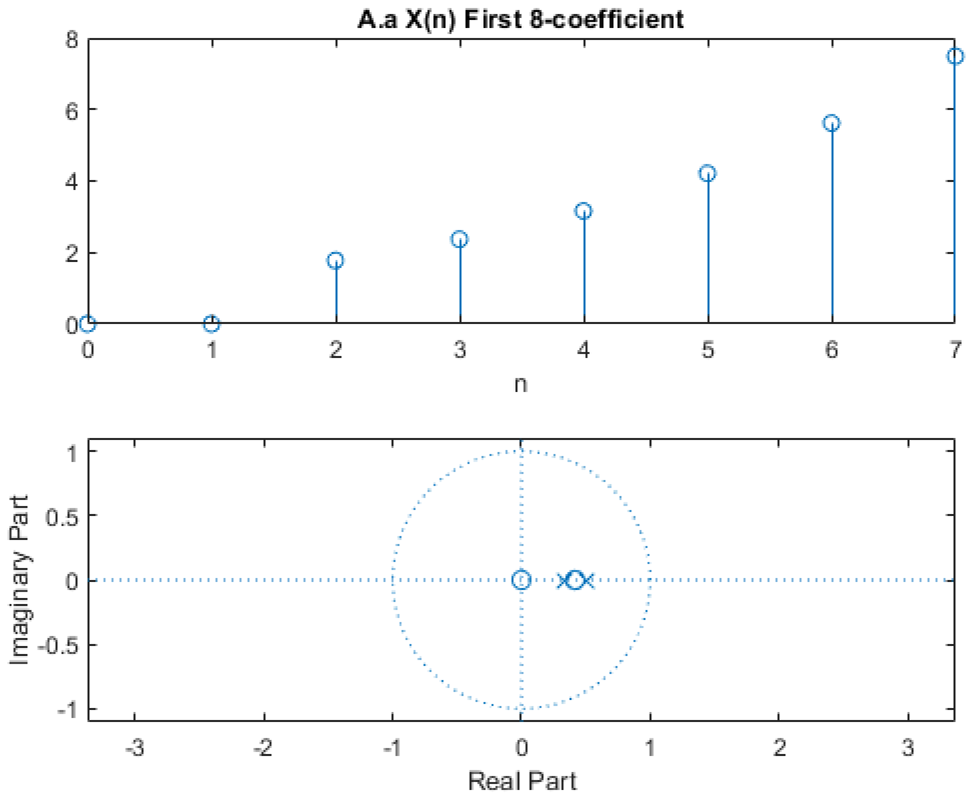
$$X(z) = \frac{2}{2-z^{-1}} + \frac{3}{3-z^{-1}}$$

$$X(z) = \frac{12-5z^{-1}}{(2-z^{-1})(3-z^{-1})},\ \mid z \mid \ > \ \frac{1}{3} \ \cap \ \mid z \mid \ > \ \frac{1}{2}$$

$ X(z) = \frac{12-5z^{-1}}{6-5z^{-1}+z^{-2}},\ \mid z \mid \ > \ \frac{1}{3} \ \cap \ \mid z \mid \ > \ \frac{1}{2}$

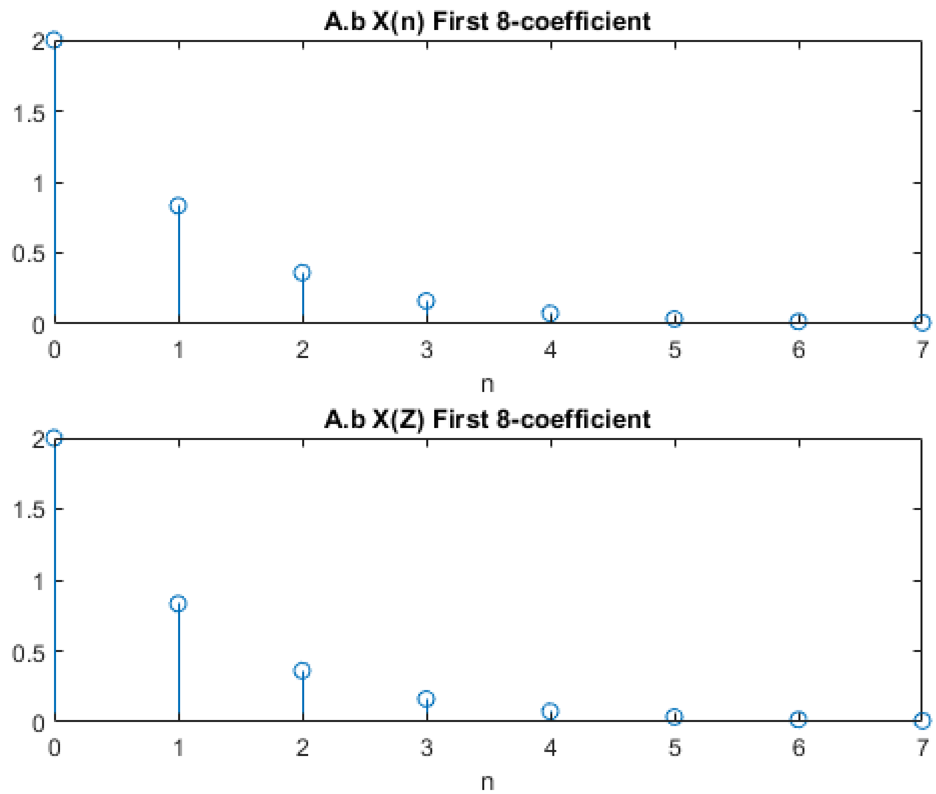
z-plane for 1.(b)

```
b_a_coef=[6 -5 1];
b_b_coef=[12 -5 0];
zplane(b_b_coef,b_a_coef);
```



Verification of z-transform v. original sequence with first 8-coef.

```
[delta,n]= impseq(0,0,7);
A_b_Xz=filter(b_b_coef,b_a_coef,delta) %A_b_Xz is z-transform sequence
A_b_Xn=((2).^(-abs(n)))+((1/3).^(abs(n))) %A_b_Xn is the original
 sequence
figure;
subplot 211
stem(n,A_b_Xn)
title('A.b X(n) First 8-coefficient')
xlabel('n')
subplot 212
stem(n,A_b_Xz)
title('A.b X(Z) First 8-coefficient')
xlabel('n')
```



*The first 8-coefficient values from $X(z)$ and $x(n)$ are the same Therefore, the z-transform operation for sequence(b) is correct. *

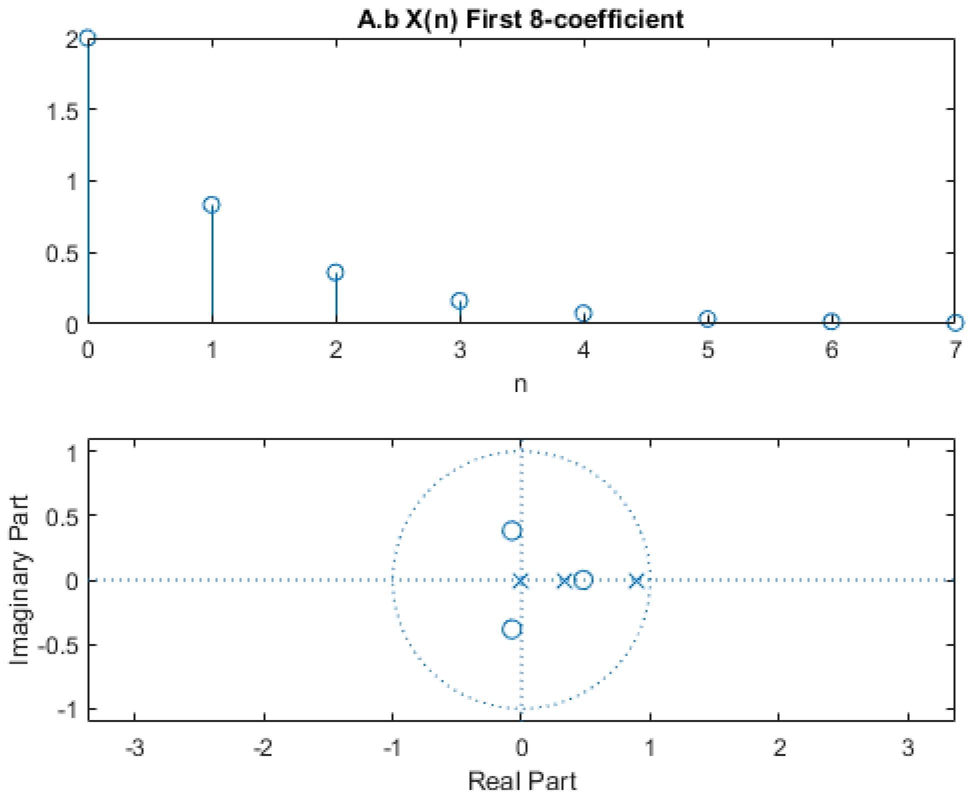
A.3. $x(n) = (\frac{1}{3})^n u(n-2) + (0.9)^{n-3} u(n)$

$$X(z) = \frac{3z^{-2}}{27-9z^{-1}} + \frac{1.3717}{1-0.9z^{-1}}$$

$$X(z) = \frac{37.036-12.34z^{-1}+3z^{-2}-2.7z^{-3}}{27-33.3z^{-1}+8.1z^{-2}} \quad |\, z \,| > \ \tfrac{1}{3} \cap \ |\, z \,| > \ 0.9$$

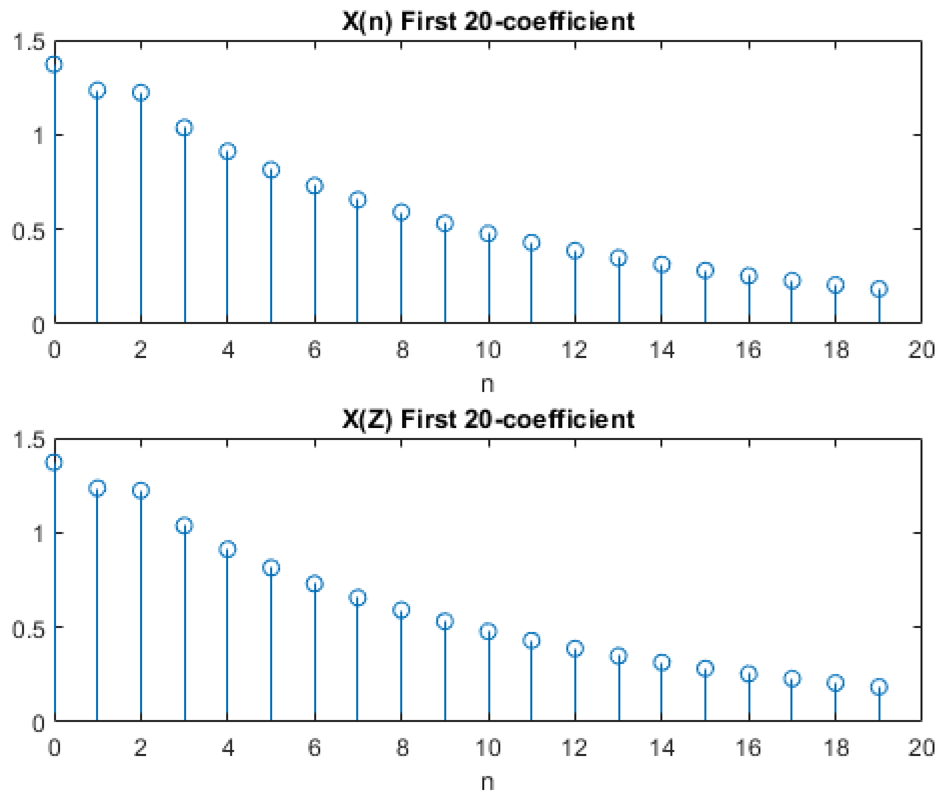
z-plane for A.3

```
A3_b_coef=[37.036, -12.34, 3, -2.7];
A3_a_coef=[27, -33.3, 8.1];
zplane(A3_b_coef,A3_a_coef);
```



Verification of z-transform vs original sequence using the first 20-coef.

```
[delta,n]= impseq(0,0,19);
A3_Xz=filter(A3_b_coef,A3_a_coef,delta) %A3_Xz is z-transform sequence
A3_Xn=(((1/3).^n).*(stepseq_n(2,0,19))+(((0.9).^(n-3)).*(stepseq_n(0,0,19))))
%A3_Xn is the original sequence, see stepseq_n.m
figure;
subplot 211
stem(n,A3_Xn)
title('X(n) First 20-coefficient')
xlabel('n')
subplot 212
stem(n,A3_Xz)
title('X(Z) First 20-coefficient')
xlabel('n')


A3_Xz =

  Columns 1 through 7

    1.3717    1.2347    1.2224    1.0373    0.9125    0.8143    0.7305

  Columns 8 through 14

    0.6567    0.5908    0.5316    0.4784    0.4306    0.3875    0.3488
...
```



*The first 20-coefficient values from $X(z)$ and $x(n)$ are the same Therefore, the z-transform operation for sequence(A 3) is correct. *

B.4. Inverse Z-Transform

Sequence(c) $X(z) = \frac{1-z^{-1}-4z^{-2}+4z^{-3}}{1-\frac{11}{4}z^{-1}+\frac{13}{8}z^{-2}-\frac{1}{4}z^{-3}}$

```
B4_b_coef=[1, -1, -4, 4];
B4_a_coef=[1, (-11/4), (13/8), (-1/4)];
[B4_zeros, B4_poles, B4_C]=residuez(B4_b_coef,B4_a_coef);
```

$$X(z) = \frac{0z}{z-2} - \frac{10z}{z-0.5} + \frac{27z}{z-0.25} - 16$$

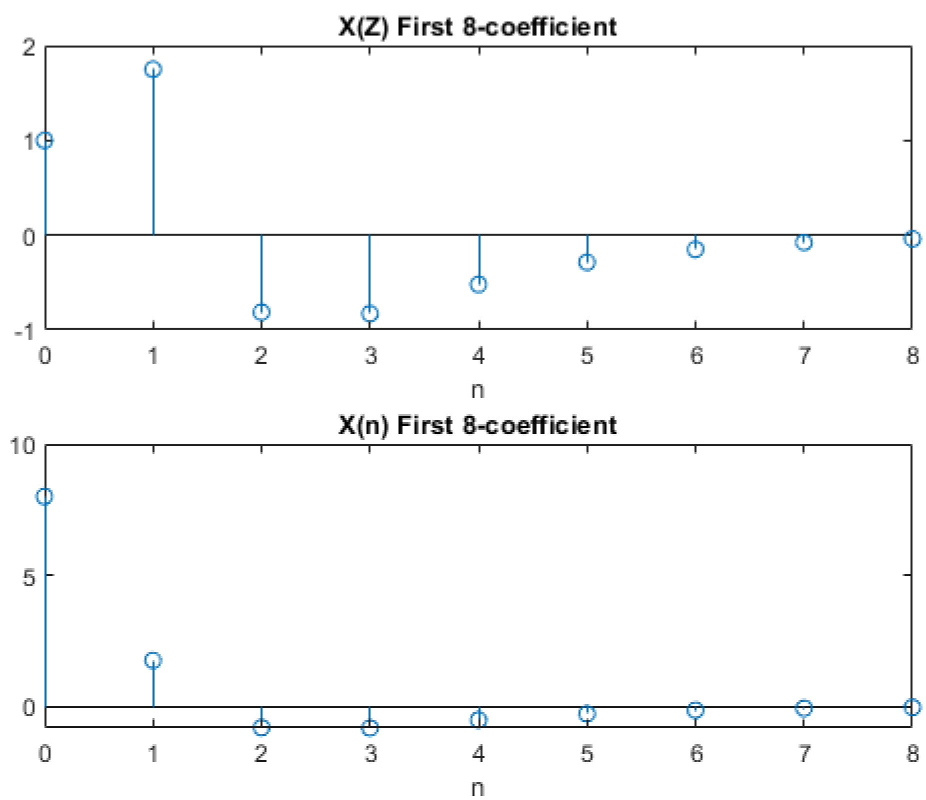
$$X(n) = u(-n) - (2^{-2n}(5 \times 2^{n+1} - 27)(1 - u(-n)))$$

Verification of z-transform v. ans sequence with first 8-coef.

```
[delta,n]= impseq(0,0,8);
B4_Xz=filter(B4_b_coef,B4_a_coef,delta); %B4_Xz is z-transform
 sequence
%B4_Xn is inv. ztrans sequence
B4_Xn=-heaviside(-n)-((2.^(-2*n)).*(5.*(2.^(n+1))-27).*(1-heaviside(-
n)));
```

```
B4_Xz(2:8)% First 8 coef of B4_Xz - Z-transf
B4_Xn(2:8)% First 8 coef of B4_Xn - Inv. Z-transf
figure;
title('Inverse Z-Transform')
subplot 211
stem(n,B4_Xz)
title('X(Z) First 8-coefficient')
xlabel('n')
subplot 212
stem(n,B4_Xn)
title('X(n) First 8-coefficient')
xlabel('n')
```

```
ans =

    1.7500   -0.8125   -0.8281   -0.5195   -0.2861   -0.1497   -0.0765


ans =

    1.7500   -0.8125   -0.8281   -0.5195   -0.2861   -0.1497   -0.0765
```

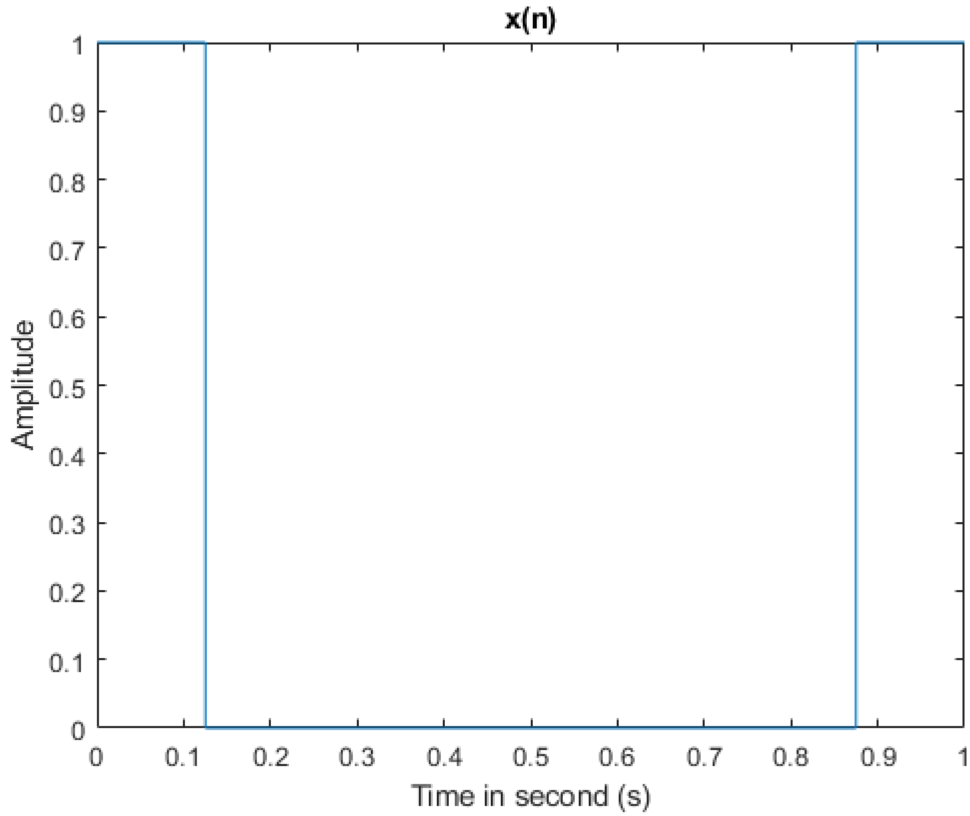


C.5. Signal Generation

Generate the periodic even symmetric square pulse x(n) from [0, 1]. The period of the pulse is 1 second and a pulse with of 250ms with sampling freq. of 8KHz. Plot one period of x(n) and verify if you have the correct waveform.

```
period = 1
pulse_width=0.250
fs=8000
C5_time=0:period/fs:period; % time from frequenzy 8kHz
C5_x=square((2*pi*C5_time),(pulse_width/(period*2))*100); % generate
 x(n)

C5_x(end)=[]
C5_time(end)=[]
C5_x=(abs(C5_x)+C5_x)/2; % remove -1 samples to make x(n) [0,1]
C5_x = C5_x + flip(C5_x)

figure;
plot(C5_time, C5_x);
title("x(n)"); % 2 periods w/ 250ms pw each 1,0.
xlabel("Time in second (s)");
ylabel("Amplitude");
```

```
period =

     1


pulse_width =

    0.2500

...
```



5.a. How many samples in one period?

```
sampled_period=(length(C5_x)) % samples in one period
```

```
sampled_period =

        8000
```

Since the Sampling frequency is 8kHz, therefore there are 8000 samples in one period.

5.b. How many samples with a value of 1?

```
value1_period=(sum(C5_x(:)==1)) % in one period
```

```
value1_period =

        2000
```

Since the generated signals has a period of 1 second and a pulse width of 0.250 seconds, thereforce 25% of the sampled datapoints contains a value of one which is equal to 2000.

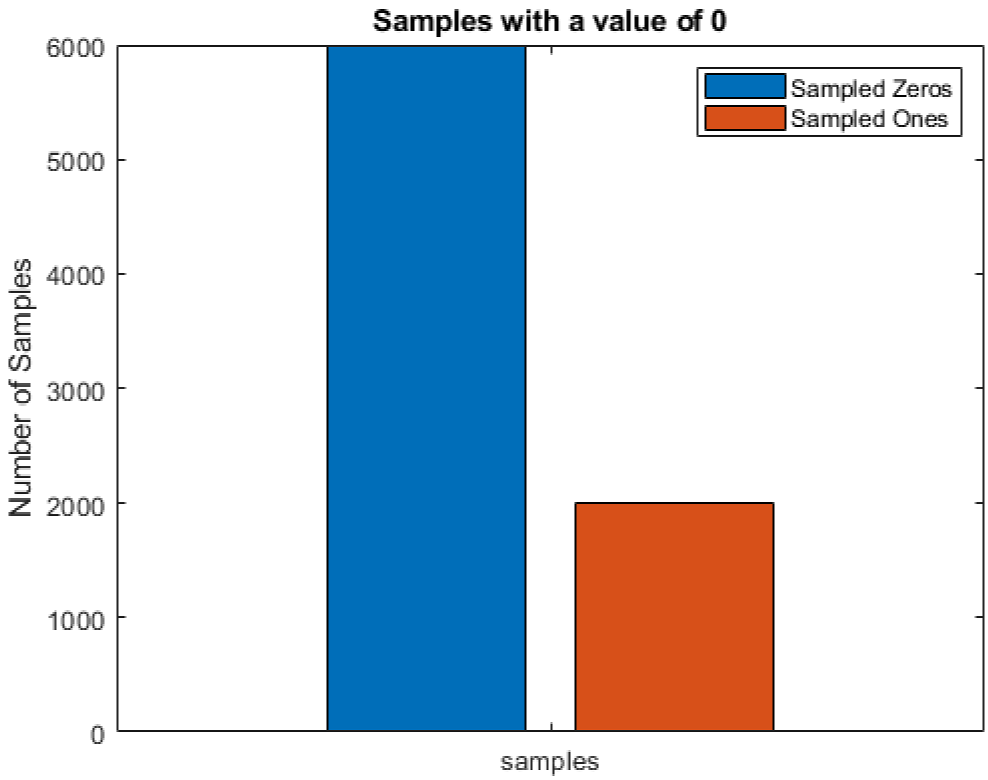
5.c. How many zeros?

```
value0_period=sum(C5_x(:)==0) % in one period
```

```
value0_period =

        6000
```

Since the generated signals has a period of 1 second and a pulse width of 0.250 seconds, thereforce 75% of the sampled datapoints contains a value of zero which is equal to 6000.

```
figure;
bar(categorical({'samples'}),[value0_period;value1_period])
title("Samples with a value of 0")
ylabel('Number of Samples')
legend('Sampled Zeros','Sampled Ones')
```



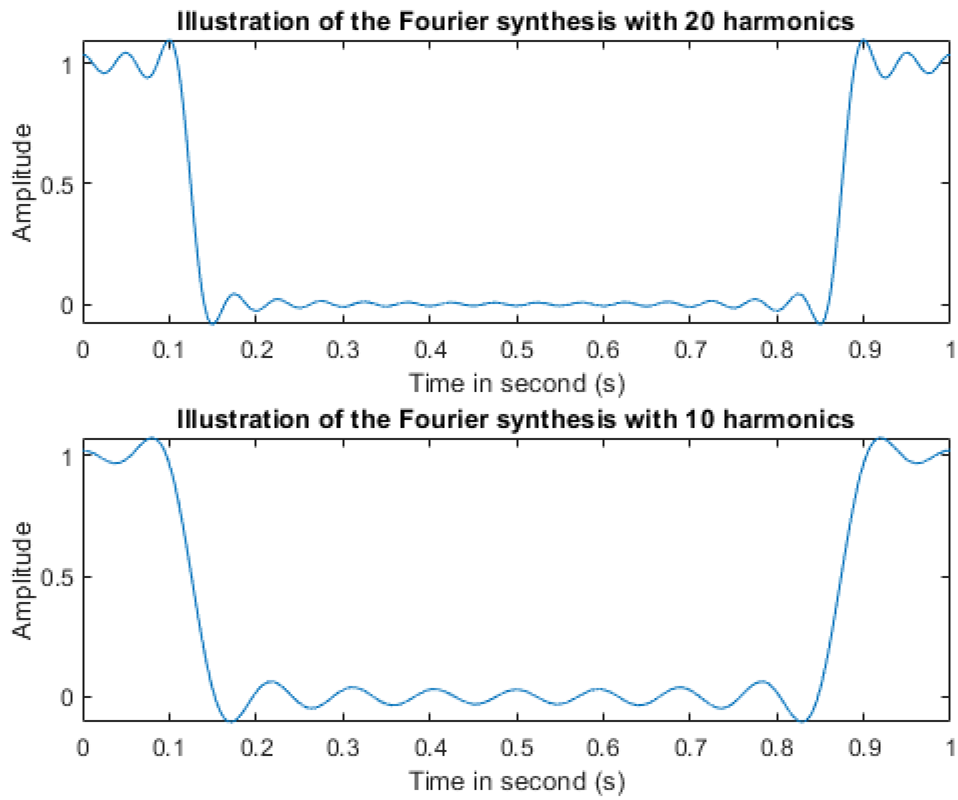
C.6.Fourier Series Analysis Equation

Using the analysis equation of the Fourier series, write a program that will compute the Fourier series coefficients of the periodic square pulse signal. Plot the magnitude and phase of the first 10 Fourier coef.

plot of 100000 harmonic

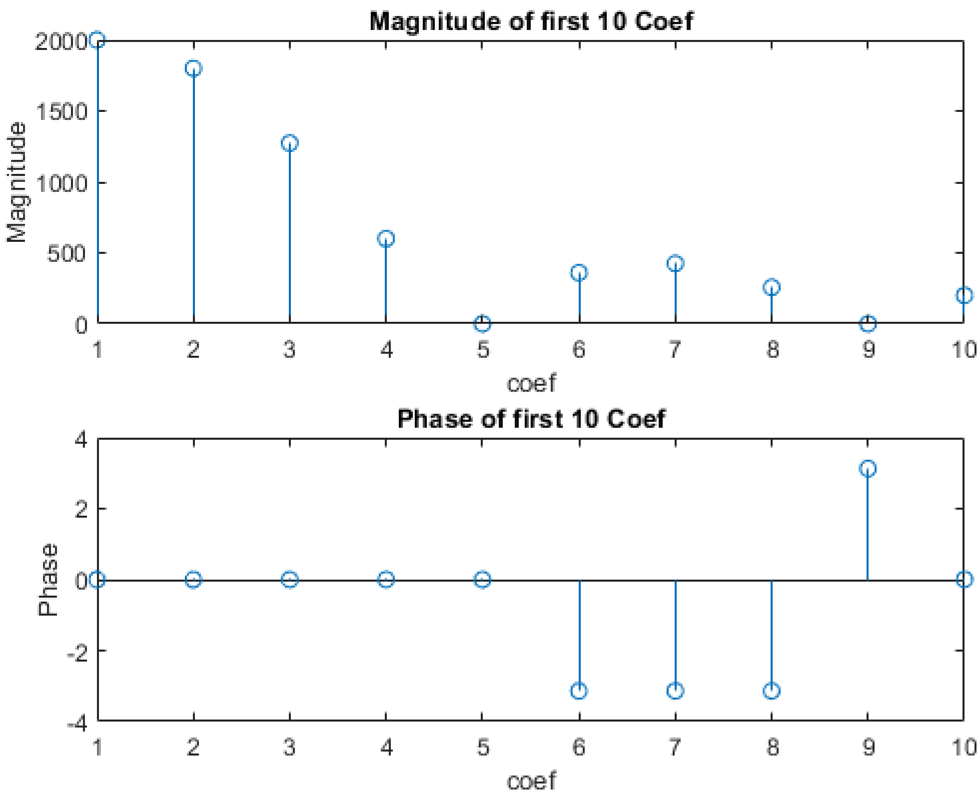
```
C6_t = (0:1/8000:1);
```

```
C6_NHarmonics=20;
cycles=1;
C6_Nsamples=8000;
C6_y(1:C6_Nsamples)=pulse_width;
C6_j=1:C6_Nsamples;
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(0.25*pi*C6_k)/(pi*C6_k))*cos(C6_k*2*pi*cycles*C6_j/
C6_Nsamples);
 C6_y=C6_y+C6_x;
end

C6_NHarmonics=10;
cycles=1;
C6_Nsamples=8000;
C6_y_harmonic(1:C6_Nsamples)=pulse_width;
C6_j=1:C6_Nsamples;
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(0.25*pi*C6_k)/(pi*C6_k))*cos(C6_k*2*pi*cycles*C6_j/
C6_Nsamples);
 C6_y_harmonic=C6_y_harmonic+C6_x;
end

figure()
subplot 211
plot(C6_t(1:8000),C6_y);
title("Illustration of the Fourier synthesis with 20 harmonics");
xlabel("Time in second (s)");
ylabel("Amplitude");
subplot 212
plot(C6_t(1:8000),C6_y_harmonic)
title("Illustration of the Fourier synthesis with 10 harmonics");
xlabel("Time in second (s)");
ylabel("Amplitude");
% plot of 20 harmonic and 10 harmonic
```



compute the magnitude and phase of 10 harmonics signal

```
C6y_fft = fft(C6_y_harmonic);
C6_magnitude = abs(C6y_fft);
C6_phaseangle = angle(C6y_fft);

figure()
subplot 211
stem(C6_magnitude(1:10));
title("Magnitude of first 10 Coef");
xlabel("coef");
ylabel("Magnitude");

subplot 212
stem(C6_phaseangle(1:10));
title("Phase of first 10 Coef");
xlabel("coef");
ylabel("Phase");
```



6.a. What is the fundamental frequency of the square pulse?

The fundamental frequency is defined by f=1/T. which indicates that one complete period is 1s with frequency =1Hz wherein 250ms is alloted for on and 750ms for off.

6.b. Enumerate the Magnitude and Phase of first 10 coef.

```
disp("Magnitude");
C6_magnitude(1:10)

disp("Phase");
C6_phaseangle(1:10)
```

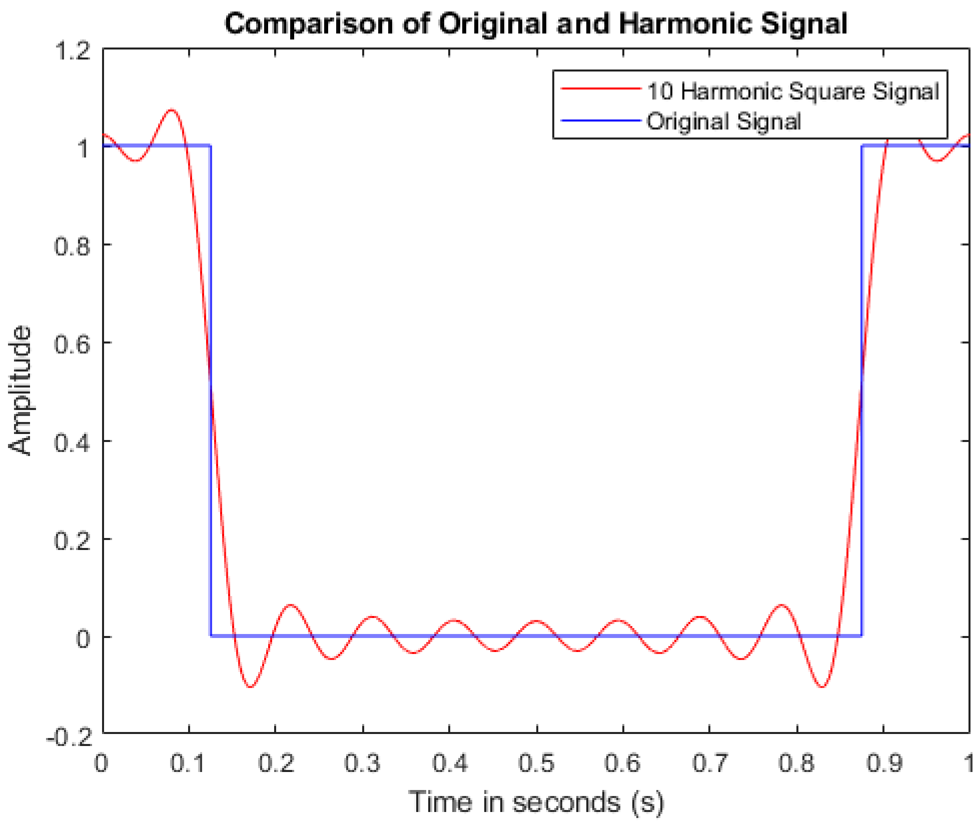
```
Magnitude

ans =

   1.0e+03 *

  Columns 1 through 7
```

```
    2.0000    1.8006    1.2732    0.6002    0.0000    0.3601    0.4244

...
```

C.7. Fourier Series Synthesis Equation

Using the synthesis equatin for the Foyrier series, synthesiez the original square pulse using the first 10 Fourier coefficients. Generate a plot of the original square pulse and the synthesized square pulse.

```
figure()
plot(C6_t(1:8000),C6_y_harmonic, 'color', 'r');
hold on; plot(C6_t(1:8000),C5_x(1:8000), 'color', 'b');
hold off;
title("Comparison of Original and Harmonic Signal");
xlabel("Time in seconds (s)");
ylabel("Amplitude");
legend("10 Harmonic Square Signal","Original Signal")
```



7.a. What is the average MSE of original square pulse vs synthesized pulse?

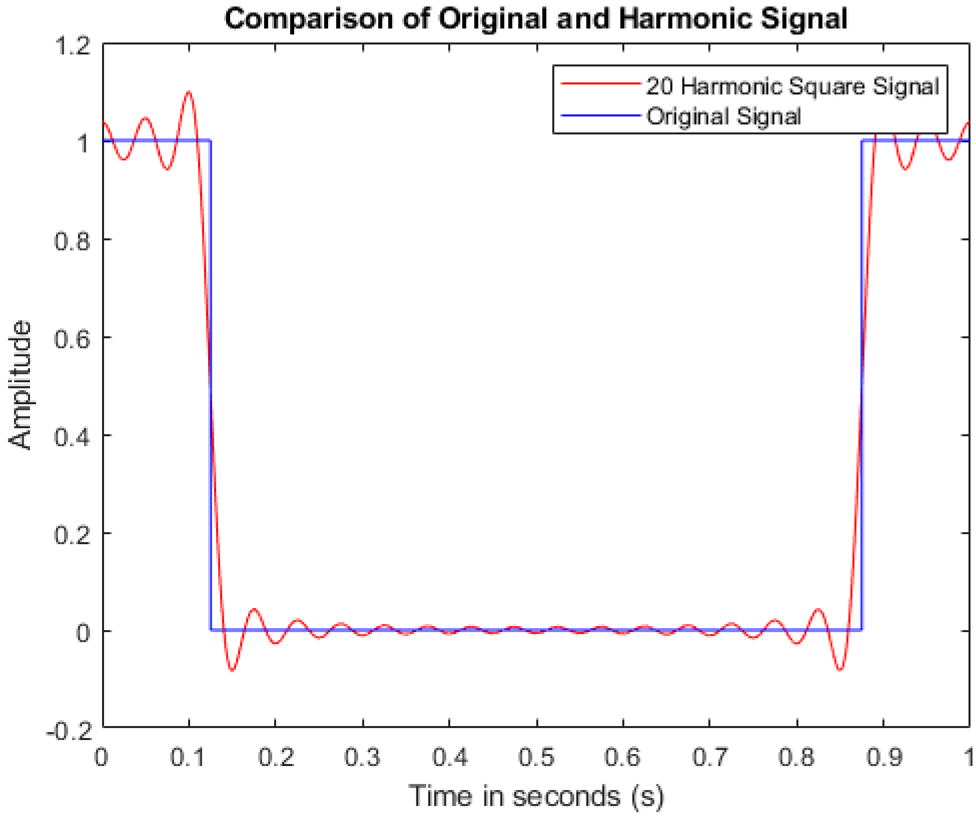
```
C7_mse_10harm = immse(C5_x(1:8000),C6_y_harmonic)
% *MSE is 0.92%*
```

```
C7_mse_10harm =

    0.0092
```

7.b. If you use 20 Fourier coef, what will be the MSE?

```
C6_NHarmonics=20;
cycles=1;
C6_Nsamples=8000;
C6_y_harmonic20(1:C6_Nsamples)=pulse_width;
C6_j=1:C6_Nsamples;
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(0.25*pi*C6_k)/(pi*C6_k))*cos(C6_k*2*pi*cycles*C6_j/
C6_Nsamples);
 C6_y_harmonic20=C6_y_harmonic20+C6_x;
end

figure()
plot(C6_t(1:8000),C6_y_harmonic20, 'color', 'r');
hold on; plot(C6_t(1:8000),C5_x(1:8000), 'color', 'b');
hold off;
title("Comparison of Original and Harmonic Signal");
xlabel("Time in seconds (s)");
ylabel("Amplitude");
legend("20 Harmonic Square Signal","Original Signal")

C7_mse_20harm = immse(C5_x(1:8000),C6_y_harmonic20)
% *MSE will be 0.51%*
```

```
C7_mse_20harm =

    0.0051
```



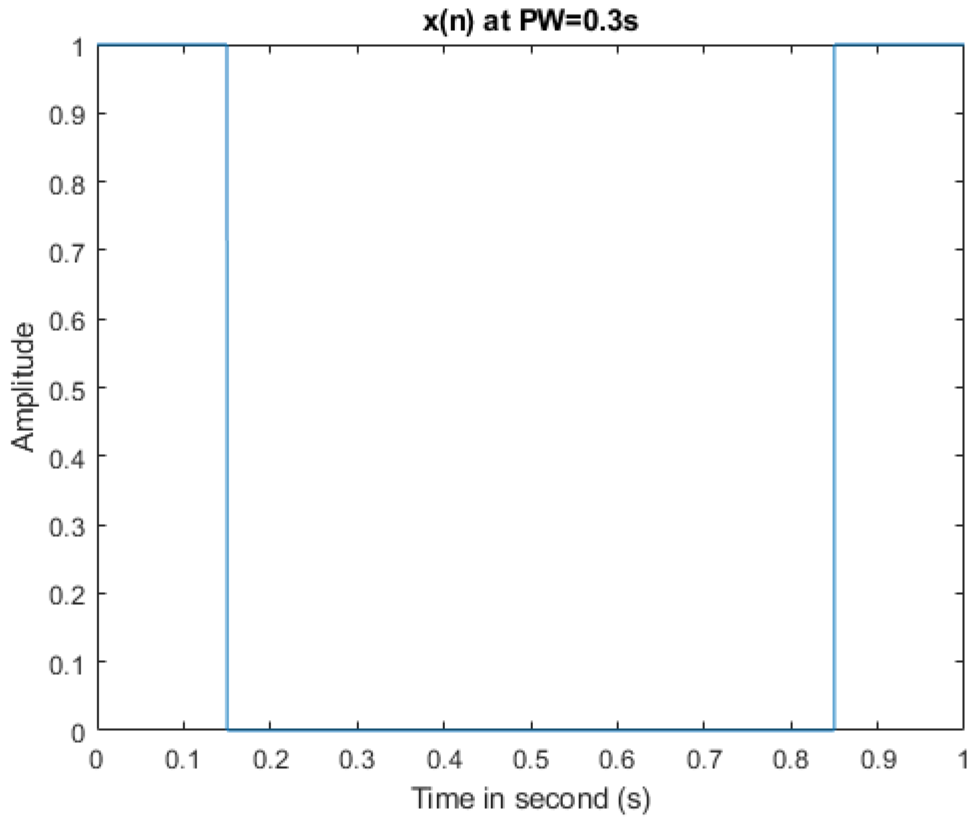
7.c. What is the effect on the fundamental freq if I increase the pulse width to 300ms? Explain.

```
period = 1
pulse_width=0.3
fs=8000
C5_time=0:period/fs:period; % time from frequenzy 8kHz
C5_x=square((2*pi*C5_time),(pulse_width/(period*2))*100); % generate
 x(n)

C5_x(end)=[]
C5_time(end)=[]
C5_x=(abs(C5_x)+C5_x)/2; % remove -1 samples to make x(n) [0,1]
C5_x = C5_x + flip(C5_x)

figure;
plot(C5_time, C5_x);
title("x(n) at PW=0.3s"); % 2 periods w/ 250ms pw each 1,0.
xlabel("Time in second (s)");
ylabel("Amplitude");
```

period =

1

```
pulse_width =

    0.3000

...
```



The fundamental frequency stays the same given that the increase in pulse width only increase the duty cycle of the signal, while the period stays the same.

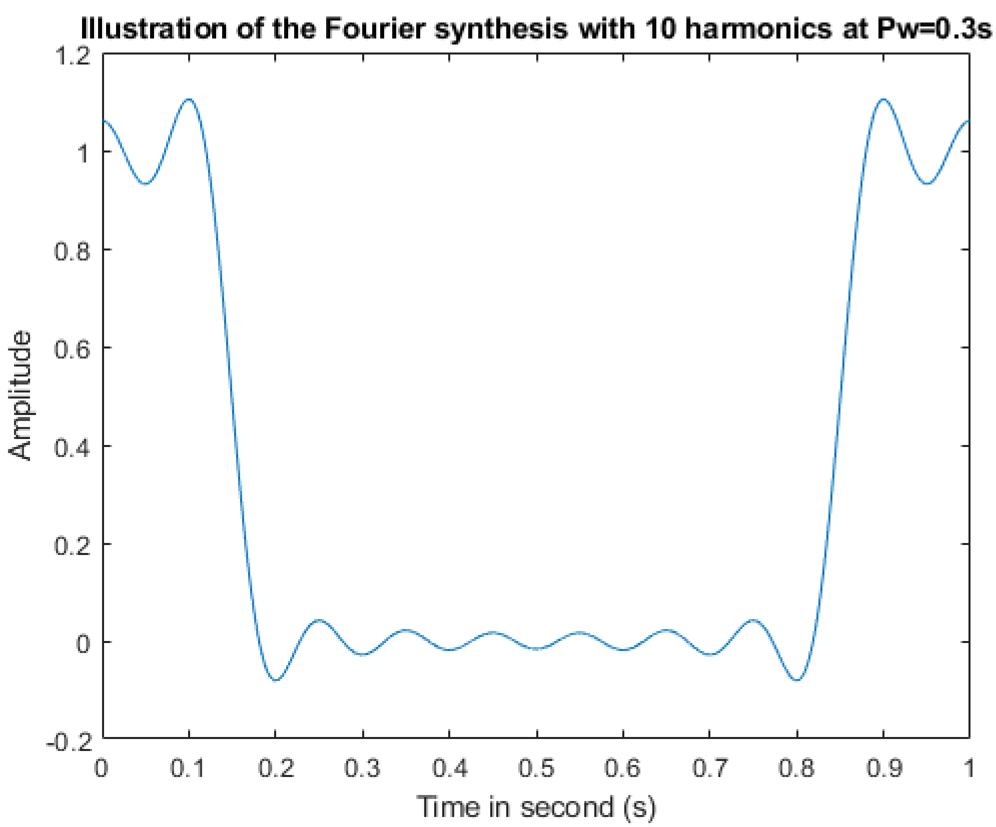
7.d. What is the effect on the Fourier coefficients if I change the pulse width

*Depending on the change applied to the pulse width, the fourier coefficient will be shifted proportional to the change in the pulse width of the square wave.

```
C6_NHarmonics=10;
cycles=1;
C6_Nsamples=8000;
C6_y_harmonic_new_width(1:C6_Nsamples)=pulse_width;
C6_j=1:C6_Nsamples;
width=0.3
for C6_k=1:C6_NHarmonics
```

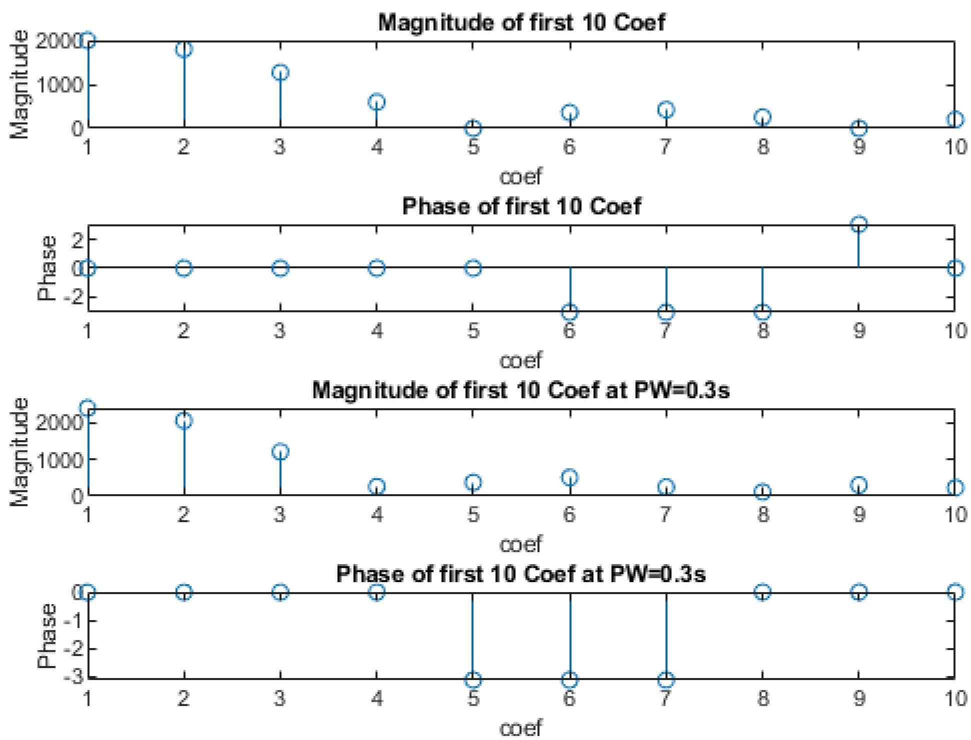
```
 C6_x(C6_j)=(2*sin(width*pi*C6_k)/
(pi*C6_k))*cos(C6_k*2*pi*cycles*C6_j/C6_Nsamples);
 C6_y_harmonic_new_width=C6_y_harmonic_new_width+C6_x;
end

figure()
plot(C6_t(1:8000),C6_y_harmonic_new_width)
title("Illustration of the Fourier synthesis with 10 harmonics at
 Pw=0.3s");
xlabel("Time in second (s)");
ylabel("Amplitude");
```

```
width =

    0.3000
```




```
C6y_fft_newwidth = fft(C6_y_harmonic_new_width);
C6_magnitude_newwidth = abs(C6y_fft_newwidth);
C6_phaseangle_newwidth = angle(C6y_fft_newwidth);

figure()
subplot 411
stem(C6_magnitude(1:10));
title("Magnitude of first 10 Coef");
xlabel("coef");
```

```
ylabel("Magnitude");

subplot 412
stem(C6_phaseangle(1:10));
title("Phase of first 10 Coef");
xlabel("coef");
ylabel("Phase");
subplot 413
stem(C6_magnitude_newwidth(1:10));
title("Magnitude of first 10 Coef at PW=0.3s");
xlabel("coef");
ylabel("Magnitude");

subplot 414
stem(C6_phaseangle_newwidth(1:10));
title("Phase of first 10 Coef at PW=0.3s");
xlabel("coef");
ylabel("Phase");
```

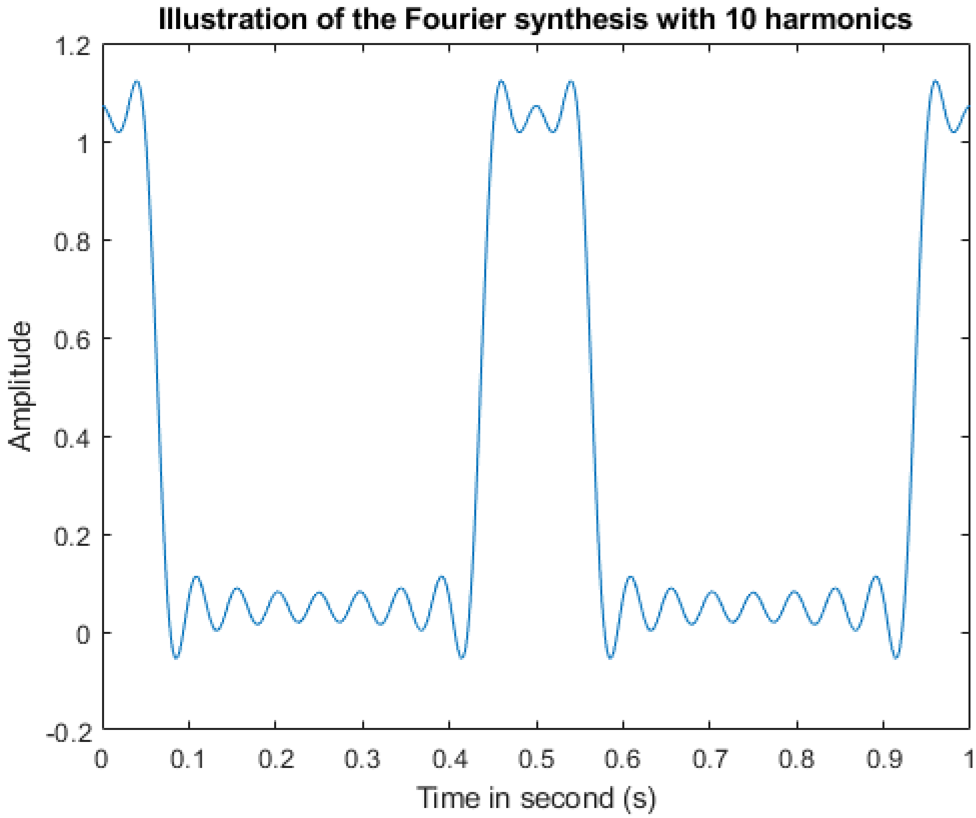


7.e. What is the effect on the Fourier coefficients if I change the period?

*Depending on the change applied to the values of the period, the changes will alter the values of Fourier coefficient inversely.

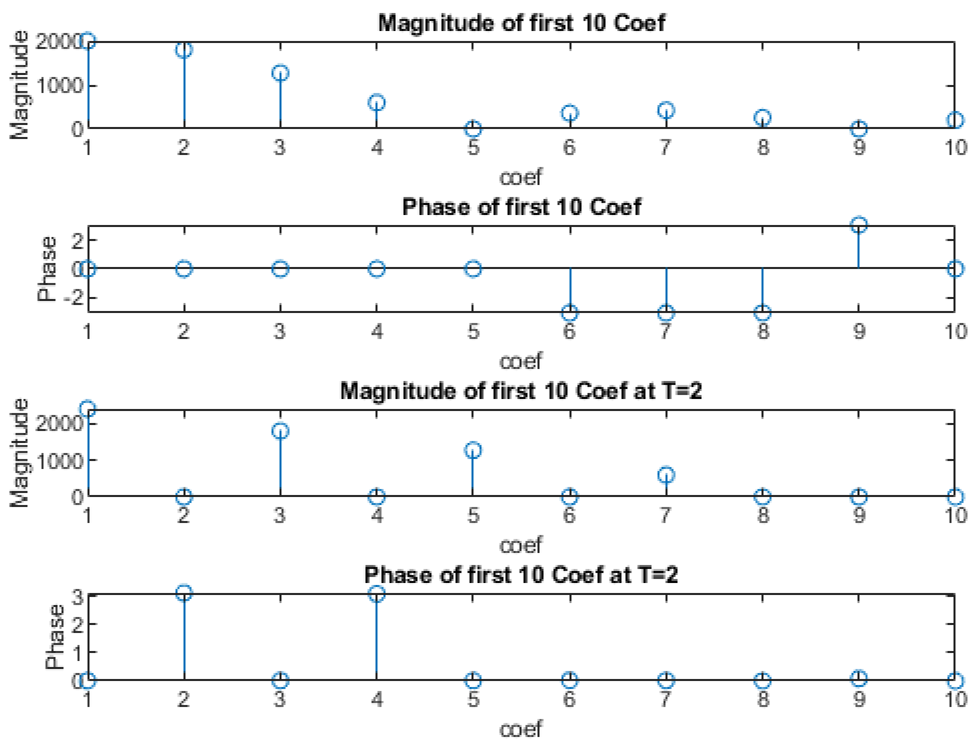
```
C6_NHarmonics=10;
cycles=2;
C6_Nsamples=8000;
C6_y_harmonic_new_width(1:C6_Nsamples)=pulse_width;
C6_j=1:C6_Nsamples;
width=0.25
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(width*pi*C6_k)/
(pi*C6_k))*cos(C6_k*2*pi*cycles*C6_j/C6_Nsamples);
 C6_y_harmonic_new_width=C6_y_harmonic_new_width+C6_x;
end

figure()
plot(C6_t(1:8000),C6_y_harmonic_new_width)
title("Illustration of the Fourier synthesis with 10 harmonics");
xlabel("Time in second (s)");
ylabel("Amplitude");
```

```
width =

    0.2500
```




```
C6y_fft_newwidth = fft(C6_y_harmonic_new_width);
C6_magnitude_newwidth = abs(C6y_fft_newwidth);
C6_phaseangle_newwidth = angle(C6y_fft_newwidth);
```

```
figure()
subplot 411
stem(C6_magnitude(1:10));
title("Magnitude of first 10 Coef");
xlabel("coef");
ylabel("Magnitude");

subplot 412
stem(C6_phaseangle(1:10));
title("Phase of first 10 Coef");
xlabel("coef");
ylabel("Phase");
subplot 413
stem(C6_magnitude_newwidth(1:10));
title("Magnitude of first 10 Coef at T=2");
xlabel("coef");
ylabel("Magnitude");

subplot 414
stem(C6_phaseangle_newwidth(1:10));
title("Phase of first 10 Coef at T=2");
xlabel("coef");
ylabel("Phase");
```



Functions used in the activity to generate sequence

```
function [x,n]=impseq(n0,n1,n2)
    n = [n1:n2];
    x = [(n-n0) == 0];
end
```

```
A_a_Xz =

  Columns 1 through 7

         0         0    1.7778    2.3704    3.1605    4.2140    5.6187

  Column 8

    7.4915


A_b_Xz =

  Columns 1 through 7

    2.0000    0.8333    0.3611    0.1620    0.0748    0.0354    0.0170

  Column 8

    0.0083
...
```

```
function [x,n]=stepseq(n0,n1,n2)
    n = [n1:n2];
    x = [(n0-n) < 0]; %change to less than to satisfy u(n-1)condition.
end
```

```
A_a_Xn =

  Columns 1 through 7

         0         0    1.7778    2.3704    3.1605    4.2140    5.6187

  Column 8

    7.4915
```

```
function [x,n]=stepseq_n(n0,n1,n2)
    n = [n1:n2];
    x = [(n0-n) <= 0]; %change to less than to satisfy (n)condition.
end
```

```
A3_Xn =

  Columns 1 through 7

    1.3717    1.2346    1.2222    1.0370    0.9123    0.8141    0.7304

  Columns 8 through 14

    0.6566    0.5906    0.5315    0.4783    0.4305    0.3874    0.3487
...
```

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